Common Core State Standards Integrated Pathway

Support Supplement

for Mathematics II

$f(x)$ $\angle ABC$ Q_1

Teacher Resource

Unit 2

WALCH
INTEGRATED MATH

This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

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Standards Correlations

Each lesson in the *CCSS Integrated Pathway Support Supplement for Mathematics II* was written specifically to address one or more Common Core State Standards describing prerequisite skills necessary for achieving the standards in the corresponding lesson of *CCSS Integrated Pathway: Mathematics II*. These standards are drawn from the elementary (grades 3–5) and middle-level (grades 6–8) CCSS. Each lesson lists the standards covered in all the sets of Skill Instruction, and each set of Skill Instruction lists the standards addressed in that specific part. In this section, you'll find a comprehensive list mapping the Support resources to the CCSS describing the identified prerequisite skills.

Single asterisks (*) denote Targeted Prerequisite Skills that have been addressed in previous lessons in the Support Supplement. These topics are revisited in abbreviated form to build on prior knowledge and promote skill-building. Double asterisks (**) denote grade-level skills addressed in *CCSS Integrated Pathway: Mathematics II*. These topics are not revisited; instead, references are provided to the relevant instruction in *CCSS Integrated Pathway: Mathematics II* for each grade-level skill.

The Elementary Prerequisite Skills (E-Skills) are italicized for visual distinction from the targeted skills. (*Note*: E-Skills instruction is addressed in the comprehensive appendix.)

Guide to Common Core State Standards Annotation

As you use this resource, you may come across a symbol included with the Common Core standards for some of the lessons and activities. The description of the star symbol is found below, taken verbatim from the Common Core State Standards Initiative website, at www.corestandards.org.

Symbol: ★

Denotes: Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

From http://www.walch.com/CCSS/00003

CCSS INTEGRATED PATHWAY: SUPPORT SUPPLEMENT FOR MATHEMATICS II STANDARDS CORRELATIONS

Unit 2: Quadratic Functions and Modeling			
Lesson	**Title**	**Standard(s)**	**Pages**
Lesson 1	**Analyzing Quadratic Functions: Prerequisite Skills**		
	E-Skill 1: Evaluating Expressions Using the Order of Operations	*5.OA.1*	*A-2*
	Skill 1: Graphing Functions by Creating Tables of Values	A–CED.2★	U2-2
	Skill 2: Identifying Key Features of Linear Functions and Quadratic Functions in Standard Form**	F–IF.4★	U2-19
Lesson 2	**Interpreting Quadratic Functions: Prerequisite Skills**		
	Skill 1: Knowing the Standard Form of Quadratic Functions**	F–IF.8a	U2-25
	Skill 2: Using Graphing Technology to Model and Interpret Quadratic Functions**	F–IF.4★	U2-26
	Skill 3: Understanding the Difference Between Domain and Range	F–IF.1	U2-27
	Skill 4: Evaluating Quadratic Functions for Specific Values of x	F–IF.2	U2-45
	Skill 5: Finding the Slope or Rate of Change of Linear Functions	8.F.4	U2-59
Lesson 3	**Building Functions: Prerequisite Skills**		
	Skill 1: Multiplying Linear Expressions	7.EE.1	U2-85
	Skill 2: Factoring Quadratic Equations**	F–IF.8a	U2-101
	Skill 3: Finding the Value of a in the Vertex Form of a Quadratic Equation Given the Vertex and a Point on the Parabola**	F–IF.8a	U2-102
	Skill 4: Finding the x- and y-coordinates of the Vertex of a Parabola**	F–IF.8a	U2-103
	Skill 5: Adding, Subtracting, Multiplying, and Dividing Functions	F–BF.1b★	U2-104

Lesson	Title	Standard(s)	Pages
Lesson 4	**Graphing Other Functions: Prerequisite Skills**		
	E-Skill 6: Creating Graphs Using Ordered Pairs	*5.G.1*	*A-30*
	Skill 1: Determining the Domain and Range of an Algebraic Equation*	F–IF.1	U2-121
	Skill 2: Evaluating Functions for Given Values*	F–IF.2	U2-126
	Skill 3: Finding Ordered Pairs by Evaluating Functions	8.F.1	U2-130
	Skill 4: Evaluating Squares and Cubes of Real Numbers With and Without a Calculator	8.EE.2	U2-144
	Skill 5: Graphing a Linear Function*	A–CED.2★	U2-158
	Skill 6: Finding the Absolute Value of a Quantity	6.NS.7c	U2-164
	Skill 7: Determining Restricted Domains and Ranges for Application Problems**	F–IF.5★	U2-176
Lesson 5	**Analyzing Functions: Prerequisite Skills**		
	E-Skill 1: Evaluating Expressions Using the Order of Operations	*5.OA.1*	*A-2*
	Skill 1: Identifying the Base and Power of an Exponent and Evaluating Exponential Expressions	6.EE.1	U2-181
	Skill 2: Simplifying Exponential Expressions with Integer Exponents*	8.EE.1	U2-195
	Skill 3: Finding the Vertex and *x*-intercepts of a Parabola**	F–IF.7a★	U2-200
	Skill 4: Writing an Equation for a Simple Exponential Function	A–CED.1★	U2-201
Lesson 6	**Transforming Functions: Prerequisite Skills**		
	Skill 1: Graphing Quadratic Functions	A–REI.10	U2-220
	Skill 2: Evaluating Quadratic Functions*	F–IF.2	U2-239
	Skill 3: Finding Intercepts and Vertices of Quadratic Functions**	F–IF.7a★	U2-244

Lesson	Title	Standard(s)	Pages
Lesson 7	**Finding Inverse Functions: Prerequisite Skills**		
	E-Skill 1: Evaluating Expressions Using the Order of Operations	*5.OA.1*	*A-2*
	Skill 1: Identifying Independent and Dependent Variables	6.EE.9	U2-250
	Skill 2: Determining the Domain and Range of Linear and Quadratic Functions*	F–IF.1	U2-275
	Skill 3: Applying Inverse Operations to Isolate a Variable, Including Taking Square Roots**	A–REI.4b	U2-279
	Skill 4: Using Function Notation*	F–IF.2	U2-280

Lesson 1: Analyzing Quadratic Functions

Instruction

Elementary Prerequisite Skills

This lesson requires the use of the following elementary skill(s) to meet the standards addressed in *CCSS Integrated Pathway: Mathematics II*. See the Appendix for material to address the skill(s).

- E-Skill 1: Applying the Order of Operations (5.OA.1), Appendix p. A-2

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics II*.

Skill 1: Graphing Functions by Creating Tables of Values (A–CED.2★)

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Skill 2: Identifying Key Features of Linear Functions and Quadratic Functions in Standard Form** (F–IF.4★)

Common Core State Standard

F–IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

Instruction

Skill 1: Graphing Functions by Creating Tables of Values

SMP	
1 ✓	2 ✓
3	4 ✓
5	6 ✓
7 ✓	8

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Essential Questions

1. What is the difference between the dependent variable and the independent variable?
2. How are independent and dependent variables used to create a table of values?

WORDS TO KNOW

dependent variable labeled on the *y*-axis; the quantity that is based on the input values of the independent variable; the output variable of a function

independent variable labeled on the *x*-axis; the quantity that changes based on values chosen; the input variable of a function

Recommended Resources

- IXL Learning. "Write Variable Expressions to Represent Word Problems."

  http://www.walch.com/rr/04039

  This site includes practice problems for translating real-life situations into variable expressions.

- Shmoop. "High School: Algebra—Creating Equations HSA-CED.A.2."

  http://www.walch.com/rr/04040

  This site provides an explanation of the standard that is useful for both students and teachers, along with sample assignments and aligned resources.

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Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Provide students with the equation $y = 3x$. Ask students to substitute various values for x and find the corresponding value of y, then create a table for the values. Have students graph the corresponding points on a coordinate plane and draw a line through the points. Have them create a new linear equation and again substitute various values of x to find corresponding y-values. Then, have students graph the points on a coordinate plane and draw a line through the points. This will help students see how the different x-values produce different y-values.

Suggestions for Discourse

- Ask students how they can determine which variable is the dependent variable and which is the independent variable.
- Ask students how the independent and dependent variables are used to create a table of values for a given equation.
- Ask students to explain how a line on a coordinate plane can represent the equation of a real-life situation. For example, if the equation $y = 10x$ gives the number of miles (y) Andrea can bike in x hours, how would a graph of the equation $y = 10x$ represent the different possibilities?

Making Connections

Equations in two variables can be graphed on a coordinate plane. It is often helpful to create a table of values that represent the x- and y-coordinates of a given function in order to create an accurate graph that represents the solutions to not only linear functions, but quadratic functions as well.

Skill 1: Graphing Functions by Creating Tables of Values

Introduction

Many real-life situations can be written as equations. The equations can then be used to analyze the situations or predict future results. Sometimes, the situation has more than one changeable factor, in which case the equation has more than one variable. For example, the owner of a hot dog cart could create an equation that tells the amount of money she will earn based on the number of hot dogs she sells.

Key Concepts

- When creating an equation with two variables, x is the independent variable and y is the dependent variable.
- The **independent variable** (x) is the input variable, meaning it is the value that starts the situation and produces the other variable. For example, using the hot dog example from the Introduction, the number of hot dogs sold is the independent variable, because the hot dog sales produce the money earned (and not the other way around). When an equation is graphed, the independent variable is labeled on the x-axis.
- The **dependent variable** (y) is based on the input value of the independent variable, meaning that the dependent variable is produced by (or is dependent on) the independent variable. In the hot dog example, the money earned is the dependent variable, because it depends on, and is produced by, the hot dog sales.

Tables and Graphs

- Tables can be useful for organizing information, especially when two variables are involved.
- To write an equation when given a table, first analyze the table to determine the common relationship between the x- and y-values. (For example, y may always be 5 more than the corresponding x.) Then write an equation that represents the relationship.
- After an equation is written, you can create a table by substituting values for x and solving for y. This table can then be used to create a graph of the function. The graph gives a visual representation of the relationship, and it shows various possible values for y depending on the value of x.

Guided Practice Skill 1

Example 1

Mr. Taggart is calculating his students' test scores on a 25-question test with the equation $y = 4x + 3$, where x is the number of correct questions and y is the score. Make a table of values and create a graph of this relationship.

1. Determine values that make the equation true.

   To create a graph of the given equation, at least two points are needed. To find the points, substitute values for x and solve for y. Let's use the values 0, 1, 10, and 25.

   Substitute 0 for x and solve for y.

$y = 4x + 3$	Given equation
$y = 4(0) + 3$	Substitute 0 for x.
$y = 3$	Simplify.

   When $x = 0$, $y = 3$.

   Substitute 1 for x and solve for y.

$y = 4x + 3$	Given equation
$y = 4(1) + 3$	Substitute 1 for x.
$y = 7$	Simplify.

   When $x = 1$, $y = 7$.

   Substitute 10 for x and solve for y.

$y = 4x + 3$	Given equation
$y = 4(10) + 3$	Substitute 10 for x.
$y = 43$	Simplify.

   When $x = 10$, $y = 43$.

   Substitute 25 for x and solve for y.

$y = 4x + 3$	Given equation
$y = 4(25) + 3$	Substitute 25 for x.
$y = 103$	Simplify.

   When $x = 25$, $y = 103$.

2. Organize these values in a table.

   Create a table of the values determined in the previous step, with *x*-values in one column and *y*-values in the other.

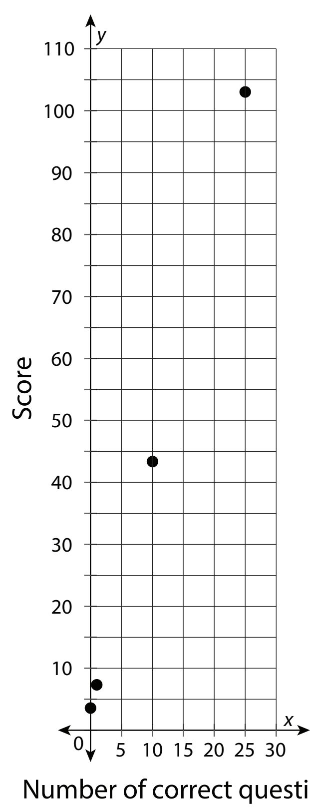
x	*y*
0	3
1	7
10	43
25	103

3. Plot the ordered pairs from the table on a coordinate plane.

   The ordered pairs from the table are (0, 3), (1, 7), (10, 43), and (25, 103).

   Plot these ordered pairs on a coordinate plane.

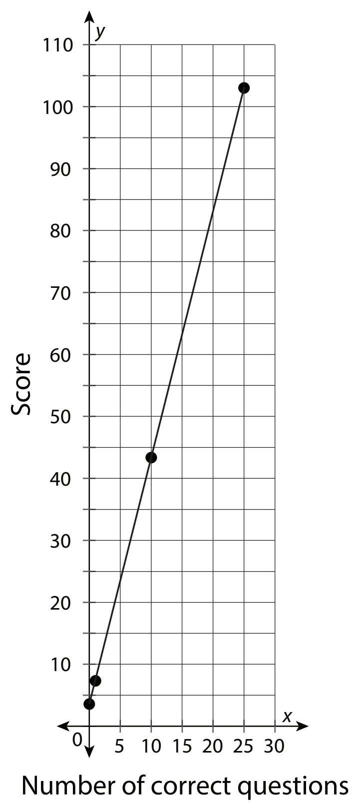
   Label the *x*-axis "Number of correct questions" and the *y*-axis "Score."



4. Draw a line through the points.

   Since the given equation, $y = 4x + 3$, is a linear equation, draw a line through the plotted points.

   Because there are only 25 questions on the test, a student could get from 0 to 25 answers correct. Therefore, the line must start at $x = 0$ and end at $x = 25$.



Example 2

Abby is saving to buy a new bike. She has $14 in her change jar and plans to save $10 per week for the next 16 weeks. She can model her savings with the equation $y = 10x + 14$, where x represents the number of weeks that she saves money, and y represents the total amount of money saved. Make a table of values and create a graph of this relationship.

1. Determine values that make the equation true.

   To create a graph of the given equation, at least two points are needed. To find the points, substitute values for x and solve for y. Let's use the values 0, 1, 8, and 16.

   Substitute 0 for x and solve for y.

   $y = 10x + 14$ Given equation
   $y = 10(0) + 14$ Substitute 0 for x.
   $y = 14$ Simplify.

   When $x = 0$, $y = 14$.

   Substitute 1 for x and solve for y.

   $y = 10x + 14$ Given equation
   $y = 10(1) + 14$ Substitute 1 for x.
   $y = 24$ Simplify.

   When $x = 1$, $y = 24$.

(*continued*)

Substitute 8 for x and solve for y.

$y = 10x + 14$	Given equation
$y = 10(8) + 14$	Substitute 8 for x.
$y = 94$	Simplify.

When $x = 8$, $y = 94$.

Substitute 16 for x and solve for y.

$y = 10x + 14$	Given equation
$y = 10(16) + 14$	Substitute 16 for x.
$y = 174$	Simplify.

When $x = 16$, $y = 174$.

2. Organize these values in a table.

   Create a table of the values determined in the previous step, with the x-values in one column and the y-values in the other column.

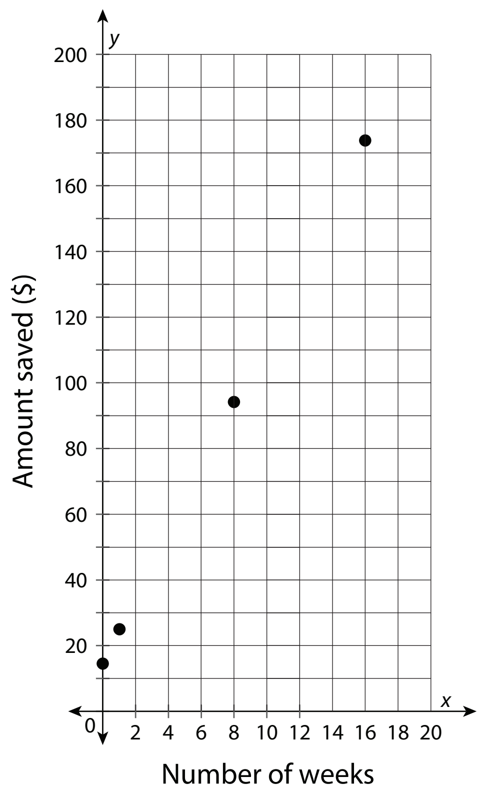
x	y
0	14
1	24
8	94
16	174

3. Plot the ordered pairs from the table on a coordinate plane.

   The ordered pairs from the table are (0, 14), (1, 24), (8, 94), and (16, 174).

   Plot these ordered pairs on a coordinate plane.

   Label the *x*-axis "Number of weeks" and the *y*-axis "Amount saved ($)."

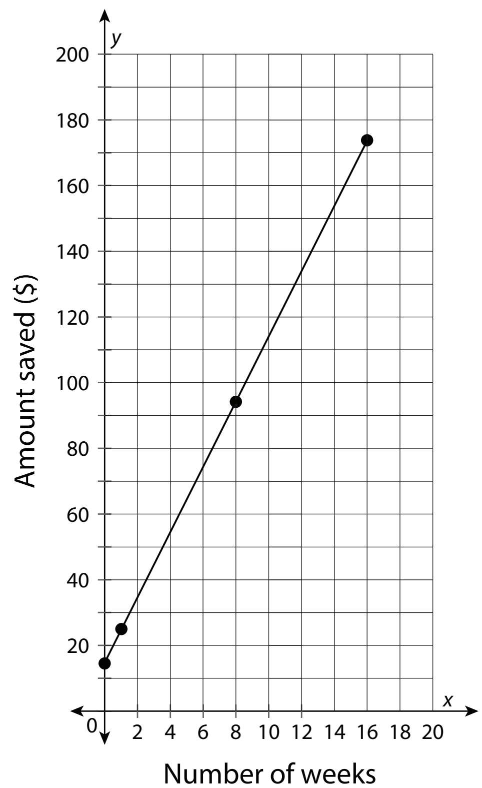


Instruction

4. Draw a line through the points.

   Since the given equation, $y = 10x + 14$, is a linear equation, draw a line through the plotted points.

   Because Abby is only saving for 16 weeks, the line must start at $x = 0$ and end at $x = 16$.



Scaffolded Practice Skill 1

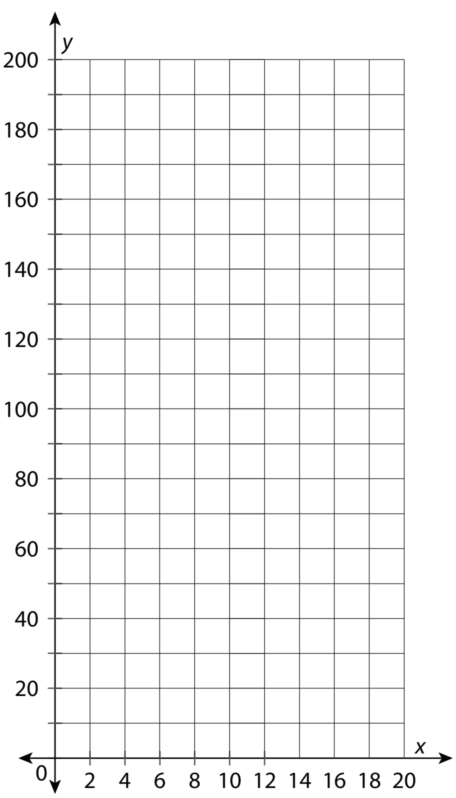
Example 1

Mr. Taggart is calculating his students' test scores on a 25-question test with the equation $y = 4x + 3$, where x is the number of correct questions and y is the score. Make a table of values and create a graph of this relationship.

1. Determine values that make the equation true.

2. Organize these values in a table.

3. Plot the ordered pairs from the table on a coordinate plane.



4. Draw a line through the points.

continued

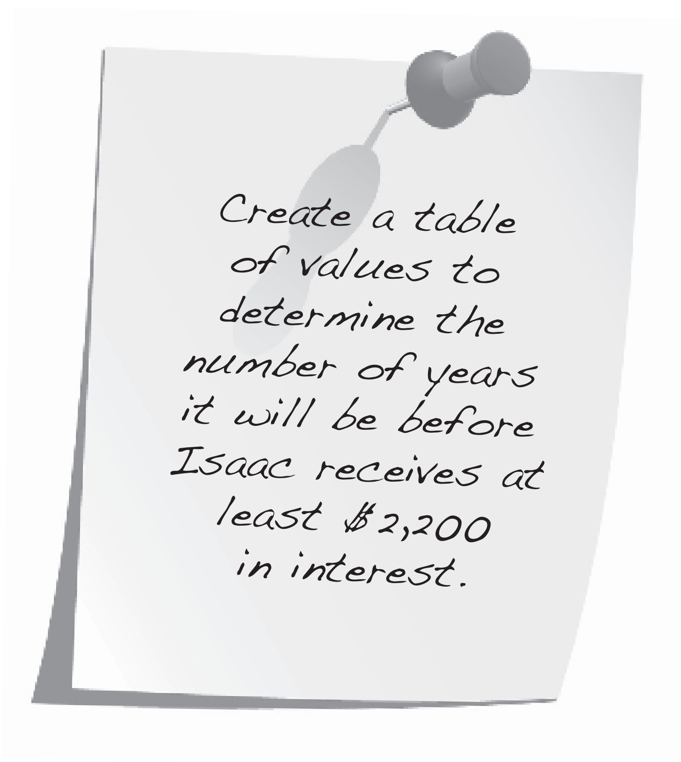
Example 2

Abby is saving to buy a new bike. She has $14 in her change jar and plans to save $10 per week for the next 16 weeks. She can model her savings with the equation $y = 10x + 14$, where x represents the number of weeks that she saves money, and y represents the total amount of money saved. Make a table of values and create a graph of this relationship.

Problem-Based Task Skill 1: Interesting Rates

SMP	
1 ✓	2 ✓
3	4 ✓
5	6 ✓
7 ✓	8

Isaac has $7,000 that he wants to invest over a period of years. He wants to determine how much interest he will make depending on how long he invests the money. Write an equation to describe the interest he will receive at a rate of 5% over a period of 10 years, with the interest paid at the end of each year. Create a table of values to determine the number of years it will be before Isaac receives at least $2,200 in interest.



Problem-Based Task Skill 1: Interesting Rates

Coaching

a. What formula should be used to create the equation?

b. What is the equation that represents this situation?

c. Which is the dependent variable, and which is the independent variable?

d. Create a table of values to represent the amount of interest Isaac will receive each year from 1 to 10 years.

e. After how many years will Isaac receive at least $2,200 in interest?

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Problem-Based Task Skill 1: Interesting Rates

Coaching Sample Responses

a. What formula should be used to create the equation?

The formula that should be used to create the equation is the interest formula $I = Prt$, where I is the interest, P is the principal (the amount of money originally invested), r is the interest rate expressed as a decimal, and t is the time in years.

b. What is the equation that represents this situation?

Using the formula $I = Prt$, substitute the known values, ($P = 7000$ and $r = 5\% = 0.05$), and then simplify the equation.

$I = Prt$

$I = (7000)(0.05)t$

$I = 350t$

The simplified equation is $I = 350t$.

c. Which is the dependent variable, and which is the independent variable?

Principal and rate are the two fixed amounts in this situation. The total amount of interest earned depends on the number of years. Therefore, the number of years, t, is the independent variable, and the interest, I, is the dependent variable.

d. Create a table of values to represent the amount of interest Isaac will receive each year from 1 to 10 years.

Recall that in a table of values, x is the independent variable and y is the dependent variable.

Start with $I = 350t$ and substitute x for t (the number of years) and y for I (the interest).

$I = 350t$

$y = 350x$

Substitute the values of 1 through 10 for the variable x in the equation $y = 350x$. Record the results in a table.

For instance, when $x = 1$, the result is $y = 350(1)$ or 350. Isaac would receive \$350 in interest the first year. When $x = 2$, the result is $y = 350(2)$ or 700. Isaac would receive \$700 in interest in the second year.

Continue in this manner until the amount of interest for each year is calculated.

Number of years (x)	**Interest in \$ (y)**
1	350
2	700
3	1,050
4	1,400
5	1,750
6	2,100
7	2,450
8	2,800
9	3,150
10	3,500

e. After how many years will Isaac receive at least \$2,200 in interest?

It can be seen in the table that Isaac would receive \$2,100 at the end of the sixth year, and \$2,450 at the end of the seventh year. The amount of \$2,200 is between these two values, but because interest is not awarded until the end of the year, Isaac would receive at least \$2,200 after year 7.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 1: Graphing Functions by Creating Tables of Values

For problems 1–6, create a table of values for each equation, and then graph the relationship on a coordinate plane.

1. $y = -6x + 2$

2. $y = \frac{1}{4}x - 8$

3. $y = 2x^2 + 1$

4. $y = \frac{x^2}{2} + 4$

5. $y = -2(x + 6)$

6. $y = 3x(x - 2)$

For problems 7–10, create a table of values for each situation, and then graph the relationship on a coordinate plane.

7. A cell phone company charges a \$20 fee for text messaging plus \$0.10 per text message. The monthly cost, y, can be found using the equation $y = 0.10x + 20$, where x is the number of text messages.

8. Landon overdrew his new bank account by \$30. His bank did not charge him a fee since it was his first time overdrawing. He decided to start saving \$15 a month to prevent future overdrafts. His account balance, y, can be found using the equation $y = 15x - 30$, where x is the number of months he has been saving.

9. A rental car company charges an \$18 fee for renting a car plus \$0.30 per mile the car is driven. The cost of renting a car, y, can be found using the equation $y = 0.30x + 18$, where x is the number of miles the car is driven.

10. John threw a ball in the air from an initial height of 6 feet at a speed of 45 feet per second. The height of the ball, y, can be found using the equation $y = -16x^2 + 45x + 6$, where x is the number of seconds since the ball was thrown.

Instruction

Skill 2: Identifying Key Features of Linear Functions and Quadratic Functions in Standard Form**

Common Core State Standard

F–IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

This skill has been addressed in previous lesson(s) in *CCSS Integrated Pathway: Mathematics II*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

Unit 2, Lesson 2, Sub-lesson 1

Supportive Instructional Strategies for Mathematics II

Unit 2 Lesson 1

Suggestions for Graphic Organizers/Manipulatives

- Provide students with a blank two-circle Venn Diagram graphic organizer from the Program Overview. Ask students to label the circle on the left "Linear function" and label the circle on the right "Quadratic function." Ask them to list specific characteristics of each type of function in the appropriate circles, and then list characteristics that both types of functions have in common in the middle. Ask volunteers to share answers for the three parts of the Venn diagram, then discuss the similarities and differences. Create a master Venn diagram so that all answers can be compiled into one organizer.
    - Possible linear function characteristics: *it has a constant rate of change, the graph is a line.*
    - Possible quadratic function characteristics: *the rate of change varies depending on the interval observed, the graph is a curve.*
    - Possible shared characteristics: *both have an independent and a dependent variable, a table of values can be created, x- and y-intercepts can be found, the rate of change can be found.*
- Provide students with the Coordinate Plane graphic organizer from the Program Overview. Ask students to write the quadratic function $f(x) = x^2 + 3$ at the top of the page. Then, ask them to create a table of values and graph the function. Have them work with a partner to identify the key features of the function, including any extrema, intercepts, vertices, and axes of symmetry. Ask students to volunteer their answers and discuss each feature of the graph and function.
- Provide students with at least 10 blank flash cards. Ask them to write the vocabulary words from the lesson on one side of each flash card. Then, have each student switch cards with a partner, and have the partner define each vocabulary word in their own words on the opposite side of the flash card. Ask students to volunteer their answers, and create a master list of vocabulary terms.

Suggestions for Discourse

- Have student pairs list real-life examples in which linear functions would apply, and then list real-life examples in which quadratic functions would apply. Ask them to think of the different scenarios in terms of restricted values, intercepts, and extrema. Then, ask for volunteers to share their ideas, and make a master list compiling all valid examples for each type of function.

- Ask students, "What are the differences in the formats and graphs of a linear function and a quadratic function?" Encourage a discussion regarding the features of each function in standard form, as well as the features displayed on a graph of each type.
- Ask students, "What is the possible number of *x*-intercepts that a quadratic function can have?" Guide them to consider the possibilities of a quadratic function that intercepts the *x*-axis once, twice, or not at all. Ask them to compare their findings to that of the possible number of *x*-intercepts that a linear function could have.

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics II*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.
- Provide a list of sentence frames in which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "I know that this is a quadratic function because the shape of the graph is a _______." Or, "The function $f(x) = x^2 + 4x - 7$ is written in _______ form." Or, "The two points where the graph crosses the <u>x-axis</u> are called the <u>x-intercepts</u>."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- incorrectly replacing *x* with 0 instead of *y* when determining the *x*-intercept (and vice versa)

  Remind students that the coordinates for the *x*-intercept are $(x, 0)$, and the coordinates for the *y*-intercept are $(0, y)$.

- using the incorrect sign when calculating the *x*-coordinate of the vertex

  Have students write the formula for finding the *x*-coordinate of the vertex, $-\frac{b}{2a}$, on the top of their paper. Remind students to substitute in the proper values for *a* and *b* first, and then apply the negative sign to the coordinate.

- confusing the attributes of different forms

  Have students make a chart listing the different forms of a quadratic function (factored form, standard form, and vertex form), and ask them to list an example under each form.

- incorrectly identifying x-intercepts of the factored form

  Remind students that in the factored form of a quadratic function, $f(x) = a(x - p)(x - q)$, the x-intercepts are p and q.

- incorrectly identifying the vertex as a maximum or minimum

  Remind students that the maximum value on a graph is defined as the largest y-value of a quadratic function, and the minimum value on a graph is defined as the smallest y-value. Remind them that *maximum* means "greatest" and *minimum* means "least."

Lesson 2: Interpreting Quadratic Functions

Instruction

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics II*.

Skill 1: Knowing the Standard Form of Quadratic Functions** (F–IF.8a)

Common Core State Standard

F–IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Skill 2: Using Graphing Technology to Model and Interpret Quadratic Functions** (F–IF.4★)

Common Core State Standard

F–IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

Skill 3: Understanding the Difference Between Domain and Range (F–IF.1)

Common Core State Standard

F–IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

Skill 4: Evaluating Quadratic Functions for Specific Values of x (F–IF.2)

Common Core State Standard

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

(*continued*)

Skill 5: Finding the Slope or Rate of Change of Linear Functions (8.F.4)

Common Core State Standard

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Instruction

Skill 1: Knowing the Standard Form of Quadratic Functions**

Common Core State Standard

F–IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

This skill has been addressed previously in *CCSS Integrated Pathway: Mathematics II*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

Unit 2, Lesson 1, Sub-lesson 2

Skill 2: Using Graphing Technology to Model and Interpret Quadratic Functions**

Common Core State Standard

F–IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★*

This skill has been addressed previously in *CCSS Integrated Pathway: Mathematics II*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

Unit 2, Lesson 2, Sub-lesson 1

Instruction

Skill 3: Understanding the Difference Between Domain and Range

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 ✓ 6 ✓
7 ✓ 8 ✓

Common Core State Standard

F–IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

Essential Questions

1. What are the differences between the domain and the range?
2. Why is it important to consider any possible limits or restrictions on the domain and/or range in a real-world scenario?

WORDS TO KNOW

domain	the set of all inputs of a function; the set of x-values that are valid for the function
function	a relation in which every element of the domain is paired with exactly one element of the range; that is, for every value of x, there is exactly one value of y
function notation	a way to name a function using $f(x)$ instead of y
range	the set of outputs of a function; the set of y-values that are valid for the function
relation	a set of ordered pairs

Recommended Resources

- freeMATHhelp.com. "Domain and Range."

  http://www.walch.com/rr/04068

  This site reviews how to find the domain and range of a function, and provides a domain and range calculator.

- IXL Learning. "Domain and Range of Relations."

  http://www.walch.com/rr/04069

  This site offers practice problems for finding the domain and range of relations, with immediate feedback for incorrect answers.

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Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Have the students create a mapping diagram of the domain and range of a set of coordinates. For example, for the set of coordinates (3, 5), (–1, –3), (0, –1), (4, 7), (–2, –5), first create two ovals, one for the domain and one for the range. Then list the domain values (in numerical order from least to greatest) in a vertical column in the first oval, and do the same with the range values in the second oval. Then, draw an arrow from each element in the domain to the matching element in the range. For example, for (3, 5), draw an arrow from the 3 in the domain to the 5 in the range.

Suggestions for Discourse

- Ask the students why, for an equation such as $f(x)=\sqrt{x}$, the domain is limited to $x \geq 0$. Students should recognize that the domain must be positive because the square root of a negative number results in an imaginary number and, therefore, cannot be represented on a coordinate plane.
- Ask students, "What are some real-life scenarios of functions in which the domain and/or range is restricted?" Encourage them to create examples and explain why, for example, there cannot be negative time or distance, or there cannot be half of an item. Create a master list of examples in which the domain and range are positive integers.

Making Connections

- In order to tell if a relation is a function, one must determine if every element of the domain is paired with only one element in the range. This process requires a clear understanding of what the domain and range are and how to find them.
- Encourage students to relate the terms *domain* and *range* to evaluating expressions for a given value. Remind them that finding a range given a set domain is the same concept.
- Discuss how different occupations require formulas (functions) that involve inputting different values and getting different output values. Help students connect this to a domain and range of a function.

Skill 3: Understanding the Difference Between Domain and Range

Introduction

A calculator is a commonly used tool that takes an input value and produces an output value. For example, if 5 + 3 is input into the calculator, the result is 8. Functions can be explained in the same manner. An input value is put into a function, and an output value is produced. For the function $f(x) = x + 3$, when the input value of 5 is entered in place of x, the output value is 8.

When a problem is put into a calculator, only one result, or output value, is produced. Similarly, an equation is a function if and only if each input value produces one and only one output value. If a particular input value produces more than one output, then the equation is not a function.

Key Concepts

- A **relation** is a set of ordered pairs. A **function** is a relation in which every input value produces exactly one output value.

Domain and Range

- Every function has a domain and a range.
- The **domain** is the set of all inputs of a function or relation. In other words, the domain is the set of x-values that are valid for the function.
- The **range** is the set of outputs of a function or relation. In other words, the range is the set of y-values that are valid for the function.
- To find the domain, list all the values of x. If an equation is given, determine if there are any numbers that cannot be a value for x. If there are no limits on the values of x, then the domain is all the real numbers. The domain can be written in words or in symbols such as "all real numbers" or {all real numbers}.
- To find the range, list all the values of y. If an equation is given, it may be necessary to graph the function in order to see what the y-values are. Or, substitute values for x to find the corresponding values for y.
- Note that, when listing the values of the domain and range, it is customary to order each set from least to greatest. This will also show any repeated values in a set. If a set has repeated values, list the value only once. For example, a domain with the values –6, 1, 0, 1, and –4 would be rewritten as {–6, –4, 0, 1}.
- If there are dozens or hundreds of values in a domain or range, rather than writing all of the values, an ellipsis (...) can be used to show that the values continue. For instance, a range of 0 to 50 can be written as {0, 1, 2, ..., 48, 49, 50}.

Instruction

Function Notation

- Functions are often written in **function notation**. This means that y (the output value) is replaced with $f(x)$, which is pronounced "f of x." For example, $f(x) = x + 2$ is the same as $y = x + 2$.
- If a problem involves more than one function, other letters will be used instead of f to describe additional functions. For example, a function may be named $g(x)$ or $h(x)$.
- For each input x, there is an output $f(x)$. For example, consider the function $f(x) = x + 2$. To find $f(3)$, substitute 3 for x and solve to find the output value. In this case, $f(3) = (3) + 2 = 5$. Thus, given the function $f(x) = x + 2$, for an input of 3, the output is 5. This can be written as an ordered pair, (3, 5).

Guided Practice Skill 3

Example 1

Find the domain and range of the following relation:

{(6, 10), (–5, –1), (0, 4), (–3, 1), (2, 6)}

1. Find the domain of the relation.

   The domain is the set of *x*-values. List the *x*-values in numerical order. If any of the values repeat, list them only once.

   The *x*-values are 6, –5, 0, –3, and 2. None of the *x*-values are repeated, but they need to be reordered from least to greatest:

   –5, –3, 0, 2, 6

   The domain of the given relation is {–5, –3, 0, 2, 6}.

2. Find the range of the relation.

   The range is the set of *y*-values. List the *y*-values in numerical order. If any of the values repeat, list them only once.

   The *y*-values are 10, –1, 4, 1, and 6. None of these are repeated, but they need to be reordered from least to greatest:

   –1, 1, 4, 6, 10

   The range of the given relation is {–1, 1, 4, 6, 10}.

Example 2

Anton is trying to sell 150 candy bars. The function $f(x) = 2x - 50$ represents the profit he will make after he sells x candy bars. Find the domain and range of the function.

1. Find the domain of the function.

   The domain is the set of x-values that are valid for the function. Since x represents the number of candy bars Anton sells, then the possible values of x must be numbers from 0 to 150. Since he cannot sell a partial candy bar, only integers are part of the domain. (Recall that integers are numbers that are not fractions or decimals.)

   The domain is {0, 1, 2, ..., 148, 149, 150}, which can also be written as $0 \le x \le 150$. (Note that the ellipsis shows that the integer values continue between 2 and 148.)

2. Find the range of the function.

   The range is the set of y-values that are valid for the function. Since the domain is all the integers from 0 to 150, the range will be the set of y-values, or outputs, that result from x-values (inputs) from 0 to 150.

   Remember that when an equation is written in function notation, $f(x)$ replaces y.

   To find the range, substitute some values from the domain for x and find the corresponding values of $f(x)$, or y-values. Then try to find a pattern.

   Let's use the first three values in the domain: 0, 1, and 2.

   Let $x = 0$.

$f(x) = 2x - 50$	Given function
$f(0) = 2(0) - 50$	Substitute 0 for x.
$f(0) = -50$	Simplify.

   For an input value of 0, the output value is –50.

   (*continued*)

Let $x = 1$.

$f(x) = 2x - 50$	Given function
$f(1) = 2(1) - 50$	Substitute 1 for x.
$f(1) = -48$	Simplify.

For an input value of 1, the output value is –48.

Let $x = 2$.

$f(x) = 2x - 50$	Given function
$f(2) = 2(2) - 50$	Substitute 2 for x.
$f(2) = -46$	Simplify.

For an input value of 2, the output value is –46.

Next, look for a pattern in the results.

Every time the domain value, x, increases by 1, the range value, $f(x)$, increases by 2. So, it can be seen from these values, as well as from the function itself, the pattern of the values in the range is adding 2 each time. Since the first output value is –50, which is an even number, and 2 is added to each output value, the range will be all even numbers, starting at –50. To find where the range ends, substitute 150 (the highest value in the domain) into the function and solve.

Let $x = 150$.

$f(x) = 2x - 50$	Given function
$f(150) = 2(150) - 50$	Substitute 150 for x.
$f(150) = 250$	Simplify.

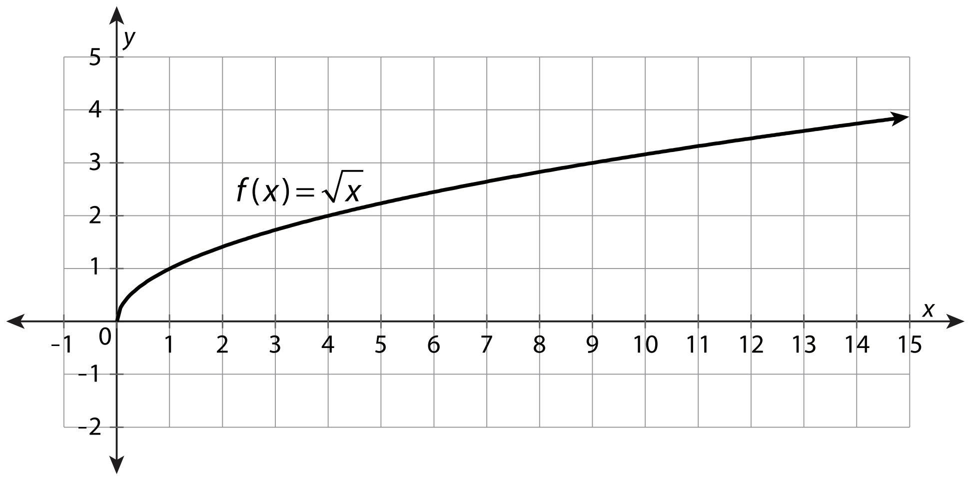
For an input value of 150, the output value is 250.

The possible range of the y-values is all the even numbers from –50 to 250.

Therefore, the range is $\{-50, -48, -46, \ldots, 246, 248, 250\}$.

Example 3

Determine the domain and range of the following graph.



1. Determine the domain of the function.

   The domain is the set of *x*-values that are valid for the function.

   The graph starts at $x = 0$, and continues infinitely in a positive direction. This means that only positive *x*-values are included in the domain because the function is only valid when *x* is greater than or equal to 0.

   Therefore, the domain is $\{x \geq 0\}$.

2. Determine the range of the function.

   The range is the set of *y*-values that are valid for the function. In this graph, the *y*-values start at $y = 0$ and continue to increase. Therefore, the range is all the numbers greater than or equal to 0.

   The range is $\{y \geq 0\}$.

Scaffolded Practice Skill 3

Example 1

Find the domain and range of the following relation:

{(6, 10), (–5, –1), (0, 4), (–3, 1), (2, 6)}

1. Find the domain of the relation.

2. Find the range of the relation.

continued

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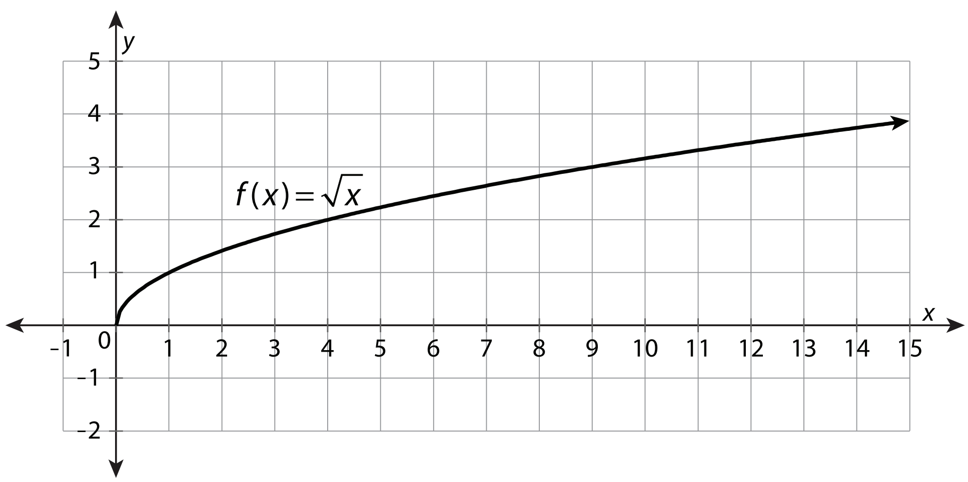
Example 2

Anton is trying to sell 150 candy bars. The function $f(x) = 2x - 50$ represents the profit he will make after he sells x candy bars. Find the domain and range of the function.

Example 3

Determine the domain and range of the following graph.



Problem-Based Task Skill 3: To the Test

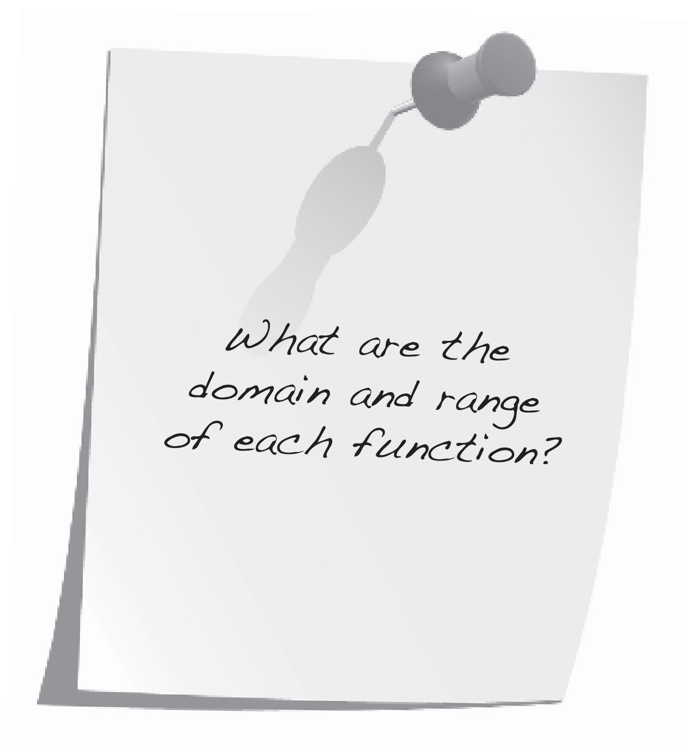
SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 ✓ 6 ✓
7 ✓ 8 ✓

Tomorrow Mrs. Peterson is giving a 20-question test to her students. Each question will be worth 5 points, so she will calculate the grade by multiplying the number of correct answers by 5; no partial credit will be given for incorrect answers. Today the whole class is working on a group assignment, and if the class completes the assignment successfully, then every student will receive 2 bonus points added to his or her test score.

