Common Core State Standards

Mathematics I

Integrated Pathway



Program Overview

WALCH
INTEGRATED MATH

This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

Developers and reviewers include:

Joyce Hale	Valerie Ackley	Doug Kühlmann
Shelly Northrop Sommer	Laura McPartland	Mike May, S.J.
Ruth Estabrook	Cameron Larkins	James Quinlan
Vanessa Sylvester	Jennifer Blair	Peter Tierney-Fife
Zachary Lien	Nancy Pierce	

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PROGRAM OVERVIEW

Introduction to the Program

Introduction

The *Common Core State Standards Integrated Pathway: Mathematics I Program* is a complete set of materials developed around the Common Core State Standards (CCSS), the overview of the Integrated Pathway for the Common Core State Mathematics Standards, and the Mathematics I content map found in Appendix A of the Common Core State Standards. Topics are built around accessible core curricula, ensuring that the *CCSS Integrated Pathway: Mathematics I Program* is useful for striving students and diverse classrooms.

This program realizes the benefits of exploratory and investigative learning and employs a variety of instructional models to meet the learning needs of students with a range of abilities.

The *CCSS Integrated Pathway: Mathematics I Program* includes components that support problem-based learning, instruct and coach as needed, provide practice, and assess students' skills. Instructional tools and strategies are embedded throughout.

The program includes:

- More than 150 hours of lessons, addressing the six units of CCSS IP: Mathematics I
- Essential Questions for each instructional topic
- Vocabulary
- Instruction and Guided Practice
- Problem-based Tasks and Coaching questions
- Step-by-step graphing calculator instructions for the TI-Nspire and the TI-83/84
- Station activities to promote collaborative learning and problem-solving skills

Purpose of Materials

The *CCSS Integrated Pathway: Mathematics I Program* has been organized to coordinate with the CCSS Integrated Pathway: Mathematics I content map and specifications from Appendix A of the Common Core State Standards.

Each lesson includes activities that offer opportunities for exploration and investigation. These activities incorporate concept and skill development and guided practice, then move on to the application of new skills and concepts in problem-solving situations. Throughout the lessons and activities, problems are contextualized to enhance rigor and relevance.

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PROGRAM OVERVIEW
Introduction to the Program

This program includes all the topics addressed in the CCSS Integrated Pathway: Mathematics I content map. These include:

- Relationships Between Quantities
- Linear and Exponential Relationships
- Reasoning with Equations
- Descriptive Statistics
- Congruence, Proof, and Constructions
- Connecting Algebra and Geometry Through Coordinates

The eight Mathematical Practices described in the Common Core are infused throughout:

- CCSS.MP.1: Make sense of problems and persevere in solving them.
- CCSS.MP.2: Reason abstractly and quantitatively.
- CCSS.MP.3: Construct viable arguments and critique the reasoning of others.
- CCSS.MP.4: Model with mathematics.
- CCSS.MP.5: Use appropriate tools strategically.
- CCSS.MP.6: Attend to precision.
- CCSS.MP.7: Look for and make use of structure.
- CCSS.MP.8: Look for and express regularity in repeated reasoning.

Structure of the Teacher Resource

The *CCSS Integrated Pathway: Mathematics I Program* is provided in an online, digital format, and/or in hard copy (Program Overview and six unit volumes). The materials, either online or hard copy, are completely reproducible. Online materials can be provided in your Learning Management System (such as Canvas or Schoology) or in Walch's proprietary system, WalchConnect. The nested folder organization in WalchConnect allows you to access the materials quickly and easily. The digital format also facilitates printing and copying student pages and/or making assignments online.

The Program Overview is the first section. This section helps you to navigate the materials, offers a collection of graphic organizers and suggested strategies for their use, and shows the correlation between the Common Core State Standards and the CCSS Integrated Pathway: Mathematics I content map found in Appendix A of the Common Core State Standards.

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The remaining materials focus on content, knowledge, and application of the six units in the CCSS Integrated Pathway Mathematics I curriculum: Relationships Between Quantities; Linear and Exponential Relationships; Reasoning with Equations; Descriptive Statistics; Congruence, Proof, and Constructions; and Connecting Algebra and Geometry Through Coordinates. The units in the *CCSS Integrated Pathway: Mathematics I Program* are designed to be flexible so that you can mix and match activities as the needs of your students and your instructional style dictate.

The Station Activities correspond to the content in the units and provide students with the opportunity to apply concepts and skills, while you have a chance to circulate, observe, speak to individuals and small groups, and informally assess and plan.

Each lesson begins with a pre-assessment and ends with a progress assessment. These allow you to assess students' progress as you move from lesson to lesson, enabling you to gauge how well students have understood the material and to differentiate as appropriate.

***Please note*:** Throughout the Teacher Resource, page references are provided on the lower, inner corner of some pages. These references indicate the corresponding page(s) in the Student Workbook (SWB) and/or Student Resource Book (SRB) as appropriate. These page references have been included here to facilitate assigning materials to students.

Glossary

The Glossary contains vocabulary terms and formulas from throughout the program, organized alphabetically. Each listing provides the term and the definition in both English and Spanish. The listings include the lesson number(s) where the terms can be found in the Words to Know.

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Correspondence to Standards for Mathematical Practice

How Do Walch Integrated Mathematics Resources Address the CCSS 8 Standards for Mathematical Practice?

Walch's programs for the *CCSS Integrated Pathway Mathematics I, II,* and *III* courses employ a problem-based model of instruction that supports and reinforces the 8 Standards for Mathematical Practice. Although the following table focuses on Problem-Based Tasks, Walch's full programs also include hundreds of additional problems in warm-ups and practices. The Implementation Guide for each PBT highlights two to three SMPs to focus on during implementation and discussion.

CCSS Standards for Mathematical Practice		Relevant Attributes of Walch Integrated Math Resources
1	Make sense of problems and persevere in solving them.	Each sub-lesson is built around a Problem-Based Task (PBT) that requires students to "make sense of problems and persevere in solving them."
2	Reason abstractly and quantitatively.	Each PBT uses a meaningful real-world context that requires students to reason both abstractly about the situation/relationships and quantitatively about the values representing the elements and relationships.
3	Construct viable arguments and critique the reasoning of others.	Since the PBT provides opportunities for multiple problem-solving approaches and varied solutions, students are required to construct viable arguments to support their approach and answer. This, in turn, provides other students the opportunity to analyze and critique their classmates' reasoning.
4	Model with mathematics.	Each PBT represents a real-world situation and requires students to model it with mathematics.
5	Use appropriate tools strategically.	PBTs require students to make choices about using appropriate tools, such as calculators, spreadsheets, graph paper, manipulatives, protractors, and compasses. The tasks do not prescribe specific tools, but instead provide opportunities for their use.
6	Attend to precision.	The real-world contexts of the PBTs require students to be precise in their solutions, both in the ways that the solutions are stated, labeled, and explained, and in the degree of precision necessary given the context (e.g., tripling chili for a crowd vs. machining a part for an airplane engine).
7	Look for and make use of structure.	The PBTs present students with complicated scenarios that must be analyzed to discern patterns and significant mathematical features.
8	Look for and express regularity in repeated reasoning.	PBTs require multiple steps, providing opportunities for students to note repeated calculations, monitor their process, and continually evaluate reasonableness of intermediate results before arriving at a solution.

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Correspondence to CCSS Publishers' Criteria

How Do Walch Integrated Mathematics Resources Address the High School Publishers' Criteria for CCSS Mathematics?

Walch's programs for the *CCSS Integrated Pathway Mathematics I, II,* and *III* courses were designed to the specifications of Appendix A and the Publishers' Criteria. Focus, Coherence, and Rigor represent the fundamental assumptions of the resources.

Publishers' Criteria	Relevant Attributes of Walch Integrated Math Resources
FOCUS Focus strongly where the standards focus.	Each course is built around the units specified in the CCSS for Mathematics Appendix A. Units are organized around important topics in mathematics and reflect the focus of the standards. Units range from 22 sub-lessons for Math I Unit 2, "Linear and Exponential Relationships," which address 21 Common Core standards, to three sub-lessons for Math I Unit 6, "Connecting Algebra and Geometry Through Coordinates," which address only three Common Core standards.
COHERENCE Think across grades/courses, and link to major topics in each course.	Walch Integrated Math programs include two features contributing to coherence. Each sub-lesson identifies necessary prerequisite skills (our forthcoming Support Supplement will point to where they were introduced, including in elementary or middle school, and provide instructional materials for remediation). Warm-Ups tie to these prerequisites in order to activate prior knowledge. Also, the major topics of each course are mapped forward to show where they are revisited and expanded upon in future courses.
RIGOR In major topics, pursue with equal intensity • conceptual understanding, • procedural skill and fluency, and • applications.	The instructional components of Walch's Integrated Math programs guide students from conceptual understanding and procedural skill developed through modeling and guided practice to the application opportunities of the PBT. The resources move students on to additional practice to refine procedural skill and fluency, and later to station activities to crystalize conceptual understanding.

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Correspondence to NCTM *Principles to Actions* Teaching Practices

How Do Walch Integrated Mathematics Resources Address the NCTM *Principles to Actions* Mathematics Teaching Practices?

Walch's programs for the *CCSS Integrated Pathway Mathematics I, II,* and *III* courses were designed by experienced educators and curriculum developers, informed by best-practice research, and refined through an iterative process of implementation and feedback. Together with professional development, these materials support and sustain good teaching practices.

NCTM Mathematics Teaching Practices	Relevant Attributes of Walch Integrated Math Resources
Establish mathematics goals to focus learning.	Each sub-lesson in Walch's programs addresses specified standards which can be used as goals to focus learning. Essential Questions offer further focus.
Implement tasks that promote reasoning and problem solving.	Each sub-lesson in Walch's programs is built around a Problem-Based Task (PBT), set in a meaningful real-world context and designed to promote reasoning and problem solving. The courses include dozens of PBTs as well as warm-up and practice problems.
Use and connect mathematical representations.	Walch's High School Math programs make frequent use of, and connections among and between, equations, tables, and graphs. PBTs often require students to use and connect two or more of these representations, and the representations are modeled through guided practice.
Facilitate meaningful mathematical discourse.	Several features of the programs support mathematical discourse, including warm-up debriefs with connections to upcoming lessons, implementation guides and optional coaching questions for the PBTs, and discussion guides for Station Activities. Explanations of PBT solutions are another opportunity for discourse. Please note: Mathematical discourse is an important topic for professional development, in conjunction with implementation of these materials.
Pose purposeful questions.	The implementation guides, coaching questions and discussion guides provide samples of purposeful questions. Note that this is another important topic for professional development.
Build procedural fluency from conceptual understanding.	The programs develop conceptual understanding through modeling, guided practice, and application, and then provide additional opportunities to practice and develop fluency.
Support productive struggle in learning mathematics.	The PBTs require "productive struggle;" implementation guides include suggestions for facilitation and monitoring, and coaching questions provide an option for additional support as appropriate, allowing students to proceed through the task and ensuring that the struggle remains productive rather than too frustrating.
Elicit and use evidence of student thinking.	Various discussions and PBTs require students to display their thinking. Implementation guides offer specific prompts and suggestions for eliciting and responding to student thinking. Professional development supports teachers in using that evidence to respond in instructionally appropriate ways.

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Unit Structure

All of the instructional units have common features. Each unit begins with a list of all the standards addressed in the lessons, and a list of one or more conceptual activities. Each lesson begins with a pre-assessment, followed by the list of standards addressed in the lesson; Essential Questions; vocabulary (titled "Words to Know"); and a list of recommended websites to be used as additional resources.

Each sub-lesson begins with a list of identified prerequisite skills that students need to have mastered in order to be successful with the new material in the upcoming sub-lesson. This is followed by an introduction, key concepts, common errors/misconceptions, guided practice examples, a problem-based task with coaching questions and sample responses, a closure activity, and practice. Each lesson ends with a progress assessment to evaluate students' learning.

All of the components are described below and on the following pages for your reference.

Standards Correlations and Connections to Future Courses

In this section, you'll find a comprehensive list of the Common Core State Standard(s) addressed in each sub-lesson, followed by a map detailing where and how future courses will build upon the topics introduced in the unit.

Conceptual Activities

Conceptual understanding serves as the foundation on which to build deeper understanding of mathematics. In an effort to build conceptual understanding of mathematical ideas and to provide more than procedural fluency and application, links to interactive open education and Desmos resources are included. (*Note*: These website links will be monitored and repaired or replaced as necessary.)

Pre-Assessment

This can be used to gauge students' prior knowledge and to inform instructional planning.

Common Core State Standards for the Lesson

All standards that are addressed in the entire lesson are listed.

Essential Questions

These are intended to guide students' thinking as they proceed through the lesson. By the end of each lesson, students should be able to respond to the questions.

Words to Know

Vocabulary terms and formulas are provided as background information for instruction or to review key concepts that are addressed in the lesson.

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Recommended Resources

This is a list of websites that can be used as additional resources. Some websites are games; others provide additional examples and/or explanations. The links for these resources are live in the PDF version of the Teacher Resource. (*Note*: These website links will be monitored and repaired or replaced as necessary.)

Common Core State Standards for the Sub-Lesson

When lessons are broken down into sub-lessons, the specific standard or standards that are addressed are presented at the beginning of the instructional portion of the lesson.

Warm-Up

Each warm-up takes approximately 5 minutes and addresses either prerequisite and critical-thinking skills or previously taught math concepts.

Warm-Up Debrief

Each debrief provides the answers to the warm-up questions, and offers suggestions for situations in which students might have difficulties. A section titled Connection to the Lesson is also included in the debrief to help answer students' questions about the relevance of the particular warm-up activity to the upcoming instruction. Warm-Ups with debriefs are also provided in PPT presentations with short video clips to enhance engagement.

Identified Prerequisite Skills

Presented at the beginning of each sub-lesson, this list cites the skills necessary to be successful with the new material.

Introduction

This brief paragraph gives a description of the concepts about to be presented and often contains some Words to Know.

Key Concepts

Provided in bulleted form, this instruction highlights the important ideas and/or processes for meeting the standard.

Common Errors/Misconceptions

This is a list of the common errors students make when applying Key Concepts. This list suggests what to watch for when students arrive at an incorrect answer or are struggling with solving the problems.

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Guided Practice

This section provides step-by-step examples of applying the Key Concepts. The three to five examples are intended to aid during initial instruction, but are also for individuals needing additional instruction and/or for use during review and test preparation. GeoGebra applets that model selected Guided Practice examples are provided on the flash drive that accompanies this program.

Problem-Based Task

This activity can serve as the centerpiece of a problem-based lesson, or it can be used to walk students through the application of the standard, prior to traditional instruction or at the end of instruction. The task makes use of critical-thinking skills.

Problem-Based Task Implementation Guides (Appendix)

This instructional overview, found in an appendix at the end of each unit, highlights connections between the task and the sub-lesson's key concepts and SMPs. It also offers suggestions for facilitating and monitoring, and provides alternative solutions.

Optional Problem-Based Task Coaching Questions

These questions scaffold the task and guide students to solving the problem(s) presented in the task. They should be used at the discretion of the teacher for students requiring additional support.

Problem-Based Task Coaching Questions Sample Responses

These are the answers and suggested appropriate responses to the coaching questions. In some cases answers may vary, but a sample answer is given for each question.

Recommended Closure Activity

Students are given the opportunity to synthesize and reflect on the lesson through a journal entry or discussion of one or more of the Essential Questions.

Practice

Each sub-lesson includes practice problems to support students' achievement of the learning objectives. These sheets are written for the student. They can be used in any combination of teacher-led instruction, cooperative learning, or independent application of knowledge.

Progress Assessment

Each lesson ends with 10 multiple-choice questions, as well as one extended-response question that incorporates critical thinking and writing components. This can be used to document the extent to which students grasp the concepts and skills addressed during instruction.

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Unit Assessment

Each unit ends with 12 multiple-choice questions and three extended-response questions that incorporate critical thinking and writing components. This can be used to document the extent to which students grasped the concepts and skills of each unit.

Answer Key

Answers for all of the Warm-Ups, Assessments, and Practice problems from the Teacher Resource and all of the problems from the Student Resource Book are provided at the end of each unit. (Student editions include odd answers for the exercises in the student book.)

Graphing Calculators

Step-by-step instructions for using a TI-Nspire and a TI-83/84 are provided whenever graphing calculators are referenced.

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Standards Correlations

Each lesson in this Integrated Pathway: Mathematics I program was written specifically to address the Common Core State Standards. Each lesson lists the standards covered in all the sub-lessons, and each sub-lesson lists the standards addressed in that particular section. In this section, you'll find a comprehensive list mapping the sub-lessons to the CCSS.

Guide to Common Core State Standards Annotation

As you use this program, you will come across a symbol included with the Common Core standards for some of the lessons and activities. These symbols are explained below.

Symbol: ⋆

Denotes: Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (⋆).

From http://www.walch.com/CCSS/00003

Symbol: (+)

Denotes: College and Career Readiness Standards

Advanced mathematics standards that are required in higher-level courses such as advanced statistics may also be included in lower-level courses. These additional standards are denoted by (+). According to the Common Core State Standards Initiative, "the evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6–8."

From http://www.walch.com/CCSS/00004

Connections to Future Courses

This section provides a map between topics introduced in each unit of this course and subsequent courses where each topic is revisited and built upon.

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CCSS INTEGRATED PATHWAY: MATHEMATICS I STANDARDS CORRELATIONS

Unit 1: Relationships Between Quantities				
Lesson	**Sub-lesson number**	**Title**	**Standard(s)**	**Pages**
Lesson 1	**Interpreting Structure in Expressions**			
	1.1.1	Identifying Terms, Factors, and Coefficients	A–SSE.1a★	U1-5
	1.1.2	Interpreting Linear and Exponential Expressions	A–SSE.1b★	U1-17
Lesson 2	**Creating Equations and Inequalities in One Variable**			
	1.2.1	Creating Linear Equations in One Variable	A–CED.1★ N–Q.2★ N–Q.3★	U1-33
	1.2.2	Creating Linear Inequalities in One Variable	A–CED.1★	U1-54
	1.2.3	Creating Exponential Equations	A–CED.1★	U1-69
Lesson 3	**Creating and Graphing Equations in Two Variables**			
	1.3.1	Creating and Graphing Linear Equations in Two Variables	A–CED.2★ N–Q.1★	U1-94
	1.3.2	Creating and Graphing Exponential Equations	A–CED.2★ N–Q.1★	U1-129
Lesson 4	**Representing Constraints**			
	1.4.1	Representing Constraints	A–CED.3★	U1-157
Lesson 5	**Rearranging Formulas**			
	1.5.1	Rearranging Formulas	A–CED.4★	U1-177

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CCSS INTEGRATED PATHWAY: MATHEMATICS I STANDARDS CORRELATIONS

Unit 2: Linear and Exponential Relationships				
Lesson	**Sub-lesson number**	**Title**	**Standard(s)**	**Pages**
Lesson 1	**Graphs As Solution Sets and Function Notation**			
	2.1.1	Graphing the Set of All Solutions	A–REI.10	U2-4
	2.1.2	Intersecting Graphs	A–REI.11★	U2-21
	2.1.3	Domain and Range	F–IF.1	U2-36
	2.1.4	Function Notation and Evaluating Functions	F–IF.2	U2-55
Lesson 2	**Solving Linear Inequalities in Two Variables and Systems of Inequalities**			
	2.2.1	Solving Linear Inequalities in Two Variables	A–REI.12	U2-79
	2.2.2	Solving Systems of Linear Inequalities	A–REI.12	U2-103
Lesson 3	**Sequences As Functions**			
	2.3.1	Sequences As Functions	F–IF.3	U2-135
Lesson 4	**Interpreting Graphs of Functions**			
	2.4.1	Identifying Key Features of Linear and Exponential Graphs	F–IF.4★ F–IF.5★	U2-164
	2.4.2	Average Rate of Change	F–IF.6★ F–LE.1a★	U2-188
	2.4.3	Recognizing Average Rate of Change	F–IF.6★ F–LE.1b★ F–LE.1c★	U2-205
Lesson 5	**Analyzing Linear and Exponential Functions**			
	2.5.1	Graphing Linear Functions	F–IF.7a★	U2-238
	2.5.2	Graphing Exponential Functions	F–IF.7e★	U2-256

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Lesson	Sub-lesson number	Title	Standard(s)	Pages
Lesson 6	**Comparing Functions**			
	2.6.1	Comparing Linear Functions	F–IF.9	U2-287
	2.6.2	Comparing Exponential Functions	F–IF.9	U2-307
	2.6.3	Comparing Linear to Exponential Functions	F–LE.3★	U2-331
Lesson 7	**Building Functions**			
	2.7.1	Building Functions from Context	F–BF.1a★	U2-358
	2.7.2	Constructing Functions from Graphs and Tables	F–LE.2★	U2-378
Lesson 8	**Operating on Functions and Transformations**			
	2.8.1	Operating on Functions	F–BF.1b★	U2-405
	2.8.2	Transformations of Linear and Exponential Functions	F–BF.3	U2-415
Lesson 9	**Arithmetic and Geometric Sequences**			
	2.9.1	Arithmetic Sequences	F–BF.2★	U2-437
	2.9.2	Geometric Sequences	F–BF.2★	U2-448
Lesson 10	**Interpreting Parameters**			
	2.10.1	Interpreting Parameters	F–LE.5★	U2-465

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CCSS INTEGRATED PATHWAY: MATHEMATICS I STANDARDS CORRELATIONS

Unit 3: Reasoning with Equations				
Lesson	**Sub-lesson number**	**Title**	**Standard(s)**	**Pages**
Lesson 1	**Solving Equations and Inequalities**			
	3.1.1	Properties of Equality	A–REI.1	U3-3
	3.1.2	Solving Linear Equations	A–REI.3	U3-19
	3.1.3	Solving Linear Inequalities	A–REI.3	U3-34
	3.1.4	Solving Exponential Equations	A–REI.1	U3-46
Lesson 2	**Solving Systems of Equations**			
	3.2.1	Solving Systems of Linear Equations by Substitution and Elimination	A–REI.5 A–REI.6	U3-68
	3.2.2	Solving Systems of Linear Equations by Graphing	A–REI.6	U3-87

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CCSS INTEGRATED PATHWAY: MATHEMATICS I STANDARDS CORRELATIONS

Unit 4: Descriptive Statistics				
Lesson	**Sub-lesson number**	**Title**	**Standard(s)**	**Pages**
Lesson 1	**Working with a Single Measurement Variable**			
	4.1.1	Representing Data Sets	S–ID.1★	U4-7
	4.1.2	Comparing Data Sets	S–ID.2★	U4-29
	4.1.3	Interpreting Data Sets	S–ID.3★	U4-50
Lesson 2	**Working with Two Categorical and Quantitative Variables**			
	4.2.1	Summarizing Data Using Two-Way Frequency Tables	S–ID.5★	U4-77
	4.2.2	Solving Problems Given Functions Fitted to Data	S–ID.6a★	U4-95
	4.2.3	Analyzing Residuals	S–ID.6b★	U4-118
	4.2.4	Fitting Linear Functions to Data	S–ID.6c★	U4-141
Lesson 3	**Interpreting Linear Models**			
	4.3.1	Interpreting Slope and *y*-intercept	S–ID.7★	U4-173
	4.3.2	Calculating and Interpreting the Correlation Coefficient	S–ID.8★	U4-194
	4.3.3	Distinguishing Between Correlation and Causation	S–ID.9★	U4-211

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Unit 5: Congruence, Proof, and Constructions				
Lesson	**Sub-lesson number**	**Title**	**Standard(s)**	**Pages**
Lesson 1	**Introducing Transformations**			
	5.1.1	Defining Terms	G–CO.1	U5-5
	5.1.2	Transformations As Functions	G–CO.2	U5-18
	5.1.3	Applying Lines of Symmetry	G–CO.3	U5-38
Lesson 2	**Defining and Applying Rotations, Reflections, and Translations**			
	5.2.1	Defining Rotations, Reflections, and Translations	G–CO.4	U5-60
	5.2.2	Applying Rotations, Reflections, and Translations	G–CO.5	U5-72
Lesson 3	**Constructing Lines, Segments, and Angles**			
	5.3.1	Copying Segments and Angles	G–CO.12	U5-94
	5.3.2	Bisecting Segments and Angles	G–CO.12	U5-118
	5.3.3	Constructing Perpendicular and Parallel Lines	G–CO.12	U5-142
Lesson 4	**Constructing Polygons**			
	5.4.1	Constructing Equilateral Triangles Inscribed in Circles	G–CO.13	U5-175
	5.4.2	Constructing Squares Inscribed in Circles	G–CO.13	U5-199
	5.4.3	Constructing Regular Hexagons Inscribed in Circles	G–CO.13	U5-219
Lesson 5	**Exploring Congruence**			
	5.5.1	Describing Rigid Motions and Predicting the Effects	G–CO.6	U5-254
	5.5.2	Defining Congruence in Terms of Rigid Motions	G–CO.6	U5-277
Lesson 6	**Congruent Triangles**			
	5.6.1	Triangle Congruency	G–CO.7	U5-312
	5.6.2	Explaining ASA, SAS, and SSS	G–CO.8	U5-332

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CCSS INTEGRATED PATHWAY: MATHEMATICS I STANDARDS CORRELATIONS

Unit 6: Connecting Algebra and Geometry Through Coordinates				
Lesson	**Sub-lesson number**	**Title**	**Standard(s)**	**Pages**
Lesson 1	**Slope and Distance**			
	6.1.1	Using Coordinates to Prove Geometric Theorems with Slope and Distance	G–GPE.4 G–GPE.5	U6-4
	6.1.2	Working with Parallel and Perpendicular Lines	G–GPE.5	U6-32
Lesson 2	**Lines and Line Segments**			
	6.2.1	Calculating Perimeter and Area	G–GPE.7★	U6-58

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TOPICS FROM FUTURE COURSES: CCSS INTEGRATED PATHWAY MATHEMATICS I

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 1	Identifying and interpreting the effects of terms, factors, and coefficients of linear, exponential, and quadratic expressions	Math II Unit 3	includes more complex quadratic expressions and introduces the standard form of a quadratic expression
		Math III Units 1 and 2A	extended to include higher-order polynomials
		Math III Unit 2B	extended to rational expressions
	Creating and graphing with linear and simple exponential equations	Math II Unit 2	creating and graphing functions by using quadratic, square root, cube root, absolute value, step, and piecewise functions
		Math II Unit 3	extended to quadratic and rational functions and to graphing higher-order polynomial functions
		Math III Unit 2A	extended to rational and radical functions
		Math III Unit 2B	introduces students to graphing logarithmic and periodic functions
		Math III Unit 4A	addressed using more complex exponential equations
		Math III Unit 4B	extended to applying the creating and graphing principles
	Constraints in terms of linear relationships	Math II Unit 3, Math III Unit 4B	addressed using linear, exponential, quadratic, and logarithmic models
	Rearranging linear formulas	Math II Unit 3	extended to rearranging quadratic expressions
		Math III Unit 4B	extended to rearranging all function families

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TOPICS FROM FUTURE COURSES: CCSS INTEGRATED PATHWAY MATHEMATICS I

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 2	Function notation (concepts regarding domain, range, intersecting functions, and graphing solution sets)	Math II Unit 2	extended to describing transformations of functions
		Math III Unit 4B	includes combinations of functions
	Linear inequalities	Math II Unit 3	extended to quadratic inequalities and rational inequalities
		Math III Unit 4B	extended to linear, exponential, quadratic, and logarithmic functions
	Sequences	Math III Unit 2A	applied to more complex sequences and series
	Analyzing the features of graphs of linear functions	Math II Unit 2	extended to exponential and quadratic functions
		Math III Unit 2A	applied to polynomial functions
		Math III Unit 2B	extended to periodic functions
		Math III Unit 4A	expanded to logarithmic functions
	Domain	Math IV	restricts the domain of trigonometric functions in order to find the inverses of these functions

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TOPICS FROM FUTURE COURSES: CCSS INTEGRATED PATHWAY MATHEMATICS I

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 3	Reasoning with equations (linear equations)	Math II Unit 3	extended to quadratic functions
		Math III Unit 2B	extended to rational and radical functions
	Systems of equations (linear-linear)	Math II Unit 3	includes linear-quadratic systems
		Math III Unit 2A	extended to linear-polynomial, polynomial-polynomial, and absolute value-polynomial systems
		Math III Unit 2B	extended to linear-rational and linear-radical systems
		Math IV	includes systems with vectors and matrices
Unit 4	Descriptive statistics	Math III Unit 1	extended to using statistics to draw conclusions

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TOPICS FROM FUTURE COURSES: CCSS INTEGRATED PATHWAY MATHEMATICS I

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 5	Transformations and connections between functions and geometric transformations; congruence	Math II Unit 5	includes similarity transformations
		Math II Unit 6	extended to coordinate geometry and proofs
	Perimeter and area	Math II Unit 6	extended to area and volume
		Math III Unit 4B	includes density and cross sections of three-dimensional objects
	Geometric figures (points, lines, and planes)	Math II Unit 5	extended to parallel lines cut by a transversal and other relationships between angle pairs, other special triangles, and parallelograms
		Math II Unit 6	includes circles and special segments
		Math III Unit 4B	extended to three-dimensional objects in modeling and design
Unit 6	Connecting algebra and geometry	Math II Unit 2	extended to transformations of quadratic functions
		Math III Unit 3	extended to proofs of the laws of sines and cosines
		Math III Units 3 and 4A	includes transformations of families of functions, evaluating trigonometric functions, and graphing trigonometric functions

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Conceptual Activities

Use these interactive open education and/or Desmos resources to build conceptual understanding of mathematical ideas. (*Note*: Activity links will be monitored and repaired or replaced as necessary.)

Unit 1

- Desmos. "Function Carnival."

  http://www.walch.com/ca/01006

  This activity focuses attention on graphs as expressing relationships between variables. It lays the informal groundwork for the more formal definitions and properties of functions.

- Desmos. "Marbleslides: Lines."

  http://www.walch.com/ca/01008

  Restrict, reposition, and rotate lines at will using slope-intercept form, and describe transformations using words and/or symbols.

- Desmos. "Put the Point on the Line."

  http://www.walch.com/ca/01009

  The focus of this activity is slope. Participants are asked to estimate, calculate, and notice proportionality as they place points on an imaginary line.

- Desmos, Inc. "Polygraph: Linear Inequalities."

  http://www.walch.com/ca/10000

  In this activity, students will engage in vocabulary-rich conversations about linear inequalities. Key vocabulary terms that may appear in student questions include *shading*, *above*, *below*, *boundary*, *solid*, *dotted*, *horizontal*, *vertical*, *slanted*, *axis*, and *quadrant*.

Unit 2

- Desmos. "Avi and Benita's Repair Shop."

  http://www.walch.com/ca/01014

  Compare linear and exponential growth in the context of daily payments. One plan increases by $100 each day, while another grows by doubling the previous day's payment. This activity is appropriate for students who have studied linear functions but may not have an experience with exponential growth.

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- Desmos. "Card Sort: Functions."

  http://www.walch.com/ca/01005

  Sort graphs, equations, and contexts according to whether each one represents a function.

- Desmos. "Card Sort: Linear Functions."

  http://www.walch.com/ca/01010

  Notice and use properties of linear functions to make groups of three. Different properties will lead to different groupings by different participants.

- Desmos. "Function Carnival, Part 2."

  http://www.walch.com/ca/01007

  This activity follows up on "Function Carnival" by using the contexts in that activity to develop an understanding of function notation.

- Desmos. "Game, Set, Flat."

  http://www.walch.com/ca/01015

  Develop understanding of the exponential relationship that describes a bouncing tennis ball. Learn to examine successive terms in a sequence to determine if it represents an exponential relationship or not, and how to construct the exponential equation itself.

- Desmos. "Marbleslides: Exponentials."

  http://www.walch.com/ca/01016

  Restrict, reposition, and otherwise transform exponential curves at will by modifying the basic form $y = b^x$, and use precision in describing these transformations using words and/or symbols.

- Desmos. "Match My Line."

  http://www.walch.com/ca/01013

  Work through a series of scaffolded linear graphing challenges to develop proficiency with direct variation, slope-intercept, point-slope, and other linear function forms.

- Desmos. "Polygraph: Exponentials."

  http://www.walch.com/ca/01019

  This Custom Polygraph is designed to spark vocabulary-rich conversations about exponentials, including how they differ from linear functions. Key vocabulary terms that may appear in student questions include *increasing*, *decreasing*, *intercept*, *rate*, *asymptote*, and *curve*.

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- Desmos. "What Comes Next?"

  http://www.walch.com/ca/01020

  Predict "what comes next" for linear and exponential functions based first on graphs and then on tables of values, then explore connections between graphs, tables, and equations of linear and exponential functions.

Unit 3

- Desmos. "Card Sort: Linear Systems."

  http://www.walch.com/ca/01000

  In this activity, students practice what they've learned about solving systems of linear equations. The activity begins with a review of the graphical meaning of a solution to a system. Later, students consider which algebraic method is most efficient for solving a given system. Finally, students practice solving equations using substitution and elimination. Prior to beginning this activity, students should have experience solving systems of linear equations graphically and algebraically.

- Desmos. "The Intersection."

  http://www.walch.com/ca/01011

  Predict the point of intersection for a system of two linear equations: first without a grid, then with one. With the grid in play, participants are able to use the slope of the lines (formally or informally) to improve the accuracy of their predictions.

- Desmos. "Solutions to Systems of Linear Equations."

  http://www.walch.com/ca/01001

  This activity will help students understand what it means for a point to be a solution to a system of equations—both graphically and algebraically.

- Desmos. "Systems of Two Linear Equations."

  http://www.walch.com/ca/01002

  This resource gives a progression of written explanations, equations, and graphs to explain what the algebraic or graphical solution to a system of equations represents.

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Unit 4

- Desmos. "LEGO Prices."

  http://www.walch.com/ca/01012

  Use the concept of linear regression to predict the cost of a LEGO set with *x* pieces. (This activity does NOT use the calculator, just the concept. Participants draw the line on the graph, and Desmos calculates the equation.)

- Desmos. "Predicting Movie Ticket Prices."

  http://www.walch.com/ca/01018

  Build a model to describe the relationship between average movie ticket prices and time, then use that model to make predictions about past and future ticket prices. Participants also interpret the parameters of their equation in context.

- Desmos. "Polygraph: Histograms."

  http://www.walch.com/ca/01024

  This activity is designed to spark vocabulary-rich conversations about histograms. Key vocabulary terms that may appear in student questions include *shape*, *center*, *spread*, *roughly symmetric*, *skew right*, *skew left*, *mean*, *median*, *range*, *peak*, *unimodal*, and *bimodal*.

Unit 5

- Desmos. "Polygraph: Transformations."

  http://www.walch.com/ca/01025

  This activity is designed to spark vocabulary-rich conversations about transformations. Key vocabulary terms that may appear in student questions include *translation*, *rotation*, *reflection*, *dilation*, *scale factor*, *preimage*, and *image*.

- Desmos. "Symmetry."

  http://www.walch.com/ca/01026

  In this activity, students develop an informal understanding of symmetry of functions. By the end, students should be able to identify the symmetry of a function (reflectional vs. rotational) by considering its graph.

Unit 6

- Illustrative Mathematics. "A Midpoint Miracle."

  http://www.walch.com/ca/10003

  Students will prove a theorem about quadrilaterals that is somewhat difficult to prove with a straightedge and ruler but relatively easy to prove using coordinates. This task requires that students be comfortable using the formula for the midpoint of a line segment and the parallel line criterion.

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Station Activities Guide

Introduction

Each unit includes a collection of station-based activities to provide students with opportunities to practice and apply the mathematical skills and concepts they are learning. You may use these activities in addition to the instructional lessons, or, especially if the pre-test or other formative assessment results suggest it, instead of direct instruction in areas where students have the basic concepts but need practice. The debriefing discussions after each set of activities provide an important opportunity to help students reflect on their experiences and synthesize their thinking. Debriefing also provides an additional opportunity for ongoing, informal assessment to guide instructional planning.

Implementation Guide

The following guidelines will help you prepare for and use the activity sets in this section.

Setting Up the Stations

Each activity set consists of four or five stations. Set up each station at a desk, or at several desks pushed together, with enough chairs for a small group of students. Place a card with the number of the station on the desk. Each station should also contain the materials specified in the teacher's notes, and a stack of student activity sheets (one copy per student). Place the required materials (as listed) at each station.

When a group of students arrives at a station, each student should take one of the activity sheets to record the group's work. Although students should work together to develop one set of answers for the entire group, each student should record the answers on his or her own activity sheet. This helps keep students engaged in the activity and gives each student a record of the activity for future reference.

Forming Groups of Students

All activity sets consist of four or five stations. You might divide the class into four or five groups by having students count off from 1 to 4 or 5. If you have a large class and want to have students working in small groups, you might set up two identical sets of stations, labeled A and B. In this way, the class can be divided into eight groups, with each group of students rotating through the "A" stations or "B" stations.

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Assigning Roles to Students

Students often work most productively in groups when each student has an assigned role. You may want to assign roles to students when they are assigned to groups and change the roles occasionally. Some possible roles are as follows:

- Reader—reads the steps of the activity aloud
- Facilitator—makes sure that each student in the group has a chance to speak and pose questions; also makes sure that each student agrees on each answer before it is written down
- Materials Manager—handles the materials at the station and makes sure the materials are put back in place at the end of the activity
- Timekeeper—tracks the group's progress to ensure that the activity is completed in the allotted time
- Spokesperson—speaks for the group during the debriefing session after the activities

Timing the Activities

The activities in this section are designed to take approximately 10 minutes per station. Therefore, you might plan on having groups change stations every 10 minutes, with a two-minute interval for moving from one station to the next. It is helpful to give students a "5-minute warning" before it is time to change stations.

Since each activity set consists of four or five stations, the above time frame means that it will take about 50 to 60 minutes for groups to work through all stations.

Guidelines for Students

Before starting the first activity set, you may want to review the following "ground rules" with students. You might also post the rules in the classroom.

- All students in a group should agree on each answer before it is written down. If there is a disagreement within the group, discuss it with one another.
- You can ask your teacher a question only if everyone in the group has the same question.
- If you finish early, work together to write problems of your own that are similar to the ones on the activity sheet.
- Leave the station exactly as you found it. All materials should be in the same place and in the same condition as when you arrived.

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Debriefing the Activities

After each group has rotated through every station, bring students together for a brief class discussion. At this time, you might have the groups' spokespersons pose any questions they had about the activities. Before responding, ask if students in other groups encountered the same difficulty or if they have a response to the question. The class discussion is also a good time to reinforce the essential ideas of the activities. The questions that are provided in the teacher's notes for each activity set can serve as a guide to initiating this type of discussion.

You may want to collect the student activity sheets before beginning the class discussion. However, it can be beneficial to collect the sheets afterward so that students can refer to them during the discussion. This also gives students a chance to revisit and refine their work based on the debriefing session. If you run out of time to hold class discussions, you might want to have students journal about their experiences and follow up with a class discussion the next day.

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Digital Enhancements Guide

Introduction

With the *Common Core State Standards Integrated Pathway: Mathematics I Program*, you have access to the following digital components, described here with guidelines and suggestions for implementation.

Digital Warm-Ups

Each of the warm-ups in this collection includes a video clip to enhance student engagement. Hard copies of all warm-ups are also included in the Teacher Resource. These optional versions of the warm-ups and debriefs run on PowerPoint. (*Please note*: Computers may render PowerPoint images differently. For best viewing and display, use a PowerPoint Viewer and adjust your settings to optimize images and text.)

We suggest that the video clip be played as students enter the class. The green "play" button on the first PowerPoint slide will link your computer to a browser and then to the video, so you will need an Internet connection. Make sure that you use the "full screen" option for display and adjust the volume appropriately. The video clips have been selected to catch students' attention by addressing the context (not the content) of the warm-up problem in an engaging way.

After the video, each warm-up takes approximately 5 minutes and addresses critical-thinking skills and/or previously taught math concepts that are prerequisites for the new content in the lesson.

The warm-ups include debriefs that reveal the answers to the warm-up questions. The answers slide into view and can be used to facilitate discussion.

In the notes section, you will find the URL for the video and "Connections to the Lesson," which describes concepts students will glean or skills they will need in the upcoming lesson. The "Connections" help transition from the warm-up to instruction.

GeoGebra Applets

In addition to the hard copy materials found in the Teacher Resource, two interactive GeoGebra applets are provided for most lessons. The applets model the mathematics in two of the Guided Practice examples. Links to all of the applets can be found on the flash drive for this program. With an Internet connection, simply click on the link identified for the lesson and example.

Please adjust your view to maximize the image. Each applet illustrates the specific problem addressed in the Guided Practice example. The applets allow you to walk through the solution by visually demonstrating the steps, such as defining points and drawing lines. Variable components of the applets (usually fill-in boxes or sliders) allow you to substitute different values in order to explore

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the mathematics. For example, "What happens to the line when we increase the amount of time?" or "What if we cut the number of students in half?" This experimentation and discussion supports development of conceptual understanding.

GeoGebra for PC/MAC

GeoGebra is not required for using the applets, but can be downloaded for free for further exploration at the following link:

http://www.geogebra.org/cms/en/download

GeoGebra Applet Troubleshooting

If you are experiencing any difficulty in using the applets in your browser, please visit the following link for our troubleshooting document.

http://www.walch.com/applethelp

ExamView Assessment Banks

These are available with sufficient items to compile up to 5 versions of each pre-, progress, and unit assessment in the program. ExamView items may also be used to generate additional worksheets for practice.

For information about ExamView, please contact Customer Service at (800) 341-6094 or customerservice@walch.com.

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Standards for Mathematical Practice Implementation Guide

Introduction

The eight Standards for Mathematical Practice describe features of lesson design, teaching pedagogy, and student actions that will lead to a true conceptual understanding of the mathematics standards. The Walch lessons, practice problems, and Problem-Based Tasks lend themselves to teaching through this framework. When the Walch resources are combined with high-level questioning and engaging teacher decisions in the classroom, it will lead to high-level math instruction and student achievement.

Here is a brief description of the SMPs and how they can be applied in the classroom:

CCSS.MP.1: Make sense of problems and persevere in solving them.

Students will read, interpret, and understand complicated mathematical and real-world problems, and they will be willing to try multiple methods with the ultimate goal of determining the correct answer. Strategies such as annotation and student discourse can lead to improvement on this standard. Presenting students with higher-level problems is essential to ensuring students achieve maximum understanding. Teacher prompts that can enhance this standard include:

- What is the problem asking you to solve?
- What are some (other) strategies you could use to solve this problem?
- Compare your answer with a classmate's answer. Who is correct? Why?

CCSS.MP.2: Reason abstractly and quantitatively.

Mathematical reasoning with numbers and variables is essential to understanding the connections among the standards. Students must be able to discover and formalize general rules using numbers and variables, and apply them to determine numerical quantities in other situations. Teacher prompts that can enhance this standard include:

- Substitute realistic numbers into the situation.
- What operation/strategy would you use?
- Will your strategy work for any number?
- For which categories of numbers (negative integers, all real numbers, etc.) will your strategy work?

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CCSS.MP.3: Construct viable arguments and critique the reasoning of others.

Many students are most concerned with the "what" aspects of mathematics, i.e. "what" do we do or "what" is the answer. However, math educators must develop the "why" of mathematics. Students must learn to question algorithms, challenge answers, and justify their reasoning in order to truly understand the concepts behind their answers. Teacher prompts that can enhance this standard include:

- How did you determine your answer?
- Why did you choose that strategy?
- Defend your answer based on a real-world situation.

CCSS.MP.4: Model with mathematics.

An important goal of mathematics instruction is for students to be able to apply mathematics to the world around them. Students should be able to link a real problem to a mathematical concept, identify quantities that are modeled well with mathematics, and use mathematics to find a solution. Emphasizing this standard will help students represent and interpret information using physical, visual, and abstract models. Encourage students to use any or all of their learning experiences to gain a deep and flexible understanding of mathematics. Teacher prompts that can enhance this standard include:

- Can you represent this situation with a visual model?
- How will it help you solve the problem?
- What information is needed to solve this problem?
- Is there another way to solve this problem?
- While working to solve this problem, what do you notice/wonder?

CCSS.MP.5: Use appropriate tools strategically.

There are many available tools suitable for mathematics, such as calculators, manipulatives, formulas, rulers, computers, and developed mathematical strategies. Choosing and using the correct tool to work through a problem is an important skill for mathematicians. Teacher prompts that can enhance this standard include:

- Can you graph this equation in the calculator to see a relationship?
- What formula or strategy might help you determine the answer to this question?
- How can you represent the situation using handheld tools (rulers, protractors, etc.) to determine an answer?

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CCSS.MP.6: Attend to precision.

When using mathematics to solve problems, an answer can be considered correct only if it is sufficiently precise and accurate for the situation to which it pertains. When applying mathematics, it is vital to clearly define the question, the reasoning, the answer, and the explanation. Vocabulary, units, numerical responses, and pictures must be represented precisely in questions and answers to ensure that the mathematical solutions represent the true answer to a question. Teacher prompts that can enhance this standard include:

- What does your answer represent in a real-world context?
- Is your answer reasonable based on your initial estimate?
- What units of measure help describe your numerical answer?

CCSS.MP.7: Look for and make use of structure.

Structure, whether geometric, algebraic, statistical, or numerical, is an important aspect of mathematical reasoning that students often overlook. Teachers often explicitly refer to geometric and other visual structures as explanations of mathematical concepts, but algebraic and numerical structures can often be just as important in analyzing and interpreting mathematical situations. These structures yield clues as to the meaning of expressions, equations, graphs, and other representations. As students interpret these structures, they will gain a greater understanding of the mathematical concepts. Teacher prompts that can enhance this standard include:

- What do the characteristics of the graph tell us about the situation?
- What do each of the variables and numbers in the equation/formula represent?
- How are these situations the same and different based on their representations?

CCSS.MP.8: Look for and express regularity in repeated reasoning.

Just as patterns appear in real life, patterns appear throughout the subject of mathematics. Recognizing and applying these patterns, and applying the reasoning contained within, is one of the most important skills teachers can instill in their students. Rather than teaching isolated algorithms to determine answers, have students discover relationships, create their own algorithms, and apply the reasoning to other situations. These skills can be applied throughout their education and will enrich their lives after high school. Teacher prompts that can enhance this standard include:

- What relationship do you notice in the graph/table/numbers?
- Why did you choose to use this process to solve this word problem/equation?
- How can you apply this process in other situations?

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Suggested Pacing Guide

Overview

The sub-lessons in Walch Integrated Math are designed to be covered over one, two, or three days, depending on the depth and complexity of the topic. The pacing guide that follows provides *suggested* pacing, but this should be viewed as a flexible guideline only. If students need an extra day to solidify understanding, a one-day lesson may be spread over two days, or a two-day lesson may be spread over three days. Conversely, if students grasp a concept more quickly than suggested by the pacing guide, it is fine to shorten the duration of the lesson and move on to the next topic.

Each sub-lesson features several components. The estimated time for each component is given in parentheses. Components include: a Warm-Up, Instruction (including an Introduction, Key Concepts, and Guided Practice), a Problem-Based Task, and Practice.

Please note: PowerPoint versions of each sub-lesson's Warm-Up and Instruction are provided. Warm-Up PowerPoints include a video clip and the Warm-Up Debrief. Instructional PowerPoints feature the Introduction, Key Concepts, and selected Guided Practice Examples with links to corresponding GeoGebra applets. Estimated times reflect presentation of these PowerPoint versions and applets.

Guided Practice, Scaffolded Practice, and Practice

For each Guided Practice in the Teacher Resource (TRB), corresponding Scaffolded Practice worksheets are provided in the Student Workbook (SWB). Practice problems may be assigned from the SWB and/or the parallel Student Resource (SRB) worksheet for in-class work or homework assignments. The SRB worksheets are designed to closely resemble the format of the SWB worksheets, but are composed of different problems for additional practice.

Please note: When a sub-lesson is spread across more than one class period, the Guided Practice Examples will also be spread across the same number of class periods. Assign Practice problems that correspond to the Guided Practice Example(s) you select for each day, to ensure appropriate prior instruction.

Suggested Progression of One-, Two-, and Three-Day Sub-Lessons

One Day

- Warm-Up PowerPoint and Debrief (10 minutes)
- Introduction and Key Concepts (10 minutes)
- One or two Guided/Scaffolded Practice Examples, including PowerPoint applet presentation as appropriate (15 minutes)
- Problem-Based Task and discussion (15 minutes)
- Homework: Practice from the SWB or SRB

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Two Days

Day 1

- Warm-Up PowerPoint and Debrief (10 minutes)
- Introduction and Key Concepts (15 minutes)
- Two or three Guided/Scaffolded Practice Examples, including PowerPoint applet presentation as appropriate (15 minutes)
- In-class practice: selected problems from the SWB Practice (10 minutes)
- Homework: selected problems from the SRB Practice

Day 2

- Review Practice problems (10 minutes)
- One or two remaining Guided/Scaffolded Practice Examples, including PowerPoint applet presentation as appropriate (10 minutes)
- Problem-Based Task and discussion (20 minutes)
- In-class practice: the remainder of the SWB Practice (10 minutes)
- Homework: the remainder of the SRB Practice

Three Days

Day 1

- Warm-Up PowerPoint and Debrief (10 minutes)
- Introduction and Key Concepts (25 minutes)
- One or two Guided/Scaffolded Practice Examples, including PowerPoint applet presentation as appropriate (15 minutes)
- Homework: selected problems from the SRB Practice

Day 2

- Review Practice problems (15 minutes)
- Two or three remaining Guided/Scaffolded Practice Examples, including PowerPoint applet presentation as appropriate (15 minutes)
- Problem-Based Task and discussion (20 minutes)
- Homework: the remainder of the SRB Practice

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Day 3

- Review Practice problems (15 minutes)
- In-class practice: selected problems from the SWB Practice (35 minutes)
- Homework: the remainder of the SWB Practice

Assessments and Station Activities

Each lesson includes a Pre-Assessment and a Progress Assessment, and each unit concludes with a Unit Assessment. Units also feature sets of Station Activities to complement instruction.

Pre-Assessments are short, multiple-choice assessments with five problems, designed to evaluate prior knowledge of the upcoming lesson. These are brief and should not factor into a student's grade. Pre-Assessments are often given on the same day as the preceding lesson's Progress Assessment, to prepare for the upcoming lesson.

Progress Assessments include 10 multiple-choice problems and one extended-response problem. Progress Assessments may not take an entire class period. The additional time may be used to review before the assessment, or to begin the next lesson after the assessment.

Unit Assessments include 12 multiple-choice problems and three extended-response problems, and generally require a full class period.

Station Activities generally require a full class period for the students to rotate through each station and then to engage in a class discussion at the end.

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Unit 1: Relationships Between Quantities (24 Days)

Unit Overview: This unit begins with students exploring the relationships between the parts that make up an expression. This leads into using individual expressions to write equations in one variable, then in two variables. Finally, students work with formulas that have multiple variables. This unit begins with definitions, calculations, and graphing, and progresses in depth to reasoning abstractly with variables instead of numbers while still applying mathematical concepts.

Lesson 1: Interpreting Structure in Expressions

Before embarking on an in-depth study of mathematics, students must have a common language for and understanding of how expressions are created, modified, and simplified. In this lesson, students reexamine the terminology used in discussing the fundamental elements of algebra. With this common language, students go on to explore how an expression is affected when its smaller elements are changed. This is the foundation of modeling in algebra—using and manipulating expressions to describe contexts. This lesson lays the groundwork for the rest of the unit, which builds on this common terminology.

Day	Area of study/content	Pages	Standard(s)
Day 1 Day 2	Lesson 1 Pre-Assessment 1.1.1: Identifying Terms, Factors, and Coefficients	**Pre-Assessment** TRB: U1-1 **Instruction/Practice** TRB: U1-5 SRB: U1-3 SWB: 7	A–SSE.1a★
Day 3	1.1.2: Interpreting Linear and Exponential Expressions	TRB: U1-17 SRB: U1-11 SWB: 17	A–SSE.1b★
Day 4	Lesson 1 Progress Assessment Lesson 2 Pre-Assessment	TRB: U1-28 TRB: U1-30	

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Lesson 2: Creating Equations and Inequalities in One Variable

From expressions, students go on to create linear equations and inequalities in one variable that model real-world situations. Students gain practice with translating contexts into symbolic representations by writing linear equations and using these equations to solve problems. Similarly, students build upon these skills as they learn how to identify and translate simple exponential contexts into exponential equations that can be solved. This lesson provides a foundation for working with linear and exponential equations in two variables as seen in the next lesson, and for exploring equations in multiple variables as encountered later in the unit.

Day	Area of study/content	Pages	Standard(s)
Day 5	Station Activities Set 1: Ratios and Proportions	TRB: U1-211 SWB: 99	N–Q.1★ A–CED.1★
Day 6 Day 7	1.2.1: Creating Linear Equations in One Variable	TRB: U1-33 SRB: U1-19 SWB: 27	A–CED.1★ N–Q.2★ N–Q.3★
Day 8	1.2.2: Creating Linear Inequalities in One Variable	TRB: U1-54 SRB: U1-33 SWB: 37	A–CED.1★
Day 9 Day 10	1.2.3: Creating Exponential Equations	TRB: U1-69 SRB: U1-42 SWB: 47	A–CED.1★
Day 11	Station Activities Set 2: Solving Inequalities	TRB: U1-224 SWB: 107	A–CED.1★
Day 12	Lesson 2 Progress Assessment Lesson 3 Pre-Assessment	TRB: U1-86 TRB: U1-89	

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Lesson 3: Creating and Graphing Equations in Two Variables

Students extend their understanding of linear and exponential equations to include equations in two variables. Students see that equations in two variables can be graphed. Students learn to set up a coordinate plane with appropriate axes, scales, and labels, which prepares them for graphing in the remainder of the course and throughout their studies in mathematics.

Day	Area of study/content	Pages	Standard(s)
Day 13 Day 14	1.3.1: Creating and Graphing Linear Equations in Two Variables	TRB: U1-94 SRB: U1-55 SWB: 57	A–CED.2★ N–Q.1★
Day 15 Day 16	1.3.2: Creating and Graphing Exponential Equations	TRB: U1-129 SRB: U1-84 SWB: 69	A–CED.2★ N–Q.1★
Day 17	Lesson 3 Progress Assessment Lesson 4 Pre-Assessment	TRB: U1-149 TRB: U1-155	

Lesson 4: Representing Constraints

Students learn that equations and inequalities can have restrictions, and that solutions may be valid under only certain conditions. This shows students that even though a solution "works" mathematically, it might not be reasonable for the given set of equations or inequalities that arise from a context. This lesson provides the basis for building functions from context and understanding domain and range, which are presented later in the course.

Day	Area of study/content	Pages	Standard(s)
Day 18	1.4.1: Representing Constraints	TRB: U1-157 SRB: U1-99 SWB: 79	A–CED.3★
Day 19	Lesson 4 Progress Assessment	TRB: U1-172	

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Lesson 5: Rearranging Formulas

In this lesson, students use what they have learned about solving equations to rearrange an equation that contains several variables in order to solve it for a given variable.

Day	Area of study/content	Pages	Standard(s)
Day 20 Day 21	Lesson 5 Pre-Assessment 1.5.1: Rearranging Formulas	**Pre-Assessment** TRB: U1-175 **Instruction/Practice** TRB: U1-177 SRB: U1-109 SWB: 89	A–CED.4★
Day 22	Lesson 5 Progress Assessment	TRB: U1-188	
Day 23	Station Activities Set 3: Solving Equations	TRB: U1-235 SWB: 113	A–CED.1★ A–CED.2★
Unit Assessment			
Day 24	Unit Assessment	TRB: U1-190	

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Unit 2: Linear and Exponential Relationships (59 days)

Unit Overview: This unit builds on the concepts of functions that were first introduced in Grade 8. Students extend their understanding of functions to include exponential relationships. Students learn how to analyze and model relationships from contexts, graphs, tables, and equations using what they know about exponential and linear relationships.

Lesson 1: Graphs As Solution Sets and Function Notation

Students investigate the concept of a graph as representing the solution set to an equation in two variables. To describe that graph, students are introduced to using function notation, evaluating functions, identifying domain and range, and analyzing the graphs of two functions that meet at one or more points. This lesson serves to familiarize students with the language and behavior of functions and how to distinguish them from relations. These notions serve as the basis for the remainder of the unit, as students go on to study functions in greater depth.

Day	Area of study/content	Pages	Standard(s)
Day 1 Day 2	Lesson 1 Pre-Assessment 2.1.1: Graphing the Set of All Solutions	**Pre-Assessment** TRB: U2-1 **Instruction/Practice** TRB: U2-4 SRB: U2-3 SWB: 7	A–REI.10
Day 3 Day 4	2.1.2: Intersecting Graphs	TRB: U2-21 SRB: U2-13 SWB: 19	A–REI.11★
Day 5 Day 6	2.1.3: Domain and Range	TRB: U2-36 SRB: U2-21 SWB: 29	F–IF.1
Day 7 Day 8	2.1.4: Function Notation and Evaluating Functions	TRB: U2-55 SRB: U2-34 SWB: 43	F–IF.2
Day 9	Station Activities Set 2: Relations Versus Functions/Domain and Range	TRB: U2-542 SWB: 263	F–IF.1 F–IF.2 F–BF.1a★
Day 10	Lesson 1 Progress Assessment Lesson 2 Pre-Assessment	TRB: U2-69 TRB: U2-73	

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Lesson 2: Solving Linear Inequalities in Two Variables and Systems of Inequalities

Students build on their knowledge of graphs as solutions to equations, and understand the concept that the solution to an inequality is a half plane. Similar to systems of equations, systems of inequalities are used to find the set of points that are valid for both inequalities. Students learn how to graph the solution set of a system of inequalities in two variables.

Day	Area of study/content	Pages	Standard(s)
Day 11 Day 12 Day 13	2.2.1: Solving Linear Inequalities in Two Variables	TRB: U2-79 SRB: U2-42 SWB: 53	A–REI.12
Day 14 Day 15 Day 16	2.2.2: Solving Systems of Linear Inequalities	TRB: U2-103 SRB: U2-57 SWB: 63	A–REI.12
Day 17	Lesson 2 Progress Assessment Lesson 3 Pre-Assessment	TRB: U2-125 TRB: U2-132	

Lesson 3: Sequences As Functions

Students learn what a sequence is and how a sequence can be described as a function either recursively or explicitly. This continues to reinforce the students' comprehension of linear and exponential relationships, and prepares them for studying arithmetic and geometric sequences later in the unit.

Day	Area of study/content	Pages	Standard(s)
Day 18 Day 19 Day 20	2.3.1: Sequences As Functions	TRB: U2-135 SRB: U2-73 SWB: 73	F–IF.3
Day 21	Lesson 3 Progress Assessment Lesson 4 Pre-Assessment	TRB: U2-156 TRB: U2-158	

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Lesson 4: Interpreting Graphs of Functions

Students learn the vocabulary used to describe functions and develop their understanding of this terminology in context. Students learn what to look for when analyzing a function in terms of its graph and the context of the situation that the graph depicts.

Day	Area of study/content	Pages	Standard(s)
Day 22 Day 23 Day 24	2.4.1: Identifying Key Features of Linear and Exponential Graphs	TRB: U2-164 SRB: U2-89 SWB: 83	F–IF.4★ F–IF.5★
Day 25 Day 26	2.4.2: Average Rate of Change	TRB: U2-188 SRB: U2-106 SWB: 99	F–IF.6★ F–LE.1a★
Day 27 Day 28	2.4.3: Recognizing Average Rate of Change	TRB: U2-205 SRB: U2-115 SWB: 109	F–IF.6★ F–LE.1b★ F–LE.1c★
Day 29	Lesson 4 Progress Assessment Lesson 5 Pre-Assessment	TRB: U2-224 TRB: U2-232	

Lesson 5: Analyzing Linear and Exponential Functions

Students learn how to graph linear and exponential functions, as well as how to identify the intercepts and asymptotes (for exponential functions). This lays the foundation for comparing the attributes of two functions later in the unit.

Day	Area of study/content	Pages	Standard(s)
Day 30 Day 31	2.5.1: Graphing Linear Functions	TRB: U2-238 SRB: U2-129 SWB: 121	F–IF.7a★
Day 32 Day 33	2.5.2: Graphing Exponential Functions	TRB: U2-256 SRB: U2-140 SWB: 131	F–IF.7e★
Day 34	Station Activities Set 3: Comparing Exponential Models	TRB: U2-551 SWB: 269	F–IF.2 F–IF.7d★ (+) F–IF.7e★
Day 35	Lesson 5 Progress Assessment Lesson 6 Pre-Assessment	TRB: U2-276 TRB: U2-282	

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Lesson 6: Comparing Functions

Students compare attributes of linear functions and then exponential functions that are presented in different ways; i.e., as tables, graphs, and/or equations. Then, students analyze linear functions and compare them to exponential functions. This skill prepares students for building functions from real-world contexts in the next lesson.

Day	Area of study/content	Pages	Standard(s)
Day 36	2.6.1: Comparing Linear Functions	TRB: U2-287 SRB: U2-156 SWB: 141	F–IF.9
Day 37	2.6.2: Comparing Exponential Functions	TRB: U2-307 SRB: U2-169 SWB: 157	F–IF.9
Day 38	2.6.3: Comparing Linear to Exponential Functions	TRB: U2-331 SRB: U2-185 SWB: 173	F–LE.3★
Day 39	Lesson 6 Progress Assessment Lesson 7 Pre-Assessment	TRB: U2-347 TRB: U2-354	

Lesson 7: Building Functions

Students create linear and exponential equations to model the relationship between two quantities given a context, a graph, or a table of values. This readies students to operate on functions by using transformations.

Day	Area of study/content	Pages	Standard(s)
Day 40 Day 41	2.7.1: Building Functions from Context	TRB: U2-358 SRB: U2-196 SWB: 183	F–BF.1a★
Day 42 Day 43	2.7.2: Constructing Functions from Graphs and Tables	TRB: U2-378 SRB: U2-207 SWB: 193	F–LE.2★
Day 44	Lesson 7 Progress Assessment Lesson 8 Pre-Assessment	TRB: U2-396 TRB: U2-400	

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Lesson 8: Operating on Functions and Transformations

Students perform operations with functions and then analyze the effects of vertical translations on linear and exponential functions. This experience allows students to perform more complex operations and transformations in later units.

Day	Area of study/content	Pages	Standard(s)
Day 45	2.8.1: Operating on Functions	TRB: U2-405 SRB: U2-219 SWB: 203	F–BF.1b★
Day 46	2.8.2: Transformations of Linear and Exponential Functions	TRB: U2-415 SRB: U2-224 SWB: 213	F–BF.3
Day 47	Station Activities Set 4: Interpreting Exponential Functions	TRB: U2-567 SWB: 277	F–IF.2 F–IF.7e★
Day 48	Lesson 8 Progress Assessment Lesson 9 Pre-Assessment	TRB: U2-429 TRB: U2-434	

Lesson 9: Arithmetic and Geometric Sequences

Students build on their earlier work with sequences as functions to extend, create, and analyze arithmetic and geometric sequences.

Day	Area of study/content	Pages	Standard(s)
Day 49 Day 50	2.9.1: Arithmetic Sequences	TRB: U2-437 SRB: U2-235 SWB: 225	F–BF.2★
Day 51 Day 52	2.9.2: Geometric Sequences	TRB: U2-448 SRB: U2-240 SWB: 235	F–BF.2★
Day 53	Station Activities Set 5: Sequences	TRB: U2-575 SWB: 281	F–BF.2★ F–LE.2★
Day 54	Lesson 9 Progress Assessment Lesson 10 Pre-Assessment	TRB: U2-460 TRB: U2-462	

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Lesson 10: Interpreting Parameters

Students understand the impact of changing the parameters of linear equations in slope-intercept form and exponential functions in general form. They examine how those parameters relate to the context of the problem, the graph, and the equation.

Day	Area of study/content	Pages	Standard(s)
Day 55 Day 56	2.10.1: Interpreting Parameters	TRB: U2-465 SRB: U2-248 SWB: 245	F–LE.5★
Day 57	Lesson 10 Progress Assessment	TRB: U2-481	
Day 58	Station Activities Set 1: Comparing Linear Models	TRB: U2-529 SWB: 255	A–CED.2★ A–REI.10 A–REI.11★ F–IF.7a★
Unit Assessment			
Day 59	Unit Assessment	TRB: U2-483	

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Unit 3: Reasoning with Equations (23 Days)

Unit Overview: This unit combines the terminology developed in Unit 1 with the algebraic skills learned in Unit 2 to guide students to solve more challenging linear equations, pushing them beyond the two-step equations that were first learned in Grade 8. Students also extend what they know about linear equations and apply this understanding toward solving linear inequalities. The unit then delves deeper into linear equations by exploring how systems of linear equations are constructed, solved, and graphed.

Lesson 1: Solving Equations and Inequalities

Students further develop their ability to solve linear equations by solving equations of greater difficulty. Afterward, students go on to solve linear inequalities and exponential equations.

Day	Area of study/content	Pages	Standard(s)
Day 1 Day 2	Lesson 1 Pre-Assessment 3.1.1: Properties of Equality	**Pre-Assessment** TRB: U3-1 **Instruction/Practice** TRB: U3-3 SRB: U3-3 SWB: 7	A–REI.1
Day 3 Day 4 Day 5	3.1.2: Solving Linear Equations	TRB: U3-19 SRB: U3-11 SWB: 17	A–REI.3
Day 6 Day 7 Day 8	3.1.3: Solving Linear Inequalities	TRB: U3-34 SRB: U3-21 SWB: 27	A–REI.3
Day 9 Day 10 Day 11	3.1.4: Solving Exponential Equations	TRB: U3-46 SRB: U3-27 SWB: 37	A–REI.3
Day 12	Lesson 1 Progress Assessment Lesson 2 Pre-Assessment	TRB: U3-61 TRB: U3-63	

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Lesson 2: Solving Systems of Equations

This lesson develops students' previous work with systems of equations first learned in Grade 8. Students explore three methods for solving systems of equations: by substitution, by elimination, and by graphing. Students develop their understanding of how to write a system of equations and what the solution means in terms of the context of the problem.

Day	Area of study/content	Pages	Standard(s)
Day 13 Day 14 Day 15	3.2.1: Solving Systems of Linear Equations by Substitution and Elimination	TRB: U3-68 SRB: U3-37 SWB: 47	A–REI.5
Day 16	Station Activities Set 1: Solving Systems by Substitution and Elimination	TRB: U3-123 SWB: 67	A–REI.5
Day 17	Station Activities Set 2: Solving Systems by Graphing	TRB: U3-133 SWB: 73	A–REI.6
Day 18 Day 19 Day 20	3.2.2: Solving Systems of Linear Equations by Graphing	TRB: U3-87 SRB: U3-46 SWB: 57	A–REI.6
Day 21	Station Activities Set 3: Using Systems in Applications	TRB: U3-144 SWB: 79	A–CED.2★ A–CED.3★ A–REI.5 A–REI.6 A–REI.11★
Day 22	Lesson 2 Progress Assessment	TRB: U3-105	
Unit Assessment			
Day 23	Unit Assessment	TRB: U3-109	

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Unit 4: Descriptive Statistics (27 Days)

Unit Overview: This unit develops students' skills and understanding of how to represent, compare, and analyze data using mathematical techniques and terminology. Students move from making statements based on inspection to drawing conclusions based on mathematics. Students begin by analyzing single-count variables, and move toward analyzing multiple variables and two-way frequency tables. Students continuously practice analyzing data sets in context by describing and interpreting the sets using measures of center and spread, and later incorporate any deviations. Students also learn how to fit linear models to data and use these models to evaluate and make predictions.

Lesson 1: Working with a Single Measurement Variable

Students learn how to represent quantitative data using histograms, dot plots, and box-and-whisker plots. Students also learn how to use the graphical representations they create from data sets to analyze the data. This lesson serves as the foundation for describing and analyzing data sets—skills that will be carried through the remainder of this unit and into data analysis units in future math courses.

Day	Area of study/content	Pages	Standard(s)
Day 1 Day 2	Lesson 1 Pre-Assessment 4.1.1: Representing Data Sets	**Pre-Assessment** TRB: U4-1 **Instruction/Practice** TRB: U4-7 SRB: U4-5 SWB: 7	S–ID.1★
Day 3 Day 4	4.1.2: Comparing Data Sets	TRB: U4-29 SRB: U4-18 SWB: 19	S–ID.2★
Day 5 Day 6	4.1.3: Interpreting Data Sets	TRB: U4-50 SRB: U4-30 SWB: 33	S–ID.3★
Day 7	Station Activities Set 1: Displaying and Interpreting Data	TRB: U4-261 SWB: 135	S–ID.1★ S–ID.2★ S–ID.3★
Day 8	Lesson 1 Progress Assessment Lesson 2 Pre-Assessment	TRB: U4-68 TRB: U4-72	

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Lesson 2: Working with Two Categorical and Quantitative Variables

Students move from analyzing single-count variables to analyzing multiple variables in a two-way frequency table. Then, students learn how to examine the relationship between two variables by plotting the data points and fitting curves to the data. Students come to understand that when a relationship between two variables exists, the data are correlated, and the relationship is not necessarily causal. This lesson prepares students to further investigate linear models in the next lesson.

Day	Area of study/content	Pages	Standard(s)
Day 9 Day 10	4.2.1: Summarizing Data Using Two-Way Frequency Tables	TRB: U4-77 SRB: U4-44 SWB: 45	S–ID.5★
Day 11 Day 12	4.2.2: Solving Problems Given Functions Fitted to Data	TRB: U4-95 SRB: U4-53 SWB: 57	S–ID.6a★
Day 13 Day 14	4.2.3: Analyzing Residuals	TRB: U4-118 SRB: U4-67 SWB: 69	S–ID.6b★
Day 15 Day 16 Day 17	4.2.4: Fitting Linear Functions to Data	TRB: U4-141 SRB: U4-82 SWB: 81	S–ID.6c★
Day 18	Station Activities Set 2: Line of Best Fit	TRB: U4-276 SWB: 145	S–ID.6a★ S–ID.6b★ S–ID.6c★ S–ID.7★
Day 19	Lesson 2 Progress Assessment Lesson 3 Pre-Assessment	TRB: U4-163 TRB: U4-169	

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Lesson 3: Interpreting Linear Models

Students delve deeper into analysis and interpretation of linear models. They also examine the strength of a linear model by exploring the correlation coefficient. Students learn the difference between correlation and causation.

Day	Area of study/content	Pages	Standard(s)
Day 20 Day 21	4.3.1: Interpreting Slope and *y*-intercept	TRB: U4-173 SRB: U4-98 SWB: 95	S–ID.7★
Day 22 Day 23	4.3.2: Calculating and Interpreting the Correlation Coefficient	TRB: U4-194 SRB: U4-111 SWB: 107	S–ID.8★
Day 24 Day 25	4.3.3: Distinguishing Between Correlation and Causation	TRB: U4-211 SRB: U4-123 SWB: 121	S–ID.9★
Day 26	Lesson 3 Progress Assessment	TRB: U4-232	
Unit Assessment			
Day 27	Unit Assessment	TRB: U4-237	

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Unit 5: Congruence, Proof, and Constructions (34 Days)

Unit Overview: This unit further develops the concept of transformations presented to students in Grade 8 by applying transformations to functions. The unit also introduces students to geometric constructions in order to show the relationships between lines and angles. Students continue to develop their understanding of these relationships as they explore the concepts of congruency and congruent triangles. Throughout the unit, students are asked to prove relationships that exist in geometric figures using theorems and postulates.

Lesson 1: Introducing Transformations

Students begin the lesson by being introduced to the language of geometry. This language is the foundation of the unit. Students then apply this terminology to connect algebra and geometry through the examination of transformations as they relate to functions. Students take the familiar topic of symmetry and learn to visualize applying lines of symmetry to verbal descriptions of geometric figures. These concepts serve as a springboard for further developing the concepts of reflections, rotations, and congruency presented later in the unit.

Day	Area of study/content	Pages	Standard(s)
Day 1 Day 2	Lesson 1 Pre-Assessment 5.1.1: Defining Terms	**Pre-Assessment** TRB: U5-1 **Instruction/Practice** TRB: U5-5 SRB: U5-4 SWB: 7	G–CO.1
Day 3 Day 4	5.1.2: Transformations As Functions	TRB: U5-18 SRB: U5-12 SWB: 19	G–CO.2
Day 5 Day 6	5.1.3: Applying Lines of Symmetry	TRB: U5-38 SRB: U5-23 SWB: 29	G–CO.3
Day 7	Lesson 1 Progress Assessment Lesson 2 Pre-Assessment	TRB: U5-55 TRB: U5-57	

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Lesson 2: Defining and Applying Rotations, Reflections, and Translations

Students take the general concepts of symmetry and transformations learned in the previous lesson and apply specific rotations, reflections, and translations in a given order. Students explore the order of transformations, and verify that the order is important. Students see that these rigid transformations allow the figures to be manipulated back onto themselves while remaining congruent. This lesson sets the stage for exploring congruent triangles later in the unit.

Day	Area of study/content	Pages	Standard(s)
Day 8	5.2.1: Defining Rotations, Reflections, and Translations	TRB: U5-60 SRB: U5-34 SWB: 39	G–CO.4
Day 9 Day 10	5.2.2: Applying Rotations, Reflections, and Translations	TRB: U5-72 SRB: U5-40 SWB: 49	G–CO.5
Day 11	Lesson 2 Progress Assessment Lesson 3 Pre-Assessment	TRB: U5-86 TRB: U5-88	

Lesson 3: Constructing Lines, Segments, and Angles

Students begin by constructing copies of segments and angles without measuring them. Students explore the relationships within constructions using only a compass and a straightedge. This lesson helps students begin to examine the spatial relationships in congruency.

Day	Area of study/content	Pages	Standard(s)
Day 12 Day 13	5.3.1: Copying Segments and Angles	TRB: U5-94 SRB: U5-51 SWB: 59	G–CO.12
Day 14	5.3.2: Bisecting Segments and Angles	TRB: U5-118 SRB: U5-67 SWB: 69	G–CO.12
Day 15	5.3.3: Constructing Perpendicular and Parallel Lines	TRB: U5-142 SRB: U5-84 SWB: 79	G–CO.12
Day 16	Lesson 3 Progress Assessment Lesson 4 Pre-Assessment	TRB: U5-167 TRB: U5-172	

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Lesson 4: Constructing Polygons

Students develop their spatial exploration of geometric figures by inscribing triangles, squares, and regular hexagons in circles. This exploration further prepares students for the study of congruent triangles.

Day	Area of study/content	Pages	Standard(s)
Day 17 Day 18	5.4.1: Constructing Equilateral Triangles Inscribed in Circles	TRB: U5-175 SRB: U5-104 SWB: 89	G–CO.13
Day 19	5.4.2: Constructing Squares Inscribed in Circles	TRB: U5-199 SRB: U5-120 SWB: 99	G–CO.13
Day 20	5.4.3: Constructing Regular Hexagons Inscribed in Circles	TRB: U5-219 SRB: U5-131 SWB: 109	G–CO.13
Day 21	Lesson 4 Progress Assessment Lesson 5 Pre-Assessment	TRB: U5-243 TRB: U5-248	

Lesson 5: Exploring Congruence

Students combine their knowledge of transformations from the beginning of the unit with their understanding of congruent angles and segments in order to explore congruent figures. This lesson leads to investigating congruent triangles in the next lesson.

Day	Area of study/content	Pages	Standard(s)
Day 22 Day 23	5.5.1: Describing Rigid Motions and Predicting the Effects	TRB: U5-254 SRB: U5-150 SWB: 119	G–CO.6
Day 24	5.5.2: Defining Congruence in Terms of Rigid Motions	TRB: U5-277 SRB: U5-168 SWB: 133	G–CO.6
Day 25	Station Activities Set 1: Rotations and Reflections	TRB: U5-379 SWB: 171	G–CO.1 G–CO.3 G–CO.4 G–CO.5
Day 26	Lesson 5 Progress Assessment Lesson 6 Pre-Assessment	TRB: U5-300 TRB: U5-309	

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Lesson 6: Congruent Triangles

Students address one of geometry's central concepts: congruent triangles. Students learn what attributes triangles must have in order to be congruent. As the lesson progresses, students see what minimum requirements are necessary to determine whether two triangles are congruent, and they understand why these criteria are enough to make the determination.

Day	Area of study/content	Pages	Standard(s)
Day 27 Day 28	5.6.1: Triangle Congruency	TRB: U5-312 SRB: U5-186 SWB: 147	G–CO.7
Day 29 Day 30 Day 31	5.6.2: Explaining ASA, SAS, and SSS	TRB: U5-332 SRB: U5-198 SWB: 159	G–CO.8
Day 32	Station Activities Set 2: Corresponding Parts, Transformations, and Proof	TRB: U5-394 SWB: 181	G–CO.1 G–CO.2 G–CO.5 G–CO.6 G–CO.7 G–CO.8
Day 33	Lesson 6 Progress Assessment	TRB: U5-353	
Unit Assessment			
Day 34	Unit Assessment	TRB: U5-357	

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Unit 6: Connecting Algebra and Geometry Through Coordinates (12 Days)

Unit Overview: This unit connects the domains of algebra and geometry through coordinate proof.

Lesson 1: Slope and Distance

Students constructed parallel and perpendicular lines in the previous unit. In this lesson, they use those skills to explore how to prove that two lines are parallel or perpendicular using coordinates and algebra.

Day	Area of study/content	Pages	Standard(s)
Day 1	Lesson 1 Pre-Assessment Station Activities Set 1: Parallel Lines, Slopes, and Equations	**Pre-Assessment** TRB: U6-1 **Instruction/Practice** TRB: U6-99 SWB: 39	G–GPE.4 G–GPE.5
Day 2 Day 3	6.1.1: Using Coordinates to Prove Geometric Theorems with Slope and Distance	TRB: U6-4 SRB: U6-3 SWB: 7	G–GPE.4 G–GPE.5
Day 4	Station Activities Set 3: Coordinate Proof with Quadrilaterals	TRB: U6-121 SWB: 51	G–GPE.4 G–GPE.5
Day 5 Day 6	6.1.2: Working with Parallel and Perpendicular Lines	TRB: U6-32 SRB: U6-19 SWB: 17	G–GPE.5
Day 7	Station Activities Set 2: Perpendicular Lines	TRB: U6-109 SWB: 45	G–GPE.4 G–GPE.5
Day 8	Lesson 1 Progress Assessment Lesson 2 Pre-Assessment	TRB: U6-52 TRB: U6-56	

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Lesson 2: Lines and Line Segments

Students use the distance formula to calculate the perimeter and area of figures given the coordinates of the vertices of the figures, again connecting algebra and geometry.

Day	Area of study/content	Pages	Standard(s)
Day 9 Day 10	6.2.1: Calculating Perimeter and Area	TRB: U6-58 SRB: U6-33 SWB: 27	G–GPE.7★
Day 11	Lesson 2 Progress Assessment	TRB: U6-83	
Unit Assessment			
Day 12	Unit Assessment	TRB: U6-86	

End-of-Course Assessment	
Day 13	End-of-Course Assessment Please go to walch.com/updates and register to receive End-of-Course Assessments and other updates and additions as they become available.

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Graphic Organizers

Overview

Graphic organizers can be a versatile tool in your classroom. Organizers offer an easy, straightforward way to visually present a wide range of material. Research suggests that graphic organizers support learning in the classroom for all levels of learners. Gifted students, students on grade level, and students with learning difficulties all benefit from the use of graphic organizers. They reduce the cognitive demand on students by helping them access information quickly and easily. Using graphic organizers, learners can understand content more clearly and can take concise notes. Ultimately, learners find it easier to retain and apply what they've learned.

Graphic organizers help foster higher-level thinking skills. They help students identify main ideas and details in their reading. They make it easier for students to see patterns such as cause and effect, comparing and contrasting, and chronological order. Organizers also help students master critical-thinking skills by asking them to recall, evaluate, synthesize, analyze, and apply what they've learned. Research suggests that graphic organizers contribute to better test scores because they help students understand relationships between key ideas, and enable them to be more focused as they study.

Types of Graphic Organizers

There are four main purposes for using graphic organizers in mathematics and a variety of tools within each category:

Purpose 1: Organizing, Categorizing, and Classifying	**Purpose 2: Problem Solving**	**Purpose 3: Understanding Mathematical Information**	**Purpose 4: Communicating Mathematical Information**
Tables Flowcharts Webs Venn Diagrams	Number Lines Geometric Drawings Factor Trees Attribute Tables Cause and Effect Maps Coordinate Plane Probability Trees	Frayer Model Semantic Map/ Concept Map Compare-and-Contrast Diagram	Line Graphs Bar Charts

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Tables

A table is simply a grid with rows and columns. Tables are useful because information stored in a table is easy to find—much easier than the same information embedded in text.

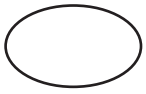
Usually, a table has a row (horizontal) for each item being listed. The columns (vertical) provide places for details about the listed items—the things they have in common. The places where the rows and columns meet are called cells. In each cell, we write information that fits both the topic of the row (the thing being listed) and the topic of the column (the aspect being examined). To create a table, we make rows and columns to fit the number of items and attributes.

Flowcharts

Flowcharts are graphic organizers that show the steps in a process. Flowcharts can be very simple—just a series of boxes with one step in each box. However, there is also a more formal type of flowchart. These flowcharts use special symbols to show different things, such as starting and stopping points, or points where decisions must be made. These symbols make flowcharts especially useful for showing complicated processes.

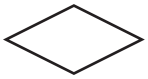
Each step in a flowchart is written in a box. The boxes are connected by arrows to show the sequence of steps. The boxes aren't all rectangular; different shapes are used to indicate different actions. The shapes and symbols are a kind of visual shorthand. Whenever a certain symbol is used, it always has the same meaning.

- Circles and ovals show starting and stopping points. They often contain the words start or stop. The "start" circle or oval has no arrows in and one arrow out. The "stop" circle or oval has one arrow in and no arrows out.



- Arrows show the direction in which the process is moving.

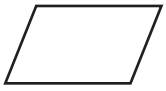
- Diamonds show points where a decision must be made or a question must be answered. The question can usually be answered either "yes" or "no."



- Rectangles and squares show steps where a process or an operation takes place.



- Parallelograms show input or output, such as writing or printing a result or solution.



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Webs

Webs are graphic organizers that help take notes, identify important ideas, and show relationships between and among pieces of information. In a web, the main idea is written in the center circle. Details are recorded in other circles with lines to connect related topics. Circles or lines can be added or deleted as necessary.

Number Lines

In its simplest form, a number line is any line that uses equally spaced marks to show numbers. Number lines are used to visualize equalities and inequalities, positive and negative numbers, and measurements of all kinds. They can "map" math problems, especially ones that involve negative numbers or distances.

Geometric Drawings

A geometric drawing is a representation on paper (or some other surface) of a geometric figure. The geometric drawings we make can never be as perfect as the geometric figures they represent, but as long as they are reasonably accurate, they can help us visualize the figures. In fact, it's often impossible to solve a geometry problem without making a drawing.

Factor Trees

There are several ways to find factors. One that helps to visually keep track of all the factors is called a factor tree. This is a diagram with a tree-like shape. It uses "branches" to show the factors of a number.

All whole numbers other than 1 can be written as the product of factors. A prime number is a number that has only two factors, itself and 1. An example of a prime number is 13. Its only factors are 13 and 1. A composite number is a number that has more than two factors. An example of a composite number is 6. Its factors include 6, 3, 2, and 1. Prime factors are factors that are also prime numbers. The greatest common factor (GCF) of two numbers is the largest number that is a factor of both numbers.

Coordinate Plane

This is the plane determined by a horizontal number line, called the x-axis, and a vertical number line, called the y-axis, intersecting at a point called the origin. A coordinate plane can be used to illustrate locations and relationships using ordered pairs of numbers.

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Venn Diagrams

A set is a list of objects in no particular order. Items in a set can be numbers, but they can also be letters or words. Venn diagrams are a visual way of showing how sets of things can include one another, overlap, or be distinct from one another.

Venn diagrams are often used to compare and contrast things. But they are also a useful tool to sort and classify information. You can use Venn diagrams to take notes on material that shows relationships between things or ideas. You can also use them to solve certain types of word problems. When a word problem names two or three different categories and asks you how many items fall into each category, a Venn diagram can be a useful problem-solving tool.

A Venn diagram begins with a rectangle representing the universal set. Then each set in the problem is represented by a circle. Circles can be separate, overlapping, or one within another. When two circles overlap, it means that the two sets intersect. Some members of one set are also members of the other set.

Venn Diagrams AND Compare-and-Contrast Diagrams

The Venn diagram is an organizing device for planning comparisons and contrasts. A completed Venn diagram helps students categorize and organize similarities and differences, and provides a blueprint for a comparison-and-contrast exercise. The compare-and-contrast diagram provides a structure to identify or list similarities and differences between two objects.

Attribute Tables

To solve logic problems, you need a way to keep track of the subjects and which attributes they have or don't have. An attribute table can help. This is a table with a row for each subject in the problem, and a column for each attribute. The rows and columns meet to form cells. Because the attributes in logic problems are usually exclusive, you can use Xs or check marks (✔) to show which attribute belongs to which subject.

Cause and Effect Maps

Cause and effect maps help you work through information to make sense of it. Write each cause in the oval. Write all its effects in the boxes. Add or delete ovals and boxes as needed.

Frayer Model

The Frayer Model is a word categorization activity that helps learners to develop their understanding of concepts. Using this model, students provide a definition, list characteristics, and provide examples and non-examples of the concept.

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Semantic Map

A semantic word map allows students to conceptually explore their knowledge of a new term or concept by mapping it with other related words, concepts, or phrases that are similar in meaning. Semantic maps portray the schematic relations that compose a concept. It assumes that there are multiple relations between a concept and the knowledge that is associated with the concept.

Line Graphs

Line graphs are often used to show how things change over time. They clearly show trends in data and can let you make predictions about future trends, too. Line graphs use two number lines, one horizontal and one vertical. The horizontal number line is called the *x*-axis. The vertical line is called the *y*-axis. The *x*-axis often shows the passage of time. The *y*-axis often shows a quantity of some kind, such as height, speed, cost, and so forth.

Bar Charts

Bar charts are useful when you want to compare things or to show how one thing changes over time. They are a good way to show overall trends. Bar charts use horizontal or vertical bars to represent data. Longer bars represent higher values. Different colors can be used to show different variables. When you look at a bar chart, it's easy to see which element has the greatest value—the one with the longest bar.

Bar charts have an *x*-axis (horizontal) and a *y*-axis (vertical). If the graph is being used to show how something changes over time, the *x*-axis has numbers for the time period. If the graph is being used to compare things, the *x*-axis shows which things are being compared. The *y*-axis has numbers that show how much of each thing there is.

Probability Trees

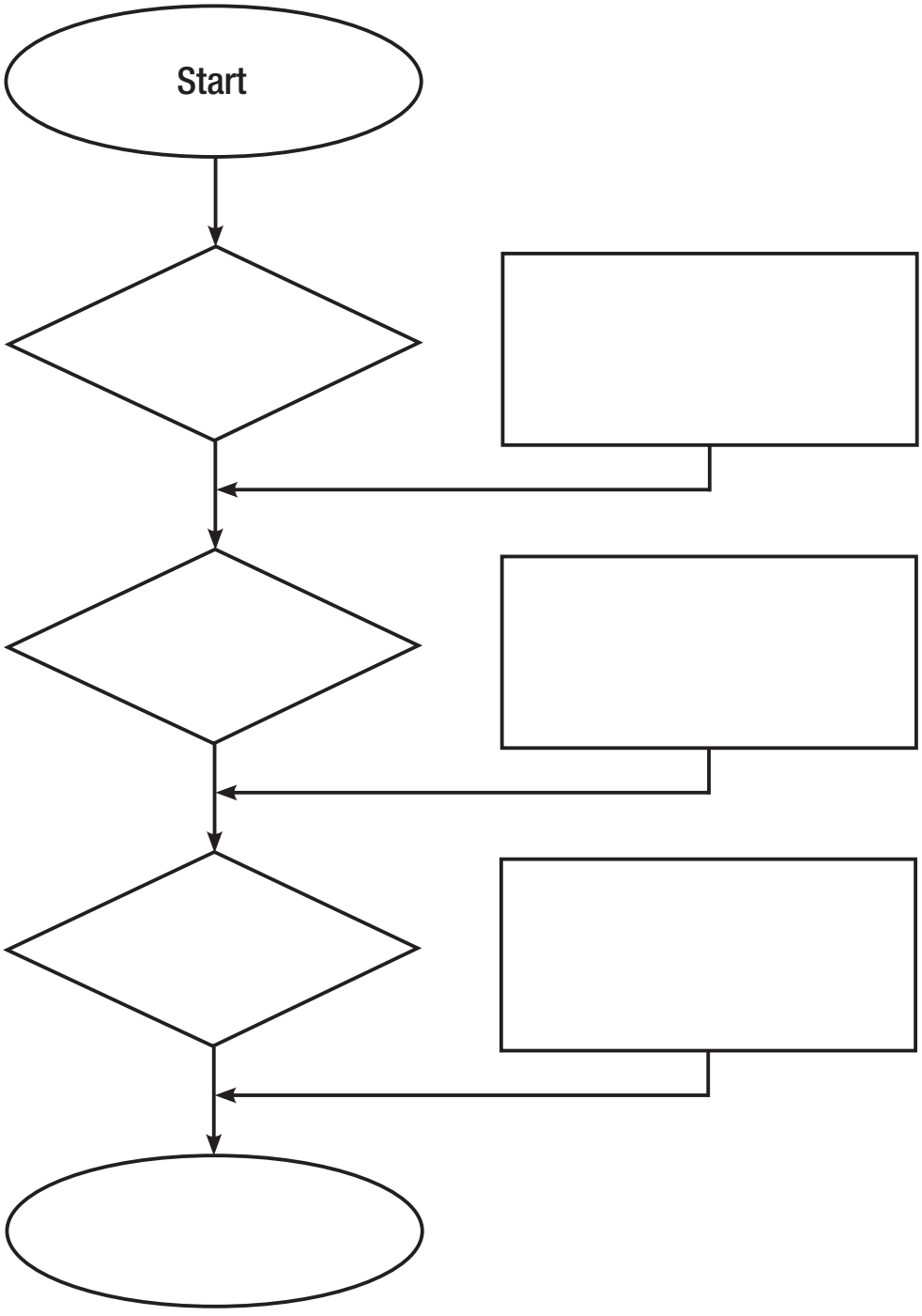
When we have probability problems with many possible outcomes, or events that depend on one another, probability trees can help. Probability trees show all the possible outcomes of an event. Whenever a problem calls for figuring out how many possible outcomes there are, and the probability that any one of them will happen, a probability tree can be useful.

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Table

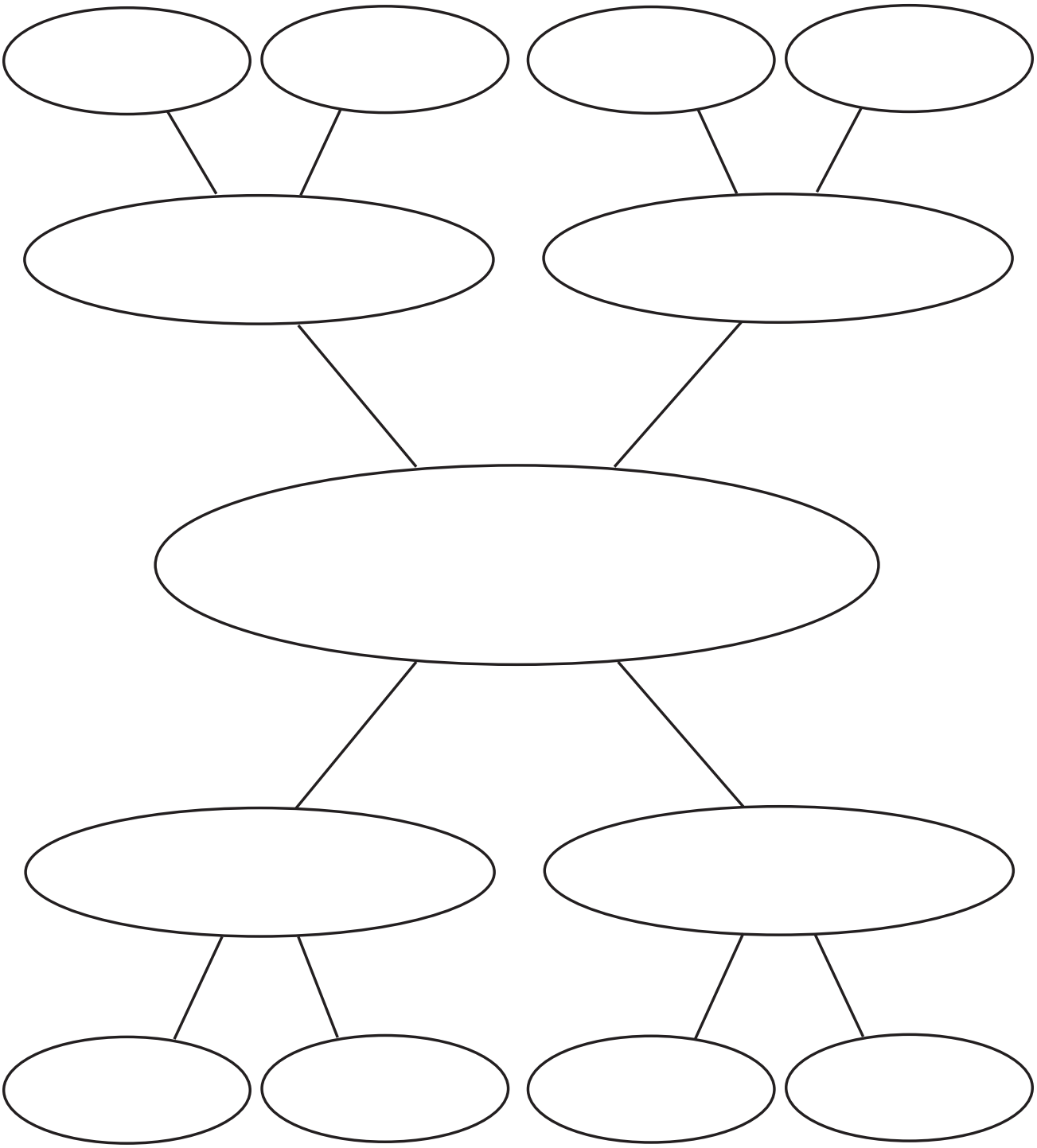
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Flowchart



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Web



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Number Line



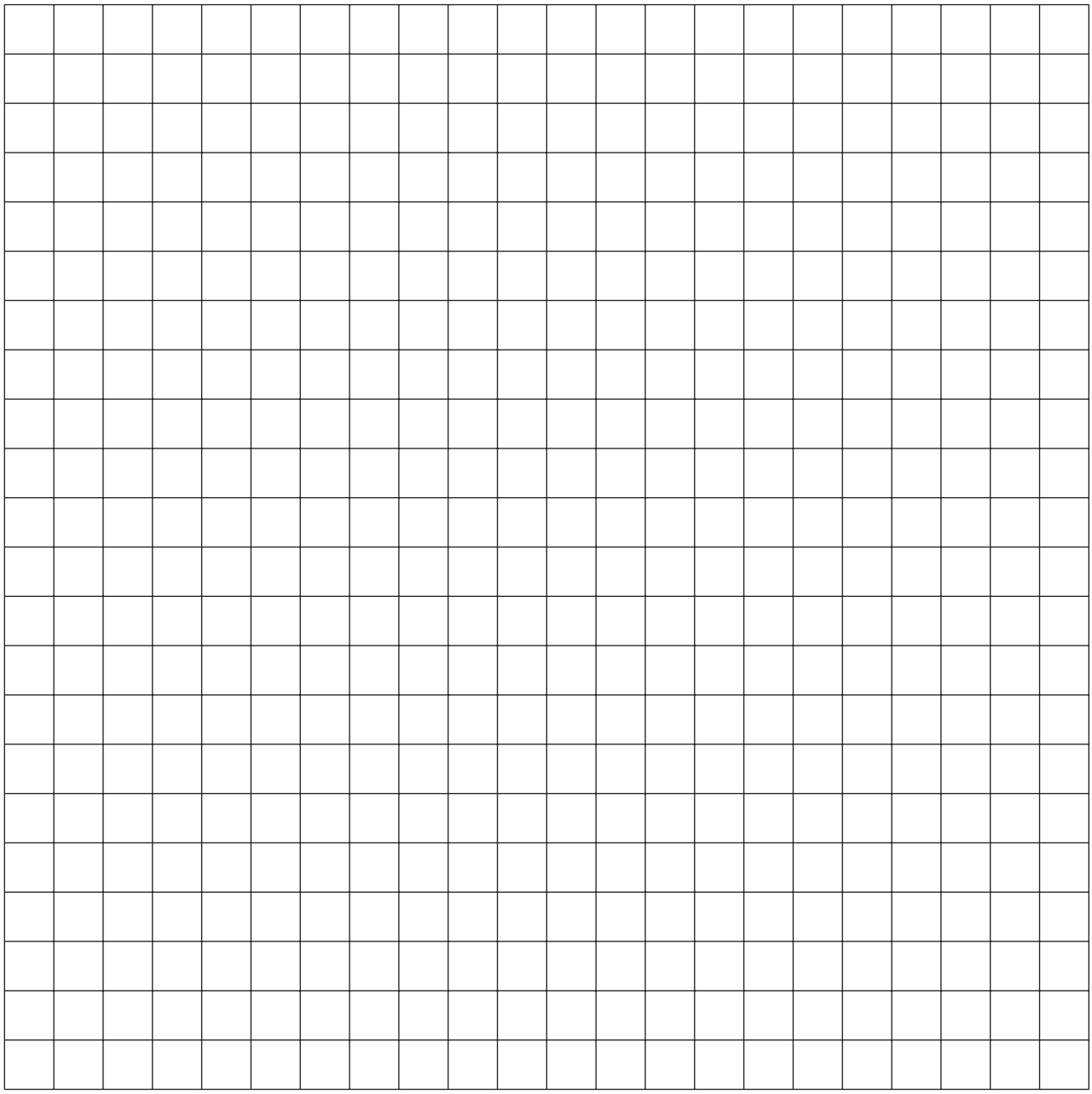
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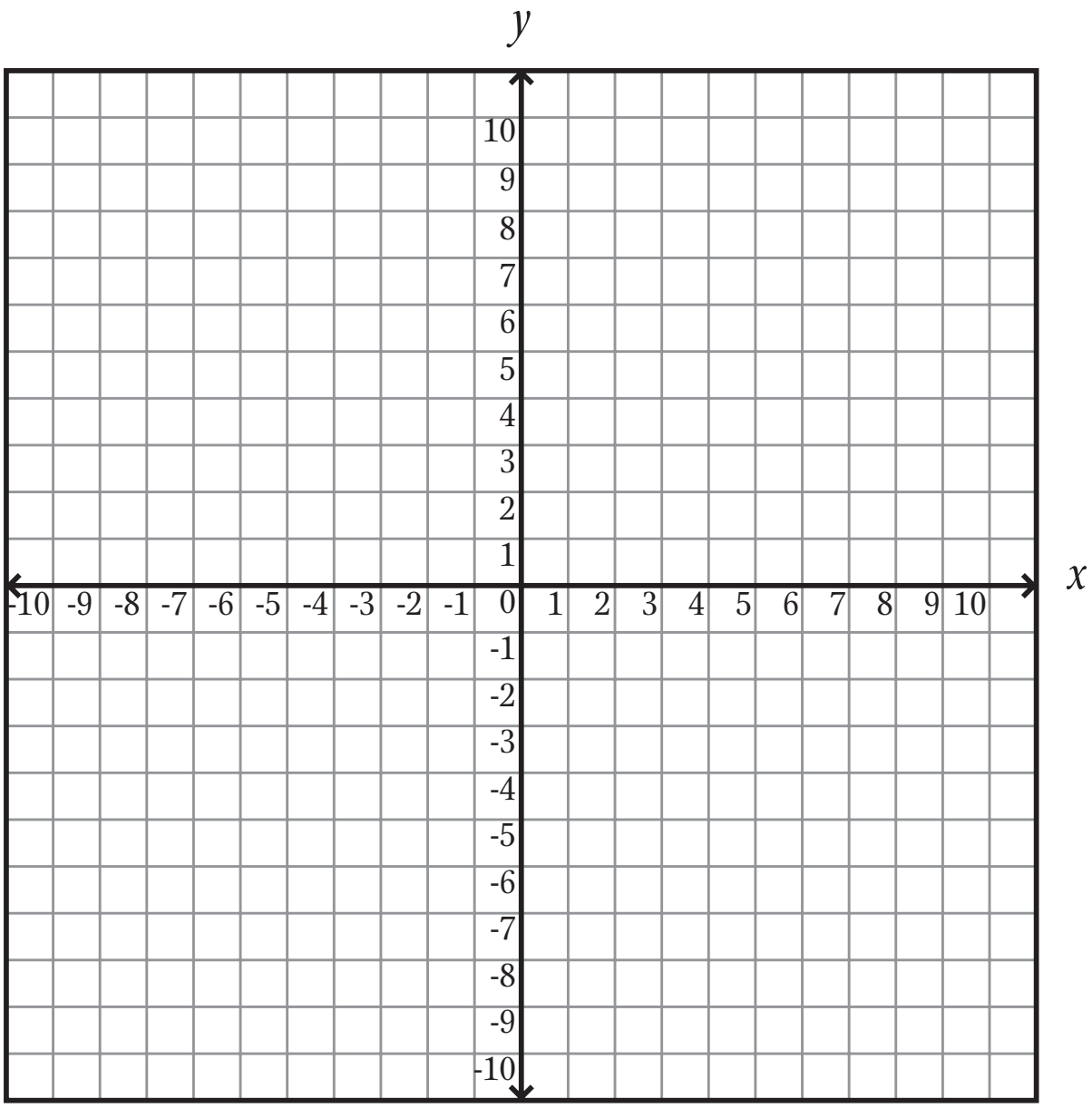
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Geometric Drawing



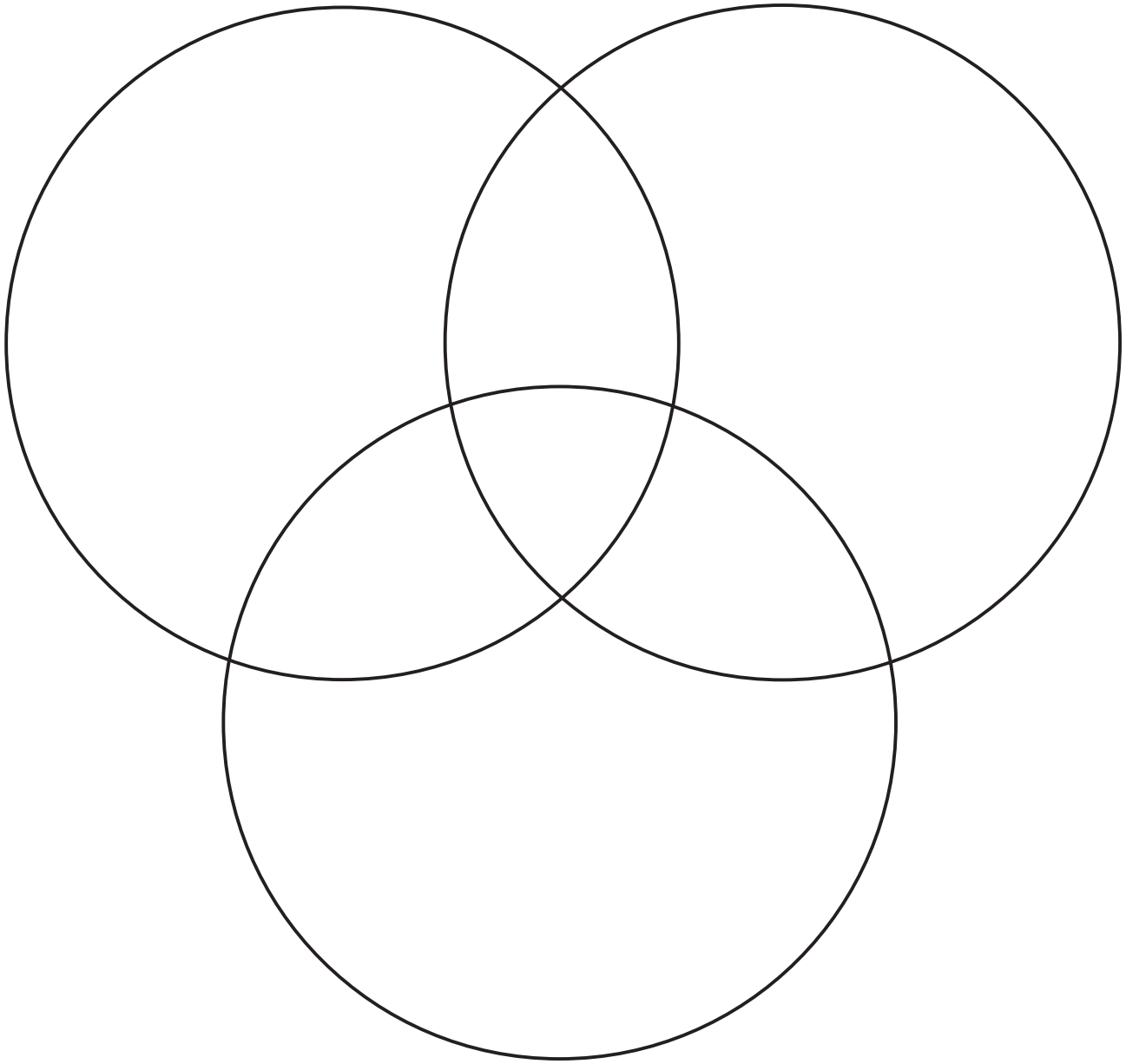
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Coordinate Plane



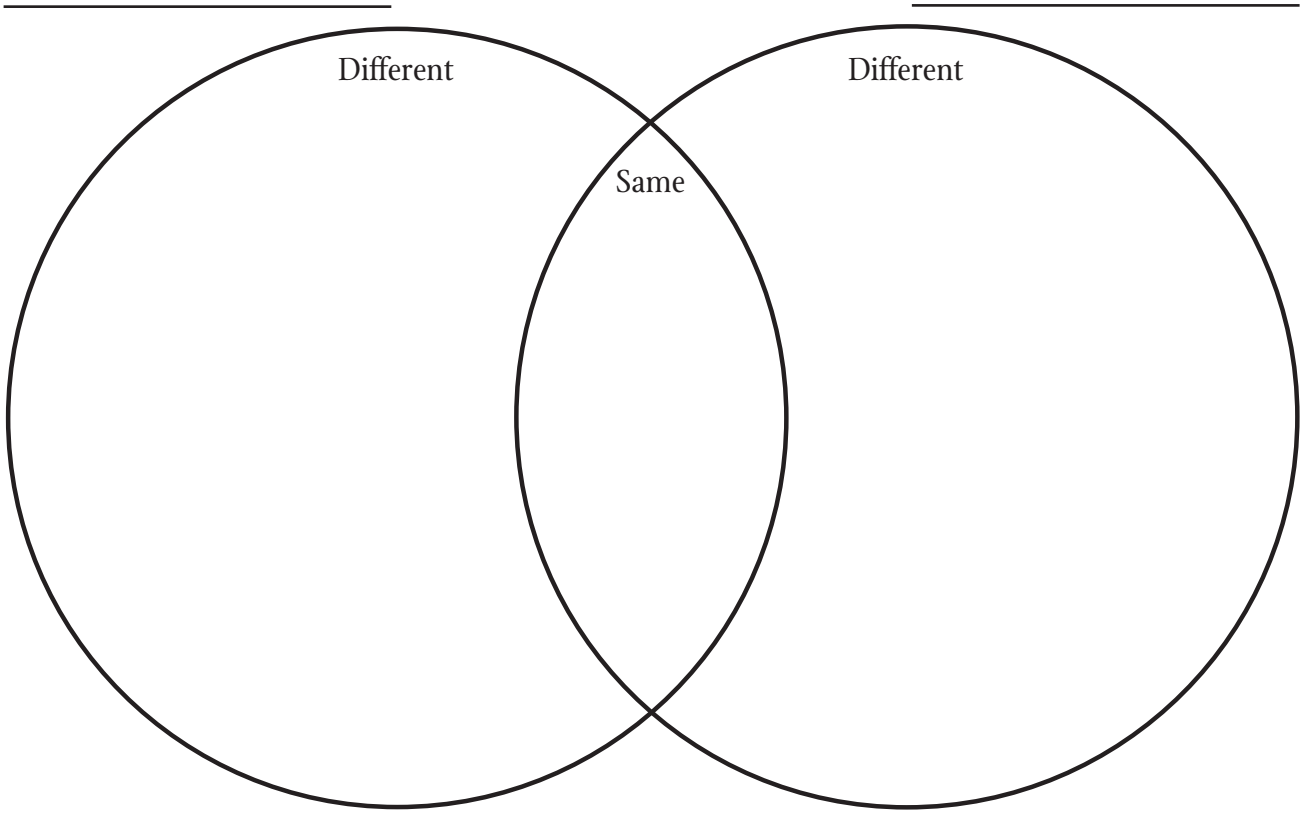
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Venn Diagram



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Venn Diagram



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Compare-and-Contrast Diagram

Item 1 _____ Item 2 _____

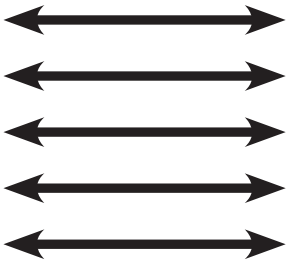
How Alike?

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How Different?

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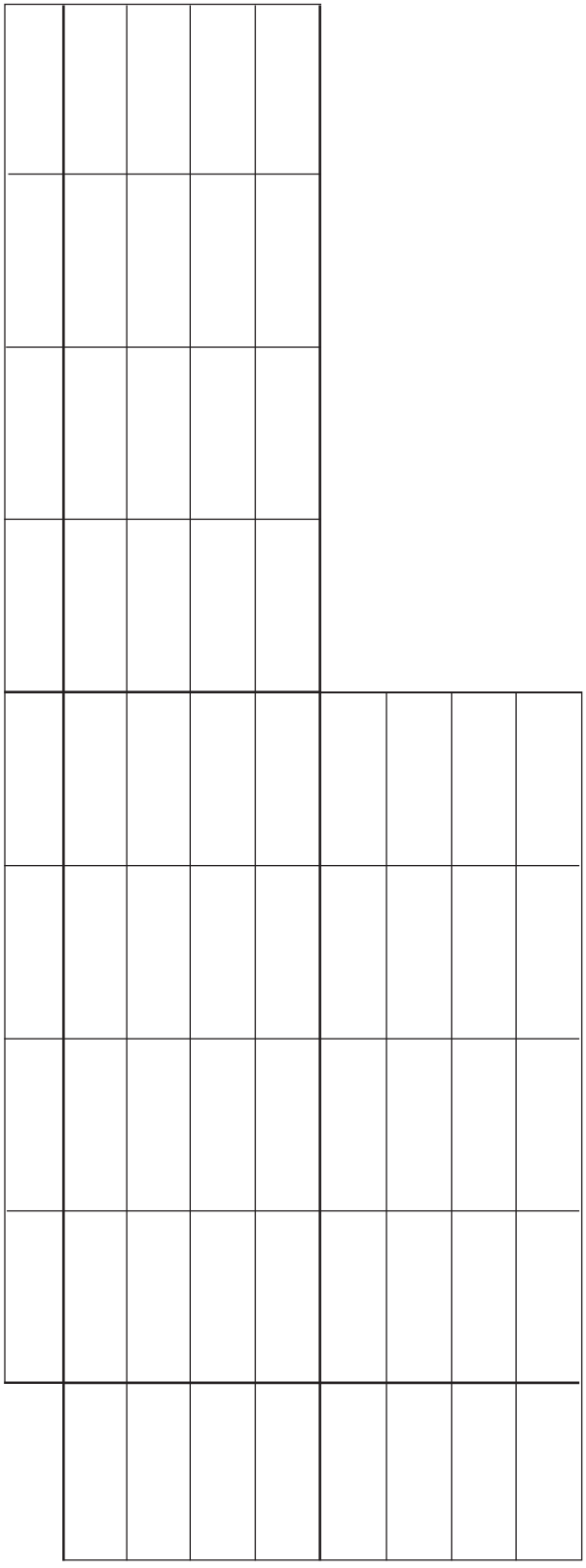
With Regard To



_____
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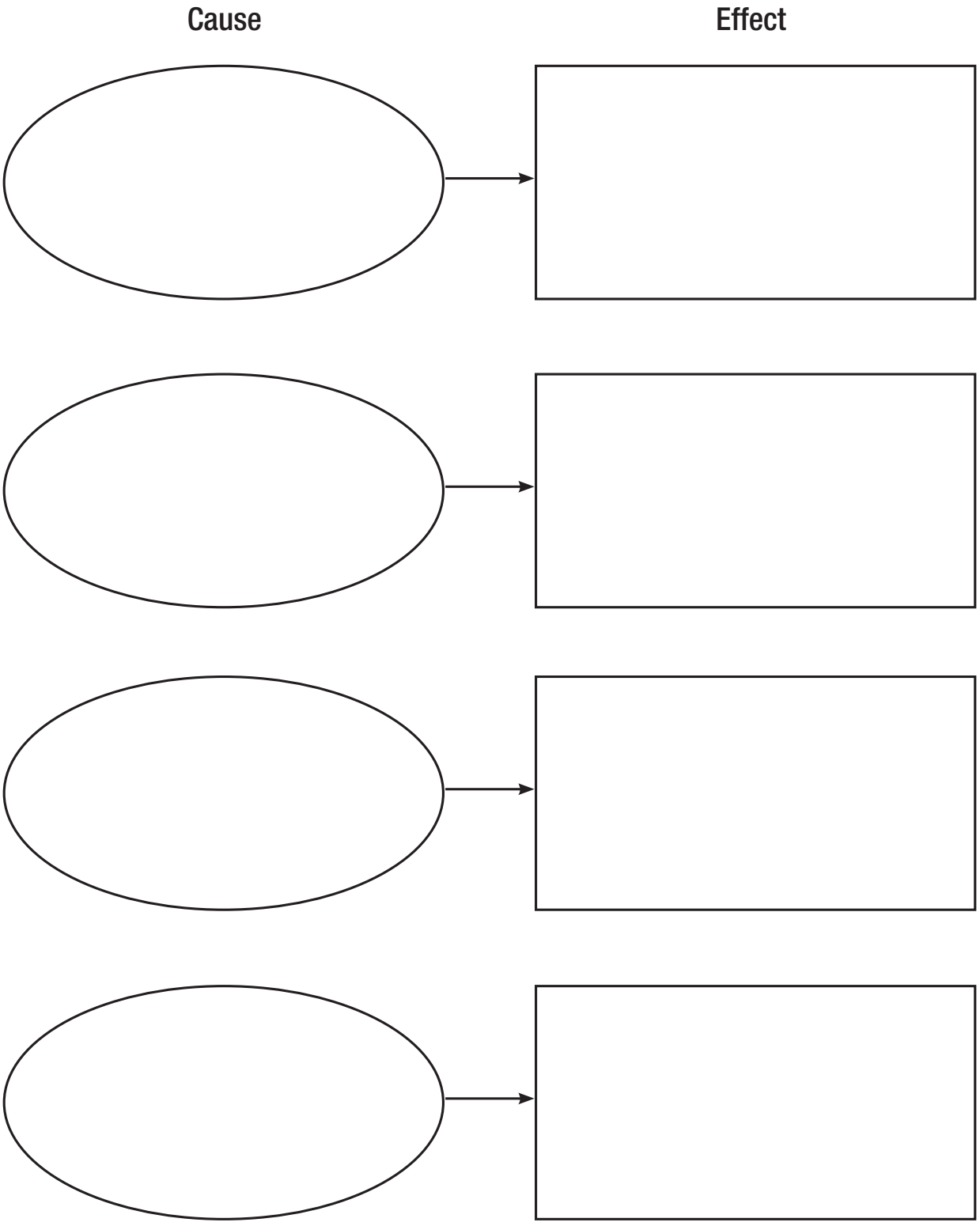
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Attribute Table



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Cause and Effect Map



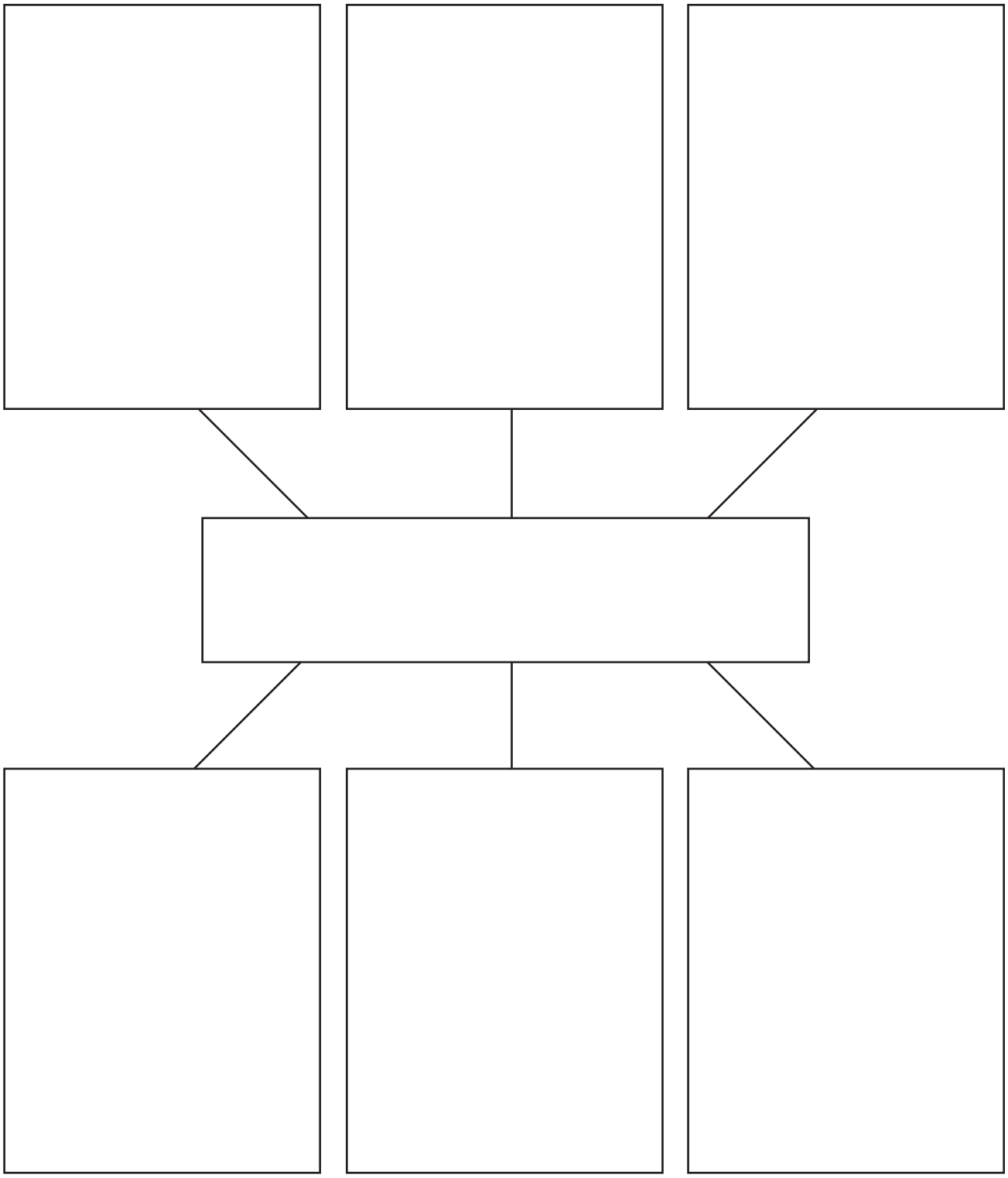
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Frayer Model

Definition	Characteristics
WORD	
Examples from Life	Non-Examples

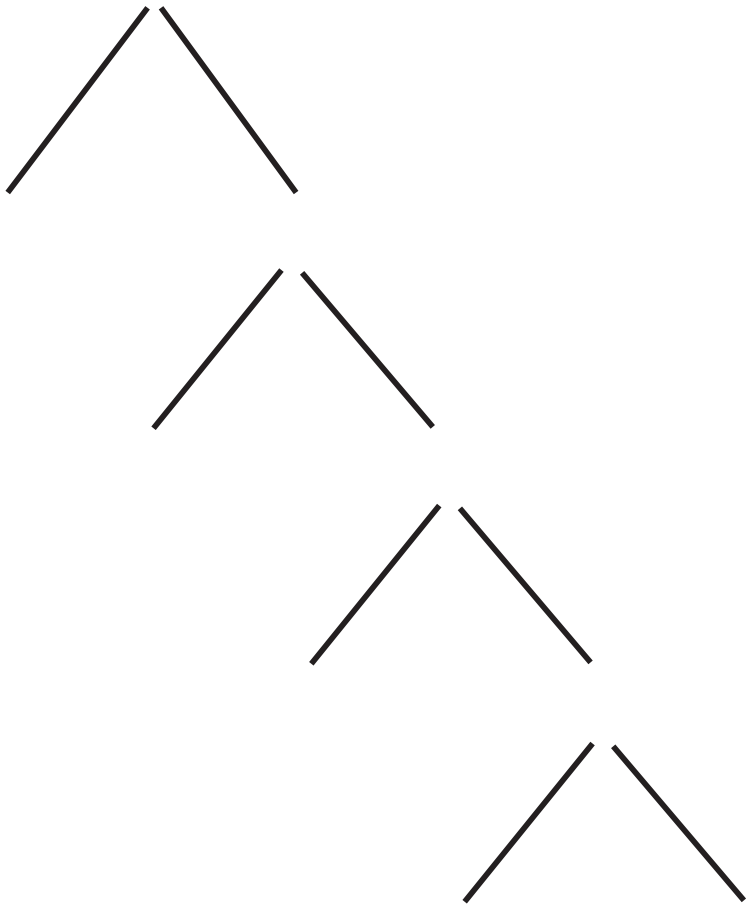
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Semantic Map/Concept Map



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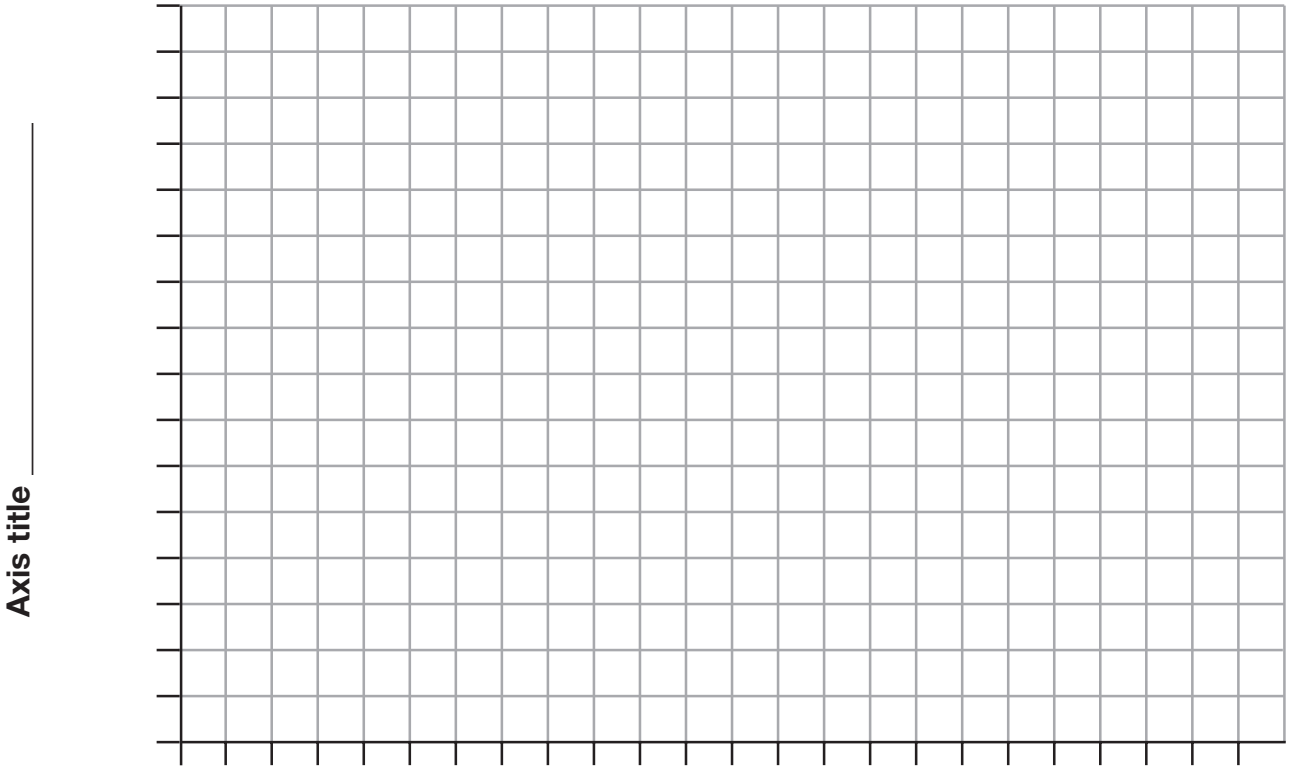
Factor Tree



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Line Graph

Graph title _____

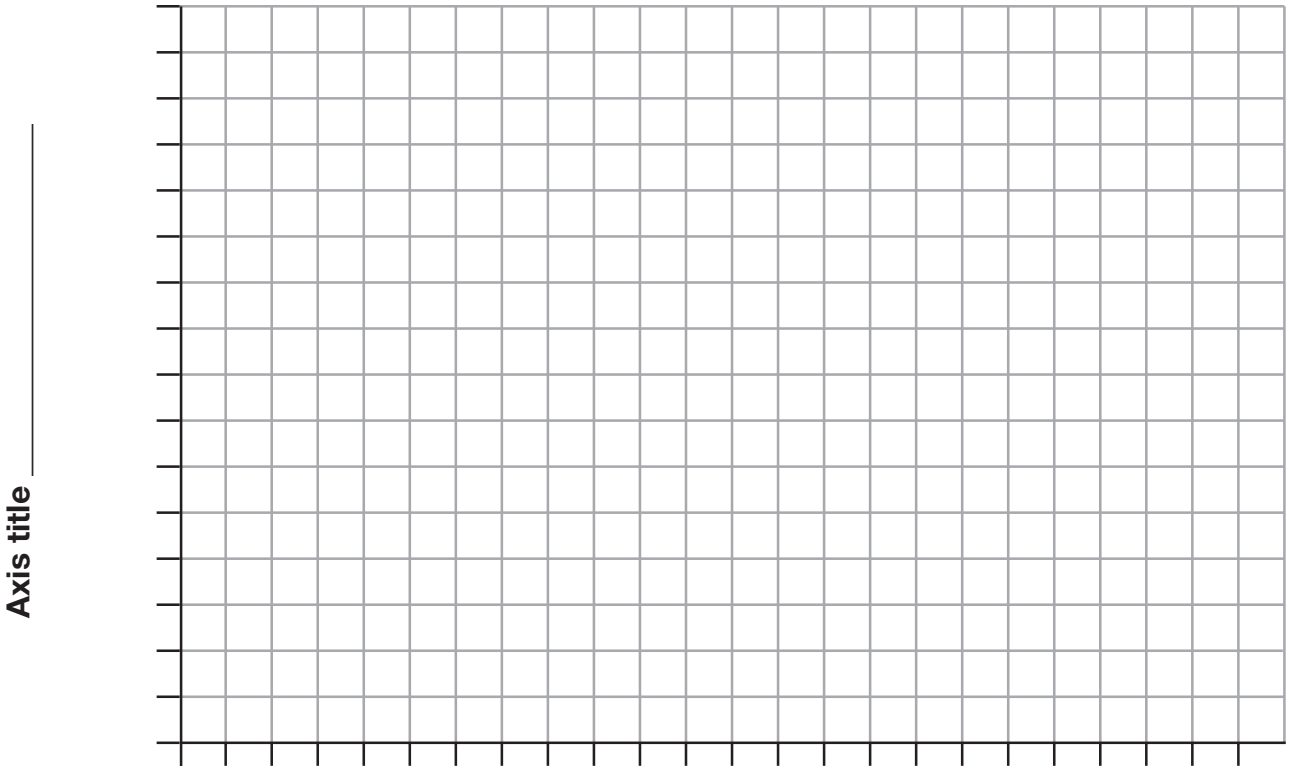


Axis title _____

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Bar Chart/Histogram

Graph title _____



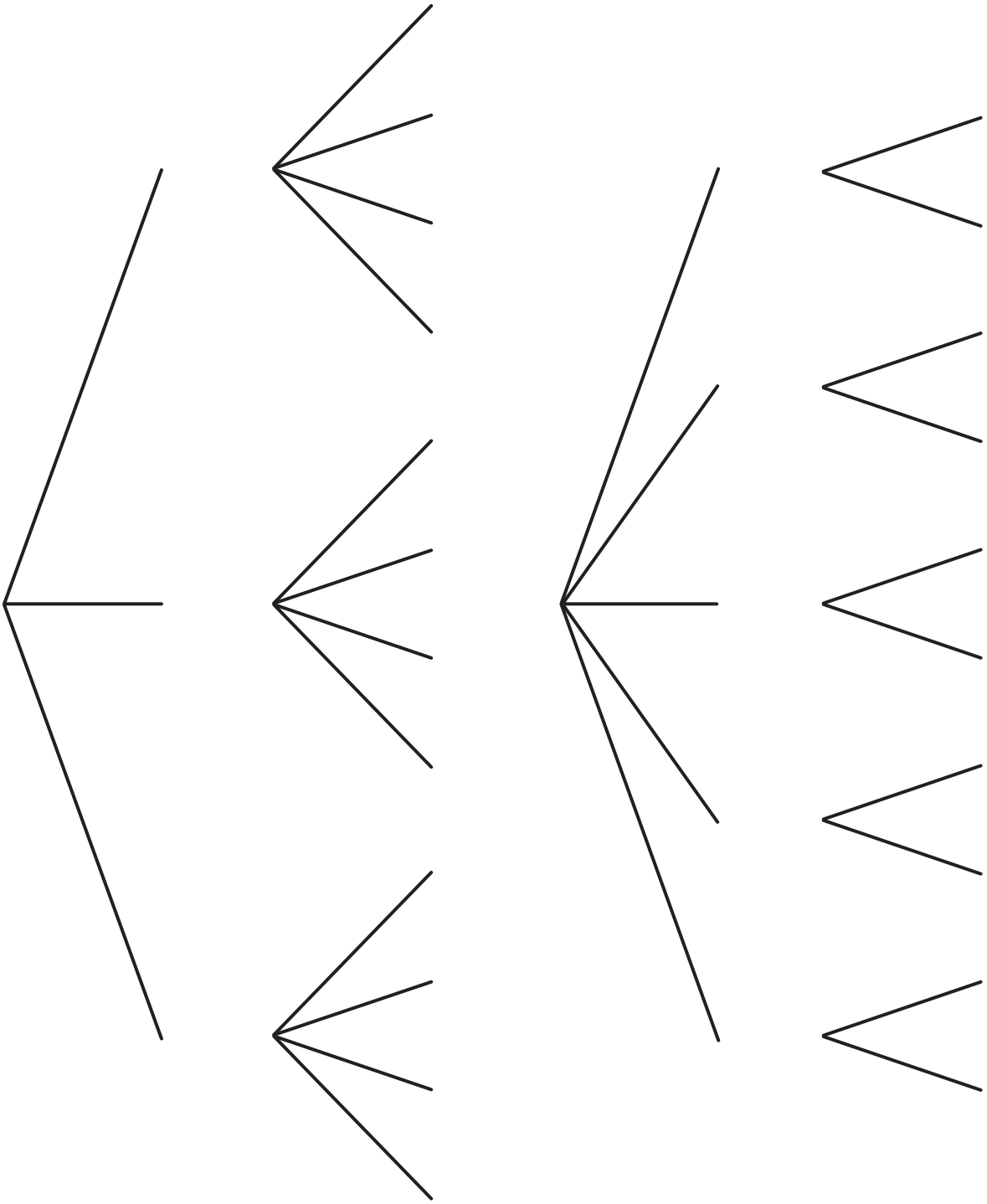
KEY

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Axis title _____

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Probability Trees



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ALGEBRA

General	
(x, y)	Ordered pair
$(x, 0)$	x-intercept
$(0, y)$	y-intercept

Symbols	
$\approx$	Approximately equal to
$\neq$	Is not equal to
$\lvert a \rvert$	Absolute value of a
$\sqrt{a}$	Square root of a

Linear Equations	
$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope
$ax + b = c$	One variable
$y = mx + b$	Slope-intercept form
$ax + by = c$	General form
$y - y_1 = m(x - x_1)$	Point-slope form

Arithmetic Sequences	
$a_n = a_1 + (n - 1)d$	Explicit formula
$a_n = a_{n-1} + d$	Recursive formula

Geometric Sequences	
$a_n = a_1 \bullet r^{n-1}$	Explicit formula
$a_n = a_{n-1} \bullet r$	Recursive formula

Exponential Equations	
$y = ab^x$	General form
$y = ab^{\frac{x}{t}}$	Exponential equation
$y = a(1 + r)^t$	Exponential growth
$y = a(1 - r)^t$	Exponential decay

$A = P\left(1 + \frac{r}{n}\right)^{nt}$	Compounded interest formula
Compounded...	n (number of times per year)
Yearly/annually	1
Semiannually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

Functions	
$f(x)$	Notation, "f of x"
$f(x) = mx + b$	Linear function
$f(x) = b^x + k$	Exponential function
$(f + g)(x) = f(x) + g(x)$	Addition
$(f - g)(x) = f(x) - g(x)$	Subtraction
$(f \bullet g)(x) = f(x) \bullet g(x)$	Multiplication
$(f \div g)(x) = f(x) \div g(x)$	Division

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Properties of Equality	
Property	**In symbols**
Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$ and $c \neq 0$, then $a \cdot c = b \cdot c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Properties of Operations	
Property	**General rule**
Commutative property of addition	$a + b = b + a$
Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of multiplication	$a \cdot b = b \cdot a$
Associative property of multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Distributive property of multiplication over addition	$a \cdot (b + c) = a \cdot b + a \cdot c$

Properties of Inequality
Property
If $a > b$ and $b > c$, then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \cdot c > b \cdot c$.
If $a > b$ and $c < 0$, then $a \cdot c < b \cdot c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Laws of Exponents	
Law	**General rule**
Multiplication of exponents	$b^m \cdot b^n = b^{m+n}$
Power of exponents	$\left(b^m\right)^n = b^{mn}$ $\left(bc\right)^n = b^n c^n$
Division of exponents	$\frac{b^m}{b^n} = b^{m-n}$
Exponents of zero	$b^0 = 1$
Negative exponents	$b^{-n} = \frac{1}{b^n}$ and $\frac{1}{b^{-n}} = b^n$

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DATA ANALYSIS

$IQR = Q_3 - Q_1$	Interquartile range
$Q_1 - 1.5(IQR)$	Lower outlier formula
$Q_3 + 1.5(IQR)$	Upper outlier formula
$y - y_0$	Residual formula

GEOMETRY

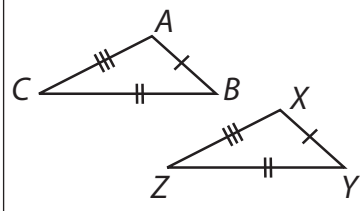
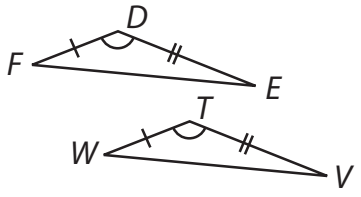
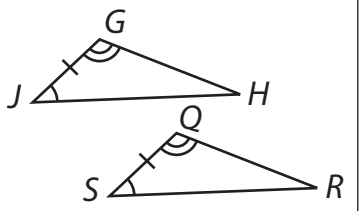
Symbols	
$d(\overset{\frown}{ABC})$	Arc length
$\angle$	Angle
$\odot$	Circle
$\cong$	Congruent
$\overleftrightarrow{PQ}$	Line
$\overline{PQ}$	Line Segment
$\overrightarrow{PQ}$	Ray
$\parallel$	Parallel
$\perp$	Perpendicular
$\bullet$	Point
$\triangle$	Triangle
A'	Prime
$^\circ$	Degrees

Translations	
$T_{(h, k)} = (x + h, y + k)$	Translation

Reflections	
$r_{x\text{-axis}}(x, y) = (x, -y)$	Through the x-axis
$r_{y\text{-axis}}(x, y) = (-x, y)$	Through the y-axis
$r_{y = x}(x, y) = (y, x)$	Through the line $y = x$

Rotations	
$R_{90}(x, y) = (-y, x)$	Counterclockwise 90° about the origin
$R_{180}(x, y) = (-x, -y)$	Counterclockwise 180° about the origin
$R_{270}(x, y) = (y, -x)$	Counterclockwise 270° about the origin

Congruent Triangle Statements

Side-Side-Side (SSS)	Side-Angle-Side (SAS)	Angle-Side-Angle (ASA)
		
$\triangle ABC \cong \triangle XYZ$	$\triangle DEF \cong \triangle TVW$	$\triangle GHJ \cong \triangle QRS$

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Pythagorean Theorem
$a^2 + b^2 = c^2$

Distance Formula	
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula

Area	
$A = lw$	Rectangle
$A = \frac{1}{2}bh$	Triangle

MEASUREMENTS

Length
Metric
1 kilometer (km) = 1000 meters (m)
1 meter (m) = 100 centimeters (cm)
1 centimeter (cm) = 10 millimeters (mm)
Customary
1 mile (mi) = 1760 yards (yd)
1 mile (mi) = 5280 feet (ft)
1 yard (yd) = 3 feet (ft)
1 foot (ft) = 12 inches (in)

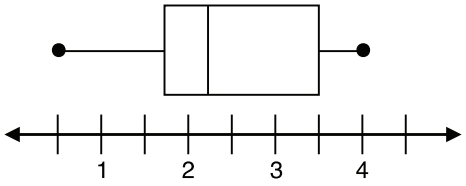
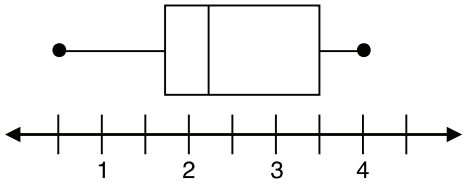
Volume and Capacity
Metric
1 liter (L) = 1000 milliliters (mL)
Customary
1 gallon (gal) = 4 quarts (qt)
1 quart (qt) = 2 pints (pt)
1 pint (pt) = 2 cups (c)
1 cup (c) = 8 fluid ounces (fl oz)

Weight and Mass
Metric
1 kilogram (kg) = 1000 grams (g)
1 gram (g) = 1000 milligrams (mg)
1 metric ton (MT) = 1000 kilograms (kg)
Customary
1 ton (T) = 2000 pounds (lb)
1 pound (lb) = 16 ounces (oz)

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English	Unit/Lesson	Español
	A	
acute angle an angle measuring less than 90° but greater than 0°	5.1	**ángulo agudo** ángulo que mide menos de 90° pero más de 0°
algebraic expression a mathematical statement that includes numbers, operations, and variables to represent a number or quantity	1.1	**expresión algebraica** declaración matemática que incluye números, operaciones y variables para representar un número o una cantidad
altitude the perpendicular line segment from a vertex of a figure to its opposite side; height	5.3	**altitud** línea perpendicular desde un vértice de una figura hasta su lado opuesto; altura
angle two rays or line segments sharing a common endpoint. Angles can be measured in degrees or radians; written as $\angle A$.	5.1 5.3	**ángulo** dos semirrectas o segmentos de línea que comparten un extremo común. Los ángulos se pueden medir en grados o radianes; se expresa como $\angle$.
angle of rotation the angle through which a figure is rotated about a center point; the measure of the angle created by connecting the center of rotation to a point on the preimage and to the corresponding point on the image	5.5	**ángulo de rotación** el ángulo a través del cual se hace girar una figura alrededor de un punto central; la medida del ángulo creado por la conexión del centro de rotación a un punto de la imagen inversa y para el punto correspondiente en la imagen
angle-side-angle (ASA) if two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent	5.6	**ángulo-lado-ángulo (ASA)** si dos ángulos y el lado incluido de un triángulo son congruentes con los dos ángulos y el lado incluido de otro triángulo, entonces los dos triángulos son congruentes
arc length the distance between the endpoints of an arc; written as $d\left(\overset{\frown}{ABC}\right)$	5.1	**longitud de arco** distancia entre los extremos de un arco; se expresa como $d\left(\overset{\frown}{ABC}\right)$
area the amount of space inside the boundary of a two-dimensional figure	6.2	**área** cantidad de espacio dentro del límite de una figura bidimensional
arithmetic sequence a linear function with a domain of positive consecutive integers in which the difference between any two consecutive terms is equal	2.3 2.9	**secuencia aritmética** función lineal con dominio de enteros consecutivos positivos, en la que la diferencia entre dos términos consecutivos es equivalente

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English	Unit/Lesson	Español
asymptote a line that a function gets closer and closer to as one of the variables increases or decreases without bound	2.4 2.5	**asíntota** una línea que una función se acerca cada vez más cerca de una de las variables aumenta o disminuye sin límite
	B	
base the factor being multiplied together in an exponential expression; in the expression a^b, a is the base	1.1	**base** factor que se multiplica en forma conjunta en una expresión exponencial; en la expresión a^b, a es la base
bisect to cut in half	5.3	**bisecar** cortar por la mitad
boundary line the graph of the line that represents a linear inequality and that divides the coordinate plane into two half planes, one of which contains all the solutions of the inequality	2.2	**línea de límite** la gráfica de la línea que representa una desigualdad lineal y que divide el plano de coordenadas en dos medios planos, uno de los cuales contiene todas las soluciones de la desigualdad
box plot a plot showing the minimum, maximum, first quartile, median, and third quartile of a data set; the middle 50% of the data is indicated by a box. Example: 	4.1	**diagrama de caja** diagrama que muestra el mínimo, máximo, primer cuartil, mediana y tercer cuartil de un conjunto de datos; se indica con una caja el 50% medio de los datos. Ejemplo: 
	C	
causation a relationship between two events where a change in one event is responsible for a change in the second event	4.3	**causalidad** relación entre dos eventos en la que un cambio en un evento es responsable por un cambio en el segundo evento
circle the set of points on a plane at a fixed distance, or radius, from a given point, the center	5.1 5.4	**círculo** conjunto de puntos en un plano a determinada distancia, o radio, de un único punto, el centro
circular arc on a circle, the set of points between the endpoints of two radii	5.1	**arco circular** en un círculo, conjunto de puntos no compartidos entre los extremos de dos radios
clockwise rotating a figure in the direction that the hands on a clock move	5.2 5.5	**sentido horario** rotación de una figura en la dirección en que se mueven las agujas de un reloj

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English	Unit/Lesson	Español
coefficient the number multiplied by a variable in an algebraic expression	1.1	**coeficiente** número multiplicado por una variable en una expresión algebraica
common difference the number added to each consecutive term in an arithmetic sequence	2.3 2.9	**diferencia común** número sumado a cada término consecutivo en una secuencia aritmética
common ratio the number that each consecutive term is multiplied by in a geometric sequence	2.3 2.9	**proporción constante** el número que cada término esta multiplicado por en una secuencia geométrica
compass an instrument for creating circles or transferring measurements that consists of two pointed branches joined at the top by a pivot	5.3	**compás** instrumento utilizado para crear círculos o transferir medidas, que consiste en dos brazos terminados en punta y unidos en la parte superior por un pivote
compression a transformation in which a figure becomes smaller; compressions may be horizontal (affecting only horizontal lengths), vertical (affecting only vertical lengths), or both	5.5	**compresión** transformación en la que una figura se hace más pequeña; las compresiones pueden ser horizontales (cuando afectan sólo la longitud horizontal), verticales (cuando afectan sólo la longitud vertical), o en ambos sentidos
conditional relative frequency a joint frequency compared to either the sum of all frequencies or the sum of the frequencies for a category of the given data	4.2	**frecuencia condicional relativa** frecuencia conjunta comparada con la suma de todas las frecuencias o la suma de las frecuencias para una categoría de los datos dados
congruency transformation a transformation in which a geometric figure moves but keeps the same size and shape	5.5	**transformación de congruencia** transformación en la que se mueve una figura geométrica pero se mantiene el mismo tamaño y la misma forma
congruent having the same shape, size, lines, and angles; the symbol for congruent is ≅	5.1 5.3 5.4 5.5 6.1	**congruente** tiene la misma forma, tamaño, líneas y anglos; el símbolo paracongruente es ≅
congruent angles two angles that have the same measure	5.6	**ángulos congruentes** dos ángulos con la misma medida

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English	Unit/Lesson	Español
congruent sides two sides that have the same length	5.6	**lados congruentes** dos lados con la misma longitud
congruent triangles triangles having the same angle measures and side lengths	5.6	**triángulos congruentes** triángulos con las mismas medidas de ángulos y longitudes de lados
consistent a system of equations with at least one ordered pair that satisfies both equations	3.2	**consistente** sistema de ecuaciones con al menos un par ordenado que satisface ambas ecuaciones
constant a quantity that does not change	1.1	**constante** cantidad que no cambia
constraint a restriction or limitation on any of the variables in an equation or inequality	1.4 2.2	**limitación** una restricción o limitación de cualquiera de las variables en una ecuación o desigualdad
construct to create a precise geometric representation using a straightedge along with either patty paper (tracing paper), a compass, or a reflecting device	5.3	**construir** crear una representación geométrica precisa mediante regla de borde recto y papel encerado (papel para calcar), compás o un dispositivo de reflexión
construction a precise representation of a figure using a straightedge and a compass, patty paper and a straightedge, or a reflecting device and a straightedge	5.3 5.4	**construcción** representación precisa de una figura mediante regla de borde recto y compás, papel encerado y una regla de borde recto, o un dispositivo de reflexión y una regla de borde recto
continuous having no breaks	2.4	**continuo** sin interrupciones
coordinate plane a plane determined by a set of two number lines, called the axes, that intersect at right angles	1.3	**plano de coordenadas** un plano determinado por un conjunto de dos líneas numéricas, llamadas los ejes, que se cruzan en ángulos rectos
correlation a relationship between two events, where a change in one event is related to a change in the second event. A correlation between two events does not imply that the first event is responsible for the change in the second event; the correlation only shows how likely it is that a change also took place in the second event.	4.3	**correlación** relación entre dos eventos en la que el cambio en un evento se relaciona con un cambio en el segundo evento. Una correlación entre dos eventos no implica que el primero sea responsable del cambio en el segundo; la correlación sólo demuestra cuán probable es que también se produzca un cambio en el segundo evento.

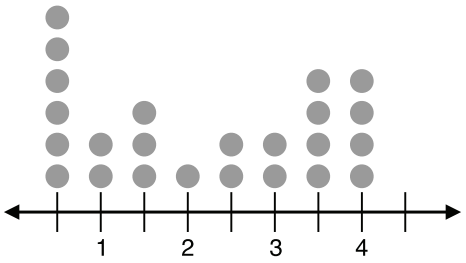
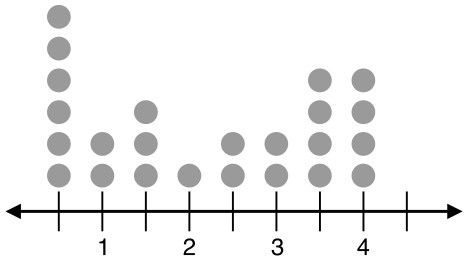
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English	Unit/Lesson	Español
correlation coefficient a quantity that assesses the strength of a linear relationship between two variables, ranging from –1 to 1; a correlation coefficient of –1 indicates a strong negative correlation, a correlation coefficient of 1 indicates a strong positive correlation, and a correlation coefficient of 0 indicates a very weak or no linear correlation	4.3	**coeficiente de correlación** cantidad que evalúa la fuerza de una relación lineal entre dos variables, que varía de –1 a 1; un coeficiente de correlación de –1 indica una fuerte correlación negativa, un coeficiente de correlación de 1 indica una fuerte correlación positiva, y un coeficiente de correlación de 0 indica una correlación muy débil o no lineal
corresponding angles 1. angles of two figures that lie in the same position relative to the figure. In transformations, the corresponding angles are the preimage and image angles, so $\angle A$ and $\angle A'$ are corresponding angles and so on. 2. a pair of angles in a similar position	5.5 5.6	**ángulos correspondientes** 1. ángulos de dos figuras que se ubican en la misma posición relativa a la figura. En las transformaciones, los vangulos correspondientes son los angulos de preimagen e imagen, de manera que $\angle A$ y $\angle A'$ son los angulos correspondientes, etc. 2. un par de ángulos en una posición similar
Corresponding Parts of Congruent Triangles are Congruent (CPCTC) if two or more triangles are proven congruent, then all of their corresponding parts are congruent as well	5.6	**Las partes correspondientes de triángulos congruentes son congruentes (CPCTC)** si se comprueba que dos o más triángulos son congruentes, entonces todas sus partes correspondientes son también congruentes
corresponding sides sides of two figures that lie in the same position relative to the figure. In transformations, the corresponding sides are the preimage and image sides, so $\overline{AB}$ and $\overline{A'B'}$ are corresponding sides and so on.	5.5 5.6	**lados correspondientes** lados de dos figuras que están en la misma posición relativa a la figura. En las transformaciones, los lados correspondientes son los de preimagen e imagen, entonces $\overline{AB}$ y $\overline{A'B'}$ son los lados correspondientes, etc.
counterclockwise rotating a figure in the opposite direction that the hands on a clock move	5.2 5.5	**en sentido antihorario** rotación de una figura en la dirección opuesta a la que se mueven las agujas de un reloj

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English	Unit/Lesson	Español
curve the graphical representation of the solution set for $y = f(x)$; in the special case of a linear equation, the curve will be a line	2.1	**curva** representación gráfica del conjunto de soluciones para $y = f(x)$; en el caso especial de una ecuación lineal, la curva será una recta
	D	
dependent a system of equations that has an infinite number of solutions; lines coincide when graphed	3.2	**dependiente** sistema de ecuaciones con una cantidad infinita de soluciones; las rectas coinciden cuando se grafican
dependent variable generally labeled on the *y*-axis**;** the quantity that is based on the input values of the independent variable	1.3	**variable dependiente** generalmente designada en el eje *y*; cantidad que se basa en los valores de entrada de la variable independiente
diameter a line segment passing through the center of a circle connecting two points on the circle; twice the radius	5.4	**diámetro** línea recta que pasa por el centro de un círculo y conecta dos puntos en el círculo; dos veces el radio
dilation a transformation in which a figure is either enlarged or reduced by a scale factor in relation to a center point	5.5	**dilatación** transformación en la que una figura se amplía o se reduce por un factor de escala en relación con un punto central
discrete individually separate and distinct	2.3	**discreto** individualmente aparte y distinto
distance along a line the linear distance between two points on a given line; written as *PQ*	5.1	**distancia a lo largo de una recta** distancia lineal entre dos puntos de una determinada línea; se expresa como *PQ*
distance formula formula that states the distance between points (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	6.1 6.2	**fórmula de distancia** fórmula que establece la distancia entre los puntos (x_1, y_1) y (x_2, y_2) equivale a $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
domain the set of all inputs values for which a function is defined; the set of *x*-values that are valid for a relation or function	2.1 2.4	**dominio** el conjunto de todos los valores de entradas para los que se define una función; el conjunto de valores *x* que son válidos para una relación o función

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English	Unit/Lesson	Español
dot plot a frequency plot that shows the number of times a response occurred in a data set, where each data value is represented by a dot. Example: 	4.1	**diagrama de puntos** diagrama de frecuencia que muestra la cantidad de veces que se produjo una respuesta en un conjunto de datos, en el que cada valor de dato está representado por un punto. Ejemplo: 
drawing a precise representation of a figure, created with measurement tools such as a protractor and a ruler	5.3	**dibujo** representación precisa de una figura, creada con herramientas de medición tales como transportador y regla
	E	
elimination method adding or subtracting the equations in the system together so that one of the variables is eliminated; multiplication might be necessary before adding the equations together	3.2	**método de eliminación** suma o sustracción conjunta de ecuaciones en el sistema de manera de eliminar una de las variables; podría requerirse multiplicación antes de la suma conjunta de las ecuaciones
end behavior the behavior of the graph as x approaches positive infinity and as x approaches negative infinity	2.5	**comportamiento final** el comportamiento de la gráfica al aproximarse x a infinito positivo o a infinito negativo
endpoint either of two points that mark the ends of a line segment; a point that marks the end of a ray	5.3	**extremo** uno de los dos puntos que marcan el final de un segmento de recta; punto que marca el final de una semirrecta
equation a mathematical sentence that uses an equal sign (=) to show that two quantities are equal	1.2 2.7	**ecuación** declaración matemática que utiliza el signo igual (=) para demostrar que dos cantidades son equivalentes
equidistant the same distance from a reference point	5.3 5.5	**equidistante** a la misma distancia de un punto de referencia
equilateral triangle a triangle with all three sides equal in length	5.4	**triángulo equilátero** triángulo con sus tres lados de la misma longitud

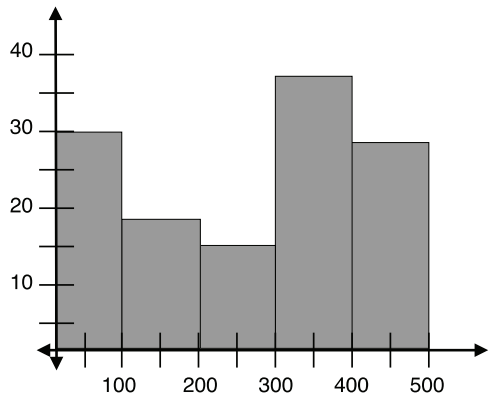
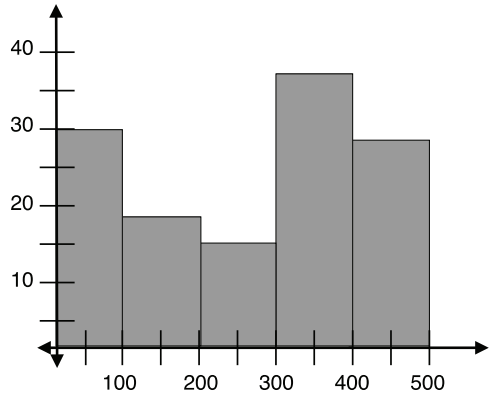
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English	Unit/Lesson	Español
explicit formula a formula used to find the *n*th term of a sequence; if *n* is the term number, then the explicit formula for the *n*th term of an arithmetic sequence is $a_n = a_1 + (n - 1)d$, and the explicit formula for the *n*th term of a geometric sequence is $a_n = a_1 \bullet r^{n-1}$	2.3 2.9	**fórmula explícita** una fórmula usada para encontrar el término *n* de una secuencia; si *n* es el número del término, entonces la fórmula explícita para el término *n* de una secuencia aritmética es $a_n = a_1 + (n - 1)d$, y la fórmula explícita para el término *n* de una secuencia geométrica es $a_n = a_1 \bullet r^{n-1}$
explicit function a function in which the dependent variable can be written in terms of the independent variable; $f(x) = 2x$ is an explicit function, where *x* is the independent variable and $f(x)$ is the dependent variable	2.7	**función explícita** una función en la que la variable dependiente se puede escribir en términos de la variable independiente; $f(x) = 2x$ es una función explícita, donde *x* es la variable independiente y $f(x)$ es la variable dependiente
exponent the number of times a factor is being multiplied together in an exponential expression; in the expression a^b, *b* is the exponent	1.1	**exponente** cantidad de veces que se multiplica un factor en forma conjunta en una expresión exponencial; en la expresión a^b, *b* es el exponente
exponential decay an exponential equation with a base, *b*, that is between 0 and 1 exclusive (that is, $0 < b < 1$); an example is the formula $y = a(1 - r)^t$, where *a* is the initial value, $(1 - r)$ is the base (with $0 < r < 1$), *t* is the variable exponent, and *y* is the final value	1.2 1.3	**decaimiento exponencial** una ecuación exponencial con una base, *b*, que está entre 0 y 1 exclusivo (es decir, $0 < b < 1$); un ejemplo es la fórmula $y = a(1 - r)^t$, donde *a* es el valor inicial, $(1 - r)$ es la base (con $0 < r < 1$), *t* es el exponente variable y *y* es el final valor
exponential equation an equation whose independent variable is in the exponent; the general form of its equation is $f(x) = ab^x + k$, where *a* is the initial value, *b* is the base, *x* is the input value, *k* is the vertical shift, and $f(x)$ is the output. Another form is $y = ab^{\frac{x}{t}}$, where *t* is the interval over which *y* changes by a factor of *b*, and *x* is measured in the same units as *t*.	1.2 1.3 2.7	**ecuación exponencial** ecuación cuya variable independiente es en el exponente; la forma general de su ecuación es $f(x) = ab^x + k$, donde *a* es el valor inicial, *b* es la base, *x* es el valor de entrada, *k* es el desplazamiento vertical y $f(x)$ es el valor de salida. Otra forma es $y = ab^{\frac{x}{t}}$, donde *t* es el intervalo en el que *y* cambia por un factor de *b*, y *x* se mide en las mismas unidades como *t*.

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English	Unit/Lesson	Español
exponential function a function whose independent variable is in the exponent; the general form of its equation is $f(x) = ab^x + k$, where a is the initial value, b is the base, x is the input value, k is the vertical shift, and $f(x)$ is the output	2.5 2.6 2.10	**función exponencial** una función cuya variable independiente es en el exponente; la forma general de su ecuación es $f(x) = ab^x + k$, donde a es el valor inicial, b es la base, x es el valor de entrada, k es el desplazamiento vertical y $f(x)$ es el valor de salida
exponential growth an exponential equation with a base, b, greater than 1 ($b > 1$); an example is the formula $y = a(1 + r)^t$, where a is the initial value, $(1 + r)$ is the base (with $r > 1$), t is the variable exponent, and y is the final value	1.2 1.3	**crecimiento exponencial** una ecuación exponencial con una base, b, mayor que 1 ($b > 1$); un ejemplo es la fórmula $y = a(1 + r)^t$, donde a es el valor inicial, $(1 + r)$ es la base (con $r > 1$), t es el exponente variable ey y es el valor final
expression a combination of variables, quantities, and mathematical operations; 4, $8x$, and $b + 10^2$ are all expressions	2.7	**expresión** combinación de variables, cantidades y operaciones matemáticas; 4, $8x$ y $b + 10^2$ son todas expresiones
extrema the minima and maxima of a function	2.4	**extremos** los mínimos y máximos de una función
	F	
factor one of two or more numbers or expressions that are multiplied to produce a product	1.1 2.6	**factor** uno de dos o más números o expresiones que se multiplican para producir un producto
first quartile the value that identifies the lower 25% of the data; the median of the lower half of the data set; written as Q_1	4.1	**primer cuartil** valor que identifica el 25% inferior de los datos; mediana de la mitad inferior del conjunto de datos; se expresa Q_1
formula a literal expression or equation that states a specific rule or relationship among quantities	1.5	**fórmula** expresión literal o ecuación que establece una regla específica o relación entre cantidades
function a relation in which each element in the domain is mapped onto exactly one element in the range; that is, for every value of x, there is exactly one value of y	2.1 2.7 2.8 4.2	**función** relación en la que cada elemento de un dominio se combina con exactamente un elemento del rango; es decir, para cada valor de x, existe exactamente un valor de y
function notation a way to name a function using $f(x)$ to represent the dependent variable instead of y	2.1	**notación de función** forma de nombrar una función con el uso de $f(x)$ como la variable dependiente en lugar de y

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English	Unit/Lesson	Español
	G	
geometric sequence an exponential function that results in a sequence of numbers separated by a common ratio	2.3 2.9	**secuencia geométrica** una función exponencial que produce como resultado una secuencia de números separados por una relación común
graphing method solving a system by graphing equations on the same coordinate plane and finding the point of intersection	3.2	**método de representación gráfica** resolución de un sistema mediante graficación de ecuaciones en el mismo plano de coordenadas y hallazgo del punto de intersección
growth factor the factor by which a quantity increases or decreases over time	2.6 2.10	**factor de crecimiento** múltiplo por el que aumenta o disminuye una cantidad con el tiempo
	H	
half plane a planar region containing all points that lie on one side of a boundary line; one-half of a plane	2.2	**semiplano** una región plana que contiene todos los puntos que se encuentran en un lado de una línea de límite; la mitad de un avión
histogram a frequency plot that shows the number of times a response or range of responses occurred in a data set. Example: 	4.1	**histograma** una diagrama de frecuencia que muestra la cantidad de veces que se produce una respuesta o rango de respuestas en un conjunto de datos. Ejemplo: 

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English	Unit/Lesson	Español
	I	
image the new, resulting figure after a transformation	5.1 5.5	**imagen** nueva figura resultante después de una transformación
included angle the angle between two sides	5.6	**ángulo incluido** ángulo entre dos lados
included side the side between two angles of a triangle	5.6	**lado incluido** lado entre dos ángulos de un triángulo
inclusive when the points in a plane that lie along the boundary line of an inequality are included in the solution	2.2	**inclusivo** cuando los puntos en un plano que se encuentran a lo largo de la línea de límite de una desigualdad se incluyen en la solución
inconsistent a system of equations with no solutions; lines are parallel when graphed	3.2	**inconsistente** sistema de ecuaciones sin soluciones; las líneas son paralelas cuando se las grafica
independent a system of equations with exactly one solution	3.2	**independiente** sistema de ecuaciones con una solución exacta
independent variable generally labeled on the *x*-axis; the quantity that changes based on values chosen	1.3	**variable independiente** generalmente designada en el eje *x*; cantidad que cambia según valores seleccionados
inequality a mathematical sentence that shows the relationship between quantities that may or may not be equivalent. An inequality contains one or more of the following symbols: <, >, ≤, ≥, or ≠.	1.2 1.4	**desigualdad** enunciado matemático que demuestra la relación entre cantidades que pueden ser o no equivalentes. Una desigualdad contiene uno o más de los siguientes símbolos: <, >, ≤, ≥ o ≠.
inscribe to draw one figure within another figure so that every vertex of the enclosed figure touches the outside figure	5.4	**inscribir** dibujar una figura dentro de otra de manera que cada vértice de la figura interior toque la exterior
integer the set of positive and negative whole numbers and 0; the set {... –3, –2, –1, 0, 1, 2, 3, ...}	2.4	**entero** el conjunto de números enteros positivos y negativos y 0; el conjunto {... –3, –2, –1, 0, 1, 2, 3, ...}
intercept the value of the *x*- or *y*-coordinate where a line or curve intersects the *x*- or *y*-axis, respectively	2.2 2.4	**intersección** valor de la coordenada *x* o *y* donde una línea o curva interseca el eje *x* o *y*, respectivamente
interquartile range the difference between the third and first quartiles; 50% of the data is contained within this range	4.1	**rango intercuartílico** diferencia entre el tercer y primer cuartil; el 50% de los datos está contenido dentro de este rango

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English	Unit/Lesson	Español
interval the continuous set of real numbers between two given numbers	2.4 2.6	**intervalo** conjunto continuo de números reales entre dos números dados
inverse operation an operation that reverses the effect of another operation. Addition and subtraction are inverse operations, and multiplication and division are inverse operations.	1.5	**operación inversa** operación que revierte el efecto de otra. La adición y la sustracción son operaciones inversas, y la multiplicación y división son operaciones inversas.
irrational number a real number that cannot be written as $\frac{m}{n}$, where m and n are integers and $n \neq 0$; a non-terminating or non-repeating decimal	2.4	**número irracional** un número real que no puede ser escrito como $\frac{m}{n}$, donde m y n son números enteros y $n \neq 0$; un no-terminación o no repetitivo decimal
isometry a transformation in which the preimage and image are congruent	5.1 5.5	**isometría** transformación en la que la preimagen y la imagen son congruentes
	J	
joint frequency the number of times a pair of characteristics appear together	4.2	**frecuencia conjunta** cantidad de veces que un par de características aparecen juntas
	L	
laws of exponents rules that must be followed when working with exponents	3.1	**leyes de los exponentes** normas que deben cumplirse cuando se trabaja con exponentes
like terms terms that contain the same variables raised to the same power	1.1	**términos semejantes** términos que contienen las mismas variables elevadas a la misma potencia
line the straight path connecting two points and extending beyond the points in both directions; written as $\overleftrightarrow{PQ}$ or ℓ	5.1 5.3	**línea recta** trayectoria recta que conecta dos puntos y que se extiende más allá de los puntos en ambas direcciones; se expresa como $\overleftrightarrow{PQ}$ o ℓ
line of reflection the perpendicular bisector of the segments that connect the corresponding vertices of the preimage and the image	5.5	**línea de reflexión** bisectriz perpendicular de los segmentos que conectan los vértices correspondientes de la preimagen y la imagen
line of symmetry a line separating a figure into two halves that are mirror images	5.1	**línea de simetría** línea que separa una figura en dos mitades que son imágenes en espejo

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English	Unit/Lesson	Español
line segment a part of a line that is between two endpoints and that includes the endpoints; written as $\overline{PQ}$	5.1 5.3	**segmento de recta** parte de una línea que se encuentra entre dos puntos finales y que incluye los puntos finales; escrito como $\overline{PQ}$
line symmetry e exists for a figure if for every point on one side of the line of symmetry, there is a corresponding point the same distance from the line of symmetry on the other side	5.1	**simetría lineal** la que existe en una figura si para cada punto a un lado de la línea de simetría, hay un punto correspondiente a la misma distancia de la línea de simetría en el otro lado
linear equation a first-degree equation that can be written in the form $ax + by = c$, where a, b, and c are rational numbers; when written as $y = mx + b$, m is the slope of the line, and b is its y-intercept. The graph of a linear equation is a straight line.	1.2 1.3 2.1 2.7	**ecuación lineal** ecuación de primer grado que puede expresarse en la forma $ax + by = c$, donde a, b y c son números racionales; cuando se expresa como $y = mx + b$, m es la pendiente de la recta y b es el intercepto de y. La representación gráfica de una ecuación lineal es una línea recta.
linear fit (or linear model) an approximation of data using a linear function	4.3	**ajuste lineal (o modelo lineal)** aproximación de datos con el uso de una función lineal
linear function a first-degree equation that can be written in the form $f(x) = mx + b$, in which m is the slope of the line and b is the y-intercept. The graph of a linear function is a straight line.	2.5 2.6 2.10	**función lineal** una ecuación de primer grado que puede expresarse en la forma $f(x) = mx + b$, en la que m es la pendiente de la recta y b es el intercepto de y. El gráfico de una función lineal es una línea recta.
literal equation an equation that involves two or more variables	1.5	**ecuación literal** ecuación que incluye dos o más variables
	M	
marginal frequency the total number of times a characteristic occurs	4.2	**frecuencia marginal** cantidad total de veces que aparece una característica
mean the average value of a data set, found by summing all values and dividing by the number of data points	4.1	**media** valor promedio de un conjunto de datos, que se determina al sumar todos los valores y dividirlos por la cantidad de puntos de datos

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English	Unit/Lesson	Español
mean absolute deviation the average distance between each data point and the mean; found by summing the absolute values of the difference between each data point and the mean, then dividing this sum by the total number of data points	4.1	**desviación media absoluta** distancia promedio entre cada punto de datos y la media; se determina al sumar los valores absolutos de la diferencia entre cada punto de datos y la media y luego dividir esta suma por la cantidad total de puntos de datos
measures of center values that describe expected and repeated data values in a data set; the mean and median are two measures of center	4.1	**medidas de centro** valores que describen los valores de datos esperados y repetidos de un conjunto de datos; la media y la mediana son dos medidas de centro
measures of spread a measure that describes the variance of data values, and identifies the diversity of values in a data set	4.1	**medidas de dispersión** medidas que describen la varianza de los valores de datos e identifican la diversidad de valores en un conjunto de datos
median the middle-most value of a data set; 50% of the data is less than this value, and 50% is greater than it	4.1	**mediana** valor medio exacto de un conjunto de datos; el 50% de los datos es menor que ese valor, y el otro 50% es mayor
median of a triangle the segment joining the vertex to the midpoint of the opposite side	5.3	**mediana de un triángulo** segmento que une el vértice con el punto medio del lado opuesto
midpoint a point on a line segment that divides the segment into two equal parts	5.3	**punto medio** punto en un segmento de recta que lo divide en dos partes iguales
midsegment a line segment joining the midpoints of two sides of a triangle	5.3	**segmento medio** segmento de recta que une los puntos medios de dos lados de una figura
multiplicative inverse a number or algebraic expression that when multiplied by the original number or algebraic expression has a product of 1; also called the *reciprocal*	1.5	**inverso multiplicativo** número o expresión algebraica que, cuando se multiplica por el número o la expresión algebraica original, tiene un producto de 1; también se llama *recíproco*

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English	Unit/Lesson	Español
	N	
natural numbers the set of positive integers {1, 2, 3, ...}	2.3 2.4	**números naturales** conjunto de enteros positivos {1, 2, 3, ...}
negative function a function or a portion of a function where the *y*-values are less than 0 for all *x*-values	2.4	**función negativa** función o porción de una función en la que los valores *y* son menores que 0 para todos los valores *x*
non-inclusive when the points in a plane that lie along the boundary line of an inequality are not included in the solution	2.2	**no inclusivo** cuando los puntos en un plano que se encuentran a lo largo de la línea de límite de una desigualdad no están incluidos en la solución
non-rigid motion a transformation done to a figure that changes the figure's shape and/or size	5.5	**movimiento no rígido** transformación hecha a una figura que cambia su forma o tamaño
	O	
obtuse angle an angle measuring greater than 90˚ but less than 180˚	5.1	**ángulo obtuso** ángulo que mide más de 90˚ pero menos de 180˚
one-to-one a relationship wherein each input is mapped to exactly one output, and each output is associated with exactly one input	5.1	**unívoca** relación en la que cada entrada se asigna a exactamente una salida, y cada salida está asociada con exactamente una entrada
order of operations the order in which expressions are evaluated from left to right (grouping symbols, evaluating exponents, completing multiplication and division, completing addition and subtraction)	1.1	**orden de las operaciones** orden en el que se evalúan las expresiones de izquierda a derecha (con agrupación de símbolos, evaluación de exponentes, realización de multiplicaciones y divisiones, sumas y sustracciones)
ordered pair the coordinates of a point in a coordinate plane, (x, y) where the order is significant	2.1	**par ordenado** coordenadas de un punto en un plano de coordenadas, (x, y), en los que el orden es significativo
outlier a data value that is much greater than or much less than the rest of the data in a data set; mathematically, any data less than $Q_1 - 1.5(IQR)$ or greater than $Q_3 + 1.5(IQR)$ is an outlier	4.1	**valor atípico** valor de datos que es mucho mayor o mucho menor que el resto de los datos de un conjunto de datos; en matemática, cualquier dato menor que $Q_1 - 1{,}5(IQR)$ o mayor que $Q_3 + 1{,}5(IQR)$ es un valor atípico

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English	Unit/Lesson	Español
	P	
parallel lines lines in a plane that do not share any points and never intersect; written as $\overleftrightarrow{AB} \parallel \overleftrightarrow{PQ}$; line segments and rays can also be parallel	5.1 5.3 6.1	**líneas paralelas** líneas en un plano que no comparten ningún punto y nunca se cortan; se expresan como $\overleftrightarrow{AB} \parallel \overleftrightarrow{PQ}$; segmentos de línea y los rayos también pueden ser paralelos
parallelogram a special type of quadrilateral with two pairs of opposite sides that are paralle	6.1	**paralelogramo** un tipo especial de cuadrilátero con dos pares de lados opuestos paralelos
parameter a constant in a function that determines the specific graph of the function but not the type of the function	2.10	**parámetro** una constante en una función que determina el gráfico específico de la función pero no el tipo de la función
perimeter the distance around a two-dimensional figure	6.2	**perímetro** distancia alrededor de una figura bidimensional
perpendicular bisector a line that intersects a segment at its midpoint at a right angle	5.3	**bisectriz perpendicular** línea que corta un segmento en su punto medio en ángulo recto
perpendicular lines two lines that intersect at a right angle (90˚); written as $\overleftrightarrow{AB} \perp \overleftrightarrow{PQ}$; line segments and rays can also be perpendicular	5.1 5.3 6.1	**líneas perpendiculares** dos líneas que se cortan en ángulo recto (90˚); se expresan como $\overleftrightarrow{AB} \perp \overleftrightarrow{PQ}$; segmentos de línea y los rayos también pueden ser perpendicular
point an exact position or location in a given plane	5.1	**punto** posición o ubicación exacta en un plano determinado
point of intersection the point at which two lines cross or meet	3.2	**punto de intersección** punto en que se cruzan o encuentran dos líneas
point of rotation the fixed location that an object is turned around; the point can lie on, inside, or outside the figure	5.5	**punto de rotación** ubicación fija en torno a la que gira un objeto; el punto puede estar encima, dentro o fuera de la figura
polygon two-dimensional figure with at least three sides	6.2	**polígono** figura bidimensional con al menos tres lados
positive function a function or a portion of a function where the *y*-values are greater than 0 for all *x*-values	2.4	**función positiva** una función o porción de una función en la que los valores *y* son mayores que 0 para todos los valores *x*
postulate a true statement that does not require a proof	5.6	**postulado** afirmación verdadera que no requiere prueba

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English	Unit/Lesson	Español
preimage the original figure before undergoing a transformation	5.1 5.5	**preimagen** figura original antes de sufrir una transformación
properties of equality rules that allow you to balance, manipulate, and solve equations	3.1	**propiedades de igualdad** normas que permiten equilibrar, manipular y resolver ecuaciones
properties of inequality rules that allow you to balance, manipulate, and solve inequalities	3.1	**propiedades de desigualdad** normas que permiten equilibrar, manipular y resolver desigualdades
	Q	
quadrant the coordinate plane is separated into four sections: • In Quadrant I, x and y are positive. • In Quadrant II, x is negative and y is positive. • In Quadrant III, x and y are negative. • In Quadrant IV, x is positive and y is negative.	5.2	**cuadrante** plano de coordenadas que se divide en cuatro secciones: • En el cuadrante I, x y y son positivos. • En el cuadrante II, x es negativo y y es positivo. • En el cuadrante III, x y y son negativos. • En el cuadrante IV, x es positivo y y es negativo.
quadrilateral a polygon with four sides	6.1	**cuadrilátero** polígono con cuatro lados
quantity a value or expression that may be expressed in numbers	1.2	**cantidad** valor o expresión que puede expresarse en números
	R	
radius a line segment that extends from the center of a circle to a point on the circle. Its length is half the diameter.	5.4	**radio** segmento de línea que se extiende desde el centro de un círculo hasta un punto de la circunferencia del círculo. Su longitud es la mitad del diámetro.
range the set of all outputs of a relation or function; the set of y-values for which a function is defined	2.1	**rango** conjunto de todas las salidas de una función; conjunto de valores de y para el que se define una función
rate a ratio that compares different kinds of units	1.2	**tasa** proporción en que se comparan distintos tipos de unidades
rate of change a ratio that describes how much one quantity changes with respect to the change in another quantity; also known as the slope of a line	2.4 2.6	**tasa de cambio** proporción que describe cuánto cambia una cantidad con respecto al cambio de otra cantidad; también se la conoce como pendiente de una recta

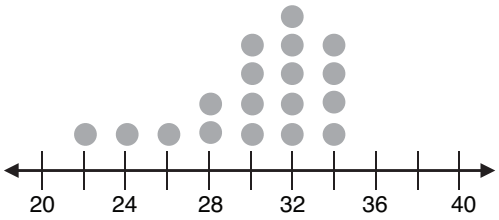
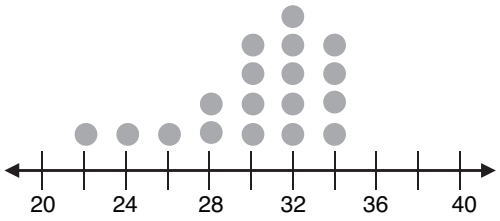
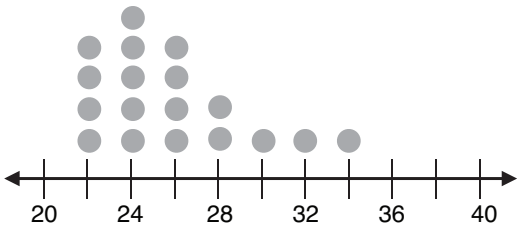
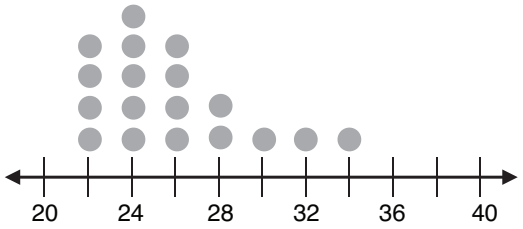
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English	Unit/Lesson	Español
ratio the relation between two quantities; can be expressed in words, or as a fraction, decimal, or percent	2.4	**proporción** relación entre dos cantidades; puede expresarse en palabras, fracciones, decimales o como porcentaje
rational number a real number that can be written as $\frac{m}{n}$, where both m and n are integers and $n \neq 0$; a terminating or repeating decimal	2.4	**número racional** en los que m y n son enteros y $n \neq 0$; cualquier número que puede escribirse como decimal finito o periódico
ray a part of a line that starts at a point and extends infinitely in one direction; written as $\overrightarrow{PQ}$	5.1 5.3	**semirrecta** parte de una línea que comienza en un punto y se extiende infinitamente en una dirección línea con un solo extremo; se expresa como $\overrightarrow{PQ}$
real numbers the set of all rational and irrational numbers	2.4	**números reales** conjunto de todos los números racionales e irracionales
reciprocal a number that when multiplied by the original number or algebraic expression has a product of 1; also called the *multiplicative inverse*	1.5	**recíproco** número que cuando se multiplica por el número original o la expresión algebraica tiene un producto de 1; también llamada la *inversa multiplicativa*
rectangle a special parallelogram with four right angles	6.1	**rectángulo** paralelogramo especial con cuatro ángulos rectos
recursive formula a formula used to find the next term of a sequence when the previous term or terms are known; the recursive formula for an arithmetic sequence is $a_n = a_{n-1} + d$; the recursive formula for a geometric sequence is $a_n = a_{n-1} \bullet r$	2.3 2.9	**fórmula recursiva** fórmula que se utiliza para encontrar el término siguiente de una secuencia cuando se conoce el o los términos anteriores; la fórmula recursiva de una secuencia aritmética es $a_n = a_{n-1} + d$; la fórmula recursiva para una secuencia geométrica es $a_n = a_{n-1} \bullet r$
reflection a transformation where a mirror image is created; also called a flip	5.1 5.2	**reflexión** transformación por la cual se crea una imagen en espejo
regular hexagon a six-sided polygon with all sides congruent and all angles measuring 120°	5.4	**hexágono regular** polígono de seis lados con todos los lados iguales y en el que todos los ángulos miden 120°
regular polygon a closed two-dimensional figure with all sides and all angles congruent	5.1 5.4	**polígono regular** figura bidimensional cerrada con todos los lados y todos los ángulos congruentes

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English	Unit/Lesson	Español
relation a set of ordered pairs	2.1	**relación** un conjunto de pares ordenados
relative maximum the greatest value of a function for a particular interval of the function	2.4	**máximo relativo** el mayor valor de una función para un intervalo particular de la función
relative minimum the least value of a function for a particular interval of the function	2.4	**mínimo relativo** el menor valor de una función para un intervalo particular de la función
residual the vertical distance between an observed data value and an estimated data value on a line of best fit	4.2	**residual** distancia vertical entre un valor de datos observado y un valor de datos estimado sobre una línea de ajuste óptimo
residual plot provides a visual representation of the residuals for a set of data; contains the points (x, residual for x)	4.2	**diagrama residual** brinda una representación visual de los residuales para un conjunto de datos; contiene los puntos (x, residual de x)
rhombus a special parallelogram with four congruent sides	6.1	**rombo** paralelogramo especial con sus cuatro lados congruente
right angle an angle measuring 90˚	5.1	**ángulo recto** ángulo que mide 90˚
rigid motion a transformation done to a figure that maintains the figure's shape and size or its segment lengths and angle measures	5.5 5.6	**movimiento rígido** transformación que se realiza a una figura que mantiene su forma y tamaño o las longitudes de sus segmentos y las medidas de ángulos
rotation a transformation that turns a figure around a fixed center point; also called a turn	5.1 5.2	**rotación** transformación que convierte una figura alrededor de un punto central fijo; también llamado un giro
	S	
scale factor a multiple of the lengths of the sides from one figure to the transformed figure. If the scale factor is larger than 1, then the figure is enlarged. If the scale factor is between 0 and 1, then the figure is reduced.	5.5	**factor de escala** múltiplo de las longitudes de los lados de una figura a la figura transformada. Si el factor de escala es mayor que 1, entonces la figura se agranda. Si el factor de escala se encuentra entre 0 y 1, entonces la figura se reduce.
scatter plot a graph of data in two variables on a coordinate plane, where each data pair is represented by a point	4.2	**diagrama de dispersión** gráfica de datos en dos variables en un plano de coordenadas, en la que cada par de datos está representado por un punto

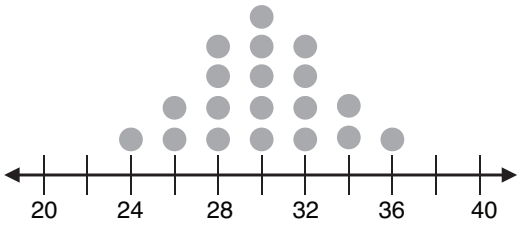
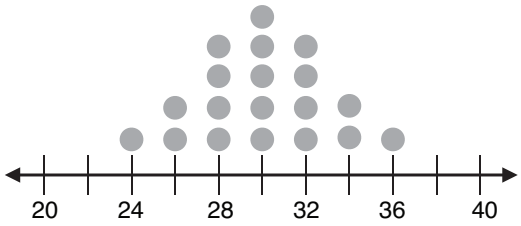
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English	Unit/Lesson	Español
sequence an ordered list of numbers	2.3	**secuencia** lista ordenada de números
side-angle-side (SAS) if two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent	5.6	**lado-ángulo-lado (SAS)** si dos lados y el ángulo incluido de un triángulo son congruentes con dos lados y el ángulo incluido de otro triángulo, entonces los dos triángulos son congruentes
side-side-side (SSS) if three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent	5.6	**lado-lado-lado (SSS)** si los tres lados de un triángulo son congruentes con los tres lados de otro triángulo, entonces los dos triángulos son congruentes
sketch a quickly done representation of a figure; a rough approximation of a figure	5.3	**bosquejo** representación de una figura realizada con rapidez; aproximación imprecisa de una figura
skewed to the left data concentrated on the higher values in the data set, which has a tail to the left. Example: 	4.1	**desviados hacia la izquierda** datos concentrados en los valores más altos del conjunto de datos, que tiene una cola hacia la izquierda. Ejemplo: 
skewed to the right data concentrated on the lower values in the data set, which has a tail to the right. Example: 	4.1	**desviados hacia la derecha** datos concentrados en los valores más bajos del conjunto de datos, que tiene una cola hacia la derecha. Ejemplo: 

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English	Unit/Lesson	Español
slope the measure of the rate of change of one variable with respect to another variable; slope = $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$; the slope in the equation $y = mx + b$ is m	1.3 2.4 2.6 2.7 4.3 6.1	**pendiente** medida de la tasa de cambio de una variable con respecto a otra; pendiente = $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$; la pendiente en la ecuación $y = mx + b$ es m
slope-intercept form of a linear equation the form $y = mx + b$, where m is the slope of the line and b is the y-intercept	2.4	**forma pendiente-intersección de una ecuación lineal** la forma $y = mx + b$, donde m es la pendiente y b es el punto de intersección con el eje y
slope-intercept method a way to use the slope-intercept form of a linear equation, $y = mx + b$, to graph the line, by first plotting the y-intercept, b, and then applying the slope, m	2.4	**método de pendiente-intersección** una manera de utilizar la forma pendiente-intersección de una ecuación lineal, $y = mx + b$, para graficar la línea, trazando primero la intersección y, b, y luego aplicando la pendiente, m
solution a value that makes an equation true	1.2	**solución** valor que hace verdadera la ecuación
solution set the value or values that make a sentence or statement true; the set of ordered pairs that represent all of the solutions to an equation or a system of equations	1.2 1.4 2.1	**conjunto de soluciones** valor o valores que hacen verdadera una afirmación o declaración; conjunto de pares ordenados que representa todas las soluciones para una ecuación o sistema de ecuaciones
solution to a system of linear inequalities the set of points that lie in the intersection of the half planes of the inequalities and which may also lie on the boundary lines; the solution set is the set of all points that satisfy the inequalities in the system	2.2	**solución a un sistema de desigualdades lineales** el conjunto de puntos que se encuentran en la intersección de los planos de la mitad de las desigualdades y que también pueden situarse en las líneas de contorno; el conjunto solución es el conjunto de todos los puntos que satisfacen las desigualdades en el sistema
square a regular polygon with four congruent sides and four right angles	5.4 6.1	**cuadrado** polígono regular con cuatro lados congruentes y cuatro ángulos rectos
straightedge a bar or strip of wood, plastic, or metal having at least one long edge of reliable straightness	5.3	**regla de borde recto** barra o franja de madera, plástico o metal que tiene, al menos, un borde largo de rectitud confiable

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English	Unit/Lesson	Español
substitution method solving one of a pair of equations for one of the variables and substituting that into the other equation	3.2	**método de sustitución** solución de un par de ecuaciones para una de las variables y sustitución de eso en la otra ecuación
symmetric situation in which data is concentrated toward the middle of the range of data; data values are distributed in the same way above and below the middle of the sample. Example: 	4.1	**simétrico** situación en la que los datos se concentran hacia el medio del rango de datos; los valores de datos se distribuyen de la misma manera por encima y por debajo del medio de la muestra. Ejemplo: 
system a set of two or more equations	2.1	**sistema** un conjunto de dos o más ecuaciones
system of equations a set of equations with the same unknowns	1.4 3.2	**sistema de ecuaciones** un conjunto de ecuaciones con las mismas incógnitas
system of inequalities a set of two or more inequalities with the same unknowns	1.4 2.2	**sistema de desigualdades** un conjunto de dos o más desigualdades con las mismas incógnitas
	T	
term a number, a variable, or the product of a number and variable(s)	1.1	**término** número, variable o producto de un número y una o más variables
third quartile value that identifies the upper 25% of the data; the median of the upper half of the data set; 75% of all data is less than this value; written as Q_3	4.1	**tercer cuartil** valor que identifica el 25% superior de los datos; mediana de la mitad superior del conjunto de datos; el 75% de los datos es menor que este valor; se expresa como Q_3
transformation a change in a geometric figure's position, shape, or size	2.8 5.1	**transformación** cambio en la posición, la forma o el tamaño de una figura geométrica

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English	Unit/Lesson	Español
translation 1. moving a graph either vertically, horizontally, or both, without changing its shape; a slide 2. a transformation that moves each point of a figure the same distance in the same direction; also called a slide	2.8 5.1 5.2	**traslación** 1. mover un gráfico verticalmente, horizontalmente o ambos, sin cambiar su forma; una diapositiva 2. transformación que mueve cada punto de una figura a la misma distancia en la misma dirección; también llamado una diapositiva
trend a pattern of behavior, usually observed over time or over multiple iterations	4.2	**tendencia** patrón de comportamiento, que se observa por lo general en el tiempo o en múltiples repeticiones
triangle a three-sided polygon with three angles	5.4	**triángulo** polígono de tres lados con tres ángulos
two-way frequency table a table that divides responses into categories, showing both a characteristic in the table rows and a characteristic in the table columns; values in cells are a count of the number of times each response was given by a respondent with a certain characteristic	4.2	**tabla de frecuencia de doble entrada** tabla que divide las respuestas en dos categorías, y muestra una característica en las filas y una en las columnas; los valores de las celdas son un conteo de la cantidad de veces que un respondedor da una respuesta con una determinada característica
	U	
undefined slope occurs when the denominator of the slope formula is equal to 0; the corresponding line is a vertical line	2.4	**pendiente indefinida** se produce cuando el denominador de la fórmula de la pendiente es igual a 0; la correspondiente línea es una línea vertical
unit rate a ratio of two measurements, the second of which is 1	1.2	**tasa unitaria** una proporción de dos medidas, de las que la segunda es 1
	V	
variable a letter used to represent an unknown value or a value that changes	1.1 1.2 2.7	**variable** una letra utilizada para representar un valor desconocido o un valor que cambia
vertical shift the number of units the graph of the function is moved up or down in a plane; a type of translation	2.8 2.10	**desplazamiento vertical** cantidad de unidades que el gráfico de la función se desplaza hacia arriba o hacia abajo en un plano; un tipo de traslación

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English	Unit/Lesson	Español
	W	
whole numbers the set of positive integers and 0: {0, 1, 2, 3, ...}	2.4	**números enteros** conjunto de enteros positivos que incluye el 0: {0, 1, 2, 3, ...}
	X	
***x*-intercept** the *x*-coordinate of the point where a line or a curve intersects the *x*-axis	1.3 2.2 2.4 2.5 2.6	**intersección *x*** la coordenada *x* del punto en que una recta o curva corta el eje *x*
	Y	
***y*-intercept** the *y*-coordinate of the point where a line or a curve intersects the *y*-axis	1.3 2.2 2.4 2.5 2.6 2.7 4.3	**intersección *y*** la coordenada *y* del punto en que una recta o curva corta el eje *y*

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Common Core State Standards

Mathematics I

Integrated Pathway



Teacher Resource

Unit 1

WALCH

INTEGRATED MATH

This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

Developers and reviewers include:

Joyce Hale	Valerie Ackley	Doug Kühlmann
Shelly Northrop Sommer	Laura McPartland	Mike May, S.J.
Ruth Estabrook	Cameron Larkins	James Quinlan
Vanessa Sylvester	Jennifer Blair	Peter Tierney-Fife
Zachary Lien	Nancy Pierce	

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Standards Correlations

Each lesson in this Integrated Pathway: Mathematics I program was written specifically to address the Common Core State Standards. Each lesson lists the standards covered in all the sub-lessons, and each sub-lesson lists the standards addressed in that particular section. In this section, you'll find a comprehensive list mapping the sub-lessons to the CCSS.