Depending on the outcome of the group assignment, one of these two functions will represent the test grades:

- If the class does not win the bonus points, the function $f(x) = 5x$ will represent the test grades, where x is the number of correct answers and $f(x)$ is the grade.
- If the class wins the bonus points, the function $g(x) = 5x + 2$ will represent the test grades, where x is the number of correct answers and $g(x)$ is the grade.

What are the domain and range of each function?



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Problem-Based Task Skill 3: To the Test

Coaching

a. What are the possible x-values for the function $f(x)$, the function used if the class does not win the bonus points?

b. What would the corresponding y-values be for the function $f(x)$?

c. What is the domain of $f(x)$?

d. What is the range of $f(x)$?

e. What are the possible x-values for the function $g(x)$, the function used if the class wins the bonus points?

f. What would the corresponding y-values be for the function $g(x)$?

g. What is the domain of $g(x)$?

h. What is the range of $g(x)$?

Problem-Based Task Skill 3: To the Test

Coaching Sample Responses

a. What are the possible x-values for the function $f(x)$, the function used if the class does not win the bonus points?

Since x represents the number of questions a student answers correctly and there are 20 questions on the test, x could be any integer from 0 to 20.

b. What would the corresponding y-values be for the function $f(x)$?

Since the possible x-values are the integers 0 through 20, determine what y-values correspond to these x-values by substituting values for x and solving for y.

$f(x) = 5x$

$f(0) = 5(0) = 0$

$f(1) = 5(1) = 5$

$f(2) = 5(2) = 10$

It can be determined from the output values as well as the function that the y-values will all be multiples of 5. This also reflects the problem statement, which specifies "no partial credit will be given," so students can only get grades that are in multiples of 5.

Substitute the highest value of x (20) to find the highest possible value of y.

$f(x) = 5x$

$f(20) = 5(20) = 100$

The highest value of y is 100.

The y-values will be all the multiples of 5 from 0 to 100.

c. What is the domain of $f(x)$?

The domain is the set of all the possible x-values for the function.

Since x could be any integer from 0 to 20, that is also the domain.

Thus, the domain is {0, 1, 2, ..., 18, 19, 20}.

d. What is the range of $f(x)$?

The range is the set of all the possible y-values for the function.

Since the possible y-values are all the multiples of 5 from 0 to 100, this is also the range.

Thus, the range is {0, 5, 10, ..., 90, 95, 100}.

e. What are the possible x-values for the function $g(x)$, the function used if the class wins the bonus points?

Since x represents the number of questions a student answers correctly and there are still 20 questions on the test, x could still be any integer from 0 to 20.

f. What would the corresponding y-values be for the function $g(x)$?

Since the values for x are the integers 0 through 20, determine what y-values correspond to these x-values by substituting values for x and solving for y.

$g(x) = 5x + 2$

$g(0) = 5(0) + 2 = 2$

$g(1) = 5(1) + 2 = 7$

$g(2) = 5(2) + 2 = 12$

It can be determined that the pattern of the output values is adding 5 every time. Substitute the highest value of x (20) to find the highest value of y:

$g(x) = 5x + 2$

$g(20) = 5(20) + 2 = 102$

The highest value of y is 102.

The possible y-values are 2, 7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, 62, 67, 72, 77, 82, 87, 92, 97, and 102.

g. What is the domain of $g(x)$?

The domain is the set of all the possible x-values for the function.

Because x could be any integer from 0 to 20, that is also the domain.

Thus, the domain is {0, 1, 2, ..., 18, 19, 20}.

h. What is the range of $g(x)$?

The range is the set of all the possible y-values for the function.

The possible y-values are 2, 7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, 62, 67, 72, 77, 82, 87, 92, 97, and 102, so this is also the range.

Thus, the range is {2, 7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, 62, 67, 72, 77, 82, 87, 92, 97, 102}.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 3: Understanding the Difference Between Domain and Range

For problems 1–5, find the domain and range of each relation or function.

1. (3, 6), (2, –5), (0, 0), (1, 6), (–1, 2)

2. (–5, 3), (–1, 6), (2, 8), (–2, 7), (–5, 6)

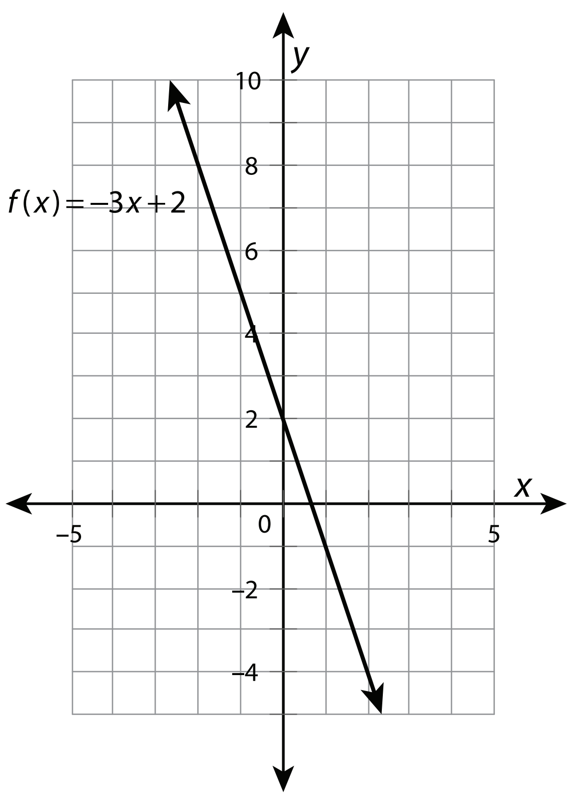
3. $f(x) = 4x - 7$

4. $y = x^2 + 1$

5. $f(x) = \sqrt{x+3}$

For problems 6–8, find the domain and range of each graphed function.

6.



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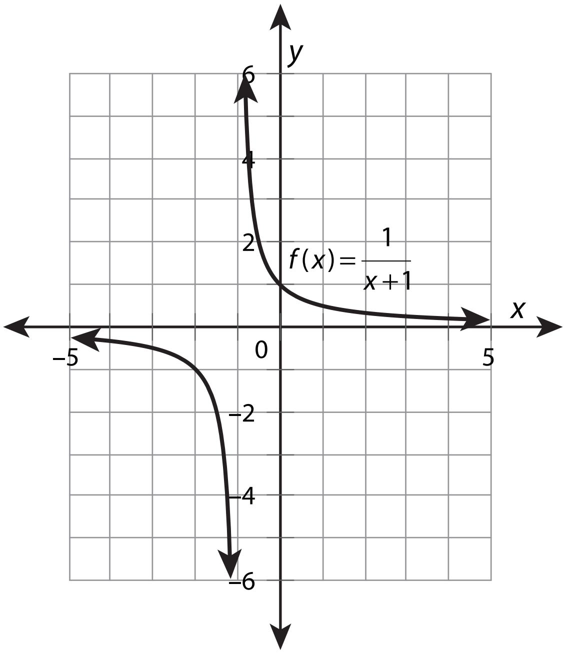
7.



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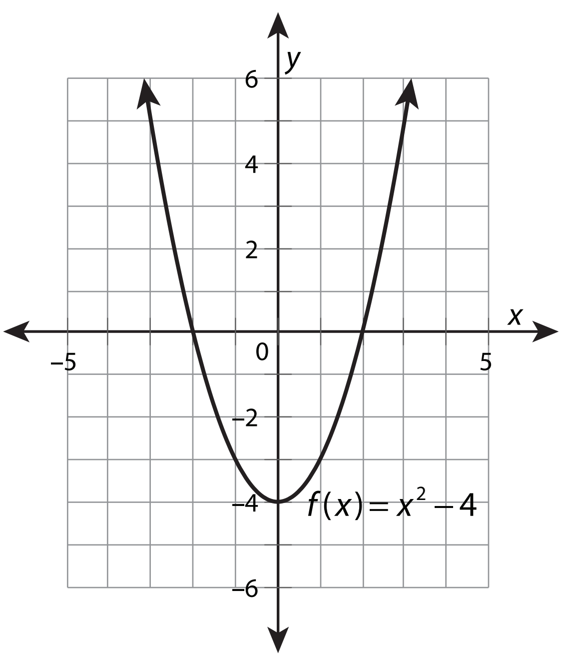

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For problems 9 and 10, use the given information to find the domain and range of the function.

9. Valerie had $20, and then she bought a novel. Find the domain and range of the function $f(x) = 20 - x$, where x represents the cost of the novel and $f(x)$ represents the money she has left.

10. A new wing has been added to a school, and several of the new classrooms need carpet. The 10 classrooms in the new wing are all the same, and each room has an area of 580 square feet. The carpet installation team is not sure of how many rooms they will install carpet in today. Find the domain and range of the function $f(x) = 580x$, where x is the number of classrooms in which the team will install carpet today and $f(x)$ is the total amount of carpet that will be installed.

Instruction

Skill 4: Evaluating Quadratic Functions for Specific Values of *x*

SMP	
1 ✓	2 ✓
3 ✓	4 ✓
5	6 ✓
7 ✓	8

Common Core State Standard

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Essential Question

1. What does it mean to evaluate a function?

WORDS TO KNOW

domain the set of all inputs of a function; the set of *x*-values that are valid for the function

evaluate to compute the value of an expression

range the set of outputs of a function; the set of *y*-values that are valid for the function

Recommended Resource

- Purplemath.com. "Evaluation: Evaluating Expressions, Polynomials, and Functions."

  http://www.walch.com/rr/04070

  This site reviews how to evaluate expressions, polynomials, and functions, with worked examples.

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Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

- Provide students with the Coordinate Plane graphic organizer from the Program Overview. Provide them with colored pencils or markers. Ask students to write the quadratic functions $f(x) = 2x^2 + 1$ and $g(x) = -2x^2 + 1$ at the top of the page. Ask them to work with a partner and first predict what the difference will be for the ranges of each function when given the domain {–2, –1, 0, 1, 2}. Then, ask them to graph each function in a different color and discuss the domain and range values based on the graph. Finally, ask students to evaluate both functions for the given domain and verify the ranges.
- Provide each student with three blank flash cards. Pair students and ask each person to create three examples of quadratic functions, along with a domain containing four values. Ask students to switch cards with their partners, and then evaluate the functions they've been given, determining the ranges for each.

Suggestions for Discourse

- Ask students, "In your own words, what does it mean to evaluate something?" Encourage a discussion about different ways to explain "evaluating." Create a master list of students' explanations, and then relate their examples to evaluating a function for particular values.
- Ask students, "What are some real-life scenarios where functions are evaluated for specific domain values?" Guide them to relate using formulas, such as area formulas, to substituting specific values to get a specific outcome. Examples could include a teacher using a formula to score a test based on the number of correct answers, or using the distance formula to determine how long it takes to jog a certain number of miles at a constant speed.

Making Connections

- Discuss with students how evaluating a function relates to graphing a function; that is, by substituting x-values into a function and simplifying, the outcome is the y-value, and the ordered pair (x, y) is graphed. Remind them that, unlike linear functions, more than two points are necessary to create an accurate graph of a quadratic function, so several domain values are needed to evaluate a quadratic function.
- Remind students that the key features of a quadratic function can be identified by evaluating the function and determining corresponding range values.

Instruction

Skill 4: Evaluating Quadratic Functions for Specific Values of *x*

Introduction

A function f of a variable x is represented by $f(x)$, and can be graphed on a coordinate plane. When graphed, the range of a function is dependent on its domain. The values for the coordinates that satisfy a function can be found by substituting domain values for x.

Key Concepts

- To **evaluate** a function means to calculate the value of the function for a given input value. In other words, substitute the values for the domain for all given occurrences of x.
- The **domain** of a function is the set of all inputs of a function, or the set of x-values that are valid for the function.
- A limited or restricted domain is a subset of the entire domain. It is common to evaluate a function over a given domain rather than over the entire domain.
- The **range** of a function is the set of outputs of a function, or the set of y-values that are valid for the function.
- For example, to evaluate $f(3)$ in the function $f(x) = x^2 + 1$, replace each x with 3 and simplify:

  $f(3) = (3)^2 + 1 = 9 + 1 = 10$
- Points on the graph of $f(x)$ are written as ordered pairs in the form $(x, f(x))$.
- In the function we just evaluated, $f(x) = 10$ when $x = 3$. Writing this information in the form $(x, f(x))$ gives us the point (3, 10), which is a point on the graph of $f(x) = x^2 + 1$.

Guided Practice Skill 4

Example 1

Evaluate $f(x) = 4x^2 - 3$ over the domain {0, 1, 2, 3}. Determine the range for this domain.

1. Evaluate the function for each of the domain values.

   To evaluate the function $f(x) = 4x^2 - 3$ over the domain {0, 1, 2, 3}, substitute the values from the domain into $f(x) = 4x^2 - 3$.

   Evaluate $f(0)$.

$f(x) = 4x^2 - 3$	Given function
$f(0) = 4(0)^2 - 3$	Substitute 0 for x.
$f(0) = -3$	Simplify.

   When 0 is substituted for x, the value of $f(0)$ is –3.

   Evaluate $f(1)$.

$f(x) = 4x^2 - 3$	Given function
$f(1) = 4(1)^2 - 3$	Substitute 1 for x.
$f(1) = 1$	Simplify.

   When 1 is substituted for x, the value of $f(1)$ is 1.

   Evaluate $f(2)$.

$f(x) = 4x^2 - 3$	Given function
$f(2) = 4(2)^2 - 3$	Substitute 2 for x.
$f(2) = 13$	Simplify.

   When 2 is substituted for x, the value of $f(2)$ is 13.

   Evaluate $f(3)$.

$f(x) = 4x^2 - 3$	Given function
$f(3) = 4(3)^2 - 3$	Substitute 3 for x.
$f(3) = 33$	Simplify.

   When 3 is substituted for x, the value of $f(3)$ is 33.

2. Determine the range of the function for the given domain.

Collect the set of outputs for the inputs. Recall that the outputs are the $f(x)$ values, and the inputs are the values we substituted for x.

The outputs from the previous step were –3, 1, 13, and 33.

Therefore, the range of the function over the given domain is {–3, 1, 13, 33}.

Example 2

Evaluate $g(x) = 3x^2 + 2x - 5$ over the domain {–1, 0, 1, 2}. Determine the range for this domain.

1. Evaluate the function for each of the domain values.

To evaluate the function $g(x) = 3x^2 + 2x - 5$ over the domain {–1, 0, 1, 2}, substitute the values from the domain into $g(x) = 3x^2 + 2x - 5$.

Evaluate $g(-1)$.

$g(x) = 3x^2 + 2x - 5$	Given function
$g(-1) = 3(-1)^2 + 2(-1) - 5$	Substitute –1 for x.
$g(-1) = -4$	Simplify.

When –1 is substituted for x, the value of $g(-1)$ is –4.

Evaluate $g(0)$.

$g(x) = 3x^2 + 2x - 5$	Given function
$g(0) = 3(0)^2 + 2(0) - 5$	Substitute 0 for x.
$g(0) = -5$	Simplify.

When 0 is substituted for x, the value of $g(0)$ is –5.

Evaluate $g(1)$.

$g(x) = 3x^2 + 2x - 5$	Given function
$g(1) = 3(1)^2 + 2(1) - 5$	Substitute 1 for x.
$g(1) = 0$	Simplify.

When 1 is substituted for x, the value of $g(1)$ is 0.

(*continued*)

Evaluate $g(2)$.

$g(x) = 3x^2 + 2x - 5$	Given function
$g(2) = 3(2)^2 + 2(2) - 5$	Substitute 2 for x.
$g(2) = 11$	Simplify.

When 2 is substituted for x, the value of $g(2)$ is 11.

2. Determine the range of the function for the given domain.

   Collect the set of outputs for the inputs. Recall that the outputs are the $f(x)$ values, and the inputs are the values we substituted for x.

   The outputs from the previous step were –4, –5, 0, and 11.

   Therefore, the range of the function over the given domain is {–5, –4, 0, 11}.

Scaffolded Practice Skill 4

Example 1

Evaluate $f(x) = 4x^2 - 3$ over the domain {0, 1, 2, 3}. Determine the range for this domain.

1. Evaluate the function for each of the domain values.

2. Determine the range of the function for the given domain.

continued

Example 2

Evaluate $g(x) = 3x^2 + 2x - 5$ over the domain {–1, 0, 1, 2}. Determine the range for this domain.

Problem-Based Task Skill 4: Long Shot

SMP	
1 ✓	2 ✓
3 ✓	4 ✓
5	6 ✓
7 ✓	8

A circus performer is being shot from a cannon. The path of his flight can be modeled by the equation $f(x) = -x^2 + 6x + 4$, where $f(x)$ is the performer's height in feet above the ground and x is the number of seconds since he was shot from the cannon. How high above the ground is the performer after 1 second? 2 seconds? 3 seconds? 4 seconds? Round answers to the nearest foot.

How high above the ground is the performer after 1 second? 2 seconds? 3 seconds? 4 seconds?

Problem-Based Task Skill 4: Long Shot

Coaching

a. What is the domain for this situation?

b. How can the performer's height above the ground be determined for the given domain?

c. What is the range for the given domain?

d. How high above the ground is the performer after 1 second?

e. How high above the ground is the performer after 2 seconds?

f. How high above the ground is the performer after 3 seconds?

g. How high above the ground is the performer after 4 seconds?

Problem-Based Task Skill 4: Long Shot

Coaching Sample Responses

a. What is the domain for this situation?

The domain for this situation is the number of seconds after the performer is shot from the cannon, or {1, 2, 3, 4}.

b. How can the performer's height above the ground be determined for the given domain?

The performer's height above the ground can be determined by substituting each of the domain values into the function for *x* and evaluating.

c. What is the range for the given domain?

To determine the range for the given domain, evaluate the function for each value of *x*.

Evaluate $f(1)$.

$f(1) = -(1)^2 + 6(1) + 4$

$f(1) = 9$

Evaluate $f(2)$.

$f(2) = -(2)^2 + 6(2) + 4$

$f(2) = 12$

Evaluate $f(3)$.

$f(3) = -(3)^2 + 6(3) + 4$

$f(3) = 13$

Evaluate $f(4)$.

$f(4) = -(4)^2 + 6(4) + 4$

$f(4) = 12$

Two of the domain values yield the same output, 12. This value should be listed only once in the range.

The range for the given domain is {9, 12, 13}.

d. How high above the ground is the performer after 1 second?

The value of $f(1)$ is 9; therefore, the height after 1 second is 9 feet.

e. How high above the ground is the performer after 2 seconds?

The value of $f(2)$ is 12; therefore, the height after 2 seconds is 12 feet.

f. How high above the ground is the performer after 3 seconds?

The value of $f(3)$ is 13; therefore, the height after 3 seconds is 13 feet.

g. How high above the ground is the performer after 4 seconds?

The value of $f(4)$ is 12; therefore, the height after 4 seconds is 12 feet.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 4: Evaluating Quadratic Functions for Specific Values of *x*

For problems 1–6, evaluate the given functions and determine the range of each.

1. Evaluate $f(x) = 3x^2 + 5x$ over the domain {0, 2, 4, 6}. What is the range?

2. Evaluate $f(x) = 2x^2 + 5x - 3$ over the domain {–2, 0, 2, 4}. What is the range?

3. Evaluate $f(x) = -x^2 - 7x$ over the domain {1, 3, 5, 7}. What is the range?

4. Evaluate $f(x) = 4x^2 + 8x - 9$ over the domain {–4, –2, 0, 2}. What is the range?

5. Evaluate $f(x) = -5x^2 - 18$ over the domain {0, 1, 2, 3}. What is the range?

6. Evaluate $f(x) = 3(x^2 - 2) + 1$ over the domain {0, 3, 6, 9}. What is the range?

continued

For problems 7–10, use the given information to determine the range of the function.

7. A hawk is descending toward its nest. The hawk's flight can be modeled by the function $h(t) = t^2 - 12t + 35$, where $h(t)$ is the hawk's height above its nest in feet and t is the time in seconds since you saw the hawk. How high above the nest is the hawk after 1 second? 2 seconds? 3 seconds?

8. A roller coaster leaves the platform on the way to a big climb. The roller coaster's path can be modeled by the function $h(t) = -t^2 + 8t + 44$, where $h(t)$ is the roller coaster's height above the ground in feet and t is the time in seconds since it began its climb. How high up is the roller coaster after 1 second? 4 seconds? 8 seconds?

9. A skateboarder is warming up on a ramp. His path can be modeled by the function $h(t) = t^2 - 6t + 10$, where $h(t)$ is the skateboarder's height in feet above the ground and t is the time in seconds since he jumped off the edge of the ramp. How high up is the skateboarder after 1 second? 3 seconds? 4 seconds?

10. A pool fountain shoots water into the air. The path of the water can be modeled by the function $h(t) = -t^2 + 5t + 2$, where $h(t)$ is the water's height in feet above the pool and t is the time in seconds since the water left the fountain. How high up is the water shot from the fountain after 1 second? 3 seconds? 5 seconds?

Instruction

Skill 5: Finding the Slope or Rate of Change of Linear Functions

SMP	
1 ✓	2 ✓
3	4 ✓
5	6 ✓
7 ✓	8

Common Core State Standard

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Essential Questions

1. How does the unit rate of a proportional relationship help determine the slope of a graph?
2. What does the slope of a linear relationship represent?

WORDS TO KNOW

proportional relationship the relationship between two quantities that vary directly with one another

slope the measure of the rate of change of one variable with respect to another variable; $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$; the slope in the equation $y = mx + b$ is m

unit rate a rate per one given unit

Recommended Resources

- IXL Learning. "Proportional Relationships: Word Problems."

  http://www.walch.com/rr/04000

  This site provides practice with solving word problems that involve proportional relationships. Immediate feedback is provided and users are shown how to correctly solve the problem when an incorrect answer is given.

- Purplemath.com. "Slope of a Straight Line."

  http://www.walch.com/rr/04001

  This site provides instruction on how to calculate the slope of a line using two points.

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Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Once students have worked through the Guided Practice, distribute the Frayer Model graphic organizer found in the Program Overview. Ask students to write the word *slope* in the center row of the organizer and then have them define the word. Be sure students are using their own words to define slope, and not simply copying the definition provided in the text. Ask students to then list specific examples of real-world uses of slope in the "Examples from Life" box. Present the leading question, "Where have you ever heard of, or seen, the word *slope* used before?" Below their list of real-life examples, have students briefly describe, or draw a picture of, their real-life slope examples. Ask volunteers to share their examples, and then discuss the examples using any slope-related words, such as *rise*, *run*, *steepness*, and *change.*

Suggestions for Discourse

Ask students to think about and list some examples of unit rates that were not used in the Guided Practice. It may be necessary to first remind students of one of the unit rates from the Guided Practice, such as cost per pound.

Making Connections

Encourage students to connect the use of the phrase *rate of change* with the word *slope* as it is used in the lesson; for example, "the rate of change in the cost of gasoline is $3.15 per gallon; this means that the slope of the line showing the total cost of gasoline bought is 3.15."

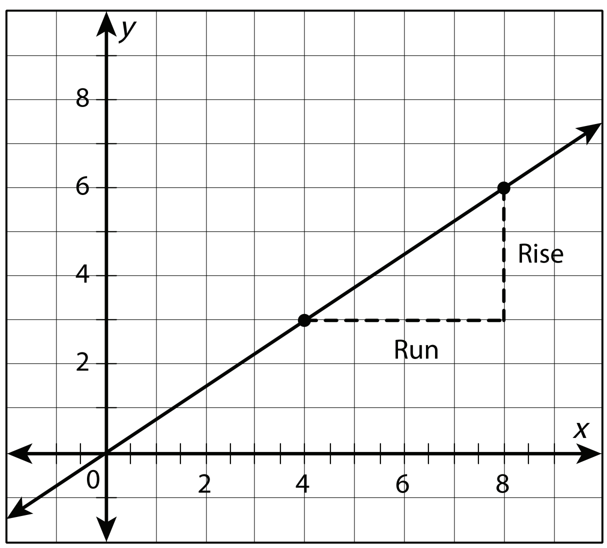
Skill 5: Finding the Slope or Rate of Change of Linear Functions

Introduction

A **proportional relationship** describes the relationship between two quantities that vary directly with one another. A few common proportional relationships that we encounter in our everyday lives include the speed a car travels (miles per hour), the amount of gas consumed on a road trip (gallons per mile), the amount of money earned at a job (dollars per hour), or the number of calories per serving of a favorite snack food (calories per serving). In all of these examples, each of the two quantities described varies directly with the other.

Key Concepts

- The quantities described by a proportional relationship are represented by a linear equation in the form $y = mx$, where m is the slope of the line that passes through the origin (0, 0).
- The **slope** of the graph of a linear equation is a measure of the rate of change of one variable with respect to another variable, and is defined by the ratio of the rise of the graph compared to the run.



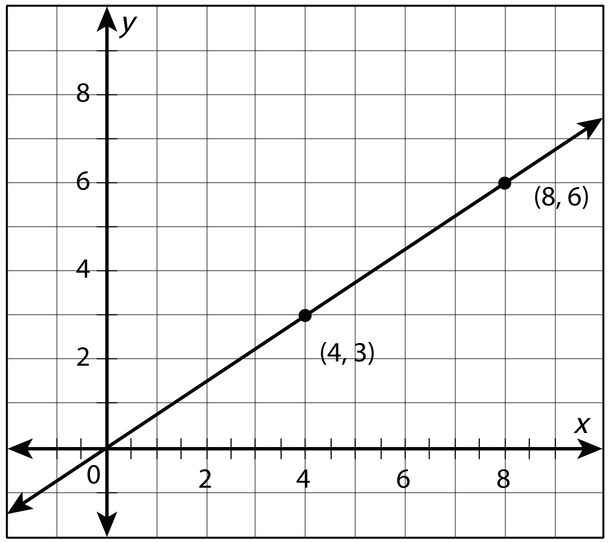
- Given two points on a line, (x_1, y_1) and (x_2, y_2), the slope is the ratio of the change in the y-values of the points (the rise) to the change in the corresponding x-values of the points (the run).

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

- The first step in calculating the slope of a line is to choose two points on the line and label the coordinates of these points as (x_1, y_1) and (x_2, y_2). Then, the rate of change can be found by applying the slope formula. Reduce any fractions to ensure the slope is in simplest form.

Instruction

- In the following graph, notice that two easily identifiable points on the line are (4, 3) and (8, 6).



- Let (x_1, y_1) be (4, 3) and (x_2, y_2) be (8, 6). Substitute these values into the slope formula and simplify to find the slope of the line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(6)-(3)}{(8)-(4)} = \frac{3}{4}$$

- The given line has a rise of 3 units and a run of 4 units; therefore, the slope of the line is $\frac{3}{4}$.
- Note that if the assignment of (x_1, y_1) and (x_2, y_2) was switched in this example, the result would still be the same. For example, let (x_1, y_1) be (8, 6) and (x_2, y_2) be (4, 3). Substitute and simplify.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(3)-(6)}{(4)-(8)} = \frac{-3}{-4} = \frac{3}{4}$$

- The resulting slope is still $\frac{3}{4}$.
- Although it does not matter which point is (x_1, y_1) and which is (x_2, y_2), it *is* important to make sure that the order in which the variables are subtracted remains the same in the numerator and denominator. In other words, $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$; however, $\text{slope} \neq \frac{y_1 - y_2}{x_2 - x_1}$.
- The slope of an equation that describes a proportional relationship is also known as the **unit rate**, or the rate per one given unit.
- The calculation of slope can be extended beyond proportional relationships to that of linear equations of the form $y = mx + b$, where b is the y-intercept.

Guided Practice Skill 5

Example 1

Marc gets paid $15 per lawn he mows. Graph the proportional relationship. Determine the slope and what it means in the context of the problem. How can the slope be used to determine how many lawns Marc mowed if he made $180? What is the equation that describes the relationship between the two quantities?

1. Create a table to show how the two quantities described vary.

   The two quantities described are the number of lawns mowed and Marc's earnings in dollars.

   For each lawn mowed, Marc earns $15; therefore, the total amount Marc earns can be determined by multiplying the number of lawns by 15.

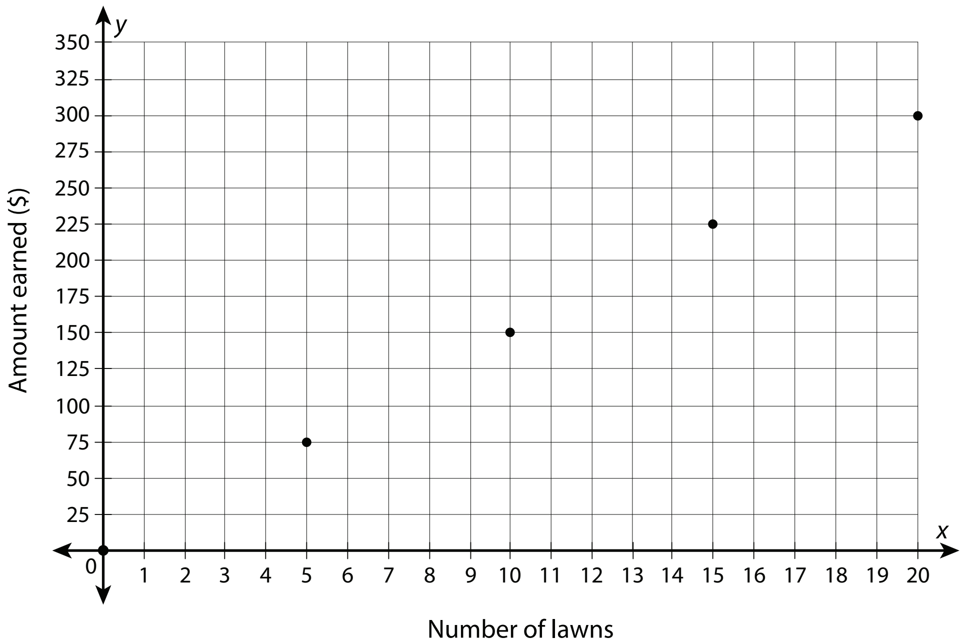
   Choose several values for the number of lawns mowed and calculate the earnings. Use a table to organize the information.

Number of lawns	0	5	10	15	20
Amount earned ($)	0	75	150	225	300

2. Graph the proportional relationship.

   Use the table of values to graph the relationship.

   Let *x* represent the number of lawns mowed and *y* represent the amount earned in dollars.



3. Determine the slope and what it means in the context of the problem.

   There are several ways to determine the slope of this proportional relationship.

   The amount of money Marc earns was given as a unit rate: he gets paid \$15 per lawn, so the slope is 15.

   The slope can also be determined by using the slope formula, $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$. Choose two points from the graph; let (x_1, y_1) be (0, 0) and (x_2, y_2) be (5, 75). Substitute these values into the slope formula to find the slope of the line.

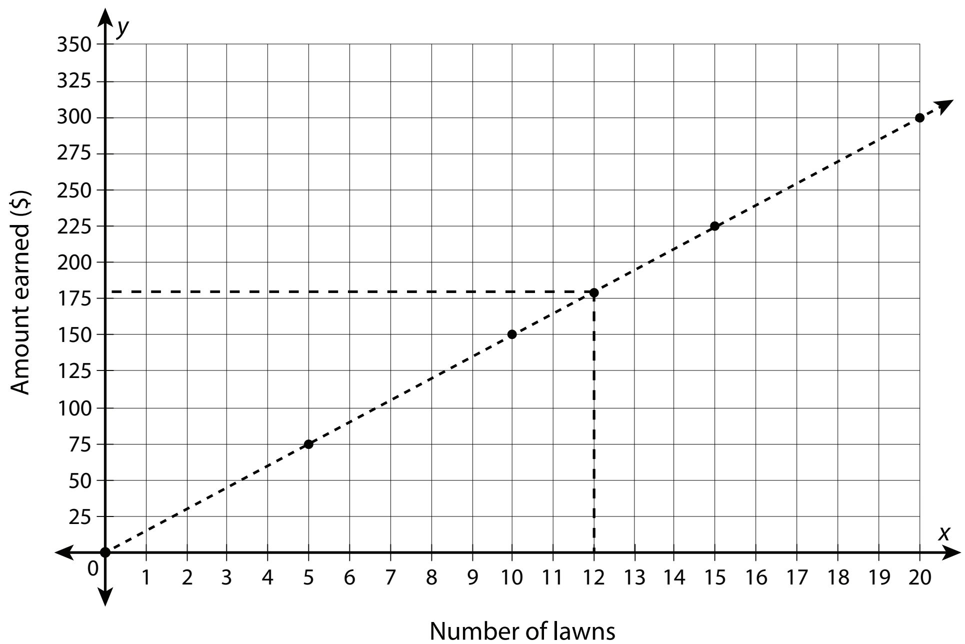
$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$\text{slope} = \frac{(75) - (0)}{(5) - (0)}$	Substitute 0 for y_1, 75 for y_2, 0 for x_1, and 5 for x_2.
$\text{slope} = \frac{75}{5}$	Subtract.
slope = 15	Simplify.

   This confirms that the slope is 15, or \$15 per lawn.

4. How can the slope be used to determine how many lawns Marc mowed if he made \$180?

   The scenario described is a proportional relationship; therefore, the number of lawns mowed for a total of \$180 can be estimated from the graphed values.

   Find 180 on the *y*-axis and then look to the right to determine the corresponding *x*-coordinate.



   From the graph, it appears that if Mark earned \$180, then he mowed 12 lawns.

   Using the unit rate of \$15 per lawn, the number of lawns can also be found by dividing 180 by 15. The result is 12 lawns for \$180.

5. Write the equation that describes the relationship between the two quantities.

   The equation that describes proportional relationships has the form $y = mx$, where m is the slope.

   The slope of this relationship is 15; therefore, the equation that describes this relationship is $y = 15x$.

Example 2

At a roadside farm stand, you can buy 5 pounds of any of the vegetables for a total cost of $6. Determine the slope of the line formed by the proportional relationship between the number of pounds purchased and the cost of the vegetables. Explain what the slope means in the context of the problem. Finally, use the slope to determine how many pounds of vegetables can be purchased for $13. Assume there is no sales tax.

1. Determine the unit rate.

   Recall that the unit rate is a rate per one given unit. If 5 pounds of vegetables sell for $6, divide 6 by 5 to determine the cost for 1 pound.

   $$\frac{6}{5}=1.2$$

   The unit rate, or cost of 1 pound, is $1.20 per pound.

2. Create a table of values and use it to graph the proportional relationship.

   The two quantities described are the number of pounds purchased and the total cost of the vegetables in dollars.

   The unit rate for the vegetables is $1.20 per pound. Therefore, the total cost can be determined by multiplying the number of pounds by 1.2.

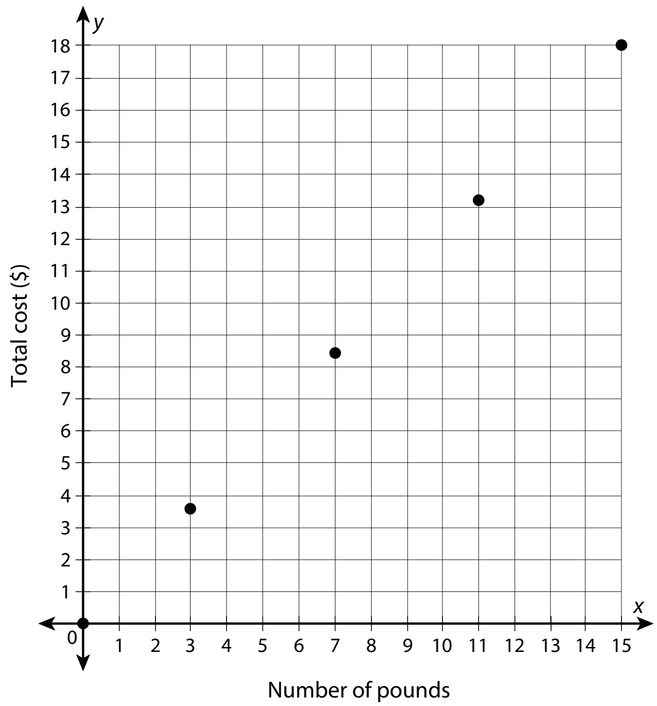
   Choose several values for the number of pounds purchased from 0 to 15 and calculate the associated cost.

Number of pounds	0	3	7	11	15
Total cost ($)	0	3.6	8.4	13.2	18

   Use these values to show the relationship between the number of pounds purchased and the total cost.

   Let x represent the number of pounds purchased and y represent the total cost in dollars.

(*continued*)



3. Determine the slope and what it means in the context of the problem.

   There are several ways to determine the slope of this proportional relationship.

   The unit rate, or cost of each pound, was determined to be $1.20; therefore, the slope is 1.2.

   The slope can also be determined by using the slope formula, $\text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$. Let (x_1, y_1) be (0, 0) and (x_2, y_2) be (3, 3.6). Substitute these values into the slope formula to find the slope of the line, and then simplify.

(continued)

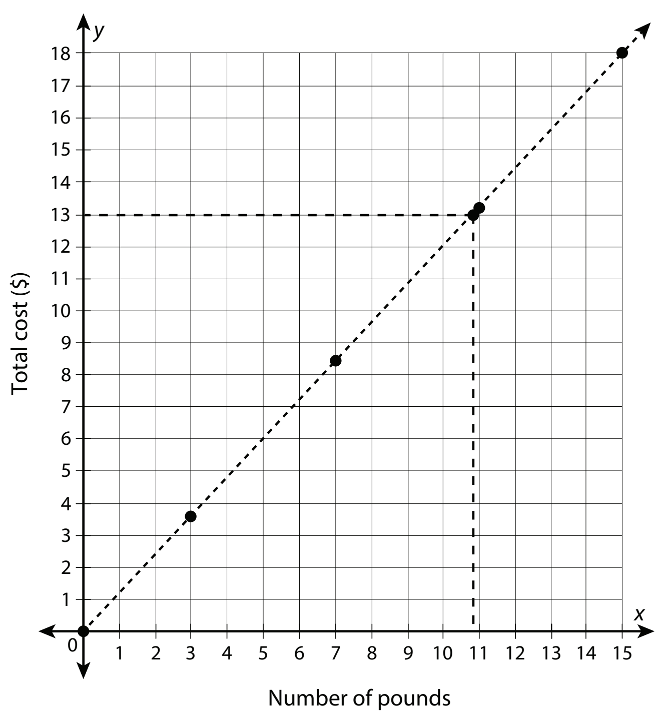
$\text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$\text{slope} = \dfrac{(3.6)-(0)}{(3)-(0)}$	Substitute 0 for y_1, 3.6 for y_2, 0 for x_1, and 3 for x_2.
$\text{slope} = \dfrac{3.6}{3}$	Subtract.
slope = 1.2	Simplify.

This confirms that the slope is 1.2, or $1.20 per pound.

4. How can the slope be used to determine how many pounds of vegetables can be purchased for $13?

   The scenario described is a proportional relationship; therefore, the number of pounds of vegetables that can be purchased for $13 can be estimated from the graphed values.

   Find 13 on the *y*-axis, and then look to the right to determine the corresponding *x*-coordinate.



(*continued*)

From the graph, it appears that approximately 11 pounds of vegetables can be purchased for $13.

Using the unit rate of $1.20 per pound, the number of pounds can also be found by dividing 13 by 1.2. The result is 10.833 or approximately 11 pounds for $13.

Example 3

A new plumber has just started his own business. In order to try and gain customers, he is running a special for his services. He charges $16 per hour, plus a standard house call fee of $25. Determine the slope of the line that passes through the points of the total cost for jobs lasting from 2 hours to 6 hours. Explain what the slope means in the context of the problem. Finally, use the slope to determine how many hours of work a customer could get for $150. Assume there is no sales tax.

1. Create a table to show how the two quantities described vary.

   The two quantities described are the number of hours and the total cost of the work.

   The total cost can be determined by multiplying the number of hours by 16 and then adding 25 to include the $25 house call fee.

   Therefore, the equation that represents this scenario is $y = 16x + 25$.

   Choose several values for the number of hours and calculate the associated cost.

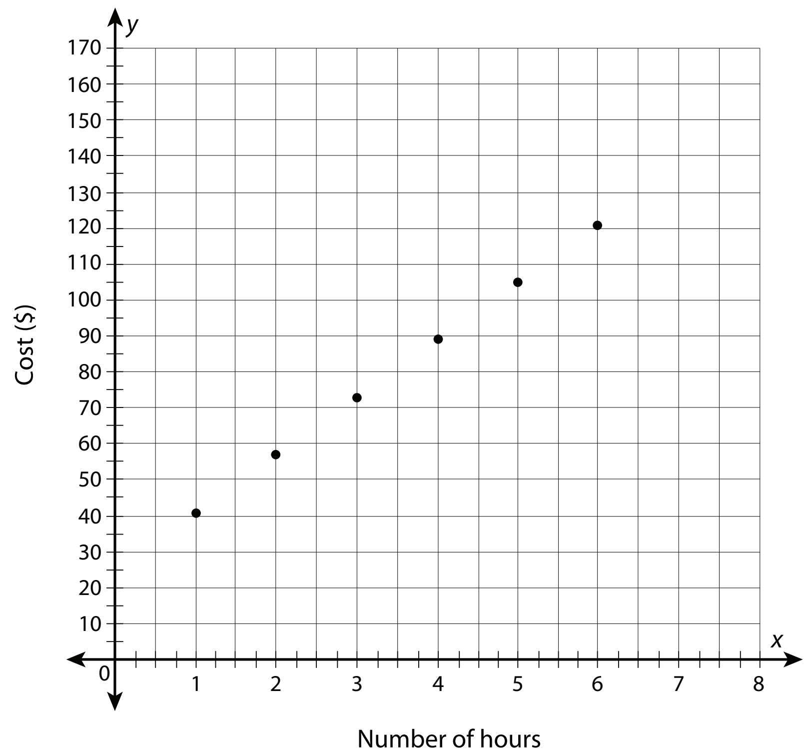
   Let's use 1, 2, 3, 4, 5, and 6. Substitute each of the values for x in the equation, and then solve for y.

Number of hours	Calculation	Cost ($)
1	16(1) + 25 = 41	41
2	16(2) + 25 = 57	57
3	16(3) + 25 = 73	73
4	16(4) + 25 = 89	89
5	16(5) + 25 = 105	105
6	16(6) + 25 = 121	121

2. Graph the relationship.

   Use the table of values to graph the relationship.

   Let *x* represent the number of hours and *y* represent the cost in dollars.



3. Determine the slope and what it means in the context of the problem.

   Determine the slope by using the slope formula, $\text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$. Let (x_1, y_1) be (1, 41) and (x_2, y_2) be (2, 57). Substitute these values into the slope formula to find the slope of the line, and then simplify.

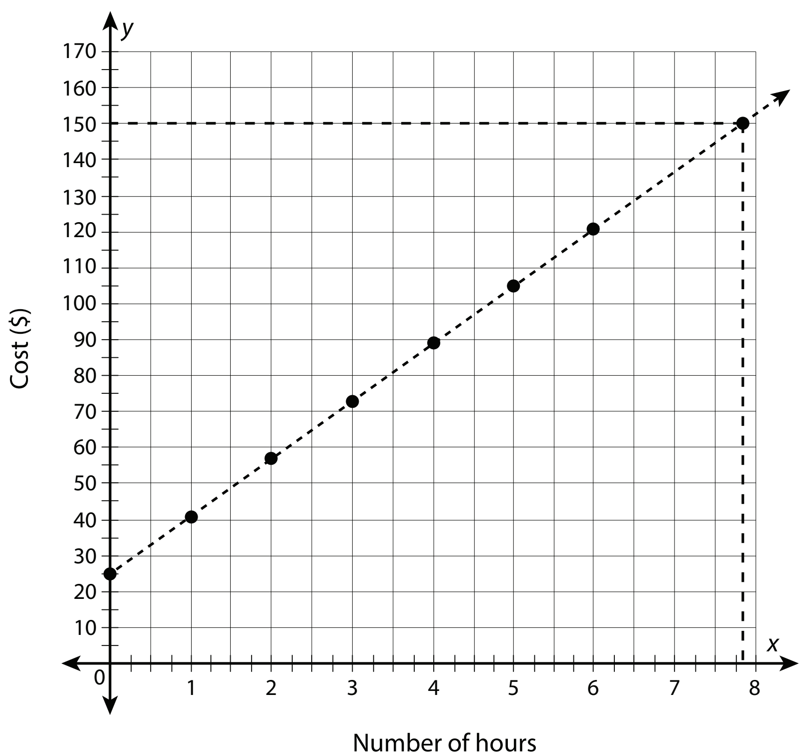
$\text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$\text{slope} = \dfrac{(57)-(41)}{(2)-(1)}$	Substitute 41 for y_1, 57 for y_2, 1 for x_1, and 2 for x_2.
$\text{slope} = \dfrac{16}{1}$	Subtract.
slope = 16	Simplify.

   The slope is 16, or $16 per hour. This verifies the given information in the problem that each hour of work costs $16.

4. How can the slope be used to determine how many hours of work a customer could get for $150?

   The number of hours of work that would cost $150 can be estimated from the graphed values.

   Find 150 on the *y*-axis, and then look to the right to determine the corresponding *x*-coordinate.



   From the graph, it appears that $150 will pay for a little less than 8 hours of work.

   Using the rate of $16 per hour, the number of hours can also be found by first subtracting 25 from 150 and then dividing the result by 16. This yields a result of 7.81; therefore, a customer could get a full 7 hours of work and part of another hour for $150.

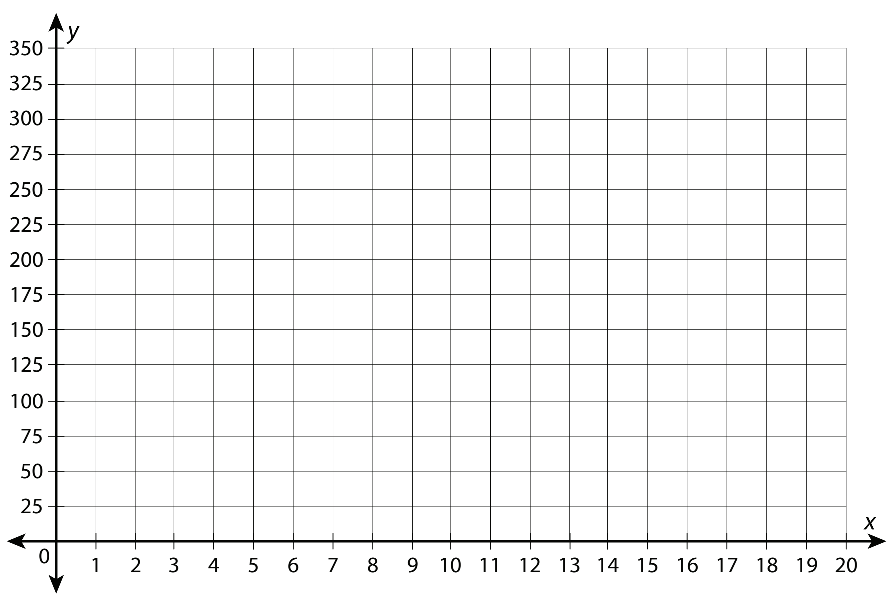
Scaffolded Practice Skill 5

Example 1

Marc gets paid $15 per lawn he mows. Graph the proportional relationship. Determine the slope and what it means in the context of the problem. How can the slope be used to determine how many lawns Marc mowed if he made $180? What is the equation that describes the relationship between the two quantities?

1. Create a table to show how the two quantities described vary.

2. Graph the proportional relationship.



3. Determine the slope and what it means in the context of the problem.

4. How can the slope be used to determine how many lawns Marc mowed if he made $180?

5. Write the equation that describes the relationship between the two quantities.

continued

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Example 2

At a roadside farm stand, you can buy 5 pounds of any of the vegetables for a total cost of $6. Determine the slope of the line formed by the proportional relationship between the number of pounds purchased and the cost of the vegetables. Explain what the slope means in the context of the problem. Finally, use the slope to determine how many pounds of vegetables can be purchased for $13. Assume there is no sales tax.

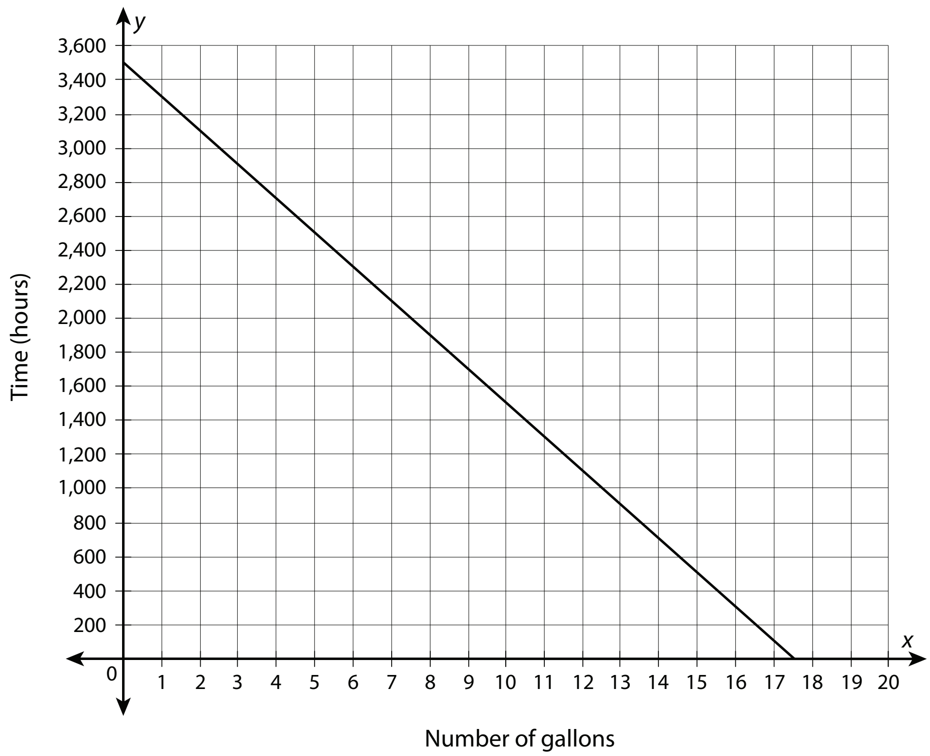
Example 3

A new plumber has just started his own business. In order to try and gain customers, he is running a special for his services. He charges $16 per hour, plus a standard house call fee of $25. Determine the slope of the line that passes through the points of the total cost for jobs lasting from 2 hours to 6 hours. Explain what the slope means in the context of the problem. Finally, use the slope to determine how many hours of work a customer could get for $150. Assume there is no sales tax.

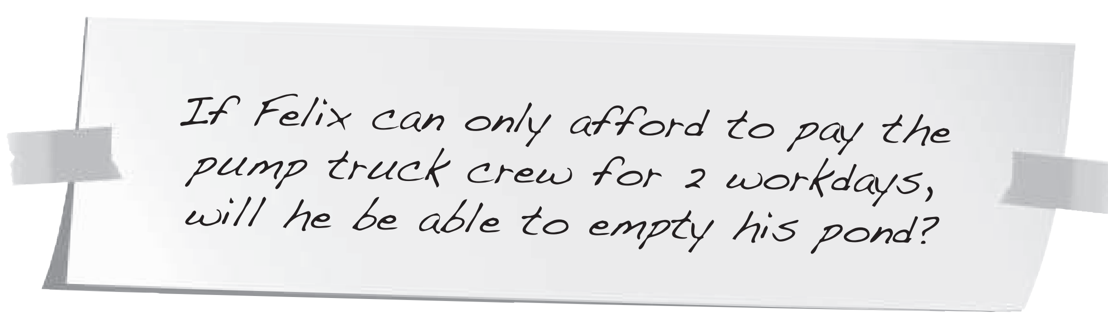
Problem-Based Task Skill 5: Pondering the Pond

SMP
1 ✓ 2 ✓
3 4 ✓
5 ✓ 6 ✓
7 ✓ 8

Felix has purchased a piece of property that has a 3,500-gallon pond. He wants to drain the pond and fill it in so he can build a garage. Felix has hired a pump truck to remove the water from his pond. The pump truck crew members take turns monitoring the level of water over the course of several hours. The following graph shows the number of gallons of water per hour being pumped out of the pond.



Determine the hourly rate at which the pond is being drained. Use this rate to write the equation of the line in the graph. Assuming the pump truck's hose is draining constantly at the same rate, how long will it take to empty the entire pond? There are 8 hours in 1 workday. If Felix can only afford to pay the pump truck crew for 2 workdays, will he be able to empty his pond? Explain your reasoning.



Problem-Based Task Skill 5: Pondering the Pond

Coaching

a. What are the two quantities described?

b. Does the graph describe a proportional relationship? Explain.

c. What is the formula for rate of change?

d. What are two points on the graphed line?

e. What is the hourly rate at which the pond is being drained?

f. What is the equation that represents this relationship?

g. Assuming the pump truck's hose is running constantly at the same rate, how long will it take to empty the entire pond?

h. If Felix can only afford to pay the pump truck crew for 2 workdays, will he be able to empty his pond? Explain your reasoning.

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Instruction

Problem-Based Task Skill 5: Pondering the Pond

Coaching Sample Responses

a. What are the two quantities described?

The quantities that are described are the number of gallons of water in the pond, and the time in hours that it takes to drain it.

b. Does the graph describe a proportional relationship? Explain.

The graph describes a proportional relationship because the graph is a straight line that decreases at a constant rate.

c. What is the formula for rate of change?

The formula for rate of change is the same as the formula for the slope of a line: $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$.

d. What are two points on the graphed line?

There are an infinite number of points on the graphed line; however, only the first quadrant is the focus in this scenario. Two points that are easily identified in the graph are (0, 3,500) and (10, 1,500).

e. What is the hourly rate at which the pond is being drained?

To find the hourly rate at which the pond is being drained, calculate the rate of change, or slope, of the graphed line.

Let (x_1, y_1) be (0, 3,500) and (x_2, y_2) be (10, 1,500).

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(1500)-(3500)}{(10)-(0)} = \frac{-2000}{10} = -200$$

The slope of the line is –200; this negative slope represents removing water from the pond at an hourly rate of 200 gallons per hour.

f. What is the equation that represents this relationship?

The relationship is proportional; therefore, the general equation of the line is $y = mx + b$, where m is the slope or rate of change, and b is the y-intercept. As determined, the slope of the equation is –200. The y-intercept is the point at which the line crosses the y-axis, or (0, 3,500). Substituting –200 for m, and 3,500 for b, the equation that represents this relationship is $y = -200x + 3500$.

g. Assuming the pump truck's hose is running constantly at the same rate, how long will it take to empty the entire pond?

Recall that y is the number of gallons of water and x is the time in hours. Therefore, to determine how long it will take to empty the entire pond, substitute 0 for y in the equation of the line, and then solve for x.

$y = -200x + 3500$

$(0) = -200x + 3500$

$200x = 3500$

$x \approx 17.5$

It will take approximately 17.5 hours to empty the 3,500-gallon pond.

h. If Felix can only afford to pay the pump truck crew for 2 workdays, will he be able to empty his pond? Explain your reasoning.

To answer this question, determine the number of workdays it will take to empty the pond. From part g, it is known that it will take approximately 17.5 hours to empty the pond. Convert this result to workdays. Since there are 8 hours in 1 workday, divide 17.5 by 8. The result is approximately 2.19 days. Therefore, it will take more than 2 days to empty the pond, so Felix will not be able to afford to empty his entire pond.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 5: Finding the Slope or Rate of Change of Linear Functions

For problems 1–4, determine the unit rate of the two quantities described and write what that rate means in the context of the problem. Then write the equation that describes the relationship between the two quantities.

1. A family pack of 12 tacos costs $8.

2. A baseball player throws the ball 180 feet in 3 seconds.

3. Charonika types 900 words in 12 minutes.

4. There are 640 chairs in 16 rows.

For problems 5–10, graph the relationship between the given quantities, and then use the slope of the line to answer the question.

5. A financial adviser meets with 8 clients each day. How many clients would she meet in 12 days?

6. Christy drove 1,088 miles on a road trip in 16 hours. How many miles did she drive per hour? Assume that she drove a constant rate of speed for the entire trip.

7. Mario rode his bike 103.6 miles in 7 days. If he rode the same number of miles daily, how many miles did he ride per day?

8. A fishing store gives customers 10 new lures for every 2 fishing poles they buy. How many lures would the store give to a customer who bought 8 fishing poles?

9. Alexander ordered 6 large pepperoni pizzas. The total cost was $82.50, which included a delivery fee of $7.50. How much did each pizza cost, not including the delivery fee?

10. Sheila started her savings account with $28, and saved the same amount of money each month. After 6 months, she had $220 in the account. How much did Sheila save each month, not including her starting amount?

Supportive Instructional Strategies for Mathematics II

Unit 2 Lesson 2

Suggestions for Graphic Organizers/Manipulatives

- Provide students with the Coordinate Plane graphic organizer from the Program Overview. Ask students to write the quadratic function $f(x) = x^2 + 3$ at the top of the page. Ask them to create a table of values and graph the function. Then, ask them to work with a partner and identify the key features of the function, including any extrema, the intercepts, the vertex, and the axis of symmetry. Ask students to volunteer their answers and discuss each feature of the graph and function.
- Provide students with at least 10 blank flash cards. Ask them to write on one side of each flash card the vocabulary words from the unit. Then ask each student to switch cards with a partner, and then to define each vocabulary word in their own words on the opposite side of the flash cards they've been given. Ask students to volunteer their answers, and then create a master list of vocabulary terms. Words should include: *concavity*, *domain*, *even function*, *odd function*, *increasing function*, *decreasing function*, *extrema*, *inflection point*, *slope*, *rate of change*, and *end behavior*.
- Provide students with a blank three-column table or chart. Ask them to title their charts "Forms of a Quadratic Function," and then label the columns "Standard," "Vertex," and "Factored." Write several examples of quadratic functions of various forms on the board, and ask students to place each example under the correct column on their charts. Discuss examples and create a master chart.

Suggestions for Discourse

- Ask students to work with a partner to create three examples of real-life scenarios of when a function might have a restricted domain. Ask volunteers to share their answers and discuss examples of quantities that cannot be negative, such as time and distance.
- Pair students, and provide each pair with a blank flash card. Ask the student pairs to list as many real-life examples as they can of where quadratic functions/parabolas are seen. Ask volunteers to share their answers. Some examples include: the path of a football or baseball as it is thrown, the cables on a suspension bridge, or a downhill curve of a roller coaster ride.
- Ask students, "How can you determine if a function is odd, even, or neither?" Ask them to work with a partner and write down explanations of what determines whether a function is odd, even, or neither. Ask volunteers to share their answers, and discuss examples of each type of function.

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics II*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.
- Provide a list of sentence frames in which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "I know that this function's graph will be concave down because the _______ is _______ in the function." Or, "The leading coefficient in this function is positive, so the graph of the function will be concave up."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- incorrectly identifying when a function is increasing or decreasing

  Remind students that a function is increasing on an interval where the output values are becoming larger as the input values become larger, and a function is decreasing on an interval when the output values are becoming smaller as the input values become larger. Remind them that *increase* means to grow larger, and *decrease* means to become smaller.

- making sign errors when determining if a function is odd, even, or neither

  Ask students to write the general forms of an odd function and an even function at the top of their papers (odd: $f(-x) = -(f(x))$; even: $f(-x) = f(x)$). Remind students that the negative sign must be distributed to the function when combining like terms and simplifying.

- identifying domain with output values rather than with input values

  Remind students that the domain of a function is the x-values (inputs), and the range of a function is the y-values (outputs).

- assuming the rate of change is the same for every interval of a quadratic function

  Remind students that a quadratic function has a varying rate of change for different intervals of the function. Remind them that it is important to determine the rate of change for a specific interval, and note that this rate of change may be different from the rate of change of another interval on the function.

Lesson 3: Building Functions

Instruction

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics II*.

Skill 1: Multiplying Linear Expressions (7.EE.1)

Common Core State Standard

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Skill 2: Factoring Quadratic Equations** (F–IF.8a)

Common Core State Standard

F–IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Skill 3: Finding the Value of *a* in the Vertex Form of a Quadratic Equation Given the Vertex and a Point on the Parabola** (F–IF.8a)

Common Core State Standard

F–IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Skill 4: Finding the *x*- and *y*-coordinates of the Vertex of a Parabola** (F–IF.8a)

Common Core State Standard

F–IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

(*continued*)

Skill 5: Adding, Subtracting, Multiplying, and Dividing Functions (F–BF.1b★)

Common Core State Standard

F–BF.1 Write a function that describes a relationship between two quantities.★

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Instruction

Skill 1: Multiplying Linear Expressions

SMP	
1 ✓	2
3	4 ✓
5	6
7 ✓	8 ✓

Common Core State Standard

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Essential Questions

1. How can linear expressions be simplified by applying properties of operations?
2. What does it mean when an expression is in simplest form?

WORDS TO KNOW

Associative Property of Addition For any quantities a, b, and c, $(a + b) + c = a + (b + c)$.

Associative Property of Multiplication For any quantities a, b, and c, $(a \bullet b) \bullet c = a \bullet (b \bullet c)$.

Commutative Property of Addition For any quantities a and b, $a + b = b + a$.

Commutative Property of Multiplication For any quantities a and b, $a \bullet b = b \bullet a$.

constant a quantity that does not change

Distributive Property For any quantities a, b, and c, $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.

like terms terms that contain the same variables raised to the same power

linear expression a mathematical phrase containing either a constant, a variable, mathematical operations (addition, subtraction, multiplication, division), or a combination of these

term a number, a variable, or the product of a number and variable(s)

variable a letter used to represent a value or unknown quantity that can change or vary

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Recommended Resources

- Khan Academy. "Multiplying Binomials Word Problem."

  http://www.walch.com/rr/04071

  This site provides step-by-step instruction on how to multiply binomials as presented in a word problem.

- Supercomputers in Schools. "Multiplying Linear Expressions."

  http://www.walch.com/rr/04072

  This site provides a step-by-step example of multiplying two linear expressions to create a quadratic expression.

Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

- Divide students into groups of three and distribute algebra tiles to each group. Explain the process of using the tiles to multiply two expressions, such as $(x + 3)(x - 2)$, and then ask students to work in their groups to create their own examples with the tiles. Ask for volunteers to share an example with the group and discuss in their own words the process of multiplying with algebra tiles.
- Provide students with a blank chart or table with three columns. Have students write the three properties of operations as column headings: "Commutative Property," "Associative Property," and "Distributive Property." Then, under each property, ask students to write the general form and create examples. Remind students to include examples for both addition and multiplication for the Commutative and Associative properties.
- Have students find a partner. Provide each student with four blank flash cards. Ask each student to write an example of two linear expressions on the front of each flash card. Have students exchange their cards with their partners, and then have the partners multiply and simplify the expressions. Ask for volunteers to share and explain their examples.

Suggestions for Discourse

- Ask students, "How do you know when an expression is simplified?" Encourage a discussion about like terms, and have students explain how to conclude when an expression has no remaining like terms to combine.
- Ask students, "How is the Distributive Property used when multiplying linear expressions? How can the Commutative and Associative properties be used in the simplification process?"

Making Connections

- Encourage students to connect the use of linear expressions to representing real-life situations. For example, calculating weekly earnings based on working a certain number of hours at a specific hourly wage, or determining the total cost of buying multiple items that are all priced the same.
- Provide examples of unsimplified linear expressions and compare them with simplified linear expressions in order to help students conclude that simplified expressions provide a more efficient way of working with linear expressions in real life. For example, demonstrate how the expression $7y + 10$ is easier to work with than $2y + 7 + 5y + 3$.

Skill 1: Multiplying Linear Expressions

Introduction

Linear expressions are mathematical phrases that are used to represent different situations. Linear expressions can be rewritten or simplified by applying the properties of operations. They can also be multiplied using the properties of operations. Simplifying the result of multiplying two linear expressions often creates a more efficient expression with which to work when determining, for example, the area of a floor.

Key Concepts

- A **linear expression** is a mathematical phrase containing either a constant, a variable, mathematical operations (addition, subtraction, multiplication, division), or a combination of these.
- A **constant** is a quantity that does not change or, in other words, a fixed number.
- A **variable** is a letter used to represent a value or unknown quantity that can change or vary, such as x or y.
- Linear expressions can only have terms with the variable raised to the first power.
- A **term** is a number, a variable, or the product of a number and variable(s). Some examples of linear expressions are $3x$, $4y + 2$, and $-8x + 5y - 1$.
- Linear expressions can be multiplied using the properties of operations. These properties are the Commutative Property, the Associative Property, and the Distributive Property.

The Commutative Properties

- The Commutative Property applies to both addition and multiplication.
- The **Commutative Property of Addition** states that for any quantities a and b, $a + b = b + a$. In other words, the order in which the numbers are added will not change the sum.
- The **Commutative Property of Multiplication** states that for any quantities a and b, $a \bullet b = b \bullet a$. That is, the order in which the numbers are multiplied will not change the product.
- When simplifying expressions, it is important to note that the order of numbers cannot be switched for subtraction and division. The Commutative Property only applies to addition and multiplication.

The Associative Properties

- The Associative Property applies to both addition and multiplication.
- The **Associative Property of Addition** states that for any quantities a, b, and c, $(a + b) + c = a + (b + c)$. In other words, the order in which the numbers are grouped will not change the sum.
- The **Associative Property of Multiplication** states that for any quantities a, b, and c, $(a \bullet b) \bullet c = a \bullet (b \bullet c)$. That is, the order in which the numbers are grouped will not change the product.
- When simplifying expressions, it is important to note that the grouping of numbers cannot be switched for subtraction and division. The Associative Property only applies to addition and multiplication.
- Having the ability to regroup addition or multiplication problems can make simplifying computations more efficient.

The Distributive Property

- The **Distributive Property** states that for any quantities a, b, and c, $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.
- The Distributive Property can be applied to the multiplication of linear expressions of the form $(a + b)(c + d)$.

Like Terms

- Once linear expressions have been multiplied by applying the properties, then the like terms need to be combined. **Like terms** are terms that contain the same variables raised to the same power.
- Constants are also like terms. For example, if the multiplication of two terms results in $17x + 9 - 8x + 4$, then the terms with the variable x can be combined using the Commutative Property of Addition, and the constant terms can be combined as well. The simplified result is $9x + 13$.
- An expression is simplified when all like terms have been combined.

Guided Practice Skill 1

Example 1

Multiply the two linear expressions $(x - 4)$ and $(x + 7)$, and then simplify the result by combining like terms.

1. Use the Distributive Property to multiply the two expressions.

   To apply the Distributive Property in order to multiply the expressions, first multiply the leftmost term in the first expression by each term in the second expression.

   The expression to be multiplied is $(x - 4)(x + 7)$.

   The first expression is $(x - 4)$ and the second expression is $(x + 7)$.

   The leftmost term in the first expression is x.

   Therefore, multiply $(x)(x + 7)$ to begin distributing.

$(x)(x + 7)$	Multiply the first term by each term in the second expression.
$= x \bullet x + x \bullet 7$	Apply the Distributive Property.
$= x^2 + 7x$	Simplify the multiplication.

   The result of distributing $(x)(x + 7)$ is $x^2 + 7x$.

   Next, multiply the rightmost term in the first expression by each term in the second expression.

   In the original multiplication, $(x - 4)(x + 7)$, the first expression is $(x - 4)$ and the second expression is $(x + 7)$.

   The rightmost term in the first expression is -4.

   Therefore, multiply $(-4)(x + 7)$.

$(-4)(x + 7)$	Multiply the second term by each term in the second expression.
$= -4 \bullet x + (-4) \bullet 7$	Apply the Distributive Property.
$= -4x + (-28)$	Simplify the multiplication.
$= -4x - 28$	Simplify.

   The result of distributing $(-4)(x + 7)$ is $-4x - 28$.

(*continued*)

Because we completed the distribution in two parts, now we need to combine the results of distributing, $x^2 + 7x$ and $-4x - 28$, into one expression.

Therefore, the result of applying the Distributive Property to $(x - 4)(x + 7)$ is $x^2 + 7x - 4x - 28$.

2. Combine the like terms.

When the leftmost term from the first expression was multiplied by each term in the second expression, the result was $x^2 + 7x$.

When the rightmost term in the first expression was multiplied by each term in the second expression, the result was $-4x - 28$.

The terms $7x$ and $-4x$ are the only like terms, because they have the same variable raised to the same power. When they are combined, the result is $7x + (-4x) = 3x$.

The expression $x^2 + 7x - 4x - 28$ is equal to $x^2 + 3x - 28$.

Therefore, when $(x - 4)$ and $(x + 7)$ are multiplied, the simplified result is $x^2 + 3x - 28$.

Example 2

Find the product of $2x$ and $(3x - y)$. Express the answer in simplest form.

1. Use the Distributive Property to find the product of the two expressions.

   The expression to be multiplied is $(2x)(3x - y)$. Notice that the first expression, $2x$, only has one term.

   Thus, to apply the Distributive Property in order to multiply the expressions, multiply the first expression, $2x$, by each term in the second expression.

$(2x)(3x - y)$	Multiply $2x$ by each term in the second expression.
$= 2x \bullet 3x + 2x \bullet -y$	Apply the Distributive Property.
$= 6x^2 + (-2xy)$	Simplify the multiplication.
$= 6x^2 - 2xy$	Simplify.

   The result of applying the Distributive Property to $(2x)(3x - y)$ is $6x^2 - 2xy$.

2. Combine the like terms.

   The result of multiplying $2x$ and $(3x - y)$ is $6x^2 - 2xy$. Recall that like terms must have the same variables raised to the same power. In this case, although both terms contain the variable x, the first term, $6x^2$, has an exponent of 2. In the second term, $-2xy$, the variable x has an understood exponent of 1. Therefore, these terms are not like terms.

   Thus, when $2x$ and $(3x - y)$ are multiplied, the simplified result is $6x^2 - 2xy$.

Example 3

Margo is a server at a restaurant. In a given night, she makes \$3 per hour, plus a set amount of \$50 in tips. If she works on a holiday, her boss will pay her twice the amount she would make in a normal night. Write an expression to represent the amount Margo normally makes in one night. Then, write a simplified expression that represents the amount she will make if she works on a holiday.

1. Write an expression that represents how much Margo makes in one night.

   Let h represent the number of hours that she works in one night. Because she makes \$3 per hour, her hourly wage can be represented by $3h$.

   Margo also makes a set amount of \$50 in tips each night, which is added to her hourly rate. Therefore, the expression that represents how much she makes in a given night is $3h + 50$.

2. Write an expression that represents how much Margo makes if she works on a holiday.

   On a holiday, Margo's boss pays her twice the amount she would normally make in a night, so the expression that represents her normal rate per night will be multiplied by 2. Therefore, the expression will be $2(3h + 50)$.

3. Simplify the resulting expression using the Distributive Property.

$2(3h + 50)$	Expression from the previous step
$= 2 \bullet 3h + 2 \bullet 50$	Apply the Distributive Property.
$= 6h + 100$	Simplify the multiplication.

   Because $6h$ and 100 are not like terms, they cannot be combined. Therefore, the expression that represents Margo's total pay for working on a holiday is $6h + 100$.

Scaffolded Practice Skill 1

Example 1

Multiply the two linear expressions $(x - 4)$ and $(x + 7)$, and then simplify the result by combining like terms.

1. Use the Distributive Property to multiply the two expressions.

2. Combine the like terms.

continued

Example 2

Find the product of $2x$ and $(3x - y)$. Express the answer in simplest form.

Example 3

Margo is a server at a restaurant. In a given night, she makes $3 per hour, plus a set amount of $50 in tips. If she works on a holiday, her boss will pay her twice the amount she would make in a normal night. Write an expression to represent the amount Margo normally makes in one night. Then, write a simplified expression that represents the amount she will make if she works on a holiday.

Name: Date:

UNIT 2 • QUADRATIC FUNCTIONS AND MODELING
Lesson 3: Building Functions

Problem-Based Task Skill 1: A Garden Delight

SMP
1 ✓ 2
3 4 ✓
5 6
7 ✓ 8 ✓

Frederick wants to plant a garden, but he's not sure how big he wants to make it. He has a rectangular space staked out in his backyard, the length of which can be represented by the expression ($4x - 1$) feet. The width of this space is 3 times the length. Write a simplified expression to represent the width of the garden space. Then, write a simplified expression that represents the total area of the garden space.

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Problem-Based Task Skill 1: A Garden Delight

Coaching

a. What unsimplified expression represents the width of Frederick's garden space?

b. What is the simplified expression for the width?

c. What is the formula used to find the area of a rectangle?

d. Which two expressions need to be multiplied in order to find the total area of the garden space?

e. What is the simplified expression that represents the total area of Frederick's garden space?

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Instruction

Problem-Based Task Skill 1: A Garden Delight

Coaching Sample Responses

a. What unsimplified expression represents the width of Frederick's garden space?

The length of the garden space is represented by the expression $(4x - 1)$ feet. The width of the space is 3 times this amount. Therefore, the expression for the width is found by multiplying the expression for the length by 3.

The unsimplified expression that represents the width of the garden space is $3(4x - 1)$.

b. What is the simplified expression for the width?

In order to simplify the expression for the width, multiply the two linear expressions, 3 and $(4x - 1)$.

Apply the Distributive Property and then simplify the multiplication.

$(3)(4x - 1)$

$= 3 \bullet 4x + 3 \bullet -1$

$= 12x + (-3)$

$= 12x - 3$

The simplified expression that represents the width of the garden space is $12x - 3$.

c. What is the formula used to find the area of a rectangle?

The formula used to find the area of a rectangle is *length* times *width*, or $l \bullet w$.

d. Which two expressions need to be multiplied in order to find the total area of the garden space?

The length of the garden space is represented by the expression $(4x - 1)$, and the width of the garden space is represented by $(12x - 3)$. Therefore, the two expressions that need to be multiplied in order to find the total area are $(4x - 1)$ and $(12x - 3)$.

e. What is the simplified expression that represents the total area of Frederick's garden space?

To find the simplified expression that represents the total area of Frederick's garden space, use the Distributive Property to multiply the two linear expressions from part d, $(4x - 1)$ and $(12x - 3)$, and then combine like terms.

Multiply the leftmost term in the first expression, $4x$, by each term in the second expression, $(12x - 3)$, and simplify.

$(4x)(12x - 3)$

$= 4x \bullet 12x + 4x \bullet -3$

$= 48x^2 + (-12x)$

$= 48x^2 - 12x$

Next, multiply the rightmost term in the first expression, -1, by each term in the second expression, $(12x - 3)$, and simplify.

$-1(12x - 3)$

$= -1 \bullet 12x + (-1) \bullet -3$

$= -12x + 3$

Combine the results of distributing, $48x^2 - 12x$ and $-12x + 3$, into one expression.

The result of applying the Distributive Property to $(4x - 1)(12x - 3)$ is $48x^2 - 12x - 12x + 3$.

Now, combine like terms.

The terms $-12x$ and $-12x$ are the only like terms, because they have the same variable raised to the same power. When they are combined, the result is $-12x + (-12x) = -24x$.

Thus, the expression $48x^2 - 12x - 12x + 3$ can be simplified to $48x^2 - 24x + 3$.

The simplified expression that represents the total area of Frederick's garden space is $48x^2 - 24x + 3$.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 1: Multiplying Linear Expressions

For problems 1– 3, simplify each expression by combining like terms.

1. $2x + 3 + 5x + 6$

2. $(8a + 2b - 4) + (3b - 5)$

3. $3xy - 2x + 4xy - 7y$

For problems 4–8, multiply the linear expressions, combine like terms, and write a simplified expression.

4. $-3y(5x - 2y)$

5. $2a(-12a + 7)$

6. $-4y(2y - y + 3)$

7. $(x - 5)(x - 3)$

8. $(3x + 1)(3x - 4)$

Use the following information to complete problems 9 and 10.

Samantha bought a picture frame. The length of the picture frame can be represented by the expression $(2x + 3)$ inches, and the width of the frame is twice as long as the length.

9. Write a simplified expression that represents the width of the picture frame.

10. Write a simplified expression that represents the total area of the picture frame.

Instruction

Skill 2: Factoring Quadratic Equations**

Common Core State Standard

F–IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

This skill has been addressed previously in *CCSS Integrated Pathway: Mathematics II*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

Unit 2, Lesson 1, Sub-lesson 2

Instruction

Skill 3: Finding the Value of *a* in the Vertex Form of a Quadratic Equation Given the Vertex and a Point on the Parabola**

Common Core State Standard

F–IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

This skill has been addressed previously in *CCSS Integrated Pathway: Mathematics II*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

Unit 2, Lesson 1, Sub-lesson 2

Instruction

Skill 4: Finding the *x*- and *y*-coordinates of the Vertex of a Parabola**

Common Core State Standard

F–IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

This skill has been addressed previously in *CCSS Integrated Pathway: Mathematics II*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

Unit 2, Lesson 1, Sub-lesson 2

Instruction

Skill 5: Adding, Subtracting, Multiplying, and Dividing Functions

SMP	
1 ✓	2 ✓
3 ✓	4 ✓
5	6 ✓
7 ✓	8

Common Core State Standard

F–BF.1 Write a function that describes a relationship between two quantities.★

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Essential Question

1. How can functions be combined by adding, subtracting, multiplying, or dividing?

WORD TO KNOW

function a relation in which every element of the domain is paired with exactly one element of the range; that is, for every value of *x*, there is exactly one value of *y*

Recommended Resource

- Purplemath.com. "Operations on Functions."

  http://www.walch.com/rr/04073

  This site demonstrates how to add, subtract, multiply, or divide functions.

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Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Provide students with a blank four-column chart or table. Have students write the functions $f(x) = 2x - 1$ and $g(x) = 5x + 4$ at the top of the page. Then have students label the columns with the four operations for the functions: "Addition: $(f + g)(x)$," "Subtraction: $(f - g)(x)$," "Multiplication: $(f \bullet g)(x)$," and "Division: $(f \div g)(x)$." Have students perform each of the four function operations to get a new resulting function, $h(x)$, for each column. Have students pair up or form small groups to discuss their answers and then ask for volunteers to share their answers with the class. Students should provide the following answers: $(f + g)(x) = 7x + 3$, $(f - g)(x) = -3x - 5$, $(f \bullet g)(x) = 10x^2 + 3x - 4$, and $(f \div g)(x) = \frac{2x-1}{5x+4}$.

Suggestions for Discourse

- Ask students, "How is adding, subtracting, multiplying, and dividing functions similar to performing these same operations on algebraic expressions?" Guide students in a discussion about how the notation used for combining functions is different, but the process is the same.
- Ask students, "When combining functions, which of the four operations could possibly have a restriction on the domain, and why?" Encourage students to conclude that when dividing functions, the denominator cannot be equal to 0.
- Have students work with a partner to create three examples of real-life scenarios of when a function might have a restricted domain. Ask volunteers to share their answers and discuss examples of quantities that cannot be negative, such as time and distance.

Making Connections

Create a list of real-life scenarios in which functions can be combined, then demonstrate examples of such scenarios. For example, computing the area of a two-dimensional figure in which the length and width of the figure can be represented by separate functions which are then multiplied to create a new function representing the area. Or, using a business profit model in which a profit function is created by finding the difference of the revenue and cost of an item.

Skill 5: Adding, Subtracting, Multiplying, and Dividing Functions

Introduction

Functions are relations in which every element in the domain is paired with exactly one element of the range; that is, for every value of x, there is exactly one value of y. Functions can be combined by performing arithmetic operations such as adding, subtracting, multiplying, or dividing.

Key Concepts

- Combine linear and exponential expressions using addition: $(f + g)(x) = f(x) + g(x)$. In other words, add the two functions together by combining like terms.
- Combine linear and exponential expressions using subtraction: $(f - g)(x) = f(x) - g(x)$. In other words, subtract the second function from the first while making sure to distribute the negative across all terms of the second function.
- Combine linear and exponential expressions using multiplication: $(f \bullet g)(x) = f(x) \bullet g(x)$. In other words, multiply the two functions together.
- Combine linear and exponential expressions using division: $(f \div g)(x) = f(x) \div g(x)$. In other words, divide the first function by the second function. Use a fraction bar to display the final function.

Guided Practice Skill 5

Example 1

Given $f(x) = 4x + 1$ and $g(x) = 3x - 6$, build a new function $h(x)$, for which $h(x) = (f + g)(x)$.

1. Add the two functions.

   $h(x) = (f + g)(x) = f(x) + g(x)$

   Because $f(x) = 4x + 1$ and $g(x) = 3x - 6$, adding $f(x)$ and $g(x)$ results in $h(x) = (f + g)(x) = (4x + 1) + (3x - 6)$.

2. Combine like terms.

   Clear the parentheses and reorder the expression, then combine like terms.

$h(x) = (4x + 1) + (3x - 6)$	Equation for $h(x)$
$h(x) = 4x + 1 + 3x - 6$	Remove the parentheses.
$h(x) = 4x + 3x + 1 - 6$	Rearrange to group like terms.
$h(x) = 7x - 5$	Combine like terms.

   The result of adding $f(x) = 4x + 1$ and $g(x) = 3x - 6$ is $h(x) = 7x - 5$.

Example 2

Given $f(x) = 2^x$ and $g(x) = 7$, build a new function $h(x)$, for which $h(x) = (f \bullet g)(x)$.

1. Multiply the two functions.

   $h(x) = (f \bullet g)(x) = f(x) \bullet g(x)$

   Because $f(x) = 2^x$ and $g(x) = 7$, multiplying $f(x)$ and $g(x)$ results in $h(x) = (f \bullet g)(x) = (2^x) \bullet (7)$.

2. Simplify the resulting equation.

$h(x) = (2^x) \bullet (7)$	Equation for $h(x)$
$h(x) = 7(2^x)$	Simplify the multiplication.

   The result of multiplying $f(x) = 2^x$ and $g(x) = 7$ is $h(x) = 7(2^x)$.

Example 3

Given $f(x) = 6x - 9$ and $g(x) = 8x - 14$, build a new function $h(x)$, for which $h(x) = (f - g)(x)$.

1. Subtract the two functions.

   $h(x) = (f - g)(x) = f(x) - g(x)$

   Because $f(x) = 6x - 9$ and $g(x) = 8x - 14$, subtracting $g(x)$ from $f(x)$ results in $h(x) = (f - g)(x) = (6x - 9) - (8x - 14)$.

2. Combine like terms.

   Clear the parentheses and reorder the expression. Be careful to correctly distribute the negative sign.

$h(x) = (6x - 9) - (8x - 14)$	Equation for $h(x)$
$h(x) = 6x - 9 - 8x + 14$	Distribute the negative.
$h(x) = 6x - 8x - 9 + 14$	Rearrange to group like terms.
$h(x) = -2x + 5$	Combine like terms.

   The result of subtracting $f(x) = 6x - 9$ and $g(x) = 8x - 14$ is $h(x) = -2x + 5$.

Example 4

Given $f(x) = 4^x$ and $g(x) = 9$, build a new function $h(x)$, for which $(f \div g)(x)$.

1. Divide the two functions.

   $h(x) = (f \div g)(x) = f(x) \div g(x)$

   Because $f(x) = 4^x$ and $g(x) = 9$, dividing $f(x)$ by $g(x)$ results in $h(x) = (f \div g)(x) = 4^x \div 9$.

2. Simplify the resulting equation.

$h(x) = 4^x \div 9$	Equation for $h(x)$
$h(x) = \frac{4^x}{9}$	Rewrite the division as a fraction.
$h(x) = \frac{1}{9}(4^x)$	Rewrite the fraction as multiplication.

The result of dividing $f(x) = 4^x$ by $g(x) = 9$ is $h(x) = \frac{1}{9}(4^x)$.

Scaffolded Practice Skill 5

Example 1

Given $f(x) = 4x + 1$ and $g(x) = 3x - 6$, build a new function $h(x)$, for which $h(x) = (f + g)(x)$.

1. Add the two functions.

2. Combine like terms.

continued

Example 2

Given $f(x) = 2^x$ and $g(x) = 7$, build a new function $h(x)$, for which $h(x) = (f \bullet g)(x)$.

Example 3

Given $f(x) = 6x - 9$ and $g(x) = 8x - 14$, build a new function $h(x)$, for which $h(x) = (f - g)(x)$.

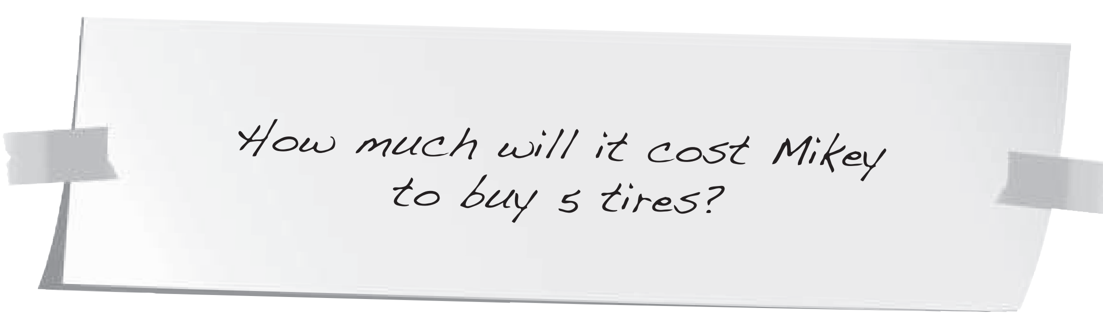
Example 4

Given $f(x) = 4^x$ and $g(x) = 9$, build a new function $h(x)$, for which $(f \div g)(x)$.

Problem-Based Task Skill 5: Mikey's Tires

Mikey is shopping for tires for his new truck and will need to buy 5 so he can have a spare. The tires normally cost $192 each, but are on sale for 25% off. Use function notation to write a simplified expression that represents the sale price of the tires. How much will it cost Mikey to buy 5 tires?

SMP	
1 ✓	2 ✓
3 ✓	4 ✓
5	6 ✓
7 ✓	8



Problem-Based Task Skill 5: Mikey's Tires

Coaching

a. What is the original price of the tires?

b. What is an expression that represents buying *x* tires at the original price?

c. How is the 25% discount calculated?

d. Write an expression to represent the sale price for *x* tires.

e. Write the expression in function notation.

f. Simplify the function.

g. How much will it cost Mikey to buy 5 tires?

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Problem-Based Task Skill 5: Mikey's Tires

Coaching Sample Responses

a. What is the original price of the tires?

Each tire normally costs $192.

b. What is an expression that represents buying x tires at the original price?

The expression that represents buying x tires at the original price is $192x$.

c. How is the 25% discount calculated?

There are multiple ways to calculate the 25% discount. One way is to multiply the original price by 0.25, then subtract this amount from the original price.

d. Write an expression to represent the sale price for x tires.

An expression that represents the sale price is $192x - 0.25(192x)$.

e. Write the expression in function notation.

The rule written in function notation is $f(x) = 192x - 0.25(192x)$.

f. Simplify the function.

Use the order of operations to simplify the function. Multiply 0.25 by $192x$ and then combine like terms.

$f(x) = 192x - 0.25(192x)$

$f(x) = 192x - 48x$

$f(x) = 144x$

g. How much will it cost Mikey to buy 5 tires?

Substitute 5 for x and evaluate the function.

$f(x) = 144x$

$f(5) = 144(5) = 720$

Mikey's total cost for 5 new tires will be $720.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 5: Adding, Subtracting, Multiplying, and Dividing Functions

For problems 1 and 2, find the result of each operation using the functions $f(x) = 3x - 8$ and $g(x) = 5x + 4$.

1. Find $(f + g)(x)$.
2. Find $(f - g)(x)$.

For problems 3 and 4, find the value of each operation using the functions $f(x) = 6^x$ and $g(x) = 7$.

3. Find $(f \bullet g)(x)$.
4. Find $(f \div g)(x)$.

For problems 5–8, find the value of each operation using the functions $f(x) = -6$ and $g(x) = 2x + 9$.

5. Find $(f + g)(x)$.
6. Find $(f - g)(x)$.
7. Find $(f \bullet g)(x)$.
8. Find $(f \div g)(x)$.

Use what you know about functions to complete problems 9 and 10.

9. Last summer, Baxter made $12 per hour mowing lawns, plus a flat fee of $10 to pay for the gasoline in his mower. Write a function, $f(x)$, to represent this scenario. This summer, to account for a change in the price of gasoline, Baxter will now charge a flat fee of $15 in addition to his $12 hourly fee. What is the new function, $g(x)$, that represents Baxter's earnings? How has the function rule changed?
10. LaDarius puts $50 of his money into an investment account that promises to double his money every 8 months. Write a function, $f(x)$, that represents this scenario. His friend Dugan invests money in the same investment account, but Dugan starts with $100 instead. What function, $g(x)$, represents the amount of money in Dugan's account? How are the two function rules different?