Guide to Common Core State Standards Annotation

As you use this program, you will come across a symbol included with the Common Core standards for some of the lessons and activities. These symbols are explained below.

Symbol: ⋆

Denotes: Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (⋆).

From http://www.walch.com/CCSS/00003

Symbol: (+)

Denotes: College and Career Readiness Standards

Advanced mathematics standards that are required in higher-level courses such as advanced statistics may also be included in lower-level courses. These additional standards are denoted by (+). According to the Common Core State Standards Initiative, "the evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6–8."

From http://www.walch.com/CCSS/00004

Connections to Future Courses

This section provides a map between topics introduced in each unit of this course and subsequent courses where each topic is revisited and built upon.

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CCSS INTEGRATED PATHWAY: MATHEMATICS I STANDARDS CORRELATIONS

Unit 1: Relationships Between Quantities				
Lesson	**Sub-lesson number**	**Title**	**Standard(s)**	**Pages**
Lesson 1	**Interpreting Structure in Expressions**			
	1.1.1	Identifying Terms, Factors, and Coefficients	A–SSE.1a★	U1-5
	1.1.2	Interpreting Linear and Exponential Expressions	A–SSE.1b★	U1-17
Lesson 2	**Creating Equations and Inequalities in One Variable**			
	1.2.1	Creating Linear Equations in One Variable	A–CED.1★ N–Q.2★ N–Q.3★	U1-33
	1.2.2	Creating Linear Inequalities in One Variable	A–CED.1★	U1-54
	1.2.3	Creating Exponential Equations	A–CED.1★	U1-69
Lesson 3	**Creating and Graphing Equations in Two Variables**			
	1.3.1	Creating and Graphing Linear Equations in Two Variables	A–CED.2★ N–Q.1★	U1-94
	1.3.2	Creating and Graphing Exponential Equations	A–CED.2★ N–Q.1★	U1-129
Lesson 4	**Representing Constraints**			
	1.4.1	Representing Constraints	A–CED.3★	U1-157
Lesson 5	**Rearranging Formulas**			
	1.5.1	Rearranging Formulas	A–CED.4★	U1-177

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TOPICS FROM FUTURE COURSES: CCSS INTEGRATED PATHWAY MATHEMATICS I

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 1	Identifying and interpreting the effects of terms, factors, and coefficients of linear, exponential, and quadratic expressions	Math II Unit 3	includes more complex quadratic expressions and introduces the standard form of a quadratic expression
		Math III Units 1 and 2A	extended to include higher-order polynomials
		Math III Unit 2B	extended to rational expressions
	Creating and graphing with linear and simple exponential equations	Math II Unit 2	creating and graphing functions by using quadratic, square root, cube root, absolute value, step, and piecewise functions
		Math II Unit 3	extended to quadratic and rational functions and to graphing higher-order polynomial functions
		Math III Unit 2A	extended to rational and radical functions
		Math III Unit 2B	introduces students to graphing logarithmic and periodic functions
		Math III Unit 4A	addressed using more complex exponential equations
		Math III Unit 4B	extended to applying the creating and graphing principles
	Constraints in terms of linear relationships	Math II Unit 3, Math III Unit 4B	addressed using linear, exponential, quadratic, and logarithmic models
	Rearranging linear formulas	Math II Unit 3	extended to rearranging quadratic expressions
		Math III Unit 4B	extended to rearranging all function families

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Conceptual Activities

Use these interactive open education and/or Desmos resources to build conceptual understanding of mathematical ideas. (*Note*: Activity links will be monitored and repaired or replaced as necessary.)

- Desmos. "Function Carnival."

  http://www.walch.com/ca/01006

  This activity focuses attention on graphs as expressing relationships between variables. It lays the informal groundwork for the more formal definitions and properties of functions.

- Desmos. "Marbleslides: Lines."

  http://www.walch.com/ca/01008

  Restrict, reposition, and rotate lines at will using slope-intercept form, and describe transformations using words and/or symbols.

- Desmos. "Put the Point on the Line."

  http://www.walch.com/ca/01009

  The focus of this activity is slope. Participants are asked to estimate, calculate, and notice proportionality as they place points on an imaginary line.

- Desmos, Inc. "Polygraph: Linear Inequalities."

  http://www.walch.com/ca/10000

  In this activity, students will engage in vocabulary-rich conversations about linear inequalities. Key vocabulary terms that may appear in student questions include *shading*, *above*, *below*, *boundary*, *solid*, *dotted*, *horizontal*, *vertical*, *slanted*, *axis*, and *quadrant*.

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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. How many terms are in the expression $36x^3 + 27x^2 - 18x - 9$?

   a. 3
   b. 7
   c. 4
   d. 9

2. What are the factors of the non-constant terms of the expression $11x^2 + 7x - 4$?

   a. 11 and x^2, 7 and x
   b. 11 and 7
   c. There aren't any factors in these terms.
   d. x

3. What are the term(s), coefficient, and constant term described by the phrase, "the cost of 4 tickets to the football game, where the price per ticket is t dollars, and a service charge of $10"?

   a. term: $4t$, coefficient: 4, constant term: 10
   b. terms: $4t$ and 10, coefficient: 10, constant term: 4
   c. term: $14t$, coefficient: 14, constant term: none
   d. terms: $4t$ and 10, coefficient: 4, constant term: 10

continued

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Assessment

4. Newer books can be downloaded for $8 each, while older books can be downloaded for $4 each. The total cost, in dollars, of 12 books is represented by the expression $8n + 4(12 - n)$, where n represents the number of new books downloaded. How does changing the value of n change the value of the term $4(12 - n)$?

   a. For values of n less than 12, the term $4(12 - n)$ will be negative; for values of n greater than 12, the term will be positive; when n equals 12, the term will equal 0.

   b. For values of n less than 12, the term $4(12 - n)$ will be positive; for values of n greater than 12, the term will be positive; when n equals 12, the term will equal 0.

   c. For values of n less than 12, the term $4(12 - n)$ will be positive; values of n greater than 12 do not make sense because we are only buying 12 books total; when n equals 12, the term will equal 0.

   d. For values of n less than 12, the term $4(12 - n)$ will be negative; values of n greater than 12 do not make sense because we are only buying 12 books total; when n equals 12, the term will equal 0.

5. An amount of money in a bank account earns interest according to the following expression: $P(1 + r)^t$, where P represents the initial deposit, r represents the interest rate, and t represents the period of time. How does increasing the period of time change the value of P?

   a. Increasing the period of time will increase the value of P.

   b. Increasing the period of time does not change the value of P.

   c. Increasing the period of time will decrease the value of P.

   d. It is not possible to increase the period of time.

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Lesson 1: Interpreting Structure in Expressions

Instruction

Common Core State Standards

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 ✓ 6 ✓
7 ✓ 8

A–SSE.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.*

Essential Questions

1. What are the different parts of an algebraic expression?
2. How are real-world scenarios translated into algebraic expressions?
3. How does changing one part of an expression affect the value of the expression?

WORDS TO KNOW

algebraic expression	a mathematical statement that includes numbers, operations, and variables to represent a number or quantity
base	the factor being multiplied together in an exponential expression; in the expression a^b, a is the base
coefficient	the number multiplied by a variable in an algebraic expression
constant	a quantity that does not change
exponent	the number of times a factor is being multiplied together in an exponential expression; in the expression a^b, b is the exponent
factor	one of two or more numbers or expressions that are multiplied to produce a product
like terms	terms that contain the same variables raised to the same power
order of operations	the order in which expressions are evaluated from left to right (grouping symbols, evaluating exponents, completing multiplication and division, completing addition and subtraction)

SRB p. U1-1

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term	a number, a variable, or the product of a number and variable(s)
variable	a letter used to represent a value or unknown quantity that can change or vary

Recommended Resources

- Math-Play.com. "Algebraic Expressions Millionaire Game."

  http://walch.com/rr/CAU1L1Expressions

  "Algebraic Expressions Millionaire Game" can be played alone or in two teams. For each question, players have to identify the correct mathematical expression that models a given expression.

- Quia. "Algebraic Symbolism Matching Game."

  http://walch.com/rr/CAU1L1AlgSymbolism

  In this matching game, players pair each statement with its algebraic interpretation. There are 40 matches to the provided game.

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Lesson 1.1.1: Identifying Terms, Factors, and Coefficients

Warm-Up 1.1.1

Ella purchased 2 DVDs and 3 CDs from Tyler's Electronics at the prices listed in the given table. After taxes, her total cost increased by $5.60. Use this information and the table to complete each problem.

Item	Cost ($)
CD	c
DVD	d

1. How can you write the cost of 2 DVDs as an algebraic expression?

2. How can you write the cost of 2 DVDs and 3 CDs as an algebraic expression?

3. How can you write the cost of 2 DVDs and 3 CDs, increased by $5.60 for taxes, as an algebraic expression?

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Instruction

Lesson 1.1.1: Identifying Terms, Factors, and Coefficients

Common Core State Standard

A–SSE.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

Warm-Up 1.1.1 Debrief

1. How can you write the cost of 2 DVDs as an algebraic expression?

   The cost of 2 DVDs can be written as an expression of multiplication, $2d$.

2. How can you write the cost of 2 DVDs and 3 CDs as an algebraic expression?

   The cost of 2 DVDs and 3 CDs can be written as the expression $2d + 3c$.

3. How can you write the cost of 2 DVDs and 3 CDs, increased by $5.60 for taxes, as an algebraic expression?

   "Increased by $5.60 for taxes" means "to add 5.60" to the expression $2d + 3c$ in order to represent the total cost of the DVDs and CDs including taxes.

   Therefore, $2d + 3c + 5.60$ represents the cost of 2 DVDs and 3 CDs increased by $5.60.

Connection to the Lesson

- Students will identify parts of algebraic expressions.
- Students will translate verbal scenarios into algebraic expressions, then identify the parts of the expressions.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- translating verbal expressions into algebraic expressions (6.EE.2a)
- evaluating expressions following the order of operations (6.EE.2c)

Introduction

Thoughts or feelings in language are often conveyed through expressions; similarly, mathematical ideas are conveyed through algebraic expressions. **Algebraic expressions** are mathematical statements that include numbers, operations, and variables to represent a number or quantity. **Variables** are letters used to represent values or unknown quantities that can change or vary. One example of an algebraic expression is $3x - 4$. Notice the variable, x.

Key Concepts

- Expressions are made up of terms. A **term** is a number, a variable, or the product of a number and variable(s). An addition or subtraction sign separates each term of an expression.
- For example, in the expression $4x^2 + 3x + 7$, there are 3 terms: $4x^2$, $3x$, and 7.
- The **factors** of each term are the numbers or expressions that when multiplied produce a given product. In the expression $4x^2 + 3x + 7$, the factors of $4x^2$ are 4 and x^2. The factors of $3x$ are 3 and x.
- Numbers have factors as well. For example, the factors of 7 are 1 and 7. *Note*: In this lesson, when asked for the "factors of a term," you are being asked to find the factors that make up an *algebraic* (non-constant) term.
- A **coefficient** is the number multiplied by a variable in an algebraic expression. In the terms $4x^2$ and $3x$, 4 is the coefficient of $4x^2$ and 3 is the coefficient of $3x$.
- The term $4x^2$ also has an exponent. **Exponents** indicate the number of times a factor is being multiplied by itself. In this term, 2 is the exponent and indicates that x is multiplied by itself 2 times.
- Terms that do not contain a variable are called **constant** terms because the quantity does not change. In this example, 7 is the constant term.

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Instruction

- The table that follows summarizes the parts of the expression $4x^2 + 3x + 7$.

Expression	$4x^2 + 3x + 7$		
Terms	$4x^2$	$3x$	7
Factors	4 and x^2	3 and x	1 and 7
Coefficients	4	3	—
Constants	—	—	7

- Terms with the same variable raised to the same exponent are called **like terms**. In the example $5x + 3x - 9$, $5x$ and $3x$ are like terms. Like terms can be combined following the **order of operations** by evaluating grouping symbols, evaluating exponents, completing multiplication and division, and completing addition and subtraction from left to right. In this example, the sum of $5x$ and $3x$ is $8x$.

Common Errors/Misconceptions

- incorrectly following the order of operations
- incorrectly identifying like terms
- incorrectly combining terms involving subtraction

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Instruction

Guided Practice 1.1.1

Example 1

Simplify the expression $2(3 + x) + x(1 - 4x) + 5$, then identify each term, coefficient, and constant, and name the factors of each term of the polynomial.

1. Simplify the expression.

   The expression can be simplified by following the order of operations and combining like terms.

$2(3 + x) + x(1 - 4x) + 5$	Given expression
$6 + 2x + x(1 - 4x) + 5$	Distribute 2 over $3 + x$.
$6 + 2x + x - 4x^2 + 5$	Distribute x over $1 - 4x$.
$6 + 5 + 2x + x - 4x^2$	Rearrange to group like terms.
$11 + 3x - 4x^2$	Combine like terms: add 6 and 5, then add $2x$ and x.

   It is common to rearrange the expression so the powers are in descending order; that is, from the largest power to the smallest power:

   $-4x^2 + 3x + 11$

2. Identify each term in the simplified expression.

   There are three terms in the simplified expression: $-4x^2$, $3x$, and 11.

3. Identify any factors of the non-constant term(s).

   The numbers or expressions that, when multiplied, produce the product $-4x^2$ are -4 and x^2. The numbers or expressions that, when multiplied, produce the product $3x$ are 3 and x.

4. Identify any coefficients of the non-constant term(s).

   The number multiplied by a variable in the term $-4x^2$ is -4, and the number multiplied by a variable in the term $3x$ is 3. Therefore, -4 and 3 are coefficients.

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Instruction

5. Identify any constant terms.

   The term that does not change in the expression is 11. Therefore, 11 is the constant term.



Example 2

A smartphone is on sale for 25% off its regular price. The sale price of the smartphone is $149.25. What expression can be used to represent the regular price of the smartphone? Identify each term, the constant term, and the factors and coefficients of the terms that contain a variable.

1. Translate the verbal expression into an algebraic expression.

   First, describe the situation. The regular price is found by adding the discount amount to the sale price:

   sale price + discount amount

   We know that the sale price is $149.25. The discount amount is found by multiplying the discount percent by the unknown regular price. The discount percent is 25%, or written as a decimal, 0.25. Let x represent the unknown regular price. Therefore, the algebraic expression that represents the regular price of the smartphone is $149.25 + 0.25x$.

2. Identify each term in the algebraic expression.

   There are two terms in the expression: 149.25 and $0.25x$. The term 149.25 represents the sale price, and $0.25x$ represents the discount.

3. Identify any factors of the non-constant term(s).

   $0.25x$ is the product of the factors 0.25 and x.

4. Identify any coefficients of the non-constant term(s).

   0.25 is multiplied by the variable, x; therefore, 0.25 is a coefficient.

5. Identify any constant terms.

   The term that does not change in the expression is 149.25. Therefore, 149.25 is the constant term.



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Instruction

Example 3

Helen purchased 3 books from an online bookstore and received a 20% discount on her total order. Each book cost the same amount. The shipping cost was $10 and was not discounted. Write an expression that can be used to represent the total amount Helen paid for 3 books plus the shipping cost. Simplify the expression, and then identify each term, the constant term, and the factors and coefficients of the terms that contain a variable.

1. Translate the verbal expression into an algebraic expression.

   First, describe the situation. The total amount Helen paid can be found by subtracting the discount amount from the price of the 3 books, then adding the shipping cost:

   price of 3 books – discount amount + shipping cost

   Let x represent the unknown price of 1 book. Multiply x by the number of books purchased. Helen bought 3 books, so the term $3x$ represents the price of the books.

   Helen received 20% off the price of the books. Written as a decimal, 20% is equal to 0.20. The amount of the discount Helen received off the price of the 3 books can be found by multiplying 0.20 by $3x$. So, $0.20(3x)$ represents the discount amount.

   Finally, Helen paid an additional $10 for shipping. This represents the shipping cost.

   The algebraic expression used to represent the total amount Helen paid for the 3 books plus shipping is $3x - 0.20(3x) + 10$.

2. Simplify the expression.

   The expression can be simplified by following the order of operations and combining like terms.

$3x - 0.20(3x) + 10$	Expression from the previous step
$3x - 0.60x + 10$	Multiply 0.20 and $3x$.
$2.4x + 10$	Combine like terms: add $3x$ and $-0.60x$.

   The simplified expression representing the total amount Helen paid for the books is $2.4x + 10$.

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Instruction

3. Identify each term in the simplified expression.

   There are two terms in the simplified expression: $2.4x$ and 10. The term $2.4x$ represents the price of the books minus the discount, and 10 represents the shipping cost.

4. Identify any factors of the non-constant term(s).

   $2.4x$ is the product of the factors 2.4 and x.

5. Identify any coefficients of the non-constant term(s).

   2.4 is multiplied by the variable, x; therefore, 2.4 is a coefficient.

6. Identify any constant terms.

   The term that does not change in the expression is 10. Therefore, 10 is the constant term.

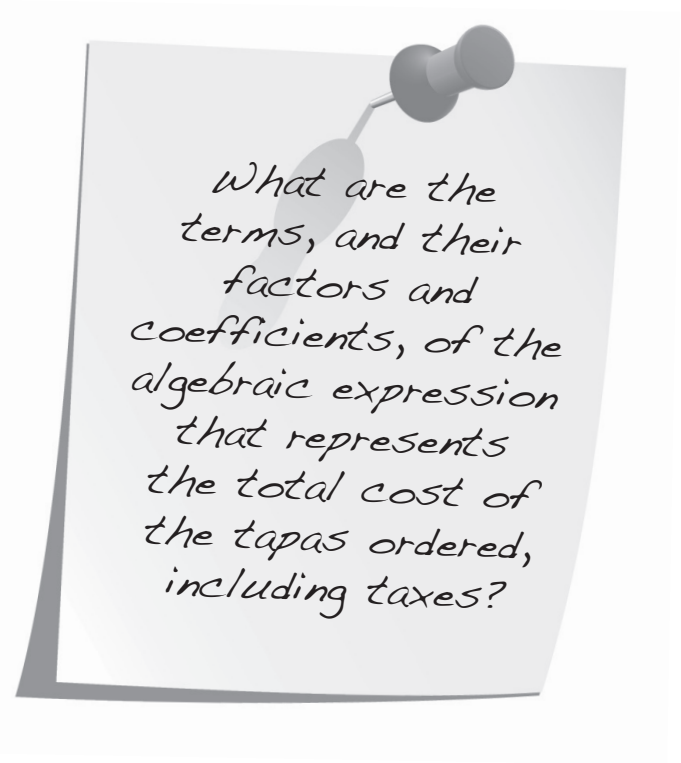


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Problem-Based Task 1.1.1: Identifying Parts of an Expression in Context

SMP	
1 ✓	2 ✓
3	4 ✓
5	6
7	8

Tara and two friends had dinner at a Spanish tapas restaurant that charged $6 per tapa, or appetizer. The three of them shared several tapas. The total bill included taxes of $4.32. What are the terms, and their factors and coefficients, of the algebraic expression that represents the total cost of the tapas ordered, including taxes?



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Problem-Based Task 1.1.1: Identifying Parts of an Expression in Context

Coaching

a. What was the cost of each tapa without including taxes?

b. What algebraic expression can be used to represent the total cost of the tapas ordered if each one cost $6?

c. What algebraic expression can be used to represent the cost of the tapas ordered including taxes?

d. How many terms does the expression from part c include and what are they?

e. What are the factors that make up each of the non-constant terms?

f. What are the coefficients of each non-constant term?

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Instruction

Problem-Based Task 1.1.1: Identifying Parts of an Expression in Context

Coaching Sample Responses

a. What was the cost of each tapa without including taxes?

Each tapa cost $6.

b. What algebraic expression can be used to represent the total cost of the tapas ordered if each one cost $6?

The number of tapas ordered is unknown. Let n represent the unknown number of tapas. The total cost of n tapas is represented by $6n$.

c. What algebraic expression can be used to represent the cost of the tapas ordered including taxes?

The total bill included $4.32 in taxes. The cost of the tapas is $6n$. The expression that represents the cost of the tapas ordered and the taxes is $6n + 4.32$.

d. How many terms does the expression from part c include and what are they?

There are two terms in the expression $6n + 4.32$: $6n$ and 4.32.

e. What are the factors that make up each of the non-constant terms?

The factors are the products of the quantities of a term. The product of $6n$, the non-constant term, is made up of the factors 6 and n.

f. What are the coefficients of each non-constant term?

The coefficient of the term $6n$ is 6.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 1.1.1: Identifying Terms, Factors, and Coefficients

For problems 1 and 2, simplify each expression if possible, and then list the terms of the simplified expression. Identify the constant term and the factors and coefficients of non-constant terms.

1. $12a^3 + 16a + 4$

2. $21x^2 + 3x - 15x^2 + 9$

For problems 3 and 4, translate each verbal expression into an algebraic expression. Then, list the terms of the given expressions, and identify the constant term and the factors and coefficients of non-constant terms.

3. half the sum of x and y, decreased by one-third y

4. the product of 5 and the cube of x, increased by the difference of 6 and x^3

For problem 5, write an expression that has the given terms and coefficients.

5. Write an expression with 4 terms, containing the coefficients 3, 6, and 9.

For problems 6–10, write an algebraic expression to describe each situation. Then, list the terms of the expressions. Identify the constant term and the factors and coefficients of non-constant terms.

6. Gavin agrees to buy a 6-month package deal of monthly gym passes, and in turn receives a 15% discount. Write an algebraic expression to represent the total cost of the monthly passes with the discount, if x represents the cost of each monthly pass.

7. Andre purchased 10 packs of trading cards online and received a 20% discount off each pack. Shipping cost $3.99. Write an algebraic expression to represent the total cost of the trading cards with the shipping cost, if x represents the cost of each pack of cards.

8. Nadia and some friends went to a movie. Their total cost was $30.24, which included taxes of $2.24. Write an algebraic expression to represent the price of each movie ticket, not including taxes. Let x represent the number of Nadia's friends who went to the movies.

9. Write an expression to represent the area of a trapezoid, which can be found by multiplying the height of the trapezoid by half of the sum of $base_1$ and $base_2$.

10. The surface area of a cylinder with radius r and height h is twice the product of π and the square of the radius plus twice the product of π, the radius, and the height.

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Lesson 1.1.2: Interpreting Linear and Exponential Expressions

Warm-Up 1.1.2

Javier deposited $750 in a bank account that earns interest at a rate of 3% of his initial deposit each year. He left the money in the account for 5 years. Use this information to complete the problems that follow. Explain your answers.

1. How much interest did Javier earn in 5 years?

2. How much money was in Javier's account after 5 years?

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Instruction

Lesson 1.1.2: Interpreting Linear and Exponential Expressions

Common Core State Standard

A–SSE.1 Interpret expressions that represent a quantity in terms of its context.★

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.*

Warm-Up 1.1.2 Debrief

1. How much interest did Javier earn in 5 years?

   It is not necessary at this point for students to create and solve an equation to represent this situation, but it is important that the interest amount is calculated based on the initial deposit of $750. Look out for students who try to calculate compound interest. There are several methods to calculating interest; compound interest will be discussed later. Focus the discussion on calculating 3% simple interest over 5 years.

   $5(0.03 \bullet 750) = 112.50$

   Javier earned $112.50 in interest in 5 years.

2. How much money was in Javier's account after 5 years?

   The total amount of money in Javier's account after 5 years is the sum of the simple interest ($112.50) and the initial deposit ($750).

   $112.50 + 750.00 = 862.50$

   Javier's account had $862.50 in it after 5 years.

Connection to the Lesson

- Students will interpret the parts of given expressions and how changes to each part affect the expression.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- evaluating expressions using the order of operations (6.EE.2c)
- evaluating expressions for a given value (6.EE.2c)
- identifying parts of an expression (6.EE.2b)
- translating verbal expressions into algebraic expressions (6.EE.2a)

Introduction

Algebraic expressions, used to describe various situations, contain variables. It is important to understand how each term of an expression works and how changing the value of variables impacts the resulting quantity.

Key Concepts

- If a situation is described verbally, it is often necessary to first translate each expression into an algebraic expression. This will allow you to see mathematically how each term interacts with the other terms.
- As variables change, it is important to understand that constant terms will always remain the same. The change in the variable will not change the value of the constant term.
- Similarly, changing the value of a constant term will not change terms containing variables.
- It is also important to follow the order of operations, as this will help guide your awareness and understanding of each term.
- When working with exponents, recall that the **base** is the factor being multiplied together in an exponential expression. In the expression a^b, a is the base.

Common Errors/Misconceptions

- incorrectly translating given verbal expressions

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Instruction

Guided Practice 1.1.2

Example 1

A new car loses an average value of $1,800 per year. When Nia bought her new car, she paid $25,000. The expression $25{,}000 - 1800y$ represents the current value of the car, where y represents the number of years since Nia bought it. What effect, if any, does the change in the number of years since Nia bought the car have on the original price of the car?

1. Refer to the expression given: $25{,}000 - 1800y$.

   The expression has two terms. The term $1800y$ represents the amount of value the car loses each year, y. The term 25,000 represents the price of the new car.

2. Determine the effect that the number of years has on the original price of the car.

   As y increases, the original price is not affected. The term 25,000 is a constant term and remains unchanged.

Example 2

To calculate the perimeter of an isosceles triangle, the expression $2s + b$ is used, where s represents the length of the two congruent sides and b represents the length of the base. What effect, if any, does increasing the length of the congruent sides have on the expression?

1. Refer to the expression given: $2s + b$.

   Changing only the length of the congruent sides, s, will not impact the length of base b since b is independent of s.

2. Determine what effect, if any, increasing the length has.

   If the length of the congruent sides, s, is increased, the product of $2s$ will also increase. Likewise, if the value of s is decreased, the value of $2s$ will also decrease. In either case, the value of b will not change.

   Therefore, increasing the value of s increases the value of the expression by $2s$, while the value of b remains the same.

SWB p. 19; SRB p. U1-12

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Instruction

Example 3

Money deposited in a bank account earns interest on the initial amount deposited as well as any interest earned as time passes. This compound interest can be described by the expression $P(1 + r)^n$, where P represents the initial amount deposited, r represents the interest rate, and n represents the number of years that pass. How does a change in each variable affect the value of the expression?

1. Refer to the expression given: $P(1 + r)^n$.

   Notice the expression is made up of one term containing the factors P and $(1 + r)^n$.

2. Determine what effect, if any, changing the value of P has.

   Changing the value of P does not change the value of the factor $(1 + r)^n$, but it will change the value of the expression by a factor of P. In other words, if P alone is doubled, then the value of the whole expression doubles.

3. Determine what effect, if any, changing the value of r has.

   Changing r changes the base of the exponent (the number that will be multiplied by itself), but does not change the value of P. If r alone is doubled, this means that the interest rate is twice as high, so the balance in the account will increase somewhat, but it will not double.

4. Determine what effect, if any, changing the value of n has.

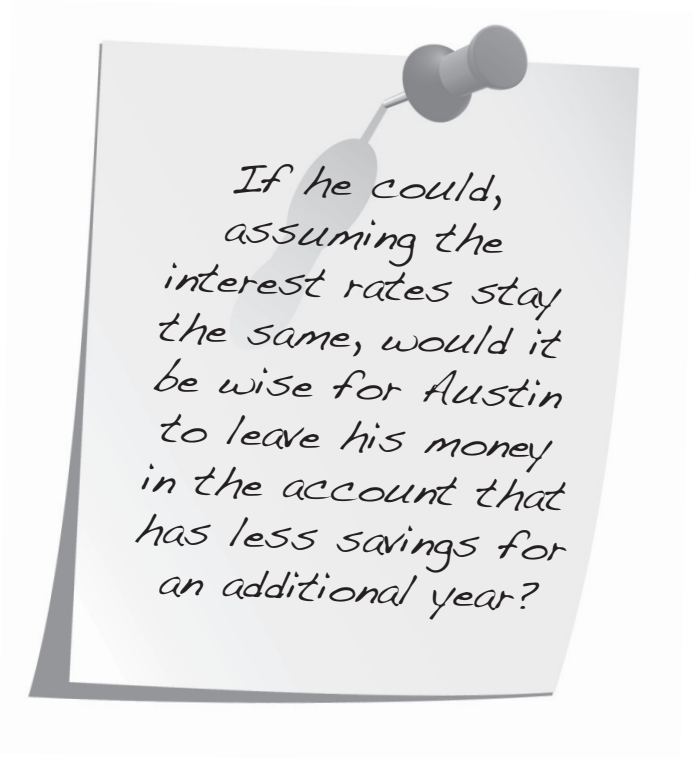
   Changing n changes the number of times $(1 + r)$ will be multiplied by itself, but does not change the value of P. Doubling the number of years that the money is in the account will increase the balance. Whether or not the value of the entire expression doubles depends on the number of years that were doubled.

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Problem-Based Task 1.1.2: Searching for a Greater Savings

SMP	
1	2 ✓
3 ✓	4
5 ✓	6 ✓
7	8

Austin plans to open a savings account. The amount of money in a savings account can be found by using the equation $s = p \bullet (1 + r)^t$, where p is the principal, or the original amount deposited into the account; r is the rate of interest; and t is the amount of time. Austin is considering two savings accounts. He will deposit $1,000 as the principal into either account. In Account A, the interest rate will be 0.015 per year for a term of 5 years. In Account B, the interest rate will be 0.02 per year for a term of 3 years. Which account has more money at the end of its term? If he could, assuming the interest rates stay the same, would it be wise for Austin to leave his money in the account that has less savings for an additional year? Explain your reasoning.



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Problem-Based Task 1.1.2: Searching for a Greater Savings

Coaching

a. What is the total amount in Austin's savings if he chooses Account A?

b. What is the total amount in Austin's savings if he chooses Account B?

c. Which account has more money at the end of its term?

d. If he could, assuming the interest rates stay the same, would it be wise for Austin to leave his money in the account that has less savings for an additional year? Explain your answer.

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Instruction

Problem-Based Task 1.1.2: Searching for a Greater Savings

Coaching Sample Responses

a. What is the total amount in Austin's savings if he chooses Account A?

Use the equation $s = p \bullet (1 + r)^t$ to determine the amount in Account A. Identify p, r, and t from the problem statement.

The principal, p, is \$1,000; the interest rate, r, is 0.015; and the time, t, is 5 years. Replace the variables in the equation with the given quantities.

$s = p \bullet (1 + r)^t$

$s = (1000) \bullet [1 + (0.015)]^{(5)}$

Evaluate the resulting equation, $s = 1000 \bullet (1 + 0.015)^5$, using the order of operations.

$s = 1000 \bullet (1 + 0.015)^5$

$s = 1000 \bullet (1.015)^5$

$s \approx 1000 \bullet 1.077$

$s \approx 1077$

If he selects Account A, after 5 years Austin will have approximately \$1,077 in savings.

b. What is the total amount in Austin's savings if he chooses Account B?

Use the equation $s = p \bullet (1 + r)^t$ to determine the amount in Account B. Identify p, r, and t from the problem statement.

The principal, p, is \$1,000; the interest rate, r, is 0.02; and the time, t, is 3 years. Replace the variables in the equation with the given quantities.

$s = p \bullet (1 + r)^t$

$s = (1000) \bullet [1 + (0.02)]^{(3)}$

Evaluate the resulting equation, $s = 1000 \bullet (1 + 0.02)^3$, using the order of operations.

$s = 1000 \bullet (1 + 0.02)^3$

$s = 1000 \bullet (1.02)^3$

$s \approx 1000 \bullet 1.061$

$s \approx 1061$

If he selects Account B, after 3 years Austin will have approximately \$1,061 in savings.

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c. Which account has more money at the end of its term?

Compare the results from parts a and b.

If Austin leaves the money in Account A for 5 years, he will have approximately \$1,077.

If Austin leaves the money in Account B for 3 years, he will have approximately \$1,061.

Account A has more money at the end of its term.

d. If he could, assuming the interest rates stay the same, would it be wise for Austin to leave his money in the account that has less savings for an additional year? Explain your answer.

Re-evaluate the total amount in savings if Austin left the money in Account B, the account with the lower total savings, for one extra year.

The new value of t in the equation $s = p \bullet (1 + r)^t$ would be 4. The values of p and r will be the same as the ones used in part b: $p = 1000$ and $r = 0.02$. Replace the values of p, r, and t in the equation $s = p \bullet (1 + r)^t$ to find the total amount in Account B after 4 years.

$s = p \bullet (1 + r)^t$

$s = (1000) \bullet [1 + (0.02)]^{(4)}$

$s = 1000 \bullet (1 + 0.02)^4$

$s = 1000 \bullet (1.02)^4$

$s \approx 1000 \bullet 1.082$

$s \approx 1082$

Assuming the interest rate remains constant for a fourth year, Austin will have approximately \$1,082 in savings in Account B. This is about \$5 more than the amount that would be in Account A after five years, and the money would be available one year sooner. Austin should put his money in Account B.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 1.1.2: Interpreting Linear and Exponential Expressions

Use your understanding of terms, coefficients, factors, exponents, and the order of operations to answer each of the following questions.

1. Is the expression $\frac{5 + 3x}{2}$ always equal to the expression $4x$? Explain your answer.

2. Is the expression $2 \cdot 4^x$ equal to the expression 8^x? Explain your answer.

3. Is the expression $(5 \cdot 2)^x$ equal to the expression 10^x? Explain your answer.

4. A transfer station charges \$15 for a waste disposal permit and an additional \$5 for each cubic yard of garbage it disposes of. This relationship can be described using the expression $15 + 5x$. What effect, if any, does changing the value of x have on the cost of the permit?

5. Absolute Cable company bills on a monthly basis. Each bill includes a \$30.00 service fee plus \$4.75 in taxes and \$2.99 for each movie purchased. The following expression describes the cost of the cable service per month: $34.75 + 2.99m$. If Absolute Cable lowers the service fee, how will the expression change?

continued

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6. In order for a pet to lose weight in a healthy manner, a veterinarian suggested an overweight large-breed dog lose 2 pounds per week. If the expression $x - 2y$ represents this situation, what must be true about the value of y?

7. The product of 7, x, and y is represented by the expression $7xy$. If the value of x is negative, what can be said about the value of y in order for the product to remain positive?

8. A bank account balance for an account with an initial deposit of P dollars earns interest at an annual rate of r. The amount of money in the account after n years is described using the following expression: $P(1 + r)^n$. What effect, if any, does decreasing the value of r have on the amount of money after n years?

9. For what values of x will the result of 5^x be greater than 1?

10. A tire can hold C cubic feet of air. It loses a percentage of its air during each period of time, t. This rate of loss, written as a decimal, is r. This situation can be described using the following formula: $C(1 - r)^t$. What effect, if any, does increasing the value of r have on the value of C?

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Progress Assessment

Circle the letter of the best answer.

1. How many terms are in the expression $14x^2 - 12x + 5$?

   a. 3
   b. 7
   c. 2
   d. 4

2. What are the factors in the non-constant terms in the expression $8x^2 + 10x - 3$?

   a. 8 and x^2
   b. x
   c. 8 and x^2, 10 and x
   d. There are no factors in these terms.

3. What are the coefficients in the expression $5x^3 + 14x^2 - 2x + 6$?

   a. 5, 14, 2, and 6
   b. 5, 14, and 2
   c. 5, 14, and –2
   d. 5, 14, –2, and 6

4. What is the constant in the expression $6x^3 + 4x - 3$?

   a. 6
   b. 4
   c. 3
   d. –3

5. Timely Taxi Service charges $3.50 for pickup, plus an additional $0.20 per mile. What is the constant in this situation?

   a. $0.20
   b. $3.50
   c. $3.70
   d. $3.30

6. A shirt that costs C dollars with 7% sales tax can be described using the expression $C + 0.07C$. Which expression that follows is NOT the same as this expression?

   a. $1.07C$
   b. $C(1 + 0.07)$
   c. $1.7C$
   d. $0.07C + C$

continued

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Assessment

7. The product of 5, a, and b is represented by the expression $5ab$. If the value of a is positive, what must be said about the value of b in order for the product to remain positive?

   a. b must be positive.
   b. b must be negative.
   c. b must be 0.
   d. The value of b does not matter.

8. The expression $\frac{5}{9}(F-32)$ is used to convert a Fahrenheit temperature to Celsius. What values of F will result in a Celsius temperature greater than 0°?

   a. $F > 0$
   b. $F < 0$
   c. $F > 32$
   d. $F < 32$

9. The population of bacteria in a Petri dish is 550 and increases according to the expression $550(3.4^{0.005t})$, where t is the number of hours. What effect would increasing the initial population of the bacteria have on the percentage rate at which the number of bacteria increases?

   a. An increase in the initial population will lower the percentage rate at which the population increases.
   b. An increase in the initial population does not affect the percentage rate at which the population increases.
   c. An increase in the initial population will increase the percentage rate at which the population increases.
   d. An increase in the initial population increases the time the bacteria are increasing.

10. For what values of x will the result of 3^x be greater than 1?

    a. x can be any number greater than 1.
    b. x can be any number greater than 3.
    c. x can be any number less than 0.
    d. x can be any number greater than 0.

Use what you know about the structure of expressions to complete the following problem.

11. Explain the differences among each of these expressions: $a^x b$, ab^x, and $(ab)^x$.

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Pre-Assessment

Circle the letter of the best answer.

1. Mina bought a plane ticket to New York City and used a coupon for 15% off the ticket price. The total cost of her ticket, with the discount, was $253.30. What equation could she use to find the price of the ticket without the discount?

   a. $0.15x = 253.30$
   b. $x - 0.15(x) = 253.30$
   c. $x = 253.30 + 0.15$
   d. $x + 0.15(x) = 253.30$

2. Lucas bought a refrigerator. His total cost of $1,331 included sales tax at the rate of 8% and an additional, untaxed delivery charge of $35. How much sales tax did he pay?

   a. $72
   b. $106
   c. $120
   d. $96

3. Your cell phone plan allows you 400 minutes to talk per month. So far this month, you have used 265 minutes and you have 7 days left on this month's plan. Which inequality could you use to determine how many minutes at most you can use per day so that you don't go over your monthly plan minutes?

   a. $7x + 265 < 400$
   b. $7x + 265 > 400$
   c. $7x + 265 \leq 400$
   d. $7x + 265 \geq 400$

4. Sydney has a $75 mall gift card. She wants to buy a sundress and a movie ticket. The movie ticket with tax costs $11.50. The sales tax on the sundress will be 4%. How much can the ticketed price of the sundress be?

   a. less than or equal to $61.06
   b. less than $61.06
   c. less than or equal to $45.36
   d. less than $45.36

5. A population of rabbits doubles every 4 months. If the population starts out with 8 rabbits, how many rabbits will there be in 1 year?

   a. 128 rabbits
   b. 64 rabbits
   c. 32 rabbits
   d. 16 rabbits

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Lesson 2: Creating Equations and Inequalities in One Variable

Instruction

Common Core State Standards

SMP	
1 ✓	2 ✓
3 ✓	4 ✓
5 ✓	6 ✓
7	8

N–Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

N–Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

A–CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Essential Questions

1. How are quantities modeled with equations and inequalities?
2. How are equations and inequalities alike and different?
3. What makes creating an exponential equation different from creating a linear equation?

WORDS TO KNOW

equation a mathematical sentence that uses an equal sign (=) to show that two quantities are equal

exponential decay an exponential equation with a base, b, that is between 0 and 1 exclusive (that is, $0 < b < 1$); an example is the formula $y = a(1 - r)^t$, where a is the initial value, $(1 - r)$ is the base (with $0 < r < 1$), t is the variable exponent, and y is the final value

exponential equation an equation whose independent variable is in the exponent; the general form of its equation is $f(x) = ab^x + k$, where a is the initial value, b is the base, x is the input value, k is the vertical shift, and $f(x)$ is the output. Another form is $y = ab^{\frac{x}{t}}$, where t is the interval over which y changes by a factor of b, and x is measured in the same units as t.

exponential growth an exponential equation with a base, b, greater than 1 ($b > 1$); an example is the formula $y = a(1 + r)^t$, where a is the initial value, $(1 + r)$ is the base (with $r > 0$), t is the variable exponent, and y is the final value

SRB p. U1-16

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inequality	a mathematical sentence that shows the relationship between quantities that may not be equivalent. An inequality contains one or more of the following symbols: $<$, $>$, $\leq$, $\geq$, or $\neq$.
linear equation	a first-degree equation that can be written in the form $ax + by = c$, where a, b, and c are numbers; when written as $y = mx + b$, m is the slope of the line, and b is its y-intercept. The graph of a linear equation is a straight line.
quantity	a value or expression that may be expressed in numbers
rate	a ratio that compares different kinds of units
solution	a value that makes an equation true
solution set	the value or values that make a sentence or statement true
unit rate	a ratio of two measurements, the second of which is 1
variable	a letter used to represent an unknown value or a value that changes

Recommended Resources

- APlusMath. "Algebra Planet Blaster."

  http://walch.com/rr/CAU1L2LinEquations

  Players solve each multi-step linear equation to find the correct planet to "blast." Incorrect answers cause players to destroy their own ship. *Note*: The applet requires Java to run.

- Figure This! Math Challenges for Families. "Challenge 24: Gasoline Tanks."

  http://walch.com/rr/CAU1L2Rates

  This website features a description of the math involved, related occupations, a hint to get started, complete solutions, and a "Try These" section, as well as additional related problems with answers, questions to think about, and resources for further exploration. These resources could be used to lead a group discussion or as an investigation. Users could work in small groups to create a presentation related to each challenge.

- Purplemath.com. "Exponential Functions: Introduction."

  http://walch.com/rr/CAU1L2ExpEquations

  This website gives an introduction of exponential equations and provides a few examples of tables of input and output values, with integers as inputs. The introduction goes into more depth about the shapes of the graphs of exponential functions and continues on to develop the concept of compound interest. It also introduces the number e.

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Lesson 1.2.1: Creating Linear Equations in One Variable

Warm-Up 1.2.1

Read the scenario and answer the questions that follow.

> Andrew is practicing for a tennis tournament and needs more tennis balls. He bought 10 cans of tennis balls online and received a 25% discount. The shipping cost was $5.99. Let x represent the cost of each can.

1. Write an algebraic expression to represent the cost of the tennis balls before taxes and shipping.

2. Write an algebraic expression to represent the cost of the tennis balls with the discount, and simplify your expression.

3. Write an algebraic expression to represent the total cost of the tennis balls with the shipping cost and the discount. Simplify the expression.

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Instruction

Lesson 1.2.1: Creating Linear Equations in One Variable

Common Core State Standards

N–Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

N–Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

A–CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Warm-Up 1.2.1 Debrief

1. Write an algebraic expression to represent the cost of the tennis balls before taxes and shipping.

   Andrew purchased 10 cans of tennis balls at an unknown price, x. Therefore, the expression to represent the cost of the tennis balls is $10x$.

2. Write an algebraic expression to represent the cost of the tennis balls with the discount, and simplify your expression.

   First, Andrew will be charged the cost of the tennis balls ($10x$).

   Then, 25% will be discounted or taken off the cost of the tennis balls, so $-0.25(10x)$.

   Add these amounts to arrive at the price of the tennis balls.

$10x - 0.25(10x)$	Write the expression.
$10x - 2.5x$	Multiply 0.25 and $10x$.
$7.5x$	Combine like terms.

   The algebraic expression that represents the price of the tennis balls is $7.5x$.

   Students might have trouble remembering to convert the percentage to a decimal. Students might not have simplified.

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3. Write an algebraic expression to represent the total cost of the tennis balls with the shipping cost and the discount. Simplify the expression.

   The shipping cost was $5.99. Add this to the expression found in problem 2.

   The algebraic expression that represents the total cost of the tennis balls including the shipping cost and the discount is $7.5x + 5.99$.

   Students might try to apply the discount to the shipping cost, but the discount does not apply to shipping.

   Encourage students to read the problem scenario carefully before and while writing their expressions.

Connection to the Lesson

- Students will extend writing expressions to writing equations.
- The process of translating an expression is the same for translating an equation.
- The warm-up is scaffolded to help students think about breaking down the context into smaller pieces, providing an example for how to think about doing this with an equation context.
- Students will be asked to take a scenario like this a step further and solve for the unknown given a total cost.

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Prerequisite Skills

This lesson requires the use of the following skills:

- applying the order of operations (5.OA.1)
- creating ratios (6.RP.2)
- translating verbal sentences into algebraic expressions (6.EE.2a)
- solving linear equations (8.EE.7b)

Introduction

Creating equations from context is important since most real-world scenarios do not involve the equations being given. An **equation** is a mathematical sentence that uses an equal sign (=) to show that two quantities are equal. A **quantity** is something that can be compared because it has a numerical value. In this lesson, contexts will be given and equations must be created from them and then used to solve the problems. Since these problems are all in context, units are essential because without them, the numbers have no meaning.

Key Concepts

- A **linear equation** is an equation in which the highest power of any variable is 1. This lesson focuses on solving linear equations that have just one variable and that can be written in the form $ax + b = c$, where a, b, and c are numbers and x is the variable. Often, the most difficult task in turning a context into an equation is determining what the variable is and how to represent it in an equation.
- A variable is a letter used to represent an unknown value or a value which changes. Once the equation is determined, solving for the variable follows the steps for isolating a variable that were learned previously.
- The **solution** is the value that makes the equation true.
- In some cases, the solution must be converted into different units. Multiplying by a unit rate or a ratio can do this.
- A **rate** is a ratio that compares different kinds of measurements. A **unit rate** is a ratio of two measurements, the second of which is 1, such as miles per (1) gallon.
- Use units that make sense, such as when reporting time; for example, if the time is less than 1 hour, report the time in minutes.

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- Make sure the units match when you begin the problem. For example, do not include a value that is measured in meters per second with a value that uses centimeters per second in the same equation. Convert one of these to match the other.
- Think about rounding and precision. The more significant digits you include, the more precise the number is.
- When using measurement in calculations, only report to the nearest decimal place of the least precise measurement.

Creating Equations from Context

1. Read the problem statement carefully.
2. Reread the scenario and make a list of the known quantities.
3. Read the statement again, identifying the unknown quantity or variable.
4. Create an equation from the known quantities and variable(s).
5. Solve the equation for the variable.
6. Interpret the solution of the equation in terms of the context of the problem and convert units if necessary.

Common Errors/Misconceptions

- attempting to solve the problem without first reading/understanding the problem statement
- incorrectly setting up the equation
- misidentifying the variable
- forgetting to convert to the correct units
- setting up the ratio inversely when converting units
- reporting too many or too few decimal places

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Guided Practice 1.2.1

Example 1

James earns $15 per hour as a teller at a bank. In one week he pays 17% of his earnings in state and federal taxes. His take-home pay for the week is $460.65. How many hours did James work?

1. Read the problem statement carefully.

2. Reread the scenario and make a list of the known quantities.

   James earns $15 per hour.

   James pays 17% of his earnings in taxes.

   After taxes, his take-home pay for the week is $460.65.

3. Read the statement again, identifying the unknown quantity or variable.

   The scenario asks for James's hours for the week. The variable to solve for is hours.

4. Create an equation from the known quantities and variable(s).

   James's pay for the week was $460.65.

   _____ = 460.65

   He earned $15 an hour. Let h represent hours.

   $15h$

   He paid 17% in taxes. Written as a decimal, 17% = 0.17. Since taxes are subtracted from pay, use a negative sign: –0.17. Multiply $15h$ by –0.17.

   $-0.17(15h)$

   Put this information all together.

   $15h - 0.17(15h) = 460.65$

SWB p. 29; SRB p. U1-21

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5. Solve the equation for the variable.

$15h - 0.17(15h) = 460.65$	Equation from the previous step
$15h - 2.55h = 460.65$	Multiply –0.17 and 15h.
$12.45h = 460.65$	Combine like terms 15h and –2.55h.
$\frac{12.45h}{12.45} = \frac{460.65}{12.45}$	Divide both sides by 12.45.
$h = 37$	Simplify.

6. Interpret the solution in the context of the problem and convert to the appropriate units if necessary.

   In the equation, h represents hours worked. The scenario asked for hours and the quantity given was in terms of hours. No unit conversions are necessary.

   James worked 37 hours.



Example 2

Brianna has saved $600 to buy a new TV. If the TV she wants costs $1,800 and she saves $20 a week, how many years will it take her to buy the TV?

1. Read the problem statement carefully.

2. Reread the scenario and make a list of the known quantities.

   The TV costs $1,800.

   Brianna has already saved $600.

   Brianna saves $20 per week.

3. Read the statement again, identifying the unknown quantity or variable.

   The scenario asks for the number of years. This is tricky because the quantity is given in terms of weeks. The variable to solve for first, then, is weeks.

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4. Create an equation from the known quantities and variable(s).

   Brianna's goal is to save $1,800.

   _____ = 1800

   Brianna has saved $600 so far and has to save more to reach her goal.

   600 + _____ = 1800

   Brianna is saving $20 a week for some unknown number of weeks to reach her goal. Let x represent the number of weeks.

   $600 + 20x = 1800$

5. Solve the equation for the variable.

$$600+20x=1800$$
$$-600 \qquad -600$$
$$20x=1200$$
$$\frac{20x}{20}=\frac{1200}{20}$$
$$x=60$$

6. Interpret the solution in the context of the problem and convert to the appropriate units if necessary.

   In the equation, x represents weeks. The problem statement asks for the number of years it will take Brianna to save for the TV, so convert weeks to years. There are 52 weeks in a year.

$$\frac{1 \text{ year}}{52 \text{ weeks}}$$
$$60 \text{ weeks} \bullet \frac{1 \text{ year}}{52 \text{ weeks}}$$
$$60 \ \cancel{\text{weeks}} \bullet \frac{1 \text{ year}}{52 \ \cancel{\text{weeks}}} \approx 1.15 \text{ years}$$

   Brianna will need approximately 1.15 years, or a little over a year, to save for her TV.

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Example 3

Suppose two brothers who live 55 miles apart decide to have lunch together. To prevent either brother from driving the entire distance, they agree to leave their homes at the same time, drive toward each other, and meet somewhere along the route. The older brother drives cautiously at an average speed of 60 miles per hour. The younger brother drives faster, at an average speed of 70 mph, but still within the speed limit. How long will it take the brothers to meet each other?

1. Read the problem statement carefully.

2. Reread the scenario and list the known quantities.

   Problems involving "how fast," "how far," or "how long" can often be solved with the distance equation, $d = rt$, where d is distance, r is the rate of speed, and t is time.

   From the scenario, we know each brother's rate of speed, and the total distance they have to drive to meet each other. Make a list:

   The older brother's rate is 60 mph.

   The younger brother's rate is 70 mph.

   The sum of their distances is 55 miles.

3. Read the statement again, identifying the unknown quantity or variable.

   The scenario asks "how long" it will take, so the variable is time, t.

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Instruction

4. Create an equation from the known quantities and variable(s).

Step 2 showed that the distance equation is $d = rt$ or $rt = d$. Together the brothers will travel a distance, d, of 55 miles.

(older brother's rate)(t) + (younger brother's rate)(t) = 55

The rate r of the older brother is 60 mph and the rate of the younger brother is 70 mph. Substitute these values into the expression.

$60t + 70t = 55$

Create a table to see this another way.

	Rate (r)	**Time (t)**	**Distance (d)**
Older brother	60 mph	t	$d = 60t$
Younger brother	70 mph	t	$d = 70t$

Together, they traveled 55 miles, so add the distance equations based on each brother's rate.

$60t + 70t = 55$

5. Solve the equation for the variable.

$60t + 70t = 55$	Equation from the previous step
$130t = 55$	Combine the like terms, $60t$ and $70t$.
$\frac{130t}{130} = \frac{55}{130}$	Divide both sides by 130.
$t \approx 0.423$	Simplify.

Note: The answer was rounded to the nearest thousandth. Although we do not need this precision, it is good practice to retain more digits than are needed until the last step to avoid rounding errors. When talking about meeting someone, it is highly unlikely that anyone would report a time that is broken down into decimals, which is why the next step will convert the units.

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Instruction

6. Interpret the solution in the context of the problem and convert to the appropriate units if necessary.

   In the equation, t represents hours. Automobile speeds in the United States are typically given in miles per hour (mph). Therefore, this unit of measurement is appropriate. However, typically portions of an hour are reported in minutes unless the time given is $\frac{1}{2}$ of an hour. Convert 0.423 hours to minutes using 60 minutes = 1 hour.

   $$60 \text{ min} = 1 \text{ hr}$$

   $$0.423 \text{ hr} \bullet \frac{60 \text{ min}}{1 \text{ hr}}$$

   $$0.423 \text{ hr} \bullet \frac{60 \text{ min}}{1 \text{ hr}} \approx 25.38 \text{ minutes}$$

   Here again, rarely would a person report that they are meeting someone in 25.38 minutes. In this case, there is a choice of rounding to either 25 or 26 minutes. Either answer makes sense.

   The two brothers will meet each other in 25 or 26 minutes.

Example 4

Think about the following scenarios. In what units should they be reported? Explain the reasoning.

- The rate at which water fills up a swimming pool
- The cost of tiling a kitchen floor
- The average speed of a falling object
- The rate at which a snail travels across a sidewalk
- The rate at which a room is painted

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Instruction

1. The rate at which water fills up a swimming pool

   A swimming pool, depending on the size, has between several gallons and hundreds of thousands of gallons of water.

   Think about water flowing out of a faucet and picture filling up a milk jug. How long does it take? Less than a minute? The point is that gallons of water can be filled in minutes.

   A small swimming pool could be reported in terms of gallons per minute, and a large swimming pool could be reported in terms of gallons per hour.

2. The cost of tiling a kitchen floor

   Think about how big rooms are. They can be small or rather large, but typically they are measured in feet. When calculating the area, the measurement units are square feet.

   Tiles cost in the dollar range, and are priced by the square foot.

   Report the cost of tiling a kitchen floor in dollars per square foot.

3. The average speed of a falling object

   Think about how fast an object falls when you drop it from shoulder-height. How far is it traveling from your shoulder to the ground? It travels several feet (or meters).

   How long does it take before the object hits the ground? It only takes a few seconds.

   Report the average speed of a falling object in terms of feet or meters per second.

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4. The rate at which a snail travels across a sidewalk

   The context of the problem will determine the correct units. Think about how slowly a snail moves. Would a snail be able to travel at least one mile in an hour? Perhaps it makes more sense to report the distance in a smaller unit. Report the snail traveling across the sidewalk in feet per minute.

   If comparing speeds of other animals to the snail's rate, and the animals' rates are being reported in miles per hour, then it makes sense to report the snail's rate in miles per hour, too.

5. The rate at which a room is painted

   Think about how long it takes to paint a room. It takes longer than several minutes. It would probably take hours.

   How is the surface area of a wall typically measured? It's usually measured in square feet.

   Report the painting of a room in square feet per hour. ✓

Example 5

Ernesto built a wooden car for a soap box derby. He is painting the top of the car blue and the sides black. He already has enough black paint, but needs to buy blue paint. He needs to know the approximate area of the top of the car to determine the size of the container of blue paint he should buy. He measured the length to be 9 feet $11\frac{1}{4}$ inches, and the width to be $\frac{1}{2}$ inch less than 3 feet. What is the surface area of the top of the car? What is the most accurate area measurement Ernesto can use to buy his paint?

1. Read the problem statement carefully.

2. Reread the scenario and make a list of the known quantities.

   The length is 9 feet 11.25 inches.
   The width is 35.5 inches (3 feet = 36 inches; $36 - \frac{1}{2}$ inch = 35.5 inches).

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Instruction

3. Read the statement again, identifying the unknown quantity or variable.

   The scenario asks for the surface area of the car's top.

   Work with the accuracy component after calculating the surface area.

4. Create an equation from the known quantities and variable(s).

   The surface area will require some assumptions. A soap box derby car is tapered, meaning it is wider at one end than it is at another. To be sure Ernesto has enough paint, he assumes the car is rectangular with the width being measured at the widest location. Use the formula for area:

   A = length • width = lw

   For step 2, we listed the length and width, but they are not in the same units. The length is given in feet and inches; the width is given in inches.

   Convert the length, 9 feet 11.25 inches, to inches.

   $9(12) + 11.25 = 119.25$ inches

   The width is 35.5 inches.

   Substitute the length and width into the formula $A = lw$.

   $A = lw$

   $A = (119.25) \bullet (35.5)$

5. Solve the equation for the variable.

   $A = 119.25 \bullet 35.5$

   $A = 4233.375$

   This gives a numerical result for the surface area, but the problem asks for the most accurate surface area measurement that can be calculated based on Ernesto's initial measurements. It is good practice to retain all three digits after the decimal place until the problem is complete to avoid inaccuracies introduced by rounding.

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6. Interpret the solution in the context of the problem and convert to the appropriate units if necessary.

   In the equation, A represents area in square inches. When buying paint, the hardware store associate will ask how many square feet need to be covered. Ernesto has his answer in terms of square inches. Convert to square feet.

   There are 144 square inches in a square foot.

   $$1\,\text{ft}^2 = 144\,\text{in}^2$$

   $$4233.375\,\text{in}^2 \bullet \frac{1\,\text{ft}^2}{144\,\text{in}^2}$$

   $$4233.375\,\cancel{\text{in}^2} \bullet \frac{1\,\text{ft}^2}{144\,\cancel{\text{in}^2}} = 29.3984375\,\text{ft}^2$$

7. Round the result to the appropriate number of places.

   Ernesto's initial measurement with the fewest digits after the decimal is 35.5, which is reported to the nearest tenth. Thus, the final answer should also be rounded to the nearest tenth.

   $$29.3984375\,\text{ft}^2 \approx 29.4\,\text{ft}^2$$

   The surface area of the top of Ernesto's car is approximately 29.4 ft^2. This is the most accurate area measurement that Ernesto can use to buy his paint.

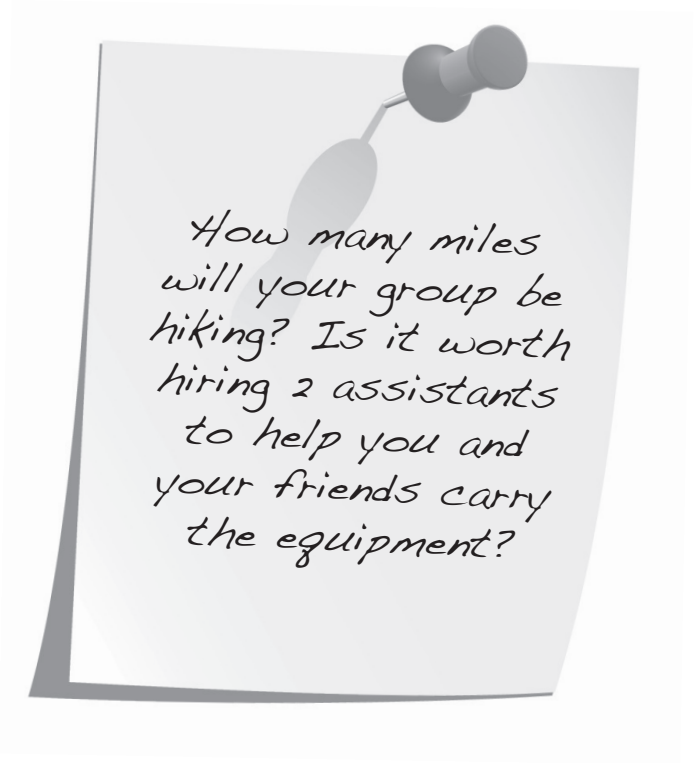
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Problem-Based Task 1.2.1: Rafting and Hiking Trip

SMP	
1 ✓	2 ✓
3 ✓	4
5	6
7	8

To celebrate graduation, you and 4 of your closest friends have decided to take a 5-day white-water rafting and hiking trip. During your 5-day trip, 2 days are spent rafting. If the rafting trip covers a distance of 60 miles and you are expected to raft 8 hours each day, how many miles must you raft each hour?

For the hiking portion of your trip, you and your friends carry the same amount of equipment, which works out to 35 pounds of equipment each. For extra money, you can hire an assistant, who will carry 50 pounds of equipment. Each assistant charges a flat fee of $50 and an additional $22 for each mile. The total amount you would have to pay 1 assistant is $512. How many miles will your group be hiking? Is it worth hiring 2 assistants to help you and your friends carry the equipment? Justify your answers.



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Problem-Based Task 1.2.1: Rafting and Hiking Trip

Coaching

a. If the rafting trip covers a distance of 60 miles and you are expected to raft 8 hours each day, how many miles must you raft each hour?

What is the ratio of miles to days?

What is the ratio you are looking for?

What is the ratio of days to hours spent rafting?

How do you convert the original ratio of miles to days into miles per hour?

b. How many miles will your group be hiking?

What is the equation of the cost of hiring 1 assistant?

What is the solution to this equation?

c. Is it worth hiring two assistants to help you and your friends carry the equipment?

How much weight will each of you carry without assistants?

How much weight will each of you carry with 2 assistants?

What is the difference in the cost per day?

Are you willing to pay more money to have someone carry your equipment?

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Instruction

Problem-Based Task 1.2.1: Rafting and Hiking Trip

Coaching Sample Responses

a. If the rafting trip covers a distance of 60 miles and you are expected to raft 8 hours each day, how many miles must you raft each hour?

What is the ratio of miles to days?

$$\frac{60 \text{ miles}}{2 \text{ days}}$$

What is the ratio you are looking for?

$$\frac{\text{miles}}{\text{hour}}$$

What is the ratio of days to hours spent rafting?

$$\frac{1 \text{ day}}{8 \text{ hours}}$$

How do you convert the original ratio of miles to days into miles per hour?

Multiply the two numeric ratios together.

$$\frac{60 \text{ miles}}{2 \cancel{\text{ days}}} \bullet \frac{1 \cancel{\text{ day}}}{8 \text{ hours}} = \frac{60 \text{ miles}}{16 \text{ hours}} = 3.75 \text{ miles/hour}$$

b. How many miles will your group be hiking?

What is the equation of the cost of hiring 1 assistant?

$22x + 50 = 512$, where x = miles

What is the solution to this equation?

$x = 21$ miles

c. Is it worth hiring 2 assistants to help you and your friends carry the equipment?

How much weight will each of you carry without assistants?

Each person will carry 35 pounds. There are 5 people in your group.

$35 \bullet 5 = 175$ pounds

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How much weight will each of you carry with 2 assistants?

First, determine how much less there would be to carry among the 5 members of your group if you hire 2 assistants who each carry 50 pounds.

$175 - 2(50) = 175 - 100 = 75$ pounds

Now, divide 75 pounds by 5 people.

$$\frac{75}{5} = 15 \text{ pounds}$$

$35 - 15 = 20$

If you hire 2 assistants, you and each of your friends will carry 15 pounds, which is 20 pounds less than the original amount.

What is the difference in the cost per day?

First, determine the cost for 2 assistants if each will be paid $512.

$2(512) = 1024$

The cost is $1,024 for 2 assistants.

Next, determine the cost per person.

$$\frac{\$1024}{5 \text{ people}} = \$204.80 / \text{person}$$

The difference is $204.80 per person or almost $205 per person.

Are you willing to pay more money to have someone carry your equipment?

Answers will vary, but should reflect and be justified by students' calculations. Sample answer: I am willing to pay about an extra $205 to be free of 20 pounds of equipment during my 21-mile hike, so that I can enjoy my experience without being weighed down.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 1.2.1: Creating Linear Equations in One Variable

For problem 1, read each scenario and give the units you would use to work with each situation.

1. What units would you use for the rate of each scenario that follows?
   a. riding a bicycle

   b. rainfall during a storm

   c. water coming from a fire hydrant

   d. watching caloric intake

For problems 2–10, read each scenario, write an equation, and then use the solution to the equation to complete the problem. Remember to include the appropriate units.

2. You need to buy new tile for your kitchen. The kitchen measures 13.25 feet by 7.5 feet. What is the area of the kitchen that you calculated? What is the most accurate area you can report to your hardware store in order to purchase enough tile?

3. Zach watches TV 3 times as much as Joel. Joel watches TV 2 hours a day. How many hours a day does Zach watch TV?

4. It costs Raquel $5 in tolls to drive to work and back each day, plus she uses 3 gallons of gas. It costs her a total of $15.50 to drive to work and back each day. How much per gallon is Raquel paying for her gas? How do you know?

continued

SWB p. 33; SRB Practice 1.2.1 p. U1-31

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5. Hayden bought 4 tickets to a football game. He paid a 5% service charge for buying them from a broker. His total cost was $105.00. What was the price of each ticket, not including the service charge?

6. It cost Justin $100 to have cable TV installed in his house. Each month he pays an access fee plus a tax of 7% of his monthly bill. After 6 months, Justin had paid a total of $350.38 for his access fee, taxes, and his initial installation. What is Justin's monthly access fee not including taxes?

7. You and 3 friends divide the proceeds of a garage sale equally. The garage sale earned $412. How much money did you receive?

8. The area of Sofia's herb garden is $\frac{1}{8}$ the area of her vegetable garden. The area of her herb garden is 6 square feet. What is the area of her vegetable garden?

9. Driving to your friend's house, you travel at an average rate of 35 miles per hour. On your way home, you travel at an average rate of 40 miles per hour. If the round trip took you 45 minutes, how far is it from your house to your friend's house?

10. Two trains heading toward each other are 400 miles apart. One train travels 15 miles per hour faster than the other train. If they arrive at the same station in 5 hours, how fast is each train traveling?

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Lesson 1.2.2: Creating Linear Inequalities in One Variable

Warm-Up 1.2.2

Read the scenario, write an equation that models the situation, and then use the equation to answer the questions that follow.

> Two people can balance on a seesaw even if they are different weights. The balance will occur when the following equation, $w_1d_1 = w_2d_2$, is satisfied or true. In this equation, w_1 is the weight of the first person, d_1 is the distance the first person is from the center of the seesaw, w_2 is the weight of the second person, and d_2 is the distance the second person is from the center of the seesaw.

1. Eric and his little sister Amber enjoy playing on the seesaw at the playground. Amber weighs 65 pounds. Eric and Amber balance perfectly when Amber sits about 4 feet from the center and Eric sits about $2\frac{1}{2}$ feet from the center. About how much does Eric weigh?

2. Their little cousin Aleah joins them and sits right next to Amber. Can Eric balance the seesaw with both Amber and Aleah on one side, if Aleah weighs about the same as Amber? If so, where should he sit? If not, why not?

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Lesson 1.2.2: Creating Linear Inequalities in One Variable

Common Core State Standard

A–CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Warm-Up 1.2.2 Debrief

1. Eric and his little sister Amber enjoy playing on the seesaw at the playground. Amber weighs 65 pounds. Eric and Amber balance perfectly when Amber sits about 4 feet from the center and Eric sits about $2\frac{1}{2}$ feet from the center. About how much does Eric weigh?

   First set up the equation. Students might struggle with which weight is represented by w_1 and which is represented by w_2. If time allows, show students that as long as the associated weight and distance stay together on the same side of the equation, it doesn't matter which weight is w_1 and which is w_2.

   Given: $w_1d_1 = w_2d_2$

   w_1 = Amber's weight = 65 pounds

   d_1 = Amber's distance = 4 feet

   w_2 = Eric's weight = x pounds

   d_2 = Eric's distance = $2\frac{1}{2}$ feet = 2.5 feet

   The unknown is Eric's weight.

   Now, make the substitutions.

$w_1d_1 = w_2d_2$	Given equation
$(65)(4) = (x)(2.5)$	Substitute 65 for w_1, 4 for d_1, x for w_2, and 2.5 for d_2.
$260 = 2.5x$	Simplify.
$x = 104$	Divide both sides by 2.5.

   Interpret the solution.

   In this equation, x represented Eric's weight. Therefore, Eric weighs about 104 pounds.

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2. Their little cousin Aleah joins them and sits right next to Amber. Can Eric balance the seesaw with both Amber and Aleah on one side, if Aleah weighs about the same as Amber? If so, where should he sit? If not, why not?

   Set up the equation using $w_1d_1 = w_2d_2$.

   w_1 = Amber and Aleah's combined weight = 2(65) pounds = 130 pounds

   d_1 = Amber and Aleah's distance = 4 feet

   w_2 = Eric's weight = 104 pounds

   d_2 = Eric's distance = x feet

   This time, the unknown is Eric's distance.

   Now, make the substitutions.

$w_1d_1 = w_2d_2$	Given equation
$(130)(4) = 104(x)$	Substitute 130 for w_1, 4 for d_1, 104 for w_2, and x for d_2.
$520 = 104x$	Simplify.
$x = 5$	Divide both sides by 104.

   Interpret the solution.

   In this equation, x represented Eric's distance from the center of the seesaw. The question asks if it's possible for Eric to balance with Amber and Aleah. It's possible if each side of the seesaw extends at least 5 feet from the center, because Eric needs to sit at least 5 feet away from the center. If each side of the seesaw is shorter than 5 feet, then he cannot balance with his sister and his cousin. Therefore, if each side of the seesaw is 5 feet or longer, then Eric can balance the seesaw.

Connection to the Lesson

- Students are asked to create equations much like they will be asked to create inequalities.
- Solving equations is similar to solving inequalities with a few exceptions.
- The second question's response deals with the inequality concept "at least."

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- solving simple linear equations (8.EE.7b)
- comparing rational numbers (6.NS.7a)

Introduction

Inequalities are similar to equations in that they are mathematical sentences. They are different in that they are not equal all the time. An inequality has infinite solutions, instead of only having one solution like a linear equation. Setting up the inequalities will follow the same process as setting up the equations did. Solving them will be similar, with two exceptions, which will be described later.

Key Concepts

- The prefix *in-* in the word *inequality* means "not." Inequalities are sentences stating that two things may not be equal. For example, $12 > 2$ and $1 < 7$.
- Remember that the symbols $>$, $<$, $\geq$, $\leq$, and $\neq$ are used with inequalities.
- Use the table that follows to review the meanings of the inequality symbols and the provided examples with their **solution sets**, or the value or values that make a sentence or statement true.

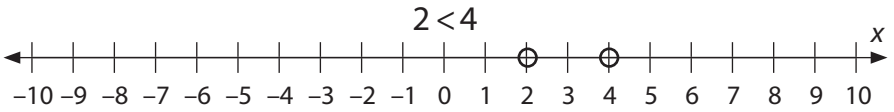
Symbol	Description	Example	Solution set
$>$	greater than, more than	$x > 3$	all numbers greater than 3; does not include 3
$\geq$	greater than or equal to, at least	$x \geq 3$	all numbers greater than or equal to 3; includes 3
$<$	less than	$x < 3$	all numbers less than 3; does not include 3
$\leq$	less than or equal to, no more than	$x \leq 3$	all numbers less than or equal to 3; includes 3
$\neq$	not equal to	$x \neq 3$	includes all numbers except 3

- Solving a linear inequality is similar to solving a linear equation. The processes used to solve inequalities are the same processes that are used to solve equations.

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Instruction

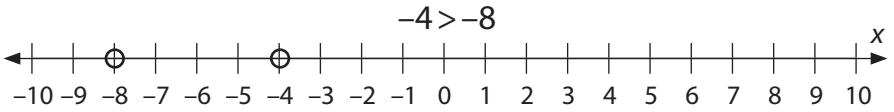
- Multiplying or dividing both sides of an inequality by a negative number requires reversing the inequality symbol. Here is a number line to show the process.
  - First, look at the example of the inequality $2 < 4$.



  - Multiply both sides by –2 and the inequality becomes $2(–2) < 4(–2)$ or $–4 < –8$.



  - Is –4 really less than –8?
  - To make the statement true, you must reverse the inequality symbol: $–4 > –8$



Creating Inequalities from Context

1. Read the problem statement carefully.
2. Reread the scenario and make a list of the known quantities.
3. Read the statement again, identifying the unknown quantity or variable.
4. Create expressions and inequalities from the known quantities and variable(s).
5. Solve the resulting inequality.
6. Interpret the solution of the inequality in terms of the context of the problem.

Common Errors/Misconceptions

- not translating the words into the correct symbols, especially with the phrases *no fewer than*, *no more than*, *at least as many*, and so on
- forgetting to switch the inequality symbol when multiplying or dividing by a negative
- not interpreting the solution in terms of the context of the problem—students can often mechanically solve the problem but don't know what the solution means

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Instruction

Guided Practice 1.2.2

Example 1

Juan has no more than $50 to spend at the mall. He wants to buy a pair of jeans and some juice. If the sales tax on the jeans is 4% and the juice with tax costs $2, what is the maximum price of jeans Juan can afford?

1. Read the problem statement carefully.

2. Reread the scenario and make a list of the known quantities.

   Sales tax is 4%.

   Juice costs $2.

   Juan has no more than $50.

3. Read the statement again, identifying the unknown quantity or variable.

   The unknown quantity is the cost of the jeans.

4. Create expressions and inequalities from the known quantities and variable(s).

   The price of the jeans + the tax on the jeans + the price of the juice must be less than or equal to $50.

   $x + 0.04x + 2 \leq 50$

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5. Solve the resulting inequality.

$x + 0.04x + 2 \leq 50$	Inequality from the previous step
$1.04x + 2 \leq 50$	Add like terms.
$1.04x \leq 48$	Subtract 2 from both sides.
$x \leq 46.153846$	Divide both sides by 1.04.

Regardless of the digit in the thousandths place, round down to 46.15; rounding up will increase the price to a value that is not in Juan's budget.

$x \leq 46.15$

6. Interpret the solution of the inequality in terms of the context of the problem.

Juan should look for jeans that are priced at or below $46.15.

Example 2

Alexis is saving to buy a laptop that costs $1,100. So far she has saved $400. She makes $12 an hour babysitting. What's the least number of hours she needs to work in order to reach her goal?

1. Read the problem statement carefully.

2. Reread the scenario and make a list of the known quantities.

Alexis has saved $400.

She makes $12 an hour.

She needs at least $1,100.

3. Read the statement again, identifying the unknown quantity or variable.

You need to know the least number of hours Alexis must work to make enough money. Solve for hours.

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Instruction

4. Create expressions and inequalities from the known quantities and variable(s).

   The sum of Alexis's saved money and her earned money must be greater than or equal to the cost of the laptop.

   $400 + 12h \geq 1100$

5. Solve the resulting inequality.

$400 + 12h \geq 1100$	Inequality from the previous step
$12h \geq 700$	Subtract 400 from both sides.
$h \geq 58.3\overline{3}$	Divide both sides by 12.

6. Interpret the solution of the inequality in terms of the context of the problem.

   In this situation, it makes sense to round up to the nearest half hour since babysitters usually get paid by the hour or half hour. Therefore, Alexis needs to work at least 58.5 hours to make enough money to save for her laptop.

Example 3

A radio station is giving away 40 concert tickets. The station gives away 2 tickets every hour on the hour for a number of hours until there are at most 4 tickets left for a grand prize. If the contest runs from 11:00 A.M. to 1:00 P.M. each day, for how many days will the contest last?

1. Read the problem statement carefully.

2. Reread the scenario and make a list of the known quantities.

   The contest starts with 40 tickets.

   The station gives away 2 tickets every hour.

   The contest ends with at most 4 tickets left.

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Instruction

3. Read the statement again, identifying the unknown quantity or variable(s).

   For how many days will the contest last?

   This is tricky because the tickets are given away in terms of hours.

   First, solve for hours.

4. Create expressions and inequalities from the known quantities and variable(s).

   The difference of the initial 40 tickets and the "2 tickets given away every hour" must be less than or equal to 4 tickets.

   $40 - 2h \leq 4$

5. Solve the resulting inequality.

$40 - 2h \leq 4$	Inequality from the previous step
$-2h \leq -36$	Subtract 40 from both sides.
$h \geq 18$	Divide both sides by –2 and switch the inequality symbol.

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Instruction

6. Interpret the solution of the inequality in terms of the context of the problem.

   The inequality is solved for the number of hours the contest will last. The contest will last at least 18 hours; that is, for 18 hours or more.

   The problem asks for the number of days the contest will last. If the contest lasts from 11:00 A.M. to 1:00 P.M. each day, and tickets are given out on the hour, that is 3 hours per day. Convert the units.

$$18 \text{ hours} \cdot \frac{1 \text{ day}}{3 \text{ hours}}$$

$$18 \ \cancel{\text{hours}} \cdot \frac{1 \text{ day}}{3 \ \cancel{\text{hours}}} = 6 \text{ days}$$

   The contest will run for at least 6 days. Note that there are a total of 40 tickets to give away, so if the station gives away 6 tickets per day until there are at most 4 tickets left for a grand prize, then the longest the contest can last is 7 days. Thus, the contest will last for 6 or 7 days.

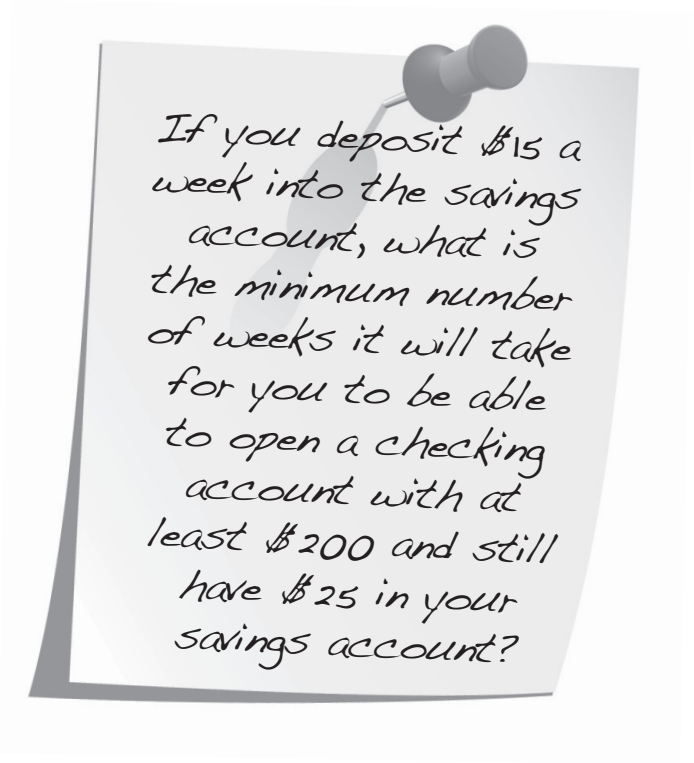


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Problem-Based Task 1.2.2: Free Checking Accounts

SMP	
1 ✓	2 ✓
3	4 ✓
5	6
7	8

The time has come for you to open a checking account. A local bank is offering you a free checking account if you maintain a minimum balance of $200. You already have a savings account with this bank and you have $60 saved. You decide to keep saving money until you have enough to open a checking account, plus keep some money in savings. If you deposit $15 a week into the savings account, what is the minimum number of weeks it will take for you to be able to open a checking account with at least $200 and still have $25 in your savings account?



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Problem-Based Task 1.2.2: Free Checking Accounts

Coaching

a. What is the minimum amount of money you want to have to open your checking account?

b. How much are you depositing each week?

c. What inequality can be written to model the scenario?

d. What is the solution to the inequality?

e. What does the solution of the inequality represent in terms of the context of the problem?

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Instruction

Problem-Based Task 1.2.2: Free Checking Accounts

Coaching Sample Responses

a. What is the minimum amount of money you want to have to open your checking account?

$225

Students at first might be inclined to write $200 because this is the minimum amount needed for a free checking account, but the problem also states that there needs to be $25 left in the savings account.

b. How much are you depositing each week?

$15/week

This is given in the problem.

c. What inequality can be written to model the scenario?

$60 + 15x \geq 225$

You start with $60 and deposit $15 each week and need at least $225.

d. What is the solution to the inequality?

$60 + 15x \geq 225$

$15x \geq 165$

$x \geq 11$

e. What does the solution of the inequality represent in terms of the context of the problem?

Any time after 11 weeks, there will be enough money to open the checking account.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 1.2.2: Creating Linear Inequalities in One Variable

For problems 1 and 2, translate each phrase into an algebraic inequality.

1. A tour bus can seat 55 passengers.

2. An energy-efficient lamp can only be used with light bulbs that are 60 watts or less.

For problems 3–9, read each scenario, write an inequality to model the scenario, and then use the inequality to solve the problem.

3. Camilla is saving to purchase a new pair of bowling shoes that will cost at least $39. She has already saved $19. What is the least amount of money she needs to save for the shoes?

4. Suppose you earn $20 per hour working part time at a tax office. You want to earn at least $1,800 this month, before taxes. How many hours must you work?

5. Hiram earned a score of 83 on his semester algebra test. He needs to have a total of at least 180 points from his semester and final tests to receive an A for his grade. What score must Hiram earn on his final test to ensure his A?

6. Claire buys DVDs online. Each DVD that she orders costs $15 and shipping for her order is $10. Claire can spend no more than $100. How many DVDs can she buy?

continued

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7. The temperature outside is dropping at a rate of 1.5 degrees each hour. It is raining, and the weather station predicts a changeover from rain to sleet when the temperature reaches 32°F or below. Currently the temperature is 53°F and the time is 8:30 A.M. When do you predict the changeover to sleet will occur?

8. A recreation center holds a soccer game every Saturday morning for older teens. The group agreed that there should be at least 5 players on each team. One team started out with 17 players. After an hour of playing, 3 players started leaving every 5 minutes. At most, for how long can the team keep playing?

9. The owners of a website have no more than $15,000 to give away for a contest. They have decided to give away $1,000 twice a day every day until there is no more than $3,000 left to award as a grand prize. How many days will the contest run?

For problem 10, create your own context for the given inequality, and then solve the inequality. Be sure to express your solution in terms of the context of the problem.

10. $2x - 5 \leq 9$

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Lesson 1.2.3: Creating Exponential Equations

Warm-Up 1.2.3

This year, Zachary has been babysitting his young cousins after school for $70 a month. His uncle also gave him an extra bonus of $100 for his excellent work. Since school started, Zachary has earned more than $500. How many months ago did school start? Write and solve an inequality that represents this situation.

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Lesson 1.2.3: Creating Exponential Equations

Common Core State Standard

A–CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Warm-Up 1.2.3 Debrief

- Students will hopefully recognize this scenario as modeling a linear inequality: $70x + 100 > 500$, where x is the number of months Zachary has been babysitting.
- Solving this equation for x gives an approximate value of $x > 5.71$ months. However, in the context of the problem, we need to round to a whole number of months. In this case, it would not be accurate to round x up from 5.71 to 6, because doing so would exclude "6 months" from the possible solutions for the number of months since school started. Therefore, we must round down, so that $x > 5$.
- Encourage students to make meaning of the numerical answer, "School started more than 5 months ago."

Connection to the Lesson

- Students will be required to create equations from context, except the equations will be exponential instead of linear.
- Students will also be required to interpret the solutions in context, rather than just stopping at solving the equation for the numerical answer.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- working with exponents (raising a base to a power) (6.EE.1)
- applying the order of operations (5.OA.1)

Introduction

Exponential equations are equations that have the variable in the exponent. Exponential equations are found in science, finance, sports, and many other areas of daily living. Some equations are complicated, but some are not.

Key Concepts

- The general form of an exponential equation is $y = a \bullet b^x$, where a is the initial value, b is the base, and x is the exponent. The final output value will be y.
- Since the equation has an exponent, the value increases or decreases rapidly.
- The base, b, must always be greater than 0 ($b > 0$).
- If the base is greater than 1 ($b > 1$), then the exponential equation represents **exponential growth**.
- If the base is between 0 and 1 exclusive (that is, $0 < b < 1$), then the exponential equation represents **exponential decay**.
- If the time is given in units other than 1 (e.g., 1 month, 1 hour, 1 minute, 1 second), use the equation $y = ab^{\frac{x}{t}}$, where t is the interval over which y changes by a factor of b, and x is the interval under consideration, measured in the same units as t.
- Another form of the exponential equation is $y = a(1 \pm r)^t$, where a is the initial value, $(1 \pm r)$ is the base, t is the variable exponent, and y is the final value.
- Use $y = a(1 + r)^t$ for exponential growth (notice the plus sign). For example, if a population grows by 2% then r is 0.02, but this is less than 1 and by itself does not indicate growth. Adding this to 1 produces the full growth rate, 1.02, or 102%.
- Since r is the percentage increase in a specific time interval, the initial value is multiplied by $(1 + r)$ in each time interval. Therefore, the base $(1 + r)$ indicates exponential growth.

SRB p. U1-42

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Instruction

- Use $y = a(1 - r)^t$ for exponential decay (notice the minus sign). For example, if a population decreases by 3%, then 97% is the factor being multiplied over and over again. The population from year to year is always 97% of the population from the year before (a 3% decrease). Think of this as 100% minus the rate, or in decimal form $(1 - r)$.
- Look for words such as *double*, *triple*, *half*, and *quarter*—such words give the number of the base. For example, if an experiment begins with 1 bacterium whose number doubles (splits itself in two) every hour, determining how many bacteria will be present after x hours is solved with the following equation: $y = (1)2^x$, where 1 is the starting value, 2 is the base that indicates doubling, x is the number of hours, and y is the final value.
- Look for the words *initial* or *starting* to find the value to substitute in for a.
- Look for the words *ended with* and *after*—these words will be near the final value given.
- Follow the same procedure as with setting up linear equations and inequalities in one variable:

Creating Exponential Equations from Context

1. Read the problem statement carefully.
2. Reread the scenario and make a list of the known quantities.
3. Read the statement again, identifying the unknown quantity or variable.
4. Create expressions and equations from the known quantities and variable(s).
5. Substitute the values into the general form of an exponential equation.
6. Follow the order of operations to solve the resulting equation.
7. Interpret the solution of the exponential equation in terms of the context of the problem.

Common Errors/Misconceptions

- multiplying a and b together before raising to the power
- misidentifying the base
- creating a linear equation instead of an exponential equation

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Guided Practice 1.2.3

Example 1

A population of mice quadruples every 6 months. If a mouse nest started out with 2 mice, how many mice would there be after 2 years? Write an equation and then use it to solve the problem.

1. Read the problem statement carefully and then reread it, this time identifying the known quantities.

   The initial number of mice is 2.

   The base is 4 because the population quadruples each time interval.

   The amount of time is every 6 months for 2 years.

2. Read the statement again, identifying the unknown quantity or variable.

   The unknown quantity is the number of mice after 2 years. Solve for the final amount of mice after 2 years.

3. Create expressions and equations from the known quantities and variable(s).

   The general form of an exponential equation is $y = a \cdot b^x$, where y is the final value, a is the initial value, b is the base, and x is the number of time intervals.

   $a = 2$

   $b = 4$

   Since the problem is given in months, to find x, you need to convert 2 years into 6-month time periods. How many 6-month time periods are there in 2 years?

   To determine this, think about how many 6-month time periods there are in 1 year. There are 2. Multiply that by 2 for each year. Therefore, there are four 6-month time periods in 2 years.

(*continued*)

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Instruction

Another way to determine the period is to set up ratios.

$$2 \text{ years} \cdot \frac{12 \text{ months}}{1 \text{ year}} = 24 \text{ months}$$

$$24 \text{ months} \cdot \frac{1 \text{ time period}}{6 \text{ months}} = 4 \text{ time periods}$$

Therefore, $x = 4$.

4. Substitute the values into the general form of an exponential equation, $y = a \cdot b^x$.

$y = a \cdot b^x$ OR $y = ab^{\frac{x}{t}}$

$y = (2) \cdot (4)^4$ OR $y = (2)(4)^{\frac{24}{6}}$

5. Follow the order of operations to solve the resulting equation.

$y = 2 \cdot 4^4$	Equation from the previous step
$y = 2 \cdot 256$	Raise 4 to the fourth power.
$y = 512$	Multiply 2 and 256.

6. Interpret the solution in terms of the context of the problem.

There will be 512 mice after 2 years if the population quadruples every 6 months.

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Instruction

Example 2

In sporting tournaments, teams are eliminated after they lose. The number of teams in the tournament then decreases by half with each round. If there are 16 teams left after 3 rounds, how many teams started out in the tournament?

1. Read the problem statement carefully and then reread it, this time identifying the known quantities.

   The final number of teams is 16.

   The reduction is $\frac{1}{2}$ per round.

   The number of rounds is 3.

2. Read the statement again, identifying the unknown quantity or variable.

   The unknown quantity is the number of teams with which the tournament began. Solve for the initial or starting value, a.

3. Create expressions and equations from the known quantities and variable(s).

   The general form of the exponential equation is $y = a \bullet b^x$, where y is the final value, a is the initial value, b is the base (the factor by which it is reduced each round), and x is the number of rounds.

   $y = 16$

   $b = \frac{1}{2}$

   $x = 3$ rounds

4. Substitute the values into the general form of an exponential equation, $y = a \bullet b^x$.

   $y = a \bullet b^x$

   $(16) = a \bullet \left(\frac{1}{2}\right)^{(3)}$

   $16 = a \bullet \left(\frac{1}{2}\right)^3$

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Instruction

5. Follow the order of operations to solve the resulting equation.

$16 = a \cdot \left(\frac{1}{2}\right)^3$	Equation from the previous step
$16 = a \cdot \frac{1}{8}$	Raise the base to the power of 3.
$a = 128$	Multiply by the reciprocal of $\frac{1}{8}$.

6. Interpret the solution in terms of the context of the problem.

   The tournament started with 128 teams.



Example 3

The population of a small town is increasing at a rate of 4% per year. If there are currently about 6,000 residents, about how many residents will there be in 5 years at this growth rate?

1. Read the problem statement carefully and then reread it, this time identifying the known quantities.

   The initial number of residents is 6,000.

   The growth rate is 4%.

   The amount of time is 5 years.

2. Read the statement again, identifying the unknown quantity or variable.

   The unknown quantity is the number of residents after 5 years. Solve for the final value after 5 years.

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Instruction

3. Create expressions and equations from the known quantities and variable(s).

   The general form of the exponential growth equation with a percent increase is $y = a(1 + r)^t$, where y is the final value, a is the initial value, r is the rate of growth, and t is the number of years that have elapsed.

   $a = 6000$

   $r = 4\% = 0.04$

   $t = 5$

4. Substitute the values into the general form of an exponential growth equation, $y = a(1 + r)^t$.

   $y = a(1 + r)^t$

   $y = (6000)[1 + (0.04)^{(5)}]$

   $y = 6000(1 + 0.04)^5$

5. Follow the order of operations to solve the resulting equation.

$y = 6000(1 + 0.04)^5$	Equation from the previous step
$y = 6000(1.04)^5$	Add inside the parentheses first.
$y \approx 6000(1.21665)$	Raise the base to the power of 5.
$y \approx 7300$	Multiply.

6. Interpret the solution in terms of the context of the problem.

   If this percentage growth rate continues for 5 years, the population will increase by more than 1,000 residents to about 7,300 people.

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Example 4

You want to reduce the size of a picture to place in a small frame. You aren't sure what size to choose on the photocopier, so you decide to reduce the picture by 15% each time you scan it until you get it to the size you want. If the picture was 10 inches long at the start, how long is it after 3 scans?

1. Read the problem statement carefully and then reread it again, this time identifying the known quantities.

   The initial length is 10 inches.

   The reduction is 15% per scan.

   The number of scans is 3.

2. Read the statement again, identifying the unknown quantity or variable.

   The unknown quantity is the length of the picture after 3 scans. Solve for the final value after 3 scans.

3. Create expressions and equations from the known quantities and variable(s).

   The general form of the exponential growth equation with a percent decrease is $y = a(1 - r)^t$, where y is the final value, a is the initial value, r is the rate of decay, and t is the number of intervals.

   $a = 10$

   $r = 15\% = 0.15$

   $t = 3$

4. Substitute the values into the general form of an exponential decay equation, $y = a(1 - r)^t$.

   $y = a(1 - r)^t$

   $y = (10)[1 - (0.15)^{(3)}]$

   $y = 10(1 - 0.15)^3$

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Instruction

5. Follow the order of operations to solve the resulting equation.

$y = 10(1 - 0.15)^3$	Equation from the previous step
$y = 10(0.85)^3$	Subtract inside the parentheses first.
$y = 10(0.614125)$	Raise the base to the power of 3.
$y \approx 6.14$	Multiply.

6. Interpret the solution in terms of the context of the problem.

   After 3 scans, the length of the picture is about 6 inches.



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Problem-Based Task 1.2.3: Population Change

SMP	
1	2 ✓
3	4 ✓
5 ✓	6 ✓
7	8

On opposite sides of a major city two suburban towns are experiencing population changes. One town, Town A, is growing rapidly at 5% per year and has a current population of 39,000. Town B has a declining population at a rate of 2% per year. Its current population is 55,000. Economists predict that in 5 years the populations of these two towns will be about the same, but the residents of both towns are in disbelief. The economists also claim that ten years after that, Town A will be double the size of Town B. Can you verify the predictions based on the data given? Do you think these predictions will come true?



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Problem-Based Task 1.2.3: Population Change

Coaching

a. Is Town A experiencing population growth or decay? What is the equation for Town A's population after 5 years?

b. What is the solution to the equation in part a?

c. Is Town B experiencing population growth or decay? What is the equation for Town B's population after 5 years?

d. What is the solution to the equation in part c?

e. Are your solutions to parts b and d similar? What can you conclude about the economists' prediction about the populations of the two towns being about the same after 5 years?

f. What will the variable, t, equal if the towns experience the same rates of growth or decline for 10 more years after that?

g. What are the equations for each town's population after 10 more years?

h. Based on your calculations, was the economists' prediction for the town populations after 10 more years correct?

i. What factors might influence whether or not the economists' predictions come true?

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Problem-Based Task 1.2.3: Population Change

Coaching Sample Responses

a. Is Town A experiencing population growth or decay? What is the equation for Town A's population after 5 years?

Town A is experiencing growth. Fill in the information we know to find the equation:

$y = a(1 + r)^t$

- $a = 39{,}000$
- $r = 5\% = 0.05$
- $t = 5$

The equation for Town A is $y = 39{,}000(1 + 0.05)^5$.

b. What is the solution to the equation in part a?

$y = 39{,}000(1 + 0.05)^5$

$y \approx 49{,}775$ people

Round to the nearest whole person.

c. Is Town B experiencing population growth or decay? What is the equation for Town B's population after 5 years?

Town B is experiencing decay. Fill in the information we know to find the equation:

$y = a(1 - r)^t$

- $a = 55{,}000$
- $r = 2\% = 0.02$
- $t = 5$

The equation for Town B is $y = 55{,}000(1 - 0.02)^5$.

d. What is the solution to the equation in part c?

$y = 55{,}000(1 - 0.02)^5$

$y \approx 49{,}716$ people

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e. Are your solutions to parts b and d similar? What can you conclude about the economists' prediction about the populations of the two towns being about the same after 5 years?

The numbers are similar, especially when considering the magnitude. If rounded to the nearest 100 people, Town A would have 49,800 and Town B would have 49,700 people. A difference in 100 people compared to tens of thousands is a small number. Based on these calculations, it would seem that the economists are correct that the populations will be about equal in 5 years.

f. What will the variable, t, equal if the towns experience the same rates of growth or decline for 10 more years after that?

The amount of time passed since the initial population count will be 5 years plus 10, which equals 15 years. Therefore, $t = 15$.

g. What are the equations and solutions for each town's population after 10 more years?

The equations that follow use the town's current populations, with $t = 15$.

Town A	Town B
$y = a(1 + r)^t$	$y = a(1 - r)^t$
$y = 39{,}000(1 + 0.05)^{15}$	$y = 55{,}000(1 - 0.02)^{15}$
$y \approx 81{,}079$ people	$y \approx 40{,}622$ people

Alternatively, students could use the value $t = 10$ and then add the population values they calculated at the 5-year mark for a.

h. Based on your calculations, was the economists' prediction for the town populations after 10 more years correct?

Yes, 81,079 is about twice as much as 40,622.

i. What factors might influence whether or not the economists' predictions come true?

Many factors might contribute to the economists' predictions coming true or not, such as the economy itself. If the economy starts to decline, the population of Town A might not continue to grow as fast. Or, a new industry might relocate to Town B, bringing jobs, and perhaps people will start moving back into that town. Or, Town A might reach its capacity before the population reaches 81,079.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 1.2.3: Creating Exponential Equations

Use what you know about linear and exponential equations to complete problems 1 and 2.

1. Determine whether each scenario can be modeled by a linear or an exponential equation.
   a. The price of a loaf of bread increases by $0.25 each week.

   b. Each week, a loaf of bread costs twice as much as it did the week before.

2. Determine whether each scenario can be modeled by a linear or an exponential equation.
   a. 10 people leave a football game every minute after the third quarter.

   b. $\frac{1}{4}$ of the people leave a football game every minute after the third quarter.

For problems 3–10, write an equation to model each scenario. Then use the equation to solve the problem.

3. A population of insects doubles every month. If there are 100 insects to start with, how many will there be after 7 months?

4. A type of bacteria in a Petri dish doubles every hour. If there were 1,073,741,824 bacteria after 24 hours, how many were there to start with?

continued

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5. A stock loses half its value every week. If the stock was worth $125 starting out, what is it worth after 4 weeks at this rate of decline?

6. A certain tablet computer depreciates (loses its value) at a rate of $\frac{1}{2}$ the original value every 2 years. If you bought a tablet 6 years ago and it's now worth $75, how much did it cost when you bought it?

7. A new car depreciates as soon as you drive it out of the parking lot. A certain car depreciates to half its original value in 4 years. If you bought a car 8 years ago and it is now worth $5,000, how much did you pay for the car originally?

8. The number of dandelions growing on your lawn triples every 3 days. If your lawn started out with 15 dandelions, how many dandelions would you have after 3 weeks?

9. The population of a town is increasing by 2% each year. The current population is 12,000. How many people will there be in 4 years?

10. The population of a city doubles during the workday. At the end of the workday, the population decreases by half. If the population of the city is roughly 4,000,000 people before "rush hour," and the morning rush hour actually lasts 3 hours, what would be the population 12 hours after the start of rush hour if the population continued to double every 3 hours? People leave the city for home at the same rate as they come into the city. If there are 8,000,000 people in the city during the workday, what would be the population 12 hours after the start of the evening rush hour, which lasts 3 hours, if the population continued to be cut in half every 3 hours?

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Progress Assessment

Circle the letter of the best answer.

1. You are redecorating your room and the only thing left to paint is your door. You're only going to paint the side that faces the inside of your room. The door is 6 feet 8 inches tall and 30 inches wide. You need to know the surface area of the side of the door to determine how much paint to buy. The hardware store sells paint by how much covers a square foot. What is the surface area you should report to the hardware store? (*Hint*: First find the number of square inches in 1 square foot.)

   a. about 16 square feet
   b. 16.67 square feet
   c. 16.7 square feet
   d. about 17 square feet

2. You are participating in a fund-raiser in which you read for donations. People can donate money based on a flat fee or based on the number of minutes you read. So far, you have two donors. Your grandma has agreed to donate \$10 and your mom has agreed to donate \$0.05 per minute. If together they donated \$22.35, and x represents the number of minutes read, what equation represents this situation?

   a. $(10 + 0.05)x = 22.35$
   b. $10x + 0.05 = 22.35$
   c. $0.05x + 10 = 22.35$
   d. $22.35x = 10 + 0.05$

3. Two friends live 253 miles apart. They drive toward each other on the same highway to meet for a weekend. The first friend drives 5 miles per hour below the speed limit and it takes her 2 hours to meet her friend. The second friend leaves 15 minutes late, and she drives at the speed limit. What was the second friend's speed?

   a. about 55 mph
   b. about 60 mph
   c. about 65 mph
   d. about 70 mph

4. A carafe at a banquet holds about 12 cups of coffee. When the carafe has 1 cup or less of coffee left, the waiter dumps the coffee out and pours in a fresh pot. Each coffee mug at the banquet holds 6 ounces of coffee. Assuming each person fills his or her mug, what inequality represents the number of people who can fill their mugs before the carafe needs to be refilled? (*Hint*: 1 cup = 8 ounces)

   a. $12 - 0.75x \leq 1$
   b. $12 - x \leq 1$
   c. $12 - 0.75x \geq 1$
   d. $12 - x \geq 1$

continued

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Assessment

5. Geraldine brings a large printing job to her local print shop. She doesn't want to sit and wait for the job to be completed, so she asks when she should come back to pick up her order. The machines print 30 pages per minute and 225 pages printed while she paid for her order. She has less than 1,000 pages left to go. How much longer will it take her order to print?

   a. slightly less than 33 minutes
   b. slightly less than 41 minutes
   c. slightly more than 33 minutes
   d. slightly more than 41 minutes

6. Juan is saving for a new computer and has saved $213. He is saving $3 a day and needs at least $654 for the computer he wants. How many weeks will it take him to save at least enough for the computer he wants?

   a. at most 147 weeks
   b. at most 21 weeks
   c. at least 147 weeks
   d. at least 21 weeks

7. A new grocery store is giving away four $20 gift certificates to the store every hour to celebrate its grand opening. The store manager wants to save at least 24 of the certificates to give to a local food bank. If there are 120 gift certificates to start with, for how many hours should the store continue giving them away?

   a. greater than or equal to 24 hours
   b. less than or equal to 24 hours
   c. less than 24 hours
   d. greater than 24 hours

8. A certain type of bacteria doubles every 7 hours. If you started with 36 bacteria, how many would you have after 28 hours?

   a. 9,663,676,416 bacteria
   b. 576 bacteria
   c. 26,873,856 bacteria
   d. 4,608 bacteria

9. A soccer tournament starts out with 128 teams. After each round, half of the teams are eliminated. After 6 rounds, how many teams are left?

   a. 2 teams
   b. 4 teams
   c. 8 teams
   d. 16 teams

continued

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10. A rural town is declining in population at a rate of 2% per year. If the current population is 7,000 people, what will the population be in 6 years?

    a. about 6,201 people
    b. about 7,884 people
    c. about 1,835 people
    d. about 6,160 people

Use the given information to solve the problem.

11. Your car broke down, and the final bill for the repairs was $261.50. The part that was replaced cost $99, and the charge for the mechanic's labor is $65 per hour. Write an equation to model this situation, then solve the equation for the number of hours the mechanic worked on your car.

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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. It costs $80 to buy an air conditioner and about $0.40 per minute to run it. Which equation models the total cost of using an air conditioner?

   a. $x + y = 80.40$

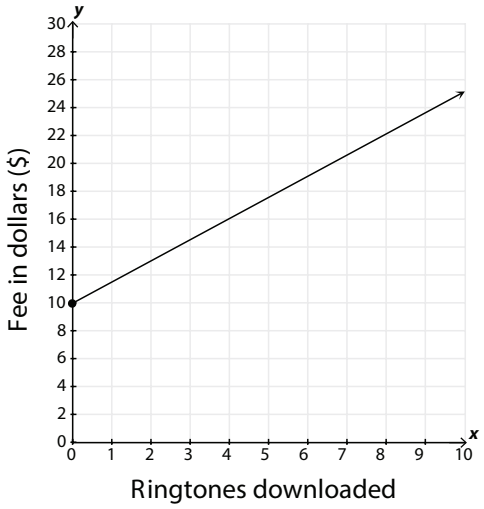
   b. $y = 80.40x$

   c. $y = 80x + 0.40$

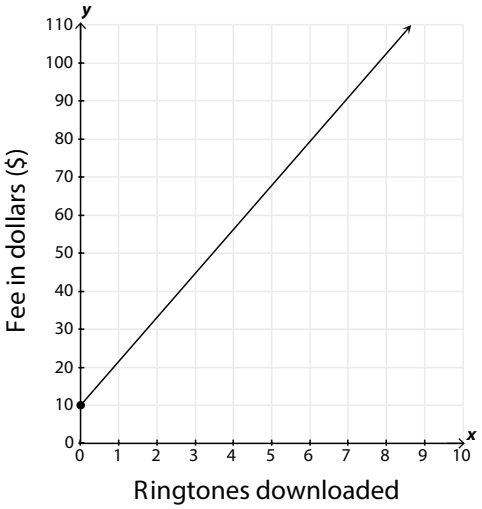
   d. $y = 0.40x + 80$

2. A ringtone company charges $10 a month for the service plus $1.50 for each ringtone downloaded. What is the graph of the equation that models the total fees?

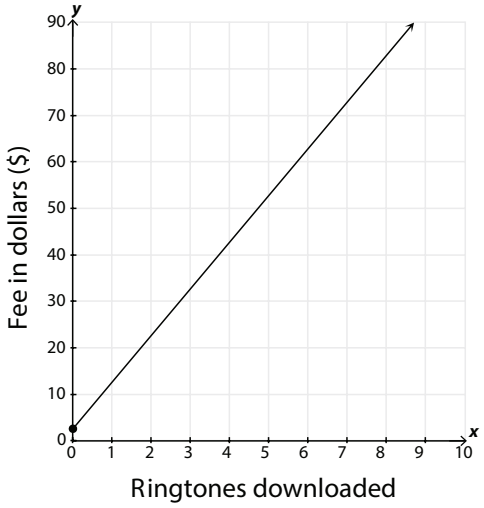
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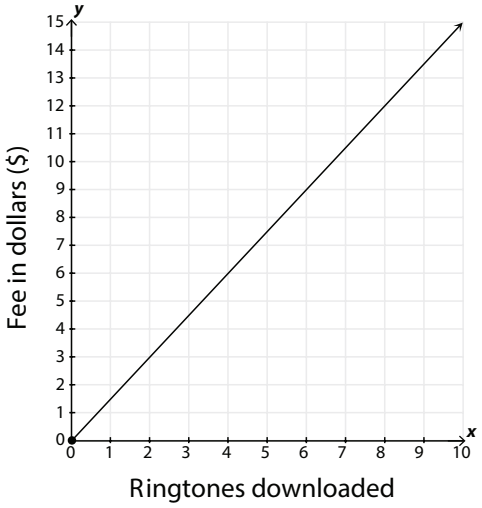
c.



b.



d.

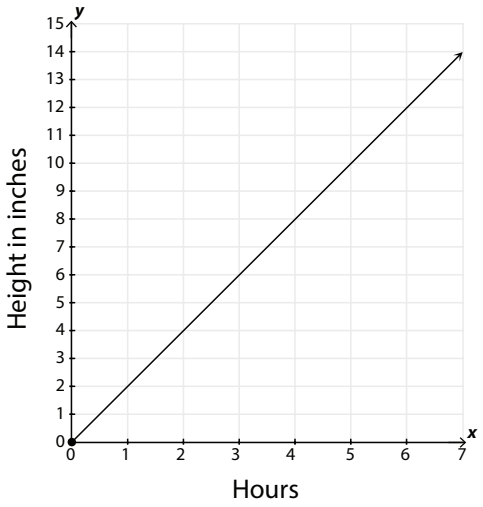


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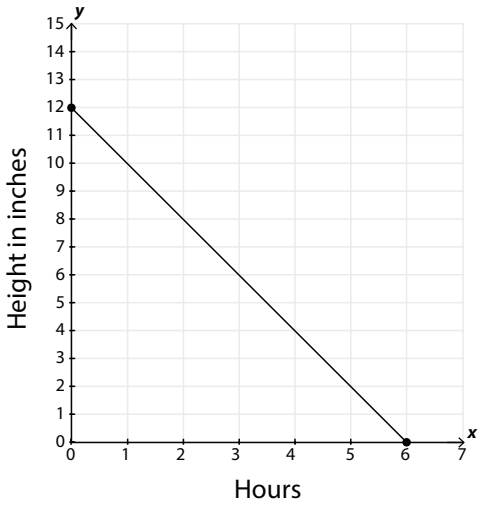
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3. A 12-inch candle burns down at a rate of 2 inches per hour. What is the graph of the equation that models the height of the candle over time?

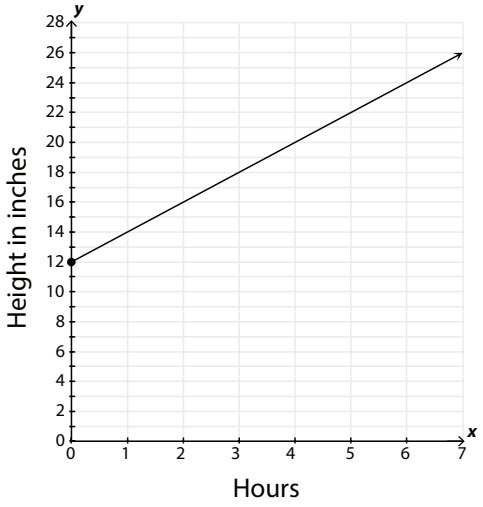
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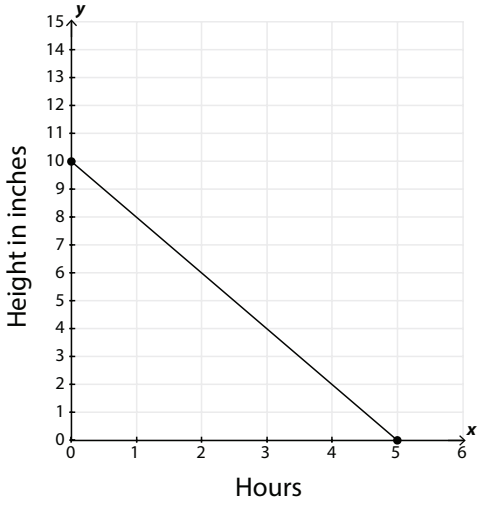
c.



b.



d.



4. A town's population increases at a rate of 2.3% every year. The current population is 7,500 people. Which equation models the population of the town over time?

a. $y = 7500(1.23)^x$

b. $y = 7500(1.023)^x$

c. $y = 7500(0.023)^x$

d. $y = 7500(0.23)^x$

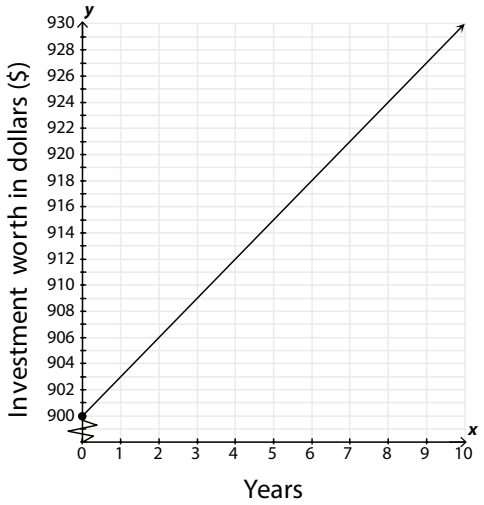
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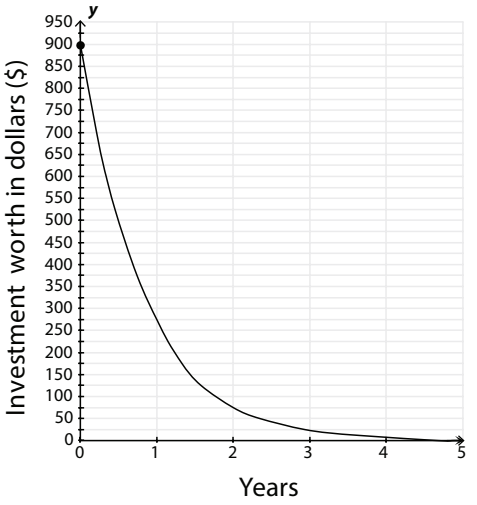
Assessment

5. An investment of $900 earns 3% interest, which is compounded semiannually. Which graph models the worth of the investment over time?

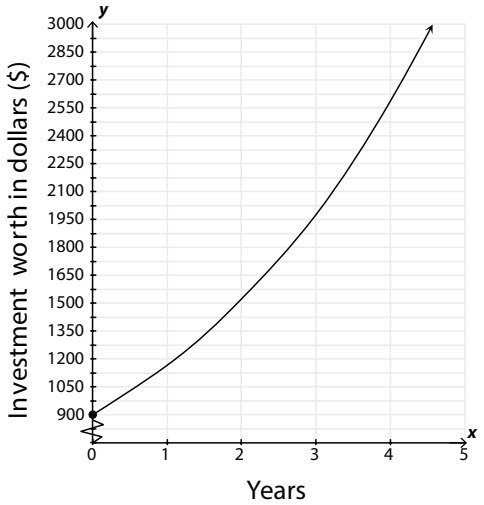
a.



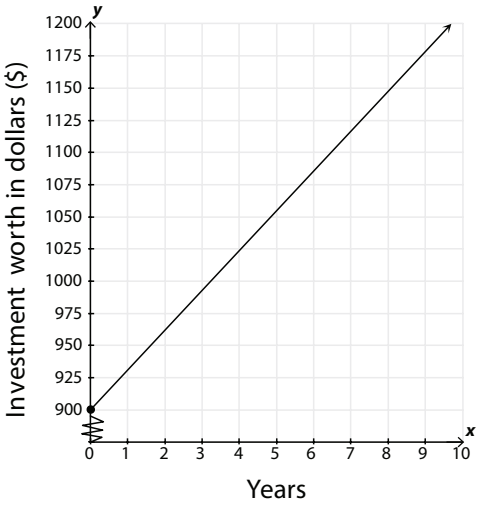
c.



b.



d.



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Lesson 3: Creating and Graphing Equations in Two Variables

Instruction

Common Core State Standards

SMP	
1	2 ✓
3 ✓	4 ✓
5 ✓	6
7 ✓	8

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

N–Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

Essential Questions

1. What do the graphs of equations in two variables represent?
2. How do you determine the scales to use for the *x*- and *y*-axes on any given graph?
3. How do the graphs of linear equations and exponential equations differ? How are they similar?
4. How can graphing equations help you to make decisions?

WORDS TO KNOW

coordinate plane a plane determined by a set of two number lines, called the axes, that intersect at right angles

dependent variable generally labeled on the *y*-axis; the quantity that is based on the input values of the independent variable

exponential decay an exponential equation with a base, *b*, that is between 0 and 1 exclusive (that is, $0 < b < 1$); an example is the formula $y = a(1 - r)^t$, where *a* is the initial value, $(1 - r)$ is the base (with $0 < r < 1$), *t* is the variable exponent, and *y* is the final value

exponential equation an equation whose independent variable is in the exponent; the general form of its equation is $f(x) = ab^x + k$, where *a* is the initial value, *b* is the base, *x* is the input value, *k* is the vertical shift, and $f(x)$ is the output. Another form is $y = ab^{\frac{x}{t}}$, where *t* is the interval over which *y* changes by a factor of *b*, and *x* is measured in the same units as *t*.

SRB p. U1-53

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Instruction

exponential growth	an exponential equation with a base, b, greater than 1 ($b > 1$); an example is the formula $y = a(1 + r)^t$, where a is the initial value, $(1 + r)$ is the base (with $r > 0$), t is the variable exponent, and y is the final value
independent variable	generally labeled on the x-axis; the quantity that changes based on values chosen
linear equation	a first-degree equation that can be written in the form $ax + by = c$, where a, b, and c are numbers; when written as $y = mx + b$, m is the slope of the line, and b is its y-intercept. The graph of a linear equation is a straight line.
slope	the measure of the rate of change of one variable with respect to another variable; slope $= m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$
***x*-intercept**	the x-coordinate of the point where a line or a curve intersects the x-axis
***y*-intercept**	the y-coordinate of the point where a line or a curve intersects the y-axis

Recommended Resources

- GeoGebra. "Exponential Functions (Graph & Equation Anatomy)."

  http://www.walch.com/rr/05000

  This applet provides sliders for the various values in an exponential equation, and shows how changing the values results in graph changes.

- Math-Play.com. "Hoop Shoot."

  http://walch.com/rr/CAU1L3SlopeandIntercept

  This one- or two-player game includes 10 multiple-choice questions about slope and y-intercept. Correct answers result in a chance to make a 3-point shot in a game of basketball.

- Oswego City School District Regents Exam Prep Center. "Equations and Graphing."

  http://walch.com/rr/CAU1L3GraphLinear

  This site summarizes the methods used to graph linear equations.

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Lesson 1.3.1: Creating and Graphing Linear Equations in Two Variables

Warm-Up 1.3.1

Read the information that follows and use it to complete the problems.

A cell phone company charges a $20 flat fee plus $0.05 for every minute used for calls.

1. Make a table of values from 0 to 60 minutes in 10-minute intervals that represents the total amount charged.

2. Write an algebraic equation that could be used to represent the situation.

3. What do the unknown values in your equation represent?

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Instruction

Lesson 1.3.1: Creating and Graphing Linear Equations in Two Variables

Common Core State Standards

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

N–Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

Warm-Up 1.3.1 Debrief

1. Make a table of values from 0 to 60 minutes in 10-minute intervals that represents the total amount charged.

Minutes used	Total amount charged ($)
0	20 + 0(0.05) = 20.00
10	20 + 10(0.05) = 20.50
20	20 + 20(0.05) = 21.00
30	20 + 30(0.05) = 21.50
40	20 + 40(0.05) = 22.00
50	20 + 50(0.05) = 22.50
60	20 + 60(0.05) = 23.00

2. Write an algebraic equation that could be used to represent the situation.

   $y = 0.05x + 20$

3. What do the unknown values in your equation represent?

   x represents the number of minutes used, and y represents the total amount charged.

Connection to the Lesson

- Students will be creating equations just like these in the upcoming lesson but will be given the option of skipping the step of creating the table of values.
- Students gain exposure to working with input and output pairs in the warm-up.
- Students will take this type of problem a step further and graph the equation.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- plotting points in all four quadrants (5.G.1)
- understanding slope as a rate of change (8.EE.5)

Introduction

Many relationships can be represented by linear equations. **Linear equations** in two variables can be written in the form $y = mx + b$, where m is the slope and b is the y-intercept. The **slope** of a linear graph is a measure of the rate of change of y with respect to x. The ***y*-intercept** of the equation is the point at which the graph intersects the y-axis, which means that the value of x is 0.

Creating a linear equation in two variables from context follows the same procedure at first for creating an equation in one variable. Once you have created the equation, the equation can be graphed on the coordinate plane. The **coordinate plane** is a plane defined by a set of two number lines, called the axes, that intersect at right angles. The axes allow every point in the plane to be described by an ordered pair of coordinates.

Key Concepts

Reviewing Linear Equations

- The slope of a linear equation is also defined by the ratio of the rise of the graph compared to the run. Given two points on a line, (x_1, y_1) and (x_2, y_2), the slope is the ratio of the change in the y-values of the points (rise) to the change in the corresponding x-values of the points (run).

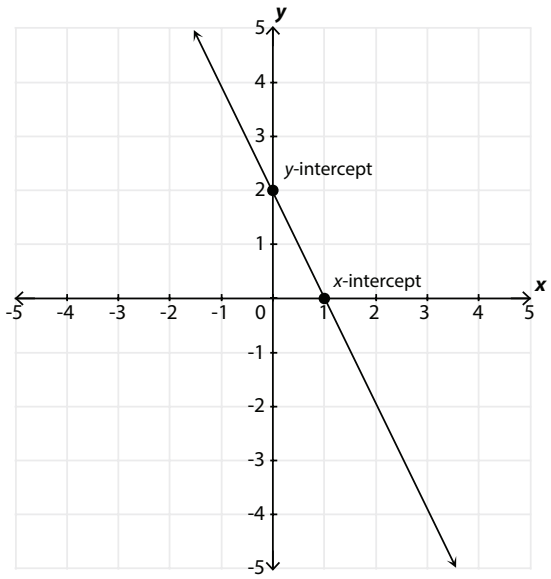
$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$$

- The slope-intercept form of an equation of a line is often used to easily identify the slope and y-intercept, which then can be used to graph the line. The slope-intercept form of an equation is $y = mx + b$, where m represents the slope of the line and b represents the y-value of the point where the line intersects the y-axis. The coordinates of the y-intercept are $(0, y)$.
- Horizontal lines have a slope of 0. They have a run but no rise. Vertical lines have an undefined slope; we cannot assign a numerical value to the slope of a vertical line.
- The ***x*-intercept** of a line is the point where the line intersects the x-axis. The coordinates of the x-intercept are $(x, 0)$.

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Instruction

- If a point lies on a line, its coordinates make the equation of the line true.
- The graph of a line is the collection of all points that satisfy the equation. The graph of the linear equation $y = -2x + 2$ is shown, with its x- and y-intercepts plotted.



Creating Equations

1. Read the problem statement carefully before doing anything.
2. Look for the information given and make a list of the known quantities.
3. Determine which information tells you the rate of change, or the slope, m. Look for words such as *each*, *every*, *per*, or *rate*.
4. Determine which information tells you the y-intercept, or b. This could be an initial value or a starting value, a flat fee, and so forth.
5. Substitute the slope and y-intercept into the linear equation formula, $y = mx + b$.

Determining the Scale and Labels When Graphing

- Adjust the units according to what you need. For example, if the y-intercept is 10,000, each square might represent 2,000 units on the y-axis. Be careful when plotting the slope to take into account the value each grid square represents.

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- Sometimes you need to skip values on the *y*-axis. It makes sense to do this if the *y*-intercept is very large (positive) or very small (negative). For example, if your *y*-intercept is 10,000, you could start your *y*-axis numbering at 0 and "skip" to 10,000 at the next *y*-axis number. Use a short, zigzag line starting at 0 to about the first grid line to show that you've skipped values. Then continue with the correct numbering for the rest of the axis. For an illustration, see Guided Practice Example 3, step 4.
- Only use *x*- and *y*-values that make sense for the context of the problem. Ask yourself if negative values make sense for the *x*-axis and *y*-axis labels in terms of the context. If negative values don't make sense (for example, time and distance can't have negative values), only use positive values.
- Determine the independent and dependent variables.
- The independent variable will generally be labeled on the *x*-axis. The **independent variable** is the quantity for which you choose values.
- The dependent variable will generally be labeled on the *y*-axis. The **dependent variable** is the quantity that is based on the input values of the independent variable.

Graphing Equations Using a Table of Values

Any equation can be graphed using a table of values. For an example, see Guided Practice Example 1, step 7.

1. Choose inputs or values of *x*.
2. Substitute those values in for *x* in the given equation and solve for *y*.
3. The result is an ordered pair (*x*, *y*) that can be plotted on the coordinate plane.
4. Plot at least 3 ordered pairs on the line.
5. Connect the points, making sure that they lie in a straight line.
6. Add arrows to the end(s) of the line to show when the line continues infinitely (if continuing infinitely makes sense in terms of the context of the problem).
7. Label the line with the equation.

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Instruction

Graphing Equations Using the Slope and *y*-intercept

For an example, see Guided Practice Example 2, step 6.

1. Plot the *y*-intercept first. The *y*-intercept will be on the *y*-axis.
2. Recall that slope is $\frac{\text{rise}}{\text{run}}$. Change the slope into a fraction if it is not already a fraction.
3. To graph the rise when the slope is positive, begin at the *y*-intercept and count up by the number of units that corresponds to the rise in the slope. (So, if your slope is $\frac{3}{5}$, count up 3 units on the *y*-axis.)
4. For the run, count over to the right the number of units that corresponds to the run in the slope, and plot the second point. (For the slope $\frac{3}{5}$, beginning where you arrived in step 3, count 5 to the right and plot the point.)
5. To find the rise when the slope is negative, begin at the *y*-intercept and count down the number of units that corresponds to the rise in the slope. For the run, you still count over to the right the number of units that corresponds to the run in the slope, and plot the second point. (For a slope of $-\frac{4}{7}$, begin at the *y*-intercept and count down 4 units and right 7 units, and plot the point.)
6. Connect the points and place arrows at one or both ends of the line when it makes sense to have arrows within the context of the problem.
7. Label the line with the equation.

Graphing Equations Using a TI-83/84:

Step 1: Press [Y=] and key in the equation using [X, T, θ, *n*] for *x*.

Step 2: Press [WINDOW] to change the viewing window, if necessary.

Step 3: Enter in appropriate values for Xmin, Xmax, Xscl, Ymin, Ymax, and Yscl, using the arrow keys to navigate.

Step 4: Press [GRAPH].

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Instruction

Graphing Equations Using a TI-Nspire:

Step 1: Press the home key.

Step 2: Arrow over to the graphing icon (the picture of the parabola or the U-shaped curve) and press [enter].

Step 3: At the blinking cursor at the bottom of the screen, enter in the equation and press [enter].

Step 4: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom, and click the center button of the navigation pad.

Step 5: Choose 1: Window settings by pressing the center button.

Step 6: Enter in the appropriate XMin, XMax, YMin, and YMax fields.

Step 7: Leave the XScale and YScale set to auto.

Step 8: Use [tab] to navigate among the fields.

Step 9: Press [tab] to "OK" when done and press [enter].

Common Errors/Misconceptions

- switching the slope and *y*-intercept when creating the equation from context
- switching the *x*- and *y*-axis labels
- incorrectly graphing the line with the wrong *y*-intercept or the wrong slope

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Instruction

Guided Practice 1.3.1

Example 1

A local convenience store owner spent $10 on 144 pencils to resell at the store. What is the equation for the store's profit if each pencil sells for $0.50? Graph the equation using a table of values.

1. Read the problem and then reread the problem, determining the known quantities.

   Initial cost of pencils: $10

   Charge per pencil: $0.50

2. Identify the slope and the *y*-intercept.

   The slope is a rate. Notice the word "each."

   The slope is 0.50.

   The *y*-intercept is a starting value. The store *paid* $10. The starting profit then is –$10 (and actually represents a loss because it is negative).

   The *y*-intercept is –10.

3. Substitute the slope and *y*-intercept into the equation $y = mx + b$, where m is the slope and b is the *y*-intercept.

   $m = 0.50$

   $b = -10$

   $y = 0.50x - 10$

4. Change the slope into a fraction in preparation for graphing.

   $$0.50 = \frac{50}{100} = \frac{1}{2}$$

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Instruction

5. Rewrite the equation using the fraction.

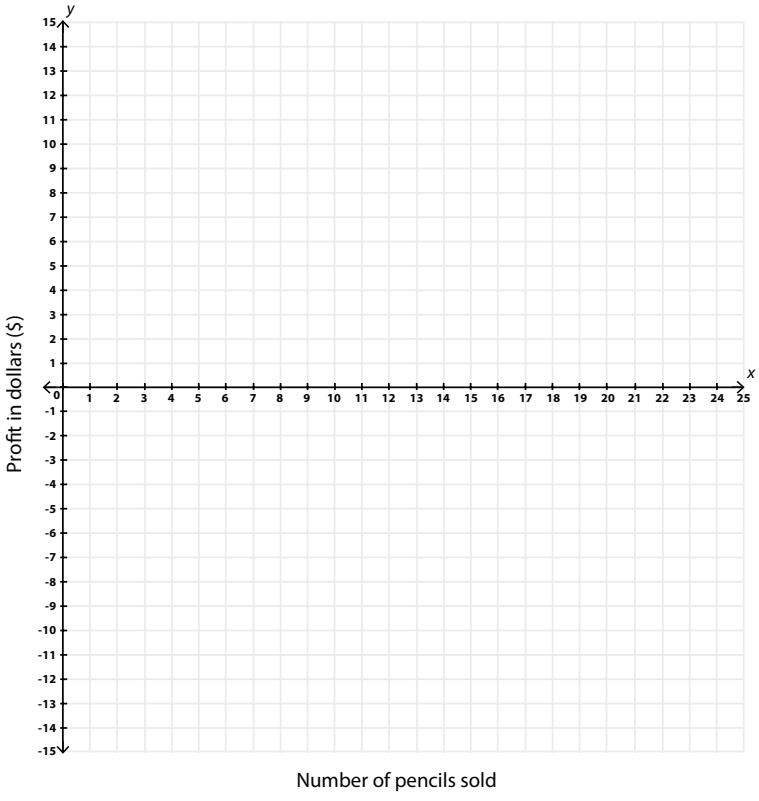
$$y=\frac{1}{2}x-10$$

6. Set up the coordinate plane and identify the independent and dependent variables.

In this scenario, x represents the number of pencils sold and is the independent variable. The x-axis label is "Number of pencils sold."

The dependent variable, y, represents the profit the store will make based on the number of pencils sold. The y-axis label is "Profit in dollars ($)."

Determine the scales to be used. Since the slope's rise and run are within 10 units and the y-intercept is –10 units, a scale of 1 on each axis is appropriate. Label the x-axis from 0 to 25 since you will not sell a negative amount of pencils, and you want to include enough pencils so that you can see the profit become positive. Label the y-axis from –15 to 15, to allow space to plot the $10 the store owner paid for the pencils (–10).



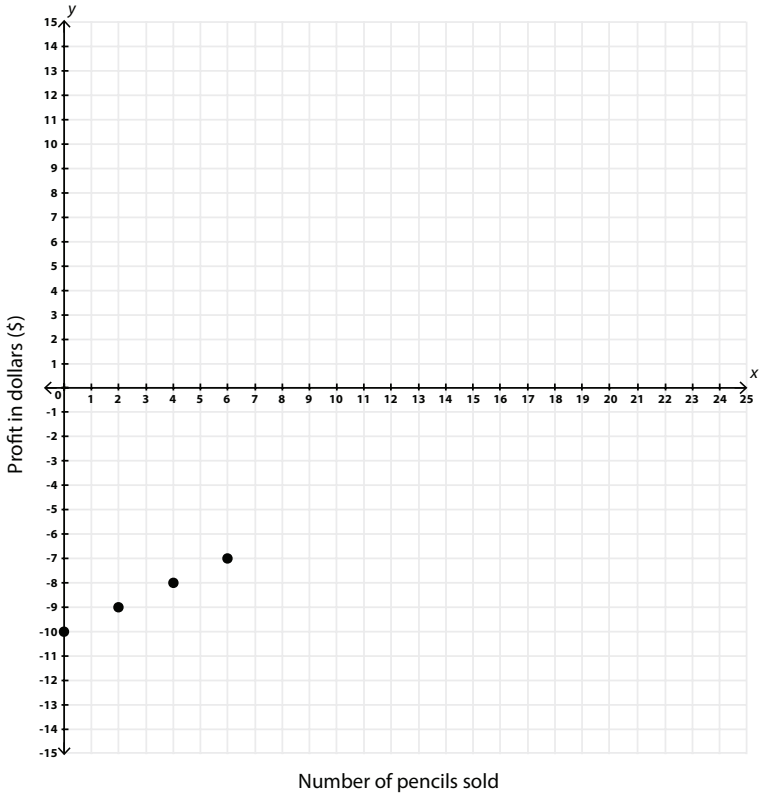
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Instruction

7. Plot points using a table of values.

Substitute x values into the equation $y=\frac{1}{2}x-10$ and solve for y. Choose any positive values of x to substitute. Here, it's easiest to use values of x that are even numbers since after substituting you will be multiplying by $\frac{1}{2}$. Using even-numbered x values will keep the numbers integers after you multiply.

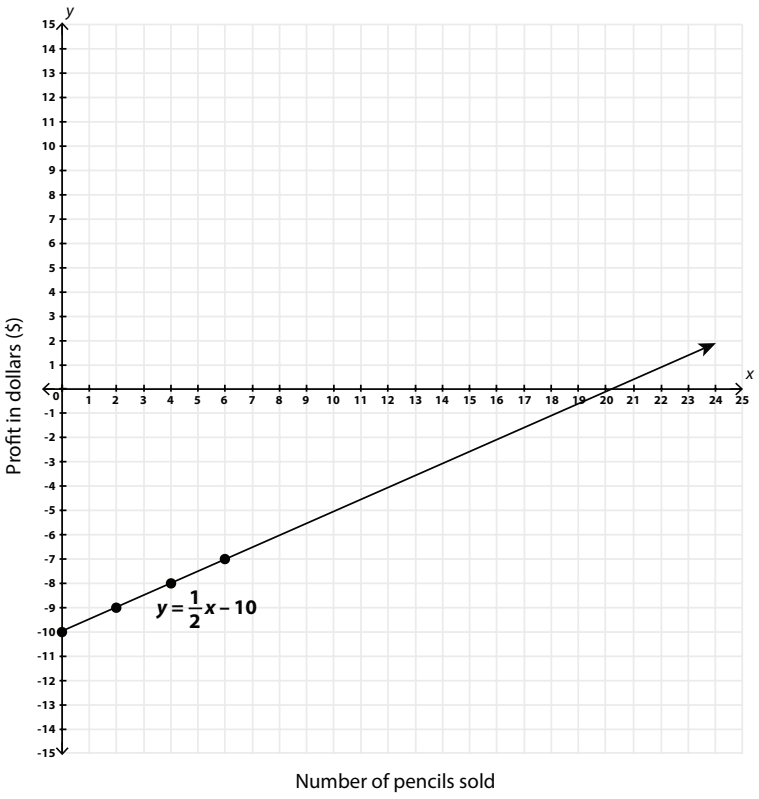
x	y
0	$\frac{1}{2}(0)-10=-10$
2	–9
4	–8
6	–7



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Instruction

8. Connect the points with a line and add an arrow at the right end of the line to show that the line of the equation continues in that direction. Be sure to write the equation of the line next to the line on the graph.



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Instruction

Example 2

A taxi company in Kansas City charges $2.50 per ride plus $2 for every mile driven. Write and graph the equation that models the cost of a taxi ride. Use the slope and the *y*-intercept to draw the graph.

1. Read the problem statement and then reread the problem, determining the known quantities.

   Initial cost of taking a taxi: $2.50

   Charge per mile: $2

2. Identify the slope and the *y*-intercept.

   The slope is a rate. Notice the word "every."

   The slope is 2.

   The *y*-intercept is a starting value. It costs $2.50 initially to hire a taxi driver.

   The *y*-intercept is 2.50.

3. Substitute the slope and *y*-intercept into the equation $y = mx + b$, where m is the slope and b is the *y*-intercept.

   $m = 2$

   $b = 2.50 = 2.5$

   $y = 2x + 2.5$

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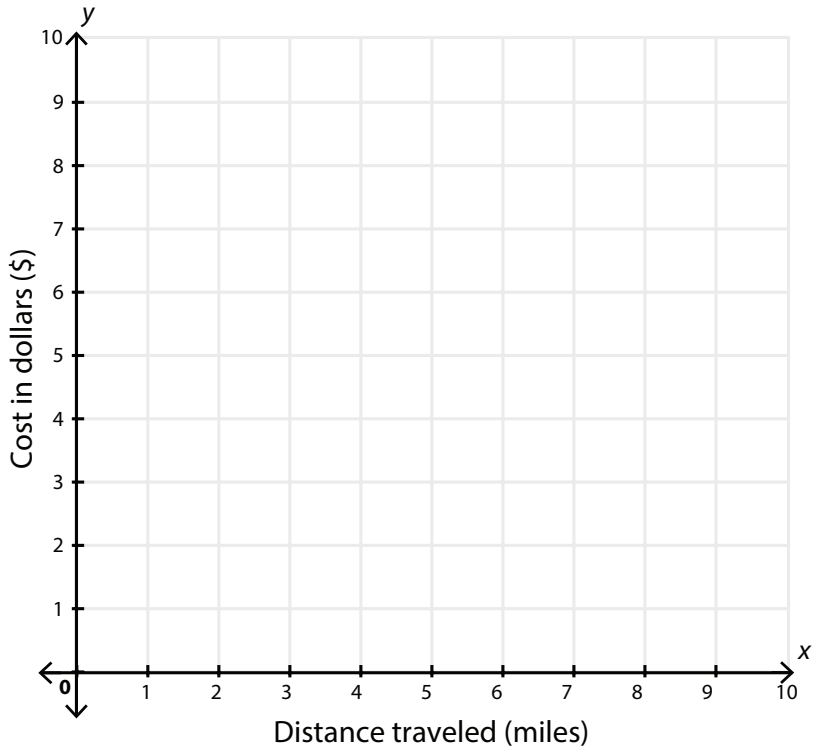
Instruction

4. Set up the coordinate plane.

   In this scenario, x represents the number of miles traveled in the taxi and is the independent variable. The x-axis label is "Distance traveled (miles)."

   The dependent variable, y, represents the cost of taking a taxi based on the number of miles traveled. The y-axis label is "Cost in dollars ($)."

   Determine the scales to be used. Since the slope's rise and run are within 10 units of each other and the y-intercept is within 10 units of 0, a scale of 1 on each axis is appropriate. Label the x-axis from 0 to 10, since miles traveled will only be positive. Label the y-axis from 0 to 10, since cost will only be positive.

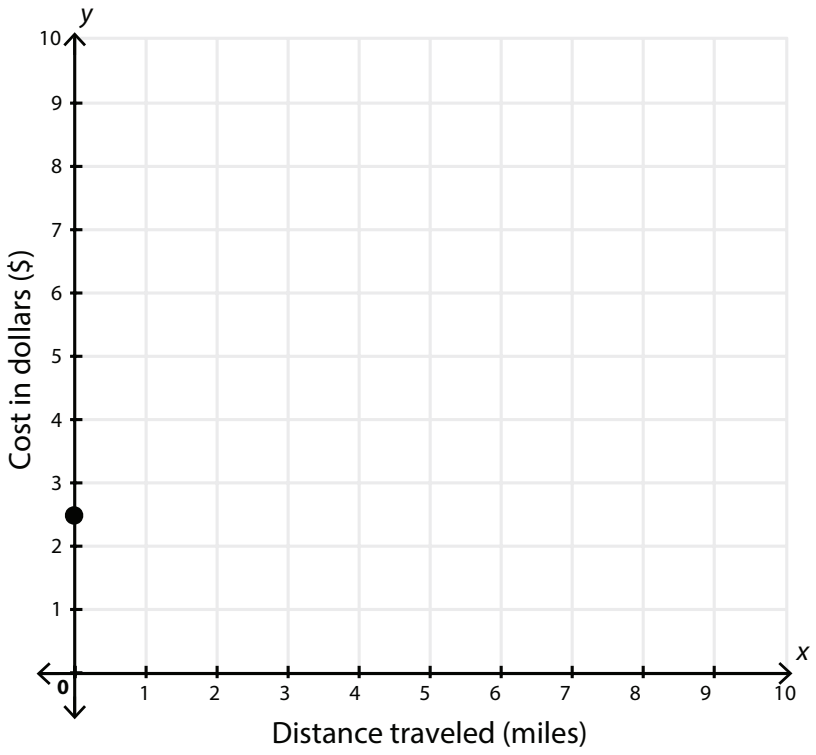


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5. Graph the equation using the slope and *y*-intercept. Plot the *y*-intercept first.

   The *y*-intercept is 2.5. Remember that the *y*-intercept is where the graph intersects the *y*-axis, so the value of *x* is 0. Therefore, the *x*-coordinate of the *y*-intercept will always be 0. In this case, the coordinates of the *y*-intercept are (0, 2.5).

   To plot points that lie in between grid lines, use estimation. Since 2.5 is halfway between 2 and 3, plot the point halfway between 2 and 3 on the *y*-axis. Estimate the halfway point.



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Instruction

6. Graph the equation using the slope and y-intercept. Use the slope to find a second point.

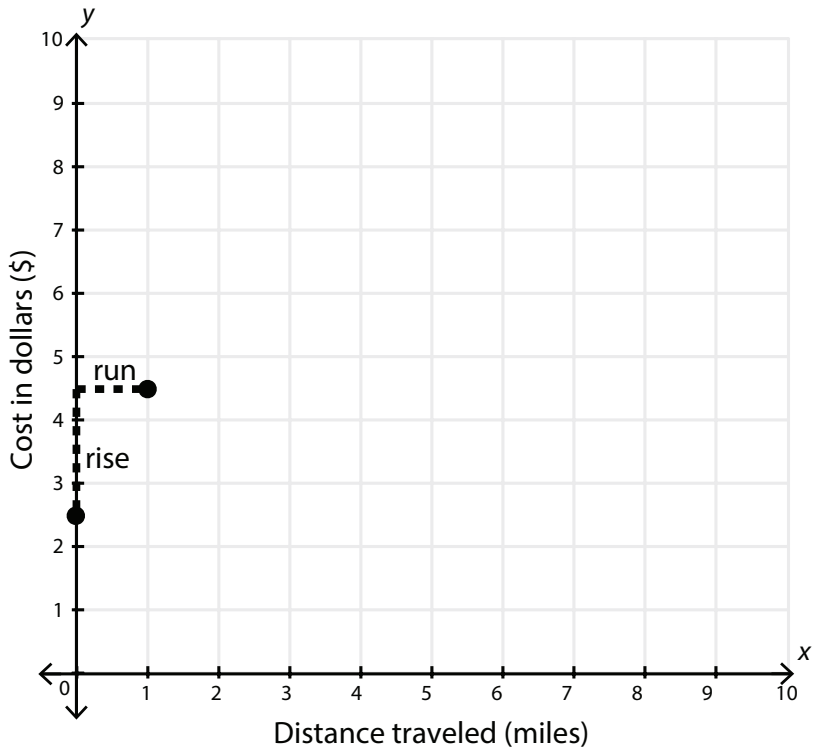
   Remember that the slope is $\frac{\text{rise}}{\text{run}}$. In this case, the slope is 2. Write 2 as a fraction.

   $$2 = \frac{2}{1} = \frac{\text{rise}}{\text{run}}$$

   The rise is 2 and the run is 1.

   Point your pencil at the y-intercept. Move the pencil up 2 units, since the slope is positive. Remember that the y-intercept was halfway between grid lines. Be sure that you move your pencil up 2 complete units by first going to halfway between 3 and 4 (3.5) and then halfway between 4 and 5 (4.5) on the y-axis.

   Now, move your pencil to the right 1 unit for the run and plot a point. This is a second point on the graph.

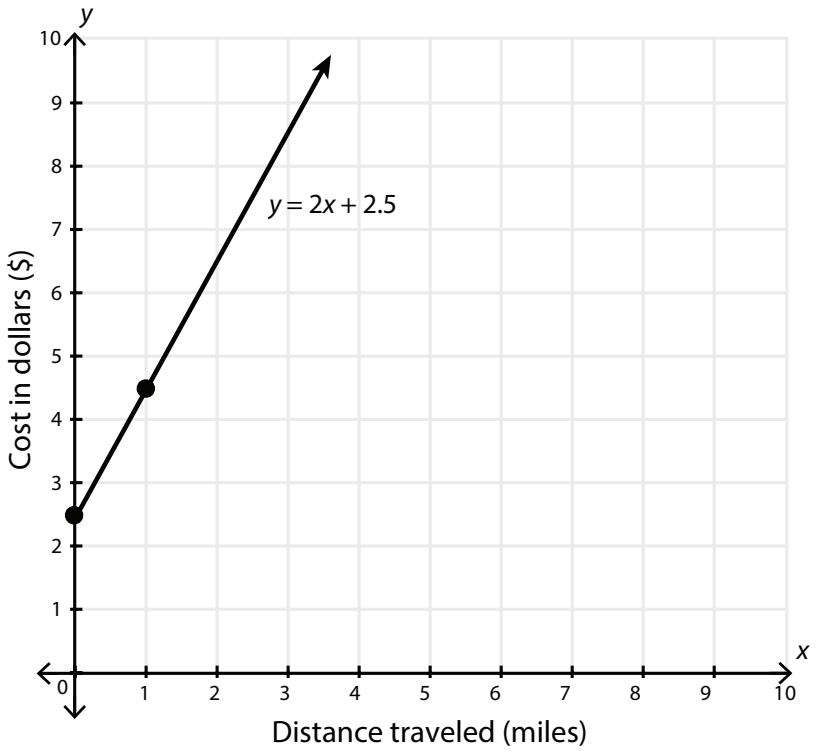


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Instruction

7. Connect the points and extend the line. Then, label your line.

   Draw a line through the two points and add an arrow to the right end of the line to show that the line of the equation continues infinitely in that direction. Label the line with the equation, $y = 2x + 2.5$.



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Instruction

Example 3

Miranda gets paid $300 each week to deliver groceries. She also earns 5% commission on the total cost of each order she delivers. Write an equation that represents her weekly pay and then graph the equation.

1. Read the problem statement and then reread the problem, determining the known quantities.

   Weekly payment: $300

   Commission: 5% = 0.05

2. Identify the slope and the y-intercept.

   The slope is a rate. Notice the symbol "%," which means *percent*, or *per 100*.

   The slope is 0.05.

   The y-intercept is a starting value. She gets paid $300 a week to start with before taking any orders.

   The y-intercept is 300.

3. Substitute the slope and y-intercept into the equation $y = mx + b$, where m is the slope and b is the y-intercept.

   $m = 0.05$

   $b = 300$

   $y = 0.05x + 300$

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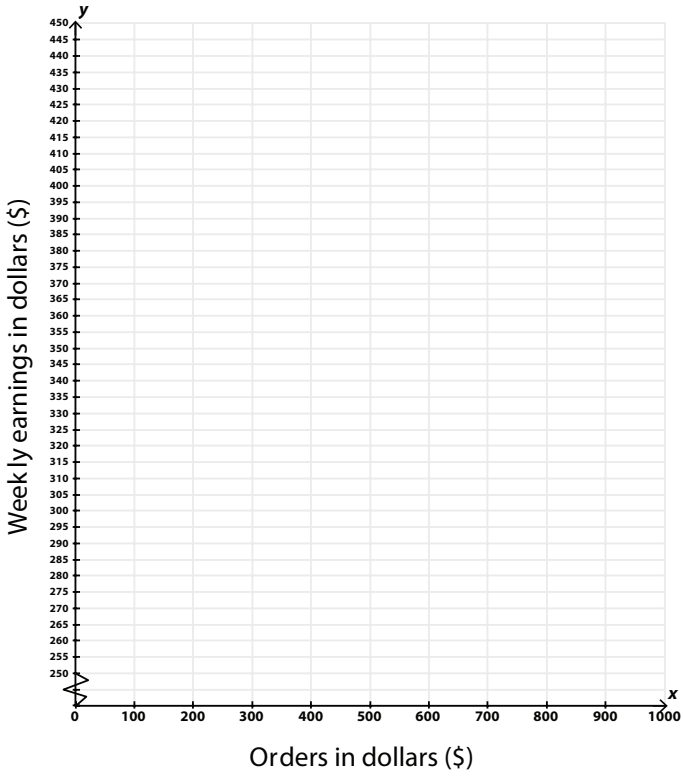
Instruction

4. Set up the coordinate plane.

   In this scenario, x represents the total value of the grocery orders. The x-axis label is "Orders in dollars ($)."

   The dependent variable, y, represents her total earnings in a week. The y-axis label is "Weekly earnings in dollars ($)."

   Determine the scales to be used. The y-intercept is in the hundreds and the slope is in decimals. Work with the slope first. The slope is 0.05 or $\frac{5}{100}$. The rise is a small number, but the run is large. The run is represented by values on the x-axis, so that should be in increments of 100. Start at 0 since the order amounts will be positive and continue to 1,000. The rise is represented by values on the y-axis and is small, but remember that the y-intercept is \$300. Since there's such a large gap before the y-intercept, you can skip values on the y-axis so the graph doesn't become too large. Start the y-axis at 0, then skip to 250 and label the rest of the axis in increments of 5 until you reach 450. Use a zigzag line to show you skipped values between 0 and 250.

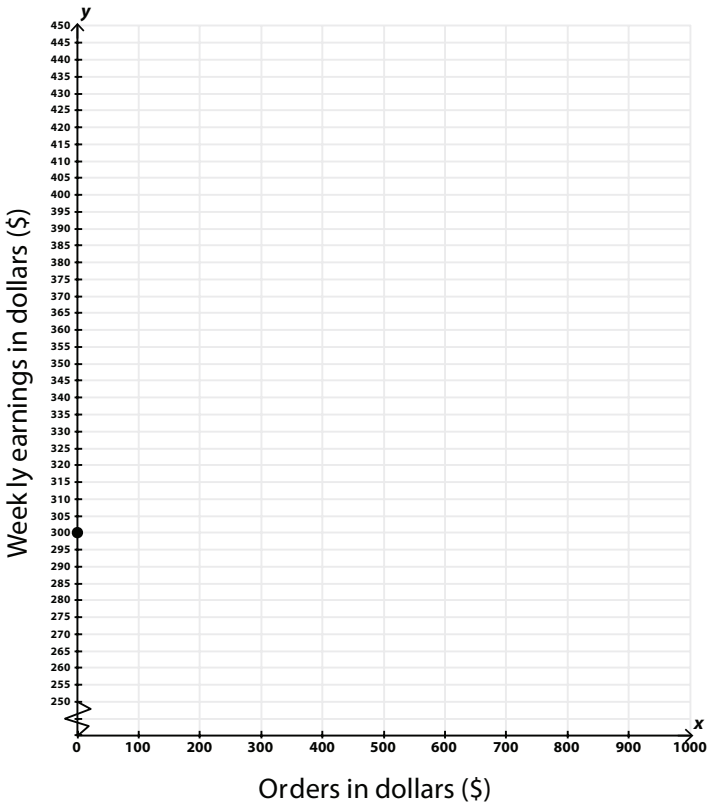


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Instruction

5. Graph the equation using the slope and y-intercept. Plot the y-intercept first.

   The y-intercept is 300. Remember that the y-intercept is where the graph intersects the y-axis, so the value of x is 0. Therefore, the x-coordinate of the y-intercept will always be 0. In this case, the coordinates of the y-intercept are (0, 300).



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6. Graph the equation using the slope and *y*-intercept. Use the slope to find a second point.

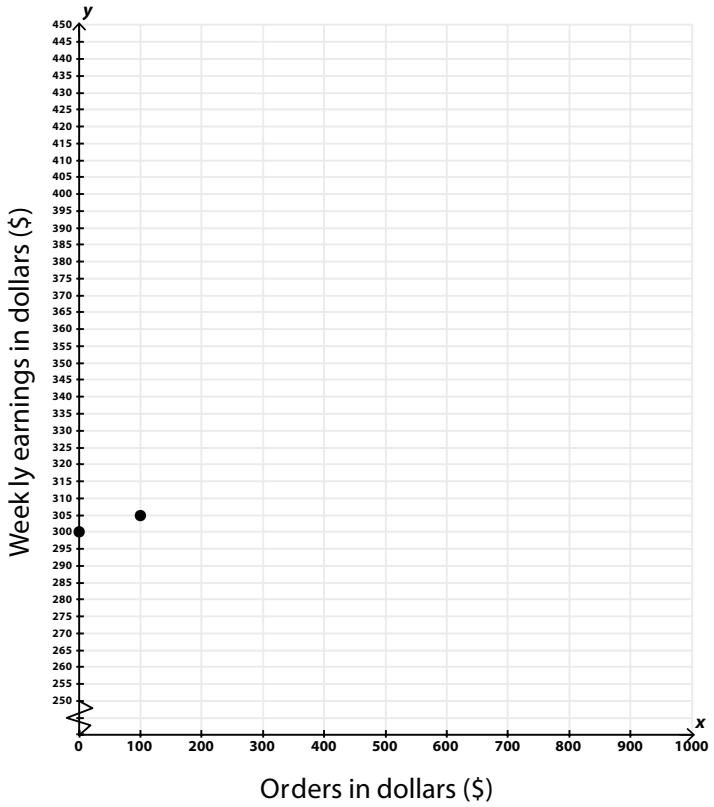
   Remember that the slope is $\frac{\text{rise}}{\text{run}}$. In this case, the slope is 0.05. Rewrite 0.05 as a fraction.

   $$0.05 = \frac{5}{100} = \frac{\text{rise}}{\text{run}}$$

   The rise is 5 and the run is 100.

   Place your pencil on the *y*-intercept. Move the pencil up 5 units, since the slope is positive. On this grid, 5 units is one tick mark.

   Now, move your pencil to the right 100 units for the run and plot a point. On this grid, 100 units to the right is one tick mark. This is a second point on the graph.

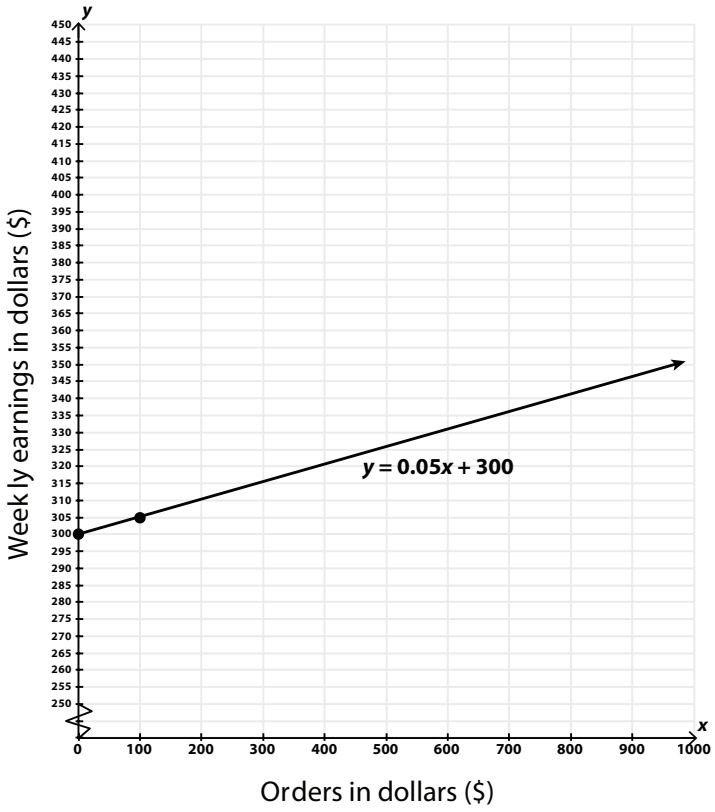


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Instruction

7. Connect the points and extend the line. Then, label your line.

   Draw a line through the two points and add an arrow to the right end of the line to show that the line continues infinitely in that direction. Label your line with the equation, $y = 0.05x + 300$.



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Instruction

Example 4

The velocity of a ball thrown directly upward can be modeled with the following equation: $v = -gt + v_0$, where v is the velocity, g is the acceleration due to gravity, t is the elapsed time, and v_0 is the initial velocity at time 0. If the acceleration due to gravity is equal to 32 feet per second per second, and the initial velocity of the ball is 96 feet per second, what is the equation that represents the velocity of the ball? Graph the equation.

1. Read the problem statement and then reread the problem, determining the known quantities.

   Initial velocity: 96 ft/s

   Acceleration due to gravity: 32 ft/s^2

   Notice that in the given equation, the acceleration due to gravity is negative. This is due to gravity acting on the ball, pulling it back to Earth and slowing the ball down from its initial velocity.

2. Identify the slope and the y-intercept.

   Notice the form of the given equation for velocity is the same form as $y = mx + b$, where $y = v$, $m = -g$, $x = t$, and $b = v_0$. Therefore, the slope is -32 and the y-intercept is 96.

3. Substitute the slope and y-intercept into the equation $y = mx + b$, where m is the slope and b is the y-intercept.

   $m = -g = -32$

   $b = v_0 = 96$

   $y = -32x + 96$

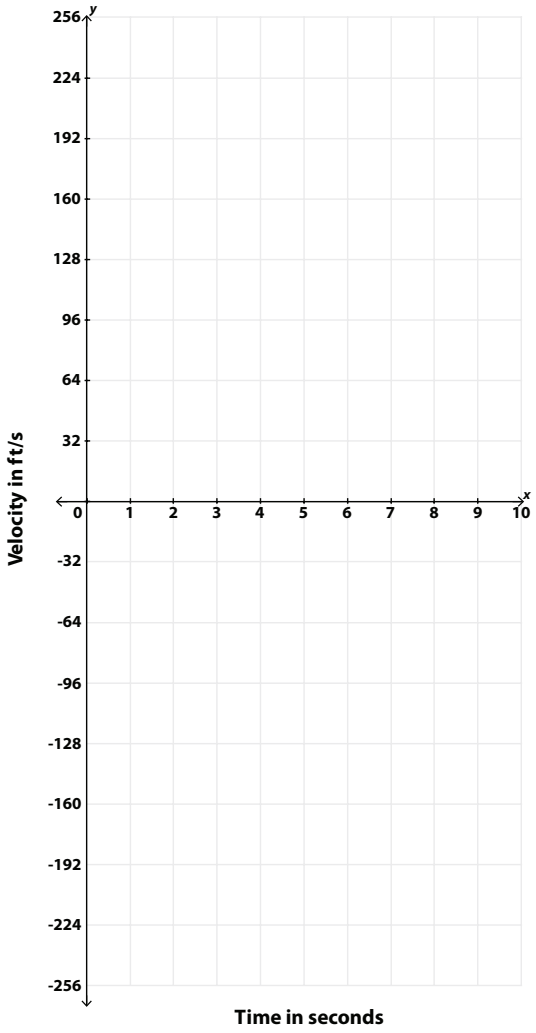
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4. Set up the coordinate plane.

   In this scenario, x represents the number of seconds after the ball was tossed upward. The x-axis label is "Time in seconds."

   The dependent variable, y, represents the velocity of the ball. The y-axis label is "Velocity in ft/s."

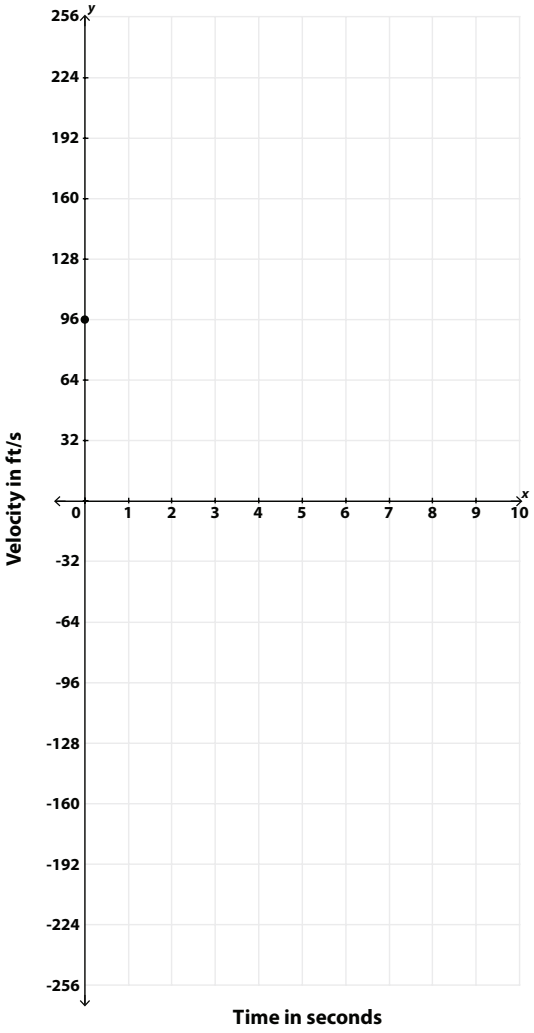
   Determine the scales to be used. The y-intercept is close to 100 and the slope is –32. Notice that 96 (the y-intercept) is a multiple of 32. The y-axis can be labeled in units of 32 to make the equation easier to graph. Since the x-axis is in seconds, it makes sense that these units are in increments of 1. Because negative x-values would indicate time before the ball was tossed, use only a positive scale for the x-axis.



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5. Graph the equation using the slope and *y*-intercept. Plot the *y*-intercept first.

   The *y*-intercept is 96. Remember that the *y*-intercept is where the graph intersects the *y*-axis, so the value of *x* is 0. Therefore, the *x*-coordinate of the *y*-intercept will always be 0. In this case, the coordinates of the *y*-intercept are (0, 96).



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6. Graph the equation using the slope and *y*-intercept. Use the slope to find the second point.

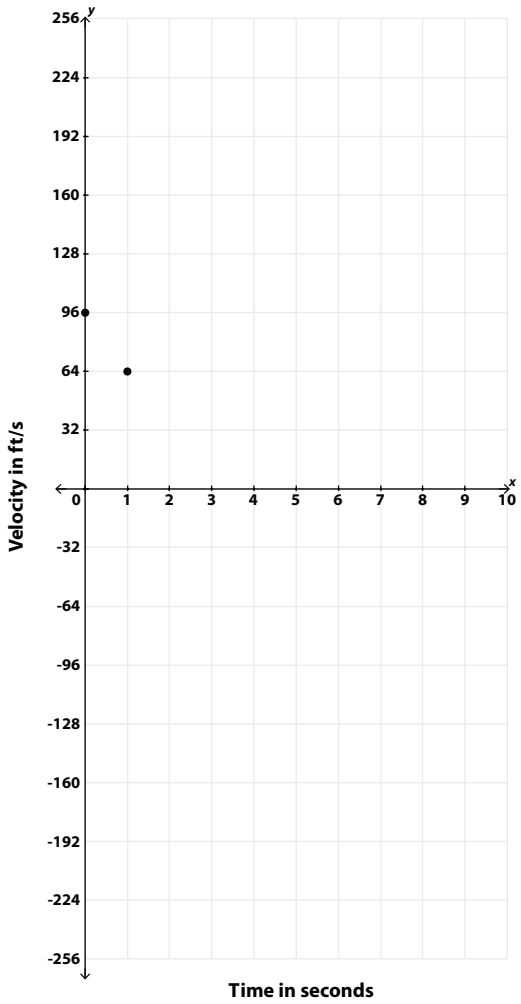
   Remember that the slope is $\frac{\text{rise}}{\text{run}}$. In this case, the slope is –32. Rewrite –32 as a fraction.

   $$-32 = \frac{-32}{1} = \frac{\text{rise}}{\text{run}}$$

   The rise is –32 and the run is 1.

   Place your pencil on the *y*-intercept. Move the pencil down 32 units, since the slope is negative. On this grid, 32 units is one tick mark.

   Now, move your pencil to the right 1 unit for the run and plot a point. This is a second point on the graph.

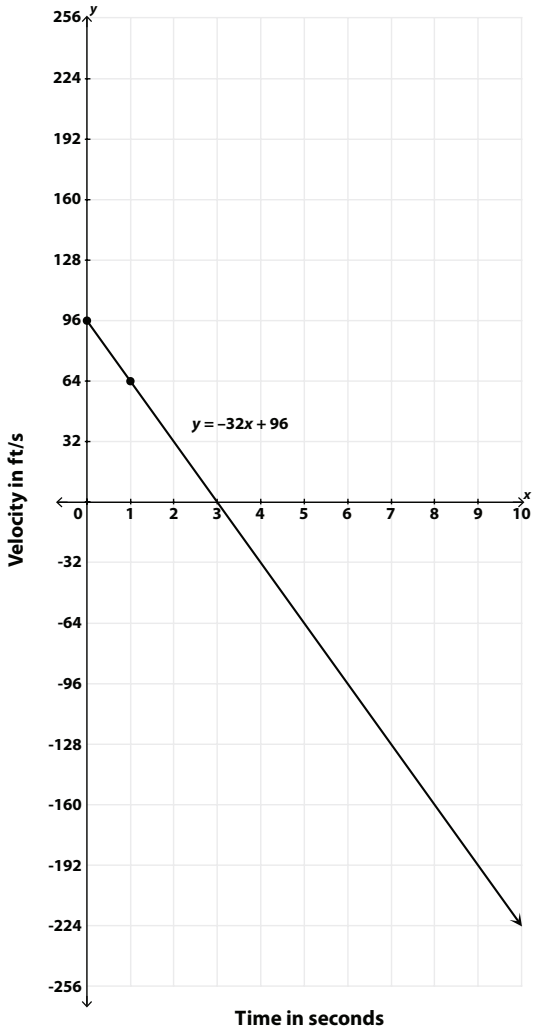


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Instruction

7. Connect the points and extend the line toward the right. Then, label your line.

   Draw a line through the two points and add an arrow to the right end of the line to show that the line of the equation continues in that direction. Label your line with the equation $y = -32x + 96$.



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Instruction

Example 5

A Boeing 747 starts out a long flight with about 57,260 gallons of fuel in its tank. The airplane uses an average of 5 gallons of fuel per mile. Write an equation that models the amount of fuel remaining in the tank over the course of the flight. Graph the equation using a graphing calculator, and then draw the resulting graph on graph paper.

1. Read the problem statement and then reread the problem, determining the known quantities.

   Starting fuel tank amount: 57,260 gallons

   Rate of fuel consumption: 5 gallons per mile

2. Identify the slope and the y-intercept.

   The slope is a rate. Notice the word "per" in the phrase "5 gallons of fuel per mile." Since the total number of gallons left in the fuel tank is decreasing at this rate, the slope is negative.

   The slope is –5.

   The y-intercept is a starting value. The airplane starts out with 57,260 gallons of fuel.

   The y-intercept is 57,260.

   Substitute the slope and y-intercept into the equation $y = mx + b$, where m is the slope and b is the y-intercept.

   $m = -5$

   $b = 57{,}260$

   $y = -5x + 57{,}260$

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Instruction

3. Graph the equation on your calculator.

On a TI-83/84:

Step 1: Press [Y=].

Step 2: At Y_1, type in [(–)][5][X, T, θ, *n*][+][57260].

Step 3: Press [WINDOW] to change the viewing window.

Step 4: At Xmin, enter [0] and arrow down one level to Xmax.

Step 5: At Xmax, enter [3000] and arrow down one level to Xscl.

Step 6: At Xscl, enter [100] and arrow down one level to Ymin.

Step 7: At Ymin, enter [40000] and arrow down one level to Ymax.

Step 8: At Ymax, enter [58000] and arrow down one level to Yscl.

Step 9: At Yscl, enter [1000].

Step 10: Press [GRAPH].

On a TI-Nspire:

Step 1: Press the [home] key.

Step 2: Arrow over to the graphing icon and press [enter].

Step 3: At the blinking cursor at the bottom of the screen, enter in the equation [(–)][5][x][+][57260] and press [enter].

Step 4: Change the viewing window by pressing [menu], arrowing down to number 4: Window/Zoom, and clicking the center button of the navigation pad.

Step 5: Choose 1: Window settings by pressing the center button.

Step 6: Enter in the appropriate XMin value, [0], then press [tab].

Step 7: Enter in the appropriate XMax value, [3000], then press [tab].

Step 8: Leave the XScale set to "Auto." Press [tab] twice to navigate to YMin and enter [40000].

Step 9: Press [tab] to navigate to YMax. Enter [58000]. Press [tab] twice to leave YScale set to "auto" and to navigate to "OK."

Step 10: Press [enter].

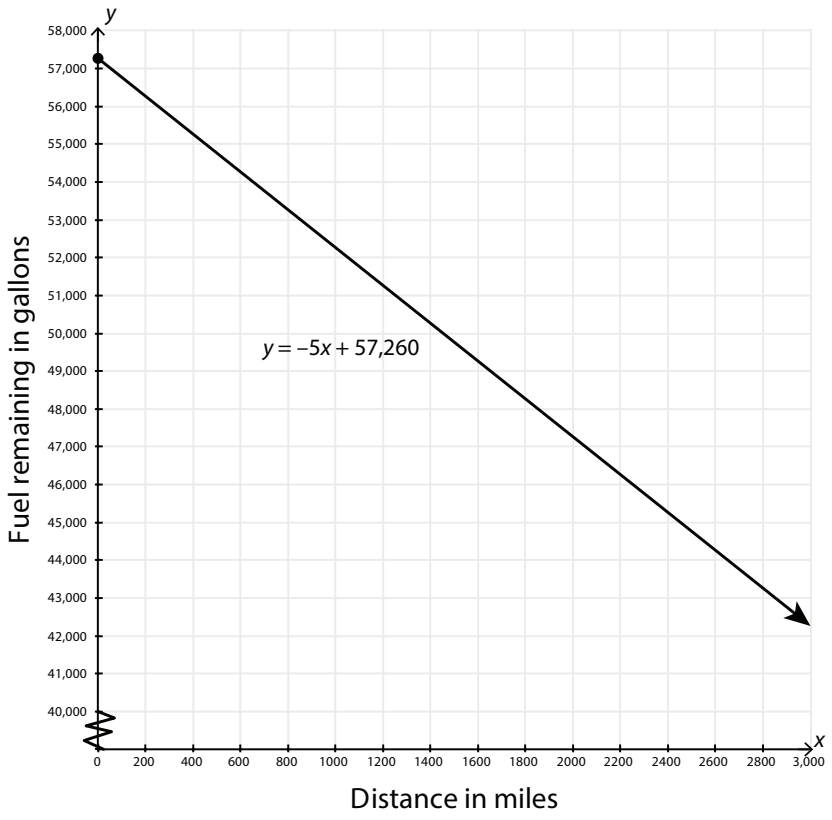
Step 11: Press [menu] and select 2: View and 5: Show Grid.

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Instruction

4. Draw the resulting graph on graph paper.

   On the TI-83/84, the scale was entered in [WINDOW] settings. The X scale was 100 and the Y scale was 1,000. Set up the graph paper using these scales. Label the *y*-axis "Fuel remaining in gallons." Show a break in the graph from 0 to 40,000 using a zigzag line. Label the *x*-axis "Distance in miles." To show the table on the calculator so you can plot points, press [2nd][GRAPH]. The table shows two columns with values; the first column holds the *x*-values, and the second column holds the *y*-values. Pick a pair to plot, and then draw the line that passes through this point and the *y*-intercept. Remember to label the line with the equation. (*Note*: It may take you a few tries to get the window settings the way you want. The graph that follows shows an X scale of 200 so that you can see a larger portion of the graphed line.)



(*continued*)

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Instruction

If you used a TI-Nspire calculator, determine the scale that was used by counting the dots on the grid from your minimum *y*-value to your maximum *y*-value. In this case, there are 18 dots vertically between 40,000 and 58,000. The difference between the YMax and YMin values is 18,000. Divide that by the number of dots (18). The result (1,000) is the scale.

$$\frac{\text{Y Max} - \text{Y Min}}{\text{Number of dots}} = \frac{58{,}000 - 40{,}000}{18} = \frac{18{,}000}{18} = 1000$$

This means each dot represents 1,000 units vertically. Label the *y*-axis "Fuel remaining in gallons." Use a zigzag line to show a break in the graph from 0 to 40,000.

Repeat the same process for determining the *x*-axis scale, using 0 for XMin, 3,000 for XMax, and 30 for the number of dots.

$$\frac{\text{X Max} - \text{X Min}}{\text{Number of dots}} = \frac{3000 - 0}{30} = \frac{3000}{30} = 100$$

This means each dot represents 100 units horizontally.

Set up your graph paper accordingly. Label the *x*-axis "Distance in miles."

On your calculator, look at the table in order to plot points. To show the table, press [tab][T]. To navigate within the table, use the navigation pad. The table shows two columns with values; the first column holds the *x*-values, and the second column holds the *y*-values. Pick a pair to plot and then draw the line that passes through this point and the *y*-intercept. Remember to label the line with the equation. To hide the table, navigate back to the graph by pressing [ctrl][tab], then press [ctrl][T].

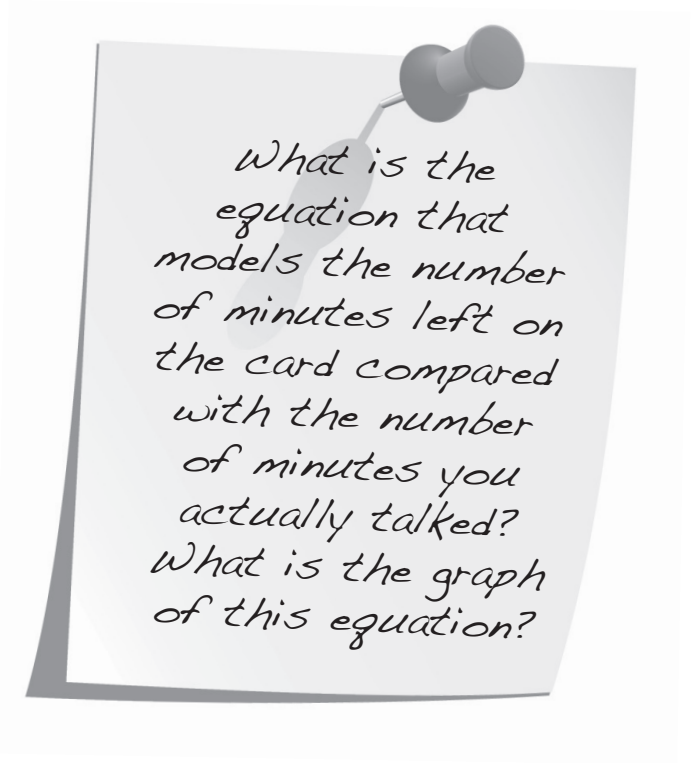
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Problem-Based Task 1.3.1: Phone Card Fine Print

SMP	
1	2 ✓
3 ✓	4 ✓
5	6
7	8

Write and graph the equation that models the following scenario.

You can buy a 6-hour phone card for $5, but the fine print says that each minute you talk actually costs you 1.5 minutes of time. What is the equation that models the number of minutes left on the card compared with the number of minutes you actually talked? What is the graph of this equation?



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Problem-Based Task 1.3.1: Phone Card Fine Print

Coaching

a. What are the slope and the *y*-intercept?

b. What is the equation of the line?

c. What are the labels of the *x*- and *y*-axes?

d. What are the scales of the *x*- and *y*-axes?

e. Which point do you plot first?

f. How can you use the equation to plot a second point on the line?

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Instruction

Problem-Based Task 1.3.1: Phone Card Fine Print

Coaching Sample Responses

a. What are the slope and the *y*-intercept?

The slope is the rate. Notice the word "each" in the phrase "each minute you talk actually costs you 1.5 minutes of time." Therefore, the rate at which the time on the card is *decreasing* is 1.5 minutes. The slope is –1.5.

$m = -1.5$

The *y*-intercept is 6 hours. That's the amount of time the card started with, in hours, but the rate at which the card's balance is decreasing is given in minutes. You need to convert hours into minutes.

1 hour = 60 minutes

$$6 \text{ hours} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} = 360 \text{ minutes}$$

$b = 360$

b. What is the equation of the line?

$y = -1.5x + 360$

c. What are the labels of the *x*- and *y*-axes?

The *x*-axis label is "Minutes used" and the *y*-axis label is "Minutes left."

d. What are the scales of the *x*- and *y*-axes?

Since the minutes on the card are in the hundreds and the slope's rise and run are in the single digits, it is best to keep the scale size the same on both axes so that you can easily use the slope to plot the points. Choose the scale on the *y*-axis first. The *y*-intercept occurs at 360. Choose a scale that starts at 0 and continues to 380 in increments of 20. This way, the *y*-intercept will be easy to plot.

For the *x*-axis, since the rate of decreasing minutes is faster than 1, the scale doesn't need to be as long. Start at 0 and continue to 300, again in increments of 20. This will let you count the rise and the run using the grid marks to plot the second point.

e. Which point do you plot first?

Plot the *y*-intercept first. The *y*-intercept is 360.

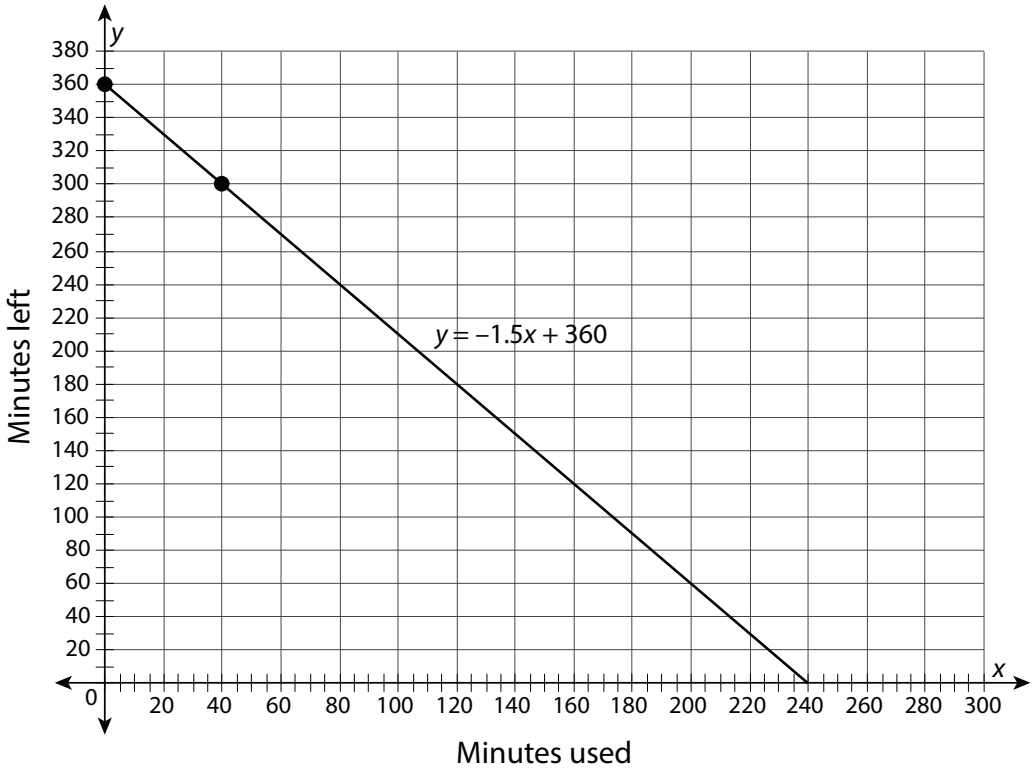
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f. How can you use the equation to plot a second point on the line?

Rewrite the slope as a fraction.

$$-1.5 = \frac{-3}{2} = \frac{\text{rise}}{\text{run}}$$

Since the scales are the same on the *x*- and *y*-axes, you can count the tick marks for the slope. Beginning at the *y*-intercept, count down 3 units and to the right 2 units, then plot this point. Draw the line segment that passes through the *y*-intercept and this point and that extends from the *y*-axis to the *x*-axis. The completed graph is shown.



Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 1.3.1: Creating and Graphing Linear Equations in Two Variables

For problems 1 and 2, graph each equation on graph paper.

1. $y = x + 2$
2. $y = \frac{1}{3}x + 2$

For problems 3–10, use the given information to write an equation, then graph the equation on graph paper.

3. A gear on a machine turns at a rate of 2 revolutions per second. Let x represent time in seconds and let y represent the number of revolutions. What is the equation that models the number of revolutions over time?

4. The relationship between degrees Celsius and degrees Fahrenheit is linear. To convert a temperature from degrees Celsius to degrees Fahrenheit, multiply the temperature by a rate of $\frac{9}{5}$ and add 32. What is the equation that models the conversion from degrees Celsius to degrees Fahrenheit?

5. A cab company charges an initial rate of $2.50 for a ride, plus $0.40 for each mile driven. What is the equation that models the total fee for using this cab company?

6. Matthew receives a base weekly salary of $300 plus a commission of $50 for each vacuum he sells. What is the equation that models his weekly earnings?

7. A water company charges a monthly fee of $6.70 plus a usage fee of $2.60 per 1,000 gallons used. What is the equation that models the water company's total fees?

8. Maddie borrowed $1,250 from a friend to buy a new TV. Her friend doesn't charge any interest, and Maddie makes $40 payments each month. What is the equation that models the money Maddie owes?

9. A company started with 3 employees and after 8 months grew to 19. The growth was steady. What is the equation that models the growth of the company's employees?

10. You and some friends are hiking the Appalachian Trail. You started out with 70 pounds of food for the group, and the group eats about 8 pounds of food each day. What is the equation that models the food you have left?

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Lesson 1.3.2: Creating and Graphing Exponential Equations

Warm-Up 1.3.2

Read the scenario and answer the questions that follow.

> One form of the element beryllium, beryllium-11, has a half-life of about 14 seconds and decays to the element boron. A chemist starts out with 128 grams of beryllium-11. She monitors the element for 70 seconds.

1. What is the equation that models the amount of beryllium-11 over time?

2. How many grams of beryllium-11 does the chemist have left after 70 seconds?

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Instruction

Lesson 1.3.2: Creating and Graphing Exponential Equations

Common Core State Standards

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

N–Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

Warm-Up 1.3.2 Debrief

1. What is the equation that models the amount of beryllium-11 over time?

   The equation is $y = ab^x$, where y is the final value, a is the initial value, b is how much of the substance remains per time period, and x is the number of time periods.

   y = unknown

   a = 128 grams

   b = 0.5

   The time is 70 seconds, but this must be converted to the number of time periods in order to find the correct value to substitute for x. Therefore, convert 70 seconds into 14-second time periods. 1 time period = 14 seconds.

   $$70 \ \cancel{\text{seconds}} \cdot \frac{1 \text{ time period}}{14 \ \cancel{\text{seconds}}} = 5 \text{ time periods}$$

   $x = 5$

   Substitute all the known values into the equation.

   $y = ab^x$

   $y = (128)(0.5)^{(5)}$

   The equation that models the amount of beryllium-11 over time is $y = 128(0.5)^5$.

2. How many grams of beryllium-11 does the chemist have left after 70 seconds?

   Apply the order of operations to evaluate the equation from the end of problem 1.

   $y = 128(0.5)^5 = 4$

   There are 4 grams of beryllium-11 left after 70 seconds.

Connection to the Lesson

- Students will create exponential equations.
- Students will deepen their understanding of exponential equations by graphing their solution sets.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- plotting points in all four quadrants (5.G.1)
- applying the order of operations (5.OA.1)

Introduction

Exponential equations in two variables are similar to linear equations in two variables in that there are an infinite number of solutions. The equation expresses the relationship between the values of the two variables. **Exponential equations** are equations that have a variable in the exponent. This means the *y*-values of exponential equations are going to increase or decrease very quickly.

Key Concepts

Reviewing Exponential Equations

- The general form of an exponential equation is $y = a \bullet b^x$, where a is the initial value, b is the base, and x is the exponent. The final output value will be y.
- Since the equation has an exponent, the *y*-value increases or decreases rapidly.
- The base, b, must always be greater than 0 ($b > 0$).
- If the base is greater than 1 ($b > 1$), then the exponential equation represents exponential growth.
- If the base is between 0 and 1 exclusive (that is, $0 < b < 1$), then the exponential equation represents exponential decay.
- If the time is given in units other than 1 (e.g., 1 month, 1 hour, 1 minute, 1 second), use the equation $y = ab^{\frac{x}{t}}$, where t is the interval over which y changes by a factor of b, and x is the interval under consideration, measured in the same units as t. For example, if an initial quantity of 100 doubles every 3 months, the equation would be $y = 100(2)^{\frac{x}{3}}$.
- Another formula for exponential growth or decay is $y = a(1 \pm r)^t$, where a is the initial value, $(1 \pm r)$ is the base, t is the variable exponent, and y is the final value. If $r > 0$, the formula represents growth, and if $0 < r < 1$, the formula represents decay.

SRB p. U1-84

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Instruction

Introducing the Compound Interest Formula

- The general form of the compounding interest formula is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A is the initial value, r is the annual interest rate, n is the number of times the investment is compounded in a year, and t is the number of years the investment is left in the account to grow.
- Use this chart for reference:

Compounded…	*n* (number of times per year)
Yearly/annually	1
Semiannually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

- Remember to change the percentage rate into a decimal by dividing the percentage by 100.
- Apply the order of operations and divide r by n, then add 1. Raise that value to the power of the product nt. Multiply that value by the principal, P.

Graphing Exponential Equations Using a Table of Values

1. Create a table of values by choosing x-values and substituting them into the given equation, then solving for y.
2. Determine the labels by reading the context. The x-axis will most likely be time and the y-axis will be the units of whatever y represents in the problem.
3. Determine the scales. The scale on the y-axis must be large since the values will grow or decline quickly. The scale on the x-axis must be large enough to show the resulting growth or decay.

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Instruction

Graphing Equations Using a TI-83/84:

Step 1: Press [Y=] and key in the equation using [^] for the exponent and [X, T, θ, *n*] for *x*.

Step 2: Press [WINDOW] to change the viewing window, if necessary.

Step 3: Enter in appropriate values for Xmin, Xmax, Xscl, Ymin, Ymax, and Yscl, using the arrow keys to navigate.

Step 4: Press [GRAPH].

Graphing Equations Using a TI-Nspire:

Step 1: Press the home key.

Step 2: Arrow over to the graphing icon (the picture of the parabola or the U-shaped curve) and press [enter].

Step 3: At the blinking cursor at the bottom of the screen, enter in the equation using [^] before entering the exponents, and press [enter].

Step 4: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom and click the center button of the navigation pad.

Step 5: Choose 1: Window settings by pressing the center button.

Step 6: Enter in the appropriate XMin, Xmax, YMin, and YMax fields.

Step 7: Leave the XScale and YScale set to auto.

Step 8: Use [tab] to navigate among the fields.

Step 9: Press [tab] to "OK" when done and press [enter].

Common Errors/Misconceptions

- incorrectly applying the order of operations: multiplying *a* and *b* before raising *b* to the exponent in the equation $y = ab^x$
- incorrectly identifying the rate—forgetting to add 1 or subtract from 1
- using the model for exponential growth instead of exponential decay
- forgetting to calculate the number of time periods it takes for a given rate of growth or decay to occur, and simply substituting in the time given

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Instruction

Guided Practice 1.3.2

Example 1

If a pendulum swings to 90% of its previous height on each swing and starts out at a height of 60 cm, what is the equation that models this scenario? What is its graph?

1. Read the problem statement and then reread the scenario, identifying the known quantities.

   The initial height is 60 cm.

   The percent remaining after one time period is 90% or 0.90.

2. Substitute the known quantities into the general form of the exponential equation $y = ab^x$.

   In this formula, a is the initial value; b is the amount remaining after each time period, expressed as a decimal; x is the number of intervals (in this case, the number of swings); and y is the final value.

   Let $a = 60$ and $b = 0.90$.

$y = ab^x$	Equation
$y = (60)(0.90)^x$	Substitute 60 for a and 0.90 for b.
$y = 60(0.90)^x$	Simplify.

   The equation $y = 60(0.90)^x$ models the height of the swing.

3. Create a table of values using the resulting equation.

x	y
0	60
1	54
2	48.6
3	43.74
5	35.43
10	20.92
20	7.29
40	0.89

SWB p. 71; SRB p. U1-87

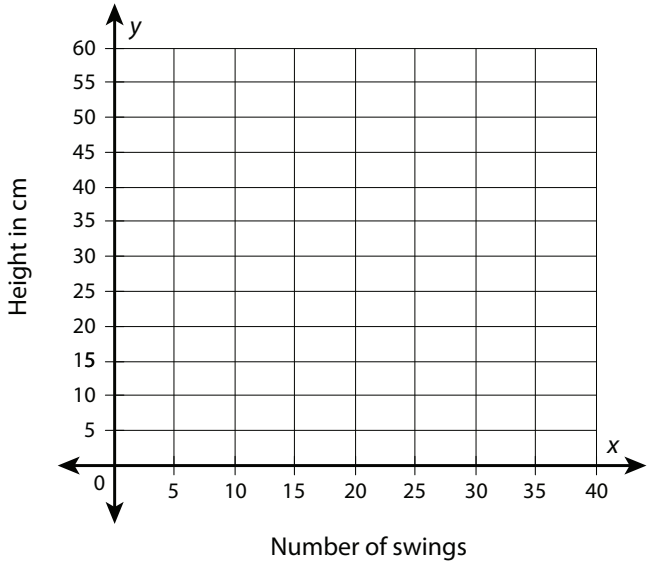
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Instruction

4. Set up the coordinate plane.

   Determine the labels by reading the problem again. The independent variable is the number of swings. This is the label of the *x*-axis. The *y*-axis label is the height of the pendulum. The height is the dependent variable because it depends on the number of swings.

   To determine the scales, examine the table of values. The *x*-axis should have a scale that goes from 0 to 40. Counting to 40 in increments of 1 would cause the axis to be very long or the increments to be very small. Use increments of 5. For the *y*-axis, start with 0 and go to 60 in increments of 5. This will make plotting numbers like 43.74 a little easier than if you chose increments of 10.

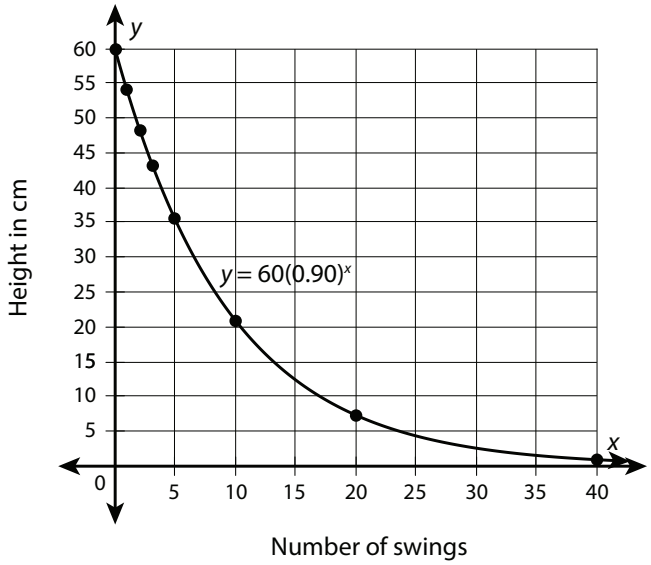


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Instruction

5. Plot the points on the coordinate plane and connect the points with a curve.

   When the points do not lie on a grid line, use estimation to approximate where the point should be plotted. Add an arrow to the right end of the line to show that the curve continues in that direction toward infinity.



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Instruction

Example 2

The bacteria *Streptococcus lactis* doubles every 26 minutes in milk. If a container of milk has 4 bacteria in it to start, write an equation that models this scenario and then graph the equation.

1. Read the problem statement and then reread the scenario, identifying the known quantities.

   The initial bacteria count is 4.

   The base that indicates doubling is 2. (If a quantity doubles, there is 200% of the previous quantity, and 200% expressed as a decimal is 2.)

   The time period required for doubling to occur is 26 minutes.

2. Substitute the known quantities into the general form of the exponential equation $y = ab^x$.

   In this formula, a is the initial value, b is the base, x is the number of time intervals (in this case, one time interval is 26 minutes), and y is the final value. Since the time interval is not equal to 1, use the equation $y=ab^{\frac{x}{t}}$, where $a = 4$, $b = 2$, and $t = 26$.

   $y=ab^{\frac{x}{t}}$ Equation

   $y=(4)(2)^{\frac{x}{(26)}}$ Substitute 4 for a, 2 for b, and 26 for t.

   $y=4(2)^{\frac{x}{26}}$ Simplify.

   The equation $y=4(2)^{\frac{x}{26}}$ models the number of bacteria.

3. Create a table of values using the resulting equation.

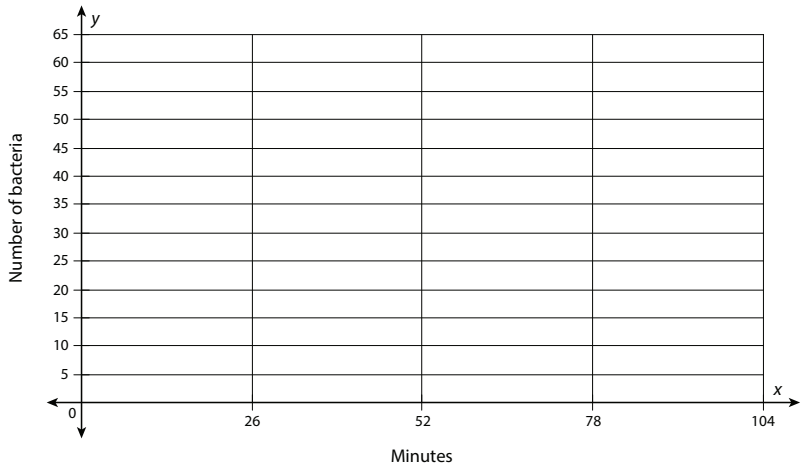
x	y
0	4
26	8
52	16
78	32
104	64

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4. Set up the coordinate plane.

   Determine the labels by reading the problem again. The independent variable is the number of time intervals. The time intervals are given in minutes. Therefore, "Minutes" will be the *x*-axis label. The *y*-axis label will be the "Number of bacteria." The number of bacteria is the dependent variable because it depends on the number of minutes that have passed.

   The *x*-axis should have a scale that reflects the fact that each time interval is 26 minutes. The table of values showed 4 time periods. Each time interval is 26 minutes, and so 4 time intervals is 4(26) = 104 minutes. This means the *x*-axis scale should go from 0 to 104. Use increments of 26 for easy plotting of the points. For the *y*-axis, start with 0 and go to 65 in increments of 5. This will make plotting numbers like 32 a little easier than if you chose increments of 10.

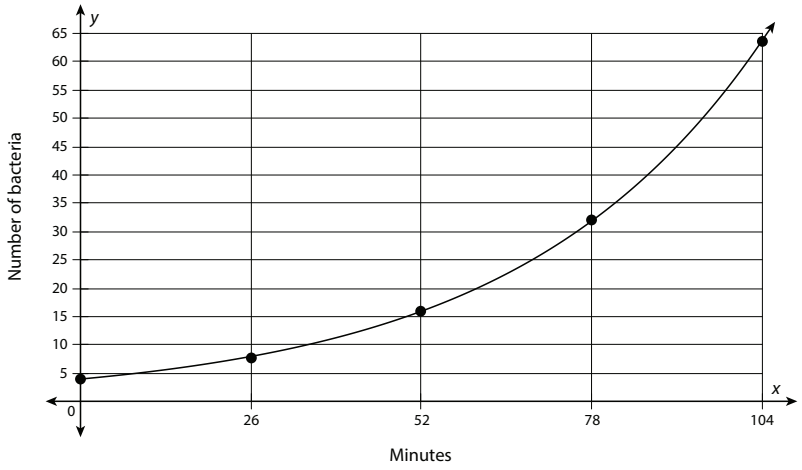


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Instruction

5. Plot the points on the coordinate plane and connect the points with a curve.

   When the points do not lie on a grid line, use estimation to approximate where the point should be plotted. Add an arrow to the right end of the line to show that the curve continues in that direction toward infinity.



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Instruction

Example 3

An investment of $500 is compounded monthly at an annual rate of 3%. What is the equation that models the value of the investment? Graph the equation both on a calculator and on graph paper.

1. Read the problem statement and then reread the scenario, identifying the known quantities.

   The initial investment is $500.

   The annual interest rate is 3%.

   Compounded monthly means 12 times per year.

2. Substitute the known quantities into the general form of the compound interest formula, $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

   In this formula, P is the initial value, r is the annual interest rate, n is the number of times the investment is compounded in a year, and t is the number of years the investment is left in the account to grow.

   Let $P = 500$, $r = 3\% = 0.03$, and $n = 12$.

   $A = P\left(1 + \frac{r}{n}\right)^{nt}$ Compound interest formula

   $A = (500)\left[1 + \frac{(0.03)}{(12)}\right]^{(12)t}$ Substitute 500 for P, 0.03 for r, and 12 for n.

   $A = 500(1.0025)^{12t}$ Simplify.

   Notice that, after simplifying, the resulting equation is of the form $y = ab^x$. To graph on the x- and y-axes, rewrite the compound interest formula in the form $y = ab^x$, where $A = y$, $P = a$, $\left(1 + \frac{r}{n}\right) = b$, and $t = x$.

   $A = 500(1.0025)^{12t}$ becomes $y = 500(1.0025)^{12x}$.

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3. Graph the equation using a graphing calculator.

On a TI-83/84:

Step 1: Press [Y=].

Step 2: Type in the equation as follows: [500][×][1.0025][^][12][X, T, θ, *n*]

Step 3: Press [WINDOW] to change the viewing window.

Step 4: At Xmin, enter [0] and arrow down one level to Xmax.

Step 5: At Xmax, enter [10] and arrow down one level to Xscl.

Step 6: At Xscl, enter [1] and arrow down one level to Ymin.

Step 7: At Ymin, enter [500] and arrow down one level to Ymax.

Step 8: At Ymax, enter [700] and arrow down one level to Yscl.

Step 9: At Yscl, enter [15].

Step 10: Press [GRAPH].

On a TI-Nspire:

Step 1: Press the [home] key.

Step 2: Arrow over to the graphing icon and press [enter].

Step 3: At the blinking cursor at the bottom of the screen, enter in the equation [500][×][1.0025][^][12x] and press [enter].

Step 4: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom, and click the center button of the navigation pad.

Step 5: Choose 1: Window settings by pressing the center button.

Step 6: Enter in the appropriate XMin value, [0], and press [tab].

Step 7: Enter in the appropriate XMax value, [10], and press [tab].

Step 8: Leave the XScale set to "Auto." Press [tab] twice to navigate to YMin and enter [500].

Step 9: Press [tab] to navigate to YMax. Enter [700]. Press [tab] twice to leave YScale set to "Auto" and to navigate to "OK."

Step 10: Press [enter].

Step 11: Press [menu] and select 2: View and 5: Show Grid.

Note: To determine the *y*-axis scale, show the table to get an idea of the values for *y*. To show the table, press [ctrl] and then [T]. To turn the table off, press [ctrl][tab] to navigate back to the graphing window and then press [ctrl][T] to turn off the table.

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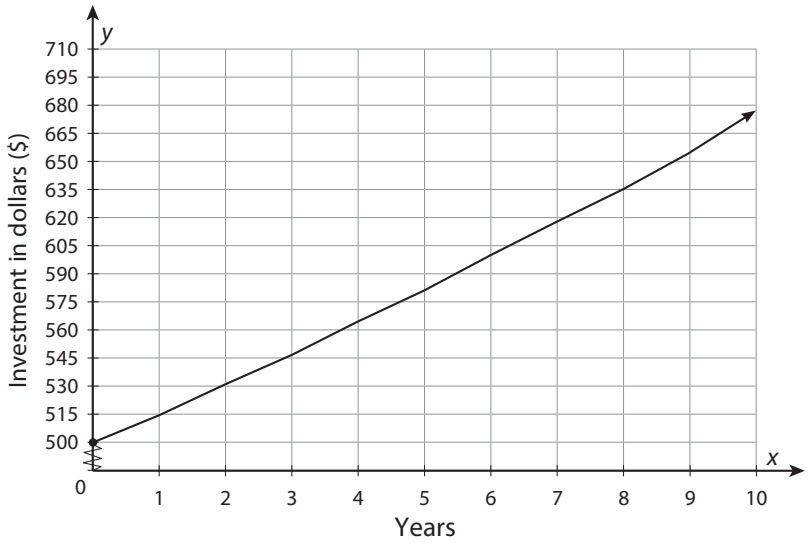
Instruction

4. Transfer your graph from the screen to graph paper.

   Use the same scales that you set for your viewing window.

   The scale on the *x*-axis is from 0 to 10 years in increments of 1 year.

   The scale on the *y*-axis is from \$500 to \$700 in increments of \$15. Show a break in the graph from 0 to 500 with a zigzag line.



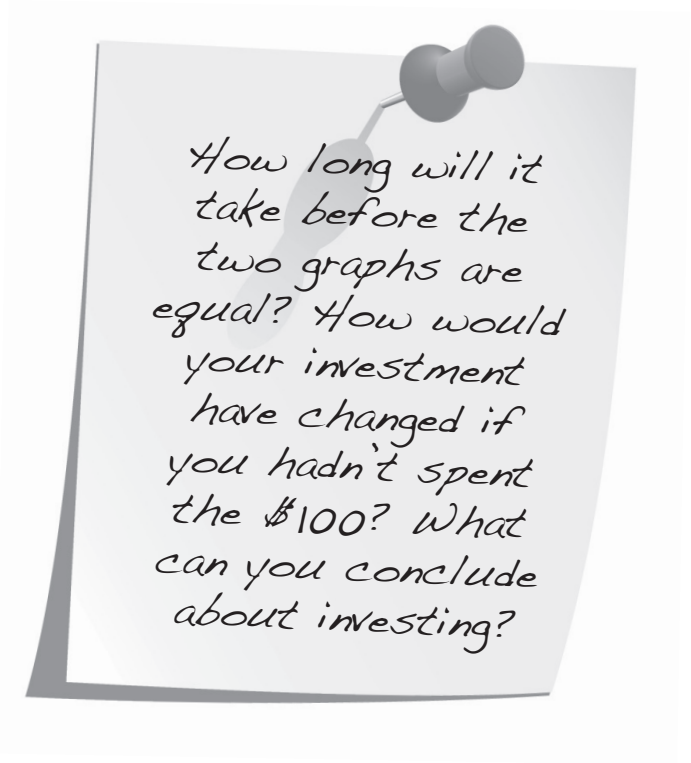
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Problem-Based Task 1.3.2: Investing Money

SMP	
1	2 ✓
3	4
5 ✓	6
7 ✓	8

You want to invest some money in a savings account. One bank offers an account that compounds the money annually at a rate of 3%. You have $2,000 to invest. As you are about to sign the papers, your friend texts you that a different bank offers a rate of 3.2% and this bank will compound the interest monthly. You decide to check out the second bank, but on your way there you spend $100. You end up choosing the second bank with the higher interest rate, but you want to know how spending $100 along the way affected your investment.

Create a graph showing how much interest you would have earned on $2,000 at the first bank, then create another graph showing how much interest you will earn on the money you invested in the second bank. Use the graphs to help you determine about how long it will take to earn back the $100 you spent. How long will it take before the two graphs are equal? How would your investment have changed if you hadn't spent the $100? What can you conclude about investing?



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Problem-Based Task 1.3.2: Investing Money

Coaching

a. What is the equation for the investment at the first bank?

b. What is the equation for the investment at the second bank? Keep in mind that you spent $100 of the money you initially planned to invest.

c. Graph the equations on the same set of axes, and be sure to label each equation.

d. Looking at the graph of the investment you actually made, how many years does it take to earn back the $100 you spent?

e. How many years does it take before the investment you made is equal to the investment you almost made?

f. What would be the equation of the investment at the second bank if you had not spent the $100?

g. Graph the equation from part f on the same set of axes as the equation from part b.

h. Look at various points along the graph and use the equations. What is the difference in investments after 10 years? 20 years?

i. Compare the investments of all three graphs and make observations. What conclusions can you draw about the amount you invest initially or the principal amount? What can you conclude about the number of times the interest is compounded in a year? What effect does this have on the investment?

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Instruction

Problem-Based Task 1.3.2: Investing Money

Coaching Sample Responses

a. What is the equation for the investment at the first bank?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 2000\left(1 + \frac{0.03}{1}\right)^{1t}$$

$$A = 2000(1.03)^{t}$$

b. What is the equation for the investment at the second bank? Keep in mind that you spent \$100 of the money you initially planned to invest.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 1900\left(1 + \frac{0.032}{12}\right)^{12t}$$

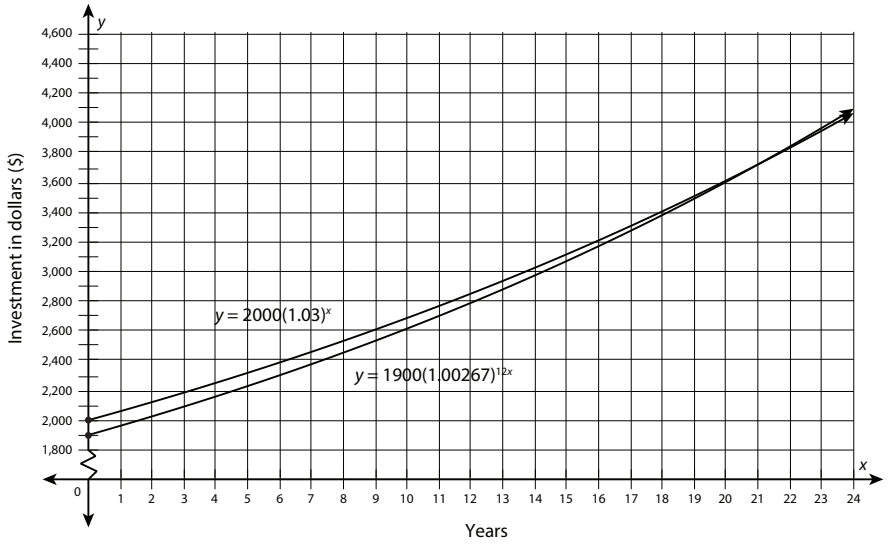
$$A = 1900(1.00267)^{12t}$$

c. Graph the equations on the same set of axes, and be sure to label each equation.

To do this, first rewrite each equation in the form $y = ab^x$.

$A = 2000(1.03)^t$ becomes $y = 2000(1.03)^x$.

$A = 1900(1.00267)^{12t}$ becomes $y = 1900(1.00267)^{12x}$.



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d. Looking at the graph of the investment you actually made, how many years does it take to earn back the $100 you spent?

It looks like the investment earns back $100 and reaches $2,000 after a little more than a year and a half, or about 18 months.

e. How many years does it take before the investment you made is equal to the investment you almost made?

The graphs intersect at about 21 years, so the investments will be equal in about 21 years.

f. What would be the equation of the investment at the second bank if you had not spent the $100?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

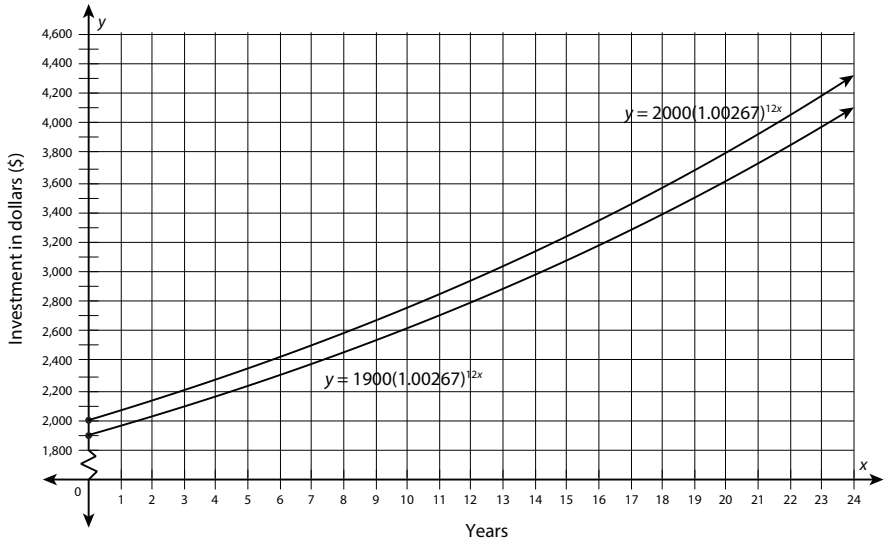
$$A = 2000\left(1 + \frac{0.032}{12}\right)^{12t}$$

$$A = 2000(1.00267)^{12t}$$

g. Graph the equation from part f on the same set of axes as the equation from part b.

Before graphing, rewrite the equation in the form $y = ab^x$.

$A = 2000(1.00267)^{12t}$ becomes $y = 2000(1.00267)^{12x}$.



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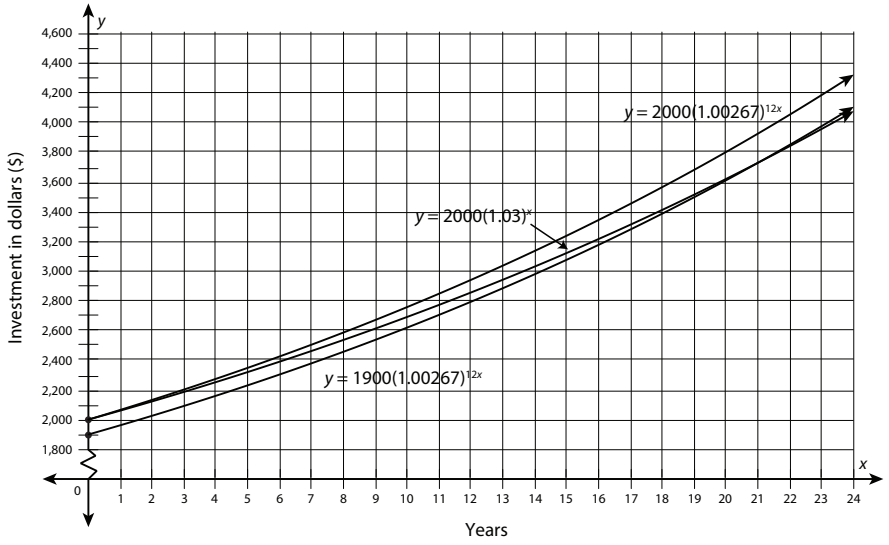
h. Look at various points along the graph and use the equations. What is the difference in investments after 10 years? 20 years?

The investment of the principal amount of $2,000 will always be greater than the investment with the principal amount of $1,900. After 10 years, the investment of $2,000 grows to $2,754.18, and the investment of $1,900 grows to $2,616.47, a difference of $137.71.

After 20 years, the investment of $2,000 grows to $3,792.76, and the investment of $1,900 grows to $3,603.12. The difference is $189.64. The gap between the larger and smaller investments is slowly widening.

i. Compare the investments of all three graphs and make observations. What conclusions can you draw about the amount you invest initially or the principal amount? What can you conclude about the number of times the interest is compounded in a year? What effect does this have on the investment?

The more you invest to begin with, the more your investment will grow. The more times the interest is compounded in a year, the faster the investment will grow for the same annual interest rate. If two banks are offering the same rate but one bank is compounding the interest more frequently, invest in the bank that compounds more often. If the rates are different, draw graphs to compare the investments.



Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 1.3.2: Creating and Graphing Exponential Equations

For problems 1 and 2, use a table of values to graph the exponential equations.

1. $y = 2(3)^x$

2. $y = 1000(0.25)^x$

For problems 3–10, write an equation to model growth or decay in each scenario, and then graph the equation.

3. A population of insects doubles every month. This particular population started out with 20 insects.

4. The half-life of rhodium, Rh-106, is about 30 seconds. You start with 500 grams.

5. A stock is declining at a rate of 25% of its value every 2 weeks. The stock started at $225.

6. An invasive weed species triples its numbers in 6 days. A field started with 12 weeds in the early spring.

7. The population of a big city is increasing at a rate of 2.5% per year. The city's current population is 67,000.

8. An investment of $1,000 earns 3.7% interest and is compounded semiannually.

9. An investment of $600 earns 2.9% interest and is compounded quarterly.

10. An investment of $3,000 earns 1.4% interest and is compounded weekly.

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Progress Assessment

Circle the letter of the best answer.

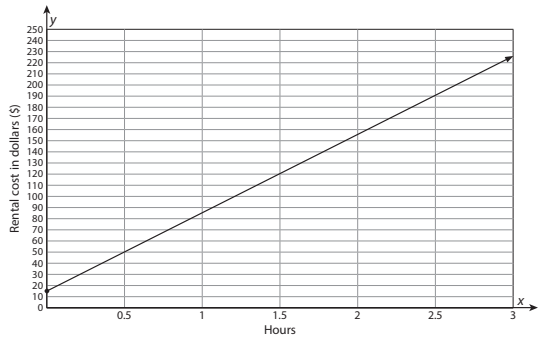
1. The cost of having your car towed is $45 to hook up the car and then $3.50 per mile towed. Which equation models your total cost?

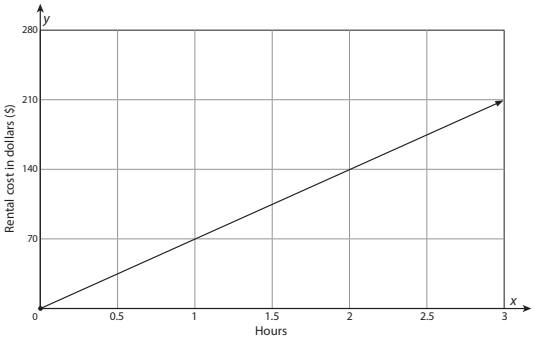
   a. $y = 40x + 3.50$
   b. $y = 3.50x + 45$
   c. $y = 48.50x$
   d. $x + y = 48.50$

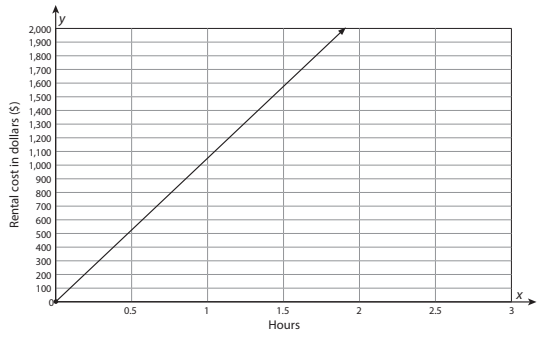
2. A store is giving away 150 gift cards valued at $20 each. They give away 1 gift card per hour for every hour that the store is open. What equation models the number of remaining gift cards?

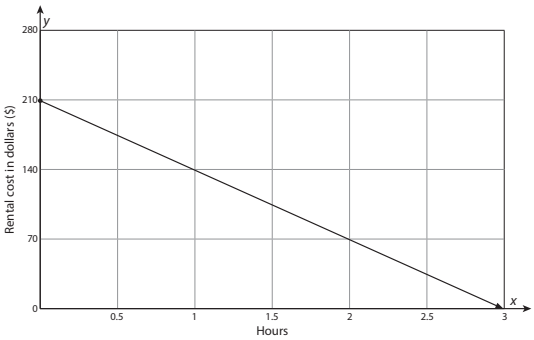
   a. $y = –20x + 3000$
   b. $y = –x + 150$
   c. either a or b
   d. neither a nor b

3. A company rents personal watercraft for $70 per hour plus an initial $15 fee. What is the graph of the equation that models this scenario?

   a.
   

   c.
   

   b.
   

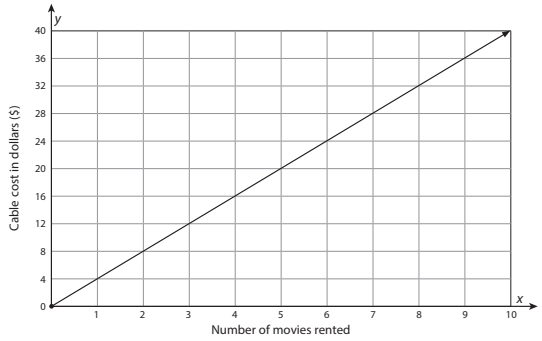
   d.
   

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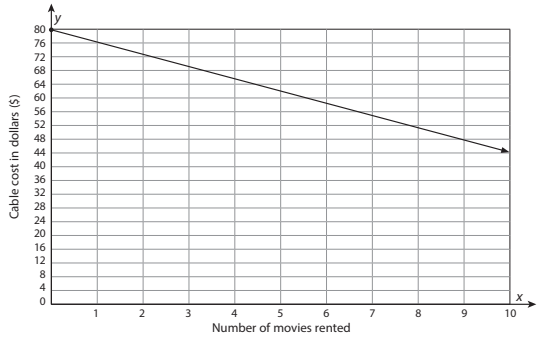
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4. A cable company charges $80 a month for service and $4 for each on-demand movie watched. What is the graph of the equation for this scenario?

a.



b.



c.



d.



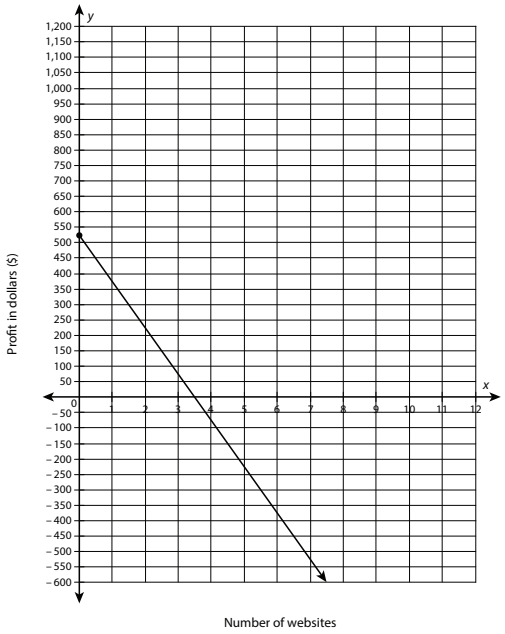
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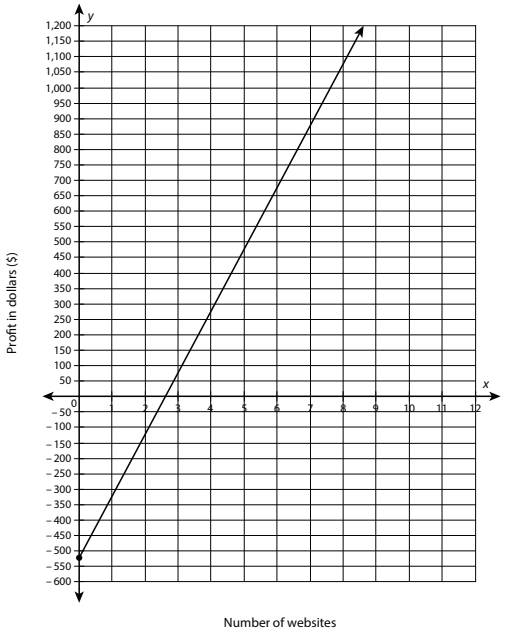
Assessment

5. You are starting your own business making websites. You spend $525 to get started, and will charge each customer $150 to build their website. Which graph represents the equation of your profit?

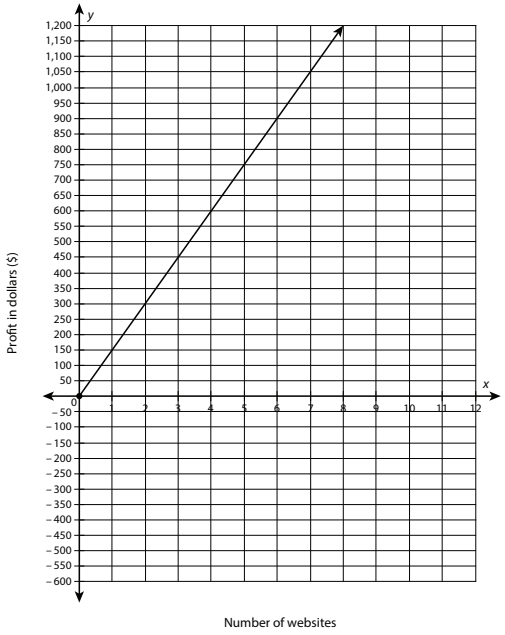
a.



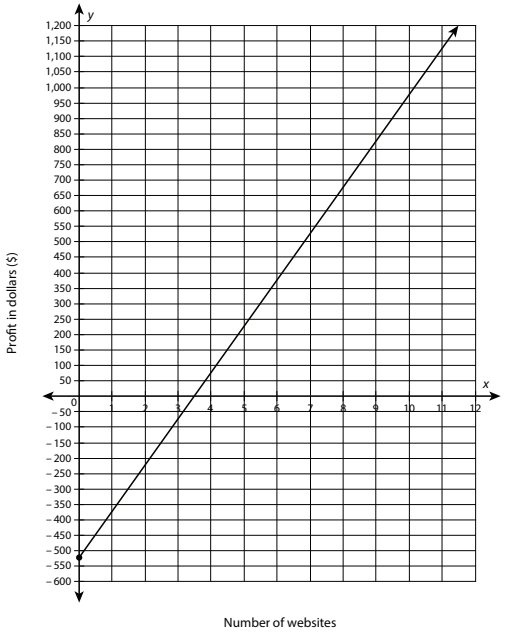
c.



b.



d.



continued

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6. The half-life of niobium-97m is 1 minute. If an experiment started with 200 grams, which equation represents the amount of niobium-97m remaining?

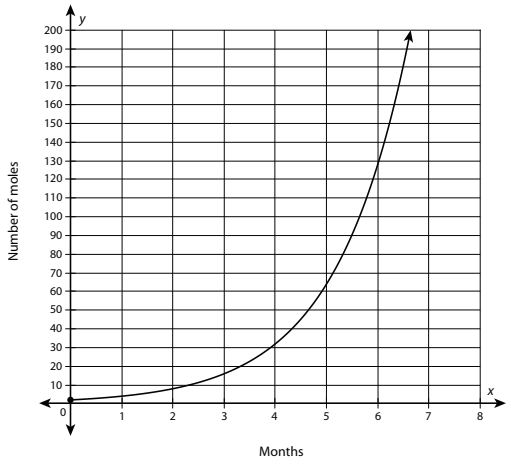
   a. $y = 200(2)^x$
   b. $y = 200(0.5)^x$
   c. $y = -0.5x + 200$
   d. $y = -2x + 200$

7. An investment of $750 earns 3.3% interest compounded monthly. What is the equation that models the value of the investment?

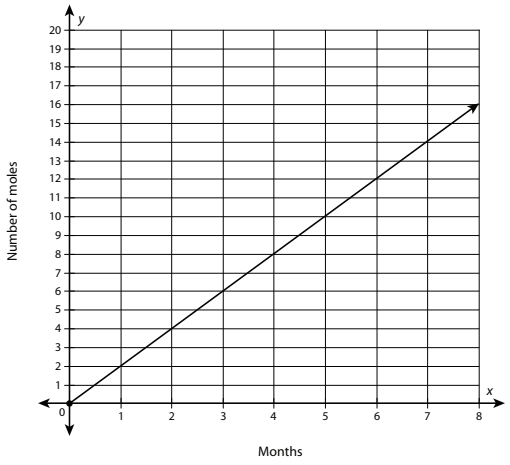
   a. $A = 750(1.033)^x$
   b. $A = 750(1.033)^{12x}$
   c. $A = 750(1.00275)^{12x}$
   d. $A = 750(1.00275)^x$

8. A mole population doubles every month. If the population started with 2 moles, what is the graph of the equation representing the total number of moles?

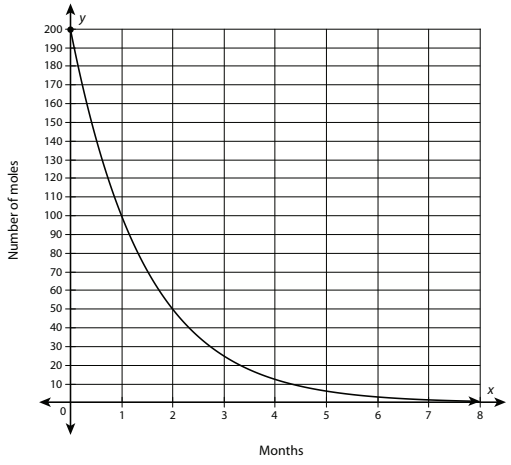
   a.



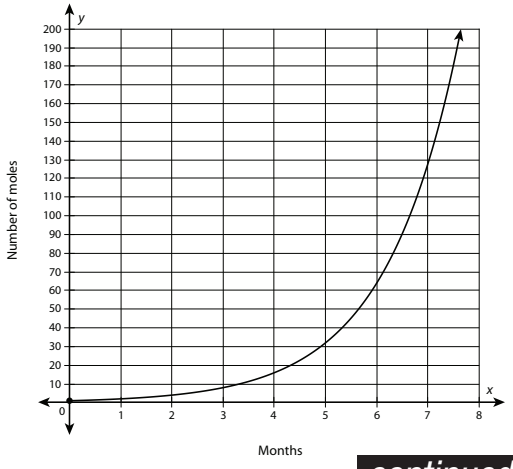
   c.



   b.



   d.

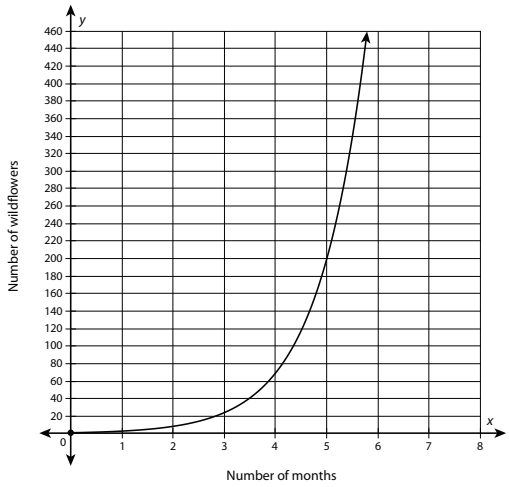


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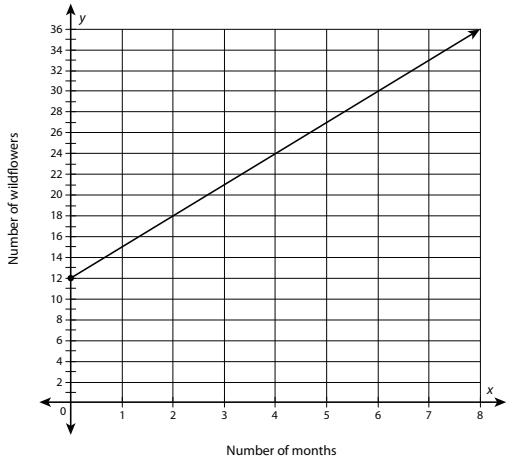
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9. A wildflower population triples every 2 months. If a meadow starts out with 12 wildflowers, what is the graph of the equation modeling the total number of wildflowers in this meadow?

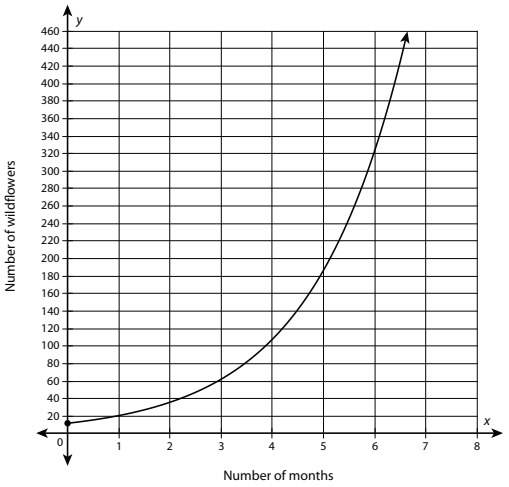
a.



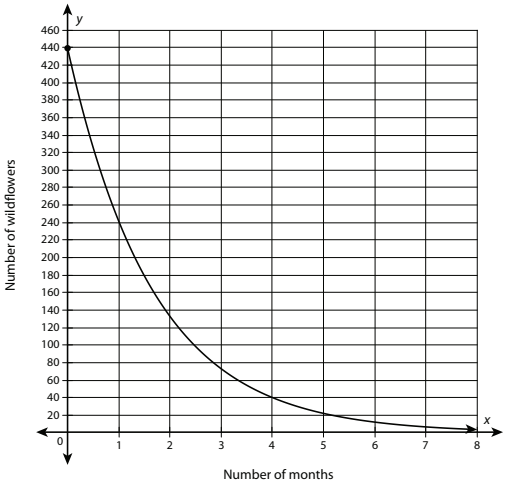
b.



c.



d.

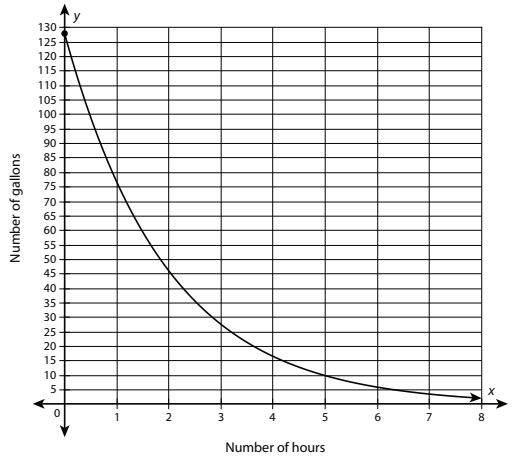


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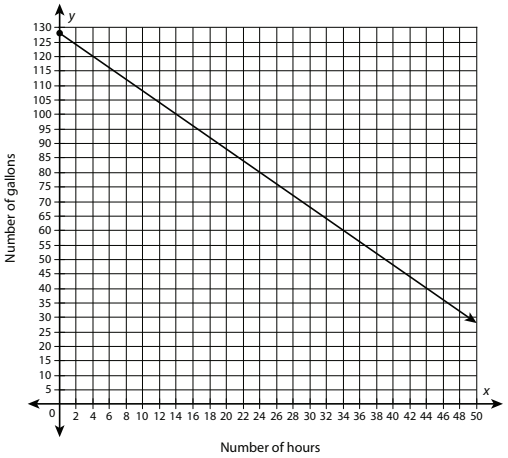
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10. Water is draining out of a large tank. The initial volume of water is 128 gallons, and the volume is decreased by half every hour. What is the graph of the equation that represents the amount of water remaining in the tank?

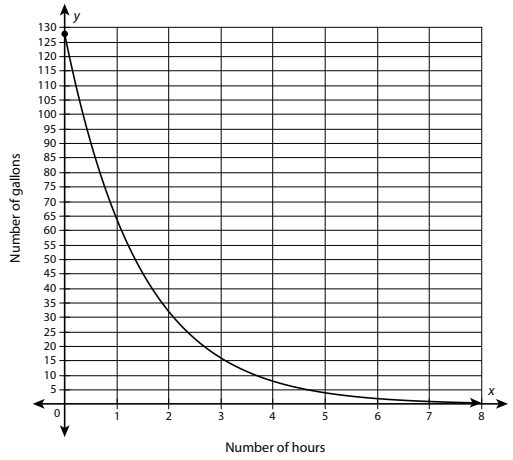
a.



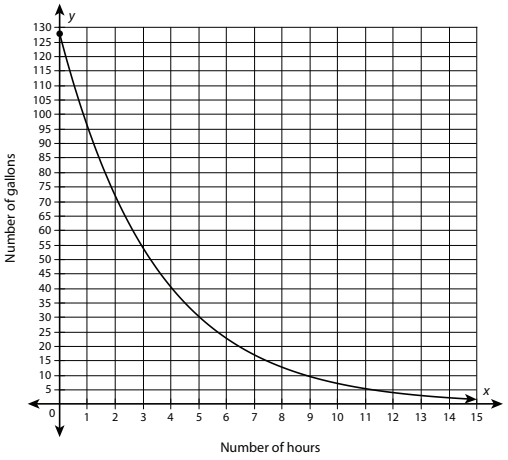
c.



b.



d.



Use what you've learned about comparing graphs of different interest rates to solve.

11. Compare two investments of $500 that each earn 3% interest. The first investment earns simple interest. (Remember, simple interest earns money only on the principal amount.) The second investment is compounded quarterly. Compare the two investments using equations and graphs. Which is the better investment and why?

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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. Given the equation $y = 5x - 7$, which point is a solution?

   a. (1, 2)
   b. (0, 7)
   c. (–1, 2)
   d. (–2, –17)

2. Given the inequality $y \leq -3x + 6$, which point is NOT a solution?

   a. (1, –3)
   b. (0, –2)
   c. (–1, –9)
   d. (2, 3)

3. Julia has $6.50 to spend on peaches and apples at the farmer's market. She bought 4 peaches at $0.75 each. How much money can she spend on apples? Determine which system of inequalities represents this situation.

   a. $\begin{cases} a + 4(0.75) \leq 6.50 \\ a \geq 0 \end{cases}$
   b. $\begin{cases} a + 4(0.75) \leq 6.50 \\ a \leq 0 \end{cases}$
   c. $\begin{cases} a + 4(0.75) \geq 6.50 \\ a \leq 0 \end{cases}$
   d. $\begin{cases} a + 4(0.75) \geq 6.50 \\ a \geq 0 \end{cases}$

4. Your cell phone company charges $29.99 a month plus $0.25 for each text message sent. You have budgeted no more than $35.00 for cell phone service each month. Given this situation, determine the minimum and maximum number of texts you can send without going over budget. Let x represent the number of texts.

   a. $x < 20.04$
   b. $x \geq 0$ and $x \leq 20.04$
   c. $x > 0$ and $x < 20$
   d. $x \geq 0$ and $x \leq 20$

5. Your doctor recommends that you eat at least 46 grams of protein each day. One serving of peanuts contains 9 grams of protein, while one egg contains 6 grams of protein. Let x represent the number of servings of peanuts and let y represent the number of servings of eggs. Determine which system of inequalities represents the number of servings of eggs and peanuts you must eat in order to reach the minimum recommendation.

   a. $\begin{cases} 9x + 6y \leq 46 \\ x \leq 0 \\ y \leq 0 \end{cases}$
   b. $\begin{cases} 9x + 6y \leq 46 \\ x \geq 0 \\ y \geq 0 \end{cases}$
   c. $\begin{cases} 9x + 6y \geq 46 \\ x \geq 0 \\ y \geq 0 \end{cases}$
   d. $\begin{cases} 9x + 6y \geq 46 \\ x \leq 0 \\ y \leq 0 \end{cases}$

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Lesson 4: Representing Constraints

Instruction

Common Core State Standard

A–CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★

SMP
1 2
3 ✓ 4 ✓
5 6
7 ✓ 8

Essential Questions

1. How can you model real-world applications using equations?
2. How can you solve real-world applications by graphing systems of equations?
3. How can you model real-world applications using inequalities?
4. Why are there constraints when solving and graphing real-world applications?

WORDS TO KNOW

constraint	a restriction or limitation on any of the variables in an equation or inequality
inequality	a mathematical sentence that shows the relationship between quantities that may or may not be equivalent. An inequality contains one or more of the following symbols: <, >, ≤, ≥, or ≠.
solution set	the value or values that make a sentence or statement true
system of equations	a set of equations with the same unknowns
system of inequalities	a set of two or more inequalities with the same unknowns

Recommended Resources

- NCTM Illuminations. "Dirt Bike Dilemma."

  http://walch.com/rr/CAU1L4SysEquations

  This teacher resource includes a printable student activity in which, with the help of graphing calculators, students use a system of equations to maximize profits for a dirt bike manufacturer.

- Purplemath.com. "Linear Programming: Word Problems."

  http://walch.com/rr/CAU1L4SysInequalities

  This site offers a review of systems of inequalities and constraints associated with real-world situations. It also includes graphs of feasible regions.

SRB p. U1-97

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Lesson 1.4.1: Representing Constraints

Warm-Up 1.4.1

Read the scenario and answer the questions that follow.

> Roshanda pays $5 in tolls and uses 3 gallons of gasoline each day she drives to work. In one day, Roshanda spent a total of $15.23 on tolls and gasoline.

1. How much did each gallon of gas cost? Explain how you found your answer.

2. Roshanda needs to work 5 days next week and has set aside $75 for tolls and gas. Will Roshanda have enough money for her workweek? Explain how you found your answer.

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Instruction

Lesson 1.4.1: Representing Constraints

Common Core State Standard

A–CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★

Warm-Up 1.4.1 Debrief

1. How much did each gallon of gas cost? Explain how you found your answer.

   Write an equation to represent this situation. Let x represent the cost of a gallon of gas.

   $5 + 3x = 15.23$

   Solve for x.

   $5 + 3x = 15.23$ Subtract 5 from both sides of the equation.

   $3x = 10.23$ Divide both sides by 3.

   $x = 3.41$

   Interpret the solution.

   The cost of 1 gallon of gas is \$3.41.

2. Roshanda needs to work 5 days next week and has set aside \$75 for tolls and gas. Will Roshanda have enough money for her workweek? Explain how you found your answer.

   Multiply the amount of money Roshanda spends on her travels for 1 day of work by the number of days she will be working.

   $15.23(5) = 76.15$

   Compare this amount to the amount that Roshanda has saved.

   Roshanda has saved \$75 and will need \$76.15. She will not have enough money.

Connection to the Lesson

- The upcoming lesson will have students examining limitations and constraints on equations and inequalities. Students will need to think about their answers and determine if they are appropriate given the constraints.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- reading and writing inequalities (6.NS.7b)
- creating and evaluating inputs and outputs of equations and inequalities (8.F.1)

Introduction

Situations in the real world often determine the types of values we would expect as answers to equations and inequalities. Remember that an inequality shows the relationship between quantities that may or may not be equivalent. An inequality contains at least one inequality symbol ($<$, $>$, $\leq$, $\geq$, or $\neq$) and may contain one or more variables.

Sometimes there are limits or restrictions on the values that can be substituted into an equation or inequality; other times, limits or restrictions are placed on answers to problems involving equations or inequalities. These limits or restrictions are called **constraints**.

Key Concepts

- Many real-world situations can be modeled using an equation, an inequality, a **system of equations**, or a **system of inequalities**. A system is a set of equations or inequalities with the same unknowns.
- When creating a system of equations or inequalities, it is important to understand that the solution set is the value or values that make all sentences in the system true statements.
- Being able to translate real-world situations into algebraic sentences will help with the understanding of constraints.

Common Errors/Misconceptions

- incorrectly translating verbal descriptions into algebraic sentences
- not including appropriate constraints as related to the situation

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Instruction

Guided Practice 1.4.1

Example 1

Determine whether the coordinate pair (2, 9) is a solution to the following system of inequalities.

$$\begin{cases} x + y \leq 11 \\ 2x + y < 16 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

1. Substitute the values for x and y into the original inequalities and solve.

$$\begin{cases} x + y \leq 11 \\ 2x + y < 16 \\ x \geq 0 \\ y \geq 0 \end{cases}$$ Given system of inequalities

$$\begin{cases} (2) + (9) \leq 11 \\ 2(2) + (9) < 16 \\ (2) \geq 0 \\ (9) \geq 0 \end{cases}$$ Substitute 2 for x and 9 for y.

$$\begin{cases} 11 \leq 11 \\ 13 \leq 16 \\ 2 \geq 0 \\ 9 \geq 0 \end{cases}$$ Simplify each inequality.

2. Interpret the results.

   Each of these inequalities is a true statement; therefore (2, 9) is a solution to the system of inequalities.

SWB p. 81; SRB p. U1-100

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Instruction

Example 2

A taxi company charges $2.50 plus $1.10 for each mile driven. Write an equation to represent this situation. Use this equation to determine how far you can travel if you have $10.00. What is the minimum amount of money you will spend?

1. Translate the verbal description into an algebraic equation.

   Let m represent the number of miles driven and let C represent the total cost of the trip.

   $2.50 + 1.10m = C$

2. Substitute in the total amount you have to spend.

   The total cost of the trip can't be more than $10.00 because that is all you have to spend. Substitute 10.00 for C.

   $2.50 + 1.10m = 10.00$

3. Change the equal sign to an inequality symbol.

   Although you have $10.00 to spend, you could also spend less than that, so use a less than or equal to symbol.

   $2.50 + 1.10m \leq 10.00$

4. Solve the inequality by isolating the variable.

   $2.50 + 1.10m \leq 10.00$ Subtract 2.50 from both sides.

   $1.10m \leq 7.50$ Divide both sides by 1.10.

   $m \leq 6.82$

   You can travel up to 6.82 miles and not pay more than $10.00. Because the company charges by the mile, you can travel no more than 6 miles.

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5. Determine the minimum amount you will spend.

   The minimum amount the taxi driver charges is \$2.50, but it is unlikely that he or she will charge you if you get in the cab and get right back out without going anywhere. You will pay \$1.10 if you travel 1 mile or less; add this to the minimum charge of \$2.50 to arrive at \$3.60.

   You will spend a minimum of \$3.60, but no more than \$10.00.

Example 3

A school supply company produces wooden rulers and plastic rulers. The rulers must first be made, and then painted.

- It takes 20 minutes to make a wooden ruler. It takes 15 minutes to make a plastic ruler. There is a maximum amount of 480 minutes per day set aside for making rulers.
- It takes 5 minutes to paint a wooden ruler. It takes 2 minutes to paint a plastic ruler. There is a maximum amount of 180 minutes per day set aside for painting rulers.

Write a system of inequalities that models the making and then painting of wooden and plastic rulers.

1. Identify the information you know.

   There is a maximum of 480 minutes for making rulers.

   - It takes 20 minutes to make a wooden ruler.
   - It takes 15 minutes to make a plastic ruler.

   There is a maximum of 180 minutes for painting rulers.

   - It takes 5 minutes to paint a wooden ruler.
   - It takes 2 minutes to paint a plastic ruler.

2. Write an inequality using variables to represent the amount of time needed to make the rulers.

   Let w represent the wooden rulers and p represent the plastic rulers.

   $20w + 15p \leq 480$

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Instruction

3. Use the same variables to write an inequality to represent the amount of time needed to paint the rulers.

   $5w + 2p \leq 180$

4. Determine the constraints on this situation, then write inequalities to represent these constraints.

   It is not possible to produce a negative amount of either wooden rulers or plastic rulers; therefore, you need to limit the values of w and p to values that are greater than or equal to 0. Note also that w and p must be integers.

   $w \geq 0$

   $p \geq 0$

5. To write the system of inequalities for this situation, combine all the inequalities related to the situation and list them in a brace, {.

   The following system of inequalities models making and then painting the wooden and plastic rulers.

$$\begin{cases} 20w + 15p \leq 480 \\ 5w + 2p \leq 180 \\ w \geq 0 \\ p \geq 0 \end{cases}$$

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Instruction

Example 4

Use the system of inequalities from Example 3 to give a possible solution to the system.

1. Use what you know about the constraints on the real-world situation to determine any limits on the possible solutions.

   We know from Step 4 of Example 3 that you cannot produce a negative amount of rulers, so none of our solutions can be negative. That is, $w \geq 0$ and $p \geq 0$.

   Note: In future lessons, we discuss more precise ways of determining the solution set to a system. For now, we can use our knowledge of numbers and ability to solve algebraic sentences to find possible solutions.

2. Choose a value for w and solve the first two inequalities in the system for p.

   For example, let $w = 0$. Substitute 0 for each occurrence of w in the first two inequalities of the system and solve for p.

$20w + 15p \leq 480$	First inequality from Example 3
$20(0) + 15p \leq 480$	Substitute 0 for w.
$15p \leq 480$	Simplify.
$p \leq 32$	Divide both sides by 15.

   For the first inequality, $p \leq 32$.

$5w + 2p \leq 180$	Second inequality from Example 3
$5(0) + 2p \leq 180$	Substitute 0 for w.
$2p \leq 180$	Simplify.
$p \leq 90$	Divide both sides by 2.

   For the second inequality, $p \leq 90$.

3. Interpret the results.

   In 480 minutes, the company can make no more than 32 plastic rulers if 0 wooden rulers are produced.

   In 180 minutes, the company can paint no more than 90 plastic rulers if there are no wooden rulers to paint.

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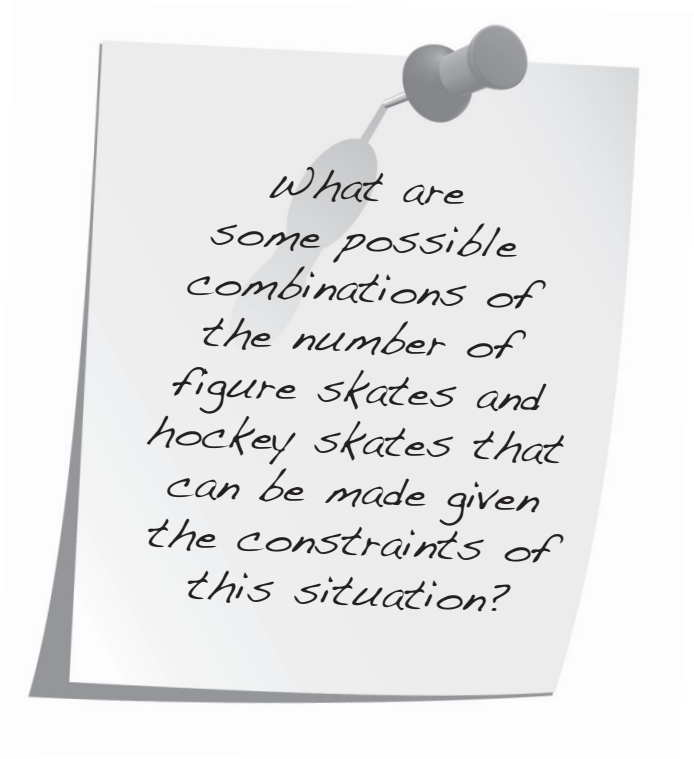
Problem-Based Task 1.4.1: Skate Constraints

SMP	
1	2
3 ✓	4 ✓
5	6
7 ✓	8

A sporting goods company makes figure skates and hockey skates. One group of workers makes the blades for both types of skates. Another group makes the boots for both types of skates.

- It takes 2 hours to make the blade of a figure skate. It takes 3 hours to make the blade of a hockey skate. There is a maximum of 40 hours per week in which the blades can be made for both types of skates.
- It takes 3 hours to make the boot of a figure skate. It takes 1 hour to make the boot for a hockey skate. There is a maximum of 20 hours per week in which boots can be made for both types of skates.

What are some possible combinations of the number of figure skates and hockey skates that can be made given the constraints of this situation?



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Problem-Based Task 1.4.1: Skate Constraints

Coaching

a. How long does it take to make the blade of a figure skate?

b. How long does it take to make the blade of a hockey skate?

c. How many hours each week can be spent making skate blades?

d. What inequality can be used to represent the amount of time it takes to make blades for both figure skates and hockey skates?

e. How long does it take to make the boot of a figure skate?

f. How long does it take to make the boot of a hockey skate?

g. How many hours each week can be spent making skate boots?

h. What inequality can be used to represent the amount of time it takes to make the boots for both figure skates and hockey skates?

i. What other constraints are needed in this situation?

j. What is the system of inequalities that represents this situation?

k. Is it possible to make 3 figure skates and 4 hockey skates given the constraints of this situation?

l. Is it possible to make 8 figure skates and 5 hockey skates given the constraints of this situation?

m. What is another possible combination of the number of figure skates and hockey skates that can be made given the constraints of this situation?

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Instruction

Problem-Based Task 1.4.1: Skate Constraints

Coaching Sample Responses

a. How long does it take to make the blade of a figure skate?

It takes 2 hours to make the blade of a figure skate.

b. How long does it take to make the blade of a hockey skate?

It takes 3 hours to make the blade of a hockey skate.

c. How many hours each week can be spent making skate blades?

No more than 40 hours can be spent making skate blades.

d. What inequality can be used to represent the amount of time it takes to make blades for both figure skates and hockey skates?

The total amount of time needed to make the blades can be represented by the sum of the time needed for each type of blade. Let f represent figure skates and h represent hockey skates.

The total number of hours needed for making the blades can't be more than 40 hours.

$2f + 3h \leq 40$

e. How long does it take to make the boot of a figure skate?

It takes 3 hours to make the boot of a figure skate.

f. How long does it take to make the boot of a hockey skate?

It takes 1 hour to make the boot of a hockey skate.

g. How many hours each week can be spent making skate boots?

No more than 20 hours can be spent making skate boots.

h. What inequality can be used to represent the amount of time it takes to make the boots for both figure skates and hockey skates?

The total amount of time needed to make the boots can be represented by the sum of the time needed for each type of boot.

The total number of hours needed for making the boots can't be more than 20 hours.

$3f + h \leq 20$

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Instruction

i. What other constraints are needed in this situation?

The number of figure skates can't be less than 0.

$f \geq 0$

The number of hockey skates can't be less than 0.

$h \geq 0$

In addition, the number of each type of skate must be an integer.

j. What is the system of inequalities that represents this situation?

To create the system of inequalities, group all of the related inequalities.

$$\begin{cases} 2f + 3h \leq 40 \\ 3f + h \leq 20 \\ f \geq 0 \\ h \geq 0 \end{cases}$$

k. Is it possible to make 3 figure skates and 4 hockey skates given the constraints of this situation?

Substitute 3 for f and 4 for h in each inequality of the system.

$2f + 3h \leq 40$	First inequality in the system
$2(3) + 3(4) \leq 40$	Substitute values for f and h, then multiply.
$6 + 12 \leq 40$	Simplify.
$18 \leq 40$	This is a true statement.
$3f + h \leq 20$	Second inequality in the system
$3(3) + (4) \leq 20$	Substitute values for f and h, then multiply.
$9 + 4 \leq 20$	Simplify.
$13 \leq 20$	This is also a true statement.
$f \geq 0$	Third inequality; substitute 3 for f.
$3 \geq 0$	This is a true statement.
$h \geq 0$	Last inequality; substitute 4 for h.
$4 \geq 0$	This is a true statement.

The substitution of 3 for f and 4 for h results in true statements for each inequality of the system. Therefore, it is possible to make 3 figure skates and 4 hockey skates given the constraints of this situation.

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Instruction

l. Is it possible to make 8 figure skates and 5 hockey skates given the constraints of this situation?

Substitute 8 for f and 5 for h in each inequality of the system.

$2f + 3h \leq 40$	First inequality in the system
$2(8) + 3(5) \leq 40$	Substitute values for f and h, then multiply.
$16 + 15 \leq 40$	Simplify.
$31 \leq 40$	This is a true statement.
$3f + h \leq 20$	Second inequality in the system
$3(8) + (5) \leq 20$	Substitute values for f and h, then multiply.
$24 + 5 \leq 20$	Simplify.
$29 \leq 20$	This is NOT a true statement.
$f \geq 0$	Third inequality; substitute 8 for f.
$8 \geq 0$	This is a true statement.
$h \geq 0$	Last inequality; substitute 5 for h.
$5 \geq 0$	This is a true statement.

The substitution of 8 for f and 5 for h results in true statements for 3 of the 4 inequalities of the system. Because one of the inequalities is not true, it is not possible to make 8 figure skates and 5 hockey skates given the constraints of this situation.

m. What is another possible combination of the number of figure skates and hockey skates that can be made given the constraints of this situation?

Students can use trial and error to find other possible combinations that will satisfy each of the inequalities in the system.

Possible solutions: 4 figure skates and 5 hockey skates; 2 figure skates and 10 hockey skates

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 1.4.1: Representing Constraints

For problems 1 and 2, determine if the given coordinate pair is a solution to the system of inequalities.

$$\begin{cases} x + y \leq 0 \\ 2x + 3y < 6 \\ y \geq -6 \end{cases}$$

1. (2, –1)

2. (–2, –4)

Read each scenario and use it to complete the parts that follow.

3. Given the inequalities $y > 5x - 8$ and $y \geq 3x + 4$, find a point that
   a. satisfies both inequalities.
   b. satisfies neither inequality.
   c. satisfies one inequality, but not the other.

4. It costs \$12 to get into the fair, plus \$3 per ticket for x ride tickets.
   a. Write an equation to find the total cost of attending the fair.
   b. Now write an inequality and solve it to determine the maximum number of tickets you can buy if you have \$24 to spend.
   c. Assuming you go to the fair, what is the minimum amount of money you will spend?

5. Charlie borrowed \$500 from his aunt. He has already paid back \$75. His aunt doesn't charge any interest and he is planning on paying her back \$15 every Friday.
   a. Write an equation that represents the number of weeks it will take Charlie to repay his aunt if he pays \$15 every Friday.
   b. Is the solution to the equation the actual number of weeks it will take Charlie to repay his aunt? Explain your answer.

continued

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Use the information in each scenario to complete problems 6–10.

6. The concession stand at the football game sells cans of soda for $0.75 and bottles of water for $1.25. You have $10.00. Write an inequality to represent this situation. What can you buy?

7. A stained glass artist has a fixed cost of $150. It costs the artist $15 to produce each piece, but each piece sells for $35. The equation $C = 150 + 15n$ represents the total cost, C, for producing n pieces. The total revenue for n pieces is determined by the equation $R = 35n$. What constraints are necessary to include when modeling this situation?

8. Your dad needs to rent a chain saw to cut down trees in your yard. The rental company charges $20 plus $6.50 per hour or part of an hour to rent the chain saw. Your dad wants to spend no more than $50. What constraints apply to this situation? What is the maximum number of hours your dad can rent the chain saw?

9. Jermaine has $10.00 to spend on ice cream. Three scoops cost $5.99, plus $0.75 for each topping. He always leaves a 20% tip for the cashier. Write an inequality and use it to determine if Jermaine can afford to buy a three-scoop ice cream with three toppings plus tip the cashier.

10. The local florist never has more than a combined total of 40 daisy bouquets and carnation bouquets, and she never has more than 12 carnation bouquets. Write a system of inequalities that represents this situation. Be sure to include all constraints.

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Assessment

Progress Assessment

Circle the letter of the best answer.

1. Which point is a solution to the system of inequalities $\begin{cases} y \geq -7x + 3 \\ y \geq \frac{2}{3}x - 3 \\ y \leq -x + 7 \end{cases}$?

   a. (2, 6)
   b. (–1, 8)
   c. (0, –3)
   d. (1, 4)

2. Given the system of inequalities in problem 1, which point is NOT a solution?

   a. (3, –1)
   b. (1, –3)
   c. (5, 1)
   d. (2, 4)

3. Given the inequalities $y < 2x - 6$ and $y \leq 4x + 5$, the point (2, 3) is:

   a. a solution to both inequalities
   b. a solution to $y < 2x - 6$ only
   c. a solution to $y \leq 4x + 5$ only
   d. not a solution of either of the inequalities

4. An online company is advertising a subscription service for downloads of e-books. The monthly fee is $6.50, plus $1.99 for each downloaded book. You can afford to spend no more than $15.00 each month on e-books. What is the maximum number of e-books you can download?

   a. 8
   b. 7
   c. 5
   d. 4

5. Used video games are advertised for $12.00 and new video games are advertised for $20.00. You have a gift card for $100.00. Which of the following game combinations can you NOT buy?

   a. 7 used video games and 1 new video game
   b. 4 used video games and 2 new video games
   c. 1 used video game and 4 new video games
   d. 0 used video games and 5 new video games

continued

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Assessment

6. An office manager needs to buy 25 laptops and has planned on spending $30,000. Two models are available; one costs $1,100, and the other costs $1,800. Assuming that x and y are integers, determine which system of inequalities represents this situation.

a. $\begin{cases} 1100x + 1800y \le 30,000 \\ x \ge 0 \\ y \ge 0 \end{cases}$

b. $\begin{cases} 1100x + 1800y \ge 30,000 \\ x \le 0 \\ y \le 0 \end{cases}$

c. $\begin{cases} 1100x + 1800y \ge 30,000 \\ x + y = 25 \\ x \le 0 \\ y \le 0 \end{cases}$

d. $\begin{cases} 1100x + 1800y \le 30,000 \\ x + y = 25 \\ x \ge 0 \\ y \ge 0 \end{cases}$

7. Carlos has a monthly budget of $250.00 for groceries. He spent $57.63 the first week, $46.89 the second week, and $68.13 the third week. Using g for groceries, how much can Carlos spend on groceries during the final week of the month and still stay within budget?

a. $g \le 77.35$

b. $g \ge 0$ and $g \le 77.35$

c. $g < 77.35$

d. $g > 0$ and $g < 77.35$

8. A science test worth 100 points has 10 questions. The test consists of fill-in-the-blank questions worth 6 points each and short-answer questions worth 16 points each. How many fill-in-the-blank questions are on the test?

a. 4

b. 6

c. 10

d. There is not enough information to determine the correct answer.

continued

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Assessment

9. Madison works part time at a local farm. She makes $10.00 an hour for cleaning the animals and $7.00 an hour for feeding them. Because Madison is a student, she cannot work more than 15 hours a week. Which system of inequalities represents the number of hours Madison will work if she wants to earn at least $150 a week?

a. $\begin{cases} x + y \le 15 \\ 10.00x + 7.00y \le 150 \\ x \ge 0 \\ y \ge 0 \end{cases}$

c. $\begin{cases} x + y \le 15 \\ 10.00x + 7.00y \le 150 \end{cases}$

b. $\begin{cases} x + y < 15 \\ 10.00x + 7.00y < 150 \\ x \ge 0 \\ y \ge 0 \end{cases}$

d. $\begin{cases} x + y \le 15 \\ 10.00x + 7.00y \ge 150 \\ x \ge 0 \\ y \ge 0 \end{cases}$

10. Jason loves going to the movies. Sometimes he is able to catch a matinee for $6.00, but otherwise he has to go at night for $9.50. If Jason went to see 22 movies this past month, what is the minimum and maximum amount of money he could have spent?

a. minimum: $0; maximum: $209

b. minimum: $0; maximum: $132

c. minimum: $132; maximum: $209

d. minimum: $60; maximum: $114

Use the information that follows to complete problem 11.

A computer company produces desktop and laptop computers. It takes 3 hours to build a desktop computer. It takes 2 hours to build a laptop computer. The maximum time allotted to this task is 60 hours per week.

The computer company then packages the computers to mail them to customers. It takes 2 hours to package a desktop computer. It takes 1 hour to package a laptop. The maximum time allotted for this task is 45 hours per week.

11. Write a system of inequalities to represent the number of computers that can be made and packaged in one week. Be sure to include all constraints.

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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. Solve the equation $8x + 4y = 12$ for y.

   a. $y = 2x - 3$
   b. $y = -2x + 3$
   c. $y = -3x + 2$
   d. $y = 3x - 2$

2. Solve the equation $-\frac{1}{5}y + 3x = 7$ for y.

   a. $y = 15x - 35$
   b. $y = -15x + 35$
   c. $y = 35 - 15x$
   d. $y = 35 + 15x$

3. The formula $P = 2l + 2w$ is used to calculate the perimeter of a rectangle. Solve this formula for l.

   a. $l = \frac{2}{2w - P}$
   b. $l = \frac{2}{P - 2w}$
   c. $l = \frac{2w - P}{2}$
   d. $l = \frac{P - 2w}{2}$

4. The formula $V = lwh$ is used to calculate the volume of a prism. Solve this formula for l.

   a. $l = \frac{wh}{V}$
   b. $l = V - wh$
   c. $l = \frac{V}{wh}$
   d. $l = wh - V$

5. The speed, v, of a point on the edge of a spinning disk is found using the formula $v = \frac{2\pi r}{T}$. Solve this formula for T.

   a. $T = \frac{v}{2\pi r}$
   b. $T = \frac{2\pi r}{v}$
   c. $T = 2\pi r - v$
   d. $T = v - 2\pi r$

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Lesson 5: Rearranging Formulas

Instruction

Common Core State Standard

SMP
1 ✓ 2 ✓
3 4
5 6
7 ✓ 8

A–CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*★

Essential Questions

1. How is solving a literal equation for a specific variable similar to solving an equation with one variable?
2. How could solving a literal equation or formula for a specific variable be helpful?
3. How do you determine for which variable a literal equation or formula should be solved?

WORDS TO KNOW

formula	a literal expression or equation that states a specific rule or relationship among quantities
inverse operation	an operation that reverses the effect of another operation. Addition and subtraction are inverse operations, and multiplication and division are inverse operations.
literal equation	an equation that involves two or more variables
multiplicative inverse	a number or algebraic expression that when multiplied by the original number or algebraic expression has a product of 1; also called the *reciprocal*
reciprocal	a number or algebraic expression that when multiplied by the original number or algebraic expression has a product of 1; also called the *multiplicative inverse*

Recommended Resources

- CRCTLessons.com. "Solving Equations Game."

  http://walch.com/rr/CAU1L5SolvingEquations

  Practice solving equations for a given variable with this online basketball game.

- Purplemath.com. "Solving Literal Equations."

  http://walch.com/rr/CAU1L5LitEquations

  This site has an overview of literal equations, with worked examples on how to solve equations and formulas for a given variable.

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Lesson 1.5.1: Rearranging Formulas

Warm-Up 1.5.1

Read the scenario below. Write an equation and use it to answer the questions that follow.

In January 2011, the national average price for 5 gallons of gasoline was $17.20.

1. What was the national average price for 1 gallon of gas?

2. What was the price for 18 gallons of gas?

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Instruction

Lesson 1.5.1: Rearranging Formulas

Common Core State Standard

A–CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*★

Warm-Up 1.5.1 Debrief

1. What was the national average price for 1 gallon of gas?

   Set up the equation.

   Let x represent the price per gallon of gasoline.

   $5x = 17.20$

   Using inverse operations, solve this equation for x.

   $5x = 17.20$ Original equation

   $\frac{5x}{5} = \frac{17.20}{5}$ Divide each side by 5.

   $x = 3.44$

   Interpret the solution.

   The average price of 1 gallon of gasoline was $3.44.

2. What was the price for 18 gallons of gas?

   To find the price of 18 gallons of gasoline, multiply the number of gallons by the price per gallon.

   $18 \cdot 3.44 = \$61.92$

   Interpret the solution.

   The price of 18 gallons of gasoline at $3.44 a gallon is $61.92.

Connection to the Lesson

- Students are asked to solve equations, much like they will be asked to solve a literal equation or formula for a specified variable.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- applying the order of operations (5.OA.1)
- solving multi-step equations (8.EE.7b)

Introduction

Literal equations are equations that involve two or more variables. Sometimes it is useful to rearrange or solve literal equations for a specific variable in order to find a solution to a given problem. In this lesson, literal equations, including common formulas, will be examined. A **formula** is a literal expression or equation that states a specific rule or relationship among quantities, such as the formula for volume or distance.

Key Concepts

- Literal equations such as formulas can be solved for a specific variable by isolating that variable.
- To isolate the specified variable, use inverse operations. **Inverse operations** reverse the effect of given operations. Addition and subtraction are inverse operations, and multiplication and division are inverse operations.
- When coefficients are fractions, multiply both sides of the equation by the reciprocal to eliminate the coefficient. The **reciprocal** of a number, also known as the **multiplicative inverse** of a number, can be found by flipping a fraction so the numerator and denominator switch places. Think of an integer as a fraction with a denominator of 1. The number 2 can be thought of as the fraction $\frac{2}{1}$. To find the reciprocal, flip the fraction: $\frac{2}{1}$ becomes $\frac{1}{2}$. You can check that you have the correct reciprocal because the product of a number and its reciprocal is always 1.

Common Errors/Misconceptions

- solving for the wrong variable
- improperly isolating the specified variable by using the incorrect inverse operation
- incorrectly simplifying terms

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Instruction

Guided Practice 1.5.1

Example 1

Solve $6y - 12x = 18$ for y.

1. Begin isolating y by adding $12x$ to both sides of the equation.

$$6y - 12x = 18$$
$$+12x \qquad +12x$$
$$6y = 18 + 12x$$

2. Divide both sides of the resulting equation by the coefficient of y.

The coefficient of y is 6.

$6y = 18 + 12x$	Equation from the previous step
$\frac{6y}{6} = \frac{18+12x}{6}$	Divide both sides of the equation by 6.
$\frac{6y}{6} = \frac{18}{6} + \frac{12x}{6}$	Separate the fraction on the right side.
$y = 3 + 2x$	Simplify each fraction.

$6y - 12x = 18$ solved for y is $y = 3 + 2x$.

Example 2

Solve $15x - 5y = 25$ for y.

1. Begin isolating y by subtracting $15x$ from both sides of the equation.

$$15x - 5y = 25$$
$$-15x \qquad -15x$$
$$-5y = 25 - 15x$$

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2. Divide both sides of the resulting equation by the coefficient of y.

The coefficient of y is –5.

$-5y = 25 - 15x$	Equation from the previous step
$\frac{-5y}{-5} = \frac{25-15x}{-5}$	Divide both sides of the equation by –5.
$\frac{-5y}{-5} = \frac{25}{-5} - \frac{15x}{-5}$	Separate the fraction on the right.
$y = -5 + 3x$	Simplify each fraction.

$15x - 5y = 25$ solved for y is $y = -5 + 3x$.

Example 3

Solve $4y + 3x = 16$ for y.

1. Begin isolating y by subtracting $3x$ from both sides of the equation.

$$\begin{array}{r} 4y+3x=16 \\ \underline{-3x \quad -3x} \\ 4y=16-3x \end{array}$$

2. Divide both sides of the resulting equation by the coefficient of y.

The coefficient of y is 4.

$4y = 16 - 3x$	Equation from the previous step
$\frac{4y}{4} = \frac{16-3x}{4}$	Divide both sides of the equation by 4.
$y = \frac{16}{4} - \frac{3x}{4}$	Separate the fraction on the right.
$y = 4 - \frac{3}{4}x$	Simplify each fraction.

$4y + 3x = 16$ solved for y is $y = 4 - \frac{3}{4}x$.

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Instruction

Example 4

The formula for finding the area of a triangle is $A=\frac{1}{2}bh$, where b is the length of the base and h is the height of the triangle. Suppose you know the area and height of the triangle, but need to find the length of the base. In this case, solving the formula for b would be helpful. Solve the area formula for b, then write a sentence describing how to find the base of a triangle given the area and height.

1. Begin isolating b by multiplying both sides of the equation by the reciprocal of $\frac{1}{2}$, or 2.

$A=\frac{1}{2}bh$	Formula for the area of a triangle
$2\bullet A=2\bullet\left(\frac{1}{2}bh\right)$	Multiply both sides by 2.
$2A=bh$	Simplify.

Multiplying both sides of the equation by the reciprocal is the same as dividing both sides of the equation by $\frac{1}{2}$. The result will be the same because dividing by a fraction is the same as multiplying by its reciprocal.

2. Divide both sides of the resulting equation by h.

$2A = bh$	Equation from the previous step
$\frac{2A}{h}=\frac{bh}{h}$	Divide both sides by h.
$\frac{2A}{h}=b$ or $b=\frac{2A}{h}$	Simplify.

The formula $A=\frac{1}{2}bh$ solved for the base, b, is $b=\frac{2A}{h}$.

3. Summarize your results.

The base of a triangle can be found by doubling the area and dividing the result by the height of the triangle.

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Example 5

The distance, d, that a train can travel is found by multiplying the speed, r, by the amount of time that it is travelling, t, or $d = rt$. Solve this formula for t to find the amount of time the train will travel given a specific distance at a given speed.

1. Isolate t by dividing both sides of the equation by r.

$d = rt$	Formula for distance
$\frac{d}{r} = \frac{rt}{r}$	Divide both sides of the equation by r.
$t = \frac{d}{r}$	Simplify.

The formula $d = rt$ solved for t is $t = \frac{d}{r}$.

2. Summarize your results.

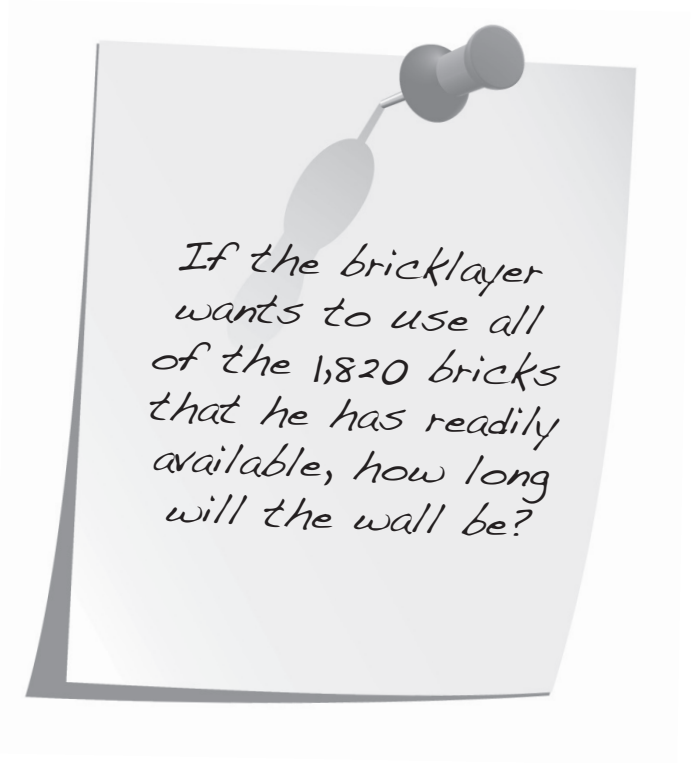
The formula for finding the amount of time it will take a train to travel a given distance at a given speed is $t = \frac{d}{r}$.

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Problem-Based Task 1.5.1: Bricklayers

SMP	
1 ✓	2 ✓
3	4
5	6
7 ✓	8

The formula $N = 7LH$ is used to determine N, the number of bricks needed to build a wall that is L feet in length and H feet high. A customer would like a wall constructed that is 4 feet high. If the bricklayer wants to use all of the 1,820 bricks that he has readily available, how long will the wall be?



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Problem-Based Task 1.5.1: Bricklayers

Coaching

a. What does each of the variables represent?

b. What variable is the given formula solved for?

c. Which variable does the formula need to be solved for?

d. Solve the given formula for the unknown variable.

e. What values are given in the problem statement?

f. How long can the wall be if the bricklayer has 1,820 bricks and the wall must be 4 feet high?

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Instruction

Problem-Based Task 1.5.1: Bricklayers

Coaching Sample Responses

a. What does each of the variables represent?

N represents the number of bricks. *L* represents the length of the wall in feet. *H* represents the height of the wall.

b. What variable is the given formula solved for?

The formula is solved for *N*, the number of bricks.

c. Which variable does the formula need to be solved for?

The formula needs to be solved for *L*, the length of the wall.

d. Solve the given formula for the unknown variable.

Solve $N = 7LH$ for L.

$$N = 7LH$$
$$\frac{N}{7H} = \frac{7LH}{7H}$$
$$L = \frac{N}{7H}$$

e. What values are given in the problem statement?

The number of bricks, *N*, is 1,820. The height of the wall, *H*, is 4 feet.

f. How long can the wall be if the bricklayer has 1,820 bricks and the wall must be 4 feet high?

Substitute the known values into the formula from part d.

$$L = \frac{N}{7H}$$
$$= \frac{1820}{7(4)}$$
$$= \frac{1820}{28}$$
$$= 65$$

The length of the wall, *L*, can be 65 feet.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 1.5.1: Rearranging Formulas

For problems 1–4, solve each equation for y.

1. $9y + 18 = 27x$

2. $6y + 24x = 66$

3. $10x - 77 = 7y$

4. $44 - 4y = 20x$

For problems 5–10, read each scenario and solve for the given variable.

5. To convert degrees Celsius to Kelvin, the formula $K = C + 273.15$ is used. Solve this formula for C.

6. The formula $C = 2\pi r$ is used to calculate the circumference of a circle. Solve this formula for r.

7. The formula $V = lwh$ is used to calculate the volume of a rectangular prism. Solve this formula for w.

8. The formula $S = 2\pi r^2 + 2\pi rh$ is used to find the surface area of a right circular cylinder. Solve this formula for h.

9. The formula for converting degrees Celsius to degrees Fahrenheit is $F = \frac{9}{5}C + 32$. Solve this formula for C.

10. The formula for calculating the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Solve this formula for h.

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Progress Assessment

Circle the letter of the best answer.

1. In the equation $4x - 2y = 19$, a possible first step to isolating y is:
   a. Divide both sides of the equation by $2y$.
   b. Subtract $4x$ from both sides of the equation.
   c. Divide both sides of the equation by $4x$.
   d. Subtract y from both sides of the equation.

2. Solve the equation $\frac{2}{3}y + x = 4$ for y.
   a. $y = \frac{10}{3} - x$
   b. $y = x - \frac{10}{3}$
   c. $y = \frac{3}{2}(x - 4)$
   d. $y = \frac{3}{2}(4 - x)$

3. Solve the equation $7x - 3y = 16$ for y.
   a. $y = \frac{7x - 16}{3}$
   b. $y = \frac{16 - 7x}{3}$
   c. $y = \frac{3x - 16}{7}$
   d. $y = \frac{16 - 3x}{7}$

4. Solve the equation $6y + 12x = 18$ for y.
   a. $y = -3x + 2$
   b. $y = 2x - 3$
   c. $y = -2x + 3$
   d. $y = 3x - 2$

5. Solve the equation $6x + \frac{3}{4}y = 9$ for y.
   a. $y = \frac{27x - 18}{4}$
   b. $y = 8x - 12$
   c. $y = \frac{-27x + 18}{4}$
   d. $y = -8x + 12$

continued

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Assessment

6. The formula for calculating simple interest is $I = prt$. Solve this formula for r.

   a. $r = I - pt$
   b. $r = \frac{I}{pt}$
   c. $r = \frac{pt}{I}$
   d. $r = pt - I$

7. The formula for calculating the volume of a circular cylinder is $V = \pi r^2 h$. Solve this formula for h.

   a. $h = \frac{\pi r^2}{V}$
   b. $h = V - \pi r^2$
   c. $h = \frac{V}{\pi r^2}$
   d. $h = \pi r^2 - V$

8. The formula for calculating the surface area of a cone is $A = \pi r^2 + \pi rl$. Solve this formula for l.

   a. $l = \frac{A - \pi r^2}{\pi r}$
   b. $l = \frac{\pi r}{A - \pi r^2}$
   c. $l = \frac{A}{\pi r^2} - \pi r$
   d. $l = \frac{\pi r^2}{A} - \pi r$

9. Solve the formula $B = \frac{(R + M)}{2}$ for R.

   a. $R = 2B + M$
   b. $R = \frac{2B}{M}$
   c. $R = \frac{M}{2B}$
   d. $R = 2B - M$

10. Solve the formula $r^3 = \frac{3V}{4\pi}$ for V.

    a. $V = \frac{4\pi r^3}{3}$
    b. $V = \frac{3r^3}{4\pi}$
    c. $V = 4\pi r^3 - 3$
    d. $V = 12r^3\pi$

Use your knowledge of rearranging formulas to answer the question that follows.

11. The formula for calculating the number of calories burned while swimming at a moderate rate is $C = 2.9wt$, where C represents the number of calories burned, w represents your weight in pounds, and t represents the time you swim in hours. How could you use this formula to help you determine how long you should swim in order to burn a specific number of calories?

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Unit 1 Assessment

Circle the letter of the best answer.

1. Once simplified, how many terms are in the expression $22x^3+14x^2-10x^2+3x+7$?

   a. 5
   c. 3
   b. 4
   d. 2

2. The product of –3, a, and b is represented by the expression $-3ab$. If the value of a is negative, what must be said about the value of b in order for the product to remain negative?

   a. b must be 0.
   c. b must be negative.
   b. b must be positive.
   d. The value of b does not matter.

3. A family's cell phone plan costs $70 per month for 1,300 minutes and 40 cents per minute over the limit. This month, the family paid $118.40. By how much time did they exceed their plan?

   a. 121 minutes
   c. 20 minutes
   b. 471 minutes
   d. 76 minutes

4. You have no more than $60 to spend. You want a drink that costs $1.50 including tax, and you want to buy a pair of pants, which will have 4% sales tax. What is the inequality that represents the pre-tax price of a pair of pants that you can afford if you buy the drink?

   a. $x + 0.04x + 1.50 > 60$
   c. $x + 0.04x + 1.50 < 60$
   b. $x + 0.04x + 1.50 \geq 60$
   d. $x + 0.04x + 1.50 \leq 60$

5. A store has a display with pencils that are for sale. The owner typically sells 6 pencils a day. The display currently has 50 pencils. The owner insists that there be no fewer than 32 pencils in the display. When should the owner restock the display?

   a. in more than 3 days
   c. in 3 days or less
   b. in less than 3 days
   d. in 3 days or more

6. A type of bacteria doubles its numbers every 7 hours. If you started with 16 bacteria in a Petri dish, how many bacteria would you have after 56 hours?

   a. 2,048 bacteria
   c. $7.2 \bullet 10^{16}$ bacteria
   b. 4,096 bacteria
   d. $1.15 \bullet 10^{18}$ bacteria

continued

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7. A form of the element actinium, Ac-225, has a half-life of 10 days. At the end of an experiment that lasted 40 days, there were 2 grams of Ac-225 left. How much Ac-225 was there at the beginning of the experiment?

   a. $2.2 \bullet 10^{12}$ grams

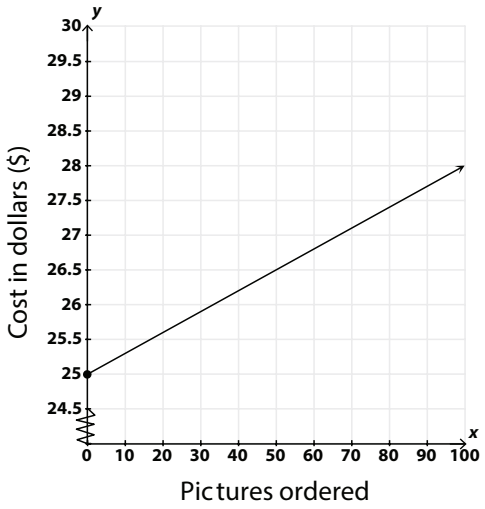
   b. 32 grams

   c. 0.125 grams

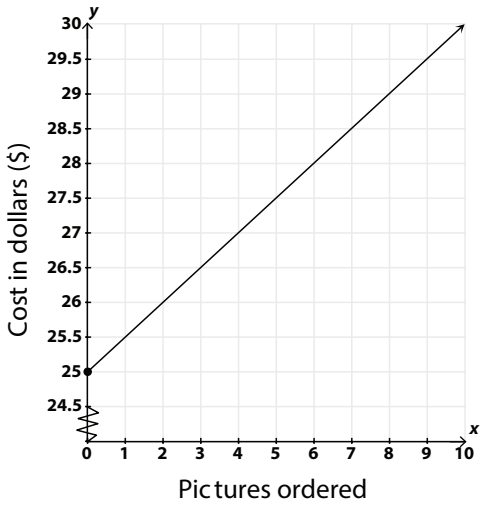
   d. 16 grams

8. A photo service charges $25.00 a year as well as $0.05 for each photo ordered. Which graph models the total cost of ordering pictures for a year?

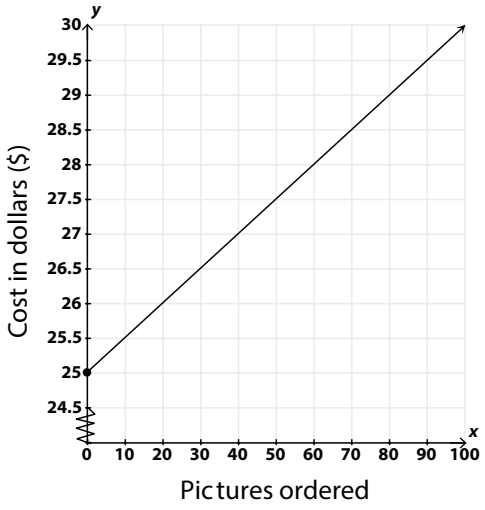
a.



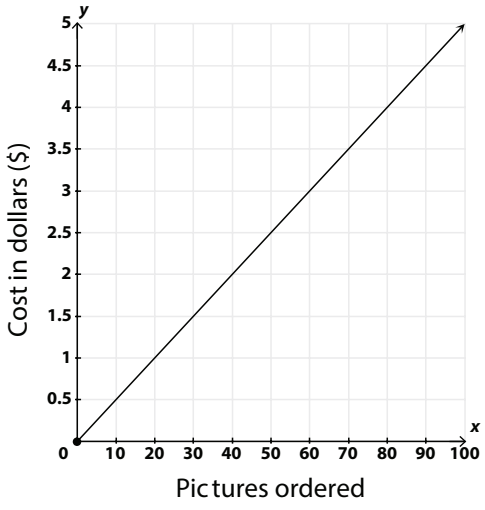
c.



b.



d.

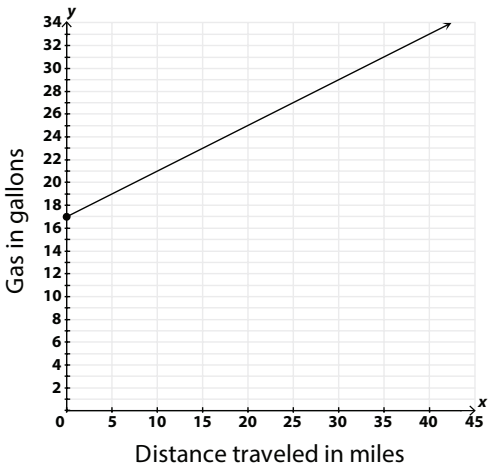


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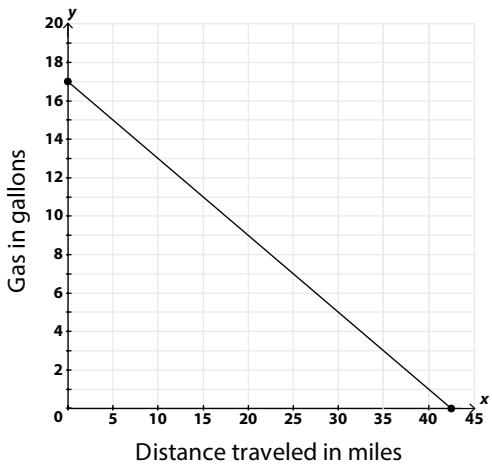
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9. A 4-door sedan holds 17 gallons of gas and the tank averages 0.04 gallons per mile. Which graph models the amount of gas left in the tank, assuming the tank was full at the beginning?

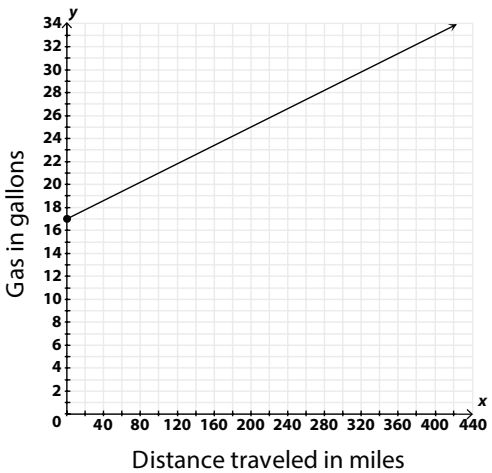
a.



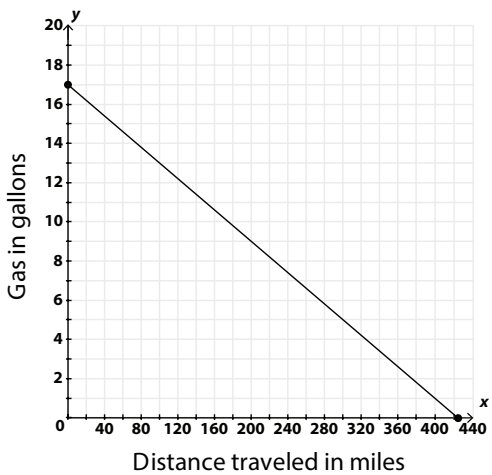
c.



b.



d.

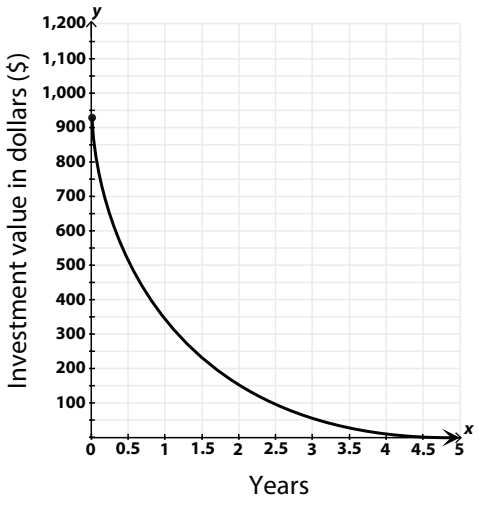


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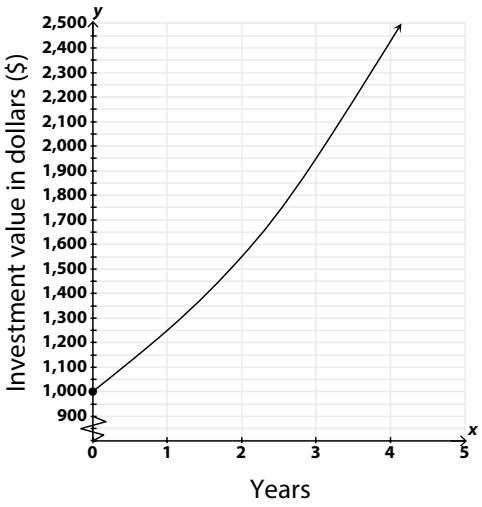
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10. An investment of $1,000 is compounded monthly at an annual rate of 2.5%. Which graph models the change of the investment over time?

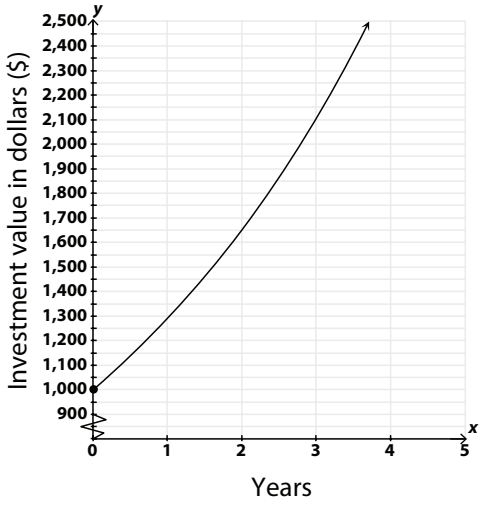
a.



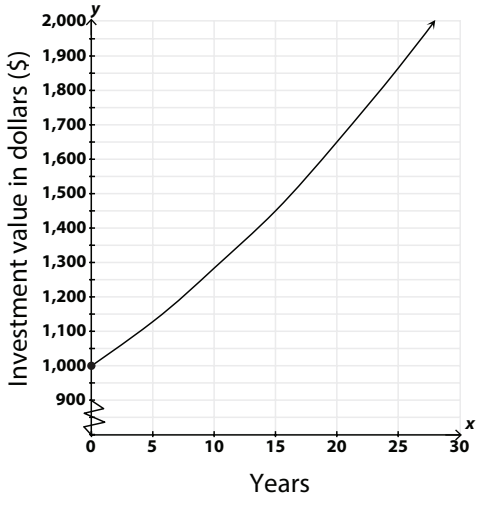
c.



b.



d.



continued

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11. Your doctor told you to eat at least 70 milligrams of vitamin C each day. One tomato contains 16 milligrams of vitamin C, while one potato contains 17 milligrams. Determine which system of inequalities represents the number of tomatoes and potatoes you must eat in order to reach your minimum recommended amount of vitamin C.

a. $\begin{cases} 16x+17y \le 70 \\ x \le 0 \\ y \le 0 \end{cases}$

c. $\begin{cases} 16x+17y \ge 70 \\ x \ge 0 \\ y \ge 0 \end{cases}$

b. $\begin{cases} 16x+17y \le 70 \\ x \ge 0 \\ y \ge 0 \end{cases}$

d. $\begin{cases} 16x+17y \ge 70 \\ x \le 0 \\ y \le 0 \end{cases}$

12. The formula for calculating a person's body mass index is $B=\frac{w}{h^2}$, for which w represents weight in kilograms and h represents height in meters. Solve this formula for w.

a. $w=Bh^2$

c. $w=\frac{B}{h^2}$

b. $w=B-h^2$

d. $w=(Bh)^2$

For problems 13–15, read each scenario and answer the questions that follow. Write your answers below each lettered part and show your work in the space provided.

13. Keisha bought 6 tickets to an indoor water park. She paid a 5% service charge for buying them online. Her total cost was $252.

    a. What equation can be used to model the total cost of the tickets as a function of the number of tickets purchased?

    b. What was the price of each ticket?

    c. How much did Keisha pay in service charges?

continued

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14. Min-Ji injured her elbow during a varsity volleyball game. Her doctor has recommended physical therapy several times a week. Min-Ji's parents want to plan for the potential cost of therapy over the course of a month. They pay $160 a month for insurance and then another $20 office fee each time Min-Ji goes to physical therapy.

    a. What equation models the total cost for insurance and physical therapy? Let x be the number of physical therapy visits and let y be the total cost.

    b. What does the graph of the equation look like? Graph the equation on the provided grid. Be sure to label the axes.



    c. How much will it cost for one month (4 weeks) of physical therapy if Min-Ji goes to physical therapy 3 times per week? Explain your answer.

continued

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15. The population of Georgia is growing at a yearly rate of about 1.3%. The current population is about 9,815,210 people. The population of North Carolina is growing at a yearly rate of about 1.25%. North Carolina's current population is about 9,656,401 people.

    a. What is the equation that models Georgia's population growth?

    b. What is the equation that models North Carolina's population growth?

    c. What do the graphs of the equations look like? Graph the equations on the provided grid. Be sure to label the axes.



    d. Write a few sentences comparing the population models of Georgia and North Carolina. What can you conclude based on your models?

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Answer Key

Lesson 1: Interpreting Structure in Expressions

Pre-Assessment, p. U1-1

1. c
2. a
3. d
4. c
5. b

Warm-Up 1.1.1, p. U1-5

1. $2d$
2. $2d + 3c$
3. $2d + 3c + 5.60$

Practice 1.1.1: Identifying Terms, Factors, and Coefficients, p. U1-16

1. terms: $12a^3$, $16a$, 4
   factors: 12 and a^3, 16 and a
   coefficients: 12, 16
   constant term: 4
2. terms: $6x^2$, $3x$, 9
   factors: 6 and x^2, 3 and x
   coefficient: 6, 3
   constant term: 9
3. expression: $((x + y)/2) – y/3 = (1/2)x + (1/6)y$
   terms: $(1/2)x$, $(1/6)y$
   factors: 1/2 and x, 1/6 and y
   coefficients: 1/2, 1/6
   constant term: none
4. expression: $5x^3 + (6 – x^3) = 4x^3 + 6$
   terms: $4x^3$, 6
   factors: 4 and x^3
   coefficient: 4
   constant term: 6
5. Answers may vary. Sample answer: $3x^3 + 6x^2 + 9x + 4$
6. expression: $6x – 0.15(6x) = 5.1x$
   term: $5.1x$
   factors: 5.1 and x
   coefficient: 5.1
   constant term: none
7. expression: $10x – 0.20(10x) + 3.99 = 8x + 3.99$
   terms: $8x$, 3.99
   factors: 8 and x
   coefficient: 8
   constant term: 3.99
8. expression: $(30.24 – 2.24)/(x + 1) = 28/(x + 1)$
   term: $28/(x + 1)$
   factors: 28 and $1/(x + 1)$
   coefficient: 28
   constant term: none
9. expression: $1/2(b_1 + b_2)h$ or $1/2(b_1)h + 1/2(b_2)h$
   terms: $1/2(b_1)h$, $1/2(b_2)h$
   factors: 1/2, b_1, h and 1/2, b_2, h
   coefficients: 1/2, 1/2
   constant term: none
10. expression: $2\pi r^2 + 2\pi rh$
    terms: $2\pi r^2$, $2\pi rh$
    factors: 2, π, r^2 and 2, π, r, h
    coefficients: 2π, 2π
    constant term: none

Warm-Up 1.1.2, p. U1-17

1. \$112.50
2. \$862.50

Practice 1.1.2: Interpreting Linear and Exponential Expressions, p. U1-26

1. No; the only time the expressions will be equal is when $x = 1$.
2. No; exponents must be applied before carrying out multiplication.
3. Yes; the operations within the parentheses must be carried out before applying the exponent.
4. The cost of the permit is not affected by the number of cubic yards. The cost of the permit remains the same and is described using a constant in the expression given.
5. A lower service fee will result in a constant less than 34.75.
6. The value of y must be greater than or equal to 0 as it represents the number of weeks. There is also an upper limit on the value of y: the number of weeks it will take for the dog to reach a healthy weight.
7. The value of y must also be negative in order for the product of 7, x, and y to be positive.
8. The amount will be decreased.
9. x must be greater than 0
10. Changing the value of r does not have an effect on the value of C because the amount of air the tire can hold is not affected by the rate at which it loses air.

Progress Assessment, p. U1-28

1. a
2. c
3. c
4. d
5. b
6. c
7. a
8. c
9. b
10. d
11. The expression a^xb has x factors of a and one factor of b. The expression ab^x has one factor of a and x factors of b. The expression $(ab)^x$ has x factors of a and x factors of b.

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Lesson 2: Creating Equations and Inequalities in One Variable

Pre-Assessment, p. U1-30

1. b
2. d
3. c
4. a
5. b

Warm-Up 1.2.1, p. U1-33

1. $10x$
2. $10x - 0.25(10x)$
3. $10x - 0.25(10x) + 5.99$

Practice 1.2.1: Creating Linear Equations in One Variable, p. U1-52

1. Answers may vary. Possible answers:
   a. miles per hour
   b. inches per hour
   c. gallons per second
   d. calories per day
2. 99.4 ft^2
3. 6 hours per day
4. $3.50 per gallon
5. $25 per ticket
6. $39 per month
7. $103
8. 48 ft^2
9. $35t = 40(0.75 - t)$; $t = 0.4$; $d = (35)(0.4) = 14$ miles. The distance from your house to your friend's house is 14 miles.
10. $400 = 5r + 5(r + 15)$; $r = 32.5$ mph. One train is traveling at 32.5 miles per hour and the other is traveling at 47.5 miles per hour.

Warm-Up 1.2.2, p. U1-54

1. Eric weighs about 104 pounds.
2. If the seesaw is 5 feet or longer, then Eric can balance the seesaw.

Practice 1.2.2: Creating Linear Inequalities in One Variable, p. U1-67

1. $x \leq 55$
2. $x \leq 60$
3. $19 + x \geq 39$; $x \geq 20$; She needs to save $20 or more.
4. $20x \geq 1800$; $x \geq 90$; You must work 90 hours or more.
5. $83 + x \geq 180$; $x \geq 97$; Hiram must get a score of 97 points or more.
6. $15x + 10 \leq 100$; $x \leq 6$; Claire can buy up to 6 DVDs.
7. $53 - 1.5h \leq 32$; $h \geq 14$ hours. The rain will likely change over to sleet at or after 14 hours from 8:30 A.M., which means the changeover will occur at or after 10:30 P.M.
8. $17 - 3x \geq 5$, where x is time in 5-minute increments; $x \leq 4$, so 5(4) = 20 minutes. After the first hour, the players will be able to continue for 20 minutes or less.
9. $15{,}000 - 2000x \geq 3000$; $x \leq 6$ days; the giveaway can last for 6 days or less.
10. Answers may vary. Sample answer: You have no money left in your wallet but found a gift card for the local deli that has $9. You want to make sure you spend less than that for 2 lunch specials for you and your friend. Your friend gave you $5 toward the bill. In this scenario, each lunch special must cost $7 or less; $x \leq 7$.

Warm-Up 1.2.3, p. U1-69

Inequality: $70x + 100 > 500$
Solution: $x > 5$
School started more than 5 months ago.

Practice 1.2.3: Creating Exponential Equations, p. U1-84

1. a. linear; b. exponential
2. a. linear; b. exponential
3. $y = 100(2)^7$; 12,800 insects
4. $1{,}073{,}741{,}824 = a(2)^{24}$; 64 bacteria
5. $y = 125(0.5)^4$; $7.81
6. $75 = a(0.5)^3$; $600
7. $5000 = a(0.5)^2$; $20,000
8. $y = 15(3)^7$; 32,805 dandelions
9. $y = 12{,}000(1.02)^4$; about 12,990 people
10. $y = 4(2)^4$; 64,000,000 people if rush hour continued in the morning, and $y = 8(0.5)^4$; 500,000 people if rush hour continued in the evening

Progress Assessment, p. U1-86

1. d
2. c
3. d
4. a
5. a
6. d
7. b
8. b
9. a
10. a
11. $65h + 99 = 261.50$; $h = 2.5$ hours. The mechanic worked on your car for 2 1/2 hours.

Lesson 3: Creating and Graphing Equations in Two Variables

Pre-Assessment, p. U1-89

1. d
2. a
3. c
4. b
5. d

Warm-Up 1.3.1, p. U1-94

1. Table of values:

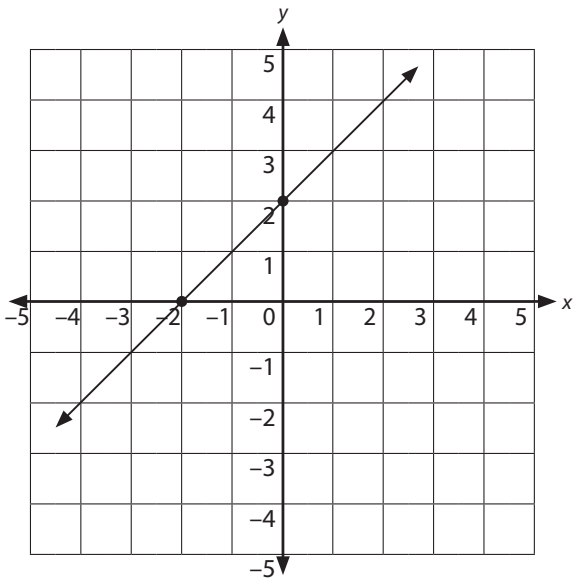
Minutes used	Total amount charged ($)
0	20 + 0(0.05) = 20.00
10	20 + 10(0.05) = 20.50
20	20 + 20(0.05) = 21.00
30	20 + 30(0.05) = 21.50
40	20 + 40(0.05) = 22.00
50	20 + 50(0.05) = 22.50
60	20 + 60(0.05) = 23.00

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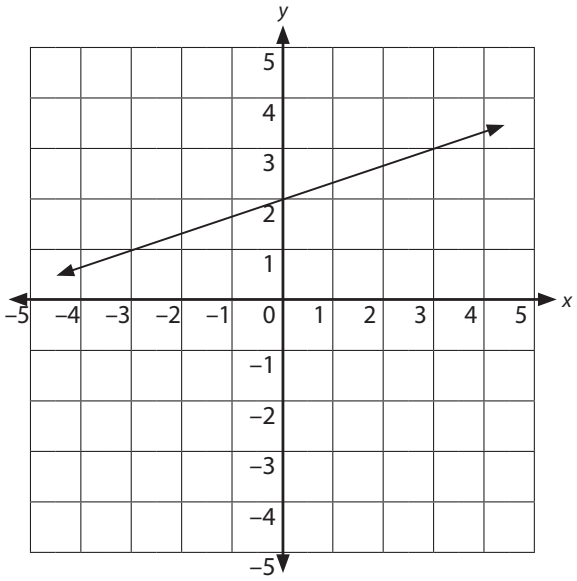
2. $y = 0.05x + 20$
3. x represents the number of minutes used, and y represents the total amount charged.

Practice 1.3.1: Creating and Graphing Linear Equations in Two Variables, p. U1-128

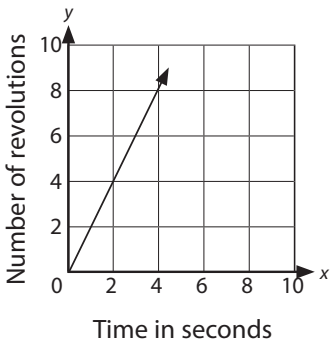
1.



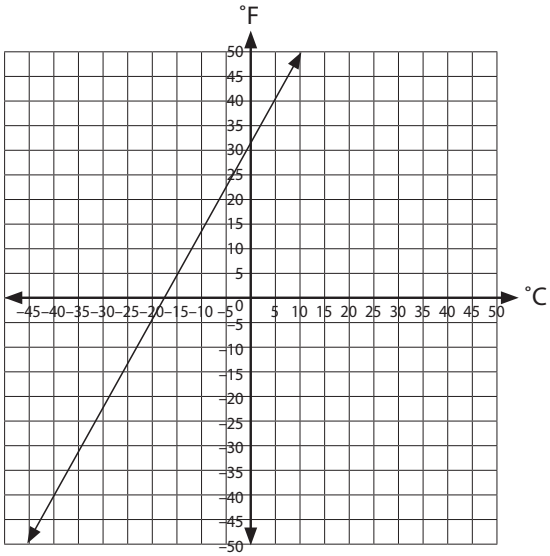
2.



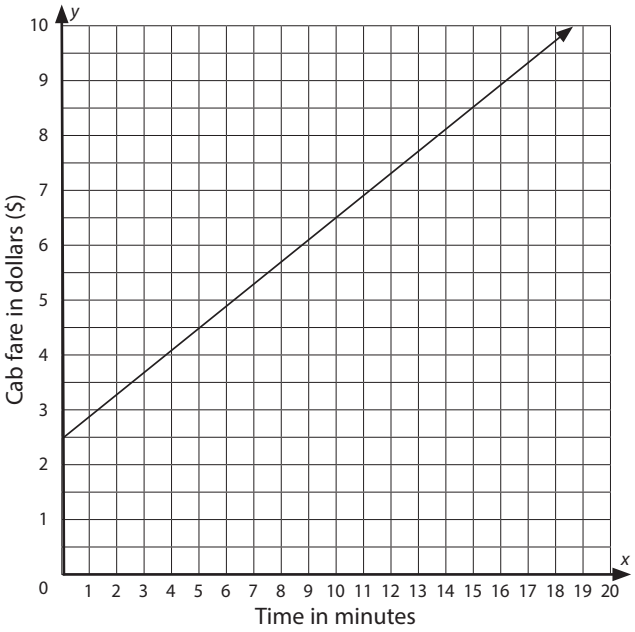
3. $y = 2x$; points could include (0, 0) and (1, 2)



4. $y = (9/5)x + 32$; points could include (0, 32) and (5, 41)

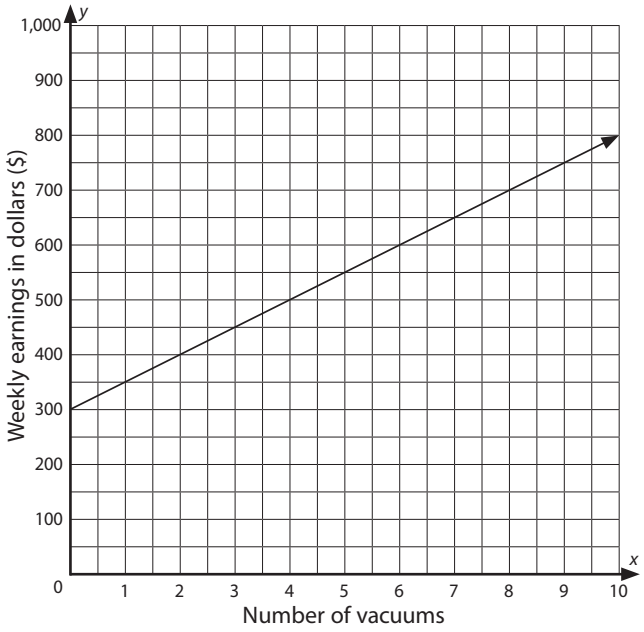


5. $y = 0.4x + 2.5$; points could include (0, 2.5) and (5, 4.5)

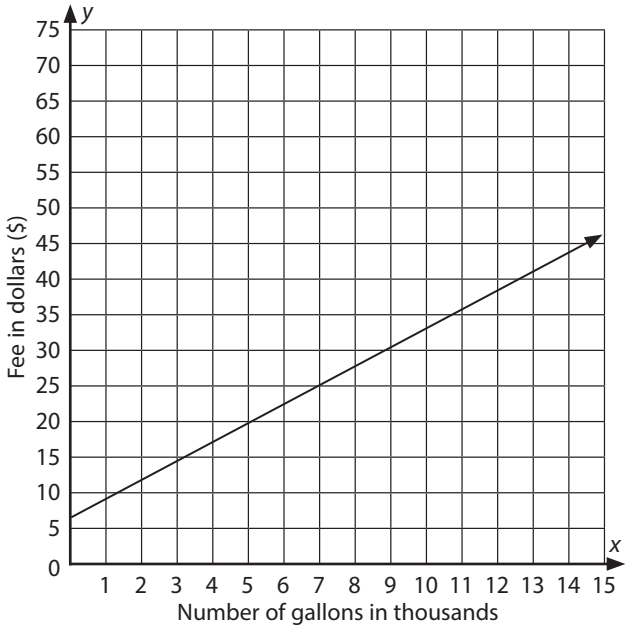


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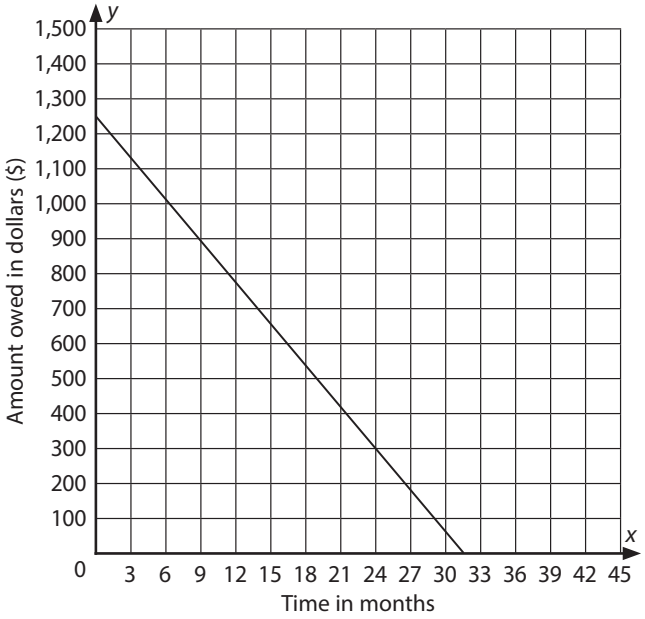
6. $y = 50x + 300$; points could include (0, 300) and (1, 350)



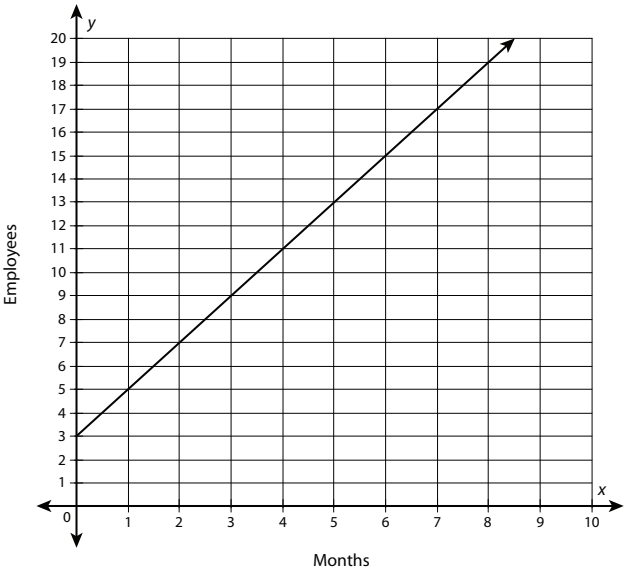
7. $y = 2.6x + 6.7$; points could include (0, 6.7) and (10, 32.7)



8. $y = -40x + 1250$; points could include (0, 1250) and (3, 1130)

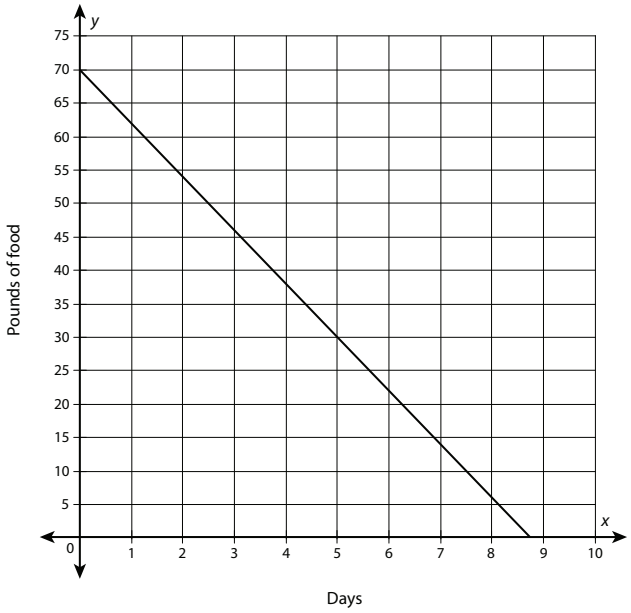


9. $y = (16/8)\,x + 3 = 2x + 3$



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10. $y = -8x + 70$

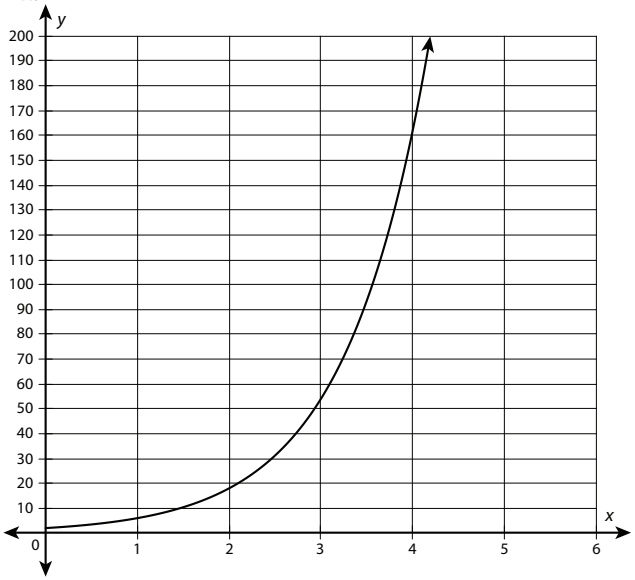


Warm-Up 1.3.2, p. U1-129

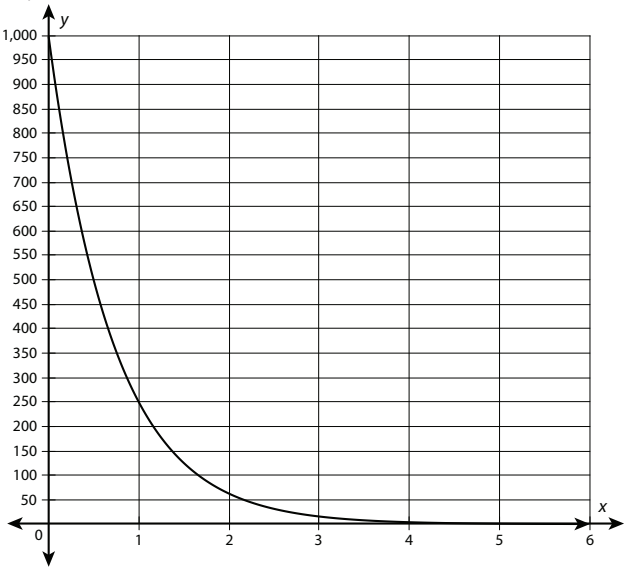
1. $y = 128(0.5)^5$
2. $y = 4$ grams

Practice 1.3.2: Creating and Graphing Exponential Equations, p. U1-148

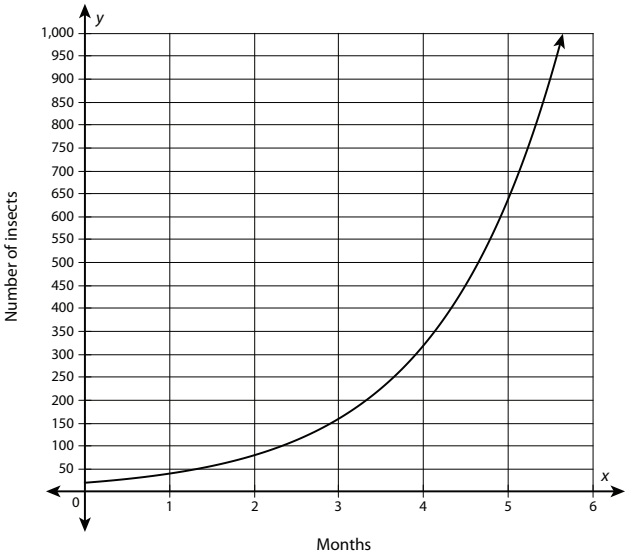
1.



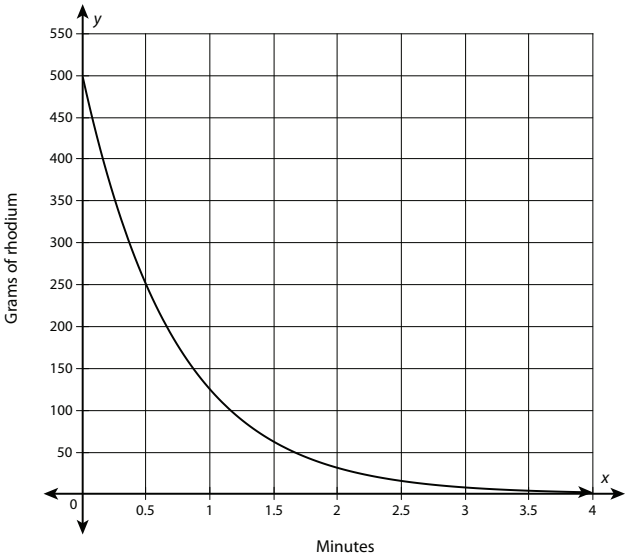
2.



3. $y = 20(2)^x$

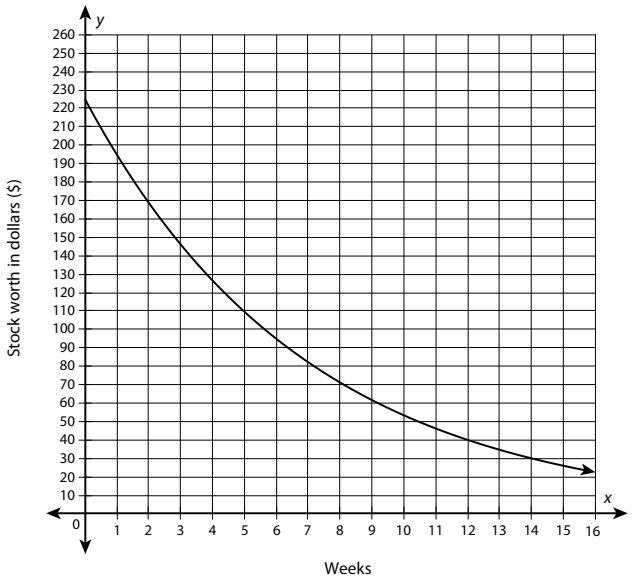


4. $y = 500(0.5)^{2x}$, for which x is in minutes

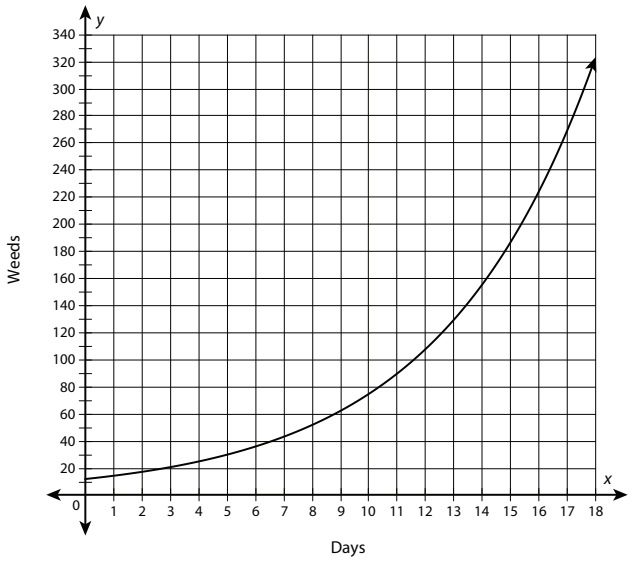


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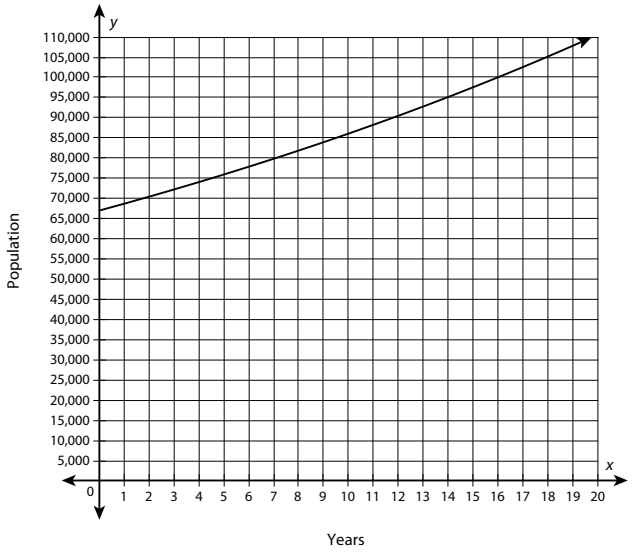
5. $y = 225(0.75)^{x/2}$, for which x is in weeks



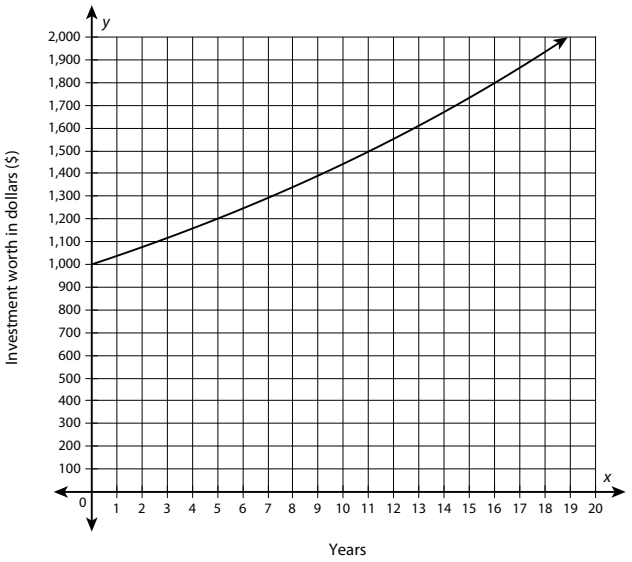
6. $y = 12(3)^{x/6}$, for which x is in days



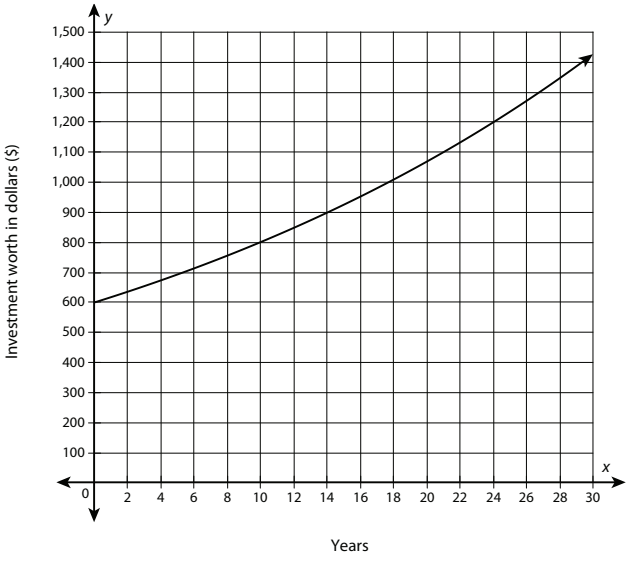
7. $y = 67{,}000(1.025)^{x}$



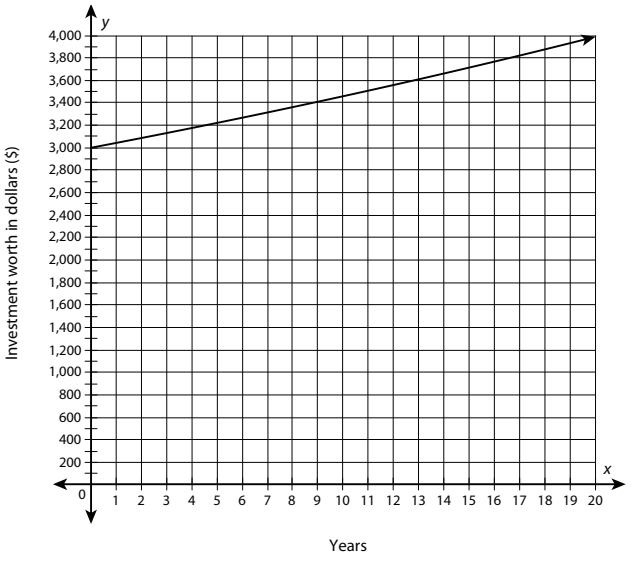
8. $y = 1000(1.0185)^{2x}$



9. $y = 600(1.00725)^{4x}$



10. $y = 3000(1.00027)^{52x}$

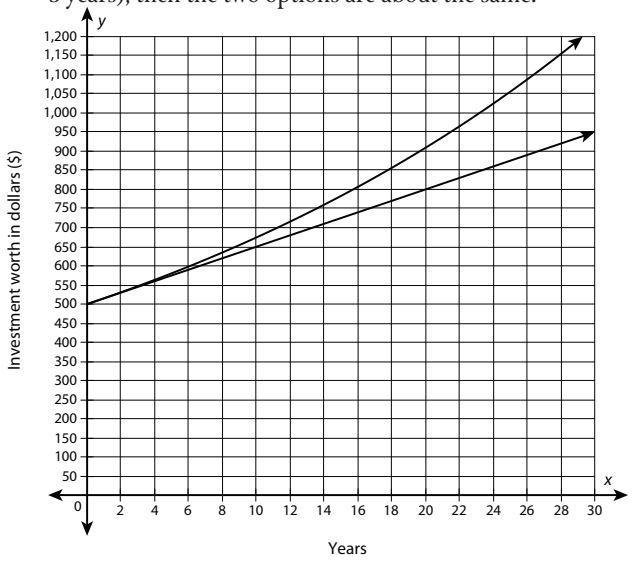


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Progress Assessment, p. U1-149

1. b
2. b
3. a
4. d
5. d
6. b
7. c
8. a
9. c
10. b
11. The first investment equation is $y = 500 + 0.03(500)x$. The second investment equation is $y = 500(1 + 0.03/4)^{4x} = 500(1.0075)^{4x}$. The first investment has a linear rate of growth, while the second equation has an exponential rate of growth. Since the second equation has the variable in the exponent, the investment should grow more quickly, but the rate is very small, 3%. Graphing the equations illustrates the growth.

    From the graph, the investments are about the same for the first 3 years, but after the fourth year the second investment starts to take off and grow much more rapidly. If this is going to be a long-term investment, then the better choice is the second investment option that compounds quarterly. If it will be a short-term investment (less than 3 years), then the two options are about the same.



Lesson 4: Representing Constraints

Pre-Assessment, p. U1-155

1. d
2. d
3. a
4. d
5. c

Warm-Up 1.4.1, p. U1-157

1. $3.41 per gallon
2. No, she will not have enough money.

Practice 1.4.1: Representing Constraints, p. U1-170

1. no
2. yes
3. Answers will vary. Sample responses: a. (–1, 2); b. (4, 2); c. (1, 3)
4. a. $y = 3x + 12$; b. 4 tickets; c. You will spend no less than $12.
5. a. $15x = 425$
   b. The solution to the equation ($x = 28.3\overline{3}$) is not the actual number of weeks it will take Charlie to repay his aunt. Charlie makes payments each Friday, so he will have to pay her the remaining balance on the 29th week.
6. Inequality: $0.75s + 1.25w \leq 10.00$. Answers will vary. You can buy a maximum of 13 cans of soda if you don't buy any water. You can buy a maximum of 8 bottles of water if you don't buy any soda. There are many combinations of cans and bottles that can be purchased.
7. n must be greater than or equal to 0 because items created cannot be negative, and n must be an integer because it represents whole pieces.
8. Your dad can spend no more than $50. The number of hours must be greater than 0. Your dad can rent the chain saw for no more than 4 hours.
9. $1.2(5.99 + 0.75x) \leq 10.00$. After solving for $x = 3$, he will have $0.11 remaining.
10. $\begin{cases} x + y \leq 40 \\ x \leq 12 \\ x \geq 0 \\ y \geq 0 \end{cases}$; in addition, x and y must be integers.

Progress Assessment, p. U1-172

1. d
2. b
3. c
4. d
5. a
6. d
7. b
8. b
9. d
10. c
11. $\begin{cases} 3x + 2y \leq 60 \\ 2x + y \leq 45 \\ x \geq 0 \\ y \geq 0 \end{cases}$; in addition, x and y must be integers.

Lesson 5: Rearranging Formulas

Pre-Assessment, p. U1-175

1. b
2. a
3. d
4. c
5. b

Warm-Up 1.5.1, p. U1-177

1. $3.44
2. $61.92

Practice 1.5.1: Rearranging Formulas, p. U1-187

1. $y = 3x - 2$
2. $y = 11 - 4x$
3. $y = (10/7)x - 11$
4. $y = -5x + 11$
5. $C = K - 273.15$
6. $r = C/(2\pi)$
7. $w = V/(lh)$
8. $h = (S - 2\pi r^2)/(2\pi r)$
9. $C = (5/9)(F - 32)$
10. $h = (3V)/(\pi r^2)$

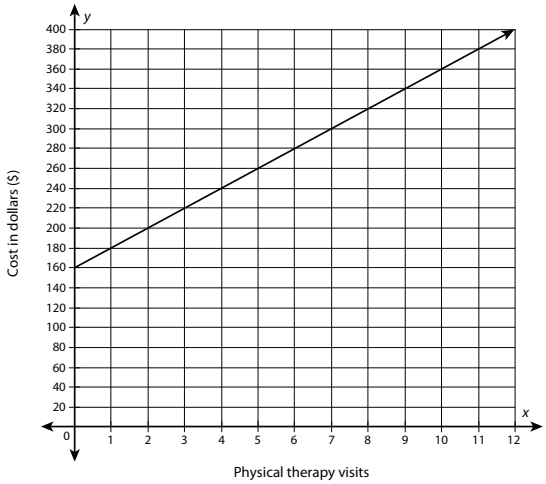
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Progress Assessment, p. U1-188

1. b
2. d
3. a
4. c
5. d
6. b
7. c
8. a
9. d
10. a
11. Rearrange the formula so that it is solved for t, the number of hours you should swim. Your weight, w, will not change in the equation, leaving just the number of calories burned, C, and time.

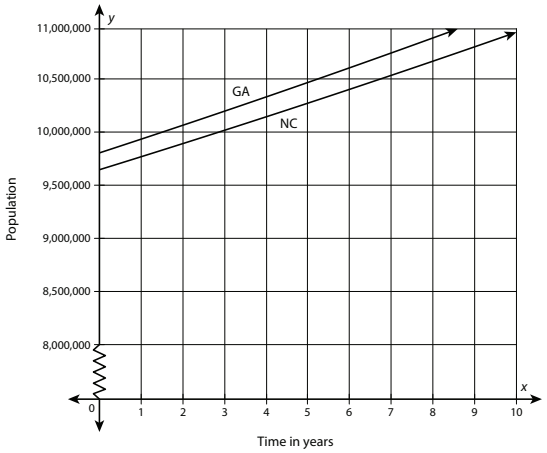
Unit Assessment p. U1-190

1. b
2. c
3. a
4. d
5. c
6. b
7. b
8. b
9. d
10. d
11. c
12. a
13. a. $252 = 6x + 0.05(6x)$
    b. $40
    c. $12
14. a. $y = 20x + 160$
    b. Graph:



    c. If she goes to physical therapy 3 times per week for one month (4 weeks), it will cost $160 for insurance, plus $20 per visit for 12 visits; $y = 160 + 20(12) = 400$, so it will cost $400.

15. a. $y = 9{,}815{,}210(1.013)^x$
    b. $y = 9{,}656{,}401(1.0125)^x$
    c. Graphed equations:



    d. Georgia starts out with more people than North Carolina. This is seen in the y-intercepts. Georgia starts out higher on the y-axis. Georgia also has a higher growth rate, 1.3% versus 1.25%. The different growth rates cause the graphs to grow farther apart as time goes on. If the current growth rates continue, Georgia's population will always be greater than North Carolina's population.

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Student Book Answer Key

Lesson 1: Interpreting Structure in Expressions

Practice 1.1.1: Identifying Terms, Factors, and Coefficients, p. U1-9

1. terms: $14x^2$, $2x$, -9
   factors: 14 and x^2, 2 and x
   coefficients: 14, 2
   constant term: -9
2. terms: $13x$, 20
   factors: 13 and x
   coefficient: 13
   constant term: 20
3. terms: $(4x^3)/5$, $9x$
   factors: 4/5 and x^3, 9 and x
   coefficients: 4/5 and 9
   constant term: none
4. expression: $(x^2)/3 + 4$
   terms: $(x^2)/3$, 4
   factors: x^2, 1/3
   coefficient: 1/3
   constant term: 4
5. expression: $x^6 + 3x$
   terms: x^6, $3x$
   factors: 3 and x
   coefficient: 3
   constant term: none
6. Answers may vary. Sample answer:
   $12x^4 + 15x^3 + 18x^2 - 21x + 3$
7. expression: $2x + 0.05(x) = 2.05x$
   terms: $2.05x$
   factors: 2.05 and x
   coefficient: 2.05
   constant term: none
8. expression: $4x - 0.15(4x) + 4.85 = 3.4x + 4.85$
   terms: $3.4x$, 4.85
   factors: 3.4 and x
   coefficient: 3.4
   constant term: 4.85
9. expression: $x + x + (x - 4) + (x - 4) = 2(x) + 2(x - 4) = 4x - 8$
   terms: $4x$, -8
   factors: 4 and x
   coefficient: 4
   constant term: -8
10. expression: $5/9(F - 32) = (5/9F) - (160/9)$
    terms: $5/9(F)$, $-160/9$
    factors: 5/9 and F
    coefficient: 5/9
    constant term: $-160/9$

Practice 1.1.2: Interpreting Linear and Exponential Expressions, p. U1-14

1. The order of operations indicates that exponents must be applied before multiplying.
2. The order of operations indicates that the parentheses must be cleared before applying exponents.
3. The number of books does not affect the value of m; the number of books is a constant and remains unchanged by the number of magazines.
4. Lowering the service fee will result in a constant less than 23.90.
5. The value of the expression will be greater than 9.
6. x must be less than 0
7. The amount will be increased.
8. Changing the value of r has no effect on the value of d. d represents the initial dose; changing the rate at which it loses effectiveness will not change the initial amount taken.
9. The values of $(1 + r)$ would be less than 1.
10. The number of inactive months has no effect on the rate. The rate will still be 1% for each month that the card is inactive.

Lesson 2: Creating Equations and Inequalities in One Variable

Practice 1.2.1: Creating Linear Equations in One Variable, p. U1-31

1. Answers may vary. Possible answers:
   a. miles per hour
   b. inches per minute or miles per hour
   c. meters per second or feet per second
   d. dollars per pound
2. 40.50π cm^3 or 127.23 cm^3
3. 287.88 ft^2
4. 960 calories per day
5. \$15
6. 6 pairs
7. \$17 per lunch
8. 25 minutes
9. 76,800 square feet
10. $1.5(r - 5) = 1r$; $r = 15$. Brian's rate is 15 mph, while Alex's rate is 10 mph.

Practice 1.2.2: Creating Linear Inequalities in One Variable, p. U1-40

1. $x \leq 8$
2. $x \leq 250$
3. $w \leq 2400$
4. $32 + x \geq 88$; $x \geq 56$; Jeff needs \$56 or more.

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5. $15x \geq 950$; $x \geq 63\ 1/3$; You must work at least 63 1/3 hours.
6. $79 + x \geq 160$; $x \geq 81$; Mackenzie needs 81 points or more.
7. $22x + 9 \leq 75$; $x \leq 3$; Arianna can buy up to 3 computer games.
8. $16 - 2x \geq 6$, where x is time in 7-minute increments; $x \leq 5$, so 5(7) = 35 minutes. After the first hour, the team can play for no more than 35 minutes before there are too few players.
9. $25{,}000 - 3000x \geq 4000$; $x \leq 7$ days. The giveaway should end in 7 days or less in order to have $4,000 or more to give away for the grand prize.
10. Answers may vary. Sample answer: You have a gift card for $6 to the office supply store and you want to use it all up. You want to buy 3 separate notebooks that cost x dollars and you have a coupon for $3 off your purchase. To satisfy these criteria, the notebooks must cost $3 or more each; $x > 3$.

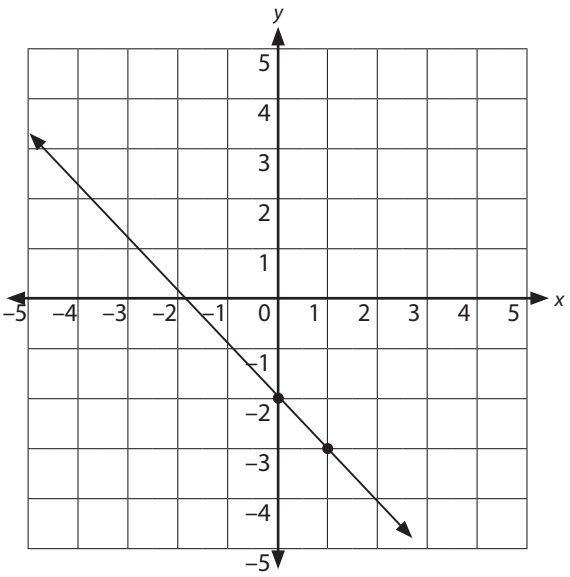
Practice 1.2.3: Creating Exponential Equations, p. U1-51

1. a. linear; b. exponential
2. a. linear; b. exponential
3. a. exponential; b. linear
4. $1920 = a(2)^6$; 30 bacteria
5. $4800 = a(2)^3$; $600
6. $y = 24(3)^4$; 1,944 insects
7. $y = 100(0.5)^8$; 0.39 grams
8. $8 = a(0.5)^3$; 64 teams
9. $y = 63{,}000(1.01)^4$; about 65,559 people
10. $y = 3000(0.99)^5$; about 2,853 people and $y = 3000(0.99)^{10}$; about 2,714 people

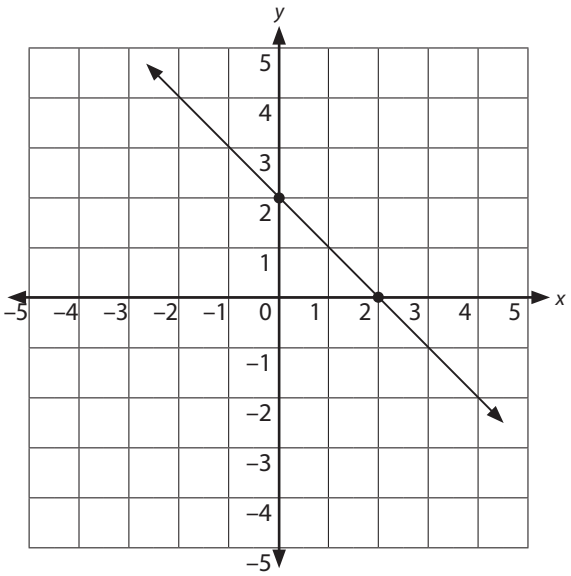
Lesson 3: Creating and Graphing Equations in Two Variables

Practice 1.3.1: Creating and Graphing Linear Equations in Two Variables, p. U1-83

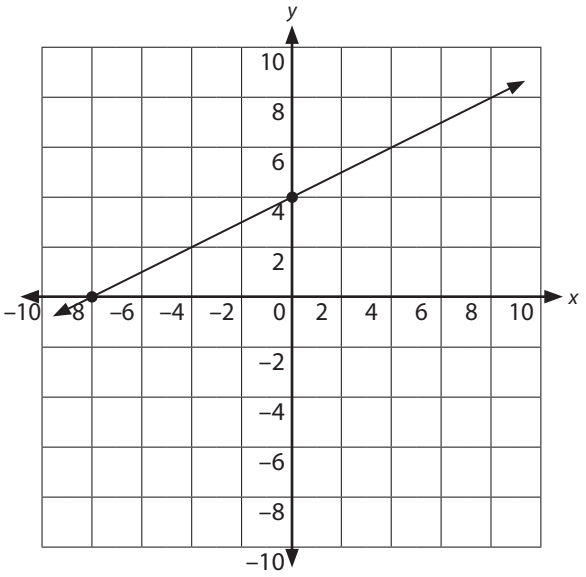
1.



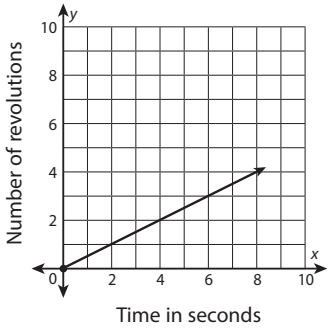
2.



3.

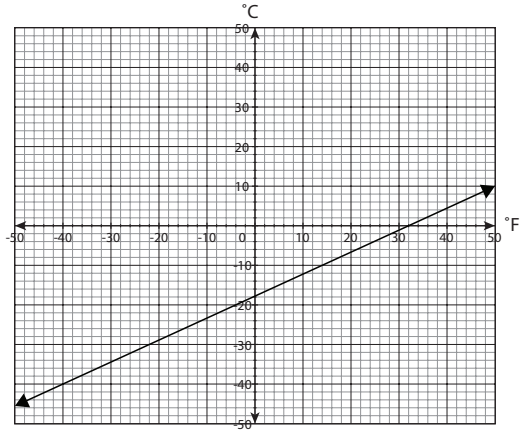


4. $y = 1/2x$; slope = 1/2; y-intercept: (0, 0)

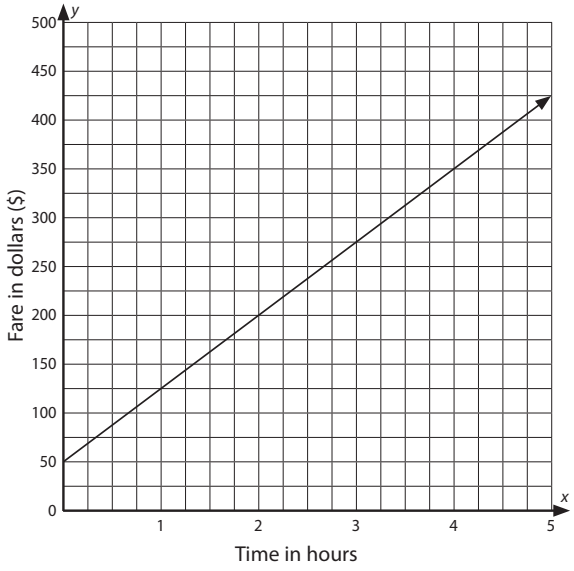


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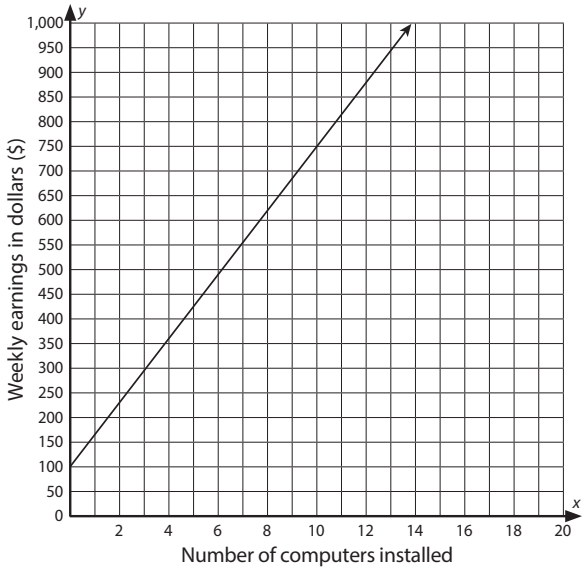
5. $y = 5/9(x - 32)$; slope = 5/9; y-intercept: (0, –17 7/9)



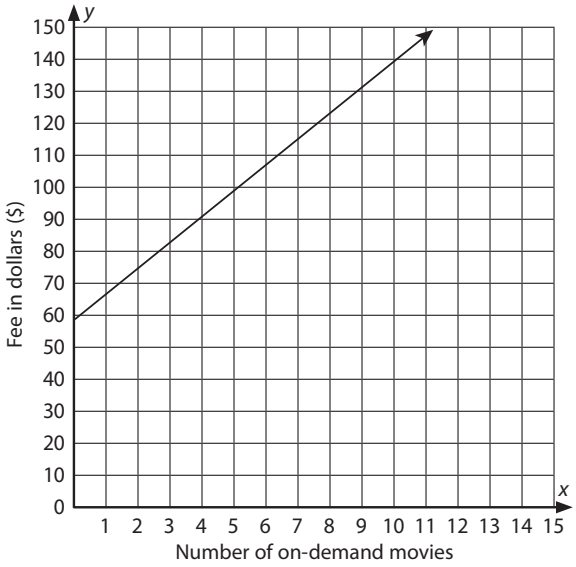
6. $y = 75x + 50$; slope = 75; y-intercept: (0, 50)



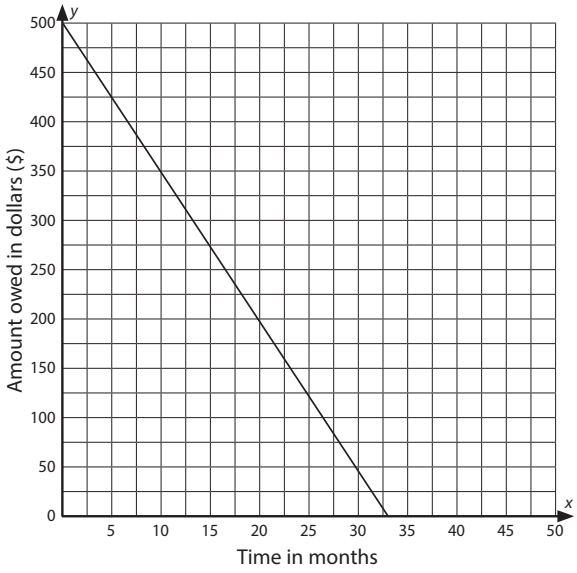
7. $y = 65x + 100$; slope = 65; y-intercept: (0, 100)



8. $y = 8x + 59$; slope = 8; y-intercept: (0, 59)

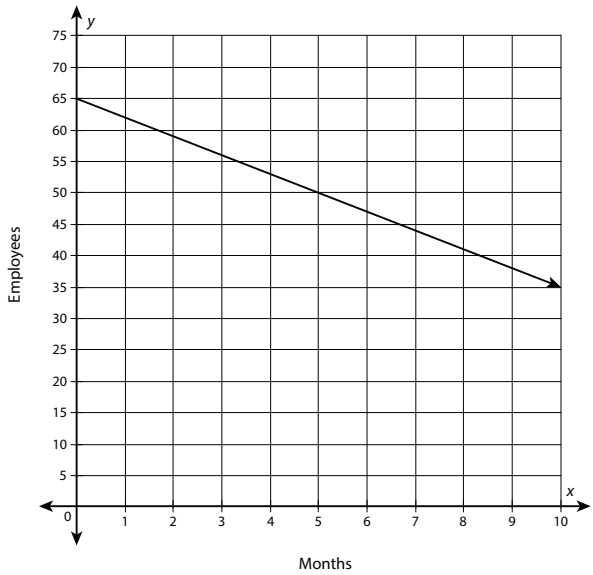


9. $y = –15x + 500$; slope = –15; y-intercept: (0, 500)



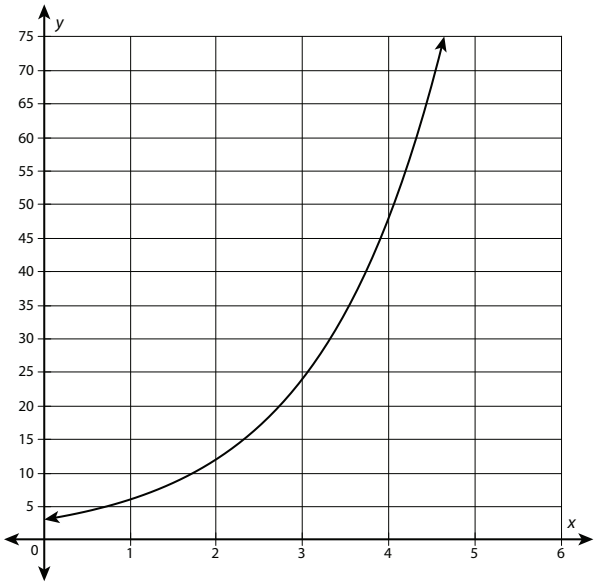
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10. $y = -36/12\,x + 65 = -3x + 65$

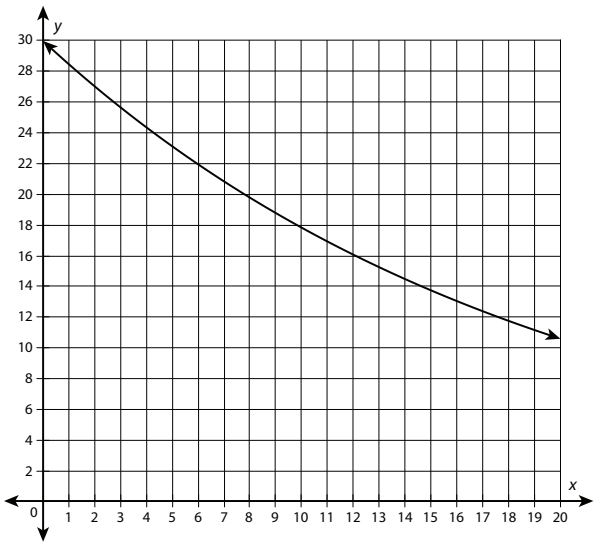


Practice 1.3.2: Creating and Graphing Exponential Equations, p. U1-96

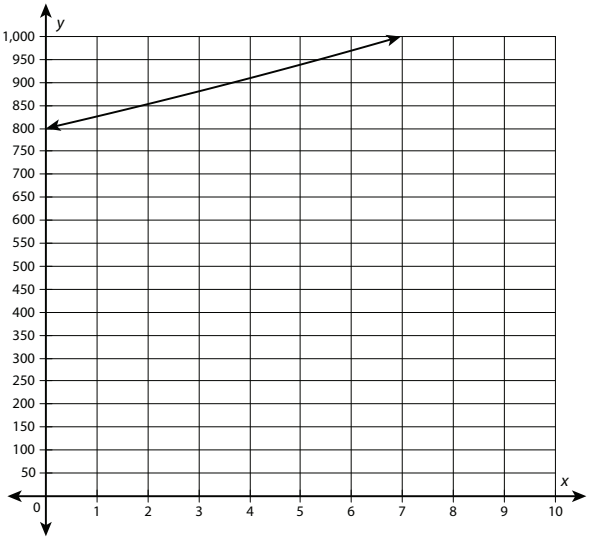
1.



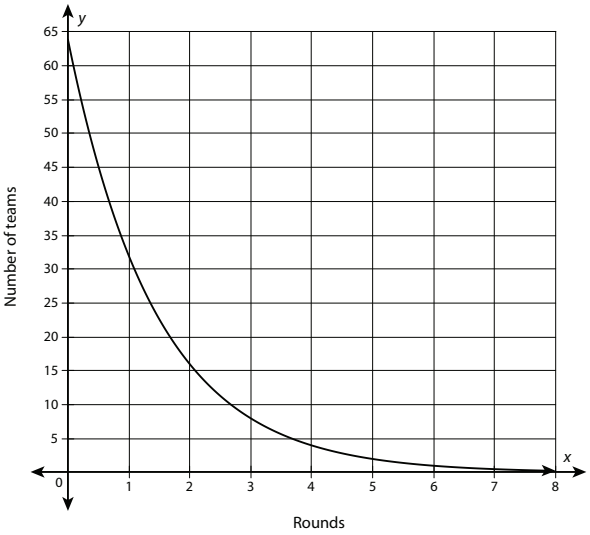
2.



3.

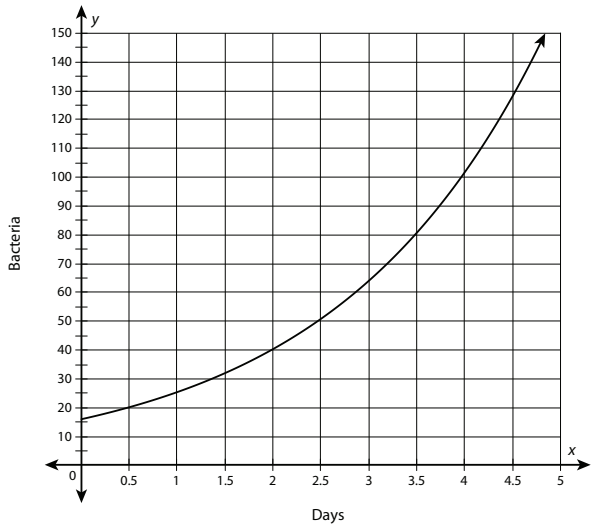


4. $y = 64(0.5)^x$

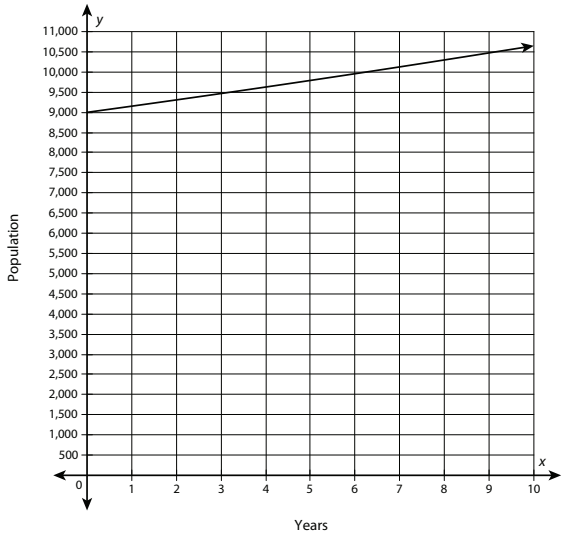


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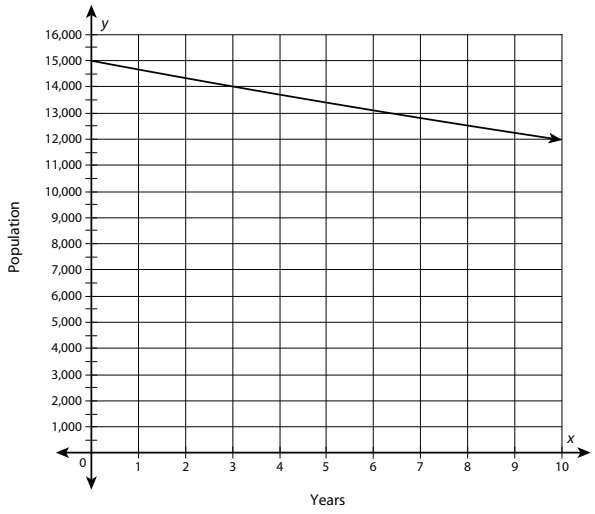
5. $y = 16(2)^{24x/36} = 16(2)^{2x/3}$, for which x is in days



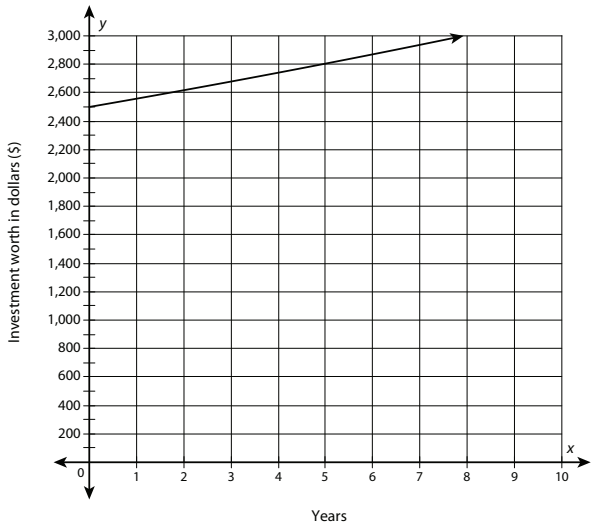
6. $y = 9000(1.017)^{x}$



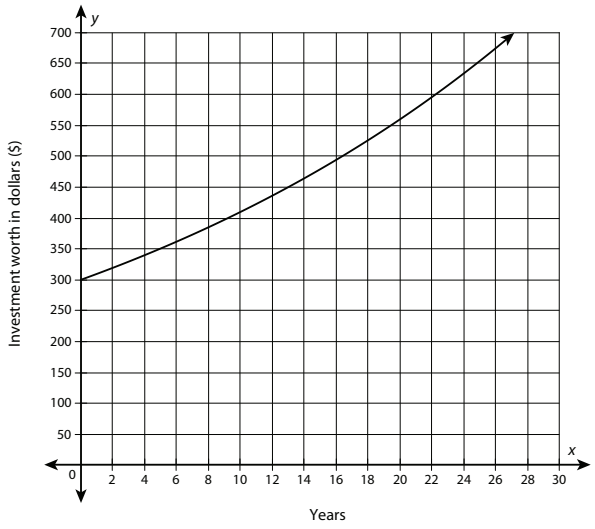
7. $y = 15{,}000(0.978)^{x}$



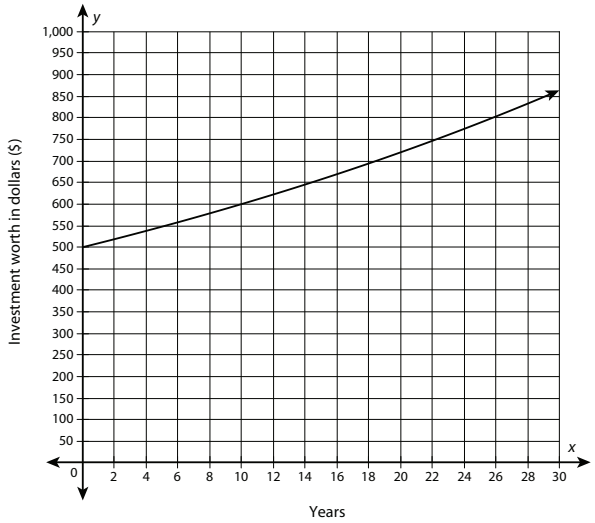
8. $y = 2500(1.00192)^{12x}$



9. $y = 300(1.0006)^{52x}$



10. $y = 500(1.00005)^{365x}$



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Lesson 4: Representing Constraints

Practice 1.4.1: Representing Constraints, p. U1-105

1. no
2. yes
3. no
4. Answers will vary. Sample responses: a. (–2, 3); b. (5, 5); c. (2, 1)
5. a. $y = 2.60x + 7.90$; b. $x = 8.5$; maximum = 8,500 gallons
6. Inequality: $7.50g + 5.00s \leq 30.00$. Answers will vary. You can buy a maximum of 4 graphic tees if you don't buy any solid tees. You can buy a maximum of 6 solid tees if you don't buy any graphic tees. There are many other combinations of graphic and solid tees that you can purchase.
7. $\begin{cases} y = 7x + 15 \\ y = 5x + 20 \\ x \geq 1 \end{cases}$
8. The length must be greater than 0, but less than or equal to 80 feet.
9. $\begin{cases} x + y \leq 32 \\ y \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$
10. $\begin{cases} x + y \leq 15 \\ x \geq 4 \\ y \geq 0 \end{cases}$

Lesson 5: Rearranging Formulas

Practice 1.5.1: Rearranging Formulas, p. U1-114

1. $y = 10x - 6$
2. $y = 9 - 2x$
3. $y = 3x - 9$
4. $y = -5x + 13$
5. $d = C/\pi$
6. $r = d/t$
7. $t = I/(pr)$
8. $l = (A - s^2)/(2s)$
9. $F = (9/5)C + 32$
10. $h = (3V)/(b^2)$

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Instruction

Goal: To provide opportunities for students to develop concepts and skills related to unit conversion, finding percents, simplifying algebraic ratios, and solving algebraic proportions

Common Core State Standards

N–Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

A–CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Student Activities Overview and Answer Key

Station 1

Students will be given 12 index cards with pairs of equivalent units of measurement written on them. They will work together to match the cards that are an equivalent unit of measurement. Then they will perform unit conversions.

Answers

1. 10 mm = 1 cm; 12 in = 1 ft; 3 ft = 1 yd; 2 pints = 1 quart; 4 quarts = 1 gallon; 1 ton = 2,000 pounds
2. 8 pints in a gallon; 2 pints = 1 quart and 4 quarts = 1 gallon, so 2(4) = 8 pints
3. 18 inches; 1/2 yard = 1.5 feet and 12 inches = 1 foot, so 12(1.5) = 18 inches
4. 5,000 pounds
5. 850 mm
6. 13.5 feet
7. 3 quarts = 0.75 gallons
8. Answers will vary. Possible answers include: cooking, when modifying recipes for more or fewer people; carpentry, when creating custom-size cabinetry

Station 2

Students will be given a calculator to help them solve the problems. They will work as a group to solve real-world applications of unit conversions.

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Answers

1. His friend measures temperature in Celsius, and Evan measures it in Fahrenheit. $F = 95°$
2. $P = 36.67$ yards; $P = 1{,}320$ inches, $A = 77.78$ yds^2; $A = 100{,}800$ in^2
3.

	Feet	Yards	Meters	Time
Tim	300	100	91.44	12 seconds
Jeremy	400	133.33	121.95	12 seconds
Martin	229.66	76.55	70	12 seconds

Jeremy, Tim, Martin; Tim = 25 feet/sec, Jeremy = 33.33 feet/sec; Martin = 19.14 feet/sec

Station 3

Students will be given a bag containing 24 green marbles and 16 yellow marbles. They will use the marbles to create ratios and percents. They will then solve percent problems.

Answers

1. Answers will vary. Possible answers include: green = 1; yellow = 7; total = 8. Find 1/8 = 0.125 = 12.5%; 12.5% were green. Subtract 12.5% from 100% to get 87.5% or 7/8 = 87.5%; 87.5% were yellow.
2. There are 40 marbles so 24/40 = 60% green marbles; 100% – 60% = 40% or 16/40 = 40%
3. 9 yellow marbles; student drawings should depict 9 yellow marbles and 12 green marbles.
4. 24(1/4) = 6 or 24(0.25) = 6
5. 17(2/1) = 34 or 17(2.0) = 34
6. 10(14) = 140 in^2; I increased the dimensions by 200% and then found the area of the photo.

Station 4

Students will be given 8 large blue algebra tiles and 20 small yellow algebra tiles. Students will visually depict ratios and proportions with the algebra tiles. Then they will solve proportions for a specified variable including a real-world application.

Answers

1. $\dfrac{8 \text{ blue}}{20 \text{ yellow}} = \dfrac{2}{5}$
2. $\dfrac{2 \text{ blue}}{3 \text{ yellow}} = \dfrac{4 \text{ blue}}{6 \text{ yellow}}$

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3. $8/20 = x/100$, so $x = 40$ blue
4. $8/20 = x/15$, so $x = 6$ blue
5. $x = 4$
6. $x = 40$
7. $\frac{\text{blue}}{\text{yellow}} = \frac{6}{10} = \frac{3}{5}$

   Let x represent the number of blue pencils and $24 - x$ represent the number of yellow pencils.

   $\frac{3}{5} = \frac{x}{(24 - x)}$, so $x = 9$ blue pencils and $24 - x = 15$ yellow pencils

Materials List/Setup

Station 1 12 index cards with the following written on them:

10 millimeters, 12 inches, 3 feet, 2 pints, 4 quarts, 1 ton, 1 centimeter, 1 foot, 1 yard, 1 quart, 1 gallon, 2,000 pounds

Station 2 calculator

Station 3 24 green marbles; 16 yellow marbles

Station 4 8 large blue algebra tiles; 20 small yellow algebra tiles

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. How do you perform unit conversion?
2. When would you use unit conversion in the real world?
3. What are two ways to find the percent of a number?
4. What is a ratio?
5. How do you know if two ratios are equivalent?
6. What is a proportion?
7. When would you use ratios and proportions in the real world?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. Use ratios and proportions to convert units.
2. Answers will vary. Possible answers include: creating scale models of buildings; using the metric system instead of U.S. customary units; converting Celsius to degrees Fahrenheit and vice versa
3. Multiply the number by a decimal or fraction that represents the percentage.
4. A ratio is a comparison of two numbers by division.
5. Two ratios are equivalent if, when simplified, they are equal.
6. A proportion is when two ratios are set equal to each other.
7. Answers will vary. Possible answers include: enlarging photos; scale models; modifying quantities of ingredients in a recipe

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Instruction

Possible Misunderstandings/Mistakes

- Not keeping track of units and using incorrect unit conversions
- Not recognizing that terms must have the same units in order to compare them
- Setting up proportions with one of the ratios written with the incorrect numbers in the numerator and denominator
- Not recognizing simplified forms of ratios in order to find equivalent ratios

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Station 1

You will be given 12 index cards with the following written on them:

10 millimeters, 12 inches, 3 feet, 2 pints, 4 quarts, 1 ton, 1 centimeter, 1 foot, 1 yard, 1 quart, 1 gallon, 2,000 pounds

Shuffle the index cards and deal a card to each student in your group until all the cards are gone. As a group, show your cards to each other and match the cards that are an equivalent unit of measurement.

1. Write your answers on the lines. The first match is shown:

10 mm = 1 cm _____ _____

_____ _____

_____ _____

2. Find the number of pints in a gallon. Explain how you can use your answers in problem 1 to find the number of pints in a gallon.

3. Find the number of inches in half of a yard. Explain how you can use your answers in problem 1 to find the number of inches in half of a yard.

continued

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Perform the following unit conversions by filling in the blanks.

4. 2.5 tons = _____ pounds

5. 85 cm = _____ mm

6. 4.5 yd = _____ ft

7. 6 pints = _____ quarts = _____ gallons

8. When would you use unit conversions in the real world?

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Station 2

You will be given a calculator to help you solve the problems. Work as a group to solve these real-world applications of unit conversions.

1. Evan has a friend in England. His friend said the temperature was very hot at 35°. Evan thought he heard his friend incorrectly since 35° is cold. What caused his misunderstanding? (*Hint*: $C = (F - 32)\frac{5}{9}$)

   Find the equivalent temperature in the United States that would make the claim of Evan's friend valid.

2. Anna is going to build a patio. She wants the patio to be 20 feet by 35 feet. What is the perimeter of the patio in yards?

   What is the perimeter of the patio in inches?

   What is the area of the patio in yards?

continued

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What is the area of the patio in inches?

3. Tim claims he can run the 100-yard dash in 12 seconds. Jeremy claims he can run 400 feet in 12 seconds. Martin claims he can run 70 meters in 12 seconds. (*Hint*: 1 yard = 0.9144 meters and 1 yard = 3 feet.)

   Fill in the table to create equivalent units of measure.

	Feet	Yards	Meters	Time (seconds)
Tim				
Jeremy				
Martin				

   List the three boys in order from fastest to slowest.

   How fast did each boy run in feet/second?

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Station 3

You will be given a bag containing 24 green marbles and 16 yellow marbles. You will use the marbles to create ratios and percents. You will then solve percent problems. Work together as a group to solve the following problems.

1. Shake the bag of green and yellow marbles so that the colors are mixed. Have each student select 2 marbles from the bag without looking. Group all your marbles together by color.

   How many green marbles did you draw?

   How many yellow marbles did you draw?

   What was the total number of marbles drawn?

   How can you determine the percentage of marbles that were green?

   Find the percentage of marbles you drew that were green.

   Name two ways you can find the percentage of marbles you drew that were yellow.

   Find the percentage of marbles you drew that were yellow.

continued

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2. Take all the marbles out of the bag. How can you determine what percentage of all the marbles are green?