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Supportive Instructional Strategies for Mathematics II

Unit 2 Lesson 3

Suggestions for Graphic Organizers/Manipulatives

- Provide students with a blank four-column chart or table. Have students write the functions $f(x) = 12x + 9$ and $g(x) = 6x + 3$ at the top of the page. Then have students label the columns with the four operations for the functions: "Addition: $(f + g)(x)$," "Subtraction: $(f - g)(x)$," "Multiplication: $(f \bullet g)(x)$," and "Division: $(f \div g)(x)$." Have students perform each of the four function operations to get a new resulting function, $h(x)$, for each column. Have students pair up or form small groups to discuss their answers and then ask for volunteers to discuss their answers with the class. Students should provide the following answers: $(f + g)(x) = 18x + 12$, $(f - g)(x) = 6x + 6$, $(f \bullet g)(x) = 72x^2 + 90x + 27$, and $(f \div g)(x) = \dfrac{4x+3}{2x+1}$.
- Provide students with the Coordinate Plane graphic organizer found in the Program Overview and two colored pencils or highlighters of different colors. Ask students to write the quadratic functions $f(x) = 2x^2 + 1$ and $g(x) = -2x^2 + 1$ at the top of the page. Have students work with a partner to predict what the difference will be in the shapes of the graphs. Then, ask each student to graph each function in a different color. Discuss the concavity of each graph and ask students to draw conclusions about which part of each function determines the concavity. Encourage students to discuss the role of the sign of the leading coefficient in determining the shape of the graph.

Suggestions for Discourse

- Pair students with a partner and ask them to list real-life examples of parabolas. Have the pairs come up with as many examples as they can. Ask for volunteers to share their answers. Examples include the path of a ball as it is thrown, the cables on a suspension bridge, a downhill curve of a roller coaster ride, and a rainbow.
- Ask students, "What are possible real-life scenarios in which there are restrictions on the domain of the quadratic function?" Encourage a discussion about specific situations in which there cannot be negative values for the domain, or else examples in which the function has a fraction. Examples of real-life values that cannot be negative include the time it takes for a thrown ball to hit the ground, or the length of a piece of wood needed to maximize the area of a dog pen.

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics II*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.
- Provide a list of sentence frames in which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "I know that this function's graph will be concave down because the _______ is _______ in the function." Or, "The <u>leading coefficient</u> in this function is <u>positive</u>, so the graph of the function will be <u>concave up</u>."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- thinking that when given a fixed perimeter, the corresponding area of the figure is constant

  Remind students that the perimeter of a figure is the distance around the figure, and the area is the space inside the figure.
- thinking that a positive leading coefficient yields a maximum value instead of a minimum value

  Have students draw a "U" on their papers and write the words "concave up (+)," and then draw an upside-down "U" and write the words "concave down (–)."
- mistakenly believing that adding linear expressions will build quadratics; they must be multiplied

  Remind students that when two linear expressions are added, the terms with the variables remain at the first power, and that only multiplying variables causes the exponent to raise to a power greater than 1.

- forgetting to restrict the domain when dividing functions

  Remind students to look for a variable in the denominator, and that the denominator cannot be equal to 0. Also remind them to consider the parameters of a real-life scenario; for example, the distance traveled during a road trip cannot be a negative value.

- not realizing that functions must be of the same variable for like terms to be combined

  Remind students that the rules of combining like terms dictate that the variables must be the same and raised to the same power, and that the process of combining functions follows the rules of combining like terms.

- having difficulty moving from the formal notation to a workable problem where functions can be used with operations

  Remind students that the formal function notation of $f(x) = x$ can be changed to $y = x$ when working through a problem that requires creating a table or a graph.

Lesson 4: Graphing Other Functions

Instruction

Elementary Prerequisite Skills

This lesson requires the use of the following elementary skill(s) to meet the standards addressed in *CCSS Integrated Pathway: Mathematics II*. See the Appendix for material to address the skill(s).

- E-Skill 6: Creating Graphs Using Ordered Pairs (5.G.1), Appendix p. A-30

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics II*.

Skill 1: Determining the Domain and Range of an Algebraic Equation* (F–IF.1)

Common Core State Standard

F–IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

Skill 2: Evaluating Functions for Given Values* (F–IF.2)

Common Core State Standard

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Skill 3: Finding Ordered Pairs by Evaluating Functions (8.F.1)

Common Core State Standard

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]

(*continued*)

[1] Function notation is not required for Grade 8.

Skill 4: Evaluating Squares and Cubes of Real Numbers With and Without a Calculator (8.EE.2)

Common Core State Standard

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Skill 5: Graphing a Linear Function* (A–CED.2★)

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Skill 6: Finding the Absolute Value of a Quantity (6.NS.7c)

Common Core State Standard

6.NS.7 Understand ordering and absolute value of rational numbers.

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*

Skill 7: Determining Restricted Domains and Ranges for Application Problems** (F–IF.5★)

Common Core State Standard

F–IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

Instruction

Skill 1: Determining the Domain and Range of an Algebraic Equation*

Common Core State Standard

F–IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 2, Skill 3

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Guided Practice Skill 1

Example 1

Payton is selling T-shirts as a fund-raiser for the Car Club. He has 60 T-shirts to sell, and the profit he will make after he sells x T-shirts can be modeled by the function $y = 12x - 180$. Find the domain and range of the function.

1. Find the domain of the function.

   The domain is the set of x-values that are valid for the function. Because x represents the number of T-shirts Payton sells, and he can't sell a negative number of shirts, the possible values of x must be from 0 up to and including 60. Because he cannot sell a partial T-shirt, only integers are part of the domain. (Recall that integers are numbers that are not fractions or decimals.)

   Therefore, the domain is $0 \le x \le 60$, where x is an integer.

2. Find the range of the function.

   The range is the set of y-values that are valid for the function. The domain is all the integers from 0 to 60, so the range will be the set of y-values that are produced from numbers from 0 to 60.

   Recall that when an equation is written in function notation, $f(x)$ replaces y.

   To find the range, substitute values for x and find the corresponding values for $f(x)$ or y. Then try to determine a pattern.

   Let's use the first three values in the domain: 0, 1, and 2.

   Let $x = 0$.

$f(x) = 12x - 180$	Given function
$f(0) = 12(0) - 180$	Substitute 0 for x.
$f(0) = -180$	Simplify.

   For an input value of 0, the output value is –180.

(*continued*)

Let $x = 1$.

$f(x) = 12x - 180$	Given function
$f(1) = 12(1) - 180$	Substitute 1 for x.
$f(1) = -168$	Simplify.

For an input value of 1, the output value is –168.

Let $x = 2$.

$f(x) = 12x - 180$	Given function
$f(2) = 12(2) - 180$	Substitute 2 for x.
$f(2) = -156$	Simplify.

For an input value of 2, the output value is –156.

It can be seen from these values, as well as from the equation of the function, that the pattern of values in the range is to add 12 each time the input increases by 1. To find where the range ends, substitute 60 (the highest value in the domain) into the function and solve for $f(x)$.

Let $x = 60$.

$f(x) = 12x - 180$	Given function
$f(60) = 12(60) - 180$	Substitute 60 for x.
$f(60) = 540$	Simplify.

For an input value of 60, the output value is 540.

Note that we already found where the range begins when we calculated the output value for an input of 0. The result was –180.

Therefore, the possible range of the function in this scenario is all the integers from –180 to 540 that are in increments of 12.

The range is {–180, –168, –156, ..., 516, 528, 540}.

Scaffolded Practice Skill 1

Example 1

Payton is selling T-shirts as a fund-raiser for the Car Club. He has 60 T-shirts to sell, and the profit he will make after he sells x T-shirts can be modeled by the function $y = 12x - 180$. Find the domain and range of the function.

1. Find the domain of the function.

2. Find the range of the function.

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Practice Skill 1: Determining the Domain and Range of an Algebraic Equation*

For problems 1–3, find the domain and range of each function.

1. $y = 2x^2 - 5$

2. $y = 2(x + 2)$

3. $y = \sqrt{x} + 4$

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Instruction

Skill 2: Evaluating Functions for Given Values*

Common Core State Standard

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 2, Skill 4

SWB p. 77

Guided Practice Skill 2

Example 1

Evaluate $g(x) = 4x^2 + 3x - 8$ over the domain {–2, –1, 0, 1}. What is the range?

1. Evaluate the function for each of the domain values.

   To evaluate the function $g(x) = 4x^2 + 3x - 8$ over the domain {–2, –1, 0, 1}, substitute the values from the domain into $g(x) = 4x^2 + 3x - 8$.

   Evaluate $g(-2)$.

$g(x) = 4x^2 + 3x - 8$	Original function
$g(-2) = 4(-2)^2 + 3(-2) - 8$	Substitute –2 for x.
$g(-2) = 2$	Simplify.

   When –2 is substituted for x, the value of $g(-2)$ is 2.

   Evaluate $g(-1)$.

$g(x) = 4x^2 + 3x - 8$	Original function
$g(-1) = 4(-1)^2 + 3(-1) - 8$	Substitute –1 for x.
$g(-1) = -7$	Simplify.

   When –1 is substituted for x, the value of $g(-1)$ is –7.

   Evaluate $g(0)$.

$g(x) = 4x^2 + 3x - 8$	Original function
$g(0) = 4(0)^2 + 3(0) - 8$	Substitute 0 for x.
$g(0) = -8$	Simplify.

   When 0 is substituted for x, the value of $g(0)$ is –8.

   Evaluate $g(1)$.

$g(x) = 4x^2 + 3x - 8$	Original function
$g(1) = 4(1)^2 + 3(1) - 8$	Substitute 1 for x.
$g(1) = -1$	Simplify.

   When 1 is substituted for x, the value of $g(1)$ is –1.

2. Determine the range of the function for the given domain.

   Collect the set of outputs for the inputs.

   The range is {2, –7, –8, –1}.

Scaffolded Practice Skill 2

Example 1

Evaluate $g(x) = 4x^2 + 3x - 8$ over the domain {–2, –1, 0, 1}. What is the range?

1. Evaluate the function for each of the domain values.

2. Determine the range of the function for the given domain.

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Practice Skill 2: Evaluating Functions for Given Values*

For problems 1–3, find the range of each function over the given domain.

1. $f(x) = 6x^2 - 7x$; domain: {–1, 0, 1, 2}

2. $f(x) = -3(x^2 + 2x - 1)$; domain: {–3, –1, 1, 3}

3. $f(x) = x^2 + 12$; domain: {–2, 0, 2, 4}

Skill 3: Finding Ordered Pairs by Evaluating Functions

SMP	
1 ✓	2 ✓
3	4 ✓
5 ✓	6 ✓
7	8

Common Core State Standard

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]

Essential Questions

1. What are the characteristics of a function?
2. How can the range of a function be determined given the domain?

WORDS TO KNOW

domain the set of all inputs of a function; the set of *x*-values that are valid for the function

function a relation in which every element of the domain is paired with exactly one element of the range; that is, for every value of *x*, there is exactly one value of *y*

function notation a way to name a function using $f(x)$ instead of *y*

ordered pair a pair of values (x, y) where the order is significant

range the set of outputs of a function; the set of *y*-values that are valid for the function

Recommended Resources

- freeMATHhelp.com. "Functions."

  http://www.walch.com/rr/04020

  This site defines functions, domain, and range. It also explains function notation and how to find the output of a function for a specific input. A step-by-step example is given, showing an input-output pair for the function $y = 4x - 7$.

- IXL Learning. "Complete a Function Table."

  http://www.walch.com/rr/04021

  This site provides multiple inputs of a function and prompts users to enter the corresponding outputs into a table. Users are immediately notified whether all of the entered outputs are correct; if any are incorrect, a detailed explanation is given on how to arrive at all of the correct outputs.

[1] Function notation is not required for Grade 8.

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Instruction

Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Once students have worked through the Guided Practice, distribute the Table graphic organizer found in the Program Overview. In the five cells of the first row, have students write "Function," "Output for $x = 1$," "Output for $x = 2$," "Output for $x = 3$," and "Output for $x = 4$." Then, in the rest of the first column, have students write the following functions: $f(x) = 2x - 1$, $g(x) = 5x + 3$, $h(x) = -4x$, $k(x) = 10x - 7$, and $l(x) = -8x + 2$. Finally, have students fill in the rest of the table with the appropriate outputs for each function.

Suggestions for Discourse

Tell students that a function is a rule that assigns to each input exactly one output. Ask them if they think a function can assign more than one input to the same output, and if so, have them give an example of such a function. If the students are mistaken, point out that a function can assign more than one input to the same output, and give an example, such as the function $f(x) = 3x^2$, for which $x = 1$ and $x = -1$ have the same output, $f(x) = 3$.

Making Connections

When creating and evaluating inputs and outputs of functions, let students know that while an inequality is not a function, the process of creating and evaluating inputs and outputs is the same. In both cases, a value for the input variable is substituted in, and the resulting expression is simplified to produce the corresponding output.

Instruction

Skill 3: Finding Ordered Pairs by Evaluating Functions

Introduction

Specific values can be substituted into equations for variables. Sometimes an equation into which values are substituted is a **function**, or a relation in which every element of the domain is paired with exactly one element of the range. That is, each input is assigned to exactly one output. For example, once a cat is 2 years old, its "age" in human years can be found by using the function $y = 4x + 24$, where x is the number of years since the cat turned 2. Different values can be substituted into the function for x and, for each, the cat's corresponding age in human years is the output. That is, for each value of x, or input, the cat has only one possible age in human years, or output.

Key Concepts

- Given a function's **domain**, or the set of all inputs of a function, it is possible to find the function's **range**, or the set of outputs of a function.
- Functions are often written in **function notation**, which is a way to describe a function that gives the function's name and the input variable. An example of a function written in function notation is $f(x) = x + 1$.
- Inputting a value into a function produces an **ordered pair**, or an input of a rule and the corresponding output. The ordered pair is written in the form (x, y), where the order is significant.
- It is possible to check if an ordered pair satisfies a function or an inequality by using substitution.
- At least two ordered pairs are necessary to graph a linear function.
- A function can be graphed by plotting ordered pairs that satisfy it and drawing either a line or curve through the points, depending on the type of function.

Guided Practice Skill 3

Example 1

The domain of the function $f(x) = 4x + 3$ is restricted to the set of integers {1, 3, 5}. What is the set of ordered pairs that corresponds to this function?

1. Find $f(1)$.

   Substitute 1 into the function for x to find $f(1)$.

$f(x) = 4x + 3$	Given function
$f(1) = 4(1) + 3$	Substitute 1 for x.
$f(1) = 4 + 3$	Multiply 4 by 1.
$f(1) = 7$	Add 4 and 3.

   The output value when $x = 1$ is 7.

2. Find $f(3)$.

   Substitute 3 into the function for x to find $f(3)$.

$f(x) = 4x + 3$	Given function
$f(3) = 4(3) + 3$	Substitute 3 for x.
$f(3) = 12 + 3$	Multiply 4 by 3.
$f(3) = 15$	Add 12 and 3.

   The output value when $x = 3$ is 15.

3. Find $f(5)$.

   Substitute 5 into the function for x to find $f(5)$.

$f(x) = 4x + 3$	Given function
$f(5) = 4(5) + 3$	Substitute 5 for x.
$f(5) = 20 + 3$	Multiply 4 by 5.
$f(5) = 23$	Add 20 and 3.

   The output value when $x = 5$ is 23.

4. Determine the ordered pairs that correspond to this function.

   Because inputting the values of the domain into the function produces the outputs 7, 15, and 23, the ordered pairs that correspond to this function are (1, 7), (3, 15), and (5, 23).

Example 2

Does the ordered pair (5, 19) satisfy the function $g(x) = 5x - 6$? Does it satisfy the inequality $y > 5x - 6$?

1. Rewrite the function $g(x) = 5x - 6$ so that it is not in function notation.

   To rewrite the function $g(x) = 5x - 6$ so that it is not in function notation, replace $g(x)$ with y to get $y = 5x - 6$.

2. Substitute the ordered pair into the function and simplify.

   The ordered pair is (5, 19). Substitute 5 for x and 19 for y.

$y = 5x - 6$	Rewritten function
$(19) = 5(5) - 6$	Substitute 5 for x and 19 for y.
$19 = 25 - 6$	Multiply 5 by 5.
$19 = 19$	Subtract 6 from 25.

   Once simplified, the equation is 19 = 19. Because this is a true statement, the ordered pair (5, 19) satisfies the function $g(x) = 5x - 6$.

3. Substitute the ordered pair into the inequality $y > 5x - 6$ and simplify.

   Use the same ordered pair, (5, 19). Again, substitute 5 for x and 19 for y.

$y > 5x - 6$	Given inequality
$(19) > 5(5) - 6$	Substitute 5 for x and 19 for y.
$19 > 25 - 6$	Multiply 5 by 5.
$19 > 19$	Subtract 6 from 25.

   Once simplified, the inequality is 19 > 19. Because this is not a true statement, the ordered pair (5, 19) does not satisfy the inequality $y > 5x - 6$.

Example 3

The equation $c = 0.4m + 3$ models the cost, c, in dollars, of using a vacuum at a self-serve car wash for m minutes. Graph the equation, with m (the input variable) along the horizontal axis and c along the vertical axis.

1. Determine an ordered pair that satisfies the equation.

   To determine an ordered pair that satisfies the equation, substitute any value of m such that $m \geq 0$ into the equation and solve for c. The value of m must be greater than or equal to 0 because it is impossible to use the vacuum for a negative number of minutes. However, as soon as the vacuum starts, which is represented by 0 minutes, the time begins. In this case, 0 will be substituted for m.

$c = 0.4m + 3$	Given equation
$c = 0.4(0) + 3$	Substitute 0 for m.
$c = 0 + 3$	Multiply 0.4 by 0.
$c = 3$	Add 3 to 0.

   An ordered pair that satisfies the equation $c = 0.4m + 3$ is (0, 3).

2. Determine another ordered pair that satisfies the equation.

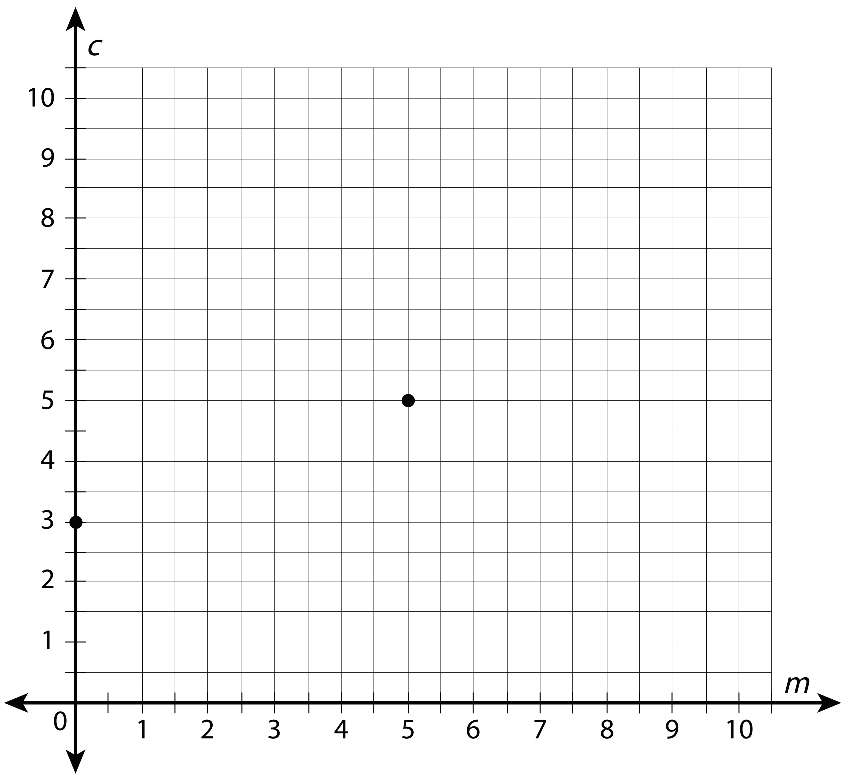
   To determine another ordered pair that satisfies the equation, substitute any value of m such that $m > 0$ into the equation and solve for c. The value of m must be greater than 0 because, again, m cannot be negative and 0 was already substituted into the equation in the previous step. In this case, 5 will be substituted for m.

$c = 0.4m + 3$	Given equation
$c = 0.4(5) + 3$	Substitute 5 for m.
$c = 2 + 3$	Multiply 0.4 by 5.
$c = 5$	Add 2 to 3.

   Another ordered pair that satisfies the equation $c = 0.4m + 3$ is (5, 5).

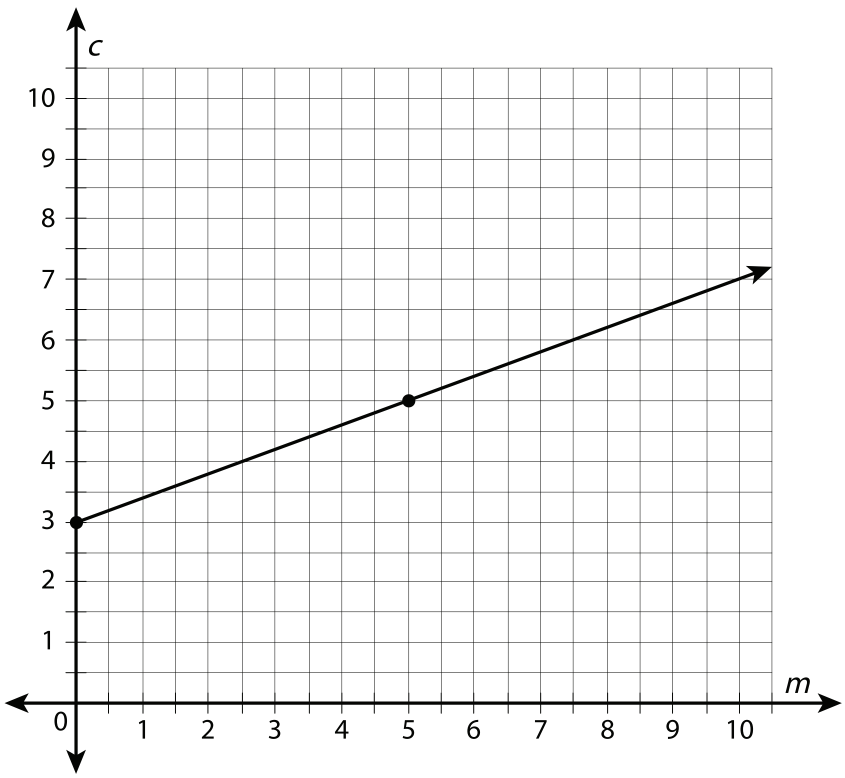
3. Plot the two ordered pairs on a coordinate plane.

   Recall that it is only necessary to have at least two ordered pairs to graph a linear function. The ordered pairs (0, 3) and (5, 5) can be plotted on a coordinate plane as shown.



4. Draw a line through the two points.

Because the number of minutes must be greater than or equal to 0, the line should not extend to the left of the vertical axis. However, it can extend infinitely to the right of the vertical axis.



The completed graph models the cost, *c*, in dollars, of using a vacuum at a self-serve car wash for *m* minutes.

Scaffolded Practice Skill 3

Example 1

The domain of the function $f(x) = 4x + 3$ is restricted to the set of integers {1, 3, 5}. What is the set of ordered pairs that corresponds to this function?

1. Find $f(1)$.

2. Find $f(3)$.

3. Find $f(5)$.

4. Determine the ordered pairs that correspond to this function.

continued

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Example 2

Does the ordered pair (5, 19) satisfy the function $g(x) = 5x - 6$? Does it satisfy the inequality $y > 5x - 6$?

Example 3

The equation $c = 0.4m + 3$ models the cost, c, in dollars, of using a vacuum at a self-serve car wash for m minutes. Graph the equation, with m (the input variable) along the horizontal axis and c along the vertical axis.

Problem-Based Task Skill 3: Tallying Taxes

SMP	
1 ✓	2 ✓
3	4 ✓
5 ✓	6 ✓
7	8

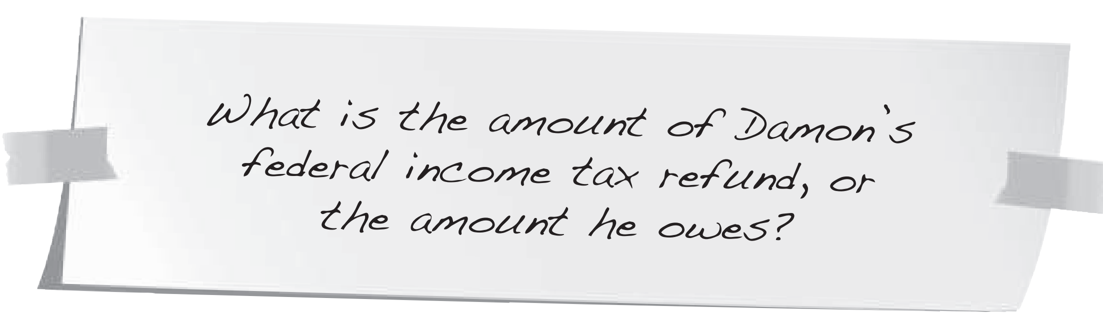
Tessa prepares tax returns. She is helping her client, Damon, determine whether he will get a federal income tax refund or owe money by performing the following steps:

1. First she will determine Damon's total income.
2. Then she will find Damon's taxable income using the formula *taxable income* = *total income* – *standard deduction* – *personal exemption*, where the standard deduction is $6,300 and the personal exemption is $4,000.
3. Next, she will substitute Damon's taxable income, n, into the correct branch of the function $T(n)$ that follows to find his tax liability.

$$T(n)=\begin{cases} 0.1n & \text{if } n\leq 9225 \\ 0.15(n-9225)+922.50 & \text{if } 9225<n\leq 37{,}450 \\ 0.25(n-37{,}450)+5156.25 & \text{if } 37{,}450<n\leq 90{,}750 \end{cases}$$

4. Finally, she will find the difference between the amount of tax Damon has already paid by having money withheld from his paychecks and Damon's tax liability. If the difference is positive, Damon will get a refund. If it is negative, he will owe money.

Damon's income consisted of $51,280 from his job and $90 in interest from his bank. He had $7,250 in federal income tax withheld from his paychecks. What is the amount of Damon's federal income tax refund, or the amount he owes?



Problem-Based Task Skill 3: Tallying Taxes

Coaching

a. What is Damon's total income?

b. What is Damon's taxable income?

c. Into which branch of the function $T(n)$ should Tessa substitute Damon's taxable income?

d. What is Damon's tax liability?

e. What is the amount of Damon's federal income tax refund, or the amount he owes?

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Problem-Based Task Skill 3: Tallying Taxes

Coaching Sample Responses

a. What is Damon's total income?

Damon's total income is the sum of his income from his job and the interest he earned from his bank.

$51,280 + $90 = $51,370

b. What is Damon's taxable income?

Damon's taxable income is his total income minus the standard deduction of $6,300 and the personal exemption of $4,000.

$51,370 – $6,300 – $4,000 = $41,070

c. Into which branch of the function $T(n)$ should Tessa substitute Damon's taxable income?

Because Damon's taxable income is greater than $37,450 and less than $90,750, Tessa should substitute it into the third branch, $T(n) = 0.25(n - 37{,}450) + 5156.25$.

d. What is Damon's tax liability?

To find Damon's tax liability, substitute his taxable income, $41,070, for n in the function $T(n) = 0.25(n - 37{,}450) + 5156.25$, and then solve.

$T(n) = 0.25(n - 37{,}450) + 5156.25$

$T(41{,}070) = 0.25[(41{,}070) - 37{,}450] + 5156.25$

$T(41{,}070) = 0.25(3620) + 5156.25$

$T(41{,}070) = 905 + 5156.25$

$T(41{,}070) = 6061.25$

Damon's tax liability is $6,061.25.

e. What is the amount of Damon's federal income tax refund, or the amount he owes?

Damon had $7,250 in federal income tax withheld from his paychecks. This is greater than his liability of $6,061.25, so he will get a refund in the amount of $7,250 – $6,061.25, which is equal to $1,188.75.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 3: Finding Ordered Pairs by Evaluating Functions

For problems 1–4, find the range of each function given the restricted domain.

1. $f(x) = 7x - 3$; domain: {1, 3, 5}
2. $g(x) = -4x + 12$; domain: {2, 3, 5}
3. $h(x) = 5(x + 2)$; domain: {–3, 0, 3}
4. $k(x) = 2x + 9$; domain: {6, 8, 11}

For problems 5–8, determine whether the ordered pair satisfies the given function or inequality. Write "yes" or "no" depending on your results.

5. (4, 2); $m(x) = 2x - 7$
6. (2, 2); $y > -5x + 9$
7. (3, 17); $n(x) = 3x + 8$
8. (–1, –13); $y < 9x - 4$

For problems 9 and 10, graph the equation for each scenario and determine whether the equation represents a function. Explain why or why not.

9. The equation $h = 0.10m + 4$ models the height, h, in feet, of an oak tree m months after it was planted.
10. The equation $w = 3s + 10$ models the weight, w, in ounces of a bird feeder filled with s cubic inches of seeds.

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Instruction

Skill 4: Evaluating Squares and Cubes of Real Numbers With and Without a Calculator

SMP	
1 ✓	2 ✓
3 ✓	4 ✓
5	6 ✓
7 ✓	8

Common Core State Standard

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Essential Questions

1. What is the square root or cube root of a number?
2. How do you find the square root or cube root of a number using a calculator?

WORDS TO KNOW

cube root For any real numbers a and b, if $a^3 = b$, then a is the cube root of b. The cube root of b is written using a radical: $\sqrt[3]{b}$.

perfect cube the product of an integer raised to the third power

perfect square the product of an integer and itself

radical expression an expression containing a root, such as $\sqrt[5]{9}$

square root For any real numbers a and b, if $a^2 = b$, then a is a square root of b. The square root of b is written using a radical: $\sqrt{b}$.

Recommended Resources

- IXL Learning. "Cube Roots of Perfect Cubes."

  http://www.walch.com/rr/04074

  This website provides practice in finding cube roots of perfect cubes with immediate and thorough feedback for any incorrect answers.

- IXL Learning. "Square Roots of Perfect Squares."

  http://www.walch.com/rr/04075

  This site offers interactive practice problems for finding square roots of perfect squares, with immediate feedback for incorrect answers.

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Recommended Instructional Strategies for Skill Development

Suggestions for Manipulatives

Have students use unit square tiles to make several larger squares. For each square, have students identify the total number of unit squares used and the side length of the square, recording their results in a two-column table. Have students think about how they can find the side length of the square if they know how many unit squares were used. Students can repeat the activity using unit cubes, preferably linking ones, to build several larger cubes, once more recording their results in a two-column table. Students should consider how to find the length of the side of the cube if the number of unit cubes used is known.

Suggestions for Discourse

- Ask students to think about the relationship between squares and square roots as well as that between cubes and cube roots. Lead them to identify the relationships as inverse operations and have students justify their conclusions.
- Ask students to write a step-by-step explanation of how to estimate square roots, using their own example. Encourage and guide a discussion about the process of using known square roots to create a range of possible values, and then narrowing down the choices until the correct value is found.

Making Connections

Have students consider for what values the square root or cube root is undefined. Students should understand that the square root function is undefined for negative values, but that the cube root function is defined for negative values. Ask students to explain their reasons and provide examples to support their explanations.

Instruction

Skill 4: Evaluating Squares and Cubes of Real Numbers With and Without a Calculator

Introduction

When a number is squared, it is multiplied by itself. When that number is an integer, the result is called a **perfect square**. Taking the square root of a number is helpful in many real-world situations; for example, if you know the area of a square rug you're thinking of buying, but need to know the length of one side in order to make sure the rug will fit where you want to put it.

Similarly, when a number is cubed, the number appears in the multiplication three times. When that number is an integer, the product is a **perfect cube**. Taking the cube root of a number can also be helpful. In terms of geometric figures, suppose you know the volume of a box that is shaped like a cube. You can find the length of a side of the cubic box by taking the cube root of the volume.

Key Concepts

Squares and Square Roots

- For any real numbers a and b, if $a^2 = b$, then a is the **square root** of b. Any positive real number has two square roots, a positive and a negative.
- A radical symbol, $\sqrt{}$, is used to represent the positive square root. A **radical expression** is an expression containing a root.
- For instance, $\sqrt{49}$, read as "the square root of 49," is equal to 7 because 7^2 or 7 • 7 is equal to 49.
- There is no square root of a negative number. For example, $\sqrt{-49}$ is not a real number because there is no real number that, when multiplied by itself, is equal to –49.
- Perfect squares are integers that are the square of integers. For instance, 1, 4, 9, 16, and 25 are all perfect squares because they are the squares of the integers 1, 2, 3, 4, and 5. That is, $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, and $5^2 = 25$.
- Perfect squares can be used to estimate the square root of a number that isn't a perfect square. For example, 10 is not a perfect square. If we compare 10 to the perfect squares already discussed, we can determine that the square root of 10 is greater than 3, but less than 4. This is because $3^2 = 9$ and $4^2 = 16$; notice that 10 falls somewhere in between 9 and 16.
- The square root of a number can be found using a calculator.

On a TI-83/84:

Step 1: Press [2ND][x^2].

Step 2: Enter the number.

Step 3: Press [ENTER].

On a TI-Nspire:

Step 1: From the home screen, select A: Calculate.

Step 2: Press [ctrl][x^2].

Step 3: Enter the number.

Step 4: Press [enter].

- The calculator will display the square root of the entered number.
- To find the square root of a fraction, find the square root of the numerator and then find the square root of the denominator.

Cubes and Cube Roots

- The symbol for **cube root** is also a radical, $\sqrt[3]{}$. Note that a small 3 on the "shelf" of the radical symbol shows that it is for a cube root.
- The cube root of a positive number is always positive.
- For instance, $\sqrt[3]{27}$, read as "the cube root of 27," is equal to 3 because 3^3 or 3 • 3 • 3 is equal to 27.
- The cube root of a negative number is always negative. For instance, $\sqrt[3]{-27}$ is equal to –3 because $(-3)^3$ or –3 • –3 • –3 is equal to –27.
- If there is an expression under the radical symbol, simplify the expression before finding the root. For example, given $\sqrt[3]{10-2}$, first subtract to get $\sqrt[3]{8}$. This can now be simplified to 2, because $2^3 = 8$.
- The cube root of a number can be found using a calculator.

On a TI-83/84:

Step 1: Press [MATH].

Step 2: Choose 4 to select $\sqrt[3]{\ }$.

Step 3: Enter the number.

Step 4: Press [ENTER].

On a TI-Nspire:

Step 1: From the home screen, select A: Calculate.

Step 2: Press [ctrl][^][3].

Step 3: Enter the number.

Step 4: Press [enter].

- The calculator will display the cube root of the entered number.
- To find the cube root of a fraction, first find the cube root of the numerator and then find the cube root of the denominator.

Guided Practice Skill 4

Example 1

Use estimation to find $\sqrt{256}$, and then verify your results with a calculator.

1. Use estimation to determine the square root.

   Because 10^2 is equal to 100 and 20^2 is equal to 400, and 256 is greater than 100 but less than 400, it can be determined that $\sqrt{256}$ is between 10 and 20.

   Choose another integer, such as 15. When 15 is squared, or 15^2, the result is 225, so it can be determined that $\sqrt{256}$ is greater than 15 and still less than 20.

   Choose another integer, such as 16. The result of 16^2 is 256, so $\sqrt{256}$ is equal to 16.

2. Use a calculator to verify the square root.

   Follow the directions specific to your calculator model.

   On a TI-83/84:

   Step 1: Press [2ND][x^2].

   Step 2: Enter 256.

   Step 3: Press [ENTER].

   On a TI-Nspire:

   Step 1: From the home screen, select A: Calculate.

   Step 2: Press [ctrl][x^2].

   Step 3: Enter 256.

   Step 4: Press [enter].

   Either calculator will display an answer of 16.

   Therefore, $\sqrt{256} = 16$.

Example 2

Use estimation to find $\sqrt[3]{64}$, and then verify your results with a calculator.

1. Use estimation to determine the cube root.

   Because 2^3 is equal to 8 and 5^3 is equal to 125, and 64 is greater than 8 but less than 125, it can be determined that $\sqrt[3]{64}$ is between 2 and 5.

   Choose another integer, such as 3. When 3 is cubed, or 3^3, the result is 27, so it can be determined that $\sqrt[3]{64}$ is greater than 3 and still less than 5.

   Choose another integer, such as 4. The result of 4^3 is 64, so $\sqrt[3]{64}$ is equal to 4.

2. Use a calculator to verify the cube root.

   Follow the directions specific to your calculator model.

   On a TI-83/84:

   Step 1: Press [MATH].

   Step 2: Choose 4 to select $\sqrt[3]{\ }$.

   Step 3: Enter 64.

   Step 4: Press [ENTER].

   On a TI-Nspire:

   Step 1: From the home screen, select A: Calculate.

   Step 2: Press [ctrl][^][3].

   Step 3: Enter 64.

   Step 4: Press [enter].

   Either calculator will display an answer of 4.

   Therefore, $\sqrt[3]{64} = 4$.

Example 3

A 13-foot ladder leans against the side of a building. The base of the ladder is 5 feet from the building. Risa calculates that the top of the ladder rests $\sqrt{13^2-5^2}$ feet up the side of the building. How high does the ladder reach?

1. Simplify the expression under the radical symbol.

   The expression under the radical symbol is $13^2 - 5^2$. Follow the order of operations to simplify this expression.

$13^2 - 5^2$	Expression under the radical symbol
$= 169 - 5^2$	Simplify 13^2.
$= 169 - 25$	Simplify 5^2.
$= 144$	Subtract.

   The expression $13^2 - 5^2$ simplifies to 144.

2. Find the square root of the simplified expression.

   The simplified expression, 144, is a perfect square because $12 \bullet 12 = 144$. Therefore, $\sqrt{144} = 12$.

   Therefore, the top of the ladder reaches 12 feet up the side of the building.

Scaffolded Practice Skill 4

Example 1

Use estimation to find $\sqrt{256}$, and then verify your results with a calculator.

1. Use estimation to determine the square root.

2. Use a calculator to verify the square root.

continued

Example 2

Use estimation to find $\sqrt[3]{64}$, and then verify your results with a calculator.

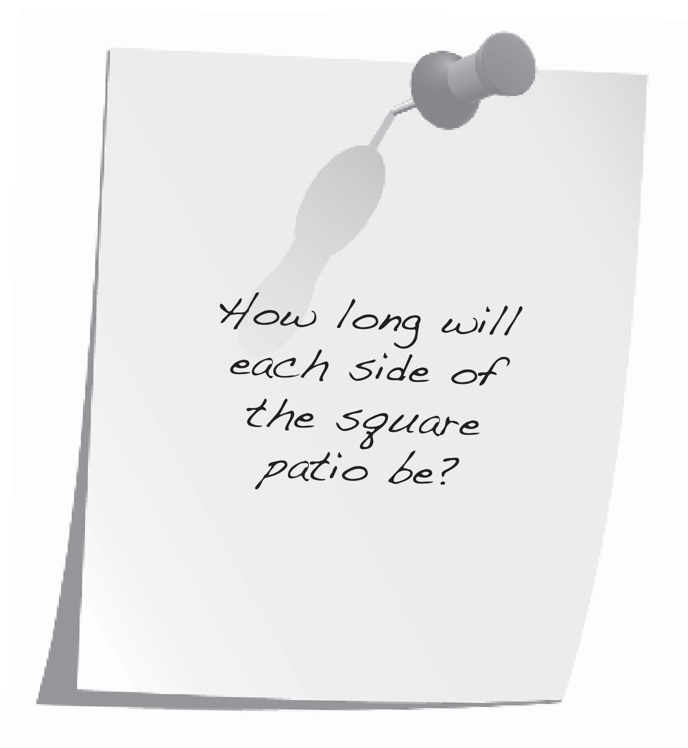
Example 3

A 13-foot ladder leans against the side of a building. The base of the ladder is 5 feet from the building. Risa calculates that the top of the ladder rests $\sqrt{13^2 - 5^2}$ feet up the side of the building. How high does the ladder reach?

Problem-Based Task Skill 4: Designing a Patio

Todd is designing a patio. He wants to cover it in 1-foot square tiles. His first design has 5 rows of 20 tiles per row. However, he wants to redesign it so that the patio is square but still uses the same number of tiles. How long will each side of the square patio be?

SMP
1 ✓ 2 ✓
3 4 ✓
5 ✓ 6 ✓
7 ✓ 8



Problem-Based Task Skill 4: Designing a Patio

Coaching

a. How many total tiles did Todd use for his original patio design?

b. What does the number of tiles represent?

c. If the patio is square, how can its side length be found?

d. What expression represents the side length of the square patio?

e. How long will each side of the square patio be?

Problem-Based Task Skill 4: Designing a Patio

Coaching Sample Responses

a. How many total tiles did Todd use for his original patio design?

According to the given information, the original patio design had 5 rows of 20 tiles per row. The total number of tiles could be determined by finding the product of 5 and 20.

$5 \bullet 20 = 100$

Todd's original patio design used 100 tiles.

b. What does the number of tiles represent?

The number of tiles represents the area of the patio.

c. If the patio is square, how can its side length be found?

To find the side length of the patio, find the square root of the area.

d. What expression represents the side length of the square patio?

The patio uses 100 tiles, and each tile is 1 foot long. Therefore, the area of the patio is 100 square feet. To find the side length, take the square root of the area.

The side length of the patio is equal to $\sqrt{100}$ feet.

e. How long will each side of the square patio be?

Using perfect squares, determine $\sqrt{100}$.

The result of 10^2 is 100; therefore, $\sqrt{100} = 10$.

Each side of the square patio will be 10 feet long.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 4: Evaluating Squares and Cubes of Real Numbers With and Without a Calculator

For problems 1–4, simplify each square root expression.

1. $\sqrt{121}$

2. $-\sqrt{225}$

3. $\sqrt{10^2 - 8^2}$

4. $\sqrt{\dfrac{25}{81}}$

For problems 5–8, simplify each cube root expression.

5. $\sqrt[3]{8}$

6. $\sqrt[3]{-125}$

7. $\sqrt[3]{4^2 + 11}$

8. $\sqrt[3]{\dfrac{343}{1000}}$

Use the given information to solve problems 9 and 10.

9. Casey dug a cubical hole to bury a memory box as a time capsule. The volume of the dirt he removed is 729 cubic inches. How deep is the hole?

10. The radius of a circle, in centimeters, is represented by the expression $\sqrt{3^2 + 4^2}$. What is the measure of the radius?

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Instruction

Skill 5: Graphing a Linear Function*

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 1

Guided Practice Skill 5

Example 1

Demarcus is saving to buy a new stereo for his truck. He has $85 in his savings account and he plans to save $60 per month for the next 5 months. He can model his savings with the equation $y = 60x + 85$, where x is the number of months for which he has been saving and y is the amount of money in his savings account. Make a table of values and create a graph of this relationship.

1. Determine values that make the equation true.

   To create a graph of the given equation, at least two points are needed. To find the points, substitute values for x and solve for y. Let's use the values 0, 1, 4, and 5.

   Substitute 0 for x and solve for y.

$y = 60x + 85$	Given equation
$y = 60(0) + 85$	Substitute 0 for x.
$y = 85$	Simplify.

   When $x = 0$, $y = 85$.

   Substitute 1 for x and solve for y.

$y = 60x + 85$	Given equation
$y = 60(1) + 85$	Substitute 1 for x.
$y = 145$	Simplify.

   When $x = 1$, $y = 145$.

   Substitute 4 for x and solve for y.

$y = 60x + 85$	Given equation
$y = 60(4) + 85$	Substitute 4 for x.
$y = 325$	Simplify.

   When $x = 4$, $y = 325$.

   Substitute 5 for x and solve for y.

$y = 60x + 85$	Given equation
$y = 60(5) + 85$	Substitute 5 for x.
$y = 385$	Simplify.

   When $x = 5$, $y = 385$.

2. Organize these values in a table.

   Create a table of the values determined in the previous step, with *x*-values in one column and *y*-values in the other.