   How can you determine what percentage of all the marbles are yellow?

3. Place 12 green marbles on the table. How many yellow marbles do you need to have 75% as many yellow marbles as green marbles on the table?

   Draw a picture of the number of green marbles and yellow marbles you have placed on the table.

4. Use equations to show two ways that you can find 25% of 24.

5. Use equations to show two ways that you can find 200% of 17.

6. Real-world application: Bryan is a photographer. He has a 5 inch by 7 inch photo that he wants to enlarge by 200%. What is the area of the new photo? Explain your answer.

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Station 4

You will be given 8 large blue algebra tiles and 20 small yellow algebra tiles.

1. Work as a group to arrange the algebra tiles so they visually depict the ratio of the number of blue tiles to yellow tiles. What is this ratio?

2. Rearrange the tiles to visually depict the following ratios:

$$\frac{\text{2 blue}}{\text{3 yellow}} \qquad \frac{\text{1 blue}}{\text{10 yellow}} \qquad \frac{\text{4 blue}}{\text{6 yellow}} \qquad \frac{\text{1 blue}}{\text{1 yellow}}$$

   Which ratios are equivalent ratios?

3. If there are 100 yellow algebra tiles, and the ratio of yellow to blue tiles is the same as your original set of tiles, how many blue algebra tiles are there? Use a proportion to solve this problem. Show your work. (*Hint*: A proportion is two ratios that are equal to each other.)

4. Keeping the same ratio of yellow to blue tiles, if there are 15 yellow algebra tiles, how many blue algebra tiles are there? Use a proportion to solve this problem. Show your work.

continued

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Work together to solve the following proportions for the variable.

5. $\frac{2}{7} = \frac{x}{14}$; $x =$

6. $\frac{8}{x} = \frac{2}{10}$; $x =$

Use the following information to answer problem 7.

> Allison has 6 blue pencils and 10 yellow pencils. Sadie has 24 pencils that are either blue or yellow. The ratio of blue pencils to yellow pencils is the same for both Allison and Sadie.

7. How many blue pencils and yellow pencils does Sadie have? Set up a proportion using the variable x to solve this problem. Show your work.

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Instruction

Goal: To provide opportunities for students to develop concepts and skills related to solving inequalities

Common Core State Standard

A–CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Student Activities Overview and Answer Key

Station 1

Students will be given a series of inequalities and a number cube. For each inequality, they will roll the number cube and work together to decide if the number shown on the cube is a solution of the inequality. Students will explain the strategies they used to decide whether each value was a solution.

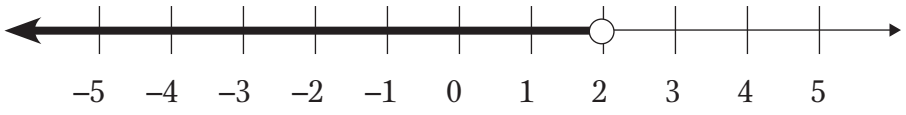
Answers: 1–3. Answers will depend upon numbers rolled. 4. Yes; 5. No; 6. Possible strategies: Substitute the value for the variable. Simplify and check to see if the resulting inequality is true.

Station 2

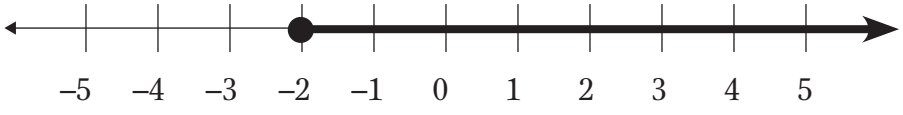
Students will work together to use number lines to help them solve inequalities. They will test various values of the variable in the given inequalities, and check to see whether each value is a solution. They will keep track of the values that are solutions by marking them on a number line. After testing enough values to see a pattern, students will shade the values that represent all solutions of the inequality. Then they will write the solution algebraically.

Answers

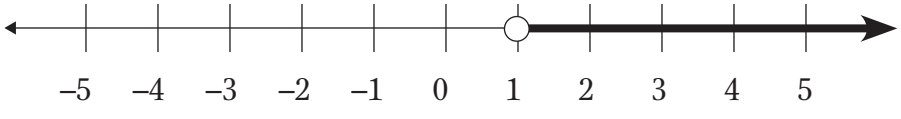
1. $x < 2$



2. $x \geq -2$



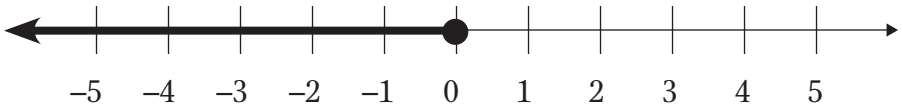
3. $x > 1$



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Instruction

4. $x \leq 0$



5. $x < 2$



Station 3

Students will work together to match a set of given inequalities with a set of given solutions. Once students have paired each inequality with its correct solution, they will discuss the strategies they used to solve the problem.

Answers: The cards should be paired as follows: $5x + 2 < 12$ and $x < 2$; $4x + 3 < -5$ and $x < -2$; $-3x < 6$ and $x > -2$; $-\frac{x}{2} > 2$ and $x < -4$; $3x + 1 > -11$ and $x > -4$; $\frac{x}{4} + 1 > 2$ and $x > 4$.

Possible strategies: Solve each inequality using inverse operations and look for the solution among the given choices.

Station 4

Students will be given a set of inequalities and a set of real-world situations. They will work together to match each situation to an inequality. Then they will solve the inequality. Students will explain the strategies they used to match the inequalities to the situations.

Answers

1. $10x + 5 \leq 105$, $x \leq 10$
2. $5x + 10 < 105$, $x < 19$
3. $10x - 5 \geq 105$, $x \geq 11$
4. $5(x - 10) < 105$, $x < 31$
5. Possible strategies: Use the words or phrases that refer to arithmetic operations as clues to identifying the corresponding inequalities. For example, a decrease corresponds to subtraction. Match words to inequalities. For example, "no more than" refers to a "less than or equal to" inequality (≤).

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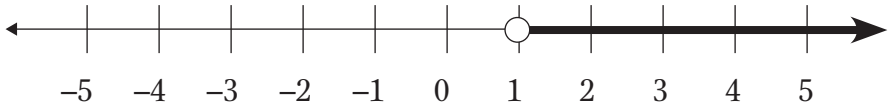
Instruction

Station 5

Students will work together to match inequalities with their solution graphs. Then they will discuss the strategies they used.

Answers

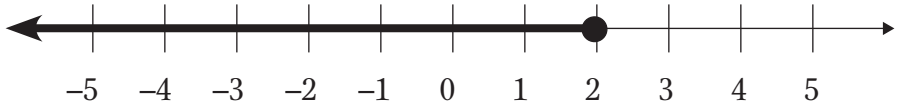
$2x - 5 > -3$



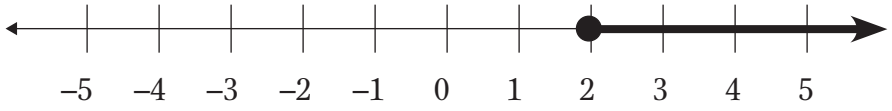
$4 - 3x \leq 7$



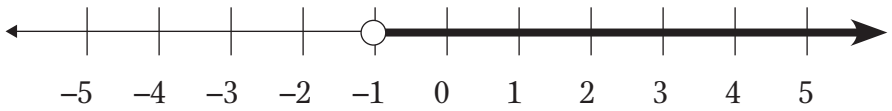
$-2x + 7 \geq 3$



$4x - 5 \geq 3$



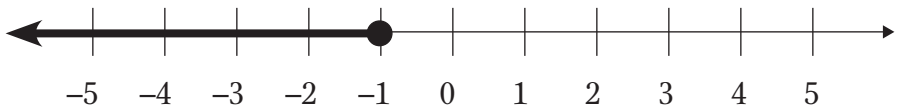
$-11 < 6x - 5$



$7 - 7x \leq 0$



$5x + 12 \leq 7$



Possible strategies: Test points in the inequalities to determine the solution, or solve for the variable and then match the graph.

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Instruction

Materials List/Setup

Station 1 number cube (numbers 1–6)

Station 2 none

Station 3 set of index cards with the following inequalities written on them:

$5x + 2 < 12$ $\quad -\frac{x}{2} > 2$

$4x + 3 < -5$ $\quad 3x + 1 > -11$

$-3x < 6$ $\quad \frac{x}{4} + 1 > 2$

set of index cards with the following solutions written on them:
$x < -4$, $x > -4$, $x < -2$, $x > -2$, $x < 2$, $x > 4$

The two sets of cards should be placed in two piles, face-up, on a table or desk at the station.

Station 4 none

Station 5 7 note cards with the following inequalities on them:

$5x + 12 \leq 7$ $\quad 2x - 5 > -3$

$4 - 3x \leq 7$ $\quad -2x + 7 \geq 3$

$4x - 5 \geq 3$ $\quad -11 < 6x - 5$

$7 - 7x \leq 0$

7 note cards with the graphs from the Station 5 answers on them

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. What is the difference between < and ≤?
2. How do you check to see if a value of the variable is a solution of an inequality?
3. How is the solution of an inequality different from the solution of an equation?
4. How do you solve an inequality using algebra?
5. What strategies can you use to prevent confusion when the variable of an inequality is on the right side of the inequality symbol? (Example: $2 < x$)

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. The symbol < means "less than." The symbol ≤ means "less than or equal to."
2. Substitute the value for the variable in the inequality. Check to see if the resulting inequality is true. If it is, the value is a solution.
3. In general, the solution of an inequality is itself an inequality (a range of values). The solution of an equation is usually a single value (or several discrete values).
4. Use inverse operations, as when solving an equation, to isolate the variable on one side of the inequality. If you multiply or divide by a negative number, reverse the direction of the inequality.
5. Rearrange the inequality so that the variable is on the left. (Using the given example, $x > 2$.) Students could also read the inequality from right to left. (Using the given example, "x is greater than 2.")

Possible Misunderstandings/Mistakes

- Using an incorrect operation to solve an inequality (e.g., solving $x + 2 < 5$ by adding 2 to both sides)
- Incorrectly translating verbal expressions to inequalities (e.g., representing the phrase "no more than" by < rather than ≤)
- Forgetting to reverse the direction of the inequality when multiplying or dividing by a negative number

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Station 1

You will find a number cube at this station.

For each inequality, roll the number cube and write the number in the box. Then work together to decide if this value of the variable is a solution of the inequality. Write "yes" or "no" on the line provided.

1. $2x + 1 < 7$ ☐ Solution? _____

2. $3x - 4 \geq 5$ ☐ Solution? _____

3. $-3x < -12$ ☐ Solution? _____

4. $\frac{x}{2} + 1 < 5$ ☐ Solution? _____

5. $1 - x > 0$ ☐ Solution? _____

6. Explain the strategies you used to decide whether each value was a solution of the inequality.

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Station 2

You can use number lines to help you solve inequalities.

For each inequality, work together to test different values of the variable to see if they are solutions of the inequality. If a value is a solution, draw a solid dot at that value on the number line. Test at least five different values for each inequality.

When you think you know what the solution set of an inequality looks like, shade the correct part of the number line to show all the solutions.

Finally, write the solution in the space provided.

1. $2x - 3 < 1$



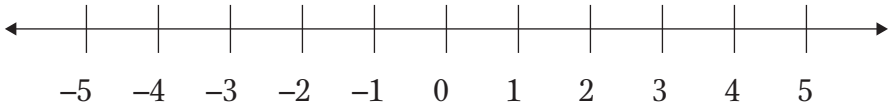
Solution: _____

2. $3x + 1 \geq -5$



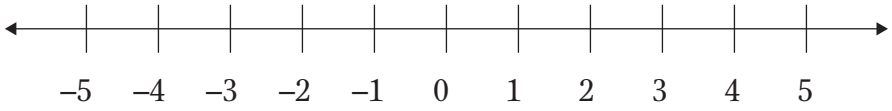
Solution: _____

3. $-2x < -2$



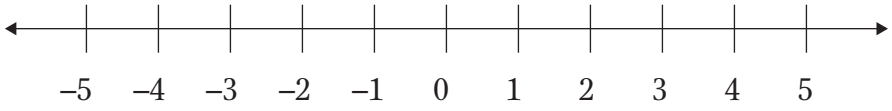
Solution: _____

4. $-3x \geq 0$



Solution: _____

5. $\frac{x}{2} + 3 < 4$



Solution: _____

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Station 3

At this station, you will work with other students to match inequalities to their solutions. You will find a set of cards with the following inequalities written on them:

$5x + 2 < 12$ $4x + 3 < -5$ $-3x < 6$ $-\frac{x}{2} > 2$ $3x + 1 > -11$ $\frac{x}{4} + 1 > 2$

You will also find a set of cards with these solutions written on them:

$x < -4$ $x > -4$ $x < -2$ $x > -2$ $x < 2$ $x > 4$

Work together to match each inequality with its solution. When everyone agrees on the answers, write the matching pairs.

Explain the strategies you used to match up the cards.

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Station 4

At this station, you will match inequalities to real-world situations and then solve the inequalities.

Work with other students to match each situation to one of the following inequalities. When everyone agrees on the correct inequality, write it on the line provided. Then work together to solve it.

$5x + 10 < 105$ $10x + 5 \leq 105$ $5(x - 10) < 105$ $10x - 5 \geq 105$

1. Mai rents DVDs by mail. There is a one-time membership fee of $5 and the service costs $10 per month. Mai wants to spend no more than $105. For how many months can she rent DVDs with this service?

   Inequality: _____

   Solution: _____

2. Tyrone bought 5 trays of food for a party. The price of each tray of food was the same. He also spent $10 on paper plates, napkins, and utensils. Altogether, he spent less than $105. What was the price of each tray of food?

   Inequality: _____

   Solution: _____

continued

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3. Mr. Garcia ordered 10 copies of a novel for students in his English class. He had a coupon for $5 off the total price of the order. The total cost of the order, before tax, came to no less than $105. What was the price of each novel?

   Inequality: _____

   Solution: _____

4. Rachel bought 5 pairs of jeans. Each pair of jeans was the same price. She had a coupon for $10 off the price of each pair of jeans. The total cost of the jeans, before tax, came to less than $105. What was the price of each pair of jeans before the coupon was applied?

   Inequality: _____

   Solution: _____

5. Explain the strategies you used to match the inequalities to the situations.

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Station 5

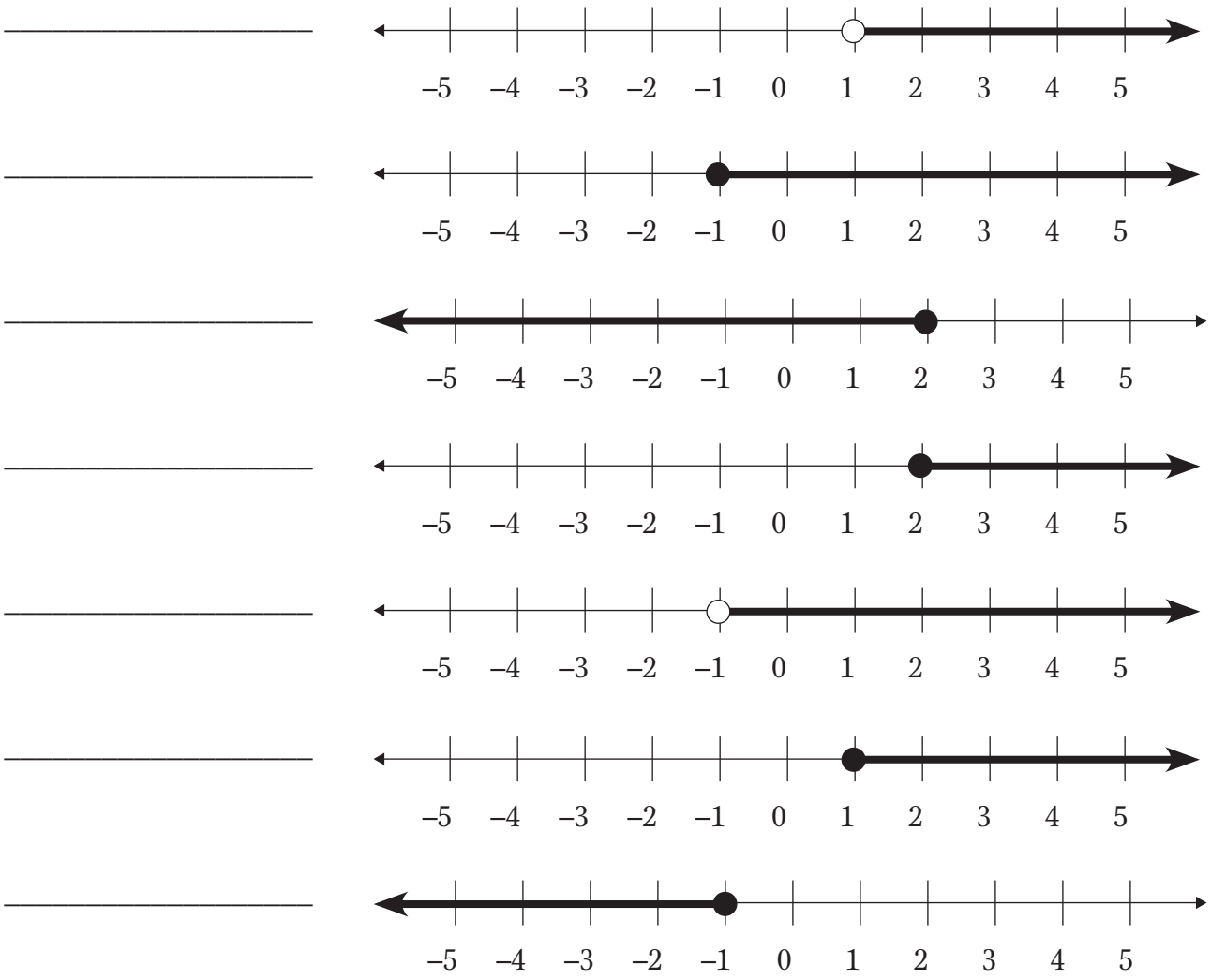
At this station, you will match inequalities to their solution graphs. Write the correct inequality on the line next to each solution. At the end, discuss the strategies that you used.

Inequalities:

$-11 < 6x - 5$ $\quad 5x + 12 \leq 7$ $\quad 4x - 5 \geq 3$ $\quad 7 - 7x \leq 0$

$-2x + 7 \geq 3$ $\quad 2x - 5 > -3$ $\quad 4 - 3x \leq 7$

Solutions:



What strategies did you use to match the inequalities to their solution graphs?

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Instruction

Goal: To provide opportunities for students to develop concepts and skills related to solving equations and creating and interpreting graphs representing real-world situations

Common Core State Standards

A–CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Student Activities Overview and Answer Key

Station 1

Students will use cups and counters to model linear equations. In the given diagrams, each cup is holding an unknown number of counters. Students will use this idea to write the equation that is modeled by each diagram. Then they will use actual cups and counters, as well as logical reasoning, to help them find the unknown number of counters in each cup. This is equivalent to solving the corresponding equation.

Answers

1. $x + 1 = 10$, $x = 9$
2. $2x = 12$, $x = 6$
3. $2x + 3 = 7$, $x = 2$
4. $10 = 3x + 1$, $x = 3$

Station 2

Students will be given a set of equations and a set of real-life situations. They will work together to match each situation to an equation. Then they will solve the equation. At the end of the activity, students will explain the strategies they used to match the equations to the situations.

Answers

1. $2x + 3 = 25$, $x = 11$
2. $3x – 25 = 2$, $x = 9$
3. $2x –3 = 25$, $x = 14$
4. $3x + 2 = 25$, $x = 7\ ^2/_3$
5. Possible strategies: Use the words or phrases that refer to arithmetic operations as clues to identifying the corresponding equations. For example, gathering equal groups of objects corresponds to multiplication.

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Instruction

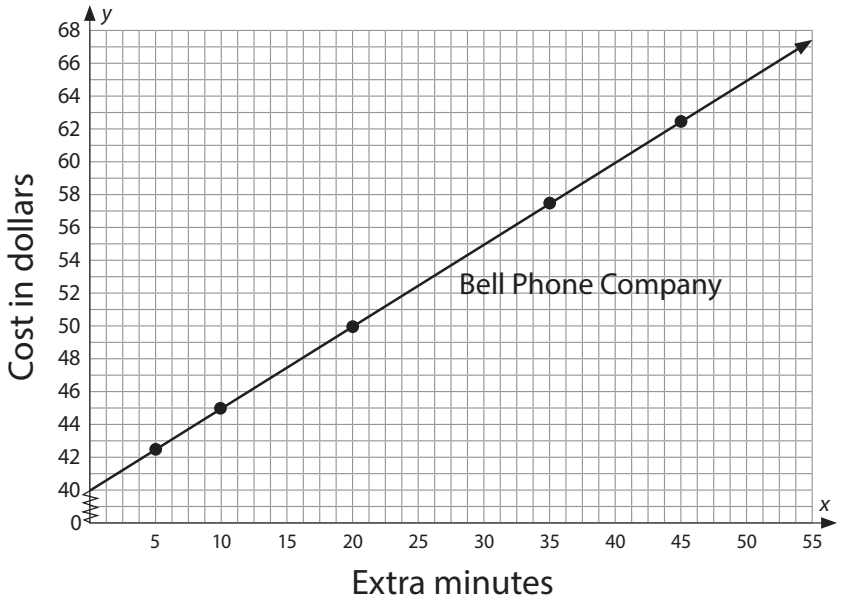
Station 3

Students will be given a ruler and graph paper. They will work together to graph the linear equation of two cell phone company plans. Then they will use the graph to compare the two cell phone plans.

Answers

1. Equation: $y = 0.5x + 40$. Table: Answers will vary; sample answers:

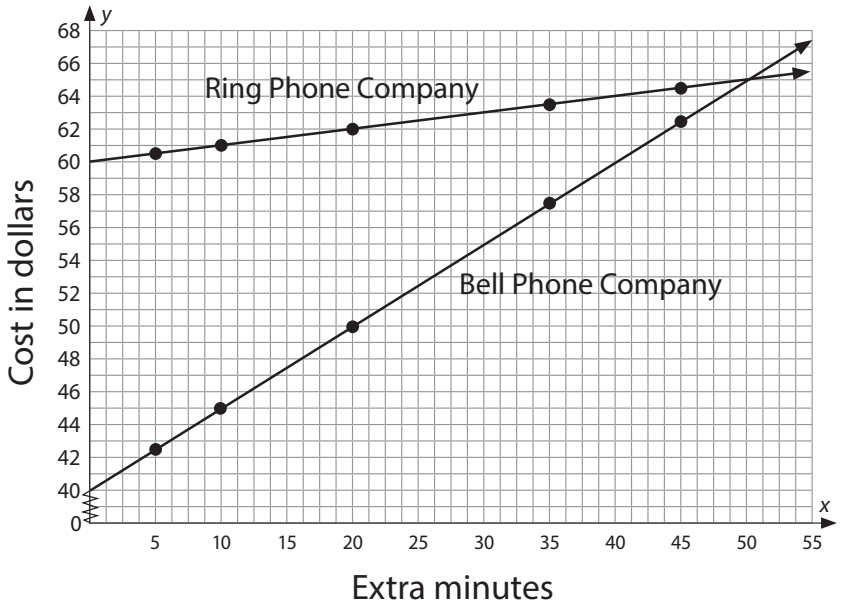
Extra minutes (x)	5	10	20	35	45
Cost in dollars (y)	42.5	45	50	57.50	62.50



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2. Equation: $y = 0.1x + 60$. Table: Answers will vary; sample answers:

Extra minutes (x)	5	10	20	35	45
Cost in dollars (y)	60.50	61	62	63.50	64.50



3. They should choose Bell Phone's plan because it only costs $55 versus $63.
4. They should choose Ring Phone's plan because it only costs $68 versus $80.
5. At 50 minutes, it doesn't matter which plan the customer chose because both plans cost $65.

Station 4

Students will be given a ruler and graph paper. They will work together to complete a table of values for a given equation, and then graph the equation. They will analyze the slope of the graph as it applies to a real-world value.

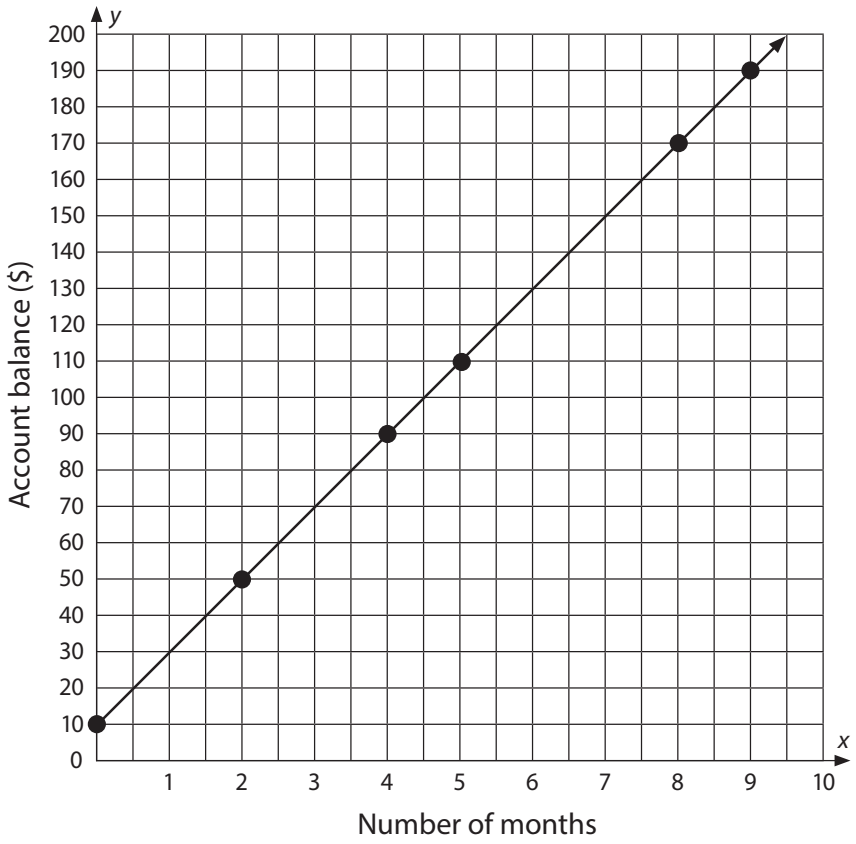
Answers

1. $y = 20x + 10$
2. Answers will vary. Possible table of values:

Number of months	Account balance ($)
0	10
2	50
4	90
5	110
8	170
9	190

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Instruction



3. He will have $310 in his savings account after 15 months. This will allow him to buy the $300 stereo.
4. The slope of the graph would be steeper because the amount he saves each month represents the slope.
5. The slope of the graph would not be as steep because the amount he saves each month represents the slope.

Materials List/Setup

Station 1 3 paper cups; 12 counters or other small objects, such as pennies or beans

Station 2 none

Station 3 graph paper; ruler

Station 4 graph paper; ruler

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. How do you know which operation to use first when you solve a two-step equation?
2. How can you check your solution to an equation?
3. What strategies can you use to create an equation in one variable?
4. What strategies can you use to create an equation in two variables?
5. How can you use an equation to plot its graph?
6. What are examples of real-world situations in which you could construct a graph to represent the data?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. You usually add or subtract on both sides of the equation before you multiply or divide on both sides of the equation. (You reverse the order of operations to "undo" the operations on the variable.)
2. Substitute the value for the variable in the equation and simplify. If the solution is correct, the two sides of the equation should be equal.
3. Read the problem first before doing any writing. Identify the variable and the operations being performed with that variable. Look for keywords indicating the four operations.
4. Identify the independent and dependent variables. Look for keywords indicating the rate or slope. Look for flat fees or constants.
5. Create a table of values that are solutions to the equation. Graph these ordered pairs and draw a line through these points.
6. Answers will vary. Possible answers: Business: yearly revenues; Biology: growth rate; Finance: savings account balance

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Instruction

Possible Misunderstandings/Mistakes

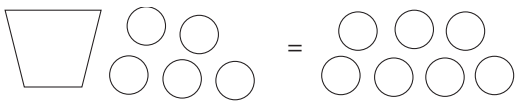
- Using an incorrect operation to solve an equation (e.g., solving $x + 3 = 12$ by adding 3 to both sides)
- Attempting to solve an equation such as $x + 4 = 9$ by subtracting x from both sides
- Applying an operation that does not isolate the variable (e.g., solving $9 = 3x$ by dividing both sides by 9)
- Reversing the x-values and the y-values when reading the graph
- Incorrectly reading the graph by matching up the wrong x- and y-values
- Reversing the x-values and y-values when constructing the graph
- Incorrectly plugging x-values into the given equation to find the y-values

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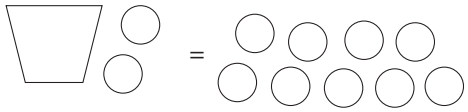
Station 1

In each diagram, the cup is holding an unknown number of items, x. If there is more than one cup, every cup is holding the same number of items.

Each diagram shows an equation. This diagram shows $x + 5 = 7$. To make the two sides equal, there must be 2 items in the cup. This means $x = 2$.

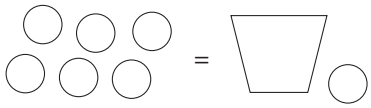


Work with other students to write an equation for each diagram. Then find the number of items in each cup. You can use the cups and items at the station to help you.

1. 

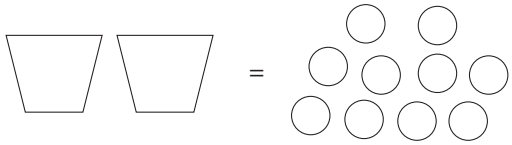
Equation: _____

Solution: _____

2. 

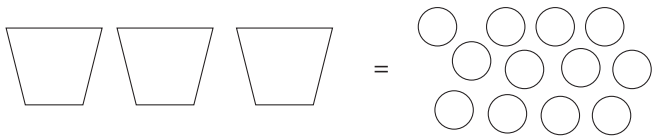
Equation: _____

Solution: _____

3. 

Equation: _____

Solution: _____

4. 

Equation: _____

Solution: _____

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Station 2

At this station, you will match equations to real-life situations and then solve the equations.

Work with other students to match each situation to one of the following equations. When everyone agrees on the correct equation, write it on the line. Then work together to solve it.

$2x - 3 = 25$ $2x + 3 = 25$ $3x + 2 = 25$ $3x - 25 = 2$

1. Rosa bought some notebooks that cost \$2 each. She also bought a compass that cost \$3. She spent a total of \$25. How many notebooks did she buy? Let x represent the cost of a notebook.

   Equation: _____

   Solution: _____

2. Ms. Chen brought 3 packages of pencils for her class. Each package contained the same number of pencils. The 25 students in her class each took one pencil. There were 2 pencils left over. How many pencils were in each package? Let x represent the number of pencils in a package.

   Equation: _____

   Solution: _____

3. Tyler bought two copies of a book to give as gifts. He had a coupon for \$3 off his total purchase. The final cost of the books was \$25. What was the price of each book? Let x represent the cost of a book.

   Equation: _____

   Solution: _____

4. A bowl can hold 25 fluid ounces of liquid. Omar empties a full teacup of water into the bowl 3 times. Then he adds another 2 fluid ounces of water to fill the bowl. How many fluid ounces of liquid does the teacup hold? Let x represent the number of ounces in the teacup.

   Equation: _____

   Solution: _____

5. Explain the strategies you used to match the equations to the situations.

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Station 3

You will be given a ruler and graph paper. Work together to analyze data from the real-world situation described in the following passage. Then, as a group, answer the questions that follow.

> You are going to get a new cell phone and need to choose between two cell phone companies. Bell Phone Company charges $40 per month. It costs $0.50 per minute after you have gone over the monthly number of minutes included in the plan. Ring Phone Company charges $60 per month. It costs $0.10 per minute after you go over the monthly number of minutes included in the plan. The monthly number of minutes for both plans is the same.

For each company's plan, let x represent the number of minutes used that exceeded the plan. Let y represent the cost of the plan.

1. Write an equation that represents the monthly cost of Bell Phone Company's plan.

Complete the table by selecting values for x and calculating y.

Extra minutes (x)					
Cost in dollars (y)					

Use your graph paper to graph the ordered pairs. Use your ruler to draw a straight line through the points and complete the graph.

2. Write an equation that represents the monthly cost of Ring Phone Company's plan.

Complete the table by selecting values for x and calculating y.

Extra minutes (x)					
Cost in dollars (y)					

Plot the ordered pairs from this table on the same graph you created for problem 1. Use your ruler to draw a straight line through the points to complete the graph.

continued

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Use your graphs from problems 1 and 2 to complete problems 3–5.

3. Which plan should a customer choose if he or she uses 30 minutes of extra time each month? Explain.

4. Which plan should a customer choose if he or she uses 80 minutes of extra time each month? Explain.

5. At what number of extra minutes per month would it not matter which phone plan the customer chose since the cost would be the same? Explain.

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Station 4

You will be given a ruler and graph paper. Use the information from the problem scenario that follows to answer the questions. Let x represent the number of months and let y represent the savings account balance.

Marcus is going to start saving $20 every month to buy a stereo. His parents gave him $10 for his birthday to open his savings account.

1. Write an equation that represents Marcus's savings account balance x months after he began saving.

2. Complete the table by selecting variables for x and calculating y.

Number of months	Account balance ($)

Use your graph paper to define the scale of the x- and y-axis and graph the ordered pairs. Use your ruler to draw a straight line through the points and complete the graph.

3. Use your graph to estimate the number of months it will take Marcus to save enough money for a $300 stereo. Explain.

continued

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4. If Marcus saved $40 per month instead of $20, how would you expect the slope of the graph to change? Explain.

5. If Marcus saved $10 per month instead of $20, how would you expect the slope of the graph to change? Explain.

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APPENDIX

Problem-Based Task Implementation Guides

Introduction

This addendum contains a collection of Implementation Guides (IGs) designed to support the use of the Problem-Based Tasks found throughout this unit.

Each Implementation Guide includes an overview and suggestions for introducing and facilitating the task. These contain specific recommendations for selected Standards for Mathematical Practice (SMPs) to highlight, guidance on addressing common errors and misconceptions associated with the task, and ideas for monitoring and coaching students as they work through the task. Finally, there is guidance for debriefing the task, with special attention to the SMPs and relevant Key Concepts from the instruction, as well as information on suitable extensions and alternate strategies.

Throughout the Implementation Guides, suggestions are provided in a question/prompt format to support your efforts to elicit student thinking and facilitate classroom discourse.

We developed these Implementation Guides for our Problem-Based Tasks at the request of math educators, and with advice and feedback from mathematics supervisors and teachers. Please let us know if they are helpful. Visit us at www.walch.com, follow us on Twitter (@WalchEd), or e-mail suggestions to customerservice@walch.com.

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Guide

Problem-Based Task: Identifying Parts of an Expression in Context

Common Core State Standard

A–SSE.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

Task Overview

Focus

When purchasing more than one of the same item, how can you write an expression to determine the cost of any number of items purchased? How do taxes affect the bill when purchasing multiples of the same item? How do you take this information and create an expression? Students will use a scenario to set up an expression that will help illustrate the meaning of terms commonly used in algebra to describe symbolic representations of real-world situations.

This activity will provide practice with:

- modeling a real-world situation with an algebraic expression or equation
- identifying and interpreting the parts of an expression

Introduction

This task should be used to explore or apply the skill of interpreting expressions that represent a quantity in terms of its context. It should be implemented after students have been introduced to translating verbal statements into algebraic expressions. This activity will provide students with a means of deepening their understanding of this skill.

Begin by reading the problem and clarifying the meaning of *tapas*:

tapas a wide variety of Spanish snacks or appetizers

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students might not know how to begin and will want to give up without trying. Some will write down the numbers they see and attach a variable randomly to one of the numbers. Encourage students to slow down and to apply what they know from their own experiences with buying multiple items that cost the same amount. Ask them to think about how they know how much they would spend if they bought 1 of these items, 2 of these items, 3 of these items, etc. Allow the students to extrapolate to *n* items.

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- **SMP 2:** Reason abstractly and quantitatively.

  Students might struggle with which operations to use in setting up the appropriate expression. Encourage them to reason about the concept of multiplication being repeated addition, and ask them how taxes are calculated and combined with the cost of the items purchased.

- **SMP 4:** Model with mathematics.

  Students are asked to model the situation symbolically. Math is a language, and translating the verbal communication into symbols is the task being asked of students. Students are often able to work with both symbols and verbal contexts, but have trouble converting between the two. Students might need explicit guidance to see that they do indeed possess the symbolic vocabulary to express the words in the context. This might require asking the students guiding questions about how specific words are translated into mathematical symbols.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- interpreting "\$6 per tapa" as division of \$6 by the number of tapas $\left(\frac{6}{n}\right)$ instead of multiplication of \$6 by the number of tapas ($6n$)

  To clarify this for students, ask, "What would be the price for 3 tapas?" (**Answer:** \$18) "Did you multiply or divide to get this answer?" (**Answer:** multiply)

- inserting an equal sign into the expression to make it an equation

  Point out that in order to have an equation, there must be two quantities that are equal. In this scenario, there is just one quantity being described.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- If students are having difficulty writing the expression:

  Ask what the unknown is in this situation. (**Answer:** the number of tapas ordered)

  Ask what the constant is in this situation. (**Answer:** the taxes, \$4.32)

  Ask students to explain verbally how they would find the total cost of the bill. (**Answer:** multiply the number of tapas by \$6, then add the tax)

- If students ask if they need to use a variable, ask in return, "What is the purpose of using a variable in an expression?" (**Sample answer:** "Using a variable allows us the freedom to change the quantity in different situations. If we want to know how much money the tapas would cost in general, we can write an expression: $6n$. With this expression, the value of n can change depending on the number of tapas we buy.")

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- To reinforce vocabulary, ask students to identify the coefficient, constant, variable, and number of terms in their expression.
- If students attempt to write an equation instead of an expression, ask: "Should there be an equal sign in the expression?" (**Answer:** No, because this is not an equation. We are modeling a situation, not solving for a variable.)
- Ask, "What is the first term in your expression?" (**Answer:** If students write their expression as $6n + 4.32$, then the first term is $6n$. If students write their expression as $4.32 + 6n$, then the first term is 4.32.)
- Ask, "Is there more than one way to write this expression?" (**Answer:** Students may switch the order of the two terms, or they may write the expression using repeated addition instead of multiplication.)

Debriefing the Task

Compare different strategies the groups used to identify and write the expression. One possible strategy is to create a table of values for purchasing increasing numbers of tapas until a pattern emerges and a variable can be substituted. (Note, however, that the amount of tax stated in the problem is only true for one specific number of tapas and not for any value of the variable.) Another possibility is students recognizing the translation of verbal statements into symbolic representations.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Expressions are made up of terms. A term is a number, a variable, or the product of a number and variable(s). An addition or subtraction sign separates each term of an expression. In the expression found in this task, the terms are $6n$ and 4.32.

  The factors of each term are the numbers or expressions that are being multiplied. In this task, 6 and n are factors.

- A coefficient is the number multiplied by a variable in an algebraic expression. In this task, the coefficient is 6.

  Terms that do not contain a variable are called constants because the quantity does not change. In the expression found in this task, the constant is 4.32.

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Extending the Task

To extend the task, change the expression and ask students to write a verbal statement to support the algebraic expression, or have them turn it into an equation to solve for n given the taxes. For example:

- Have students write a context for the expression $6(n + 4.32)$. Ask students to describe the similarities and differences between this expression and the original expression in terms of the vocabulary, algebraic expression, coefficient, constant, factor, term, and variable. (**Sample answer:** "In this expression, the number of tapas, n, is being added to 4.32 and then multiplied by 6, but in the original expression, the number of tapas was multiplied by 6 and then added to 4.32. After this new expression is simplified by distributing the 6, it becomes $6n + 25.92$. In this expression, the coefficient is still 6, the factors are still 6 and n, there are still two terms, and the variable is still n. However, the constant is now 25.92 instead of 4.32.")
- Ask students to write an equation and solve it to determine the cost of the tapas before tax if the taxes were \$4.32, calculated at a tax rate of 8%. Then, ask how many tapas were ordered. (**Answer:** Let x represent the total bill. $0.08x = 4.32$; $x = \$54$. Each tapa costs \$6, and $\frac{54}{6} = 9$, so they ordered 9 tapas.)

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK: "**How did you demonstrate perseverance?" (**Answer:** "I kept rereading the problem when I didn't understand what they were asking the first time," or "I wrote down the information I knew even though I didn't know what the answer was.")
- **For SMP 2, ASK: "**How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** writing the algebraic expressions)

  "Which of your strategies represent quantitative reasoning?" (**Answer:** calculating the price of the tapas before tax)
- **For SMP 4, ASK: "**How did you use mathematics to model this particular scenario?" (**Answer:** "I chose the variable n to represent the number of tapas, multiplied it by the price per tapa, \$6, and then added the constant \$4.32 for the tax to get $6n + 4.32$.")

Alternate Strategies or Solutions

- Addition is commutative. Therefore, the expression can be written as $6n + 4.32$ or $4.32 + 6n$.
- Without combining like terms, students could have also written $n + n + n + n + n + n + 4.32$.
- Using factoring, students can also write $2(3n + 2.16)$.

Technology

Students do not need any technology for this task.

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Guide

Problem-Based Task: Searching for a Greater Savings

Common Core State Standard

A–SSE.1 Interpret expressions that represent a quantity in terms of its context.★

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.*

Task Overview

Focus

How do interest rates and time affect the overall savings of an initial amount deposited into a savings account? Can two different accounts with different interest rates yield the same amount of savings for a different number of years? Students will compare two savings accounts with different interest rates and analyze the savings over different numbers of years.

This activity will provide practice with:

- substituting numbers for variables in an equation
- applying the order of operations
- comparing the results of calculations
- analyzing and making decisions about a real-world scenario
- interpreting the effect of changing the quantities of variables in an expression

Introduction

This task should be used to explore or to apply interpreting expressions that represent a quantity in terms of its context. This task should be implemented after students have learned how to substitute numbers for variables, apply the order of operations, and operate with exponents. Students may elect to solve this problem by graphing; in this case, students should be able to graph exponential equations and interpret points of intersection. Graphing may be done using a graphing utility.

Begin by reading the problem and clarifying the meaning of the following terms:

interest rate	for a savings account, money paid by a bank to the account holder; expressed as a percentage of the account balance
principal	the initial amount of money loaned, invested, or deposited in an account to earn interest
savings account	a bank account that earns interest

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Guide

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 2:** Reason abstractly and quantitatively.

  Some students might want to jump right to the conclusion that Account B is the better choice. Encourage students to support and justify this reasoning with explicit quantitative evidence. In helping students to work through the computational process, students will also be able to reason more generally about the effect that the exponent has on the base.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students might be content with substituting and calculating without making any interpretations or decisions. Encourage students to summarize their findings about the two accounts as they determine whether Austin should leave his money in the account with less savings for an extra year. If students disagree on their findings, ask students to explain their thinking. If all students agree that one account is best, ask them to consider a reason that Austin might have for choosing the other account and ask them to support this position.

- **SMP 5:** Use appropriate tools strategically.

  Students might struggle with using the calculator effectively and/or efficiently. Students at the high school level will most likely be using a scientific calculator that stores the latest calculation as the "answer" or "ans." Be prepared to show students how to efficiently use this feature for ease in calculating the savings. Also, students often struggle with raising a quantity to a power other than 2 or 3 since there are not typically buttons for powers of 4, 5, 6, and so on.

- **SMP 6:** Attend to precision.

  Students have a tendency to use equality symbols for every calculation. Encourage students to think about differentiating between the use of "approximately equal to" and "equal to." Additionally, students might want to write down the number exactly as it appears in their calculator. Ask students to think about what level of precision makes sense for the context of the problem.

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Guide

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly applying the order of operations

  Make sure students apply the exponent to the sum in the parentheses *before* they multiply by p.

- making decisions about the account with greater savings before carrying out the calculations

  Let students know that any decisions made without calculations are estimations, and that they should back up their answers with proof through calculations.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

- If students have trouble getting started:

  Ask, "To simplify the expression, which operation will you perform first and why?" (**Answer:** The first step is to find the sum of 1 and the interest rate, because this follows the order of operations by attending to grouping symbols first.)

  Ask for the most efficient way to use their calculators to perform these calculations. (**Sample answers:** Using the calculator's "ans" feature to recall the last calculation to use in the next calculation avoids the need to write down intermediate results; using parentheses to key in the expression as a whole in order to perform a single calculation.)

  Ask how to raise a quantity to a power other than 2 or 3 on the calculator. (**Answer:** Use the "^" button or the "y^x" button.)

- If students attempt to distribute the p before applying the exponent, remind them of the order of operations.

- Ask students, "By how much do the interest rates differ?" (**Answer:** 0.5% or 0.005)

- If students are having difficulty with the equation, suggest that they analyze the problem using a table of values.

  Ask, "What part of the expression does the interest rate affect: the base, the exponent, or the term as a whole? There is more than one correct answer. Explain how each part is affected." (**Answer:** The base and the term as a whole are affected. The base is increased by half a percentage point from Account A to Account B. This rate is then added to 1. The sum is raised to a power. Note that changing the interest rate does not affect the power. The power is a different variable—the number of years the money is left in savings. The principal is then multiplied by the result of raising the sum to the power. This affects the term as a whole.)

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Ask, "By how much does the savings increase from year 3 to year 4 in Account B?" (**Answer:** by about $20)

Ask students if this change in Account B from year 3 to 4 makes sense and why. (**Answer:** Yes, because 2% of $1,000 is $20, so it makes sense that by leaving the money in the account for one more year, the account would gain about another $20. We can make this estimation of 2% of the principal, $1,000, because we are looking at the investment after just a few years, so it has not grown by very much. As the balance grows and the interest is calculated based on the accumulated balance and not the principal, this estimate would be too low.)

Ask how the difference in the interest rate affects the balance of each savings account in one year. (**Answer:** After one year, Account A would have $1,015 and Account B would have $1,020.)

Debriefing the Task

- Compare the methods and strategies used by various groups. Possible methods include making a table of values for each function and comparing the two, substituting the years given in the problem for each function, or graphing the two functions.
- Have students who resisted completing the calculations because they could "see" why Account B would be better explain their reasoning. Follow up with a discussion of why we need to justify our decisions in real life; e.g., in business or at a job, any decision-making must be supported with evidence as to why the decision is sound.
- If no students graphed the functions for each account, ask them to graph the functions on a graphing calculator or with graphing software in order to analyze the two scenarios. Ask students how increasing the number of years changes the difference between the ending balances of the two accounts at the end of their investment terms. Discuss how the growth of Account B is much faster than Account A as the number of years increases, and ask students to think about why that is.

Connecting to Key Concepts

Make explicit connections to key concepts:

- As variables change, it is important to understand that constant terms will always remain the same. The change in the variable will not change the value of the constant term.

  Note that changing the principal does not change the interest rate or the number of years. Likewise, altering the number of years that the money is in an account will not change the amount of the principal. Altering these values does not change any of the other values in the equation other than the ending balance of money in the account.
- It is important to follow the order of operations, as this will help guide your awareness and understanding of each term.

  In this task, if the order of operations is not observed, students will not be able to correctly calculate the amount of money in each savings account at the end of each investment term.

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Extending the Task

To extend the task, have students graph the two functions and analyze the accounts based on the graphs of functions, and/or have students manipulate the variables to analyze the effects of each change. For example:

- Ask students to comment about the change in growth between the functions as the number of years increases, and have them quantify this change in growth.
- Have students calculate and compare average rates of change over several intervals.
- Have students manipulate the principal amounts while maintaining the same given interest rates so that the accounts yield the same savings after the same number of years. In other words, keep the given values of r for each account ($r = 0.015$ for Account A and $r = 0.02$ for Account B), and choose a value of t equal to some number of years, such as 6 years. Then, have students manipulate the principal amounts for each account separately, until they find the principal amounts that will result in both accounts having the same ending balance after 6 years. This can be repeated for different values of r and/or t.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "I interpreted what my answers meant in the context of the problem, and I guessed what would happen if I changed the value of certain variables.") "Which of your strategies represent quantitative reasoning?" (**Answer:** "I substituted values for the variables, and I calculated various account balances.")
- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "I explained my answer using numbers and calculations to support my reasoning.")
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" Ask students how they keyed in the expressions and compare the different methods students used. (**Answer:** "I used the '^' key to enter the exponent." Or, "I used repeated multiplication to calculate in place of the exponent.") Ask students which method they prefer/which is more efficient, and why. (**Answer:** "The '^' key is more efficient because I don't have to count the number of times I repeat the multiplication.") Ask students if one method of keying in the expressions is more susceptible to human error than another. (**Answer:** "Using repeated multiplication is more susceptible to human error because I could enter more or fewer instances of multiplication.")

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- **For SMP 6, ASK:** "How did you make sure you attended to precision?" Ask students to reveal their rounding choices and the reasoning behind them. (**Answer:** "I rounded my final answer to the nearest dollar because it wasn't necessary to determine the amount to the nearest cent in order to determine which account would have more money.") Ask students to think about money and the number of decimal places that can be used in this situation, and then ask them to think about how precisely they would report their answers if they were communicating the information to a family member or a friend; e.g., whether they would round to the nearest cent or the nearest dollar. (**Answer:** "I would round my answer to the nearest dollar because it isn't necessary to round to the nearest cent when communicating the information to a family member or friend.")

Alternate Strategies or Solutions

- Students can graph the two functions using graphing technology and compare the y-values for both functions when $x = 3$, $x = 4$, and $x = 5$. Students would see that the ending balance is about the same (approximately \$1,080) when $x = 5$ for the Account A function and when $x = 4$ for the Account B function.
- Students may choose to analyze the two accounts with an online interest calculator, such as this one from The Calculator Site, which allows users to input the principal, interest rate, number of years, and the compound interval for a savings account, and then calculate the resulting balance. Users may experiment by adjusting any of these values to see the effect on the outcome in the "Standard Calculator" tab. For more advanced calculations, the "Regular Deposit/ Withdrawal" tab lets users specify regular deposits or withdrawals and observe the effects.

  The Calculator Site. "Compound Interest Calculator."

  http://www.walch.com/rr/07000
- Students can create a table of values for each account where they use the interest rate to find that percent of the principal, add that interest to the principal, take the percent of the new amount, add it, and continue this process until they've reached the designated number of years. Sample tables for each account follow.

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Account A

Year	Account balance ($)	Interest rate	Interest to add ($)	Account balance after interest ($)
1	1,000.00	0.015	15.00	1,015.00
2	1,015.00	0.015	15.225	1,030.225
3	1,030.225	0.015	15.453	1,045.678
4	1,045.678	0.015	15.685	1,061.364
5	1,061.364	0.015	15.920	1,077.284

Account B

Year	Account balance ($)	Interest rate	Interest to add ($)	Account balance after interest ($)
1	1,000.00	0.02	20.00	1,020.00
2	1,020.00	0.02	20.40	1,040.40
3	1,040.40	0.02	20.808	1,061.208
4	1,061.208	0.02	21.224	1,082.432

Technology

Students can use scientific calculators for computations. Students could also use graphing utilities to arrive at the solution, as opposed to calculating. Graphing can also be a strategy used in justifying the solution to the task.

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Problem-Based Task: Rafting and Hiking Trip

Common Core State Standards

N–Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

N–Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

A–CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Task Overview

Focus

How does knowing how to write and solve an algebraic equation help in the real world? Aside from possibly becoming math or science teachers, when might students ever use algebraic concepts in their lives? Sometimes, calculations are used to help make decisions, and often these calculations involve algebra. In this task, students will use algebra to write and solve equations to make a decision about whether to hire assistants to help carry equipment on a hiking trip.

This activity will provide practice with:

- setting up ratios
- converting units
- writing and solving linear equations
- interpreting solutions in terms of the context
- making and justifying decisions based on the interpretations of the solutions

Introduction

This task can be used to explore or apply the skill of modeling a situation mathematically. This task should be implemented after students have learned how to translate verbal sentences into algebraic expressions, set up and simplify ratios, convert rates, and solve two-step linear equations.

Begin by reading the problem and clarifying the meaning of rates as they relate to the rafting trip—paddling in miles per day or cost in dollars per day. Suggest to students that they answer one question at a time before moving on to the next part of the task.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Some students will become overwhelmed with the amount of information presented in the task. Encourage students to break the task into smaller pieces. Also, suggest that students to write and label the known information as well as the missing information.

- **SMP 2:** Reason abstractly and quantitatively.

  Some students will be able to calculate the rafting mileage per hour mentally. Encourage these students to communicate their thinking clearly to another person. Others will need to see the concrete unit conversion on paper. Encourage these students to also explain their thinking.

  Some students will be able to determine the number of miles for the hiking trip without setting up an equation. Encourage these students to make their abstract thoughts concrete by writing an equation.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Some students will generate an argument for or against hiring assistants without any supporting evidence. Be sure that students use mathematics as part of their justifications for their decision.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- forgetting to convert to miles per hour from miles per day

  Remind students to check the units they are looking for (miles per hour) and the units they are given (miles per day). Explain that they must convert the given rate to miles per hour.

- incorrectly setting up the equation or not being able to set up the equation to determine the number of miles for the hiking trip

  Explain that the assistant would charge a flat fee of $50 (the constant term) plus $22 per mile (the variable term, where x is the number of miles), and this is equal to $512, the amount of money that would be paid to one assistant.

- not thinking about combining the equipment weight as a whole

  Rather than working with each person's equipment weight separately, explain to students that it is useful to combine all of the equipment weight together when looking into hiring assistants.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- If you hear students reporting that they need to travel 30 miles per day for the rafting portion of the trip (which is true), ask them what units the question is asking for. (**Answer:** miles per hour, not miles per day)
- If students are struggling with how to proceed for the hiking portion of the task, you might hear students wondering what to do with the 35 pounds and the 50 pounds and/or the other numbers in the task. Ask students to break down the information they already know. (**Answer:** There are 5 people in the group; each person will carry 35 pounds; each assistant charges a flat $50 fee and an additional $22 per mile; it will cost $512 to hire an assistant to carry equipment.)
- Again, if students are stuck, ask them to identify the question they are trying to answer from the task. (Assuming students have answered the rafting mileage question, the next question in the task asks how many miles the group will hike.)

  Ask students if they see any information that could be used to set up an equation to determine the number of miles the group will hike. (**Answer:** They know that the total cost of hiring one assistant for their trip is $512. They also know that each assistant charges a flat fee of $50 and an additional $22 per mile.)

  If students are able to identify this information, ask them what the variable is in the equation. (**Answer:** the number of miles)

  Ask how much it will cost to hire one assistant. (**Answer:** $512)

  Ask what contributes to this total fee. (**Answer:** the flat fee of $50 plus $22 per mile)

  Ask students if they can see the equation that can be set up from this information. (**Answer:** $50 + 22x = 512$)
- If students are unable to determine how to answer the question about hiring assistants:

  Ask students to think about the information given about how much weight each assistant can carry. (**Answer:** 50 pounds)

  Ask students to think about how they could evenly distribute the weight of the group's equipment among the members after giving two assistants 50 pounds each to carry.

  (**Answer:** Determine the total weight of the group's equipment: 35 • 5 = 175. Then subtract 100 pounds: 175 – 100 = 75. Divide the result by 5: $\frac{75}{5} = 15$.)

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- If students are able to perform the calculations but are confused about what to do with the results, ask them to retrace their steps in their calculations, and ask them what each number in the calculation represents. (**Answer:** 35 is the weight of the equipment each person carries without an assistant. 5 is the number of people in the group. 100 is the amount of weight that can be removed from the group to be carried by the assistants because there are 2 assistants who can each carry 50 pounds. 75 is the amount of weight left to be distributed evenly among each member of the group of 5, and that means that each group member will need to carry 15 pounds if the assistants are hired.)
- Because the process and the steps for solving the problem presented in the task are long, students might lose track of how to use the result of "15 pounds" to help complete the task. You might hear students say things like, "Now what?" Ask students to compare the 15 pounds to the original weight each group member would have to carry without assistants. (**Answer:** 15 pounds is 20 pounds less than the 35 pounds each group member would have to carry without assistants.)
- Students might stop and make their decision based on the weight alone without considering the added cost. Ask students to consider the cost of hiring two assistants and how they would distribute that cost among the members of the group. (**Answer:** Two assistants would cost 2 • $512 = $1024. This would be divided by 5 so that each member would need to pay an additional $204.80 for the trip.)
- Ask students why they chose their answer for the final question about whether it's worth it to hire two assistants. If students are not able to give justifications for their decision, ask them to go back and use some of the results of their calculations to justify their decision. (**Sample answer:** "We should not hire an assistant because the added cost is too much for three days of hiking." Or, "It's worth it to hire two assistants because for about $205 we could reduce the amount of equipment we have to hike with each day by 20 pounds.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

Compare strategies used by the various groups to solve the problems presented in the task. Ask students to demonstrate how they used their strategies and ask other students to comment about what they liked and/or disliked about that strategy. Possible strategies include:

- breaking the task into smaller sections
- creating a data table to organize the given information
- working backward
- listing any information they need or want to know before attempting to write and solve an equation
- asking questions about what the task requires them to find or do

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Connecting to Key Concepts

Make explicit connections to key concepts:

- A rate is a ratio that compares different kinds of measurements. A unit rate is a ratio of two measurements.

  In this task, students are required to convert miles per day into miles per hour using unit rates.

- Often, the most difficult task in translating a verbal context into an equation is determining what the variable is and how to represent that variable.

  Encourage students to reread the task as many times as necessary to understand what information has been given and what information is sought.

- The solution is the value of the variable that makes the equation true.

  Understanding what to do next with the solution to each part of a task, or how to interpret each solution, is a key to problem solving. Completing this task is a multistep process and students need to consider what each solution means in terms of the context of the task, and how each solution can be used to facilitate the process of completing the task.

Extending the Task

To extend the task, add a third component to the trip, such as biking. For example:

- Set up the extension problem to resemble the rafting portion of the trip, so that the members must travel a certain number of miles within a certain number of days. In the task, a day of travel is considered to be 8 hours. Ask students to determine how fast the group members must bike in order to complete the given distance in the given number of days.
- For the hiking portion of the trip, ask students to determine the ideal group size and number of assistants for which they would be willing to pay. Assume the same number of miles is hiked, 21 miles. How does increasing the group size affect students' decisions? At what point would they want to add another assistant? What is the ideal group size and number of assistants?

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I kept rereading the problem until I was able to work with at least one piece of information.") Ask students if they ever found themselves questioning whether their answers made sense in the context of the problem, and discuss whether or not they reworked their equation or calculations because their answer didn't seem to make sense.

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- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "Writing the expressions and the equations from context.") "Which of your strategies represent quantitative reasoning?" (**Answer:** "Analyzing the weight difference when using assistants versus not using assistants and comparing that to the additional cost per person for the trip if we hire assistants.")
- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Sample answer:** "After arriving at my answer, I was able to justify my decision about hiring the assistants by explaining my calculations.") Ask students if there were disagreements within or between the groups about which decision was the better one, and how any disagreements were handled.

Alternate Strategies or Solutions

- Students don't necessarily have to write equations to solve the problems presented in the task. Students could use reasoning and logic to determine quantities.
- Students may find it helpful to use manipulatives when determining how much weight each group member would carry if they hire two assistants. If possible, use bags of rice or sugar to fill two separate backpacks or bags with the indicated weights.

Technology

Students may wish to use a calculator.

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Problem-Based Task: Free Checking Accounts

Common Core State Standard

A–CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Task Overview

Focus

How do constant weekly savings accumulate? How much time does it take to reach a savings goal? Students will set up an inequality to determine the minimum number of weeks required to reach a savings goal by starting with an initial amount and setting aside a constant amount each week.

This activity will provide practice with:

- writing linear inequalities from context
- solving linear inequalities
- interpreting the solution to an inequality in terms of the context of the problem

Introduction

This task should be used to explore or apply the concept of creating inequalities from a context. It is best to implement this task after students have learned how to translate inequalities from verbal statements into symbolic representations and how to solve linear inequalities.

Begin by reading the problem and clarifying the difference between savings accounts and checking accounts, and explain the meaning of a minimum balance. Explain that checking accounts often require account holders to carry a minimum balance in order for the account holder to enjoy the benefit of free checking. Review the following terms:

bank account	an arrangement made with a bank whereby a person (the account holder) can deposit money and withdraw it
checking account	a bank account from which the account holder can withdraw money by writing checks, using debit cards, or transferring funds electronically
minimum balance	the least amount of money a bank requires an account holder to have in a bank account in order for maintenance fees to be waived
savings account	a bank account that earns interest

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Some students will not know how to translate verbal statements into symbolic representations. Encourage them to break verbal statements down into smaller pieces and, if necessary, review the statements word-by-word to translate them into symbols. Encourage students to think about what inequality is represented by the word *minimum*.

- **SMP 2:** Reason abstractly and quantitatively.

  Some students will be able to reason the number of weeks necessary to save $15 to reach the goal without setting up an equation. Allow this method as long as students can explain and describe the process. Help students translate the verbal process into symbolic representation. Other students might use a table of values and increase the savings account balance by $15 each week. Look for varying strategies to be discussed when debriefing the task.

- **SMP 4:** Model with mathematics.

  Students have the option of modeling the situation with a linear inequality. Some students will prefer this strategy because this is the most efficient method. Ask students how the model can be modified to fit other scenarios. For example, ask students how the model could be adapted to determine the number of weeks needed to save for a different amount of money, or to depict changing the amount of money saved each week but keeping the goal amount the same.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- using the less than symbol instead of the greater than or equal to symbol

  Remind students that "at least" means that values must be greater than or equal to the given value.

- using a non-inclusive inequality symbol (< or >) instead of an inclusive symbol (≤ or ≥)

  Advise students that each time they write an inequality, they should reread the problem to see whether the boundary value is an acceptable solution. Use this information to determine whether the inequality should be inclusive or not.

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- forgetting to reverse the inequality when necessary

  Make sure that students follow the same rules that are used when solving equations, and remind them that if they multiply or divide by a negative number while solving, they must remember to reverse the direction of the inequality.

- misidentifying the savings goal amount as $200 instead of $225

  Point out that there must be $25 in the savings account and $200 in the checking account, for a total of $225.

- writing a coefficient of 60 rather than 15

  Emphasize that they are depositing $15 per week and the number of weeks is represented by x, so the coefficient of x must be 15. The value 60 is a constant and does not change week to week.

- misinterpreting or not being able to interpret the solution of the inequality in terms of the context of the problem

  After solving the inequality, suggest that students reread the problem to remind themselves of what they were trying to find. (**Answer:** the minimum number of weeks to save $225)

- determining that it takes 12 weeks to save and not counting week 0 as the starting point if using a table of values

  Ask students who are using a table how much money is saved after week 1. (**Answer:** $75) Remind them to make sure their tables don't have $60 for week 1; $60 is the starting amount (week 0) before they begin to save.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- If students incorrectly identify the targeted savings amount:

  Remind them to consider the minimum amount of money that they are striving to save. (**Answer:** $225)

  Encourage students to think about the minimum balance required to maintain a free checking account. (**Answer:** $200)

  Ask students if there are any other criteria for the goal. (**Answer:** leaving $25 in the savings account after using $200 to open a checking account)

  Ask, "What does that mean you need to have in total?" (**Answer:** $225)

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- If students know what they are trying to find but are unsure of how to do so, ask them to review the information they have been given. Prompt students with questions such as:

  "How much money do you already have in the account?" (**Answer:** $60)

  "How much money are you depositing and how often?" (**Answer:** $15 per week)

- If students are unsure of how to set up the inequality, ask them the following:

  "What the rate is in this situation?" (**Answer:** $15 per week)

  "What is the variable?" (**Answer:** the number of weeks)

  "What quantities contribute to the overall savings?" (**Answer:** the initial $60 and the weekly deposits)

- If students use an equal sign instead of an inequality symbol:

  Ask them to identify the specific word or phrase from the problem statement that they used to translate into the equal sign. (**Answer:** Students won't be able to find such a word or phrase, or they will incorrectly identify a word or phrase as requiring an equal sign.)

  Ask students to think about how they might rephrase "the minimum number of weeks" using more familiar terms that they could translate into an inequality symbol. (**Answer:** Students could rephrase it as "the least number of weeks.")

  Ask students what inequality symbol represents the phrase "the minimum number of weeks." (**Answer:** greater than or equal to; ≥)

- If students are unsure of how to put the inequality statement ($60 + 15x \geq 225$) all together, review the task's Coaching questions with them, and ask students to write down the numbers and symbols with each step.

- Once students have the inequality solved ($x \geq 11$), ask them to think about why the inequality is inclusive; in other words, why it includes the parameter "equal to." (**Answer:** Answers may vary. Sample answer: "The inequality is inclusive because the minimum number of weeks it would take to save $225 is 11. By saving for 11 weeks, you can reach the goal of $225, which was the least amount of money required for your goal. This goal of $225 is included as part of the solution because the word minimum means 'the least amount allowed.'")

- Ask students to think about how the problem would be worded differently if the inequality were *not* inclusive; that is, if it did not include the parameter "equal to" as part of the solution. (**Answer:** Answers may vary. Sample answer: "What is the minimum number of weeks for which you will need to save money if you want to have more than $225?")

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- If students used a table of values to solve the problem and incorrectly determined that it will take 12 weeks to save the required amount, ask them how much money they had during week 1 if they deposited $15 during that first week, and ask them to compare that to the amount they had given in their tables for their first week of savings. (**Answer:** $75; note that some students will incorrectly start with $60 for their first week.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

Compare the strategies used by various groups. Ask students to share the strategies they used and ask why they used these methods.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Inequalities are sentences stating that two things are not equal. The two expressions created on either side of the inequality symbol in this task are sometimes equal, but not always. When they are not equal, the left side must be greater than the right side in the inequality $60 + 15x \geq 225$.
- Remember that the symbols $>$, $<$, $\geq$, $\leq$, and $\neq$ are used with inequalities. There is a tendency to use $=$ with all word problems.
- In this task, the phrase "the minimum number of weeks" is used and must be translated into "at least."
- Solving a linear inequality is similar to solving a linear equation. The processes used to solve inequalities are the same processes that are used to solve equations.

Extending the Task

To extend the task, change any of the parameters of the task. For example:

- Change the amount deposited per week.
- Change the minimum balance.
- Change the desired amount to have left over in savings.
- Change the rate of deposit after a certain time frame.

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Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you demonstrate perseverance?" (**Answer:** "I kept rereading the problem to find the information I needed," or "I broke up the problem into smaller phrases.")
- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** writing the expressions and the inequality from context) "Which of your strategies represent quantitative reasoning?" (**Answer:** making a table of values)
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** In the inequality $60 + 15x \geq 225$, 60 represents the initial savings and 15 in $15x$ represents the rate of savings per week or the deposit per week, where x is the number of weeks. \$225 represents the goal savings, and the "≥" symbol represents that we are looking for the minimum number of weeks to reach our goal savings.)

Alternate Strategies or Solutions

This task could be explored graphically, by analyzing the graph of $y \geq 15x + 60$. Students could also create a table of values that might look something like this:

Week number	Total savings ($)	Deposit amount ($)
0	60	15
1	75	15
2	90	15
3	105	15
4	120	15
5	135	15
6	150	15
7	165	15
8	180	15
9	195	15
10	210	15
11	225	15

Technology

Students can use tables or spreadsheets created with a graphing calculator or appropriate software to calculate the number of weeks of saving necessary to reach the goal of \$225.

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Problem-Based Task: Population Change

Common Core State Standard

A–CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Task Overview

Focus

How do populations change? Are they modeled by linear or exponential equations? Students will model two towns' populations and compare their predictions for the towns' populations at certain years.

This activity will provide practice with:

- writing exponential equations
- solving exponential equations without the use of logarithms
- comparing results
- analyzing the potential reasons for population growth and decline
- justifying conclusions

Introduction

This task should be used to explore or to apply the skill of creating exponential functions and using them to solve problems. It should be implemented after students have learned how to apply the order of operations, especially as this relates to exponential expressions, and after they have learned to write exponential equations. Calculators may be used and graphing may be done using technology, including graphing calculators or online graphing utilities. The use of spreadsheets is optional.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 2:** Reason abstractly and quantitatively.

  Encourage students to apply the formulas for population growth and decay and to ask themselves what variables they need to substitute and therefore find in the problem statement. Ask students if they can think of another way to solve the problem without using equations, such as through the use of tables.

- **SMP 4:** Model with mathematics.

  Students can model the scenario with two equations of the same structure, one for each town. Help students to see that the equations that they are creating model each town's population growth or decline over time for any given number of years at the constant growth or decay rate as given in the problem.

- **SMP 5:** Use appropriate tools strategically.

  Students might struggle with using calculators effectively. Be prepared to show students how to efficiently use the calculator feature that stores the latest calculation ("ans") for ease in calculating the new population after t years. If students are trying to type in the entire expression all at once, they might avoid using parentheses, which will result in an incorrect answer. Encourage students to think about the order of operations and why parentheses are necessary.

- **SMP 6:** Attend to precision.

  Students have a tendency to use equality symbols for every calculation. Encourage students to think about differentiating between the use of "approximately equal to" and "equal to." Ask students to think about what level of precision makes sense for the context of the problem. Encourage students to become comfortable with rounding, estimating, and making general statements about large numbers, especially when using a model to predict a future situation.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- misidentifying the base in the exponential model by forgetting to add or subtract the percent to or from 1

  Explain that adding the percent change to 1 ensures that the amount of change is added to the initial amount. For example, if an initial value of 100 increases by 25%, you multiply 100 by (1 + 0.25). In other words, you add 100 plus 25% of 100.

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- incorrectly applying the order of operations by multiplying the initial population by the base before raising the base to the power

  Remind students that in the order of operations, exponents must be applied before multiplying.

- forgetting to convert the growth/decay rate into a decimal

  Review with students that percentages are always written as decimals when carrying out operations.

- modeling the situations with linear equations instead of exponential equations

  Point out that the amount that the population increases each year is not a constant. It increases each year by a percentage of the previous year's population.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.
- If students are struggling with how to set up the initial equations, ask the following:

  "What is the general equation for exponential growth or decay?" (**Answer:** $y = ab^x$)

  "What is another way to write this formula if the growth rate is given as a percentage of the initial value?" (**Answer:** $y = a(1 + r)^t$)

- If students arrive at unusually large or small populations, check to see that students converted the percentages into decimals.

  Ask, "What conversion is important to make before entering the numbers into the formula?" (**Answer:** converting the percentages into decimals)

- Another way students might arrive at an unusually large or small population is if they multiply the current population by the base before raising the base to the power. Ask the following:

  "What is the order of operations?" (**Answer:** grouping symbols, powers, multiplication and/or division left to right, addition and/or subtraction left to right)

  "How does this order apply to your exponential models?" (**Answer:** "For Town A, we raise (1 + 0.05) to the fifth power before multiplying by the initial population of 39,000. For Town B, we raise (1 – 0.02) to the fifth power before multiplying by the initial population of 55,000.")

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- If students want to use the same equation to model both towns, ask the following:

  "Are both towns growing?" (**Answer:** No; one town is growing, and the other is shrinking.)

  "How does this fact affect the equations you write for each town?" (**Answer:** The Town A equation will model growth, and the Town B equation will model decay. The equations will not be the same.)

- If students try using a linear model for the situation, ask the following:

  "As you increase the x-values by 1, what happens to the y-values in a linear equation?" (**Answer:** The y-values increase or decrease by the same amount each time.)

  "What happens to the value of a number if you consistently take a percentage of that number, like in an exponential model?" (**Answer:** The y-values increase or decrease by a constant factor, not a constant difference.)

- If students wonder why it's necessary to add 1 to the growth rate for Town A, or if they forget to add 1 to the growth rate, ask the following:

  "What number in decimal form represents 100%?" (**Answer:** 1)

  "When you have growth, do you have more or less than 100%?" (**Answer:** more than 100%)

  Ask students if they can reason why the growth rate must be added to 1. (**Sample answer:** "In order to express growth, you must have more than 100%, so you must indicate an added growth rate that is greater than 100%. To calculate the base, you must use a decimal. 100% written as a decimal is equal to 1. Therefore, you add the growth rate in decimal form to 1.")

- If students wonder why it's necessary to subtract the decay rate from 1 for Town B, or forget to subtract the decay rate from 1, ask the following:

  "What number in decimal form represents 100%?" (**Answer:** 1)

  "When you have decay, do you have more or less than 100%?" (**Answer:** less than 100%)

  Ask students if they can reason why the decay rate must be subtracted from 1. (**Sample answer:** "In order to express decay, you must have less than 100%, so you must indicate a rate subtracted from 100%. To calculate the base, you must use a decimal. 100% written as a decimal is equal to 1. Therefore, you subtract the decay rate in decimal form from 1.")

- Remind students of the general form, $y = ab^x$, and ask them which parameter indicates growth or decay. (**Answer:** The variable b indicates growth or decay.)

  Ask students how to determine if an exponential function represents growth or decay based on that parameter. (**Answer:** If $b > 1$, then the function represents growth. If $0 < b < 1$, then the function represents decay.)

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- If students argue that the two populations are not the same after 5 years, or that one is not twice the population of the other after 15 years, ask them to think about the following:

  "What are the limitations of these models?" (**Sample answer:** The models do not account for any unforeseeable changes in growth or decline due to changes in trends based on employment/unemployment, disease, national economic changes, etc. Each model is also only a prediction and not a certainty or a guarantee.)

  "To what precision can we truly predict the population?" (**Sample answer:** The less specific we try to be, the more accurate our prediction will be. The calculated answer would be rounded up to the nearest whole person. However, we can round more than this. A safe estimate would be in the thousands place; when we round to the thousands place, the predictions are true.)

- If students are struggling with the second part of the task, ask them what is meant by "10 years after that." (**Answer:** This refers to 10 years after the populations are the same, which is 5 years from now. This means the economists are searching for the populations based on the model when $t = 15$.)

  Ask students how they can use this information and the models they set up in the first part of the task to complete the second part. (**Answer:** They can use the equations of exponential growth for Town A and exponential decay for Town B and use 15 years for the time, t.)

Debriefing the Task

Compare methods used by the groups and discuss advantages and disadvantages of the varying strategies.

Connecting to Key Concepts

Make explicit connections to key concepts:

- The general form of an exponential equation is $y = a \cdot b^x$, where a is the initial value, b is the base, and x is the exponent. The final output value will be y.

  In this task, b is first expressed as $(1 \pm r)$.

- Since the equation has an exponent, the value increases or decreases rapidly.

  The value increases or decreases by a constant factor, b.

- If the base is greater than 1 ($b > 1$), then the exponential equation represents exponential growth. If the base is between 0 and 1 exclusive (that is, $0 < b < 1$), then the exponential equation represents exponential decay. Another form of the exponential equation is $y = a(1 \pm r)^t$, where a is the initial value, $(1 \pm r)$ is the base, t is the variable exponent, and y is the final value. Use $y = a(1 + r)^t$ for exponential growth (notice the plus sign).

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In this task, in Town A, the population grows by 5%. This means that $r = 0.05$. The product of the initial population and 0.05 must be added to the initial population to show the increased population. Substituting 0.05 for b in the formula $y = a \bullet b^x$ requires the expression $(1 + r)$ in place of b to arrive at the full growth rate of 105%, or 1.05.

- Use $y = a(1 - r)^t$ for exponential decay (notice the minus sign).

  In this task, use the exponential decay formula to model the population changes in Town B. Because the population of Town B is decreasing by 2% each year, 98% is the factor by which the population is multiplied each year. The population is always 98% of the population from the year before (a 2% decrease). Think of this as 100% minus the rate, or in decimal form, $(1 - r)$.

Extending the Task

- To extend the task, give students "actual" populations of the towns over several years and ask them to choose a rate and model the situations. Ask students to use their chosen model to predict the populations of the towns in 15 years. Have students justify the rates they chose and the implications for choosing one rate over another.
- This task could also be extended by requiring students to solve the original task or the first extension both by creating a table of values using a spreadsheet and by graphing through calculators or software. Be sure that students arrive at the same solutions even though they are using different representations.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "Writing the expressions and the equations from context.") "Which of your strategies represent quantitative reasoning?" (**Sample answer:** "I created a table of values to arrive at the solutions rather than creating equations and models.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I translated the words into symbols to represent the calculations that were happening given the context of the situation. The scenario involved growth and decay rates of the towns' populations, so I knew it would be modeled with exponential equations.")

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- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Sample answers:** "When entering the equation in the calculator, I used the parentheses to make sure the order of operations was followed." Or, "I used the 'ans' feature to recall previous answers when used in intermediate calculations.") "How did you use spreadsheets strategically?" (**Sample answers:** "I used a table of values instead of using equations." Or, "I set up a spreadsheet to perform repeated calculations.")
- **For SMP 6, ASK:** "How did you make sure you attended to precision? (**Sample answer:** "When rounding, I rounded to the thousands place because that made the numbers easier to work with and made more sense to me.") "What level of precision makes sense for the context of the problem and why?" (**Sample answer:** "Rounding to the nearest 1,000 or even 10,000 makes sense because estimating future populations produces general predictions only.")

Alternate Strategies or Solutions

- Students may use repeated multiplication and/or set up a table of values on a graphing calculator or a spreadsheet.
- Students might elect to calculate 5% of the population of Town A and then add the result to the initial population to get 40,950. They would follow this same procedure for the remaining years. These same students might multiply Town B's population of 55,000 by 2% and then subtract that result to get 53,900. They would continue this same procedure for the remaining years.
- Other students might jump right to multiplying Town A's initial population by 1.05 to get 40,950 and similarly multiplying Town B's population by 0.98 to get 53,900, then repeating the process to find the populations for years 5 and 15.
- Alternatively, students could choose to solve this problem by graphing. Once the models are generated, students can plot the equations and compare the results for 5 years and 15 years.

Technology

Students can use scientific calculators, graphing utilities, and/or spreadsheets.

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Problem-Based Task: Phone Card Fine Print

Common Core State Standards

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

N–Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

Task Overview

Focus

Graphs can be powerful images that give us a great deal of information quickly. How do we set up and graph a linear equation? What units are best to use on each axis to display the information in a problem? In this task, students will write a linear equation from context, and then graph the equation.

This activity will provide practice with:

- writing a linear equation
- setting up a coordinate plane
- graphing a linear equation

Introduction

This task should be used to explore or apply the skill of creating and graphing linear equations and interpreting units. Students' knowledge of graphing should include familiarity with setting up a coordinate plane with the appropriate units and axes. Students may verify their graphs using graphing technology.

Begin by reading the problem and clarifying the meaning of the following terms:

fine print a part of an agreement or document that lists restrictions and important details; often written in small type or language that's hard to understand

phone card a prepaid card used to pay for phone calls

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 2:** Reason abstractly and quantitatively.

  Students might have difficulty writing the equation, which is abstract, from the context, which is quantitative. Encourage students to break down the problem into smaller parts and to pick out words they can translate into symbols.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Some students will disagree about what the correct equation is that models the scenario. Encourage students to justify their thinking. Justifications should include concrete examples of applying the model. Students should also be encouraged to think about a common-sense approach (without using an equation) and to create a few examples with this approach.

- **SMP 4:** Model with mathematics.

  Students are trying to translate the context of the problem into a mathematical model that works for all values of "Minutes used," represented by x. Students might struggle with deciding whether to use a linear or an exponential model. Encourage students to think about what is happening to the minutes left on the card with each minute spent on the phone. Ask students to talk about the differences between linear and exponential models, and help them to see that this situation is linear.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- switching the slope with the y-intercept from the context of the problem

  Remind students that the slope represents the rate. A phone call will reduce the number of available minutes on the card by 1.5 minutes for every actual minute spent on the phone. Because the number of minutes on the card is decreasing, the rate of change is negative: –1.5. The y-intercept tells the starting number of minutes available on the card, 360, before any have been used. This is a constant.

  Remind students that in the slope-intercept form of a line, $y = mx + b$, the variable m represents slope and the variable b represents the y-intercept.

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- forgetting to convert "6 hours" into minutes

  Emphasize the fact that the units must be the same throughout the problem. Because the problem asks us to graph how many minutes are left on the card in relation to how many minutes have been used talking on the phone, 6 hours should be converted to minutes.

- graphing a positive slope instead of a negative slope

  Point out that the number of minutes left on the phone card is decreasing, so the line should have a negative slope.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.
- Refer to the following sections for specific approaches while students are writing the model and graphing it.

Writing the Model

- If students are struggling with where to start:

  Ask, "What kind of equation is described here: linear, exponential, or other?" (**Answer:** linear)

  Follow up by asking, "How do you know?" (**Sample answer:** "There is a rate of constant decrease given and a starting value given. This sounds like the slope-intercept form of a line.")

- If students are trying to use 5 or 6 as the slope:

  Ask them to identify what *slope* means. (**Sample answer:** "Slope is a rate of change.")

  Ask students to identify what is changing. (**Answer:** "As you talk on the phone, the number of minutes you have left on the card is decreasing.")

  Ask students if they can identify by how much the time left on the card is decreasing with each minute spent on the phone. (**Answer:** 1.5 minutes)

- If students use a positive slope in their model:

  Ask if the number of minutes on their phone card is increasing. (**Answer:** No, it's decreasing.)

  Ask students how decreasing minutes affects the model. (**Answer:** Because the card is losing minutes, the slope is negative.)

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- If students are struggling with determining what the *y*-intercept is:

  Ask them to consider what the *y*-intercept means in terms of the context of the problem. (**Answer:** The *y*-intercept is the starting number of minutes on the card, 360.)

  If students have trouble identifying the context, ask them what the *y*-intercept means in math terms. (**Answer:** It's the value of y when $x = 0$.)

  Ask students to identify the meaning of the *y*-intercept when $x = 0$. (**Answer:** It's the amount of time on the card before using it at all, which is given in the problem as 6 hours.)

- If students jump to substituting 6 for b in the model and then try graphing the result:

  Ask them to reread the problem statement and again identify what the problem is asking for. (**Answer:** The problem is asking for a model that shows the number of minutes left on the phone card based on the number of minutes used.)

  Ask students why using the given time of "6 hours" is problematic. (**Answer:** This amount of time is not in terms of minutes.)

- If students are struggling with converting hours to minutes, ask them how many minutes are in an hour and then ask how many hours they have to convert. (**Answer:** There are 60 minutes in 1 hour and there are 6 hours on the card, so $6 \cdot 60 = 360$ minutes in 6 hours.)

- If students set up the model incorrectly, ask them to review what we found to be the rate and the *y*-intercept. (**Answer:** The rate is –1.5 minutes per minute and the *y*-intercept is 360 minutes.)

Graphing the Model

- If students are struggling with which variable is placed on each axis:

  Ask students to identify the dependent variable; this variable belongs on the vertical axis. (**Answer:** The dependent variable is the number of minutes left on the card, y, because this amount *depends* on the number of minutes used.)

  If students have difficulty identifying the dependent variable, focus their attention on first identifying the independent variable, x. Ask them what variable or quantity they have control over; this is the independent variable. (**Answer:** "I control how much time I talk on the phone.")

  Ask students on which axis the x or the independent variable belongs. (**Answer:** the horizontal axis)

- If students are struggling with what units to use for the tick marks, you might see them counting by ones. Ask students what the largest number is that they have to count to on the *y*-axis. (**Answer:** 360)

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Ask students if they have enough room on their paper to count by ones, twos, or fives. (**Answer:** The paper probably isn't large enough to count by any of those numbers.)

Ask students what number they would like to use and what considerations they might take into account when choosing their scale. (**Answer:** Students might want to consider the *x*-intercept.)

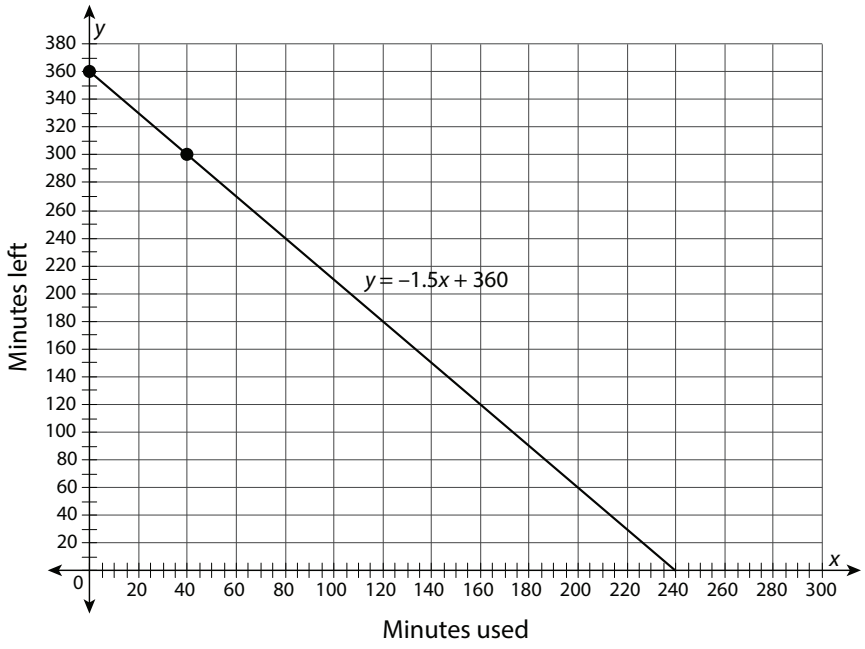
Ask students if they have considered where the *x*-intercept is located and how to calculate the *x*-intercept. (**Answer:** To calculate the *x*-intercept, set $y = 0$ and solve for x: the *x*-intercept is 240 minutes.)

Note: At this point, student graphs may differ; students might choose a scale of 20, 40, 50, or 60, or some other large increment.

- Ask students about how they drew their graphs:

  Ask which points they plotted on the graph to be sure that they graphed the model they created to fit the situation. (**Sample answer:** (0, 360) and (240, 0))

  Ask students how to use the slope to graph the equation. (**Answer:** Plot the *y*-intercept and then count down 1.5 units and over to the right 1 unit. Since the units used in the graph are large, this will be difficult to do unless the slope $-\frac{3}{2}$ is written in larger units. For example, students might have counted by 20s on their graphs. In this case, they could count down 60 units and over 40 units, then plot the second point, as shown in the following graph.)



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Debriefing the Task

- Ask volunteers to share their graphs with the class for comparison.

  Ask the class which graph is easiest to read and why.

  Ask which graph shows the most information and why.

  Ask if there is enough information on each graph (labels, equation, title, tick marks, etc.) that a person outside the class would be able to read and interpret it clearly.

- Compare and discuss the strategies each student used to write the equation.

Connecting to Key Concepts

Make explicit connections to key concepts:

- The slope of a linear graph is a measure of the rate of change of one variable with respect to another variable.

  In this task, the amount of minutes left on the phone card decreases by 1.5 minutes for each minute the user talks on the phone. The rate is $-\frac{1.5}{1}$ or $-\frac{3}{2}$.

- The slope-intercept for of an equation, $y = mx + b$, is often used to easily identify the slope and y-intercept, which then can be used to graph the line.

  In this task, students must determine the slope and y-intercept in order to write and then graph the equation $y = -1.5x + 360$, where -1.5 is the slope and 360 is the y-intercept.

- The x-intercept of a line is the point where the line intersects the x-axis, $(x, 0)$.

  The x-intercept is found by substituting 0 for y in the equation. The x-intercept in the task is (240, 0).

- Only use positive values of x and y when this makes sense for the context of the problem.

  In this task, both of the graph's axes are in units of time, which is not negative. Therefore, only positive units are used on the x- and y-axes.

- Determine the independent and dependent variables. The independent variable will be labeled on the x-axis. The dependent variable will be labeled on the y-axis.

  In this task, the independent variable, x, represents the number of minutes that the phone card is used. The dependent variable, y, is the number of minutes left on the card.

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Extending the Task

To extend the task, choose a different rate and/or a different start value, or ask students to determine how much each minute of phone time costs since the card costs $5. For example:

- Ask students to write and graph the equation for a card that loses 1.2 minutes for each minute used. (**Answer:** $y = -1.2x + 360$)
- Ask students to write and graph the equation for a card that loses 1.2 minutes for each minute used and that starts with 4 hours of minutes. (**Answer:** $y = -1.2x + 240$)
- Ask students to determine about how much each minute of use of the 4-hour card costs the user if the card initially cost $5. (**Answer:** $\frac{\$5}{240} \approx 0.0208\overline{3}$, or a little more than $0.02 per minute)

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** writing the equation from context) "Which of your strategies represent quantitative reasoning?" (**Answer:** converting to the appropriate units)
- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Sample answer:** "Yes, I used my equation to show how many minutes would be left on the card after 6 minutes of use. Then I discussed with others to determine that 6 minutes of use would result in a decrease of 9 minutes on the card.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I modeled the scenario by using the equation $y = -1.5x + 360$. In this model, the y represents the dependent variable, which is the number of minutes left on the card. The value of –1.5 represents the rate at which the minutes on the card decrease for each minute of use of the card, x. The value of 360 represents the initial number of minutes on the card.")

Alternate Strategies or Solutions

- Students may use different scales on the x- and y-axes, so their graphs might look different.
- Students could use a table of values to find how many minutes of use it takes to use up all the card's minutes. Students may recognize this as the x-intercept. They could then plot the x- and y-intercepts and connect the two points, thereby creating the graph without the equation.
- Students might list the slope in fraction form as $-\frac{3}{2}$.

Technology

Students can use graphing technology to verify that their graph is correct. They can also use a spreadsheet or the table feature of a graphing calculator to create a table of values to verify the x-intercept.

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Problem-Based Task: Investing Money

Common Core State Standards

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

N–Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

Task Overview

Focus

How do interest rates and compounding schedules affect the growth of an investment? Does it matter if the interest is compounded monthly or yearly? In this activity, students will compare two investments by modeling them with exponential equations.

This activity will provide practice with:

- writing exponential equations from a context
- graphing exponential equations using technology
- comparing functions by looking at tables
- analyzing and making decisions about a real-world scenario
- interpreting units in exponential equations and graphs
- interpreting the significance of the intersection of two graphs in a real-world context
- examining the effect on the outcome if one part of an equation is changed

Introduction

This task should be implemented after students have learned how to identify whether a situation is linear or exponential, and after they have learned how to write and graph exponential equations. Graphing may be done using technology, including graphing calculators or online graphing programs.

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Begin by reading the problem and clarifying the meaning of the following terms:

balance	the amount of money currently in a bank account
interest compounded annually	a percentage of the balance that is added to the account once per year
interest compounded monthly	a percentage of the balance that is added to the account once per month
investing	putting money into something to make a profit
principal	the initial amount of money loaned, invested, or deposited in an account to earn interest

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 2:** Reason abstractly and quantitatively.

  Some students will quickly write the exponential equations to model the two investment strategies, but other students will rely first on making tables, trying different values, and discussing possible outcomes. Allow and encourage these "non-standard" methods, as they often lead to valid and insightful solutions. Look for students using differing strategies so that their strategies can be discussed during the debrief.

- **SMP 5:** Use appropriate tools strategically.

  Students will need to find the intersection of two curves to calculate when the two investment options would yield the same results. Students will see the usefulness of graphing technology in finding the intersection of these two exponential curves.

- **SMP 7:** Look for and make use of structure.

  The task presents "what if" questions that lead the students to view the separate pieces of the equations as discrete units, so they can analyze what happens to the outcome if one part of the equation changes. Pose questions that allow students to make these connections. For example, ask, "Does changing the principal affect the shape of the curve?" "What happens to the shape of the curve if the interest rate increases?" "Which number in the equation corresponds to the *y*-intercept of the graph?"

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Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly setting up the equation

  Do not provide the formula for compounding interest unless no one in the class is able to do this. If necessary, stop the task and provide a mini-lesson on the formula and what the various parts represent. The formula for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A is the current balance, P is the principal, r is the interest rate written in decimal form, n is the number of compounding periods per year, and t is the number of years.

- writing the exponent for monthly compounding as $\frac{1}{12}t$ instead of $12t$

  Ask students how many compounding periods there are in 1 year (12), in 2 years (24), in 3 years (36), and so on. Point out that in each of these cases, the number of years is multiplied by 12 to find the number of monthly compounding periods.

- incorrectly modeling this situation with a linear equation

  Discuss the fact that the amount of money in the account does not increase by a constant amount each year; rather, each year the balance is multiplied by a percentage of the previous year's balance and then increased by that amount. This describes an exponential function, not a linear function.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.
- Ask what the shape of the graph tells you about the amount of money in the account. (**Answer:** Both graphs are increasing, and they are getting steeper and steeper with time. The longer the money is in the account, the faster it will grow.)
- If students are struggling with how to determine whether the equation should be linear or exponential, ask how they can determine that the graph shouldn't be a line. (**Answer:** The amount of money in the account is not increasing by the same amount each year. The amount by which it increases is a percentage of the total, so it increases by a greater amount with each passing year.)

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- For students using tables instead of graphs, ask how the table can be used to find the year in which the two investment possibilities have the same amount of money. (**Answer:** Scroll through the table to find two *y*-values that are approximately equal. The corresponding *x*-value is the number of years.)
- Ask, "What does the value of *P* tell you about the graph?" (**Answer:** The value of *P*, which is the principal, is the *y*-intercept of the graph.)
- Ask whether it would be possible to have the same amount of savings as the first bank in the problem if the principal is lower and the interest rate is higher. (**Answer:** Yes.)
- Point to the value of *n* in the student's equation and ask, "If I change *this* value to 4, what would that mean in the context of the problem?" (**Answer:** The value of *n* tells how many times the interest is compounded each year, so changing this to 4 would mean that the interest is compounded quarterly.)

Debriefing the Task

- Compare the methods used by different students. Possible methods include the strategy outlined in the coaching questions, as well as creating tables for the various scenarios (either by hand or with a graphing calculator) and comparing balances at various time intervals. Discuss all the strategies the students used, and ask students if they can think of any other ways to solve the problem.
- Have students share and explain their work. Encourage students to ask each other questions.

Connecting to Key Concepts

Make explicit connections to key concepts:

- The general form of an exponential equation is $y = a \bullet b^x$, where *a* is the initial value, *b* is the rate of decay or growth, and *x* is the time. The final output value will be *y*.

  Compare this general form to the formula for compound interest, $A = P\left(1 + \frac{r}{n}\right)^{nt}$. Ask students to explain why there is a "1" in this formula. (**Answer:** Adding "1" ensures that the result for *A* is not just the interest, but the principal *plus* the interest.)

- If the base is greater than 1 ($b > 1$), then the exponential equation represents exponential growth.

  In the formula used for this problem, the base is $\left(1 + \frac{r}{n}\right)$. Because we're adding 1 to the term containing the interest rate, the base will be greater than 1, which indicates growth.

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- Remember to change the percentage rate into a decimal by dividing the percentage by 100.

  In this task, we changed 3% to 0.03 and we changed 3.2% to 0.032.

- Remember to apply the order of operations and divide *r* by *n*, then add 1. Raise that value to the power of the product of *nt*. Multiply that value by the principal, *P*.

  Follow this procedure when simplifying $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

Extending the Task

To extend the task, provide students with prompts that can be answered using their equations, tables, or graphs. For example:

- "Predict and then calculate the amount of money in each account after 40 years." (**Answer:** approximately $6,524 in the first account, and $6,833 in the second account)
- "If you had spent $200 on your way to the second bank instead of $100, how long would it take for the balances in the two accounts to be the same? What would the balances be?" (**Answer:** approximately 43 years; approximately $7,173)

To further extend the task, have students research savings accounts and interest rates at a local bank and choose an account in which to invest a hypothetical $2,000. Then, compare what the account balances would be after 30 years.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** writing equations) "Which of your strategies represent quantitative reasoning?" (**Answer:** making a table and looking at numbers/data)
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** "I used a graphing calculator to find the intersection of the two exponential curves.")
- **For SMP 7, ASK:** "How did you look for and make use of structure when solving this problem?" (**Answer:** "I recognized that the equations that represent the two bank accounts have the same structure, and I looked at the impact that changes in the different parts of the equation would have on the result.") "Which of the variables in the formula make the graph rise more steeply or more gradually?" (**Answer:** the interest rate, *r*) "Which part changes the *y*-intercept of the graph?" (**Answer:** the principal, *P*)

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Alternate Strategies or Solutions

- Along with the method described in the Coaching questions, students may solve the problem without making graphs. This is a valid method, although students may see through class discussion that there are advantages to looking at the graphs of the functions.
- To solve the problem using the table feature of a graphing calculator instead of graphs, students can set up the two equations for the two different banks: $A = 2000(1.03)^x$ and $A = 1900(1.00267)^{12x}$. Students can enter both equations into their graphing calculators, then set up the table so that x begins at 0 and increases by ones.
- To find how long it will take to earn back the \$100 spent before opening the account at the second bank, have students scroll through the y-values for the second equation until they find the year when this value reaches \$2,000. This occurs during year 2. To find a more precise answer, set the table so that the x-values are counting up by $\frac{1}{12}$; each row of the table will now represent one month. It is during the nineteenth month that the balance in the account reaches \$2,000.
- To determine how long it will take for the balances in the two accounts to be equal, have students scroll through the table until they find the point where the two balances (y-values) are approximately the same. The x-value when this happens is 21, so the accounts have equal balances after 21 years.

Technology

Students can use graphing technology and spreadsheets to help them solve this problem.

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Problem-Based Task: Skate Constraints

Common Core State Standard

A–CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★

Task Overview

Focus

How do you set up constraints to model feasible regions, or the set of ordered pairs that satisfy the constraints, for manufacturing multiple items using the same equipment? How do you know if your manufacturing goal meets the criteria and can be manufactured given the constraints? In this task, students will write a system of inequalities with constraints given a real-world context, and test various scenarios for feasibility.

This activity will provide practice with:

- writing a system of linear inequalities from context
- analyzing numbers of items that can be manufactured given the system of inequalities
- interpreting the results of various test points

Introduction

This task should be used to explore or apply the concept of constraints. It should be implemented after students have learned how to make substitutions and interpret whether inequalities are true for a given point.

Begin by reading the problem and clarifying the relevant parts of hockey skates and figure skates—the boot and the blade. For students unfamiliar with the differences between the two types of skates, provide pictures. Review the meaning of the term *constraint*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Some students may argue with each other about what the correct system of inequalities is that models the scenario. Encourage students to justify their thinking. Justifications should include concrete examples of applying the model. Listen for students who justify their reasoning using the phrase, "Because it just makes sense" or something similar. Encourage these students to detail their thinking. Encourage students to ask themselves questions about why they chose the model or method they used.

- **SMP 4:** Model with mathematics.

  Students are trying to translate the context of the problem into a mathematical model that works for the manufacturing of both figure and hockey skates. Students might try to group the information by the type of skate as opposed to grouping the information by component (blade and boot). Encourage students to think about what the final time restriction is and how it has been given—by skate or by component.

- **SMP 7:** Look for and make use of structure.

  Students might randomly choose x- and y-values to substitute into the system of inequalities. Remind students to think about how choosing a value for the number of figure skates (or hockey skates) in the first two-variable inequality affects which two-variable inequality becomes the limiting inequality. Also, encourage students to think about how increasing or decreasing the value for one variable affects the value for the other variable in that inequality. Encourage students to think about what values will not work in the system given the real-world context.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly translating verbal descriptions into algebraic sentences

  Students may try to write one inequality about figure skates and the other about hockey skates. Emphasize that the constraints presented in the problem refer to blades and boots, not the type of skate.

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- using the incorrect inequality symbol

  Point out that because there is a maximum of 40 hours available to make blades, the number of hours must be less than or equal to 40. Therefore, the correct symbol to use is ≤. Likewise, there is a maximum of 20 hours available to make the boots, so the number of hours must be less than or equal to 20. Again, the correct symbol to use is ≤.

- forgetting the two constraints, $f \geq 0$ and $h \geq 0$

  Emphasize the importance of these two constraints, as they ensure that negative values are not considered.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- If students are struggling with how to start:

  Ask them what kind of situation we are trying to model. (**Answer:** a system of inequalities)

  Ask students how they know the situation can be modeled using inequalities. (**Answer:** "I know this because we have a maximum amount of hours to spend on creating blades and boots, but we could spend anywhere up to and including that maximum, which indicates that this situation is modeled by an inequality.")

  Ask students how they know the situation is modeled by a system of inequalities. (**Answer:** "There are several criteria to be met for two variables. When multiple equations or inequalities work together with the same variables, the result is a system. Because the problem gives a different maximum number of hours for each part, there needs to be one inequality for blades and a second inequality for boots.")

- If students try to set up the inequalities so that the information is grouped by skate type instead of by component, ask them to think about the terms in which the criteria for maximum number of hours were given. (**Answer:** "The maximum numbers of hours is given first in terms of creating blades and then in terms of creating boots.")

- If students struggle with writing an inequality representing skate blades, ask:

  "What are the variables in the system? What letters can represent these variables?" (**Answer:** "The variables are the number of figure skates made and the number of hockey skates made. Let f represent figure skates and h represent hockey skates.")

  "How many hours does it take to make a blade for each type of skate?" (**Answer:** "It takes 2 hours to make the blade of a figure skate and 3 hours to make the blade of a hockey skate.")

  "How many hours a week can be spent making blades for both types of skates?" (**Answer:** no more than 40 hours a week)

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"Which inequality symbol should be used?" (**Answer:** "The ≤ symbol, because the amount of hours can be equal to 40, but cannot be greater than 40. Therefore, the total number of hours has to be less than or equal to 40.")

"Based on this information, what inequality can you write to express the time constraint for making skate blades?" (**Answer:** $2f + 3h \leq 40$)

Direct students to follow the same process when setting up the inequality for making boots, but provide less coaching. If students struggle with the second inequality, guide them through reading the problem scenario to determine the variables (f and h), coefficient values (3 and 1), and time limit (≤ 20 hours) for making boots. The resulting inequality is $3f + h \leq 20$.

- If students forget to specify that the values of both variables cannot be negative, ask them to think about what else must be true about the number of skates made. (**Answer:** "The number of each type of skate made can be equal to 0, but cannot be less than 0. Therefore, the values of the variables cannot be negative.")

  Ask students how to represent these limits with inequalities. (**Answer:** $f \geq 0$ and $h \geq 0$)

- Students might have all four inequalities written on their papers, but not grouped together as a system. Ask students how to write the four inequalities as a system. (**Answer:** $\begin{cases} 2f + 3h \leq 40 \\ 3f + h \leq 20 \\ f \geq 0 \\ h \geq 0 \end{cases}$)

- If students are unsure of how to use the system of inequalities to complete the task:

  Ask students to reread the problem statement to help them determine what else the task asks them to find. (**Answer:** "The problem statement asks us to identify possible combinations of figure and hockey skates that can be made given the constraints of the system.")

  Ask students how they can determine possible solutions. (**Answer:** "Pick a test point—a sample number of figure skates and a sample number of hockey skates—and substitute those values into the system.")

  Ask students to identify how they will know if their test point is a solution to the system. (**Answer:** "The test point will make all four of the inequalities true.")

  Ask students to identify how they will know if their test point is *not* a solution. (**Answer:** "The test point will make at least one of the inequalities false.")

- If students struggle with how to choose a test point, ask them to start with 1 figure skate and 1 hockey skate (1, 1). Ask them to increase each value in increments of 1 until a test point fails.

- As students work through the task using test points:

  Ask students to find several combinations that would work for the system. (**Sample answers:** (0, 3), (1, 12), (2, 11), (3, 10), and (4, 8). *Note*: There are 40 possible solutions.)

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Ask students to explain why their points are solutions to the inequality. (**Answer:** Students should show their work for the values they substituted for f and h, and demonstrate that each inequality is true for these values.)

Ask students to find some counterexamples, or test points that are *not* solutions. (**Sample answers:** (3, 12), (4, 9), (2, 15), and (6, 4))

Ask students to explain why their counterexamples are not solutions. (**Answer:** "The point makes one or more of the inequalities false.")

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Compare strategies used by the groups.
- Ask if anyone used graphing to find the solutions. Ask volunteers to share their work and explain their thinking.
- Ask if anyone tested points systematically to determine the viable solutions. Ask volunteers to explain their thinking.
- Ask students who used trial and error to explain how they chose their test points.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Sometimes there are limits or restrictions on the values that can be substituted for the variables into an equation or inequality; other times, limits or restrictions are placed on answers to problems involving equations or inequalities. These limits or restrictions are called constraints.

  In this task, the constraints are the number of hours it takes to make each component of each type of skate, and the number of hours per week available to make each component.

- A system is a set of equations or inequalities with the same unknowns.

  In this task, the unknowns are the number of figure skates and the number of hockey skates that could be made in a week.

- When creating a system of equations or inequalities, it is important to understand that the solution set is the value or values that make all sentences in the system true statements.

  In this task, students test possible solutions by substituting them into each inequality to see if they make the inequality true.

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Extending the Task

Students might want to learn more about how skate components are actually made. The following is a short video (2:49) about the process of making blades for hockey skates. This video can also be used for engagement prior to implementing the task.

- Discovery Science Channel. "How It's Made: Hockey Blades."

  http://www.walch.com/rr/07001

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "After finding solutions by substitution, I discovered that other students disagreed with my answers. I convinced them that my solutions are correct by returning to the original problem to show that my solutions make sense in the context of the problem.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I modeled the scenario using a system of inequalities.")
- **For SMP 7, ASK:** "How did you look for and make use of structure when solving this problem?" (**Sample answer:** "I began by setting the number of both types of skates equal to 0. I then increased the number of hockey skates in increments of 1 until one of the two-variable inequalities in the system became false. Then I returned to $f \geq 0$ and $h \geq 0$. This time, I increased the number of figure skates by 1 until one of the constraint inequalities became false. Then I used those boundaries to find solutions in between the maximum number of figure skates and hockey skates by themselves.")

Alternate Strategies or Solutions

- Students can make a table of values to determine which combinations of skates satisfy the constraints.
- Students can graph the inequalities and analyze the graphs to determine some possible combinations.

Technology

Students can use graphing technology. Students can also use spreadsheets or tables to create a systematic list of which test points work and which do not.

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Guide

Problem-Based Task: Bricklayers

Common Core State Standard

A–CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*★

Task Overview

Focus

How do you rearrange an equation with multiple variables so that it is solved for a particular variable? How is this useful? Students will take an equation with multiple variables, rearrange the equation so that it is solved for a particular variable, and substitute values to answer the question posed in the task.

This activity will provide practice with:

- solving a linear equation for a given variable
- substituting constants for variables
- simplifying expressions by multiplication and division
- interpreting the solution in the context of the problem

Introduction

This task should be implemented after students have learned how to perform substitution and solve one-step linear equations with multiplication and division.

Begin by reading the problem and making sure the context is understood. Before students begin working, ask for guesses as to how long the wall will be.

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students might not know how to begin, or will want to use substitution before rearranging the given equation ($N = 7LH$). Although the answer can be found by substituting values into the given equation first, encourage students to begin by rearranging the equation to solve for L, since this skill is the focus of the sub-lesson. Tell students to use what they know about solving one-variable equations and inverse operations if they are stuck.

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- **SMP 2:** Reason abstractly and quantitatively.

  Students reason abstractly as they rearrange the equation before substituting in the values of the variables. A quantitative method of solving the problem uses common sense rather than an equation. A student might reason that the wall uses 28 bricks for every foot of wall length, and then divide 1,820 by 28 to find the length. Encourage students to think about and draw comparisons between both the abstract and quantitative methods.

- **SMP 7:** Look for and make use of structure.

  Students who immediately want to make the substitutions before rearranging the given formula could use that work to show how rearranging the equation follows the same structure, steps, and procedures for simplifying the equation first and then rearranging. Help students to create a step-by-step procedure for each method in a two-column table and illuminate the similarities and benefits in rearranging first versus substituting first. Ask which method they prefer and why.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- multiplying both sides of the equation by 7 and H instead of dividing by 7 and H

  Point out to students that 7 and H are being *multiplied* by L, and to undo the multiplication, they must divide.

- performing some operation other than division with 7 and H

  Students may try subtraction to eliminate 7 and H from the right side of the equation. Again, point out that 7 and H are being multiplied by L, and to undo the multiplication, they must divide.

- incorrectly substituting

  Encourage students to double-check the meaning of each variable before substituting, to verify that the correct value is substituted for each. (The value 4 should be substituted for the height, H, and 1,820 should be substituted for the number of bricks, N.)

- incorrectly simplifying terms

  To simplify the expression $\frac{1820}{7(4)}$, make sure students first multiply 7 by 4, then divide 1,820 by this result. If they use a calculator and they enter $1820 \div 7 \times 4$, they will not get the correct answer. They must instead enter $1820 \div (7 \times 4)$ or $1820 \div 28$.

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Guide

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- If students start to make the substitutions before rearranging the given formula, encourage them to rearrange the equation first, or encourage them to use their first set of steps as a guide to rearranging the equation without substitution. Note that doing the substitutions first will also lead to the correct answer, but for this problem, the goal is to practice rearranging formulas.

  Ask students what two factors in the given formula are being multiplied by *L*. (**Answer:** 7 and *H*)

  Ask students what to do with those factors so that *L* can be isolated. (**Answer:** Divide both sides by those factors.)

  Ask, "What does your equation look like after you divide both sides by the factors that are being multiplied by *L*?" (**Answer:** $\frac{N}{7H} = L$)

- If students are confused by all the variables in $N = 7LH$ and don't know how to begin rearranging the formula:

  Ask students to analyze the given formula for the operations being used. (**Answer:** "The operation used on the right side is multiplication.")

  Ask students how to solve for a variable that is being multiplied by something. (**Answer:** "Perform the inverse operation, division.")

- After substituting values into the equation $L = \frac{N}{7H}$, if students are confused about how to handle the multiple factors in the expression being solved for *L*, $\frac{1820}{7(4)}$:

  Ask students about the factors of a number like 24. (**Sample answer:** "Three factors that make up 24 are 2, 3, and 4.") Using these factors, suggest that if you were trying to isolate the 2, you could you divide by 3 and 4: $\frac{2 \bullet 3 \bullet 4}{3 \bullet 4} = \frac{2 \bullet \cancel{3} \bullet \cancel{4}}{\cancel{3} \bullet \cancel{4}} = 2$. Ask students to draw the parallel between isolating 2 and using common factors to solve $\frac{1820}{65}$.

- If students perform division on one factor of $N = 7LH$ at a time, they might be confused with how to handle dividing by another factor where a fraction already exists. Students' equations might look like this: $\frac{N}{7} = LH$. Remind students that dividing by *H* is the same as multiplying by $\frac{1}{H}$.

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- If students are confused about what to do after they have isolated the desired variable, L:

  Ask them to reread the problem statement and write down the information they've been given. (**Answer:** "The height is 4 feet and the number of bricks is 1,820.")

  Ask students how many variables are in the equation. (**Answer:** 3)

  Ask students how many variables they have numbers for. (**Answer:** 2)

  Ask students if they have enough information to solve the problem and why. (**Answer:** "Yes, because we only have one unknown in the equation after we substitute the known values.")

- Ask students to think about how the answer would change if they divided 1,820 by 7 first and then divided that result by 4. Have students try this on their calculators and ask them to reason why the answer is the same. (**Answer:** "The answer is the same because in the end, you are still dividing 1,820 by 28.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

Compare methods for solving the problem. For example:

- Ask students who substituted values into $N = 7LH$ first to explain how they were able to use a parallel process to rearrange the formula without numbers. Ask students to discuss how the process for solving $N = 7LH$ is the same as that for solving $L = \frac{N}{7H}$, both before and after values are substituted.
- Ask if anyone used a method not yet discussed to solve the problem and discuss this method as a class.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Literal equations such as formulas can be solved for a specific variable by isolating that variable.

  In this task, students isolate L.

- To isolate the specified variable, use inverse operations.

  In this task, students use division, the inverse of multiplication, to solve $N = 7LH$ for L.

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Extending the Task

- To extend the task, ask students to create a two-column poster illustrating how solving this or any other literal equation for a specific variable follows the same process as solving any single-variable equation (isolating the variable by performing inverse operations in the reverse order of operations).
- The task may also be extended by solving the formula $N = 7LH$ for a different variable, and/or by creating new parameters. For example, imagine a different wall that uses 1,680 bricks and is 40 feet long. How high would the wall be?

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I asked for help and had some coaching from the teacher and from using the Coaching questions. I saw parallels in solving this literal equation with other equations that I'm used to working with." Or, "When I rearranged the formula, I thought about what the steps needed to rearrange it would be if the formula used numbers instead of variables.")
- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "Rearranging the equation without numbers and then putting the solution back into context.") "Which of your strategies represent quantitative reasoning?" (**Answer:** "I simplified the expression on the right side of the equation once it was solved for L." Or, "I solved the problem without using the formula, by dividing 1,820 by 4, and then dividing the result by 7.")
- **For SMP 7, ASK:** "How did you look for and make use of structure when solving this problem?" (**Answer:** "I applied inverse operations and the reverse of the order of operations to isolate and solve for L.")

Alternate Strategies or Solutions

Some students will be able to do the reasoning in their heads and solve the problem by just using a calculator. Encourage these students to formalize their thoughts by writing them down in a structured way. Other students might find it useful to draw a diagram of the wall, labeling the height 4 and the length L.

Technology

No technology is required for this task, but students may wish to use calculators.

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Common Core State Standards

Mathematics I

Integrated Pathway



Teacher Resource

Unit 2

WALCH
INTEGRATED MATH

This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

Developers and reviewers include:

Joyce Hale	Valerie Ackley	Doug Kühlmann
Shelly Northrop Sommer	Laura McPartland	Mike May, S.J.
Ruth Estabrook	Cameron Larkins	James Quinlan
Vanessa Sylvester	Jennifer Blair	Peter Tierney-Fife
Zachary Lien	Nancy Pierce	

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Appendix: Implementation Guides for Problem-Based Tasks IG-1

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Standards Correlations

Each lesson in this Integrated Pathway: Mathematics I program was written specifically to address the Common Core State Standards. Each lesson lists the standards covered in all the sub-lessons, and each sub-lesson lists the standards addressed in that particular section. In this section, you'll find a comprehensive list mapping the sub-lessons to the CCSS.

Guide to Common Core State Standards Annotation

As you use this program, you will come across a symbol included with the Common Core standards for some of the lessons and activities. These symbols are explained below.

Symbol: ⋆

Denotes: Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (⋆).

From http://www.walch.com/CCSS/00003

Symbol: (+)

Denotes: College and Career Readiness Standards

Advanced mathematics standards that are required in higher-level courses such as advanced statistics may also be included in lower-level courses. These additional standards are denoted by (+). According to the Common Core State Standards Initiative, "the evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6–8."

From http://www.walch.com/CCSS/00004

Connections to Future Courses

This section provides a map between topics introduced in each unit of this course and subsequent courses where each topic is revisited and built upon.

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CCSS INTEGRATED PATHWAY: MATHEMATICS I STANDARDS CORRELATIONS

Unit 2: Linear and Exponential Relationships				
Lesson	**Sub-lesson number**	**Title**	**Standard(s)**	**Pages**
Lesson 1	**Graphs As Solution Sets and Function Notation**			
	2.1.1	Graphing the Set of All Solutions	A–REI.10	U2-4
	2.1.2	Intersecting Graphs	A–REI.11★	U2-21
	2.1.3	Domain and Range	F–IF.1	U2-36
	2.1.4	Function Notation and Evaluating Functions	F–IF.2	U2-55
Lesson 2	**Solving Linear Inequalities in Two Variables and Systems of Inequalities**			
	2.2.1	Solving Linear Inequalities in Two Variables	A–REI.12	U2-79
	2.2.2	Solving Systems of Linear Inequalities	A–REI.12	U2-103
Lesson 3	**Sequences As Functions**			
	2.3.1	Sequences As Functions	F–IF.3	U2-135
Lesson 4	**Interpreting Graphs of Functions**			
	2.4.1	Identifying Key Features of Linear and Exponential Graphs	F–IF.4★ F–IF.5★	U2-164
	2.4.2	Average Rate of Change	F–IF.6★ F–LE.1a★	U2-188
	2.4.3	Recognizing Average Rate of Change	F–IF.6★ F–LE.1b★ F–LE.1c★	U2-205
Lesson 5	**Analyzing Linear and Exponential Functions**			
	2.5.1	Graphing Linear Functions	F–IF.7a★	U2-238
	2.5.2	Graphing Exponential Functions	F–IF.7e★	U2-256

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Lesson	Sub-lesson number	Title	Standard(s)	Pages
Lesson 6	**Comparing Functions**			
	2.6.1	Comparing Linear Functions	F–IF.9	U2-287
	2.6.2	Comparing Exponential Functions	F–IF.9	U2-307
	2.6.3	Comparing Linear to Exponential Functions	F–LE.3★	U2-331
Lesson 7	**Building Functions**			
	2.7.1	Building Functions from Context	F–BF.1a★	U2-358
	2.7.2	Constructing Functions from Graphs and Tables	F–LE.2★	U2-378
Lesson 8	**Operating on Functions and Transformations**			
	2.8.1	Operating on Functions	F–BF.1b★	U2-405
	2.8.2	Transformations of Linear and Exponential Functions	F–BF.3	U2-415
Lesson 9	**Arithmetic and Geometric Sequences**			
	2.9.1	Arithmetic Sequences	F–BF.2★	U2-437
	2.9.2	Geometric Sequences	F–BF.2★	U2-448
Lesson 10	**Interpreting Parameters**			
	2.10.1	Interpreting Parameters	F–LE.5★	U2-465

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TOPICS FROM FUTURE COURSES: CCSS INTEGRATED PATHWAY MATHEMATICS I

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 2	Function notation (concepts regarding domain, range, intersecting functions, and graphing solution sets)	Math II Unit 2	extended to describing transformations of functions
		Math III Unit 4B	includes combinations of functions
	Linear inequalities	Math II Unit 3	extended to quadratic inequalities and rational inequalities
		Math III Unit 4B	extended to linear, exponential, quadratic, and logarithmic functions
	Sequences	Math III Unit 2A	applied to more complex sequences and series
	Analyzing the features of graphs of linear functions	Math II Unit 2	extended to exponential and quadratic functions
		Math III Unit 2A	applied to polynomial functions
		Math III Unit 2B	extended to periodic functions
		Math III Unit 4A	expanded to logarithmic functions
	Domain	Math IV	restricts the domain of trigonometric functions in order to find the inverses of these functions

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Conceptual Activities

Use these interactive open education and/or Desmos resources to build conceptual understanding of mathematical ideas. (*Note*: Activity links will be monitored and repaired or replaced as necessary.)

- Desmos. "Avi and Benita's Repair Shop."

  http://www.walch.com/ca/01014

  Compare linear and exponential growth in the context of daily payments. One plan increases by $100 each day, while another grows by doubling the previous day's payment. This activity is appropriate for students who have studied linear functions but may not have an experience with exponential growth.

- Desmos. "Card Sort: Functions."

  http://www.walch.com/ca/01005

  Sort graphs, equations, and contexts according to whether each one represents a function.

- Desmos. "Card Sort: Linear Functions."

  http://www.walch.com/ca/01010

  Notice and use properties of linear functions to make groups of three. Different properties will lead to different groupings by different participants.

- Desmos. "Function Carnival, Part 2."

  http://www.walch.com/ca/01007

  This activity follows up on "Function Carnival" by using the contexts in that activity to develop an understanding of function notation.

- Desmos. "Game, Set, Flat."

  http://www.walch.com/ca/01015

  Develop understanding of the exponential relationship that describes a bouncing tennis ball. Learn to examine successive terms in a sequence to determine if it represents an exponential relationship or not, and how to construct the exponential equation itself.

- Desmos. "Marbleslides: Exponentials."

  http://www.walch.com/ca/01016

  Restrict, reposition, and otherwise transform exponential curves at will by modifying the basic form $y = b^x$, and use precision in describing these transformations using words and/or symbols.

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- Desmos. "Match My Line."

  http://www.walch.com/ca/01013

  Work through a series of scaffolded linear graphing challenges to develop proficiency with direct variation, slope-intercept, point-slope, and other linear function forms.

- Desmos. "Polygraph: Exponentials."

  http://www.walch.com/ca/01019

  This Custom Polygraph is designed to spark vocabulary-rich conversations about exponentials, including how they differ from linear functions. Key vocabulary terms that may appear in student questions include *increasing*, *decreasing*, *intercept*, *rate*, *asymptote*, and *curve*.

- Desmos. "What Comes Next?"

  http://www.walch.com/ca/01020

  Predict "what comes next" for linear and exponential functions based first on graphs and then on tables of values, then explore connections between graphs, tables, and equations of linear and exponential functions.

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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. Which ordered pair falls on the line described by $y = 5x - 1$?
   a. (3, 2)
   b. (5, 1)
   c. (0, –1)
   d. (2, 8)

2. Which ordered pair is a solution for both $y = 3^x$ and $y = 3x + 3$?
   a. (1, 1)
   b. (0, 3)
   c. (1, 3)
   d. (2, 9)

3. If the domain of $f(x) = x + 2$ is {0, 2, 4}, what is the range of $f(x)$?
   a. {2, 4, 6}
   b. all real numbers
   c. {–2, 0, 2}
   d. {0, 4, 8}

4. If $f(x) = 2x + 5$, what is $f(x + 1)$?
   a. undefined
   b. $2(x + 1) + 5$
   c. $2x + 6$
   d. $2(x + 1) - 5$

5. Evaluate $f(x) = 5(3^x) + 1$ for $x = 3$.
   a. 46
   b. 136
   c. 31
   d. 109

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Lesson 1: Graphs As Solution Sets and Function Notation

Instruction

Common Core State Standards

SMP	
1 ✓	2 ✓
3	4 ✓
5	6 ✓
7 ✓	8 ✓

A–REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A–REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

F–IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Essential Questions

1. How can we represent the set of all solutions of a function?
2. What condition must be met for $y = f(x)$ and $y = g(x)$ to intersect?
3. What is the relationship between the domain and range of a function?
4. What does it mean to evaluate a function?
5. What does it mean to evaluate a function over a given domain?

WORDS TO KNOW

curve	the graphical representation of the solution set for $y = f(x)$; in the special case of a linear equation, the curve will be a straight line
domain	the set of all input values for which a function is defined; the set of x-values that are valid for a relation or function

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Instruction

function	a relation in which each element in the domain is mapped onto exactly one element in the range; that is, for every value of x, there is exactly one value of y
function notation	a way to name a function using $f(x)$ to represent the dependent variable instead of y
linear equation	a first-degree equation that can be written in the form $ax + by = c$, where a, b, and c are numbers; when written as $y = mx + b$, m is the slope of the line, and b is its y-intercept. The graph of a linear equation is a straight line.
ordered pair	the coordinates of a point in a coordinate plane, (x, y), where the order is significant
range	the set of all outputs of a relation or function; the set of y-values for which a function is defined
relation	a set of ordered pairs
solution set	the set of ordered pairs that represent the solutions of an equation or system of equations
system	a set of two or more equations

Recommended Resources

- Algebrahelp.com. "Function Graphing Calculator."

  http://walch.com/rr/CAU3L1FunctionGrapher

  Users can enter a function, specify a domain and range, and see the function graphed.

- Interactivate. "Sequencer."

  http://walch.com/rr/CAU3L1Sequencer

  Users can devise multiple sequences by changing the starting number, multiplier, and add-on values. The Sequencer will then calculate the sequence and show its graph.

- MathPlayground. "Function Machine."

  http://walch.com/rr/CAU3L1FunctionMachine

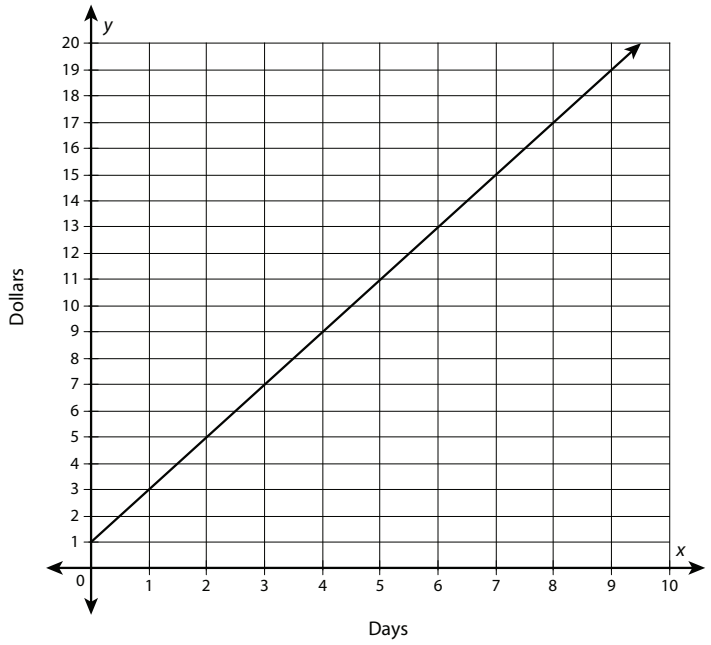
  Users can input values into the function machine, which shows the correct output. Users then determine the function rule that produced the output.

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Lesson 2.1.1: Graphing the Set of All Solutions

Warm-Up 2.1.1

Mallory had $1 this morning and asked her mom for another dollar. Instead of giving Mallory more money, her mom said she would give Mallory $2 the next day if she still had the dollar she was holding. Plus, she would continue to give Mallory $2 each day as long as she saved it all. Mallory agreed to the deal and wondered how much money she might have at the end of the week. To find out, Mallory graphed the equation $y = 2x + 1$, as shown, where x represents the number of days and y represents the amount of money she would have.



1. If Day 0 is Monday, how much could Mallory have on Wednesday?

2. How much could Mallory have on Friday?

3. Explain how the equation and graph represent Mallory receiving $2 each day.

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Instruction

Lesson 2.1.1: Graphing the Set of All Solutions

Common Core State Standard

A–REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Warm-Up 2.1.1 Debrief

1. If Day 0 is Monday, how much could Mallory have on Wednesday?

   Monday is Day 0, so Tuesday is Day 1 and Wednesday is Day 2. Substitute 2 for x into the equation and solve for y.

   $y = 2x + 1$ Given equation

   $y = 2(2) + 1 = \$5$ Substitute 2 for x and simplify.

   Mallory could have \$5 on Wednesday.

2. How much could Mallory have on Friday?

   Friday is Day 4. Substitute this value into the equation.

   $y = 2x + 1$ Given equation

   $y = 2(4) + 1 = \$9$ Substitute 4 for x and simplify.

   Mallory could have \$9 on Friday.

3. Explain how the equation and graph represent Mallory receiving \$2 each day.

   The equation of the line is in slope-intercept form, so $m = 2$, which means the slope of the line is 2. The slope represents a unit rate; in this case, dollars received per day. Therefore, a slope of 2 means that Mallory received \$2 each day.

   On the graph, it can be seen that as the value of x (days) increases by 1, the value of y (dollars) increases by 2. Therefore, for every additional day, Mallory receives an additional \$2.

Connection to the Lesson

- Students will be graphing linear and exponential equations in two variables.
- Students will be asked to read the coordinates of points from a graph.
- Students will be asked to determine values of y for given values of x.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- evaluating linear and exponential equations to complete a table of values (6.EE.2c)
- connecting a table of values to a set of ordered pairs in the solution of an equation (5.G.1)
- plotting ordered pairs in the coordinate plane (5.G.1)
- connecting plotted points with a curve that represents all solutions to an equation (A–CED.2★)
- expressing linear equations in slope-intercept form (A–CED.4★)
- creating equations from context (A–CED.2★)
- evaluating negative exponents (8.EE.1)

Introduction

In a linear equation with one variable, x, the solution is the value that makes the equation true. For example, 1 is the solution for the equation $x = 1$, and 2 is the solution for the equation $2^x = 4$.

The solution of an equation with two variables, x and y, is an ordered pair of values (x, y) that make the equation true. For example:

$(1, 2)$ is a solution to the equation $y = 2x$ because the statement $2 = 2(1)$ is true.

$(1, 3)$ is not a solution for $y = 2x$ because the statement $3 = 2(1)$ is false.

The pairs of values (x, y) are called **ordered pairs**, and the set of all ordered pairs that satisfy an equation is called the **solution set**. Each ordered pair in a solution set represents a point in the coordinate plane. When we plot these points, they usually form a curve. A **curve** is a graphical representation of the solution set for an equation. In the special case of a linear equation, the curve will be a straight line. A **linear equation** is a first-degree equation that can be written in the form $ax + by = c$, where a, b, and c are real numbers. A linear equation can also be written in slope-intercept form, $y = mx + b$, where m is the slope of the line and b is the y-intercept.

It is important to understand that the solution set for most equations is infinite; therefore, it is impossible to plot every point when graphing. There are several reasons why a solution set is infinite. One reason is because given any two x-values in the solution of an equation, there is another number between them. For that value of x, there will be a corresponding value of y that satisfies the equation. So when we graph the solution set for an equation, we can plot several points and then typically connect them with the appropriate curve with arrows at both ends. The curve that connects the points represents the infinite solution set to the equation.

SRB p. U2-3

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Instruction

Key Concepts

- The solution to an equation with two variables is the set of ordered pairs (x, y) that satisfies the given equation.
- Ordered pairs can be plotted in the coordinate plane.
- If the plotted points in the coordinate plane are connected, the path they describe is called a curve.
- A curve may be a straight line.
- An equation whose graph is a straight line is called a linear equation.
- The solution set of an equation is usually infinite.
- When we graph the solution set of an equation, we can connect the plotted points with a curve that represents the solution set.

Common Errors/Misconceptions

- believing that the solutions of an equation are limited to the points plotted in the coordinate plane
- incorrectly evaluating an equation for given values of a variable
- incorrectly plotting ordered pairs in the coordinate plane

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Guided Practice 2.1.1

Example 1

Graph the solution set for the linear equation $-3x + y = -2$.

1. Solve the equation for y.

   $-3x + y = -2$

   $y = 3x - 2$

2. Make a table. Choose at least three values for x and find the corresponding values of y using the given equation.

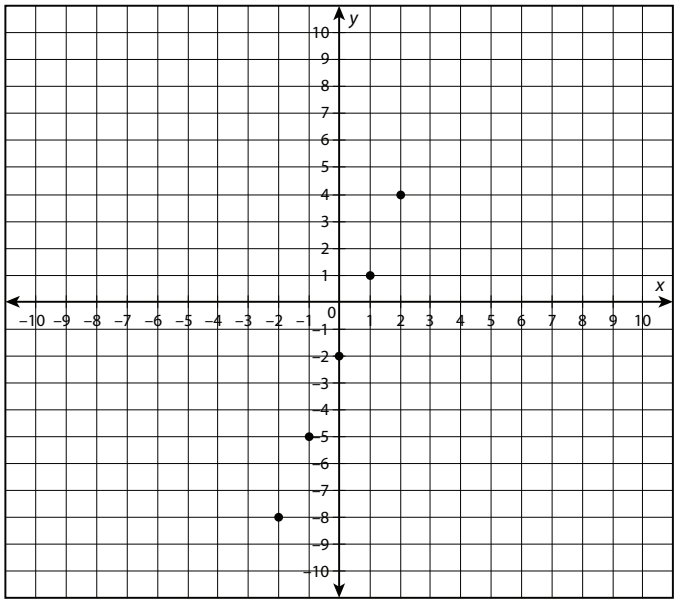
x	y
–2	–8
–1	–5
0	–2
1	1
2	4

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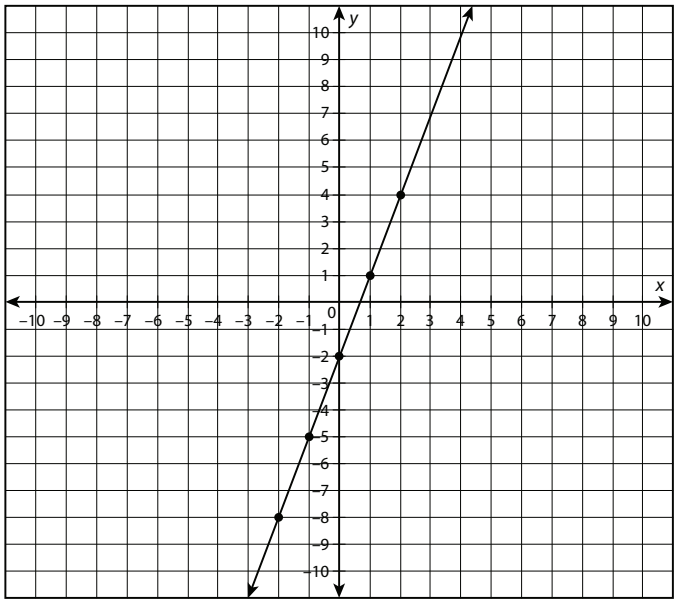
Instruction

3. Plot the ordered pairs on the coordinate plane.

   Notice that the points appear to lie in a straight line.



4. Connect the points by drawing a line through them. Use arrows at each end of the line to show that a line continues infinitely in each direction. This represents all of the solutions for the equation.



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Instruction

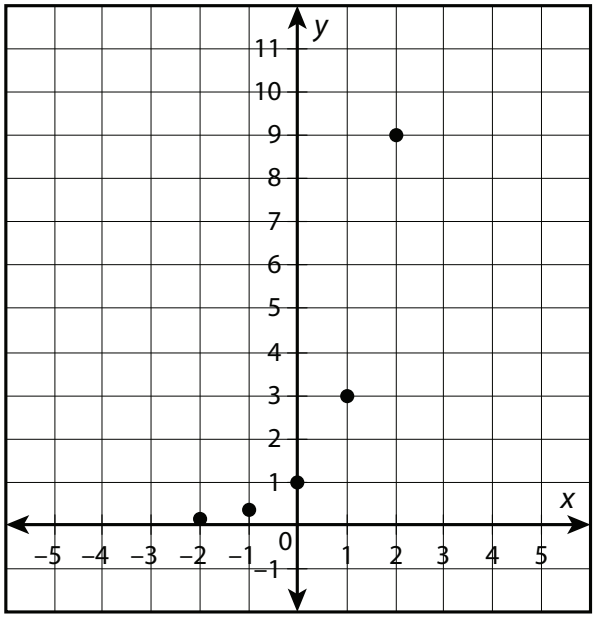
Example 2

Graph the solution set for the exponential equation $y = 3^x$.

1. Make a table. Choose at least three values for x and find the corresponding values of y using the given equation.

x	y
–2	$\frac{1}{9}$
–1	$\frac{1}{3}$
0	1
1	3
2	9

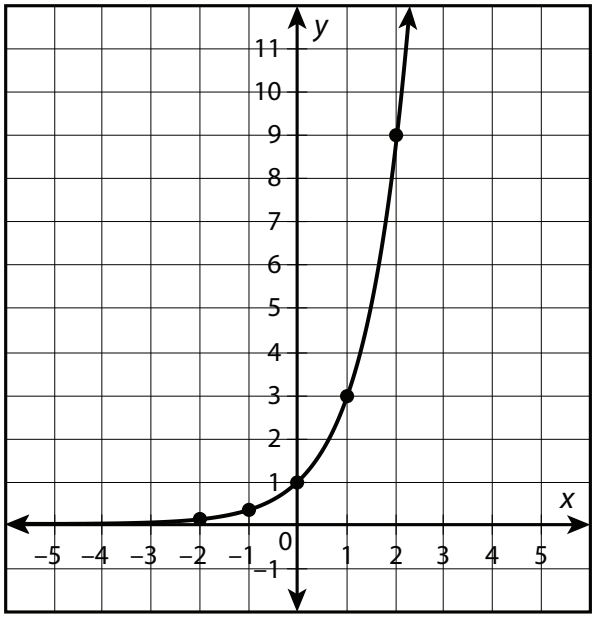
2. Plot the ordered pairs in the coordinate plane.



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Instruction

3. Notice the points do not appear to fall on a straight line. The solution set for $y = 3^x$ is an exponential curve. Connect the points by drawing the curve through them. Use arrows at each end of the curved line to demonstrate that the curve continues infinitely in each direction. This represents all of the solutions for the equation.



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Instruction

Example 3

The Russell family is driving 1,000 miles to the beach for their summer vacation. Mr. Russell drives at an average rate of 60 miles per hour and plans on stopping four times to break up the journey. Let t represent the number of hours the Russells will travel before they reach their destination, and let d represent the remaining distance after each stop. Write an equation in terms of d and t that represents the 1,000-mile trip. Next, draw a graph that represents the number of miles traveled for each hour of the trip.

1. Write an equation that represents how far the Russells are from the beach for each hour traveled.

   $d = 1000 - 60t$, where d is the distance in miles and t is the time in hours.

2. Make a table. Choose values for t and find the corresponding values of d.

   The trip begins at time 0, so let $t = 0$ be the first value of t.

   The problem states that the Russells plan to stop four times during their trip, so choose four additional values for t. Let's use 2, 5, 10, and 15.

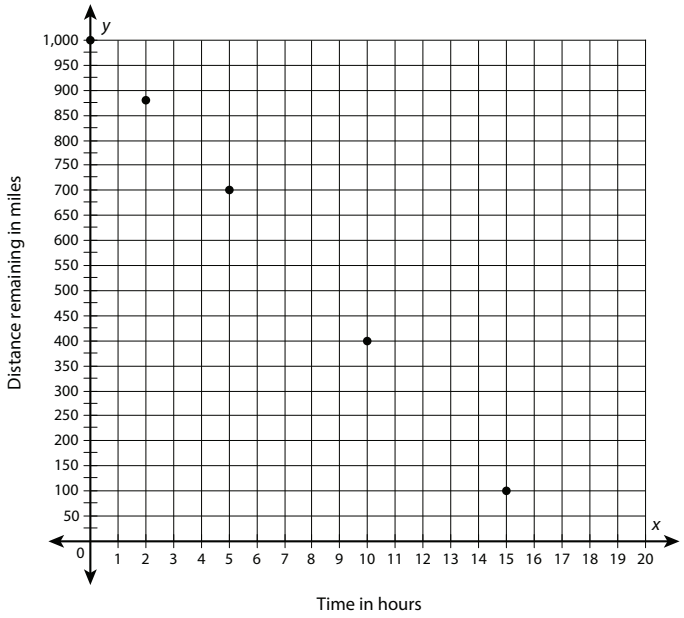
   Use the equation $d = 1000 - 60t$ to find d for each value of t. Fill in the table.

t	d
0	1,000
2	880
5	700
10	400
15	100

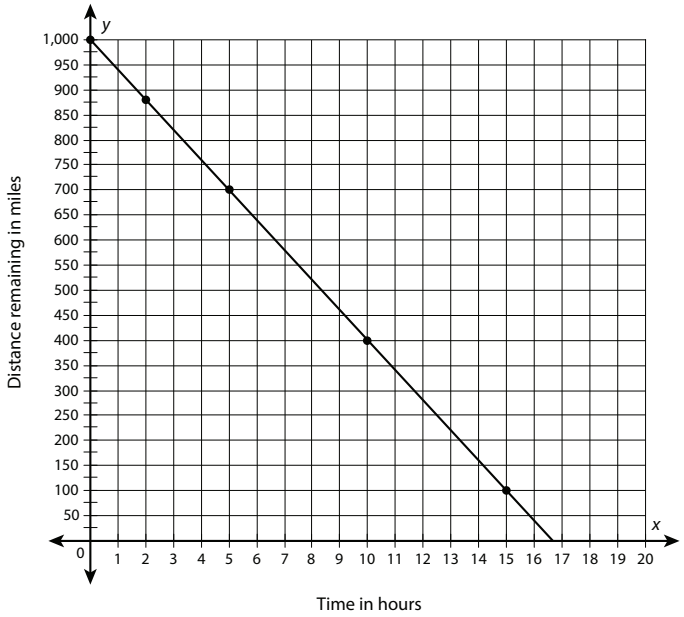
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Instruction

3. Plot the ordered pairs on a coordinate plane.



4. Connect the points by drawing a line segment. Do not use arrows at each end of the line segment because it does not continue in each direction. This line segment represents all of the possible stopping points in distance and time.

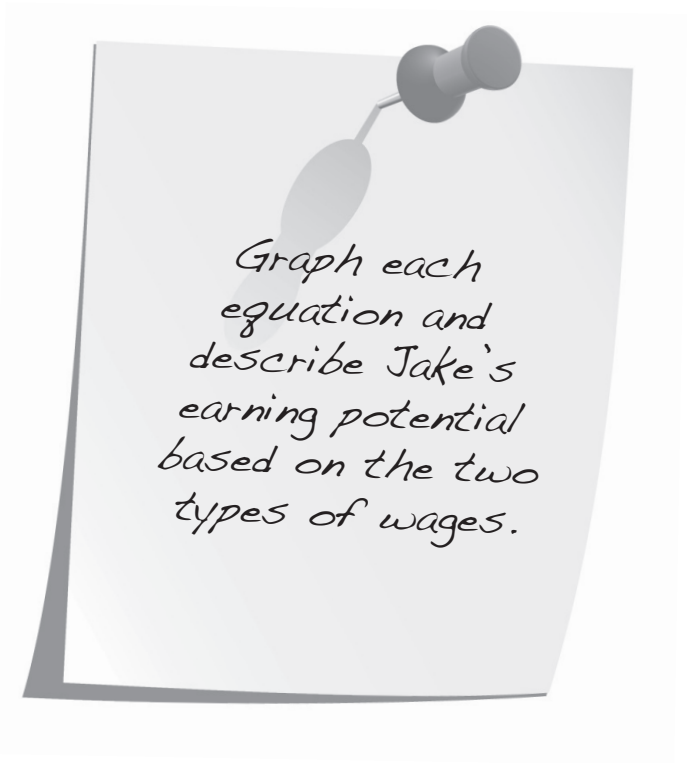


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Problem-Based Task 2.1.1: Saving for College

SMP	
1 ✓	2 ✓
3	4 ✓
5	6
7	8

Jake graduated from high school and is working at the family furniture store to save for college. He can either be paid $12.50 per hour or earn a commission of 15% on all his sales above $500. Jake's commission is represented by the equation $c = (0.15)(s - 500)$, where c is Jake's commission in dollars and s is the amount of Jake's total sales. Create a second equation describing Jake's hourly wages, w, in terms of the number of hours he works, h. Graph each equation and describe Jake's earning potential based on the two types of wages.



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Problem-Based Task 2.1.1: Saving for College

Coaching

a. Write an equation that models what Jake can earn based on an hourly wage of $12.50.

b. Graph the equation from part a.

c. On a separate set of axes, graph the equation that represents what Jake can earn based solely on commission, $c = (0.15)(s - 500)$.

d. Use each graph to estimate what Jake would earn if he were paid an hourly wage for 20 hours and what he would earn if he were paid on commission and sold $3,000 worth of furniture.

e. Estimate what Jake would earn if he worked 40 hours for an hourly wage and what he would earn if he were paid on commission and sold $6,000 worth of furniture.

f. Which method of payment—getting paid an hourly wage or getting paid on commission—results in a guaranteed wage?

g. About how much in furniture sales does Jake need to make during a 40-hour week for his commission to match his hourly wage?

h. Describe Jake's earning potential based on the two possible payment options.

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Instruction

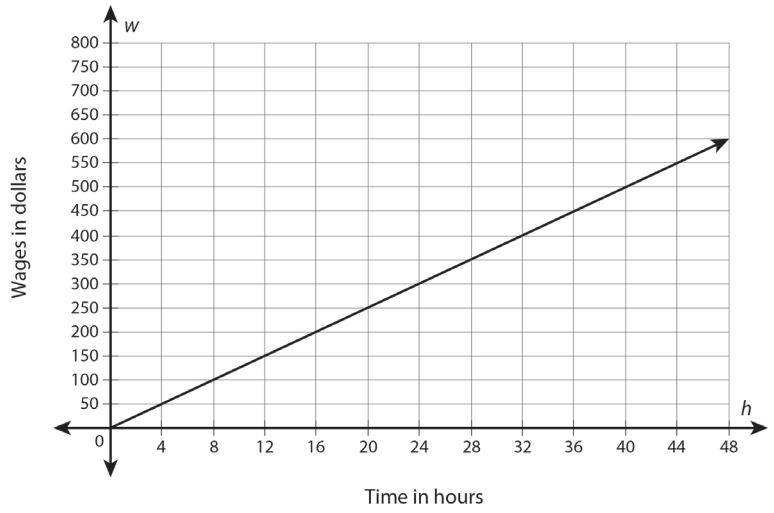
Problem-Based Task 2.1.1: Saving for College

Coaching Sample Responses

a. Write an equation that models what Jake can earn based on an hourly wage of $12.50.

$w = 12.50h$, where w is Jake's wage in dollars and h is the number of hours he works.

b. Graph the equation from part a.



c. On a separate set of axes, graph the equation that represents what Jake can earn based solely on commission, $c = (0.15)(s - 500)$.



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d. Use each graph to estimate what Jake would earn if he were paid an hourly wage for 20 hours and what he would earn if he were paid on commission and sold $3,000 worth of furniture.

Jake would earn $250 in hourly wages versus $375 on commission.

e. Estimate what Jake would earn if he worked 40 hours for an hourly wage and what he would earn if he were paid on commission and sold $6,000 worth of furniture.

Jake would earn $500 in hourly wages versus $825 in commission.

f. Which method of payment—getting paid an hourly wage or getting paid on commission—results in a guaranteed wage?

Getting paid hourly results in a guaranteed wage.

g. About how much in furniture sales does Jake need to make during a 40-hour week for his commission to match his hourly wage?

Jake earns $500 for working 40 hours. To earn $500 in commission, he will have to sell about $3,834 worth of furniture.

h. Describe Jake's earning potential based on the two possible payment options.

Jake's earning potential for hourly work tops out at $500 for a 40-hour week.

There is theoretically no limit to how much Jake could make on commission. If he believes he can sell more than $3,834 worth of furniture each week, commission would be the best way for him to be paid.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.1.1: Graphing the Set of All Solutions

For problems 1–4, draw the graph that represents the solution set of the equation.

1. $2x + y = -1$

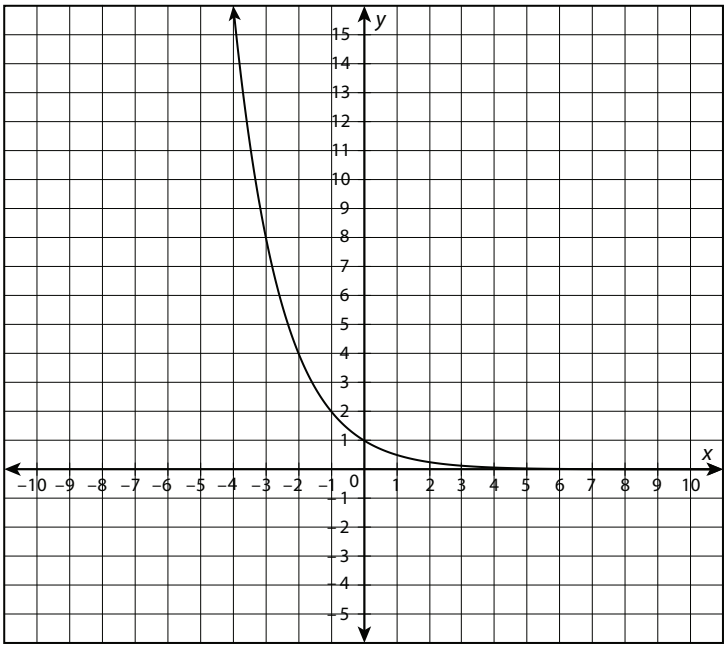
2. $4x - 2y = -6$

3. $y = 2^x$

4. $y = 3^x$

For problems 5 and 6, use each given graph to find three solutions that will satisfy the equation.

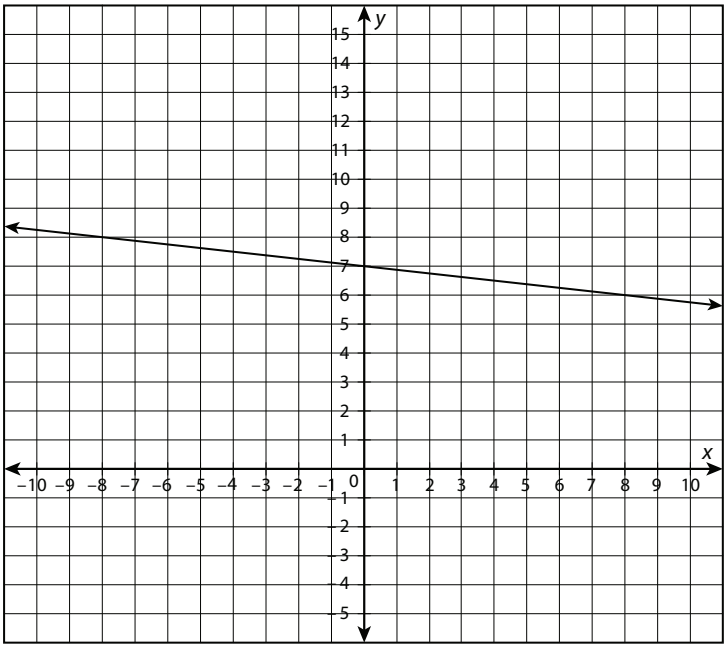
5. $y = \left(\frac{1}{2}\right)^x$



continued

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6. $y = -\frac{1}{8}x + 7$



For problems 7–10, use the given information to answer the questions.

7. A company's yearly profit during its first 5 years of operation can be modeled by the equation $P = 225(1.13)^x + 400$, where x is the number of years since the company started and P is the profit in dollars. Draw a graph to represent this situation. If this pattern continues, what would the company's profit be in year 7?

8. Katya is a caterer. She has a cookie recipe that calls for 2 eggs per batch. Katya wants to know the number of eggs she needs according to how many batches she cooks. What equation can be used to represent the number of eggs Katya needs for any number of batches? Draw a graph to represent this situation. How many eggs would Katya need for 4 batches of cookies?

continued

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9. The price of a certain company's stock grew at the same rate during the first 6 months of the year. The following table shows the price in dollars per share of stock, P, for each month, m.

m	P
1	35.75
2	36
3	36.25
4	36.5
5	36.75
6	37

What equation can be used to represent this situation? Draw a graph to represent the growth of the stock's price per share during this period. If the pattern continues, what will the price per share be after 12 months?

10. Gas costs \$3 per gallon, which can be modeled by the equation $C = 3x$, where x is the number of gallons needed and C is the total cost in dollars. Your car has a 15-gallon gas tank. Draw a graph that represents how much you might have to pay depending on the number of gallons of gas you buy. How much will you have to pay for 7.5 gallons of gas?

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Lesson 2.1.2: Intersecting Graphs

Warm-Up 2.1.2

Brian is 2 years older than his brother Andrew. Brian likes to remind Andrew that he is both older and taller and tells Andrew that it will always be this way. But one day their father overhears Brian and makes a prediction. "Andrew, your brother will always be older than you, but one day you will be taller—and I can prove it mathematically." He writes two equations, one describing Brian's growth rate, the other describing Andrew's growth rate. Both equations are in terms of Brian's age in years, x, and each boy's height, y, in feet. Assume that the boys will finish growing at age 18. Use the equations to complete the problems.

Brian's growth rate: $y_{\text{Brian}} = \frac{1}{5}x + 2.25$

Andrew's growth rate: $y_{\text{Andrew}} = \frac{1}{4}(x-2) + 2$

1. Make a table of values for the two equations.

2. Graph the equations on the same coordinate plane.

3. According to the graph, will Andrew ever be taller than Brian? Explain your answer.

4. If Andrew does outgrow Brian, how old will Andrew be when he does?

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Lesson 2.1.2: Intersecting Graphs

Common Core State Standard

A–REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

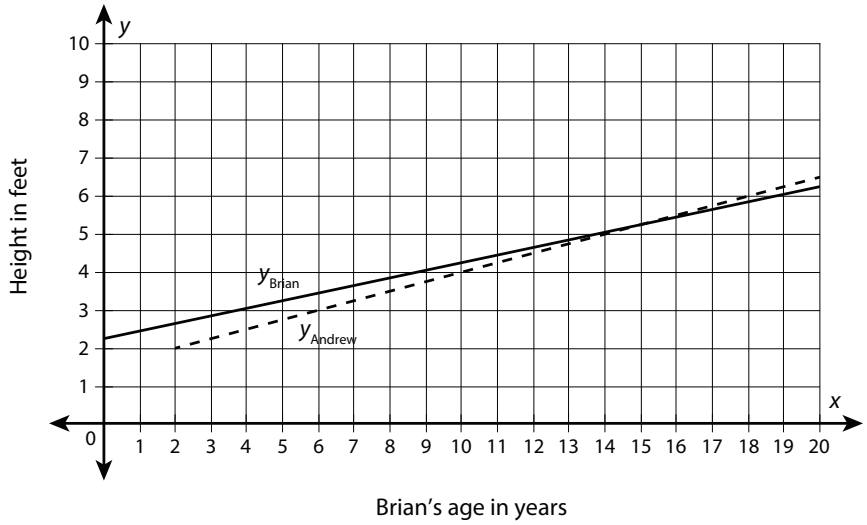
Warm-Up 2.1.2 Debrief

1. Make a table of values for the two equations.

   List values for x, and then find the corresponding values for y for each boy.

x	$y_{\text{Brian}} = \frac{1}{5}x + 2.25$	$y_{\text{Andrew}} = \frac{1}{4}(x-2)+2$
0	2.25	not born yet
2	2.65	2
4	3.05	2.5
10	4.25	4
15	5.25	5.25
16	5.45	5.5
20	6.25	6.5

2. Graph the equations on the same coordinate plane.



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3. According to the graph, will Andrew ever be taller than Brian? Explain your answer.

   Yes, Andrew will eventually be taller. The two graphs intersect when the boys have reached the same height. To the right of this intersection, the graph representing Andrew's height has a greater *y*-value than the graph representing Brian's height, indicating that Andrew is taller.

4. If Andrew does outgrow Brian, how old will Andrew be when he does?

   Andrew will outgrow Brian shortly after Brian turns 15. Andrew is 2 years younger than Brian. Therefore, Andrew will be 15 – 2 or 13 years old when he outgrows Brian.

Connection to the Lesson

- Students will be asked to estimate where the graphs of two equations intersect.
- Students will draw graphs that represent given equations.
- Students will create tables of ordered pairs.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- reading coordinates of points in a coordinate plane (5.G.1)
- plotting points in a coordinate plane (5.G.1)
- evaluating negative exponents (8.EE.1)

Introduction

Recall that the graph of an equation, such as $y = x + 1$, represents the set of solutions for that equation. The values for y are the result of a process applied to x. To make a general statement, we represent the process by a letter, such as f, and we call the results of that process "f of x." We write the expression "f of x" as $f(x)$.

The process f is a **function** if each element in the domain is mapped onto exactly one element in the range. That is, for every value of x for which the function is defined, there is exactly one value of y. Both y and $f(x)$ represent the outcome of applying the function f to x. So, for the equation $y = x + 1$, adding 1 to x is the function f. Therefore, $f(x)$ is $x + 1$, where $y = f(x)$. This way of writing equations is called **function notation**.

A set of two or more equations is called a **system**. If one function is called f and another is called g, then we can graph f and g on the same axes. By graphing the functions on the same axes, we can more easily see the functions' differences or similarities. Functions can be named using any letter, but f and g are used most often. In this lesson, we will work with systems of functions, such as $f(x)$ and $g(x)$, and find the solution where $f(x) = g(x)$. In other words, we will find all values of x for which $f(x)$ and $g(x)$ are equal.

Key Concepts

Graphing Solutions of Functions

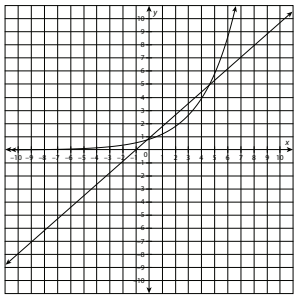
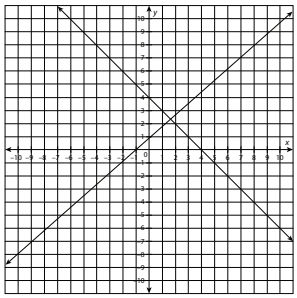
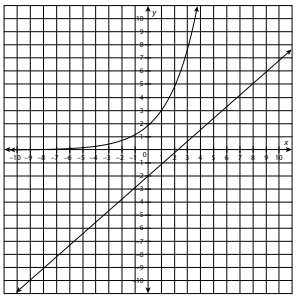
- When graphing functions $f(x)$ and $g(x)$, the solution set for $f(x) = g(x)$ is the coordinates of the point(s) of intersection of the graphs of the two functions.
- If $f(x) = g(x) = b$ for a particular value of x, such as a, then the point (a, b) will lie on the curves defined by both f and g.
- If the point (a, b) lies on both curves, the curves intersect at the point (a, b).
- To determine the solution of the system of functions f and g, look for the point(s) where the curves intersect.
- To determine if a point (a, b) is a solution of the system, substitute a for x and b for y into the equation of each function to see if the results are true.
- Note that it is possible for the graphs of a system of equations to intersect at more than one point, at exactly one point, or to not intersect at all.

SRB p. U2-13

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Points of Intersection

More than one	One	None
		
Both points of intersection are solutions to the system.	The point of intersection is the system's only solution.	There is no solution to the system.

Using a Table of Values to Find Solutions

- To create a table of values for a system of equations, list the inputs for each equation and determine the corresponding outputs in both equations.
- In the first column of a four-column table, list values of x to substitute into the system's equations.
- In the second column, enter the first equation and calculate the corresponding outputs for the x-values in the first column.
- In the third column, enter the second equation and calculate the corresponding outputs for the x-values in the first column.
- In the fourth column, list the difference, $f(x) - g(x)$, using values from the second and third columns.
- Look for the difference that has the least absolute value, or for a sign change between successive output values; that is, where the values of $f(x) - g(x)$ change from negative to positive or from positive to negative.
- The solution to the system is found where the sign change occurs and the difference between the two outputs is equal to 0 or has the least absolute value.
- If the difference between outputs in the fourth column is 0, the corresponding value of x is the x-coordinate of the solution, and the corresponding output is the y-coordinate of the solution.

Common Errors/Misconceptions

- thinking that the graphs of all functions will intersect
- substituting input values incorrectly
- incorrectly evaluating output values for given input values

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Guided Practice 2.1.2

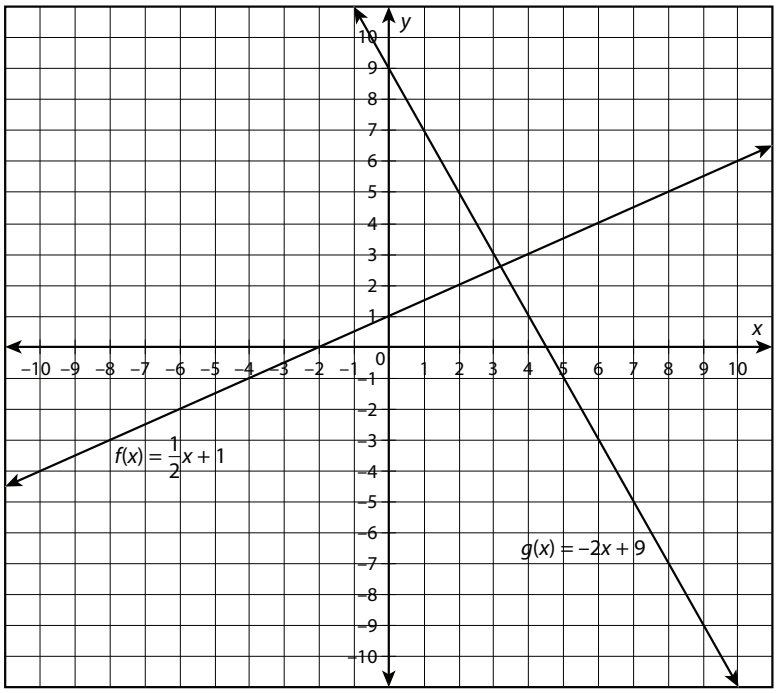
Example 1

Use a graph to approximate the x-coordinate of the solution for the following system of equations. Find the difference in outputs, $f(x) - g(x)$, for your estimate.

$$f(x) = \frac{1}{2}x + 1$$

$$g(x) = -2x + 9$$

1. Graph $f(x) = \frac{1}{2}x + 1$ and $g(x) = -2x + 9$ on the same coordinate axes.



2. Approximate the values for x where $f(x) = g(x)$.

   From the graph, $x = 3$ appears to be a good estimate, because the functions intersect when x is approximately 3.

SWB p. 21; SRB p. U2-15

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3. Evaluate $f(x) = \frac{1}{2}x + 1$ for $x = 3$.

$y = \frac{1}{2}x + 1$	Change "$f(x) =$" to "$y =$."
$y = \frac{1}{2}(3) + 1$	Substitute 3 for x.
$y = 1.5 + 1 = 2.5$	Simplify.

4. Evaluate $g(x) = -2x + 9$ for $x = 3$.

$y = -2x + 9$	Change "$g(x) =$" to "$y =$."
$y = -2(3) + 9$	Substitute 3 for x.
$-6 + 9 = 3$	Simplify.

5. Find the difference of the y-values of the equations.

$f(x) - g(x)$	Subtract $g(x)$ from $f(x)$.
$2.5 - 3 = -0.5$	Substitute the calculated values for $f(x)$ and $g(x)$ and subtract.

The difference is close to 0, so we can say that the functions intersect when x is approximately equal to 3.

Example 2

Use a graph to approximate the solution set for the following system of equations. Find $f(x) - g(x)$ for your estimates.

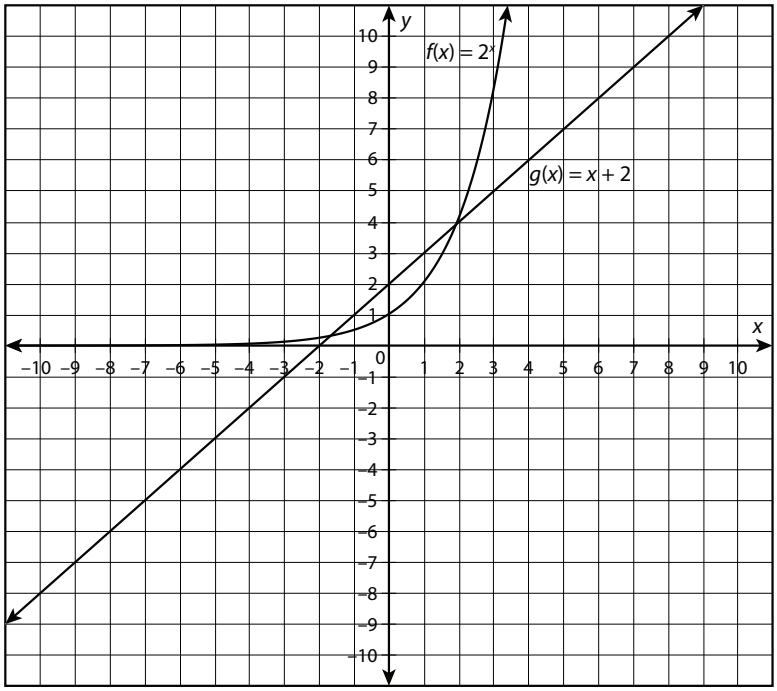
$f(x) = 2^x$

$g(x) = x + 2$

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Instruction

1. Graph $f(x) = 2^x$ and $g(x) = x + 2$ on the same coordinate axes.



2. Approximate the values for x where $f(x) = g(x)$.

   The graphs intersect in two points. From the graph, $x = 2$ and $x = -2$ appear to be good estimates.

3. Evaluate $f(x) = 2^x$ and $g(x) = x + 2$ for $x = 2$.

   Change "$f(x) =$" and "$g(x) =$" to "$y =$" and substitute 2 for x.

   $y = 2^x = 2^{(2)} = 4$

   $y = x + 2 = (2) + 2 = 4$

4. Find the difference in the y-values for the equations.

   $4 - 4 = 0$; therefore, $x = 2$ satisfies $f(x) = g(x)$ and is the x-coordinate of a solution to the system.

   The point whose ordered pair is (2, 4) is in the solution set of this system.

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Instruction

5. Evaluate $f(x) = 2^x$ and $g(x) = x + 2$ for $x = -2$.

   Change "$f(x) =$" and "$g(x) =$" to "$y =$" and substitute -2 for x.

   $y = 2^x = 2^{(-2)} = 0.25$

   $y = x + 2 = (-2) + 2 = 0$

6. Find the difference in the y-values for the equations.

   $0.25 - 0 = 0.25$

   0.25 is very close to 0; therefore, $f(x) = g(x)$ when x is approximately equal to -2.

   From the graph, we can see that the coordinates of the second point of intersection are approximately (–1.69, 0.31). Therefore, the solutions of this system are (2, 4) and (–1.69, 0.31).

Example 3

Use a table of values to approximate the solution set for the following system of equations:

$f(x) = 3^x$

$g(x) = 2^x + 1$

1. Create a table of values.

x	$f(x) = 3^x$	$g(x) = 2^x + 1$	$f(x) - g(x)$
–1	$0.3\overline{3}$	1.5	–1.17
0	1	2	–1
1	3	3	0
2	9	5	4
3	27	9	18

2. In column $f(x) - g(x)$, look for sign changes.

   There is a sign change from $x = 0$ to $x = 2$, and at $x = 1$, $f(x) - g(x) = 0$. This tells us the curves f and g intersect at $x = 1$. The corresponding y-value is 3. Therefore, the solution set of this system is (1, 3).

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Problem-Based Task 2.1.2: Senior Trip

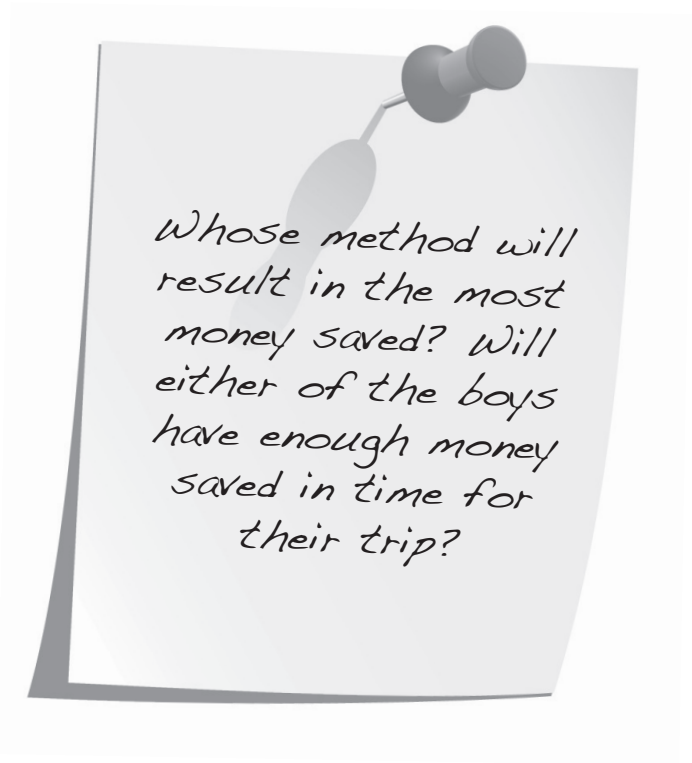
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Jacob and Antonio are incoming high school freshmen, with big plans for a trip to Europe after college graduation in 8 years. They each need to save $3,500 for their trip. Jacob has $1,100 saved and hidden in his room because he doesn't like to go to the bank. He earns $75 a month doing landscaping on the weekends. Jacob decides he will put $25 in his sock drawer every month for the next 8 years. Antonio likes his weekends for himself, so instead of working he sold his baseball card collection for $1,000 and put the money in a bank account earning 18% interest, compounded monthly. Whose method will result in the most money saved? Will either of the boys have enough money saved in time for their trip?

The following equations represent the boys' saving techniques, for which x is the time in years, and y_{Jacob} and y_{Antonio} are the amounts saved in dollars. Note that if Jacob saves $25 per month, this is equivalent to $300 per year.

$$y_{\text{Jacob}} = 1100 + 300x$$

$$y_{\text{Antonio}} = 1000\left(1+\frac{0.18}{12}\right)^{12x}$$



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Problem-Based Task 2.1.2: Senior Trip

Coaching

a. Who will have saved more money in 2 years?

b. Who will have saved more money in 4 years?

c. Who will have saved more money in 6 years?

d. Who will have saved more money in 8 years?

e. Will either of the boys have enough money saved at the end of 8 years?

f. Starting at $x = 0$, graph the two equations for y_{Jacob} and $y_{Antonio}$, noting where the graphs of the equations intersect.

g. Choose values for x and create a table of values to approximate when one boy's savings surpasses the other's.

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Problem-Based Task 2.1.2: Senior Trip

Coaching Sample Responses

a. Who will have saved more money in 2 years?

Remember that x is in years.

y_{Jacob} at 2 years = 1100 + 300(2) = \$1,700

y_{Antonio} at 2 years = $1000\left(1+\frac{0.18}{12}\right)^{12(2)}$ = \$1,429.50

After 2 years, Jacob will have saved more money than Antonio.

b. Who will have saved more money in 4 years?

y_{Jacob} at 4 years = 1100 + 300(4) = \$2,300

y_{Antonio} at 4 years = $1000\left(1+\frac{0.18}{12}\right)^{12(4)}$ = \$2,043.48

After 4 years, Jacob will have saved more money than Antonio.

c. Who will have saved more money in 6 years?

y_{Jacob} at 6 years = 1100 + 300(6) = \$2,900

y_{Antonio} at 6 years = $1000\left(1+\frac{0.18}{12}\right)^{12(6)}$ = \$2,921.16

After 6 years, Antonio will have saved more money than Jacob.

d. Who will have saved more money in 8 years?

y_{Jacob} at 8 years = 1100 + 300(8) = \$3,500

y_{Antonio} at 8 years = $1000\left(1+\frac{0.18}{12}\right)^{12(8)}$ = \$4,175.80

After 8 years, Antonio will have saved more money than Jacob.

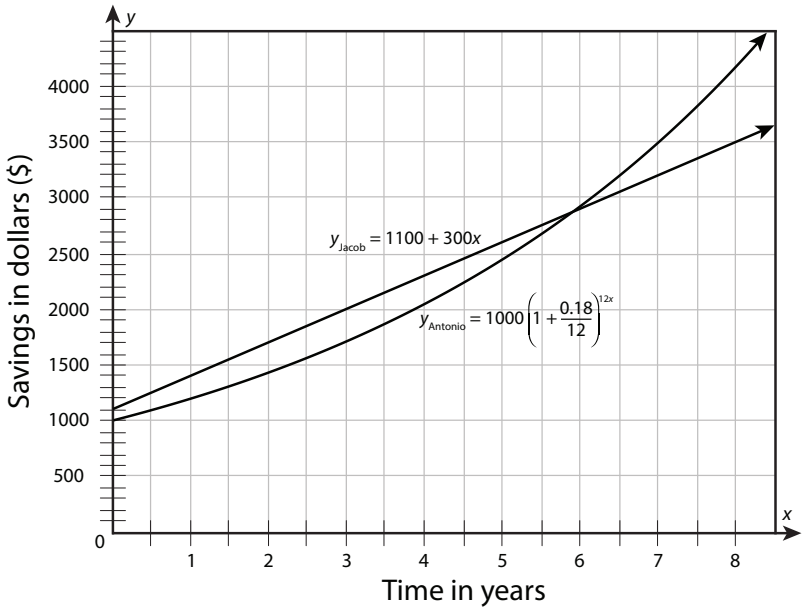
e. Will either of the boys have enough money saved at the end of 8 years?

Yes; both boys will have saved enough for the trip. Jacob will have saved \$3,500 and Antonio will have saved \$4,175.80.

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f. Starting at $x = 0$, graph the two equations for y_{Jacob} and y_{Antonio}, noting where the graphs of the equations intersect.

The graphs intersect at (5.9, 2871).



g. Choose values for x and create a table of values to approximate when one boy's savings surpasses the other's.

Suggested table:

Years	Savings in dollars		
x	$y_{\text{Jacob}} = 1100 + 300x$	$y_{\text{Antonio}} = 1000\left(1 + \frac{0.18}{12}\right)^{12x}$	$y_{\text{Jacob}} - y_{\text{Antonio}}$
5.5	2750.00	2671.52	78.48
5.6	2780.00	2719.68	60.32
5.7	2810.00	2768.71	41.29
5.8	2840.00	2818.62	21.38
5.9	2870.00	2869.40	0.57
6.0	2900.00	2921.16	–21.16

From the table, we can see Antonio's savings will surpass Jacob's savings between 5.9 and 6 years, or shortly before the 6-year mark.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.1.2: Intersecting Graphs

Use what you know about graphing functions to complete each problem.

1. Given the graphs of $y = f(x)$ and $y = g(x)$, what is the value of $f(x) - g(x)$ where the two graphs intersect?

2. If you are using the graphs of $y = f(x)$ and $y = g(x)$ to estimate x where $f(x) = g(x)$, and you see that the graphs intersect at two different points, how many values of x should you attempt to find?

3. If you are trying to estimate the value of x where $f(x) = g(x)$ using a table of values, and all the values for $f(x) - g(x)$ are positive, what might this imply?

4. While using a table of values to estimate the value of x where $f(x) = g(x)$, you see that there are three sign changes in the $f(x) - g(x)$ column. How many possible solutions for x are there?

5. Estimate the solution(s) to the following system by graphing.

   $f(x) = 2x + 5$

   $g(x) = -\frac{1}{2}x + 1$

6. Estimate the solution(s) to the following system by graphing.

   $h(x) = 3^x$

   $k(x) = x + 4$

7. Use a table of values to estimate the solution(s) to the following system on the interval $-3 \le x \le 3$.

   $s(x) = 2^x + 1$

   $r(x) = x + 3$

continued

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8. Use a table of values to estimate the solution(s) to the following system on the interval $-3 \le x \le 3$.

   $f(x) = x$

   $g(x) = \left(\frac{1}{2}\right)^x$

9. Marisol and Amy go for a 5-mile run every Monday morning at 6:00 A.M. However, on one particular Monday, Amy was going to be 5 minutes late and asked Marisol if she would slow her pace so Amy would be able to catch up to her during the run. Marisol will run at 10 minutes per mile and Amy will start 5 minutes later, running at 8 minutes per mile. The equations for the runners are given as follows, where $M(x)$ represents the distance that Marisol runs, and $A(x)$ represents the distance that Amy runs.

   $M(x) = \frac{1}{10}x$ and $A(x) = \frac{1}{8}(x-5)$, where x is in minutes from when Marisol starts her run and $M(x)$ and $A(x)$ are in miles.

   Use a graph to find how many minutes will have passed before Amy catches Marisol. How far will the girls have run when they meet?

10. Tickets for the school play cost $10 for adults and $5 for students. 120 tickets were sold in all for $800. The equations for total profit and total tickets are as follows:

    $5x + 10y = 800$ and $x + y = 120$ tickets, where x is the number of students and y is the number of adults.

    Graph the two equations. At what point do the graphs intersect? What do the coordinates of the intersection tell us about the ticket sales at the school play?

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Lesson 2.1.3: Domain and Range

Warm-Up 2.1.3

It's Hat Day at the ballpark and every child 10 years old and younger receives a team baseball cap at Gate 7. The policies at the ballpark are very strict:

- Every child entering Gate 7 must receive a cap.
- Every child entering Gate 7 must wear the cap.
- Only children age 10 or younger can enter Gate 7.
- No child shall wear a different cap than the one received at the gate.

Use this information to answer the questions.

1. What is the gate's input?

2. What is the gate's output?

3. What might be implied if all the rules were followed but there were still children 10 years old and younger in the ballpark without caps?

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Lesson 2.1.3: Domain and Range

Common Core State Standard

F–IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

Warm-Up 2.1.3 Debrief

1. What is the gate's input?

   The input is who enters through the gate. Children who haven't yet received caps are entering the gate, so the input is "children without caps."

2. What is the gate's output?

   The children who come out of the gate all have caps. Therefore, the output of the gate is "children with caps."

3. What might be implied if all the rules were followed but there were still children 10 years old and younger in the ballpark without caps?

   It could be implied that children in the ballpark without caps did not enter the park through Gate 7.

Connection to the Lesson

- The warm-up describes a function with a specific set of potential inputs and a specific set of potential outputs.
- In the sub-lesson, the domain and range of a function will be defined.
- The warm-up provides students practice with inputs and outputs in a purely contextual sense.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

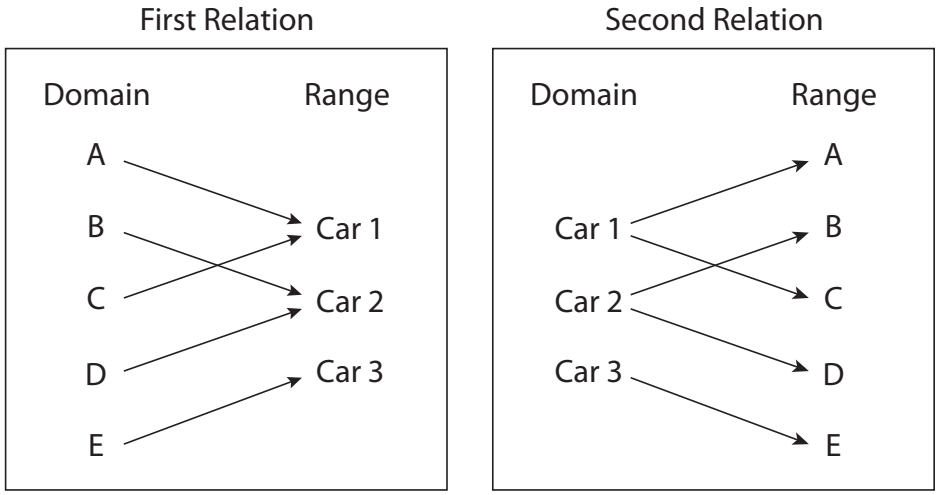
- substituting values for variables (6.EE.2c)
- applying the order of operations to simplify expressions (5.OA.1)
- evaluating negative exponents (8.EE.1)

Introduction

Earlier we saw that the graph of $y = f(x)$ is the set of all solutions for the function f. We also found that when $f(x) = g(x)$, the functions f and g have a common solution, and the graphs of these functions intersect at that point. In this lesson, we learn what defines a function and what its properties are.

Before we define what it is to be a function, we must first define a relation. A **relation** is a relationship between two sets of data. If you pair any two sets of data, you create a relation between the sets. For example, you could define a relation between the neighbors on your block and the cars they drive such that if you are given a neighbor's name, you can properly assign that person to his or her car. And, you could define another relation between the same sets such that if you are given a car's make and model, you can assign that car to the correct neighbor.

Remember that a function is a special relation in which each input is mapped to only one output. So, as with the first example, say we are given a name of a neighbor and we assign that neighbor to a make and model of a car. Even if some of your neighbors drive the same make and model car, each person's name is assigned to only one type of car. This relation is a function. But, what if we go the other way and look at the make and model of the cars and map them to the neighbors who drive them? Then a particular make and model of a car may be assigned to two or more neighbors. Both relations are pictured in the following diagram:



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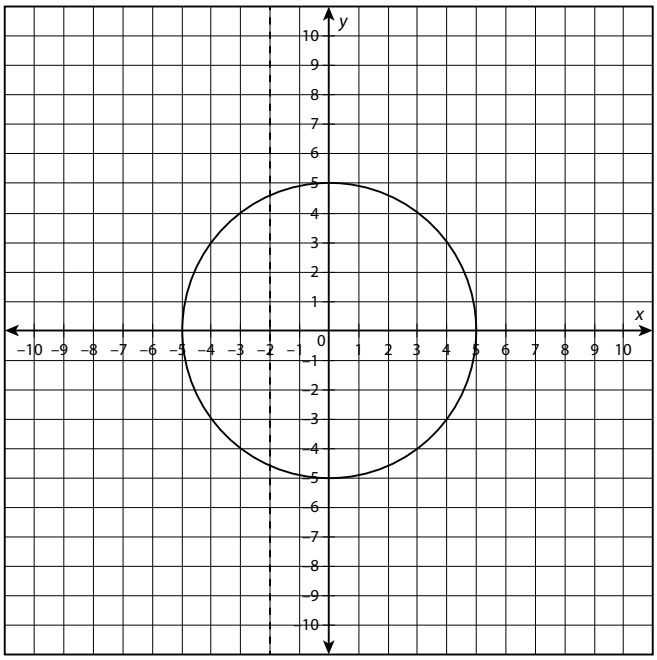
Instruction

In the first relation, neighbors A and C are both mapped onto Car 1 and neighbors B and D are both mapped onto Car 2. But what is important is that all the neighbors A through E are each mapped onto only one car. Therefore, this relation is a function. In the second relation, we see Car 1 is mapped onto neighbors A and C, and Car 2 is mapped onto neighbors B and D. The second relation is not a function because the same car is mapped onto more than one neighbor.

We call the set of all potential inputs the **domain** of a function. We call the set of potential outputs the **range** of the function. So, a function f maps an element x from the domain and creates $f(x)$, an element in the range. It is important to understand that $f(x)$ is strictly one value; each element in the domain of a function can be mapped to exactly one element in the range. That is, for every value of x, there is exactly one value of $f(x)$.

One way to determine whether a relation is a function is to graph the relation and perform a vertical line test. A vertical line in the coordinate plane is described by the equation $x = a$, where a is the value of x where the line intersects the x-axis. So, if a vertical line intersects a graph at two unique points, this implies the graphed relation has two unique values of $f(x)$ for the given x. In other words, if a vertical line intersects the graph of a relation in only one point, the relation is a function. If the vertical line intersects the graph of a relation in two or more points, the relation is not a function.

Take a look at the following graph of a circle. If we draw the vertical line whose equation is $x = -2$, we see that the line intersects the circle at two points. This means the relation that describes this circle will map $x = -2$ to more than one value of y. Therefore, a circle is not a function.

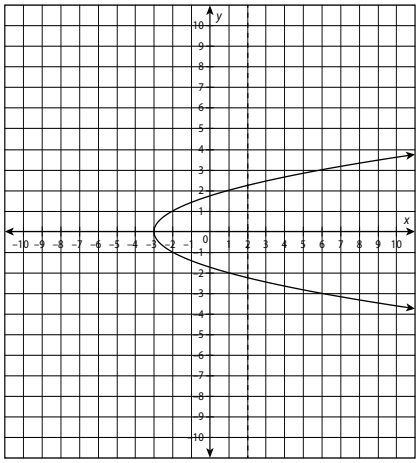


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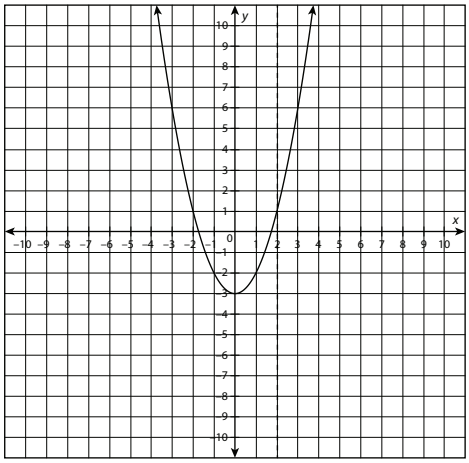
Instruction

The following graphs are very similar, but the graph of the relation on the left fails the vertical line test, and the graph of the relation on the right passes the vertical line test. Therefore, the relation on the left is not a function and the relation on the right is a function.

Is not a function:



Is a function:



Key Concepts

- The domain is the set of *x*-values that are valid for the function.
- The range is the set of *y*-values that are valid for the function.
- A function maps elements from the domain of a function to elements from the range of the function.
- Each value of *x* in the domain of a function can be mapped to only one value of *f*(*x*) in the range. In other words, if an element in the domain maps to more than one element in the range, then the relation is not a function.
- To create a mapping, list the elements of the domain in one column and the elements of the range in a second column. Then draw lines that match the elements in the domain to the corresponding elements in the range.
    - In the mapping, if one line goes from each element in the domain to only one element in the range, then the relation is a function.
    - In the mapping, if more than one line goes from an element in the domain to multiple elements in the range, then the relation is not a function.
- The vertical line test can also be used to determine if a relation is a function. If you pass an imaginary vertical line across the graph of a relation, look to see if the line ever intersects more than one point on the graph of the relation.

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Instruction

- If a vertical line intersects only one point on the graph of the relation, then the relation is a function.
- If a vertical line intersects more than one point on the graph of a relation, then the relation is not a function.

Common Errors/Misconceptions

- not understanding the difference between a relation and a function
- confusing the domain and range of a function or relation
- thinking that if a range value repeats or is the same for different values in the domain, then the relation is not a function; that is, not understanding that different *x*-values can map onto the same *y*-value

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Guided Practice 2.1.3

Example 1

Use a mapping diagram to determine if the given relation is a function.

{(–2, 4), (–1, 1), (0, 0), (1, 1), (2, 4), (3, 9)}

1. Find the domain of the first relation.

   The domain is the set of *x*-values of the relation. List the domain in numerical order. If any of the *x*-values repeat, list them only once.

   Domain: {–2, –1, 0, 1, 2, 3}

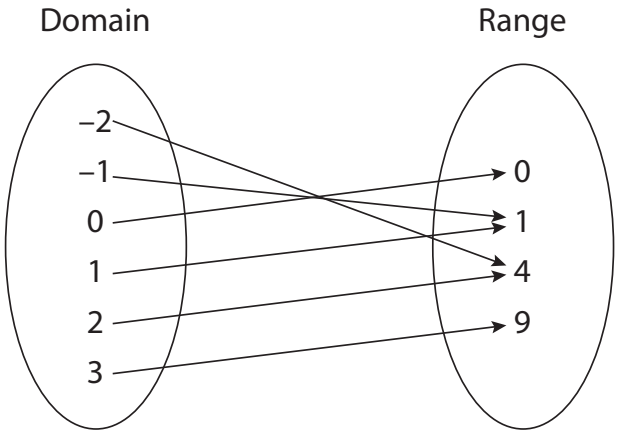
2. Find the range of the relation.

   The range is the set of *y*-values of the relation. List the range in numerical order. If any of the *y*-values repeat, list them only once.

   Range: {0, 1, 4, 9}

   The *y*-values 1 and 4 each appear in more than one of the ordered pairs, but they are only listed once.

3. Map the elements in the domain to the corresponding elements in the range.



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4. Analyze the mapping.

   If there is only one line coming from each element of the domain, the relation is a function. In this case, each element of the domain is paired with exactly one element in the range, so the relation is a function. Notice that some elements in the range have two arrows coming to them. The relation is still a function because each element in the domain has only one line from it.

Example 2

Use a mapping diagram to determine if the given relation is a function.

{(4, –5), (1, –3), (0, 0), (1, 1), (4, 5), (9, 3)}

1. Find the domain of the relation.

   The domain is the set of *x*-values of the relation. List the domain in numerical order. List repeating values only once.

   Domain: {0, 1, 4, 9}

   1 and 4 repeat, but they are only listed once.

2. Find the range of the relation.

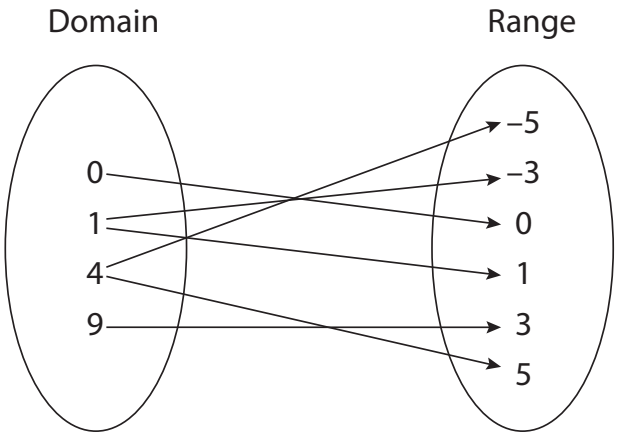
   The range is the set of *y*-values of the relation. List the range in numerical order. List repeating values only once.

   Range: {–5, –3, 0, 1, 3, 5}

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Instruction

3. Map the elements in the domain to the corresponding elements in the range.



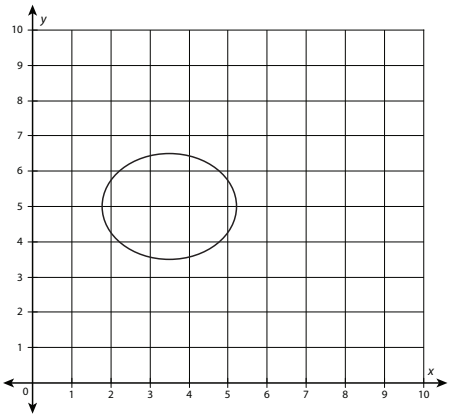
4. Analyze the mapping.

If there is only one line coming from each element of the domain, the relation is a function. In this case, two elements of the domain have multiple lines going to elements in the range. The relation is not a function.

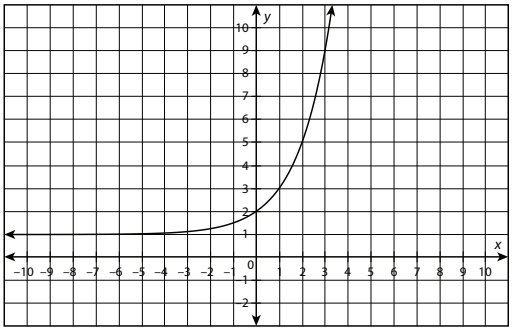
Example 3

Use the vertical line test to determine if each relation is a function.

Graph A



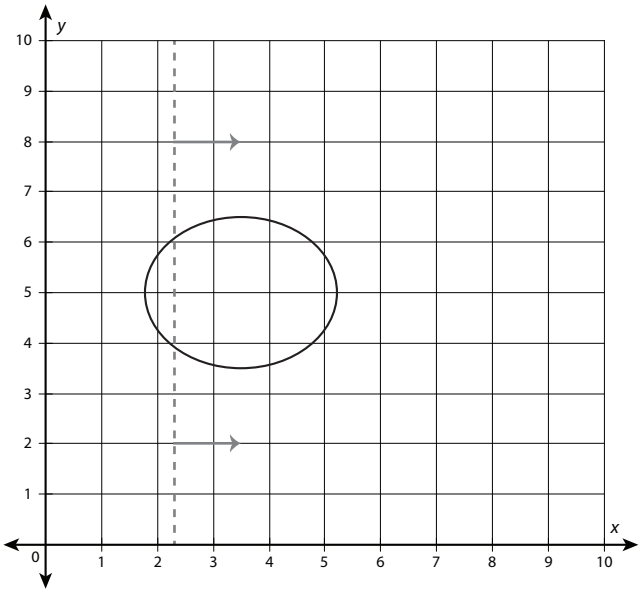
Graph B



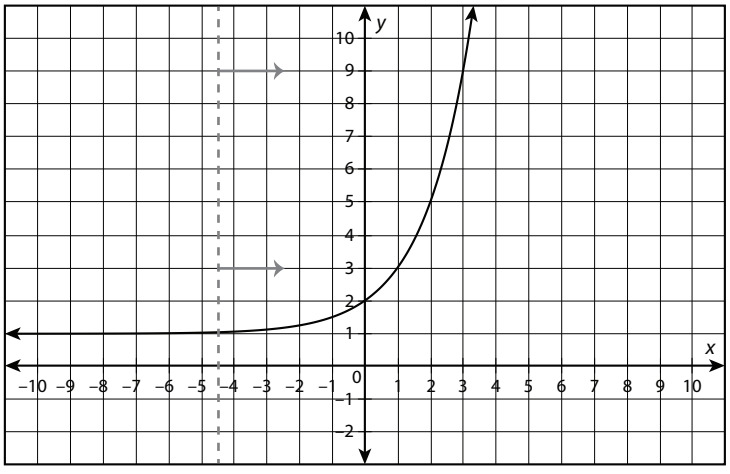
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Instruction

1. Pass a vertical line over the figure in Graph A. Notice that the line intersects the figure at two different points. The relation is not a function.



2. Pass a vertical line over the curve in Graph B. Notice that the vertical line only intersects the curve at one point at a time. The relation is a function.



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Instruction

Example 4

Omar has decided to take yoga classes for one year. It costs $10 to join the yoga studio, and each yoga class costs $5. Omar's fees can be represented by the function $f(x) = 5x + 10$, where x represents the number of classes and $f(x)$ represents the total cost. What are the domain and range of the function?

1. Identify the domain.

   The domain is the set of x-values that are valid for the function. Theoretically, Omar can take 0, 1, 2, 3, 4, ..., etc., classes up to infinity. However, it is unlikely that he will take more than one yoga class per day. If Omar never takes a class, the least value in the domain will be 0. If he takes a class every day for a year (and the studio is open every day), the greatest value in the domain will be 365.

   Domain: {0, 1, 2, 3, ..., 363, 364, 365}

2. Identify the range.

   The range is the values of $f(x)$ that are valid for the function. These values will be the outputs of the function when values in the domain are substituted into the equation of the function. The function is $f(x) = 5x + 10$. Substitute a set of domain values for x into the function to find values for the range.

   Range: {10, 15, 20, 25, ..., 1825, 1830, 1835}

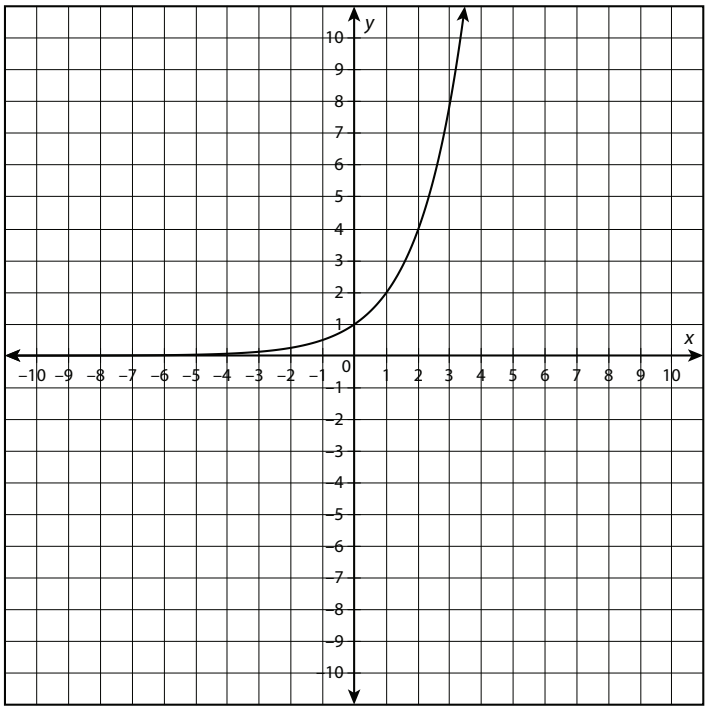
   These values represent Omar's possible fees for taking yoga classes for a year.

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Instruction

Example 5

Use the following graph to identify the domain and range of the function $f(x) = 2^x$.



1. Identify the domain.

   The domain is the set of *x*-values that are valid for the function. This graph goes on infinitely; so, the domain can be any real *x*-value.

   Domain: {all real numbers}

2. Identify the range.

   The range is the set of *y*-values that are valid for the function.

   The range will never go below 0. In fact, the range never actually reaches 0. The upper end of the range, however, is limitless.

   Range: $\{y > 0\}$

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Problem-Based Task 2.1.3: The Private Library

SMP	
1 ✓	2
3	4
5	6 ✓
7 ✓	8

In 2012, Terrence was hired to organize a private library. He decided to sort the books alphabetically by title first, and then by year of publication.

First, Terrence placed a sticker with the first letter of the title on the spine of the book. Then he sorted the books by year of publication. Terrence already knew that the oldest book in the collection was published in 1800, which meant that none of the books were published before that year. Therefore, he decided to subtract 1800 from the year of publication for each book, and include the resulting number on each book's sticker. For example, Terrence sorted and added stickers to the following books:

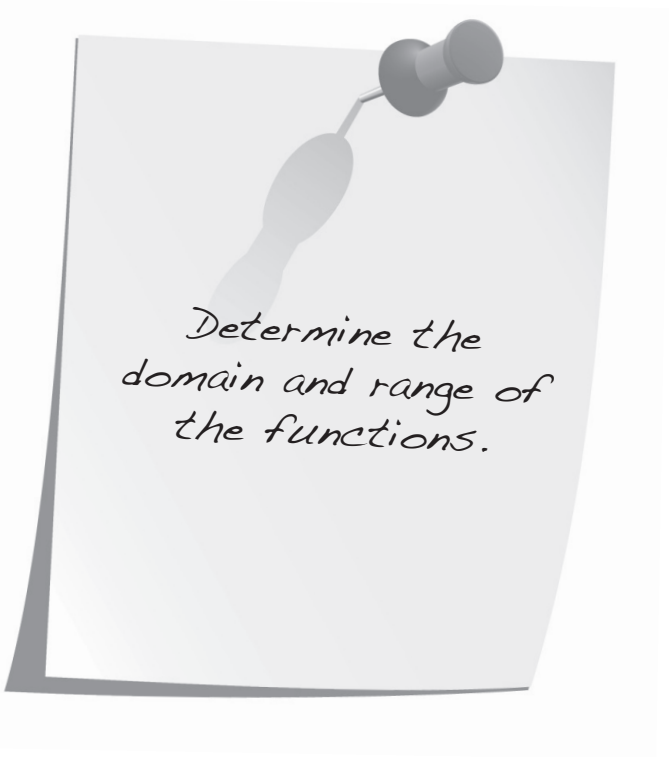
- *All Is Fair*, 1960: Sticker = A160
- *Newts and Squirmers*, 2003: Sticker = N203
- *Zanzibar's Wetlands*, 1890: Sticker = Z90

Terrence's sorting processes can be written as functions. Given the function T for the title, the function Y for the year of publication, and the function S for the sticker number, determine the domain and range of the following functions:

- T(title) = first letter of title
- Y(year) = year of publication – 1800
- S(book) = T(title)Y(year)

What would be the range of the function S if the library only had the following books?

- *Georgian Love Story*, published in 1822
- *Panama and Other Canals*, published in 1904
- *Likely Story*, published in 1965
- *Jitterbug*, published in 1920
- *Donkey's Wild Adventure*, published in 2008
- *Wormholes and Quasars*, published in 1989
- *Ben and the Art of Stagecoach Maintenance*, published in 1835



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Problem-Based Task 2.1.3: The Private Library

Coaching

a. What are the domain and range of the title function, *T*?

b. What are the domain and range of the year function, *Y*?

c. What are the domain and range of the sticker function, *S*?

d. What would be the range of the function *S* if the library only had the following books?

Georgian Love Story, published in 1822

Panama and Other Canals, published in 1904

Likely Story, published in 1965

Jitterbug, published in 1920

Donkey's Wild Adventure, published in 2008

Wormholes and Quasars, published in 1989

Ben and the Art of Stagecoach Maintenance, published in 1835

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Instruction

Problem-Based Task 2.1.3: The Private Library

Coaching Sample Responses

a. What are the domain and range of the title function, T?

The domain of T is all possible book titles in the library.

The range of T is {A, B, C, ..., Y, Z}.

b. What are the domain and range of the year function, Y?

The domain of Y is all publication dates in the library, and the publication dates range from 1800 to possibly 2012. Thus the domain is [1800, 2012].

The range of Y is the number of years that have passed since 1800.

At least one book in the library was published in 1800, so the least value in the range is $1800 - 1800 = 0$.

Terrence was hired in 2012. The greatest possible value in the range is $2012 - 1800 = 212$.

The range of Y is {0, 1, 2, ..., 211, 212}.

c. What are the domain and range of the sticker function, S?

The domain of S is all books in the library.

The range of S is all possible combinations of letters T and numbers Y.

The range of S is {A0, A1, A2, ..., A212, B0, B1, B2, ..., B212, C0, C1, C2, ..., C212, ..., Z0, Z1, Z2, ..., Z212}.

d. What would be the range of the function S if the library only had the following books?

Georgian Love Story, published in 1822

Panama and Other Canals, published in 1904

Likely Story, published in 1965

Jitterbug, published in 1920

Donkey's Wild Adventure, published in 2008

Wormholes and Quasars, published in 1989

Ben and the Art of Stagecoach Maintenance, published in 1835

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Determine the first letter and (year of publication – 1800) for each book.

S of *Georgian Love Story*, published in 1822, is G22.

S of *Panama and Other Canals*, published in 1904, is P104.

S of *Likely Story*, published in 1965, is L165.

S of *Jitterbug*, published in 1920, is J120.

S of *Donkey's Wild Adventure*, published in 2008, is D208.

S of *Wormholes and Quasars*, published in 1989, is W189.

S of *Ben and the Art of Stagecoach Maintenance*, published in 1835, is B35.

In alphabetical order, the range of *S* is {B35, D208, G22, J120, L165, P104, W189}.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.1.3: Domain and Range

Use what you know about functions, domain, and range to answer each question.

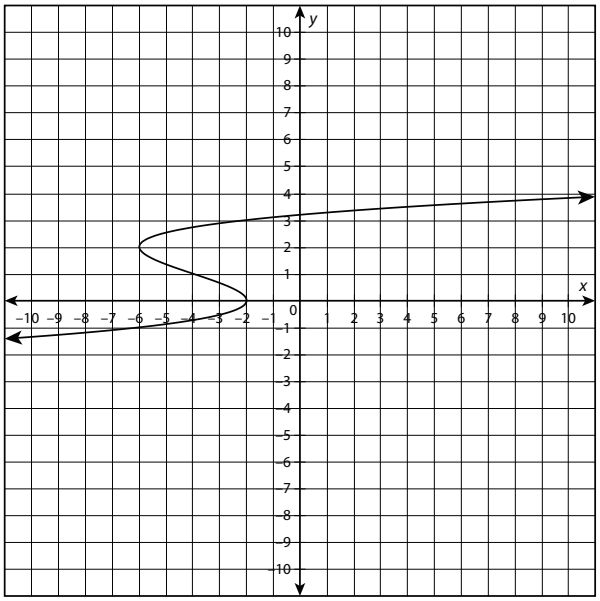
1. Could the following table represent a function? Why or why not?

x	*y*
1	7
2	6
3	5
4	4
5	3
6	2

2. Could the following table represent a function? Why or why not?

x	*y*
0	1
2	3
4	5
6	7
8	9
10	1

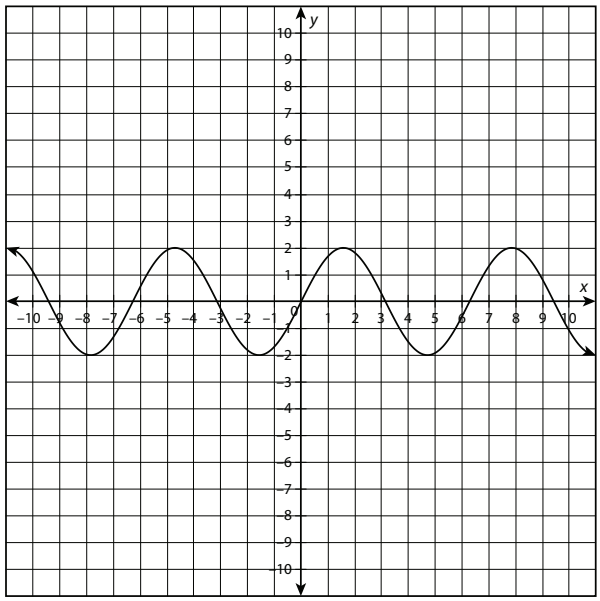
3. Could the following graph be a function? Why or why not?



continued

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4. Could the following graph be a function? Why or why not?



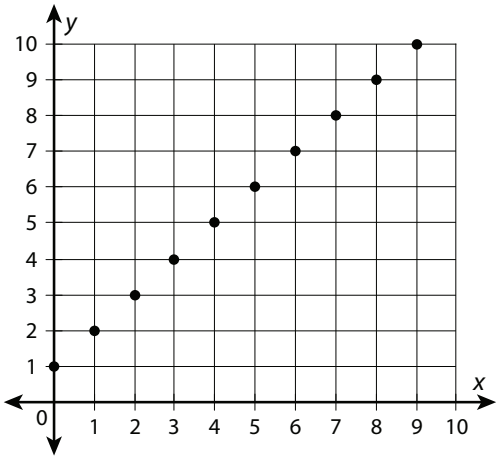
5. Does the following set of ordered pairs make up a relation? If so, is the relation also a function? Why or why not?

   {(2, 4), (3, 6), (4, 8), (5, 10), (6, 12), (7, 14)}

6. Does the following set of ordered pairs make up a relation? If so, is the relation also a function? Why or why not?

   {(2, 2), (3, 3), (4, 4), (5, 5), (5, 6), (7, 7)}

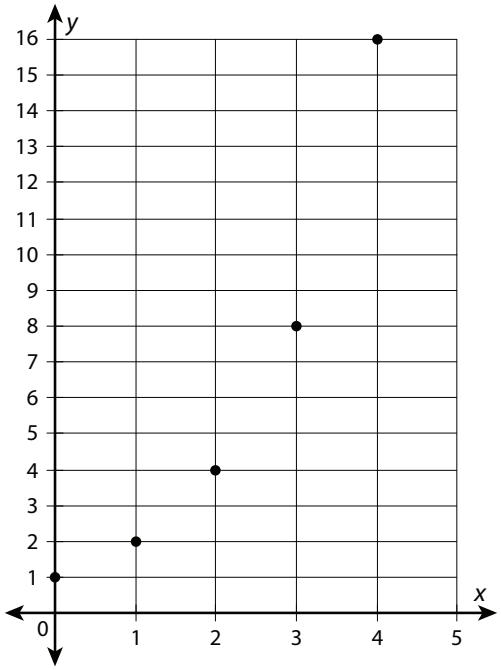
7. What are the domain and range of the graphed function?



continued

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8. What are the domain and range of the graphed function?



9. A candle burns down at a rate of 1 inch per hour. The candle was originally 12 inches tall. The function that describes the height of the candle as it burns can be represented as $f(x) = -x + 12$, where x represents the number of hours the candle burns and $f(x)$ is the height of the candle. Draw a graph of the function. What are the domain and range?

10. The distance a trucker travels on the highway at 65 mph can be modeled by the function $f(x) = 65x$, where x is the time in hours and $f(x)$ is the distance in miles. What are the domain and range of the function?

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Lesson 2.1.4: Function Notation and Evaluating Functions

Warm-Up 2.1.4

A farmer wants to convert three different-sized greenhouses to solar power. He needs 1,500 square feet of solar panels for his small greenhouse, 2,100 square feet of panels for his medium greenhouse, and 2,800 square feet of panels for his large greenhouse. The farmer gets bids from three different companies, each with different pricing. The bids are as follows, where x is the number of square feet of panels needed for each greenhouse, and y is the cost:

- Company A charges \$2,000 for installation per greenhouse, and an additional \$2.00 per square foot of panels. The function for this situation is $y_A = 2000 + 2(x)$.
- Company B charges \$3,000 for installation per greenhouse, and an additional \$1.50 per square foot of panels. The function for this situation is $y_B = 3000 + 1.5(x)$.
- Company C charges \$4,200 for installation per greenhouse, and an additional \$1.00 per square foot of panels. The function for this situation is $y_C = 4200 + x$.

1. Which company will charge the least for the small greenhouse?

2. Which company will charge the least for the medium greenhouse?

3. Which company will charge the least for the large greenhouse?

4. If the farmer decides to hire a single company to convert all three greenhouses, which company will be the least expensive?

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Instruction

Lesson 2.1.4: Function Notation and Evaluating Functions

Common Core State Standard

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Warm-Up 2.1.4 Debrief

1. Which company will charge the least for the small greenhouse?

   The small greenhouse requires 1,500 square feet of solar panels, so evaluate y_A, y_B, and y_C for $x = 1500$:

   $y_A = 2000 + 2(1500) = 2000 + 3000 = 5000$

   $y_B = 3000 + 1.5(1500) = 3000 + 2250 = 5250$

   $y_C = 4200 + (1500) = 5700$

   Company A's bid is the lowest, at \$5,000. Therefore, Company A will charge the least for the small greenhouse.

2. Which company will charge the least for the medium greenhouse?

   The medium greenhouse requires 2,100 square feet of solar panels, so evaluate y_A, y_B, and y_C for $x = 2100$:

   $y_A = 2000 + 2(2100) = 2000 + 4200 = 6200$

   $y_B = 3000 + 1.5(2100) = 3000 + 3150 = 6150$

   $y_C = 4200 + (2100) = 6300$

   Company B's bid is the lowest, at \$6,150. Therefore, Company B will charge the least for the medium greenhouse.

3. Which company will charge the least for the large greenhouse?

   The large greenhouse requires 2,800 square feet of solar panels, so evaluate y_A, y_B, and y_C for $x = 2800$:

   $y_A = 2000 + 2(2800) = 2000 + 5600 = 7600$

   $y_B = 3000 + 1.5(2800) = 3000 + 4200 = 7200$

   $y_C = 4200 + (2800) = 7000$

   Company C's bid is the lowest, at \$7,000. Therefore, Company C will charge the least for the large greenhouse.

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4. If the farmer decides to hire a single company to convert all three greenhouses, which company will be the least expensive?

   Add the three greenhouse bids for each company, as found in problems 1–3.

   $y_A = 5000 + 6200 + 7600 = 18{,}800$

   $y_B = 5250 + 6150 + 7200 = 18{,}600$

   $y_C = 5700 + 6300 + 7000 = 19{,}000$

   Company B's total bid is lowest, at $18,600, so it is the least expensive option if the farmer decides to hire a single company to convert all three greenhouses to solar power.

Connection to the Lesson

- Students will evaluate functions, which requires substitution.
- Students will learn how to evaluate functions using function notation, which is an easier method to keep track of multiple functions with varying inputs.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- substituting values for the variables (6.EE.2c)
- evaluating expressions (6.EE.2c)
- understanding the domain and range of a function (F–IF.1)

Introduction

So far, we have seen a function f of a variable x represented as $f(x)$. We have graphed $f(x)$ and learned that its range is dependent on its domain. In this lesson, we will explore function notation and the versatility of functions.

For example, let f be a function with the domain {1, 2, 3}, and let $f(x) = 3x + 1$. To evaluate f over the domain {1, 2, 3}, we write the following equations by substituting each value in the domain for x:

$f(1) = 3(1) + 1 = 4$

$f(2) = 3(2) + 1 = 7$

$f(3) = 3(3) + 1 = 10$

{4, 7, 10} is the range of $f(x)$.

Key Concepts

- Functions can be evaluated for values of variables.
- To evaluate a function such as $f(x)$, substitute values in the domain for all occurrences of x.
- To evaluate $f(2)$ in $f(x) = x + 4$, replace x with 2 and simplify: $f(2) = (2) + 4 = 6$. This means that $f(2) = 6$.
- The coordinates of a point on the graph of a function can be written as an ordered pair: $(x, f(x))$.

Common Errors/Misconceptions

- thinking that $f(x)$ means "$f \bullet x$"
- not understanding how to use the domain in order to find the range

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Guided Practice 2.1.4

Example 1

Evaluate $f(x) = 4x - 7$ over the domain of f, {1, 2, 3, 4}. What is the range of f?

1. To evaluate $f(x) = 4x - 7$ over the domain {1, 2, 3, 4}, substitute the values from the given domain into $f(x) = 4x - 7$.

2. Evaluate $f(1)$.

$f(x) = 4x - 7$	Original function
$f(1) = 4(1) - 7$	Substitute 1 for x.
$f(1) = 4 - 7 = -3$	Simplify.

3. Evaluate $f(2)$.

$f(x) = 4x - 7$	Original function
$f(2) = 4(2) - 7$	Substitute 2 for x.
$f(2) = 8 - 7 = 1$	Simplify.

4. Evaluate $f(3)$.

$f(x) = 4x - 7$	Original function
$f(3) = 4(3) - 7$	Substitute 3 for x.
$f(3) = 12 - 7 = 5$	Simplify.

5. Evaluate $f(4)$.

$f(x) = 4x - 7$	Original function
$f(4) = 4(4) - 7$	Substitute 4 for x.
$f(4) = 16 - 7 = 9$	Simplify.

6. Collect the set of outputs from the inputs.
   The range of f is {–3, 1, 5, 9}.

SWB p. 45; SRB p. U2-35

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Instruction

Example 2

Evaluate $g(x) = 3^x + 1$ over the domain of g, {0, 1, 2, 3}. What is the range of g?

1. To evaluate $g(x) = 3^x + 1$ over the domain {0, 1, 2, 3}, substitute the values from the given domain into $g(x) = 3^x + 1$.

2. Evaluate $g(0)$.

$g(x) = 3^x + 1$	Original function
$g(0) = 3^0 + 1$	Substitute 0 for x.
$g(0) = 1 + 1 = 2$	Simplify.

3. Evaluate $g(1)$.

$g(x) = 3^x + 1$	Original function
$g(1) = 3^1 + 1$	Substitute 1 for x.
$g(1) = 3 + 1 = 4$	Simplify.

4. Evaluate $g(2)$.

$g(x) = 3^x + 1$	Original function
$g(2) = 3^2 + 1$	Substitute 2 for x.
$g(2) = 9 + 1 = 10$	Simplify.

5. Evaluate $g(3)$.

$g(x) = 3^x + 1$	Original function
$g(3) = 3^3 + 1$	Substitute 3 for x.
$g(3) = 27 + 1 = 28$	Simplify.

6. Collect the set of outputs from the inputs.

   The range of g is {2, 4, 10, 28}.



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Instruction

Example 3

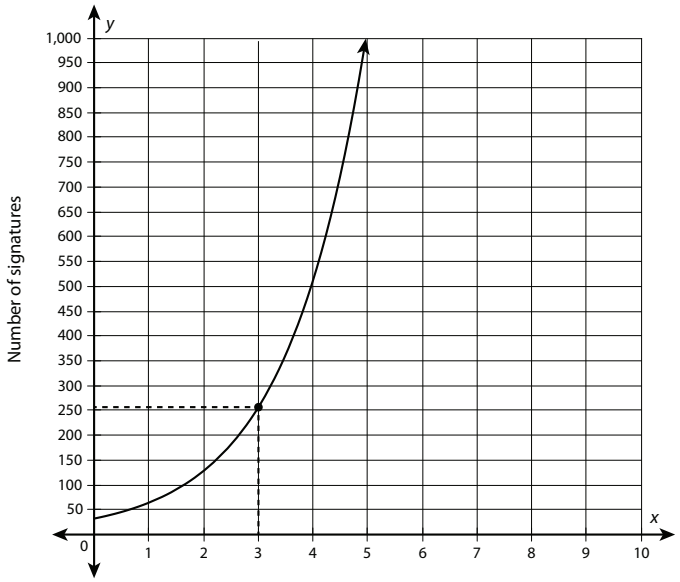
Raven started an online petition calling for more vegan options in the school cafeteria. So far, the number of signatures has doubled every day. She started with 32 signatures on the first day. Raven's petition can be modeled by the exponential function $f(x) = 32(2)^x$. Evaluate $f(3)$ and interpret the results in terms of the petition.

1. Evaluate the function.

$f(x) = 32(2)^x$	Original function
$f(3) = 32(2)^{(3)}$	Substitute 3 for x.
$f(3) = 32(8)$	Apply the exponent.
$f(3) = 256$	Simplify.

2. Interpret the results.

On day 3, the petition has 256 signatures. This is a point on the graph, (3, 256), of the function $f(x) = 32(2)^x$.



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Problem-Based Task 2.1.4: Party Time

SMP	
1 ✓	2
3	4
5	6 ✓
7	8 ✓

A company is throwing a large party requiring a venue, tables, chairs, and waiters. There will be between 50 and 200 guests, so the party coordinator has been asked to predict the total cost of the event for 50, 100, 150, and 200 guests. The functions that follow describe the number of tables, chairs, and waiters needed, as well as the size of the venue in square feet. In each equation, x represents the number of guests.

- Number of tables: $t(x)=\frac{1}{10}x$
- Number of chairs: $c(x)=x+10$
- Number of waiters: $w(x)=\frac{1}{5}x$
- Square feet of venue: $s(x)=1000\left(2^{\frac{x}{50}}\right)$

Tables are \$50 each, chairs are \$3 each, waiters earn \$100 each, and venues charge \$2 per square foot. Therefore, the total estimate for the party, $P(x)$, is given by this equation:

$$P(x) = (50 \bullet t(x)) + (3 \bullet c(x)) + (100 \bullet w(x)) + (2 \bullet s(x))$$

How much will the party cost?



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Problem-Based Task 2.1.4: Party Time

Coaching

a. What is the domain of $t(x)$?

b. What is the domain of $c(x)$?

c. What is the domain of $w(x)$?

d. What is the domain of $s(x)$?

e. What is the domain of $P(x)$?

f. What is the most the venue can cost?

g. What is the cost for tables and chairs for 100 guests?

h. What is the entire cost of the party for 150 guests?

i. What is the least amount the party can cost?

j. Complete a table of values for the costs of the party for the numbers of possible guests.

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Instruction

Problem-Based Task 2.1.4: Party Time

Coaching Sample Responses

a. What is the domain of $t(x)$?

The domain is the number of guests planned for: {50, 100, 150, 200}.

b. What is the domain of $c(x)$?

The domain is the number of guests planned for: {50, 100, 150, 200}.

c. What is the domain of $w(x)$?

The domain is the number of guests planned for: {50, 100, 150, 200}.

d. What is the domain of $s(x)$?

The domain is the number of guests planned for: {50, 100, 150, 200}.

e. What is the domain of $P(x)$?

The domain is the number of guests planned for: {50, 100, 150, 200}.

f. What is the most the venue can cost?

The size of the venue is $s(x)=1000\left(2^{\frac{x}{50}}\right)$, the greatest number of guests is 200, and the price per square foot is \$2, so evaluate $2 \bullet s(200)$:

$$2 \bullet s(200)=2 \bullet 1000\left(2^{\frac{200}{50}}\right)=2 \bullet 100(2^4)=2 \bullet 1000(16)=2 \bullet 16{,}000=32{,}000$$

The maximum price of the venue is \$32,000.

g. What is the cost for tables and chairs for 100 guests?

The number of tables is $t(x)=\frac{1}{10}x$, and they cost \$50 each. The number of chairs is $c(x) = (x + 10)$, and they cost \$3 each.

Evaluate $(50 \bullet t(100)) + (3 \bullet c(100))$.

$$\left(50 \bullet t(100)\right)+\left(3 \bullet c(100)\right)=\left(50 \bullet \frac{1}{10}(100)\right)+\left(3 \bullet (100+10)\right)=500+330=830$$

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h. What is the entire cost of the party for 150 guests?

Evaluate $P(x) = (50 \bullet t(x)) + (3 \bullet c(x)) + (100 \bullet w(x)) + (2 \bullet s(x))$ for $x = 150$.

$$P(x)=\left(50\bullet t(x)\right)+\left(3\bullet c(x)\right)+\left(100\bullet w(x)\right)+\left(2\bullet s(x)\right)$$

$$P(150)=\left(50\bullet\left(\frac{1}{10}x\right)\right)+\left(3\bullet(x+10)\right)+\left(100\bullet\left(\frac{1}{5}x\right)\right)+\left(2\bullet\left(1000\left(2^{\frac{x}{50}}\right)\right)\right)$$

$$P(150)=\left(50\bullet\left(\frac{1}{10}(150)\right)\right)+\left(3\bullet(150+10)\right)+\left(100\bullet\left(\frac{1}{5}(150)\right)\right)+\left(2\bullet\left(1000\left(2^{\frac{150}{50}}\right)\right)\right)$$

$$P(150)=\left(50\bullet 15\right)+\left(3\bullet 160\right)+\left(100\bullet 30\right)+\left(2\bullet 1000\left(2^{3}\right)\right)$$

$$P(150)=\left(50\bullet 15\right)+\left(3\bullet 160\right)+\left(100\bullet 30\right)+\left(2\bullet 1000\left(8\right)\right)$$

$$P(150)=750+480+3000+16{,}000=20{,}230$$

The cost of the party for 150 guests is \$20,230.

i. What is the least amount the party can cost?

The least number of guests is 50.

Evaluate $P(x) = 50 \bullet t(x) + 3 \bullet c(x) + 100 \bullet w(x) + 2 \bullet s(x)$ for $x = 50$.

$$P(x)=\left(50\bullet t(x)\right)+\left(3\bullet c(x)\right)+\left(100\bullet w(x)\right)+\left(2\bullet s(x)\right)$$

$$P(50)=\left(50\bullet\left(\frac{1}{10}x\right)\right)+\left(3\bullet(x+10)\right)+\left(100\bullet\left(\frac{1}{5}x\right)\right)+\left(2\bullet\left(1000\left(2^{\frac{x}{50}}\right)\right)\right)$$

$$P(50)=\left(50\bullet\left(\frac{1}{10}(50)\right)\right)+\left(3\bullet(50+10)\right)+\left(100\bullet\left(\frac{1}{5}(50)\right)\right)+\left(2\bullet\left(1000\left(2^{\frac{50}{50}}\right)\right)\right)$$

$$P(50)=\left(50\bullet 5\right)+\left(3\bullet 60\right)+\left(100\bullet 10\right)+\left(2\bullet 1000\left(2\right)\right)$$

$$P(50)=250+180+1000+4000$$

$$P(50)=5430$$

The least amount the party can cost is \$5,430.

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j. Complete a table of values for the costs of the party for the numbers of possible guests.

Guests (x)	Party costs in dollars				Total cost
	Tables $t(x)$ $\left(50 \bullet \frac{1}{10}x\right)$	Chairs $c(x)$ $(3 \bullet (x+10))$	Waiters $w(x)$ $\left(100 \bullet \frac{1}{5}x\right)$	Venue $s(x)$ $\left(2 \bullet 1000\left(2^{\frac{x}{50}}\right)\right)$	
50	250	180	1,000	4,000	5,430
100	500	330	2,000	8,000	10,830
150	750	480	3,000	16,000	20,230
200	1,000	630	4,000	32,000	37,630

The total cost of the party ranges from \$5,430 to \$37,630 depending on the number of guests.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.1.4: Function Notation and Evaluating Functions

For problems 1–4, evaluate the given functions and determine the range of each.

1. Evaluate $f(x) = 2x - 8$ over the domain {0, 1, 3, 8}. What is the range of $f(x)$?

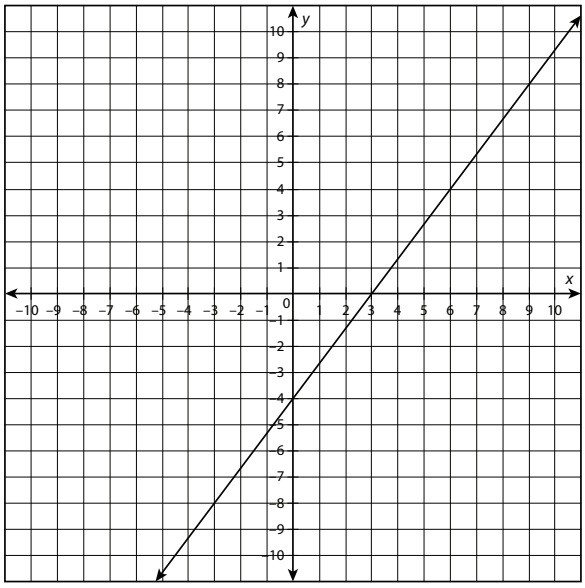
2. Evaluate $g(x) = x - 13$ over the domain {2, 4, 6, 8}. What is the range of $g(x)$?

3. Evaluate $f(x) = 3^x + 1$ over the domain {1, 2, 3, 4}. What is the range of $f(x)$?

4. Evaluate $r(x) = 2^x - 1$ over the domain {0, 1, 2, 3}. What is the range of $r(x)$?

Use what you know about function notation and graphing functions to complete problems 5–10.

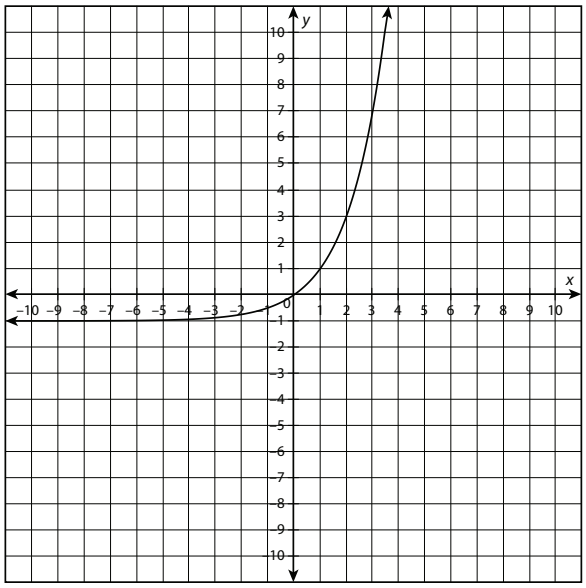
5. Given the graph of $f(x)$, what is $f(-3)$?



continued

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6. Given the graph of $f(x)$, what is $f(2)$?



7. A growing company has been hiring employees at a steady rate of 1 new hire per month. The company started with 2 employees. The growth of the company can be modeled by the function $g(x) = x + 2$, where x represents the number of months, and $g(x)$ represents the number of employees. Evaluate the function over the domain {3, 6, 18, 24}. Interpret the results and use a graph to explain your answer.

8. A population of insects doubles every 3 days. The population started with 8 insects. The function that models this growth is $f(x) = 8(2)^{\frac{x}{3}}$. Evaluate the function over the domain {0, 3, 6, 12}.

   Interpret the results and use a graph to explain your answer.

9. A postal delivery service charges \$3.40 per package and an additional \$0.50 for each ounce the package weighs. The function can be modeled by $f(x) = 0.5x + 3.4$, where x represents the number of ounces and $f(x)$ represents the total delivery charge. Tom ships four packages with the following weights: 2 ounces, 3.5 ounces, 15 ounces, and 21.3 ounces. Write four statements using function notation that you can use to evaluate the function given each of these weights. Interpret the results in terms of the context of the function.

10. An investment promises a return of 12% per year. Brody wants to figure out how much money he will have if he invests \$1,000 for 5, 10, or 15 years. The investment's growth can be modeled using the exponential function $f(x) = 1000(1.12)^x$, where x represents the number of years and $f(x)$ represents the return on the investment. Write three statements using function notation that you can use to evaluate the function for each time frame Brody wants to know about. Interpret the results in terms of the context of the function.

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Progress Assessment

Circle the letter of the best answer.

1. What is true about the set of all solutions for $y = f(x)$?
   a. It can be graphed.
   b. It is infinite.
   c. It is a set of ordered pairs.
   d. all of the above

2. The graphical representation of the solution set for $y = f(x)$ is _____.
   a. always a straight line
   b. called a curve
   c. a plane
   d. a circle

3. Which of the following is true at the intersection of $y = f(x)$ and $y = g(x)$?
   a. $f(x) = g(x)$
   b. $x = 0$
   c. $f(x) = 0$
   d. $g(x) < f(x)$

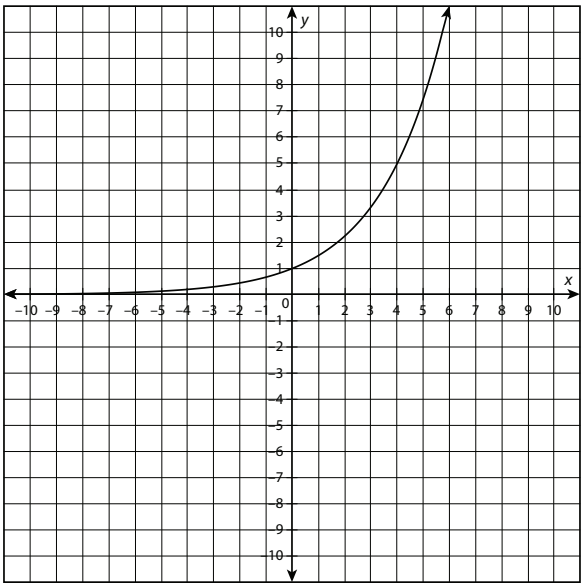
4. What does a sign change in the $f(x) - g(x)$ column of a table of values imply?
   a. f and g intersect.
   b. f and g do not intersect.
   c. $f = g$
   d. f and g are parallel.

continued

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Assessment

5. If the domain of the graphed function is all real numbers, what is the range of the function?



a. all real numbers
b. $x < 6$
c. $f(x) > 0$
d. $f(x) \geq 0$

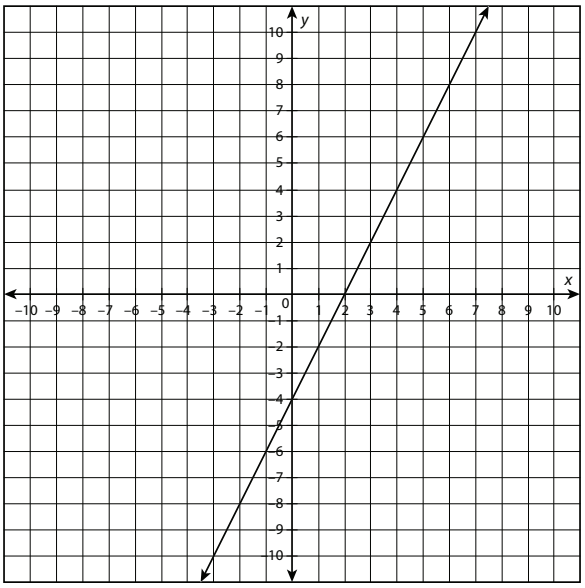
6. If $f(x) = 2x + 1$, and the domain of f is {5, 25, 50}, what is the range of f?

a. {5, 25, 50}
b. {10, 50, 100}
c. {11, 51, 101}
d. {0, 5, 10}

continued

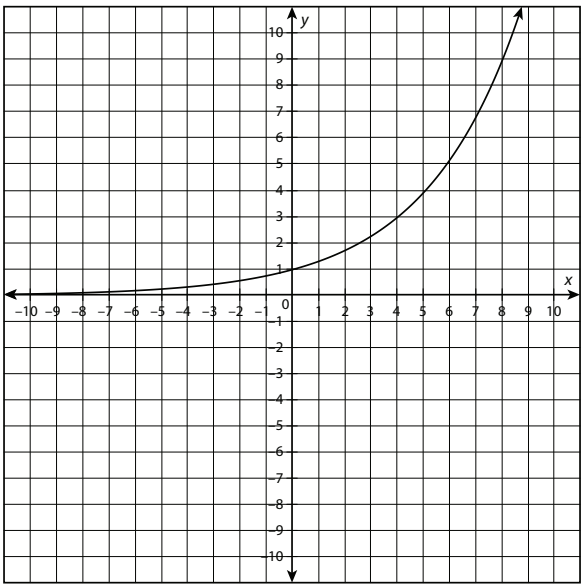
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7. Given the graph of the linear function $f(x)$, what is $f(6)$?



a. 5
b. 8
c. –4
d. 0

8. Given the graph of $f(x)$, what is $f(4)$?



a. 3
b. 5
c. 2.25
d. 8

continued

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9. DeAndre modeled the growth of his ant farm's population using the function $a(x) = 2(4)^{\frac{x}{3}}$, where x represents the number of days and $a(x)$ represents the population of ants. He started with 2 ants, and the population has quadrupled every 3 days. He evaluated the function at $a(12)$ and calculated $a(12) = 512$. What does his calculation say about the ant population?

   a. After 4 days, DeAndre will have 512 ants.

   b. After 12 days, DeAndre will have 512 ants.

   c. After 512 days, DeAndre will have 12 ants.

   d. After 512 days, DeAndre will have about 171 ants.

10. The contents of the fuel tank of Sierra's car can be modeled by the function $g(x) = -0.04x + 15$, where x represents the number of miles driven and $g(x)$ represents the number of gallons of gas in the tank. Sierra started her trip with a full tank of gas and traveled 200 miles. Which equation represents the amount of gas that Sierra had left in her car's fuel tank at the end of the trip?

   a. $g(200) = 7$

   b. $g(200) = 8$

   c. $g(7) = 200$

   d. $g(8) = 200$

Read the scenario and use the information to complete the problem that follows.

11. Luisa is saving money to buy a new smartphone that costs $430. She started with $150 and every month she saves $20. Her savings can be modeled by the function $s(x) = 20x + 150$, where x represents the number of months she saves and $s(x)$ represents how much she has saved.

   a. What are the domain and range of the function?

   b. Evaluate the function for $x = 4$, 8, and 12, then represent these results graphically.

   c. Interpret part b in the context of the problem.

   d. Write a statement using function notation that shows when Luisa will reach her goal. Then use words to describe the symbolic statement.

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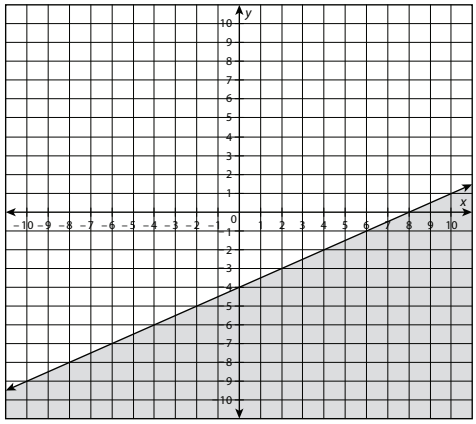
Assessment

Pre-Assessment

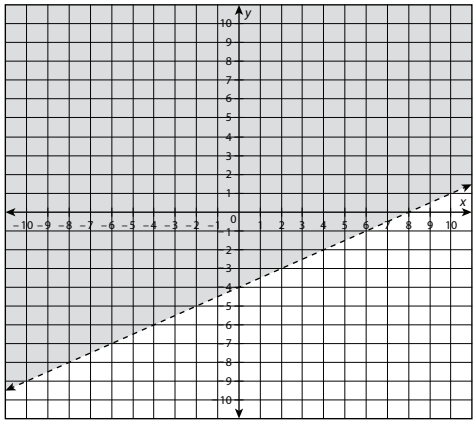
Circle the letter of the best answer.

1. Which graph represents the solution to the inequality $y < \frac{1}{2}x - 4$?

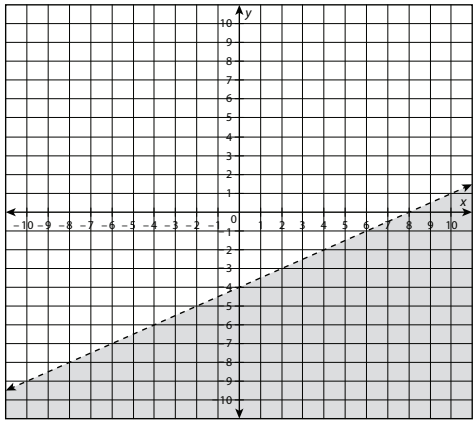
a.



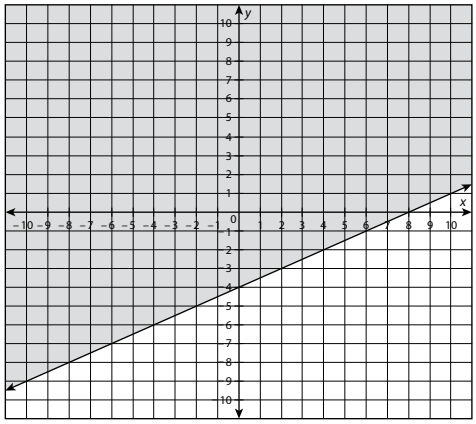
c.



b.



d.

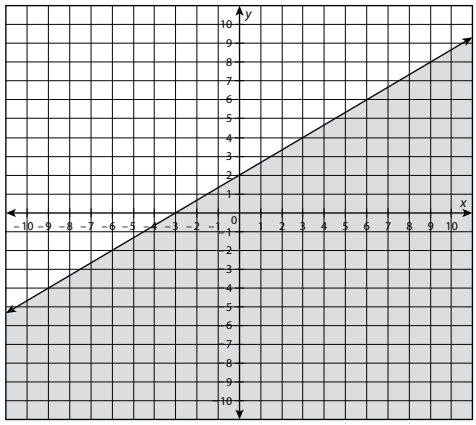


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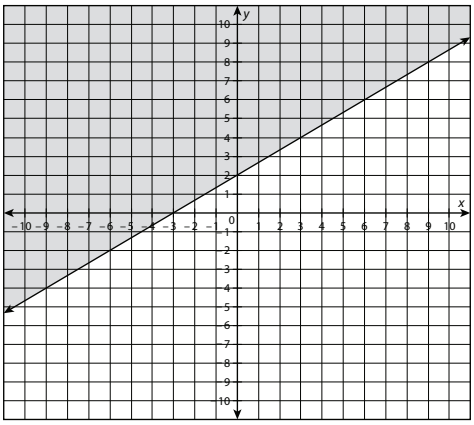
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2. Which graph represents the solution to the inequality $2x - 3y \leq -6$?

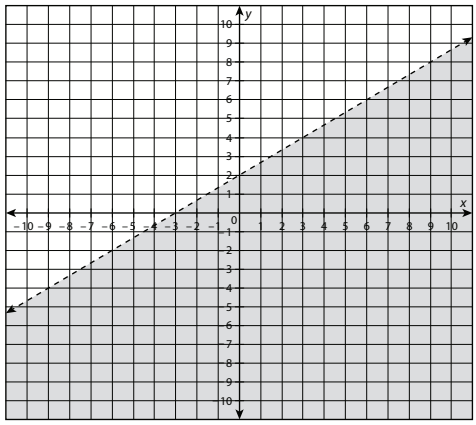
a.



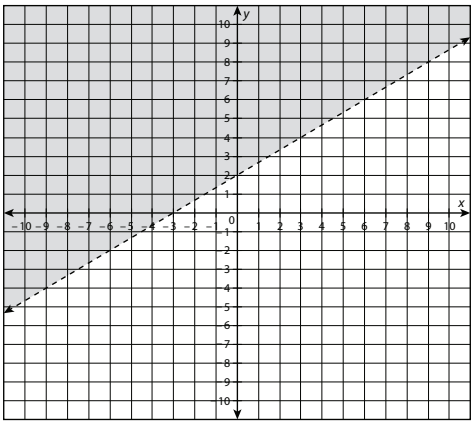
c.



b.



d.

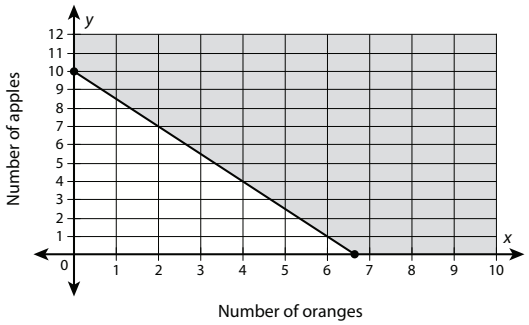


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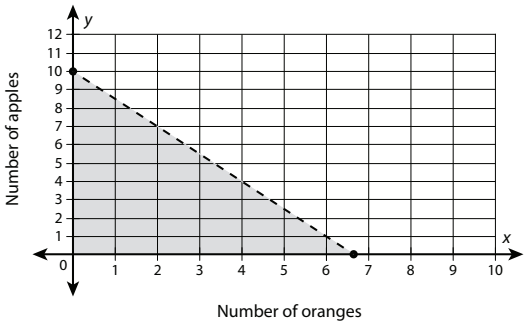
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3. Mauricio's mother gave him $5 to spend at the concession stand to buy fruit for himself and his friends. Oranges cost $0.75 each, and apples cost $0.50 each. What are the solutions to the inequality that represents the number of oranges and apples Mauricio can buy if x represents the number of oranges and y represents the number of apples?

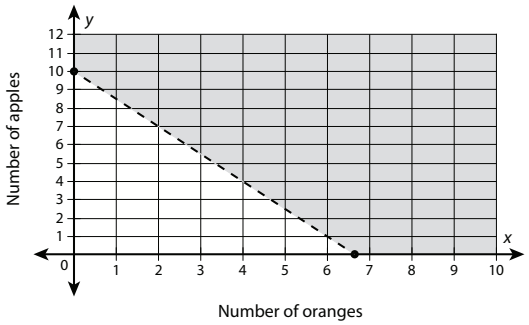
a.



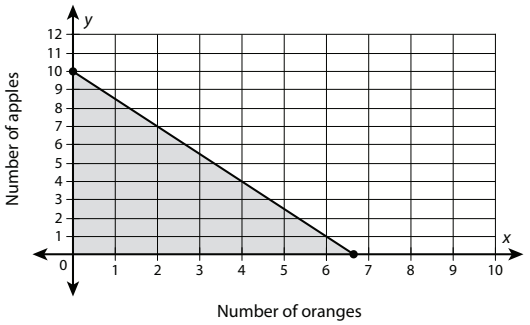
c.



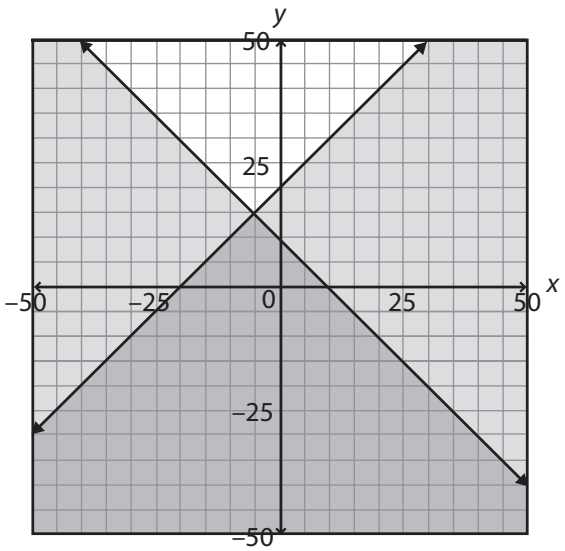
b.



d.



4. Which system of inequalities corresponds to this graph?



a. $\begin{cases} -x - y \leq 10 \\ x + y \geq 20 \end{cases}$

b. $\begin{cases} x + y \leq 10 \\ -x + y \geq 20 \end{cases}$

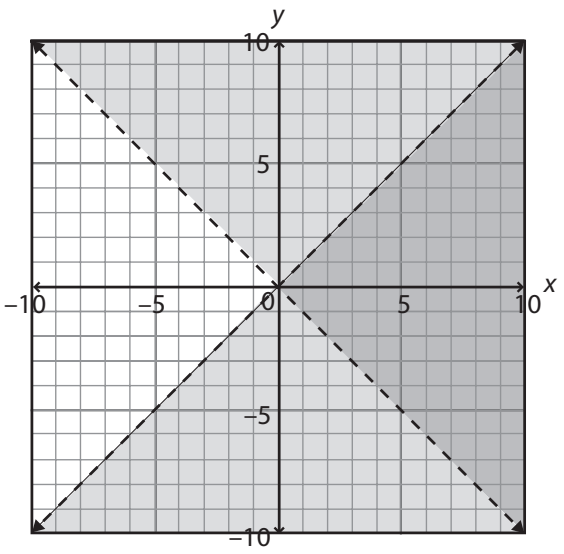
c. $\begin{cases} x + y > 10 \\ -x + y \geq 20 \end{cases}$

d. $\begin{cases} x + y \leq 10 \\ -x + y \leq 20 \end{cases}$

continued

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5. Which system of inequalities corresponds to this graph?



a. $\begin{cases} y > x \\ y < -x \end{cases}$

b. $\begin{cases} y \geq -x \\ y \leq x \end{cases}$

c. $\begin{cases} y > -2x \\ y < 3x \end{cases}$

d. $\begin{cases} y > -x \\ y < x \end{cases}$

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Lesson 2: Solving Linear Inequalities in Two Variables and Systems of Inequalities

Instruction

Common Core State Standard

A–REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

SMP	
1 ✓	2 ✓
3 ✓	4 ✓
5 ✓	6
7	8

Essential Questions

1. How can you represent the solution to a linear inequality in two variables?
2. How can you graph the boundary of a linear inequality in two variables?
3. How are linear equations and inequalities in two variables similar? How are they different?
4. How can you represent the solution to a system of inequalities?
5. How are systems of linear equations and inequalities similar? How are they different?

WORDS TO KNOW

boundary line	the graph of the line that represents a linear inequality and that divides the coordinate plane into two half planes, one of which contains all of the solutions of the inequality
constraint	a restriction or limitation on any of the variables in an equation or inequality
half plane	a planar region containing all points that lie on one side of a boundary line; one-half of a plane
inclusive	when the points in a plane that lie along the boundary line of an inequality are included in the solution
intercept	the value of the x- or y-coordinate where a line or curve intersects the x- or y-axis, respectively
non-inclusive	when the points in a plane that lie along the boundary line of an inequality are not included in the solution

SRB p. U2-40

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solution to a system of linear inequalities	the set of points that lie in the intersection of the half planes of the inequalities and which may also lie on the boundary lines; the solution set is the set of all points that satisfy the inequalities in the system
system of inequalities	a set of two or more inequalities with the same unknowns
***x*-intercept**	the x-coordinate of the point where a line or a curve intersects the x-axis
***y*-intercept**	the y-coordinate of the point where a line or a curve intersects the y-axis

Recommended Resources

- Interactivate. "Linear Inequalities."

  http://walch.com/rr/CAU2L3SystemsLesson

  This site provides an activity for students to experiment with systems of linear inequalities. Students enter up to four inequalities and analyze the resulting graph.

- Math Planet. "Systems of Linear Inequalities."

  http://walch.com/rr/CAU2L3GraphingSystems

  This site gives an overview of graphing systems of linear inequalities. It includes a short video clip with an example of how to graph one of these systems.

- Purplemath.com. "Graphing Linear Inequalities: $y > mx + b$."

  http://walch.com/rr/CAU2L3GraphingInequalities

  This site includes brief instruction and examples of graphing linear inequalities in forms such as $y > mx + b$.

- Quia.com. "Rags to Riches."

  http://walch.com/rr/CAU2L3InequalityGame

  Players match the given inequality with the graph in this quiz show-style game. The game provides extra practice and immediate feedback.

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Lesson 2.2.1: Solving Linear Inequalities in Two Variables

Warm-Up 2.2.1

Read the scenario and use the information to complete the problems that follow.

> Sanibel wants to sell wallets that she makes out of duct tape at the farmer's market. She bought $20 worth of tape to get started. To pay for both the tape and the time she spends making the wallets, she plans to charge $2.50 for each wallet she sells.

1. Let x represent the number of wallets that Sanibel sells, and let y represent her profit. Write an equation in slope-intercept form to show Sanibel's profit for the wallets she sells.

2. Graph the equation and explain what the graph shows.

3. What happens if Sanibel decides to charge $4 per wallet? Write a new equation in slope-intercept form to represent her profit, and graph the equation. Explain what the graph shows.

4. The graph of each of these equations is a line. However, Sanibel is selling entire wallets, not parts of wallets. Explain how you could modify the graphs to better represent the situations.

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Instruction

Lesson 2.2.1: Solving Linear Inequalities in Two Variables

Common Core State Standard

A–REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Warm-Up 2.2.1 Debrief

1. Let x represent the number of wallets that Sanibel sells, and let y represent her profit. Write an equation in slope-intercept form to show Sanibel's profit for the wallets she sells.

   Use the information from the scenario to write an equation in slope-intercept form ($y = mx + b$).

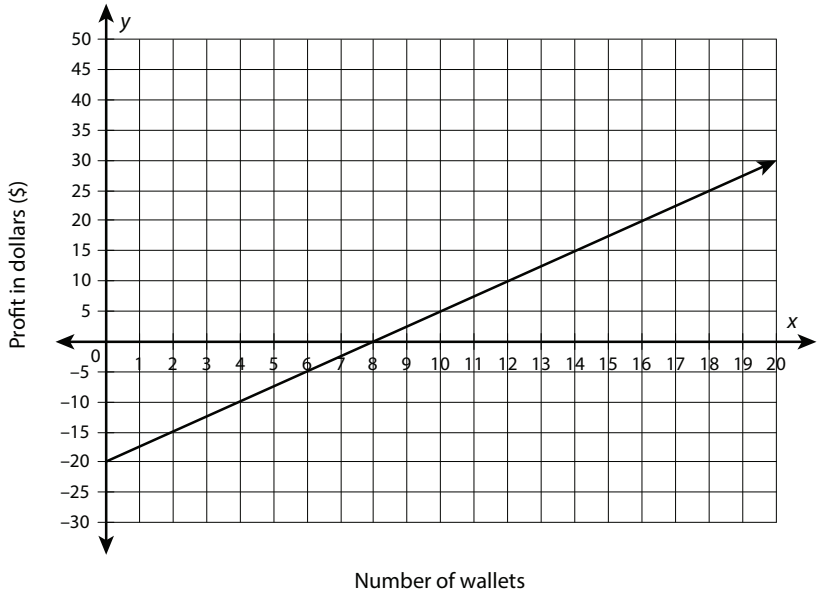
   Sanibel plans to charge \$2.50 per wallet. So, the amount of money she earns from the sale of x wallets at \$2.50 each is $2.50x$.

   Sanibel spent \$20 on supplies. She hasn't sold any wallets at the outset, so show this amount as a negative. Her initial cost in dollars for tape can be represented by –20.

   The profit, y, is based on the money Sanibel earns from selling wallets minus the initial cost of buying the duct tape.

   The equation for Sanibel's profit is $y = 2.50x - 20$.

2. Graph the equation and explain what the graph shows.



   The graph shows that there is no profit until Sanibel sells 9 wallets. She breaks even when she sells 8 wallets. Her profit is negative when she sells less than 8 wallets.

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3. What happens if Sanibel decides to charge $4 per wallet? Write a new equation in slope-intercept form to represent her profit, and graph the equation. Explain what the graph shows.

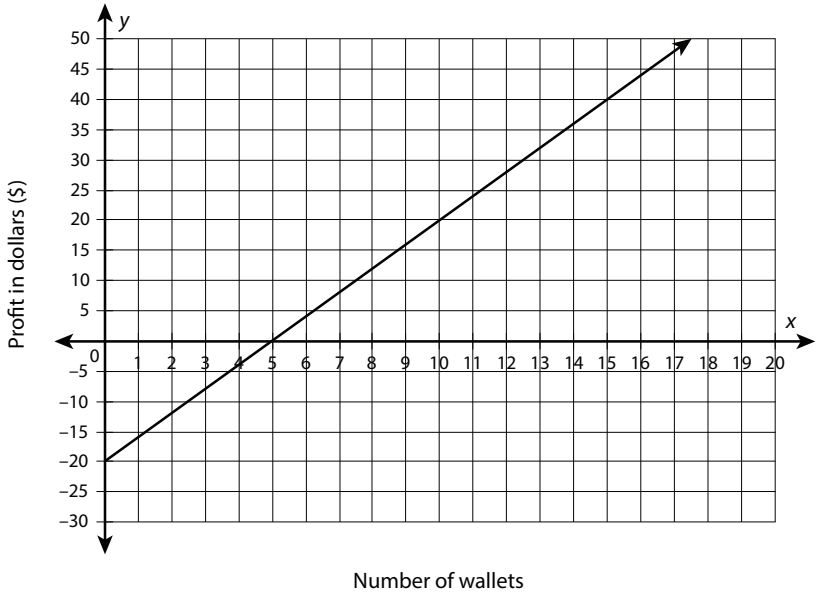
   Modify the equation you wrote for problem 1 to reflect the new price per wallet.

   Original equation: $y = 2.50x - 20$

   Substitute the new price, $4, for the old price, $2.50.

   New equation: $y = 4x - 20$

   Now graph the new equation and explain what it shows.



   The graph shows that there is no profit until Sanibel sells 6 wallets. She breaks even when she sells 5 wallets. Her profit is negative when she sells less than 5 wallets.

4. The graph of each of these equations is a line. However, Sanibel is selling entire wallets, not parts of wallets. Explain how you could modify the graphs to better represent the situations.

   In both cases, a more realistic graph would include only points on the line where the number of wallets is an integer, and not the entire line. Each point represents the number of wallets and the corresponding profit. For example, in the graph of $y = 2.50x - 20$ created for problem 1, the point (12, 10) represents the sale of 12 wallets and a profit of $10.

Connection to the Lesson

- Students will graph the line that represents the boundary for the graph of a linear inequality.
- Students will create inequalities similar to the equations created in the warm-up.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

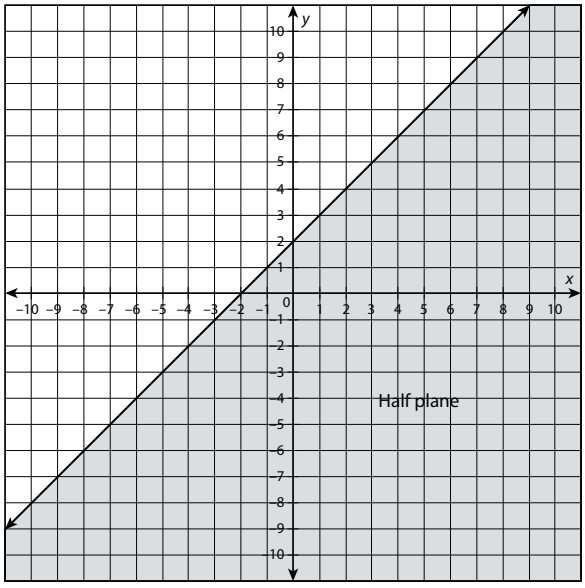
- graphing linear equations in two variables (A–CED.2★)
- verifying whether inequalities are true or false for given values of the variable(s) (6.EE.5)

Introduction

Solving a linear inequality in two variables is similar to using a graph to find the solution to a linear equation, with a few extra steps that will be explained in this lesson. Remember that, like equations, inequalities have an infinite number of solutions, and all the solutions need to be represented. For the graphs of inequalities, this is done through the use of shading.

Key Concepts

- The graph of a linear inequality is a straight line. This line is the **boundary line** of all the solutions of the inequality.
- The graph of the solution set of a linear inequality in two variables is the set of all points in a half plane, and may or may not include the points along its boundary line.
- A **half plane** is a planar region containing all points that lie on one side of a boundary line.



- To determine the solution set of an inequality, first graph the boundary line.

SRB p. U2-42

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Instruction

- When the points on the boundary line are included in the solution, the graph is **inclusive**. Inequalities that have "greater than or equal to" (≥) or "less than or equal to" (≤) symbols are inclusive.
- Use a solid boundary line when graphing the solution set of an inclusive inequality.
- Other times the points on the line or boundary are NOT part of the solution; in other words, the inequality is **non-inclusive**. Inequalities that have "greater than" (>) or "less than" (<) symbols are non-inclusive.
- Use a dashed boundary line when graphing the solution set of a non-inclusive inequality.
- The solution set of the inequality will be all of the points in a half plane, on one side of the boundary line.
- To find out which side of a boundary line contains the solution, choose a point that is clearly on one side of the line or the other and substitute the coordinates of this test point into the inequality. A good test point to choose is (0, 0), unless the boundary line contains the origin.
- If the coordinates of the test point satisfy the inequality, shade the half plane that contains the test point. If the coordinates of the test point do not make the inequality true, shade the opposite half plane. Shading indicates that the coordinates of all points in that region are in the solution set of the inequality.

Graphing Equations Using a TI-83/84:

Step 1: Press [Y=] and arrow over to the left two times so that the cursor is blinking on the "\".

Step 2: Press [ENTER] two times for the greater than icon "◥" and three times for the less than icon "◣".

Step 3: Arrow over to the right two times so that the cursor is blinking after the equal sign.

Step 4: Key in the equation using [X, T, θ, *n*] for *x*.

Step 5: Press [WINDOW] to change the viewing window, if necessary.

Step 6: Enter in appropriate values for Xmin, Xmax, Xscl, Ymin, Ymax, and Yscl, using the arrow keys to navigate.

Step 7: Press [GRAPH].

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Graphing Equations Using a TI-Nspire:

Step 1: Press the home key.

Step 2: Arrow over to the graphing icon (the picture of the parabola or the U-shaped curve) and press [enter].

Step 3: At the blinking cursor at the bottom of the screen, press the backspace key once (a left-facing arrow). A menu pops up that gives choices for less than or equal to (≤), less than (<), greater than (>), and greater than or equal to (≥). Choose the appropriate symbol by using the arrow keys to navigate to the desired symbol and press the center button of the navigation pad. Alternatively, enter the number that is associated with the symbol.

Step 4: Enter the equation and press [enter].

Step 5: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom, and click the center button of the navigation pad.

Step 6: Choose 1: Window Settings by pressing the center button.

Step 7: Enter in the appropriate XMin, XMax, YMin, and YMax fields.

Step 8: Leave the XScale and YScale set to auto.

Step 9: Use [tab] to navigate among the fields.

Step 10: Press [tab] to navigate to "OK" when done and press [enter].

Graphing a Linear Inequality in Two Variables

1. Determine the symbolic representation (write the inequality using symbols) of the scenario if given a context.
2. Graph the linear equation that represents the boundary line.
3. If the inequality is inclusive (≤ or ≥), draw a solid line.
4. If the inequality is non-inclusive (< or >), draw a dashed line.
5. Pick a test point on one side of the line and substitute the coordinates of the point into the inequality.
6. If the coordinates of the test point satisfy the inequality, shade the half plane that contains the test point.
7. If the coordinates of the test point do not satisfy the inequality, shade the half plane that does NOT contain the test point.

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Instruction

Standard Form of Linear Equations and Inequalities

- Linear equations in standard form are written as $ax + by = c$, where a, b, and c are integers.
- Similarly, an inequality can be written in the same form but with an inequality symbol ($<$, $>$, $\leq$, or $\geq$) instead of an equal sign.
- To convert to slope-intercept form ($y = mx + b$), solve the equation or inequality for y.
- Remember to reverse the inequality symbol if you multiply or divide both sides by a negative number.

Intercepts

- An **intercept** is the x- or y-coordinate of the point where a line or a curve intersects (or intercepts) the x- or y-axis, respectively.
- You have dealt with the ***y*-intercept**, which is the y-coordinate of the point where a line intersects the y-axis. When an equation is in slope-intercept form, $y = mx + b$, b is the y-intercept.
- To find the y-intercept of an equation, set x equal to 0 and solve for y.
- The coordinates of the y-intercept are $(0, y)$.
- The ***x*-intercept** is the x-coordinate of the point where a line intersects the x-axis.
- To find the x-intercept in an equation, set y equal to 0 and solve for x.
- The coordinates of the x-intercept are $(x, 0)$.
- You can plot a line that is not horizontal or vertical by using its intercepts. Plot the two points, $(x, 0)$ and $(0, y)$, and then connect them.
- Plotting a line using the intercepts is helpful in graphing linear inequalities that are derived from contexts.
- A **constraint** is a restriction or limitation on any of the variables in an equation or inequality. Generally, linear inequalities in context have one or more constraints such that the variables may not be negative. This means the endpoints of a line segment are its intercepts.

Common Errors/Misconceptions

- using a solid line for a non-inclusive inequality or a dashed line for an inclusive inequality
- shading the region that makes the inequality false
- forgetting to reverse the inequality symbol when multiplying or dividing both sides by a negative number
- forgetting to shade the half plane that contains the solution set of the inequality

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Guided Practice 2.2.1

Example 1

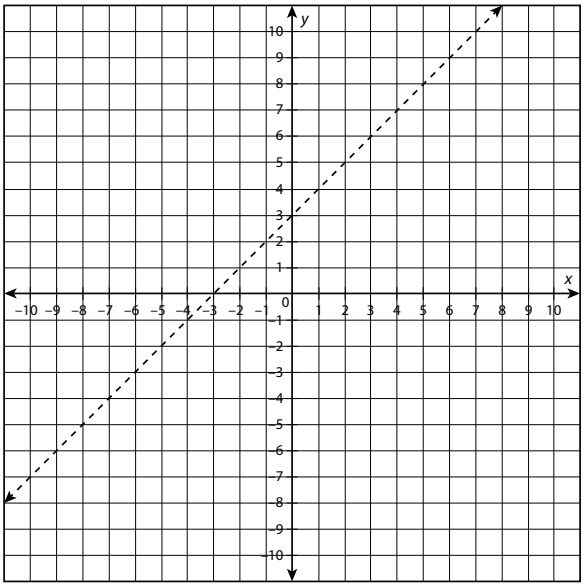
Graph the solution to the following inequality.

$y > x + 3$

1. Graph the linear equation that represents the boundary line.

   The inequality uses >, so it is non-inclusive. Because the inequality is non-inclusive, represent the boundary line, $y = x + 3$, with a dashed line.

   To graph the line, first plot the *y*-intercept, (0, 3), and then plot the *x*-intercept, (–3, 0). Or, use the slope to find a second point. The slope is 1. To use the slope, start at the *y*-intercept, count up by 1 unit and then to the right by 1 unit, then plot the second point. Connect the two points and extend the line to the edges of the coordinate plane.



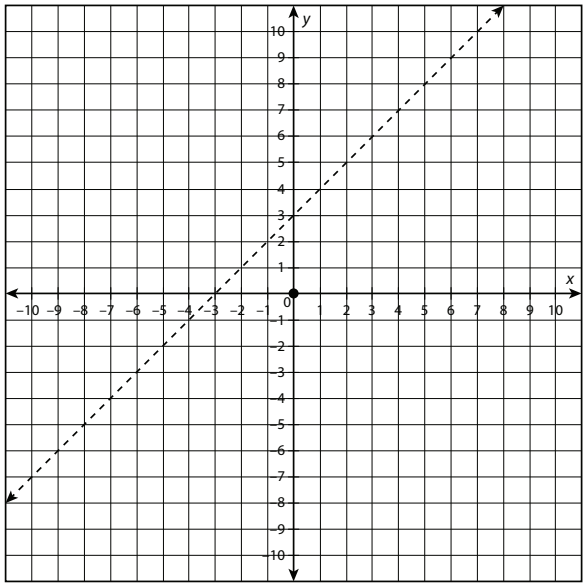
SWB p. 55; SRB p. U2-46

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2. Pick a test point on one side of the line and substitute the coordinates of the point into the inequality.

   Choose (0, 0) because the boundary line does not contain the origin, and this point is easy to substitute into the inequality.

$y > x + 3$	Given inequality
$(0) > (0) + 3$	Substitute 0 for x and 0 for y.
$0 > 3$	Simplify. This statement is false.

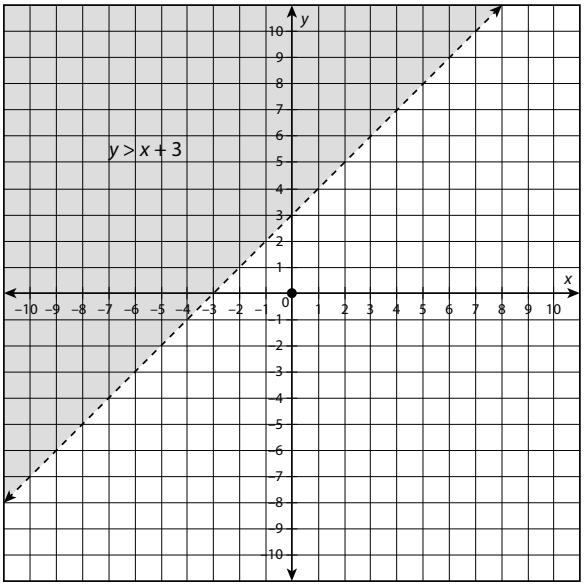


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Instruction

3. Shade the appropriate half plane.

   The coordinates of the test point make the inequality false, so none of the points on that side of the line are in the solution set of the inequality. Shade the other side of the line instead. In other words, shade the half plane that does NOT contain the test point.



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Example 2

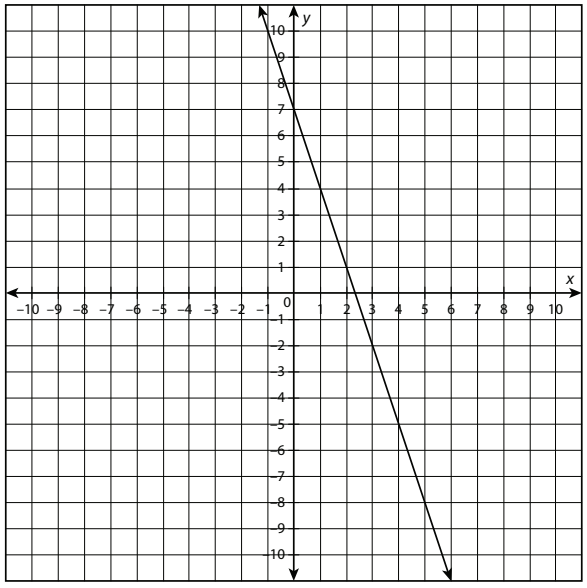
Graph the solution to the following inequality.

$y \leq -3x + 7$

1. Graph the linear equation that represents the boundary line.

   The inequality uses ≤, so it is inclusive. Because the inequality is inclusive, represent the boundary line, $y = -3x + 7$, with a solid line.

   To graph the line, first plot the *y*-intercept, (0, 7). To plot a second point, either find the *x*-intercept or use the slope. Let's use the slope, which is –3. To use the slope, start at the *y*-intercept, count down by 3 units and to the right by 1 unit, then plot the second point. Connect the two points and extend the line to the edges of the coordinate plane.

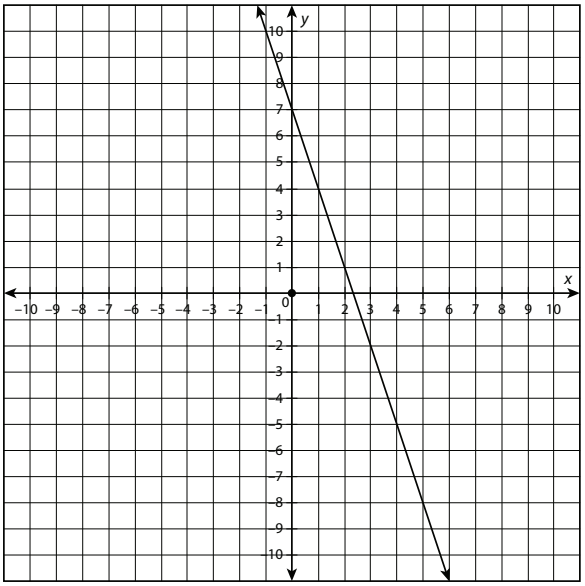


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2. Pick a test point on one side of the line and substitute the coordinates of the point into the inequality.

   Choose (0, 0) because the boundary line does not contain the origin, and this point is easy to substitute into the inequality.

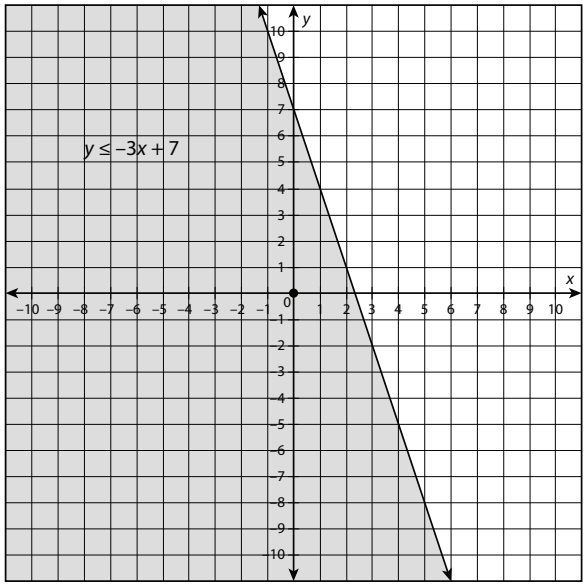
$y \leq -3x + 7$	Given inequality
$(0) \leq -3(0) + 7$	Substitute 0 for x and 0 for y.
$0 \leq 7$	Simplify. This statement is true.



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3. Shade the appropriate half plane.

   Because the coordinates of the test point make the inequality true, all points on that side of the line make the inequality true. Shade the half plane that contains the test point.



Example 3

A company that manufactures MP3 players needs to hire more workers to keep up with an increase in orders. Some workers will be assembling the players, and others will be packaging them. The company can hire no more than 15 new employees. Write and graph an inequality that represents the number of new workers who can be hired.

1. Write an inequality using symbols from the context.

   There are two jobs to perform: assembling players, and packaging them.

   Let x represent the number of workers who will assemble the MP3 players.

   Let y represent the number of workers who will package the MP3 players.

   The number of new workers must be less than or equal to 15.

   Write an inequality that expresses the relationship in this situation.

   $x + y \leq 15$

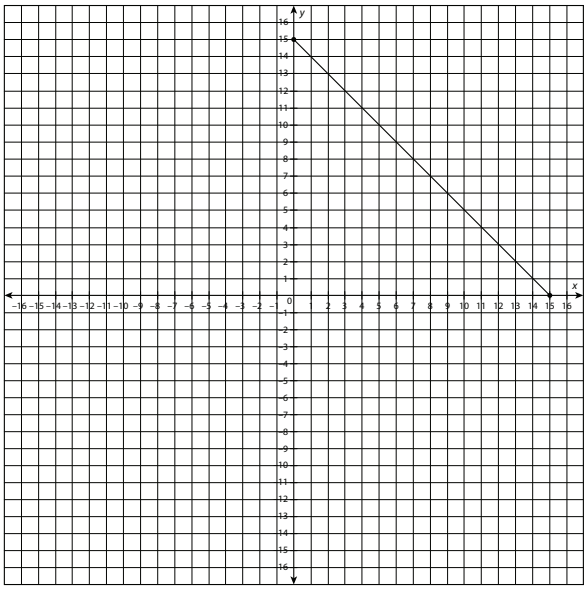
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2. Graph the linear equation that represents the boundary line.

   The inequality uses ≤, so it is inclusive. Because the inequality is inclusive, represent the boundary line, $x + y = 15$, with a solid line.

   To graph the line, find the x- and y-intercepts of the line.

   First, plot the coordinates of the y-intercept, (0, 15), and then plot the coordinates of the x-intercept, (15, 0). Connect the two points. Draw the connecting segment only between these two intercepts because there cannot be a negative number of employees.

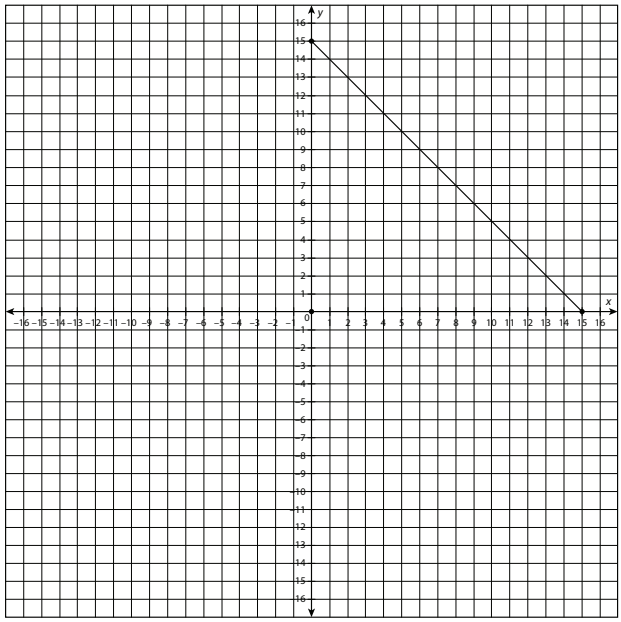


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3. Pick a test point on one side of the line and substitute the coordinates of the point into the inequality.

   Choose (0, 0) because this point does not lay on the boundary line and the coordinates are easy to substitute into the inequality.

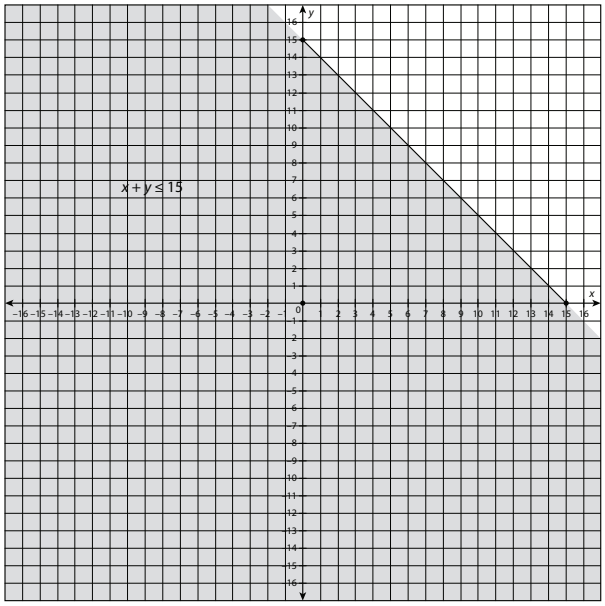
$x + y \leq 15$	Given inequality
$(0) + (0) \leq 15$	Substitute 0 for x and 0 for y.
$0 \leq 15$	Simplify. This statement is true.



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4. Shade the appropriate half plane.

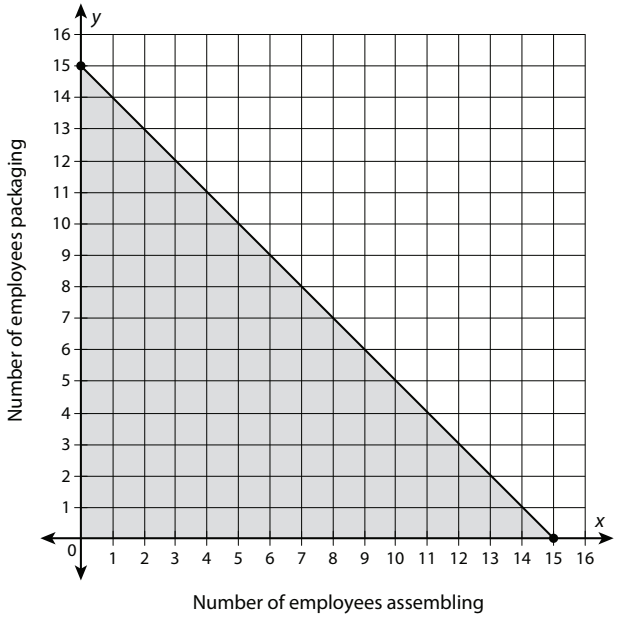
   Because the test point makes the inequality true, all points on that side of the line make the inequality true. Shade the half plane that contains the test point.



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5. Restrict the shading to fit the context of the problem and add labels.

   Having a negative number of employees doesn't make sense, so restrict the graph to the region of the half plane that's bounded by the two axes and contains the boundary line. Label the graph; recall from step 1 that x represents the number of employees assembling MP3 players, and y represents the number of employees packaging MP3 players.



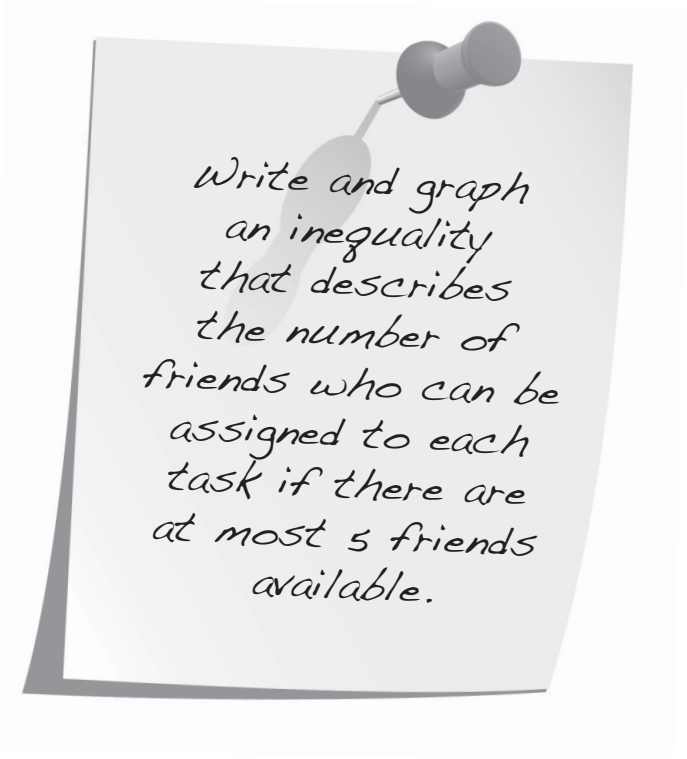
   Keep in mind that the number of employees must be an integer, so the ideal graph includes only integer pairs within the shaded region and on the boundaries of the shaded region.

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Problem-Based Task 2.2.1: Cupcakes

SMP	
1 ✓	2 ✓
3 ✓	4
5	6
7	8

The class president has asked you to make your delicious cupcakes for the next student council fund-raiser. With the fund-raiser fast approaching, you have asked your friends to help you out. Some friends will frost the cupcakes, and others will decorate the cupcakes. At most, 5 friends have agreed to help. Write and graph an inequality that describes the number of friends who can be assigned to each task if there are at most 5 friends available.



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Problem-Based Task 2.2.1: Cupcakes

Coaching

a. What tasks are available for friends to do?

b. How many friends at most can help you?

c. If x represents the number of friends frosting the cupcakes and y represents the number of friends decorating the cupcakes, what is the inequality written in standard form that represents the number of friends performing each task?

d. What is the inequality written in slope-intercept form?

e. How can you represent the boundary line graphically?

f. Where should the boundary stop? Explain.

g. What are the coordinates of a test point that you can use? Explain.

h. What part of the graph should you shade? Explain.

i. Where should the shading stop? Explain.

j. Why is the actual solution to the inequality that represents this situation a set of discrete points, instead of a shaded region? How can you redraw the graph so that it more accurately represents this situation?

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Problem-Based Task 2.2.1: Cupcakes

Coaching Sample Responses

a. What tasks are available for friends to do?

The two tasks are frosting the cupcakes and decorating them.

b. How many friends at most can help you?

5

c. If x represents the number of friends frosting the cupcakes and y represents the number of friends decorating the cupcakes, what is the inequality written in standard form that represents the number of friends performing each task?

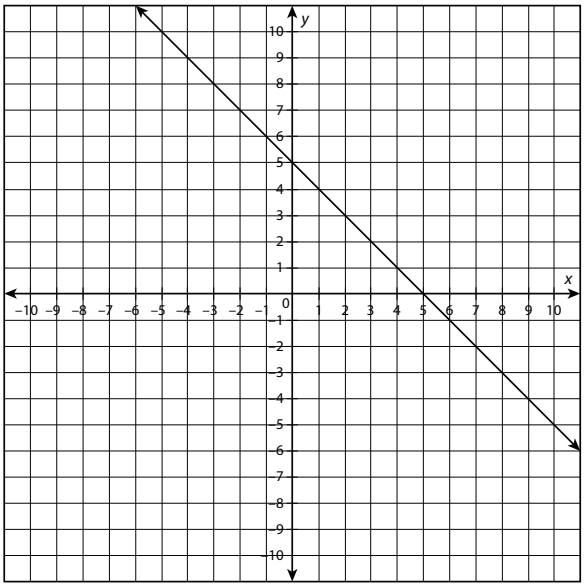
$x + y \leq 5$

d. What is the inequality written in slope-intercept form?

$y \leq -x + 5$

e. How can you represent the boundary line graphically?

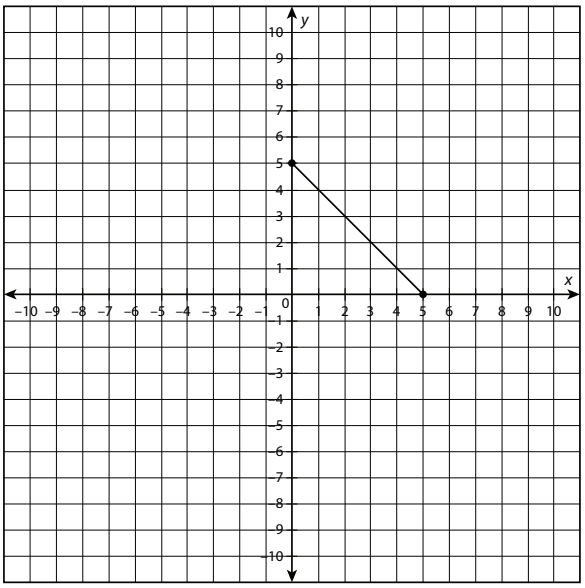
The boundary line for this situation should be solid because it's possible for the number of friends who participate to equal 5. Therefore, this inequality is inclusive. Draw the boundary line, $y = -x + 5$, as a solid line.



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f. Where should the boundary stop? Explain.

Because the number of friends will never exceed 5 nor be a negative number, the boundary line should stop on the axes.



g. What are the coordinates of a test point that you can use? Explain.

Use the coordinates of any point on one side of the line or the other. Let's use (0, 0) as a test point.

h. What part of the graph should you shade? Explain.

Substituting the coordinates of the test point, (0, 0), into the inequality gives the following result.

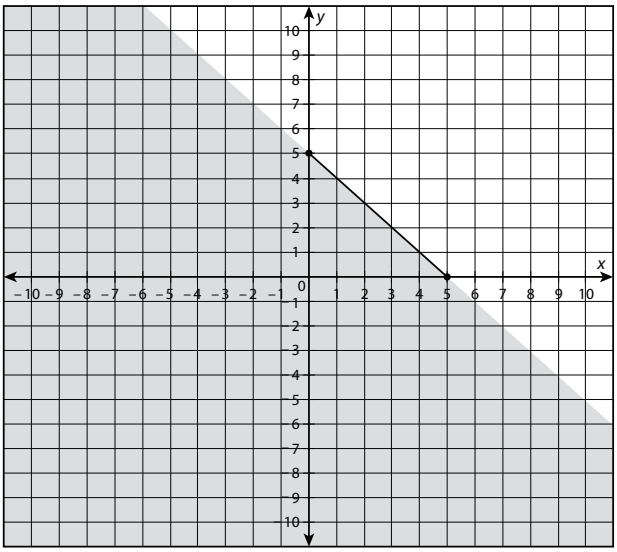
$y \leq -x + 5$

$(0) \leq (0) + 5$

$0 \leq 5$ This is a true statement.

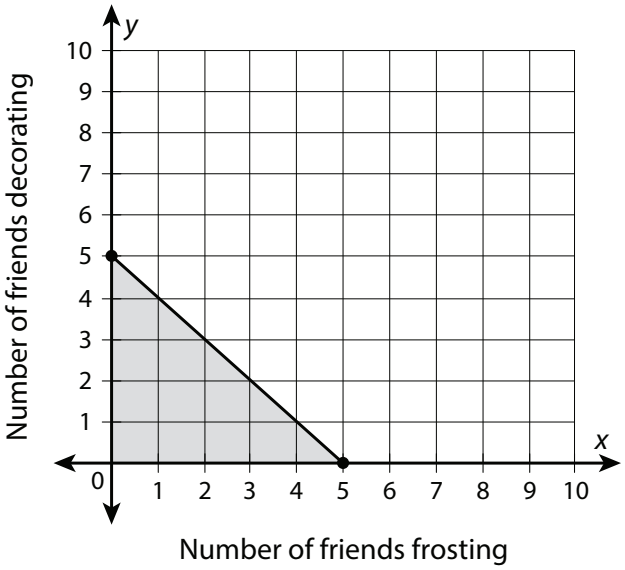
Because the coordinates of the test point make the inequality true, all points on that side of the boundary (as well as points on the boundary) make the inequality true. Shade the region that contains the point.

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i. Where should the shading stop? Explain.

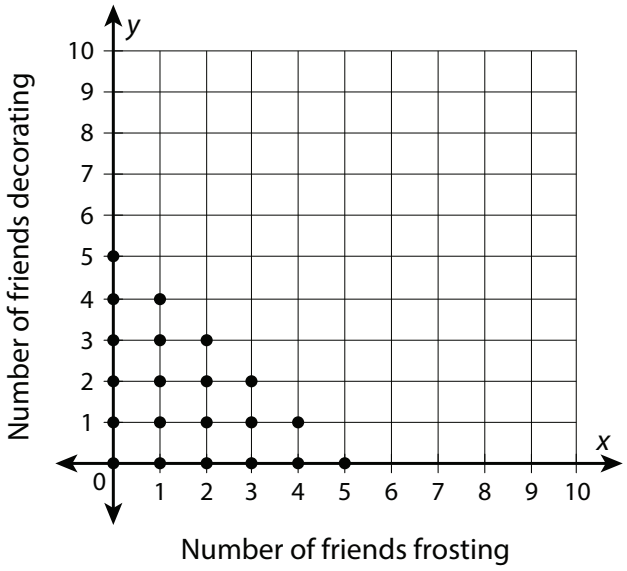
Because you cannot have a negative number of friends, x and y will never be negative. Stop the shading at the x- and y-axes.



j. Why is the actual solution to the inequality that represents this situation a set of separate points, instead of a shaded region? How can you redraw the graph so that it more accurately represents this situation?

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The situation involves the number of friends who can help you. Therefore, the domain and range of this relation consist of the whole numbers {0, 1, 2, 3, 4, 5}. That is, the coordinates of possible solutions must be whole numbers. A more accurate graph of this situation includes only points whose coordinates are whole numbers and whose sum is less than or equal to 5.



Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.2.1: Solving Linear Inequalities in Two Variables

For problems 1–7, graph the solution to each inequality.

1. $y > 2x$

2. $y > -3x + 1$

3. $y < -x + 3$

4. $y \leq \frac{1}{4}x - 4$

5. $3x + y \geq 5$

6. $2x - y < 1$

7. $x > 4$

Use the given information to complete problems 8–10.

8. Gisele runs a company that makes tablet computers. Each tablet requires an employee to assemble it and an employee to test it. There are 25 employees or fewer available, depending on who is out sick or on vacation. Write an inequality that represents the number of employees Gisele has available to do the work and then graph the solution set.

9. At Binh's Greenhouse in early spring, there are many greenhouse plants to repot and many outside plants to water. If it takes Binh 5 minutes to repot each plant and 2 minutes to water each plant, and he has at most 2 hours before the greenhouse opens, what inequality represents the time Binh has to repot and water the plants before the greenhouse opens? What is the graph of the solution set?

10. Toya is training for a triathlon and wants to bike and run on the same day. She has less than 3 hours to spend on her workouts. What inequality represents the time Toya has to bike and run? What is the graph of the solution set?

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Lesson 2.2.2: Solving Systems of Linear Inequalities

Warm-Up 2.2.2

Read the scenario. Use the information to complete the problems that follow.

> Taylor's mother is fed up with all the dirty socks and T-shirts scattered on his bedroom floor. She made a bet with Taylor that if she finds 20 or more socks and T-shirts on the floor, Taylor would be grounded. If she finds fewer than that, Taylor doesn't have to do dishes for a week.

1. What inequality can you write to represent the number of socks and T-shirts found on the floor that will result in Taylor being grounded?

2. Graph this inequality.

3. Taylor's pretty sure he won't have to do dishes for a week. What is the inequality that represents how many socks and T-shirts Taylor can have on the floor and still win the bet?

4. Graph this inequality on a separate set of axes.

5. What happens if there are exactly 20 socks and T-shirts on the floor? Who wins the bet?

6. Explain why a more accurate graph for either of these situations would be a set of discrete points rather than a shaded region.

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Lesson 2.2.2: Solving Systems of Linear Inequalities

Common Core State Standard

A–REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Warm-Up 2.2.2 Debrief

1. What inequality can you write to represent the number of socks and T-shirts found on the floor that will result in Taylor being grounded?

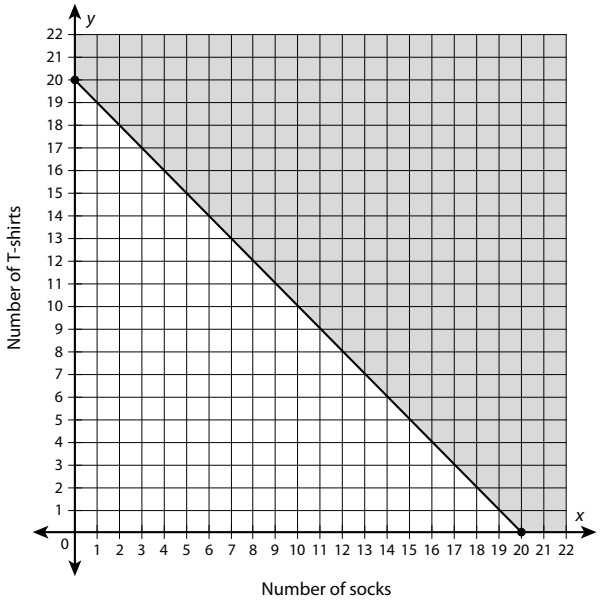
   Taylor's mother thinks she'll find 20 or more socks and T-shirts.

   The total number of items needs to be 20 or more. Use the inequality symbol for "greater than or equal to" (≥) for this situation.

   Let x represent the number of socks and let y represent the number of T-shirts.

   $x + y \geq 20$

2. Graph this inequality.



3. Taylor's pretty sure he won't have to do dishes for a week. What is the inequality that represents how many socks and T-shirts Taylor can have on the floor and still win the bet?

   Taylor must have fewer than 20 socks and T-shirts on his floor to win the bet.

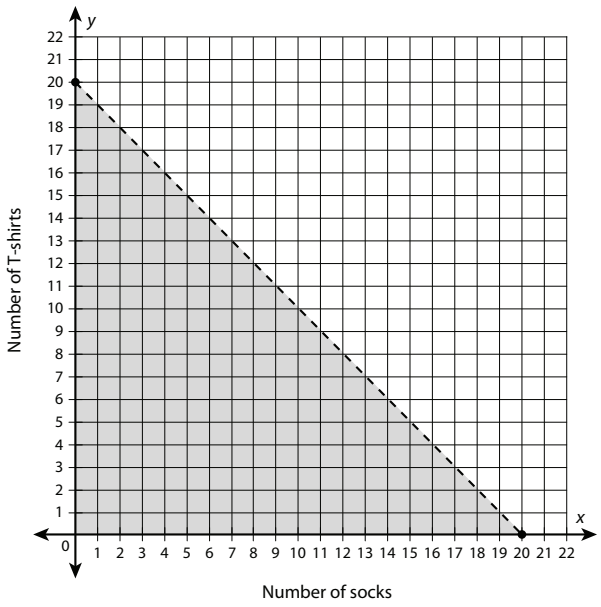
   Use the same variables, x and y.

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The sum of x and y must be less than 20 because Taylor's mother said she could find "20 or more." Use the inequality symbol for "less than" (<) for this situation.

$x + y < 20$

4. Graph this inequality on a separate set of axes.



5. What happens if there are exactly 20 socks and T-shirts on the floor? Who wins the bet?

   Taylor's mother set up the situation so that if she finds "20 or more" socks and T-shirts she wins; if there are exactly 20 of them, Taylor's mother wins, and Taylor is grounded. This can also be seen on the graph for problem 2. The boundary for Taylor's mother winning is a solid line because it includes the cases where the total is 20. In the graph for problem 4, the boundary for Taylor winning is non-inclusive; therefore, the boundary is a dashed line because it does not include a possible total of 20 items.

6. Explain why a more accurate graph for either of these situations would be a set of discrete points rather than a shaded region.

   The domain and range for these relations is the set of socks and T-shirts. These must be whole numbers. So, more accurate graphs would be sets of discrete points, such as (3, 9), (10, 8), and (15, 1).

Connection to the Lesson

- Students will be creating inequalities in two variables and graphing their solution sets.
- Students will be graphing multiple linear inequalities on the same coordinate plane and finding the intersection of the different regions.

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Prerequisite Skills

This lesson requires the use of the following skills:

- creating equations from context (A–CED.2★)
- graphing linear inequalities in two variables (A–REI.12)

Introduction

Finding the solution to a system of linear equations requires graphing multiple linear inequalities on the same coordinate axes. Most real-world applications dealing with linear inequalities are actually systems of linear inequalities where at least one of the constraints is that one or both of the variables must be greater than or equal to 0, and frequently, the variables must represent integers.

Key Concepts

- A **system of inequalities** is a set of two or more inequalities with the same unknowns.
- The **solution to a system of linear inequalities** is the set of points that lie in the intersection of the half planes of the inequalities. The points may also lie on the boundary lines. The solution set is the set of all points that satisfy the inequalities in the system.
- Graph each inequality and shade the corresponding half plane.
- Look for the region where the half planes overlap; this is the solution.
- Remember to add constraints to the system that aren't explicitly stated in the problem context but that make sense for the problem.
- For many real-world problems, the solution set will be all points in the shaded region whose coordinates are integers, because fractions of items or people do not make sense in the context of the problem.

Common Errors/Misconceptions

- incorrectly graphing one or more of the inequalities in the system
- not seeing the solution or identifying where the intersections may occur
- forgetting to include constraints or boundaries that make sense for the context of the problem

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Guided Practice 2.2.2

Example 1

Solve the following system of inequalities graphically:

$$\begin{cases} x + y > 10 \\ 2x - 4y > 5 \end{cases}$$

1. Graph the boundary line for the first inequality.

   The equation of the first boundary line is $x + y = 10$. Use a dashed line because the inequality is non-inclusive (strictly greater than).

2. Determine the half plane that includes the solution set.

   Pick a test point. Choose a point that is on one side of the boundary line.

   Test point: (0, 0)

3. Substitute the coordinates of the test point into the inequality.

   Substitute the coordinates (0, 0) into the inequality $x + y > 10$. If the coordinates of the test point make the inequality true, shade the region that contains that point. If the coordinates of the test point make the inequality false, shade on the opposite side of the line.

$x + y > 10$	First inequality
$(0) + (0) > 10$	Substitute 0 for x and 0 for y.
$0 > 10$	Simplify. This statement is false.

4. Shade the appropriate half plane.

   Because the coordinates of the point (0, 0) make the inequality false, shade the opposite side of the line. The shaded region represents the solution for $x + y > 10$.

5. Graph the boundary line for the second inequality.

   The equation of the second boundary line is $2x - 4y = 5$. Graph this line on the same coordinate axes as the first line. Use a dashed line because the inequality is non-inclusive (strictly greater than).

SWB p. 65; SRB p. U2-58

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6. Determine the half plane that contains the solution set.

   Pick a test point. Choose a point that is on one side of the line.

   Test point: (0, 0)

7. Substitute the coordinates of the test point into the inequality.

   Substitute the coordinates (0, 0) into the inequality $2x-4y>5$. If the coordinates make the inequality true, shade the region that contains that point. If not, shade on the opposite side of the line.

$2x-4y>5$	Second inequality
$2(0)-4(0)>5$	Substitute 0 for x and 0 for y.
$0>5$	Simplify. This statement is false.

8. Shade the appropriate half plane.

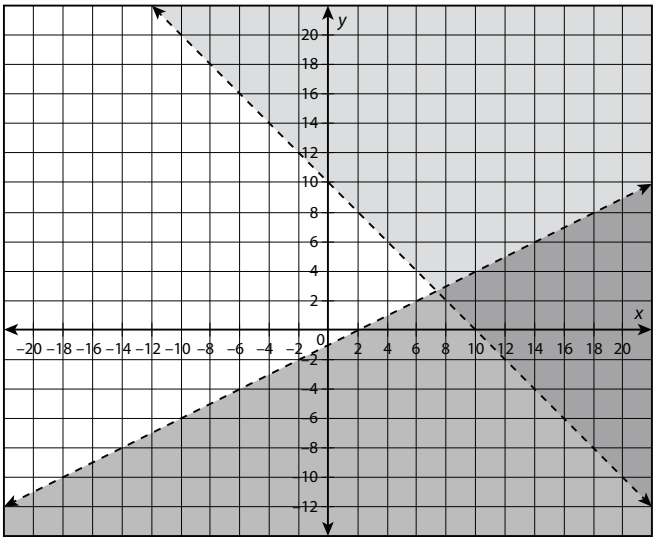
   The coordinates (0, 0) make the inequality false, so shade the opposite side of the line. The second shaded region represents the solution for $2x-4y>5$.

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9. Find the solution to the system.

The darker shaded region is where the half planes overlap and is the solution to the system $\begin{cases} x + y > 10 \\ 2x - 4y > 5 \end{cases}$.

One solution to this system is (14, 2) because the coordinates satisfy both inequalities.



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Example 2

Solve the following system of inequalities graphically:

$$\begin{cases} -4x + 2y \geq 20 \\ x - y \geq 10 \end{cases}$$

1. Graph the boundary line for the first inequality.

   The equation of the first boundary line is $-4x + 2y = 20$. Use a solid line because the inequality is inclusive (greater than *or* equal to).

2. Determine the half plane that includes the solution set.

   Pick a test point. Choose a point that is on one side of the boundary line.

   Test point: (0, 0)

3. Substitute the coordinates of the test point into the inequality.

   Substitute (0, 0) into the inequality $-4x + 2y \geq 20$. If the coordinates make the inequality true, shade the region that contains that point. If not, shade the opposite side of the line.

$-4x + 2y \geq 20$	First inequality
$-4(0) + 2(0) \geq 20$	Substitute 0 for x and 0 for y.
$0 \geq 20$	Simplify. This statement is false.

4. Shade the appropriate half plane.

   The coordinates of the point (0, 0) make the inequality false, so shade the opposite side of the line. The shaded region represents the solution for $-4x + 2y \geq 20$.

5. Graph the boundary line for the second inequality.

   The equation of the second boundary line is $x - y = 10$. Use a solid line because the inequality is inclusive (greater than *or* equal to).

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6. Determine the half plane that includes the solution set.

   Pick a test point. Choose a point that is on one side of the line.

   Test point: (0, 0)

7. Substitute the coordinates of the test point into the inequality.

   Substitute the coordinates (0, 0) into the inequality $x - y \geq 10$. If the coordinates make the inequality true, shade the region that contains that point. If not, shade the opposite side of the line.

$x - y \geq 10$	Second inequality
$(0) - (0) \geq 10$	Substitute 0 for x and 0 for y.
$0 \geq 10$	Simplify. This statement is false.

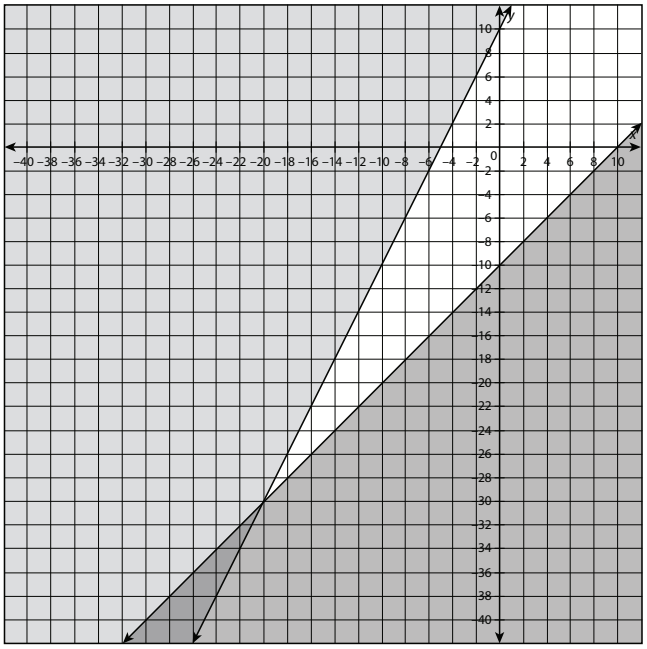
8. Shade the appropriate half plane.

   The coordinates of the point (0, 0) make the inequality false, so shade the opposite side of the line. The second shaded region represents the solution for $x - y \geq 10$.

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9. Find the solution to the system.

The darker shaded region is where the half planes overlap and is the solution to the system $\begin{cases} -4x + 2y \geq 20 \\ x - y \geq 10 \end{cases}$. One solution is (–24, –36) because the coordinates satisfy both inequalities.



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Example 3

Solve the following system of inequalities graphically:

$$\begin{cases} y \le x + 2 \\ y \ge 2x - 10 \\ x \le 0 \\ y \ge 0 \end{cases}$$

1. Graph the boundary line for the first inequality.

   The equation of the first boundary line is $y = x + 2$. Use a solid line because the inequality is inclusive (less than *or* equal to).

2. Determine the half plane that includes the solution set.

   Pick a test point. Choose a point that is on one side of the line.

   Test point: (0, 0)

3. Substitute the coordinates of the test point into the inequality.

   Substitute (0, 0) into the inequality $y \le x + 2$. If the coordinates of the test point make the inequality true, shade the region that contains that point. If not, shade on the opposite side of the line.

$y \le x + 2$	First inequality
$0 \le 0 + 2$	Substitute 0 for x and 0 for y.
$0 \le 2$	Simplify. This statement is true.

4. Shade the appropriate half plane.

   The coordinates (0, 0) make the inequality true, so shade the region that contains the point. The shaded region represents $y \le x + 2$.

5. Graph the boundary line for the second inequality.

   The equation of the second boundary line is $y = 2x - 10$. Use a solid line because the inequality is inclusive (greater than *or* equal to).

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6. Determine the half plane that includes the solution set.

   Pick a test point. Choose a point that is on one side of the line.

   Test point: (0, 0)

7. Substitute the coordinates of the test point into the inequality.

   Substitute (0, 0) into the inequality $y \geq 2x - 10$. If the coordinates of the test point make the inequality true, shade the region that contains that point. If not, shade on the opposite side of the line.

$y \geq 2x - 10$	Second inequality
$(0) \geq 2(0) - 10$	Substitute 0 for x and 0 for y.
$0 \geq -10$	Simplify. This statement is true.

8. Shade the appropriate half plane.

   The coordinates (0, 0) make the inequality true, so shade the region that contains the point.

9. Find the intersection of the two shaded regions.

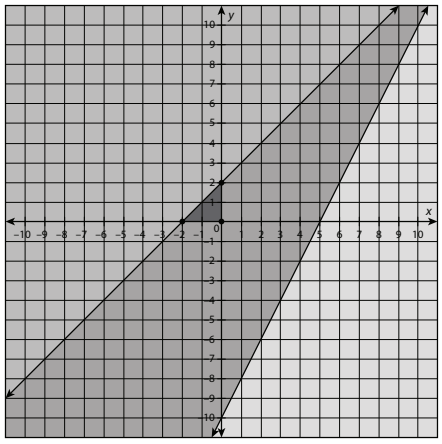
   The darker shaded region represents where the solutions to $y \leq x + 2$ and $y \geq 2x - 10$ overlap.

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10. Consider any remaining inequalities in the system.

The final two inequalities, $x \leq 0$ and $y \geq 0$, represent additional constraints of the system. These constraints decrease the size of the overlapping region that satisfies the inequalities in the system: x must be less than or equal to 0, and y must be greater than or equal to 0.



Therefore, the solutions to the system are within the dark triangular region bounded by points (–2, 0), (0, 0), and (0, 2). One solution to this system is (–0.5, 0.5), because these coordinates satisfy all four inequalities.

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Example 4

An artist wants to analyze the time that he spends creating his art. He makes oil paintings and watercolor paintings. The artist takes 8 hours to paint an oil painting. He takes 6 hours to paint a watercolor painting. He has set aside a maximum of 24 hours per week to paint his paintings. The artist then takes 2 hours to frame and put the final touches on his oil paintings. He takes 3 hours to frame and put the final touches on his watercolor paintings. He has set aside a maximum of 12 hours per week for framing and final touch-ups. Write a system of inequalities that represents the time per week that the artist has to complete his paintings. Graph the solution.

1. Write a system of inequalities to represent the situation and constraints.

   Let x represent the number of oil paintings.

   Let y represent the number of watercolor paintings.

   The total number of hours spent painting must be less than or equal to 24.

   The total number of hours spent framing and touching up the paintings must be less than or equal to 12.

   It might be helpful to create a table:

	Oil (x)	Watercolor (y)	Inequality	Total
Paint	8	6	≤	24
Frame	2	3	≤	12

   The system of inequalities for painting and framing is $\begin{cases} 8x+6y \le 24 \\ 2x+3y \le 12 \end{cases}$.

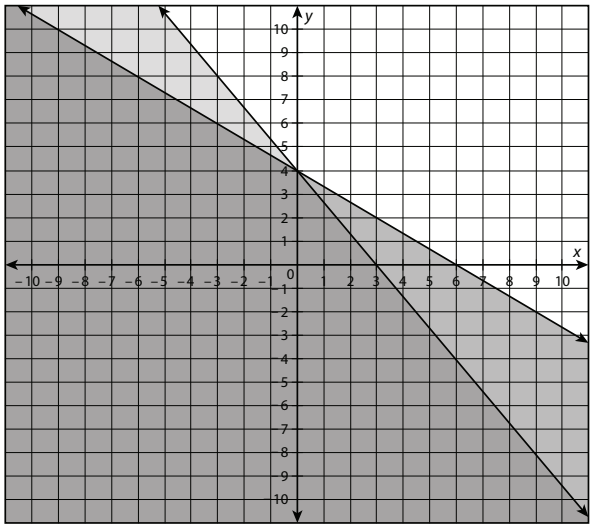
   Now, think about what must always be true about the paintings: the number of paintings will never be negative, but there could be zero oil paintings or watercolor paintings per week. Add these two constraints to the system: $x \ge 0$ and $y \ge 0$:

$$\begin{cases} 8x+6y \le 24 \\ 2x+3y \le 12 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

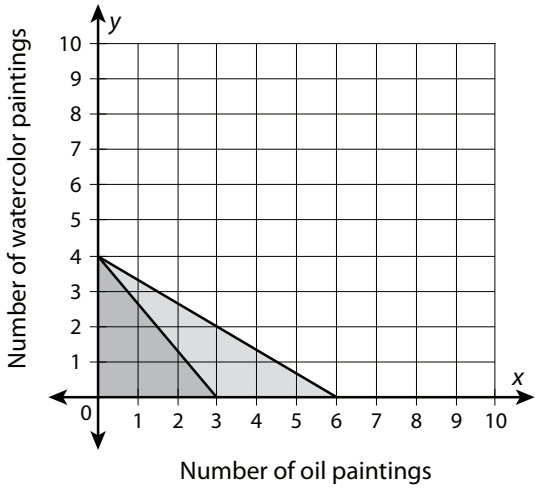
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2. Graph the system on the same coordinate axes.

Start by graphing the two inequalities in two variables.



Now apply the two constraints, $x \geq 0$ and $y \geq 0$. This means that the solution lies in the first quadrant and along parts of the x- and y-axes.



The solution is the darker shaded region; any points that lie within it and along its boundary lines, including the x- and y-axes, represent the solutions to the system. The point (1, 1) is a solution because it satisfies all of the inequalities. The artist can create 1 oil painting and 1 watercolor painting given the time constraints he has. Or, he can create 0 oil paintings and 4 watercolor paintings, (0, 4). However, he cannot create 4 oil paintings and 1 watercolor painting, because the point (4, 1) does not satisfy one inequality and therefore does not lie in solution.

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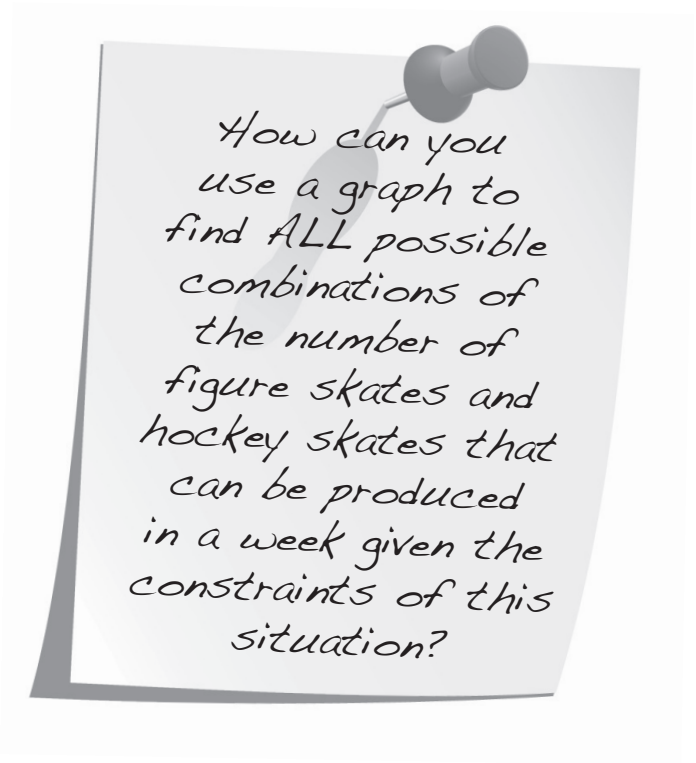
Problem-Based Task 2.2.2: Skate Constraints Revisited

SMP	
1	2 ✓
3	4 ✓
5 ✓	6
7	8

A sporting goods company produces figure skates and hockey skates. One group of workers makes the blades for both types of skates. Another group makes the boots for both types of skates.

- It takes 2 hours to make the blade of a figure skate. It takes 3 hours to make the blade of a hockey skate. Both sets of workers work a maximum of 40 hours per week.
- It takes 3 hours to make the boot for a figure skate. It takes 1 hour to make the boot for a hockey skate. There is a maximum of 20 hours per week in which boots can be made for both types of skates.

How can you use a graph to find ALL possible combinations of the number of figure skates and hockey skates that can be produced in a week given the constraints of this situation?



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Problem-Based Task 2.2.2: Skate Constraints Revisited

Coaching

a. What is an inequality that represents the time it takes to make the blades of the two different types of skates?

b. What is an inequality that represents the amount of time it takes to make the boots of the two different types of skates?

c. What other two constraints are necessary to represent this situation?

d. What is a system of inequalities that represents this situation?

e. How can you visually represent the inequality that represents the time it takes to make the blades for the two types of skates?

f. How can you visually represent the inequality that represents the time it takes to make the two types of boots? Add this inequality to the same graph you created for part e.

g. How can you visually represent the remaining constraints for this situation? Add these constraints to the same graph you created for part f.

h. How can you describe the graph of all possible combinations of figure skates and hockey skates that can be produced given the constraints of this situation?

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Problem-Based Task 2.2.2: Skate Constraints Revisited

Coaching Sample Responses

a. What is an inequality that represents the time it takes to make the blades of the two different types of skates?

The total amount of time needed to make the blades can be represented by the sum of the time needed to make each type of blade. Let x represent the number of hours to make blades for figure skates and let y represent the number of hours to make blades for hockey skates.

It takes two hours to make a blade for a figure skate, and 3 hours to make a blade for a hockey skate. The total number of hours needed for making the blades can't be more than 40 hours. The inequality can be written as follows.

$2x + 3y \leq 40$

b. What is an inequality that represents the amount of time it takes to make the boots for the two different types of skates?

The total amount of time needed to make the boots can be represented by the sum of the time needed to make each type of boot. Again, use x for figure skates and y for hockey skates.

It takes 3 hours to make a boot for a figure skate, and 1 hour to make a boot for a hockey skate. The total number of hours needed for making the boots can't be more than 20 hours. The inequality can be written as follows.

$3x + y \leq 20$

c. What other two constraints are necessary to represent this situation?

The number of figure skates or hockey skates made is never negative, so include these two constraints.

$x \geq 0$

$y \geq 0$

d. What is a system of inequalities that represents this situation?

Put together all the inequalities that represent the system.

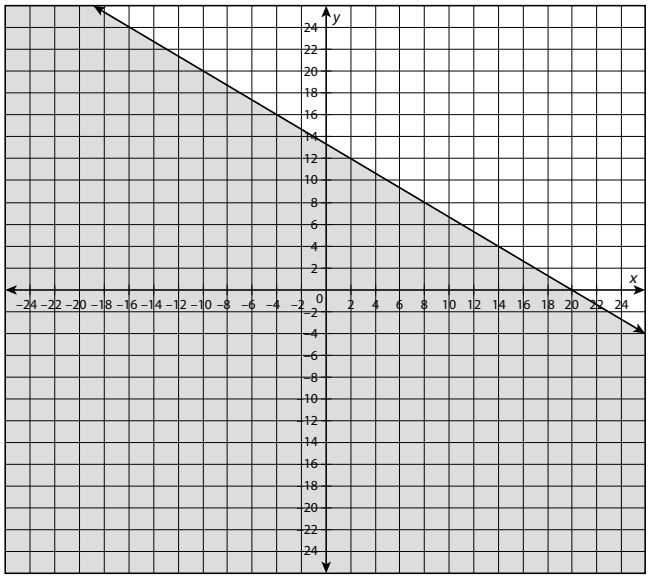
The system is as follows:

$$\begin{cases} 2x + 3y \leq 40 \\ 3x + y \leq 20 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

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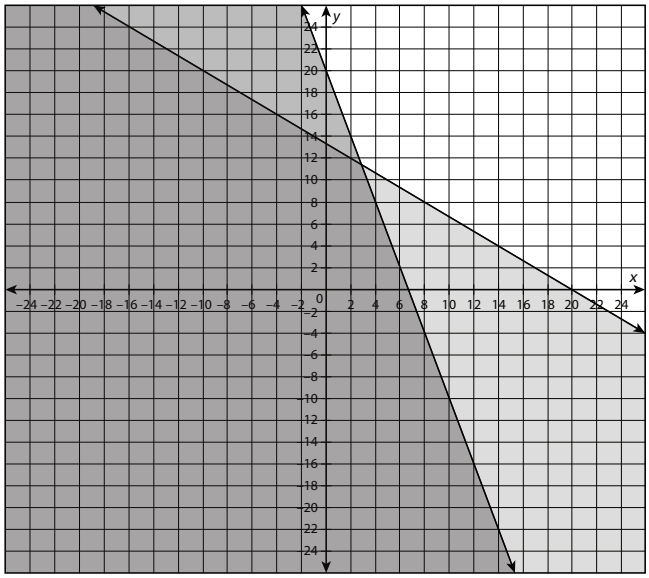
e. How can you visually represent the inequality that represents the time it takes to make the blades for the two types of skates?

The inequality that represents making the blades is $2x + 3y \leq 40$. The graph is shown.



f. How can you visually represent the inequality that represents the time it takes to make the two types of boots? Add this inequality to the same graph you created for part e.

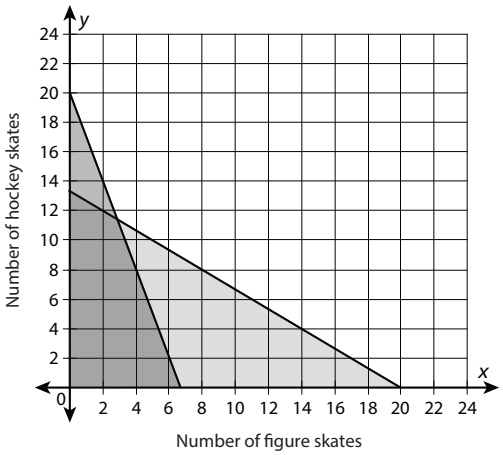
The inequality that represents making the boots is $\begin{cases} 2x + 3y \leq 40 \\ 3x + y \leq 20 \end{cases}$. The graph is shown.



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g. How can you visually represent the remaining constraints for this situation? Add these constraints to the same graph you created for part f.

The graph that follows reflects the complete system, $\begin{cases} 2x+3y \le 40 \\ 3x+y \le 20 \\ x \ge 0 \\ y \ge 0 \end{cases}$.



h. How can you describe the graph of all possible combinations of figure skates and hockey skates that can be produced given the constraints of this situation?

The solution to the system of linear inequalities is the set of all possible combinations of skates that can be produced. This is represented by the darker shaded region, which is bounded by the graphs of the four inequalities. The actual solution for this situation, however, is the set of points in the darker shaded region and along the boundary lines whose coordinates are whole numbers. This is because the number of skates made are whole skates and not fractional parts.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.2.2: Solving Systems of Linear Inequalities

For problems 1–7, solve the given system by creating a graph.

1. $\begin{cases} y > x \\ y > -x \end{cases}$

2. $\begin{cases} y \leq x \\ y \geq -x \end{cases}$

3. $\begin{cases} y > x - 4 \\ y < 2x + 10 \end{cases}$

4. $\begin{cases} y > x - 10 \\ y > -3x + 4 \end{cases}$

5. $\begin{cases} 3x + y \leq 3 \\ x + y \geq 0 \\ x \geq 0 \\ y \geq 0 \end{cases}$

6. $\begin{cases} 2x - y > 0 \\ 2x - y \geq 2 \\ x \leq 5 \\ y \geq 0 \end{cases}$

7. $\begin{cases} x + y \leq 7 \\ x - y \leq 7 \\ x \geq 0 \\ y \leq 0 \end{cases}$

continued

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For problems 8–10, use the given information to answer the questions that follow.

8. A school supply company produces wooden rulers and plastic rulers. It takes 30 minutes to cut, shape, and sand a wooden ruler. It takes 15 minutes to mold, dry, and cut a plastic ruler. A maximum of 480 minutes per day is set aside for these tasks.

   It takes 6 minutes to paint the lines and numbers on both types of rulers. A maximum of 180 minutes per day is set aside for this task.

   a. What is a system of inequalities that models this scenario?

   b. What is the graph of the solution to this system?

9. A bicycle shop sells road bikes and town cruisers. It takes 2 hours to assemble a road bike. It takes 1 hour to assemble a town cruiser. A maximum of 60 hours a week is set aside to complete the assembly for both types of bike.

   After assembly, for safety reasons the assembly needs to be checked and tested. It takes $\frac{1}{2}$ hour to check and test the road bikes, and $\frac{1}{4}$ hour to check and test the town cruisers. A maximum of 20 hours per week is set aside to complete these tasks for both types of bike.

   a. What is a system of inequalities that models this scenario?

   b. What is the graph of the solution to this system?

10. Jana is a caterer. She's making lasagna cupcakes and Gorgonzola onion tarts for a party, and only has a limited time to finish. She already has all her fillings ready and just needs to assemble and bake the appetizers. It takes 10 minutes to assemble each pan of lasagna cupcakes. It takes 15 minutes to roll out the Gorgonzola onion tarts and fill each pan. She has a maximum of 2 hours to assemble the appetizers.

    Next she needs to bake the appetizers. Each pan of lasagna cupcakes takes 20 minutes to bake. Each pan of Gorgonzola tarts takes 6 minutes to bake. She has 60 minutes to bake the appetizers.

    a. What is a system of inequalities that models this scenario?

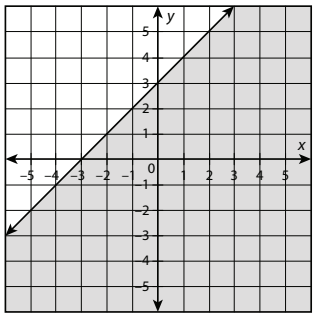
    b. What is the graph of the solution to this system?

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Progress Assessment

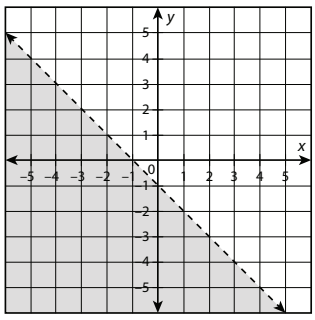
Circle the letter of the best answer.

1. Which inequality corresponds to this graph?



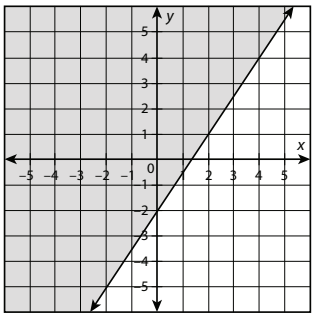
a. $y \leq x - 3$
b. $y \leq x + 3$
c. $y > -x + 3$
d. $y \geq x + 3$

2. Which inequality corresponds to this graph?



a. $y < -x - 1$
b. $y > x - 1$
c. $y \leq -x - 1$
d. $y \leq x + 2$

3. Which inequality corresponds to this graph?



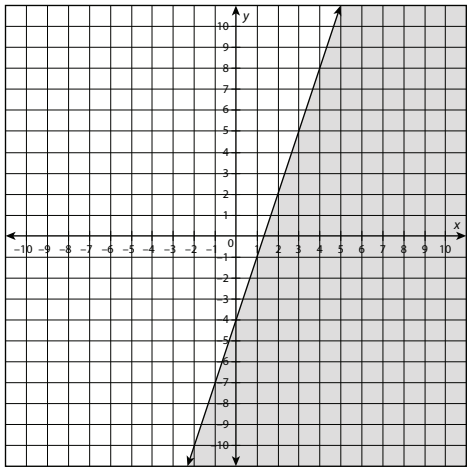
a. $-3x + 2y \leq 4$
b. $3x + 2y \geq -4$
c. $-3x - 2y \geq -4$
d. $3x - 2y \leq 4$

continued

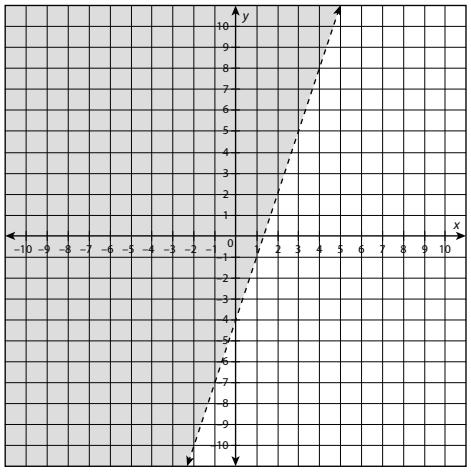
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4. Which graph represents the solution to the inequality $3x - y > 4$?

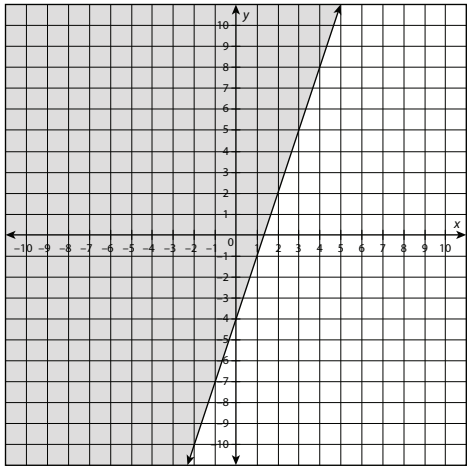
a.



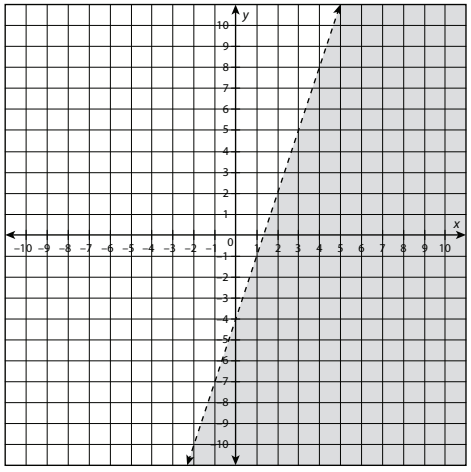
c.



b.



d.

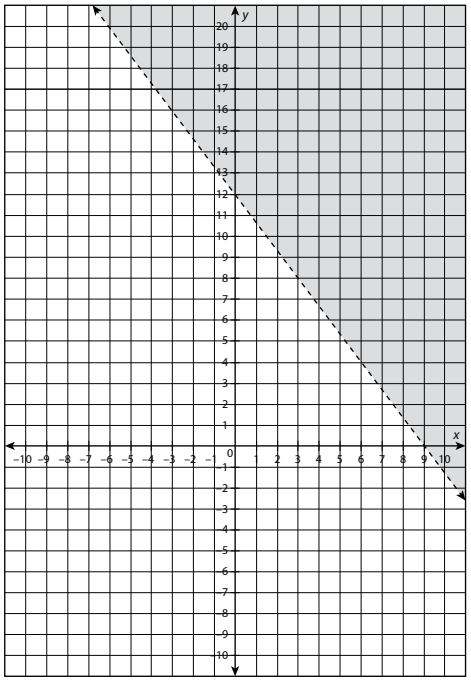


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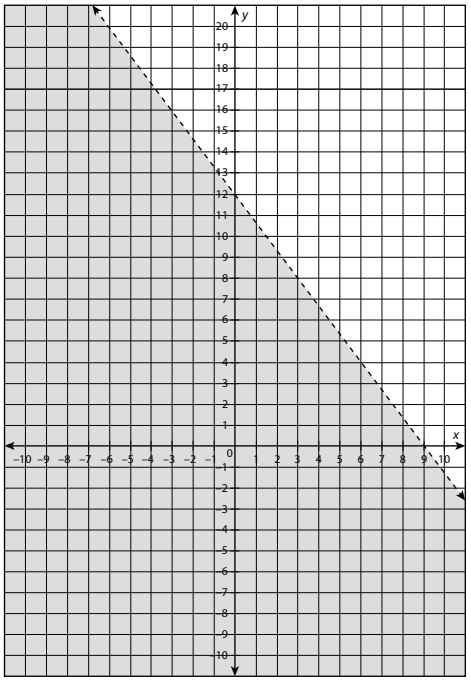
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5. Which graph represents the solution to the inequality $4x + 3y \geq 36$?

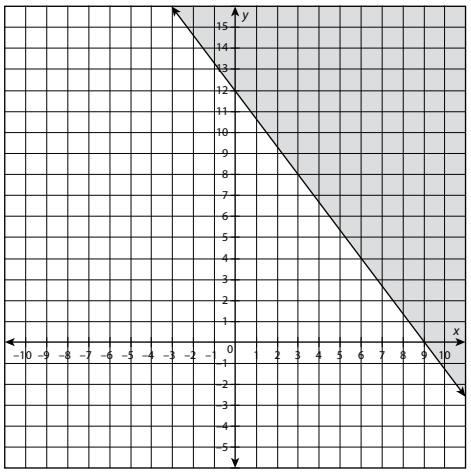
a.



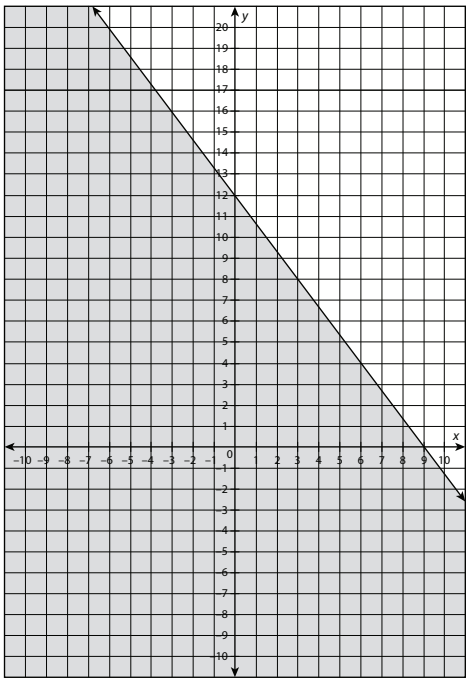
c.



b.



d.

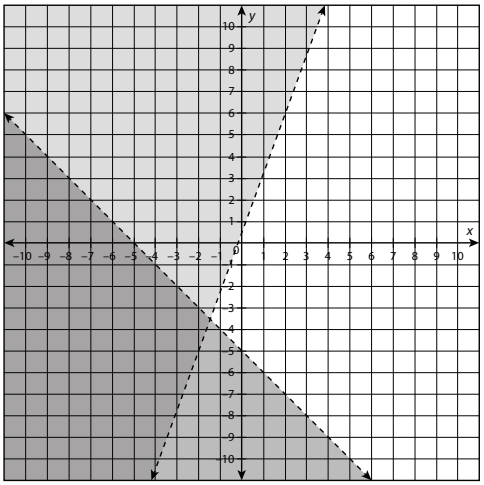


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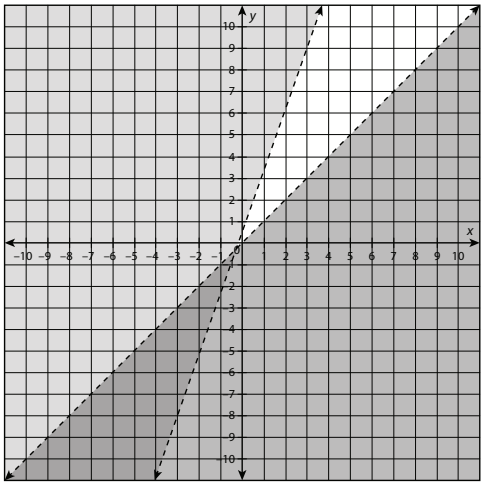
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6. Which graph represents the solution to the system of inequalities $\begin{cases} y > 3x - 10 \\ y < x - 5 \end{cases}$?

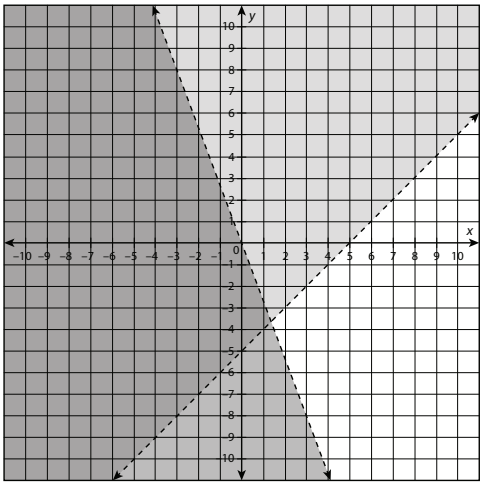
a.



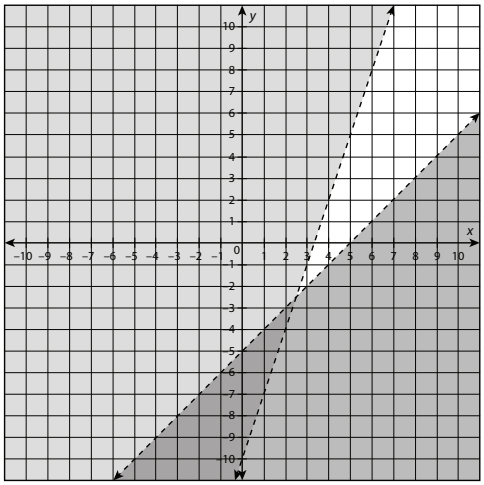
c.



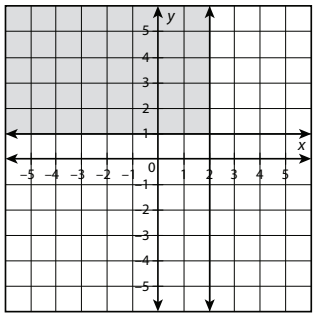
b.



d.



7. Which system of inequalities corresponds to this graph?



a. $\begin{cases} x > 2 \\ y \leq 1 \end{cases}$

b. $\begin{cases} x \leq 2 \\ y > 1 \end{cases}$

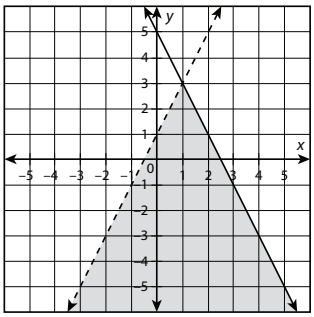
c. $\begin{cases} x \geq 2 \\ y \geq 1 \end{cases}$

d. $\begin{cases} x \leq 2 \\ y \geq 1 \end{cases}$

continued

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8. Which system of inequalities corresponds to this graph?



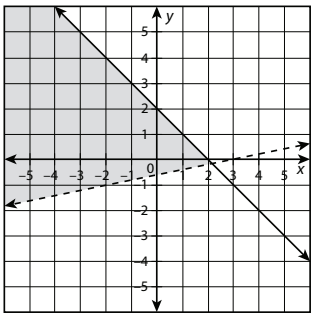
a. $\begin{cases} y \leq 2x+1 \\ y > -2x+5 \end{cases}$

b. $\begin{cases} y > 2x+1 \\ y > -2x+5 \end{cases}$

c. $\begin{cases} y > 2x+1 \\ y \leq -2x+5 \end{cases}$

d. $\begin{cases} y < 2x+1 \\ y \leq -2x+5 \end{cases}$

9. Which system of inequalities corresponds to this graph?



a. $\begin{cases} y < \frac{1}{5}x - \frac{1}{2} \\ y \geq -x+2 \end{cases}$

b. $\begin{cases} y > \frac{1}{5}x - \frac{1}{2} \\ y \leq -x+2 \end{cases}$

c. $\begin{cases} y > \frac{1}{5}x - \frac{1}{2} \\ y \geq -x+2 \end{cases}$

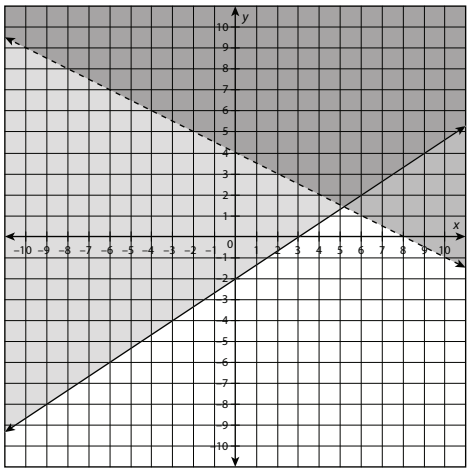
d. $\begin{cases} y < \frac{1}{5}x - \frac{1}{2} \\ y \leq -x+2 \end{cases}$

continued

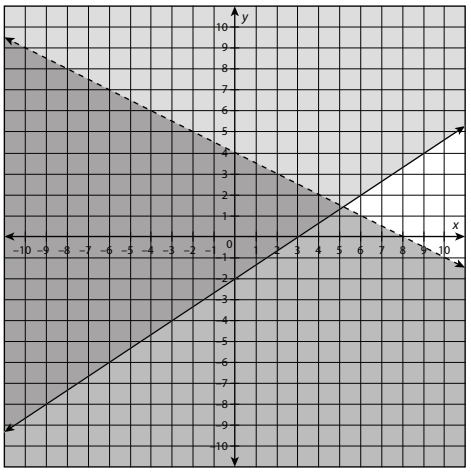
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10. Which graph represents the solution to the system of inequalities $\begin{cases} 3x+6y>24 \\ 2x-3y\leq 6 \end{cases}$?

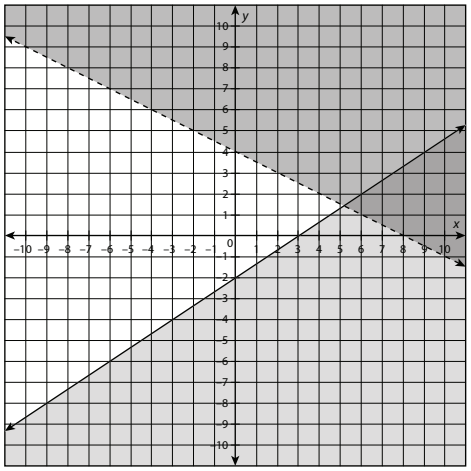
a.



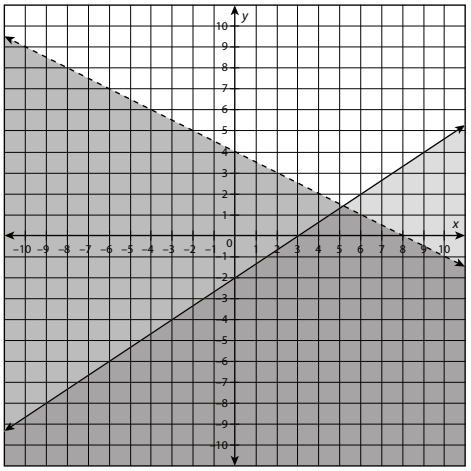
c.



b.



d.



continued

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Assessment

Read the scenario and answer the questions that follow.

11. Rory is an independent filmmaker and has great ideas for several movies. She has to raise money for each short film or full-length movie she wants to make. She figures that it will take 6 months to raise money for each short film, and 9 months to raise money for each full-length movie. She has given herself at most 4.5 years (54 months) to raise money for the films.

    While Rory is raising money, she also has to write the scripts. It will take her $\frac{1}{2}$ of a month to write each short-film script, and 2 months to write each full-length movie script. She is committed to spending at most 10 months on writing scripts.

    a. What is a system of inequalities that models this scenario?

    b. What is the graph of the solution to this system?

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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. Identify the pattern in the sequence of numbers and choose the next number in the sequence.

   1, 1, 2, 3, 5, ___

   a. 7
   b. 8
   c. 9
   d. 10

2. Identify the pattern in the sequence of numbers and choose the missing number in the sequence.

   4, 7, 12, ___, 28, 39

   a. 19
   b. 17
   c. 21
   d. 20

3. A sequence is generated by the formula $a_n = 3n + 19$. What is the value of the ninth term?

   a. 27
   b. 37
   c. 21
   d. 46

4. Which of the following is an acceptable domain for a sequence?

   a. {–2, –1, 0, 1, 2, 3}
   b. $1 \leq n \leq 100$
   c. {1, 1.5, 2, 2.5, 3, 3.5}
   d. {2, 3, 4, 5, ..., ∞}

5. What type of function is a sequence?

   a. unique
   b. positive
   c. discrete
   d. negative

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Lesson 3: Sequences As Functions

Instruction

Common Core State Standard

F–IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n + 1) = f(n) + f(n – 1) for n ≥ 1.*

SMP	
1 ✓	2 ✓
3	4
5	6
7	8 ✓

Essential Questions

1. How can you determine the *n*th term in a sequence?
2. Is a sequence always a function?
3. What is the domain of a sequence that is a function?
4. What is recursion?
5. How can you add and subtract sequences?
6. How can sequences be scaled?

WORDS TO KNOW

arithmetic sequence	a linear function with a domain of positive consecutive integers in which the difference between any two consecutive terms is equal
common difference	the number added to each consecutive term in an arithmetic sequence
common ratio	the number that each consecutive term is multiplied by in a geometric sequence
discrete	individually separate and distinct
explicit formula	a formula used to find the *n*th term of a sequence; if *n* is the term number, then the explicit formula for the *n*th term of an arithmetic sequence is $a_n = a_1 + (n - 1)d$, and the explicit formula for the *n*th term of a geometric sequence is $a_n = a_1 \bullet r^{n-1}$
geometric sequence	an exponential function that results in a sequence of numbers separated by a common ratio
natural numbers	the set of positive integers {1, 2, 3, ...}

SRB p. U2-71

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recursive formula	a formula used to find the next term of a sequence when the previous term or terms are known; the recursive formula for the *n*th term of an arithmetic sequence is $a_n = a_{n-1} + d$, and the recursive formula for the *n*th term of a geometric sequence is $a_n = a_{n-1} \cdot r$
sequence	an ordered list of numbers

Recommended Resources

- MathIsFun Advanced. "Sequences."

  http://walch.com/rr/CAU3L2Sequences

  This site introduces and defines arithmetic and geometric sequences, and illustrates notation of sequences.

- OEIS.org. "The On-Line Encyclopedia of Integer Sequences."

  http://walch.com/rr/CAU3L2SequencesEncyclopedia

  This quirky site will complete and define an entered sequence of integers. It will also provide any historical or quasi-relevant links to the defined sequence.

- Purplemath.com. "Sequences and Series."

  http://walch.com/rr/CAU3L2SequencesAndSeries

  This site provides a more in-depth look at sequences and series.

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Lesson 2.3.1: Sequences As Functions

Warm-Up 2.3.1

Sophia has a greenhouse. She's trying to grow a demanding species of orchid that requires water every 3 days—no more, no less. The first day Sophia waters the orchids is Sunday, May 1. Use this information to answer the following questions.

1. What is the third date on which Sophia will need to water the orchids?

2. What is the fifth date on which Sophia will need to water the orchids?

3. What is the seventh date on which Sophia will need to water the orchids?

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Instruction

Lesson 2.3.1: Sequences As Functions

Common Core State Standard

F–IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n + 1) = f(n) + f(n – 1) for n ≥ 1.*

Warm-Up 2.3.1 Debrief

1. What is the third date on which Sophia will need to water the orchids?

   Set up a calendar using what we know to find the date.

   The first day Sophia waters the orchids is May 1. Fill in the calendar with this information.

Date	May 1	May 2	May 3	May 4	May 5	May 6	May 7
Day of week	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Instances of watering the orchids	First time						

   The problem tells us that Sophia waters the orchids every three days. Three days from May 1 is Wednesday, May 4, so the second time Sophia waters is May 4. Three days after May 4 is May 7.

Date	May 1	May 2	May 3	May 4	May 5	May 6	May 7
Day of week	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Instances of watering the orchids	First time			Second time			Third time

   The third time Sophia waters the orchids is May 7. How can we use this information to find the fifth and seventh dates? What connection can we make between the third instance of watering and the date being May 7?

   From the calendar, we see that at each instance of watering, the date has increased by 3 days. The first watering day is on May 1, the second is on May 4, and the third is on May 7. So we can say that the number associated with the date should be 3 times the number associated with the instance of watering, or $3n$, where n is the nth day that Sophia needs to water the orchids. So, if we let x represent the number of the date, then we may be able to find the date using the formula $3n = x$.

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Instruction

Check to see if the formula $3n = x$ is accurate by substituting values for n. Use 1, 2, and 3 for the first, second, and third times Sophia waters the orchids.

$3n = x$

$3(1) = 3$; the corresponding date is May 3.

$3(2) = 6$; the corresponding date is May 6.

$3(3) = 9$; the corresponding date is May 9.

These dates don't match what we see on the calendar. We need to adjust our formula so that the first watering is on May 1.

May 1 is two days before May 3. If we subtract 2 from $3n$, we get the formula $3n - 2 = x$, where n represents which time the orchids are watered, and x represents the corresponding dates.

$3n - 2 = x$

$3(1) - 2 = 1$; the corresponding date is May 1.

$3(2) - 2 = 4$; the corresponding date is May 4.

$3(3) - 2 = 7$; the corresponding date is May 7.

The formula $3n - 2 = x$, where n is the nth watering day, and x is the date, is correct for the first, second, and third times Sophia needs to water the orchids.

2. What is the fifth date on which Sophia will need to water the orchids?

Substitute 5 for n in the formula $3n - 2 = x$.

$3n - 2 = x$

$3(5) - 2 = 15 - 2 = 13$

The fifth time Sophia needs to water the orchids is on May 13. Check this using the calendar.

We need to expand our calendar table for another week.

Add a week to the calendar and fill it out to check what we've found.

Date	May 1	May 2	May 3	May 4	May 5	May 6	May 7
Day of week	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Instances of watering the orchids	First time			Second time			Third time

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Date	May 8	May 9	May 10	May 11	May 12	May 13	May 14
Day of week	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Instances of watering the orchids			Fourth time			Fifth time	

The formula correctly predicted the fifth watering day on May 13.

3. What is the seventh date on which Sophia will need to water the orchids?

   Substitute 7 for n in the formula $3n - 2 = x$.

   $3n - 2 = x$

   $3(7) - 2 = 21 - 2 = 19$

   The formula predicts that Sophia will water the plants on May 19. Add another week to the calendar and confirm.

Date	May 1	May 2	May 3	May 4	May 5	May 6	May 7
Day of week	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Instances of watering the orchids	First time			Second time			Third time

Date	May 8	May 9	May 10	May 11	May 12	May 13	May 14
Day of week	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Instances of watering the orchids			Fourth time			Fifth time	

Date	May 15	May 16	May 17	May 18	May 19	May 20	May 21
Day of week	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Instances of watering the orchids		Sixth time			Seventh time		

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Instruction

The formula correctly predicted the seventh watering day on May 19.

This confirms that the formula $3n - 2 = x$ can predict when Sophia needs to water her orchids.

The third, fifth, and seventh dates on which Sophia must water the orchids are May 7, May 13, and May 19.

Connection to the Lesson

- The watering schedule is an arithmetic sequence, and arithmetic sequences will be introduced in the lesson.
- Students might find the counting to be laborious and wonder if there is an easier way to solve the problem mathematically.
- The exercise shows a process for building a sequence.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- recognizing patterns (3.OA.9)
- understanding the properties of a function (8.F.4)
- understanding function notation (F–IF.2)

Introduction

A **sequence** is an ordered list of numbers. The numbers, or terms, in the ordered list can be determined by a formula that is a function of the position of each term in the list. So if we have a sequence A determined by a function f, the terms of the sequence will be:

$$A = a_1, a_2, a_3, ..., a_n, \text{ where } a_1 = f(1), a_2 = f(2), a_3 = f(3), ..., a_n = f(n)$$

Unlike a typical function on a variable, there are no fractional elements in the domain of a sequence. Each value of n is a whole number. Just as no one can place 1.25th in a race, there is no 1.25th term in a sequence. Therefore, the domain of f includes elements from the set {1, 2, 3, ...}. These numbers are a subset of the set of integers, and are called the **natural numbers**. Natural numbers are the numbers we use for counting. Every element of the domain of a sequence is individually separate and distinct, so we say that a sequence is a **discrete** function.

Key Concepts

- Sequences are ordered lists that can be expressed as functions.
- The domain of a function that generates a sequence is the set of all natural numbers.
- There are two ways sequences are generally defined—recursively and explicitly.
- An **explicit formula** is a formula used to find the nth term of a sequence. A sequence is defined explicitly (that is, with an explicit formula) if a function is given.
- A **recursive formula** is a formula used to find the next term of a sequence when the previous term or terms are known. In a recursive sequence (that is, one with a recursive formula), the next term is based on the preceding term and the commonality between terms.
- For this lesson, most sequences have either a common difference or a common ratio.

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Instruction

- Sequences that have a common difference between consecutive terms are called **arithmetic sequences**. Sequences that have a common ratio among consecutive terms are called **geometric sequences**.
- Some sequences, such as 4, 7, 12, 19, 28, ..., are neither arithmetic nor geometric.
- A **common difference** is a number that is added to each consecutive term in an arithmetic sequence. To determine the common difference in an arithmetic sequence, subtract the first term from the second term. Then subtract the second term from the third term, and so on.
- If a common difference exists, $a_n - a_{n-1}$ is a constant. So, $a_4 - a_3 = a_3 - a_2 = a_2 - a_1$.
- If no common difference exists, then the sequence may be geometric, and the commonality might be a ratio between terms. A **common ratio** is a number by which each consecutive term in a geometric sequence is multiplied.
- To find a common ratio, divide the second term by the first term. Then, divide the third term by the second term, and so on.
- If a common ratio exists, then $\frac{a_n}{a_{n-1}}$ is a constant. So, $\frac{a_4}{a_3} = \frac{a_3}{a_2} = \frac{a_2}{a_1}$.

Explicit Formulas

- Explicitly defined sequences provide a formula, or function, that will generate each term. For example, $a_n = 2n + 3$ or $a_n = 5(3)^{n-1}$. Explicit formulas do not rely on knowing a previous term.
- In each case, simply substitute the value of n (representing the nth term) to find a_n.

Recursive Formulas

- A second way to define a sequence is with a recursive formula. In a sequence that is defined recursively, each term is a function of the term or terms that come before it. For example, $a_n = a_{n-1} + 2$ or $a_n = a_{n-1} \bullet 3$, where n is the number of the term and a_n is the value of that term.

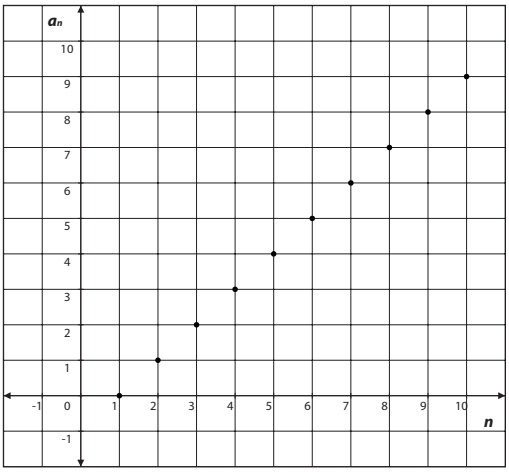
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Instruction

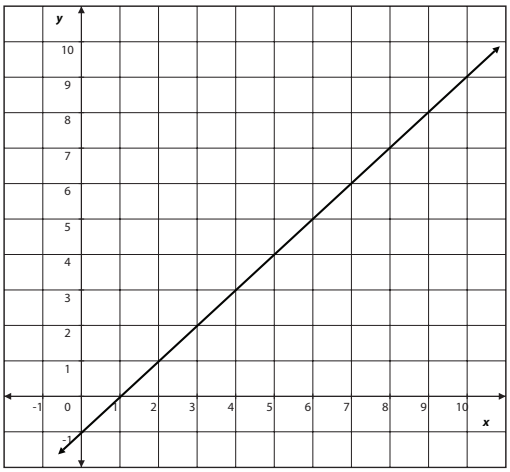
Graphing Sequences

- A sequence can be graphed. Its domain is the set of natural numbers. Compare the following graphs. The first graph represents the sequence $a_n = n - 1$, and the second graph represents the line $f(x) = x - 1$.

Graph of a sequence



Graph of a linear function



- Notice that the sequence only has values for $n = 1, 2, 3, 4$, etc., and consists of discrete points.

Common Errors/Misconceptions

- not realizing that a sequence can be written as a function
- not understanding that the graph of a sequence consists of a set of discrete points

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Guided Practice 2.3.1

Example 1

What is the fourth term of the sequence given by the formula $a_n = a_{n-1} + 5$, where a_n is the value of the term, n is the term number, and $a_1 = 2$?

1. Determine whether the sequence is explicit or recursive.

   The formula defines a term in the sequence, a_n, with regard to the term before it, a_{n-1}. Therefore, this is a recursive sequence.

2. Use the given formula to find the first 4 terms of the sequence.

   The first term of the sequence is given: $a_1 = 2$.

   Use this term to find the second term in the sequence.

$a_n = a_{n-1} + 5$	Given formula
$a_2 = a_{2-1} + 5$	Substitute 2 for n.
$a_2 = a_1 + 5$	Simplify the subscript.
$a_2 = (2) + 5$	Substitute 2 for a_1.
$a_2 = 7$	Simplify.

   The second term of the sequence is 7.

   Use this term to find the third term in the sequence.

$a_n = a_{n-1} + 5$	Given formula
$a_3 = a_{3-1} + 5$	Substitute 3 for n.
$a_3 = a_2 + 5$	Simplify the subscript.
$a_3 = (7) + 5$	Substitute 7 for a_2.
$a_3 = 12$	Simplify.

   The third term of the sequence is 12.

   Use this term to find the fourth term in the sequence.

$a_n = a_{n-1} + 5$	Given formula
$a_4 = a_{4-1} + 5$	Substitute 4 for n.
$a_4 = a_3 + 5$	Simplify the subscript.
$a_4 = (12) + 5$	Substitute 12 for a_3.
$a_4 = 17$	Simplify.

   The fourth term of the sequence is 17.

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Instruction

Example 2

Identify the missing terms in the sequence using recursion.

$A = \{8, 13, 18, 23, a_5, a_6, a_7\}$

1. Determine the pattern by looking for a common difference or common ratio. Use this information to write a formula to find a_n.

   Subtract the first term from the second term and then continue that pattern to see if the difference between each pair of terms is the same.

   $a_2 - a_1 = 13 - 8 = 5$

   $a_3 - a_2 = 18 - 13 = 5$

   $a_4 - a_3 = 23 - 18 = 5$

   The terms are separated by a common difference of 5. This means that to find each term (a_n), add 5 to the term before it (a_{n-1}).

   From this, we can deduce that $a_n = a_{n-1} + 5$.

2. Use the formula to find the missing terms.

   Find each missing term, a_n, by substituting the term immediately before it for a_{n-1} and solving for a_n.

   The missing terms are a_5, a_6, and a_7.

   $a_n = a_{n-1} + 5$

   $a_5 = a_4 + 5 = 23 + 5 = 28$

   $a_6 = a_5 + 5 = 28 + 5 = 33$

   $a_7 = a_6 + 5 = 33 + 5 = 38$

   The missing terms are 28, 33, and 38.

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Instruction

Example 3

Identify the missing terms in the sequence using recursion.

$B = \{6, 18, 54, 162, b_5, b_6, 4374, b_8\}$

1. Determine the pattern by looking for a common difference or common ratio.

   Subtract the first term from the second term and then continue that pattern to see if the difference between each pair of terms is the same.

   $b_2 - b_1 = 18 - 6 = 12$

   $b_3 - b_2 = 54 - 18 = 36$

   $b_4 - b_3 = 162 - 54 = 108$

   We can quickly see there is not a common difference. But, is there a common ratio?

   Divide each term by the term before it to determine if there is a common ratio.

   $$\frac{b_2}{b_1} = \frac{18}{6} = 3$$

   $$\frac{b_3}{b_2} = \frac{54}{18} = 3$$

   $$\frac{b_4}{b_3} = \frac{162}{54} = 3$$

   The terms share a common ratio of 3.

2. Consider what function is performed to get from one term to the next. Use this information to write a formula to find the missing terms.

   The common ratio is 3.

   The terms increase in value as the sequence progresses.

   To get to the next term, multiply by 3.

   $b_n = b_{n-1} \cdot 3$

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Instruction

3. Use the recursive formula to find the missing terms.

   Find each missing term, b_n, by substituting the term immediately before it for b_{n-1} and solving for b_n.

   The missing terms are b_5, b_6, and b_8.

   $b_n = b_{n-1} \cdot 3$

   $b_5 = b_4 \cdot 3 = 162 \cdot 3 = 486$

   $b_6 = b_5 \cdot 3 = 486 \cdot 3 = 1458$

   $b_8 = b_7 \cdot 3 = 4374 \cdot 3 = 13{,}122$

   The missing terms in the sequence are 486, 1,458, and 13,122.

Example 4

Find the ninth term in the sequence given by the formula $a_n = 3n + 1$, where a_n is the value of the term and n is the term number. Then, find and graph the first 5 terms in the sequence.

1. Determine whether this is an explicit formula or a recursive formula, then use the formula to find the desired term.

   The formula is explicit because plugging in the given term number (9 in this case) will produce the desired term. Substitute 9 for n and solve.

$a_n = 3n + 1$	Given formula
$a_9 = 3(9) + 1$	Substitute 9 for n.
$a_9 = 27 + 1$	Multiply.
$a_9 = 28$	Add.

   The ninth term in the sequence is 28.

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Instruction

2. Find the first 5 terms of the sequence.

Substitute 1, 2, 3, 4, and 5, respectively, for n, then solve for the term.

$a_n = 3n + 1$

$a_1 = 3(1) + 1 = 4$

$a_2 = 3(2) + 1 = 7$

$a_3 = 3(3) + 1 = 10$

$a_4 = 3(4) + 1 = 13$

$a_5 = 3(5) + 1 = 16$

The first 5 terms are 4, 7, 10, 13, and 16.

3. Create ordered pairs from the sequence.

n is the first coordinate, and a_n is the second coordinate.

(n, a_n)

(1, 4)

(2, 7)

(3, 10)

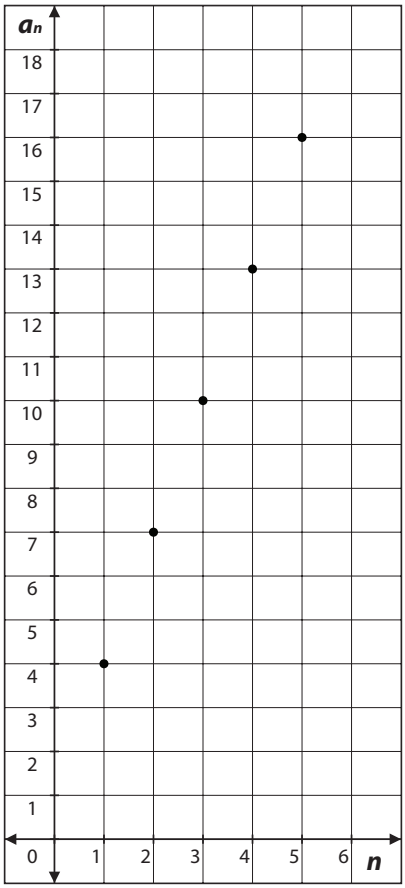
(4, 13)

(5, 16)

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Instruction

4. Plot the ordered pairs.



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Instruction

Example 5

Find the seventh term in the sequence given by the formula $a_n = 3 \cdot 2^{n-1}$, where a_n is the value of the term and n is the term number. Then, find and graph the first 5 terms in the sequence.

1. Determine whether this is an explicit formula or a recursive formula, then use the formula to find the desired term.

   The formula is explicit because plugging in the given term number (7 in this case) will produce the desired term. Substitute 7 for n.

   $a_n = 3 \cdot 2^{n-1}$

   $a_7 = 3 \cdot 2^{(7)-1} = 3 \cdot 2^6 = 3 \cdot 64 = 192$

   The seventh term in the sequence is 192.

2. Find the first 5 terms of the sequence.

   Substitute 1, 2, 3, 4, and 5, respectively, for n, then solve for the term.

   $a_n = 3 \cdot 2^{n-1}$

   $a_1 = 3 \cdot 2^{(1)-1} = 3 \cdot 2^0 = 3 \cdot 1 = 3$

   $a_2 = 3 \cdot 2^{(2)-1} = 3 \cdot 2^1 = 3 \cdot 2 = 6$

   $a_3 = 3 \cdot 2^{(3)-1} = 3 \cdot 2^2 = 3 \cdot 4 = 12$

   $a_4 = 3 \cdot 2^{(4)-1} = 3 \cdot 2^3 = 3 \cdot 8 = 24$

   $a_5 = 3 \cdot 2^{(5)-1} = 3 \cdot 2^4 = 3 \cdot 16 = 48$

   The first 5 terms are 3, 6, 12, 24, and 48.

3. Create ordered pairs from the sequence.

   n is the first coordinate, and a_n is the second coordinate.

   (n, a_n)

   (1, 3)

   (2, 6)

   (3, 12)

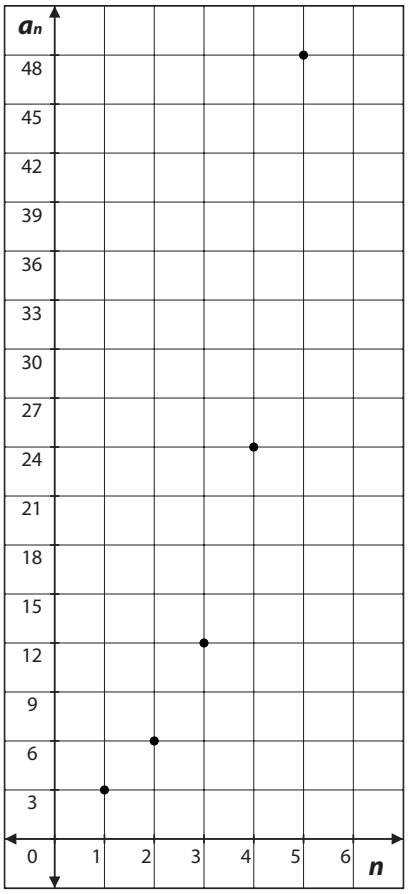
   (4, 24)

   (5, 48)

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Instruction

4. Plot the ordered pairs.



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Problem-Based Task 2.3.1: Which Tree Will It Be?

SMP	
1 ✓	2 ✓
3	4
5	6
7	8 ✓

Tree rings show the growth pattern of a tree. Each year a new ring forms and increases the diameter of the tree. Naturally, different species of trees have different growth rates. The growth formulas for three different tree species are shown. These formulas can be used to calculate the diameter in centimeters of each tree species in any given year, n.

Growth formula of Species A:

$a_n = a_1 \bullet 1.05^{n-1}$

a_1, the diameter of a first-year sapling, is 2 cm.

Growth formula of Species B:

$b_n = b_1 \bullet 1.04^{n-1}$

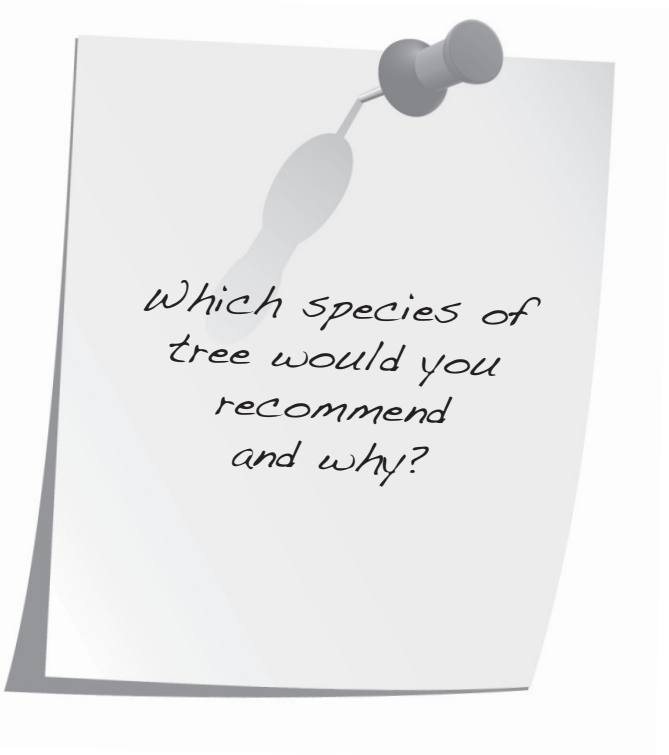
b_1, the diameter of a first-year sapling, is 4 cm.

Growth formula of Species C:

$c_n = c_1 \bullet 1.03^{n-1}$

c_1, the diameter of a first-year sapling, is 6 cm.

A city needs to purchase dozens of new trees for a downtown development project. Different interest groups are arguing about which species of tree to plant. City officials want small trees, so they prefer Species A. Historians want the state tree, Species B, which is medium sized. Community activists want larger trees to help provide shade and fresh air, and prefer Species C. You have been hired to end the argument by writing a report predicting the growth of the three tree species over the years. Which species of tree would you recommend and why? Consider the diameters of the trees after 5, 10, 25, 50, and 100 years.



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Problem-Based Task 2.3.1: Which Tree Will It Be?

Coaching

a. What will be the diameter of each tree species after 5, 10, 25, 50, and 100 years?

b. Which tree species will have the largest diameter after 5 years?

c. Which tree species will have the largest diameter after 10 years?

d. Which tree species will have the largest diameter after 25 years?

e. Which tree species will have the largest diameter after 50 years?

f. Which tree species will have the largest diameter after 100 years?

g. Which tree species would you recommend and why?

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Instruction

Problem-Based Task 2.3.1: Which Tree Will It Be?

Coaching Sample Responses

a. What will be the diameter of each tree species after 5, 10, 25, 50, and 100 years?

Remember, the diameter is in centimeters and n represents years.

Diameter after 5 years:

$a_5 = 2(1.05)^{5-1} = 2(1.05)^4 \approx 2.43$ cm

$b_5 = 4(1.04)^{5-1} = 4(1.04)^4 \approx 4.68$ cm

$c_5 = 6(1.03)^{5-1} = 6(1.03)^4 \approx 6.75$ cm

Diameter after 10 years:

$a_{10} = 2(1.05)^{10-1} = 2(1.05)^9 \approx 3.10$ cm

$b_{10} = 4(1.04)^{10-1} = 4(1.04)^9 \approx 5.69$ cm

$c_{10} = 6(1.03)^{10-1} = 6(1.03)^9 \approx 7.83$ cm

Diameter after 25 years:

$a_{25} = 2(1.05)^{25-1} = 2(1.05)^{24} \approx 6.45$ cm

$b_{25} = 4(1.04)^{25-1} = 4(1.04)^{24} \approx 10.25$ cm

$c_{25} = 6(1.03)^{25-1} = 6(1.03)^{24} \approx 12.20$ cm

Diameter after 50 years:

$a_{50} = 2(1.05)^{50-1} = 2(1.05)^{49} \approx 21.84$ cm

$b_{50} = 4(1.04)^{50-1} = 4(1.04)^{49} \approx 27.33$ cm

$c_{50} = 6(1.03)^{50-1} = 6(1.03)^{49} \approx 25.54$ cm

Diameter after 100 years:

$a_{100} = 2(1.05)^{100-1} = 2(1.05)^{99} \approx 250.48$ cm

$b_{100} = 4(1.04)^{100-1} = 4(1.04)^{99} \approx 194.25$ cm

$c_{100} = 6(1.03)^{100-1} = 6(1.03)^{99} \approx 111.95$ cm

b. Which tree species will have the largest diameter after 5 years?

Species C

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c. Which tree species will have the largest diameter after 10 years?

Species C

d. Which tree species will have the largest diameter after 25 years?

Species C

e. Which tree species will have the largest diameter after 50 years?

After 50 years, both Species A and B are growing faster than Species C, and Species B has the greatest diameter.

f. Which tree species will have the largest diameter after 100 years?

After 100 years, Species A has pulled away from the pack with an astonishing 250.4 cm diameter. That's more than 8 feet!

g. Which tree species would you recommend and why?

Species C seems to be the most reasonable choice because it meets the needs of the most people. In the short term, the community activists will be pleased to have a shade-producing tree, and the city officials can be satisfied that the tree will eventually grow to be the smallest of the three species. The other trees may grow too large for a city setting.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.3.1: Sequences As Functions

Use what you know about sequences to complete each problem.

1. What is the fourth term in the sequence given by the formula $a_n = 10n - 12$?

2. What is the fourth term in the sequence given by the formula $a_n = a_{n-1} + 3$ if $a_1 = -4$?

3. Graph the first 5 terms of the sequence given by the formula $a_n = 5n - 7$.

4. Graph the first 5 terms of the sequence given by the formula $a_n = 2^{n-1} - 2$.

5. What is the third term in the sequence given by the formula $a_n = a_{n-1} \bullet 4$ if $a_1 = 2$?

6. What is the fourth term in the sequence given by the formula $a_n = 13(2)^{n-1}$?

7. Complete and graph the sequence: 2, 6, 18, 54, a_5, 486.

8. Complete and graph the sequence: 13, 21, 29, 37, a_5, a_6.

9. A radio show breaks for news every 30 minutes. After every fourth news report, the newscaster reads the daily sports highlights. If the radio show began at 12:01 P.M. and the first news report was read at 12:31 P.M., at what time will the daily sports highlights be read?

10. Water stations are set up periodically along a marathon route. The water stations are set up every 3.5 miles. If the first station is at the 5-mile mark, at what mile mark will the fifth water station be?

SWB p. 79; SRB Practice 2.3.1 p. U2-84

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Progress Assessment

Circle the letter of the best answer.

1. Identify the pattern in the sequence of numbers and choose the next number in the sequence.

   1, 2, 4, 8, 16, ___

   a. 24
   b. 32
   c. 8
   d. 34

2. Identify the pattern in the sequence of numbers and choose the missing number in the sequence.

   3, 11, 19, ___, 35, 43

   a. 22
   b. 24
   c. 27
   d. 30

3. Identify the pattern in the sequence of numbers and choose the missing number in the sequence.

   4, 7, 12, ___, 28, 39

   a. 17
   b. 15
   c. 21
   d. 19

4. A sequence is generated by the formula $a_n = 2n + 3$. What is the value of the tenth term?

   a. 13
   b. 25
   c. 32
   d. 23

5. A sequence is generated by the formula $a_n = 5n - 2$. What is the value of the ninth term?

   a. 43
   b. 53
   c. 38
   d. 48

continued

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Assessment

6. A sequence is generated by the formula $a_n = 3(2^n) + 1$. What is the value of the fourth term?

   a. 49
   c. 25
   b. 47
   d. 33

7. If $a_n = a_{n-1} + 5$ and $a_3 = 9$, what is a_5?

   a. 29
   c. 24
   b. 19
   d. 14

8. If $a_n = a_{n-1} \bullet 2$ and $a_2 = 6$, what is a_4?

   a. 12
   c. 48
   b. 18
   d. 24

9. If $b_n = b_{n-1} \bullet 4$ and $b_3 = 16$, what is b_5?

   a. 64
   c. 256
   b. 1024
   d. 512

10. If $a_n = a_{n-1} + 7$ and $a_1 = 10$, what is a_4?

    a. 10
    c. 31
    b. 16
    d. 24

Use what you know about sequences to solve the problem.

11. Shelly is on a nature hike. She starts out in a valley and climbs up a hill, stopping at regular intervals to take pictures. The elevation of the land in feet below or above sea level at each place Shelly stops follows the function $a_n = a_{n-1} + 5$. The first place Shelly stops to take a picture is 4 feet below sea level, so $a_1 = -4$. What is the elevation of the sixth place Shelly stops?