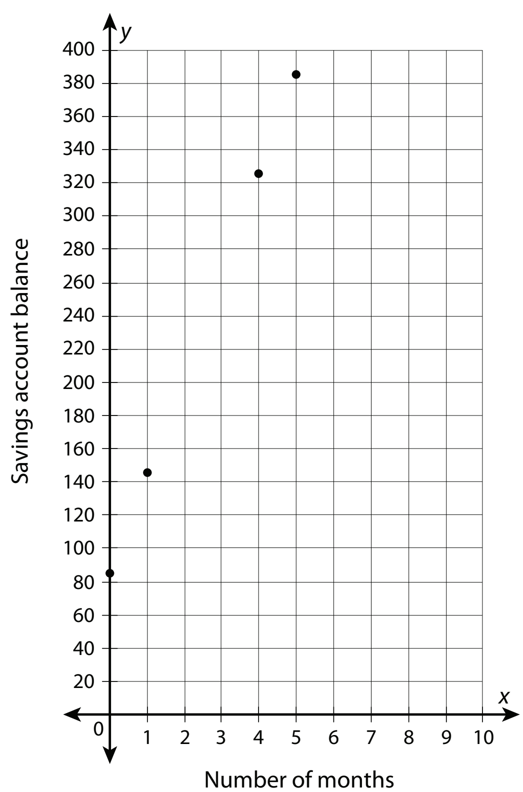
x	*y*
0	85
1	145
4	325
5	385

3. Plot the ordered pairs from the table on a coordinate plane.

   The ordered pairs from the table are (0, 85), (1, 145), (4, 325), and (5, 385).

   Plot these ordered pairs on a coordinate plane.

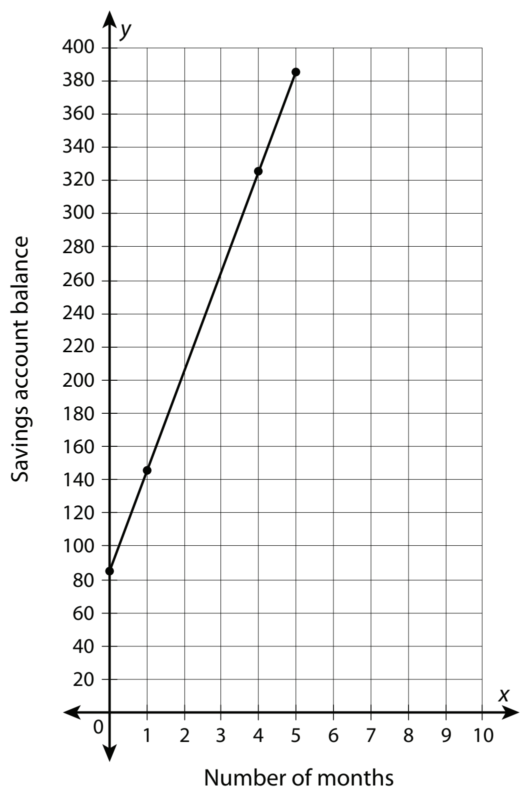
   Label the *x*-axis "Number of months" and the *y*-axis "Savings account balance."



4. Draw a line through the points.

   Because the given equation, $y = 60x + 85$, is a linear equation, draw a line through the plotted points.

   Demarcus is only saving for 5 months, so the line must start at $x = 0$ and end at $x = 5$.



Scaffolded Practice Skill 5

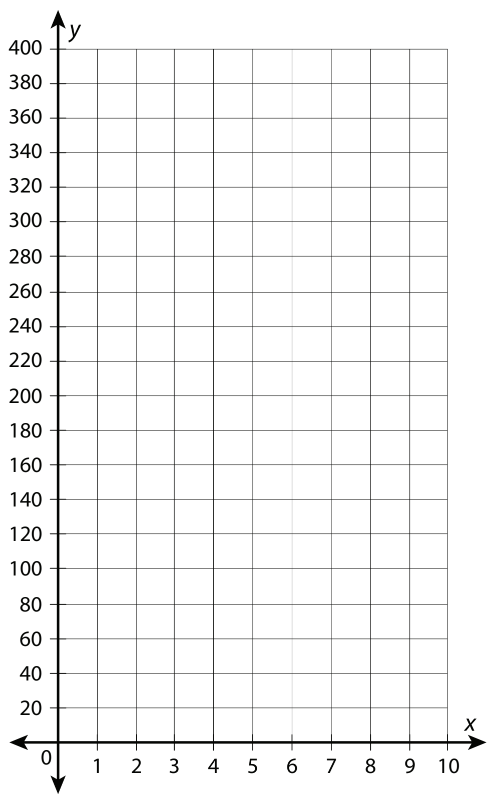
Example 1

Demarcus is saving to buy a new stereo for his truck. He has \$85 in his savings account and he plans to save \$60 per month for the next 5 months. He can model his savings with the equation $y = 60x + 85$, where x is the number of months for which he has been saving and y is the amount of money in his savings account. Make a table of values and create a graph of this relationship.

1. Determine values that make the equation true.

2. Organize these values in a table.

3. Plot the ordered pairs from the table on a coordinate plane.



4. Draw a line through the points.

Practice Skill 5: Graphing a Linear Function*

For problems 1–3, graph each linear function on a coordinate plane.

1. $y = -4x + 7$

2. $y = \frac{1}{2}x + 6$

3. $y = 3(x - 2)$

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Skill 6: Finding the Absolute Value of a Quantity

SMP	
1 ✓	2 ✓
3	4 ✓
5 ✓	6 ✓
7 ✓	8

Common Core State Standard

6.NS.7 Understand ordering and absolute value of rational numbers.

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*

Essential Questions

1. What is absolute value?
2. How is the absolute value of a number used in real-world situations?

WORDS TO KNOW

absolute value | |; a number's distance from 0 on a number line

magnitude the size of something

Recommended Resource

- IXL Learning. "Absolute Value of Rational Numbers."

  http://www.walch.com/rr/04076

  This site offers extensive practice in finding the absolute value of rational numbers. For incorrect answers, the feedback reteaches the skill and provides the correct answer.

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Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers

- Provide students with the Coordinate Plane graphic organizer from the Program Overview. Using different colors for each function, have students write the functions $f(x) = |x|$ and $g(x) = -|x|$ at the top of the page. Ask students to work with a partner to first create a table of values for each function, using {–3, –2, –1, 0, 1, 2, 3} as the domain, and then graph each function, in its respective color, on the same graph. Then, guide a discussion about the differences between the two graphs. Help students draw conclusions about the effect of the negative sign on the $g(x)$ function.
- Provide students with a blank three-column chart or table. Ask students to title the chart "Comparing Absolute Value Expressions." Then, have them label three column headings from left to right as follows: "First expression," "<, >, or =," and "Second expression." Have each student create three expressions in the first column (labeled A, B, and C) and three expressions in the third column (labeled D, E, and F), leaving room between the expressions for simplifying work. Then, ask students to switch charts with a partner. Have each student simplify his or her partner's expressions in each column. Then, have students compare expressions A and D, B and E, and C and F, filling in the symbols <, >, or = in the middle column for each pair of expressions. Ask students to volunteer one of their comparisons.

Suggestions for Discourse

- Ask students to think about measuring distance and the sign (+ or –) that measures of distance always have. Have students explain why distance is always positive. Then, have students relate this to absolute value, and ask them to explain how the definition of absolute value necessitates it always being positive.
- Ask students to create a list of real-life examples in which negative amounts are used, such as debts/money owed and temperatures below 0°F. Also ask students to list some words that are commonly used to represent these negative amounts, i.e.,"below" and "owe." Ask students to volunteer their answers. Encourage and guide a discussion about how the absolute values of these amounts are used to represent the magnitude of the amount.

Making Connections

Discuss with students how an absolute value function is a type of function that relates to other functions, such as linear and quadratic functions. An absolute value function can be thought of as two linear functions, with restricted domains, that meet at a critical point (extreme point). Also, discuss how the extreme point of an absolute value function is similar to a maximum or minimum of a quadratic function.

Instruction

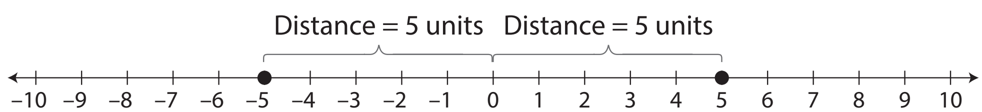
Skill 6: Finding the Absolute Value of a Quantity

Introduction

A scuba diver started at sea level and dove to –22 meters, or 22 meters below sea level. A mountain climber also started at sea level and climbed up 22 meters, or 22 meters above sea level. Each adventurer went a distance of 22 meters, and each is the same distance from sea level. This distance can be represented by the absolute value of their elevations.

Key Concepts

- The **absolute value** of a number is a number's distance from 0 on a number line. Because it is a distance, the absolute value of a number is always positive.
- The notation for taking the absolute value of a number is two vertical bars: $|n|$. For example, the absolute value of –5 is written as $|-5|$. The result of taking the absolute value of –5 is 5, because –5 is 5 units from 0 on a number line. The absolute value of 5 is also equal to 5, because positive 5 is also 5 units from 0 on a number line.



- Because opposite numbers are the same distance from 0 on a number line, their absolute values are equal.
- Absolute value can also represent the **magnitude**, or size, of a quantity in the real world.
- Simplifying an expression between absolute value symbols is similar to simplifying an expression inside parentheses. The order of operations is still followed to simplify the expression before the absolute value is found.
- You can use a calculator to find the absolute value.

On a TI-83/84:

Step 1: Press [MATH].

Step 2: Scroll to highlight "NUM" and select "abs(."

Step 3: Enter the number and press [)].

Step 4: Press [ENTER].

On a TI-Nspire:

Step 1: From the home screen, select A: Calculate.

Step 2: Type [a][b][s][(], enter the number, and then press [)].

Step 3: Press [enter].

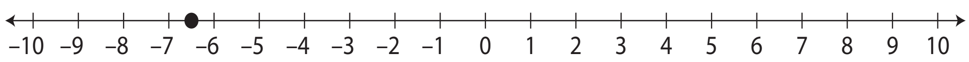
Guided Practice Skill 6

Example 1

Use a number line to find $|-6.5|$. Verify your results with a calculator.

1. Graph the value on a number line.

   The number within the absolute value bars is –6.5. Graph this value on a number line.



2. Find the graphed point's distance from 0.

   The distance between –6.5 and 0 is 6.5 units. Therefore, $|-6.5| = 6.5$.

3. Verify the absolute value using a calculator.

   Follow the directions specific to your calculator model.

   On a TI-83/84:

   Step 1: Press [MATH].

   Step 2: Scroll to highlight "NUM" and select "abs(."

   Step 3: Enter –6.5 and press [)].

   Step 4: Press [ENTER].

   On a TI-Nspire:

   Step 1: From the home screen, select A: Calculate.

   Step 2: Type [a][b][s][(], enter –6.5, and then press [)].

   Step 3: Press [enter].

   Either calculator will display a result of 6.5. This confirms that $|-6.5| = 6.5$.

Example 2

What is the value of $\left|\frac{3}{4}-\frac{2}{5}\right|$?

1. Simplify the expression inside the absolute value bars.

   The absolute value bars act as a grouping symbol, so it is necessary to simplify the expression within them before trying to find the absolute value.

$\frac{3}{4}-\frac{2}{5}$	Expression inside the absolute value bars
$=\frac{15}{20}-\frac{8}{20}$	Rewrite the fractions with a common denominator.
$=\frac{7}{20}$	Subtract.

   The expression inside absolute value bars, $\frac{3}{4}-\frac{2}{5}$, simplifies to $\frac{7}{20}$. Therefore, $\left|\frac{3}{4}-\frac{2}{5}\right|$ is equal to $\left|\frac{7}{20}\right|$.

2. Find the absolute value of the simplified expression.

   The distance from $\frac{7}{20}$ to 0 on a number line is $\frac{7}{20}$, so the absolute value is $\frac{7}{20}$.

   Symbolically, this is written as $\left|\frac{7}{20}\right|=\frac{7}{20}$.

3. Verify the absolute value using a calculator.

   Follow the directions specific to your calculator model.

   On a TI-83/84:

   Step 1: Press [MATH].

   Step 2: Scroll to highlight "NUM" and select "abs(."

   Step 3: Enter [3][÷][4][–][2][÷][5], then press [)].

   Step 4: Press [ENTER].

   On a TI-Nspire:

   Step 1: From the home screen, select A: Calculate.

   Step 2: Type [a][b][s][(], enter [3][÷][4][–][2][÷][5], and then press [)].

   Step 3: Press [enter].

   Either calculator will display 0.35, which is equivalent to $\frac{7}{20}$. Therefore, the value of $\left|\frac{3}{4}-\frac{2}{5}\right|$ is $\frac{7}{20}$ or 0.35. ✓

Example 3

Hank borrowed money from his parents to buy a new bike. His parents have determined his balance is –$55. What is the magnitude of his debt?

1. Find the absolute value.

   The absolute value of –55 can be written as $|-55|$, which is equal to 55.

2. Find the magnitude of Hank's debt.

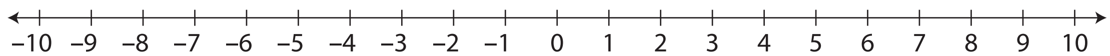
   The magnitude of his debt is represented by the absolute value of his debt; therefore the magnitude, or size, of Hank's debt is $55. ✓

Scaffolded Practice Skill 6

Example 1

Use a number line to find $|-6.5|$. Verify your results with a calculator.

1. Graph the value on a number line.



2. Find the graphed point's distance from 0.

3. Verify the absolute value using a calculator.

continued

Example 2

What is the value of $\left|\frac{3}{4}-\frac{2}{5}\right|$?

Example 3

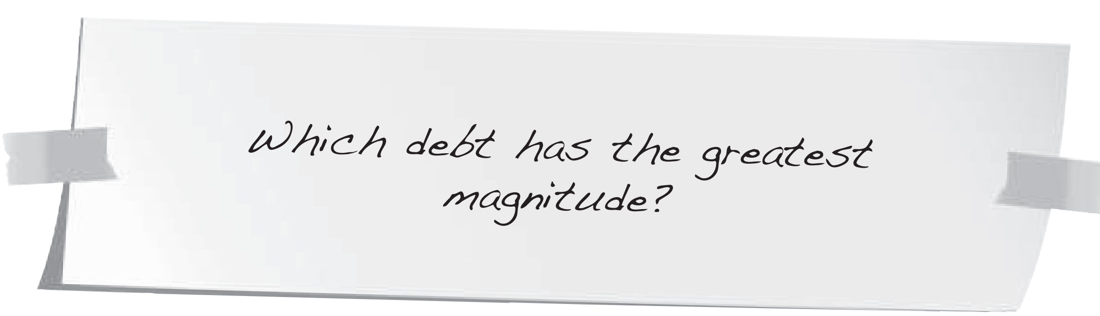
Hank borrowed money from his parents to buy a new bike. His balance with his parents is –$55. What is the magnitude of his debt?

Problem-Based Task Skill 6: Which Debt Is Greatest?

SMP
1 ✓ 2 ✓
3 4 ✓
5 ✓ 6 ✓
7 ✓ 8

Robert is reviewing his finances, and he has listed his outstanding debts in the following table. Which debt has the greatest magnitude?

Debt	Amount ($)
Car loan	–1,029.55
Credit card	–450.00
Medical bills	–375.37
Student loan	–698.23



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Problem-Based Task Skill 6: Which Debt Is Greatest?

Coaching

a. How is the magnitude of each debt determined?

b. What is the magnitude of the car loan?

c. What is the magnitude of the credit card debt?

d. What is the magnitude of the medical bills?

e. What is the magnitude of the student loan?

f. Which debt has the greatest magnitude?

Problem-Based Task Skill 6: Which Debt Is Greatest?

Coaching Sample Responses

a. How is the magnitude of each debt determined?

To determine the magnitude of each debt, find the absolute value of each debt amount.

b. What is the magnitude of the car loan?

The balance of the car loan is –$1,029.55.

$|-1029.55| = 1029.55$

The magnitude of the car loan is 1,029.55.

c. What is the magnitude of the credit card debt?

The balance of the credit card debt is –$450.00.

$|-450.00| = 450.00$

The magnitude of the credit card debt is 450.00.

d. What is the magnitude of the medical bills?

The balance of the medical bills is –$375.37.

$|-375.37| = 375.37$

The magnitude of the medical bills is 375.37.

e. What is the magnitude of the student loan?

The balance of the student loan is –$698.23.

$|-698.23| = 698.23$

The magnitude of the student loan is 698.23.

f. Which debt has the greatest magnitude?

The debt with the greatest magnitude is the car loan because it has the greatest absolute value.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 6: Finding the Absolute Value of a Quantity

For problems 1–4, find the absolute value.

1. $|45.028|$

2. $\left|-\frac{31}{78}\right|$

3. $|13.1-24.09|$

4. $\left|\frac{3}{4}-\frac{7}{8}\right|$

For problems 5–7, draw a >, <, or = symbol between the expressions to compare the absolute values.

5. $|-28.75|$ $|-29.075|$

6. $|-56|$ $|56|$

7. $\left|-4\frac{2}{9}\right|$ $\left|3\frac{7}{10}\right|$

Use your knowledge of absolute values to answer problems 8–10.

8. What two temperatures have an absolute value of 10?

9. Dan's debt is –$15. Tory's debt is –$22. Who has less debt? Explain.

10. The following table shows the yardage in each football team's first play of the game. Which team had the greater yardage magnitude? Explain.

Team	Yardage
Tigers	–12
Lions	9

Instruction

Skill 7: Determining Restricted Domains and Ranges for Application Problems**

Common Core State Standard

F–IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

This skill has been addressed previously in *CCSS Integrated Pathway: Mathematics II*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

Unit 2, Lesson 2, Sub-lesson 2

Supportive Instructional Strategies for Mathematics II

Unit 2 Lesson 4

Suggestions for Graphic Organizers/Manipulatives

- Provide students with the Coordinate Plane graphic organizer from the Program Overview. Ask students to write the functions $f(x) = x^2$, $g(x) = x^3$, and $h(x) = |x|$ at the top of the page, using a different color for each function. Have students work with a partner to create a table of values for each function, using {–3, –2, –1, 0, 1, 2, 3} as the domain, and then graph the functions, in their respective colors, on the same graph. Then, guide a discussion about the differences and similarities among the three graphs, including the domains and ranges for each function.
- Provide each student with a blank five-column chart or table. Ask students to label the chart "Types of Functions," then have them label the columns with the following headings from left to right: "Function name," "General format," "Graph sketch," "Domain," and "Range." Divide students into small groups and ask them to work together to complete their charts. First, have students list the different types of functions covered in the lesson. These functions should include square root, cube root, absolute value, radical, ceiling/least integer, floor/greatest integer, and piecewise. Next, have them fill out the respective corresponding columns for each function type. Ask for volunteers to share their results. Create a master list of the functions and their characteristics.

Suggestions for Discourse

- Ask students, "Why is the domain for a cube root function all real numbers, whereas the domain for a square root function is restricted to all real numbers greater than or equal to 0?" Ask students to provide examples, and lead them to conclude that it is not possible to find the square root of a negative value in the real number system.
- Have students work with a partner to create real-life applications of piecewise functions. Ask them, "What type of scenario would call for separate domains?" Have volunteers share their answers, and encourage and guide a discussion about examples in which different domains need to be considered when evaluating a function.
- Ask students, "Why can't the domains of a piecewise function overlap?" Encourage and guide a discussion about the basic definition of a function. Guide students to realize that for each input there is only one unique output for a function, and if there were two outputs for one input, then it would not be a function.
- Ask students, "In your own words, what is a critical, or extreme, point of a function? Why is it an important characteristic that represents a function?" Encourage and guide a discussion about a critical point being a point where the behavior of a function changes.

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics II*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.
- Provide a list of sentence frames in which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "A _______ function is a series of disconnected _______ functions." Or, "The domain of a square root function is _______, and the domain of a cube root function is _______." Or, "In a decreasing function, the dependent values decrease as the independent values increase."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- plotting too few points on a graph to understand the behavior of the function on the graphed domain and range

  Remind students that although only two points are needed to graph a linear function, because the graph will always be a straight line, other types of functions have different characteristics that require several points in order to get an accurate graph, including both negative and positive values.
- incorrectly calculating square roots and cube roots when identifying points on a graph

  Have students write the radical notation at the top of their papers, pointing out that the square root function is represented by only the radical symbol and that a cube root function has the radical symbol with a power of 3 on the "shelf."
- forgetting the relationship between an algebraic function and critical points on the graph

  Remind students that a critical point on a graph is a point where the graph changes direction, such as the vertex on a quadratic function. Remind students that the only function that has a constant rate of change and does not change direction is a linear function.

- incorrectly identifying integers greater than or less than negative values

  Remind students to check the direction of an inequality sign when comparing values. Remind them that negative values are to the left of 0 on the number line and positive values are to the right of 0 on the number line.

- entering items into the calculator with incorrectly placed parentheses

  Remind students to enter each value and symbol carefully, and also to double-check their entries before executing any operations.

- connecting steps on a step function instead of showing the jump between output values

  Remind students that a step function is a series of disconnected constant functions and that, as a result, there will be an open circle to represent the disconnection of the output values.

- drawing a curved graph of an absolute value function

  Remind students that an absolute value function differs from a quadratic function in terms of the shape of the graphs, as the absolute value function has a V-shaped graph (or an upside-down V) and the shape of a quadratic function is a curve.

- incorrectly determining the extreme value of an absolute value function, potentially by inverting *x*- and *y*-values

  Remind students that the table of values for an absolute value function is created in the same manner as other functions, and that the extreme value is the point where the graph changes behavior/direction. Remind students to include several negative and positive values in the table of an absolute value function.

- defining one point on a domain with two different expressions, even if the expressions are equivalent at that point

  Remind students to carefully check the inequality symbol on a given domain of a piecewise function. For example, a domain value of 1 would not work for the piece of the function with a domain restriction of $x > 1$, but it would work for the piece of the function with a domain restriction of $x \geq 1$.

- connecting graphs of different components of a piecewise function that should be disconnected

  Remind students that, on a graph of a piecewise function, a function with a ≤ or ≥ symbol will have a closed circle, indicating the value is included in that piece. A piecewise function with a < or > symbol will have an open circle on the graph, indicating the particular value is not included in that piece.

Lesson 5: Analyzing Functions

Instruction

Elementary Prerequisite Skills

This lesson requires the use of the following elementary skill(s) to meet the standards addressed in *CCSS Integrated Pathway: Mathematics II*. See the Appendix for material to address the skill(s).

- E-Skill 1: Evaluating Expressions Using the Order of Operations (5.OA.1), Appendix p. A-2

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics II*.

Skill 1: Identifying the Base and Power of an Exponent and Evaluating Exponential Expressions (6.EE.1)

Common Core State Standard

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

Skill 2: Simplifying Exponential Expressions with Integer Exponents* (8.EE.1)

Common Core State Standard

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Skill 3: Finding the Vertex and *x*-intercepts of a Parabola** (F–IF.7a★)

Common Core State Standard

F–IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Skill 4: Writing an Equation for a Simple Exponential Function (A–CED.1★)

Common Core State Standard

A–CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Skill 1: Identifying the Base and Power of an Exponent and Evaluating Exponential Expressions

SMP
1 ✓ 2 ✓
3 4
5 6 ✓
7 ✓ 8 ✓

Common Core State Standard

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

Essential Questions

1. How can expressions with exponents be simplified?
2. How are expressions with exponents written?

WORDS TO KNOW

base	the factor being multiplied together in an exponential expression; in the expression a^b, a is the base
cube	the product of a number multiplied by itself three times, indicated with an exponent of 3
expanded form	an expression written in the form of $b \bullet b \bullet b \bullet b \ldots$
exponent	the number of times a factor is being multiplied together in an exponential expression; in the expression a^b, b is the exponent
exponential expression	an expression written in the form of a^b
square	the product of a number multiplied by itself, indicated with an exponent of 2

Recommended Resources

- CK-12 Foundation. "Whole Number Exponents."

  http://www.walch.com/rr/04016

  This site provides additional explanation on exponents and their uses. Resources include a video tutorial and additional practice problems.

- IXL Learning. "Evaluate Exponents."

  http://www.walch.com/rr/04017

  This site has a wide range of interactive practice problems that contain whole number exponents and whole number bases.

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Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Provide each student with three blank flash cards. Ask students to create three varieties of exponential expressions, one on each flash card. One expression should have a fractional base, one should be written in words, and one should contain a variable. Have students exchange their flash cards with a partner, and ask the partner to write on the opposite side of the flash card the following information about each expression: Base: _______; Exponent: _______; Expanded form: ___________________. Ask for volunteers to share their answers and explain each part of the expression.

Suggestions for Discourse

- Ask students, "Why do you think it is important to be able to simplify and evaluate exponential expressions?" Encourage and guide a discussion about working with exponential expressions that contain large numbers and exponents, and how simplifying will yield a much simpler expression that can be evaluated more efficiently.
- Ask students, "When comparing several exponential expressions, what are the benefits of writing all of the bases in the same format? That is, why is it advantageous to have the bases as either all fractions or all decimals?" Discuss how making comparisons of amounts is more simple when the amounts are in the same numerical format.
- Guide students to discuss how the value of the exponent can vastly change the result when evaluating two seemingly similar expressions. Present students with the following problem: "Given the exponential expressions 4^3 and 3^4, which expression do you predict has the greater value when evaluated? Explain your reasoning, and then evaluate each expression to determine the value for each."

Making Connections

- Discuss with students the importance of understanding the parts of an exponential expression, such as which part is the base and which part is the exponent. Explain how exponents are commonly used in formulas and models of functions, and that understanding how to simplify expressions containing exponents is important for evaluating values of functions.
- Discuss why understanding how to perform operations with exponents is an important component in working with function models. For example, exponents indicate that a function is not increasing or decreasing at a steady rate, so it is important to simplify exponential expressions in order to obtain the correct function model.

Instruction

Skill 1: Identifying the Base and Power of an Exponent and Evaluating Exponential Expressions

Introduction

Exponents are used to write the repeated product of the same number in a shorter form. Exponents have a wide range of applications, including calculating interest earned for savings accounts, determining the amount of depreciation of a car, and predicting future population levels.

Key Concepts

- An **exponential expression** is a mathematical expression written in the form of a^b, where a is called the **base** and b is called the **exponent**.
- The base can be any rational number, meaning whole number, fraction, or decimal.
- The exponent indicates the number of times the base is being multiplied by itself in an exponential expression.
- The **expanded form** refers to the product written out in long form, while **exponential form** uses the base and the exponent. For instance, 3 • 3 • 3 • 3 • 3 is the expanded form of 3^5 and indicates that 3 is multiplied by itself 5 times. The exponential form of this expression is 3^5.
- Exponential form is typically the form used more often in order to calculate the value of an expression.
- When a number is **squared**, it means that the base is multiplied by itself 2 times; therefore, the exponent is 2.
- When a number is **cubed**, it means that the base is multiplied by itself 3 times; therefore, the exponent is 3.
- Any number raised to the 0 power is always equal to 1.
- It is important to note that the base and exponent are *not* multiplied together.

Instruction

Guided Practice Skill 1

Example 1

A cube has a side length of 9 inches. Write the volume of the cube in expanded form, and rewrite that expression in exponential form. Identify the base and the exponent of the exponential expression. Then, evaluate the exponential form to find the volume of the cube.

1. Write the volume of the cube in expanded form.

   The volume of a cube is found by multiplying the length of the cube by the width of the cube by the height of the cube, *volume* = *length* • *width* • *height*. The length, width, and height of the cube each measure 9 inches, because all sides of a cube are the same measure; therefore, the volume of the cube is found by multiplying 9 by 9 by 9. This can be written in expanded form as follows.

volume = length • width • height	Volume formula for a cube
volume = (9) • (9) • (9)	Substitute 9 for the length, width, and height of the cube.
volume = 9 • 9 • 9	Expanded form

   The volume of the cube written in expanded form is 9 • 9 • 9.

2. Write the volume of the cube in exponential form.

   The volume of the cube is 9 • 9 • 9. The value 9 is multiplied by itself 3 times; therefore, the exponent will be 3.

volume = 9 • 9 • 9	Expanded form
volume = 9^3	Exponential form

   The volume of the cube written in exponential form is 9^3.

3. Identify the base and the exponent of the exponential expression.

   The exponential expression is 9^3. The base of this expression is 9 and the exponent is 3.

4. Evaluate the exponential expression for the volume of the cube.

   To find the numeric result of the volume, cube 9. That is, find the product of 9 times 9 times 9.

   $9^3 = 729$

   The volume of the cube is 729 in^3.

Example 2

Consider the expression "four-fifths to the third power." Rewrite this expression using decimals, first in exponential form and then in expanded form. Then, write the original expression using fractions, first in exponential form and then in expanded form. Identify the base and the exponent of both exponential expressions. Finally, evaluate both the decimal version and the fraction version of the expression.

1. Write the expression "four-fifths to the third power" as a numeric expression using decimals in exponential form.

   In the expression "four-fifths to the third power," "four-fifths" written as a decimal is 0.8. This represents the base of the exponential expression. "The third power" is written as an exponent of 3.

   "Four-fifths to the third power" written in exponential form is $(0.8)^3$.

2. Write the expression in expanded form.

   The exponent of 3 in the expression $(0.8)^3$ indicates that the base, 0.8, is multiplied by itself 3 times. The decimal expression written in expanded form is 0.8 • 0.8 • 0.8.

3. Write the expression “four-fifths to the third power” as a numeric expression using fractions in exponential form.

   In the expression, “four-fifths” written as a fraction is $\frac{4}{5}$. This represents the base of the exponential expression. “The third power” is written as an exponent of 3.

   “Four-fifths to the third power” written in exponential form is $\left(\frac{4}{5}\right)^3$.

4. Write the expression in expanded form.

   The exponent of 3 in the expression $\left(\frac{4}{5}\right)^3$ indicates that the base, $\frac{4}{5}$, is multiplied by itself 3 times. The fractional expression written in expanded form is $\frac{4}{5} \bullet \frac{4}{5} \bullet \frac{4}{5}$.

5. Identify the base and the exponent of both exponential expressions.

   The base of the exponential expression $(0.8)^3$ is 0.8 and the exponent is 3.

   The base of the exponential expression $\left(\frac{4}{5}\right)^3$ is $\frac{4}{5}$ and the exponent is 3.

6. Evaluate the decimal and fraction versions of the expression.

   As a decimal, the numeric result of the expression 0.8 • 0.8 • 0.8 is 0.512.

   As a fraction, the numeric result of the expression $\frac{4}{5} \bullet \frac{4}{5} \bullet \frac{4}{5}$ is $\frac{64}{125}$.

Scaffolded Practice Skill 1

Example 1

A cube has a side length of 9 inches. Write the volume of the cube in expanded form, and rewrite that expression in exponential form. Identify the base and the exponent of the exponential expression. Then, evaluate the exponential form to find the volume of the cube.

1. Write the volume of the cube in expanded form.

2. Write the volume of the cube in exponential form.

3. Identify the base and the exponent of the exponential expression.

4. Evaluate the exponential expression for the volume of the cube.

continued

Example 2

Consider the expression "four-fifths to the third power." Rewrite this expression using decimals, first in exponential form and then in expanded form. Then, write the original expression using fractions, first in exponential form and then in expanded form. Identify the base and the exponent of both exponential expressions. Finally, evaluate both the decimal version and the fraction version of the expression.

Problem-Based Task Skill 1: Mineral Weights

SMP	
1 ✓	2 ✓
3 ✓	4
5	6 ✓
7 ✓	8 ✓

A geologist has hired several science students to help him weigh various minerals in his lab. The students recorded the weight of each mineral in grams, but they did so inconsistently. In order to analyze the data collected, first write each expression for the weight of the minerals in exponential form. Identify the base and the exponent of each exponential expression. Then, evaluate the exponential forms to determine which mineral is the heaviest and which mineral is the lightest.

Mineral A: one to the fourth power

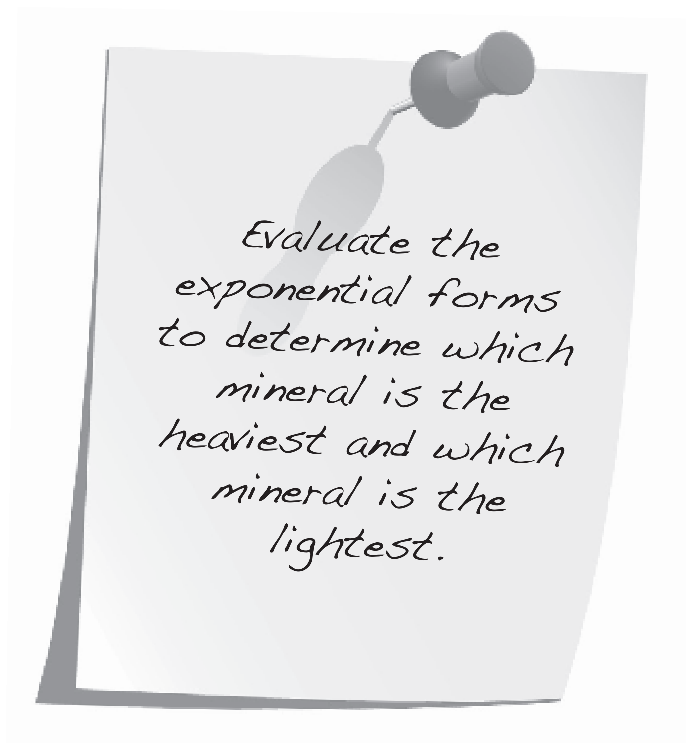
Mineral B: two-fifths squared

Mineral C: three-halves cubed

Mineral D: six-fifths to the fourth power

Mineral E: one-eighth to the fifth power

Mineral F: two squared



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Problem-Based Task Skill 1: Mineral Weights

Coaching

a. What is the weight of Mineral A in exponential form? Identify the base and the exponent.

b. What is the simplified weight of Mineral A?

c. What is the weight of Mineral B in exponential form? Identify the base and the exponent.

d. What is the simplified weight of Mineral B?

e. What is the weight of Mineral C in exponential form? Identify the base and the exponent.

f. What is the simplified weight of Mineral C?

g. What is the weight of Mineral D in exponential form? Identify the base and the exponent.

h. What is the simplified weight of Mineral D?

i. What is the weight of Mineral E in exponential form? Identify the base and the exponent.

j. What is the simplified weight of Mineral E?

k. What is the weight of Mineral F in exponential form? Identify the base and the exponent.

l. What is the simplified weight of Mineral F?

m. List the minerals in order from heaviest to lightest.

n. Which mineral is the heaviest?

o. Which mineral is the lightest?

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Problem-Based Task Skill 1: Mineral Weights

Coaching Sample Responses

a. What is the weight of Mineral A in exponential form? Identify the base and the exponent.

Mineral A has a weight of one to the fourth power.

The base of this expression is 1 and the exponent is 4.

In exponential form, one to the fourth power is written as 1^4.

The weight of Mineral A in exponential form is 1^4 g.

b. What is the simplified weight of Mineral A?

The weight of Mineral A is 1^4 g and can be simplified to 1 g.

c. What is the weight of Mineral B in exponential form? Identify the base and the exponent.

Mineral B has a weight of two-fifths squared.

The base of this expression is $\frac{2}{5}$ and the exponent is 2.

In exponential form, two-fifths squared is written as $\left(\frac{2}{5}\right)^2$.

The weight of Mineral B in exponential form is $\left(\frac{2}{5}\right)^2$ g.

d. What is the simplified weight of Mineral B?

The weight of Mineral B is $\left(\frac{2}{5}\right)^2$ g and can be simplified to 0.16 g.

e. What is the weight of Mineral C in exponential form? Identify the base and the exponent.

Mineral C has a weight of three-halves cubed.

The base of this expression is $\frac{3}{2}$ and the exponent is 3.

In exponential form, three-halves cubed is written as $\left(\frac{3}{2}\right)^3$.

The weight of Mineral C in exponential form is $\left(\frac{3}{2}\right)^3$ g.

f. What is the simplified weight of Mineral C?

The weight of Mineral C is $\left(\frac{3}{2}\right)^3$ g and can be simplified to $\frac{27}{8}$ g or approximately 3.38 g.

g. What is the weight of Mineral D in exponential form? Identify the base and the exponent.

Mineral D has a weight of six-fifths to the fourth power.

The base of this expression is $\frac{6}{5}$ and the exponent is 4.

In exponential form, six-fifths to the fourth power is written as $\left(\frac{6}{5}\right)^4$.

The weight of Mineral D in exponential form is $\left(\frac{6}{5}\right)^4$ g.

h. What is the simplified weight of Mineral D?

The weight of Mineral D is $\left(\frac{6}{5}\right)^4$ g and can be simplified to $\frac{1296}{625}$ g or approximately 2.07 g.

i. What is the weight of Mineral E in exponential form? Identify the base and the exponent.

Mineral E has a weight of one-eighth to the fifth power.

The base of this expression is $\frac{1}{8}$ and the exponent is 5.

In exponential form, one-eighth to the fifth power is written as $\left(\frac{1}{8}\right)^5$.

The weight of Mineral E in exponential form is $\left(\frac{1}{8}\right)^5$ g.

j. What is the simplified weight of Mineral E?

The weight of Mineral E is $\left(\frac{1}{8}\right)^5$ g and can be simplified to $\frac{1}{32{,}768}$ g or approximately 0.000031 g.

k. What is the weight of Mineral F in exponential form? Identify the base and the exponent.

Mineral F has a weight of two squared.

The base of this expression is 2 and the exponent is 2.

In exponential form, two squared is written as 2^2.

The weight of Mineral F in exponential form is 2^2 g.

l. What is the simplified weight of Mineral F?

The weight of Mineral F is 2^2 g and can be simplified to 4 g.

m. List the minerals in order from heaviest to lightest.

Use the simplified values to determine the order. The findings are summarized as follows:

- Mineral A is 1 g.
- Mineral B is 0.16 g.
- Mineral C is about 3.38 g.
- Mineral D is about 2.07 g.
- Mineral E is about 0.000031 g.
- Mineral F is 4 g.

From heaviest to lightest, the order is Mineral F, Mineral C, Mineral D, Mineral A, Mineral B, and Mineral E.

n. Which mineral is the heaviest?

The heaviest mineral is Mineral F, with a weight of 4 g.

o. Which mineral is the lightest?

The lightest mineral is Mineral E, with a weight of about 0.000031 g.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 1: Identifying the Base and Power of an Exponent and Evaluating Exponential Expressions

For problems 1 and 2, identify the base and exponent and then evaluate.

1. 7^3

2. $-2(0.52)^4$

For problems 3 and 4, write each expression in exponential form, identify the base and the exponent, and then evaluate.

3. $\frac{1}{3} \bullet \frac{1}{3} \bullet \frac{1}{3} \bullet \frac{1}{3}$

4. $2 \bullet 2 \bullet 2 \bullet 2 \bullet 5 \bullet 5 \bullet 5$

For problems 5–7, simplify each pair of expressions. Then, determine which of the two expressions has the larger value.

5. 0^3 and 30

6. $(0.75)^4$ and $(0.6)^5$

7. $\left(\frac{4}{5}\right)^3$ and $\left(\frac{5}{4}\right)^2$

For problems 8–10, write the expression in exponential form and simplify.

8. The lowest point in Spring River is about four to the third power feet deep. What is the depth of the lowest point?

9. The size of a bacterium is two-fiftieth micrometers squared. How big is the bacterium?

10. The number of flowers in a local botanical garden can be represented by seven raised to the eighth power. How many flowers are in the botanical garden?

Instruction

Skill 2: Simplifying Exponential Expressions with Integer Exponents*

Common Core State Standard

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 1, Lesson 1, Skill 1

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Guided Practice Skill 2

Example 1

Write the simplified expression of $\dfrac{\left(x^2\right)^4 x^{-5} y^0}{y^{-7}}$ using only positive exponents.

1. Use the exponent rule $a^0 = 1$ to simplify the expression.

   Any value to the 0 power is 1. Thus, the term y^0 is equal to 1.

   Substitute 1 for y^0.

$\dfrac{\left(x^2\right)^4 x^{-5} y^0}{y^{-7}}$	Original expression
$=\dfrac{\left(x^2\right)^4 x^{-5}(1)}{y^{-7}}$	Substitute 1 for y^0.
$=\dfrac{\left(x^2\right)^4 x^{-5}}{y^{-7}}$	Simplify the fraction.

   The expression $\dfrac{\left(x^2\right)^4 x^{-5} y^0}{y^{-7}}$ can be simplified to $\dfrac{\left(x^2\right)^4 x^{-5}}{y^{-7}}$.

2. Use the exponent rule $(a^m)^n = a^{m \bullet n}$ to further simplify the expression.

   When a power is raised to another power, multiply the exponents.

$\dfrac{\left(x^2\right)^4 x^{-5}}{y^{-7}}$	Expression from the previous step
$=\dfrac{x^{2\bullet 4}x^{-5}}{y^{-7}}$	Rewrite $\left(x^2\right)^4$ as a multiplication of the exponents.
$=\dfrac{x^8 x^{-5}}{y^{-7}}$	Multiply the exponents.

   The expression $\dfrac{\left(x^2\right)^4 x^{-5}}{y^{-7}}$ can be simplified to $\dfrac{x^8 x^{-5}}{y^{-7}}$.

3. Use the exponent rule $a^m \bullet a^n = a^{m+n}$ to further simplify the expression.

   Notice that x^8 and x^{-5} have the same base, x. When powers with the same base are multiplied, add the exponents.

$\dfrac{x^8 x^{-5}}{y^{-7}}$	Expression from the previous step
$=\dfrac{x^{8+(-5)}}{y^{-7}}$	Rewrite $x^8 \bullet x^{-5}$ as an addition of exponents.
$=\dfrac{x^3}{y^{-7}}$	Add the exponents.

   The expression $\dfrac{x^8 x^{-5}}{y^{-7}}$ can be simplified to $\dfrac{x^3}{y^{-7}}$.

4. Use the exponent rule $a^{-m}=\dfrac{1}{a^m}$ to write the negative exponent as a positive exponent.

   To rewrite a negative exponent as a positive one, take its reciprocal.

   Written with a positive exponent, the reciprocal of $\dfrac{1}{y^{-7}}$ is $\dfrac{y^7}{1}$.

$\dfrac{x^3}{y^{-7}}$	Expression from the previous step
$=x^3 \bullet \left(\dfrac{y^7}{1}\right)$	Substitute $\dfrac{y^7}{1}$ for $\dfrac{1}{y^{-7}}$.
$=x^3y^7$	Simplify the expression.

   The expression $\dfrac{x^3}{y^{-7}}$ can be simplified to x^3y^7; therefore, the expression $\dfrac{\left(x^2\right)^4 x^{-5} y^0}{y^{-7}}$ can be simplified to x^3y^7.

Scaffolded Practice Skill 2

Example 1

Write the simplified expression of $\frac{\left(x^2\right)^4 x^{-5} y^0}{y^{-7}}$ using only positive exponents.

1. Use the exponent rule $a^0 = 1$ to simplify the expression.

2. Use the exponent rule $(a^m)^n = a^{m \cdot n}$ to further simplify the expression.

3. Use the exponent rule $a^m \bullet a^n = a^{m+n}$ to further simplify the expression.

4. Use the exponent rule $a^{-m} = \frac{1}{a^m}$ to write the negative exponent as a positive exponent.

Practice Skill 2: Simplifying Exponential Expressions with Integer Exponents*

Simplify each expression and write it with only positive exponents.

1. $(a^3)^{-6}$

2. $\dfrac{x^3y^{-5}x^{-2}}{xy^{-1}}$

3. $x^4 \bullet \dfrac{y^3x^{-6}}{x^5y^{-8}}$

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Instruction

Skill 3: Finding the Vertex and *x*-intercepts of a Parabola**

Common Core State Standard

F–IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

This skill has been addressed previously in *CCSS Integrated Pathway: Mathematics II*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

Unit 2, Lesson 1, Sub-lesson 2

Instruction

Skill 4: Writing an Equation for a Simple Exponential Function

Common Core State Standard

A–CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 ✓ 6 ✓
7 ✓ 8 ✓

Essential Questions

1. How are quantities modeled with functions?
2. How is creating an exponential function different from creating a linear function?

WORDS TO KNOW

exponential decay an exponential equation with a base, b, that is between 0 and 1 ($0 < b < 1$); can be represented by the formula $y = a(1 - r)^t$, where a is the initial value, r is the decay rate, t is time, and y is the final value

exponential function a function that has a variable in the exponent; the general form is $y = ab^t$, where a is the initial value, b is the base, t is the time, and y is the final output value

exponential growth an exponential equation with a base, b, greater than 1 ($b > 1$); can be represented by the formula $y = a(1 + r)^t$, where a is the initial value, r is the growth rate, t is time, and y is the final value

Recommended Resource

- Purplemath.com. "Exponential Functions: Introduction."

  http://www.walch.com/rr/04077

  This website gives an introduction of exponential equations and provides a few examples of tables of input and output values, with integers as inputs. The introduction goes into more depth about the shapes of the graphs of exponential functions and continues on to develop the concept of compound interest. It also introduces the number *e*.

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Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

- Provide students with a blank two-circle Venn diagram. Ask students to label the left circle "Quadratic function" and the right circle "Exponential function." Have students list the features of each type of function in the corresponding circles. In the overlapping center part of the diagram, ask students to list features that both functions share. Possible similarities include: *the graph is a curve, both have an exponent in the function*, and *there is not a constant rate of change*. Possible differences include: *exponential functions have asymptotes* and *the graphs of quadratic functions are U-shaped.*
- Provide each student with two blank flash cards. Ask each student to write a scenario that would require an exponential growth model on one flash card, and then write a scenario that would require an exponential decay model on the other flash card. Have students exchange their flash cards with a partner, who should then write which type of model represents each scenario on the opposite side of each flash card. Next, ask students to work with their partners to create equations for their functions. Have volunteers share their answers.

Suggestions for Discourse

- Ask students, "What are synonyms for the words *growth* and *decay*?" Ask students to create a list of as many words as they can that mean the same as *growth* and then do the same for *decay*. Ask students to volunteer their answers, and use their responses to create a master list. Examples for *growth* include *increase, grow, double,* and *gain*. Examples for *decay* include *decrease, depreciate, decline,* and *lose*.
- Ask students, "Why do you think it is important to be able to use exponential growth and decay models to be able to predict future values? What are some real-life scenarios in which it might be necessary to be able to predict the future value of something?" Encourage and guide a discussion about how different professions study patterns of behavior over time and use growth and decay models to make predictions based on these patterns. Examples include financial and banking reports, scientific and medical research, sales and marketing trends, and forestry and conservation efforts.

Making Connections

- Discuss with students the importance of being able to write a function based on a given scenario, as well as being able to determine which type of function represents the scenario. Guide them to realize that being able to write the correct function will allow for creating the proper graph, from which key features can be determined.
- Remind students that another important result of being able to correctly write a function for a given scenario is being able to correctly evaluate a function for a given value.

Instruction

Skill 4: Writing an Equation for a Simple Exponential Function

Introduction

Exponential functions are functions that have a variable in the exponent. Exponential functions are found in science, finance, sports, and many other areas of daily life. Writing equations for these functions is useful in terms of being able to apply them to a real-world scenario.

Key Concepts

- The general form of an exponential function is $y = ab^t$, where a is the initial value, b is the base, and t is the time. The final output value will be y.
- Since the equation has an exponent, the output value increases or decreases rapidly.
- The base, b, must always be greater than 0 ($b > 0$).
- If the base is greater than 1 ($b > 1$), then the exponential equation represents **exponential growth**.
- If the base is between 0 and 1 ($0 < b < 1$), then the exponential equation represents **exponential decay**.
- If the time is given in units other than 1 (e.g., 1 month, 1 hour, 1 minute, 1 second), use the equation $y = ab^{\frac{x}{t}}$, where t is the time it takes for the base to repeat.
- Another form of the exponential equation is $y = a(1 \pm r)^t$, where a is the initial value, r is the rate of growth or decay, and t is the time.
- Use $y = a(1 + r)^t$ for exponential growth (notice the plus sign). For example, if a population grows by 2% then r is 0.02, but this is less than 1 and by itself does not indicate growth.
- Substituting 0.02 for b into the formula $y = a \bullet b^x$ requires the expression $(1 + r)$ to arrive at the full growth rate of 102%, or 1.02.
- Use $y = a(1 - r)^t$ for exponential decay (notice the minus sign). For example, if a population decreases by 3%, then 97% is the factor being multiplied over and over again. The population from year to year is always 97% of the population from the year before (a 3% decrease). Think of this as 100% minus the rate, or in decimal form $(1 - r)$.
- Look for words such as *double*, *triple*, *half*, and *quarter*—such words give the number of the base. For example, if an experiment begins with 1 bacterium that doubles (splits itself in two) every hour, determining how many bacteria will be present after x hours is solved with the following equation: $y = (1)2^x$, where 1 is the starting value, 2 is the rate, x is the number of hours, and y is the final value.

- Look for the words *initial* or *starting* to substitute in for *a*.
- Look for the words *ended with* and *after*—these words will be near the final value given.
- Follow the same procedure as with setting up linear equations and inequalities in one variable:

Creating Exponential Equations from Context

1. Read the problem statement first.
2. Reread the scenario and make a list or a table of the known quantities.
3. Read the statement again, identifying the unknown quantity or variable.
4. Create expressions and inequalities from the known quantities and variable(s).
5. Solve the problem.
6. Interpret the solution of the exponential equation in terms of the context of the problem.

Guided Practice Skill 4

Example 1

Stanley planted a tree in his backyard. The manager of the plant nursery he purchased it from told Stanley that the tree would double in height every 4 years. If Stanley's tree started out only 1.5 feet tall, how tall will it be in 20 years? Write an exponential equation of the general form $y = ab^t$, where a is the initial value, b is the base, t is the time, and y is the final value. Then use it to solve the problem.

1. Identify the known quantities.

   The initial height of the tree = 1.5 feet.

   The growth rate = doubles, so that means 2.

   The amount of time = every 4 years for 20 years.

2. Identify the unknown quantity or variable.

   The unknown quantity is the height of the tree after 20 years. Solve for the final height after 20 years.

3. Create expressions and equations from the known quantities and variable(s).

   The general form of an exponential equation is $y = ab^t$, where a is the initial value, b is the base, t is the time, and y is the final value.

   $a = 1.5$

   $b = 2$

   x = every 4 years for 20 years

   There are 5 time periods in 20 years since each time period is 4 years.

   Another way to determine the time period is to set up a ratio.

   $$20\ \cancel{\text{years}} \bullet \frac{1\text{ time period}}{4\ \cancel{\text{years}}} = 5\text{ time periods}$$

   Therefore, $x = 5$.

4. Substitute the values into the general exponential formula, $y = ab^t$.

   $y = ab^t$

   $y = (1.5) \bullet (2)^5$

5. Follow the order of operations to solve the problem.

$y = 1.5 \bullet 2^5$	Equation from the previous step
$y = 1.5 \bullet 32$	Raise 2 to the fifth power.
$y = 48$	Multiply 1.5 and 32.

6. Interpret the solution in terms of the context of the problem.

   In 20 years, the tree will be 48 feet tall if its height doubles every 4 years. ✓

Example 2

The population of ants in a newly formed anthill increases at a rate of 25% per day. If there are currently 500 ants, about how many ants will there be in the anthill after 12 days at this growth rate? Use the general growth function $y = a(1 + r)^t$, where y is the final value, a is the initial value, r is the rate of growth, and t is the amount of time.

1. Identify the known quantities.

   The initial number of ants = 500.

   The growth rate = 25%.

   The amount of time = 12 days.

2. Identify the unknown quantity or variable.

   The unknown quantity is the number of ants after 12 days. Solve for the final value after 12 days.

3. Create expressions and equations from the known quantities and variable(s).

   The general form of the exponential growth equation with a percent increase is $y = a(1 + r)^t$, where y is the final value, a is the initial value, r is the rate of growth, and t is the amount of time.

   $a = 500$

   $r = 25\% = 0.25$

   $t = 12$ days

4. Substitute the values into the general exponential growth equation, $y = a(1 + r)^t$.

$y = a(1 + r)^t$	General growth equation
$y = (500)[1 + (0.25)]^{(12)}$	Substitute 500 for a, 0.25 for r, and 12 for t.
$y = 500(1 + 0.25)^{12}$	Simplify.

5. Follow the order of operations to solve the problem.

$y = 500(1 + 0.25)^{12}$	Equation from the previous step
$y = 500(1.25)^{12}$	Add inside the parentheses.
$y = 500(14.5519)$	Raise the base to the power of 12.
$y \approx 7276$	Multiply.

6. Interpret the solution in terms of the context of the problem.

   If this growth rate continues for 12 days, the population will increase from 500 ants to about 7,276 ants, which is an increase of about 6,776 ants.

Scaffolded Practice Skill 4

Example 1

Stanley planted a tree in his backyard. The manager of the plant nursery he purchased it from told Stanley that the tree would double in height every 4 years. If Stanley's tree started out only 1.5 feet tall, how tall will it be in 20 years? Write an exponential equation of the general form $y = ab^t$, where a is the initial value, b is the base, t is the time, and y is the final value. Then use it to solve the problem.

1. Identify the known quantities.

2. Identify the unknown quantity or variable.

3. Create expressions and equations from the known quantities and variable(s).

4. Substitute the values into the general exponential formula, $y = ab^t$.

5. Follow the order of operations to solve the problem.

6. Interpret the solution in terms of the context of the problem.

continued

Example 2

The population of ants in a newly formed anthill increases at a rate of 25% per day. If there are currently 500 ants, about how many ants will there be in the anthill after 12 days at this growth rate? Use the general growth function $y = a(1 + r)^t$, where y is the final value, a is the initial value, r is the rate of growth, and t is the amount of time.

Problem-Based Task Skill 4: Population Problem

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 ✓ 6 ✓
7 ✓ 8 ✓

On opposite sides of a river, two cities are experiencing population changes. One city, Riverview, is growing rapidly at 2.7% per year and has a current population of 140,000. The other city, Cliffside, has a declining population at rate of 4% per year. Its current population is 190,000. Economists predict that in 5 years the populations of these two cities will be about the same, but the residents of both cities are in disbelief. The economists also claim that 10 years after that, the population of Riverview will be double the size of the population of Cliffside. Verify the predictions based on the data given. Do you think these predictions will come true?

Do you think these predictions will come true?

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Problem-Based Task Skill 4: Population Problem

Coaching

a. Is Riverview experiencing population growth or decay? What is the equation for Riverview's population change after 5 years?

b. What is the solution to the equation in part a, rounded up to a whole person?

c. Is Cliffside experiencing population growth or decay? What is the equation for Cliffside's population change after 5 years?

d. What is the solution to the equation in part c, rounded up to a whole person?

e. Are your solutions to parts b and d similar? What can you conclude about the economists' prediction that the populations of the two cities will be about the same in 5 years?

f. What will the variable t equal if the cities experience the same rates of growth or decline for 10 more years after that?

g. What are the equations and solutions for each city's population change after 10 more years?

h. Based on your calculations, was the economists' prediction for the cities' populations after 10 more years correct?

i. What factors might influence whether this second prediction comes true?

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Problem-Based Task Skill 4: Population Problem

Coaching Sample Responses

a. Is Riverview experiencing population growth or decay? What is the equation for Riverview's population change after 5 years?

Riverview is experiencing growth because the words "growing rapidly" were used in the original problem. The equation $y = a(1 + r)^t$ can be used to model this situation, where y is the final value, a is the initial value, r is the rate of growth, and t is the amount of time.

Let a = 140,000 people, r = 2.7% = 0.027, and t = 5 years.

Substitute these values into the equation.

$y = a(1 + r)^t$

$y = (140{,}000)[1 + (0.027)]^{(5)}$

The equation is $y = 140{,}000(1 + 0.027)^5$.

b. What is the solution to the equation in part a, rounded up to a whole person?

To find the solution to the equation, follow the order of operations and solve for y.

$y = 140{,}000(1 + 0.027)^5$

$y = 140{,}000(1.027)^5$

$y \approx 140{,}000(1.142)$

$y \approx 159{,}949$

The solution is approximately 159,949 people. Therefore, at a 2.7% growth rate, the population of Riverview after 5 years will be approximately 159,949.

c. Is Cliffside experiencing population growth or decay? What is the equation for Cliffside's population change after 5 years?

Cliffside is experiencing decay because the words "declining population" were used in the original problem. The equation $y = a(1 - r)^t$ can be used to model this situation, where y is the final value, a is the initial value, r is the rate of decay, and t is the amount of time.

Let a = 190,000 people, r = 4% = 0.04, and t = 5 years.

Substitute these values into the equation.

$y = a(1 - r)^t$

$y = (190{,}000)[1 - (0.04)]^{(5)}$

The equation is $y = 190{,}000(1 - 0.04)^5$.

d. What is the solution to the equation in part c, rounded up to a whole person?

To find the solution to the equation, follow the order of operations and solve for y.

$y = 190{,}000(1 - 0.04)^5$

$y = 190{,}000(0.96)^5$

$y \approx 190{,}000(0.815)$

$y \approx 154{,}921$

The solution is approximately 154,921 people. Therefore, at a 4% decay rate, the population of Cliffside after 5 years will be approximately 154,921.

e. Are your solutions to parts b and d similar? What can you conclude about the economists' prediction that the populations of the two cities will be about the same in 5 years?

The numbers are similar, especially when considering the magnitude. A difference of about 5,000 people is much smaller when compared to the original difference of 50,000 people. Based on these calculations, it would seem that the economists are correct that the cities' populations will be about equal in 5 years.

f. What will the variable t equal if the cities experience the same rates of growth or decline for 10 more years after that?

The amount of time passed since the initial population count will be 5 years plus 10, which equals 15 years.

$t = 5 + 10$

$t = 15$

g. What are the equations and solutions for each city's population change after 10 more years?

The following equations use the city's current populations, with $t = 15$.

Riverview	Cliffside
$y = a(1 + r)^t$	$y = a(1 - r)^t$
$y = 140{,}000(1 + 0.027)^{15}$	$y = 190{,}000(1 - 0.04)^{15}$
$y \approx 208{,}778$	$y \approx 102{,}996$

Thus, if the rates continue unchanged for 15 years, Riverview's population will be about 208,778 people and Cliffside's population will be about 102,996 people.

h. Based on your calculations, was the economists' prediction for the cities' populations after 10 more years correct?

Yes, because 208,778 is about twice as much as 102,996.

i. What factors might influence whether this second prediction comes true?

Many factors might contribute to the economists' prediction coming true or not, such as the economy itself. If the economy starts to decline, the population of Riverview might not continue to grow as fast. Or, a new industry might relocate to Cliffside, bringing jobs, and perhaps people will start moving back into that city. Furthermore, the predictions assume that the rate of change for each city will remain the same for 15 years, which cannot be guaranteed.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 4: Writing an Equation for a Simple Exponential Function

For problems 1–3, determine whether each scenario is modeled by a linear equation or an exponential equation.

1. The price of gasoline increases by $0.10 every week.
2. Every year, there are a third as many newspapers printed as the year before.
3. A pine tree produces 25 pinecones each week during the spring.

For problems 4–10, write an equation to model each scenario. Then, use the equation to solve the problem.

4. A rose bush doubles in size each month. If it is 0.5 feet tall when planted, how tall will it be in 6 months?
5. A magazine loses 5% of its subscribers each month. If the magazine currently has 850 subscribers, how many will it have after 1 year?
6. A stock loses half its value every week. If the stock was initially worth $480, what will it be worth after 6 weeks at this rate of decline?
7. A new car depreciates (loses its value) at a rate of 10% each year. If a car is worth $18,000 when it is 5 years old, how much did it cost when it was brand new?
8. The number of weeds in a garden triples every 2 days. If the garden started out with only 5 weeds, how many weeds will it have after 2 weeks?
9. The population of a city is decreasing by 3.8% each year. The current population is 98,000. How many people will live there in 6 years?
10. The number of new voters in a small town increases by 12% each year. If there are currently 116 people registered to vote, how many people will be registered to vote in 9 years?

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Supportive Instructional Strategies for Mathematics II

Unit 2 Lesson 5

Suggestions for Graphic Organizers/Manipulatives

- Provide students with 14 blank flash cards, one for each of the Words to Know from the lesson. Ask students to write on one side of each flash card a single vocabulary word from the lesson. Then, have students define each vocabulary word in their own words on the opposite side of each flash card. Ask students to compare their definitions with a partner, and then volunteer their answers. Use their responses to create a master list of vocabulary terms. Terms should include: *decay factor, decay rate, exponential decay, exponential decay model, exponential function, exponential growth, exponential growth model, first difference, growth factor, growth rate, interval, linear function, quadratic function,* and *second difference*.
- Provide students with a blank three-column chart or table. Ask students to title their charts "Types of Functions" and label the three column headings as "Linear," "Quadratic," and "Exponential." Ask students to fill in the respective columns with key features about each type of function, including the format of the general equation, the type of graph, and the rate of change. Ask for volunteers to share their information, and then create a master chart.
- Provide students with the Coordinate Plane graphic organizer from the Program Overview, along with colored pencils or markers. Using a different color for each function, ask students to write the following functions at the top of the page: $f(x) = 2x + 3$, $g(x) = x^2 + 2x + 3$, and $h(x) = 3(2)^x$. Ask students to work with a partner to create a table of values for each function and then to graph each function, in its respective color, on the same graph. Then, guide a discussion about the differences and similarities among the three graphs.

Suggestions for Discourse

- Pair students with a partner. Ask each pair to list as many real-life examples as they can of scenarios involving linear functions, and then scenarios that involve quadratic or exponential functions. Encourage a discussion about how the rates of changes of each scenario help determine which type of function is necessary.
- Ask students, "How do you calculate a percent rate of change on an interval of a function? How is a percent rate of change for an exponential function different than a percent rate of change for a linear function?"

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics II*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.
- Provide a list of sentence frames in which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "I know that this is a linear function because the rate of change is _______." Or, "This function is an exponential _______ function because of the word _______ in the problem." Or, "The word '<u>depreciates</u>' in this problem indicates that this function is an exponential <u>decay</u> function."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- interpreting the rate r as a constant linear rate

  Remind students that a linear function has the general form $f(x) = mx + b$, where m is the slope, or the rate of change, and this rate of change will be the same on any interval of the graph.

- incorrectly applying the order of operations to evaluate the expression ab^t

  Ask students to write the acronym PEMDAS; remind students that any operation in parentheses is performed before evaluating an exponent and then, finally, multiplication is performed.

- identifying $r > 1$ as exponential decay and $r < 1$ as exponential growth

  Remind students that an exponential decay expression will have a decay factor of less than 1, since the decay factor is $(1 - r)$, and that an exponential growth expression will have a growth factor greater than 1, since the growth factor is $(1 + r)$.

- misidentifying the appropriate type of function (linear, quadratic, or exponential)

  Review the main features of each type of function, reminding students that a linear function will not have an exponent higher than 1, a quadratic function will have a term with an exponent of 2, and an exponential function will have a variable as an exponent.

- thinking that an equation with a higher *y*-intercept has a higher maximum value

  Remind students that the maximum value of a quadratic function is determined by the vertex, not the *y*-intercept.

- incorrectly determining the rate of change

  Remind students that the rate of change for a linear function is the slope of the line, the rate of change for an exponential function is determined by the variable *r* in the exponential growth or decay model, and the rate of change for a quadratic function is determined over a specific interval on the function.

- assuming that the rate of change of a function is linear by only referencing one interval

  Remind students that the only type of function that has a constant rate of change is a linear function, and that with quadratic and exponential functions, the rate of change can vary from one interval to the next.

Lesson 6: Transforming Functions

Instruction

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics II*.

Skill 1: Graphing Quadratic Functions (A–REI.10)

Common Core State Standard

A–REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Skill 2: Evaluating Quadratic Functions* (F–IF.2)

Common Core State Standard

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Skill 3: Finding Intercepts and Vertices of Quadratic Functions** (F–IF.7a★)

Common Core State Standard

F–IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Instruction

Skill 1: Graphing Quadratic Functions

Common Core State Standard

A–REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

SMP
1 ✓ 2 ✓
3 4 ✓
5 ✓ 6 ✓
7 ✓ 8

Essential Questions

1. How do you create a table of values for a quadratic function?
2. How do you graph a quadratic function?

WORDS TO KNOW

ordered pair	a pair of values (x, y) where the order is significant
parabola	the U-shaped graph of a quadratic equation
quadratic function	a function that can be written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. The graph of any quadratic function is a parabola.
table of values	a listing of the values of the variables of a function
***x*-coordinate**	the first number in an ordered pair
***y*-coordinate**	the second number in an ordered pair

Recommended Resources

- Khan Academy. "Graphing a Parabola with a Table of Values."

  http://www.walch.com/rr/04078

  This video tutorial shows how to create a table of values for a quadratic function by substituting different values of x into the function and solving for y. It then shows how to plot the points on a coordinate plane and draw a parabola through the points. In addition, given the equation of a quadratic function, the video tells how to determine whether the graph of the function will open upward or downward.

- Purplemath.com. "Graphing Quadratic Functions."

  http://www.walch.com/rr/04079

  This site points out the differences between graphing a linear function and graphing a quadratic function. It gives an example of how to graph the most basic quadratic function, $y = x^2$, by creating a table of values, plotting points, and drawing a parabola through the points.

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Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Once students have worked through the Guided Practice, distribute copies of the Coordinate Plane graphic organizer found in the Program Overview. At the bottom of the graphic organizer, have students create a table of values for the quadratic function represented by the equation $y = x^2 - 2x - 8$ using the integers from $x = -2$ to $x = 2$. Students should produce the ordered pairs (–2, 0), (–1, –5), (0, –8), (1, –9), and (2, –8). Have students plot the points on the coordinate plane and draw a parabola through the points.

Suggestions for Discourse

Point out to students that in the examples and exercises for graphing quadratic functions contained in this skill, they are told the values of x for which to create tables of values. Thus, in all of the examples and exercises, it is clear what the line of symmetry is for each of the parabolas they have to draw. Remind them, however, that when asked to choose their own values of x, sometimes they might guess incorrectly, which results in not being able to identify the axis of symmetry clearly. Ask them what they think they should do in this case. Tell them that one option is to choose more values of x for their table of values. Also, inform them that in future lessons, they will review a way to identify the axis of symmetry of a quadratic function just by looking at its equation.

Making Connections

Inform students that when graphing a quadratic function, it may not always be necessary to first create a table of values if the equation of the function is in a certain form. For example, if you already know what the graph of the most basic quadratic function, $y = x^2$, looks like, and you are asked to graph the quadratic function $y = x^2 - 3$, rather than first creating a table of values for $y = x^2 - 3$, you could just shift the graph of $y = x^2$ down 3 units. The end result will be the same as if you had created a table of values for $y = x^2 - 3$, plotted the points, and drawn a parabola through them.

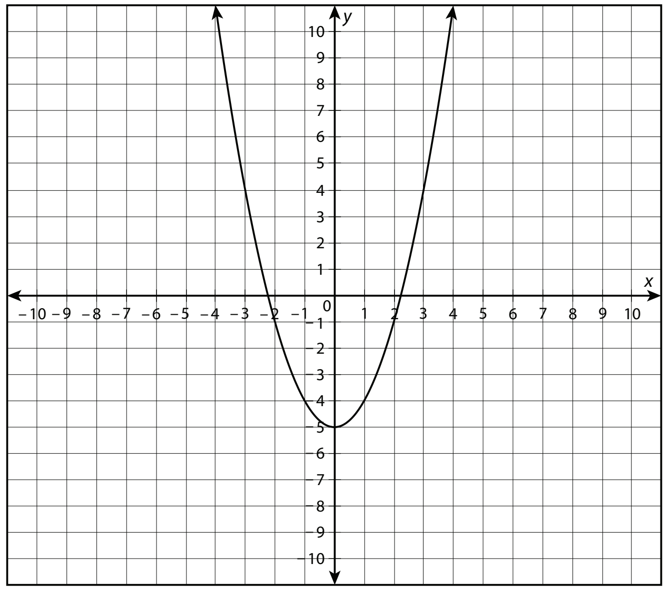
Skill 1: Graphing Quadratic Functions

Introduction

A **quadratic function** is a function that can be written in the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. The graph of any quadratic function is a **parabola**, or U-shaped curve. The parabola may be facing up or it may be facing down, depending on the curve. Graphing a quadratic function produces a visual representation of the relationship between two variables. For example, the relationship between a car's speed in miles per hour and its fuel economy in miles per gallon is often quadratic. The graph of the quadratic function that relates speed to fuel economy shows that speeding up improves fuel economy up until a certain point. After that, driving faster decreases fuel economy. Quadratic functions have many other similar applications in the real world.

Key Concepts

- To graph a quadratic function, first create a **table of values**, which is a listing of the values of the variables of a function.
- To create a table of values, choose values of x and then solve for y.
- Each of the solutions to a quadratic equation can be represented as an ordered pair. An **ordered pair** is two numbers written inside parentheses and separated by a comma, such as (1, 4).
- An ordered pair consists of an ***x*-coordinate**, which is the first number in the ordered pair, and a ***y*-coordinate**, which is the second number. These coordinates are written as (x, y).
- After creating a table of values, plot the points corresponding to the ordered pairs contained in the table on a coordinate plane.
- Once the points have been plotted, a parabola, or the U-shaped curve that is the graph of a quadratic function, can be drawn through them. The following is an example of a parabola.



- Quadratic functions can also be graphed with a graphing calculator.

On a TI-83/84:

Step 1: Press the [Y=] button.

Step 2: Type the equation into Y1, using the [X, T, θ, n] button for the variable *x* and [^][2] for the exponent 2. Press [ENTER].

On a TI-Nspire:

Step 1: Press the [home] key.

Step 2: Arrow over to the graphing icon and press [enter].

Step 3: Type the equation next to *f1*(*x*), using the [X] button for the letter *x* or the [x^2] button for a square. Press [enter].

Step 4: To change the viewing window, press [menu]. Select 4: Window/Zoom and select A: Zoom – Fit.

Guided Practice Skill 1

Example 1

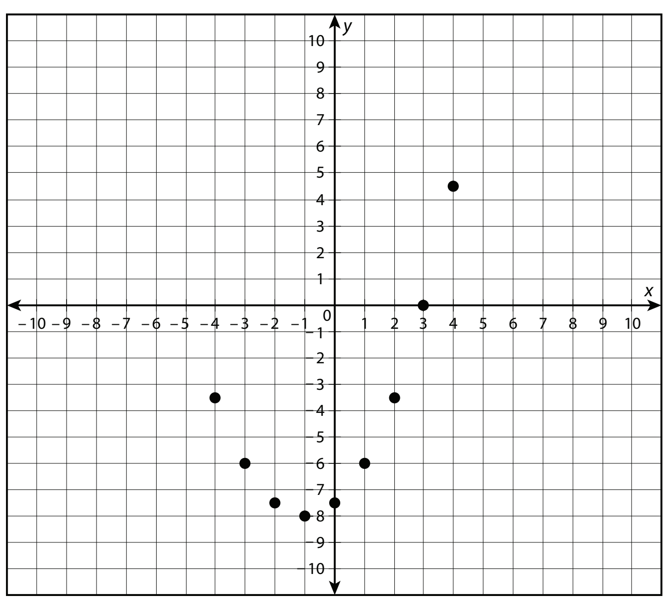
Some of the solutions to a quadratic equation are shown in the following table of values. Use the table to create a graph of the equation.

x	y
–4	–3.5
–3	–6
–2	–7.5
–1	–8
0	–7.5
1	–6
2	–3.5
3	0
4	4.5

1. Plot the solutions to the quadratic equation on a coordinate plane.

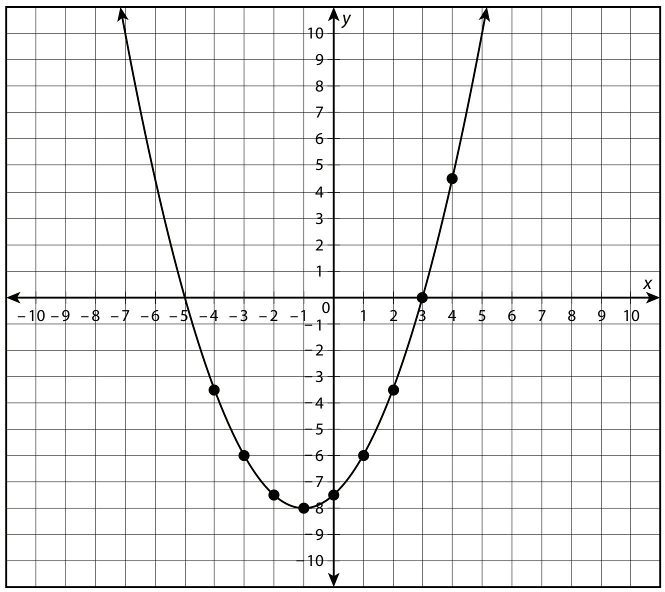
   Each of the solutions to the quadratic equation can be represented as an ordered pair, where the first number is the x-coordinate of the corresponding point, and the second number is the y-coordinate. For example, the first solution given in the table can be represented as (–4, –3.5), where –4 is the x-coordinate and –3.5 is the y-coordinate.

   The following graph shows the solutions from the table.



2. Draw a smooth curve through the points.

   The smooth curve drawn through the points should be extended upward on both sides to form a parabola.



   Each point on the parabola, not just the plotted points, represents a solution to the quadratic equation.

Example 2

Create a table of values for the quadratic function $y = -x^2 + 2x + 7$ using all of the integers from $x = -2$ to $x = 2$, and then graph the function.

1. Determine the corresponding *y*-values for each of the given *x*-values.

   The given *x*-values are the integers from –2 to 2. Recall that integers are whole numbers. So, the *x*-values for our table of values are –2, –1, 0, 1, and 2.

   Substitute each of these values for *x* into the function and solve for *y*.

   Let $x = -2$.

$y = -x^2 + 2x + 7$	Given function
$y = -(-2)^2 + 2(-2) + 7$	Substitute –2 for *x*.
$y = -(4) + 2(-2) + 7$	Square –2.
$y = -4 + (-4) + 7$	Multiply.
$y = -1$	Add.

   When $x = -2$, $y = -1$.

   Let $x = -1$.

$y = -x^2 + 2x + 7$	Given function
$y = -(-1)^2 + 2(-1) + 7$	Substitute –1 for *x*.
$y = -(1) + 2(-1) + 7$	Square –1.
$y = -1 + (-2) + 7$	Multiply.
$y = 4$	Add.

   When $x = -1$, $y = 4$.

   Let $x = 0$.

$y = -x^2 + 2x + 7$	Given function
$y = -(0)^2 + 2(0) + 7$	Substitute 0 for *x*.
$y = -(0) + 2(0) + 7$	Square 0.
$y = 0 + 0 + 7$	Multiply.
$y = 7$	Add.

   When $x = 0$, $y = 7$.

(*continued*)

Let $x = 1$.

$y = -x^2 + 2x + 7$	Given function
$y = -(1)^2 + 2(1) + 7$	Substitute 1 for x.
$y = -(1) + 2(1) + 7$	Square 1.
$y = -1 + 2 + 7$	Multiply.
$y = 8$	Add.

When $x = 1$, $y = 8$.

Let $x = 2$.

$y = -x^2 + 2x + 7$	Given function
$y = -(2)^2 + 2(2) + 7$	Substitute 2 for x.
$y = -(4) + 2(2) + 7$	Square 2.
$y = -4 + 4 + 7$	Multiply.
$y = 7$	Add.

When $x = 2$, $y = 7$.

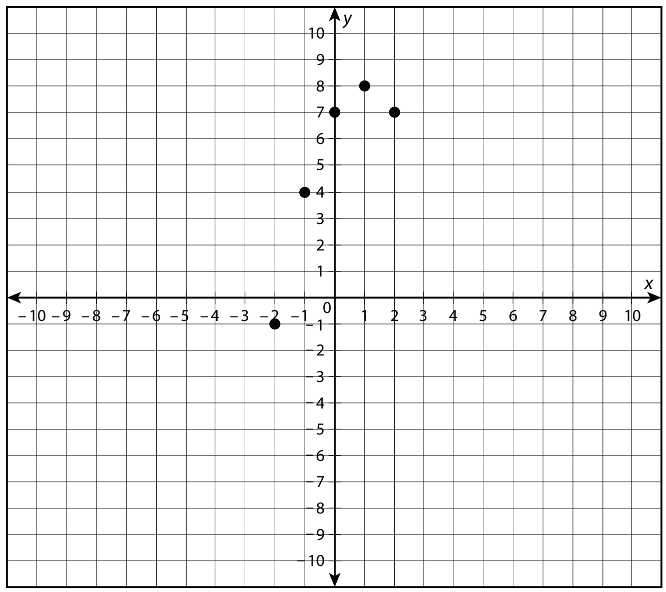
2. Create a table of values for the quadratic function.

   The table of values is created from the set of ordered pairs that were found by substituting each x-value into the function to produce each corresponding y-value. The ordered pairs are (–2, –1), (–1, 4), (0, 7), (1, 8), and (2, 7).

x	y
–2	–1
–1	4
0	7
1	8
2	7

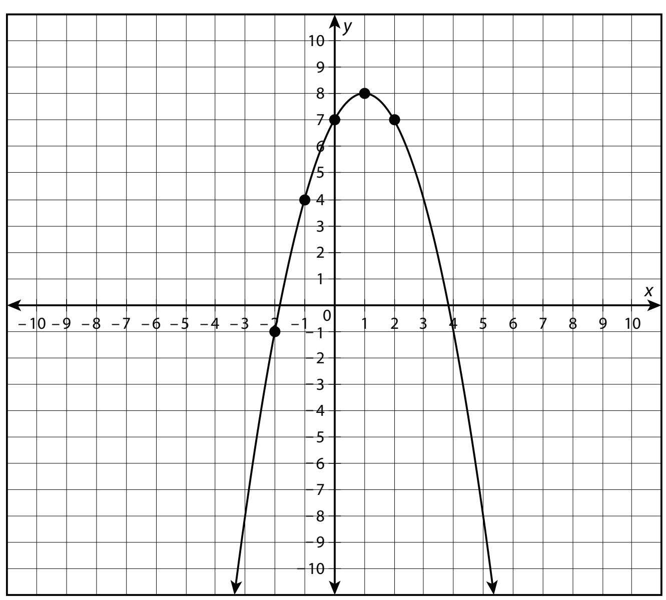
3. Plot the ordered pairs on a coordinate plane.

The following graph shows the points from the table.



4. Draw a smooth curve through the points.

The smooth curve drawn through the points should be extended downward on both sides.



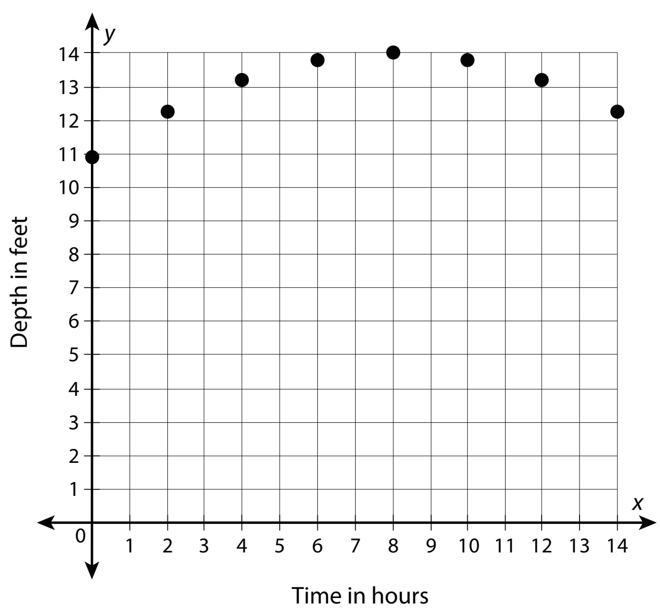
Example 3

The depth of a river since the beginning of a long, heavy rainstorm can be modeled by a quadratic function. The following table of values shows the river's depth at various times, where t is the time in hours since the rainstorm began and d is the depth of the river in feet. Graph the function from $t = 0$ to $t = 14$, in increments of 2 hours.

Time in hours (t)	Depth in feet (d)
0	10.8
2	12.2
4	13.2
6	13.8
8	14
10	13.8
12	13.2
14	12.2

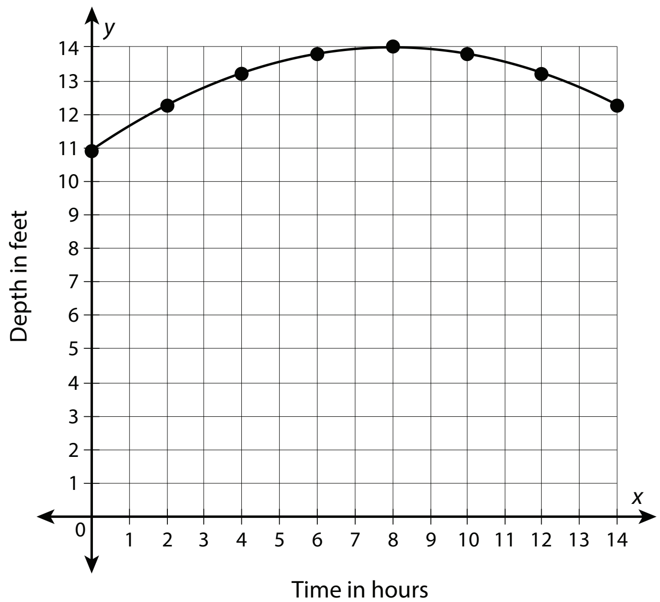
1. Plot the data on a coordinate plane.

   Each of the data points can be represented as an ordered pair, where the first number is the x-coordinate, and the second number is the y-coordinate. For example, the first data point given in the table can be represented as (0, 10.8), where 0 is the x-coordinate and 10.8 is the y-coordinate.



2. Draw a smooth curve through the points.

   The smooth curve drawn through the points should not extend past the plotted points.



   The river's depth increased for the first 8 hours of the storm. However, sometime after hour 8, the river started going down.

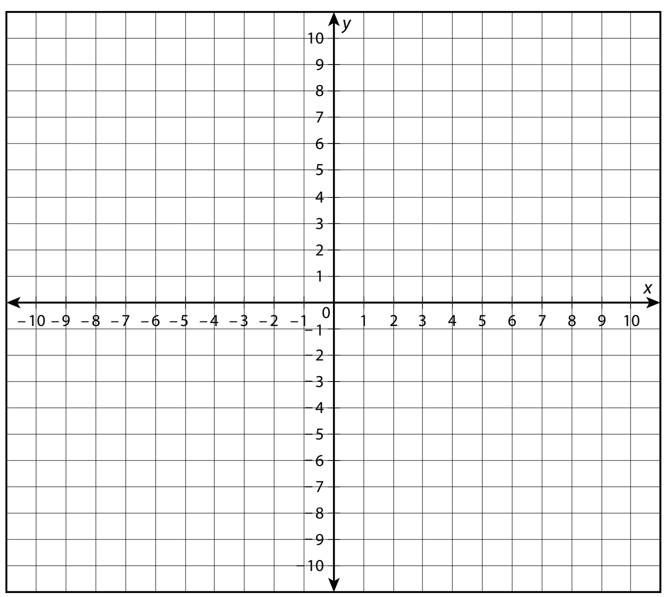
Scaffolded Practice Skill 1

Example 1

Some of the solutions to a quadratic equation are shown in the following table of values. Use the table to create a graph of the equation.

x	*y*
–4	–3.5
–3	–6
–2	–7.5
–1	–8
0	–7.5
1	–6
2	–3.5
3	0
4	4.5

1. Plot the solutions to the quadratic equation on a coordinate plane.



2. Draw a smooth curve through the points.

continued

Example 2

Create a table of values for the quadratic function $y = -x^2 + 2x + 7$ using all of the integers from $x = -2$ to $x = 2$, and then graph the function.

Example 3

The depth of a river since the beginning of a long, heavy rainstorm can be modeled by a quadratic function. The following table of values shows the river's depth at various times, where t is the time in hours since the rainstorm began and d is the depth of the river in feet. Graph the function from $t = 0$ to $t = 14$, in increments of 2 hours.

Time in hours (*t*)	Depth in feet (*d*)
0	10.8
2	12.2
4	13.2
6	13.8
8	14
10	13.8
12	13.2
14	12.2

Problem-Based Task Skill 1: The Parabola of Doom

Piper is a mechanical engineer at a company that designs roller coasters. Piper's team is designing the first segment of a new roller coaster, The Parabola of Doom. Piper is using computer simulation to predict how far above ground level a rider will be at different times during the ride. The Parabola of Doom's height, h, in feet above the ground in the first segment can be modeled by the quadratic function $h = -20t^2 + 80t + 70$, where t is the time in seconds after the ride begins. Use this function to complete the table of values, then graph the function on a coordinate plane from $t = 0$ to $t = 4$.

SMP
1 ✓ 2 ✓
3 4 ✓
5 ✓ 6 ✓
7 ✓ 8

Time in seconds (t)	**Height in feet (h)**
0	
1	
2	
3	
4	

Use this function to complete the table of values, then graph the function on a coordinate plane from t = 0 to t = 4.

Problem-Based Task Skill 1: The Parabola of Doom

Coaching

a. What is the roller coaster's height above ground level when the ride begins?

b. What is the roller coaster's height above ground level after 1 second?

c. What is the roller coaster's height above ground level after 2 seconds?

d. What is the roller coaster's height above ground level after 3 seconds?

e. What is the roller coaster's height above ground level after 4 seconds?

f. Use your answers for parts a–e to complete the table of values.

g. Use the table of values to graph the quadratic function for the first segment of the roller coaster.

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Problem-Based Task Skill 1: The Parabola of Doom

Coaching Sample Responses

a. What is the roller coaster's height above ground level when the ride begins?

When the ride begins, the value for time, *t*, is 0 seconds.

Substitute 0 into the equation for *t* and solve for *h*.

$h = -20t^2 + 80t + 70$

$h = -20(0)^2 + 80(0) + 70$

$h = -20(0) + 80(0) + 70$

$h = 0 + 0 + 70$

$h = 70$

The roller coaster's height above ground level when the ride begins is 70 feet.

b. What is the roller coaster's height above ground level after 1 second?

The value for time, *t*, after 1 second is 1.

Substitute 1 into the equation for *t* and solve for *h*.

$h = -20t^2 + 80t + 70$

$h = -20(1)^2 + 80(1) + 70$

$h = -20(1) + 80(1) + 70$

$h = -20 + 80 + 70$

$h = 130$

The roller coaster's height above ground level 1 second after the ride begins is 130 feet.

c. What is the roller coaster's height above ground level after 2 seconds?

The value for time, *t*, after 2 seconds is 2.

Substitute 2 into the equation for *t* and solve for *h*.

$h = -20t^2 + 80t + 70$

$h = -20(2)^2 + 80(2) + 70$

$h = -20(4) + 80(2) + 70$

$h = -80 + 160 + 70$

$h = 150$

The roller coaster's height above ground level 2 seconds after the ride begins is 150 feet.

d. What is the roller coaster's height above ground level after 3 seconds?

The value for time, *t*, after 3 seconds is 3.

Substitute 3 into the equation for *t* and solve for *h*.

$h = -20t^2 + 80t + 70$

$h = -20(3)^2 + 80(3) + 70$

$h = -20(9) + 80(3) + 70$

$h = -180 + 240 + 70$

$h = 130$

The roller coaster's height above ground level 3 seconds after the ride begins is 130 feet.

e. What is the roller coaster's height above ground level after 4 seconds?

The value for time, *t*, after 4 seconds is 4.

Substitute 4 into the equation for *t* and solve for *h*.

$h = -20t^2 + 80t + 70$

$h = -20(4)^2 + 80(4) + 70$

$h = -20(16) + 80(4) + 70$

$h = -320 + 320 + 70$

$h = 70$

The roller coaster's height above ground level 4 seconds after the ride begins is 70 feet.

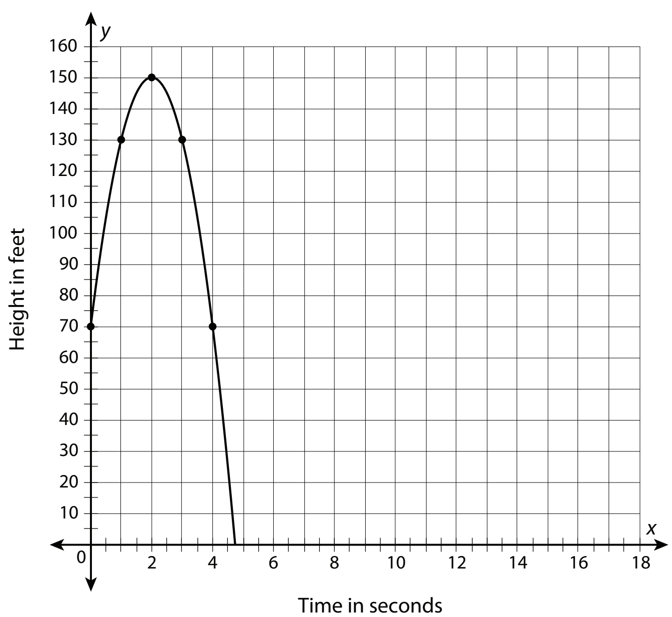
f. Use your answers for parts a–e to complete the table of values.

The previous steps show that the table of values contains the ordered pairs (0, 70), (1, 130), (2, 150), (3, 130), and (4, 70).

Time in seconds (*t*)	Height in feet (*h*)
0	70
1	130
2	150
3	130
4	70

g. Use the table of values to graph the quadratic function for the first segment of the roller coaster.

Plot the data on a coordinate plane, and draw part of a parabola through the points.



Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 1: Graphing Quadratic Functions

For problems 1–4, complete a table of values for the given quadratic function using the x-values –2, –1, 0, 1, and 2.