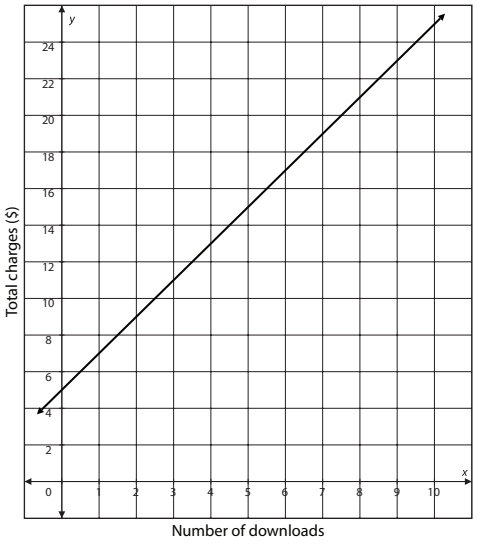
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Pre-Assessment

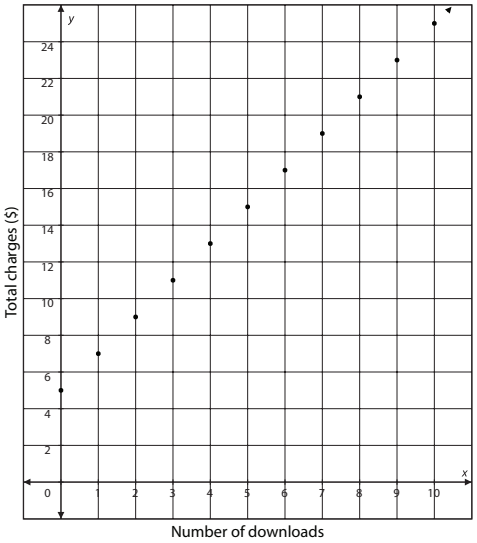
Circle the letter of the best answer.

1. An online company charges $5 a month plus $2 for each movie you decide to download. Which of the following graphs best represents this scenario?

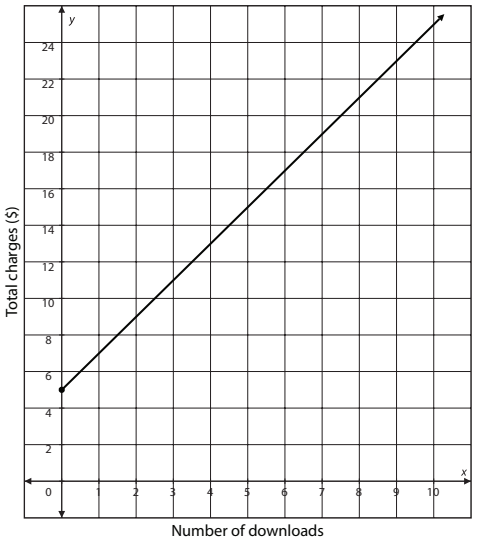
a.



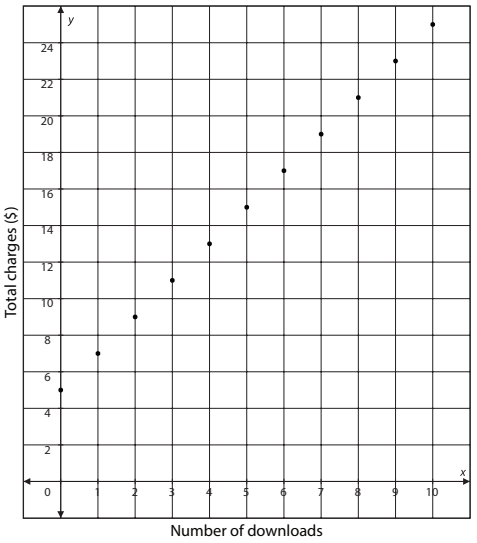
c.



b.



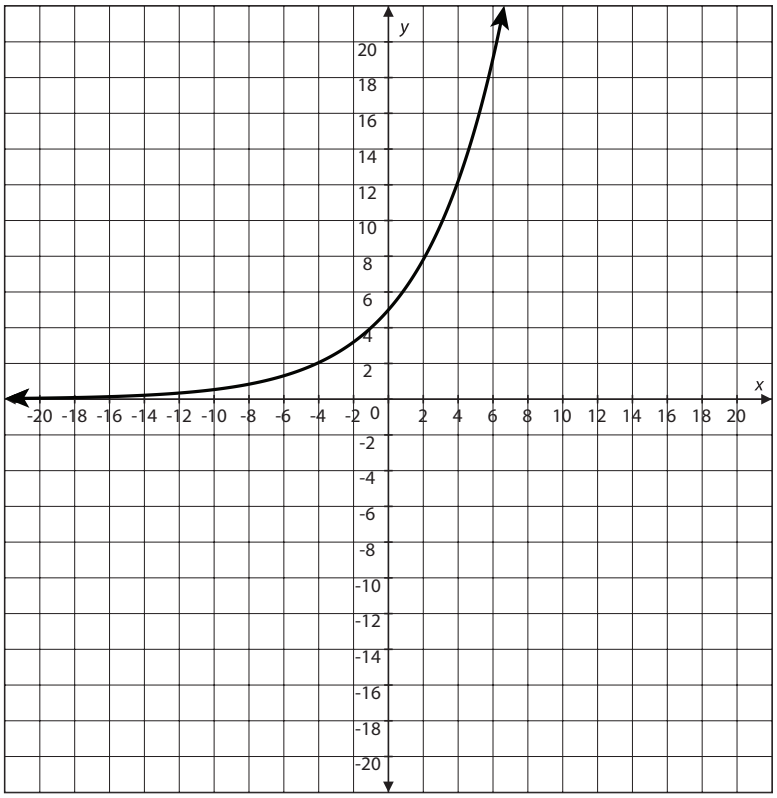
d.



continued

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2. The following graph can be described as:



a. a positive function that is increasing
b. a negative function that is decreasing
c. a positive function that is decreasing
d. a negative function that is increasing

3. Use the following table to determine the rate of change for the interval [1, 5].

Weeks (x)	Amount saved in dollars ($f(x)$)
1	16
2	38
3	60
4	82
5	104

a. \$17.60 per week
b. \$88 per week
c. \$0.05 per week
d. \$22 per week

continued

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4. Use the following table to determine the approximate rate of change for the interval [0, 3].

Years (x)	Amount invested in dollars ($f(x)$)
0	1000.00
1	1060.90
2	1125.51
3	1194.05
4	1266.77

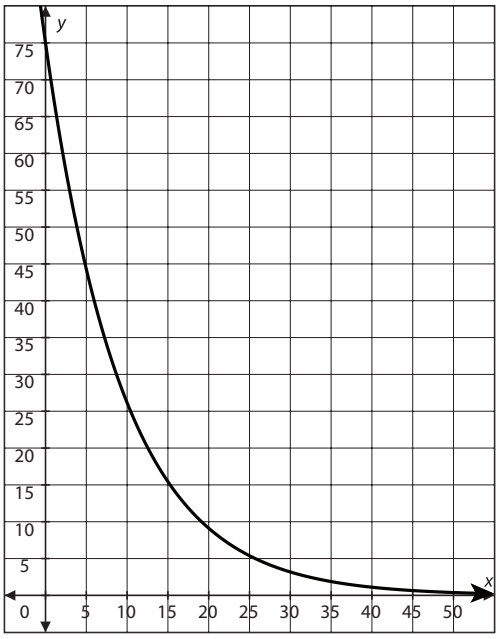
   a. –$64.68 per year

   b. $64.68 per year

   c. $0.02 per year

   d. The rate of change cannot be determined.

5. Use the following graph to determine the approximate rate of change for the interval [0, 30].



   a. –2.4

   b. 2.4

   c. –0.42

   d. The rate of change cannot be determined.

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Lesson 4: Interpreting Graphs of Functions

Instruction

Common Core State Standards

SMP	
1 ✓	2 ✓
3 ✓	4 ✓
5	6 ✓
7 ✓	8

F–IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F–IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F–IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

F–LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Essential Questions

1. How can maximum and minimum values of a function be applied to a real-world context?
2. What is the purpose of using the rate of change to analyze real-world data?
3. For what types of real-world data can you find the rate of change?

SRB p. U2-85

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Instruction

WORDS TO KNOW

asymptote	a line that a function gets closer and closer to as one of the variables increases or decreases without bound
continuous	having no breaks
domain	the set of all input values for which a relation or function is defined; the set of x-values that are valid for a relation or function
extrema	the minima and maxima of a function
integer	the set of positive and negative whole numbers and 0; the set {... –3, –2, –1, 0, 1, 2, 3, ...}
intercept	the value of the x- or y-coordinate where a line or curve intersects the x- or y-axis, respectively
interval	the continuous set of real numbers between two given numbers
irrational number	a real number that cannot be written as $\frac{m}{n}$, where m and n are integers and $n \neq 0$; a non-terminating or non-repeating decimal
natural numbers	the set of positive integers {1, 2, 3, ...}
negative function	a function or a portion of a function where the y-values are less than 0 for all x-values
positive function	a function or a portion of a function where the y-values are greater than 0 for all x-values
rate of change	a ratio that describes how much one quantity changes with respect to the change in another quantity; also known as the slope of a line
ratio	the relation between two quantities; can be expressed in words, or as a fraction, decimal, or percent
rational number	a real number that can be written as $\frac{m}{n}$, where m and n are integers and $n \neq 0$; a terminating or repeating decimal
real numbers	the set of all rational and irrational numbers
relative maximum	the greatest value of a function for a particular interval of the function

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Instruction

relative minimum	the least value of a function for a particular interval of the function
slope	the measure of the rate of change of one variable with respect to another variable; slope $= m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$
slope-intercept form of a linear equation	the form $y = mx + b$, where m is the slope of the line and b is the y-intercept
slope-intercept method	a way to use the slope-intercept form of a linear equation, $y = mx + b$, to graph the line, by first plotting the y-intercept, b, and then applying the slope, m
undefined slope	occurs when the denominator of the slope formula is equal to 0; the corresponding line is a vertical line
whole numbers	the set of positive integers and 0: {0, 1, 2, 3, ...}
***x*-intercept**	the x-coordinate of the point where a line or a curve intersects the x-axis
***y*-intercept**	the y-coordinate of the point where a line or a curve intersects the y-axis

Recommended Resources

- Illuminations. "Changing Cost per Minute."

  http://walch.com/rr/CAU3L3ChangingCost

  This interactive applet of cell phone charges allows users to view how changing the graph of the cost per minute affects the graph of the total cost. *Note*: Requires Java.

- Illuminations. "Constant Cost per Minute."

  http://walch.com/rr/CAU3L3ConstantCost

  This interactive applet of cell phone charges allows users to view how the total cost of service changes when a constant cost per minute is manipulated. *Note*: Requires Java.

- Oswego City School District Regents Exam Prep Center. "Exponential Growth and Decay."

  http://walch.com/rr/CAU3L3ExponentialDecay

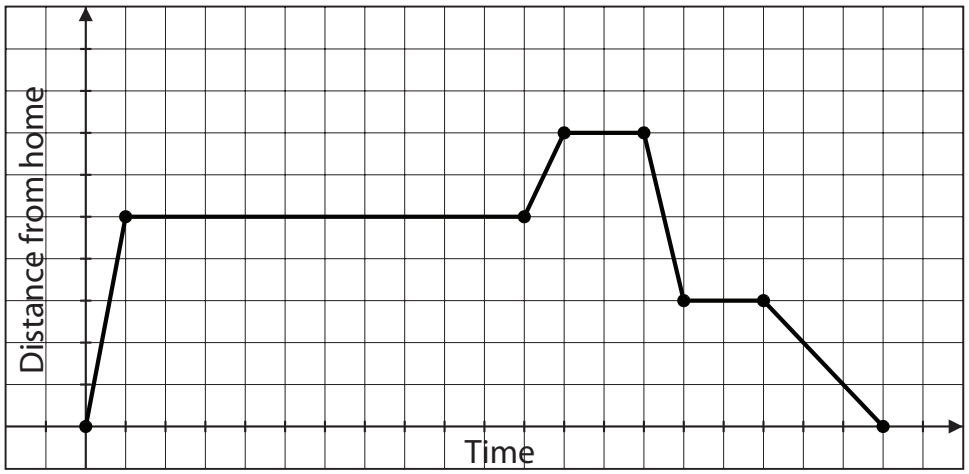
  This site summarizes exponential growth and decay using various examples to describe the key features of graphs including rates of change.

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Lesson 2.4.1: Identifying Key Features of Linear and Exponential Graphs

Warm-Up 2.4.1

The following graph represents Kim's distance from home one day as she rode her bike to meet a friend and to do a couple of errands for her mom before returning home.



1. Use the graph to describe Kim's journey.

2. What do the horizontal segments on the graph represent?

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Instruction

Lesson 2.4.1: Identifying Key Features of Linear and Exponential Graphs

Common Core State Standards

F–IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F–IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

Warm-Up 2.4.1 Debrief

1. Use the graph to describe Kim's journey.

   Answers will vary. One possible response: Kim left her house and rode her bike to her friend's house. She stayed at her friend's house for a while. Then she left her friend's house and rode to a store, which is even farther away from her house. She stayed at the store for a short time and bought a couple of items. Kim then headed back toward her house, stopping once more to take a picture of a beautiful statue along the way. She then biked the rest of the way back home.

2. What do the horizontal segments on the graph represent?

   The horizontal segments represent times when Kim stayed at one location. Her distance from home did not change, but time continued to pass.

Connection to the Lesson

- Students will extend their understanding of graphs and begin to more formally describe various features.
- Students will be asked to interpret key features from graphs and tables as well as sketch graphs given a verbal description.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

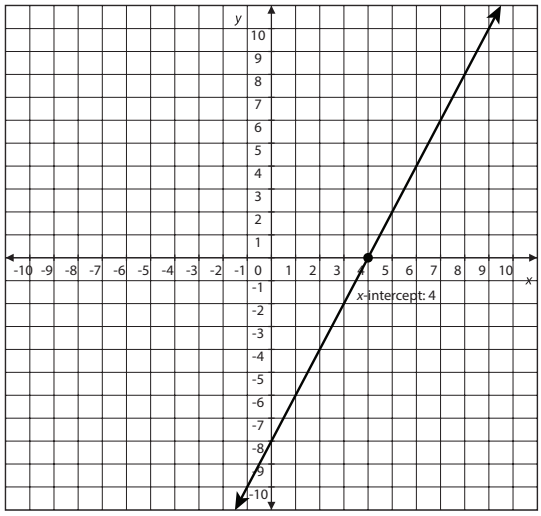
- graphing a linear function from a table or equation (A–CED.2★)
- graphing an exponential function from a table or equation (A–CED.2★)
- having knowledge of function notation, domain, and independent and dependent variables (F–IF.1)

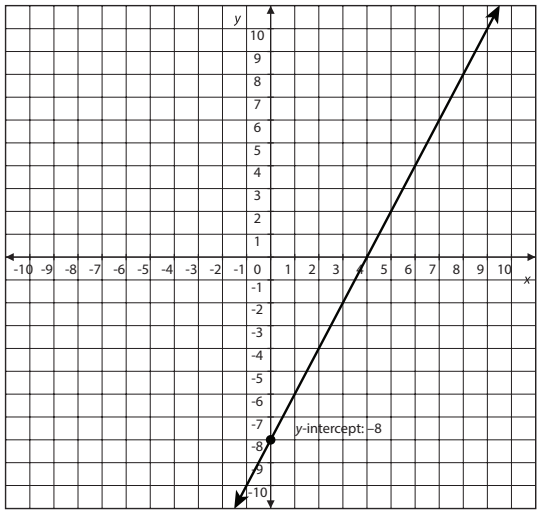
Introduction

Many real-world contexts can be represented by two variables, and these situations can be graphed on a coordinate plane. There are many characteristics of functions and their graphs that can provide a great deal of information. These characteristics can be analyzed so that the real-world context can be better understood.

Key Concepts

- One of the characteristics of a function's graph that we can observe are the **intercepts**, where a function intersects the *x*-axis and/or the *y*-axis.
  - The *y*-intercept is the value of *y* where the graph intersects the *y*-axis. The coordinates of the point are (0, *y*).
  - The *x*-intercept is the value of *x* where the graph intersects the *x*-axis. The coordinates of the point are (*x*, 0).

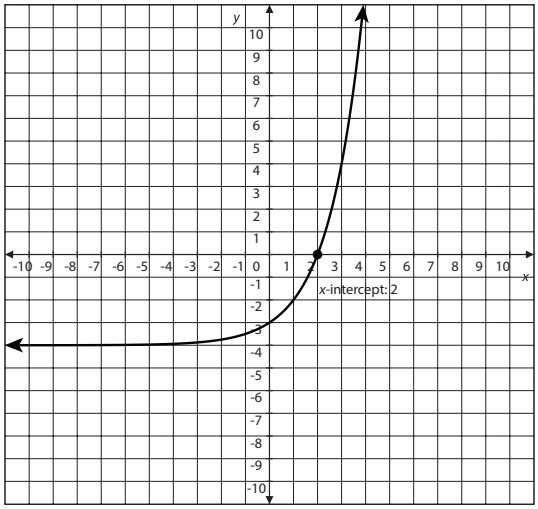


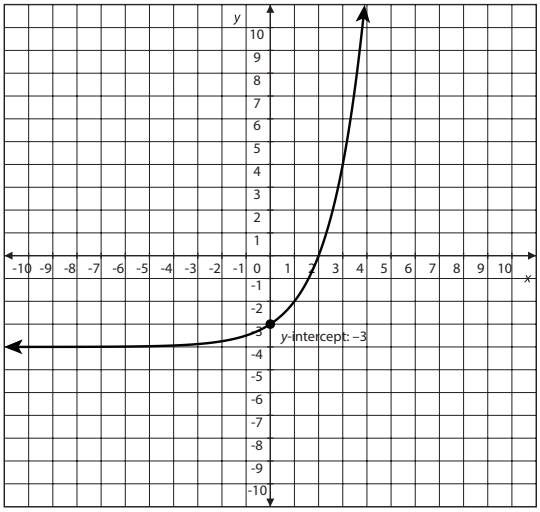


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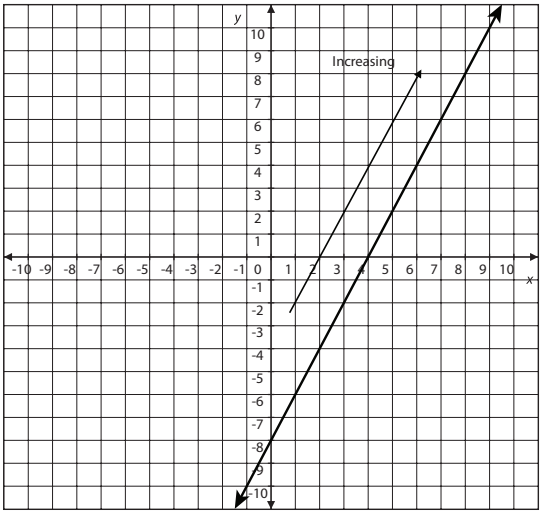
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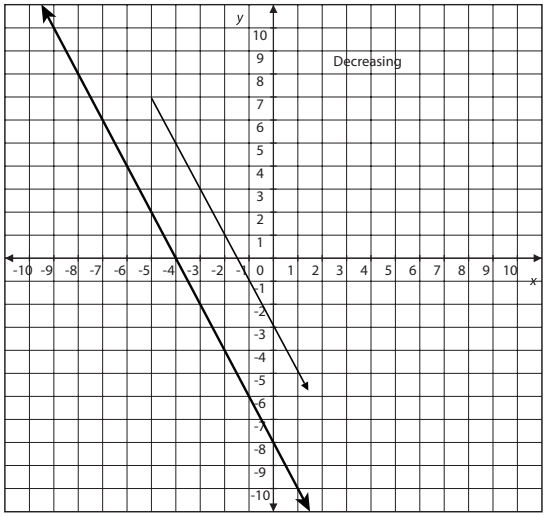
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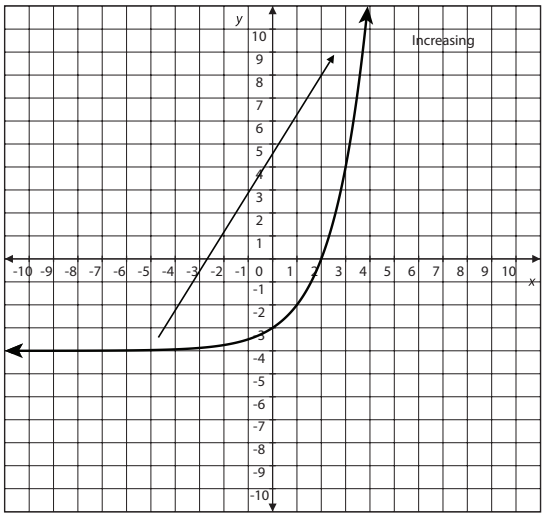
- Another characteristic of a function's graph that we can observe is whether the function is increasing or decreasing.
- When determining whether a function is increasing or decreasing, focus on the *y*-values.
- Read the graph from left to right and notice what happens to the graphed line. If the line rises from left to right, then the function is increasing. If the line falls from left to right, then the function is decreasing.
- The graph of a constant function is horizontal; it neither rises nor falls over its domain.

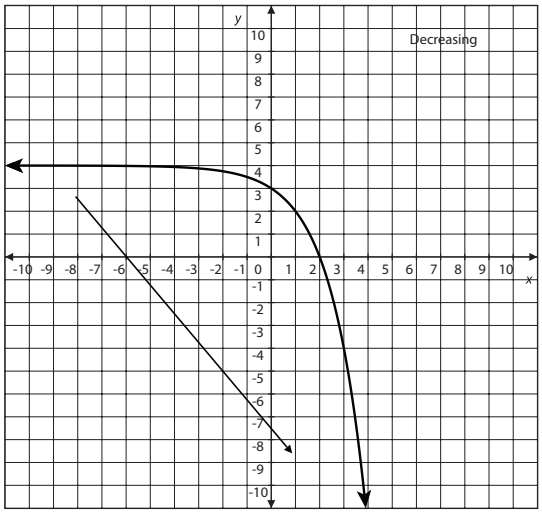




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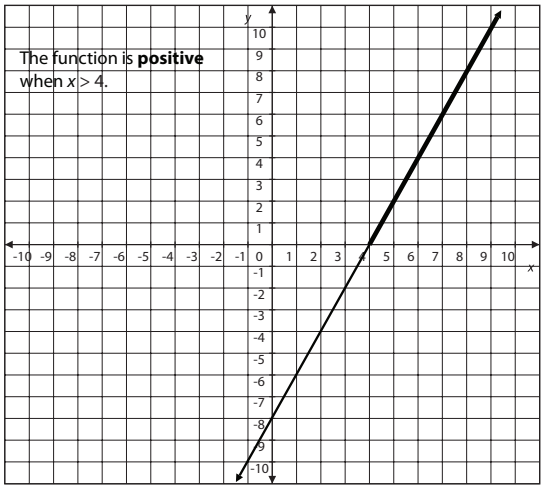


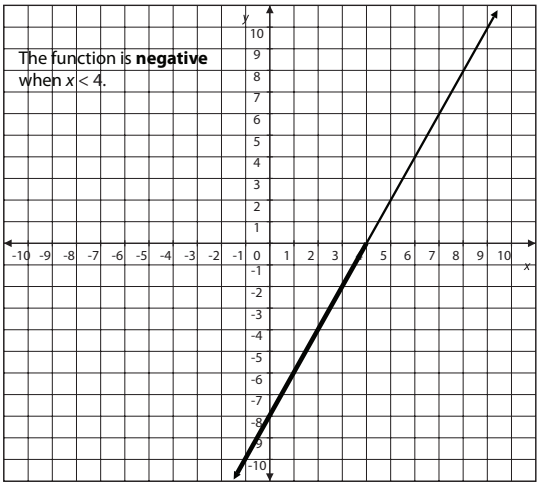


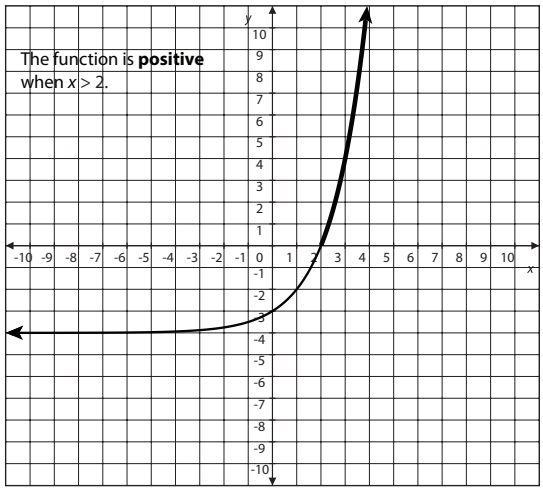
- An **interval** is the continuous set of real numbers between two given numbers. (**Continuous** means "having no breaks.")
- A function is **positive** on an interval if the *y*-values are greater than 0 for all *x*-values in that interval.
- A function is positive when its graph is above the *x*-axis.
- Use an inequality to define the values of *x* that make up the interval where the function is positive.
- A function is **negative** on an interval if the *y*-values are less than 0 for all *x*-values in that interval.
- A function is negative when its graph is below the *x*-axis.
- Use an inequality to define the values of *x* that make up the interval where the function is negative.

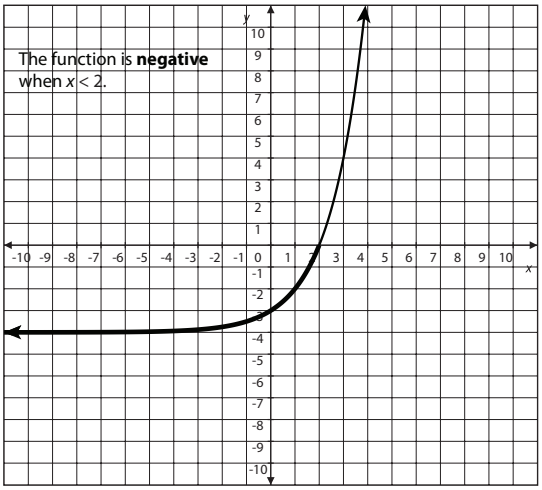
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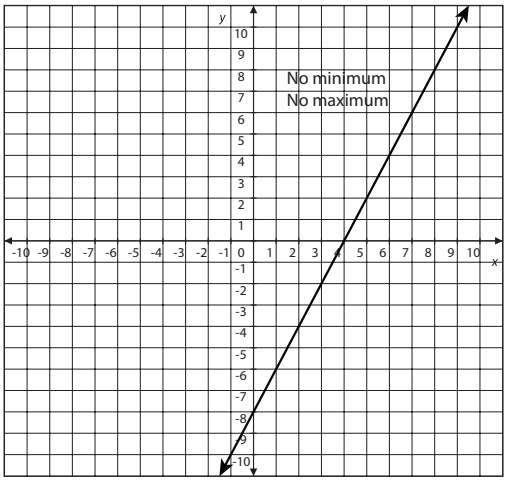


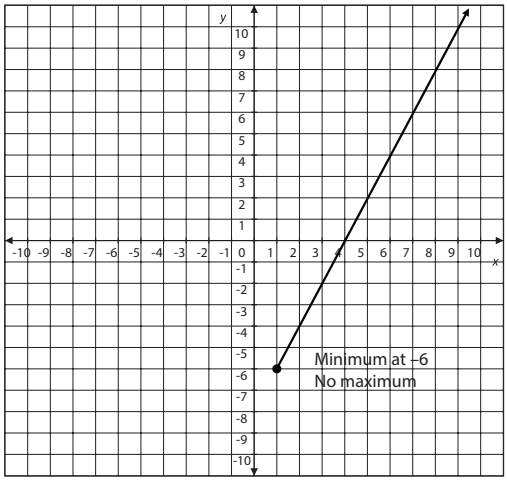


- The graph of a function may contain **extrema**, or the minima or maxima for the function.
- A **relative minimum** is the least value of a function for a particular interval of the function.
- A **relative maximum** is the greatest value of a function for a particular interval of the function.
- Linear and exponential functions will only have a relative minimum or maximum if the domain of the function is restricted.
- The **domain** of a function is the set of all inputs, or x-values, that are valid for the function.
- Compare the following two graphs. The graph on the left is of the function $f(x) = 2x - 8$. It doesn't have a minimum or a maximum. The graph on the right is of the same function, but the domain is all x-values such that $x \geq 1$. The minimum of the function on the right is -6, at $x = 1$.

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- Functions that represent real-world scenarios often include domain restrictions. For example, if we were to calculate the cost to buy a number of movie tickets, we would not expect to see negative or fractional numbers as values for x.
- There are several ways to classify numbers. The following table lists commonly used sets that are within the set of real numbers.

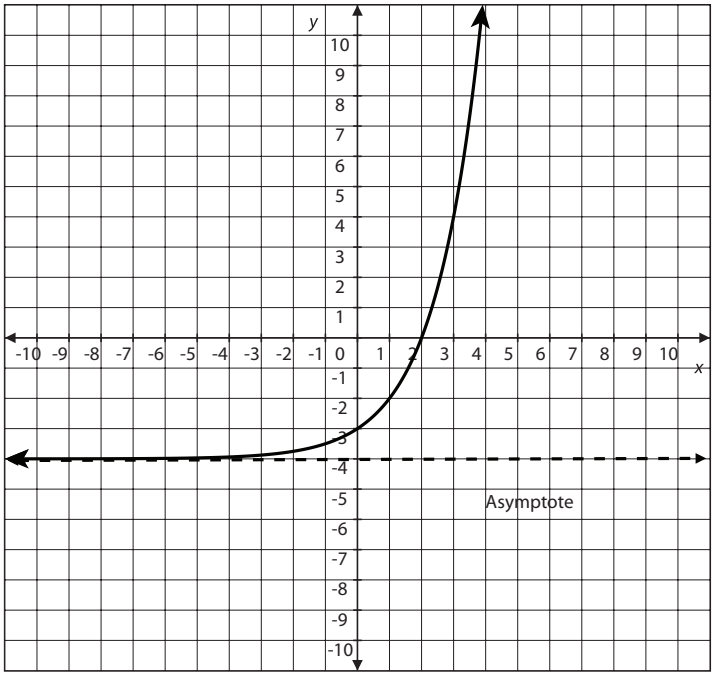
Natural numbers	{1, 2, 3, ...}
Whole numbers	{0, 1, 2, 3, ...}
Integers	{..., –3, –2, –1, 0, 1, 2, 3, ...}
Rational numbers	real numbers that can be written as $\frac{m}{n}$, where m and n are integers and $n \neq 0$; any terminating or repeating decimal
Irrational numbers	real numbers that cannot be written as $\frac{m}{n}$, where m and n are integers and $n \neq 0$; any non-terminating or non-repeating decimal
Real numbers	the set of all rational and irrational numbers

- An exponential function in the form $f(x) = a^x$, where $a > 0$ and $a \neq 1$, has an **asymptote**, which is a line that a function gets closer and closer to as one of the variables increases or decreases without bound. For equations in this form, the asymptote is the x-axis.
- The equation of the function in the following graph is $f(x) = 2^x - 4$. It has a horizontal asymptote at $y = -4$.

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- It may appear as though the graphed line touches the line $y = -4$, but it never does.



- Fairly accurate representations of functions can be sketched using the key features we have just described.

Common Errors/Misconceptions

- incorrectly identifying the type of function as either exponential or linear
- misidentifying key features on a graph
- incorrectly identifying the domain for a function

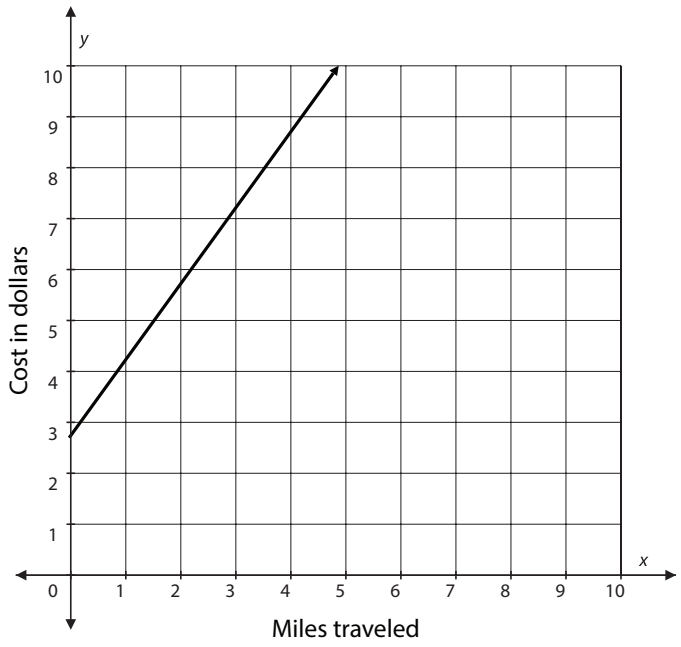
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Guided Practice 2.4.1

Example 1

A taxi company in Atlanta charges $2.75 per ride plus $1.50 for every mile driven. Determine the key features of the graph of this function.



1. Determine whether the function is linear or exponential.

   We can see by the graph that the function is increasing at a constant rate.

   The function is linear.

2. Identify the intercepts of the function.

   The graph of the function intersects the y-axis at the point (0, 2.75).

   The y-intercept is 2.75.

   The function does not intersect the x-axis.

   There is no x-intercept.

SWB p. 85; SRB p. U2-94

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3. Determine whether the graphed function is increasing or decreasing.

   Reading the graph left to right, the y-values are increasing.

   The function is increasing.

4. Determine where the function is positive and negative.

   The y-values are positive for all x-values greater than 0.

   The function is positive when $x > 0$.

   The y-values are never negative in this scenario.

   The function is never negative.

5. Determine the relative minimum and maximum of the graphed function.

   The least y-value of the function is 2.75. This is shown by the point with the coordinates (0, 2.75).

   The relative minimum is 2.75, when $x = 0$.

   The y-values increase infinitely; therefore, there is no relative maximum.

6. Identify the domain of the graphed function.

   The least x-value is 0.

   x can be any real number greater than or equal to 0.

   The domain can be written as $x \geq 0$. However, in the context of this problem, there would be a limit on x because a taxi cannot drive an infinite number of miles.

7. Identify any asymptotes of the graphed function.

   This is the graph of a linear function, not an exponential function. Therefore, there are no asymptotes for this function.



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Example 2

A pendulum begins at a height of 80 cm. After each successive swing, the pendulum reaches 90% of the previous height. This behavior can be described by the set of ordered pairs in the table. Determine the key features of this function.

Number of swings (x)	Height in cm (y)
0	80
1	72
2	64.8
3	58.32
5	47.24
10	27.89
20	9.73
40	1.18
60	0.14
80	0.02

1. Determine whether the function is linear or exponential.

   The scenario described here is that of an exponential function.

   We can see from the table that the values for y do not decrease at a constant rate. That is, when the differences between successive x-values are equal to 1, the corresponding differences between y-values are not the same.

2. Identify the intercepts of the function based on the information in the table.

   The function intersects the y-axis at the point (0, 80) as indicated in the table.

   The y-intercept is 80.

   As the x-values increase, the y-values get closer and closer to 0, but do not seem to reach 0; therefore, there is no x-intercept.

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3. Determine whether the function is increasing or decreasing.

   As the x-values increase, the y-values decrease.

   The function is decreasing.

4. Determine where the function is positive and negative.

   The y-values are positive for all x-values greater than or equal to 0.

   The function is positive when $x \geq 0$.

   The y-values are never negative in this scenario.

   The function is never negative.

5. Determine the relative minimum and maximum of the function.

   Based on the information given in the problem and the values in the table, we know that this is an exponential function, not a linear function.

   Exponential functions do not have a relative minimum because the graph continues to approach 0 as x increases.

   The height of the pendulum is never greater than its initial height of 80 cm; therefore, the relative maximum of this function is 80, when $x = 0$.

6. Identify the domain of the function.

   The least x-value is 0, the starting height.

   x can be any whole number, because we are not calculating the height of partial swings.

   Theoretically, the domain is all whole numbers. In reality, the pendulum would eventually stop swinging.

7. Identify any asymptotes of the function.

   The y-values appear to approach 0.

   The equation of the asymptote of this function is $y = 0$, which is the x-axis.

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Example 3

A ringtone company charges $15 a month plus $2 for each ringtone downloaded. Create a graph for this function and then determine its key features.

1. Create an equation of a function to represent this scenario.

   Let x represent the number of ringtones downloaded and $f(x)$ represent the total monthly fee.

   $f(x) = 15 + 2x$

2. Graph the function.

   When creating the graph of the function, consider the domain of the function.

   In past lessons, we have graphed the equations of functions that were not in context. In this lesson, and in the future, we want to consider the function as it relates to the scenario.

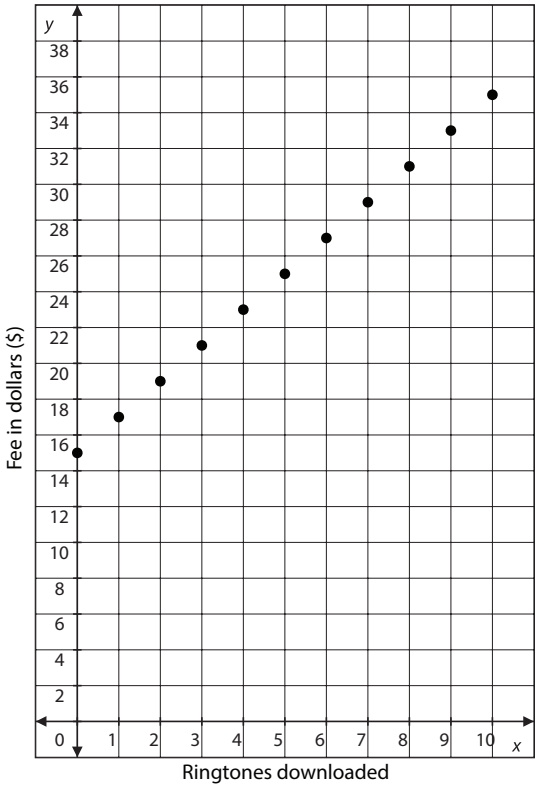
   We can't download a partial ringtone, and can't be charged for a partial download. Also, we cannot download a negative number of ringtones.

   Therefore, it does not make sense to consider values for x other than those that are whole numbers.

(*continued*)

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The points in the graph are not connected.



3. Identify the type of function described.

   The function is still a linear function; it just has a restricted domain. We can see this in the equation of the function, $f(x) = 15 + 2x$, and in the graph of the function.

4. Identify the intercepts of the graphed function.

   The graphed function crosses the y-axis at the point (0, 15).

   The y-intercept is 15.

   The function does not cross the x-axis.

   There is no x-intercept.

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5. Determine whether the graphed function is increasing or decreasing.

   Reading the graph left to right, the y-values are increasing.

   The function is increasing.

6. Determine where the function is positive and negative.

   The y-values are positive for all x-values greater than or equal to 0.

   The function is positive when $x \geq 0$.

   The y-values are never negative in this scenario.

   The function is never negative.

7. Determine the relative minimum and maximum of the graphed function.

   The least y-value of the function is 15. This is shown with the point plotted at (0, 15).

   The relative minimum is 15, when $x = 0$.

   Theoretically, the x-values increase infinitely; therefore, there is no relative maximum. In reality, there would be a maximum number of ringtones that could be downloaded.

8. Identify the domain of the graphed function.

   The least x-value is 0.

   x can be any whole number greater than or equal to 0.

9. Identify any asymptotes of the graphed function.

   The function is a linear function, and linear functions do not have asymptotes; therefore, there are no asymptotes for this function.

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Problem-Based Task 2.4.1: Careful Calculations

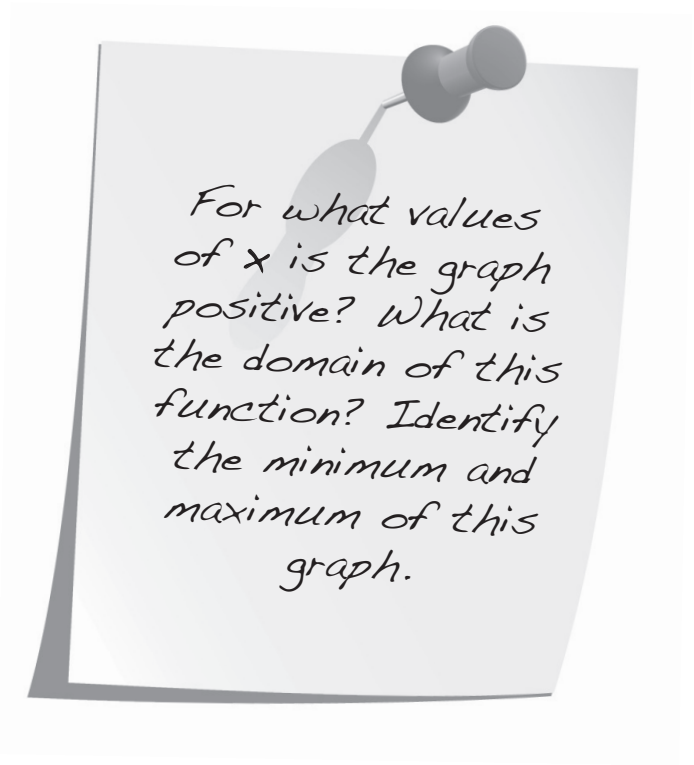
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There are many things to think about when purchasing a new car: year, make, model, included options, and price. As important as these considerations are, it is also important to consider the decreasing value of the car over time.

Use the following information to graph the value of a car over time:

- The average price of a new car in the United States is approximately $30,750.
- The value decreases each year at an exponential rate and approaches an asymptote whose equation is $y = 0$.
- The average car loses nearly half its original value after 5 years.

For what values of x is the graph positive? What is the domain of this function? Identify the minimum and maximum of this graph.



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Problem-Based Task 2.4.1: Careful Calculations

Coaching

a. What is the independent variable for this function?

b. What is the minimum value of this variable?

c. What is the maximum value of this variable?

d. What is an appropriate scale to use to represent this variable on your graph?

e. What is the dependent variable?

f. What is the minimum value of this variable?

g. What is the maximum value of this variable?

h. What is an appropriate scale to use to represent this variable on your graph?

i. Is this situation modeled by a linear function or an exponential function?

j. How does the value after 5 years help you determine the shape of this graph?

k. Why does this graph have an asymptote whose equation is $y = 0$?

l. For what values of x is the graph positive?

m. Is the graph of the function ever negative? Why or why not?

n. What is the minimum of this graph?

o. What is the maximum of this graph?

p. What is the domain of this function?

q. Use these key features to sketch the graph of the function.

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Problem-Based Task 2.4.1: Careful Calculations

Coaching Sample Responses

a. What is the independent variable for this function?

The independent variable represents the number of years.

b. What is the minimum value of this variable?

The minimum value of this variable is 0.

c. What is the maximum value of this variable?

There is not a maximum value because, theoretically, the number of years continues forever.

d. What is an appropriate scale to use to represent this variable on your graph?

An appropriate scale for this variable could be from 0 to 20 years in increments of 1, because 20 years is longer than most people would keep a car.

e. What is the dependent variable?

The dependent variable is the value of the car in dollars.

f. What is the minimum value of this variable?

The minimum value of this variable is 0.

g. What is the maximum value of this variable?

The maximum value of this variable is 30,750.

h. What is an appropriate scale to use to represent this variable on your graph?

An appropriate scale for this variable should be from 0 to at least 30,750. It would be advisable to count by 2,000 or 2,500 in order to maintain a readable graph.

i. Is this situation modeled by a linear function or an exponential function?

This situation is modeled by an exponential function. We are told in the description that the value decreases exponentially.

j. How does the value after 5 years help you determine the shape of this graph?

The value after 5 years gives us a place to plot another point. We know the graph is going to curve toward the x-axis because the car depreciates over time. This gives us one more point to go through in order to create a more accurate graph.

k. Why does this graph have an asymptote whose equation is $y = 0$?

The value of the car will never be $0. The car may not be worth much, but it will always be worth something.

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l. For what values of x is the graph positive?

The graph is positive for all x-values greater than or equal to 0, or $x \geq 0$.

m. Is the graph of the function ever negative? Why or why not?

The graph is never negative because the value of the car is never negative.

n. What is the minimum of this graph?

There is no minimum because the graph is an exponential function.

o. What is the maximum of this graph?

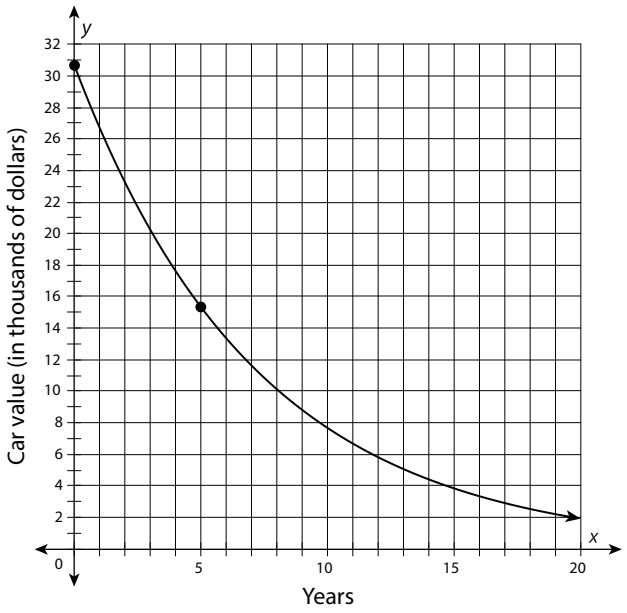
The maximum of the graph occurs at the point (0, 30,750).

p. What is the domain of this function?

The domain of this function is all real numbers greater than or equal to 0, or where $x \geq 0$.

q. Use these key features to sketch the graph of the function.

The maximum, (0, 30,750), represents the price of a new car. The point (5, 15,375) represents the car's value at year 5.



Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.4.1: Identifying Key Features of Linear and Exponential Graphs

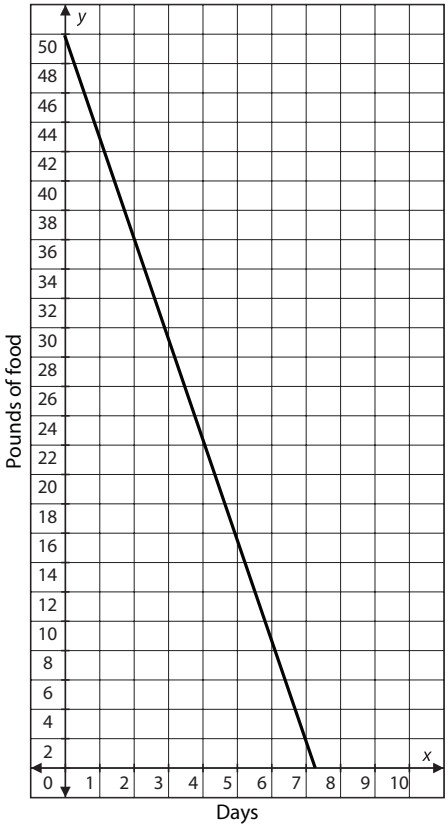
For problems 1 and 2, determine the domain of each function, then graph the function on graph paper.

1. Mikhail receives a base weekly salary of $350 plus a commission of $50 for each vacuum he sells. His weekly earnings can be modeled by the function $f(x) = 50x + 350$.

2. A population of insects begins with 15 insects and doubles every month. The population can be modeled by the function $f(x) = 15(2)^x$.

For problems 3–10, determine the key features of each function. Include the x- and y-intercepts, if they exist, and determine whether the function is increasing or decreasing, whether it's negative or positive, the minimum and maximum values, the asymptotes, and the domain. Find the equations of any asymptotes, assuming that the domain consists of real numbers.

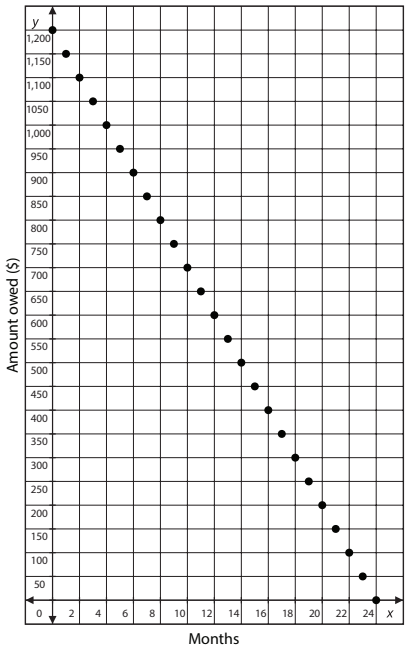
3. You and your friends are out hiking. You start the hike with 50 pounds of food for the group, and eat about 7 pounds each day. Identify the key features of the graph of this function.



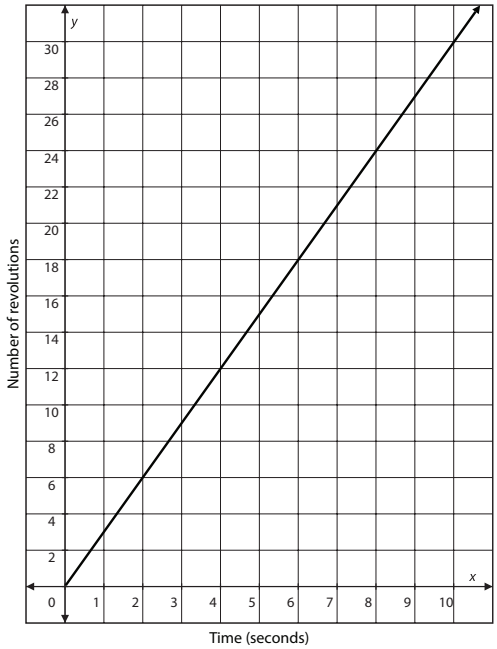
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4. Mason borrowed $1,200 from a friend to buy a new hot tub. His friend doesn't charge any interest and Mason makes $50 payments each month. Identify the key features of the graph of this function.



5. A gear on a machine turns at a rate of 3 revolutions per second. Identify the key features of the graph of this function.



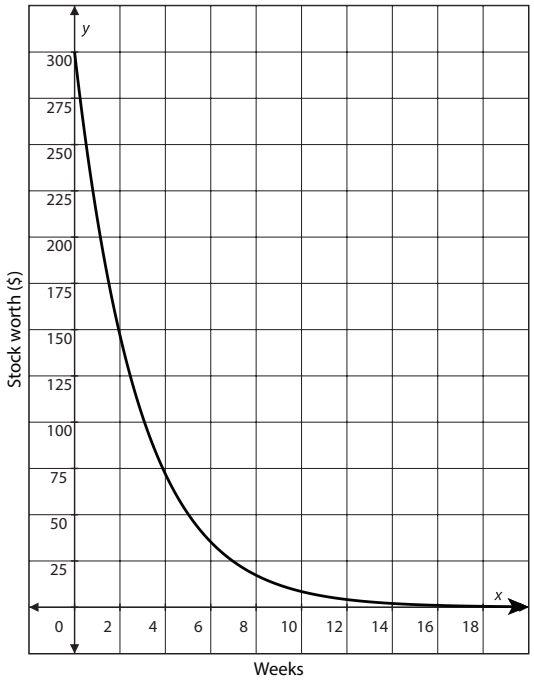
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6. The cost of an air conditioner is $110. The cost to run the air conditioner is $0.35 per minute. Identify the key features of this function.

Minutes (x)	Cost in dollars ($f(x)$)
0	110.00
30	120.50
60	131.00
90	141.50
120	152.00

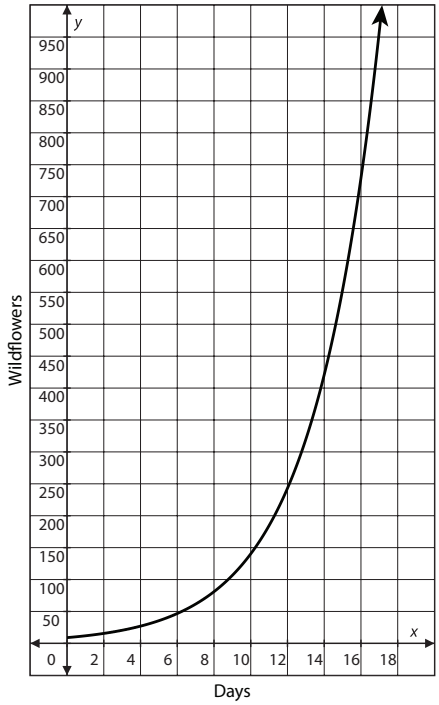
7. A stock is declining at a rate of 50% of its value every 2 weeks. The stock started at $300. Identify the key features of the graph of this function.



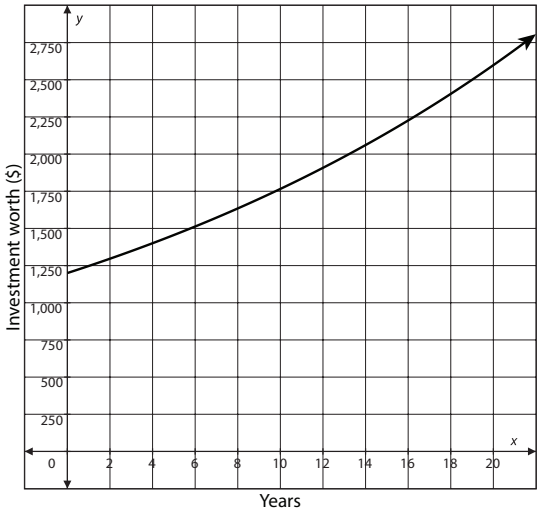
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8. A wildflower species triples in 4 days. A field started with 9 wildflowers in the early spring. Identify the key features of the graph of this function.



9. An investment of $1,200 earns 3.9% interest and is compounded semiannually. Identify the key features of the graph of this function.



continued

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10. An investment of $3,000 earns 1.5% interest and is compounded weekly. Identify the key features of this function.

Year (x)	Investment value in dollars ($f(x)$)
0	3000
1	3045.58
2	3091.84
3	3138.82
4	3186.50
5	3234.91
6	3281.06

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Lesson 2.4.2: Average Rate of Change

Warm-Up 2.4.2

Lupita wants to buy a boat that will have the best resale value after 3 years.

1. At one boat dealer, she found a boat she likes that sells for $15,000 and depreciates at a rate of 30% per year. What will be the value of the boat after 3 years?

2. At another dealer, she found a boat that costs $12,000 and depreciates at a rate of 20% per year. What will be the value of the boat after 3 years?

3. Which boat will have the greater value in 3 years?

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Instruction

Lesson 2.4.2: Average Rate of Change

Common Core State Standards

F–IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

F–LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. ★

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

Warm-Up 2.4.2 Debrief

1. At one boat dealer, she found a boat she likes that sells for $15,000 and depreciates at a rate of 30% per year. What will be the value of the boat after 3 years?

   When the boat is new, it is worth $15,000; then it depreciates at a rate of 30% each year. If the boat decreases in value by 30% per year, then it retains 70% of its value each year. The equation of this function is $f(x) = 15{,}000(0.70^x)$.

   Create a table to show the decrease in value for the first 3 years.

Year	Value of Boat A, in dollars ($)
0	15,000
1	15,000(0.70) = 10,500
2	10,500(0.70) = 7350
3	7350(0.70) = 5145

   Boat A will be worth $5,145 after 3 years.

2. At another dealer, she found a boat that costs $12,000 and depreciates at a rate of 20% per year. What will be the value of the boat after 3 years?

   When the boat is new, it is worth $12,000; then it depreciates at a rate of 20% each year. This means that the boat retains 80% of its value each year. The equation of this function is $f(x) = 12{,}000(0.80^x)$.

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Create a table to show the decrease in value for the first 3 years.

Year	Value of Boat B, in dollars ($)
0	12,000
1	12,000(0.80) = 9600
2	9600(0.80) = 7680
3	7680(0.80) = 6144

Boat B will be worth $6,144 after 3 years.

3. Which boat will have the greater value in 3 years?

   Boat A will be worth $5,145 after 3 years.

   Boat B will be worth $6,144 after 3 years.

   Boat B will be worth more than Boat A after 3 years.

Connection to the Lesson

- Students will be asked to calculate the average rate of change of a function over a specified interval given a graph.
- Students will compare rates of change over various intervals.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- reading and interpreting data from charts and tables (6.EE.9)
- understanding slope (8.EE.5)

Introduction

In previous lessons, we found the slope of the graph of a linear function using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$. We also identified the slope of a line from a given equation by rewriting the equation in **slope-intercept form**, $y = mx + b$, where m is the slope of the line and b is the y-intercept. By calculating the slope, we are able to determine the **rate of change**, or the ratio between the change in the y-values and the corresponding change in the x-values. The rate of change can be determined from graphs, tables, and equations. In this lesson, we will extend our understanding of slope as it applies to exponential functions.

Key Concepts

- A **ratio** is the relation between two quantities. A ratio can be expressed in words, or as a fraction, decimal, or percent.
- A rate of change is a ratio describing how one quantity changes with respect to another quantity.
- Slope is a way to describe the rate of change of a function.
- The **slope** of a line is the ratio of the change in y-values and the change in x-values between two points.
- A positive rate of change signifies that a function increases as x increases.
- A negative rate of change signifies that a function decreases as x increases.
- Linear functions have a constant rate of change, meaning that the y-value increases or decreases the same amount for a given change in x. Linear functions may also be constant, which means that they don't increase or decrease.
- Not all functions change at a constant rate.
- The rate of change of a function within an interval, or a continuous portion of a function, can be calculated.

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Instruction

- The rate of change of a function within an interval is the average rate of change for the interval, defined by values of x in the domain of a function.
- Closed intervals can be noted using the format $[a, b]$, where a represents the initial x-value of the interval and b represents the final x-value of the interval. This is a closed interval because it includes the endpoints of the interval. Another way to state that the interval is closed is $a \le x \le b$.
- A linear function that has a slope of 0 is a horizontal line.
- Vertical lines have an **undefined slope**. This occurs when the denominator of the slope formula is equal to 0.

Calculating Rate of Change from a Table

1. Choose two points from the table.
2. Assign the coordinates of one point to be (x_1, y_1) and the other point to be (x_2, y_2).
3. Substitute the values into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
4. If the difference between the x-coordinates is not 0, the result is the rate of change between the two points.

- The rate of change between any two points of a linear function will be equal.
- The rate of change between any two points of a non-linear function will not be equal for all points chosen, but will instead be an average for the interval determined by the points.

Calculating Rate of Change from an Equation of a Linear Function

1. Transform the given linear equation into slope-intercept form, $f(x) = mx + b$.
2. Identify the slope of the line as m from the equation.
3. The slope of the linear function is the rate of change between all pairs of points on the graph of the function.

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Instruction

Calculating the Average Rate of Change Within an Interval of an Exponential Function

1. Determine the interval to be observed.
2. Determine x_1 by identifying the initial or left-most x-value of the interval and substituting it into the function.
3. Solve for y_1.
4. Determine x_2 by identifying the final or right-most x-value of the interval and substituting it into the function.
5. Solve for y_2.
6. Substitute (x_1, y_1) and (x_2, y_2) into the slope formula to calculate the average rate of change for the interval.
7. The result is the rate of change for the interval between the points.

Common Errors/Misconceptions

- incorrectly choosing the values of the indicated interval to calculate the rate of change
- substituting incorrect values into the slope formula
- assuming the rate of change must remain constant regardless of the type of function
- interpreting interval notation as coordinates

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Instruction

Guided Practice 2.4.2

Example 1

Janie invests \$1,300 at a rate of 2.6%, compounded monthly. The function that models this situation is $f(x)=1300\left(1+\frac{0.026}{12}\right)^{12x}$, where x represents time in years. What is the average rate of change for the interval [1, 4]?

1. Determine the interval to be observed.

   The interval to be observed is [1, 4], or the interval where $1 \le x \le 4$.

2. Determine (x_1, y_1), the coordinates of the initial point of the interval.

   The initial x-value is 1.

   Substitute the value 1 into the given function.

   $f(x)=1300\left(1+\frac{0.026}{12}\right)^{12x}$ Given function

   $f(1)=1300\left(1+\frac{0.026}{12}\right)^{12(1)}$ Substitute 1 for the value of x.

   $f(1)=1300\left(1+\frac{0.026}{12}\right)^{12}$ Simplify as needed.

   $f(1) = 1300(1.002)^{12}$

   $f(1) \approx 1334.21$

   (x_1, y_1) is (1, 1334.21).

SWB p. 101; SRB p. U2-108

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Instruction

3. Determine (x_2, y_2), the coordinates of the endpoint of the interval.

The ending x-value is 4.

Substitute the value 4 into the given function.

$$f(x)=1300\left(1+\frac{0.026}{12}\right)^{12x}$$ Given function

$$f(4)=1300\left(1+\frac{0.026}{12}\right)^{12(4)}$$ Substitute 4 for the value of x.

$$f(4)=1300\left(1+\frac{0.026}{12}\right)^{48}$$ Simplify as needed.

$f(4) \approx 1442.32$

(x_2, y_2) is (4, 1442.32).

4. Substitute (x_1, y_1) and (x_2, y_2) into the slope formula to calculate the rate of change.

$$m=\frac{y_2-y_1}{x_2-x_1}$$ Slope formula

$$=\frac{(1442.32)-(1334.21)}{(4)-(1)}$$ Substitute (1, 1334.21) for (x_1, y_1) and (4, 1442.32) for (x_2, y_2).

$$=\frac{108.11}{3}$$ Simplify as needed.

≈ 36.04

The average rate of change for the interval [1, 4] is \$36.04 per year.

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Example 2

In 2008, about 66 million U.S. households had both landline phones and cell phones. This number decreased by an average of 5 million households per year. Use the following table to calculate the average rate of change for the interval [2008, 2011].

Year (x)	Households in millions ($f(x)$)
2008	66
2009	61
2010	56
2011	51

1. Determine the interval to be observed.

   The interval to be observed is [2008, 2011], or where $2008 \le x \le 2011$.

2. Determine (x_1, y_1), the coordinates of the initial point of the interval.

   The initial x-value is 2008 and the corresponding y-value is 66; therefore, (x_1, y_1) is (2008, 66).

3. Determine (x_2, y_2), the coordinates of the endpoint of the interval.

   The ending x-value is 2011 and the corresponding y-value is 51; therefore, (x_2, y_2) is (2011, 51).

4. Substitute (x_1, y_1) and (x_2, y_2) into the slope formula to calculate the rate of change.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(51) - (66)}{(2011) - (2008)}$$ Substitute (2008, 66) for (x_1, y_1) and (2011, 51) for (x_2, y_2).

$$= \frac{-15}{3}$$ Simplify as needed.

$$= -5$$

The average rate of change for the interval [2008, 2011] is 5 million households per year.

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Instruction

Example 3

A slow-growing strain of bacteria doubles every 36 hours. A Petri dish starts out with 12 bacteria. Use the following table to calculate the average rate of change for the interval [2, 5].

Days (x)	Number of bacteria ($f(x)$)
0	12
1	19
2	30
3	48
4	76
5	121
6	192

1. Determine the interval to be observed.

   The interval to be observed is [2, 5], or where $2 \le x \le 5$.

2. Determine (x_1, y_1), the coordinates of the initial point of the interval.

   The initial x-value is 2 and the corresponding y-value is 30; therefore, (x_1, y_1) is (2, 30).

3. Determine (x_2, y_2), the coordinates of the endpoint of the interval.

   The ending x-value is 5 and the corresponding y-value is 121; therefore, (x_2, y_2) is (5, 121).

4. Substitute (x_1, y_1) and (x_2, y_2) into the slope formula to calculate the rate of change.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(121) - (30)}{(5) - (2)}$$ Substitute (2, 30) and (5, 121) for (x_1, y_1) and (x_2, y_2).

$$= \frac{91}{3}$$ Simplify as needed.

$$\approx 30.3$$

The average rate of change for the interval [2, 5] is approximately 30 bacteria per day.



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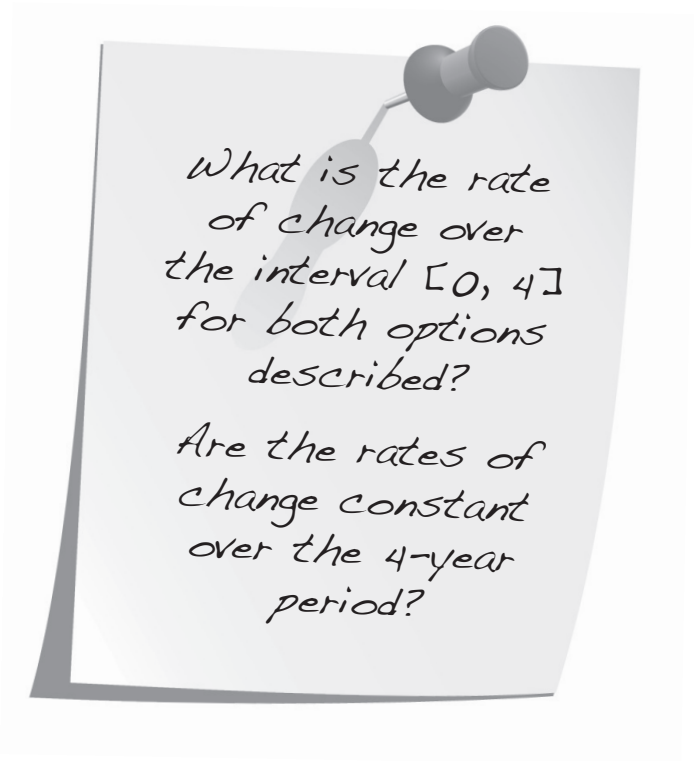
Problem-Based Task 2.4.2: Costing Up a Storm

SMP	
1	2
3	4 ✓
5	6 ✓
7 ✓	8

After a recent storm, many homeowners reported damages to their insurance company in order to receive money for their losses. One insurance company has two different methods for calculating the current value of household items. Many household items lose value, or depreciate, over time. For example, a desk originally purchased 4 years ago for $487.65 will be worth a different amount now depending on the method used to calculate its worth. The following table illustrates the changing value of the desk over a 4-year period, based on the two different calculation methods.

Year	Option A, in dollars ($)	Option B, in dollars ($)
0	487.65	487.65
1	390.12	341.36
2	292.59	238.95
3	195.06	167.26
4	97.53	117.08

What is the rate of change over the interval [0, 4] for both options described? Explain what this means in terms of the scenario. Are the rates of change constant over the 4-year period?



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Problem-Based Task 2.4.2: Costing Up a Storm

Coaching

a. What is the interval to be observed?

b. What is the initial point of the interval for Option A?

c. What is the ending point of the interval for Option A?

d. What is the rate of change for Option A over the interval specified?

e. What does the rate of change for Option A mean in terms of this scenario?

f. What is the initial point of the interval for Option B?

g. What is the ending point of the interval for Option B?

h. What is the rate of change for Option B over the interval specified?

i. What does the rate of change for Option B mean in terms of this scenario?

j. How does the rate of change for Option A compare to the rate of change for Option B over the same interval? What does this mean in terms of the real-world context?

k. Is the rate of change for Option A constant over the 4-year period?

l. Is the rate of change for Option B constant over the 4-year period?

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Instruction

Problem-Based Task 2.4.2: Costing Up a Storm

Coaching Sample Responses

a. What is the interval to be observed?

The interval to be observed is [0, 4], or where $0 \le x \le 4$.

b. What is the initial point of the interval for Option A?

The initial point of the interval is (0, 487.65).

c. What is the ending point of the interval for Option A?

The ending point of the interval is (4, 97.53).

d. What is the rate of change for Option A over the interval specified?

To calculate the rate of change for the interval [0, 4], use the slope formula. Let (0, 487.65) be (x_1, y_1) and (4, 97.53) be (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{(97.53) - (487.65)}{(4) - (0)}$$
$$= \frac{-390.12}{4}$$
$$\approx -97.53$$

The rate of change for Option A over the interval [0, 4] is –97.53.

e. What does the rate of change for Option A mean in terms of this scenario?

Option A represents a linear function, because the value decreases by the same amount each year. Over the 4 years since the desk was bought, the value decreased by $97.53 each year.

f. What is the initial point of the interval for Option B?

The initial point of the interval is (0, 487.65).

g. What is the ending point of the interval for Option B?

The ending point of the interval is (4, 117.08).

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Instruction

h. What is the rate of change for Option B over the interval specified?

To calculate the rate of change for the interval [0, 4], use the slope formula. Let (0, 487.65) be (x_1, y_1) and (4, 117.08) be (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{(117.08) - (487.65)}{(4) - (0)}$$
$$= \frac{-370.57}{4}$$
$$\approx -92.64$$

The rate of change for the interval [0, 4] is –92.64.

i. What does the rate of change for Option B mean in terms of this scenario?

Over the 4 years since the desk was bought, the value decreased by an average of $92.64 each year.

j. How does the rate of change for Option A compare to the rate of change for Option B over the same interval? What does this mean in terms of the real-world context?

The rate of change over the interval [0, 4] for Option A is –$97.53 per year, and the rate of change for Option B is –$92.64 per year. This means that the value of the desk decreases at a faster rate with Option A than with Option B.

k. Is the rate of change for Option A constant over the 4-year period?

Choose a second interval to determine if the rate is constant. One possible interval is [1, 3].

Let $(x_1, y_1) = (1, 390.12)$ and $(x_2, y_2) = (3, 195.06)$. Calculate the rate of change.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{(195.06) - (390.12)}{(3) - (1)}$$
$$= \frac{-195.06}{2}$$
$$\approx -97.53$$

The rate of change for Option A over the interval [1, 3] is –97.53. This is the same rate of change as was calculated for the interval [0, 4]. Any interval chosen for Option A will have the same rate of change because Option A represents a linear function.

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l. Is the rate of change for Option B constant over the 4-year period?

Choose a second interval to determine if the rate is constant.

One possible interval is [1, 3].

Let $(x_1, y_1) = (1, 341.36)$ and $(x_2, y_2) = (3, 167.26)$.

Calculate the rate of change.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{(167.26) - (341.36)}{(3) - (1)}$$
$$= \frac{-174.10}{2}$$
$$\approx -87.05$$

The rate of change for Option B over the interval [1, 3] is –87.05. This rate of change is different from the rate of change for the interval [0, 4]. Any interval chosen for Option B will result in a different rate of change because Option B represents an exponential function.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.4.2: Average Rate of Change

For problems 1–5, calculate the rate of change for each scenario described.

1. The Beechcraft 1900D is a commuter airplane with a fuel capacity of 665 gallons. The function that represents how the amount of fuel changes as a function of distance flown is $f(x) = -0.9x + 665$, where x represents miles flown, and $f(x)$ represents the amount of fuel remaining. What is the rate of change for this scenario?

2. The velocity of a ball thrown directly upward can be modeled by the function $f(x) = -32x + 96$, where x represents time in seconds, and $f(x)$ represents the height of the ball above the ground in feet. What is the rate of change for this scenario?

3. An investment of \$900 is invested monthly at a rate of 4%. The function that models this situation is $f(x)=900\left(1+\frac{0.04}{12}\right)^{12x}$, where x represents time in years, and $f(x)$ represents the worth of the investment. What is the rate of change for this function over the interval [3, 10]?

4. The price of a stock started out at \$150 per share and has declined to 75% of its value every 2 weeks. The function that models this decline is $f(x)=150(0.75)^{\frac{x}{2}}$, where x represents time in weeks, and $f(x)$ represents the price of the stock. What is the rate of change in the price of the stock over the interval [1, 4]?

5. The following table lists common Celsius to Fahrenheit degree conversions. What is the rate of change for this function?

°C (x)	°F ($f(x)$)
0	32
10	50
20	68
30	86
40	104

continued

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The following table represents the total cost of a cell phone plan. Use the table to answer questions 6 and 7.

Number of minutes (x)	Total cost in dollars ($f(x)$)
0	35
250	55
500	75
750	95
1000	115

6. What is the rate of change for this function over the interval [250, 1000]?

7. What is the rate of change for this function over the interval [250, 750]?

Use the given information to complete problem 8.

8. A Petri dish starts out with 15 bacteria. The number of bacteria doubles every 12 minutes. Use the following table to calculate the rate of change for the interval [24, 48].

Minutes (x)	Number of bacteria ($f(x)$)
0	15
12	30
24	60
36	120
48	240

The following table represents the worth every two years of an initial investment of $500 that earns 2.6% interest, compounded quarterly. Use the table to answer questions 9 and 10.

Years (x)	Investment value in dollars ($f(x)$)
0	500.00
2	526.60
4	554.61
6	584.12
8	615.19

9. What is the rate of change for this function over the interval [0, 4]?

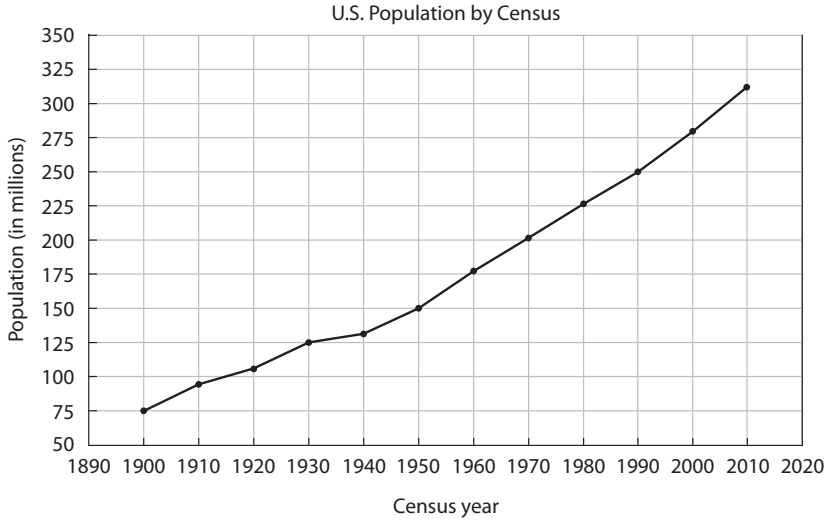
10. What is the rate of change for this function over the interval [2, 8]?

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Lesson 2.4.3: Recognizing Average Rate of Change

Warm-Up 2.4.3

The following graph shows the approximate United States population from 1900 to 2010, as recorded by the U.S. Census Bureau.



1. What was the rate of change in the population from 1900 to 2000? Is this greater or less than the rate of change in the population from 2000 to 2010?

2. Which 10-year time periods have the highest and the lowest rates of change? How did you find these?

3. What do you predict the U.S. population will be in 2020? Explain your reasoning.

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Instruction

Lesson 2.4.3: Recognizing Average Rate of Change

Common Core State Standards

F–IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

F–LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Warm-Up 2.4.3 Debrief

1. What was the rate of change in the population from 1900 to 2000? Is this greater or less than the rate of change in the population from 2000 to 2010?

   The rate of change in the population from 1900 to 2000 can be found using the slope formula, $m=\frac{y_2-y_1}{x_2-x_1}$.

   Determine the population for the years 1900 and 2000.

   According to the graph, the population in 1900 was approximately 75 million and the population in 2000 was approximately 278 million. Let (1900, 75) and (2000, 278) represent the coordinates of the two points.

   Substitute the values for (x_1, y_1) and (x_2, y_2) into the slope formula to calculate the rate of change for this interval.

   $$m=\frac{y_2-y_1}{x_2-x_1}$$

   $$\frac{(278)-(75)}{(2000)-(1900)}=\frac{203}{100}=2.03$$

   The rate of change in the population from 1900 to 2000 was approximately 2.03 million per year.

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Instruction

Determine the rate of change in the population from 2000 to 2010.

According to the graph, the population in 2010 was approximately 313 million. Let (2000, 278) and (2010, 313) represent the coordinates of the two points on the graph.

Substitute the values for (x_1, y_1) and (x_2, y_2) into the slope formula to calculate the rate of change.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{(313) - (278)}{(2010) - (2000)} = \frac{35}{10} = 3.5$$

The rate of change in the population from 2000 to 2010 was approximately 3.5 million per year.

Compare the two rates.

The rate of change for 1900 to 2000 was less than the rate of change for 2000 to 2010, so the population did not increase as fast during this earlier period.

2. Which 10-year time periods have the highest and the lowest rates of change? How did you find these?

   Calculate the rate of change for each 10-year period.

   The intervals from 1950 to 1960 and from 1990 to 2000 can be described as the 10-year periods with the highest rate of change in the population.

   The 10-year period with the lowest rate of change was 1930 to 1940.

3. What do you predict the U.S. population will be in 2020? Explain your reasoning.

   Answers will vary, but a reasonable prediction is 348,000,000 people. This will happen if the population continues to increase at a rate of 3.5 million per year.

Connection to the Lesson

- Students will be asked to calculate the average rate of change of a function over a specified interval given a graph.
- Students will compare rates of change over various intervals.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- reading the coordinates of points from a graph (5.G.1)
- applying the order of operations (5.OA.1)
- interpreting interval notation (no standard)

Introduction

Previously, we calculated the rates of change of linear functions and the rates of change over intervals of exponential functions. We observed that the rate of change of an exponential function changed when we focused on different intervals of the function. Here, we will focus on graphs of linear and exponential functions and learn how to estimate the rates of change.

Key Concepts

- To determine the rate of change of a function, first identify the coordinates of the endpoints of the interval being observed.
- Sometimes it is necessary to estimate the values for y.
- The resulting calculation may be an estimation of the rate of change for the interval identified for the given function.

Estimating Rate of Change from a Graph

1. Determine the interval to be observed.
2. Identify (x_1, y_1) as the initial point of the interval.
3. Identify (x_2, y_2) as the endpoint of the interval.
4. Substitute (x_1, y_1) and (x_2, y_2) into the slope formula to calculate the rate of change.
5. The result is the estimated rate of change for the interval between the two points identified.

Common Errors/Misconceptions

- incorrectly choosing the values of the indicated interval to estimate the rate of change
- incorrectly estimating the values for the indicated interval
- substituting incorrect values into the slope formula

SRB p. U2-115

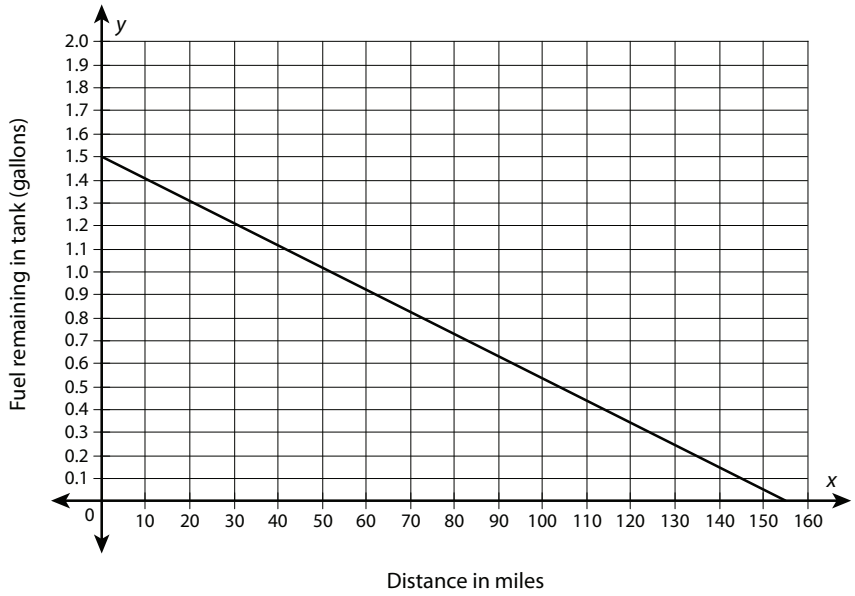
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Instruction

Guided Practice 2.4.3

Example 1

The following graph compares the distance a small motor scooter can travel in miles as a function of the number of gallons of fuel used. What is the rate of change for this scenario?



1. Determine the interval to be observed.

   The function is linear, so the rate of change will be constant for any interval of the function.

2. Choose the initial point of the interval.

   Choose a point on the graph with coordinates that are easy to estimate.

   It appears as though the line crosses the *y*-axis at the point (0, 1.5).

   Let (0, 1.5) be the initial point of the interval.

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Instruction

3. Choose an endpoint of the interval.

   Choose another point on the graph with coordinates that are easy to estimate.

   It appears as though the line crosses the x-axis at the point (155, 0).

   Let (155, 0) be the endpoint of the interval.

4. Substitute the two points into the slope formula to calculate the rate of change.

   The two points are (0, 1.5) and (155, 0).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(0) - (1.5)}{(155) - (0)}$$ Substitute (0, 1.5) for (x_1, y_1) and (155, 0) for (x_2, y_2).

$$= \frac{-1.5}{155}$$ Simplify.

$$\approx -0.01$$

   The rate of change for this function is approximately –0.01 gallons per mile. The amount of fuel decreases by 0.01 gallons per mile.

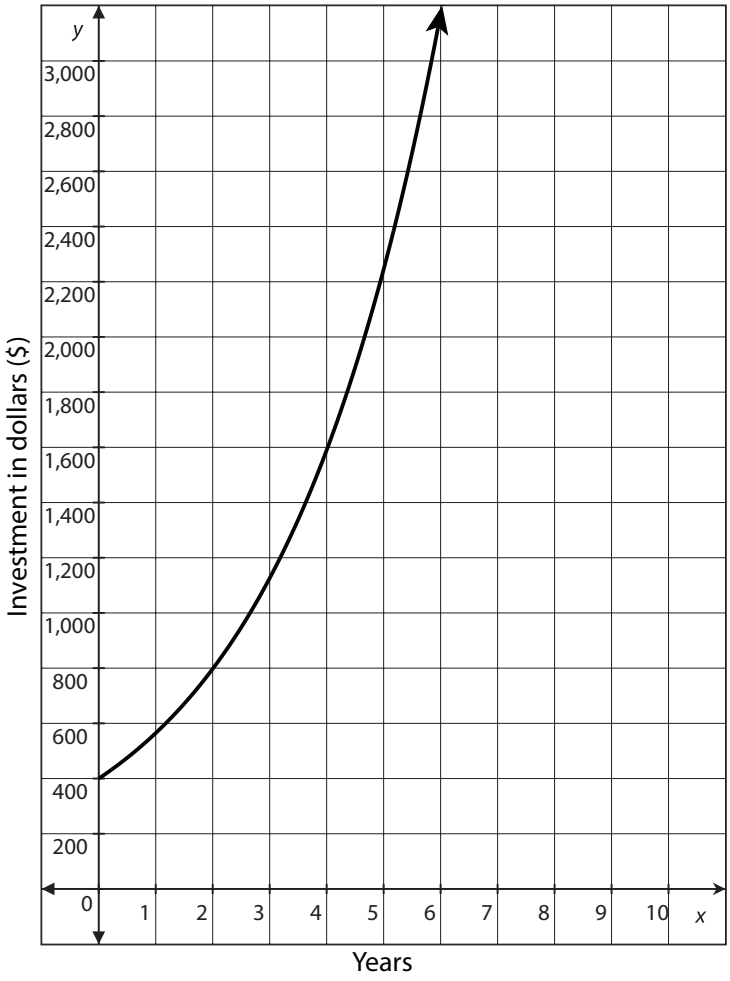


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Instruction

Example 2

Jasper has invested an amount of money in a savings account. The following graph shows the value of his investment over a period of time. What is the rate of change for the interval [1, 3]?



1. Determine the interval to be observed.

   The interval to observe is [1, 3], or where $1 \leq x \leq 3$.

2. Identify the initial point of the interval.

   The x-value of the initial point is 1. The corresponding y-value is approximately 550.

   The coordinates of the initial point of the interval are about (1, 550).

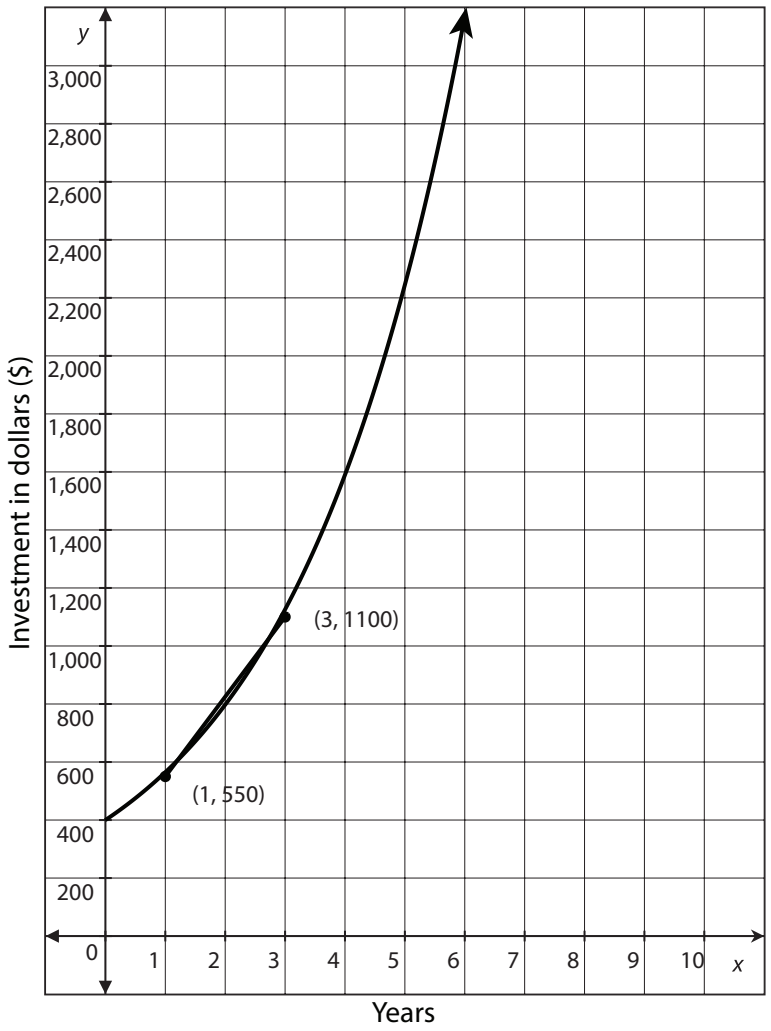
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Instruction

3. Identify the endpoint of the interval.

The x-value for the endpoint is 3. The corresponding y-value is approximately 1,100.

The coordinates of the endpoint of the interval are about (3, 1100).



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4. Substitute the coordinates of the two points into the slope formula to calculate the rate of change.

   The coordinates of the two points are (1, 550) and (3, 1100).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(1100) - (550)}{(3) - (1)}$$ Substitute (1, 550) for (x_1, y_1) and (3, 1100) for (x_2, y_2).

$$= \frac{550}{2}$$ Simplify.

$$= 275$$

   The rate of change for this function over the interval [1, 3] is approximately \$275 per year.

Example 3

Jasper is curious about how the rate of change for his savings account differs for the interval [3, 6]. Calculate the rate of change using the graph from Example 2.

1. Determine the interval to be observed.

   The interval to observe is [3, 6], or where $3 \leq x \leq 6$.

2. Identify the initial point of the interval.

   The x-value of the initial point is 3. The corresponding y-value is approximately 1,100.

   The coordinates of the initial point of the interval are about (3, 1100).

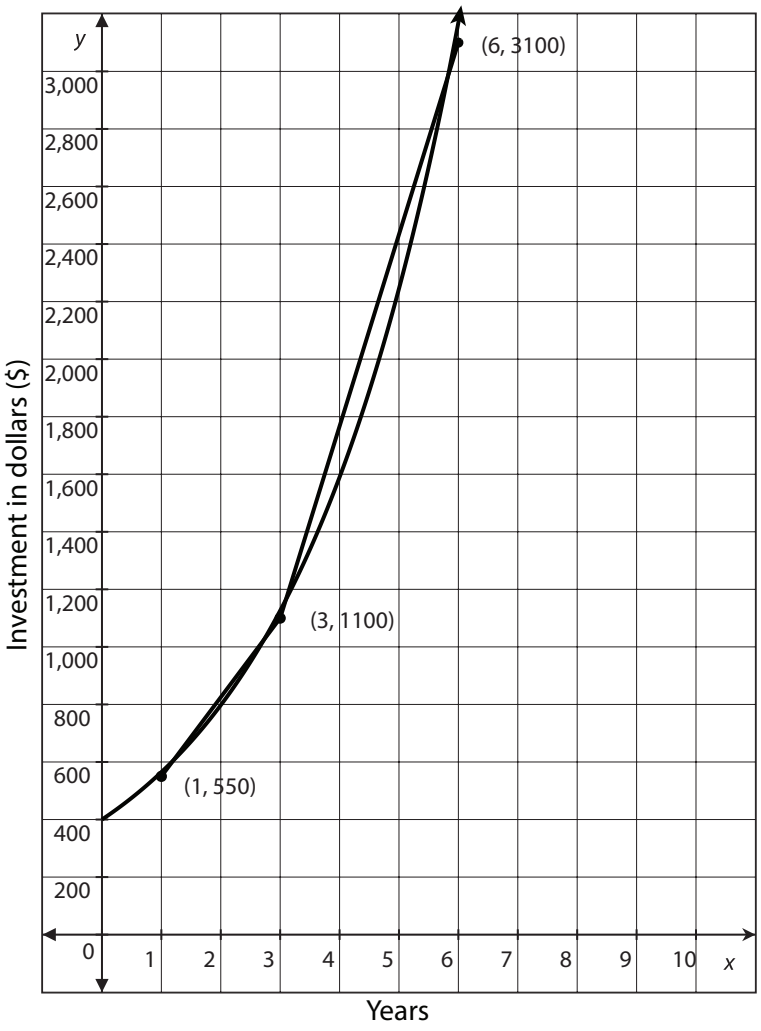
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Instruction

3. Identify the endpoint of the interval.

The x-value for the endpoint is 6. The corresponding y-value is approximately 3,100.

The coordinates of the endpoint of the interval are about (6, 3100).



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Instruction

4. Substitute the coordinates of the points into the slope formula to calculate the rate of change.

   The coordinates of the points are (3, 1100) and (6, 3100).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(3100) - (1100)}{(6) - (3)}$$ Substitute (3, 1100) for (x_1, y_1) and (6, 3100) for (x_2, y_2).

$$= \frac{2000}{3}$$ Simplify.

$$\approx 666.67$$

   The rate of change for this function over the interval [3, 6] is approximately \$666.67 per year.

   Notice that the rate of change of the worth of the investment over the interval [3, 6] is much greater than that of the interval [1, 3]. The graph is also steeper over this interval.



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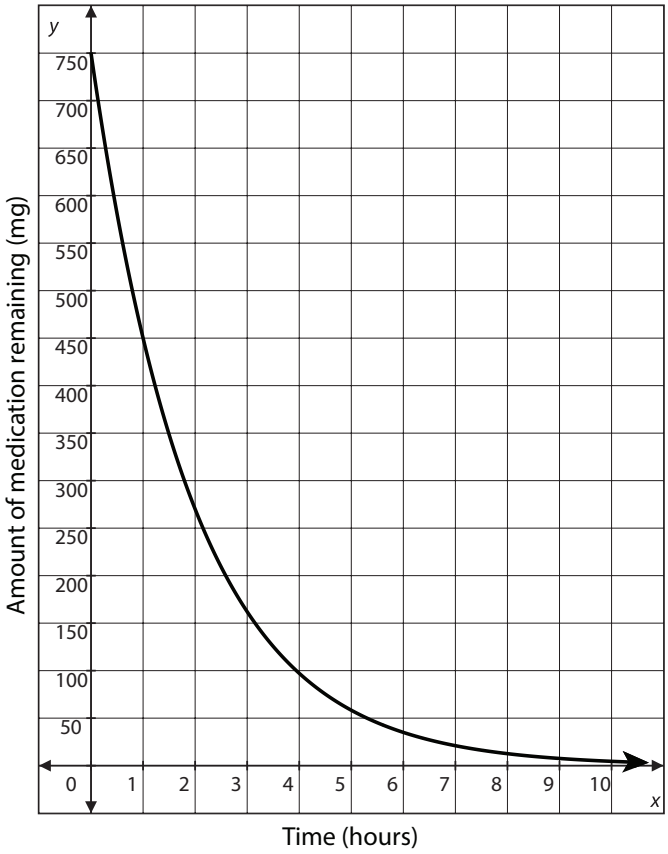
Problem-Based Task 2.4.3: Dwindling Concentrations

SMP	
1 ✓	2 ✓
3 ✓	4
5	6
7	8

The liver and kidneys work to eliminate medications from the body exponentially. It is important for doctors to understand how much of any medication remains in the body after a certain period of time in order to prescribe the correct dosage. The following graph shows the amount in milligrams of a certain medication remaining in the body each hour after one dose.

What is the rate of change in the amount of medication in the body for the interval [1, 3]? How does this rate of change compare to the rate of change for the interval [3, 10]?

It is not uncommon for more than one dose of medication to be prescribed. Based on your findings, why would a patient be instructed to take a second dose of medication 12 hours after the initial dose?



Why would a patient be instructed to take a second dose of medication 12 hours after the initial dose?

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Problem-Based Task 2.4.3: Dwindling Concentrations

Coaching

a. How much medication remains in the body after 1 hour?

b. How much medication remains in the body after 3 hours?

c. What is the rate of change in the amount of medication in the body for the interval [1, 3]?

d. How much medication remains in the body after 10 hours?

e. What is the rate of change for the interval [3, 10]?

f. How does the rate of change for the interval [1, 3] compare to the rate of change for the interval [3, 10]? What does this mean in terms of the real-world context?

g. How much medication do you predict remains in the body after 12 hours?

h. Why would a patient be instructed to take a second dose of medication 12 hours after the initial dose?

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Instruction

Problem-Based Task 2.4.3: Dwindling Concentrations

Coaching Sample Responses

a. How much medication remains in the body after 1 hour?

After 1 hour, approximately 450 milligrams of medication remain in the body.

b. How much medication remains in the body after 3 hours?

After 3 hours, approximately 160 milligrams of medication remain.

c. What is the rate of change in the amount of medication in the body for the interval [1, 3]?

To calculate the rate of change for the interval [1, 3], use the slope formula. Let (x_1, y_1) be (1, 450) and (x_2, y_2) be (3, 160).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{(160) - (450)}{(3) - (1)}$$
$$= \frac{-290}{2}$$
$$= -145$$

The rate of change for the interval [1, 3] is approximately –145 milligrams per hour.

d. How much medication remains in the body after 10 hours?

After 10 hours, approximately 5 milligrams of medication remain.

e. What is the rate of change for the interval [3, 10]?

To calculate the rate of change for the interval [3, 10], use the slope formula. Let (x_1, y_1) be (3, 160) and (x_2, y_2) be (10, 5).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{(5) - (160)}{(10) - (3)}$$
$$= \frac{-155}{7}$$
$$\approx -22$$

The rate of change for the interval [3, 10] is approximately –22 milligrams per hour.

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f. How does the rate of change for the interval [1, 3] compare to the rate of change for the interval [3, 10]? What does this mean in terms of the real-world context?

The slope for the interval [1, 3] is steeper than the slope for the interval [3, 10]. This means the medication is eliminated from the body at a much faster rate within the first 3 hours of the initial dose than over the next 7 hours.

g. How much medication do you predict remains in the body after 12 hours?

The amount of medication in the body 12 hours after the initial dose is very close to 0 milligrams.

h. Why would a patient be instructed to take a second dose of medication 12 hours after the initial dose?

There is virtually no medication left in the body 12 hours after the initial dose. In order for this particular medication to be effective, more medication would need to enter the body.

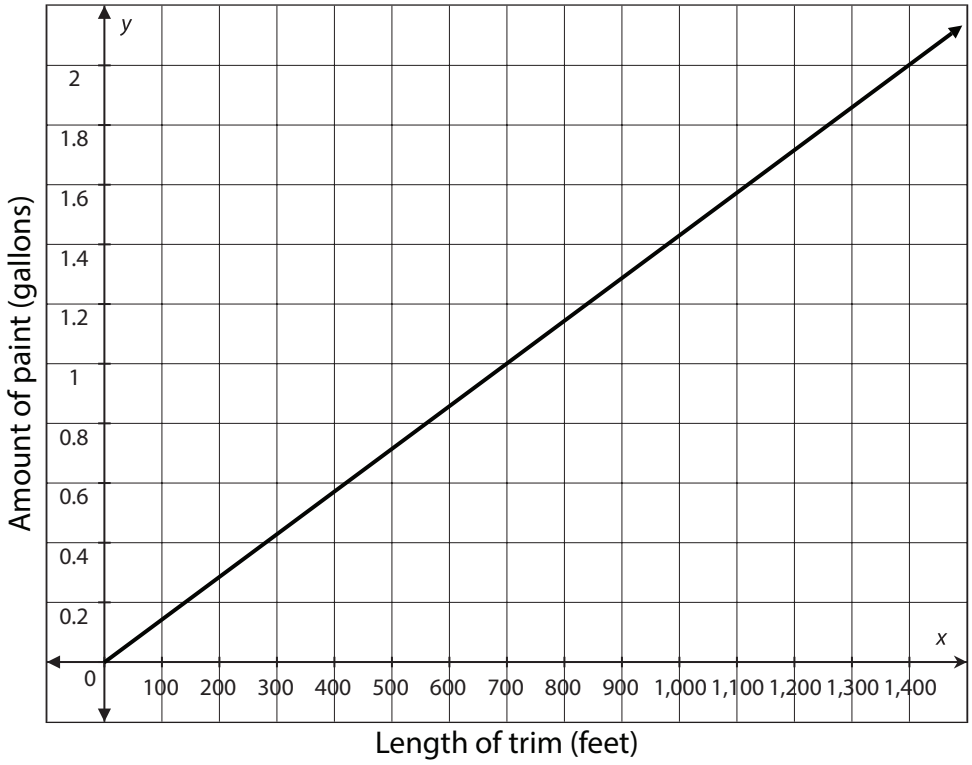
Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.4.3: Recognizing Average Rate of Change

The following graph shows the amount of paint needed to paint the trim molding of a house as a function of the length of trim to be painted. Use the graph to answer questions 1 and 2.



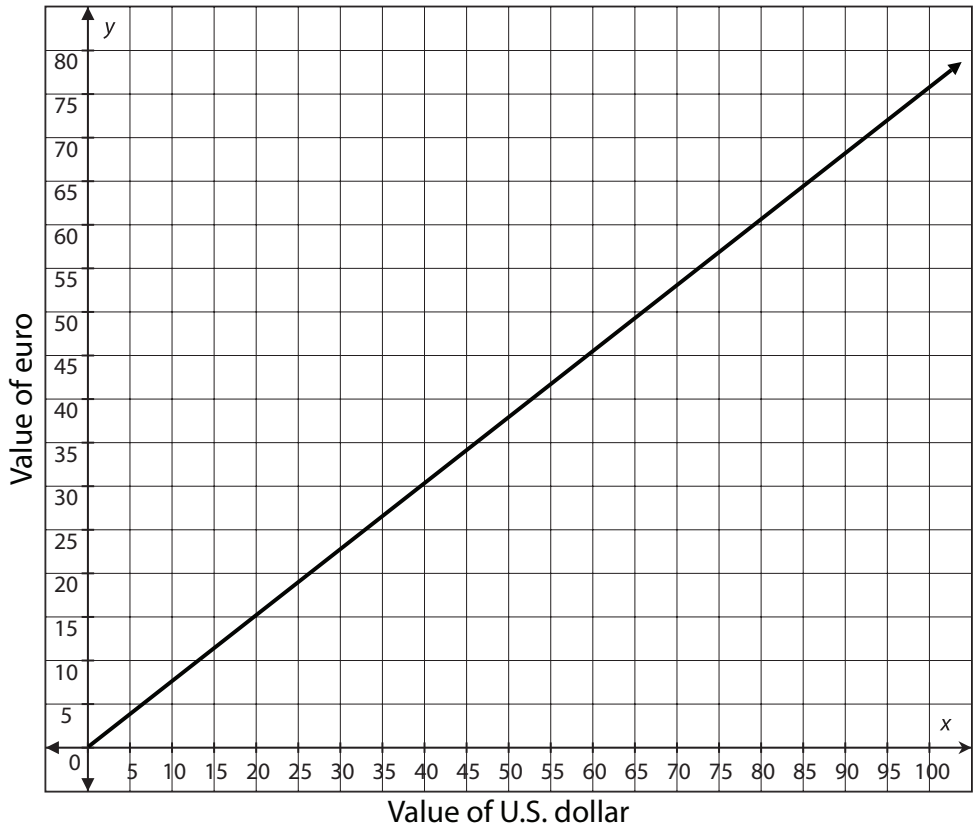
1. What is the approximate rate of change for the interval [150, 500]?

2. What is the approximate rate of change for the interval [900, 1300]?

continued

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The following graph shows the value of the U.S. dollar compared to the value of the euro on a specific day. Use the graph to answer questions 3–5.



3. What is the approximate rate of change in the number of euros per dollar for the interval [10, 20]?

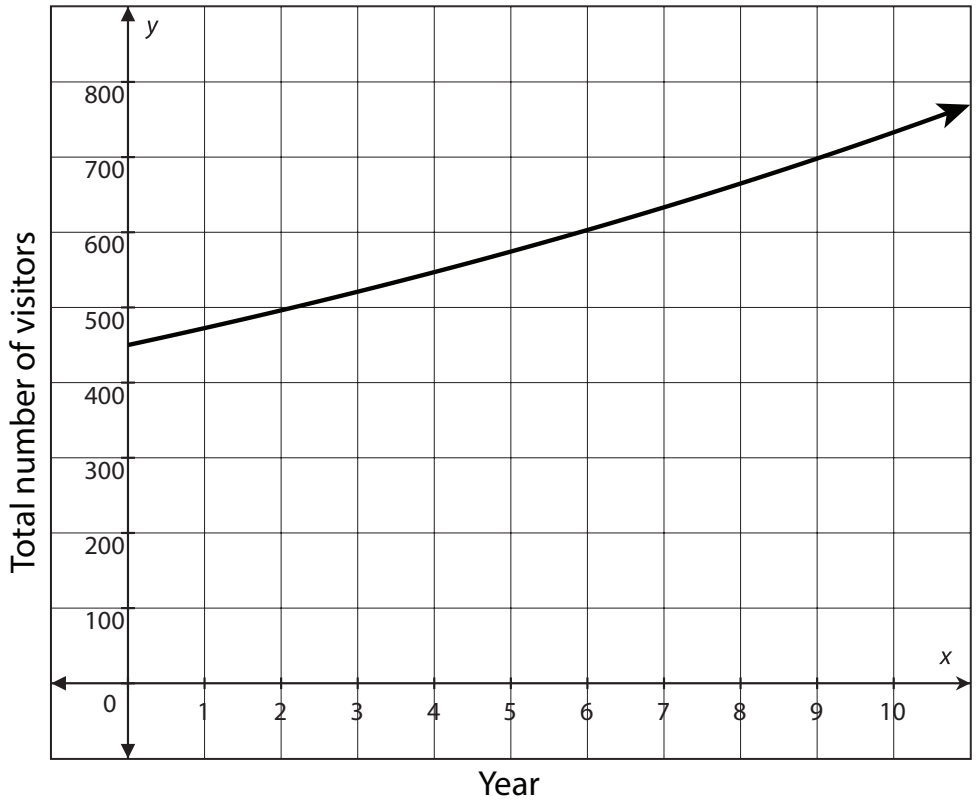
4. What is the approximate rate of change in the number of euros per dollar for the interval [40, 55]?

5. Could you predict the rate of change for a third interval on the same graph? If so, what is your prediction?

continued

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Each year, employees at a wildlife park record the total number of visitors to the park over Labor Day weekend. The following graph shows the number of visitors for each of the last 10 years. Use the graph to answer questions 6 and 7.



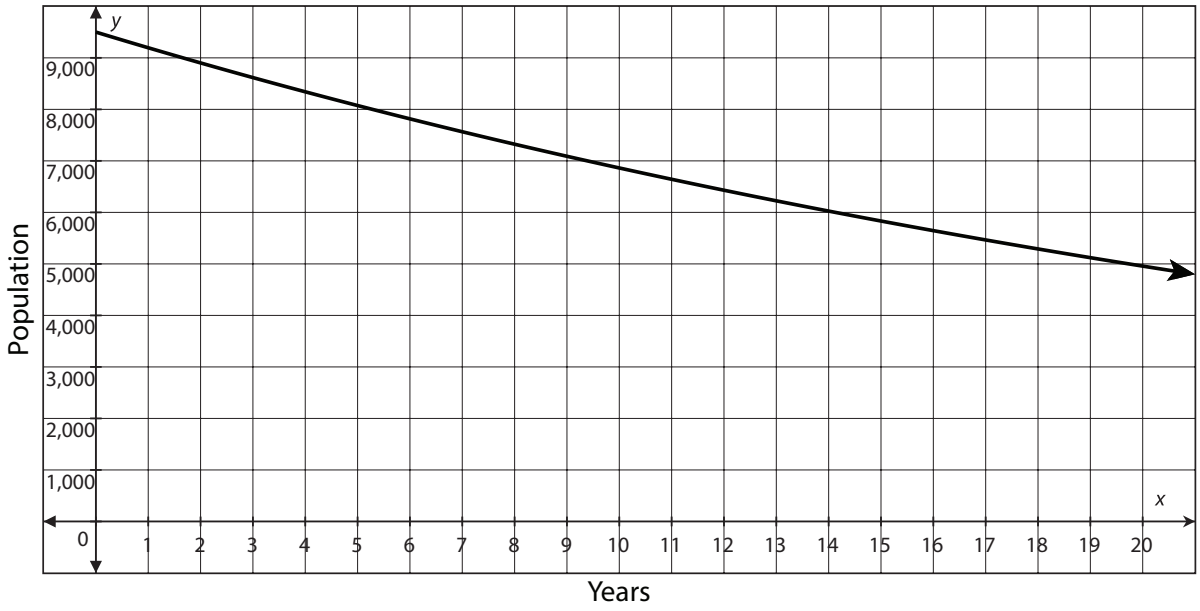
6. What is the approximate rate of change for the interval [2, 6]?

7. What is the approximate rate of change for the interval [2, 9]?

continued

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The following graph shows the yearly population of a small town. Use the graph to answer questions 8–10.



8. What is the approximate rate of change for the interval [2, 9]?

9. What is the approximate rate of change for the interval [14, 20]?

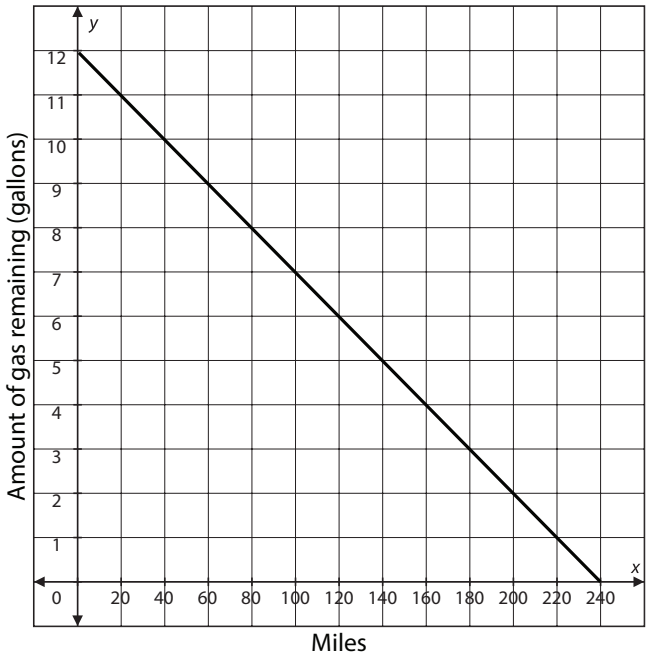
10. How does the rate of change differ for each interval?

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Progress Assessment

Circle the letter of the best answer.

1. The following graph can be described as:



a. having no maximum

b. having no minimum

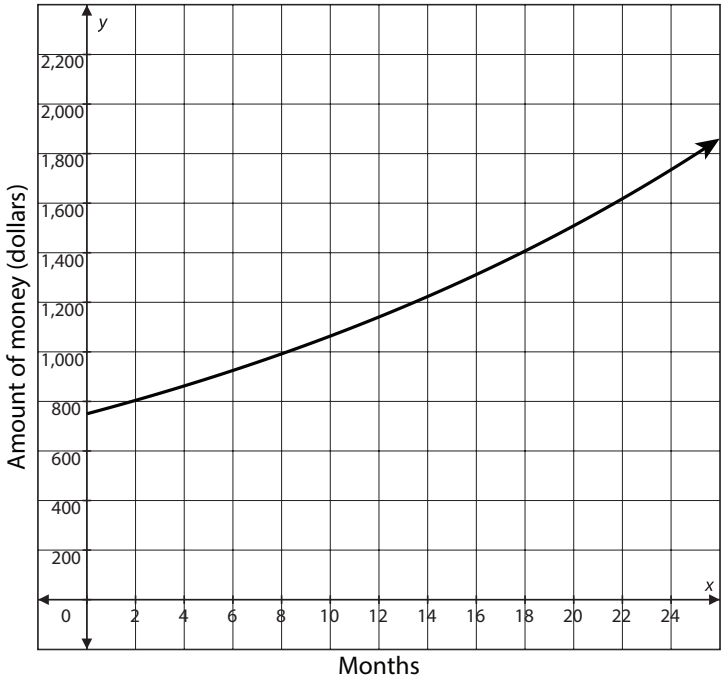
c. having a maximum of 12 and a minimum of 0

d. having a maximum of 240 and a minimum of 0

continued

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2. The following graph can be described as:



a. a positive function that is increasing

b. a positive function that is decreasing

c. a negative function that is increasing

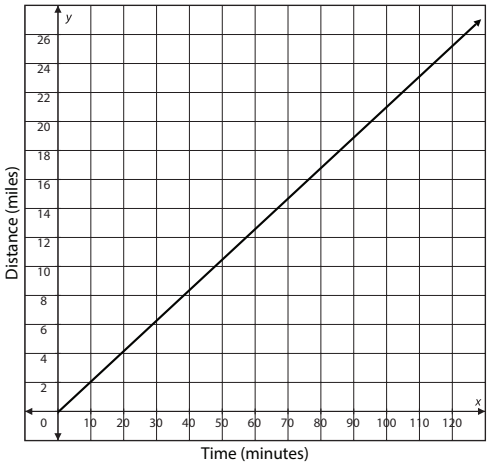
d. a negative function that is decreasing

continued

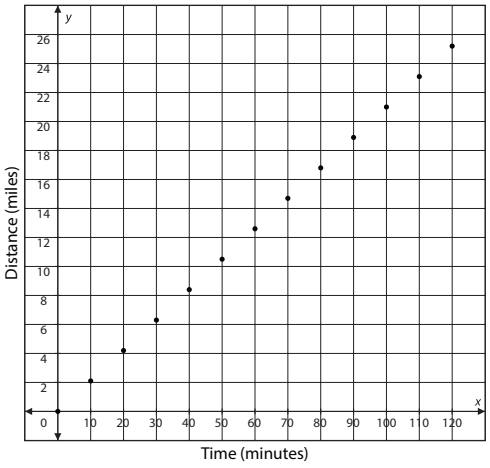
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3. Which of the following graphs best represents the number of miles biked after a period of time?

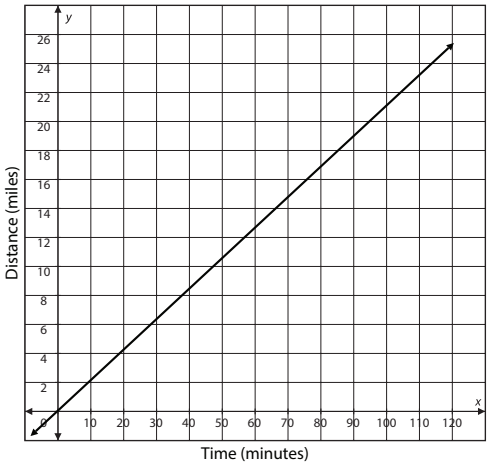
a.



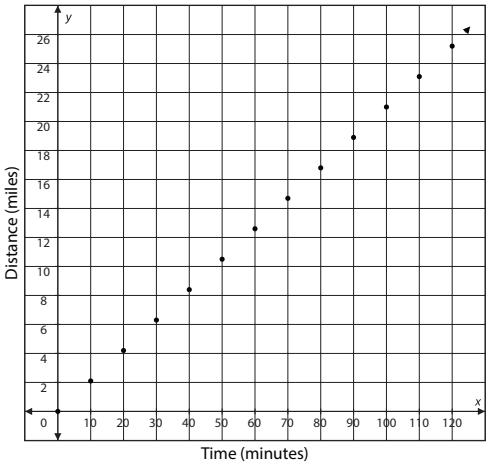
c.



b.



d.



4. Use the table to determine the rate of change for the interval [10, 15].

Weeks (x)	Amount owed in dollars ($f(x)$)
0	1500
5	1350
10	1200
15	1050
20	900

a. –$150 per week

b. $10 per week

c. –$30 per week

d. $15 per week

continued

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5. Use the table to determine the rate of change for the interval [2, 4].

Weeks (x)	Value of stock in dollars ($f(x)$)
0	610.00
1	578.70
2	549.00
3	520.83
4	494.10

   a. –$0.04 per week
   b. –$27.45 per week
   c. $27.45 per week
   d. The rate of change cannot be determined.

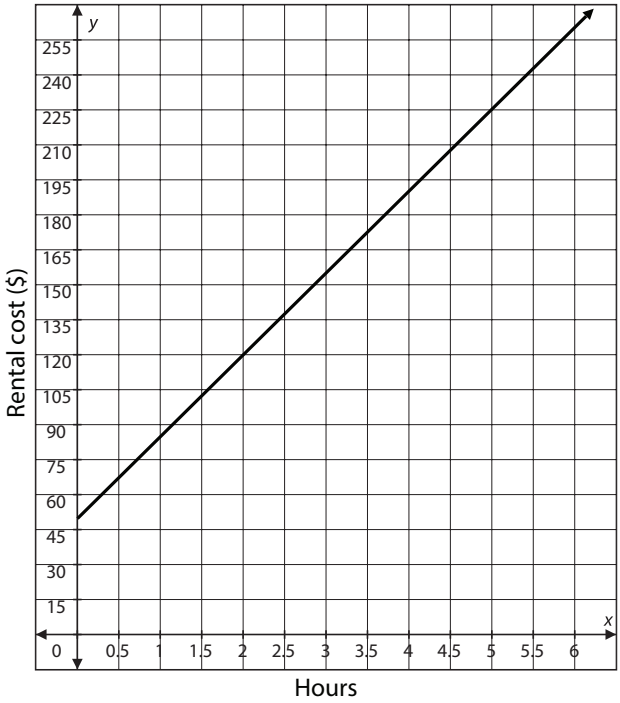
6. What is the rate of change for the function $f(x)=10(2)^{\frac{x}{2}}$ over the interval [4, 8]?
   a. 15
   b. 30
   c. 4
   d. The rate of change cannot be determined.

7. What is the rate of change for the function $f(x) = -3.2x + 5$ over the interval [10, 20]?
   a. –3.2
   b. –32
   c. –0.3125
   d. The rate of change cannot be determined.

continued

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8. The following graph represents the cost to rent mini-bikes over a period of time. What is the approximate rate of change for the function for the interval [0.5, 2.5]?



a. $72.50 per hour

b. $0.03 per hour

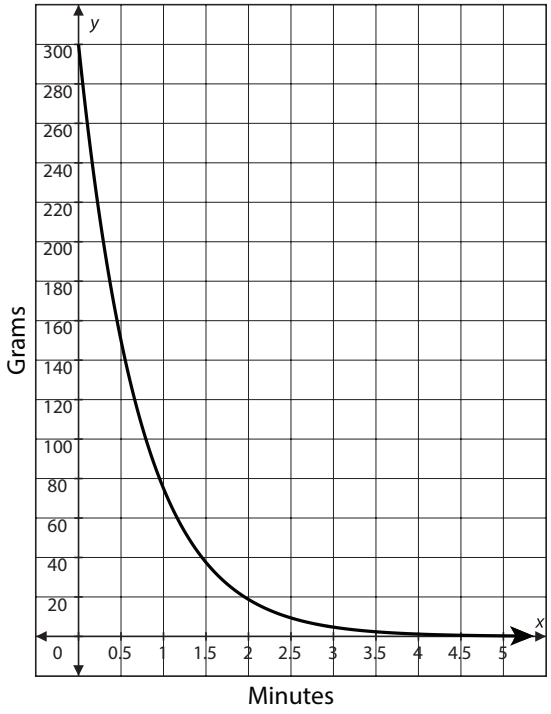
c. $36.25 per hour

d. The rate of change cannot be determined.

continued

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9. The following graph represents the amount in grams of a radioactive element that remains after a period of time. What is the approximate rate of change for the interval [0, 2.5]?



a. 116 grams per minute

b. –116 grams per minute

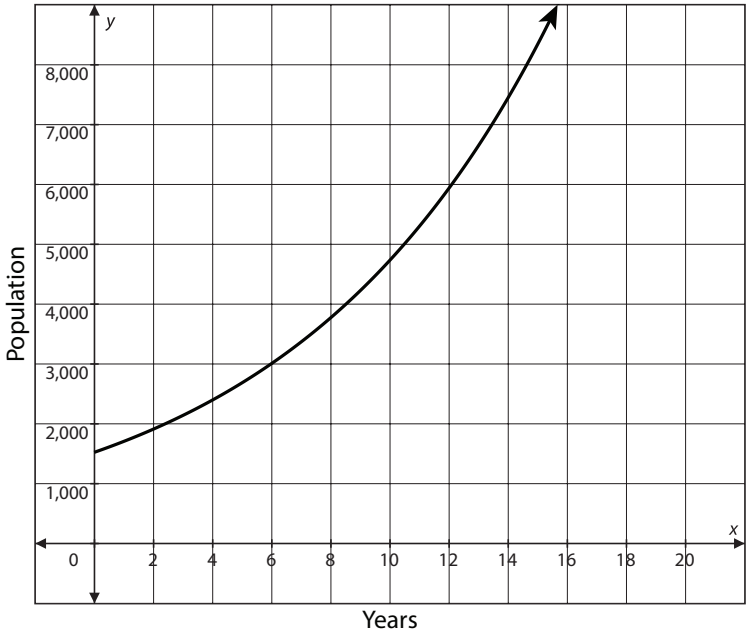
c. 290 grams per minute

d. The rate of change cannot be determined.

continued

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10. The following graph represents the population of a small town over a period of time. What is the approximate rate of change for the function for the interval [6, 12]?



a. 500 people per year

b. –500 people per year

c. –3,000 people per year

d. The rate of change cannot be determined.

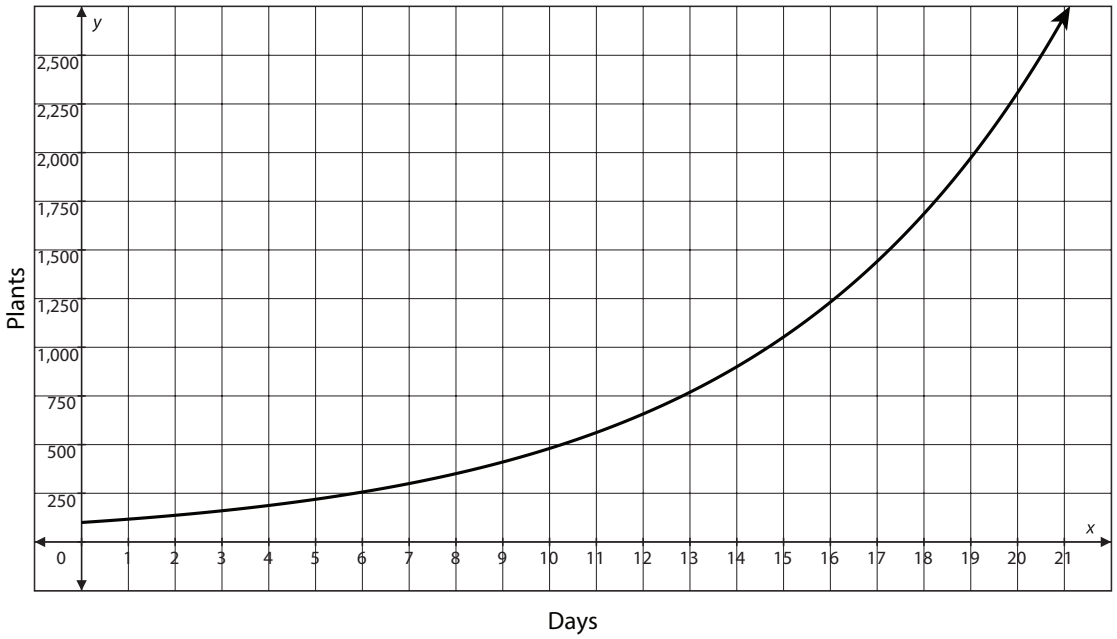
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Assessment

Use what you have learned about key features and rates of change to complete problem 11.

11. When a plant is introduced to a new outdoor environment, it can be difficult to tell whether the plant will adapt to the new conditions. Sometimes the plant will adapt too well and begin to overtake other nearby plants. When this happens, the rate at which the plant reproduces appears to explode. The following graph shows the population of a particular plant after it was introduced into a new environment.



a. What are the key features of this function and what do they mean in terms of this scenario? Be sure to include the intercepts, if any, whether the function is increasing or decreasing, and whether the function is positive or negative. Also, include any relative minima and maxima as well as the equations of any asymptotes.

b. What is the domain of this function?

c. How does the rate of change vary over time?

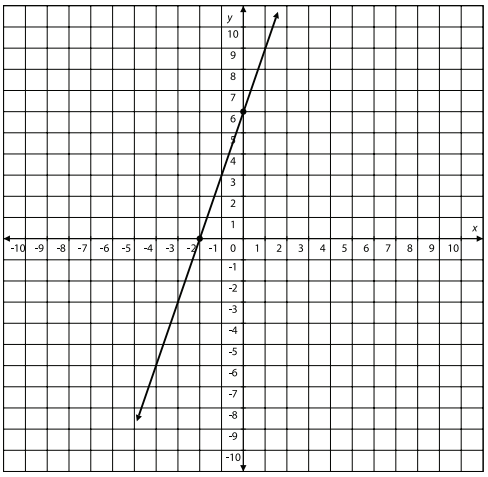
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Pre-Assessment

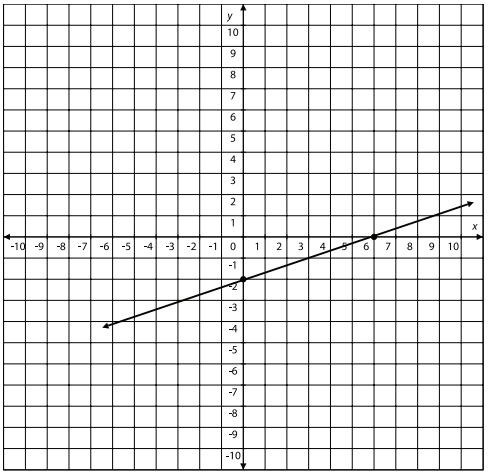
Circle the letter of the best answer.

1. Which graph shows a line with an *x*-intercept of –2?

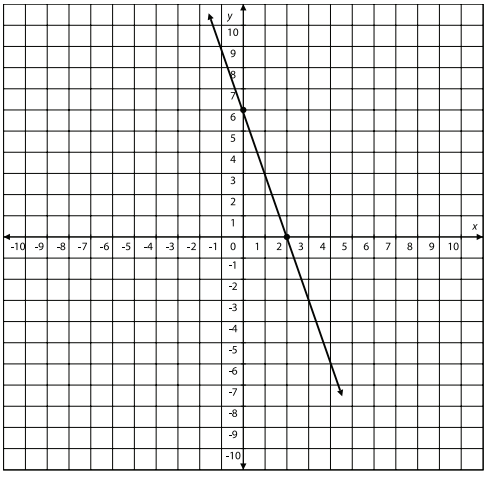
a.



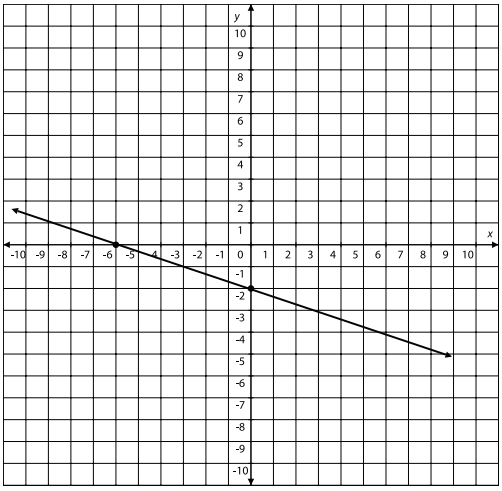
c.



b.



d.

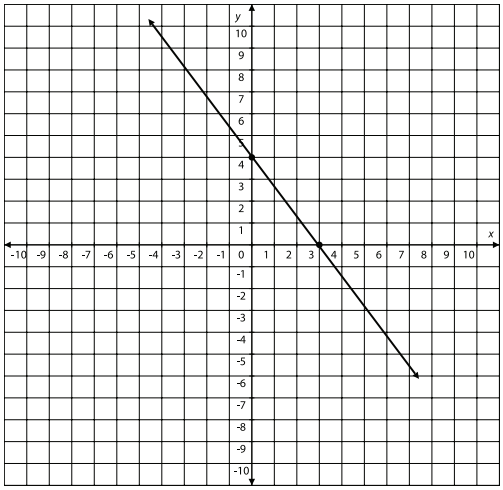


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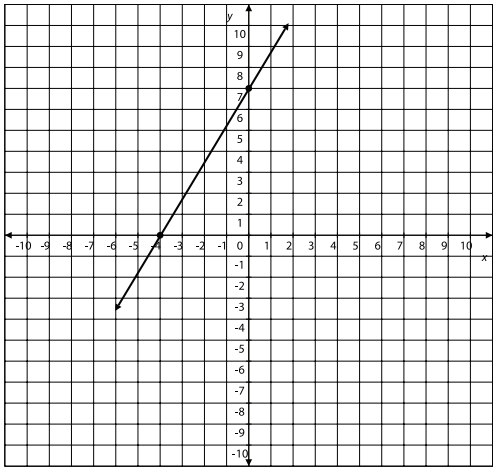
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2. Which graph shows a line with a *y*-intercept of –4?

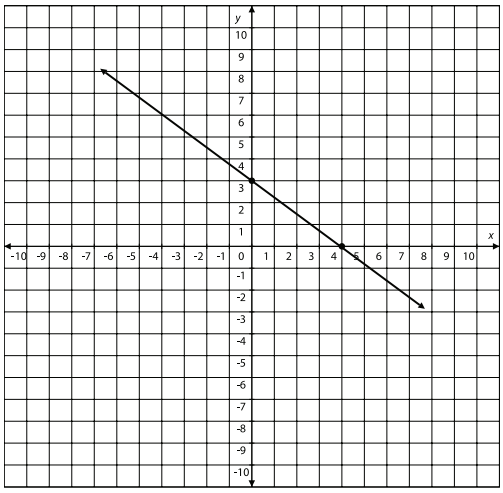
a.



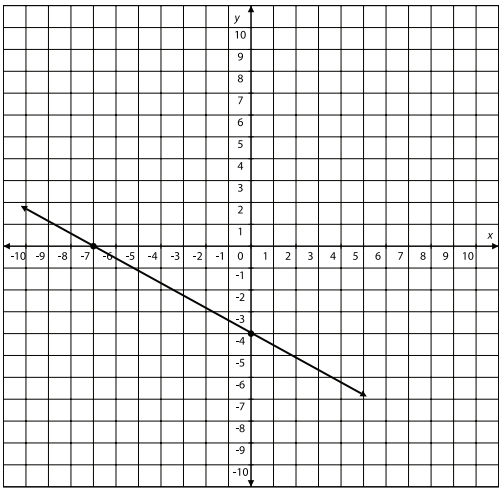
c.



b.



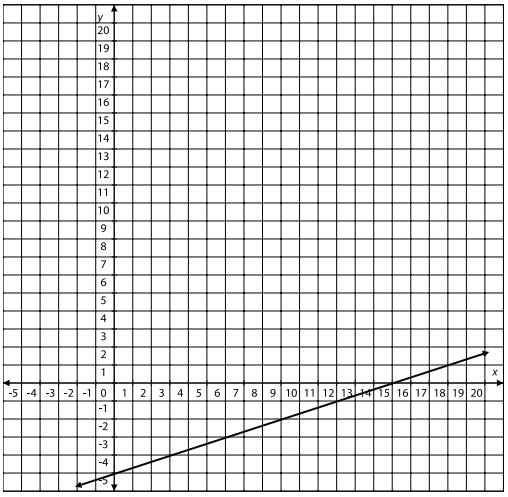
d.



continued

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3. What is the *y*-intercept of the graph?



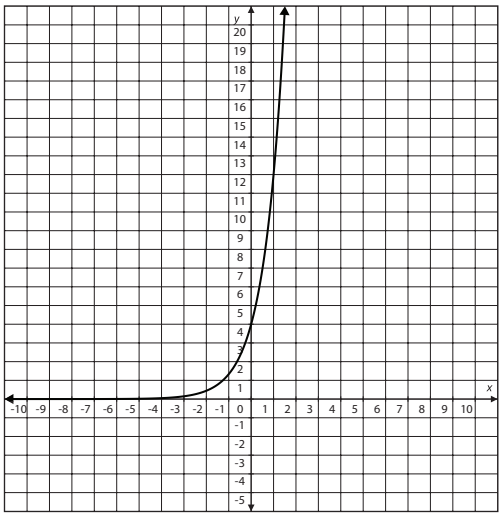
a. 5

b. 15

c. –5

d. –15

4. What is the *y*-intercept of the graph?



a. 4

b. –4

c. –5

d. There is no *y*-intercept.

continued

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Assessment

5. Describe the end behavior of the curve whose equation is $y = 2^x$ for $x \geq 0$.

   a. growth, with a horizontal asymptote of $y = 1$

   b. decay, with a horizontal asymptote of $y = 0$

   c. decay, with a horizontal asymptote of $y = 1$

   d. growth, with a horizontal asymptote of $y = 0$

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Lesson 5: Analyzing Linear and Exponential Functions

Instruction

Common Core State Standards

SMP	
1 ✓	2 ✓
3	4 ✓
5 ✓	6
7	8

F–IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Essential Questions

1. What are the key features of the graph of a linear function?
2. What are the key features of the graph of an exponential function?
3. How can technology be used to graph linear and exponential functions?
4. How can linear and exponential functions be used to model real-world scenarios?

WORDS TO KNOW

asymptote	a line that a function gets closer and closer to as one of the variables increases or decreases without bound
end behavior	the behavior of the graph as x approaches positive infinity and as x approaches negative infinity
exponential function	a function whose independent variable is in the exponent; the general form of its equation is $f(x) = ab^x + k$, where a is the initial value, b is the base, x is the input value, k is the vertical shift, and $f(x)$ is the output
linear function	a first-degree equation that can be written in the form $f(x) = mx + b$, in which m is the slope of the line and b is the y-intercept. The graph of a linear function is a straight line.
***x*-intercept**	the x-coordinate of the point at which a line or a curve intersects the x-axis
***y*-intercept**	the y-coordinate of the point at which a line or a curve intersects the y-axis

SRB p. U2-127

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Instruction

Recommended Resources

- Analyzemath.com. "Exponential Function HTML5 applet."

  http://www.walch.com/rr/05002

  This applet allows users to change the values of the variables of an expontial function and observe its effects on the associated graph.
- Math Open Reference. "Linear Function Explorer."

  http://walch.com/rr/CAU3L4LinFunctionApplet

  Use the sliders to observe the changes in the slope and intercepts of a linear function.

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Lesson 2.5.1: Graphing Linear Functions

Warm-Up 2.5.1

Geocaching is an outdoor activity that is much like a treasure hunt. Clues to the location of each treasure, or cache, are given for seekers to discover the hidden location. The following coordinates represent the locations of several caches.

A (–3, 5)	D (–7, –2)
B (2, 1)	E (0, 9)
C (–6, 0)	F (3, 0)

1. Use graph paper to graph each point on the same coordinate plane.

2. Identify the quadrant in which each point lies. If a point does not lie in a quadrant, identify if it is an x-intercept or a y-intercept.

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Instruction

Lesson 2.5.1: Graphing Linear Functions

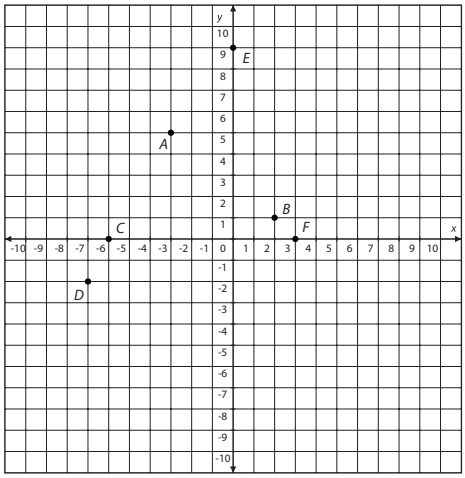
Common Core State Standard

F–IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Warm-Up 2.5.1 Debrief

1. Use graph paper to graph each point on the same coordinate plane.



2. Identify the quadrant in which each point lies. If the point does not lie in a quadrant, identify if it is an *x*-intercept or a *y*-intercept.

   Point *A* (–3, 5) is in the second quadrant.

   Point *B* (2, 1) is in the first quadrant.

   Point *C* (–6, 0) is on the *x*-axis, so –6 is an *x*-intercept.

   Point *D* (–7, –2) is in the third quadrant.

   Point *E* (0, 9) is on the *y*-axis, so 9 is a *y*-intercept.

   Point *F* (3, 0) is on the *x*-axis, so 3 is an *x*-intercept.

Connection to the Lesson

- Students will need to correctly plot points on the coordinate plane.
- Students will need to be able to identify intercepts.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- plotting points on the coordinate plane (5.G.1)
- graphing a function from a table of values (A–CED.2★)

Introduction

In this lesson, different methods will be used to graph lines and analyze the features of the graph. In a linear function, the graph is a non-vertical straight line with a constant slope. All linear functions have a *y*-intercept. The *y*-intercept is the *y*-coordinate of the point at which a line or a curve intersects the *y*-axis. If a linear equation has a slope other than 0, then the function also has an *x*-intercept. The *x*-intercept is the *x*-coordinate of the point at which a line or a curve intersects the *x*-axis.

Key Concepts

- A **linear function** is a function that can be written in the form $f(x) = mx + b$, in which m is the slope and b is the *y*-intercept, and whose graph is a straight line.
- To find the *y*-intercept in function notation, evaluate $f(0)$.
- The *y*-intercept is the value of y when x is 0.
- To locate the *y*-intercept of a function, determine the *y*-coordinate of the point where the line intersects the *y*-axis.
- To find the *x*-intercept using function notation, set $f(x) = 0$ and solve for x.
- The *x*-intercept is the value of x when y is 0.
- To locate the *x*-intercept of a function, determine the *x*-coordinate of the point where the line intersects the *x*-axis.
- To find the slope of a linear function, pick two points on the line and substitute the coordinates of the points into the equation $m=\frac{y_2-y_1}{x_2-x_1}$, where m is the slope, (x_1, y_1) are the coordinates of one point, and (x_2, y_2) are the coordinates of the other point.
- If the equation of a line is in slope-intercept form, the slope is the coefficient of x.

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Instruction

Graphing Equations Using a TI-83/84:

Step 1: Press [Y=].

Step 2: Key in the equation using [X, T, θ, *n*] for *x*.

Step 3: Press [WINDOW] to change the viewing window, if necessary.

Step 4: Enter in appropriate values for Xmin, Xmax, Xscl, Ymin, Ymax, and Yscl, using the arrow keys to navigate.

Step 5: Press [GRAPH].

Graphing Equations Using a TI-Nspire:

Step 1: Press the home key.

Step 2: Arrow over to the graphing icon (the picture of the parabola or the U-shaped curve) and press [enter].

Step 3: Enter in the equation and press [enter].

Step 4: To change the viewing window: press [menu], arrow down to 4: Window/Zoom, and click the center button of the navigation pad.

Step 5: Choose 1: Window settings by pressing the center button.

Step 6: Enter in the appropriate XMin, XMax, YMin, and YMax values.

Step 7: Leave the XScale and YScale set to auto.

Step 8: Use [tab] to navigate among the fields.

Step 9: Press [tab] to "OK" when done and press [enter].

Common Errors/Misconceptions

- incorrectly plotting points
- mistaking the *y*-intercept for the *x*-intercept and vice versa
- being unable to identify key features of a linear graph
- confusing the value of a function for its corresponding *x*-coordinate

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Instruction

Guided Practice 2.5.1

Example 1

Given the function $f(x)=\frac{3}{2}x-6$, use a table of values to graph and identify the x- and y-intercepts.

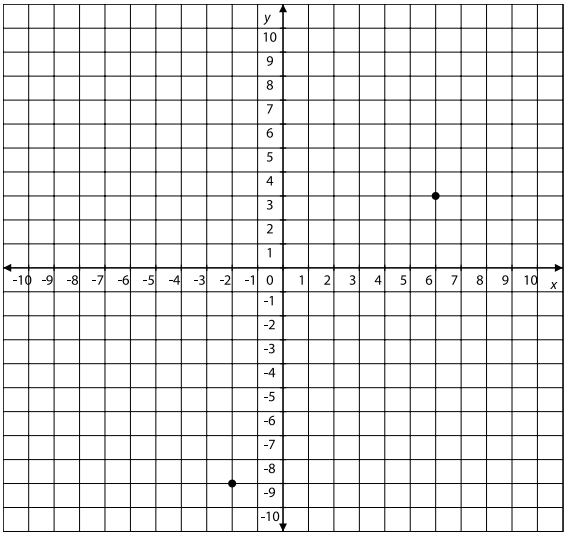
1. Create a table of values.

   Choose values for x and determine the corresponding $f(x)$ values.

x	$f(x)$
–2	–9
0	–6
2	–3
4	0
6	3

2. Plot two points from the table.

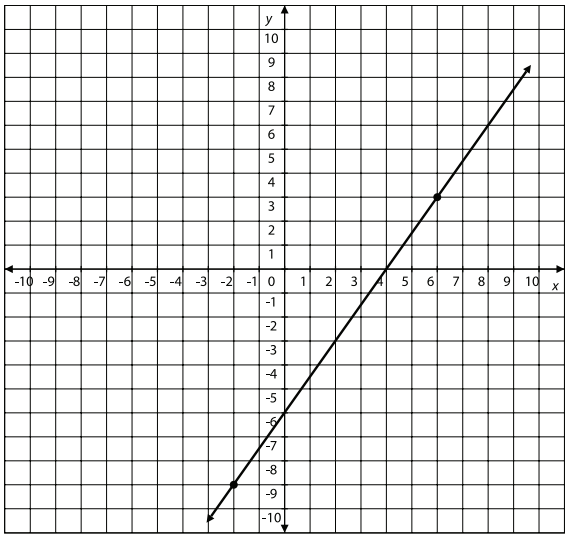
   The points (6, 3) and (–2, –9) are shown plotted in the following graph.



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Instruction

3. Draw the line connecting the two points. Be sure to extend the line so that it intersects both the *x*- and *y*-axes.



4. Identify the *x*-intercept.

   The *x*-intercept is the value of *x* where the line intersects the *x*-axis.

   The *x*-intercept is 4.

5. Identify the *y*-intercept.

   The *y*-intercept is the value of *y* where the line intersects the *y*-axis.

   The *y*-intercept is –6.

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Instruction

Example 2

Given the function $f(x)=-\frac{1}{5}x+2$, use the slope and *y*-intercept to graph the function. Then, identify the *x*-intercept of the function.

1. Identify the slope and *y*-intercept.

   The function $f(x)=-\frac{1}{5}x+2$ is written in $f(x) = mx + b$ form, where m is the slope and b is the *y*-intercept.

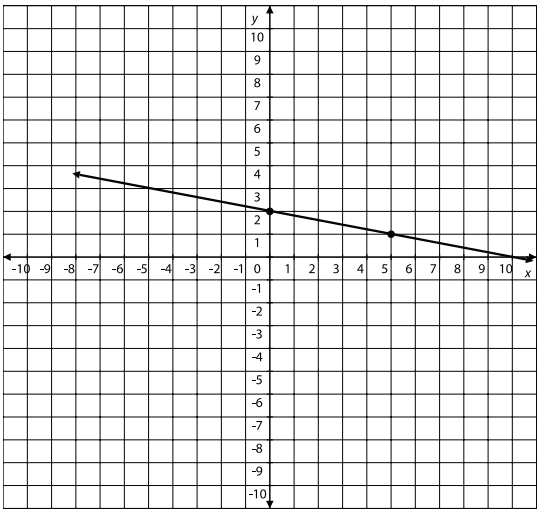
   The slope of the function is $-\frac{1}{5}$.

   The *y*-intercept is 2.

2. Graph the function on a coordinate plane.

   Use the *y*-intercept, 2, and slope, $-\frac{1}{5}$, to graph the function.

   Be sure to extend the line to intersect both the *x*- and *y*-axes.



3. Identify the *x*-intercept.

   The *x*-intercept is the value of x where the line intersects the *x*-axis.

   The *x*-intercept is 10.

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Instruction

Example 3

Given the function $f(x)=-\frac{4}{3}x+4$, solve for the x- and y-intercepts. Use the intercepts to graph the function.

1. Find the x-intercept.

   Substitute 0 for $f(x)$ in the equation and solve for x.

$f(x)=-\frac{4}{3}x+4$	Original function
$0=-\frac{4}{3}x+4$	Substitute 0 for $f(x)$.
$-4=-\frac{4}{3}x$	Subtract 4 from both sides.
$x = 3$	Multiply both sides by $-\frac{3}{4}$.

   The x-intercept is 3.

2. Find the y-intercept.

   Substitute 0 for x in the equation and solve for $f(x)$.

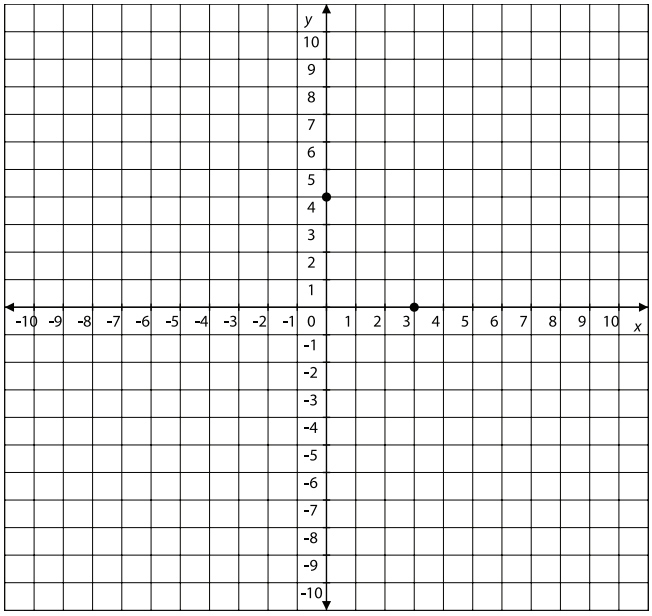
$f(x)=-\frac{4}{3}x+4$	Original function
$f(x)=-\frac{4}{3}(0)+4$	Substitute 0 for x.
$f(x) = 0 + 4$	Simplify as needed.
$f(x) = 4$	

   The y-intercept is 4.

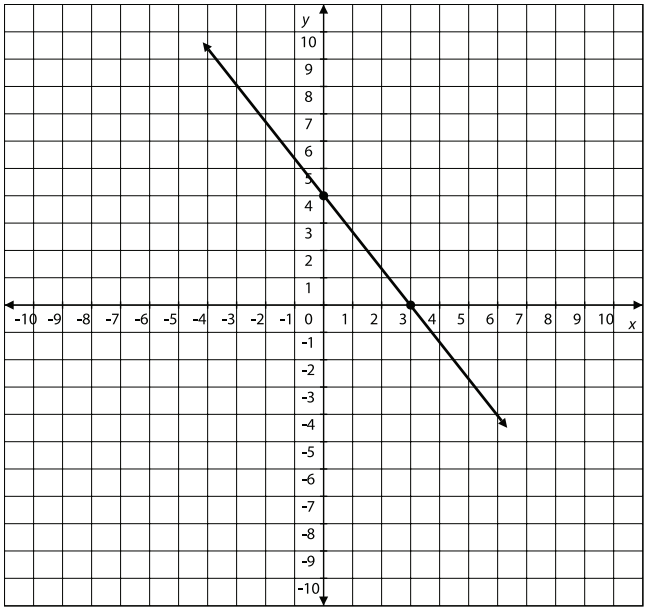
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3. Graph the function.

   Plot the *x*- and *y*-intercepts.



   Draw a line connecting the two points.



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Instruction

Example 4

Given the function $f(x)=-\frac{1}{8}x+20$, graph the function using technology. Identify the intercepts.

1. Set the viewing window of the graphing calculator.

   Use the following settings:

   Xmin = –20 Ymax = 30

   Xmax = 20 Yscal = 2

   Xscl = 2 Xres = 1

   Ymin = –20

   To Set the Window on a TI-83/84:

   Step 1: Press [WINDOW].

   Step 2: Change values accordingly. Use the arrow keys to navigate.

   Step 3: Press [ENTER].

   To Set the Window on a TI-Nspire:

   Step 1: Press [menu], arrow down to number 4: Window/Zoom, and click the center button of the navigation pad.

   Step 2: Choose 1: Window settings by pressing the center button.

   Step 3: Enter in the appropriate XMin, XMax, YMin, and YMax fields.

   Step 4: Leave the XScale and YScale set to auto.

   Step 5: Use [tab] to navigate among the fields.

   Step 6: Press [tab] to "OK" when done and press [enter].

2. Graph the function.

   Graphing Equations Using a TI-83/84:

   Step 1: Press [Y=].

   Step 2: Key in the equation using [X, T, θ, *n*] for *x*: [–1/8x+20].

   Step 3: Press [GRAPH].

(*continued*)

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Graphing Equations Using a TI-Nspire:

Step 1: Press the home key.

Step 2: Arrow over to the graphing icon (the picture of the parabola or the U-shaped curve) and press [enter].

Step 3: Enter in the equation [–1/8x+20] and press [enter].

Step 4: Press [tab] to "OK" when done and press [enter].

3. Find the intercepts using technology.

Finding the Intercepts Using a TI-83/84:

Step 1: Press [2ND] and [TRACE].
Step 2: Press 1: value.
Step 3: Type [0] and press [ENTER].
Step 4: Record the *y*-value. This is the *y*-intercept.
Step 5: Press [2ND] and [TRACE].
Step 6: Type 2: zero.
Step 7: Move the cursor so it is to the left of the *x*-intercept and press [ENTER].
Step 8: Move the cursor so it is to the right of the *x*-intercept and press [ENTER].
Step 9: Press [ENTER].
Step 10: Record the *x*-value. This is the *x*-intercept.

Finding the Intercepts Using a TI-Nspire:

Step 1: Press [menu].
Step 2: Press 6: Analyze Graph.
Step 3: Press 1: Zero.
Step 4: Move the hand to a point on the graph to the left of the *x*-intercept.
Step 5: Press [enter].
Step 6: Record the *x*-value. This is the *x*-intercept.
Step 7: Press [menu].
Step 8: Press 5: Trace.
Step 9: Press 1: Graph Trace.
Step 10: Move the cursor so it is on the *y*-axis. Use the arrow keys to navigate.
Step 11: Record the *y*-value. This is the *y*-intercept.

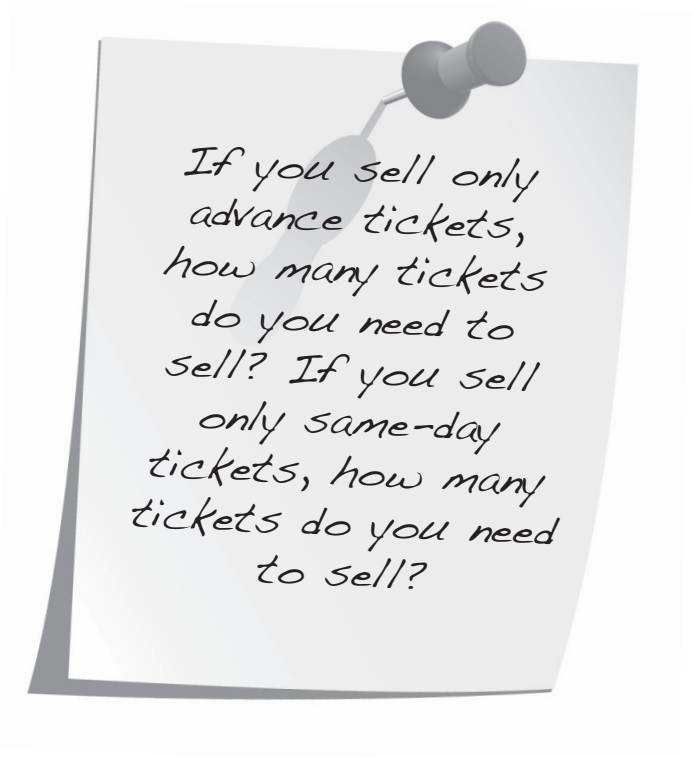
The *x*-intercept is 160 and the *y*-intercept is 20.

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Problem-Based Task 2.5.1: Fund-raising Concert

SMP	
1 ✓	2 ✓
3	4 ✓
5	6
7	8

You are helping to organize a fund-raising concert for your community awareness group. Concert tickets will sell for $20 each in advance, and for $30 each on the day of the show. Your club's goal is to raise $6,000. Write an equation in standard form for the function that represents this scenario. Draw the graph of the function that represents how many of each type of ticket you will need to sell. If you sell only advance tickets, how many tickets do you need to sell? Where do you find this on your graph? If you sell only same-day tickets, how many tickets do you need to sell? Where do you find this on your graph?



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Problem-Based Task 2.5.1: Fund-raising Concert

Coaching

a. What is the price of each ticket sold in advance?

b. Write an expression to represent the total amount of money made from selling advance tickets.

c. What is the price of each ticket sold the day of the show?

d. Write an expression to represent the total amount of money made from selling same-day tickets.

e. What is the amount your club needs to raise?

f. Write an equation in standard form to represent how many of each type of ticket you need to sell to reach your club's goal.

g. Identify the *x*-intercept.

h. Identify the *y*-intercept.

i. Plot the two intercepts.

j. Draw a line segment connecting the two points.

k. Determine the number of tickets that need to be sold if only advance tickets are sold.

l. Determine the number of tickets that need to be sold if only same-day tickets are sold.

m. Where are these points located on the graph?

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Instruction

Problem-Based Task 2.5.1: Fund-raising Concert

Coaching Sample Responses

a. What is the price of each ticket sold in advance?

The price of each ticket sold in advance is $20.

b. Write an expression to represent the total amount of money made from selling advance tickets.

Use x to represent the number of $20 tickets sold.

The amount of money made from selling $20 tickets is $20x$.

c. What is the price of each ticket sold the day of the show?

The price of each ticket sold the day of the show is $30.

d. Write an expression to represent the total amount of money made from selling same-day tickets.

Use y to represent the number of $30 tickets sold.

The amount of money made from selling $30 tickets is $30y$.

e. What is the amount your club needs to raise?

Your club needs to raise $6,000.

f. Write an equation in standard form to represent how many of each type of ticket you need to sell to reach your club's goal.

The equation that represents how many of each type of ticket you need to sell is a combination of the expressions found in parts b and d, set equal to the goal of $6,000.

$20x + 30y = 6000$

g. Identify the x-intercept.

Substitute 0 for y and solve for x.

$20x + 30(0) = 6000$

$20x + 0 = 6000$

$20x = 6000$

$x = 300$

The x-intercept is 300.

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h. Identify the y-intercept.

Substitute 0 for x and solve for y.

$20(0) + 30y = 6000$

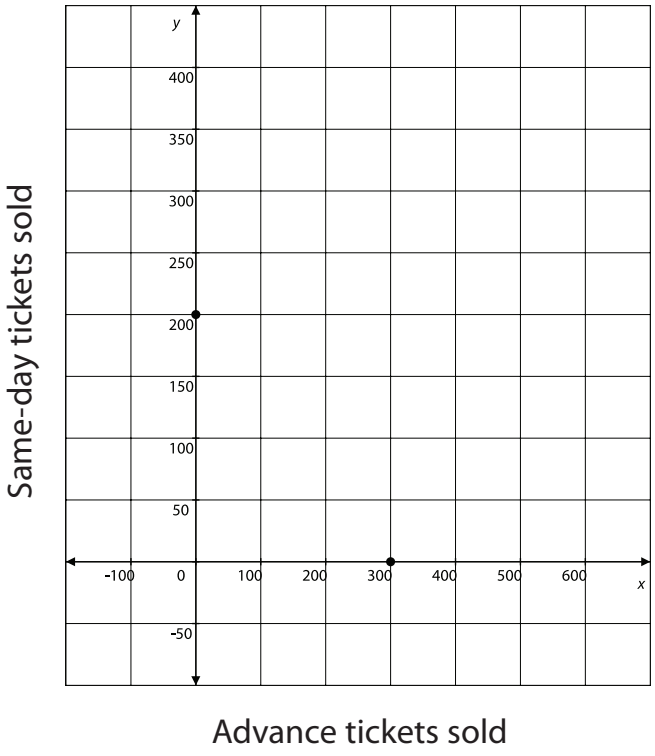
$0 + 30y = 6000$

$30y = 6000$

$y = 200$

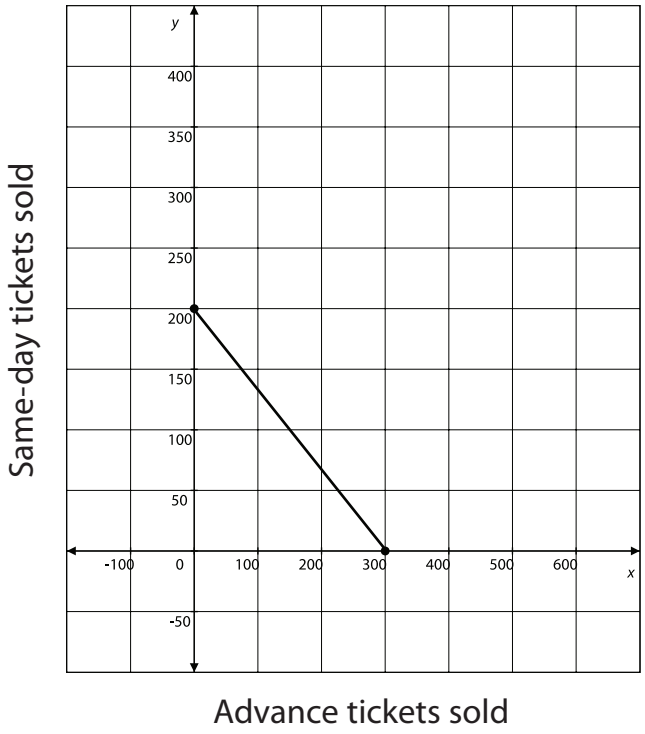
The y-intercept is 200.

i. Plot the two intercepts.



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j. Draw a line segment connecting the two points.



k. Determine the number of tickets that need to be sold if only advance tickets are sold.

The number of tickets that need to be sold if only advance tickets are sold is 300.

l. Determine the number of tickets that need to be sold if only same-day tickets are sold.

The number of tickets that need to be sold if only same-day tickets are sold is 200.

m. Where are these points located on the graph?

These points have the coordinates (300, 0) and (0, 200).

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.5.1: Graphing Linear Functions

Use what you know about linear functions to complete the following problems.

1. Given the function $f(x)=-\frac{4}{3}x+4$, use the slope and *y*-intercept to graph the function. Identify the *x*- and *y*-intercepts.

2. Given the function $f(x)=\frac{2}{7}x+2$, use the slope and *y*-intercept to graph the function. Identify the *x*- and *y*-intercepts.

3. Given the table of values, graph the function and identify the *x*- and *y*-intercepts.

x	*f*(*x*)
–3	10
0	5
3	0
6	–5

4. Given the table of values, graph the function and identify the *x*- and *y*-intercepts.

x	*f*(*x*)
–14	–2
–7	0
0	2
7	4

5. Given the function $f(x)=-\frac{2}{3}x+2$, solve for the *x*- and *y*-intercepts. Use the intercepts to graph the function.

6. Given the function $f(x)=\frac{5}{4}x-5$, solve for the *x*- and *y*-intercepts. Use the intercepts to graph the function.

continued

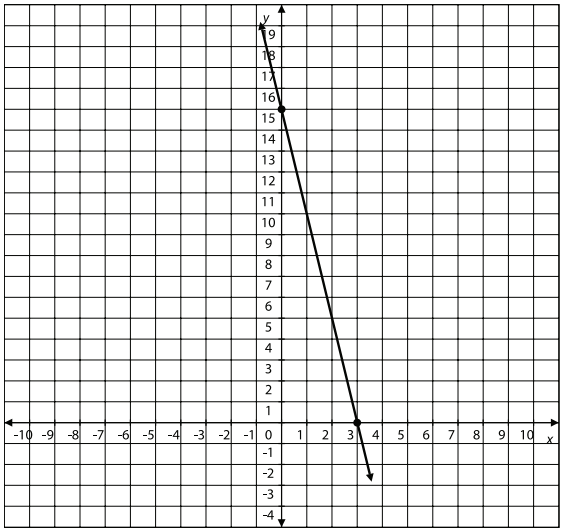
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7. Kaylee is selling candles to raise money for her lacrosse team. The large candles sell for $25 each and the small candles sell for $10 each. She needs to raise $600. Write a function to represent how many of each type of candle Kaylee needs to sell. Draw the graph of the function. If she sells only large candles, how many candles does she need to sell? If she sells only small candles, how many candles does she need to sell?

8. Jerome knits scarves and hats for charity. Scarves require 4 skeins of yarn and hats require 2 skeins of yarn. Jerome has 24 skeins of yarn. Write a function to represent the combination of scarves and hats he can knit. Draw the graph of the function. If Jerome knits only scarves, how many can he knit? If he knits only hats, how many can he knit?

9. A farmer raises goats and cows. Each goat requires 400 square feet of grazing area and each cow requires 1,200 square feet of grazing area. The farmer has 36,000 square feet for grazing area. Write a function to represent the combination of goats and cows that the farmer can raise. Use technology to graph the function. If the farmer raises only cows, how many can he raise? If the farmer raises only goats, how many can he raise?

10. The graph of a function is shown. Write a scenario that could be represented by the graph.



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Lesson 2.5.2: Graphing Exponential Functions

Warm-Up 2.5.2

Tyrone needs help understanding the order of operations. Help Tyrone evaluate each of the expressions using the correct order of operations.

1. $3(2^3) - 1$

2. $4(2)^{-1} + 3$

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Instruction

Lesson 2.5.2: Graphing Exponential Functions

Common Core State Standard

F–IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Warm-Up 2.5.2 Debrief

1. $3(2^3) - 1$

   Follow the order of operations: evaluate expressions in parentheses, apply exponents, multiply/divide, and add/subtract.

$3(2^3) - 1$	Original expression
$3(8) - 1$	Apply the exponent.
$24 - 1$	Multiply.
23	Subtract.

   After applying the order of operations to $3(2^3) - 1$, the result is 23.

2. $4(2)^{-1} + 3$

   Follow the order of operations: evaluate parentheses, apply exponents, multiply/divide, and add/subtract.

$4(2)^{-1} + 3$	Original expression
$4\left(\frac{1}{2}\right)+3$	Apply the exponent.
$2 + 3$	Multiply.
5	Add.

   After applying the order of operations to $4(2)^{-1} + 3$, the result is 5.

Connection to the Lesson

- Students need to be able to correctly evaluate expressions containing exponents along with other operations.
- Students need to be able to evaluate expressions containing negative exponents.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- plotting points on the coordinate plane (5.G.1)
- graphing a function from a table (A–CED.2★)
- understanding the rules of exponents, including negative exponents (8.EE.1)
- recognizing the general shape of an exponential function (decay or growth) (A–CED.2★)

Introduction

In this lesson, different methods will be used to graph exponential functions and analyze the key features of their graphs. In an exponential function, the graph is a smooth curve. If the domain of an exponential function is all real numbers, then the graph will have an asymptote, which is a line that the graph of a function gets closer and closer to as one of the variables increases or decreases without bound. Every exponential function has a y-intercept, where the graph intersects the y-axis. There is, at most, one x-intercept, where the function intersects the x-axis.

Key Concepts

- An **exponential function** is a function whose independent variable is in the exponent. The general form of its equation is $f(x) = ab^x + k$, where a is the initial value, b is the base, x is the input value, k is the vertical shift, and $f(x)$ is the output.
- To find the y-intercept of an exponential function, evaluate $f(0)$.
- The y-intercept is the value of y when x is 0.
- To locate the y-intercept of a function, determine the y-coordinate of the point where the line intersects the y-axis.
- To find the x-intercept in function notation, set $f(x) = 0$ and solve for x.
- The x-intercept is the value of x when y is 0.
- To locate the x-intercept of a function, determine the x-coordinate of the point where the line intersects the x-axis.
- Not all exponential functions have an x-intercept.
- The asymptote of an exponential function of the form $f(x) = ab^x$ is always the x-axis, or the line $y = 0$.
- If the exponential function is of the form $f(x) = ab^x + k$, then the function will be shifted vertically by k units.
- The equation of the asymptote for equations in the form $f(x) = ab^x + k$ is $y = k$.

SRB p. U2-140

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Instruction

- The **end behavior** is the behavior of each end of the graph as x increases or decreases without limit. The value of $f(x)$ as x approaches positive or negative infinity will approach either positive infinity, negative infinity, or the asymptote.
- It is easiest to first graph the function and then observe what happens to the value of y as the value of x increases and decreases.
- Graph complex exponential models using technology, as values can become quite large or small very quickly.

Graphing Equations Using a TI-83/84:

Step 1: Press [Y=].

Step 2: Key in the equation using [X, T, θ, *n*] for x.

Step 3: Press [WINDOW] to change the viewing window, if necessary.

Step 4: Enter in appropriate values for Xmin, Xmax, Xscl, Ymin, Ymax, and Yscl, using the arrow keys to navigate.

Step 5: Press [GRAPH].

Graphing Equations Using a TI-Nspire:

Step 1: Press the home key.

Step 2: Arrow over to the graphing icon (the picture of the parabola or the U-shaped curve) and press [enter].

Step 3: Enter in the equation and press [enter].

Step 4: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom, and click the center button of the navigation pad.

Step 5: Choose 1: Window settings by pressing the center button.

Step 6: Enter in the appropriate XMin, XMax, YMin, and YMax fields.

Step 7: Leave the XScale and YScale set to auto.

Step 8: Use [tab] to navigate among the fields.

Step 9: Press [tab] to "OK" when done and press [enter].

Common Errors/Misconceptions

- incorrectly plotting points
- mistaking the y-intercept for the x-intercept and vice versa
- not being able to identify key features of an exponential model
- confusing the value of a function for its corresponding x-coordinate

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Instruction

Guided Practice 2.5.2

Example 1

Create a table of values for the exponential function $f(x) = 3(2)^x + 4$. Identify the asymptote and y-intercept of the function. Plot the points and sketch the graph of the function, and describe the end behavior.

1. Create a table of values.

   Choose values of x and solve for the corresponding values of $f(x)$.

x	$f(x)$
–4	4.1875
–2	4.75
0	7
2	16
4	52

2. Identify the equation of the asymptote of the function.

   The equation of the asymptote of an exponential function in the form $f(x) = ab^x + k$ is always $y = k$.

   In the function $f(x) = 3(2)^x + 4$, the value of k is 4.

   The equation of the asymptote of the function is $y = 4$.

3. Determine the y-intercept of the function.

   The y-intercept of the function is the value of $f(x)$ when x is equal to 0.

   It can be seen in the table that when $x = 0$, $f(x) = 7$.

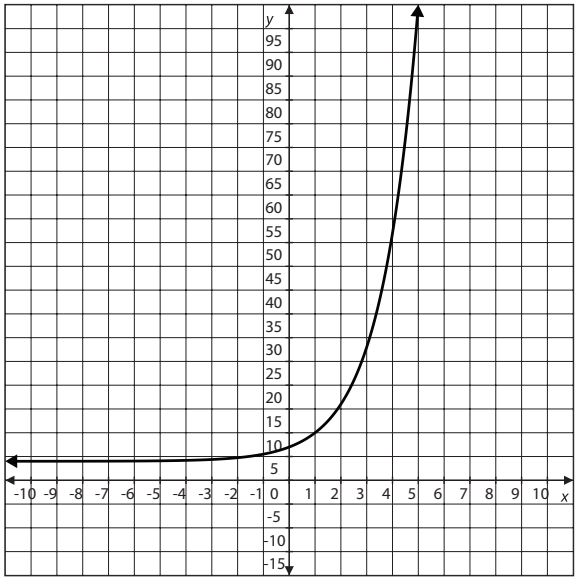
   The y-intercept is 7.

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Instruction

4. Graph the function.

Use the table of values to create a graph of the function.



5. Describe the end behavior of the graph.

The end behavior is what happens at the ends of the graph.

As x becomes larger, the values of the function approach infinity.

As x becomes smaller, the values of the function approach the asymptote, $y = 4$.

Since the function approaches infinity as x becomes larger, the graph shows exponential growth.

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Instruction

Example 2

Create a table of values for the exponential function $f(x) = -1(3)^x - 2$. Identify the asymptote and y-intercept of the function. Plot the points and sketch the graph of the function, and describe the end behavior.

1. Create a table of values.

   Choose values of x and solve for the corresponding values of $f(x)$.

x	$f(x)$
–1	–2.33
0	–3
1	–5
2	–11
3	–29

2. Identify the equation of the asymptote of the function.

   The equation of the asymptote of an exponential function in the form $f(x) = ab^x + k$ is $y = k$.

   In the function $f(x) = -1(3)^x - 2$, the value of k is –2.

   The equation of the asymptote of the function is $y = -2$.

3. Determine the y-intercept of the function.

   The y-intercept of the function is the value of $f(x)$ when x is equal to 0.

   It can be seen in the table that when $x = 0$, $f(x) = -3$.

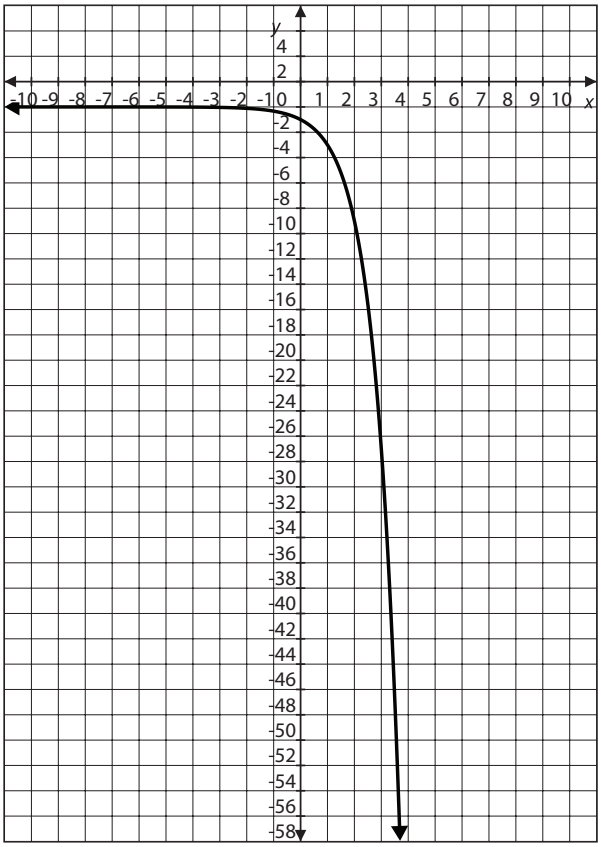
   The y-intercept is –3.

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Instruction

4. Graph the function.

   Use the table of values to create a graph of the function.



5. Describe the end behavior of the graph.

   The end behavior is what happens at the ends of the graph.

   As x becomes larger, the values of the function approach negative infinity.

   As x becomes smaller, the values of the function approach the asymptote, $y = -2$.

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Instruction

Example 3

Create a table of values for the exponential function $f(x)=4\left(\frac{2}{3}\right)^x-3$. Identify the asymptote and *y*-intercept of the function. Plot the points and sketch the graph of the function, and describe the end behavior.

1. Create a table of values.

   Choose values of x and solve for the corresponding values of $f(x)$.

x	$f(x)$
–4	17.25
–2	6
0	1
2	–1.22
4	–2.2099

2. Identify the asymptote of the function.

   The equation of the asymptote of an exponential function in the form $f(x) = ab^x + k$ is always $y = k$.

   In the function $f(x)=4\left(\frac{2}{3}\right)^x-3$, the value of k is –3.

   The equation of the asymptote of the function is $y = –3$.

3. Determine the *y*-intercept of the function.

   The *y*-intercept of the function is the value of $f(x)$ when x is equal to 0.

   It can be seen in the table that when $x = 0$, $f(x) = 1$.

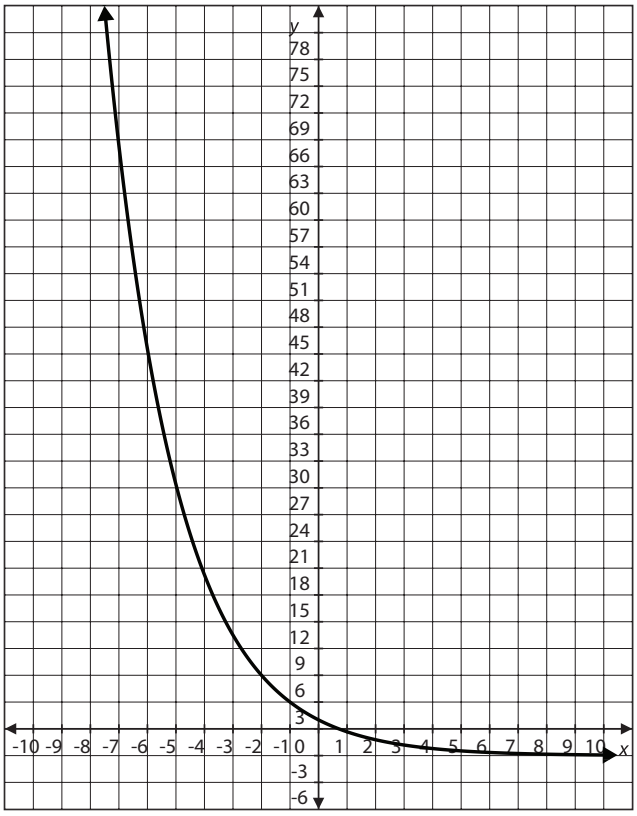
   The *y*-intercept is 1.

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Instruction

4. Graph the function.

Use the table of values to create a graph of the function.



5. Describe the end behavior of the graph.

The end behavior is what happens at the ends of the graph.

As x becomes larger, the values of the function approach the asymptote, $y = -3$.

As x becomes smaller, the values of the function approach infinity.

Since the function approaches the asymptote as x becomes larger, the graph shows exponential decay.

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Instruction

Example 4

Create a table of values for the exponential function $f(x)=-2\left(\frac{1}{4}\right)^x+5$. Identify the asymptote and *y*-intercept of the function. Plot the points and sketch the graph of the function, and describe the end behavior.

1. Create a table of values.

   Choose values of x and solve for the corresponding values of $f(x)$.

x	$f(x)$
–2	–27
–1	–3
0	3
1	4.5
2	4.875

2. Identify the asymptote of the function.

   The equation of the asymptote of an exponential function in the form $f(x) = ab^x + k$ is always $y = k$.

   In the function $f(x)=-2\left(\frac{1}{4}\right)^x+5$, the value of k is 5.

   The equation of the asymptote of the function is $y = 5$.

3. Determine the *y*-intercept of the function.

   The *y*-intercept of the function is the value of $f(x)$ when x is equal to 0.

   It can be seen in the table that when $x = 0$, $f(x) = 3$.

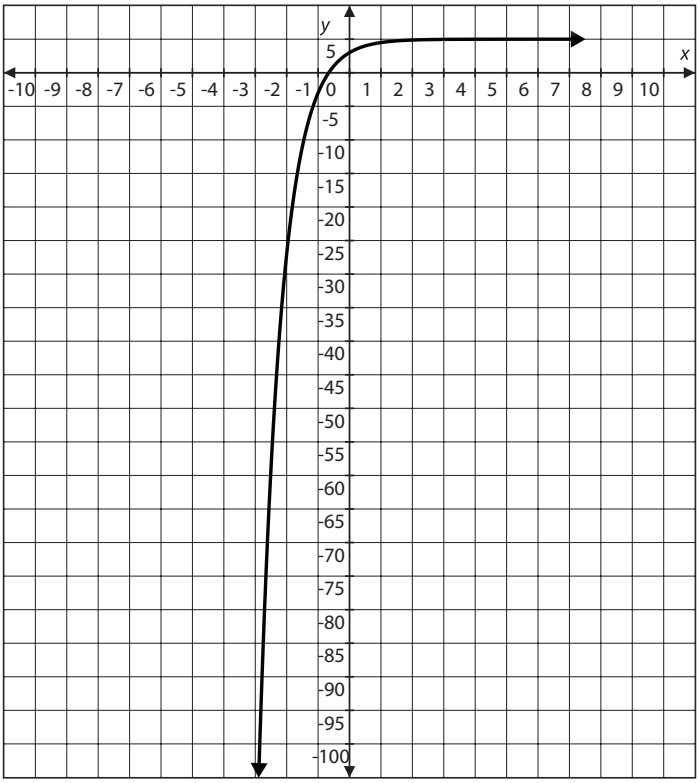
   The *y*-intercept is 3.

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Instruction

4. Graph the function.

   Use the table of values to create a graph of the function.



5. Describe the end behavior of the graph.

   The end behavior is what happens at the ends of the graph.

   As x becomes larger, the values of the function approach the asymptote, $y = 5$.

   As x becomes smaller, the values of the function approach negative infinity.

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Instruction

Example 5

Use technology to identify the *y*-intercept of the function $f(x)=-2\left(\frac{1}{4}\right)^x+5$.

1. Set the viewing window of the graphing calculator.

   Use the following settings:

Xmin = –5	Ymax = 10
Xmax = 5	Yscal = 5
Xscl = 1	Xres = 1
Ymin = –100	

2. Graph the function on a graphing calculator.

   Use the following steps:

   Graphing Equations Using a TI-83/84:

   Step 1: Press [Y=].

   Step 2: Key in the equation using [X, T, θ, *n*] for *x*, [–2(1/4)^x + 5].

   Step 3: Press [WINDOW] to change the viewing window, if necessary.

   Step 4: Enter in appropriate values for Xmin [–5], Xmax [5], Xscl [1], Ymin [–100], Ymax [10], and Yscl [5], using the arrow keys to navigate.

   Step 5: Press [GRAPH].

   Graphing Equations Using a TI-Nspire:

   Step 1: Press the home key.

   Step 2: Arrow over to the graphing icon (the picture of the parabola or the U-shaped curve) and press [enter].

   Step 3: Enter in the equation [–2(1/4)^x + 5] and press [enter].

   Step 4: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom, and click the center button of the navigation pad.

(*continued*)

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Step 5: Choose 1: Window settings by pressing the center button.

Step 6: Enter in the appropriate XMin [–5], XMax [5], YMin [–100], and YMax [10] values.

Step 7: Leave the XScale and YScale set to auto.

Step 8: Use [tab] to navigate among the fields.

Step 9: Press [tab] to "OK" when done and press [enter].

3. Find the *y*-intercept of the function.

Use the following steps:

Finding the Value When $x = 0$ Using a TI-83/84:

Step 1: Press [2ND] [CALC].

Step 2: Select 1: value.

Step 3: Type [5] and press [ENTER].

Step 4: Record the value, $y = 3$.

Finding the Value When $x = 0$ Using a TI-Nspire:

Step 1: Press [menu].

Step 2: Select 5: Trace.

Step 3: Select 1: Graph Trace.

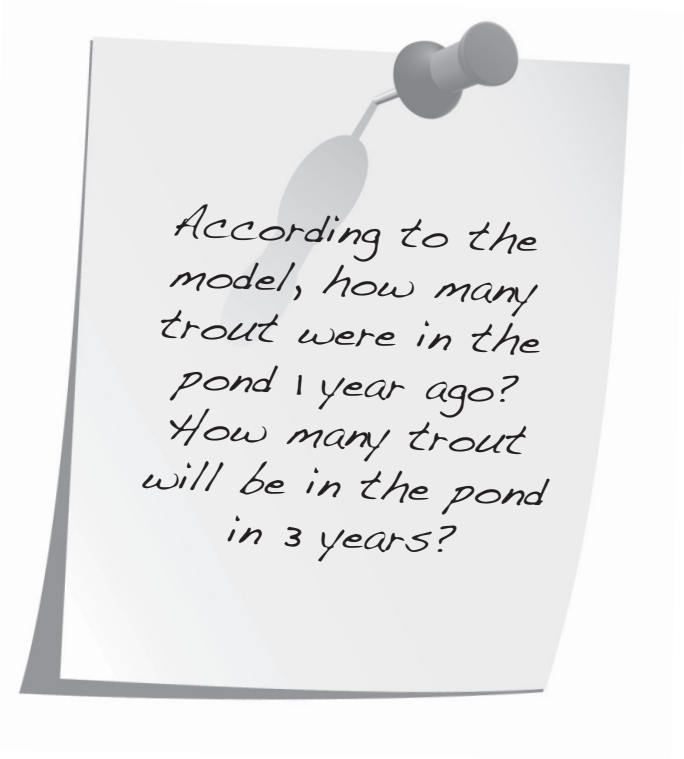
Step 4: Record the *y*-value, 3.

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Problem-Based Task 2.5.2: Trout Pond

SMP	
1 ✓	2 ✓
3	4
5 ✓	6
7	8

The population of trout in a certain pond doubles every year. The current population is 275 trout. Use technology to model this scenario. According to the model, how many trout were in the pond 1 year ago? How many trout will be in the pond in 3 years?



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Problem-Based Task 2.5.2: Trout Pond

Coaching

a. What kind of model would best fit this scenario?

b. What is the starting population?

c. Is this growth or decay?

d. What is the base that indicates this growth or decay?

e. Write the function that models this scenario.

f. Use technology to graph the function.

g. What window settings should you use to see the graph and be able to answer the questions?

h. Use technology to determine the number of trout in the pond 1 year ago. What value did you use for x? What kind of number is appropriate to answer the question?

i. Use technology to determine the number of trout in the pond in 3 years. What value did you use for x? What kind of number is appropriate to answer the question?

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Instruction

Problem-Based Task 2.5.2: Trout Pond

Coaching Sample Responses

a. What kind of model would best fit this scenario?

The population is doubling, so an exponential model would be best.

b. What is the starting population?

The starting population is 275.

c. Is this growth or decay?

The population is increasing, so this is a growth model.

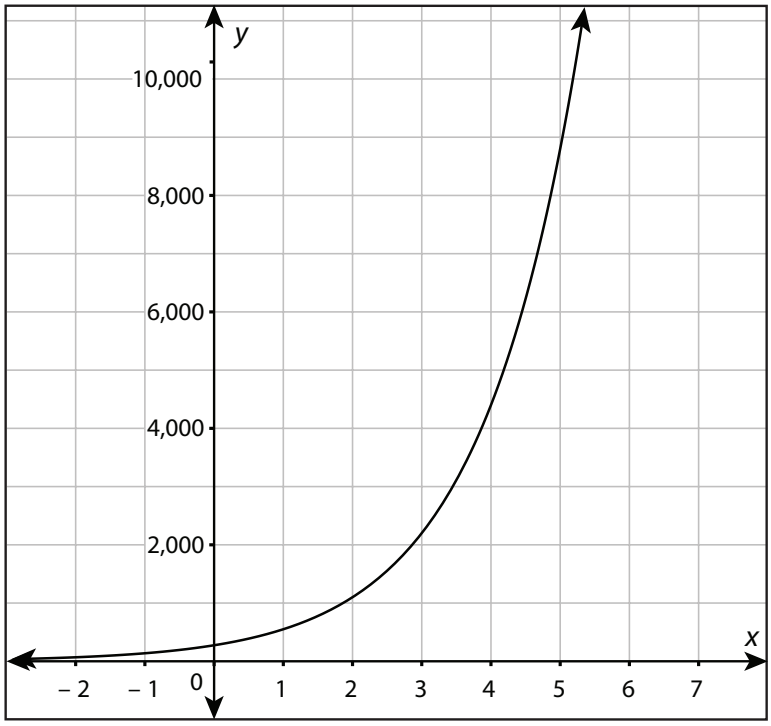
d. What is the base that indicates this growth or decay?

Since the population doubles, the base is 2.

e. Write the function that models this scenario.

The function that models this scenario is $f(x) = 275(2)^x$.

f. Use technology to graph the function.



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Instruction

g. What window settings should you use to see the graph and be able to answer the questions?

The following settings can be used to see the graph:

Xmin: –2 Ymin: 0

Xmax: 7 Ymax: 10000

h. Use technology to determine the number of trout in the pond 1 year ago. What value did you use for x? What kind of number is appropriate to answer the question?

The x-value is –1.

The number of trout 1 year ago is about 138.

The number should be an integer.

i. Use technology to determine the number of trout in the pond in 3 years. What value did you use for x? What kind of number is appropriate to answer the question?

The x-value is 3.

According to the model, the number of trout in 3 years is 2,200.

The number should be an integer.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.5.2: Graphing Exponential Functions

Use what you know about exponential functions to complete the problems that follow.

1. Given the function $f(x) = 3(2)^x + 4$, identify the *y*-intercept and describe the end behavior.

2. Given the function $f(x)=5\left(\frac{1}{4}\right)^x-4$, identify the *y*-intercept and describe the end behavior.

3. Given the function $f(x) = 4(2)^x - 6$, identify the *y*-intercept and describe the end behavior.

4. Given the function $f(x)=7\left(\frac{1}{3}\right)^x+2$, identify the *y*-intercept and describe the end behavior.

5. Given the function $f(x)=2\left(\frac{1}{4}\right)^x-3$, identify the *y*-intercept and describe the end behavior.

6. Given the function $f(x)=\frac{1}{4}(2^x)+1$, identify the *y*-intercept and describe the end behavior.

7. A certain radioactive isotope has a half-life of 6 hours. The starting amount is 100 grams. Write an exponential function to model this scenario. How much of the isotope remains after 12 hours?

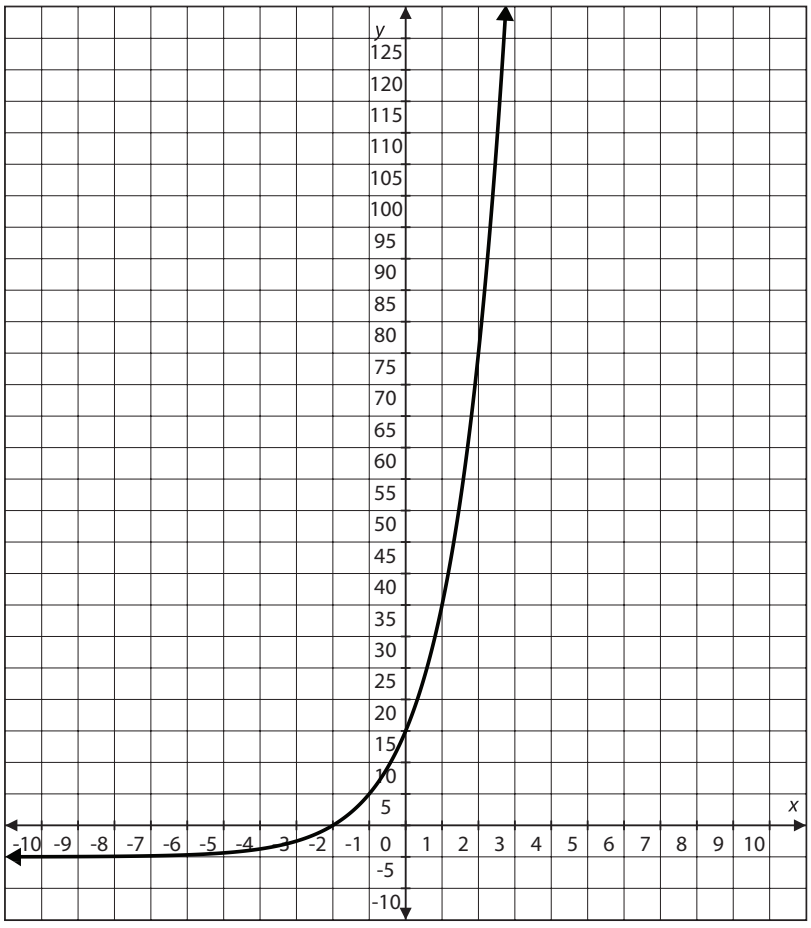
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8. You are growing bacteria for a science experiment. The number of organisms triples every 10 hours. You begin the experiment with 20 organisms. Write an exponential function to model this scenario. How many organisms will you have in 30 hours?

9. Violet invests $400 in an account that earns 2.5% interest per year. Write an exponential function to represent this scenario. How much money is in the account after 7 years?

10. The graph of a function is shown. Write a scenario that could be represented by the graph.



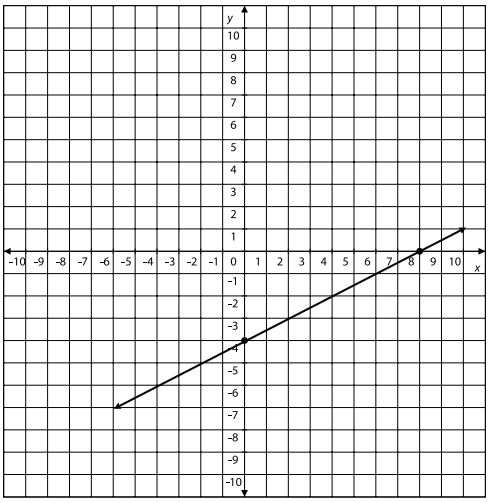
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Progress Assessment

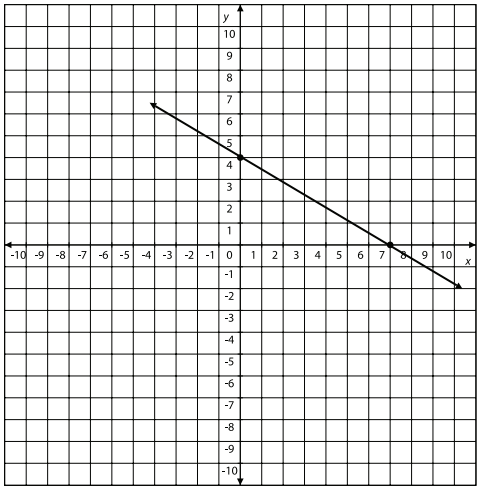
Circle the letter of the best answer.

1. Which graph shows a line with an *x*-intercept of 4?

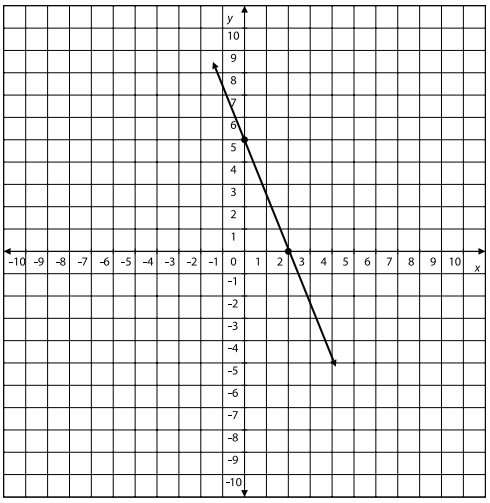
a.



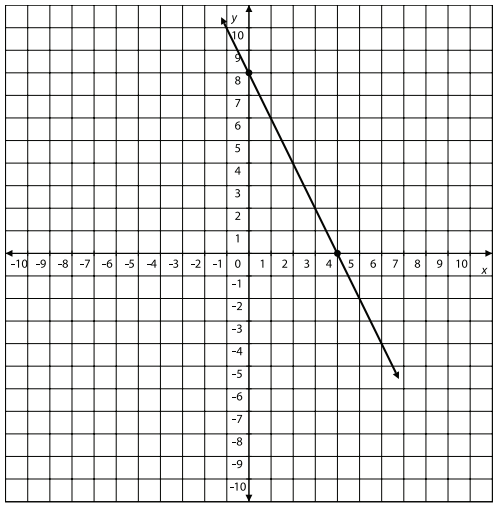
c.



b.



d.

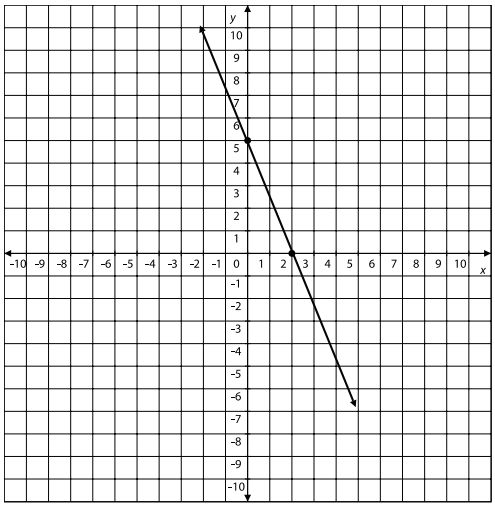


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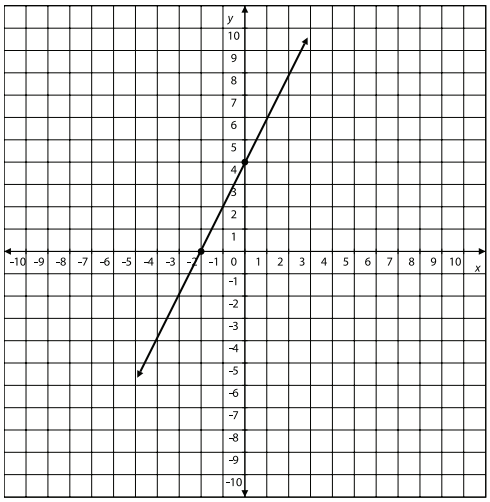
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2. Which graph shows a line with a *y*-intercept of –2?

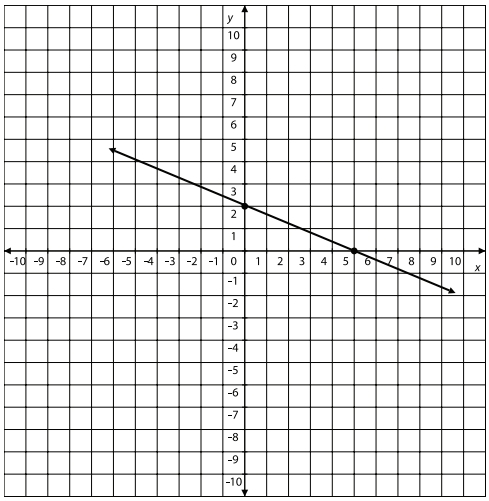
a.



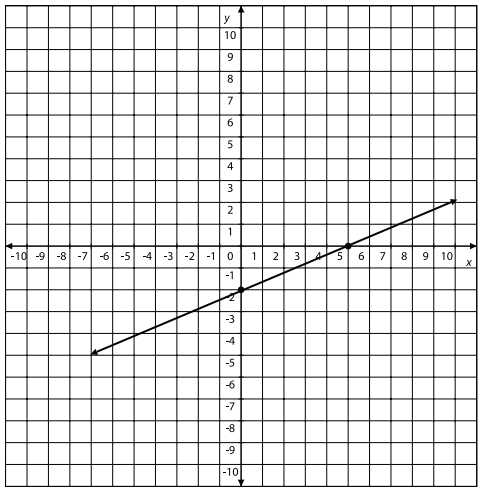
c.



b.



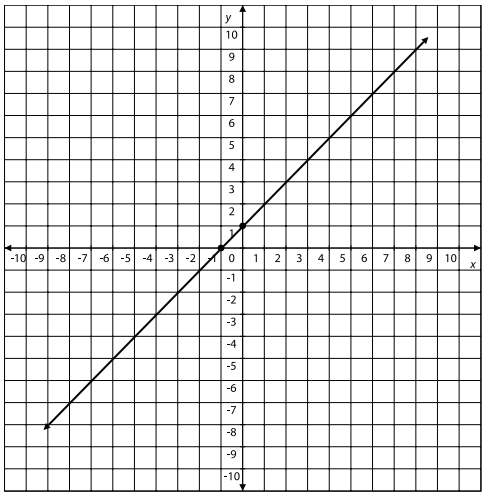
d.



continued

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3. What is the *y*-intercept of the graph?



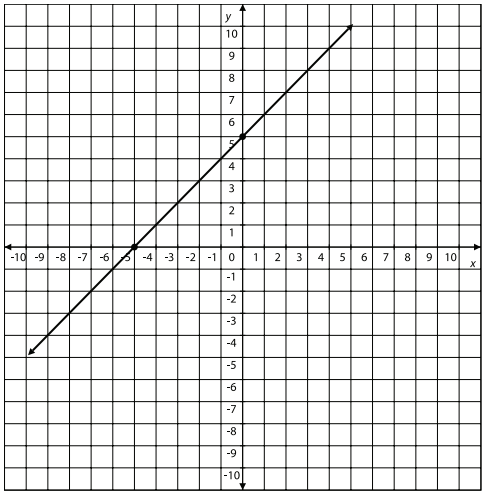
a. 1

b. –1

c. 0

d. There is no *y*-intercept.

4. What is the *x*-intercept of the graph?



a. –5

b. 5

c. 0

d. There is no *x*-intercept.

continued

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5. What are the *x*- and *y*-intercepts of the graph of $f(x)=\frac{1}{4}x-4$?

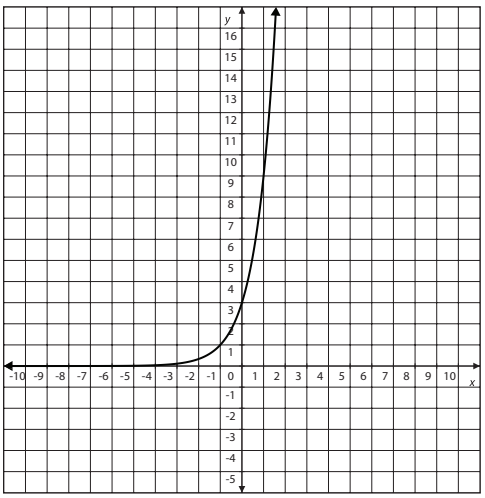
   a. 4 and 16

   b. 4 and –16

   c. –4 and –16

   d. –4 and 16

6. What is the *y*-intercept of the graph?



   a. –5

   b. 3

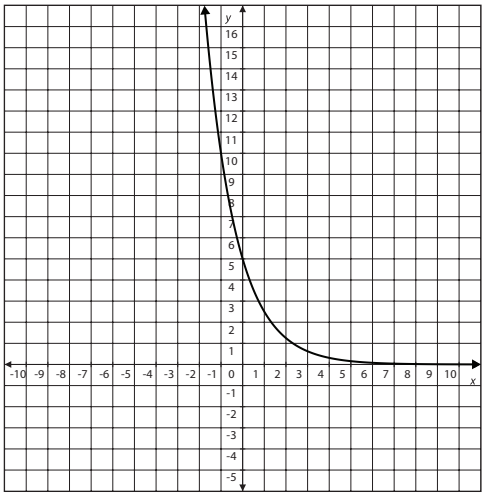
   c. 0

   d. 5

continued

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7. What is the best description of the behavior of the graph?



a. growth, with a horizontal asymptote of $y = 1$

b. decay, with a horizontal asymptote of $y = 0$

c. growth, with a horizontal asymptote of $y = 0$

d. decay, with a horizontal asymptote of $y = 1$

8. What is the y-intercept of the graph of $y=3\left(\frac{1}{5}\right)^x-2$?

a. –2

b. 0

c. 1

d. 2

9. What is the y-intercept of the graph of $f(x)=\frac{1}{4}(4)^x+2$?

a. 0

b. 0.25

c. 2

d. 2.25

continued

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10. What is the best description of the behavior of the graph of $f(x)=\frac{1}{2}(3^x)$?
    a. growth, with a horizontal asymptote of $y = 1$
    b. decay, with a horizontal asymptote of $y = 3$
    c. growth, with a horizontal asymptote of $y = 0$
    d. decay, with a horizontal asymptote of $y = 0$

Read the scenario and answer the question that follows.

11. A certain radioactive isotope has a half-life of 250 years. A scientist determines that there are 575 grams of the radioactive material present today. How much of the isotope was present 1,000 years ago?

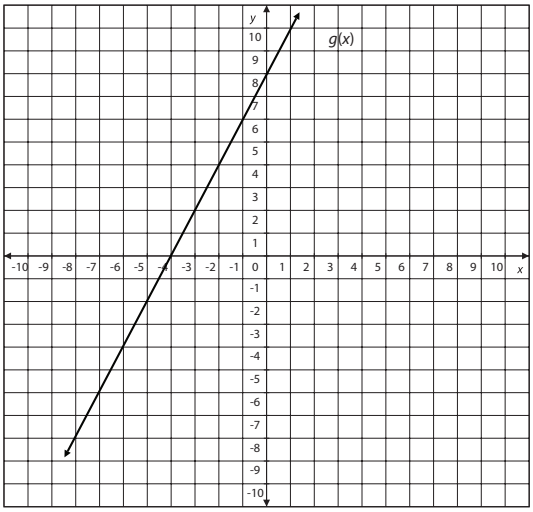
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Pre-Assessment

Circle the letter of the best answer.

1. Which of the following statements is true about the functions $f(x)$ and $g(x)$, shown in the table and graph?

x	$f(x)$
–2	1
0	7
2	13
4	19



   a. The function $f(x)$ has a greater y-intercept than the function $g(x)$.

   b. The function $g(x)$ has a greater rate of change than the function $f(x)$.

   c. The function $g(x)$ has a greater y-intercept than the function $f(x)$.

   d. The rates of change for both $f(x)$ and $g(x)$ are equal.

2. Which of the following statements is true about the functions $f(x)$ and $g(x)$?

$$f(x)=\frac{2}{3}x-6$$

x	$g(x)$
–4	10
0	7
4	4
8	1

   a. The rate of change of $f(x)$ is less than the rate of change of $g(x)$.

   b. The rate of change of $f(x)$ is greater than the rate of change of $g(x)$.

   c. The y-intercept of $f(x)$ is equal to the rate of change of $g(x)$.

   d. The y-intercept of $f(x)$ is greater than the rate of change of $g(x)$.

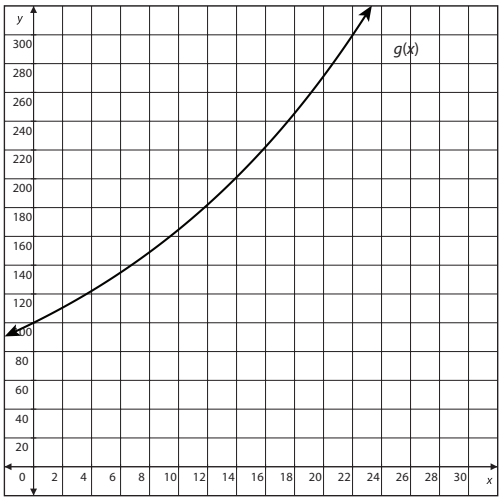
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Assessment

3. Which of the following statements is true about the functions $f(x)$ and $g(x)$ over the interval [0, 12]?

$$f(x)=200\left(1+\frac{0.05}{12}\right)^{12x}$$



a. The rates of change for the functions $f(x)$ and $g(x)$ are equal over the interval [0, 12].

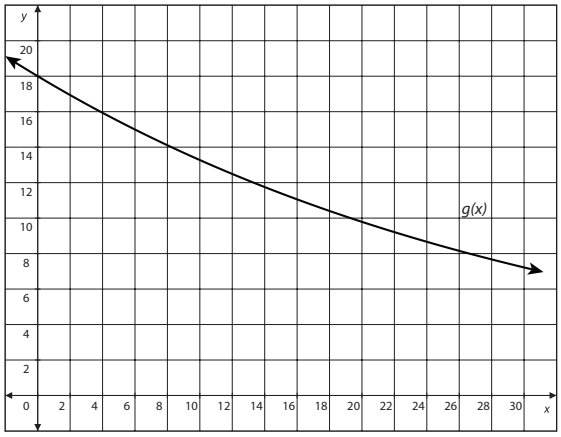
b. The rate of change for the function $f(x)$ is greater than the rate of change for the function $g(x)$ over the interval [0, 12].

c. The rate of change for the function $f(x)$ is less than the rate of change for the function $g(x)$.

d. The rate of change for the functions cannot be determined.

4. Which of the following statements is true about the functions $f(x)$ and $g(x)$?

x	$f(x)$
–1	18.95
0	18
1	17.1
2	16.25



a. The y-intercept of the function $f(x)$ is greater than the y-intercept for the function $g(x)$.

b. The y-intercept of the function $f(x)$ is less than the y-intercept for the function $g(x)$.

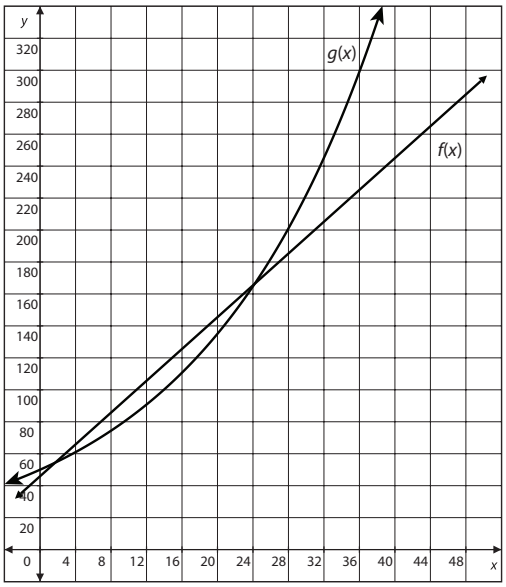
c. The y-intercepts of the functions $f(x)$ and $g(x)$ are equal.

d. The y-intercepts of the functions cannot be determined.

continued

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5. Which of the following statements is true about the functions $f(x)$ and $g(x)$ shown?



a. The rate of change of the function $f(x)$ is always greater than the rate of change of the function $g(x)$.

b. The rate of change of the function $g(x)$ will eventually be greater than the rate of change of the function $f(x)$.

c. The rate of change of the function $f(x)$ is never greater than the rate of change of the function $g(x)$.

d. The rate of change of an exponential function cannot be determined.

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Lesson 6: Comparing Functions

Instruction

Common Core State Standards

SMP	
1 ✓	2
3 ✓	4 ✓
5 ✓	6 ✓
7	8

F–IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

F–LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★

Essential Questions

1. What different interpretations can be made from different representations of functions?
2. Why is comparing functions important?
3. How can you use characteristics of functions to compare functions?

WORDS TO KNOW

exponential function a function whose independent variable is in the exponent; the general form of its equation is $f(x) = ab^x + k$, where a is the initial value, b is the base, x is the input value, k is the vertical shift, and $f(x)$ is the output

factor one of two or more numbers or expressions that are multiplied to produce a product

growth factor the factor by which a quantity increases or decreases over time

interval the continuous set of real numbers between two given numbers

linear function a first-degree equation that can be written in the form $f(x) = mx + b$, in which m is the slope of the line and b is the y-intercept. The graph of a linear function is a straight line.

rate of change a ratio that describes how much one quantity changes with respect to the change in another quantity; also known as the slope of a line

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Instruction

slope	the measure of the rate of change of one variable with respect to another variable; slope $= m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$
***x*-intercept**	the x-coordinate of the point at which a line or a curve intersects the x-axis
***y*-intercept**	the y-coordinate of the point at which a line or a curve intersects the y-axis

Recommended Resources

- Algebra 4 All. "Exponential Functions."

  http://walch.com/rr/CAU3L5ExponentialFunctions

  This site provides links to several applets that allow users to explore exponential functions.

- Interactivate. "Graphit."

  http://walch.com/rr/CAU3L5Graphit

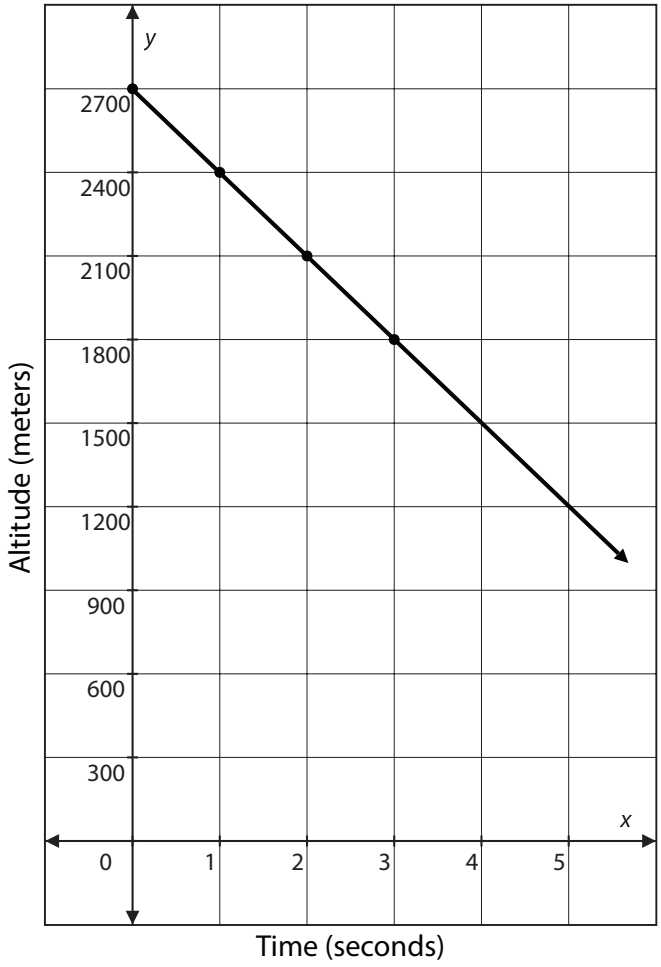
  This interactive applet allows users to compare functions using tables, graphs, and/or equations.

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Lesson 2.6.1: Comparing Linear Functions

Warm-Up 2.6.1

Cecilia took her first parachute jump lesson last weekend. Her instructor gave her the following graph, which shows her change in altitude in meters during a 5-second interval. Use the graph to complete the problems that follow.



1. Estimate Cecilia's average rate of change in altitude in meters per second.

2. What are the domain and range for this function?

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Instruction

Lesson 2.6.1: Comparing Linear Functions

Common Core State Standard

F–IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Warm-Up 2.6.1 Debrief

1. Estimate Cecilia's average rate of change in altitude in meters per second.

   Select two points from the graph and use the slope formula to calculate the rate of change.

   Let $(x_1, y_1) = (1, 2400)$ and $(x_2, y_2) = (3, 1800)$.

   $$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

   $$= \frac{(1800) - (2400)}{(3) - (1)}$$ Substitute (1, 2400) for (x_1, y_1) and (3, 1800) for (x_2, y_2).

   $$= \frac{-600}{2}$$ Simplify.

   $$= -300$$

   The average rate of change is –300 meters per second.

   Cecilia's altitude is decreasing an average of 300 meters each second.

2. What are the domain and range for this function?

   The domain is the set of all possible x-values for which the function is defined.

   In this problem, the domain is all real numbers from $x = 0$, when she jumps, to $x = 9$, when she lands on the ground: $0 \le x \le 9$.

   The range is the set of all possible y-values for the defined values of x.

   In this problem, the range is all real numbers from $y = 2700$, her initial height, to $y = 0$, her altitude when she lands: $0 \le y \le 2700$.

Connection to the Lesson

- Students will be determining the rate of change as well as the y-intercept from graphs, tables, equations, and verbal descriptions.
- Students will take this type of problem a step further and compare the rate of change and y-intercept to other linear functions.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- determining the slopes of linear functions (8.EE.5)
- determining the intercepts of linear functions (8.EE.6)

Introduction

Remember that linear functions are first-degree equations that can be written in the form $f(x) = mx + b$, where m is the slope and b is the y-intercept. The slope of a linear function is also the rate of change and can be calculated using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$. The y-intercept is the y-coordinate of the point at which the graph intersects the y-axis; it is the value of y when $x = 0$. The x-intercept, if it exists, is the x-coordinate of the point where the graph intersects the x-axis; it is the value of x when $y = 0$. The slope and both intercepts can be determined from tables, equations, and graphs. These features are used to compare linear functions to one another.

Key Concepts

- Linear functions can be represented in words or as equations, graphs, or tables.
- To compare linear functions, determine the rate of change and intercepts of each function.
- Review the following processes for identifying the rate of change and y-intercept of a linear function.

Identifying the Rate of Change and the *y*-intercept from Context

1. Read the problem statement carefully.
2. Look for the information given and make a list of the known quantities.
3. Determine which information tells you the rate of change, or the slope, m. Look for words such as *each*, *every*, *per*, or *rate*.
4. Determine which information tells you the y-intercept, or b. This could be an initial value or a starting value, a flat fee, and so forth.

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Identifying the Rate of Change and the *y*-intercept from an Equation

1. Write the equation of the function in slope-intercept form, $f(x) = mx + b$.
2. Identify the rate of change, or the slope, m, as the coefficient of x.
3. Identify the y-intercept, or b, as the constant term in the function.

Identifying the Rate of Change and the *y*-intercept from a Table

1. Choose two points from the table.
2. Assign one point to be (x_1, y_1) and the other point to be (x_2, y_2).
3. Substitute the coordinates into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
4. Identify the y-intercept as the y-coordinate in the ordered pair $(0, y)$. If this coordinate is not given, substitute the slope and the coordinates of any ordered pair from the table into the equation $f(x) = mx + b$ and solve for b.

Identifying the Rate of Change and the *y*-intercept from a Graph

1. Choose two points from the graph.
2. Assign one point to be (x_1, y_1) and the other point to be (x_2, y_2).
3. Substitute the coordinates into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
4. Identify the y-intercept as the y-coordinate of the point where the line intersects the y-axis.

- When presented with functions represented in different ways, it is helpful to rewrite the information using function notation.
- Linear functions are increasing if the rate of change is a positive value.
- Linear functions are decreasing if the rate of change is a negative value.
- The greater the absolute value of the slope, the steeper the line will appear on the graph.
- A rate of change of 0 indicates a horizontal line on a graph.

Common Errors/Misconceptions

- incorrectly determining the rate of change
- not comparing the absolute values of the slopes to determine which function is steeper
- interchanging the x- and y-intercepts

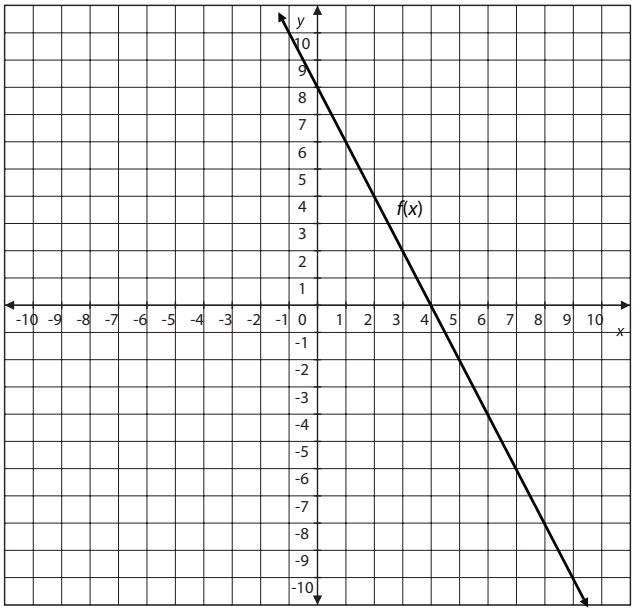
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Instruction

Guided Practice 2.6.1

Example 1

The functions $f(x)$ and $g(x)$ are shown. Compare the properties of each.



x	$g(x)$
–2	–10
–1	–8
0	–6
1	–4

1. Identify the rate of change for the first function, $f(x)$.

   Let (0, 8) be (x_1, y_1) and (4, 0) be (x_2, y_2).

   Substitute the coordinates into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(0) - (8)}{(4) - (0)}$$ Substitute (0, 8) for (x_1, y_1) and (4, 0) for (x_2, y_2).

$$= \frac{-8}{4}$$ Simplify.

$$= -2$$

   The rate of change for this function is –2.

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Instruction

2. Identify the rate of change for the second function, $g(x)$.

Choose two ordered pairs from the table. Let $(-2, -10)$ be (x_1, y_1) and let $(-1, -8)$ be (x_2, y_2).

Substitute the coordinates into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(-8) - (-10)}{(-1) - (-2)}$$ Substitute $(-2, -10)$ for (x_1, y_1) and $(-1, -8)$ for (x_2, y_2).

$$= \frac{2}{1}$$ Simplify.

$$= 2$$

The rate of change for this function is 2.

3. Identify the y-intercept of the first function, $f(x)$.

The graph intersects the y-axis at $(0, 8)$, so the y-intercept is 8.

4. Identify the y-intercept of the second function, $g(x)$.

From the table, we can determine that the function would intersect the y-axis where the x-value is 0. This happens at the point $(0, -6)$. The y-intercept is -6.

5. Compare the properties of each function.

The rate of change for the first function is -2 and the rate of change for the second function is 2. The first function is decreasing and the second is increasing, but the absolute values of the slopes are equal, so the lines are equally steep.

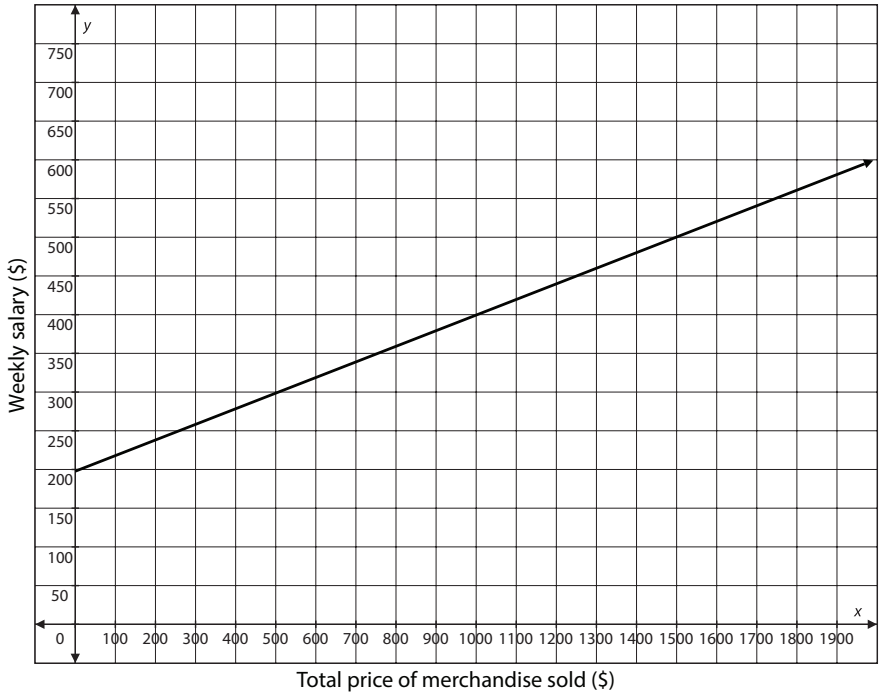
The y-intercept of the first function is 8, but the y-intercept of the second function is -6. The graph of the second function intersects the y-axis at a lower point.

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Example 2

Your employer has offered two pay scales for you to choose from. The first option is to receive a base salary of $250 a week plus 15% of the price of any merchandise you sell. The second option is represented in the graph, where x represents the price of the merchandise sold and y represents your weekly salary. Compare the properties of the functions.



1. Identify the rate of change for the first function.

   Determine which information tells you the rate of change, or the slope, m.

   You are told that your employer will pay you 15% of the price of the merchandise you sell.

   This information is the rate of change for this function and can be written as 0.15.

2. Identify the y-intercept for the first function.

   Your employer has offered a base salary of $250 per week.

   250 is the y-intercept of the function.

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Instruction

3. Identify the rate of change for the second function.

Let (0, 200) be (x_1, y_1) and (500, 300) be (x_2, y_2).

Substitute the values into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(300) - (200)}{(500) - (0)}$$ Substitute (0, 200) for (x_1, y_1) and (500, 300) for (x_2, y_2).

$$= \frac{100}{500}$$ Simplify.

$$= \frac{1}{5} = 0.2$$

The rate of change for this function is 0.2.

4. Identify the *y*-intercept as the *y*-coordinate of the point where the line intersects the *y*-axis.

The graph intersects the *y*-axis at (0, 200). The *y*-intercept is 200.

5. Compare the properties of each function.

The rate of change for the second function is greater than the first function. You will get paid more for the amount of merchandise you sell.

The *y*-intercept of the first function is greater than the second. You will get a higher base pay with the first function.

In the first function, you would receive a higher base salary, but get paid less for the amount of merchandise you sell.

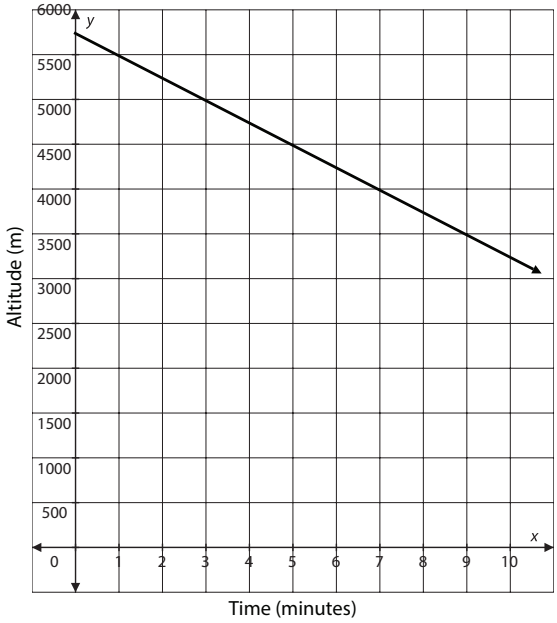
In the second function, you would receive a lower base salary, but get paid more for the merchandise you sell.

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Instruction

Example 3

Two airplanes are in flight. The function $f(x) = 400x + 1200$ represents the altitude in meters, $f(x)$, of one airplane after x minutes. The following graph represents the altitude of the second airplane after x minutes. Compare the properties of the functions.



1. Identify the rate of change for the first function.

   The function is written in $f(x) = mx + b$ form; therefore, the rate of change for the function is 400.

2. Identify the y-intercept for the first function.

   The y-intercept of the first function is 1,200, as stated in the equation.

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3. Identify the rate of change for the second function.

Choose two points from the graph.

Let (0, 5750) be (x_1, y_1) and (5, 4500) be (x_2, y_2).

Substitute the coordinates into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(4500) - (5750)}{(5) - (0)}$$ Substitute (0, 5750) for (x_1, y_1) and (5, 4500) for (x_2, y_2).

$$= \frac{-1250}{5}$$ Simplify.

$$= -250$$

The rate of change for this function is –250.

4. Identify the *y*-intercept of the second function as the *y*-coordinate of the point where the line intersects the *y*-axis.

The graph intersects the *y*-axis at (0, 5750), so the *y*-intercept is 5,750.

5. Compare the properties of each function.

The absolute value of the slope for the first function is greater than the absolute value of the slope for the second function. The slope for the first function is also positive, whereas the slope for the second function is negative. The first airplane is ascending at a faster rate than the second airplane is descending.

The *y*-intercept of the second function is greater than the first. The second airplane is higher in the air than the first airplane at that moment.

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Problem-Based Task 2.6.1: Supply and Demand

SMP	
1 ✓	2
3 ✓	4 ✓
5	6
7	8

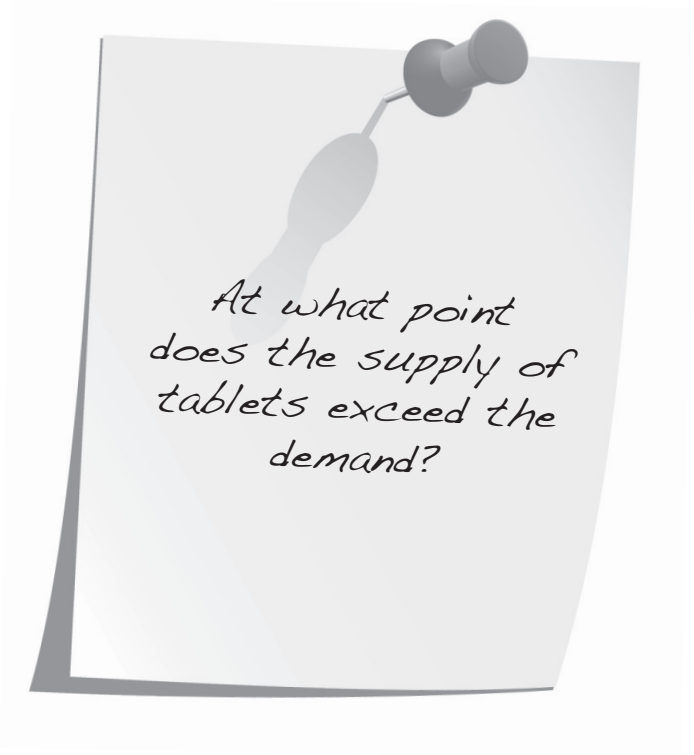
Ideal Electronics is determining the price of the newest tablet to hit the market. In an effort to make the most money and sell the most tablets, Ideal Electronics wants to price the tablet appropriately to the product's supply and demand. Supply is the number of tablets that are available and demand is the amount that buyers are willing to pay. The relationship between supply and demand often influences the price of products.

Supply is modeled by the linear function $f(x) = 0.3x + 100$, where $f(x)$ represents the price per tablet in dollars and x represents the number of tablets.

Demand is modeled in the following table, where $g(x)$ represents the price per tablet in dollars and x represents the number of tablets.

x	$g(x)$
100	490
300	370
500	250
600	190

Compare the properties of both of the functions described. At what point does the supply of tablets exceed the demand? Explain your reasoning.



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Problem-Based Task 2.6.1: Supply and Demand

Coaching

a. What is the rate of change of the supply function?

b. What is the *y*-intercept of the supply function?

c. What is the rate of change of the demand function?

d. What is the *y*-intercept of the demand function?

e. How does the rate of change of the supply function compare to the rate of change of the demand function?

f. How does the *y*-intercept of the supply function compare to the *y*-intercept of the demand function?

g. When graphing both functions, what does the *x*-axis represent?

h. What does the *y*-axis represent?

i. At what point are the supply and demand functions equal?

j. For what value of *x* does the supply function exceed the demand function?

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Instruction

Problem-Based Task 2.6.1: Supply and Demand

Coaching Sample Responses

a. What is the rate of change of the supply function?

The supply function is $f(x) = 0.3x + 100$, and is written in $f(x) = mx + b$ form.

The rate of change of the function is the coefficient of x, or 0.3.

b. What is the y-intercept of the supply function?

The y-intercept of the supply function is the constant term, 100.

c. What is the rate of change of the demand function?

To determine this, choose two points from the table.

Let (100, 490) be (x_1, y_1) and (600, 190) be (x_2, y_2).

Substitute the coordinates into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{(190) - (490)}{(600) - (100)}$$
$$= \frac{-300}{500}$$
$$= \frac{-3}{5} = -0.6$$

The rate of change for this function is –0.6.

d. What is the y-intercept of the demand function?

The y-intercept is not listed in the table.

Calculate the y-intercept using one of the points from the table and the rate of change.

Let (100, 490) be (x_1, y_1) and $(0, y_2)$ be (x_2, y_2).

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$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(-0.6) = \frac{y_2 - (490)}{(0) - (100)}$$

$$-0.6 = \frac{y_2 - 490}{-100}$$

$$60 = y_2 - 490$$

$$550 = y_2$$

The y-intercept of the demand function is 550.

e. How does the rate of change of the supply function compare to the rate of change of the demand function?

The rate of change of the supply function is a positive value of 0.3, whereas the demand function is a negative value of –0.6. That is, the supply function is increasing as the demand function is decreasing.

f. How does the y-intercept of the supply function compare to the y-intercept of the demand function?

The y-intercept of the supply function, 100, is less than the y-intercept of the demand function, 550.

g. When graphing both functions, what does the x-axis represent?

The x-axis represents the number of tablets.

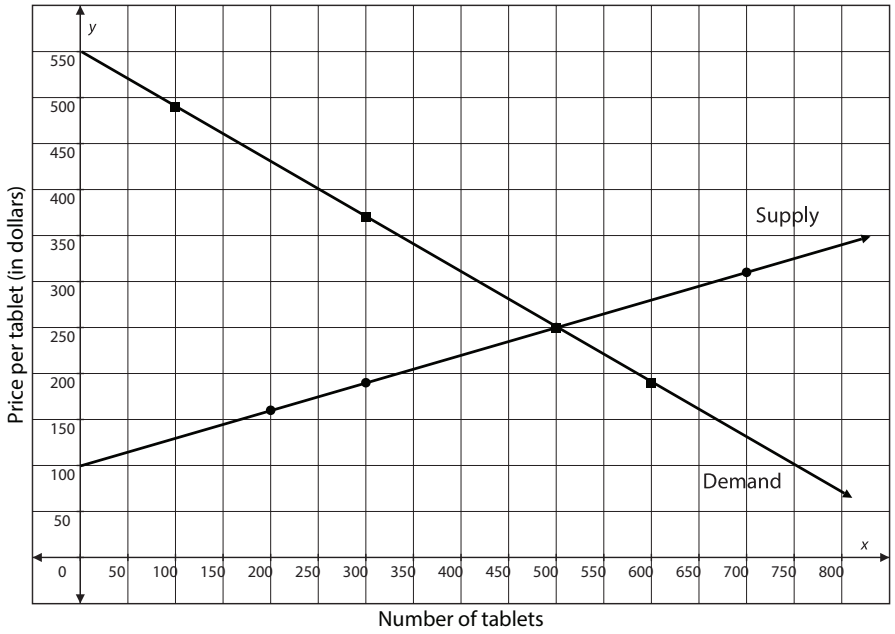
h. What does the y-axis represent?

The y-axis represents the price per tablet.

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i. At what point are the supply and demand functions equal?

Graph each function using the information from parts a–h.



From the graph, we can see that the lines intersect at the point (500, 250).

At this point, the number of tablets demanded by the buyers is equal to the number of tablets the seller has.

When priced at $250, it can be expected that 500 tablets will be sold.

j. For what value of x does the supply function exceed the demand function?

For $x > 500$, the supply function exceeds the demand function.

After that, the number of tablets that are available is greater than the number of tablets demanded by the buyers.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.6.1: Comparing Linear Functions

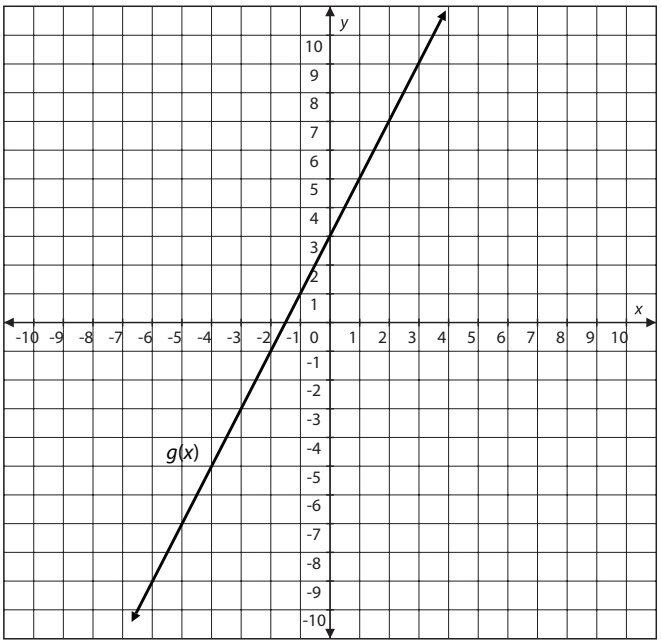
Compare the properties of the linear functions.

1. Which function has a greater rate of change? Which function has the greater *y*-intercept? Explain how you know.

Function A

x	$f(x)$
–4	12
–1	0
2	–12
3	–16

Function B

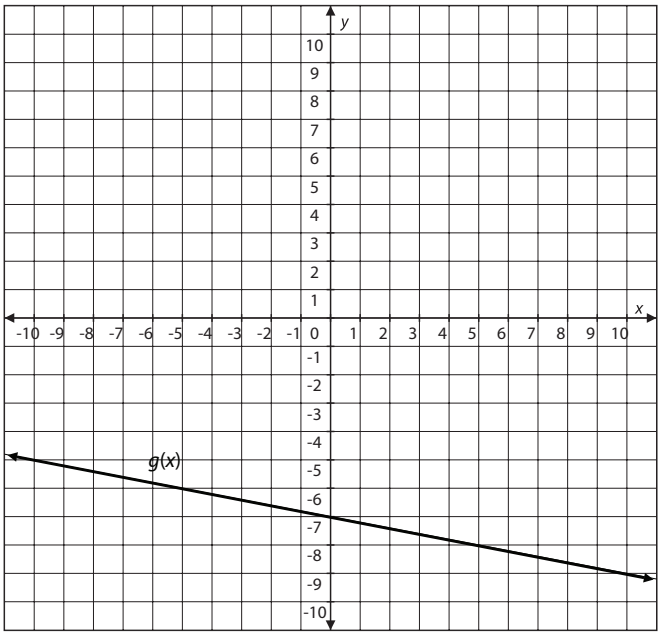


2. Which function has a greater rate of change? Which function has the greater *y*-intercept? Explain how you know.

Function A

x	$f(x)$
–8	1
0	2
4	2.5
8	3

Function B



continued

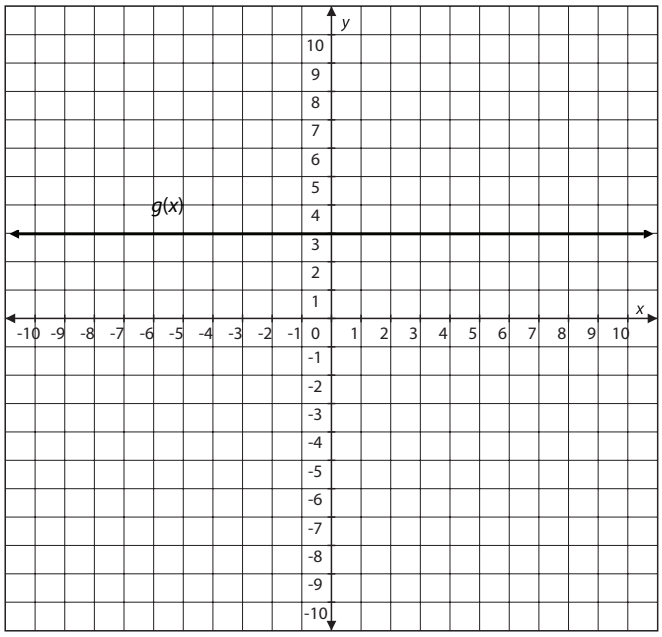
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3. Compare the properties of each function.

Function A

$f(x)=\frac{1}{4}x+3$

Function B

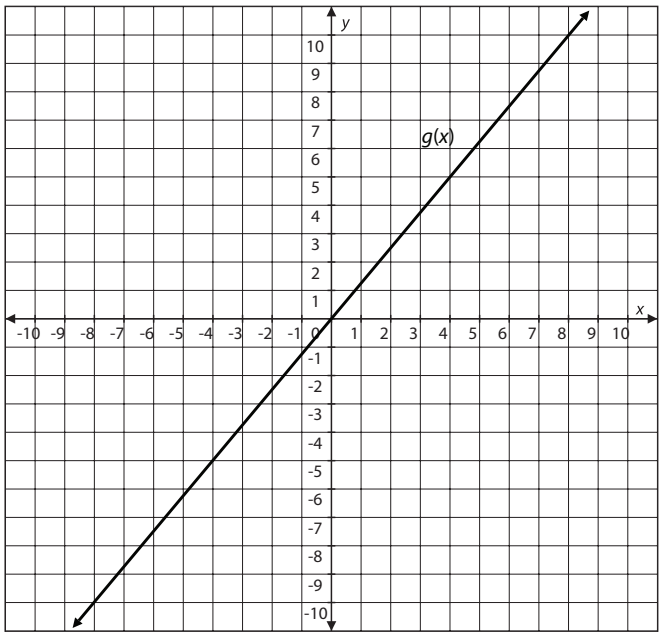


4. Compare the properties of each function.

Function A

$f(x) = -5x$

Function B



continued

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5. Compare the properties of each function.

Function A

The following table describes the profit in dollars that a restaurant makes for the number of beverages it sells.

Number of beverages sold (x)	Profit ($f(x)$)
0	0
25	29.25
50	58.50
75	87.75

Function B

For each hamburger sold, the same restaurant makes a profit of $0.40.

6. Compare the properties of each function.

Function A

A local newspaper began with a circulation of 1,300 readers in its first year. Since then, its circulation has increased by 150 readers per year.

Function B

The function $g(x) = 225x + 950$ represents the circulation of another newspaper where $g(x)$ represents total subscriptions and x represents the number of years since its first year.

7. Compare the properties of each function.

Function A

A rental store charges $40 to rent a steam cleaner, plus an additional $4 per hour.

Function B

The following table shows the total cost in dollars to rent a steam cleaner at a different rental store. $g(x)$ represents the total cost after x hours.

Hours (x)	Total cost ($g(x)$)
3	46
4	53
5	60
6	67

continued

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8. Compare the properties of each function.

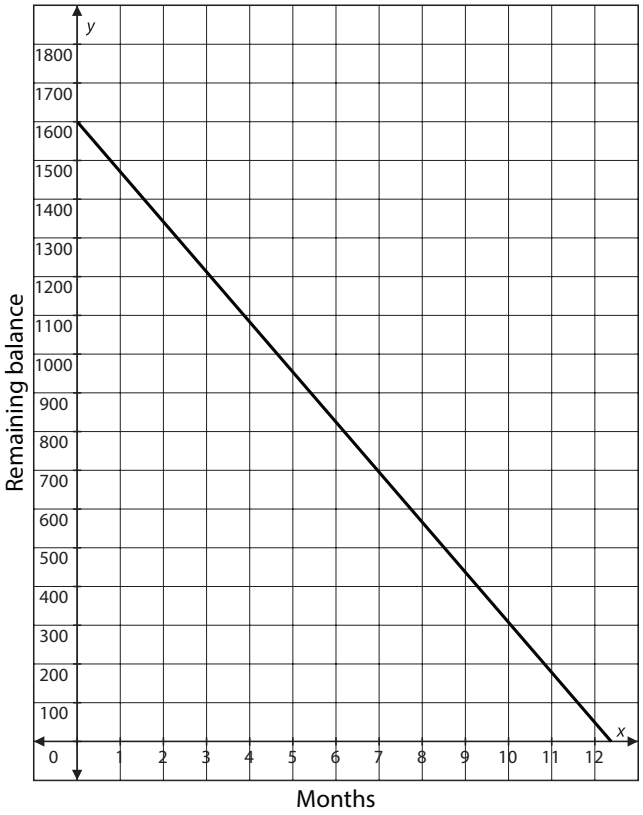
Function A

The table shows the remaining balance in dollars, $f(x)$, of the cost of car repairs after x months.

Months (x)	Remaining balance ($f(x)$)
0	1560
1	1430
2	1300
3	1170

Function B

The graph shows the remaining balance in dollars, $g(x)$, of the cost of car repairs after x months.



9. Compare the properties of each function. What do the rate of change and y-intercept mean in terms of the scenarios?

Function A

The function $f(x) = 7.5 - 0.25x$ represents the pounds of puppy food remaining, $f(x)$, when the puppy is fed the same amount each day for x days.

Function B

The table represents the amount in pounds of puppy food remaining, $g(x)$, when the puppy is fed the same amount each day for x days.

Days (x)	Remaining food ($g(x)$)
4	9
5	8.75
6	8.5
7	8.25

continued

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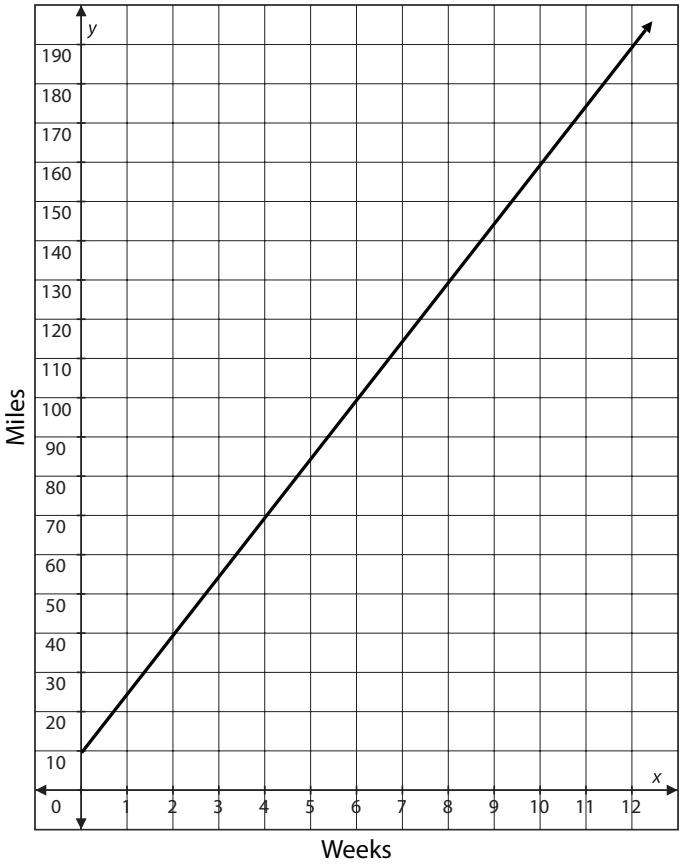
10. Compare the properties of each function. What do the rate of change and *y*-intercept mean in terms of the scenarios?

Function A

Reggie bicycled 15 miles last week and plans to bicycle 20 miles each additional week.

Function B

The graph represents the total number of miles Zac plans to have bicycled by the end of each week.



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Lesson 2.6.2: Comparing Exponential Functions

Warm-Up 2.6.2

A hard rubber ball will rebound to 75% of its height each time it bounces.

1. If the ball is dropped from a height of 200 centimeters, what will the height of each bounce be after 11 bounces? Create a table and a graph of the ball's bounce rebound height over several bounces.

2. What is the first bounce for which the height of the rebound will be less than 50 centimeters?

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Instruction

Lesson 2.6.2: Comparing Exponential Functions

Common Core State Standard

F–IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Warm-Up 2.6.2 Debrief

1. If the ball is dropped from a height of 200 centimeters, what will the height of each bounce be after 11 bounces? Create a table and a graph of the ball's bounce rebound height over several bounces.

   Begin by creating a table with the number of bounces and the height of the ball after each bounce.

   At 0 bounces, the ball is at the initial height of 200 centimeters.

   After each bounce, the height is 75% of the previous height.

   To calculate the height of the ball after 1 bounce, find 75% of 200, or 200 • 0.75.

   The height after the first bounce is 150 centimeters.

   To calculate the height of the ball after 2 bounces, find 75% of the previous height, or 150 • 0.75.

   The height after the second bounce is 112.5 centimeters.

   Continue this way until the table is completed for 11 bounces.

Number of bounces	Height of bounce (cm)
0	200
1	150
2	112.5
3	84.38
4	63.28
5	47.46
6	35.60
7	26.70
8	20.02
9	15.02
10	11.26
11	8.45

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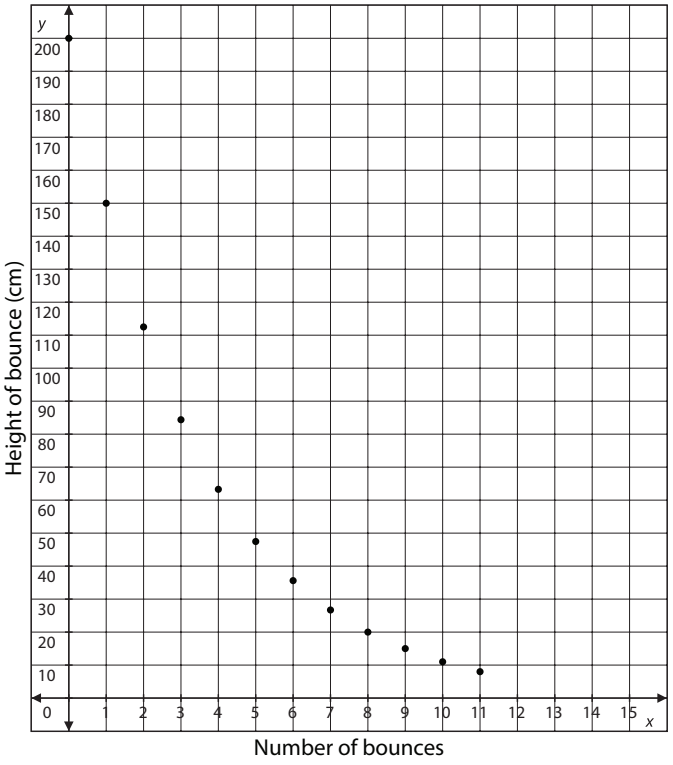
Instruction

Use the table to plot the points on a coordinate plane.

Label the *x*-axis "Number of bounces" and the *y*-axis "Height of bounce (cm)."

Plot each point.

This scenario can represented by the exponential function $f(x) = 200(0.75)^x$.



2. What is the first bounce for which the height of the rebound will be less than 50 centimeters?

   We can see from both the table and the graph that the height of the fifth bounce is the first bounce whose height is less than 50 centimeters.

Connection to the Lesson

- As in the warm-up, students will create exponential functions from context.
- Students will use their knowledge of writing exponential functions to analyze properties of exponential functions in order to make comparisons.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- determining the rate of change of exponential functions (F–IF.6★)
- determining the intercepts of exponential functions (F–IF.4★)

Introduction

Exponential functions are functions that can be written in the form $f(x) = ab^x + k$, where a and k are real numbers, b is the base, x is the input value, and $f(x)$ is the output. In a growth or decay formula, the **growth factor** is the factor by which a quantity increases or decreases over time. The rate of change of an exponential function can be calculated using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, over a specified interval. An interval is a continuous set of real numbers between two given numbers. The y-intercept is the y-coordinate of the point at which the function intersects the y-axis. Both the rate of change and y-intercept can be determined from tables, equations, and graphs. Exponential functions can also be compared to one another using these features.

Key Concepts

- Exponential functions can be represented in words or as equations, graphs, or tables.
- To compare exponential functions, determine the rate of change and the intercepts of each function. Review the following processes for identifying the rate of change and the y-intercept of an exponential function.

Identifying the Rate of Change and the *y*-intercept from Context

1. Determine the interval to be observed.
2. Create a table of values by choosing appropriate x-values, substituting them into the equation for the function, and solving for $f(x)$.
3. Choose two points from the table.
4. Assign one point to be (x_1, y_1) and the other point to be (x_2, y_2).
5. Substitute the coordinates into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
6. The result is the rate of change for the interval between the two points.
7. Determine which information tells you the y-intercept. This could be an initial value or a starting value, a flat fee, and so forth.

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Instruction

Identifying the Rate of Change and the *y*-intercept from Exponential Equations

1. Determine the interval to be observed.
2. Determine (x_1, y_1) by identifying the initial x-value of the interval and substituting it into the function.
3. Solve for $f(x)$.
4. Determine (x_2, y_2) by identifying the ending x-value of the interval and substituting it into the function.
5. Solve for $f(x)$.
6. Substitute (x_1, y_1) and (x_2, y_2) into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to calculate the rate of change.
7. Determine the y-intercept by substituting 0 for x and solving for $f(0)$.

Identifying the Rate of Change and the *y*-intercept from a Table

1. Determine the interval to be observed.
2. Assign one point to be (x_1, y_1) and the other point to be (x_2, y_2).
3. Substitute the values into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
4. The result is the rate of change for the interval between the two points.
5. Identify the y-intercept as the y-coordinate in the ordered pair $(0, y)$.

Identifying the Rate of Change and the *y*-intercept from a Graph

1. Determine the interval to be observed.
2. Identify (x_1, y_1) as the initial point of the interval.
3. Identify (x_2, y_2) as the ending point of the interval.
4. Substitute (x_1, y_1) and (x_2, y_2) into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to calculate the rate of change.
5. Identify the y-intercept as the y-coordinate in the ordered pair $(0, y)$.

- Exponential functions are increasing if the rate of change is a positive value.
- Exponential functions are decreasing if the rate of change is a negative value.
- The function whose rate of change has the greater absolute value over a given interval has a steeper graph.

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Instruction

Common Errors/Misconceptions

- incorrectly determining the rate of change
- assuming that a positive slope will be steeper than a negative slope
- not comparing the absolute values of the slopes to determine which function is steeper
- incorrectly applying the order of operations
- using the exponential growth model instead of exponential decay and vice versa

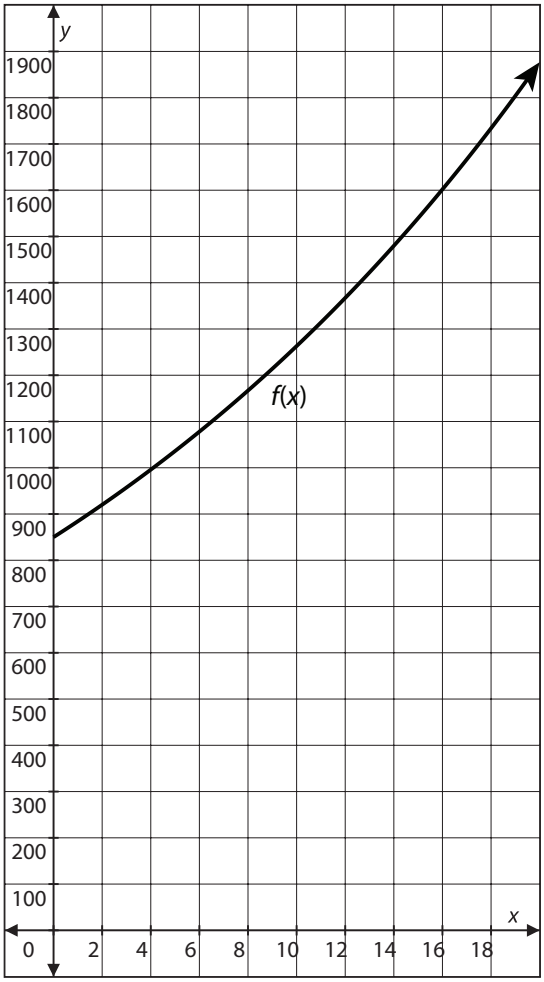
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Guided Practice 2.6.2

Example 1

Compare the properties of each of the following two functions over the interval [0, 16].

Function A



Function B

x	$g(x)$
0	850
4	976.55
8	1121.94
12	1288.98
16	1480.88

1. Compare the y-intercepts of each function.

   Identify the y-intercept of the graphed function, $f(x)$.

   The graphed function appears to cross the y-axis at the point (0, 850).

   According to the table, $g(x)$ has a y-intercept of (0, 850).

   Both functions have a y-intercept of (0, 850).

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Instruction

2. Compare the rate of change for each function over the interval [0, 16].

Calculate the rate of change over the interval [0, 16] for $f(x)$.

Let $(x_1, y_1) = (0, 850)$.

Determine (x_2, y_2) from the graph.

The value of y when x is 16 is approximately 1,600.

Let $(x_2, y_2) = (16, 1600)$.

Calculate the rate of change using the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(1600) - (850)}{(16) - (0)}$$ Substitute (0, 850) for (x_1, y_1) and (16, 1600) for (x_2, y_2).

$$= \frac{750}{16}$$ Simplify.

$$= 46.875$$

The rate of change for $f(x)$ is approximately 47.

Calculate the rate of change over the interval [0, 16] for $g(x)$.

Let $(x_1, y_1) = (0, 850)$.

Determine (x_2, y_2) from the table.

The value of y when x is 16 is 1,480.88.

Let $(x_2, y_2) = (16, 1480.88)$.

(*continued*)

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Instruction

Calculate the rate of change using the slope formula.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{(1480.88) - (850)}{(16) - (0)}$ Substitute (0, 850) for (x_1, y_1) and (16, 1480.88) for (x_2, y_2).

$= \frac{630.88}{16}$ Simplify.

$= 39.43$

The rate of change for $g(x)$ is 39.43.

The rate of change for the graphed function, $f(x)$, is greater over the interval [0, 16] than the rate of change for the function in the table, $g(x)$.

3. Summarize your findings.

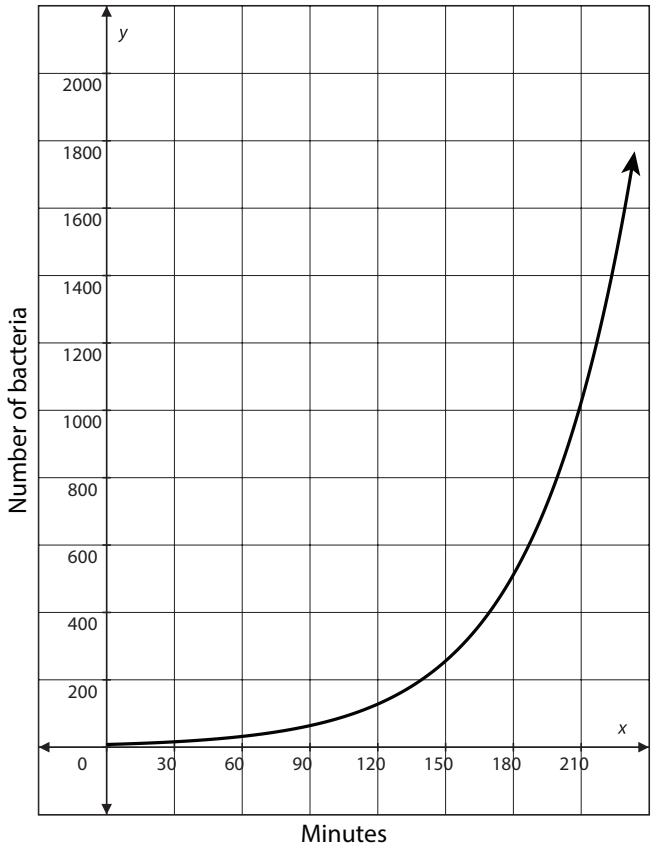
The y-intercepts of both functions are the same; however, the graphed function, $f(x)$, has a greater rate of change over the interval [0, 16].

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Example 2

A Petri dish started with a population of 8 bacteria. The population doubles every 15 minutes. A second population of bacteria, shown in the following graph, also started with 8 bacteria. Compare the properties of the functions that represent each population over the interval [150, 210].



1. Compare the *y*-intercepts of each function.

   According to the scenario, the initial number of bacteria for both functions is 8; therefore, the *y*-intercept is 8.

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2. Compare the rate of change for each function over the interval [150, 210].

Calculate the rate of change over the interval [150, 210] for the graphed function.

Determine (x_1, y_1) from the graph.

The value of y when x is 150 is approximately 275.

Let $(x_1, y_1) = (150, 275)$.

Determine (x_2, y_2) from the graph.

The value of y when x is 210 is approximately 1,000.

Let $(x_2, y_2) = (210, 1000)$.

Calculate the rate of change using the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(1000) - (275)}{(210) - (150)}$$ Substitute (150, 275) for (x_1, y_1) and (210, 1000) for (x_2, y_2).

$$= \frac{725}{60}$$ Simplify.

$$\approx 12$$

The rate of change for the graphed function is approximately 12 bacteria per minute.

To determine the rate of change for the other function described in the scenario, first write a function rule to represent the situation.

$$f(x) = 8(2)^{\frac{x}{15}}$$

Determine the value for y when x is 150 using the function.

$$f(x) = 8(2)^{\frac{x}{15}}$$ Original function

$$f(x) = 8(2)^{\frac{(150)}{15}}$$ Substitute 150 for x.

$$f(150) = 8(2)^{10}$$ Simplify.

(*continued*)

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Instruction

$f(150) = 8(1024)$

$f(150) = 8192$

$(x_1, y_1) = (150, 8192)$

Determine the value for y when x is 210 using the function.

$f(x) = 8(2)^{\frac{x}{15}}$ Original function

$f(x) = 8(2)^{\frac{(210)}{15}}$ Substitute 210 for x.

$f(210) = 8(2)^{14}$ Simplify as needed.

$f(210) = 8(16{,}384)$

$f(210) = 131{,}072$

$(x_2, y_2) = (210, 131{,}072)$

Calculate the rate of change using the slope formula.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{(131{,}072) - (8192)}{(210) - (150)}$ Substitute (150, 8192) for (x_1, y_1) and (210, 131,072) for (x_2, y_2).

$= \frac{122{,}880}{60}$ Simplify as needed.

$= 2048$

The rate of change for the function in the table is 2,048 bacteria per minute.

The rate of change for the graphed function is less steep over the interval [150, 210] than the rate of change for the other function.

3. Summarize your findings.

The y-intercepts of both functions are the same; however, the graphed function is less steep over the interval [150, 210]. The population of bacteria shown by the graphed function are doubling at a slower rate than the bacteria in the first function described.

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Example 3

A pendulum swings to 90% of its previous height. Pendulum A starts at a height of 50 centimeters. Its height at each swing is modeled by the function $f(x) = 50(0.90)^x$. The height after every fifth swing of Pendulum B is recorded in the following table. Compare the properties of each function over the interval [5, 15].

x	$g(x)$
0	100
5	59.05
10	34.87
15	20.59
20	12.16

1. Compare the y-intercepts of each function.

   Identify the y-intercept of Pendulum A.

   The problem states that the pendulum starts at a height of 50 centimeters.

   The y-intercept of the function is (0, 50).

   Identify the y-intercept of Pendulum B.

   The value of $g(x)$ is 100 when x is 0.

   The y-intercept of the function is 100.

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2. Compare the rate of change for each function over the interval [5, 15].

Calculate the rate of change over the interval [5, 15] for Pendulum A.

Determine (x_1, y_1) from the function.

$f(x) = 50(0.90)^x$	Original function
$f(5) = 50(0.90)^{(5)}$	Substitute 5 for x.
$f(5) = 29.52$	Simplify.

Let $(x_1, y_1) = (5, 29.52)$.

Determine (x_2, y_2) from the function.

$f(x) = 50(0.90)^x$	Original function
$f(15) = 50(0.90)^{(15)}$	Substitute 15 for x.
$f(15) \approx 10.29$	Simplify.

The value of y when x is 15 is approximately 10.29.

Let $(x_2, y_2) = (15, 10.29)$.

Calculate the rate of change using the slope formula.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$= \frac{(10.29) - (29.52)}{(15) - (5)}$	Substitute (5, 29.52) for (x_1, y_1) and (15, 10.29) for (x_2, y_2).
$= \frac{-19.23}{10} = -1.923$	Simplify.

The rate of change for Pendulum A's function is approximately –1.923 centimeters per swing.

(*continued*)

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Calculate the rate of change over the interval [5, 15] for Pendulum B.

Let $(x_1, y_1) = (5, 59.05)$.

Let $(x_2, y_2) = (15, 20.59)$.

Calculate the rate of change using the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(20.59) - (59.05)}{(15) - (5)}$$ Substitute (5, 59.05) for (x_1, y_1) and (15, 20.59) for (x_2, y_2).

$$= \frac{-38.46}{10} = -3.846$$ Simplify.

The rate of change for Pendulum B's function is approximately –3.846 centimeters per swing.

The rate of change for Pendulum B is greater over the interval [5, 15] than the rate of change for Pendulum A.

3. Summarize your findings.

The *y*-intercept of Pendulum A is less than the *y*-intercept of Pendulum B. This means that Pendulum B begins higher than Pendulum A. The rate of change for Pendulum A is less than the rate of change for Pendulum B. This means that Pendulum B is losing height faster than Pendulum A.

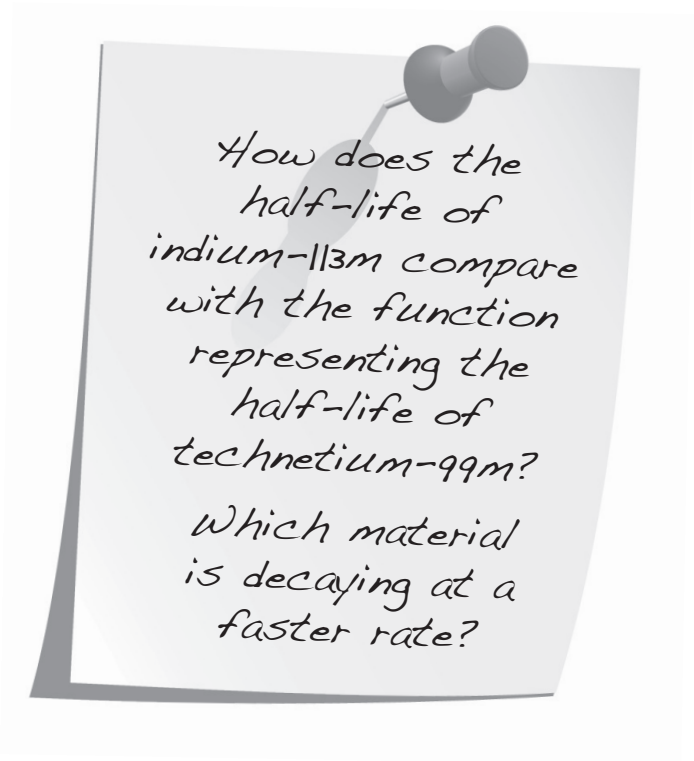
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Problem-Based Task 2.6.2: Analyzing Kidney Function

SMP	
1 ✓	2
3 ✓	4
5 ✓	6
7	8

Renal scans are common medical procedures used to measure how well a person's kidneys are working. The patient is injected with a small amount of radioactive material. Then, a series of computerized scans are taken. The type of radioactive material used for the test depends on the specific kidney function being observed. Two types of radioactive materials used are technetium-99m and indium-113m.

The half-life, or decay rate, of indium-113m is 1.7 hours. The half-life of technetium-99m is modeled by the function $f(x)=500\left(\frac{1}{2}\right)^{\frac{x}{6}}$, where $f(x)$ represents the amount of material remaining and x represents hours. There are 500 milligrams of each material. How does the half-life of indium-113m compare with the function representing the half-life of technetium-99m? Which material is decaying at a faster rate? Use a graph to help explain your reasoning.



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Problem-Based Task 2.6.2: Analyzing Kidney Function

Coaching

a. What is the initial amount of indium-113m?

b. At what rate does indium-113m decay?

c. What function represents the half-life of indium-113m?

d. What function represents the half-life of technetium-99m?

e. What is the initial amount of technetium-99m?

f. At what rate does technetium-99m decay?

g. Graph both functions on one coordinate plane.

h. Which function is decaying at a faster rate? Use the functions and your graph to explain your reasoning.

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Instruction

Problem-Based Task 2.6.2: Analyzing Kidney Function

Coaching Sample Responses

a. What is the initial amount of indium-113m?

The initial amount of indium-113m is 500 milligrams.

b. At what rate does indium-113m decay?

Indium-113m has a half-life of 1.7 hours.

c. What function represents the half-life of indium-113m?

An exponential function in the form $g(x)=ab^{\frac{x}{t}}$ represents this situation, where a represents the initial amount, b represents the rate of decay, x represents the number of hours, and t represents the time period.

The function $g(x)=500\left(\frac{1}{2}\right)^{\frac{x}{1.7}}$ represents the half-life of indium-113m.

d. What function represents the half-life of technetium-99m?

As stated in the problem, the function that represents the half-life of technetium-99m is $f(x)=500\left(\frac{1}{2}\right)^{\frac{x}{6}}$.

e. What is the initial amount of technetium-99m?

The initial amount of technetium-99m is 500 milligrams, as found in the equation.

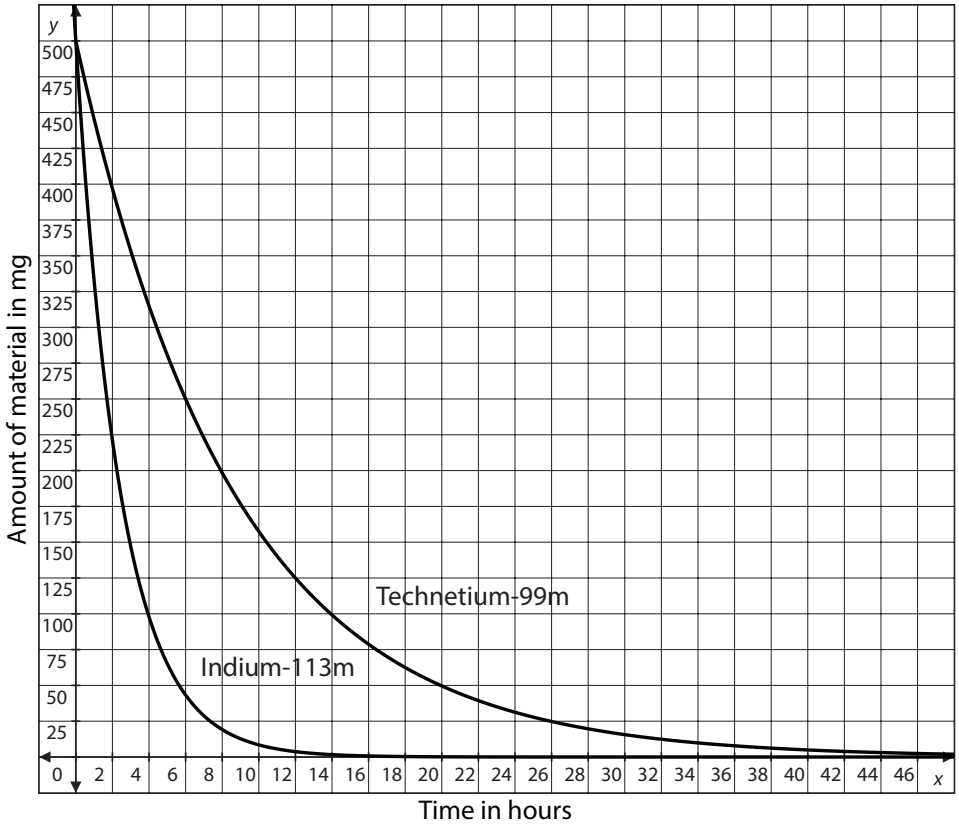
f. At what rate does technetium-99m decay?

Technetium-99m has a half-life of 6 hours.

This value is found in the denominator of the exponent in the function given.

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g. Graph both functions on one coordinate plane.



h. Which function is decaying at a faster rate? Use the functions and your graph to explain your reasoning.

It can be observed from the graph that indium-113m decays at a faster rate than technetium-99m. The initial amount of each material is the same, 500 milligrams. As time passes, the amount of indium-113m decreases more quickly than technetium-99m. For example, after 2 hours, there are approximately 220 milligrams of indium-113m compared with 400 milligrams of technetium-99m. After 6 hours, there are approximately 40 milligrams of indium-113m compared with 100 milligrams of technetium-99m. It would take longer for technetium-99m to decay than for indium-113m to decay.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.6.2: Comparing Exponential Functions

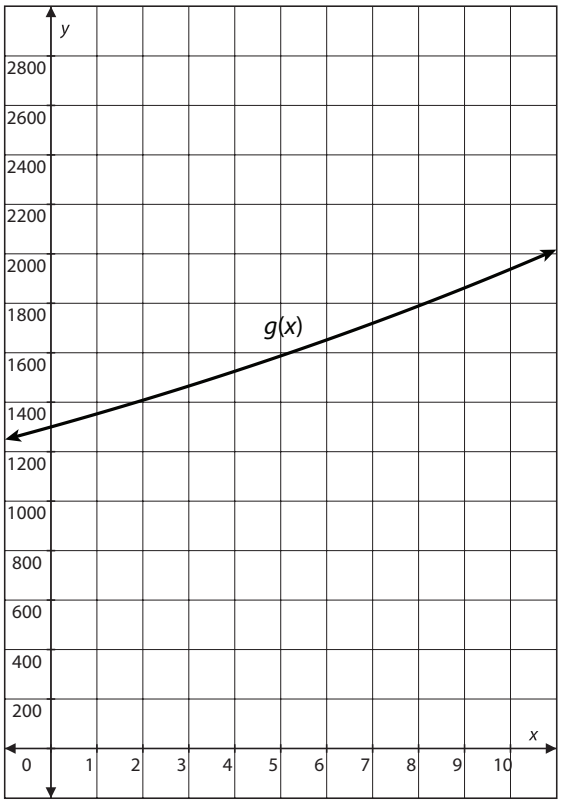
Compare the properties of the exponential functions.

1. Which function has a greater rate of change over the interval [2, 8]? Which function has the greater *y*-intercept? Explain how you know.

Function A

x	$f(x)$
0	1400
2	1546.92
4	1709.25
6	1888.62
8	2086.82

Function B



2. Which function has a greater rate of change over the interval [0, 5]? Which function has the greater *y*-intercept?

Function A

$$f(x)=\left(\frac{1}{2}\right)^x$$

Function B

$$g(x) = 2^x$$

continued

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3. Compare the properties of each function over the interval [2, 8].

Function A

$$f(x) = 400\left(1 + \frac{0.06}{12}\right)^{12x}$$

Function B

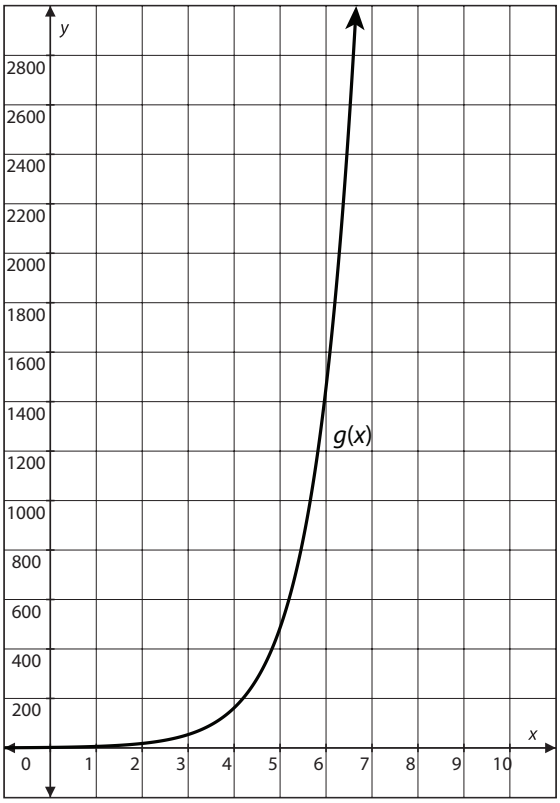
x	$g(x)$
0	350.00
2	398.45
4	453.61
6	516.40
8	587.88

4. Compare the properties of each function over the interval [0, 5].

Function A

$f(x) = 3(2)^x$

Function B



continued

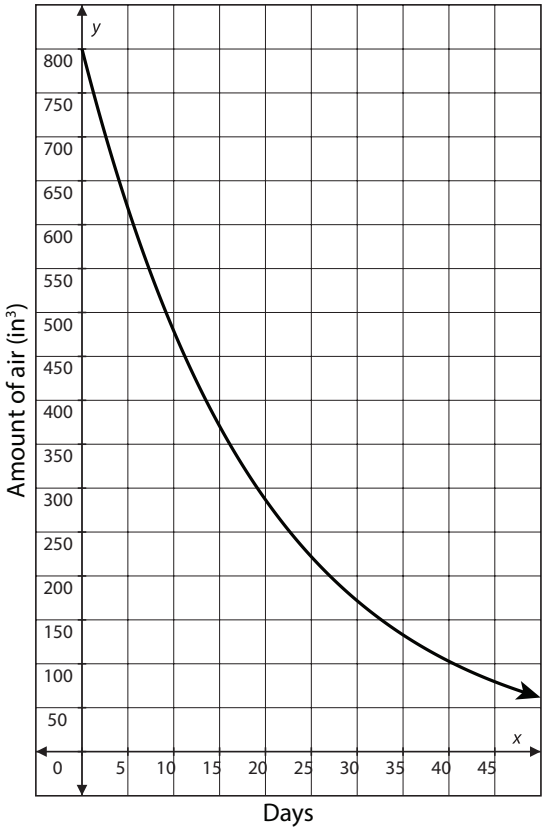
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5. Compare the properties of each exponential function over the interval [0, 10].

Function A

A fully inflated beach ball is losing 7.5% of its air every day. The beach ball originally contained 800 cubic inches of air.

Function B



6. Compare the properties of each exponential function over the interval [0, 5].

Function A

Jasmine received a job offer with a starting salary of $32,000 and a 1.5% increase every year.

Function B

A second job offer for Jasmine can be described by the function $f(x) = 30{,}000(1 + 0.02)^x$.

continued

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7. Compare the properties of each exponential function over the interval [0, 4].

Function A

The enrollment of Eastern High School, $f(x)$, after x years is modeled by the function $f(x) = 1700(1 + 0.025)^x$.

Function B

The following table shows the enrollment of a rival high school, $g(x)$, for 5 years.

x	$g(x)$
0	1900
1	1872
2	1843
3	1816
4	1789

8. Compare the properties of each exponential function over the interval [1, 3].

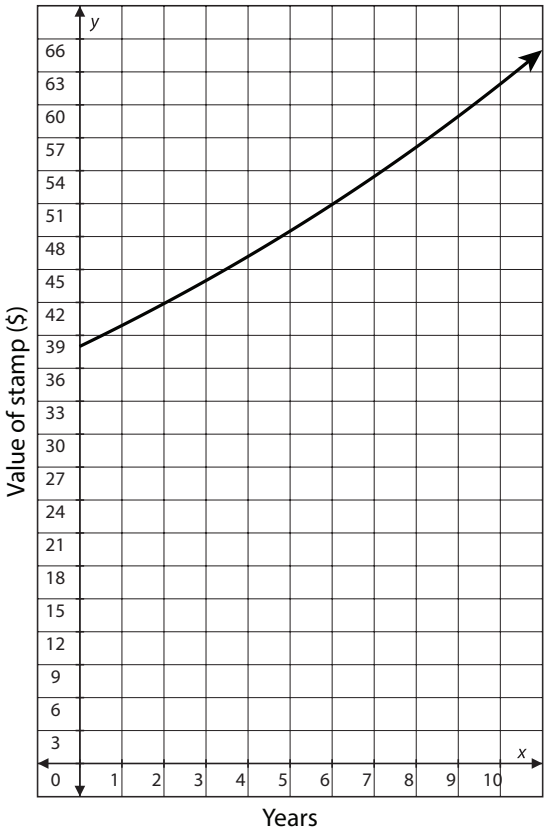
Function A

The following table shows the value in dollars of a rare stamp, $f(x)$, x years from the date purchased.

x	$f(x)$
0	52
1	54.08
2	56.24
3	58.49
4	60.83

Function B

The following graph models the value in dollars of a second rare stamp, $g(x)$, after x years.



continued

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9. Compare the properties of each exponential function over the interval [0, 4].

Function A

The value of a car in dollars, $f(x)$, depreciates after each year, x. The following table shows the value of a car for each of the first 4 years after it was purchased.

x	$f(x)$
0	22,450
1	19,307
2	16,604.02
3	14,279.46
4	12,280.33

Function B

The value of a second car is modeled by the equation $g(x) = 19{,}375(1 - 0.16)^x$, where $g(x)$ represents the value of the car x years after the date it was purchased.

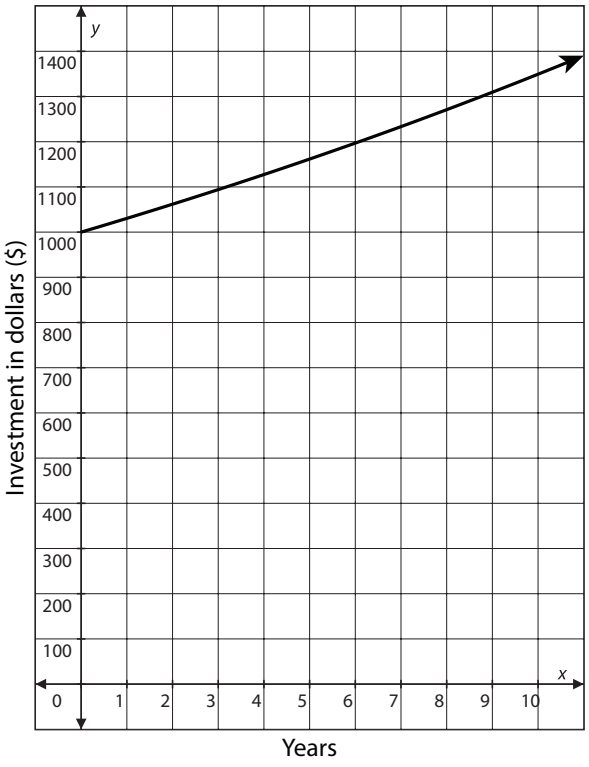
10. Compare the properties of each exponential function over the interval [0, 10].

Function A

An investment of $1,000 earns interest at a rate of 3.75%, compounded monthly.

Function B

The value of a second investment is modeled in the following graph.



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Lesson 2.6.3: Comparing Linear to Exponential Functions

Warm-Up 2.6.3

Food-poisoning bacteria have to multiply to high numbers in food before there are enough bacteria to make someone sick. When conditions such as type of food, moisture level, temperature, and amount of time passed are ideal, the bacteria can double every 20 minutes.

1. Write a function to represent this scenario.

2. What is the rate of change for the first hour?

3. How many bacteria will there be after 6 hours if the initial number of bacteria is 1?

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Instruction

Lesson 2.6.3: Comparing Linear to Exponential Functions

Common Core State Standard

F–LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★

Warm-Up 2.6.3 Debrief

1. Write a function to represent this scenario.

   This scenario is represented by an exponential function.

   - The initial number of bacteria is 1.
   - The bacteria doubles every 20 minutes, so the growth factor is 2.
   - The time period is 20 minutes.

   The function that represents this scenario is $f(x)=1(2)^{\frac{x}{20}}$.

2. What is the rate of change for the first hour?

   Identify the interval.

   The x-value for the start of the interval is 0.

   The x-value for the end of the interval is 60, because there are 60 minutes in 1 hour.

   When x is 0, $f(x)$ = 1. This is stated in the problem.

   To calculate the number of bacteria after 60 minutes, substitute 60 for x and evaluate the expression.

   $f(x)=1(2)^{\frac{x}{20}}$ Original function

   $f(60)=1(2)^{\frac{(60)}{20}}$ Substitute 60 for x.

   $f(60) = 1(2)^3$ Simplify.

   $f(60) = 1(8)$

   $f(60) = 8$

   There are 8 bacteria in the population after 60 minutes.

   Calculate the rate of change. Let (x_1, y_1) = (0, 1) and (x_2, y_2) = (60, 8).

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$m=\frac{y_2-y_1}{x_2-x_1}$ Slope formula

$=\frac{(8)-(1)}{(60)-(0)}$ Substitute (0, 1) for (x_1, y_1) and (60, 8) for (x_2, y_2).

$=\frac{7}{60}$ Simplify.

≈ 0.12

The rate of change for the first hour is approximately 0.12 bacteria per minute.

3. How many bacteria will there be after 6 hours if the initial number of bacteria is 1?

Determine how many minutes are in 6 hours.

$6 \text{ hours} \bullet \frac{60 \text{ minutes}}{1 \text{ hour}}$

$6 \cancel{\text{hours}} \bullet \frac{60 \text{ minutes}}{1 \cancel{\text{hour}}}$

$6 \bullet 60 \text{ minutes}$

360 minutes

Substitute 360 minutes into the function $f(x)=1(2)^{\frac{x}{20}}$.

$f(x)=1(2)^{\frac{x}{20}}$ Original function

$f(360)=1(2)^{\frac{(360)}{20}}$ Substitute 360 for x.

$f(360) = 1(2)^{18}$ Simplify.

$f(360) = 1(262{,}144)$

$f(360) = 262{,}144$

After 6 hours, there will be 262,144 bacteria.

Connection to the Lesson

- As in the warm-up, students will create exponential functions from context.
- Students will determine the rate of change as well as the *y*-intercepts from graphs, tables, equations, and verbal descriptions.
- Students will use their knowledge of writing exponential functions to analyze these properties in order to compare exponential functions to linear functions.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- determining the rate of change of a function (F–IF.6★)
- graphing functions (A–CED.2★)
- identifying linear and exponential functions (F–LE.1★)

Introduction

In previous lessons, linear functions were compared to linear functions and exponential functions to exponential. Here, the properties of linear functions will be compared to those of exponential functions.

Key Concepts

- The equation of a linear function can be written in the form $f(x) = mx + b$.
- A **factor** is one of two or more numbers or expressions that when multiplied produce a given product. In the equation $f(x) = mx + b$, m and x are factors.
- As the value of x increases, the value of $f(x)$ will increase at a constant rate. The rate of change of linear functions remains constant.
- Exponential functions are written in the form $g(x) = ab^x + k$, where a and k are real numbers, b is the base, and x, the input, is a real number.
- The variable of an exponential function is part of the exponent.
- As the value of x increases, the value of $g(x)$ will increase or decrease by a power of b.
- The rate of change of an exponential function varies depending on the interval observed.
- Graphs of exponential functions where the base is greater than 1 will increase at a faster rate than graphs of linear functions.
- A quantity that increases exponentially will always eventually exceed one that increases linearly.

Common Errors/Misconceptions

- incorrectly determining the rate of change
- not comparing the absolute values of the slopes to determine which function is steeper
- interchanging the x- and y-intercepts
- assuming the rate of change of a function is linear by only referencing one interval

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Guided Practice 2.6.3

Example 1

Which function increases faster, $f(x) = 4x - 5$ or $g(x) = 4^x - 5$? Justify your answer with a graph.

1. Make a general observation.

   $f(x) = 4x - 5$ is a linear function of the form $f(x) = mx + b$.

   The variable x is multiplied by the coefficient 4.

   $g(x) = 4^x - 5$ is an exponential function of the form $g(x) = ab^x + k$.

   The variable x is the exponent.

2. Create a table of values.

   Substitute values for x into each function.

$f(x) = 4x - 5$	
x	$f(x)$
–2	–13
–1	–9
0	–5
1	–1
2	3

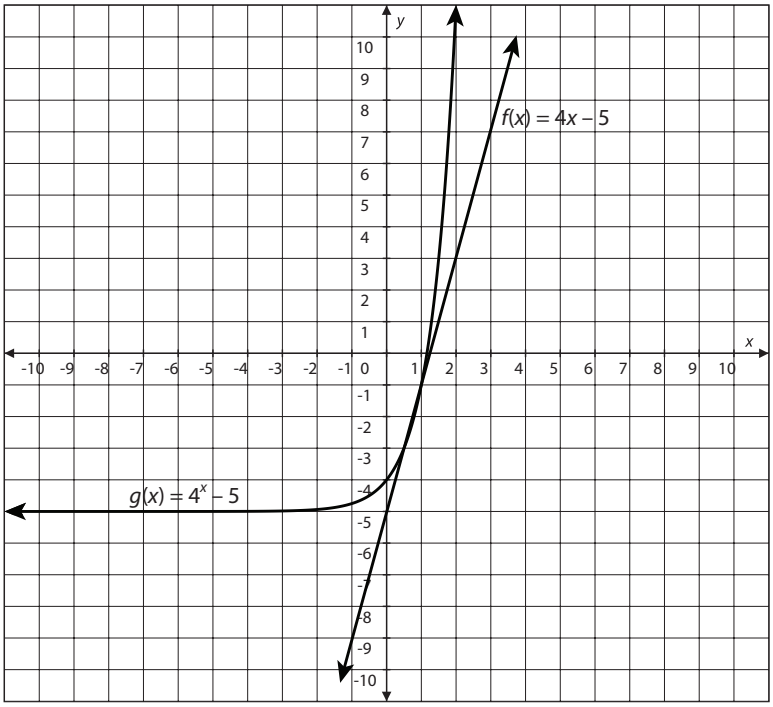
$g(x) = 4^x - 5$	
x	$g(x)$
–2	–4.9375
–1	–4.75
0	–4
1	–1
2	11

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Instruction

3. Graph both functions on the same coordinate plane.

   Use the tables of values created in order to plot both functions.



4. Compare the rate of change of each function.

   The graph of $f(x) = 4x - 5$ appears to be steeper than the graph of $g(x) = 4^x - 5$ until the point $(1, -1)$. At this point, the graphs intersect and $f(x) = g(x)$. Once x is greater than 1, the graph of $g(x) = 4^x - 5$ becomes steeper. From there, $g(x) = 4^x - 5$ increases faster than $f(x) = 4x - 5$.

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Instruction

Example 2

At approximately what point does the value of $f(x)$ exceed the value of $g(x)$ if $f(x)=2(4)^{\frac{x}{20}}$ and $g(x) = 0.5x$? Justify your answer with a graph.

1. Make a general observation.

   $f(x)=2(4)^{\frac{x}{20}}$ is an exponential function of the form $g(x) = ab^x$.

   The variable x is part of the exponent.

   $g(x) = 0.5x$ is a linear function of the form $f(x) = mx + b$.

   The variable x is multiplied by the coefficient 0.5.

2. Create a table of values.

   Substitute values for x into each function.

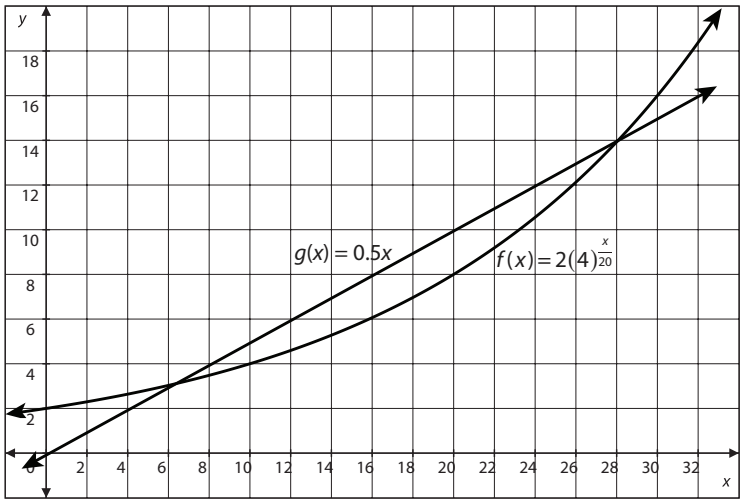
$f(x)=2(4)^{\frac{x}{20}}$		$g(x) = 0.5x$	
x	$f(x)$	x	$g(x)$
0	2	0	0
2	2.30	2	1
4	2.64	4	2
6	3.03	6	3

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Instruction

3. Graph both functions on the same coordinate plane.

   Use the tables of values created in order to plot both functions.



4. Identify the approximate point where $f(x)$ is greater than $g(x)$.

   It can be seen from the graph that both functions are equal where x is approximately equal to 28. When x is greater than 28, $f(x)$ is greater than $g(x)$.

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Instruction

Example 3

Lena has been offered a job with two salary options. The first option is modeled by the function $f(x) = 500x + 31{,}000$, where $f(x)$ is her salary in dollars after x years. The second option is represented by the function $g(x) = 29{,}000(1.04)^x$, where $g(x)$ is her salary in dollars after x years. If Lena is hoping to keep this position for at least 5 years, which salary option should she choose? Support your answer with a graph.

1. Make a general observation.

   $f(x) = 500x + 31{,}000$ is a linear function of the form $f(x) = mx + b$.

   The variable x is multiplied by the coefficient 500 and added to the constant 31,000.

   $g(x) = 29{,}000(1.04)^x$ is an exponential function of the form $g(x) = ab^x$.

   The variable x is the exponent.

   Use the two equations to create a table of values.

   Substitute the same values for x into each function.

$f(x) = 500x + 31{,}000$	
x	$f(x)$
0	31,000
2	32,000
4	33,000
6	34,000

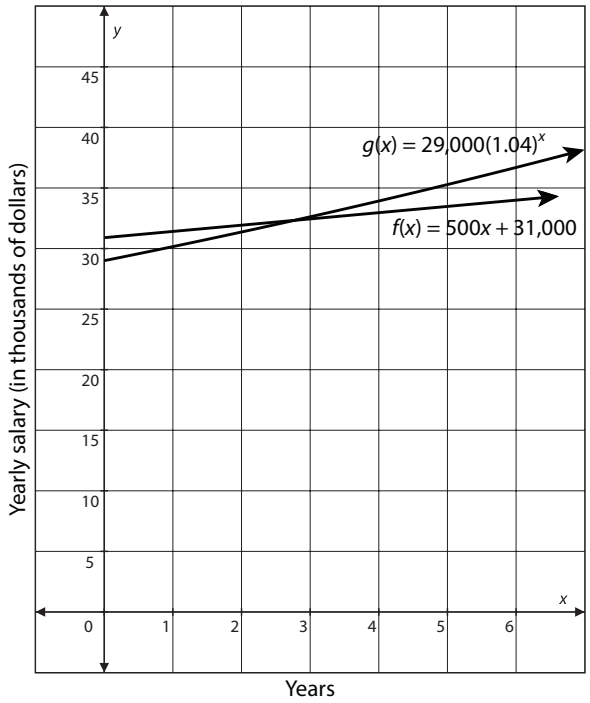
$g(x) = 29{,}000(1.04)^x$	
x	$g(x)$
0	29,000
2	31,366.40
4	33,925.90
6	36,694.25

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Instruction

2. Graph both functions on the same coordinate plane.

   Use the tables of values created in order to plot both functions.



3. Identify the approximate point where $g(x)$ is greater than $f(x)$.

   It can be seen from the graph that after 3 years, $g(x)$ is greater than $f(x)$. If Lena is hoping to keep this position for at least 5 years, it is in her best interest to choose the salary option modeled by $g(x) = 29{,}000(1.04)^x$.

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Problem-Based Task 2.6.3: Future Finances

SMP	
1	2
3 ✓	4 ✓
5	6 ✓
7	8

Cole has just graduated from high school and is going to attend college while working on the family farm. The degree program he has chosen takes an average of 24 months to complete, but it's possible that he could finish earlier or that he might need more time. His parents have offered to pay him a single payment for all his work on the farm once he graduates. The payment will be based on one of two options. The first option is to start at \$0 and earn \$3,495.25 a month for the remainder of his time on the farm. This means that if Cole were somehow able to graduate at the end of the first month, he would earn just \$3,495.25; but for every month he works on the farm while he is in school, the amount of what he'll eventually be paid increases by another \$3,495.25. When he graduates, he will collect the amount earned for all the months he worked. The second option is to earn \$0.01 for the first month, and then earn double the previous month's pay every month for the remainder of his time on the farm. With this pay plan, he can collect his payout for the month in which he graduates, but he cannot collect the money accumulated up to that point. Which option is the better choice for Cole?



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Problem-Based Task 2.6.3: Future Finances

Coaching

a. How can you determine whether earning $3,495.25 a month for the remainder of Cole's time on the farm is linear or exponential?

b. Write a function to model earning $3,495.25 a month for the length of Cole's time on the farm.

c. How can you determine whether earning $0.01 for the first month and then earning double the previous month's pay for each month afterward is linear or exponential?

d. Write a function to model earning $0.01 for the first month and then earning double the previous month's pay each month for the length of Cole's time on the farm.

e. Graph both functions on the same coordinate plane.

f. When is it a better option to choose $0.01 for the first month and then double the previous month's pay for every month after?

g. Is there a point on the graph where choosing either option results in the same payment?

h. If Cole finishes school in 21 months and collects his payment then, which is the better option?

i. If Cole finishes school in 27 months and collects his payment then, which is the better option?

j. Which is the better option for Cole?

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Instruction

Problem-Based Task 2.6.3: Future Finances

Coaching Sample Responses

a. How can you determine whether earning $3,495.25 a month for the remainder of Cole's time on the farm is linear or exponential?

Earning $3,495.25 a month for the remainder of his time on the farm is a constant pay rate.

If the rate of change of a function remains the same, the function is linear.

b. Write a function to model earning $3,495.25 a month for the length of Cole's time on the farm.

Linear functions are of the form $f(x) = mx + b$.

- The rate of change for this function is 3,495.25.
- The y-intercept is 0.

The function that models earning an increase of $3,495.25 a month is $f(x) = 3495.25x$.

c. How can you determine whether earning $0.01 for the first month and then earning double the previous month's pay for each month afterward is linear or exponential?

Analyze the rates of change over the first few intervals.

- Cole's pay doubles each month.
- The first month, his pay is $0.01.
- The second month, his pay is $0.02.
- The third month, his pay is $0.04.

The rate of change is not the same for each interval of the function, so the function can't be linear.

This is an example of an exponential growth function.

d. Write a function to model earning $0.01 for the first month and then earning double the previous month's pay each month for the length of Cole's time on the farm.

Exponential growth functions are of the form $g(x) = ab^x$.

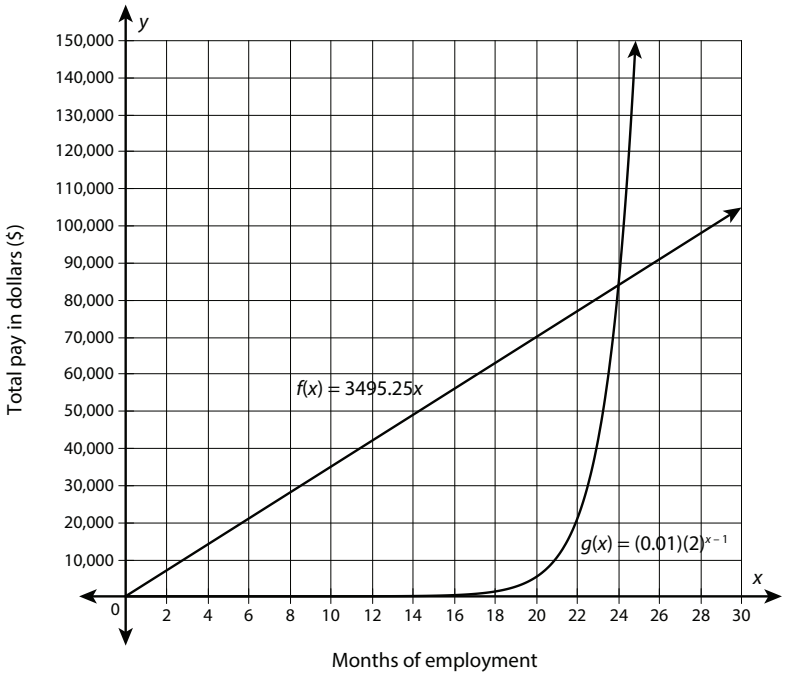
- The initial amount is 0.01.
- The rate of growth is 2 because Cole's pay is doubling.

The function that models earning $0.01 for the first month and double that for each month afterward is $g(x) = (0.01)(2)^{x-1}$ for $x \geq 1$.

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Instruction

e. Graph both functions on the same coordinate plane.



f. When is it a better option to choose \$0.01 for the first month and then double the previous month's pay for every month after?

According to the graph, after about 24 months of employment, the value of the option of earning \$0.01 for the first month and then double the previous month's salary after that becomes greater than the value of the option of being paid \$3,495.25 each month. If Cole plans to be in school for more than two years, choosing to double his pay each month becomes the better option.

g. Is there a point on the graph where choosing either option results in the same payment?

At 24 months, or 2 years, the payments are the same regardless of the option chosen. Up until this point, the better option is to earn \$3,495.25 a month. After this point, the better option is to earn \$0.01 for the first month and then earn double the previous month's pay each month afterward.

h. If Cole finishes school in 21 months and collects his payment then, which is the better option?

The better option before 24 months have passed is the linear function, $f(x) = 3495.25x$. Earnings for this option far exceed the second exponential option at this point. If Cole finishes school in 21 months, he would be better off choosing the first option, the one that increases his pay at the rate of \$3,495.25 a month.

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i. If Cole finishes school in 27 months and collects his payment then, which is the better option?

At this point, the exponential model far exceeds the linear model, so Cole should choose the second option, $g(x) = 0.01(2)^{x-1}$ for $x \geq 1$.

j. Which is the better option for Cole?

If Cole plans to take the average amount of time (2 years) to finish his schooling, either option will yield the same earnings. However, if he plans to finish early, he should choose the linear option. If he takes longer than 2 years to finish his schooling, he should choose the exponential option.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.6.3: Comparing Linear to Exponential Functions

Use what you know about linear and exponential functions to complete problems 1–6.

1. Which increases faster, $f(x) = 5x$ or $g(x) = 5^x$? Justify your answer using a graph.

2. Which increases faster, $f(x) = 2x - 4$ or $g(x) = 2^x - 4$? Justify your answer using a table of values.

3. Which decreases faster, $f(x) = 200(0.97)^x$ or $g(x) = 200 - 3.6x$? Justify your answer using a graph.

4. Which decreases faster, $f(x) = 1000(0.85)^x$ or $g(x) = 1000 - 0.15x$? Justify your answer using a table of values.

5. At what point does the value of $f(x)$ exceed the value of $g(x)$ if $f(x) = 2(8)^{\frac{x}{4}}$ and $g(x) = 8x + 4$? Justify your answer with a graph.

6. At what point does the value of $f(x)$ exceed the value of $g(x)$ if $f(x) = 50(1.35)^{\frac{x}{12}}$ and $g(x) = 0.4x + 60$? Justify your answer with a graph.

Use the following information to answer questions 7–10.

You are looking to invest \$450. One savings option follows the function $f(x) = 450 + 450(0.055)x$, where $f(x)$ is the amount of money in savings after x years. The second option is represented by the function $g(x) = 450\left(1 + \frac{0.04}{12}\right)^{12x}$, where $g(x)$ is the amount of money after x years.

7. Which increases faster, $f(x) = 450 + 450(0.055)x$ or $g(x) = 450\left(1 + \frac{0.04}{12}\right)^{12x}$? Use a graph to explain your answer.

8. At what point does the value of $g(x)$ exceed the value of $f(x)$?

9. If you were looking to invest your money for fewer than 10 years, which option would you choose? Explain your reasoning.

10. If you were looking to invest your money for more than 20 years, which option would you choose? Explain your reasoning.

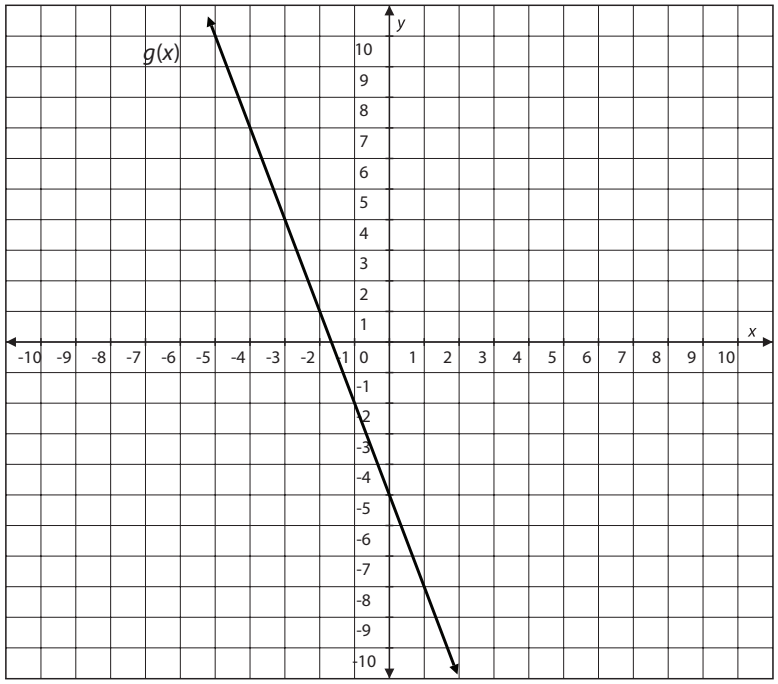
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Progress Assessment

Circle the letter of the best answer.

1. Which of the following statements is true about the functions $f(x)$ and $g(x)$?

x	$f(x)$
–2	10
0	2
2	–6
4	–14



a. The function $f(x)$ has a greater rate of change than the function $g(x)$.

b. The function $g(x)$ has a greater rate of change than the function $f(x)$.

c. The rates of change for both $f(x)$ and $g(x)$ are equal.

d. The rates of change cannot be determined.

continued

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Assessment

2. Which of the following statements is true about the functions $f(x)$ and $g(x)$?

$$f(x)=\frac{1}{4}x+8$$

x	$g(x)$
–6	4
–3	6
3	10
6	12

a. The y-intercept of the function $f(x)$ is less than the y-intercept of the function $g(x)$.

b. The y-intercept of the function $f(x)$ is greater than the y-intercept of the function $g(x)$.

c. The y-intercept of the function $f(x)$ is equal to the y-intercept of the function $g(x)$.

d. The y-intercepts cannot be determined.

3. The function $f(x)$ represents the total bill from a rental store that charges \$10 to rent a power tool plus an additional \$2 an hour. A second rental store uses the function $g(x) = 15 + 1.5x$ to represent the total bill for a similar power tool.

Which of the following statements is true about the functions $f(x)$ and $g(x)$?

a. The function $f(x)$ has a greater rate of change than the function $g(x)$.

b. The function $g(x)$ has a greater rate of change than the function $f(x)$.

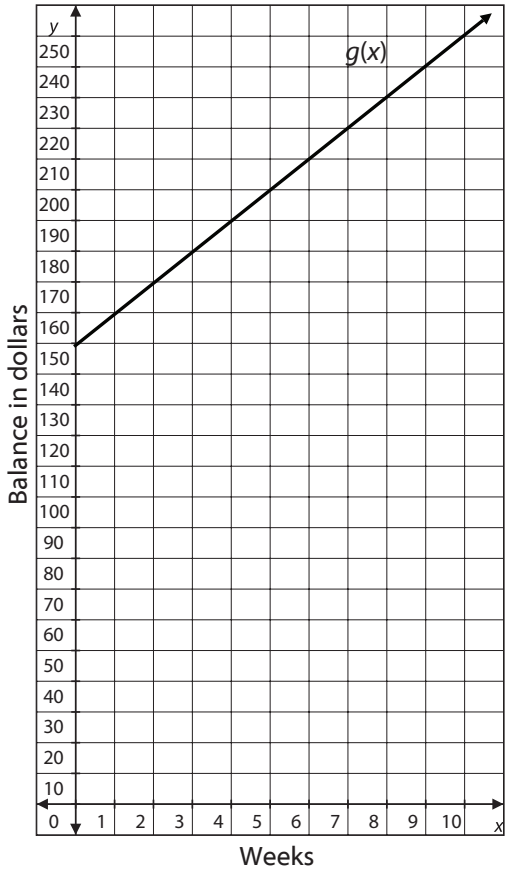
c. The rates of change for both $f(x)$ and $g(x)$ are equal.

d. The rates of change cannot be determined.

continued

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4. The function $f(x)$ represents the balance of a savings account with an initial deposit of \$500 and weekly deposits of \$25. A different savings account follows the function $g(x)$. The graph of function $g(x)$ is shown.



Which of the following statements is true about the functions $f(x)$ and $g(x)$?

a. The y-intercept of the function $f(x)$ is less than the y-intercept of the function $g(x)$.

b. The y-intercept of the function $f(x)$ is greater than the y-intercept of the function $g(x)$.

c. The y-intercept of the function $f(x)$ is equal to the y-intercept of the function $g(x)$.

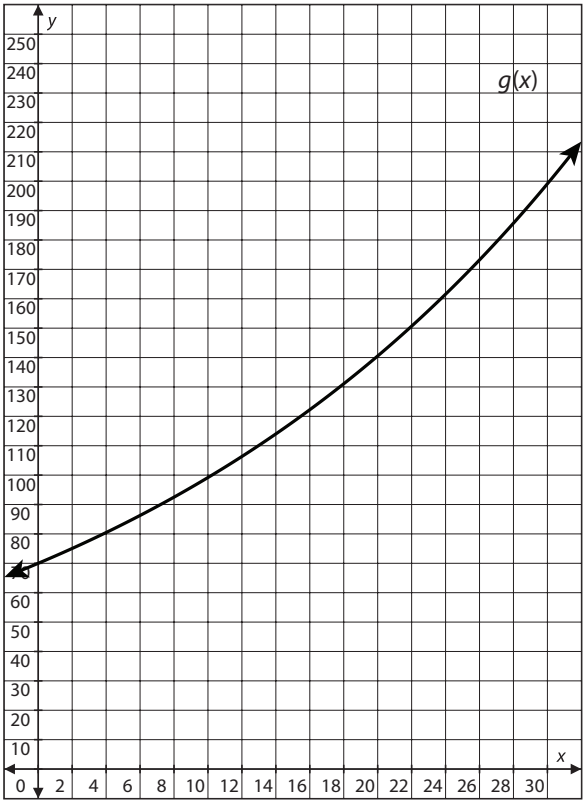
d. The y-intercepts cannot be determined.

continued

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5. Which of the following statements is true about the functions $f(x)$ and $g(x)$ over the interval [0, 4]?

$$f(x)=50\left(1+\frac{0.35}{4}\right)^{4x}$$



a. The function $f(x)$ has a greater rate of change than the function $g(x)$ over the interval [0, 4].

b. The function $g(x)$ has a greater rate of change than the function $f(x)$ over the interval [0, 4].

c. The rates of change for both $f(x)$ and $g(x)$ are equal over the interval [0, 4].

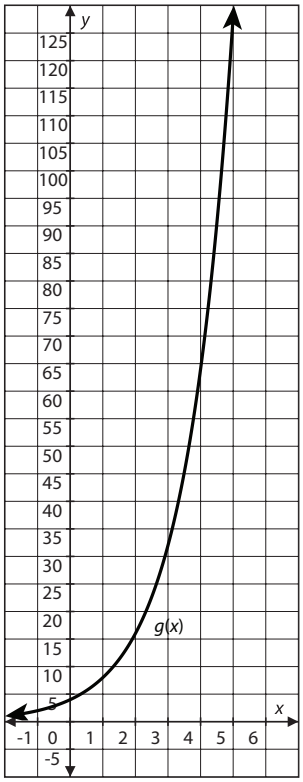
d. The rates of change cannot be determined.

continued

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6. Which of the following statements is true about the functions $f(x)$ and $g(x)$?

x	$f(x)$
–1	1.5
0	3
1	6
2	12



a. The y-intercept of the function $f(x)$ is less than the y-intercept of the function $g(x)$.

b. The y-intercept of the function $f(x)$ is greater than the y-intercept of the function $g(x)$.

c. The y-intercept of the function $f(x)$ is equal to the y-intercept of the function $g(x)$.

d. The y-intercepts cannot be determined.

continued

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Assessment

7. The function $f(x)$ represents the amount of air remaining in an exercise ball that originally had 4,500 cubic inches of air and is losing 6% of its air every minute, x. The function $g(x) = 4500(1 - 0.03)^x$ represents the amount of air remaining in a second exercise ball after x minutes.

   Which of the following statements is true about the functions $f(x)$ and $g(x)$?

   a. The function $f(x)$ has a greater rate of change than the function $g(x)$ over the interval [2, 5].

   b. The function $g(x)$ has a greater rate of change than the function $f(x)$ over the interval [2, 5].

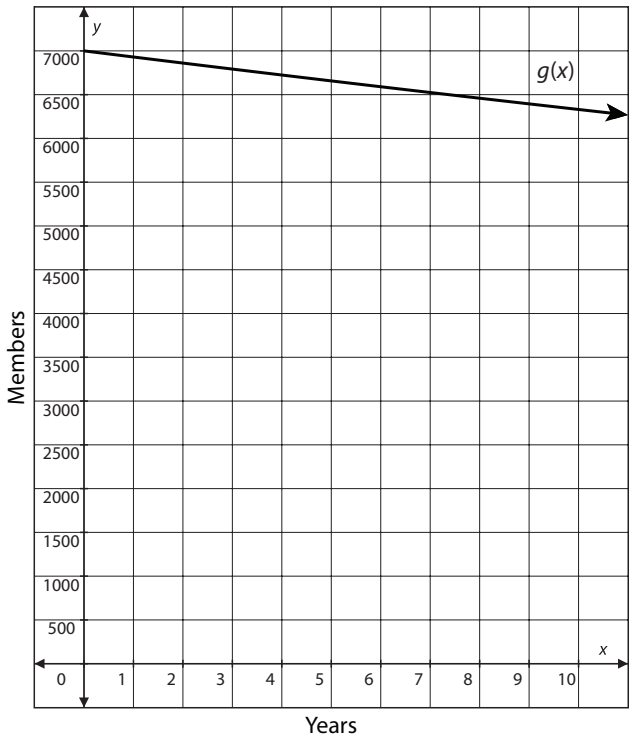
   c. The rates of change for both $f(x)$ and $g(x)$ are equal over the interval [2, 5].

   d. The rates of change cannot be determined.

8. The memberships of two music clubs have decreased each year since 2006. The following table shows the membership of the first music club, $f(x)$. The graph represents the membership of the second music club, $g(x)$.

   Which of the following statements is true about the functions $f(x)$ and $g(x)$?

x	$f(x)$
0	12,000
2	11,290
4	10,624
6	9996



   a. The y-intercept of the function $f(x)$ is less than the y-intercept of the function $g(x)$.

   b. The y-intercept of the function $f(x)$ is greater than the y-intercept of the function $g(x)$.

   c. The y-intercept of the function $f(x)$ is equal to the y-intercept of the function $g(x)$.

   d. The y-intercepts cannot be determined.

continued

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Assessment

9. Which of the following statements is true about the functions $f(x) = 5(2)^x$ and $g(x) = 5 + 2x$?

   a. The rate of change of the function $f(x)$ is always greater than the rate of change of the function $g(x)$.

   b. The rate of change of the function $f(x)$ will eventually be greater than the rate of change of the function $g(x)$.

   c. The rate of change of the function $f(x)$ is never greater than the rate of change of the function $g(x)$.

   d. The rate of change of an exponential function cannot be determined.

10. Which of the following statements is true about the functions $f(x) = 0.4x + 2$ and $g(x) = 10(1.05)^x$?

   a. The rate of change of the function $f(x)$ is always greater than the rate of change of the function $g(x)$.

   b. The rate of change of the function $g(x)$ will eventually be greater than the rate of change of the function $f(x)$.

   c. The rate of change of the function $f(x)$ is never greater than the rate of change of the function $g(x)$.

   d. The rate of change of an exponential function cannot be determined.

Use what you know about linear and exponential functions to solve all the parts of the problem that follows.

11. You are looking to invest $700. One savings option follows the function $f(x) = 700 + 49x$, where $f(x)$ is the amount of money in savings after x years. The second option is represented by the function $g(x) = 700\left(1 + \frac{0.05}{12}\right)^{12x}$, where $g(x)$ is the amount of money after x years.

   a. Which increases faster, $f(x)$ or $g(x)$? Use a graph to explain your answer.

   b. At what point does the value of $g(x)$ exceed the value of $f(x)$?

   c. If you were looking to invest your money for less than 10 years, which option would you choose? Explain your reasoning.

   d. If you were looking to invest your money for more than 25 years, which option would you choose? Explain your reasoning.

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Pre-Assessment

Circle the letter of the best answer.

1. Which explicit function represents the pattern in the table?

x	$f(x)$
0	32
1	21
2	10
3	–1
4	–12

a. $f(x) = 11x + 32$

b. $f(x) = 11x$

c. $f(x) = -11x + 32$

d. $f(x) = 32 \bullet 11^x$

2. The value of a house generally increases over time. Bailey buys a house for $200,000. After 1 year, the house is worth $220,000. After 2 years, the house is worth $240,000. After 3 years, the house is worth $260,000. Which function describes the relationship between the year and the house value?

a. $f(x) = 20{,}000x$

b. $f(x) = 20{,}000x + 200{,}000$

c. $f(x) = 200{,}000 \bullet (1.10)^x$

d. $f(x) = 220{,}000 \bullet (1.10)^x$

3. The population of a small town is 500 people. Based on growth of the population in past years, it is estimated that after 1 year the population will be 600 people. Similarly, it is estimated that after 2 and 3 years, the population will be 720 and 864 people, respectively. Which function describes the relationship between year and town population?

a. $f(x) = 500 \bullet (1.2)^x$

b. $f(x) = 500 \bullet (1.2)^{x-1}$

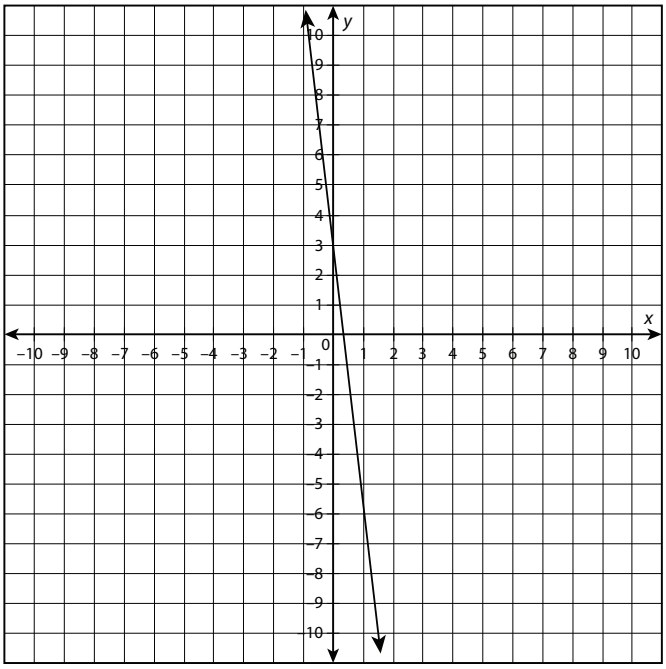
c. $f(x) = 100x + 500$

d. $f(x) = 500 \bullet (0.2)^x$

continued

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4. Which function represents the relationship between x and y shown in the graph?



a. $f(x) = -9x + 3$

b. $f(x) = 9x - 3$

c. $f(x) = (-9)^x$

d. $f(x) = -9x - 3$

5. Which function represents the relationship between x and y shown in the table?

x	y
0	3
1	33
2	363
3	3993

a. $f(x) = 3^x$

b. $f(x) = 11 \bullet 3^x$

c. $f(x) = 11^x$

d. $f(x) = 3 \bullet 11^x$

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Lesson 7: Building Functions

Instruction

Common Core State Standards

SMP	
1 ✓	2 ✓
3	4 ✓
5	6
7	8 ✓

F–BF.1 Write a function that describes a relationship between two quantities. ★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F–LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★

Essential Questions

1. How can an unknown quantity be represented in an expression or an equation?
2. What is the difference between the slope of a linear function and the slope of an exponential function?
3. What is the difference between the shape of the graph of a linear function and the shape of the graph of an exponential function?

WORDS TO KNOW

equation	a mathematical sentence that uses an equal sign (=) to show that two quantities are equal
explicit function	a function in which the dependent variable can be written in terms of the independent variable; $f(x) = 2x$ is an explicit function, where x is the independent variable and $f(x)$ is the dependent variable
exponential equation	an equation whose independent variable is in the exponent; the general form of its equation is $y = a \bullet b^x + k$, where a is the initial value, b is the base, x is the input value, k is the vertical shift, and y is the output
expression	a combination of variables, quantities, and mathematical operations; 4, $8x$, and $b + 10^2$ are all expressions.
function	a relation in which each element in the domain is mapped onto exactly one element in the range; that is, for every value of x, there is exactly one value of y

SRB p. U2-194

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Instruction

linear equation	an equation that can be written in the form $ax + by = c$, where a, b, and c are numbers; can also be written as $y = mx + b$, in which m is the slope, b is the y-intercept, and the graph is a straight line
slope	the measure of the rate of change of one variable with respect to another variable; slope $= m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$
variable	a letter used to represent a value or unknown quantity that can change or vary
***y*-intercept**	the y-coordinate of the point at which a line or a curve intersects the y-axis

Recommended Resources

- Discovery Education. "Find the Equation of a Line Given That You Know Two Points It Passes Through."

  http://walch.com/rr/CAU3L6LinearEquationGenerator

  Input two coordinate pairs to have the computer generate a linear equation. The user is then walked through a step-by-step process for the input coordinate pairs that also explains how to find the equation in slope-intercept form.

- Jim Reed's and Shannon Sookochoff's Pure Math 30. "Exponential Function ($y = ab^x$)."

  http://www.walch.com/rr/05003

  Users can manipulate this applet by using sliders to change the values of a and b in the equation $y = ab^x$. The graph is instantly modified as the sliders are moved.

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Lesson 2.7.1: Building Functions from Context

Warm-Up 2.7.1

For each problem, write an equation to represent the situation and then answer the question.

1. Willem buys 4 mangoes each week, and mango prices vary from week to week. Write an equation that represents the weekly cost of the mangoes. If each mango costs $1 this week, what is the total cost of his mangoes for this week?

2. Kerin drives at a speed of 55 miles per hour on the highway for her job. Write an equation that represents the distance she travels as a function of the time she spends driving. If one day she drives for 6 hours, how many miles did she travel?

3. Mr. Stevens teaches 4 math classes every day. Depending on absences, the number of students in each class varies. Write an equation that represents the number of students Mr. Stevens teaches in a day. If there are 30 students taking each class and one day all of the students were present, how many students did Mr. Stevens teach that day?

4. Jessica reads approximately 12 pages of her novel each hour. Depending on extracurricular activities and homework, the time that Jessica has to read varies. Write an equation that represents the number of pages Jessica reads as a function of the time spent reading. If Jessica read for 3 hours yesterday, approximately how many pages did she read?

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Instruction

Lesson 2.7.1: Building Functions from Context

Common Core State Standard

F–BF.1 Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Warm-Up 2.7.1 Debrief

1. Willem buys 4 mangoes each week, and mango prices vary from week to week. Write an equation that represents the weekly cost of the mangoes. If each mango costs $1 this week, what is the total cost of his mangoes for this week?

   Let the total cost of the mangoes = c.

   Let the price of each mango = p.

   The number of mangoes = 4.

   total cost = the number of mangoes • the price of each mango

   $c = 4p$

   If the cost of each mango is $1, substitute 1 for p.

   $c = 4p$

   $c = 4(1)$

   $c = 4$

   The total cost for Willem to buy mangoes this week is $4.

2. Kerin drives at a speed of 55 miles per hour on the highway for her job. Write an equation that represents the distance she travels as a function of the time she spends driving. If one day she drives for 6 hours, how many miles did she travel?

   Let distance = d.

   Let time = t.

   The rate = 55 mph.

   distance = rate • time

   $d = 55t$

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If the time is 6 hours, substitute 6 for t.

$d = 55t$

$d = 55(6)$

$d = 330$

Kerin drove 330 miles that day.

3. Mr. Stevens teaches 4 math classes every day. Depending on absences, the number of students in each class varies. Write an equation that represents the number of students Mr. Stevens teaches in a day. If there are 30 students taking each class and one day all of the students were present, how many students did Mr. Stevens teach that day?

   Let the total number of students Mr. Stevens teaches = t.

   Let the number of students taking each class = s.

   The number of classes = 4.

   $t = 4s$

   If Mr. Stevens has 30 students in each class and all of them are present, substitute 30 for s.

   $t = 4s$

   $t = 4(30)$

   $t = 120$

   Mr. Stevens taught 120 students on the day when all of his students were present.

4. Jessica reads approximately 12 pages of her novel each hour. Depending on extracurricular activities and homework, the time that Jessica has to read varies. Write an equation that represents the number of pages Jessica reads as a function of the time spent reading. If Jessica read for 3 hours yesterday, approximately how many pages did she read?

   Let the number of hours Jessica has to read = h.

   Let the total number of pages read = p.

   The number of pages read per hour = 12.

   $p = 12h$

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If the number of hours is 3, substitute 3 for h.

$p = 12h$

$p = 12(3)$

$p = 36$

Jessica read 36 pages of her novel.

Connection to the Lesson

- In this lesson, students will translate between verbal descriptions of scenarios and mathematical equations.
- This warm-up reminds students how to use written information to calculate specific quantities.
- Students will progress to writing explicit functions of unknown quantities.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- evaluating expressions using multiplication, division, addition, and subtraction (6.EE.2c)
- evaluating exponential expressions (8.EE.1)

Introduction

Verbal descriptions of mathematical patterns and situations can be represented using equations and expressions. A **variable** is a letter used to represent a value or unknown quantity that can change or vary in an expression or equation. An **expression** is a combination of variables, quantities, and mathematical operations; 4, $8x$, and $b + 10^2$ are all expressions. An **equation** is an expression set equal to another expression; $a = 4$, $1 + 2^3 = x + 9$, and $(2 + 3)^1 = 2c$ are all equations.

A function is a relation between two variables, such that for each value of x, there is only one value of y. One way to generalize a functional relationship is to write an equation. A linear equation is a first-degree equation in one or two variables; the equation $y = 2x - 7$ is a linear equation. The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept. Some linear equations represent functions, and can be written in the form $f(x) = mx + b$. The slope of a non-vertical line is the rate of change of the dependent variable with respect to the independent variable. In a linear equation, the slope is always constant. If the consecutive values of a dependent variable have a common difference for consecutive values of the independent variable, then the pattern is linear.

An exponential function can be represented using an exponential equation. An **exponential equation** is of the form $y = ab^x + k$, where a and k are real numbers, b is the base and not equal to 1, and x is the input value. For example, $y = 3^x + 1$ is an exponential equation. Exponential equations represent functions and can be written in the form $f(x) = ab^x + k$. If consecutive values of a dependent variable have a common ratio for consecutive values of the independent variable, then the pattern is exponential.

An **explicit function** is a function in which the dependent variable is written in terms of the independent variable. For example, $f(x) = 2x$ is an explicit function.

Key Concepts

- A variable is a letter used to represent an unknown quantity.
- An expression is a combination of variables, quantities, and mathematical operations.
- An equation is an expression set equal to another expression.

SRB p. U2-196

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Instruction

- A linear equation is a first-degree equation. That is, the greatest exponent on the variable is 1.
- An equation to represent a linear function is $f(x) = mx + b$, where m is the slope and b is the y-intercept.
- An exponential equation is of the form $f(x) = ab^x + k$, where a and k are real numbers, b is the base and is not equal to 1, and x is the input value.
- Consecutive dependent terms in a linear function have a common difference if the values for the corresponding independent variable are consecutive integers.
- If consecutive terms in a linear pattern have an independent quantity that increases by 1, the common difference between the dependent quantities is the slope of the relationship between the two quantities.
- Use the slope of a linear relationship and the coordinates of a point on the line to find the linear equation that represents the relationship. Use the equation $f(x) = mx + b$, and replace m with the slope, $f(x)$ with the dependent quantity, and x with the independent quantity. Solve for b. The equation is $f(x) = mx + b$, with m and b replaced by their values.
- Consecutive dependent terms in an exponential function have a common ratio if the corresponding independent variables are consecutive integers.
- Use the common ratio to find the exponential equation that describes an exponential relationship between two quantities. In the general equation $f(x) = ab^x + k$, b is the common ratio.
- A model can be used to analyze a situation.

Common Errors/Misconceptions

- confusing linear and exponential functions when writing a function rule for a problem situation
- believing that $f(x)$ means "f times x;" i.e., not understanding that $f(x)$ represents the dependent variable for a function
- confusing the term *explicit function* with the explicit formula for a sequence

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Instruction

Guided Practice 2.7.1

Example 1

The starting balance of Anna's account is $1,250. She takes $30 out of her account each month. How much money is in her account after 1, 2, and 3 months? Find an explicit function to represent the balance in her account at any month.

1. Use the description of the account balance to find the balance after each month.

   Anna's account has $1,250. After 1 month, she takes out $30, so her account balance decreases by $30: $1250 – $30 = $1220.

   The new starting balance of Anna's account is $1,220. After 2 months, she takes out another $30. Subtract this $30 from the new balance of her account: $1220 – $30 = $1190.

   The new starting balance of Anna's account is $1,190. After 3 months, she takes out another $30. Subtract this $30 from the new balance of her account: $1190 – $30 = $1160.

2. Determine the independent and dependent quantities.

   The month number is the independent quantity, since the account balance depends on the month. The account balance is the dependent quantity.

SWB p. 185; SRB p. U2-198

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Instruction

3. Determine if there is a common difference or common ratio that describes the change in the dependent quantity.

   Organize your results in a table. Enter the independent quantity in the first column, and the dependent quantity in the second column. The balance at zero months is the starting balance of the account, before any money has been taken out. Because the independent quantity is changing by one month at a time, analyzing the differences between the dependent quantities will determine if there is a common difference between the dependent quantities.

Month	Account balance in dollars ($)	Difference
0	1250	
1	1220	1250 –1220 = –30
2	1190	1220 –1190 = –30
3	1160	1190 –1160 = –30

   The account balance has a common difference; it decreases by $30 for every 1 month. The relationship between the month and the account balance can be represented using a linear function.

4. Use the common difference to write an explicit function.

   The slope-intercept form of a linear function is $f(x) = mx + b$, where m is the slope and b is the y-intercept. The common difference between the dependent terms in the pattern is the slope of the relationship between the independent and dependent quantities. Replace m with the slope, and replace x and $f(x)$ with an ordered pair from the table, such as (1, 1220). Solve for b.

   $1220 = (-30) \bullet (1) + b$

   $1250 = b$

   $f(x) = -30x + 1250$

   The explicit function for this scenario is $f(x) = -30x + 1250$.

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Instruction

5. Evaluate the function to verify that it is correct.

   Organize your results in a table. Use the explicit function to find each term. The terms that are calculated should match the terms in the original list.

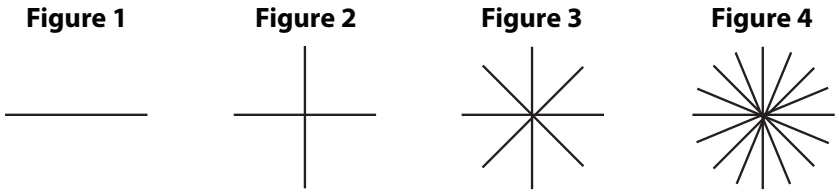
Month, x	Account balance, $f(x)$, in dollars ($)
0	(–30) • (0) + 1250 = 1250
1	(–30) • (1) + 1250 = 1220
2	(–30) • (2) + 1250 = 1190
3	(–30) • (3) + 1250 = 1160

   The pairs of dependent and independent quantities match the ones in the original pattern, so the explicit function is correct.

   The balance in Anna's account can be represented using the function $f(x) = -30x + 1250$.

Example 2

Consider that the first figure in the following diagram has two 180° angles, one on each side of the line segment. Each of these angles is then bisected or cut in half. This pattern continues, and the first 4 figures in the pattern are shown.



Write an explicit function to represent the relationship between the figure number and the number of angles in the figure.

1. Use the figures to determine the number of angles in figures 1, 2, 3, and 4.

   Count the angles in each figure, taking into consideration the note that the first figure has 2 angles.

   Figure 1: 2 angles

   Figure 2: 4 angles

   Figure 3: 8 angles

   Figure 4: 16 angles

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Instruction

2. Define the independent and dependent quantities.

   The figure number is the independent quantity, and the number of angles in the figure is the dependent quantity.

3. Determine if there is a common difference or a common ratio that describes the change in the dependent quantity.

   Organize your results in a table. Enter the independent quantity in the first column, and the dependent quantity in the second column. The pattern appears to have a common ratio. Use a table to find the ratio between the terms. Divide the current term by the previous term.

Figure	Number of angles	Ratio
1	2	
2	4	$\frac{4}{2}=2$
3	8	$\frac{8}{4}=2$
4	16	$\frac{16}{8}=2$

   The common ratio between the dependent terms is 2.

4. Use the first pair of quantities and the common ratio to write an explicit function.

   The common ratio is b, and a_1 is the value of the dependent quantity when the independent quantity is 1. The general equation to represent the relationship is $f(x) = a_1 b^{x-1}$. In this case, $a_1 = 2$ and $b = 2$, so the function to represent the relationship is $f(x) = 2 \cdot 2^{x-1}$.

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Instruction

5. Evaluate the function to verify that it is correct.

   Organize your results in a table. Use the explicit function to find each dependent term. The terms that are calculated should match the terms in the original list.

Figure	Number of angles
1	$2 \cdot 2^{1-1} = 2$
2	$2 \cdot 2^{2-1} = 4$
3	$2 \cdot 2^{3-1} = 8$
4	$2 \cdot 2^{4-1} = 16$

   The dependent terms match the ones in the original pattern, so the explicit function is correct.

   The relationship between the number of angles and the figure number can be described using the function $f(x) = 2 \cdot 2^{x-1}$.



Example 3

A video arcade charges an entrance fee, then charges a fee per game played. The entrance fee is \$5, and each game costs an additional \$1. Find the total cost for playing 0, 1, 2, or 3 games. Describe the total cost of playing x games with an explicit function.

1. Use the description of the costs to find the total costs.

   If no games are played, then only the entrance fee is paid. The total cost for playing 0 games is \$5.

   If 1 game is played, then the entrance fee is paid, plus the cost of one game. If each game is \$1, the cost of one game is \$1. The total cost is \$5 + \$1 = \$6.

   If 2 games are played, then the entrance fee is paid, plus the cost of two games. If each game is \$1, the cost of two games is \$1 • 2 = \$2. The total cost is \$5 + \$2 = \$7.

   If 3 games are played, then the entrance fee is paid, plus the cost of three games. If each game is \$1, the cost of three games is \$1 • 3 = \$3. The total cost is \$5 + \$3 = \$8.



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Instruction

2. Identify the independent and dependent quantities.

   The total cost is dependent on the number of games played, so the number of games is the independent quantity and the total cost is the dependent quantity.

3. Determine if there is a common difference or a common ratio between the dependent terms.

   There appears to be a common difference between the dependent terms. Use a table to find the difference between the dependent quantities. Subtract the current term from the previous term.

Games	Cost in dollars ($)	Difference
0	5	
1	6	6 – 5 = 1
2	7	7 – 6 = 1
3	8	8 – 7 = 1

   The common difference between the dependent terms is $1.

4. Use the common difference to write an explicit function.

   The slope-intercept form of a linear function is $f(x) = mx + b$, where m is the slope and b is the y-intercept. The common difference between the dependent terms in the pattern is the slope of the relationship between the independent and dependent quantities. Replace m with the slope, and replace x and $f(x)$ with a coordinate pair from the table, such as (1, 6). Solve for b.

   $6 = (1) \bullet (1) + b$

   $5 = b$

   The explicit function is $f(x) = 1x + 5$.

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Instruction

5. Evaluate the function to verify that it is correct.

   Organize your results in a table. Use the explicit function to find each term. The terms that are calculated should match the terms in the original list.

Games	Cost in dollars ($)
0	1 • (0) + 5 = 5
1	1 • (1) + 5 = 6
2	1 • (2) + 5 = 7
3	1 • (3) + 5 = 8

   The pairs of independent and dependent quantities match the ones in the original pattern, so the explicit function is correct.

   The total cost of any number of games, x, can be represented using the function $f(x) = x + 5$.

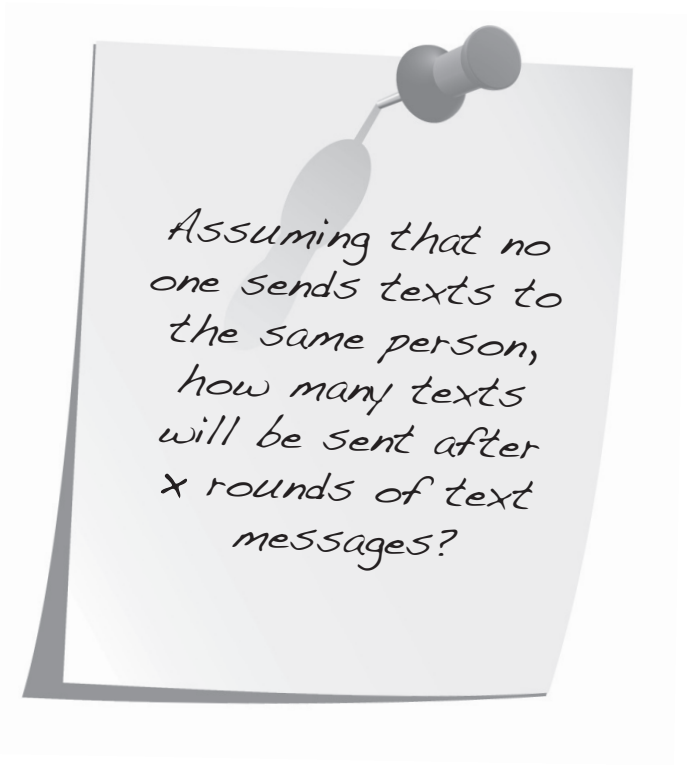


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Problem-Based Task 2.7.1: Texting for the Win

SMP	
1 ✓	2 ✓
3	4
5	6
7	8 ✓

Lucas's friend Isabel is performing in a singing competition. The winner of the competition will be determined by call-in votes. To help Isabel earn votes, Lucas sends text messages to 8 of his friends. He then asks each of his friends to send texts to 8 more friends, and asks his friends to ask each of their friends to send 8 texts. He is hoping this pattern will continue and that many people will receive text messages telling them to vote for Isabel. Call each set of text messages a "round" of text messages, where Lucas's messages are the first round, Lucas's friends messages are the second round, and so on. Assuming that no one sends texts to the same person, how many texts will be sent after x rounds of text messages?



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Problem-Based Task 2.7.1: Texting for the Win

Coaching

a. Draw a diagram to model the situation.

b. How many texts were sent the first round?

c. How many texts were sent the second round?

d. Determine how many texts were sent in rounds 3, 4, and 5.

e. Create a table showing the number of texts sent after the first five rounds.

f. If you know the number of texts sent in round $x - 1$, how many texts will be sent in round x?

g. How many texts will be sent in round x? In other words, what is the explicit function that represents the number of texts sent in any round, x?

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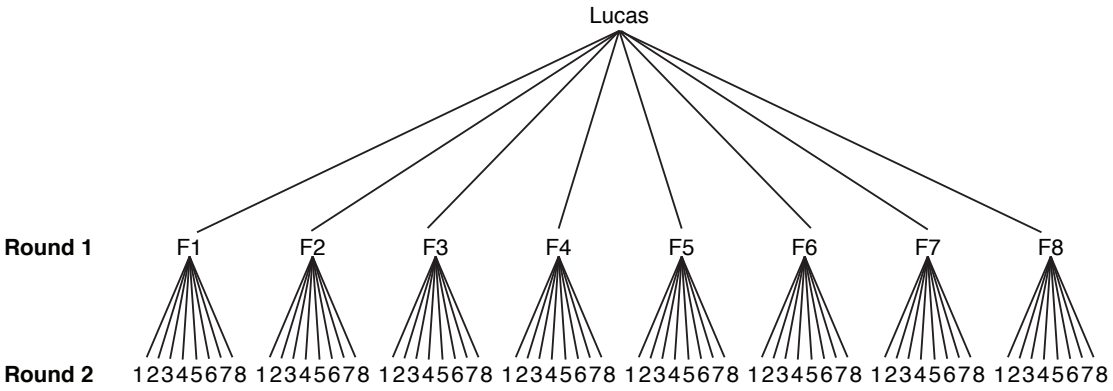
Instruction

Problem-Based Task 2.7.1: Texting for the Win

Coaching Sample Responses

a. Draw a diagram to model the situation.

Start the diagram with Lucas, and the 8 friends whom he texts in Round 1. Then show each of the 8 people his friends text in Round 2.



b. How many texts were sent the first round?

Lucas sent 8 texts the first round.

c. How many texts were sent the second round?

Each of Lucas's friends sent 8 texts. The total number of texts sent in Round 2 was 8 • 8 = 64.

d. Determine how many texts were sent in rounds 3, 4, and 5.

Each of the 64 people who received a text in Round 2 each send 8 texts, so the total number of texts in Round 3 is 64 • 8 = 512.

Each of the 512 people who received a text in Round 3 each send 8 texts, so the total number of texts in Round 4 is 512 • 8 = 4096.

Each of the 4,096 people who received a text in Round 4 each send 8 texts, so the total number of texts in Round 5 is 4096 • 8 = 32,768.

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e. Create a table showing the number of texts sent after the first five rounds.

Include the calculation in the table to help reveal the pattern.

Round	Number of texts sent
1	8
2	8 • 8 = 64
3	64 • 8 = 512
4	512 • 8 = 4096
5	4096 • 8 = 32,768

f. If you know the number of texts sent in round $x - 1$, how many texts will be sent in round x?

Look at the calculations in the table. Let $t(x)$ equal the number of texts sent in any round, x. To find the number of texts sent in Round 2, the calculation is $t(1) \cdot 8$. To find the number of texts sent in Round 3, the calculation is $t(2) \cdot 8$. This pattern continues in the table, and the number of texts sent in any round is equal to the number of texts sent in the previous round times 8. In symbols, this is $t(x) = t(x - 1) \cdot 8$.

g. How many texts will be sent in round x? In other words, what is the explicit function that represents the number of texts sent in any round, x?

To find an expression for $t(x)$ that doesn't include $t(x - 1)$, examine the pattern of the calculations in the table. Rewrite the calculations for each round, without evaluating the number of texts sent in each round.

Round	Number of texts sent
1	8
2	8 • 8
3	(8 • 8) • 8
4	(8 • 8 • 8) • 8
5	(8 • 8 • 8 • 8) • 8

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Instruction

The number of texts sent in each round is a power of 8, where the power is equal to the round number. The table shows each calculation rewritten using exponents.

Round	Number of texts sent
1	8^1
2	8^2
3	8^3
4	8^4
5	8^5

The general expression for the number of texts sent in round x is 8^x. The explicit function, where $t(x)$ is the number of texts sent in round x, is $t(x) = 8^x$.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.7.1: Building Functions from Context

Write an explicit function to represent each pattern.

1. Mr. Ramos notices a pattern in the number of people attending the weekly student government meetings. For weeks 1, 2, 3, 4, and 5, the number of students attending the meeting was 31, 43, 55, 67, and 79, respectively.

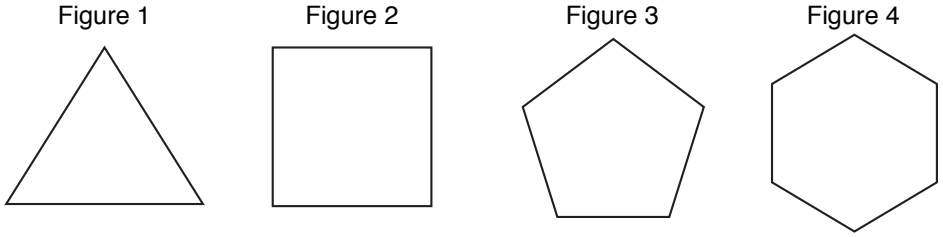
2. Hannah borrows $30 from her parents. Each week, she pays them back the same amount. The total amounts she owes her parents after weeks 0, 1, 2, 3, and 4 are $30, $25, $20, $15, and $10, respectively.

3. Angelo sells cookies in packages, where each package contains the same number of cookies. The total number of cookies he has after 1, 2, 3, 4, and 5 packages are sold are 110, 88, 66, 44, and 22, respectively.

4. Cameron tracks the number of people who read his blog. In weeks 1, 2, 3, 4, and 5, the blog had 12, 60, 300, 1,500, and 7,500 visitors, respectively.

5. As a treat, Nia eats a portion of a chocolate bar each day. She eats the same portion of the remaining bar each day. On day 0, the bar of chocolate starts with 32 pieces. After 1 day, 16 pieces remain. After days 2, 3, and 4, there are a total of 8, 4, and 2 pieces remaining.

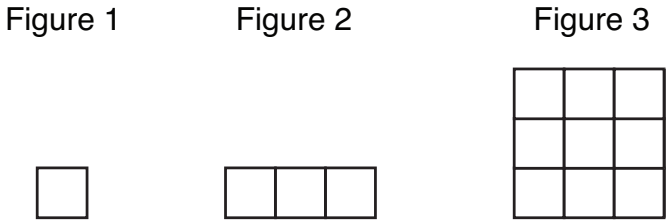
6. Given the diagram, if the pattern continues, describe the number of sides in Figure *x*.



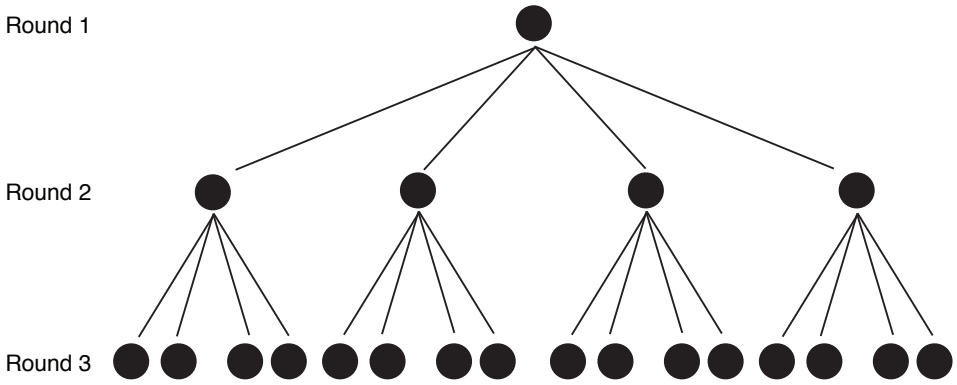
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7. Given the diagram that follows, describe the number of blocks in Figure *x* if this pattern continues.



8. A rural school uses a phone tree to reach parents when the school is closed. Each parent calls multiple parents to notify them of the school closing. These parents then each call multiple parents, and so on. The following diagram shows the number of parents called after each round of calls. Each dot represents a parent. Find an explicit function to represent the number of parents called in any round *x*.



9. A hotel charges a room fee per night, plus an additional fee if more than one guest is staying in a room. Good Nights hotel charges $150 per night for a room, plus $25 per guest if more than one guest is staying in a room. Find an explicit function to represent the nightly cost for any number of guests.

10. The population of a city is growing. Each year, the population increases by approximately 10%, or 0.10 times the previous year's population. The population this year is 10,000. Find an explicit function to represent the population of the town in any year. Consider that year 0 is this year.

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Lesson 2.7.2: Constructing Functions from Graphs and Tables

Warm-Up 2.7.2

Read the scenario and use the table to complete problem 1. For problems 2 and 3, read each scenario and create a graph of each linear function.

1. The students in Mr. Jacobs' class record their ages in months and their heights in centimeters. The table shows the data from 10 students. Create a graph of the students' ages and heights.

Age in months	Height in centimeters
162	146
165	149
164	180
158	126
156	140
166	183
160	176
161	193
167	184
156	125

2. Elliot uses the equation $f(x) = 65x$ to estimate the number of miles traveled after any number of hours, x, during a recent road trip.

3. A store is having a sale. T-shirts are \$5, and the store is offering \$3 off any purchase. The total purchase cost of any number of T-shirts can be represented by the explicit function $f(x) = 5x - 3$.

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Instruction

Lesson 2.7.2: Constructing Functions from Graphs and Tables

Common Core State Standard

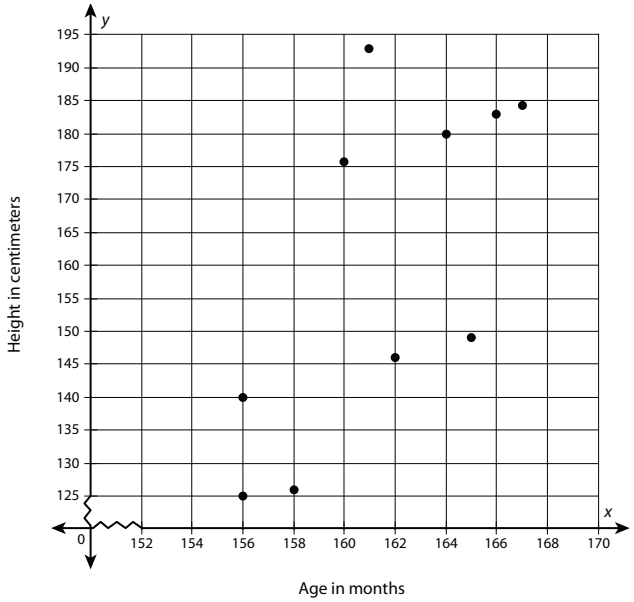
F–LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

Warm-Up 2.7.2 Debrief

1. The students in Mr. Jacobs' class record their ages in months and their heights in centimeters. The table shows the data from 10 students. Create a graph of the students' ages and heights.

Age in months	Height in centimeters
162	146
165	149
164	180
158	126
156	140
166	183
160	176
161	193
167	184
156	125

Each point will be graphed as (x, y). Find the value of x in the horizontal x-axis, then find the value of y in the vertical y-axis. Find the point at which these two values meet on the coordinate plane.



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Instruction

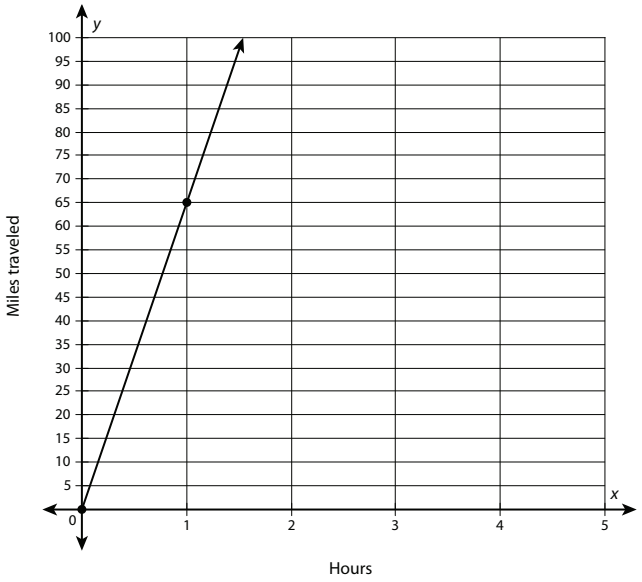
2. Elliot uses the equation $f(x) = 65x$ to estimate the number of miles traveled after any number of hours, x, during a recent road trip.

   Find two points on the line, and then draw a line through the two points. Evaluate the equation at two values of x to find the two points. Two easy values of x to use are 0 and 1.

   $f(0) = 65 \bullet (0) = 0$

   $f(1) = 65 \bullet (1) = 65$

   Two points are (0, 0) and (1, 65).



3. A store is having a sale. T-shirts are $5, and the store is offering $3 off any purchase. The total purchase cost of any number of T-shirts can be represented by the equation $f(x) = 5x - 3$.

   Find two points on the line, then draw a line through the two points. Evaluate the equation at two values of x to find the two points. Note that the number of T-shirts must be a natural number since you cannot buy a portion of a T-shirt. Therefore, two values of x that make sense in the context of the problem are 3 and 5.

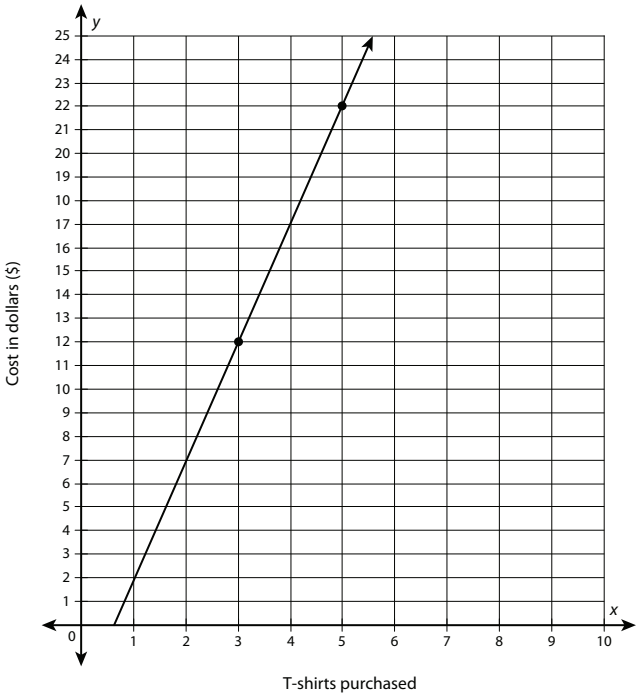
   $f(3) = 5(3) - 3 = 12$

   $f(5) = 5(5) - 3 = 22$

   Two points are (3, 12) and (5, 22).

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Instruction



Connection to the Lesson

- In this lesson, students will make connections between tables, graphs, and equations.
- In this warm-up, students will practice translating from tables to graphs and equations to graphs before writing equations based on graphs and tables.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- graphing points on the coordinate plane (5.G.1)
- understanding independent and dependent quantities (6.EE.9)

Introduction

Tables and graphs of functions can be represented by equations. Data represented in a table can either be analyzed as a pattern, or it can be graphed first. The shape of a graph helps identify which type of equation can be used to represent the data. Data that is represented using a linear equation has a constant slope and the graph is a straight line. The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept. The y-intercept is the y-coordinate of the point at which the graph intersects the y-axis. To find the equation of a line, first find the slope, then replace x and y in the slope-intercept form of the equation with x and y from an ordered pair on the graph and solve the equation for b. To write the equation, substitute the values of m and b into $y = mx + b$.

The slope of a line is the change in the independent variable divided by the change in the dependent variable. Using the points (x_1, y_1) and (x_2, y_2), the slope, m, can be calculated by finding $\frac{y_2 - y_1}{x_2 - x_1}$. The slope is similar to the common difference between terms, except the slope can be found between points that have x-values that are more than one unit apart. The slope allows us to compare the change between any two points on a graph or in a table. An exponential equation has a slope that is constantly changing, either increasing or decreasing. Graphically, exponential equations are curves. The general form of an exponential equation is $y = ab^x + k$, where a and k are real numbers, and b is the common ratio and is not equal to 1. The values of a and b determine the shape of the graph of an exponential function. The following graphs demonstrate functions with different a and b values.

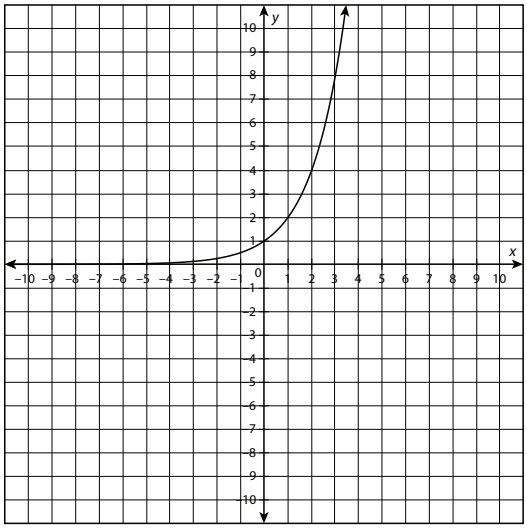
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Instruction

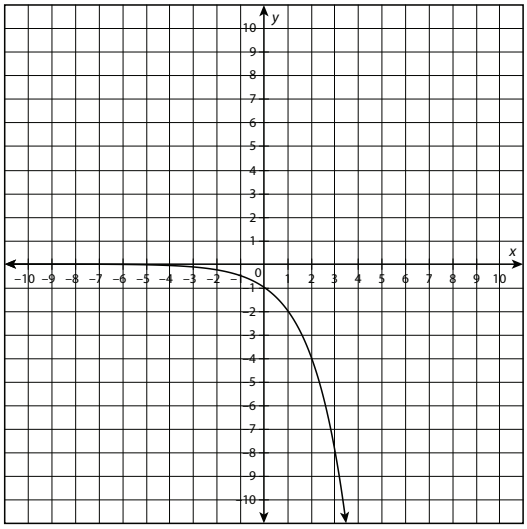
$a > 0$ and $b > 1$

$f(x) = 2^x$



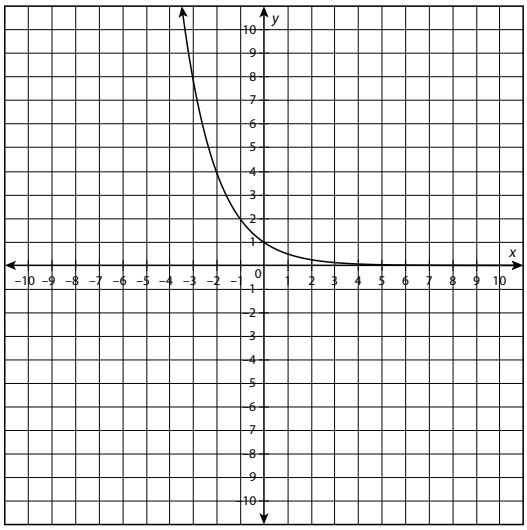
$a < 0$ and $b > 1$

$f(x) = (-1) \bullet 2^x$



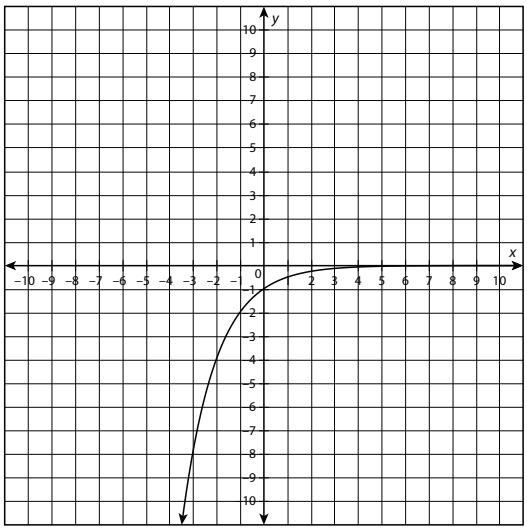
$a > 0$ and $0 < b < 1$

$f(x) = (0.5)^x$



$a < 0$ and $0 < b < 1$

$f(x) = -1 \bullet (0.5)^x$



Data that follow an exponential pattern have a common ratio between the dependent quantities. When looking for the common ratio, be sure to verify that the *x*-values, or independent quantities, are each one unit apart. The common ratio, *b*, can be used to write an equation to represent the exponential pattern. The value of *a* in an exponential function is $f(0)$. It is the *y*-intercept of the graph of the function.

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Instruction

Key Concepts

- The graph of a linear equation is a straight line.
- Linear equations have a constant slope, or rate of change.
- Linear equations can be written as functions.
- The slope-intercept form of a linear function is $f(x) = mx + b$, where m is the slope and b is the y-intercept.
- The slope of a linear function can be calculated using any two points, (x_1, y_1) and (x_2, y_2); the formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- The y-intercept is the y-coordinate of the point at which the graph of the equation crosses the y-axis. It is the value of y when $x = 0$.
- Exponential equations have a slope that is constantly changing.
- Exponential equations can be written as functions.
- The general form of an exponential function is $f(x) = ab^x + k$, where a and k are real numbers, and the base b is a real number not equal to 1.
- The graph of an exponential function is a curve.

Common Errors/Misconceptions

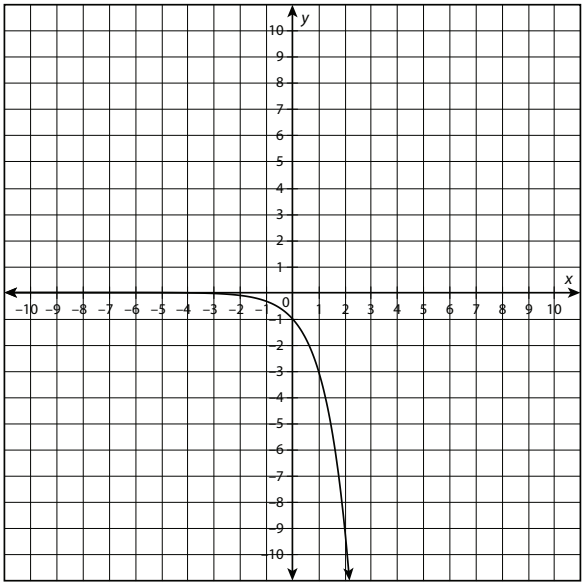
- looking for a common difference between dependent quantities when the independent values are not one unit apart
- incorrectly calculating the slope
- trying to match an exponential graph to a linear equation
- trying to match a linear graph to an exponential equation

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Guided Practice 2.7.2

Example 1

Determine the function that represents the relationship between x and y in the following graph.



1. Determine which type of equation, linear or exponential, will fit the graph.

   The graph of a linear equation is a straight line, and the graph of an exponential equation is a curve. An exponential equation can be used to represent the graph.

2. Identify at least three points from the graph, with one of the points at $x = 0$.

   From the graph, three points are (0, –1), (1, –3), and (2, –9).

3. Find the common ratio between the terms, ensuring that the x-values are one unit apart.

   The three points are at the x-values 0, 1, and 2, so the x-values are each one unit apart. Look at the pattern of the y-values: –1, –3, and –9. Each value is multiplied by 3 to find the next value, so the common ratio is 3.

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Instruction

4. Use the function value when $x = 0$ and the common ratio to write an equation to represent the graph.

   When $x = 0$, $f(0) = -1$.

   The exponential equation can be written as $f(x) = f(0) \cdot b^x$, where b is the common ratio. The equation of the graph is $f(x) = (-1) \cdot 3^x$. Check the equation by evaluating it at the values of x from previously identified points.

   $f(0) = (-1) \cdot 3^0 = -1$

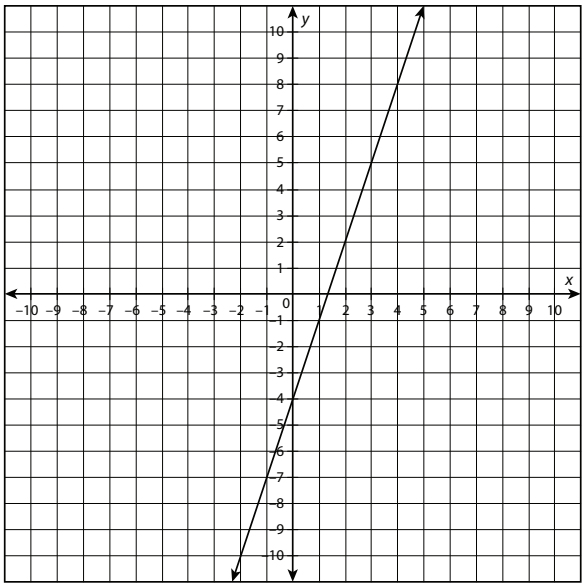
   $f(1) = (-1) \cdot 3^1 = -3$

   $f(2) = (-1) \cdot 3^2 = -9$

   The relationship between x and y can be represented using the function $f(x) = (-1) \cdot 3^x$.

Example 2

Determine the function that represents the relationship between x and y in the following graph.



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Instruction

1. Determine which type of equation, linear or exponential, will fit the graph.

   The graph of a linear equation is a straight line, and the graph of an exponential equation is a curve. A linear equation can be used to represent the graph.

2. Identify at least three points from the graph.

   From the graph, three points are (0, –4), (1, –1), and (2, 2).

3. Find the slope of the line, using any two of the points.

   The slope, m, of the line is $\frac{(y_2 - y_1)}{(x_2 - x_1)}$ for any two points (x_1, y_1) and (x_2, y_2). Using the points (0, –4) and (1, –1), the slope is $\frac{(-1-(-4))}{(1-0)} = 3$.

4. Find the y-intercept of the line.

   The y-intercept can be found by finding the value of y when $x = 0$. On the graph, we can see the point (0, –4). The y-intercept is –4.

5. Use the slope and y-intercept to find an equation of the line.

   The slope-intercept form of the linear function is $f(x) = mx + b$, where m is the slope and b is the y-intercept. The equation that represents the line is $f(x) = 3x - 4$. Check the equation by evaluating it at the values of x from previously identified points.

   $f(0) = 3(0) - 4 = -4$

   $f(1) = 3(1) - 4 = -1$

   $f(2) = 3(2) - 4 = 2$

   The relationship can be represented using the equation $f(x) = 3x - 4$.

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Instruction

Example 3

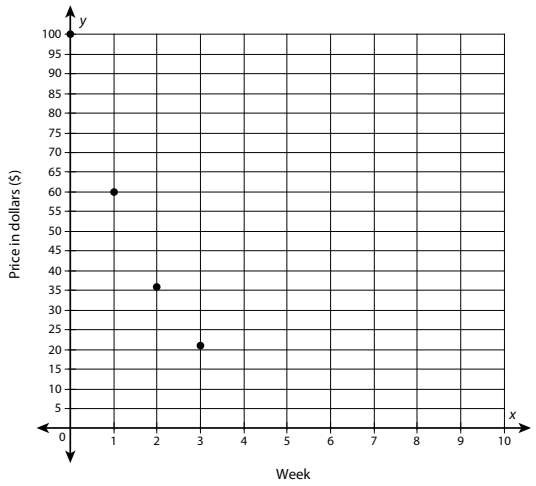
A clothing store discounts items on a regular schedule. Each week, the price of an item is reduced. The prices for one item are in the following table. Week 0 is the starting price of the item.

Week	Price in dollars ($)
0	100.00
1	60.00
2	36.00
3	21.60

Determine a linear or exponential equation that represents the relationship between the week and the price of the item.

1. Create a graph of the data.

   Let the *x*-axis represent the week, and the *y*-axis represent the price in dollars.



2. Determine if a linear or exponential equation could represent the data.

   The *x*-values of the points vary by 1. Look at the vertical distance between each pair of points. It appears to be decreasing, and is not remaining constant. An exponential equation could represent the data.

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Instruction

3. Find the common ratio between the terms.

The four points are at the x-values 0, 1, 2, and 3, so the x-values are each one unit apart. Look at the pattern of the y-values: 100, 60, 36, and 21.60. Divide each y-value by the previous y-value to identify the common ratio.

x	y	Ratio
0	100	
1	60	$\frac{60}{100} = 0.60$
2	36	$\frac{36}{60} = 0.60$
3	21.6	$\frac{21.6}{36} = 0.60$

The common ratio is 0.60.

4. Use the value of the function when $x = 0$ and the common ratio to write an equation to represent the relationship.

At week 0, the price is \$100. The common ratio is 0.60. An equation to represent the relationship is $f(x) = 100 \bullet (0.60)^x$. Evaluate the equation at the given values of x to check the equation.

$f(0) = 100 \bullet (0.60)^0 = 100$

$f(1) = 100 \bullet (0.60)^1 = 60$

$f(2) = 100 \bullet (0.60)^2 = 36$

$f(3) = 100 \bullet (0.60)^3 = 21.6$

The price of the clothing item, y, at any week, x, can be represented by the equation $f(x) = 100 \bullet (0.60)^x$.

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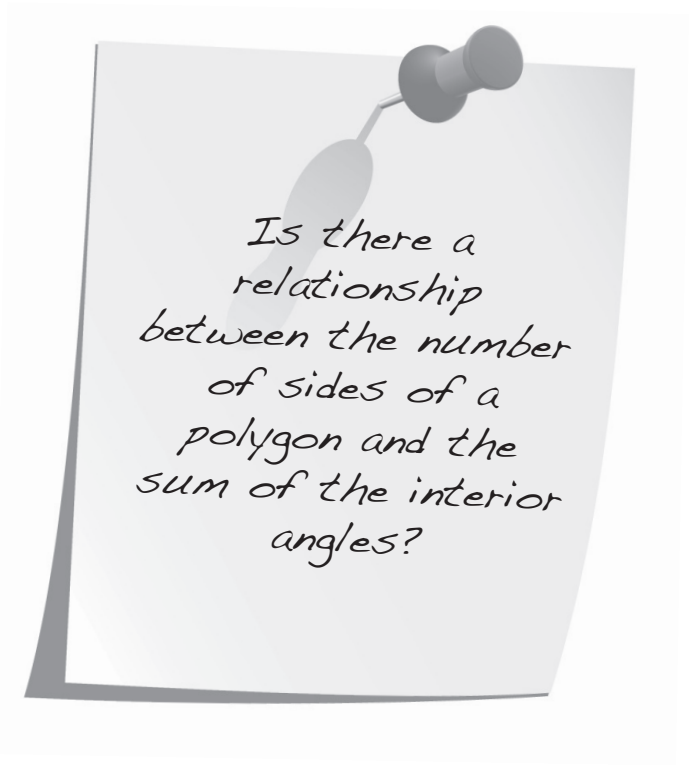
Problem-Based Task 2.7.2: Interior Angles in Polygons

SMP	
1 ✓	2 ✓
3	4 ✓
5	6
7	8

Julia is studying the sum of the measures of the interior angles in polygons. She creates polygons with 3, 5, 6, 7, and 9 sides, and records the sum of the interior angles in each polygon in the following table.

Number of sides	Sum of interior angles, in degrees
3	180
5	540
6	720
7	900
9	1260

Is there a relationship between the number of sides of a polygon and the sum of the interior angles? Write a function that can be used to determine the sum of the interior angles of a polygon with any number of sides.



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Problem-Based Task 2.7.2: Interior Angles in Polygons

Coaching

a. Create a graph of the data.

b. Should the relationship be represented using a linear or exponential function? Use the shape of the graph to explain your answer.

c. What is the slope of the graph that represents the relationship?

d. Use the slope and a pair of data values to write a function that represents the sum of the interior angles for any polygon.

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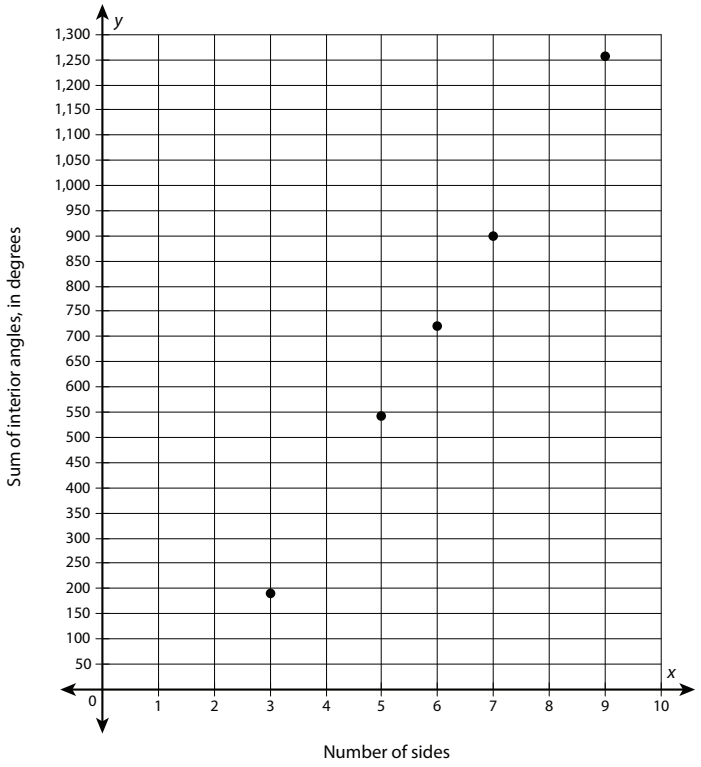
Instruction

Problem-Based Task 2.7.2: Interior Angles in Polygons

Coaching Sample Responses

a. Create a graph of the data.

Let the *x*-axis represent the number of sides, and the *y*-axis represent the sum of the interior angles.



b. Should the relationship be represented using a linear or exponential function? Use the shape of the graph to explain your answer.

The shape of the graph of a linear function is a line, with a constant rate of change. The shape of the graph of an exponential function is a curve, where the rate of change is not constant, and is either increasing or decreasing. This graph is a set of points that lie on a straight line, so the relationship can be represented using a linear function.

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c. What is the slope of the graph that represents the relationship?

To find the slope of a line, find the change in the dependent variable and divide by the corresponding change in the independent variable. In this case, the dependent variable is the sum of the measures of the interior angles. The independent variable is the number of sides of the polygon. To verify that the slope is constant, find the slope between each pair of data points.

Number of sides	Sum of interior angles	Slope
3	180	
5	540	$\frac{540-180}{5-3}=180$
6	720	$\frac{720-540}{6-5}=180$
7	900	$\frac{900-720}{7-6}=180$
9	1260	$\frac{1260-900}{9-7}=180$

The slope of the line through the points is 180.

d. Use the slope and a pair of data values to write a function that represents the sum of the interior angles for any polygon.

The slope-intercept form of a linear function is $f(x) = mx + b$, where m is the slope and b is the y-intercept. The slope is 180, as found in part c. Substitute this value for m. To find b, replace x and $f(x)$ in the equation with values from a data pair. Using the data pair (5, 540), solve for b:

$f(x) = mx + b$

$(540) = (180)(5) + b$

$-360 = b$

Use the y-intercept and the slope to write the function.

$f(x) = mx + b$

$f(x) = 180x - 360$

The function $f(x) = 180x - 360$ represents the sum of the interior angles of any polygon.

Recommended Closure Activity

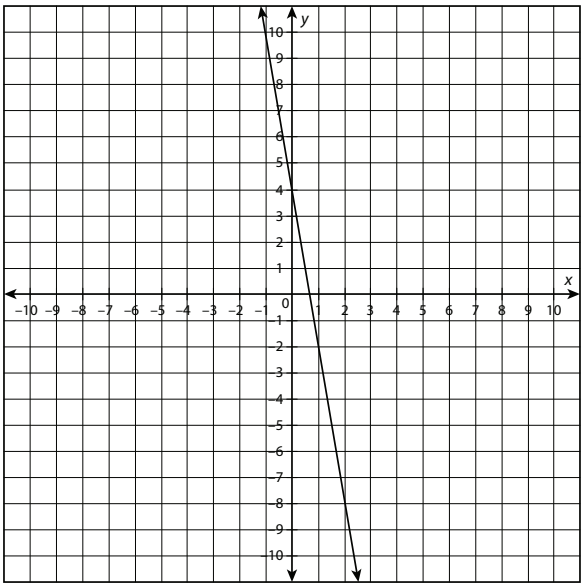
Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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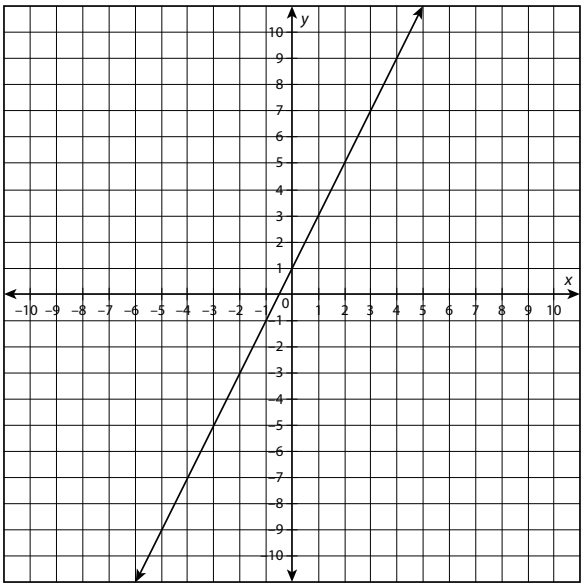
Practice 2.7.2: Constructing Functions from Graphs and Tables

Determine a linear or exponential equation that represents the relationship between x and y in each graph or table that follows.