1. $y = 2x^2 - 8$

2. $y = -x^2 + 2x + 6$

3. $y = x^2 - 2x - 5$

4. $y = -0.5x^2 - x + 4$

For problems 5–8, use the tables of values created for problems 1–4 to graph each quadratic function on a coordinate plane.

5. $y = 2x^2 - 8$

6. $y = -x^2 + 2x + 6$

7. $y = x^2 - 2x - 5$

8. $y = -0.5x^2 - x + 4$

For problems 9 and 10, graph the given quadratic function on a coordinate plane.

9. $y = 1.5x^2 + 3x - 5$

10. A certain type of garden plant is grown in rows, with the same amount of space between each row and each plant. The growth of this plant can be modeled by the quadratic function $h = -s^2 + 4s + 4$, where h is the height of the plant in inches, and s is the space in inches between each of the plants. Graph the function on a coordinate plane from $s = 0$ to $s = 4$.

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Instruction

Skill 2: Evaluating Quadratic Functions*

Common Core State Standard

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 2, Skill 4

SWB p. 161

Instruction

Guided Practice Skill 2

Example 1

Evaluate $f(x) = -x^2 - 4x + 7$ over the domain {–2, –1, 0, 1, 2}. Determine the range for this domain.

1. Evaluate the function for each of the domain values.

   To evaluate the function $f(x) = -x^2 - 4x + 7$ over the domain {–2, –1, 0, 1, 2}, substitute the values from the domain into the function and solve.

   Evaluate $f(-2)$.

$f(x) = -x^2 - 4x + 7$	Given function
$f(-2) = -(-2)^2 - 4(-2) + 7$	Substitute –2 for x.
$f(-2) = 11$	Simplify.

   When –2 is substituted for x, the value of $f(-2)$ is 11.

   Evaluate $f(-1)$.

$f(x) = -x^2 - 4x + 7$	Given function
$f(-1) = -(-1)^2 - 4(-1) + 7$	Substitute –1 for x.
$f(-1) = 10$	Simplify.

   When –1 is substituted for x, the value of $f(-1)$ is 10.

   Evaluate $f(0)$.

$f(x) = -x^2 - 4x + 7$	Given function
$f(0) = -(0)^2 - 4(0) + 7$	Substitute 0 for x.
$f(0) = 7$	Simplify.

   When 0 is substituted for x, the value of $f(0)$ is 7.

   Evaluate $f(1)$.

$f(x) = -x^2 - 4x + 7$	Given function
$f(1) = -(1)^2 - 4(1) + 7$	Substitute 1 for x.
$f(1) = 2$	Simplify.

   When 1 is substituted for x, the value of $f(1)$ is 2.

(*continued*)

Evaluate $f(2)$.

$f(x) = -x^2 - 4x + 7$	Given function
$f(2) = -(2)^2 - 4(2) + 7$	Substitute 2 for x.
$f(2) = -5$	Simplify.

When 2 is substituted for x, the value of $f(2)$ is –5.

2. Determine the range of the function for the given domain.

   Collect the set of outputs for the inputs. Recall that the outputs are the $f(x)$ values, and the inputs are the values we substituted for x.

   The outputs from the previous step were 11, 10, 7, 2, and –5.

   Therefore, the range is {–5, 2, 7, 10, 11}.

Scaffolded Practice Skill 2

Example 1

Evaluate $f(x) = -x^2 - 4x + 7$ over the domain {–2, –1, 0, 1, 2}. Determine the range for this domain.

1. Evaluate the function for each of the domain values.

2. Determine the range of the function for the given domain.

Practice Skill 2: Evaluating Quadratic Functions*

For problems 1–3, evaluate the given functions and determine the range of each.

1. Evaluate $f(x) = 6x^2 - 3x + 2$ over the domain {1, 2, 3, 4}. What is the range?

2. Evaluate $g(x) = -7x^2 + 4x - 1$ over the domain {1, 3, 5, 7}. What is the range?

3. Evaluate $h(x) = -5x^2 - 4x + 1$ over the domain {–2, –1, 0, 1, 2}. What is the range?

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Skill 3: Finding Intercepts and Vertices of Quadratic Functions**

Common Core State Standard

F–IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

This skill has been addressed previously in *CCSS Integrated Pathway: Mathematics II*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

Unit 2, Lesson 1, Sub-lesson 2

Supportive Instructional Strategies for Mathematics II

Unit 2 Lesson 6

Suggestions for Graphic Organizers/Manipulatives

- Once students have worked through the Guided Practice, distribute the Coordinate Plane graphic organizer found in the Program Overview. Ask students to graph the quadratic function $f(x) = x^2 + 6x + 3$ and identify the vertex of the function, which is (–3, –6). Next, on the same coordinate plane, have students graph $f(x - 5)$ by using a dotted parabola, and have them identify the vertex of the transformed function, which is (2, –6). Ask them to determine the relationship between the positions on the coordinate plane of the parabola representing the original function and the parabola representing the transformed function by looking at the vertices of the two functions. They should conclude that the transformed function is 5 units to the right of the original function.
- Provide students with a blank four-column table or chart. Ask students to label their chart with the title "Transforming Functions." Then, have them label the columns with the following headings, from left to right: "Horizontal stretch," "Horizontal compression," "Vertical stretch," and "Vertical compression." Ask students to work with a partner to fill in the following information for each column, then ask volunteers to share their completed columns. Column information:
    - a short written explanation of the type of transformation
    - a simple drawing of the transformation
    - the general format of the transformation equation
    - a sketch of a graph showing the comparison of the transformation to $f(x) = x^2$
- Write a parent function on the board. Then provide each student with three blank flash cards. Ask students to sketch an example of a child function on one side of each flash card. Encourage students to include a different type of transformation on each card. Then, ask students to trade cards with a partner, and have each student try to identify the type of transformation for each function, writing the name of the transformation on the other side of each flash card. Ask students to discuss their answers in pairs, and then volunteer their results to share with the class. Encourage and guide a discussion about which parts of each function indicate a transformation.

Suggestions for Discourse

- Point out to students that when performing vertical and horizontal stretches and compressions on the graphs of quadratic functions, sometimes a vertical stretch has the same effect as a horizontal compression, and sometimes a vertical compression has the same effect as a horizontal stretch. For example, a vertical stretch by a factor of 9 has the same effect on the parabola representing the quadratic function $f(x) = x^2$ as a horizontal compression by a factor of $\frac{1}{3}$, because $9 \bullet f(x) = f(3 \bullet x) = 9x^2$. Have students form groups, and ask each group to think of three other examples of a vertical stretch and a horizontal compression, or a vertical compression and a horizontal stretch, that have the same effect on the parabola representing the quadratic function $f(x) = x^2$. Then have each group report its examples to the class.
- Ask students, "What is the difference between a translation and a transformation?" Ask them to write down a few sentences explaining the differences. Then, encourage and guide a discussion about how a translation moves the graph up or down, but the size and shape of the graph remain the same. In contrast, a transformation changes the graph's position and/or shape.

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics II*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.
- Elaborate on culture-specific contexts.
- Provide a list of sentence frames in which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "I know that this graph is shifted _______ by _______ units because of the constant _______ in the equation of the function." Or, "A translation moves a function horizontally or _______, but the function's size and _______ remain the same." Or, "A <u>transformation</u> changes a function's <u>position</u> and/or <u>shape</u> by <u>adding</u> or <u>multiplying</u> a <u>constant</u>."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- incorrectly moving the graph in the direction opposite that indicated by k, especially in horizontal shifts; for example, moving the graph left when it should be moved right

  Remind students that another way to write the general format for a translation, $f(x + k)$, is $f(x - (-k))$. Ask students to write an example on their papers along with a description, such as "$f(x + 3) = f(x - (-3))$, so the graph moves to the left."

- incorrectly moving the graph left and right versus up and down (and vice versa) when operating with $f(x + k)$ and $f(x) + k$

  Provide students with a chart (such as the one used in the Suggestions for Graphic Organizers/Manipulatives) that lists horizontal and vertical transformations, specifically including summaries such as "$f(x + k)$ = left or right" and "$f(x) + k$ = up or down."

- thinking that multiplying the dependent variable by a constant k yields the same equation as multiplying the independent variable by the same constant k in all cases (i.e., that $k \bullet f(x)$ is always equal to $f(k \bullet x)$, but this is not always true)

  Have students use a specific value of k to test whether $k \bullet f(x)$ is equal to $f(k \bullet x)$ for a specific quadratic function.

- forgetting to substitute all values of x with $k \bullet x$ when working with the transformation $f(k \bullet x)$

  Have students create a table of values with three columns, labeled as x, $f(x)$, and $f(k \bullet x)$.

- forgetting to square the constant k when substituting $f(k \bullet x)$ into $ax^2 + bx + c$

  Remind students that when a value of k is substituted into a function, that value is also inside of the parentheses and therefore must be squared; ask students to write a simple example on their papers as a reminder, such as "$f(3x) = (3x)^2 = 9x^2$, NOT $3x^2$."

- confusing horizontal with vertical transformations and vice versa

  Refer students to the graphs in the text showing vertical and horizontal translations.

- confusing stretches and compressions

  Show students a graph of the function $f(x) = x^2$. Ask students to sketch a general graph showing the result of stretching this function's graph, and then to sketch the result of shrinking the function's graph. Remind students of words that relate to stretches and compressions; for example, *stretching* means to "widen" or "make bigger," and *compressing* means to "narrow" or "make smaller."

Lesson 7: Finding Inverse Functions

Elementary Prerequisite Skills

This lesson requires the use of the following elementary skill(s) to meet the standards addressed in *CCSS Integrated Pathway: Mathematics II*. See the Appendix for material to address the skill(s).

- E-Skill 1: Evaluating Expressions Using the Order of Operations (5.OA.1), Appendix p. A-2

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics II*.

Skill 1: Identifying Independent and Dependent Variables (6.EE.9)

Common Core State Standard

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

Skill 2: Determining the Domain and Range of Linear and Quadratic Functions* (F–IF.1)

Common Core State Standard

F–IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

(*continued*)

Skill 3: Applying Inverse Operations to Isolate a Variable, Including Taking Square Roots** (A–REI.4b)

Common Core State Standard

A–REI.4 Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Skill 4: Using Function Notation* (F–IF.2)

Common Core State Standard

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Instruction

Skill 1: Identifying Independent and Dependent Variables

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 ✓ 6 ✓
7 ✓ 8 ✓

Common Core State Standard

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

Essential Questions

1. How can variables be used to represent two quantities in a real-world problem that change in relationship to one another?
2. How can equations, tables, and graphs be used to represent the relationship between independent and dependent variables?

WORDS TO KNOW

dependent variable — labeled on the *y*-axis; the quantity that is based on the input values of the independent variable; the output variable of a function

equation — a mathematical sentence that uses an equal sign (=) to show that two quantities are equal

independent variable — labeled on the *x*-axis; the quantity that changes based on values chosen; the input variable of a function

variable — a letter used to represent a value or unknown quantity that can change or vary

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Recommended Resources

- IXL Learning. "Write Linear Functions."

  http://www.walch.com/rr/04052

  This site provides practice with writing linear functions given a problem scenario, and includes a table showing the relationship between independent and dependent variables. Immediate feedback is provided, with step-by-step explanations for finding the correct linear function when an incorrect answer is given.

- Khan Academy. "Dependent and Independent Variables."

  http://www.walch.com/rr/04053

  This site provides practice with understanding the difference between independent and dependent variables in real-world situations, along with how to represent each problem scenario using an equation, a table, and/or a graph.

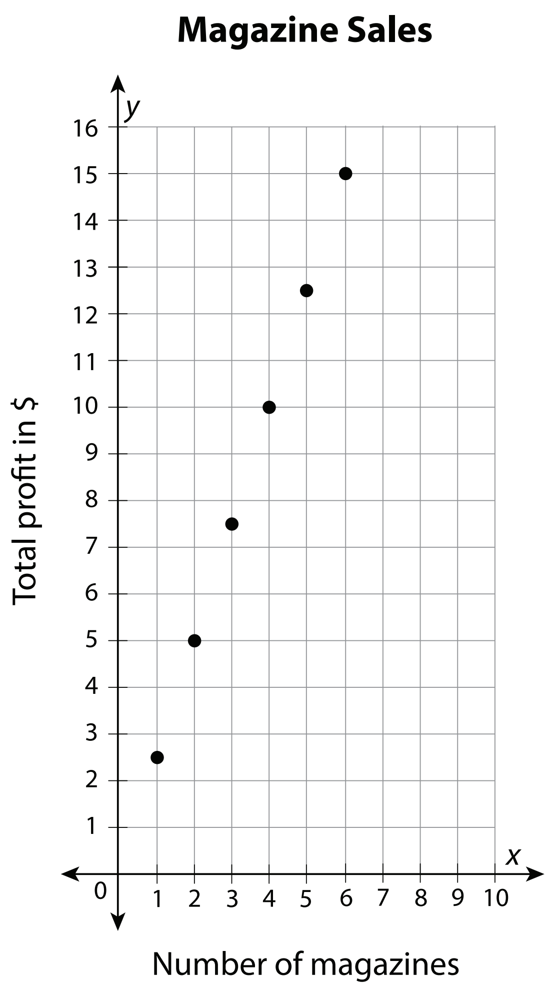
Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Once students have completed the Guided Practice, distribute the Line Graph graphic organizer. Write the following scenario on the board for the students:

> "A store sells magazines for $2.50 each. Explain the relationship between the number of magazines sold and the total profit with an equation, a table, and a graph."

First, discuss with the class how to tell which quantity is independent and which quantity is dependent. Then ask the students to choose a variable to represent each quantity. Let the students work on their own to represent the scenario with an equation, a table, and a graph. Remind the students to label the *x*-axis, the *y*-axis, and the title of the graph. The graph should look similar to the following:



Ask volunteers to share and explain their equation, which should be similar to $y = 2.5x$, their table of values, and their graph. Additional time and practice spent on identifying quantities and creating equations, tables, and graphs will enhance students' understanding of the relationships between independent and dependent quantities.

Suggestions for Discourse

Ask students to think about situations that have two quantities that change in relation to one another. Give students the example that if a person earns $8 per hour at a job, then the more hours that person works, the more money that person will earn. When comparing the number of hours worked to the total amount of money earned, discuss which quantity in the scenario is independent, which quantity is dependent, and why. Explain that the number of hours worked is the independent quantity because the number of hours worked can be any value greater than or equal to 0, but the amount of money earned is determined by multiplying $8 by the number of hours the person worked, so the amount of money earned is the dependent quantity.

Making Connections

Encourage students to connect the concept that an independent quantity in a scenario can be any reasonable value chosen for the scenario. For example, the number of movie tickets purchased must be a whole number since it is not possible to buy a negative number of tickets or a fraction of a ticket. The dependent quantity is then the result of a calculation involving the independent variable. For example, the total cost to see a movie is calculated by taking the cost per ticket and multiplying it by the number of movie tickets purchased. Therefore, the dependent quantity depends on a calculation performed with the independent quantity.

Instruction

Skill 1: Identifying Independent and Dependent Variables

Introduction

There are many situations in which two quantities being compared change in relationship to one another. For example, the distance you travel during a road trip varies over time. Or, the money a worker earns varies based on the number of hours worked. In these situations, the dependent quantity can be expressed in terms of an independent quantity. Understanding this relationship between the independent and dependent quantities, along with how to represent these situations with equations, tables, and graphs, is an important step in learning how to find solutions to real-world problems.

Key Concepts

- When two quantities in a problem scenario change in relationship to one another, one quantity, the **dependent variable**, is dependent upon the other quantity, the **independent variable**.
- In these situations, variables are used to represent the quantities being compared. **Variables** are letters used to represent values or unknown quantities that can change or vary in expressions or equations.
- The relationship between independent and dependent variables can be represented as equations, tables, and graphs.

Equations

- An **equation** is a mathematical sentence that uses an equal sign (=) to show that two quantities are equal.
- The independent variable in an equation can change, whereas the dependent variable is affected by changes made to the independent variable.
- For example, in the equation $y = 5x$, when $x = 1$, then $y = 5(1)$, which simplifies to $y = 5$. When $x = 2$, then $y = 5(2)$, which simplifies to $y = 10$. For this equation, as the value of x increases, the value of y also increases. However, the value of y depends on changes in the value of x, so y is the dependent variable and x is the independent variable.
- Other letters besides x and y can be used to represent the independent and dependent quantities in an equation to make it easier to associate each variable with a quantity.
- For example, if books cost \$4 each and we want to find the total cost for a certain number of books, let c represent the total cost and let b represent the number of books. Since each book costs \$4, the total cost will be equal to 4 times the number of books purchased, which can be written as the equation $c = 4b$.

Tables

- Tables comparing independent and dependent variables typically have two columns, with the independent quantity in the left column and the dependent quantity in the right column. The title of each column should describe the variable. For example, in the equation $c = 4b$, the independent quantity is the number of books (b), so a good title for the left column is "Number of books (b)." The dependent quantity is the total cost in dollars (c), so a good title for the right column is "Total cost in \$ (c)."

Number of books (b)	Total cost in \$ (c)

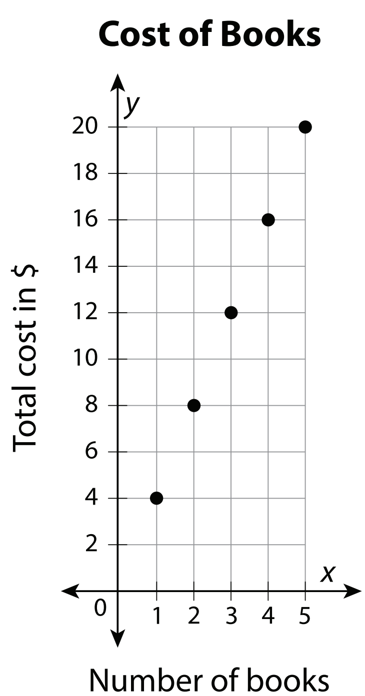
- To fill in each row of the table, start by choosing a reasonable value for the independent variable. For example, a reasonable number of books to start with is $b = 1$. Substitute 1 for b in the equation $c = 4b$ to calculate the corresponding value for c. In this case, $c = 4(1)$, which simplifies to $c = 4$. The first row of the table will then have 1 for b and 4 for c. For the second row, when $b = 2$, substitute 2 for b into the equation: $c = 4(2) = 8$. The second row of the table will then have 2 for b and 8 for c. Fill in the rest of the table following the same steps, using different values of b to calculate the corresponding value for c.

Number of books (b)	Total cost in \$ (c)
1	4
2	8
3	12
4	16
5	20

- This table of values can be used to create a graph for the equation.

Graphs

- In a graph, the independent quantity is represented along the *x*-axis and the dependent quantity is represented along the *y*-axis. The axes are labeled to describe each quantity being represented.
- For a problem scenario based on a real-world context, the graph is generally drawn in the first quadrant since the scenario more than likely deals with positive independent and dependent quantities. For example, it is impossible to buy a negative number of books, or spend a negative amount of money on the books.
- Each row in the table created for $c = 4b$ provides a coordinate point that can be graphed on the coordinate plane to represent the relationship. For example, the values in the first row, 1 and 4, can be written as the coordinate point (1, 4).
- When graphing a problem scenario for which it is possible to use fractions for the independent quantity, draw a line through the points. For example, it is possible to spend 2.5 hours on a task, so a graph with time as the independent variable would have a line through the points.
- If the only possible values for the independent quantity are whole numbers, such as with people or entire objects, the graph would only show the points, without a line through them. For example, the graph of the equation for the cost of books, $c = 4b$, does not have a line through the points because it is impossible to buy a fraction of a book. Notice that the points on this graph match the table values.



Guided Practice Skill 1

Example 1

On the way to the Grand Canyon, Eric drove his car at an average speed of 70 miles per hour. Explore the relationship between the total distance Eric traveled and the time he spent driving. Identify the independent and dependent quantities, and choose variables to represent them. Then, create an equation, a table, and a graph for this situation.

1. Identify the independent and dependent quantities.

   Eric drove his car at 70 miles per hour, which means that the total distance in miles that Eric traveled depends on the time in hours that he spent driving.

   Therefore, the time is the independent quantity, and the distance is the dependent quantity.

2. Choose variables to represent the independent and dependent quantities.

   Because "time" starts with the letter "t," let the time Eric spent driving be represented by the variable t.

   Because "distance" starts with the letter "d," let the distance Eric traveled be represented by the variable d.

3. Write an equation to represent the relationship between the independent and dependent variables.

   The distance in miles (d) that Eric traveled is calculated by multiplying his rate (70 miles per hour) by the time in hours (t) that he spent driving.

   Therefore, the relationship can be represented by the equation $d = 70t$.

4. Create a table to represent the relationship between the independent and dependent variables.

Use the left column of the table for the values of the independent variable, the time in hours (t).

Use the right column for the values of the dependent variable, the total distance in miles (d).

To fill in the table, choose values for t to substitute into the equation $d = 70t$.

In the first row of the table, let $t = 0$, so $d = 70(0)$, which simplifies to $d = 0$. In the second row, let $t = 1$, so $d = 70(1)$, which simplifies to $d = 70$, and so on, as shown.

Time in hours (t)	Total distance in miles (d)
0	0
1	70
2	140
3	210
4	280
5	350

Based on the table, as the time increases, the total distance also increases.

5. Use the table to create a graph that represents the relationship between the independent and dependent variables.

   The independent quantity, the time in hours (t), is plotted along the x-axis of the coordinate plane.

   The dependent quantity, the total distance in miles (d), is plotted along the y-axis.

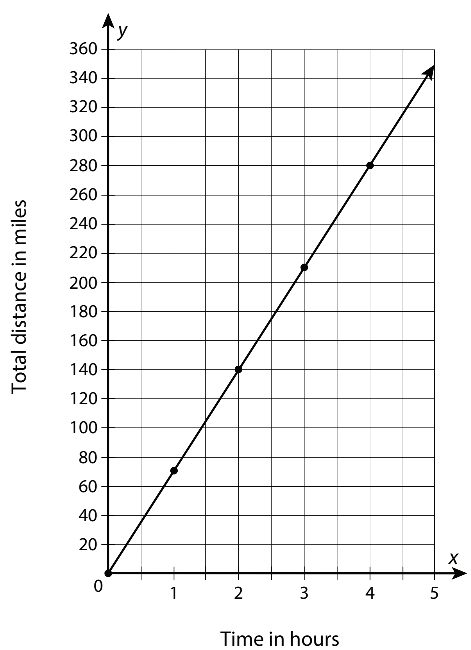
   It is not possible to have a negative amount of time, so the x-axis starts at 0. Likewise, it is not possible to travel a negative total distance, so the y-axis also starts at 0.

   Label the x-axis "Time in hours" and label the y-axis "Total distance in miles." The title of the graph can be "Distance Traveled over Time."

   Each row in the table of values provides a coordinate point that can be graphed on the coordinate plane to represent the relationship.

   Because the speed traveled is at a constant rate, and because it is possible to have a fractional number of hours (such as 2.5 hours) instead of only a whole number of hours, a line can be drawn through the coordinate points to show how the total distance traveled was constantly changing.

Distance Traveled over Time



(*continued*)

This graph shows that as the time increases, the total distance traveled also increases. This confirms the relationship shown in the table of values.

Example 2

During the summer, Sarah earns $8 per hour babysitting for her aunt. The number of hours she babysits is rounded up to a whole hour. Explore the relationship between the total amount of money Sarah earns and the number of hours she babysits. Identify the independent and dependent quantities, and choose variables to represent them. Then, create an equation, a table, and a graph for this situation.

1. Identify the independent and dependent quantities.

   Sarah makes $8 per hour, which means that the total amount of money she earns depends on the number of hours she babysits.

   Therefore, the number of hours is the independent quantity and the total amount of money she earns is the dependent quantity.

2. Choose variables to represent the independent and dependent quantities.

   Because "hours" starts with the letter "h," let the number of hours Sarah babysits be represented by the variable h.

   Because "money" starts with the letter "m," let the total amount of money Sarah earns be represented by the variable m.

3. Write an equation to represent the relationship between the independent and dependent variables.

   The total amount of money (m) is calculated by multiplying the earnings per hour ($8) by the number of hours worked (h).

   Therefore, the relationship can be represented by the equation $m = 8h$.

4. Create a table to represent the relationship between the independent and dependent variables.

   List values for the independent variable, which is the number of hours worked (h), in the left column of the table.

   List values for the dependent variable, which is the total money earned (m), in the right column.

   To fill in the table, choose values for m to substitute into the equation $m = 8h$.

   In the first row of the table, let $h = 1$, so $m = 8(1)$, which simplifies to $m = 8$. In the second row of the table, let $h = 2$, so $m = 8(2)$, which simplifies to $m = 16$. Continue filling in rows of the table in this manner, as shown.

Number of hours (h)	**Total money earned in \$ (m)**
1	8
2	16
3	24
4	32
5	40

   Based on the table, as the number of hours increases, the total amount of money earned also increases.

5. Use the table to create a graph that represents the relationship between the independent and dependent variables.

   The independent quantity, the number of hours (*h*), is plotted along the *x*-axis of the coordinate plane.

   The dependent quantity, the total money earned in dollars (*m*), is plotted along the *y*-axis.

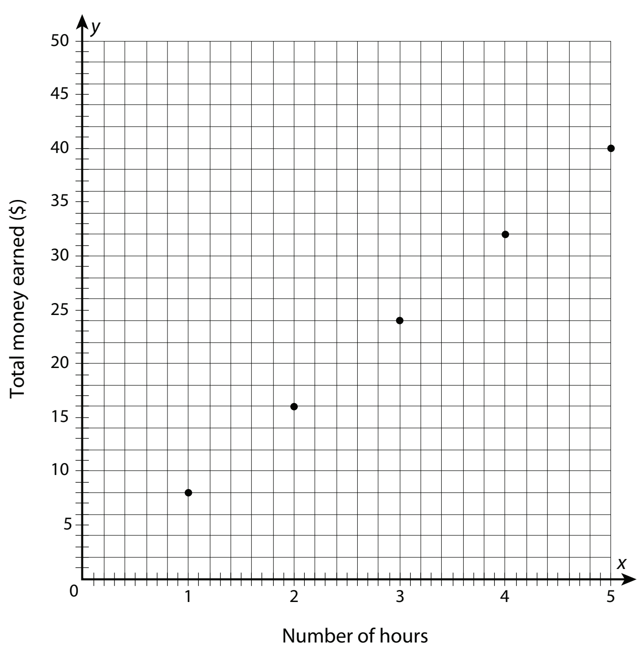
   It is not possible to work a negative number of hours, so the *x*-axis starts at 0. Likewise, it is not possible to earn a negative amount of money, so the *y*-axis also starts at 0.

   Label the *x*-axis "Number of hours" and label the *y*-axis "Total money earned ($)." The title of the graph can be "Money Earned Babysitting."

   Each row in the table of values provides a coordinate point that can be graphed on the coordinate plane to represent the relationship.

   Because Sarah's pay is calculated using only whole hours instead of fractions of hours, do not draw a line through the coordinate points plotted on the graph.

Money Earned Babysitting



   This graph shows that as the number of hours increases, the total amount of money earned also increases. This confirms the relationship shown in the table of values.

Example 3

A car dealership is going out of business and needs to sell all the cars left on the lot. There were 100 cars on the lot when the going-out-of-business sale began, and the dealership has been selling them at a constant rate of 4 cars per week since then. Explore the relationship between the total number of cars left on the lot and the number of weeks since the sale began. Identify the independent and dependent quantities, and choose variables to represent them. Then, create an equation, a table, and a graph for this situation.

1. Identify the independent and dependent quantities.

   The cars are selling at a constant rate of 4 per week. This means that the total number of cars left on the lot depends on the number of weeks since the going-out-of-business sale began.

   Therefore, the number of weeks is the independent quantity and the total number of cars left on the lot is the dependent quantity.

2. Choose variables to represent the independent and dependent quantities.

   Because "weeks" starts with the letter "w," let the number of weeks be represented by the variable w.

   Because "cars" starts with the letter "c," let the total number of cars left on the lot be represented by the variable c.

3. Write an equation to represent the relationship between the independent and dependent variables.

   The total number of cars left on the lot (c) is calculated by multiplying the rate at which cars are selling (4 cars per week) by the number of weeks (w) and then subtracting that value from the initial 100 cars.

   Therefore, the relationship can be represented by the equation $c = 100 - 4w$.

4. Create a table to represent the relationship between the independent and dependent variables.

   List values for the independent variable, which is the number of weeks (w), in the left column of the table.

   List values for the dependent variable, which is the total number of cars left (c), in the right column.

   To fill in the table, choose values for w to substitute into the equation $c = 100 - 4w$.

   In the first row of the table, let $w = 0$, so $c = 100 - 4(0)$, which simplifies to $c = 100 - 0 = 100$. In the second row of the table, let $w = 1$, so $c = 100 - 4(1)$, which simplifies to $c = 100 - 4 = 96$.

   Continue filling in rows of the table in this manner, as shown.

Number of weeks (w)	Total cars left (c)
0	100
1	96
2	92
3	88
4	84
5	80
6	76
7	72
8	68

   Based on the table, as the number of weeks increases, the total number of cars on the lot decreases.

5. Use the table to create a graph that represents the relationship between the independent and dependent variables.

   The independent quantity, the number of weeks (*w*), is plotted along the *x*-axis of the coordinate plane.

   The dependent quantity, the total number of cars left on the lot (*c*), is plotted along the *y*-axis.

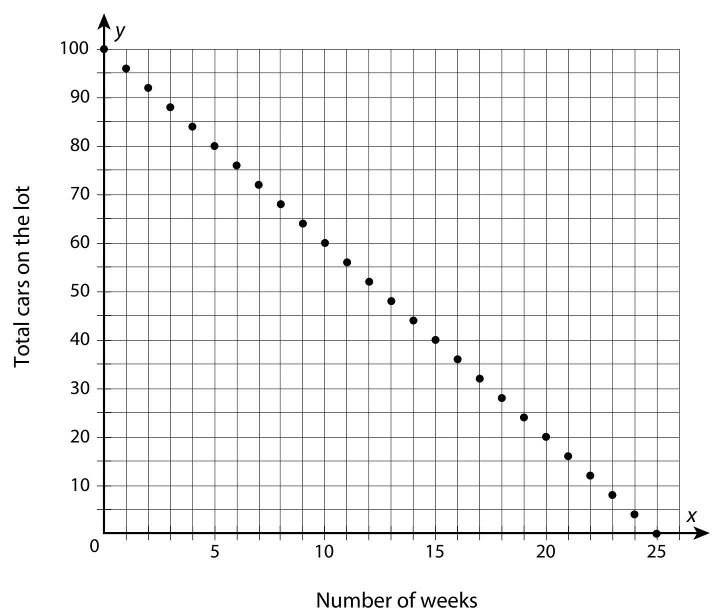
   It is not possible to have a negative number of weeks, so the *x*-axis starts at 0. Likewise, it is not possible to have a negative number of cars, so the *y*-axis also starts at 0.

   Label the *x*-axis "Number of weeks" and label the *y*-axis "Total cars on the lot." The title of the graph can be "Remaining Cars to Sell."

   Each row in the table of values provides a coordinate point that can be graphed on the coordinate plane to represent the relationship.

   Because the cars are being sold at a constant rate, and because it is only possible to sell a whole number of cars, do not draw a line through the coordinate points.

Remaining Cars to Sell



   This graph shows that as the number of weeks increases, the total number of cars on the lot decreases. This confirms the relationship shown in the table of values.

Scaffolded Practice Skill 1

Example 1

On the way to the Grand Canyon, Eric drove his car at an average speed of 70 miles per hour. Explore the relationship between the total distance Eric traveled and the time he spent driving. Identify the independent and dependent quantities, and choose variables to represent them. Then, create an equation, a table, and a graph for this situation.

1. Identify the independent and dependent quantities.

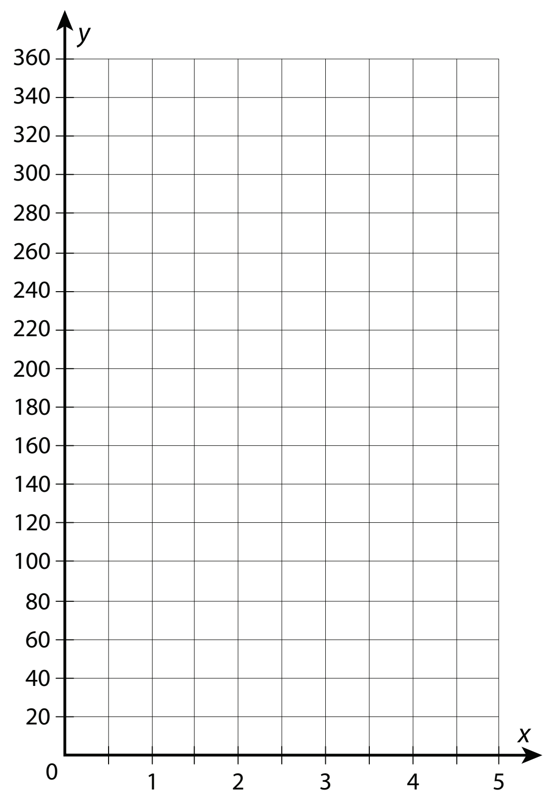
2. Choose variables to represent the independent and dependent quantities.

3. Write an equation to represent the relationship between the independent and dependent variables.

continued

4. Create a table to represent the relationship between the independent and dependent variables.

5. Use the table to create a graph that represents the relationship between the independent and dependent variables.



continued

Example 2

During the summer, Sarah earns $8 per hour babysitting for her aunt. The number of hours she babysits is rounded up to a whole hour. Explore the relationship between the total amount of money Sarah earns and the number of hours she babysits. Identify the independent and dependent quantities, and choose variables to represent them. Then, create an equation, a table, and a graph for this situation.

Example 3

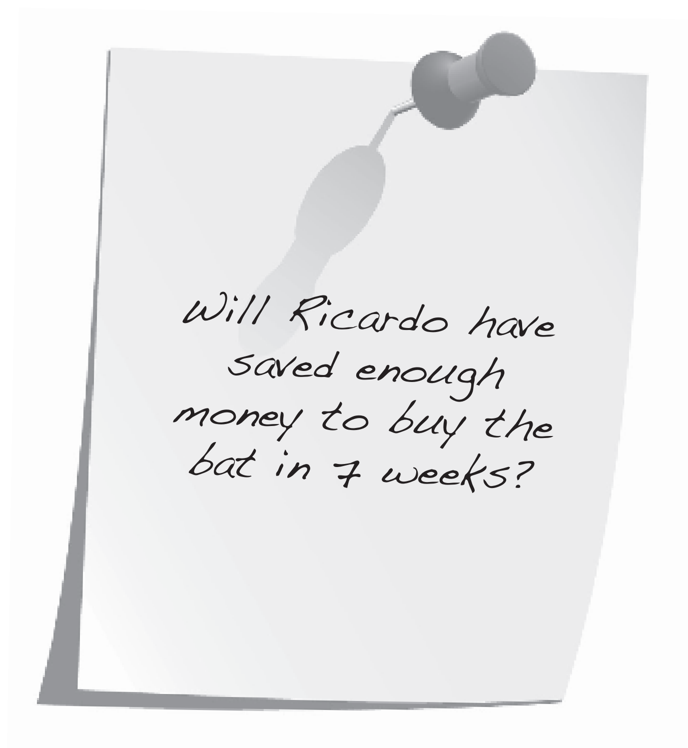
A car dealership is going out of business and needs to sell all the cars left on the lot. There were 100 cars on the lot when the going-out-of-business sale began, and the dealership has been selling them at a constant rate of 4 cars per week since then. Explore the relationship between the total number of cars left on the lot and the number of weeks since the sale began. Identify the independent and dependent quantities, and choose variables to represent them. Then, create an equation, a table, and a graph for this situation.

Problem-Based Task Skill 1: Batter Up!

SMP

1 ✓ 2 ✓
3 ✓ 4 ✓
5 ✓ 6 ✓
7 ✓ 8 ✓

Ricardo has signed up to play baseball in 7 weeks and wants to buy a new bat before the season starts. The bat that Ricardo wants to buy costs $415, including tax. He decides to start saving $65 per week to put toward the purchase of his new bat. Write an equation and create a table to represent the relationship between the number of weeks and the total amount of money saved. Will Ricardo have saved enough money to buy the bat in 7 weeks?



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Problem-Based Task Skill 1: Batter Up!

Coaching

a. What variables can be used to represent the quantities in the relationship?

b. What equation represents the relationship between the number of weeks and the total amount of money saved?

c. Use the equation to create a table that represents the relationship between the number of weeks and the total amount of money saved.

d. Use the table to determine if Ricardo will have saved enough money to buy the bat in 7 weeks.

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Problem-Based Task Skill 1: Batter Up!

Coaching Sample Responses

a. What variables can be used to represent the quantities in the relationship?

The variables for each quantity are arbitrary, but because "weeks" starts with the letter "w," let the number of weeks that Ricardo saves money be represented by the variable w.

Because "money" starts with the letter "m," let the total amount of money that Ricardo has saved be represented by the variable m.

b. What equation represents the relationship between the number of weeks and the total amount of money saved?

The total amount of money saved (m) is calculated by multiplying the amount of money saved per week ($65) by the number of weeks (w). Therefore, the relationship can be represented by the equation $m = 65w$.

c. Use the equation to create a table that represents the relationship between the number of weeks and the total amount of money saved.

List values for the independent variable, w, in the left column, and values for the dependent variable, m, in the right column. Since the problem scenario specifies a time frame of 7 weeks, fill in the left column with values of w from 1 through 7. Substitute these values into $m = 65w$ to fill in the right column, as shown.

Number of weeks (w)	Total money saved in $ (m)
1	65
2	130
3	195
4	260
5	325
6	390
7	455

d. Use the table to determine if Ricardo will have saved enough money to buy the bat in 7 weeks.

The table shows that Ricardo will have saved $455 by week 7. The bat costs $415, which is $455 – $415 = $40 less than what he will have saved, so he will have enough money.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 1: Identifying Independent and Dependent Variables

For problems 1 and 2, identify which variable is the independent variable and which is the dependent variable. Then explain the relationship between the variables.

1. Michele rides her motorcycle at a speed of 55 miles per hour on a cross-country charity ride. Let h represent the number of hours spent riding and let d represent the total distance traveled in miles.

2. A landscape company can plant 14 trees per week. Let w represent the number of weeks and let t represent the total number of trees planted.

For problems 3–6, write an equation to represent the relationship between the two variables.

3. A florist sells roses for $3 each. Let r represent the number of roses purchased and let c represent the total cost in dollars to buy those roses.

4. A tire manufacturer can produce 120 tires per hour. Let h represent the number of hours and let t represent the total number of tires made.

5. A recipe calls for $\frac{3}{4}$ cup of sugar for each pan of brownies made. Let b represent the number of pans of brownies made, and let c represent the total cups of sugar used.

continued

6. Amanda is training for a marathon, and she times herself while running to keep track of her speed. She can run at a constant speed of 0.1 mile per minute. Let *m* represent the number of minutes and let *d* represent the total distance Amanda runs.

Use the given information to complete problems 7–10.

7. A shoe store sells pairs of shoelaces for $1.20. Let *s* represent the number of pairs of shoelaces purchased and let *c* represent the total cost to buy those shoelaces. Complete the table to show the total cost in dollars based on the number of pairs of shoelaces purchased.

Pairs of shoelaces (*s*)	Total cost in $ (*c*)
1	
2	
3	
4	
8	
10	

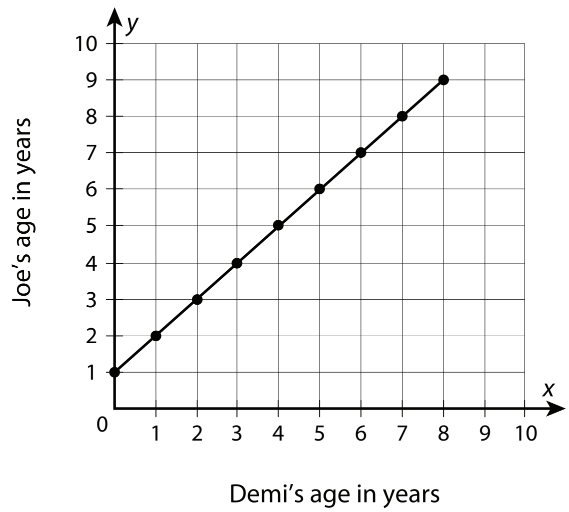
8. Lyndon bought a new boat for $38,000. Every year, the value of the boat decreases by $1,800. Let *n* represent the number of years since he bought the boat, and let *v* represent the value of the boat in dollars. Complete the table to show the value of the boat based on the number of years since Lyndon bought it.

Number of years (*n*)	Boat's value in $ (*v*)
1	
2	
3	
4	
5	

continued

9. The graph shows the relationship between Demi's age and Joe's age. Use the data in the graph to write an equation to represent this relationship. Let *d* represent Demi's age in years and let *j* represent Joe's age in years.

Ages of Demi and Joe



10. A street vendor sells baseball caps for $15 each, including tax. The table shows the total price for a given number of caps. Use the table to create a graph that represents this relationship. Use the labels "Number of caps" and "Total price in $" along each appropriate axis. Title the graph "Total Price of Caps."

Number of caps (*c*)	Total price in $ (*p*)
1	15
2	30
3	45
4	60
5	75

Skill 2: Determining the Domain and Range of Linear and Quadratic Functions*

Common Core State Standard

F–IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

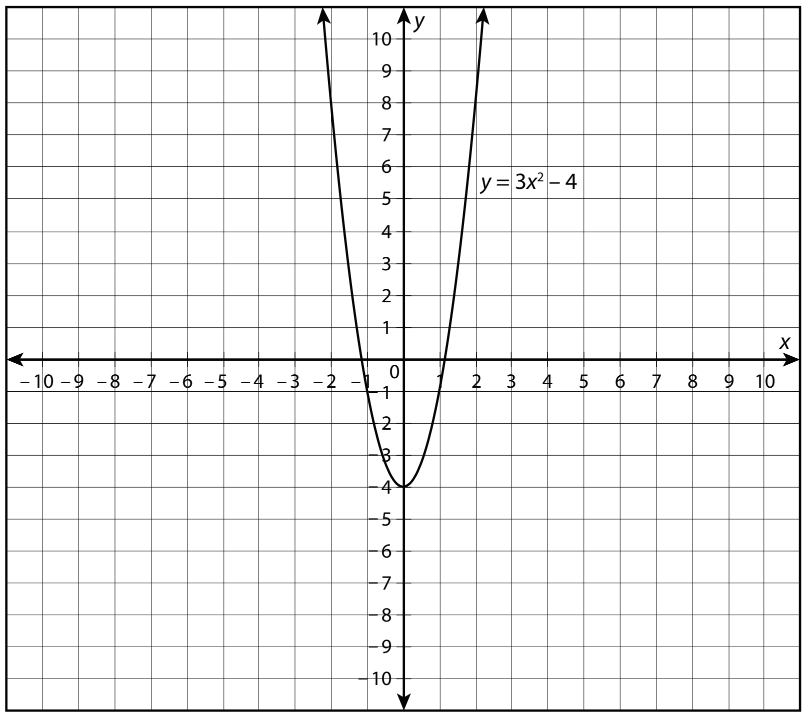
This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 2, Skill 3

Guided Practice Skill 2

Example 1

Determine the domain and range of the following graph.



1. Determine the domain of the function.

   The domain is the set of *x*-values that are valid for the function.

   It can be seen from the graph that the parabola continues infinitely in both the negative and positive directions. This means that all *x*-values are included in the domain because the function is valid when any value of *x* is used as the input.

   Therefore, the domain is $-\infty < x < \infty$.

2. Determine the range of the function.

   The range is the set of *y*-values that are valid for the function.

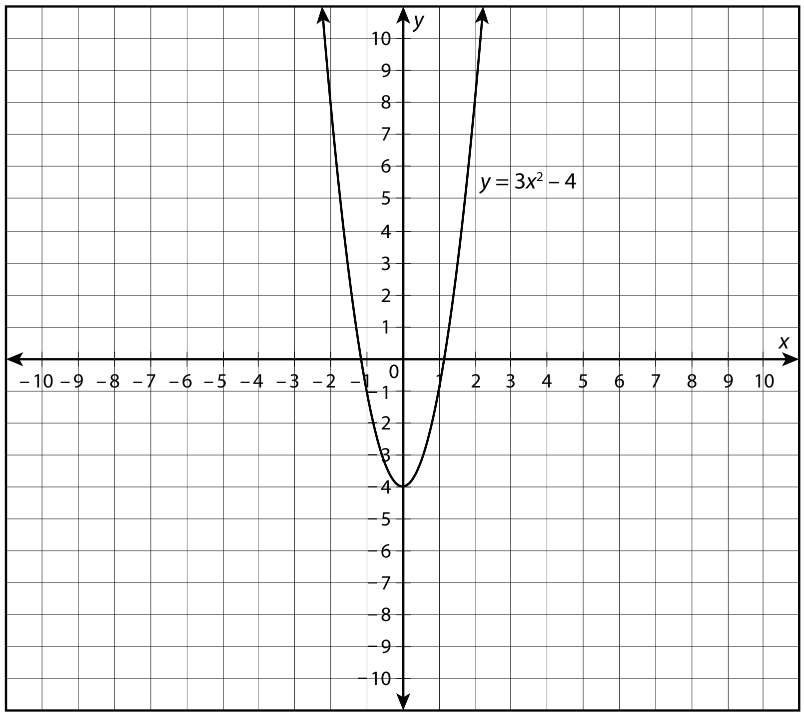
   In this graph, the *y*-values start at $y = -4$ and continue to increase. Therefore, the range is all the numbers greater than or equal to –4.

   The range is $y \geq -4$.

Scaffolded Practice Skill 2

Example 1

Determine the domain and range of the following graph.



1. Determine the domain of the function.

2. Determine the range of the function.

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Practice Skill 2: Determining the Domain and Range of Linear and Quadratic Functions*

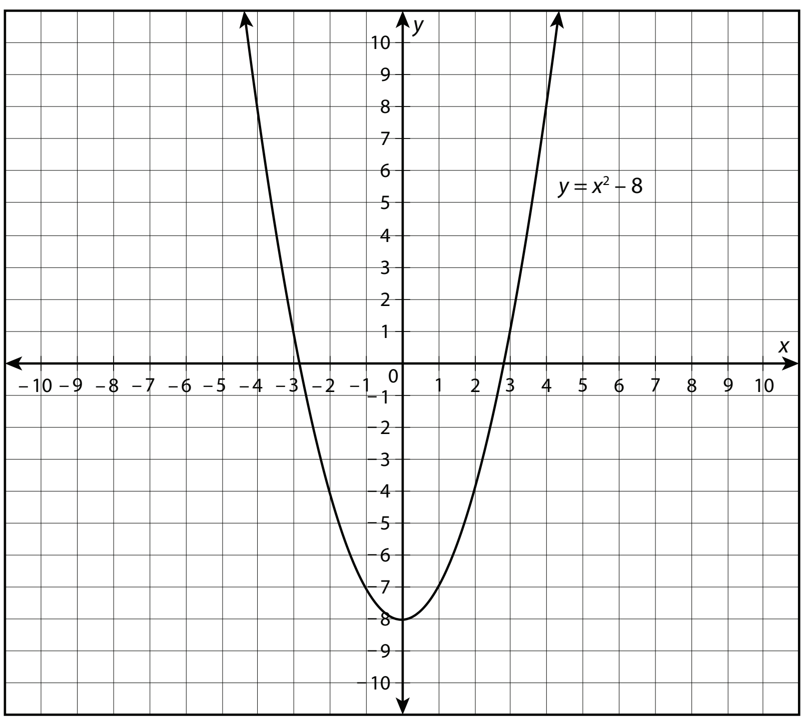
For problems 1 and 2, find the domain and range of each function.

1. $f(x) = 6(x - 2)$

2. $y = 2x^2 + 5$

For problem 3, find the domain and range of the graphed function.

3.



Instruction

Skill 3: Applying Inverse Operations to Isolate a Variable, Including Taking Square Roots**

Common Core State Standard

A–REI.4 Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

This skill has been addressed previously in *CCSS Integrated Pathway: Mathematics II*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

Unit 3, Lesson 2, Sub-lesson 1

Unit 3, Lesson 2, Sub-lesson 2

Unit 3, Lesson 2, Sub-lesson 3

Unit 3, Lesson 2, Sub-lesson 4

Unit 3, Lesson 2, Sub-lesson 5

Skill 4: Using Function Notation*

Common Core State Standard

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 2, Skill 4

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Guided Practice Skill 4

Example 1

Evaluate $f(x) = 5x^2 + 7x - 2$ over the domain {1, 2, 3, 4}. Determine the range for this domain.

1. Evaluate the function for each of the domain values.

   To evaluate the function $f(x) = 5x^2 + 7x - 2$ over the domain {1, 2, 3, 4}, substitute the values from the domain into $f(x) = 5x^2 + 7x - 2$.

   Evaluate $f(1)$.

$f(x) = 5x^2 + 7x - 2$	Given function
$f(1) = 5(1)^2 + 7(1) - 2$	Substitute 1 for x.
$f(1) = 10$	Simplify.

   When 1 is substituted for x, the value of $f(1)$ is 10.

   Evaluate $f(2)$.

$f(x) = 5x^2 + 7x - 2$	Given function
$f(2) = 5(2)^2 + 7(2) - 2$	Substitute 2 for x.
$f(2) = 32$	Simplify.

   When 2 is substituted for x, the value of $f(2)$ is 32.

   Evaluate $f(3)$.

$f(x) = 5x^2 + 7x - 2$	Given function
$f(3) = 5(3)^2 + 7(3) - 2$	Substitute 3 for x.
$f(3) = 64$	Simplify.

   When 3 is substituted for x, the value of $f(3)$ is 64.

   Evaluate $f(4)$.

$f(x) = 5x^2 + 7x - 2$	Given function
$f(4) = 5(4)^2 + 7(4) - 2$	Substitute 4 for x.
$f(4) = 106$	Simplify.

   When 4 is substituted for x, the value of $f(4)$ is 106.

2. Determine the range of the function for the given domain.

   Collect the set of outputs for the inputs. Recall that the outputs are the $f(x)$ values, and the inputs are the values we substituted for x.

   The outputs from the previous step were 10, 32, 64, and 106.

   Therefore, the range of the function over the given domain is {10, 32, 64, 106}.

Scaffolded Practice Skill 4

Example 1

Evaluate $f(x) = 5x^2 + 7x - 2$ over the domain {1, 2, 3, 4}. Determine the range for this domain.

1. Evaluate the function for each of the domain values.

2. Determine the range of the function for the given domain.

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Practice Skill 4: Using Function Notation*

For problems 1–3, evaluate the given functions and determine the range of each.

1. Evaluate $f(x) = -2x^2 + 3x$ over the domain {–2, –1, 0, 1, 2}. What is the range?

2. Evaluate $g(x) = 4x^2 - 5x + 1$ over the domain {0, 1, 2, 3}. What is the range?

3. Evaluate $h(x) = -x^2 + 7x - 2$ over the domain {0, 2, 4, 6}. What is the range?

Supportive Instructional Strategies for Mathematics II

Unit 2 Lesson 7

Suggestions for Graphic Organizers/Manipulatives

- Provide students with the Coordinate Plane graphic organizer from the Program Overview, along with colored pencils or markers. Ask students to write the linear function $f(x) = 2x + 1$ at the top of the page. Have students create a table of values, and then graph the function on the coordinate plane using one color. Then, ask students to find the inverse of the function and graph it on the same coordinate plane, in a different color. Encourage and guide a discussion about the relationship between the two graphed functions, specifically the table of values for each. Lead students toward drawing a conclusion about the line of symmetry on the graph.
- Provide each student with three blank flash cards. On one side of each flash card, ask each student to write either a linear or a quadratic function. Have students exchange their cards with a partner and ask the partner to determine the inverse of each function. Ask students to write the inverse function on the opposite side of the flash cards. If a function does not have an inverse, ask students to write a brief explanation of why. Have students volunteer and explain their answers.
- Provide each student with a blank two-column chart or table. Ask students to label the chart "Types of Functions." Then have students label the columns "Linear function" and "Quadratic function." In the "Linear function" column, ask students to list features of linear functions. Then, in the space directly to the right of each feature, have students list the corresponding feature for a quadratic function. For example, one feature of a linear function could be "the highest power is 1," and the corresponding feature for the quadratic function would be "the highest power is 2." Another example of a feature of a linear function could be "it is a one-to-one function," whereas the corresponding feature for the quadratic function would be "it is not a one-to-one function."

Suggestions for Discourse

- Ask students, "Why is a linear function one-to-one, but a quadratic function *not* one-to-one?" Encourage and guide a discussion about how each input value in a linear function is mapped to exactly one output value, but the same is not true for a quadratic function.
- Ask students, "What is the relationship between the domains and ranges of a function and its inverse function?" Encourage and guide a discussion regarding the switching of the domain and range when determining the inverse of a function.
- Ask students to work with a partner to create a list of types of inverse operations with examples of each. Examples will vary, but possible responses include: *addition and subtraction*, *multiplication and division*, and *squaring a number and taking its square root*. Ask students to volunteer their answers, and use their responses to create a master list.

- Ask students, "What are some reasons the domain of a function would be restricted?" Ask students to work with a partner to create a list of real-life scenarios in which the domain is restricted, as well as other examples of functions that have mathematical restrictions. (Some examples *not* involving real-life scenarios include restricting the domain to all real numbers greater than 0 under a square root symbol, or excluding values that would result in having a 0 in the denominator of a fraction.) Ask students to volunteer their answers.

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics II*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.
- Provide a list of sentence frames in which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "I know that a _______ function is a one-to-one function because each input value is mapped to _______ output value." Or, "The inverse operation of multiplication is _______, and the inverse operation of addition is _______." Or, "The domain of a function's inverse is the range of the function."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- misinterpreting the superscript in inverse function notation, $f^{-1}(x)$, as a power of $f(x)$

  Remind students that the correct notation of an inverse function is represented with a power of –1 in the function's name, typically on the left side of the equal sign, whereas a power of the original function, $f(x)$, will be in the function itself.

- incorrectly performing inverse operations to find the inverse of a function

  Ask students to make a list of inverse operations (e.g., addition/subtraction; multiplication/division; squaring/taking the square root). Remind students that each operation only has one inverse operation.

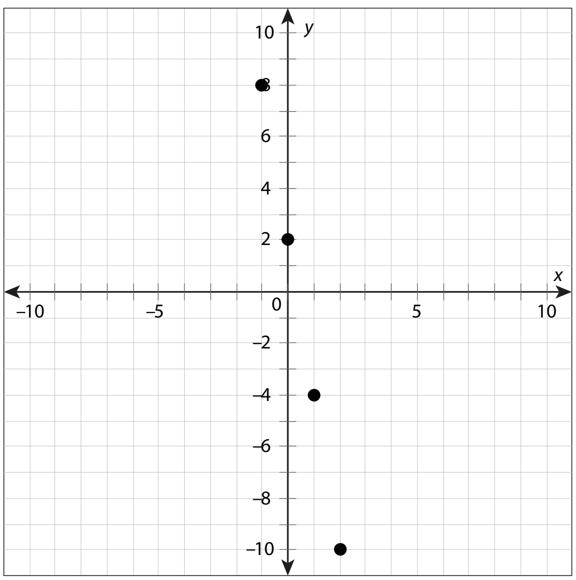
Answer Key

Lesson 1: Analyzing Quadratic Functions

Practice Skill 1: Graphing Functions by Creating Tables of Values, p. U2-18

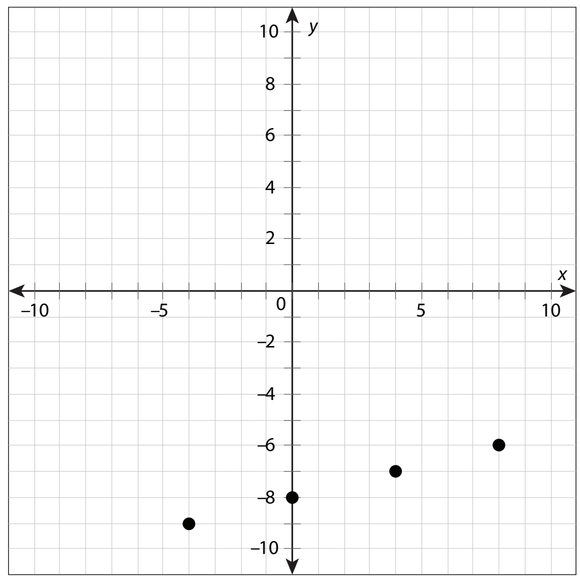
1.

x	*y*
–1	8
0	2
1	–4
2	–10



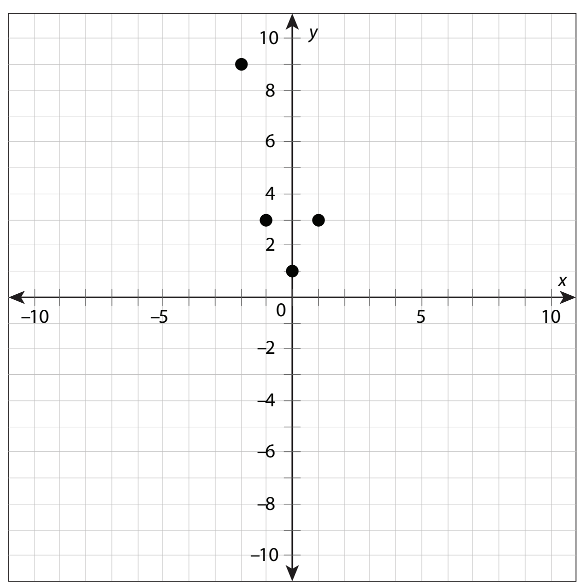
2.

x	*y*
–4	–9
0	–8
4	–7
8	–6



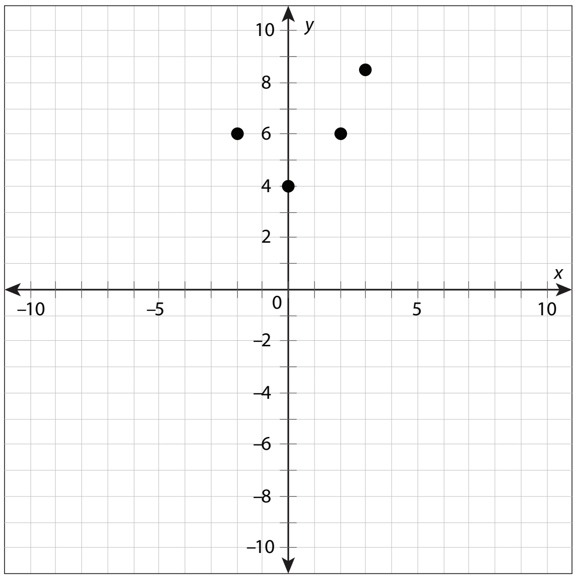
3.

x	*y*
–2	9
–1	3
0	1
1	3



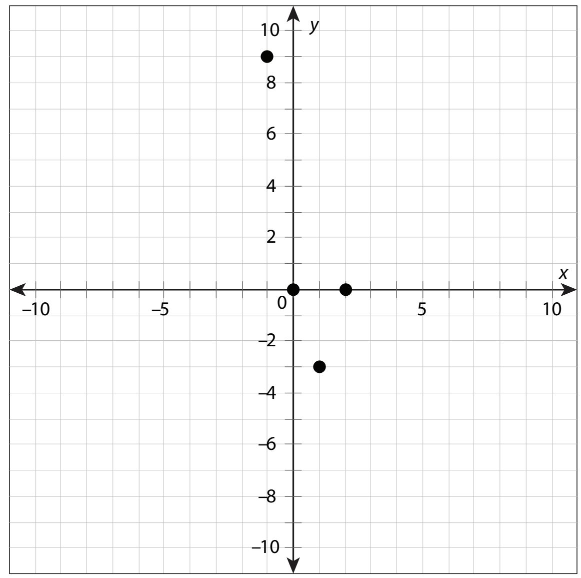
4.

x	y
–2	6
0	4
2	6
3	8.5



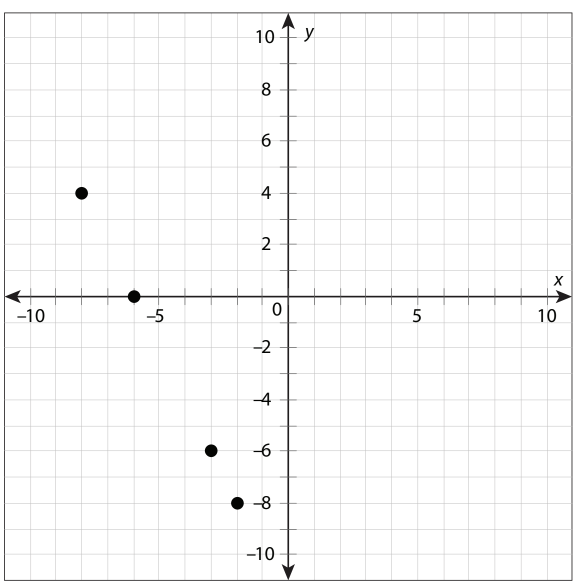
6.

x	y
–1	9
0	0
1	–3
2	0



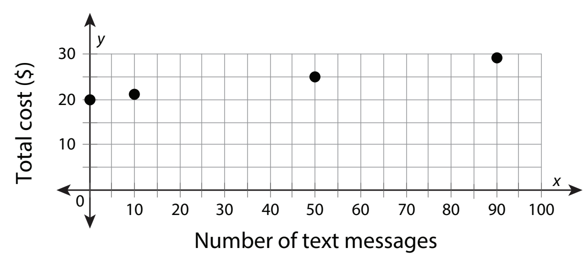
5.

x	y
–8	4
–6	0
–3	–6
–2	–8



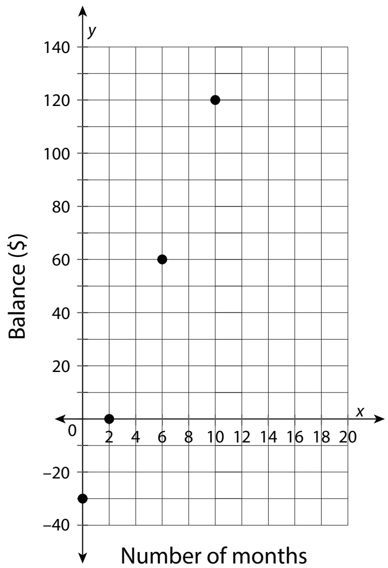
7.

x	y
0	20
10	21
50	25
90	29



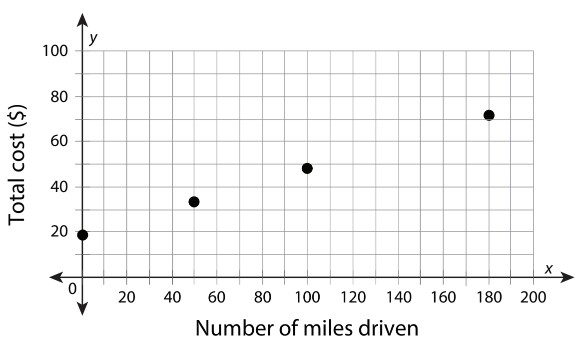
8.

x	y
0	–30
2	0
6	60
10	120



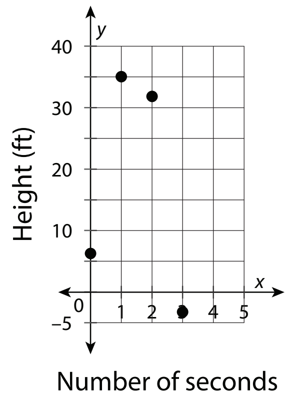
9.

x	y
0	18
50	33
100	48
180	72



10.

x	y
0	6
1	35
2	32
3	–3



Lesson 2: Interpreting Quadratic Functions

Practice Skill 3: Understanding the Difference Between Domain and Range, p. U2-42

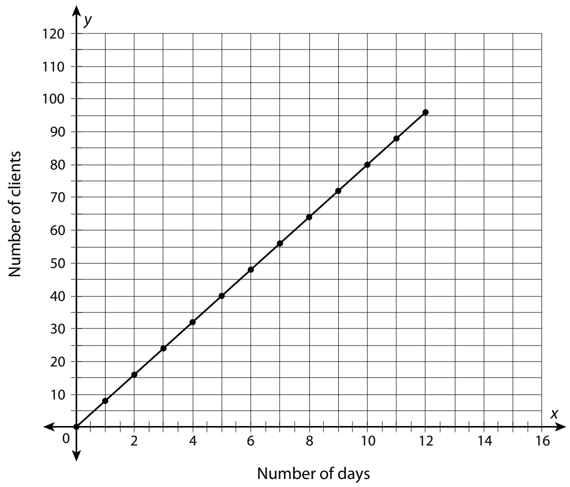
1. domain: {–1, 0, 1, 2, 3}; range: {–5, 0, 2, 6}
2. domain: {–5, –2, –1, 2}; range: {3, 6, 7, 8}
3. domain: {all real numbers}; range: {all real numbers}
4. domain: {all real numbers}; range: {$y \geq 1$}
5. domain: {$x \geq -3$}; range: {$y \geq 0$}
6. domain: {all real numbers}; range: {all real numbers}
7. domain: {all real numbers; $x \neq -1$};
   range: {all real numbers; $y \neq 0$}
8. domain: {all real numbers}; range: {$y \geq -4$}
9. domain: {$0 < x \leq 20$}; range: {$0 \leq x < 20$}
10. domain: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}; range: {0, 580, 1160, 1740, 2320, 2900, 3480, 4060, 4640, 5220, 5800}

Practice Skill 4: Evaluating Quadratic Functions for Specific Values of *x*, p. U2-57

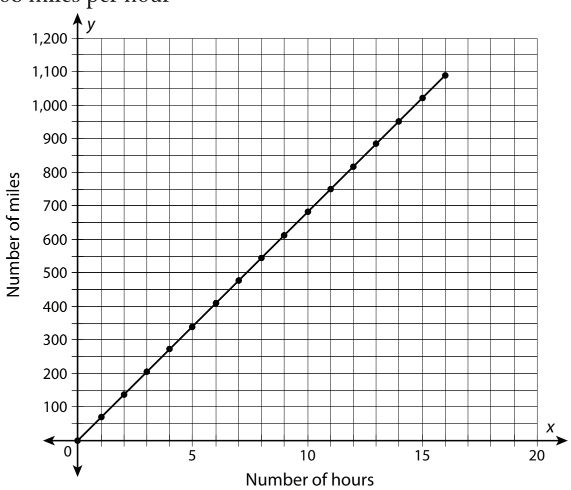
1. range: {0, 22, 68, 138}
2. range: {–5, –3, 15, 49}
3. range: {–8, –30, –60, –98}
4. range: {23, –9, 23}
5. range: {–18, –23, –38, –63}
6. range: {–5, 22, 103, 238}
7. 24 feet, 15 feet, and 8 feet
8. 51 feet, 60 feet, and 44 feet
9. 5 feet, 1 feet, and 2 feet
10. 6 feet, 8 feet, and 2 feet

Practice Skill 5: Finding the Slope or Rate of Change of Linear Functions, p. U2-80

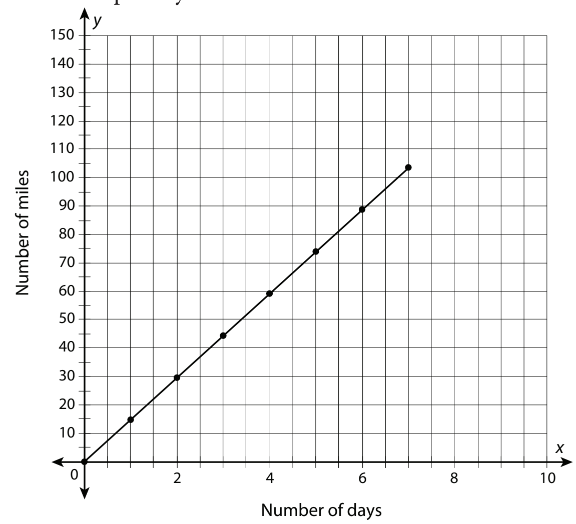
1. unit rate: \$0.67 per taco; meaning: each taco costs \$0.67; $y = 0.67x$
2. unit rate: 60 feet per second; meaning: the ball travels 60 feet every second; $y = 60x$
3. unit rate: 75 words per minute; meaning: every minute she types 75 words; $y = 75x$
4. unit rate: 40 chairs per row; meaning: each row has 40 chairs; $y = 40x$
5. 96 clients



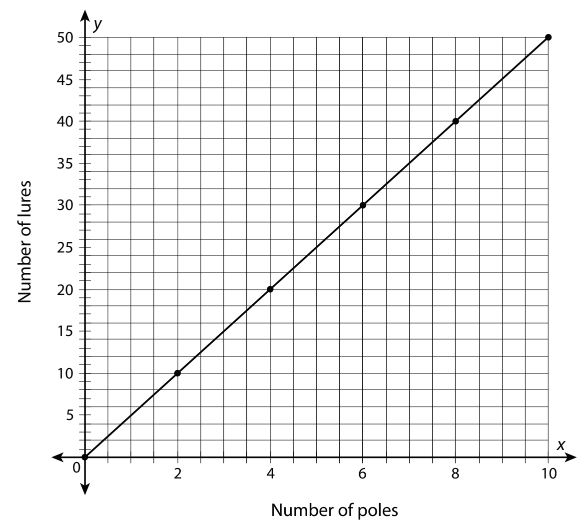
6. 68 miles per hour



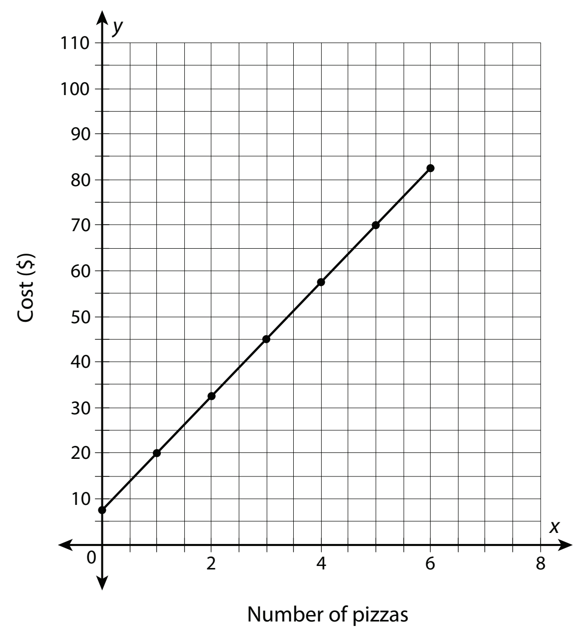
7. 14.8 miles per day



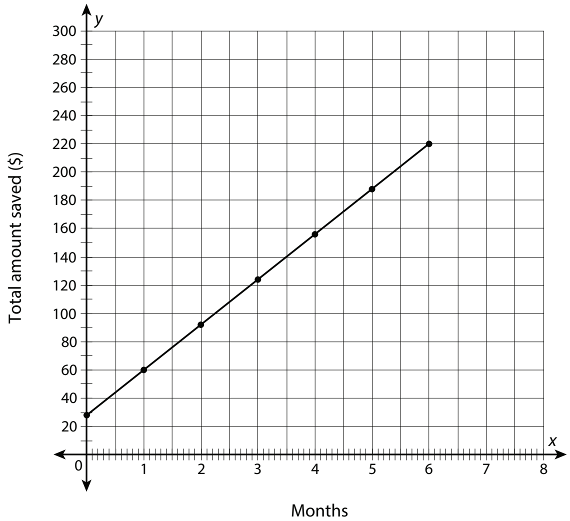
8. 40 lures



9. \$12.50 per pizza



10. \$32 each month



Lesson 3: Building Functions

Practice Skill 1: Multiplying Linear Expressions, p. U2-100

1. $7x + 9$
2. $8a + 5b - 9$
3. $7xy - 2x - 7y$
4. $-15xy + 6y^2$
5. $-24a^2 + 14$
6. $-4y^2 - 12y$
7. $x^2 - 2x - 15$
8. $9x^2 - 9x - 4$
9. $4x + 6$
10. $8x^2 + 24x + 18$

Practice Skill 5: Adding, Subtracting, Multiplying, and Dividing Functions, p. U2-115

1. $(f + g)(x) = 8x - 4$
2. $(f - g)(x) = -2x - 12$
3. $(f \bullet g)(x) = 7(6^x)$
4. $(f \div g)(x) = \frac{6^x}{7}$
5. $(f + g)(x) = 2x + 3$
6. $(f - g)(x) = -2x - 15$
7. $(f \bullet g)(x) = -12x - 54$
8. $(f \div g)(x) = -\frac{6}{2x+9}$
9. $f(x) = 12x + 10$; $g(x) = 12x + 15$; 5 is added to the first function.
10. $f(x) = 50\left(2^{\frac{x}{8}}\right)$; $g(x) = 100\left(2^{\frac{x}{8}}\right)$; the first function is multiplied by 2.

Lesson 4: Graphing Other Functions

Practice Skill 1: Determining the Domain and Range of an Algebraic Equation*, p. U2-125

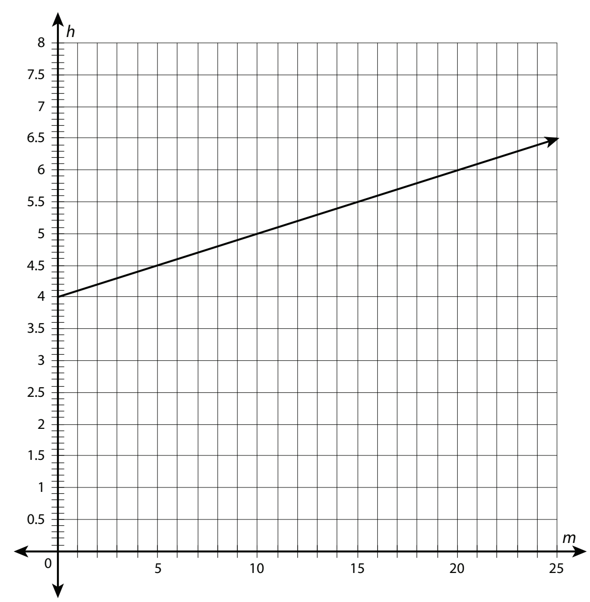
1. domain: {all real numbers}; range: $\{y \geq -5\}$
2. domain: {all real numbers}; range: {all real numbers}
3. domain: $\{x \geq 0\}$; range: $\{y \geq 4\}$

Practice Skill 2: Evaluating Functions for Given Values*, p. U2-129

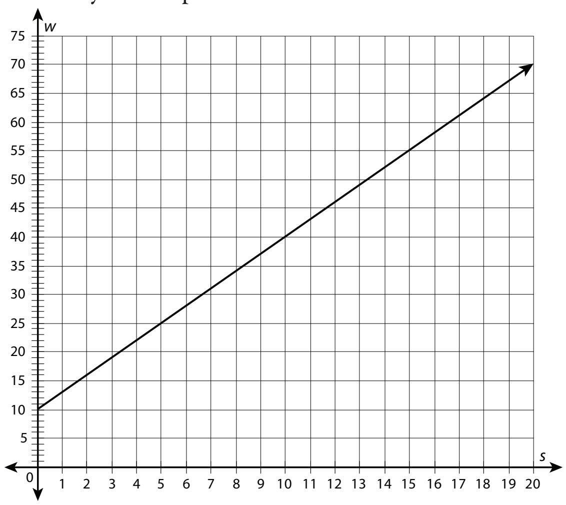
1. range: {13, 0, –1, 10}
2. range: {–6, 6, –6, –42}
3. range: {16, 12, 16, 28}

Practice Skill 3: Finding Ordered Pairs by Evaluating Functions, p. U2-143

1. range: {4, 18, 32}
2. range: {4, 0, –8}
3. range: {–5, 10, 25}
4. range: {21, 25, 31}
5. no
6. yes
7. yes
8. no
9. Yes, the equation represents a function because each input has exactly one output.



10. Yes, the equation represents a function because each input has exactly one output.

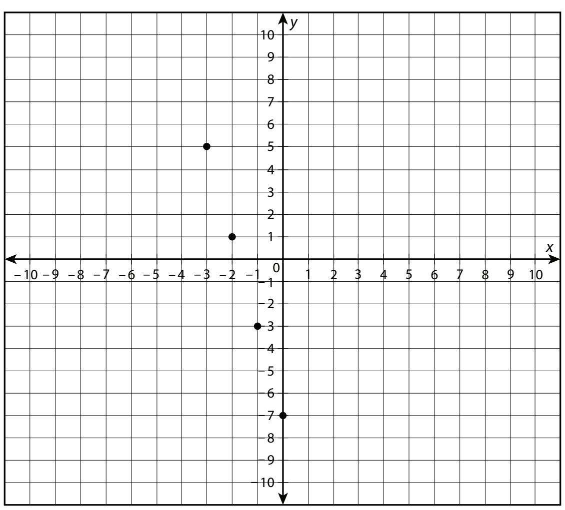


Practice Skill 4: Evaluating Squares and Cubes of Real Numbers With and Without a Calculator, p. U2-157

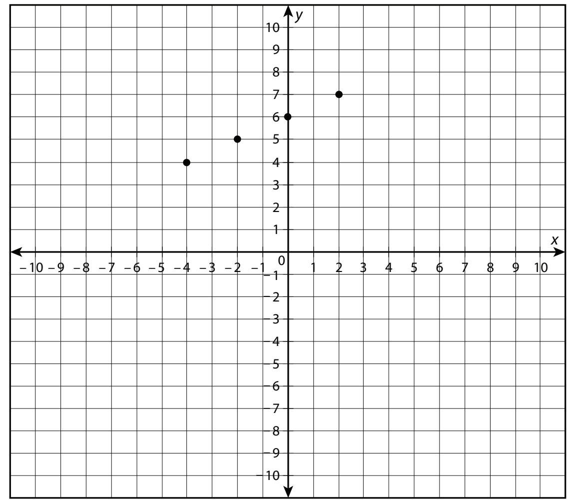
1. 11
2. –15
3. 6
4. $\frac{5}{9}$
5. 2
6. –5
7. 3
8. $\frac{7}{10}$
9. 9 in
10. 5 cm

Practice Skill 5: Graphing a Linear Function*, p. U2-163

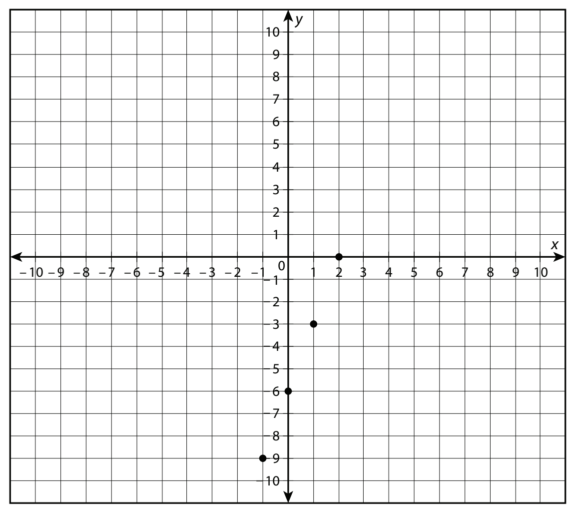
1.



2.



3.



Practice Skill 6: Finding the Absolute Value of a Quantity, p. U2-175

1. 45.028
2. $\frac{31}{78}$
3. 10.99
4. $\frac{1}{8}$
5. <
6. =
7. >
8. –10° and 10°
9. Dan has less debt because $|-15|<|-22|$.
10. The Tigers have a greater yardage magnitude because $|-12|>|9|$.

Lesson 5: Analyzing Functions

Practice Skill 1: Identifying the Base and Power of an Exponent and Evaluating Exponential Expressions, p. U2-194

1. base = 7; exponent = 3; 343
2. base = 0.52; exponent = 4; –0.146
3. $\left(\frac{1}{3}\right)^4$; base = $\frac{1}{3}$; exponent = 4; $\frac{1}{81}$
4. $2^4 \bullet 5^3$; first term: base = 2 and exponent = 4; second term: base = 5 and exponent = 3; 2,000
5. $0 < 1$
6. $0.316 > 0.078$
7. $\frac{64}{27} < \frac{25}{16}$
8. $4^3 = 64$ ft
9. $\left(\frac{2}{50}\right)^2 = 0.0016\ \mu\text{m}$
10. $7^8 = 5{,}764{,}801$ flowers

Practice Skill 2: Simplifying Exponential Expressions with Integer Exponents*, p. U2-199

1. $\frac{1}{a^{18}}$
2. $\frac{1}{y^4}$
3. $\frac{y^{11}}{x^7}$

Practice Skill 4: Writing an Equation for a Simple Exponential Function, p. U2-215

1. linear
2. exponential
3. linear
4. $y = 0.5(2)^6$; 32 feet
5. $y = 850(1 - 0.05)^{12}$; 459 subscribers
6. $y = 480\left(\frac{1}{2}\right)^6$; $7.50
7. $18{,}000 = a(1 - 0.1)^5$; a = $30,483
8. $y = 5(3)^7$; 10,935 weeds
9. $y = 98{,}000(1 - 0.038)^6$; 77,674 people
10. $y = 116(1 + 0.12)^9$; 322 registered voters

Lesson 6: Transforming Functions

Practice Skill 1: Graphing Quadratic Functions, p. U2-238

1.

x	*y*
–2	0
–1	–6
0	–8
1	–6
2	0

2.

x	*y*
–2	–2
–1	3
0	6
1	7
2	6

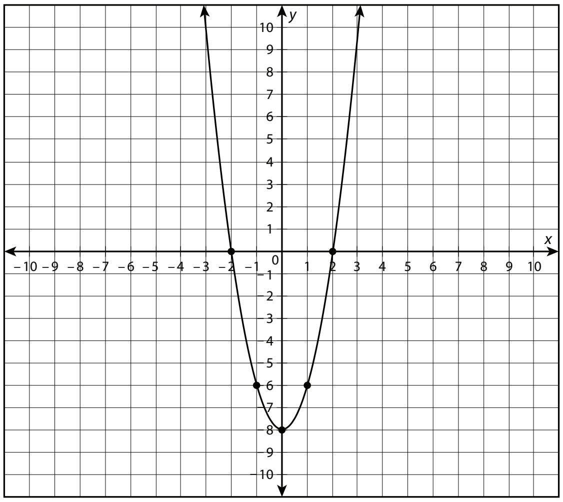
3.

x	*y*
–2	3
–1	–2
0	–5
1	–6
2	–5

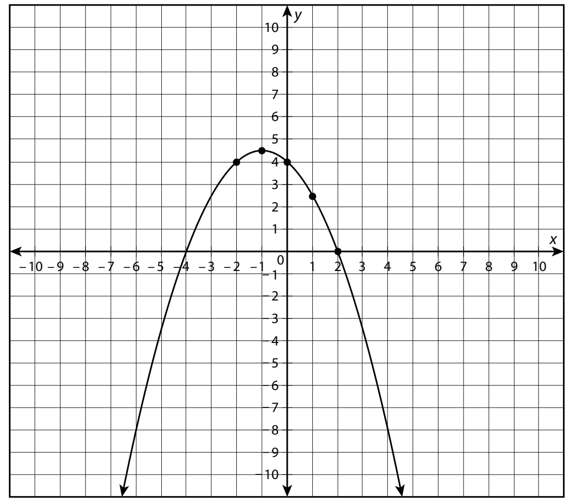
4.

x	*y*
–2	4
–1	4.5
0	4
1	2.5
2	0

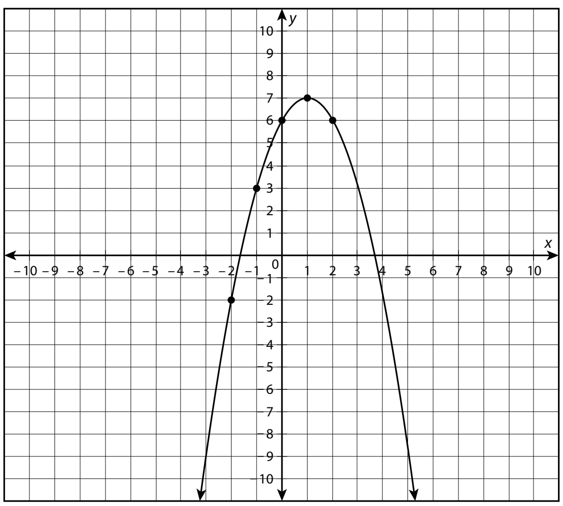
5.



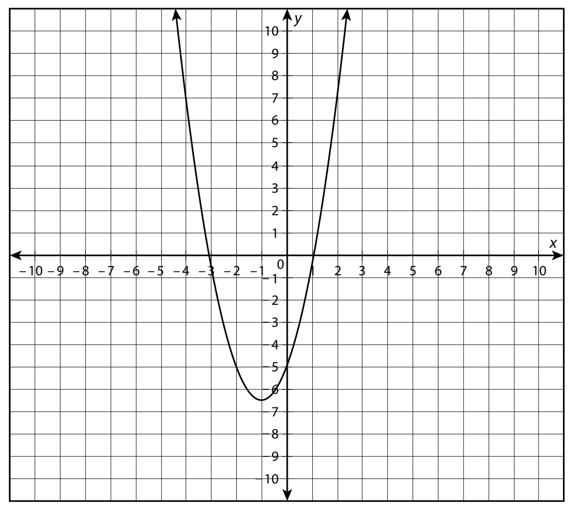
8.



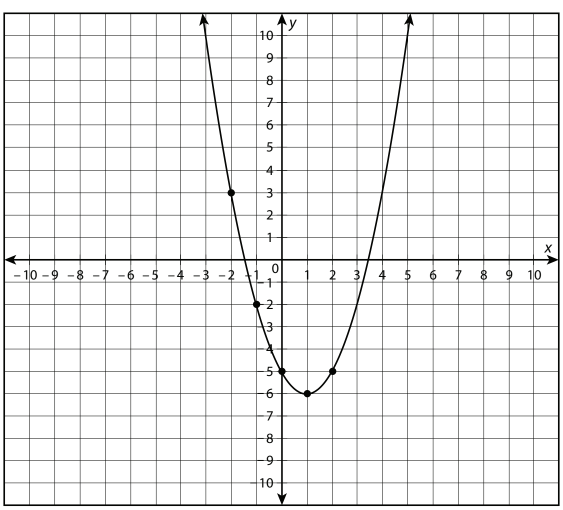
6.



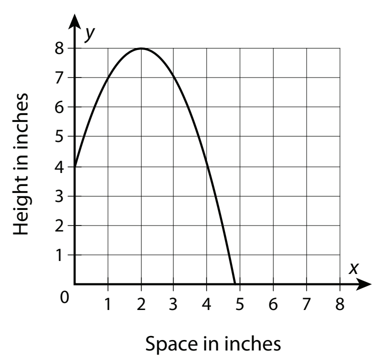
9.



7.



10.



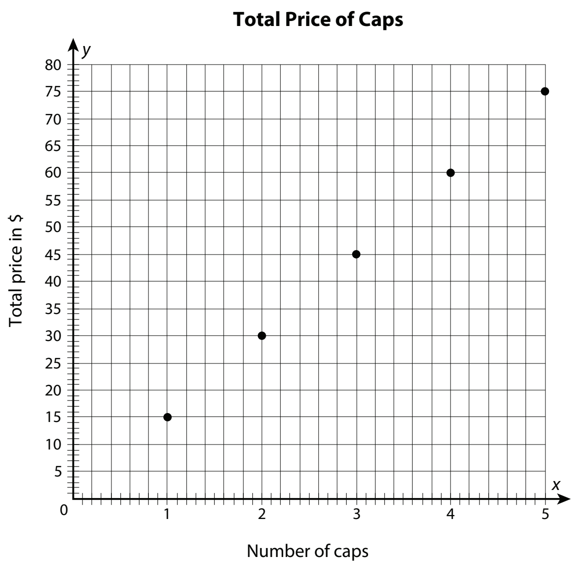
Practice Skill 2: Evaluating Quadratic Functions*, p. U2-243

1. range: {5, 20, 47, 86}
2. range: {–316, –156, –52, –4}
3. range: {–27, –11, –8, 0, 1}

Lesson 7: Finding Inverse Functions

Practice Skill 1: Identifying Independent and Dependent Variables, p. U2-272

1. h = independent variable; d = dependent variable; as the value of h increases, the value of d also increases
2. w = independent variable; t = dependent variable; as the value of w increases, the value of t also increases
3. $c = 3r$
4. $t = 120h$
5. $c = \frac{3}{4}b$
6. $d = 0.1m$
7. values for c: $1.20, $2.40, $3.60, $4.80, $9.60, $12.00
8. values for v: $36,200, $34,400, $32,600, $30,800, $29,000
9. $j = d + 1$
10.



Practice Skill 2: Determining the Domain and Range of Linear and Quadratic Functions*, p. U2-278

1. domain: $-\infty < x < \infty$; range: $-\infty < y < \infty$
2. domain: $-\infty < x < \infty$; range: $y \geq 5$
3. domain: $-\infty < x < \infty$; range: $y \geq -8$

Practice Skill 4: Using Function Notation*, p. U2-284

1. range: {–14, –5, –2, 0, 1}
2. range: {1, 0, 7, 22}
3. range: {–2, 4, 8, 10}