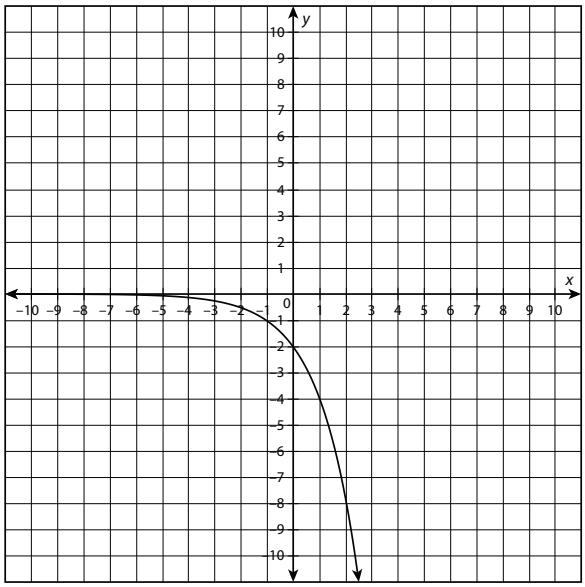
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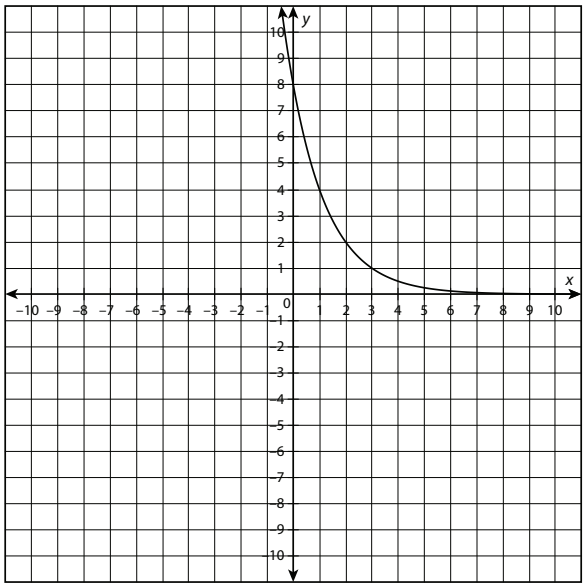
2.



3.



4.

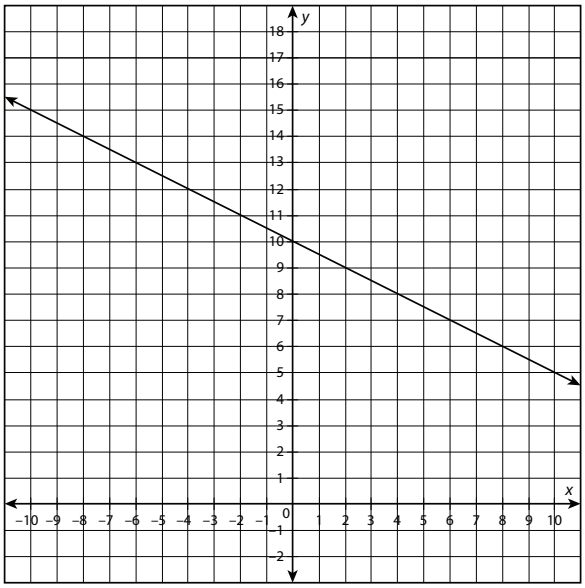


continued

SWB p. 199; SRB Practice 2.7.2 p. U2-215

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5.



6.

x	*y*
0	10
1	40
2	160
3	640
4	2560

7.

x	*y*
0	16
2	13
3	11.5
5	8.5

8.

x	*y*
0	–12
1	–4
4	20
5	28

9.

x	*y*
0	–2
1	–14
2	–98
3	–686
4	–4802

10.

x	*y*
0	–7
1	–56
2	–448
3	–3584

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Progress Assessment

Circle the letter of the best answer.

1. Terrell's car displays the number of gallons remaining in his gas tank. When he fills his tank, he has 15 gallons of gas. After traveling 1 mile, Terrell has 14.95 gallons of gas. After traveling 2 miles, Terrell has 14.90 gallons of gas. After traveling 3 miles, Terrell has 14.85 gallons of gas. Which explicit function represents the remaining gallons of gas after traveling x miles?

   a. $f(x) = 15 - 0.5x$
   b. $f(x) = 15x$
   c. $f(x) = 15 - 0.05x$
   d. $f(x) = 15 + 0.05x$

2. Which explicit function represents the pattern in the table?

x	y
1	70
2	58
3	46
4	34
5	22

   a. $f(x) = -12x + 82$
   b. $f(x) = -12x - 82$
   c. $f(x) = 12x + 82$
   d. $f(x) = 12x - 82$

3. Which explicit function represents the pattern in the table?

x	y
1	–10
2	–90
3	–810
4	–7290

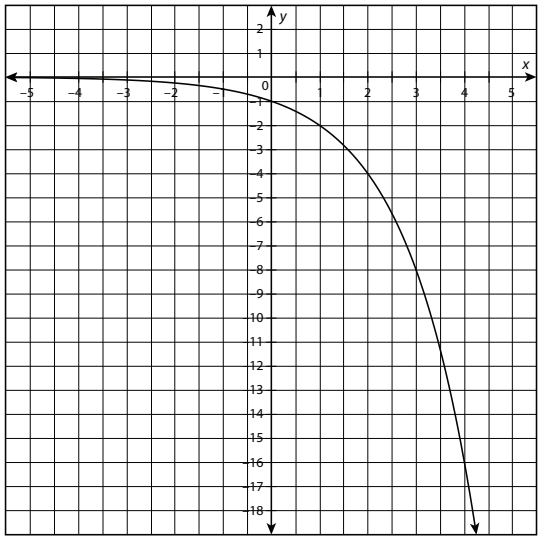
   a. $f(x)=(-10)^{x-1}$
   b. $f(x)=(-10)\bullet 9^{x-1}$
   c. $f(x)=10\bullet(-9)^{x-1}$
   d. $f(x)=-9\bullet(-10)^{x-1}$

continued

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Assessment

4. Which explicit function represents the relationship between x and y shown in the graph?



a. $f(x)=-2x-1$

b. $f(x)=2^x$

c. $f(x)=(-2)^x$

d. $f(x)=(-1)\bullet 2^x$

5. Which explicit function represents the relationship between x and y shown in the table?

x	y
1	–115
3	1
4	59
5	117

a. $f(x) = 58x - 173$

b. $f(x) = 58x - 115$

c. $f(x) = -173x$

d. $f(x) = 58x$

6. Jonathan is putting on a play. He sells tickets for $10 each. Which explicit function describes the total money he earns for any number of tickets sold?

a. $f(x)=10^x$

b. $f(x) = 10x$

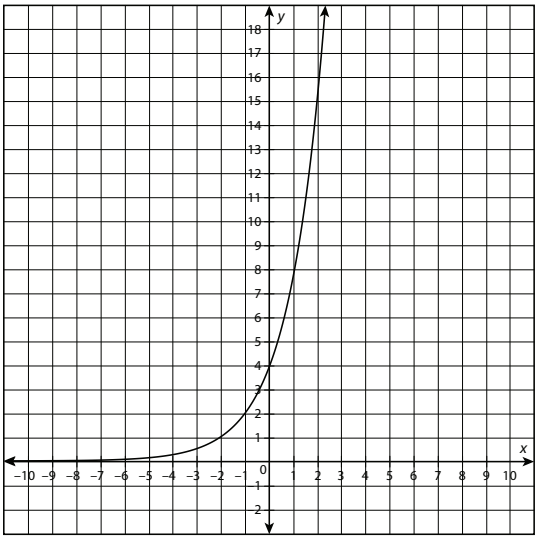
c. $f(x) = 10 + x$

d. $f(x) = -10x$

continued

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7. Which explicit function represents the relationship between x and y in the graph?



a. $f(x) = -4 \bullet 2^x$

b. $f(x) = -2 \bullet 4^x$

c. $f(x) = 4 \bullet 2^x$

d. $f(x) = 4 \bullet 4^x$

8. A population of bears is decreasing. The population this year is 200 bears. After 1 year, it is estimated that the population will be 160 bears. After 2 years, it is estimated that the population will be 128 bears. Which explicit function describes the bear population in any year x?

a. $f(x) = 200 \bullet 0.8^x$

b. $f(x) = 160 \bullet 0.8^x$

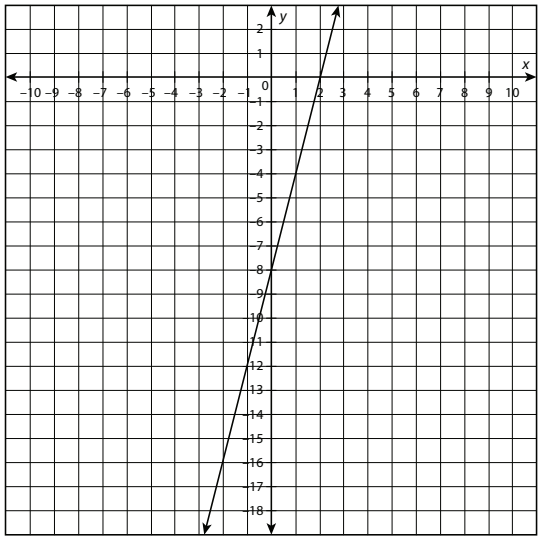
c. $f(x) = 200 \bullet 1.2^x$

d. $f(x) = 200 \bullet 0.8^{x-1}$

continued

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9. Which explicit function represents the relationship between x and y shown in the graph?



a. $f(x) = 4x - 8$

b. $f(x) = 4x$

c. $f(x) = 8x - 4$

d. $f(x) = 8x + 4$

10. Which explicit function represents the relationship between x and y shown in the table?

x	y
0	3
1	42
2	588
3	8232

a. $f(x)=14^x$

b. $f(x)=3^x$

c. $f(x)=14\bullet 3^x$

d. $f(x)=3\bullet 14^x$

Use the table to answer the question that follows.

11. What explicit function represents the relationship between x and y?

x	y
0	–13
1	–117
2	–1053
3	–9477

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Pre-Assessment

Circle the letter of the best answer.

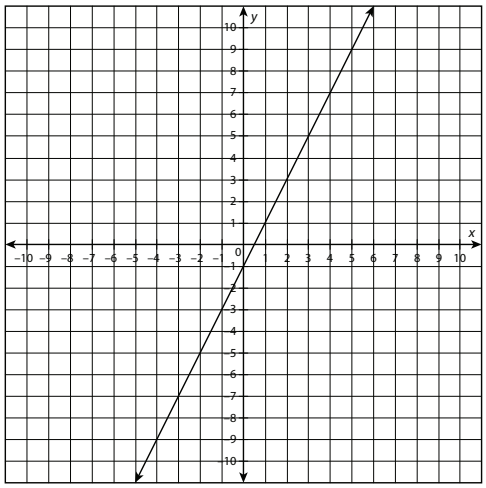
1. If $f(x) = 3x + 2$ and $g(x) = 7x - 1$, what is $(f + g)(x)$?
   a. $21x - 2$
   b. $10x + 1$
   c. $4x - 3$
   d. $-4x + 3$

2. If $f(x) = 2^x$ and $g(x) = 4$, what is $(f \bullet g)(x)$?
   a. 8^x
   b. $2^x + 4$
   c. $4(2)^x$
   d. 6^x

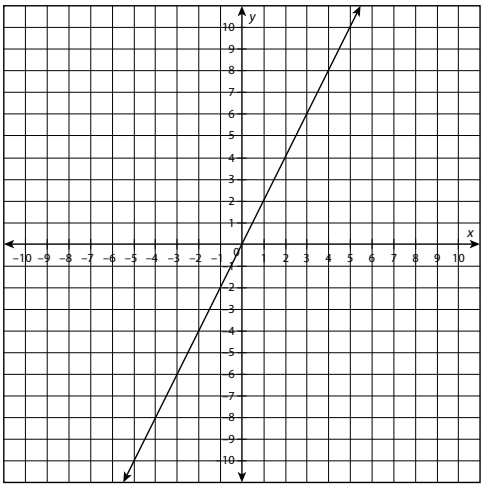
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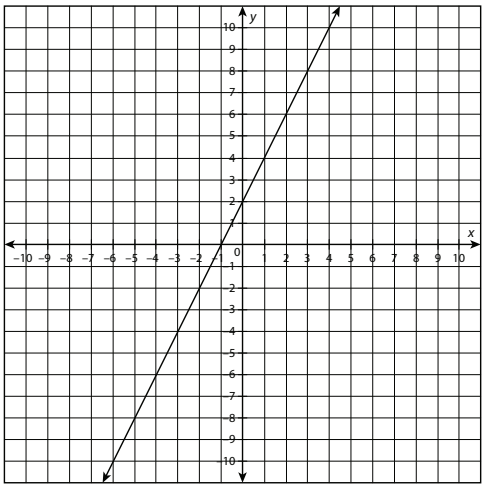
3. Given the following graph of $f(x)$, which is the graph of $f(x) + 2$?



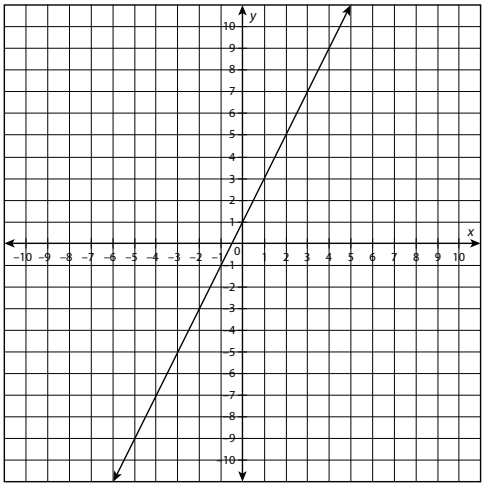
a.



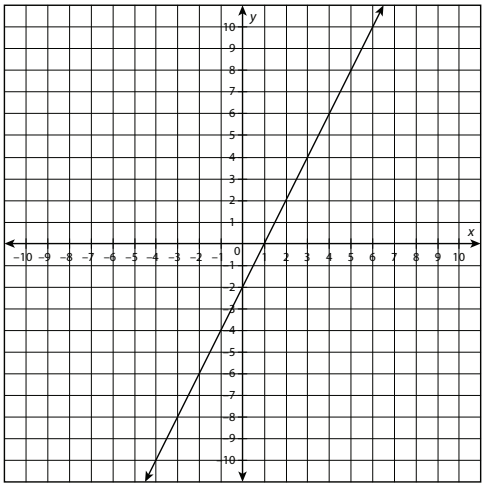
c.



b.



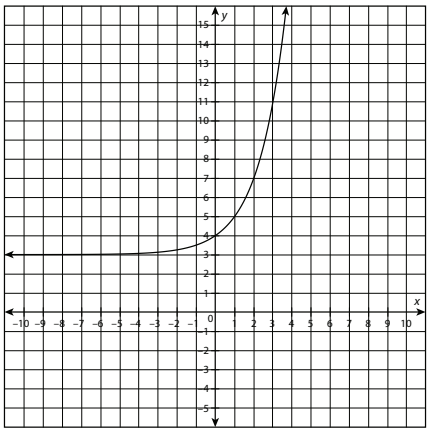
d.



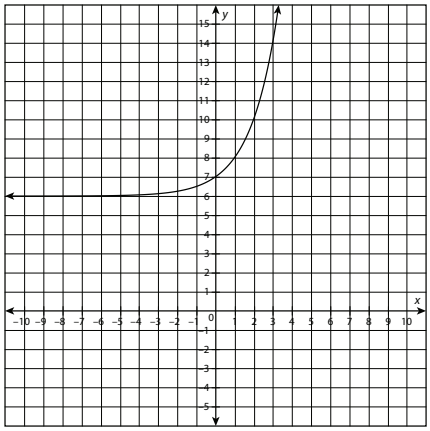
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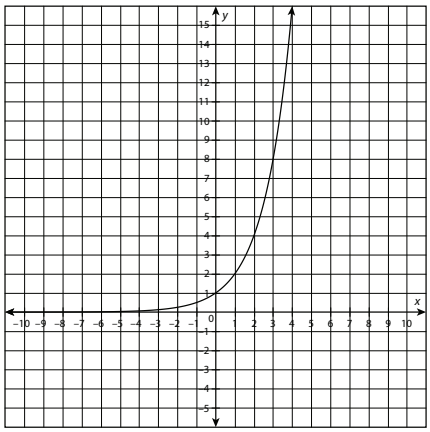
4. Given the following graph of $f(x)$, which is the graph of $f(x) - 3$?



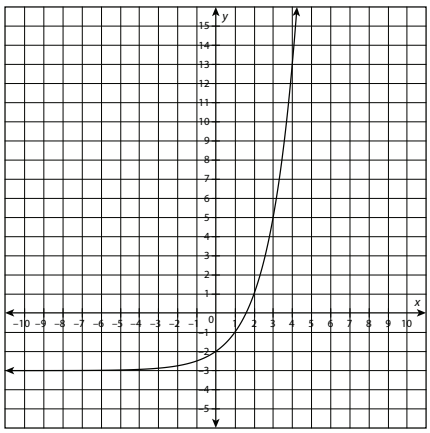
a.



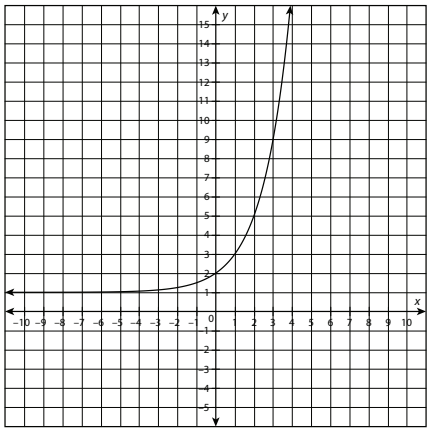
c.



b.



d.



5. $f(x) = -x + 3$ and $g(x) = -x - 2$. If $g(x)$ can be written as $f(x) + k$, what is the value of k?

a. 5

b. –5

c. 1

d. –1

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Lesson 8: Operating on Functions and Transformations

Instruction

Common Core State Standards

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 6 ✓
7 ✓ 8

F–BF.1 Write a function that describes a relationship between two quantities.★

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

F–BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Essential Questions

1. How can functions be combined by adding, subtracting, multiplying, or dividing?
2. How are the graph of a function and the equation of a function related?
3. How do translations of the graph of a function relate to the function's equation?

WORDS TO KNOW

function	a relation in which each element in the domain is mapped onto exactly one element in the range; that is, for every value of x, there is exactly one value of y
transformation	a change in a geometric figure's position, shape, or size
translation	moving a graph either vertically, horizontally, or both, without changing its shape; a slide
vertical shift	the number of units the graph of a function is moved up or down in a plane; a type of translation

SRB p. U2-217

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Instruction

Recommended Resources

- Interactivate. "Function Flyer."

  http://walch.com/rr/CAU3L7FunctionFlyer

  Users can manipulate the constants and coefficients in any function in order to explore the effects of those manipulations on the graph of the function.

- NCTM Illuminations. "Function Matching."

  http://walch.com/rr/CAU3L7FunctionMatching

  In this activity, users try to guess a function rule to match given graphs.

- Purplemath.com. "Operations on Functions."

  http://walch.com/rr/CAU3L7OperationsOnFunctions

  This site demonstrates how to add, subtract, multiply, or divide functions.

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Lesson 2.8.1: Operating on Functions

Warm-Up 2.8.1

Newspapers are printed on large sheets of paper that are folded in half to form the pages. Each side of each half is 1 page. For example, a newspaper that is made from 2 large sheets of paper has 8 pages. Use this information to solve the following problems.

1. If there are 10 sheets of paper in a particular newspaper issue, how many pages would there be?

2. What page numbers would appear in the center, or on the innermost sheet of this newspaper?

3. What is the sum of the numbers in the center?

4. What is the sum of the page numbers that would appear on ANY one side of ANY sheet of the newspaper?

5. What is the sum of all the page numbers of the newspaper?

6. Write a rule in words or symbols that would give the sum of the pages for a newspaper with n sheets of paper.

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Instruction

Lesson 2.8.1: Operating on Functions

Common Core State Standard

F–BF.1 Write a function that describes a relationship between two quantities.★

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Warm-Up 2.8.1 Debrief

1. If there are 10 sheets of paper in a particular newspaper issue, how many pages would there be?

   Since each sheet of paper creates 4 pages when folded and printed on front and back, 10 sheets of paper would create 4 • 10 or 40 pages.

2. What page numbers would appear in the center, or on the innermost sheet of this newspaper?

   Since half of 40 is 20 and the page numbering starts at 1, page numbers 20 and 21 would appear in the center.

3. What is the sum of the numbers in the center?

   $20 + 21 = 41$

4. What is the sum of the page numbers that would appear on ANY one side of ANY sheet of the newspaper?

   Since n sheets of paper result in $4n$ pages, and the first page is numbered 1 instead of 0, the sum of the page numbers of any one side of one sheet is $4n + 1$.

5. What is the sum of all the page numbers of the newspaper?

   There are 10 sheets of paper and 20 sides. The total number of pages plus 1 is 41. Multiply this number by the number of sides.

   The sum of all of the page numbers is 20(41) or 820.

6. Write a rule in words or symbols that would give the sum of the pages for a newspaper with n sheets of paper.

   $2n$ represents the number of sides of each sheet. Therefore, the sum of the pages of a newspaper with n sheets is $2n(4n + 1)$.

Connection to the Lesson

- Students will manipulate expressions, and extend their ability to write a rule or expression to represent a certain scenario to performing operations with these rules.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- simplifying expressions (5.OA.1)
- having familiarity with linear functions and their graphs (A–CED.2★)
- having familiarity with exponential functions and their graphs (A–CED.2★)

Introduction

Functions are relations in which each element in the domain is mapped onto exactly one element in the range; that is, for every value of x, there is exactly one value of y. In this lesson, we will add, subtract, multiply, and divide exponential and linear functions.

Key Concepts

- Combine linear and exponential expressions using addition: $(f + g)(x) = f(x) + g(x)$. In other words, add the two functions together by combining like terms.
- Combine linear and exponential expressions using subtraction: $(f - g)(x) = f(x) - g(x)$. In other words, subtract the second function from the first while making sure to distribute the negative across all terms of the second function.
- Combine linear and exponential expressions using multiplication: $(f \bullet g)(x) = f(x) \bullet g(x)$. In other words, multiply the two functions together.
- Combine linear and exponential expressions using division: $(f \div g)(x) = f(x) \div g(x)$, where $g(x)$ does not equal 0. In other words, divide the first function by the second function. Use a fraction bar to display the final function.

Common Errors/Misconceptions

- incorrectly using the Distributive Property when subtracting two expressions
- incorrectly using the order of operations when multiplying or dividing two expressions
- incorrectly using the Distributive Property when multiplying two expressions

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Instruction

Guided Practice 2.8.1

Example 1

If $f(x) = 3x + 2$ and $g(x) = 2x - 7$, what is $(f + g)(x)$? How do you represent this algebraically?

1. Add the two functions.

   $(f + g)(x) = f(x) + g(x)$

   Since $f(x) = 3x + 2$ and $g(x) = 2x - 7$, $(f + g)(x) = (3x + 2) + (2x - 7)$.

2. Simplify the equation.

   Clear the parentheses and reorder the terms on the right side of the equation.

$(f + g)(x) = (3x + 2) + (2x - 7)$	Equation from the previous step
$(f + g)(x) = 3x + 2 + 2x - 7$	Remove the parentheses.
$(f + g)(x) = 3x + 2x + 2 - 7$	Apply the Commutative Property of Addition.
$(f + g)(x) = 5x - 5$	Combine like terms.

   The result of adding $f(x) = 3x + 2$ and $g(x) = 2x - 7$ is $(f + g)(x) = 5x - 5$.

SWB p. 205; SRB p. U2-220

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Instruction

Example 2

If $f(x) = 3^x$ and $g(x) = 5$, what is $(f \cdot g)(x)$? How do you represent this algebraically?

1. Multiply the two functions.

   $(f \cdot g)(x) = f(x) \cdot g(x)$

   Since $f(x) = 3^x$ and $g(x) = 5$, $(f \cdot g)(x) = (3^x) \cdot (5)$.

2. Simplify the equation.

$(f \cdot g)(x) = (3^x) \cdot (5)$	Equation from the previous step
$(f \cdot g)(x) = 5(3^x)$	Apply the Commutative Property of Multiplication.

$(f \cdot g)(x) = 5(3^x)$ is the result of multiplying $f(x) = 3^x$ and $g(x) = 5$.

Example 3

If $f(x) = 2x - 3$ and $g(x) = 4x - 11$, what is $(f - g)(x)$? How do you represent this algebraically?

1. Subtract the two functions.

   $(f - g)(x) = f(x) - g(x)$

   Since $f(x) = 2x - 3$ and $g(x) = 4x - 11$, $(f - g)(x) = (2x - 3) - (4x - 11)$.

2. Simplify the equation.

   Clear the parentheses and reorder the terms on the right side of the equation. Be careful to correctly distribute the negative sign.

$(f - g)(x) = (2x - 3) - (4x - 11)$	Equation from the previous step
$(f - g)(x) = 2x - 3 - 4x + 11$	Distribute the negative.
$(f - g)(x) = 2x - 4x - 3 + 11$	Apply the Commutative Property of Addition.
$(f - g)(x) = -2x + 8$	Combine like terms.

$(f - g)(x) = -2x + 8$ is the result of subtracting $g(x) = 4x - 11$ from $f(x) = 2x - 3$.

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Instruction

Example 4

If $f(x) = 2^x$ and $g(x) = 3$, what is $(f \div g)(x)$? How do you represent this algebraically?

1. Divide the two functions.

   $(f \div g)(x) = f(x) \div g(x); g(x) \neq 0$

   Since $f(x) = 2^x$ and $g(x) = 3$, $(f \div g)(x) = (2^x) \div 3$.

2. Simplify the equation.

   $(f \div g)(x) = (2^x) \div 3$ Equation from the previous step

   $(f \div g)(x) = \frac{2^x}{3}$ Write the division as a fraction.

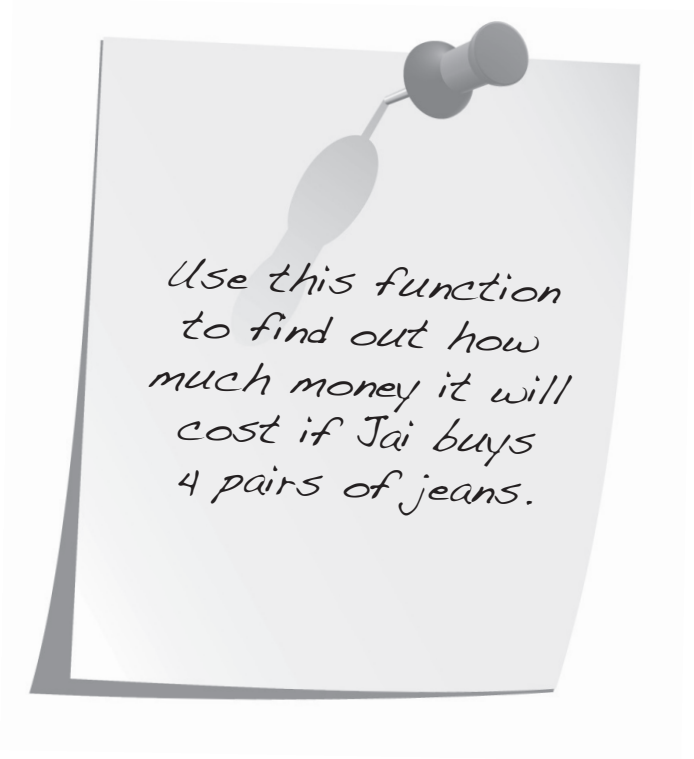
   $(f \div g)(x) = \frac{2^x}{3}$ is the result of dividing $f(x) = 2^x$ by $g(x) = 3$.

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Problem-Based Task 2.8.1: Jai's Jeans

SMP	
1 ✓	2 ✓
3	4
5	6 ✓
7	8

Jai is shopping for jeans and finds a style she likes so much that she may buy more than one pair. The jeans cost $50 each and are on sale for 30% off. Write an expression for the sale price of the jeans in such a way that the original purchase price is still evident. Then, write a function rule for the expression and simplify it. Use this function to find out how much money it will cost if Jai buys 4 pairs of jeans.



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Problem-Based Task 2.8.1: Jai's Jeans

Coaching

a. What is the original price of the jeans?

b. What is an expression to represent buying x pairs of jeans at the original price?

c. How would you calculate the 30% discount?

d. Write an expression to represent the sale price for x pairs of jeans.

e. Write an equation for the cost of x pairs of jeans using function notation.

f. Simplify the function.

g. Substitute 4 for x and evaluate.

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Instruction

Problem-Based Task 2.8.1: Jai's Jeans

Coaching Sample Responses

a. What is the original price of the jeans?

Each pair of jeans cost $50.

b. What is an expression to represent buying x pairs of jeans at the original price?

$50x$

c. How would you calculate the 30% discount?

Subtract 30% of the total cost from the total cost.

d. Write an expression to represent the sale price for x pairs of jeans.

$50x - 0.30(50x)$

e. Write an equation for the cost of x pairs of jeans using function notation.

$f(x) = 50x - 0.30(50x)$

f. Simplify the function.

$f(x) = 50x - 0.30(50x)$

$f(x) = 50x - 15x$

$f(x) = 35x$

g. Substitute 4 for x and evaluate.

$f(4) = 35(4) = 140$

Jai's total cost for 4 pairs of jeans will be $140.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.8.1: Operating on Functions

Find the value of each operation using the given functions.

For problems 1 and 2, $f(x) = 5x - 2$ and $g(x) = 3x + 5$.

1. Find $(f + g)(x)$.

2. Find $(f - g)(x)$.

For problems 3 and 4, $f(x) = 5^x$ and $g(x) = 4$.

3. Find $(f \cdot g)(x)$.

4. Find $(f \div g)(x)$.

For problems 5–8, $f(x) = -2$ and $g(x) = 7x + 4$.

5. Find $(f + g)(x)$.

6. Find $(f - g)(x)$.

7. Find $(f \cdot g)(x)$.

8. Find $(f \div g)(x)$.

Use what you know about functions to complete problems 9 and 10.

9. Jenna makes $7 per hour babysitting plus a flat fee to cover her round-trip bus fare. Round-trip bus fare is $4. Write a function to represent this scenario. Her bus fare is increased to $5 for the round trip. What is the new function to represent Jenna's take-home pay? How has the function rule changed?

10. Jorge is growing bacteria for an experiment in science class. He starts with 1 bacterium, which doubles in number every 4 hours. Write a function to represent this scenario. Jorge's classmate Jacob grows the same type of bacteria but starts with 3 bacteria. What is the function rule to represent the number of bacteria in Jacob's experiment? How are the two function rules different?

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Lesson 2.8.2: Transformations of Linear and Exponential Functions

Warm-Up 2.8.2

On a coordinate grid, Maple Street is represented by the function $f(x) = 2x - 1$, and Highland Street is represented by the function $g(x) = 2x + 3$.

1. Graph both functions on the same set of axes.

2. How are the two graphs similar?

3. How are the two graphs different?

4. How could you describe the geometric translation from $f(x)$ to $g(x)$?

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Instruction

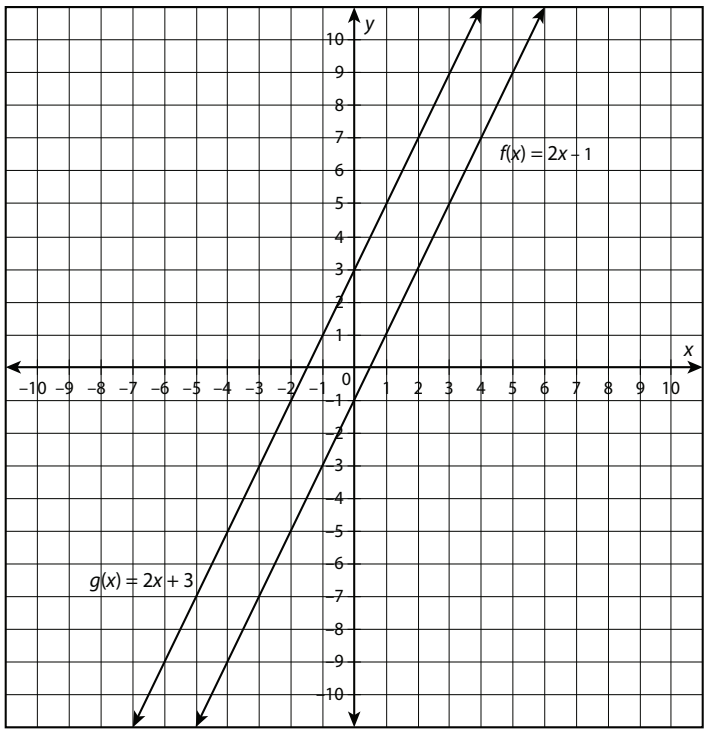
Lesson 2.8.2: Transformations of Linear and Exponential Functions

Common Core State Standard

F–BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Warm-Up 2.8.2 Debrief

1. Graph both functions on the same set of axes.



2. How are the two graphs similar?

   The lines have the same slope and are parallel.

3. How are the two graphs different?

   The lines have different y-intercepts.

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Instruction

4. How could you describe the geometric translation from $f(x)$ to $g(x)$?

   $g(x)$ is the graph of $f(x)$ translated up 4 units.

Connection to the Lesson

- Students will graph multiple functions on the same set of axes.
- Students will compare functions.
- Students will describe a function in terms of a geometric translation of another function.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- graphing linear and exponential functions (A–CED.2★)
- translating functions (G–CO.5)
- identifying *y*-intercepts of graphs of functions (8.EE.6)

Introduction

It is important to understand the relationship between the equation of a function and the graph of the function. In this lesson, we will explore how the equation and graph of a function change when a constant value is added to or subtracted from the function. When a constant value is added to or subtracted from a function, the graph undergoes a vertical shift. A **vertical shift** is a type of translation that moves the graph up or down depending on the value added to the function. A **translation** moves the graph either vertically, horizontally, or both, without changing its shape. A translation is sometimes called a slide. A translation is a specific type of transformation. A **transformation** is a change in a geometric figure's position, shape, or size. Transformations also include reflections, rotations, and dilations in addition to translations. We will also examine translations of graphs and determine how they are similar or different.

Key Concepts

- Vertical translations can be performed on linear and exponential graphs using $f(x) + k$, where k is the value of the vertical shift.
- A vertical shift moves the graph up or down k units.
- If k is positive, the graph is translated up k units.
- If k is negative, the graph is translated down k units.
- Given the graphs of two functions that are vertical translations of each other, the value of the vertical shift, k, can be found by finding the distance between the *y*-intercepts.

Common Errors/Misconceptions

- mistaking vertical shift for horizontal shift
- mistaking a *y*-intercept for the value of a vertical translation
- incorrectly graphing linear or exponential functions
- incorrectly combining like terms when changing a function rule

SRB p. U2-224

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Instruction

Guided Practice 2.8.2

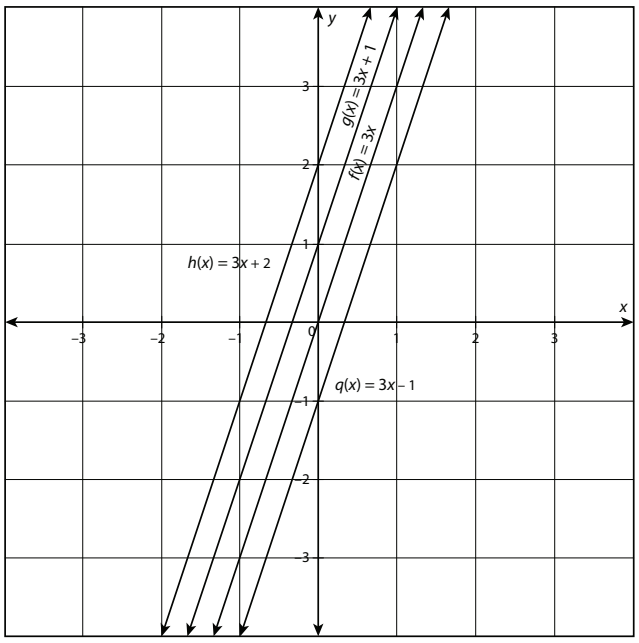
Example 1

Graph the following functions on the same set of axes:

$f(x) = 3x$ $\qquad$ $h(x) = 3x + 2$

$g(x) = 3x + 1$ $\qquad$ $q(x) = 3x - 1$

1. Graph the functions.



2. What is the y-intercept of $f(x)$? $g(x)$? $h(x)$? $q(x)$?

   The y-intercept of $f(x)$ is 0.

   The y-intercept of $g(x)$ is 1.

   The y-intercept of $h(x)$ is 2.

   The y-intercept of $q(x)$ is –1.

SWB p. 215; SRB p. U2-225

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Instruction

3. How could you describe the translation of $f(x)$ to $h(x)$?

   The graph of $f(x)$ was shifted up 2 units.

4. How could you describe the translation of $f(x)$ to $q(x)$?

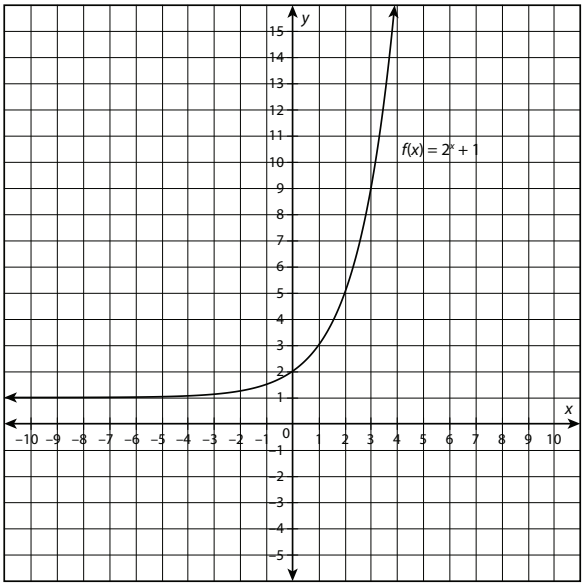
   The graph of $f(x)$ was shifted down 1 unit.

5. How could you describe the translation of $g(x)$ to $q(x)$?

   The graph of $g(x)$ was shifted down 2 units.

Example 2

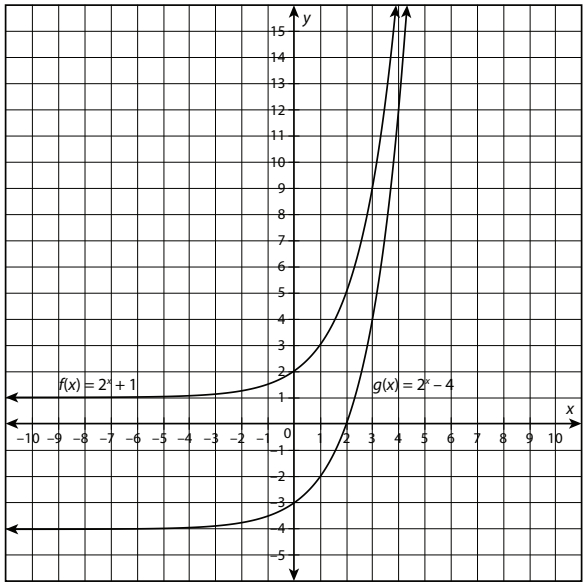
Given the graph of $f(x) = 2^x + 1$, graph $g(x) = f(x) - 5$.



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Instruction

1. Graph $g(x)$.



2. How are $f(x)$ and $g(x)$ related?

   $g(x)$ is a vertical shift of $f(x)$ down 5 units.

3. What are the steps you need to follow to graph $g(x)$?

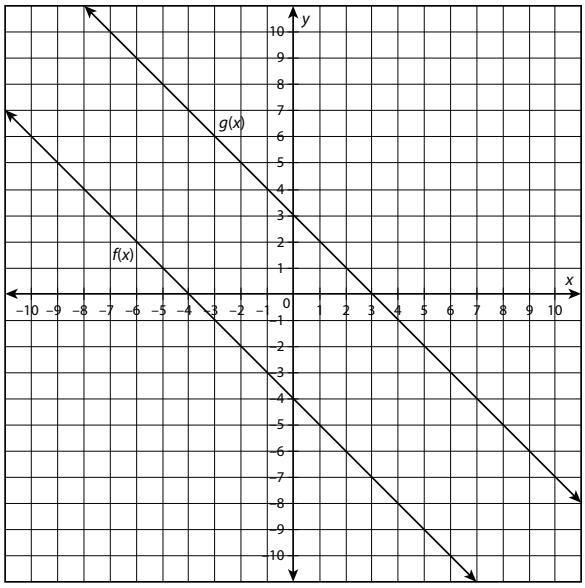
   For each point on $f(x)$, plot a point 5 units lower on the graph and connect the points.

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Example 3

The graphs of two functions $f(x)$ and $g(x)$ are shown. Write a rule for $g(x)$ in terms of $f(x)$.



1. Write a function rule for the graph of $f(x)$.

   $f(x) = -x - 4$

2. Write a function rule for the graph of $g(x)$.

   $g(x) = -x + 3$

3. How are $f(x)$ and $g(x)$ related?

   $g(x)$ is a vertical shift up 7 units from $f(x)$, since the vertical distance is the distance between the y-intercepts, –4 and 3, which is equal to 7. You could also count the units on the graph.

4. Write a function rule for $g(x)$ in terms of $f(x)$.

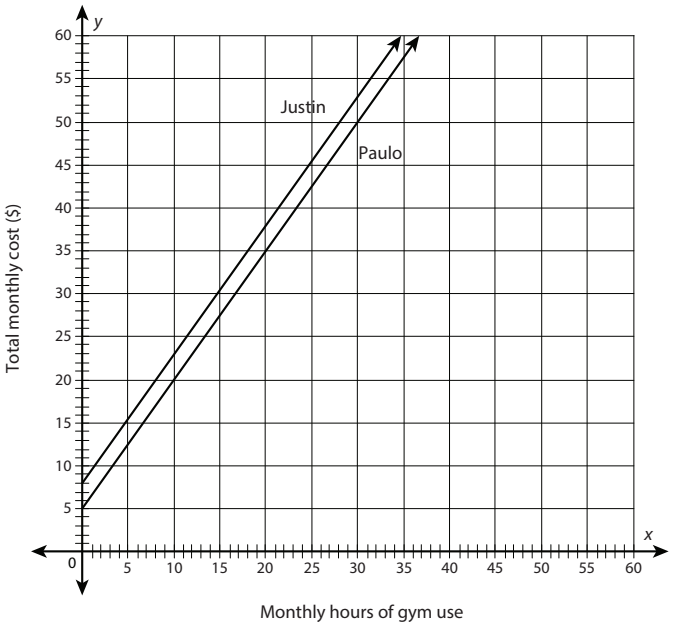
   $g(x) = f(x) + 7$

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Problem-Based Task 2.8.2: Gym Fees

SMP	
1	2
3 ✓	4 ✓
5	6
7 ✓	8

Paulo and Justin belong to the same gym. The following graph shows how much each man pays per month in gym fees. Both pay the same per-hour use fee, but Paulo gets an employee discount, so his monthly membership fee is different from Justin's membership fee. What is a function rule that represents Paulo's total monthly gym fees? What is a function rule that represents Justin's total monthly gym fees? What is the difference in their fees?



What is the difference in their fees?

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Problem-Based Task 2.8.2: Gym Fees

Coaching

a. What is the slope of Paulo's graph? What does the slope represent?

b. What is the *y*-intercept of Paulo's graph?

c. What is a function rule representing Paulo's monthly gym fees?

d. What is the slope of Justin's graph? What does the slope represent?

e. What is the *y*-intercept of Justin's graph?

f. What is a function rule representing Justin's monthly gym fees?

g. How are the two functions the same? How are they different?

h. For any given number of hours of use, what is the difference in the fees?

i. Using this difference, how can you describe the graphs of the two functions in terms of vertical shift?

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Instruction

Problem-Based Task 2.8.2: Gym Fees

Coaching Sample Responses

a. What is the slope of Paulo's graph? What does the slope represent?

The slope is 1.5, which represents the per-hour use fee of $1.50.

b. What is the *y*-intercept of Paulo's graph?

The *y*-intercept is 5.

c. What is a function rule representing Paulo's monthly gym fees?

$f(x) = 1.5x + 5$

d. What is the slope of Justin's graph? What does the slope represent?

The slope is 1.5, which represents the per-hour use fee of $1.50.

e. What is the *y*-intercept of Justin's graph?

The *y*-intercept is 8.

f. What is a function rule representing Justin's monthly gym fees?

$f(x) = 1.5x + 8$

g. How are the two functions the same? How are they different?

The functions have the same slope, but different *y*-intercepts.

h. For any given number of hours of use, what is the difference in the fees?

The difference in fees is the difference between the *y*-intercepts: $8 - 5 = 3$. The difference in fees is $3.

i. Using this difference, how can you describe the graphs of the two functions in terms of vertical shift?

This difference means that the graph of the function that represents Justin's gym fees is vertically shifted up 3 units from the graph of the function that represents Paulo's gym fees.

Recommended Closure Activity

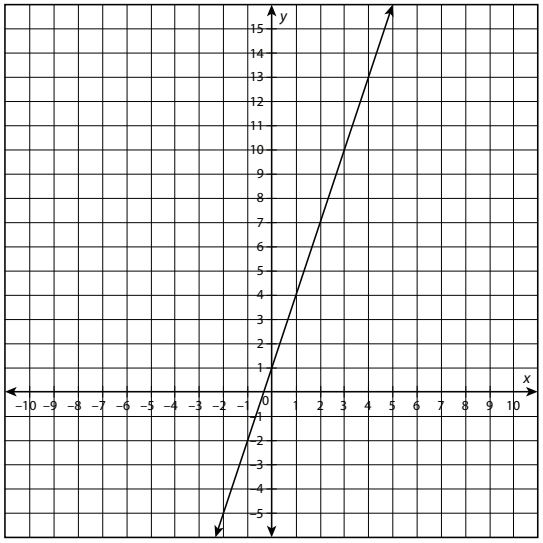
Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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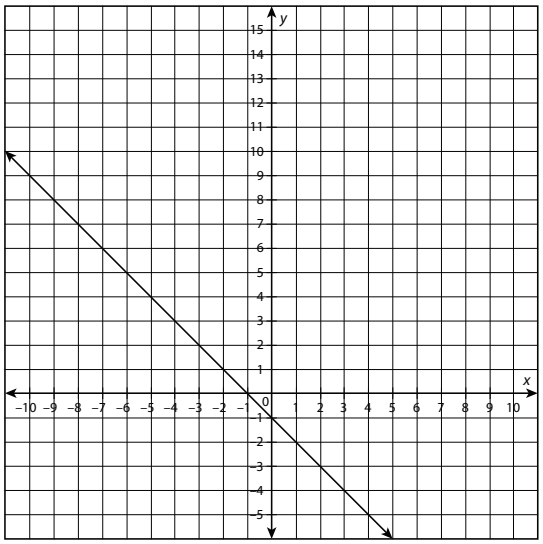
Practice 2.8.2: Transformations of Linear and Exponential Functions

For problems 1–4, graph the function $g(x) = f(x) + k$, using the given graph of $f(x)$.

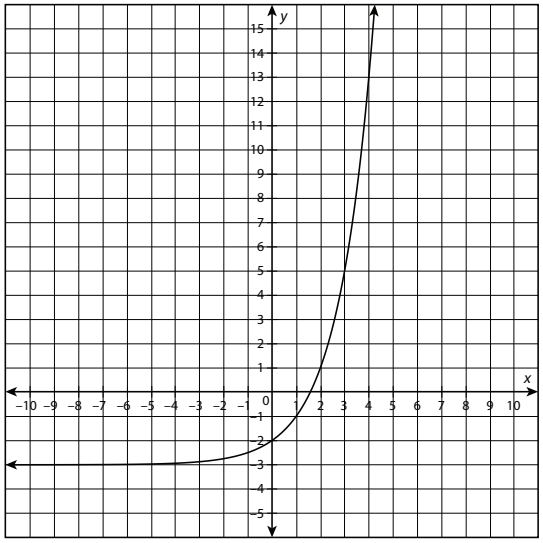
1. $g(x) = f(x) + 2$



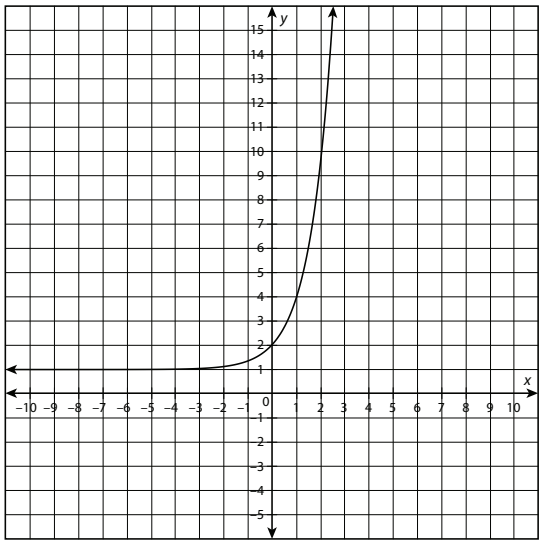
2. $g(x) = f(x) - 3$



3. $g(x) = f(x) + 4$



4. $g(x) = f(x) - 3$

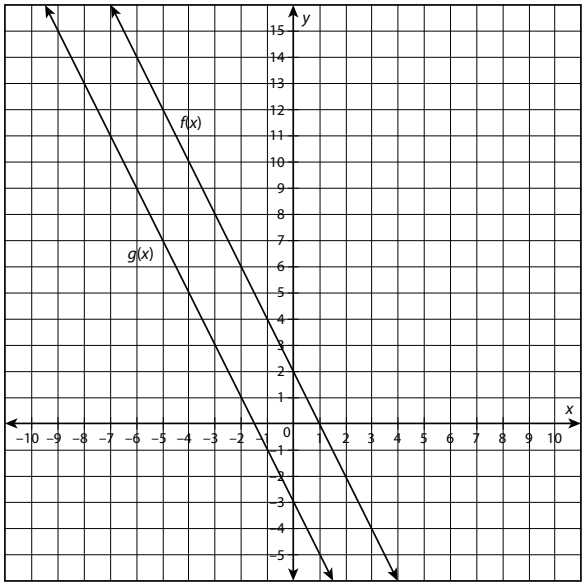


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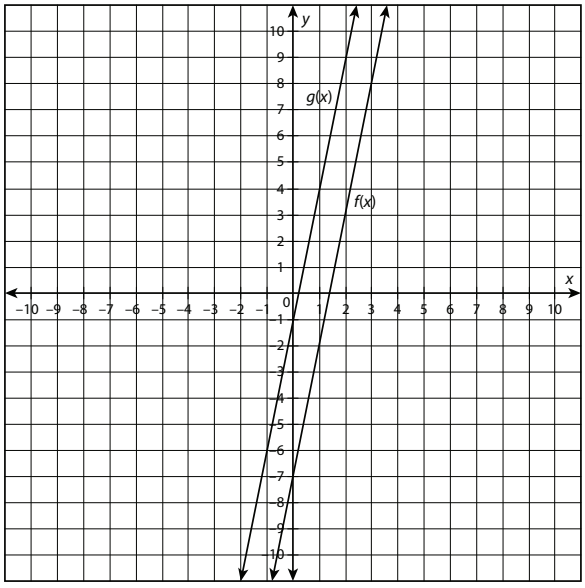
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Use the given information to complete problems 5–10.

5. Given the graphs of $f(x)$ and $g(x)$, write a function rule for $g(x)$ in terms of $f(x)$.



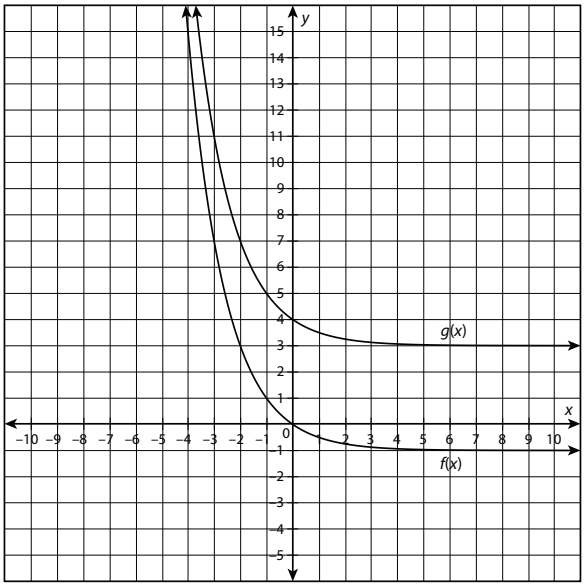
6. Given the graphs of $f(x)$ and $g(x)$, write a function rule for $g(x)$ in terms of $f(x)$.



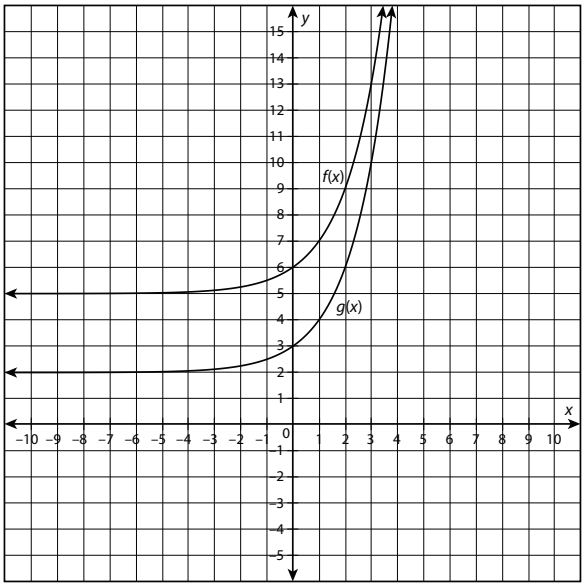
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7. Given the graphs of $f(x)$ and $g(x)$, write a function rule for $g(x)$ in terms of $f(x)$.



8. Given the graphs of $f(x)$ and $g(x)$, write a function rule for $g(x)$ in terms of $f(x)$.



9. $f(x) = 2x + 1$ and $g(x) = 2x - 2$. If $g(x)$ can be written as $f(x) + k$, what is the value of k?

10. $f(x) = 2^x - 1$ and $g(x) = 2^x + 3$. If $g(x)$ can be written as $f(x) + k$, what is the value of k?

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Progress Assessment

Circle the letter of the best answer.

1. If $f(x) = -4x + 1$ and $g(x) = 2x - 3$, what is $(f + g)(x)$?

   a. $2x + 2$
   b. $-2x - 2$
   c. $-8x - 3$
   d. $-6x - 2$

2. If $f(x) = 6x - 7$ and $g(x) = -x - 3$, what is $(f - g)(x)$?

   a. $5x + 4$
   b. $-6x + 21$
   c. $7x - 4$
   d. $-7x + 4$

3. If $f(x) = 5^x$ and $g(x) = 4$, what is $(f \bullet g)(x)$?

   a. $4(5^x)$
   b. 20^x
   c. $4x$
   d. $5^x + 4$

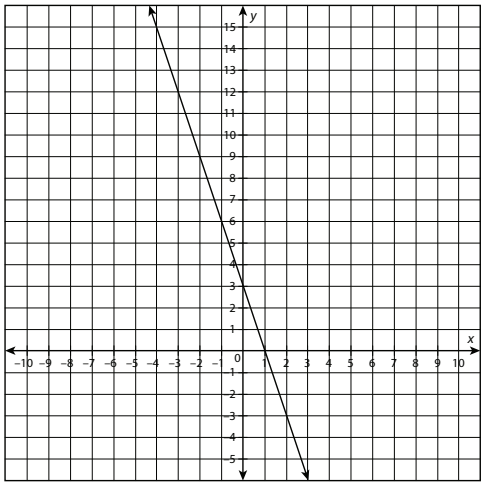
4. If $f(x) = 2^x$ and $g(x) = 3$, what is $(f \div g)(x)$?

   a. $3(2^x)$
   b. $\left(\frac{2}{3}\right)^x$
   c. $\frac{2^x}{3}$
   d. $2^x - 3$

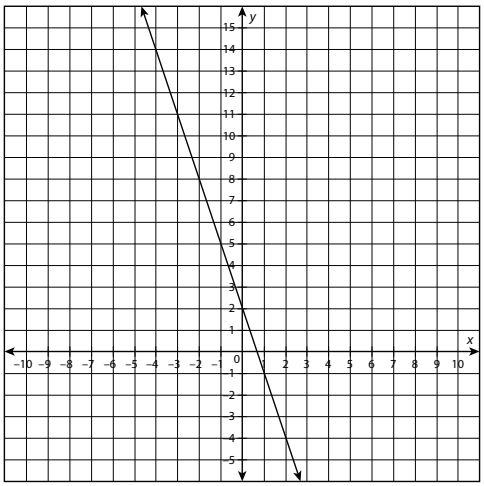
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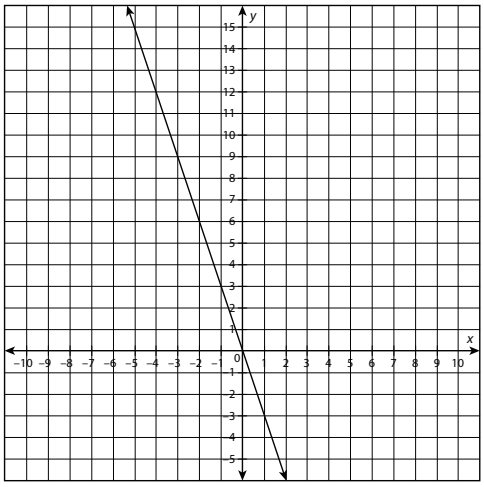
5. Given the following graph of $f(x) = -3x + 3$, which is the graph of $f(x) - 1$?



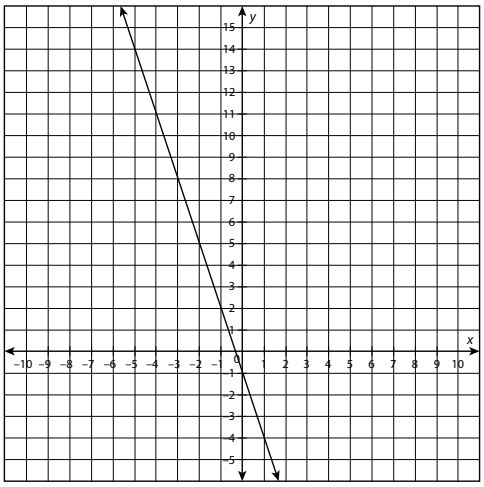
a.



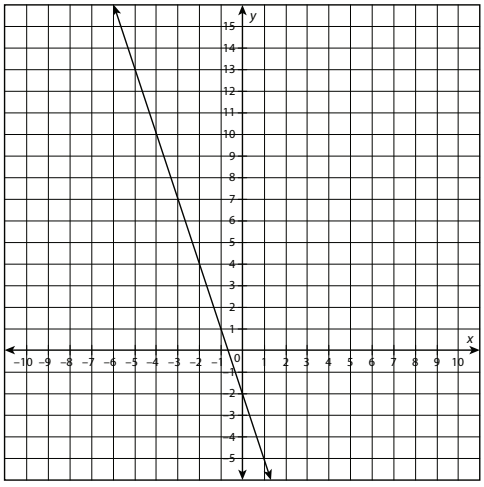
c.



b.



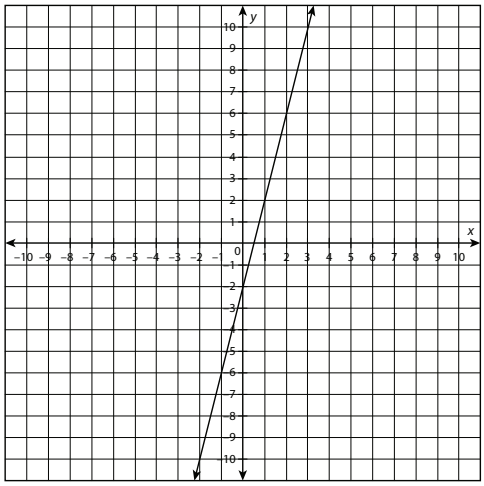
d.



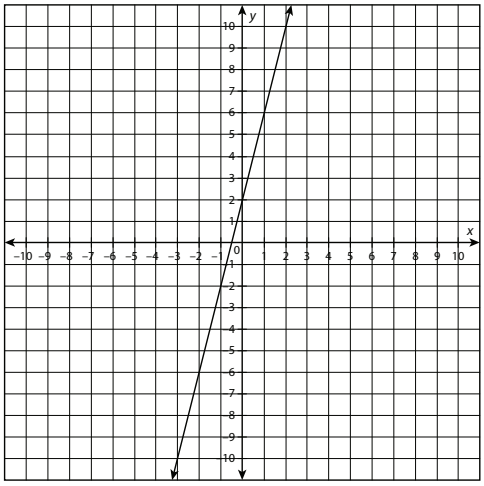
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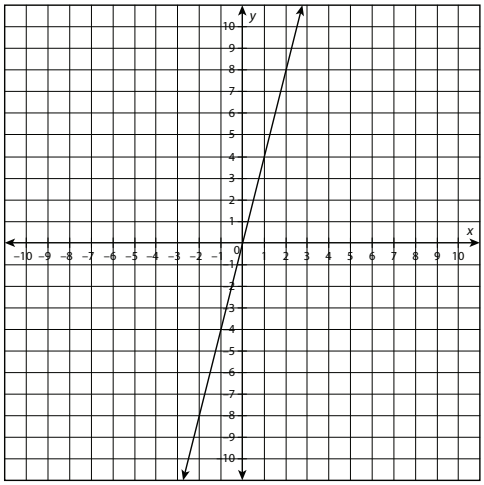
6. Given the following graph of $f(x) = 4x - 2$, which is the graph of $f(x) + 2$?



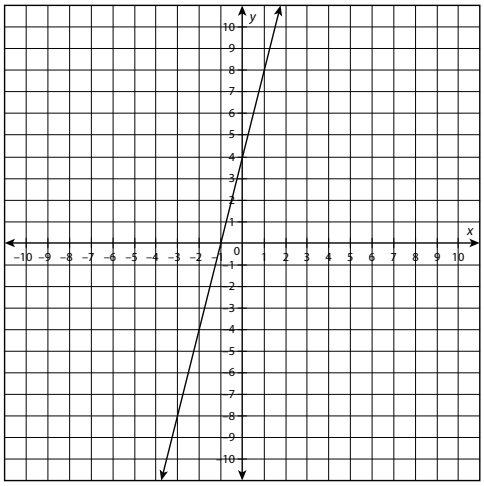
a.



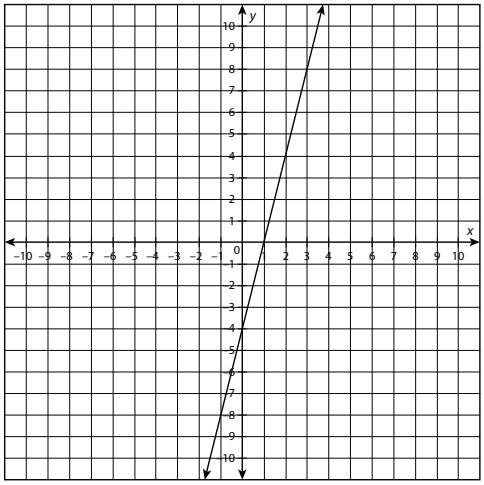
c.



b.



d.



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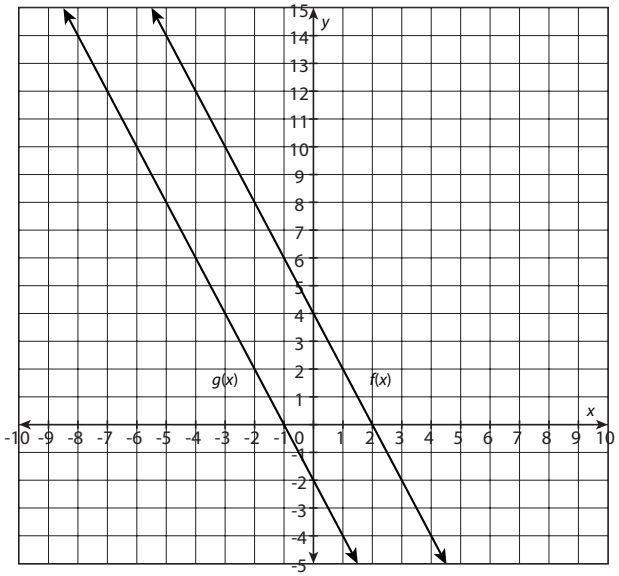
7. $f(x) = -2x - 7$ and $g(x) = -2x + 2$. If $g(x)$ can be written as $f(x) + k$, which is the value of k?

   a. 5
   c. –5
   b. 9
   d. –9

8. $f(x)=\left(\frac{1}{3}\right)^x+4$ and $g(x)=\left(\frac{1}{3}\right)^x-1$. If $g(x)$ can be written as $f(x) + k$, which is the value of k?

   a. –3
   c. 3
   b. –5
   d. 5

9. Given the following graphs of $f(x)$ and $g(x)$, which is the function rule for $g(x)$ in terms of $f(x)$?

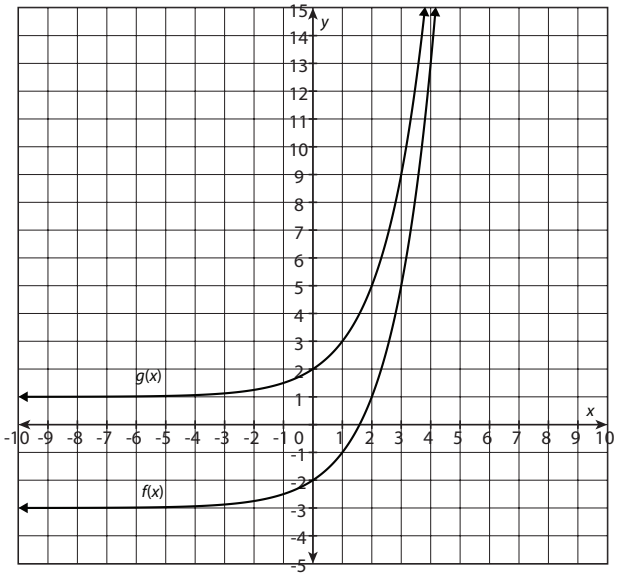


   a. $g(x) = f(x) + 6$
   c. $g(x) = f(x) - 2$
   b. $g(x) = f(x) + 4$
   d. $g(x) = f(x) - 6$

continued

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10. Given the following graphs of $f(x)$ and $g(x)$, which is the function rule for $g(x)$ in terms of $f(x)$?



a. $g(x) = f(x) - 3$

b. $g(x) = f(x) + 3$

c. $g(x) = f(x) + 1$

d. $g(x) = f(x) + 4$

Read the scenario and write the two functions.

11. Aidan and Raj both belong to a video game rental club. Both of them pay $2.75 per game. Aidan pays a $5.00 monthly access fee, while Raj joined the club during a promotion and pays only $3.75 per month for the access fee. Write a function, $f(x)$, to represent Aidan's total monthly cost for the game club. Then write a function $g(x)$ in the form $g(x) = f(x) + k$ to represent Raj's total monthly cost for the game club.

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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. The formula for an arithmetic sequence is $a_n = 5n + 2$. What is the sixth term in the sequence?

   a. 2
   c. 32
   b. 6
   d. 28

2. The formula for a geometric sequence is $a_n = 64\left(\frac{1}{8}\right)^{n-1}$. What is the eighth term in the sequence?

   a. 64
   c. 8
   b. $\frac{1}{512}$
   d. $\frac{1}{32{,}768}$

3. What is the next term in the sequence?

   3, –2, –7, –12, ...

   a. 17
   c. –18
   b. –60
   d. –17

4. What is the next term in the sequence?

   –1, 5, –25, 125, ...

   a. –625
   c. 5
   b. –5
   d. 625

5. The fourth term in an arithmetic sequence is 7. If the sequence has a common difference of 3, what is the ninth term?

   a. –2
   c. 22
   b. 3
   d. 25

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Lesson 9: Arithmetic and Geometric Sequences

Instruction

Common Core State Standard

SMP	
1 ✓	2
3	4
5	6 ✓
7 ✓	8 ✓

F–BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Essential Questions

1. How are arithmetic sequences and linear functions connected in theory?
2. What are the different ways that an arithmetic sequence can be defined?
3. How are geometric sequences and exponential functions connected in theory?
4. What are the different ways that a geometric sequence can be defined?
5. How can arithmetic and geometric sequences be used to model real-world problems?

WORDS TO KNOW

arithmetic sequence a linear function with a domain of positive consecutive integers in which the difference between any two consecutive terms is equal

common difference the number added to each consecutive term in an arithmetic sequence

common ratio the number that each consecutive term is multiplied by in a geometric sequence

explicit formula a formula used to find the *n*th term of a sequence; if *n* is the term number, then the explicit formula for the *n*th term of an arithmetic sequence is $a_n = a_1 + (n - 1)d$ and the explicit formula for the *n*th term of a geometric sequence is $a_n = a_1 \bullet r^{n-1}$

geometric sequence an exponential function that results in a sequence of numbers separated by a common ratio

recursive formula a formula used to find the next term of a sequence when the previous term is known; the recursive formula for the *n*th term of an arithmetic sequence is $a_n = a_{n-1} + d$ and the recursive formula for the *n*th term of a geometric sequence is $a_n = a_{n-1} \bullet r$

SRB p. U2-233

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Instruction

Recommended Resources

- MathIsFun.com. "Geometric Sequences and Sums."

  http://www.walch.com/rr/05001

  This site explains geometric sequences, the common ratio, and the formula for summing a geometric series.

- MSTE.Illinois.edu. "Teaching Arithmetic Sequences and Series."

  http://walch.com/rr/CAU3L8TeachingSequences

  This website for teachers has links to a detailed lesson plan including the use of manipulatives to derive the formula for arithmetic sequences.

- Oswego City School District Regents Exam Prep Center. "Practice with Arithmetic and Geometric Sequences and Series."

  http://walch.com/rr/CAU3L8SequencesPractice

  This interactive page provides problems that users answer; users can check their answers against the correct answer.

- Purplemath.com. "Arithmetic and Geometric Sequences."

  http://walch.com/rr/CAU3L8SequencesTutorial

  This tutorial introduces both arithmetic and geometric sequences.

- YouTube. "Arithmetic Sequences: A Formula for the '*n*-th' Term."

  http://walch.com/rr/CAU3L8SequencesVideo

  This video gives an excellent breakdown of how arithmetic sequences work.

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Lesson 2.9.1: Arithmetic Sequences

Warm-Up 2.9.1

Just as Julian finished his homework, he went to feed his fish and accidentally dropped his homework in the fish tank. He was working on patterns of numbers, and the last number in four of the patterns was washed away by the water in the fish tank. Julian needs help recreating his patterns. For each problem, describe the pattern and determine the next number in the list.

1. 1, 3, 5, 7, ...

2. 13, 18, 23, 28, ...

3. 7, 4, 1, –2, –5, ...

4. –22, –15, –8, –1, ...

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Instruction

Lesson 2.9.1: Arithmetic Sequences

Common Core State Standard

F–BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Warm-Up 2.9.1 Debrief

1. 1, 3, 5, 7, ...

   In this list of numbers, each number is 2 more than the previous number. Add 2 to 7 to get the next number in the list. The next number in the list is 9.

2. 13, 18, 23, 28, ...

   In this list of numbers, each number is 5 more than the previous number. Add 5 to 28 to get the next number in the list. The next number in the list is 33.

3. 7, 4, 1, –2, –5, ...

   In this list of numbers, each number is 3 less than the previous number. Subtract 3 from –5 to get the next number in the list. The next number in the list is –8.

4. –22, –15, –8, –1, ...

   In this list of numbers, each number is 7 more than the previous number. Add 7 to –1 to get the next number in the list. The next number in the list is 6.

Connection to the Lesson

- Students will be recognizing patterns to determine if a list of numbers is an arithmetic sequence.
- Students will be identifying common differences in order to write an explicit or recursive formula to represent an arithmetic sequence.
- The warm-up gives the students practice with identifying patterns and determining a common difference, which they will need to do during the lesson.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- adding and subtracting signed numbers (7.NS.1b, 7.NS.1c)
- recognizing patterns (3.OA.9)
- identifying linear relationships (8.F.4)

Introduction

Arithmetic sequences are linear functions whose domain is the set of positive consecutive integers and in which the difference between any two consecutive terms is equal. Arithmetic sequences can be represented by formulas, either explicit or recursive, and those formulas can be used to find a specific term of the sequence or the number of a specific term in the sequence. An explicit formula is a formula used to find the *n*th term of a sequence, and a recursive formula is a formula used to find the next term of a sequence when the previous term is known.

Key Concepts

- An arithmetic sequence is a list of terms separated by a **common difference**, the number added to each consecutive term in the sequence.
- An arithmetic sequence is a linear function whose domain is the set of positive consecutive integers and whose range is made up of the terms in the sequence.
- The formula for an arithmetic sequence can be expressed either explicitly or recursively.
- The explicit formula for the *n*th term of an arithmetic sequence is $a_n = a_1 + (n - 1)d$, where a_1 is the first term in the sequence, n is the term number and is a positive integer, d is the common difference, and a_n is the *n*th term in the sequence.
- The recursive formula for the *n*th term of an arithmetic sequence is $a_n = a_{n-1} + d$, where n is a positive integer, a_n is the *n*th term in the sequence, a_{n-1} is the previous term, and d is the common difference.

Common Errors/Misconceptions

- identifying a non-arithmetic sequence as arithmetic
- defining the common difference, d, in a decreasing sequence as a positive number
- incorrectly using the distributive property when finding the *n*th term with the explicit formula
- forgetting to identify the first term when defining an arithmetic sequence recursively

SRB p. U2-235

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Instruction

Guided Practice 2.9.1

Example 1

Find the common difference, write the explicit formula, and find the tenth term for the following arithmetic sequence.

3, 9, 15, 21, ...

1. Find the common difference by subtracting two successive terms.

   $9 - 3 = 6$

2. Confirm that the difference is the same between each remaining pair of consecutive terms.

   $15 - 9 = 6$ and $21 - 15 = 6$

3. Identify the first term, a_1.

   $a_1 = 3$

4. Write the explicit formula.

   $a_n = a_1 + (n - 1)d$ Explicit formula for an arithmetic sequence

   $a_n = (3) + (n - 1)(6)$ Substitute 3 for a_1 and 6 for d.

5. Simplify the explicit formula.

   $a_n = 3 + 6n - 6$ Distribute 6 over $(n - 1)$.

   $a_n = 6n - 3$ Simplify.

6. To find the tenth term, substitute 10 for n in the explicit formula.

   $a_n = 6n - 3$ Explicit formula from the previous step

   $a_{(10)} = 6(10) - 3$ Substitute 10 for n.

   $a_{10} = 60 - 3$ Multiply.

   $a_{10} = 57$ Subtract.

   The tenth term in the sequence is 57.



SWB p. 227; SRB p. U2-236

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Instruction

Example 2

Write a linear function that corresponds to the following arithmetic sequence.

8, 1, –6, –13, ...

1. Find the common difference by subtracting two successive terms.

   $1 - 8 = -7$

2. Confirm that the difference is the same between each remaining pair of consecutive terms.

   $-6 - 1 = -7$ and $-13 - (-6) = -7$

3. Identify the first term, a_1.

   $a_1 = 8$

4. Write the explicit formula.

   $a_n = a_1 + (n - 1)d$ Explicit formula for an arithmetic sequence

   $a_n = (8) + (n - 1)(-7)$ Substitute 8 for a_1 and –7 for d.

5. Simplify the explicit formula.

   $a_n = 8 - 7n + 7$ Distribute –7 over $(n - 1)$.

   $a_n = -7n + 15$ Simplify.

6. Write the explicit formula in function notation.

   $f(x) = -7x + 15$

   Note that the domain of an arithmetic sequence is positive consecutive integers.

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Instruction

Example 3

An arithmetic sequence is defined recursively by the formula $a_n = a_{n-1} + 5$, with $a_1 = 29$. Find the first 5 terms of the sequence, write an explicit formula to represent the sequence, and find the 15th term.

1. Use the recursive formula, beginning with a_1, to calculate the next 4 terms.

   We are given that the first term, a_1, is 29. Substitute 2, 3, 4, and 5, respectively, for n in the recursive formula $a_n = a_{n-1} + 5$ to find the next 4 terms.

   $a_1 = 29$

   $a_2 = 29 + 5 = 34$

   $a_3 = 34 + 5 = 39$

   $a_4 = 39 + 5 = 44$

   $a_5 = 44 + 5 = 49$

   The first 5 terms of the sequence are 29, 34, 39, 44, and 49.

2. Write the explicit formula for this sequence.

   The first term is $a_1 = 29$ and the common difference is $d = 5$, so the explicit formula is $a_n = 29 + (n - 1)5$.

3. Simplify the explicit formula.

   $a_n = 29 + 5n - 5$ Distribute 5 over $(n - 1)$.

   $a_n = 5n + 24$ Simplify.

4. Find the requested term in the sequence.

   Substitute 15 for n in the explicit formula to find the 15th term.

   $a_n = 5n + 24$ Explicit formula from the previous step

   $a_{(15)} = 5(15) + 24$ Substitute 15 for n.

   $a_{15} = 75 + 24$ Multiply.

   $a_{15} = 99$ Add.

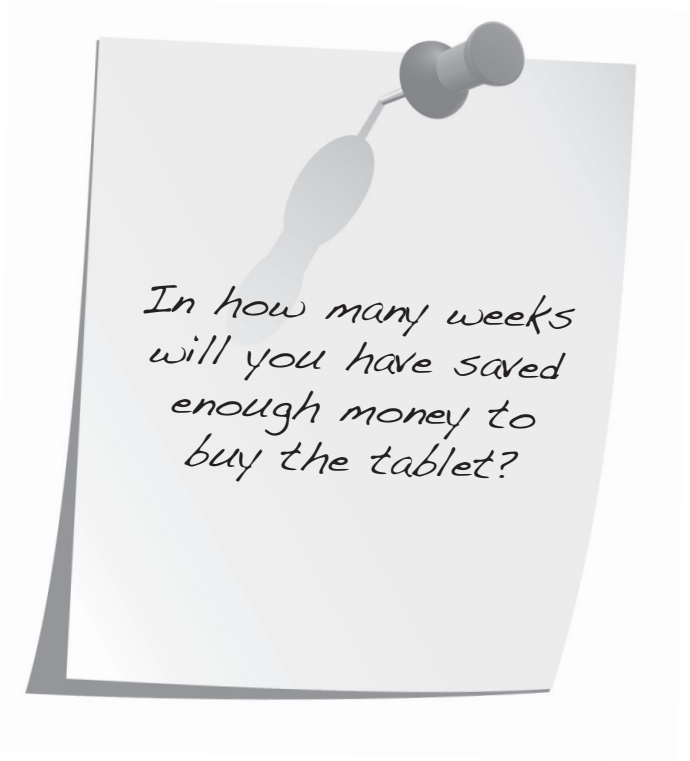
   The 15th term in the sequence is 99.

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Problem-Based Task 2.9.1: New Tablet

SMP	
1 ✓	2
3	4
5	6 ✓
7	8 ✓

You are saving money so you can buy a new tablet computer. The tablet costs $450. Your grandparents gave you $50 for your birthday. Before that $50 gift, you didn't have any money saved. You have a babysitting job and plan to save $10 each week. Write an arithmetic sequence to represent the amount of money you will have at the end of each week. In how many weeks will you have saved enough money to buy the tablet?



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Problem-Based Task 2.9.1: New Tablet

Coaching

a. How much money do you have to start with?

b. How much money do you plan to save each week?

c. How much money will you have after 1 week? 2 weeks? 3 weeks?

d. Does the amount of money you will have at the end of each week form an arithmetic sequence? How do you know?

e. Write the explicit formula for this situation.

f. How much money do you need to save to buy the tablet?

g. Use the formula to find how many weeks it will take to save enough money to buy the tablet.

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Instruction

Problem-Based Task 2.9.1: New Tablet

Coaching Sample Responses

a. How much money do you have to start with?

You have $50 that your grandparents gave you for your birthday.

b. How much money do you plan to save each week?

You plan to save $10 per week.

c. How much money will you have after 1 week? 2 weeks? 3 weeks?

After 1 week you will have $60, after 2 weeks you will have $70, and after 3 weeks you will have $80.

d. Does the amount of money you will have at the end of each week form an arithmetic sequence? How do you know?

The numbers form the sequence 60, 70, 80, ... The common difference is 10, so the numbers do form an arithmetic sequence.

e. Write the explicit formula for this situation.

Let $a_1 = 60$ and $d = 10$.

$a_n = a_1 + (n - 1)d$

$a_n = (60) + (n - 1)(10)$

$a_n = 60 + 10n - 10$

$a_n = 10n + 50$

f. How much money do you need to save to buy the tablet?

You need $450 to buy the tablet.

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g. Use the formula to find how many weeks it will take to save enough money to buy the tablet.

The ending amount is $450, and you have to find the term number, which will tell the number of weeks. Substitute 450 for a_n in the equation and solve for n.

$a_n = 10n + 50$

$(450) = 10n + 50$

$400 = 10n$

$n = 40$

After 40 weeks of saving, you will have enough money to buy the tablet.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.9.1: Arithmetic Sequences

For problems 1–4, find the common difference and write the explicit formula for the *n*th term of each arithmetic sequence.

1. 27, 31, 35, 39, ...

2. 4, –3, –10, –17, ...

3. –101, –87, –73, –59, ...

4. $\frac{1}{2}, \frac{5}{2}, \frac{9}{2}, \frac{13}{2}, \ldots$

Use the given information to complete problems 5–10.

5. Find the first five terms of the arithmetic sequence defined as follows:

   $a_n = a_{n-1} + 2.7; a_1 = 3.2$

6. Find the first five terms of the arithmetic sequence defined as follows:

   $a_n = a_{n-1} - 22; a_1 = 18$

7. You have read 25 pages of a book. You plan to read an additional 10 pages each night. Write the explicit formula to represent the number of pages you will read after *n* nights.

8. You are going on vacation. You have $105 to take with you. You expect to spend $15 each day. You want to have $30 remaining at the end of the vacation. Write an explicit formula to represent this scenario. For how many days can you spend $15 each day?

9. A bicyclist is training for a race. On the first day of training, she rides 12 miles. She increases the distance she rides by 3 miles each day. Write an explicit formula to represent this scenario. How many miles will the bicyclist ride on her ninth day of training?

10. Sofie needs to complete community service hours for her service club. She needs to complete 150 hours to earn a merit badge. Sofie has already completed 65 hours. Write an explicit formula to represent this scenario. If she volunteers 5 hours each week, in how many weeks will she have completed the hours to earn the merit badge?

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Lesson 2.9.2: Geometric Sequences

Warm-Up 2.9.2

Sondra left her homework in the back pocket of her jeans, and her dad put her jeans through the washing machine. Afterward, the last number of each sequence on her paper was illegible. For each problem, describe the pattern and determine the next number in the sequence.

1. 1, 3, 9, 27, ...

2. 256, 64, 16, 4, ...

3. $4, \frac{8}{3}, \frac{16}{9}, \frac{32}{27}, \ldots$

4. $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \ldots$

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Instruction

Lesson 2.9.2: Geometric Sequences

Common Core State Standard

F–BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Warm-Up 2.9.2 Debrief

1. 1, 3, 9, 27, ...

   In this list of numbers, each number is 3 times the previous number. Multiply 27 by 3 to get the next number in the list. The next number in the list is 81.

2. 256, 64, 16, 4, ...

   In this list of numbers, each number is $\frac{1}{4}$ of the previous number. Multiply 4 by $\frac{1}{4}$ to get the next number in the list. The next number in the list is 1.

3. $4, \frac{8}{3}, \frac{16}{9}, \frac{32}{27}, \ldots$

   In this list of numbers, each number is $\frac{2}{3}$ of the previous number. Multiply $\frac{32}{27}$ by $\frac{2}{3}$ to get the next number in the list. The next number in the list is $\frac{64}{81}$.

4. $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \ldots$

   In this list of numbers, each number is $-\frac{1}{2}$ of the previous number. Multiply $-\frac{1}{16}$ by $-\frac{1}{2}$ to get the next number in the list. The next number in the list is $\frac{1}{32}$.

Connection to the Lesson

- Students will be recognizing patterns to determine if a list of numbers is a geometric sequence.
- Students will be identifying common ratios in order to write an explicit or recursive formula to represent a geometric sequence.
- The warm-up gives the students practice with identifying patterns and determining the common ratio, which they will need to do during the lesson.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- multiplying signed numbers (7.NS.2a)
- multiplying fractions (5.NF.4a)
- applying the order of operations (5.OA.1)
- applying exponents (8.EE.1)

Introduction

Geometric sequences are exponential functions that have a domain of consecutive positive integers and in which successive terms have a common ratio. Geometric sequences can be represented by formulas, either explicit or recursive, and those formulas can be used to find a specific term of the sequence or the term number of a specific value in the sequence.

Key Concepts

- A geometric sequence is a list of terms separated by a **common ratio**, the number that each consecutive term is multiplied by in a geometric sequence.
- A geometric sequence is an exponential function whose domain is the set of positive consecutive integers and whose range is made up of the terms in the sequence.
- The formula for a geometric sequence can be expressed either explicitly or recursively.
- The explicit formula for the *n*th term of a geometric sequence is $a_n = a_1 \bullet r^{n-1}$, where a_1 is the first term in the sequence, n is the term number and is a positive integer, r is the common ratio, and a_n is the *n*th term in the sequence.
- The recursive formula for the *n*th term of a geometric sequence is $a_n = a_{n-1} \bullet r$, where a_n is the *n*th term in the sequence, a_{n-1} is the previous term, and r is the common ratio.

Common Errors/Misconceptions

- identifying a non-geometric sequence as geometric
- defining the common ratio, r, in a decreasing sequence as a number greater than 1
- incorrectly using the order of operations when finding the *n*th term in a geometric sequence
- forgetting to identify the first term when defining a geometric sequence recursively

SRB p. U2-240

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Guided Practice 2.9.2

Example 1

Find the common ratio, write the explicit formula, and find the seventh term for the following geometric sequence.

3, 1.5, 0.75, 0.375, ...

1. Find the common ratio by dividing two successive terms.

   $1.5 \div 3 = 0.5$

2. Confirm that the ratio is the same between each remaining pair of consecutive terms.

   $0.75 \div 1.5 = 0.5$ and $0.375 \div 0.75 = 0.5$

   $r = 0.5$

3. Identify the first term, a_1.

   $a_1 = 3$

4. Write the explicit formula.

   $a_n = a_1 \bullet r^{n-1}$ Explicit formula for a geometric sequence

   $a_n = (3)(0.5)^{n-1}$ Substitute 3 for a_1 and 0.5 for r.

5. To find the seventh term, substitute 7 for n.

   $a_n = (3)(0.5)^{n-1}$ Explicit formula from the previous step

   $a_{(7)} = (3)(0.5)^{(7)-1}$ Substitute 7 for n.

   $a_7 = (3)(0.5)^6$ Simplify.

   $a_7 = (3)(0.015625)$ Raise 0.5 to the 6th power.

   $a_7 = 0.046875$ Multiply.

   The seventh term in the sequence is 0.046875.



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Instruction

Example 2

The fifth term of a geometric sequence is 1,792. The common ratio is 4. Write an explicit formula for the sequence, and then write the corresponding exponential function.

1. The fifth term is 1,792; therefore $n = 5$ and $a_5 = 1792$.

2. The common ratio is 4; therefore, $r = 4$.

3. Substitute the known values into the explicit formula for a geometric sequence and solve for a_1.

$a_n = a_1 \bullet r^{n-1}$	Explicit formula for a geometric sequence
$(1792) = a_1(4)^{(5)-1}$	Substitute 1,792 for a_n, 4 for r, and 5 for n.
$1792 = a_1(256)$	Simplify.
$a_1 = 7$	

4. Write the explicit formula.

$a_n = a_1 \bullet r^{n-1}$	Explicit formula for a geometric sequence
$a_n = (7)(4)^{n-1}$	Substitute 7 for a_1 and 4 for r.

5. Write the explicit formula in function notation.

$f(x) = 7(4)^{x-1}$

Note that the domain of a geometric sequence is consecutive positive integers.



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Instruction

Example 3

A geometric sequence is defined recursively by $a_n = (a_{n-1})\left(-\frac{1}{3}\right)$, with $a_1 = 729$. Find the first 5 terms of the sequence, write an explicit formula to represent the sequence, then find the eighth term.

1. Use the recursive formula, beginning with a_1, to calculate the next 4 terms.

   We are given that the first term, a_1, is 729. Substitute 2, 3, 4, and 5, respectively, for n in the recursive formula $a_n = (a_{n-1})\left(-\frac{1}{3}\right)$ to find the next 4 terms.

   $a_1 = 729$

   $a_2 = (729)\left(-\frac{1}{3}\right) = -243$

   $a_3 = (-243)\left(-\frac{1}{3}\right) = 81$

   $a_4 = (81)\left(-\frac{1}{3}\right) = -27$

   $a_5 = (-27)\left(-\frac{1}{3}\right) = 9$

   The first five terms of the sequence are 729, –243, 81, –27, and 9.

2. Write the explicit formula for this sequence.

   The first term is $a_1 = 729$ and the common ratio is $r = -\frac{1}{3}$, so the explicit formula is $a_n = (729)\left(-\frac{1}{3}\right)^{n-1}$.

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Instruction

3. Use the formula to find the requested term.

We are asked to find the eighth term in this sequence. Substitute 8 for n and evaluate.

$a_n = (729)\left(-\frac{1}{3}\right)^{n-1}$ Explicit formula from the previous step

$a_{(8)} = (729)\left(-\frac{1}{3}\right)^{(8)-1}$ Substitute 8 for n.

$a_8 = (729)\left(-\frac{1}{3}\right)^{7}$ Simplify.

$a_8 = (729)\left(-\frac{1}{2187}\right)$ Raise $-\frac{1}{3}$ to the 7th power.

$a_8 = -\frac{1}{3}$ Multiply.

The eighth term of the sequence is $-\frac{1}{3}$.

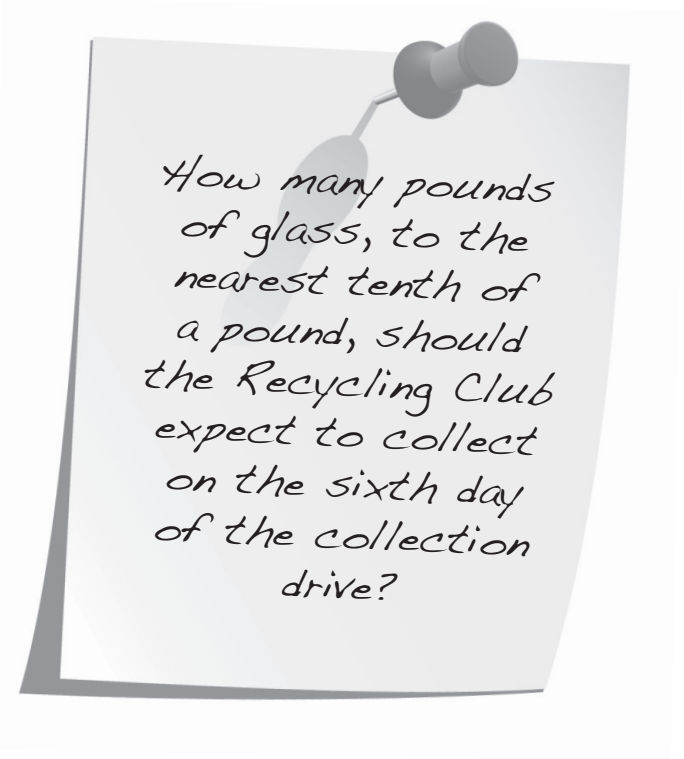


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Problem-Based Task 2.9.2: Glass Recycling

SMP	
1 ✓	2
3	4
5	6 ✓
7 ✓	8

The school Recycling Club collected 39.6 pounds of glass on the second day of a collection drive. The members collected 57.024 pounds of glass on the fourth day. The club estimates that the number of pounds collected will form a geometric sequence. Write an explicit formula to represent this scenario. How many pounds of glass, to the nearest tenth of a pound, should the Recycling Club expect to collect on the sixth day of the collection drive?



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Problem-Based Task 2.9.2: Glass Recycling

Coaching

a. What equation can you write using the information about the second day of the collection drive?

b. What equation can you write using the information about the fourth day of the collection drive?

c. How can you use the two equations to find the common ratio?

d. What is the explicit formula for the geometric sequence?

e. Use the explicit formula to estimate how many pounds of glass, to the nearest tenth of a pound, the Recycling Club should expect to collect on the sixth day of the collection drive.

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Problem-Based Task 2.9.2: Glass Recycling

Coaching Sample Responses

a. What equation can you write using the information about the second day of the collection drive?

$39.6 = a_1(r)^{2-1}$

$39.6 = a_1 r$

b. What equation can you write using the information about the fourth day of the collection drive?

$57.024 = a_1(r)^{4-1}$

$57.024 = a_1 r^3$

c. How can you use the two equations to find the common ratio?

Write each equation in terms of a_1.

$$a_1 = \frac{39.6}{r}$$

$$a_1 = \frac{57.024}{r^3}$$

Set the two ratios equal to each other and solve for r.

$$\frac{57.024}{r^3} = \frac{39.6}{r}$$

$57.024r = 39.6r^3$

$57.024 = 39.6r^2$

$$r^2 = \frac{57.024}{39.6} = 1.44$$

$r = \pm 1.2$

Only the positive square root, 1.2, makes sense in the context of the problem.

d. What is the explicit formula for the geometric sequence?

Substitute 1.2 for r to find a_1.

$$a_1 = \frac{39.6}{1.2} = 33$$

Substitute the known quantities into the explicit form of the geometric sequence.

$a_n = 33(1.2)^{n-1}$

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e. Use the explicit formula to estimate how many pounds of glass, to the nearest tenth of a pound, the Recycling Club should expect to collect on the sixth day of the collection drive.

$a_n = 33(1.2)^{n-1}$

$a_{(6)} = 33(1.2)^{(6)-1}$

$a_6 = 33(1.2)^5$

$a_6 = 82.11456$

To the nearest tenth of a pound, the Recycling Club should expect to collect 82.1 pounds of glass on the sixth day.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.9.2: Geometric Sequences

For problems 1–4, find the common ratio and write the explicit formula for the *n*th term of each geometric sequence.

1. 1, 2, 4, 8, 16, ...

2. 10, –2, $\frac{2}{5}$, $-\frac{2}{25}$, ...

3. 5, 15, 45, 135, ...

4. 320, –80, 20, –5, ...

Use the given information to complete problems 5–10.

5. Find the first five terms of the geometric sequence defined as follows:

   $a_n = a_{n-1}(3);\ a_1 = -1$

6. Find the first five terms of the geometric sequence defined as follows:

   $a_n = a_{n-1}\left(\frac{1}{4}\right);\ a_1 = 216$

7. Jade is training for a marathon. During her first week of training, each run she completes is 90 minutes long. She increases the time she runs by 10% each week. Write the explicit formula to represent how many minutes she runs after *n* weeks.

8. You are opening a savings account with $500 that you have saved. The bank offers 3.2% interest, compounded yearly. How much money will you have in your account after 7 years?

9. Nigel is participating in a read-a-thon. The number of pages he reads each night follows a geometric sequence. On the second day of the read-a-thon, Nigel read 8 pages. On the fifth day of the read-a-thon, he read 64 pages. Write an explicit formula to represent this scenario.

10. Mr. Galloway purchased a car for $20,000. The car retains 85% of its value each year. How much will the car be worth in 5 years?

SWB p. 241; SRB Practice 2.9.2 p. U2-245

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Progress Assessment

Circle the letter of the best answer.

1. What is the common difference of the sequence?

   34, 25, 16, 7, ...

   a. 9
   b. –9
   c. 11
   d. 34

2. What is the common ratio of the sequence?

   32, –16, 8, –4, ...

   a. 2
   b. –2
   c. 0.5
   d. –0.5

3. What is the value of the eighth term in the sequence?

   $\frac{1}{4}, \frac{5}{4}, \frac{9}{4}, \frac{13}{4}, \ldots$

   a. 1
   b. $\frac{29}{4}$
   c. 29
   d. $\frac{29}{64}$

4. What is the value of the sixth term in the sequence?

   –8, –40, –200, –1000, ...

   a. 25,000
   b. –25,000
   c. 5000
   d. –5000

5. The explicit formula for an arithmetic sequence is given as $a_n = x + (n-1)(4)$. The fifth term of the sequence is –27. What is the value of x?

   a. –43
   b. 43
   c. –11
   d. 11

continued

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6. The explicit formula for a geometric sequence is given as $a_n = x(8)^{n-1}$. The third term is 320. What is the value of x?

   a. 5
   b. 40
   c. 64
   d. 512

7. The recursive formula for an arithmetic sequence is given as $a_n = a_{n-1} + 12$, with $a_1 = 9$. What are the first four terms of the sequence?

   a. 12, 21, 30, 39
   b. 9, 21, 33, 45
   c. 9, 84, 1008, 12,096
   d. 1, 12, 144, 1728

8. The recursive formula for a geometric sequence is given as $a_n = a_{n-1}\left(-\frac{1}{4}\right)$, with $a_1 = 256$. What are the first four terms of the sequence?

   a. $-\frac{1}{4}, 1, -4, 16$
   b. 256, 64, 16, 4
   c. 256, –64, 16, –4
   d. –256, 64, –16, 4

9. The explicit formula for an arithmetic sequence is $a_n = 1.9 + (n-1)(2.1)$. What is the sixth term of the sequence?

   a. 14.5
   b. 12.4
   c. 16.6
   d. 1.9

10. The explicit formula for a geometric sequence is $a_n = 2(-5)^{n-1}$. What is the fourth term of the sequence?

    a. –250
    b. –125
    c. 1125
    d. 1250

Use the given information to complete problem 11.

11. A ball is dropped off a ledge. Each time, the ball bounces to 0.9 of its height. The ledge is 6 feet off the ground.

    a. Write an explicit formula for this sequence.

    b. Find the height of the ball after the fifth bounce.

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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. Identify the parameters in the function $f(x) = 2x + 5$.
   a. x and $f(x)$
   b. 2 and 5
   c. $2x$ and 5
   d. 0 and 5

2. Identify the parameters in the function $f(x) = 2^x + 3$.
   a. 2 and 3
   b. x and $f(x)$
   c. x and 3
   d. 1 and 2

3. Jamal has a membership to a magazine club. He pays \$5.00 per month for membership and \$1.25 for each magazine he purchases. What are the parameters in this scenario?
   a. $1.25x$ and 5
   b. 1.25 and 5
   c. The parameters are not defined.
   d. x and $f(x)$

4. Identify the parameters in the function $f(x) = 4(3^x)$.
   a. The parameters are not defined.
   b. x and $f(x)$
   c. 3 and 4
   d. 3 and 1

5. Sonja has invested \$250 in a stock that is expected to double every 8 years. What are the parameters in this scenario?
   a. 8 and 250
   b. 2 and 8
   c. 2 and 250
   d. x and $f(x)$

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Lesson 10: Interpreting Parameters

Instruction

Common Core State Standard

F–LE.5 Interpret the parameters in a linear or exponential function in terms of a context.★

SMP
1 ✓ 2
3 4 ✓
5 ✓ 6
7 8

Essential Questions

1. What are the parameters in a linear function?
2. What are the parameters in an exponential function?
3. How do you determine the parameters in the context of a word problem?
4. How does changing the parameter in a function change the graph of a function?

WORDS TO KNOW

exponential function	a function whose independent variable is in the exponent; the general form of its equation is $f(x) = ab^x + k$, where a is the initial value, b is the base, x is the input value, k is the vertical shift, and $f(x)$ is the output
growth factor	the factor by which a quantity increases or decreases over time
linear function	a first-degree equation that can be written in the form $f(x) = mx + b$, in which m is the slope of the line and b is the y-intercept. The graph of a linear function is a straight line.
parameter	a constant in a function that determines the specific graph of the function but not the type of function
vertical shift	the number of units the graph of a function is moved up or down in a plane; a type of translation

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Instruction

Recommended Resources

- Illuminations. "Function Matching."

  http://walch.com/rr/CAU3L9FunctionMatching

  At this site, users can match graphs to function rules on an interactive graph.

- Illuminations. "Movie Lines."

  http://walch.com/rr/CAU3L9MovieLines

  This lesson plan for teachers provides an activity in which students identify the *y*-intercept and slope and state their significance in the context of a real-world problem.

- Texas Instruments. "Characteristics of Exponential Functions."

  http://walch.com/rr/CAU3L9FunctionCharacteristics

  This lesson plan for teachers centers on investigating how the graph of an exponential function changes as the base changes.

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Lesson 2.10.1: Interpreting Parameters

Warm-Up 2.10.1

You are buying a membership to a gaming store. The membership fee is $5 per month and each game costs $2 to rent. There are no late fees.

1. Write a linear function to represent the amount of money you spend in a month.

2. What is the domain of this function?

3. Graph the function and identify the *y*-intercept.

4. What does the *y*-intercept represent in the context of the problem?

5. What does the slope represent in the context of the problem?

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Instruction

Lesson 2.10.1: Interpreting Parameters

Common Core State Standard

F–LE.5 Interpret the parameters in a linear or exponential function in terms of a context.★

Warm-Up 2.10.1 Debrief

1. Write a linear function to represent the amount of money you spend in a month.

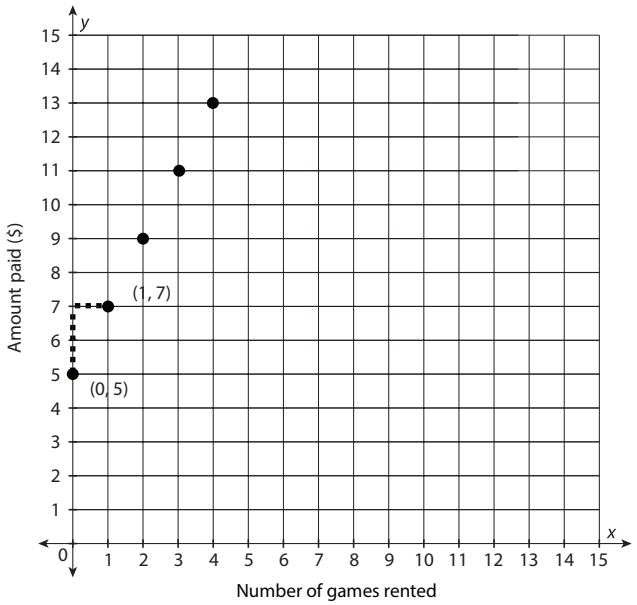
   The independent variable represents the number of games rented during the month. The price per game is $2. The constant value is the $5 monthly fee. The function in slope-intercept form is $f(x) = 2x + 5$.

2. What is the domain of this function?

   The domain is the set of non-negative integers, because you can't rent a fraction of a game.

3. Graph the function and identify the y-intercept.

   The y-intercept is 5 and the slope is 2. Plot the point (0, 5) and then count up 2 units and over to the right 1 unit to plot the next point, (1, 7). Continue plotting points in this manner.



4. What does the y-intercept represent in the context of the problem?

   The y-intercept represents the membership fee, which is constant each month.

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Instruction

5. What does the slope represent in the context of the problem?

   The slope represents the $2 rental fee per game. In other words, the amount you owe for any given month increases by $2 for each game you rent.

Connection to the Lesson

- Students will graph linear and exponential functions.
- Students will write linear and exponential functions similar to writing the function in the warm-up.
- The warm-up gives the students practice with both writing and graphing, which they will be doing in the lesson.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- graphing linear equations (A–CED.2★)
- graphing exponential equations (A–CED.2★)
- writing linear equations from context (A–CED.2★)
- writing exponential equations from context (A–CED.2★)

Introduction

In order to fully understand how various functions model real-world contexts, we need to understand how changing parameters will affect the functions. This lesson will explore the effect of parameters on linear and exponential functions. We will also interpret the effects of these parameters in a context.

Key Concepts

- An exponential function is a function that may be written in the form $f(x) = ab^x + k$, where a is the initial value, b is the base or growth factor, x is the input value, k is the vertical shift, and $f(x)$ is the output.
- A linear function may be written in slope-intercept form, $f(x) = mx + b$, where m is the slope of the line and b is the y-intercept.
- A **parameter** of a function is a constant that determines the specific graph of the function but not the type of function. It is a value that is built into the function.
- For a linear function written in slope-intercept form, $f(x) = mx + b$, the parameters are m and b. m represents the slope and b represents the y-intercept. Changing either of these parameters will change the graph of the function.
- For an exponential function in the form $f(x) = ab^x + k$, the parameters are a, b, and k. Changing any of these parameters will change the graph of the function.

Graphing Equations Using a TI-83/84:

Step 1: Press [Y=].

Step 2: Key in the equation using [X, T, θ, n] for x.

Step 3: Press [WINDOW] to change the viewing window, if necessary.

Step 4: Enter in appropriate values for Xmin, Xmax, Xscl, Ymin, Ymax, and Yscl, using the arrow keys to navigate.

Step 5: Press [GRAPH].

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Instruction

Graphing Equations Using a TI-Nspire:

Step 1: Press the home key.

Step 2: Arrow over to the graphing icon (the picture of the parabola or the U-shaped curve) and press [enter].

Step 3: Enter in the equation and press [enter].

Step 4: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom, and click the center button of the navigation pad.

Step 5: Choose 1: Window settings by pressing the center button.

Step 6: Enter in the appropriate XMin, XMax, YMin, and YMax fields.

Step 7: Leave the XScale and YScale set to auto.

Step 8: Use [tab] to navigate among the fields.

Step 9: Press [tab] to "OK" when done and press [enter].

Common Errors/Misconceptions

- not understanding the difference between variables and parameters in a function
- mistaking the slope for the y-intercept or the y-intercept for the slope
- forgetting to identify the vertical shift in the context of an exponential problem
- when reading a word problem, not being able to identify the parameters in the context of the problem

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Instruction

Guided Practice 2.10.1

Example 1

You visit a pick-your-own apple orchard. There is an entrance fee of $5.00, plus you pay $0.50 for each apple you pick. Write a function to represent this scenario. Complete a table of values to show your total cost if you pick 10, 20, 30, 40, and 50 apples. Graph the function and identify the parameters in this problem. What do the parameters represent in the context of the problem?

1. Write a function.

   This scenario is represented by a linear function.

   Identify the slope and the y-intercept.

   - The slope is the $0.50 charged for each apple picked.
   - The y-intercept is the entrance fee of $5.00.

   Substitute the slope and the y-intercept into the linear function $f(x) = mx + b$, where m is the slope and b is the y-intercept.

   The function for this scenario is $f(x) = 0.5x + 5$.

2. Create a table.

   Let x represent the number of apples picked and $f(x)$ represent the total cost.

   Use the values 0, 10, 20, 30, 40, and 50 for x.

x	$0.5x + 5$	$f(x)$
0	0.5(0) + 5	5
10	0.5(10) + 5	10
20	0.5(20) + 5	15
30	0.5(30) + 5	20
40	0.5(40) + 5	25
50	0.5(50) + 5	30

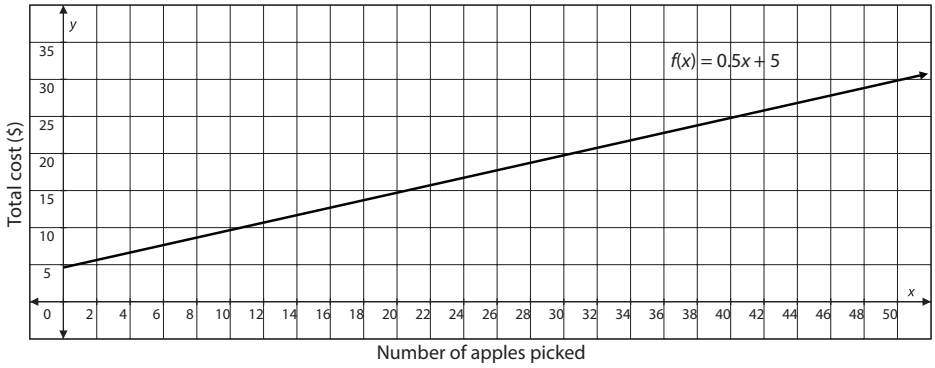
SWB p. 247; SRB p. U2-250

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Instruction

3. Identify the domain of the function and then graph the function.

The domain of the function is non-negative integers, because the *x*-values represent the number of apples picked, and you can't pick a fraction of an apple. Use the table of values to graph the function that represents this scenario, keeping in mind that the points on the graph with *x*-values that are non-negative integers are the points that make the most sense for this scenario.



4. Identify the parameters.

The parameters in this problem are the slope and the *y*-intercept. In this problem, the *y*-intercept is the entrance fee, $5.00, and the slope is the cost per apple, $0.50.

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Instruction

Example 2

You deposit $100 into a long-term certificate of deposit (CD) in which your money will double every 7 years. Write a function to show how much money you will have in total in 7, 14, 21, 28, and 35 years. Use your equation to create a table, and then graph the function. What do the parameters represent in the context of this problem?

1. Write a function.

   This scenario is represented by an exponential function.

   - The initial deposit is $100.
   - Your money doubles every 7 years, so the growth factor is 2.
   - The time period is 7 years.

   Substitute these values into the exponential function.

   The function for this scenario is $f(x) = 100(2)^{\frac{x}{7}}$.

2. Create a table.

   Let x represent the number of years and $f(x)$ represent the money earned in dollars.

   Use the values 7, 14, 21, 28, and 35 for x.

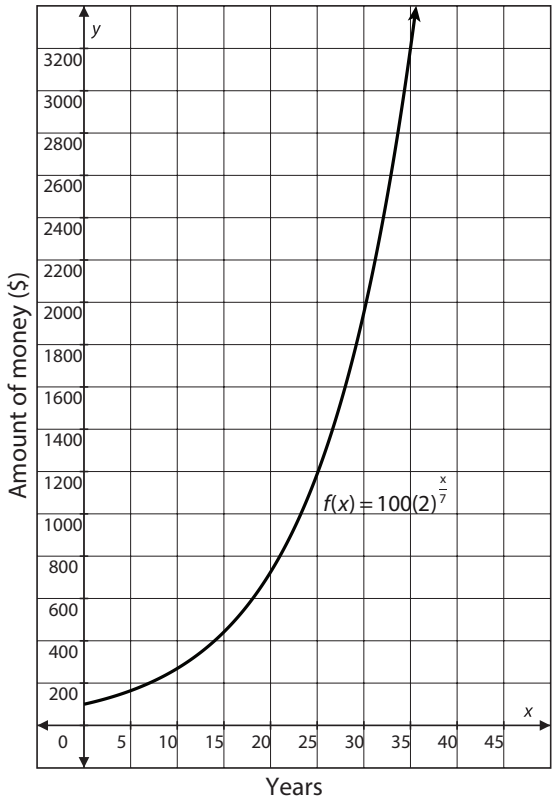
x	$100(2)^{\frac{x}{7}}$	$f(x)$
7	$100(2)^{\frac{(7)}{7}}$	200
14	$100(2)^{\frac{(14)}{7}}$	400
21	$100(2)^{\frac{(21)}{7}}$	800
28	$100(2)^{\frac{(28)}{7}}$	1600
35	$100(2)^{\frac{(35)}{7}}$	3200

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Instruction

3. Graph the function.

   Plot the ordered pairs in the table to graph the function.



4. Identify the parameters.

   The parameters in this problem are the initial amount of $100, the base of 2, and the time period of 7 years.

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Instruction

Example 3

You are growing bacteria in science class. You start with 1 bacterium, which doubles every 12 minutes. Write a function to show how many bacteria you have after 12 minutes, 24 minutes, 36 minutes, 48 minutes, and 60 minutes. Use the function to create a table, then graph the function. What are the parameters in this problem?

1. Write a function.

   This scenario is represented by an exponential function.

   - The initial number of bacteria is 1.
   - The bacteria double every 12 minutes, so the growth factor is 2.
   - The time period is 12 minutes.

   Substitute these values into the exponential function.

   The function for this scenario is $f(x)=2^{\frac{x}{12}}$.

2. Create a table.

   Let x represent the number of minutes and $f(x)$ represent the number of bacteria.

   Use the values 0, 12, 24, 36, 48, and 60 for x.

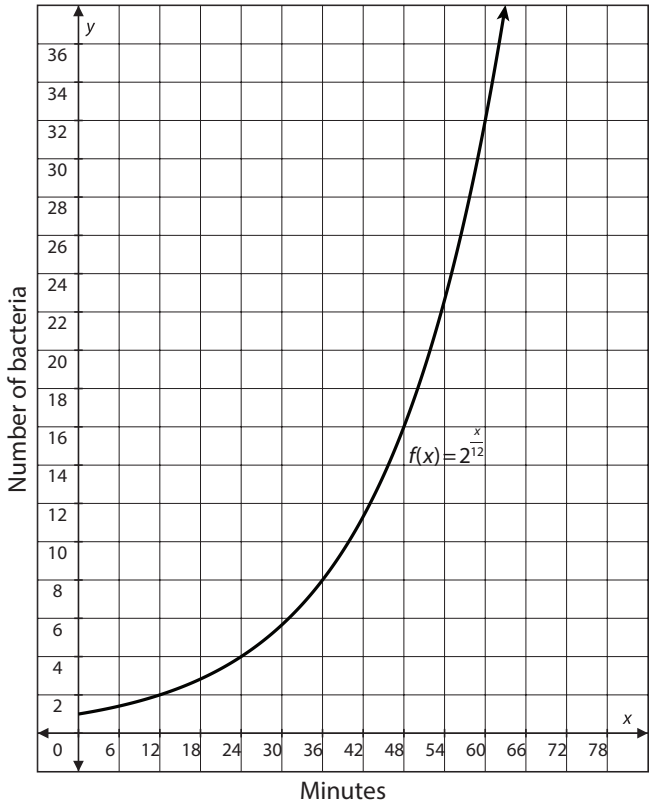
x	$1(2)^{\frac{x}{12}}$	$f(x)$
0	$1(2)^{\frac{(0)}{12}}$	1
12	$1(2)^{\frac{(12)}{12}}$	2
24	$1(2)^{\frac{(24)}{12}}$	4
36	$1(2)^{\frac{(36)}{12}}$	8
48	$1(2)^{\frac{(48)}{12}}$	16
60	$1(2)^{\frac{(60)}{12}}$	32

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Instruction

3. Graph the function.

Plot the ordered pairs in the table to graph the function.



4. Identify the parameters.

The parameters in this problem are the initial amount of 1 bacterium, the base of 2, and the time period of 12 minutes.

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Problem-Based Task 2.10.1: Cell Phone Plans

SMP	
1 ✓	2
3	4 ✓
5 ✓	6
7	8

You are comparing cell phone providers in order to determine which one offers the best deal. AT&Me offers a plan with a monthly fee of $25, plus a $0.10 per minute charge. Tracmyphone offers a plan with a monthly fee of $15, with a $0.15 per minute charge. You typically use fewer than 100 minutes per month. Which plan would be the best choice for you and why?



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Problem-Based Task 2.10.1: Cell Phone Plans

Coaching

a. What kinds of functions represent the plans in this scenario?

b. What are the parameters for these functions?

c. Write the function for each plan.

d. Graph the two functions.

e. If you use fewer than 100 minutes per month, what is a reasonable scale on the *x*-axis?

f. By analyzing the graph, how can you determine which plan to select?

g. Which plan is the best deal if you use fewer than 100 minutes a month?

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Instruction

Problem-Based Task 2.10.1: Cell Phone Plans

Coaching Sample Responses

a. What kinds of functions represent the plans in this scenario?

Both of these plans can be represented by linear functions.

b. What are the parameters for these functions?

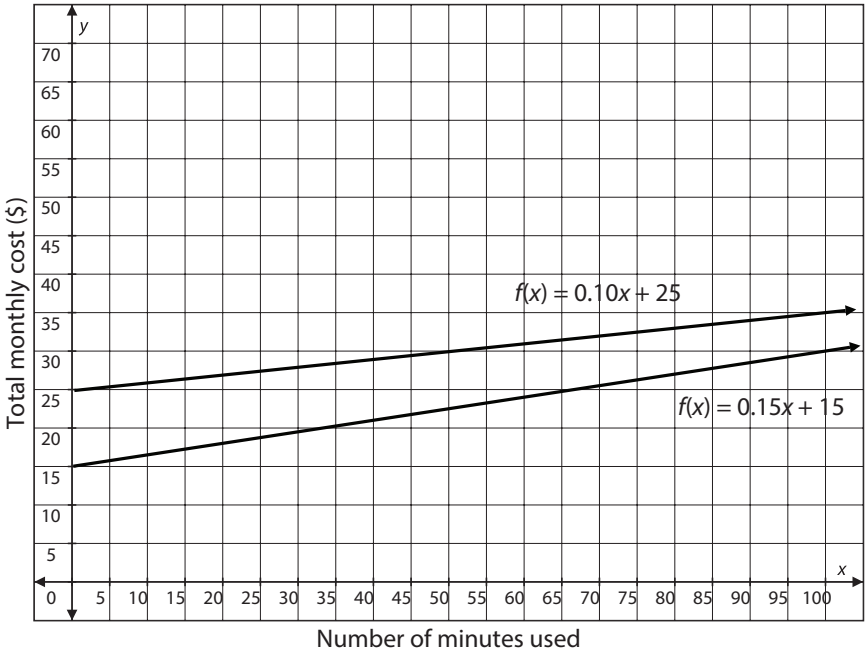
The slope and the y-intercept are the parameters in a linear function. For AT&Me, the slope is the cost per minute, \$0.10, and the y-intercept is the monthly fee, \$25. For Tracmyphone, the slope is the cost per minute, \$0.15, and the y-intercept is the monthly fee, \$15.

c. Write the function for each plan.

For AT&Me, the function is $f(x) = 0.10x + 25$.

For Tracmyphone, the function is $f(x) = 0.15x + 15$.

d. Graph the two functions.



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Instruction

e. If you use fewer than 100 minutes per month, what is a reasonable scale on the *x*-axis?

The scale on the *x*-axis could span from 0 to 100 minutes.

f. By analyzing the graph, how can you determine which plan to select?

The line that is lower represents the plan with the lower total cost per month.

g. Which plan is the best deal if you use fewer than 100 minutes a month?

If you use fewer than 100 minutes a month, Tracmyphone is the best deal because it is less expensive.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 2.10.1: Interpreting Parameters

Identify the parameters for the functions in problems 1–5.

1. $f(x) = 7x + 5$

2. $f(x) = 2^x + 3$

3. $f(x) = -2x + 10$

4. $f(x) = 2(3^x)$

5. $f(x) = 3(2^x) + 5$

Use what you know about functions to complete problems 6–10.

6. You join a gym. The monthly membership fee is \$10 and the rate per hour is \$2. What is the function that represents this scenario? What are the parameters in this scenario?

7. Claire subscribes to a movie rental program. She pays a monthly fee of \$5.00, plus \$1.25 for each movie rented. What are the parameters in this scenario?

8. Max is picking apples with his brother. The number of apples in his bag is described by $f(x) = 18x + 15$, where x is the number of minutes Max spends picking apples. What do the numbers 18 and 15 tell you about Max's apple picking?

9. The number of ants in an ant farm is described by $f(x) = 10(2)^{\frac{x}{36}}$, where x is time in hours. What do the numbers 10 and 2 tell you about the number of ants in the colony?

10. Anastasia hides \$200 in her mattress and deposits \$150 into an account that doubles every 10 years. What is the function that represents the amount of money that Anastasia has? What are the parameters?

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Progress Assessment

Circle the letter of the best answer.

1. What are the parameters in a linear function?
   a. x and $f(x)$
   b. growth factor and vertical shift
   c. slope
   d. slope and y-intercept

2. What are the parameters in an exponential function?
   a. slope and y-intercept
   b. growth factor and vertical shift
   c. vertical shift
   d. x and $f(x)$

3. Identify the parameters in $f(x) = -3x + 7$.
   a. 0 and 7
   b. x and y
   c. –3 and 7
   d. m and b

4. Identify the parameters in $f(x) = 2^x + 600$.
   a. b and k
   b. 2 and 600
   c. x and y
   d. 0 and 601

5. How does increasing the slope in a linear function from 1 to 2 change the graph of the line?
   a. The line rises more steeply.
   b. The line is less steep.
   c. The y-intercept increases.
   d. The y-intercept decreases.

6. Logan makes $8 per hour washing dishes. His boss offers to pay a flat fee of $50 for scrubbing down the kitchen after closing. What are the parameters in this scenario?
   a. 0 and 50
   b. 1 and 58
   c. 8 and 50
   d. x and $f(x)$

continued

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Assessment

7. Ted owns a valuable baseball card that increases in value according to the function $f(x)=500(2^x)$. What are the parameters in this scenario?

   a. 0 and 500
   b. x and y
   c. 1 and 600
   d. 2 and 500

8. You belong to an e-book club. The membership fee is \$10.00 per month plus a \$1.50 fee for each book you download. You just received notice that the membership fee is being raised to \$11.50 per month. How does this change your monthly cost?

   a. You pay \$1.50 more per book.
   b. You pay \$1.50 less per book.
   c. You pay \$1.50 more each month.
   d. There is not enough information given.

9. Ayala belongs to a movie rental club. When she joined the club, her monthly fee was \$5.00 and each video cost \$3.50 to rent. Ayala has just received notice that the cost to rent each movie will now be \$2.50. Which parameter is changed in this scenario?

   a. the slope
   b. the y-intercept
   c. x
   d. $f(x)$

10. Grace joins a photo printing website that charges a membership fee, but only charges \$0.15 for each photo print. She buys 20 prints, and her total cost is \$8.00. Which parameter is not defined?

    a. the y-intercept
    b. the slope
    c. x
    d. $f(x)$

Read the scenario, define the parameters, write a function to represent the problem, and then solve the problem.

11. Evan works as a lifeguard each summer at the lake. The job pays \$13 per hour. One day, the owner of a rowboat rental company offers to pay Evan a flat fee to clean out the boats after he finishes his shift. After working 8 hours and cleaning out the boats, Evan takes home \$124. How much did Evan earn cleaning out the boats?

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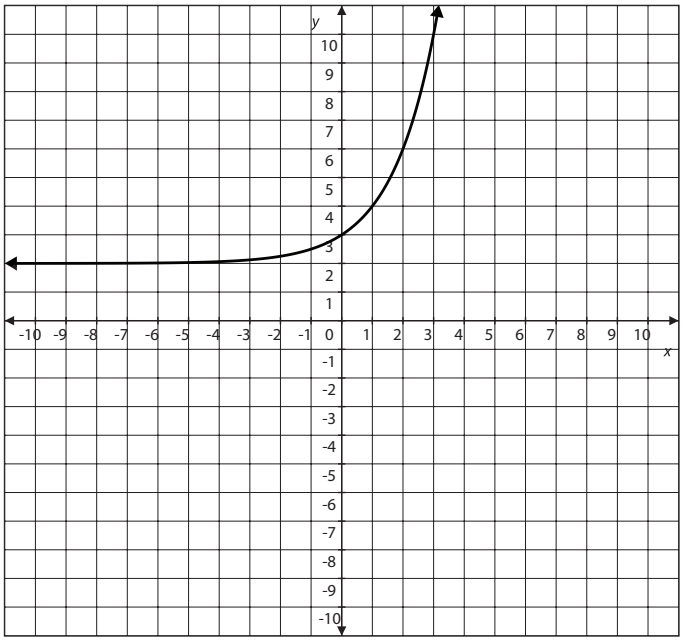
Unit 2 Assessment

Circle the letter of the best answer.

1. If $f(x) = 3x - 5$ and the domain of f is {2, 4, 6}, what is the range of $f(x)$?

   a. {11, 17, 20}
   b. {–6, –4, –2}
   c. {2, 4, 6}
   d. {1, 7, 13}

2. Given the graph of $f(x)$, what is $f(2)$?



   a. 2
   b. 0
   c. 6
   d. 10

3. If $a_n = a_{n-1} + 4$ and $a_5 = 12$, what is a_7?

   a. 11
   b. 10
   c. 20
   d. 19

4. What is the rate of change for the function $f(x) = 5(2)^{\frac{x}{4}}$ over the interval [8, 12]?

   a. 320
   b. 10
   c. 5
   d. The rate of change cannot be determined.

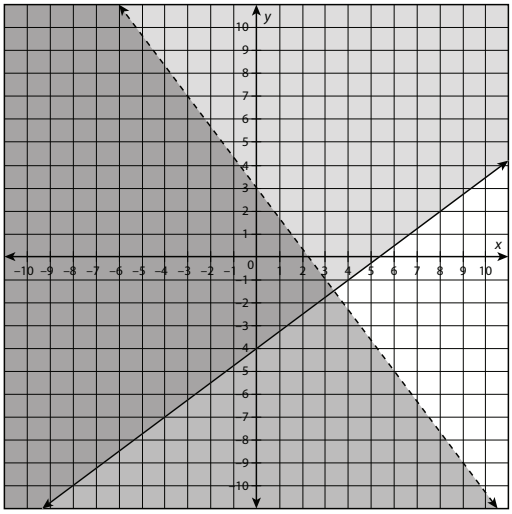
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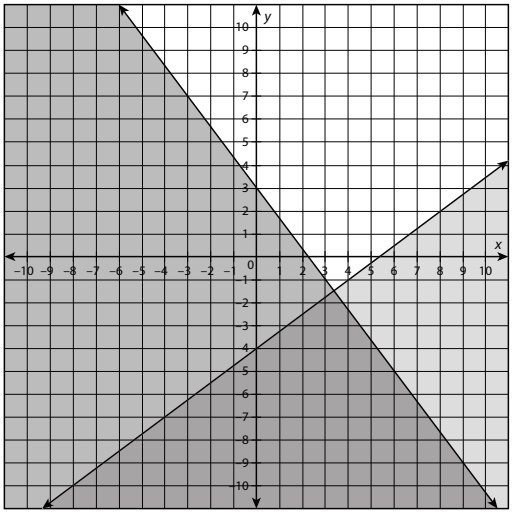
5. Which graph represents the solution to the following system of linear inequalities?

$$\begin{cases} 3x - 4y \leq 16 \\ 4x + 3y < 9 \end{cases}$$

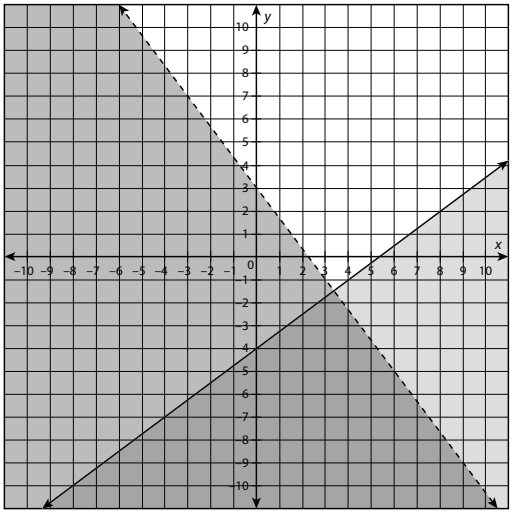
a.



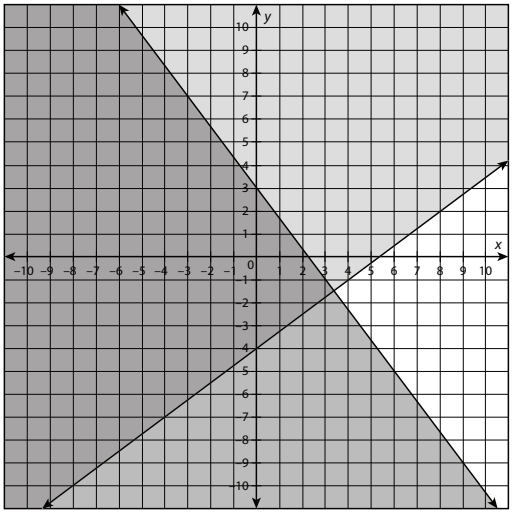
c.



b.



d.



continued

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6. Given the equation for $f(x)$ and the table that represents $g(x)$, which of the following statements is true about the functions $f(x)$ and $g(x)$?

$f(x)=\frac{2}{5}x-3$

x	$g(x)$
–4	–29
–2	–17
2	7
4	19

a. The y-intercept of the function $f(x)$ is less than the y-intercept of the function $g(x)$.

b. The y-intercept of the function $f(x)$ is greater than the y-intercept of the function $g(x)$.

c. The y-intercept of the function $f(x)$ is equal to the y-intercept of the function $g(x)$.

d. The y-intercepts cannot be determined.

7. Which explicit function represents the pattern in the table?

x	$f(x)$
1	–5
2	–35
3	–245
4	–1715

a. $f(x)=(-5)^{x-1}$

b. $f(x)=-7\bullet(5)^{x-1}$

c. $f(x)=(5)\bullet(-7)^{x-1}$

d. $f(x)=(-5)\bullet 7^{x-1}$

8. If $f(x)=5^x$ and $g(x) = 6$, what is $(f \bullet g)(x)$?

a. $(f \bullet g)(x) = 6\left(5^x\right)$

b. $(f \bullet g)(x) = 6^x$

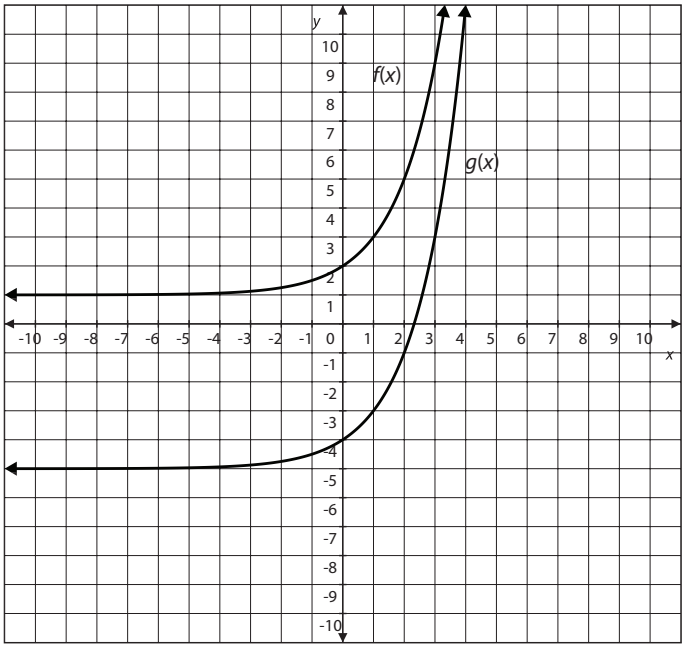
c. $(f \bullet g)(x) = 30^x$

d. $(f \bullet g)(x) = 6+5^x$

continued

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9. Given the graph of $f(x)$ and $g(x)$, which is the function rule for $g(x)$ in terms of $f(x)$?



a. $g(x) = f(x) + 6$

b. $g(x) = f(x) - 4$

c. $g(x) = f(x) - 6$

d. $g(x) = f(x) + 4$

10. The recursive formula for an arithmetic sequence is given as $a_n = a_{n-1} + 9$, with $a_1 = 3$. What is the seventh term of the sequence?

a. 13

b. 21

c. 48

d. 57

11. The explicit formula for a geometric sequence is $a_n = 3(-2)^{n-1}$. What is the fifth term of the sequence?

a. –96

b. 48

c. 192

d. –48

continued

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12. Leroy makes $9 per hour cleaning cars. His boss offers to pay him an additional $45 a week for cleaning the garage after work. What are the parameters in this scenario?

    a. 0 and 45
    b. 1 and 54
    c. 9 and 45
    d. x and $f(x)$

For problems 13 and 14, read each scenario. Then write a function and graph it to solve the problem.

13. The starting balance of Adam's savings account is $575. Each month, Adam deposits $60.

    a. Write a function to model this scenario.

    b. Create a graph to show how much money Adam has in his savings account each month for the first year.

    c. Identify the key features of the function. Determine the x- and y-intercepts, the maximum, the minimum, whether the function is increasing or decreasing, and the rate of change of the function.

14. A population of bees is decreasing. The population in a particular region this year is 1,250. After 1 year, it is estimated that the population will be 1,000. After 3 years, it is estimated that the population will be 640.

    a. Write a function to model this scenario.

    b. Create a graph to show the bee population over the next 10 years.

    c. Identify the key features of the function. Identify the x- and y-intercepts. Determine the maximum, the minimum, whether the function is increasing or decreasing, the rate of change of the function over the interval [0, 10], and any asymptotes.

continued

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Use what you know about linear and exponential functions to solve the problem.

15. You are looking to invest \$1,500. One savings option follows the function $f(x) = 52.5x + 1500$, where x is the time in years and $f(x)$ is the amount of money in savings after x years. The second option is represented by the function $g(x)=1500\left(1+\frac{0.025}{4}\right)^{4x}$, where x is the time in years and $g(x)$ is the amount of money after x years.

    a. Which increases faster, $f(x)$ or $g(x)$? Use a graph to explain your answer.

    b. What are the coordinates of the point where $f(x) = g(x)$?

    c. If you were looking to withdraw the money at age 55 and you invest your money at the age of 50, would you choose differently than if you were looking to invest your money at the age of 20? Explain your reasoning.

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Answer Key

Lesson 1: Graphs As Solution Sets and Function Notation

Pre-Assessment, p. U2-1

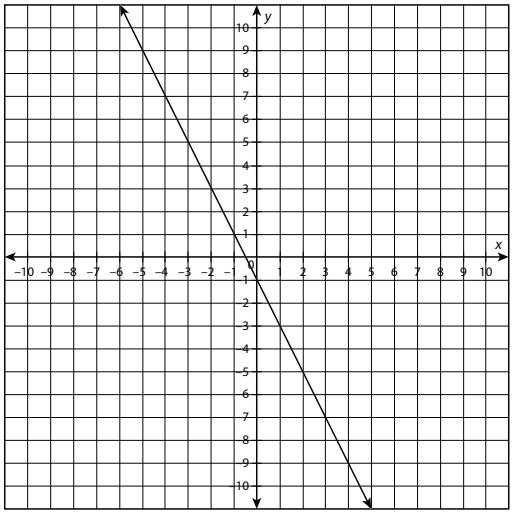
1. c
2. d
3. a
4. b
5. b

Warm-Up 2.1.1, p. U2-4

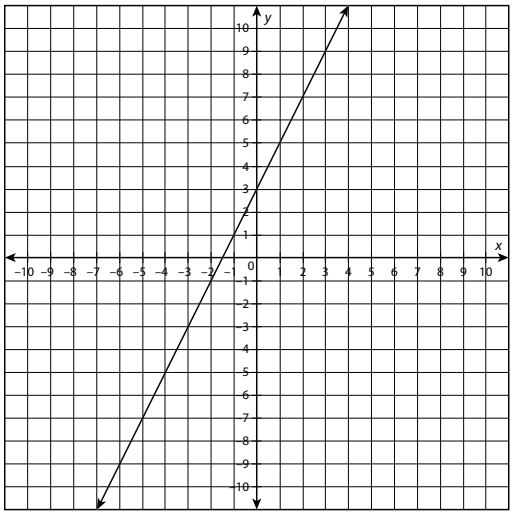
1. $5
2. $9
3. The slope of the graph is 2. The graph shows that Mallory can receive $2 on the *y*-axis for every 1 day on the *x*-axis.

Practice 2.1.1: Graphing the Set of All Solutions, p. U2-18

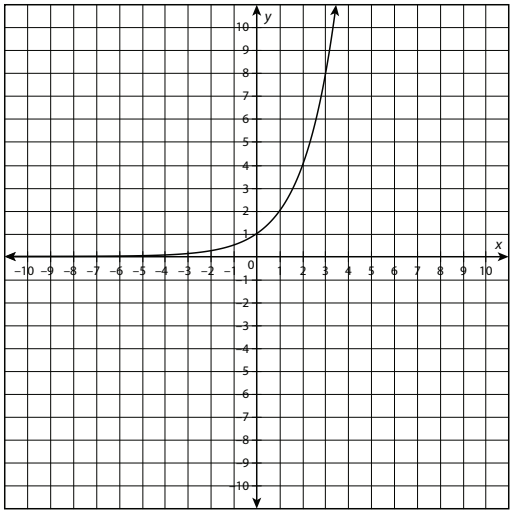
1.



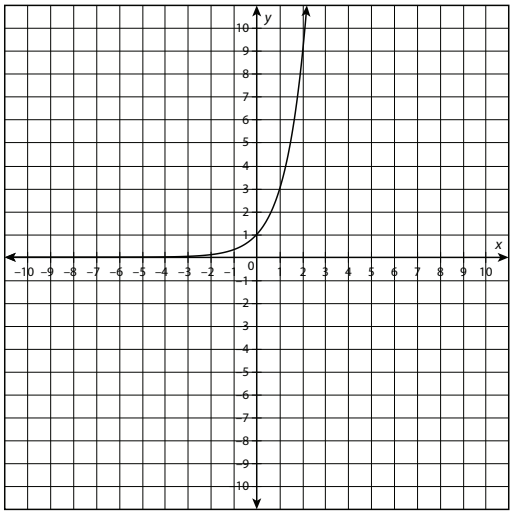
2.



3.



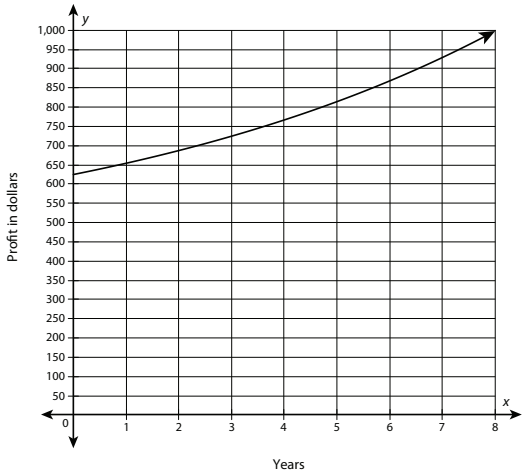
4.



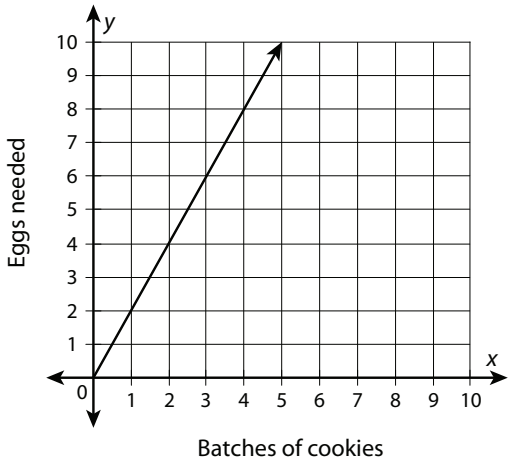
5. {(0, 1); (–2, 4); (–3, 8)}
6. {(0, 7); (8, 6); (–8, 8)}

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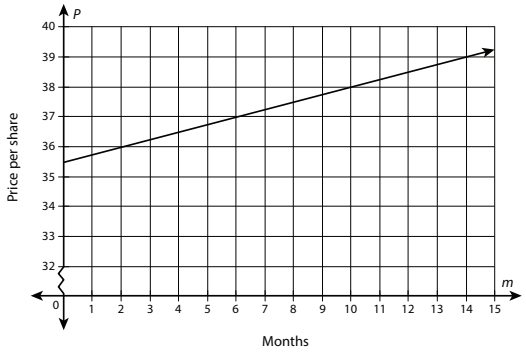
7. The profit in year 7 should be about $929.34.



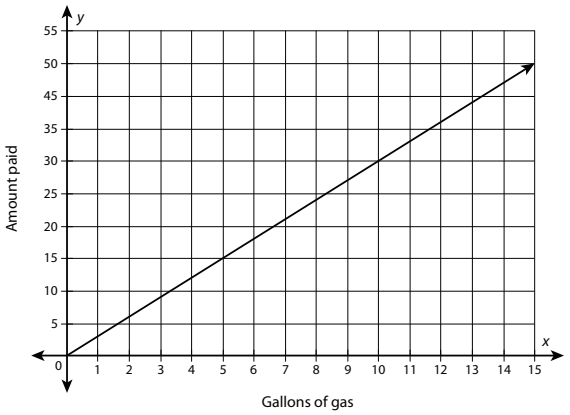
8. $y = 2x$; for 4 batches of cookies, Katya needs 8 eggs.



9. $P = \frac{1}{4}m + 35.50$; after 12 months, the price will be $38.50.



10. You would have to pay $22.50 for 7.5 gallons of gas.

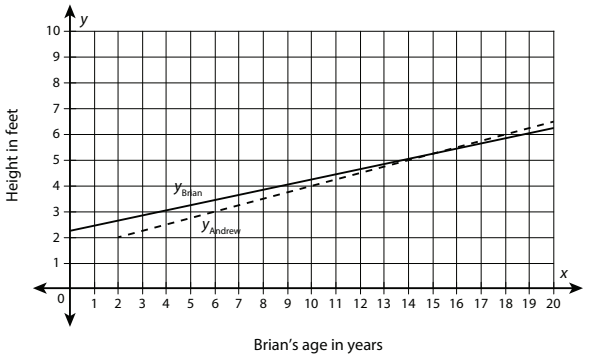


Warm-Up 2.1.2, p. U2-21

1. Table:

x	$B(x) = \frac{1}{5}x + 2.25$	$A(x) = \frac{1}{4}(x-2)+2$
0	2.25	not born yet
2	2.65	2
4	3.05	2.5
10	4.25	4
15	5.25	5.25
16	5.45	5.5
20	6.25	6.5

2. Graph:



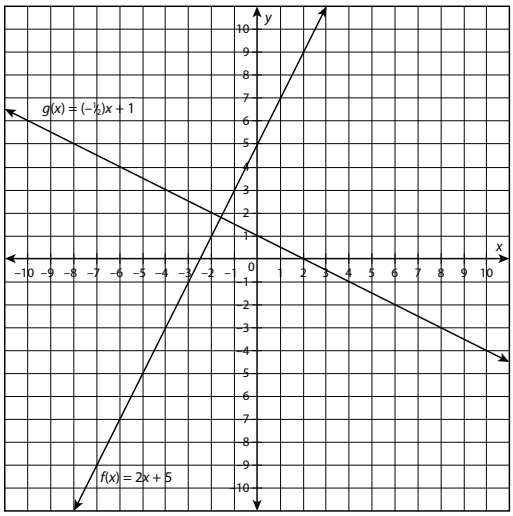
3. yes
4. 13 years old

Practice 2.1.2: Intersecting Graphs, p. U2-34

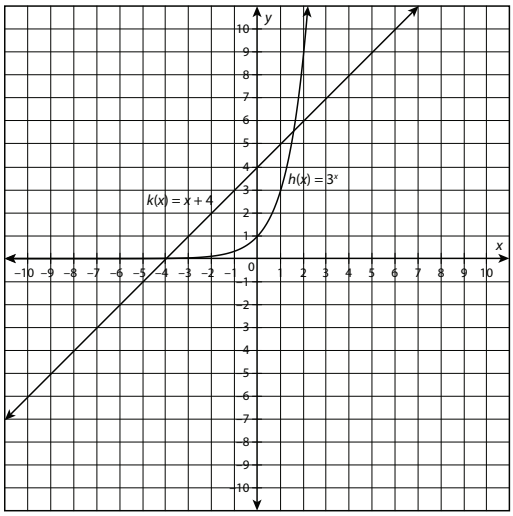
1. 0
2. 2
3. The graphs will not cross between the values of x in the table.
4. 3

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5. The graphs intersect on the interval $-2 < x < -1$.



6. $x \approx -4$ and $x \approx 1.5$



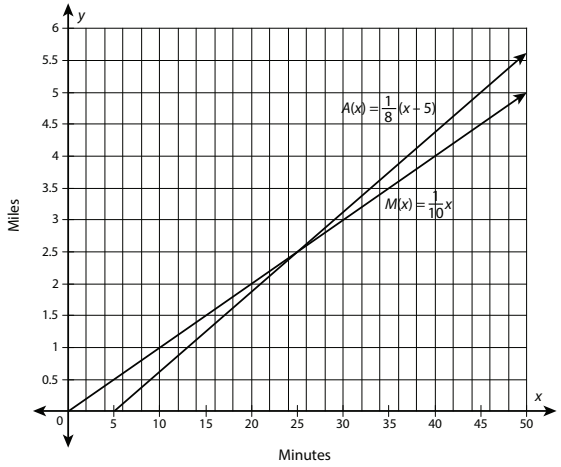
7. $x = 2$ and $-2 < x < -1$

x	$s(x) = 2^x + 1$	$r(x) = x + 3$	$s(x) - r(x)$
–3	1.125	0.00	1.125
–2	1.25	1.00	0.25
–1	1.50	2.00	–0.50
0	2.00	3.00	–1.00
1	3.00	4.00	–1.00
2	5.00	5.00	0.00
3	9.00	6.00	3.00

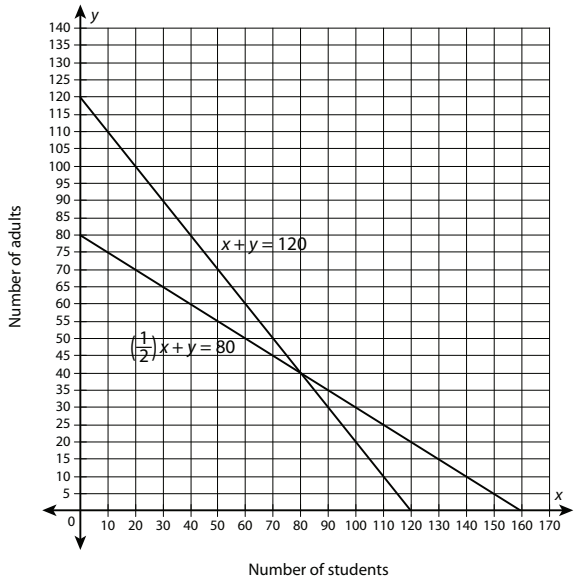
8. $0 < x < 1$

x	$f(x) = x$	$g(x) = \left(\frac{1}{2}\right)^x$	$f(x) - g(x)$
–3	–3.00	8.00	–11.00
–2	–2.00	4.00	–6.00
–1	–1.00	2.00	–3.00
0	0.00	1.00	–1.00
1	2.00	0.50	1.50
2	2.00	0.25	1.75
3	3.00	0.125	2.875

9. The girls will meet 25 minutes after Marisol begins. They will have run 2.5 miles.



10. The graphs intersect at (80, 40). This means that 80 students and 40 adults bought tickets for the school play.



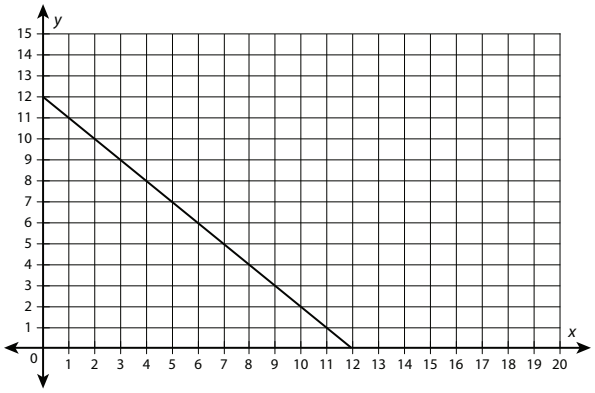
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Warm-Up 2.1.3, p. U2-36

1. children without caps
2. children with caps
3. It could be implied that children in the ballpark without caps did not enter the park through Gate 7.

Practice 2.1.3: Domain and Range, p. U2-52

1. Yes, each x is mapped to only one y.
2. Yes, each x is mapped to only one y.
3. No; the graph fails the vertical line test on the interval $-6 < x < -2$.
4. Yes; the graph passes the vertical line test.
5. Yes; each x is paired to only one y.
6. No; $x = 5$ is mapped to $y = 5$ and $y = 6$.
7. domain: {0, 1, 2, ..., 8, 9}; range: {1, 2, 3, ..., 9, 10}
8. domain: {0, 1, 2, 3, 4}; range: {1, 2, 4, 8, 16}
9. domain: $\{0 \le x \le 12\}$; range: $\{0 \le y \le 12\}$



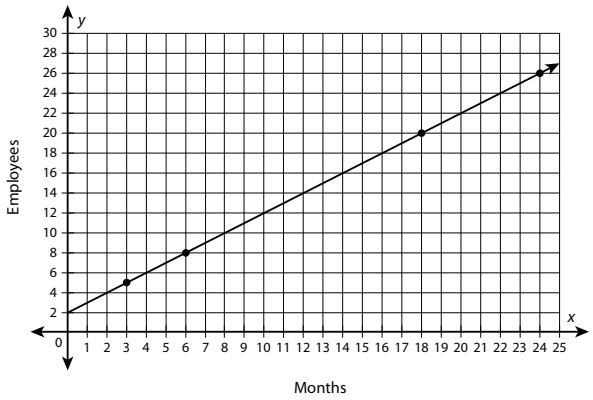
10. domain: $\{x \ge 0\}$; range: $\{y \ge 0\}$

Warm-Up 2.1.4, p. U2-55

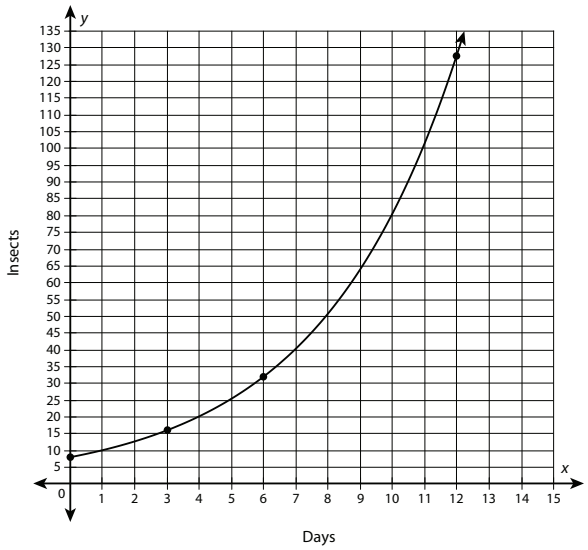
1. Company A
2. Company B
3. Company C
4. Company B

Practice 2.1.4: Function Notation and Evaluating Functions, p. U2-67

1. {–8, –6, –2, 8}
2. {–11, –9, –7, –5}
3. {4, 10, 28, 82}
4. {0, 1, 3, 7}
5. $f(-3) = -8$
6. $f(2) = 3$
7. Evaluating the function over the domain results in the range of {5, 8, 20, 26}. After 3 months, the company has 5 employees. After 6 months, the company has 8 employees. After 18 months, the company has 20 employees. After 24 months, the company has 26 employees. These are all points on the graph of the function $g(x) = x + 2$:
   $g(3) = 5$
   $g(6) = 8$
   $g(18) = 20$
   $g(24) = 26$



8. Evaluating the function over the domain results in the range of {8, 16, 32, 128}. The insect population starts out with 8 insects. After 3 days, the population doubles to 16. After 6 days, the population is 32 insects, and after 12 days the population is 128 insects. These are all points on the function, $f(x) = 8(2)^{\frac{x}{3}}$:
   $f(0) = 8$
   $f(3) = 16$
   $f(6) = 32$
   $f(12) = 128$

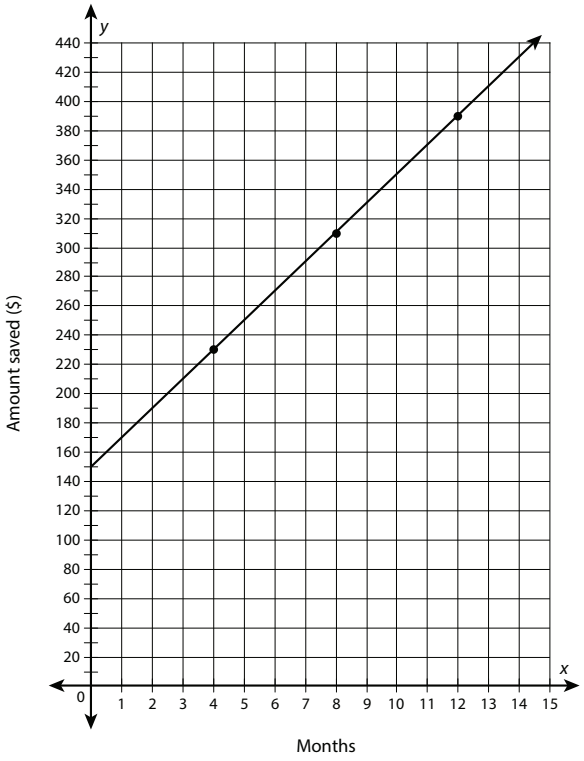


9. $f(2) = 4.40$, $f(3.5) = 5.15$, $f(15) = 10.90$, $f(21.3) = 14.05$. If a package weighs 2 ounces, the cost to mail it is \$4.40. If a package weighs 3.5 ounces, then it costs \$5.15 to mail it. If a package weighs 15 ounces, then it costs \$10.90 to mail it. If the package weighs 21.3 ounces, it costs \$14.05 to mail it.
10. $f(5) \approx 1762.34$, $f(10) \approx 3105.85$, $f(15) \approx 5473.57$. If Brody invests \$1,000 for 5 years, he will have \$1,762.34. If he invests for 10 years, he will have \$3,105.85. If he invests for 15 years, he will have \$5,473.57.

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Progress Assessment, p. U2-69

1. d
2. b
3. a
4. a
5. c
6. c
7. b
8. a
9. b
10. a
11. Answers:
    a. domain: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}; range: {150, 170, 190, 210, 230, 250, 270, 290, 310, 330, 350, 370, 390, 410, 430}
    b. $s(4) = 230$; $s(8) = 310$; $s(12) = 390$



    c. After 4 months, Luisa will have saved $230. After 8 months, Luisa will have saved $310. After 12 months, Luisa will have saved $390.
    d. $f(14) = 430$. After 14 months of saving, Luisa will reach her savings goal of $430.

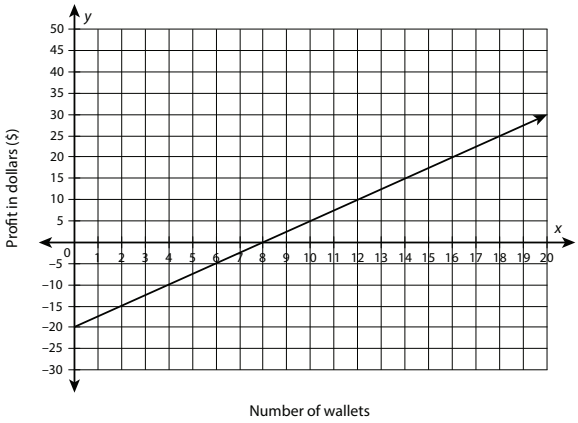
Lesson 2: Solving Linear Inequalities in Two Variables and Systems of Inequalities

Pre-Assessment, p. U2-73

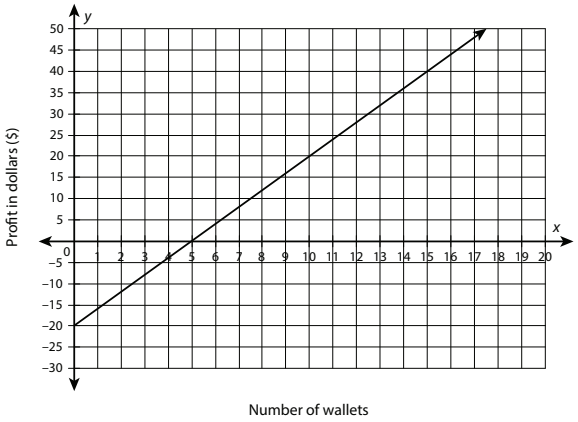
1. b
2. c
3. d
4. d
5. d

Warm-Up 2.2.1, p. U2-79

1. $y = 2.50x - 20$
2. The graph shows that there is no profit until Sanibel sells 9 wallets. She breaks even when she sells 8 wallets. Her profit is negative when she sells less than 8 wallets.



3. $y = 4x - 20$

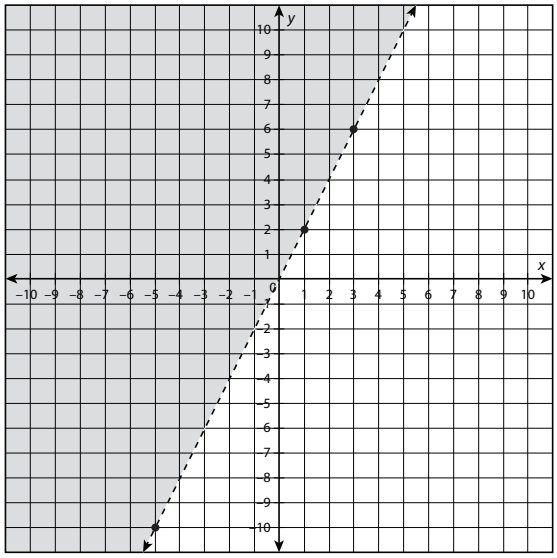


The graph shows that there is no profit until Sanibel sells 6 wallets. She breaks even when she sells 5 wallets. Her profit is negative when she sells less than 5 wallets.

4. In both cases, a more realistic graph would include only points on the line where the number of wallets is an integer, and not the entire line.

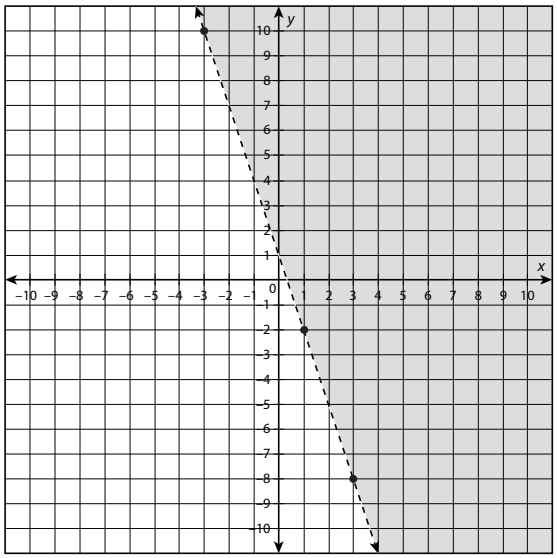
Practice 2.2.1: Solving Linear Inequalities in Two Variables, p. U2-102

1.

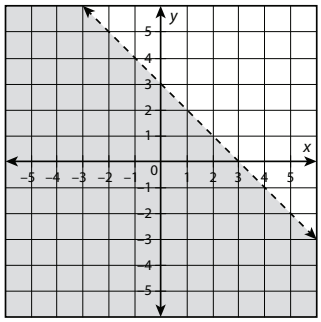


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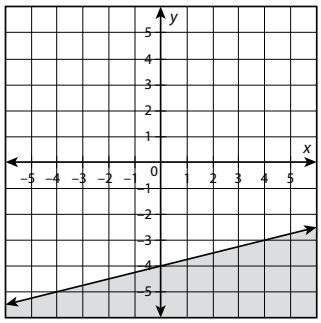
2.



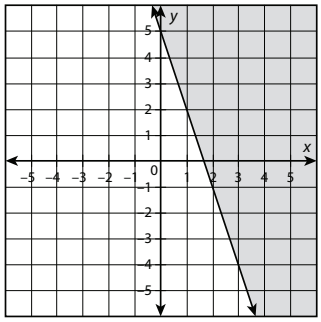
3.



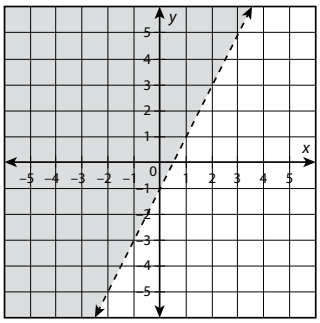
4.



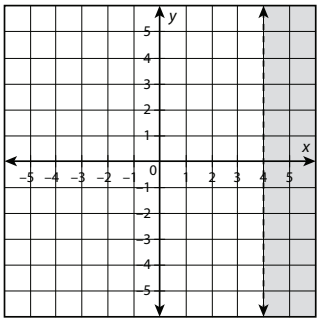
5.



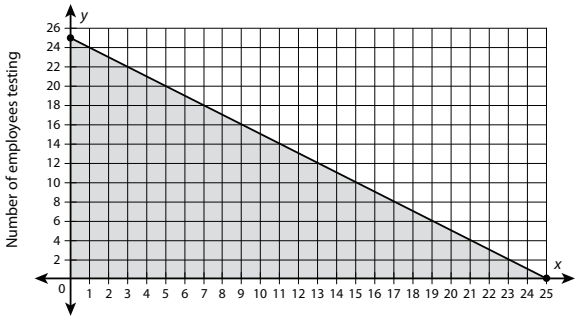
6.



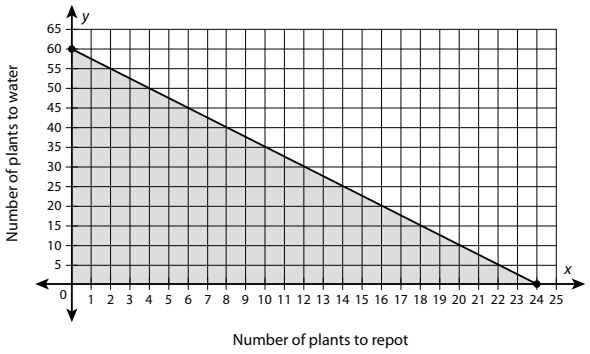
7.



8. $x + y \leq 25$ (Note that the ideal graph of this situation contains only integer pairs that are in the shaded region.)

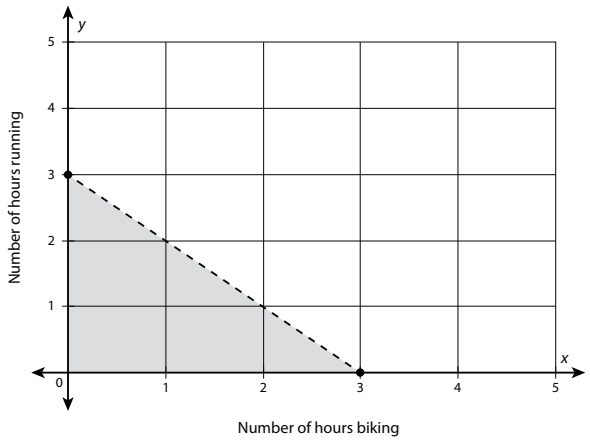


9. $5x + 2y \leq 120$ (Note that the ideal graph of this situation contains only integer pairs that are in the shaded region.)

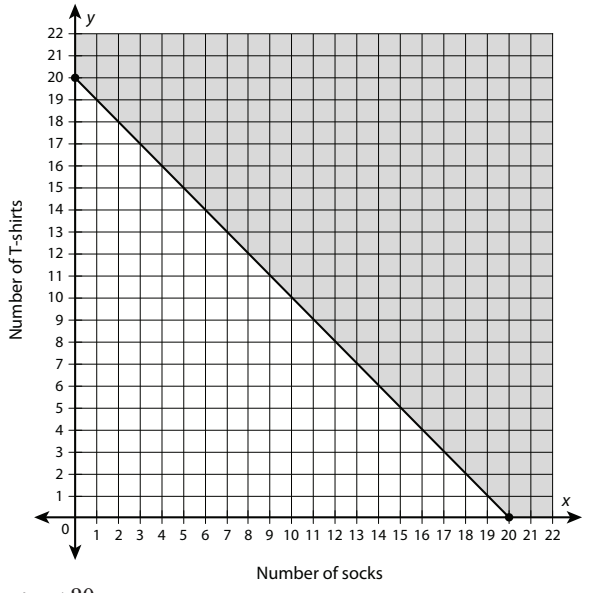


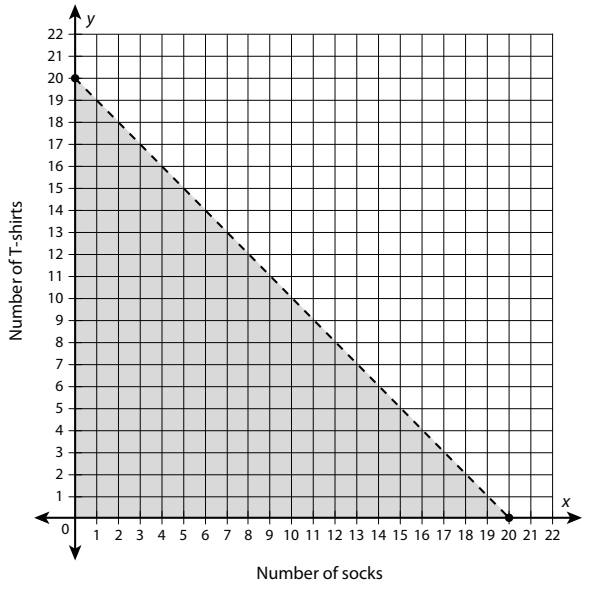
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10. $x + y < 3$



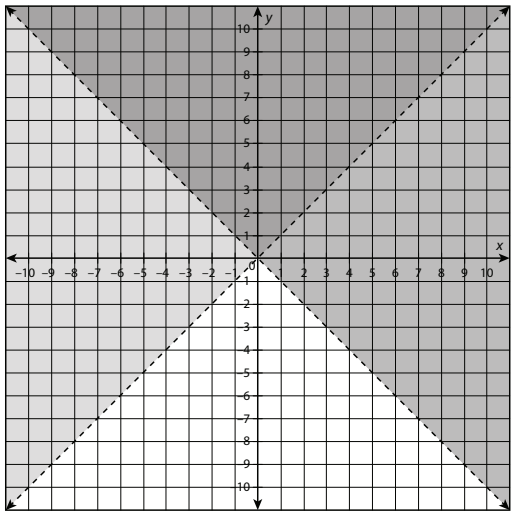
Warm-Up 2.2.2, p. U2-103

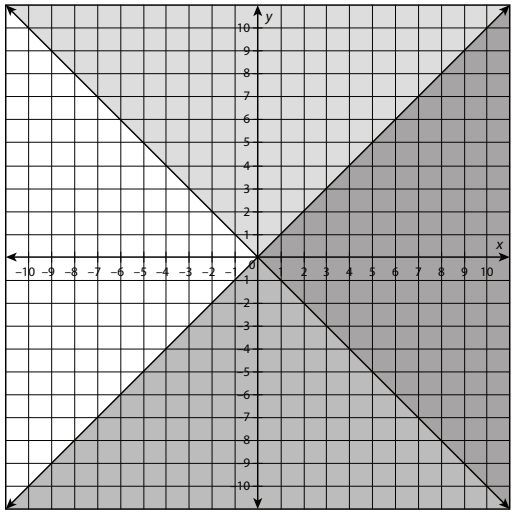
1. $x + y \geq 20$, for which x represents the number of socks and y represents the number of T-shirts
2. 

3. $x + y < 20$
4. 

5. Taylor's mother set up the situation so that if she finds "20 or more" socks and T-shirts she wins; if there are exactly 20 of them, Taylor's mother wins, and Taylor is grounded. This can also be seen on the graph for problem 2. The boundary for Taylor's mother winning is a solid line because it includes the cases where the total is 20. In the graph for problem 4, the boundary for Taylor winning is non-inclusive; therefore, the boundary is a dashed line because it does not include a possible total of 20 items.
6. The domain and range for these relations is the set of socks and T-shirts. These must be whole numbers. So, more accurate graphs would be sets of discrete points, such as (3, 9), (10, 8), and (15, 1).

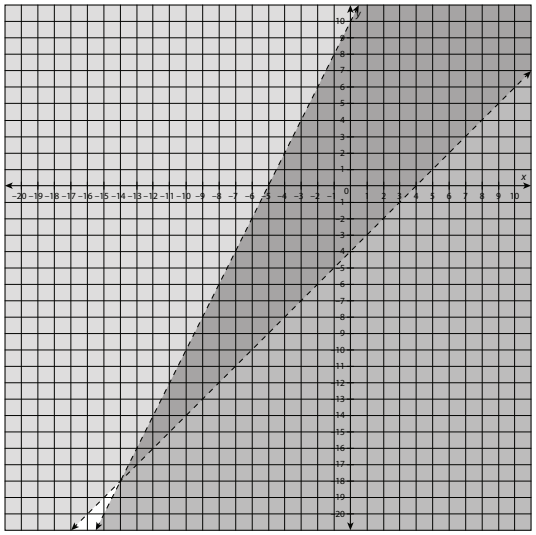
Practice 2.2.2: Solving Systems of Linear Inequalities, p. U2-123

1. 

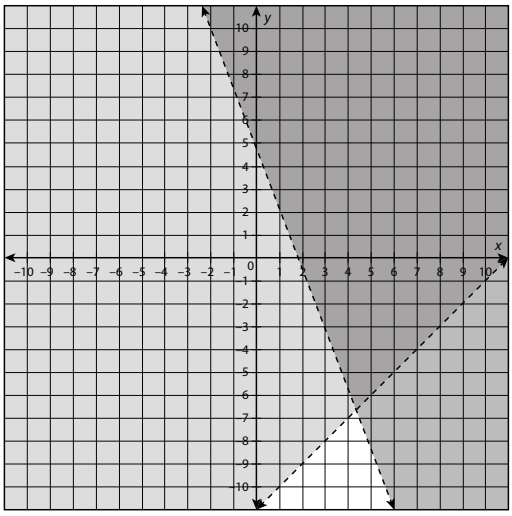
2. 

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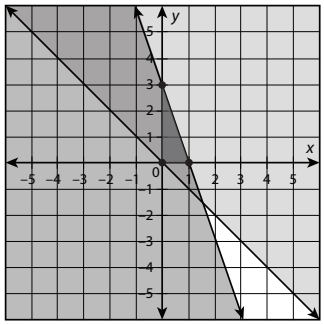
3.



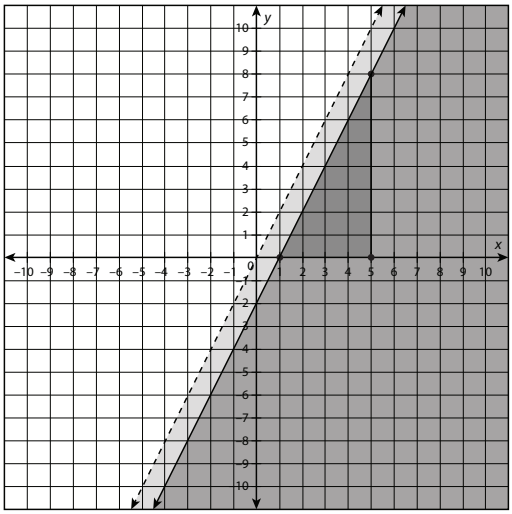
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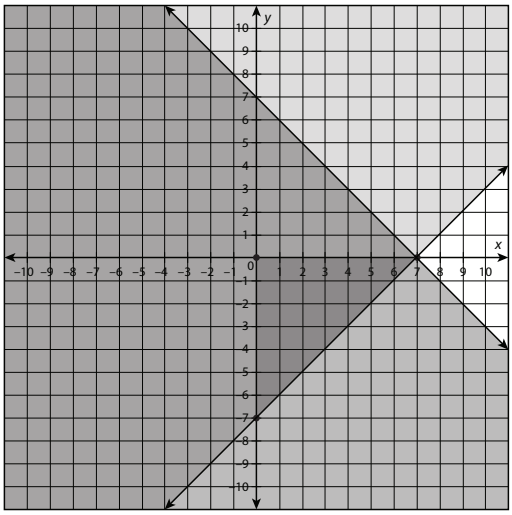
5.



6.

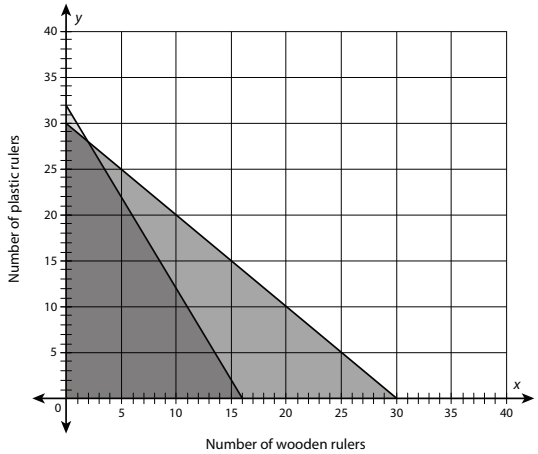


7.



8. a. $\begin{cases} 30x + 15y \le 480 \\ 6x + 6y \le 180 \\ x \ge 0 \\ y \ge 0 \end{cases}$

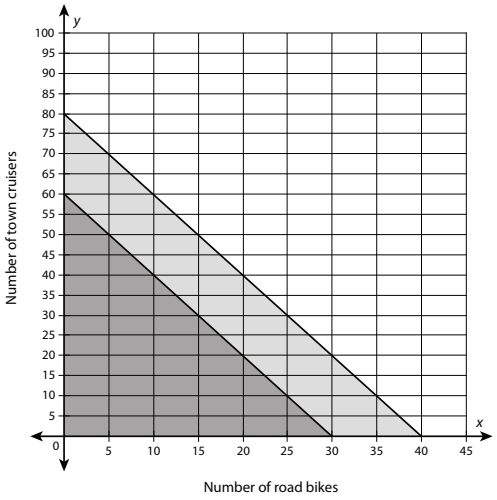
b. Graph of the system (note that the ideal graph of this situation contains only integer pairs that are in the darkly shaded region):



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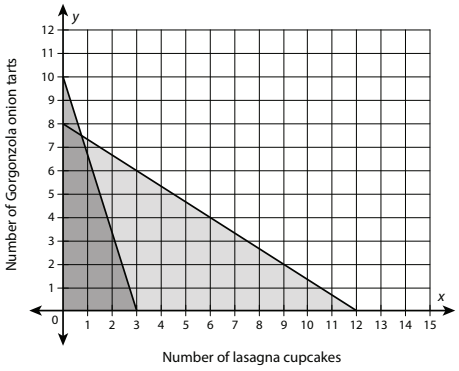
9. a. $\begin{cases} 2x + y \le 60 \\ \frac{1}{2}x + \frac{1}{4}y \le 20 \\ x \ge 0 \\ y \ge 0 \end{cases}$

b. Graph of the system (note that the ideal graph of this situation contains only integer pairs that are in the darkly shaded region):



10. a. $\begin{cases} 10x + 15y \le 120 \\ 20x + 6y \le 60 \\ x \ge 0 \\ y \ge 0 \end{cases}$

b. Graph of the system (note that the ideal graph of this situation contains only integer pairs that are in the darkly shaded region):

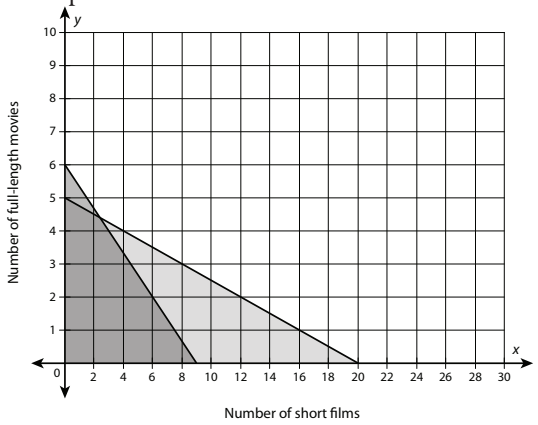


Progress Assessment, p. U2-125

1. b
2. a
3. d
4. d
5. b
6. d
7. d
8. d
9. b
10. a

11. a. $\begin{cases} 6x + 9y \le 54 \\ \frac{1}{2}x + 2y \le 10 \\ x \ge 0 \\ y \ge 0 \end{cases}$

b. Graph:



Lesson 3: Sequences As Functions

Pre-Assessment, p. U2-132

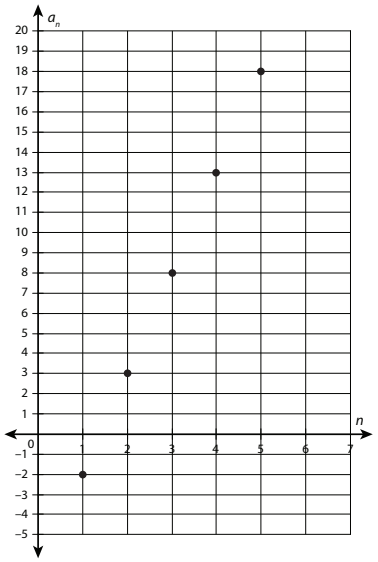
1. b
2. a
3. d
4. d
5. c

Warm-Up 2.3.1, p. U2-135

1. Saturday, May 7
2. Friday, May 13
3. Thursday, May 19

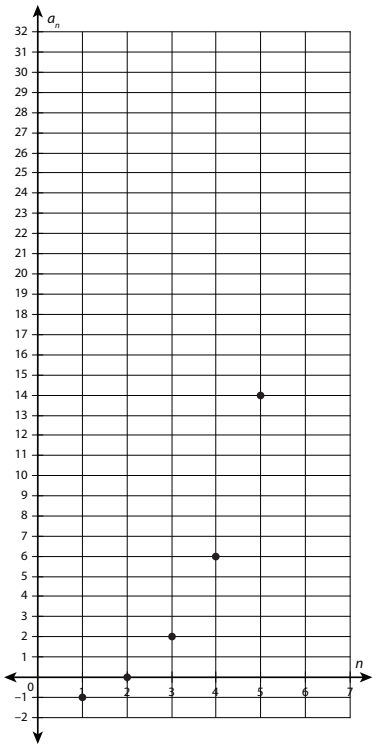
Practice 2.3.1: Sequences As Functions, p. U2-155

1. 28
2. 5
3.

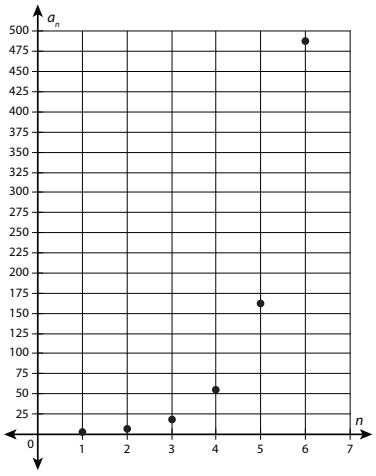


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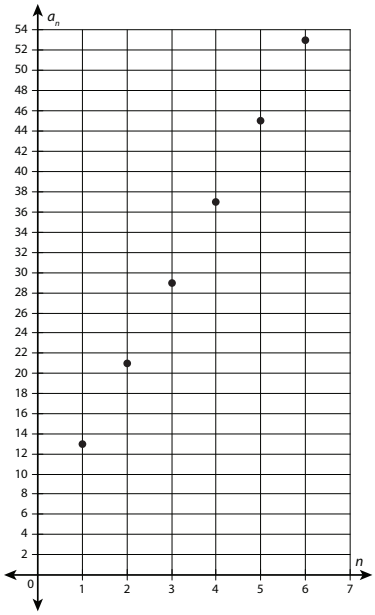
4.



5. 32
6. 104
7. $a_5 = 162$



8. $a_5 = 45$, $a_6 = 53$



9. 2:01 P.M.
10. The fifth water station will be at the 19-mile mark.

Progress Assessment, p. U2-156

1. b
2. c
3. d
4. d
5. a
6. a
7. b
8. d
9. c
10. c
11. $a_6 = 21$; the sixth place Shelly stops is 21 feet above sea level.

Lesson 4: Interpreting Graphs of Functions

Pre-Assessment, p. U2-158

1. c
2. a
3. d
4. b
5. a

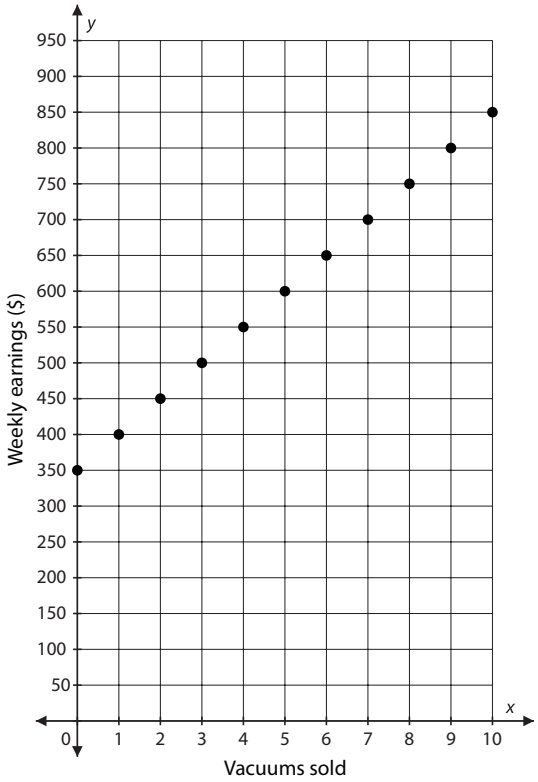
Warm-Up 2.4.1, p. U2-164

1. Answers will vary. One possible response: Kim rode her bike to her friend's house. She stayed at her friend's house for a while. Then she left her friend's house and rode to a store, which is even farther away from her house. She stayed at the store for a short time and bought a couple of items. Kim then headed back toward her house, stopping once more to take a picture of beautiful statue along the way. She then biked the rest of the way back home.
2. The horizontal lines represent times when Kim stayed at one location.

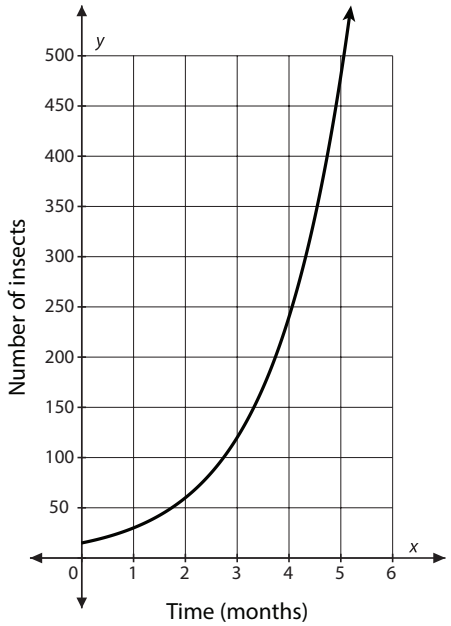
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Practice 2.4.1: Identifying Key Features of Linear and Exponential Graphs, p. U2-183

1. domain: whole numbers



2. domain: whole numbers



3. *x*-intercept: 7 1/7; *y*-intercept: (0, 50); the function is decreasing; the function is positive for $0 \le x \le 7\ 1/7$; minimum: 0; maximum: 50; asymptote: there is no asymptote; domain: $0 \le x \le 7\ 1/7$
4. *x*-intercept: (24, 0); *y*-intercept: (0, 1200); the function is decreasing; the function is positive for $0 \le x \le 24$; minimum: 0; maximum: 1200; asymptote: there is no asymptote; domain: $0 \le x \le 24$, where *x* is a whole number
5. *x*-intercept: (0, 0); *y*-intercept: (0, 0); the function is increasing; the function is positive for $x \ge 0$; minimum: 0; maximum: there is no maximum; asymptote: there is no asymptote; domain: $x \ge 0$
6. *x*-intercept: none; *y*-intercept: (0, 110); the function is increasing; the function is positive for $x \ge 0$; minimum: 110; maximum: there is no maximum; asymptote: there is no asymptote; domain: $x \ge 0$
7. *x*-intercept: none; *y*-intercept: (0, 300); the function is decreasing; the function is positive for $x \ge 0$; minimum: there is no minimum; maximum: 300; asymptote: $y = 0$; domain: $x \ge 0$
8. *x*-intercept: none; *y*-intercept: (0, 9); the function is increasing; the function is positive for $x \ge 0$; minimum: 9; maximum: there is no maximum; domain: $x \ge 0$
9. *x*-intercept: none; *y*-intercept: (0, 1200); the function is increasing; the function is positive for $x \ge 0$; minimum: 1200; maximum: there is no maximum; domain: $x \ge 0$
10. *x*-intercept: none; *y*-intercept: (0, 3000); the function is increasing; the function is positive for $x \ge 0$; minimum: 3000; maximum: there is no maximum; domain: $x \ge 0$

Warm-Up 2.4.2, p. U2-188

1. Boat A will be worth $5,145 after 3 years.
2. Boat B will be worth $6,144 after 3 years.
3. Boat B will be worth more than Boat A after 3 years.

Practice 2.4.2: Average Rate of Change, p. U2-203

1. –0.9
2. –32
3. 46.74
4. –15.18
5. 1.8
6. 0.08
7. 0.08
8. 7.5
9. 13.65
10. 14.77

Warm-Up 2.4.3, p. U2-205

1. The rate of change from 1900 to 2000 was 2.03 million per year. The rate of change from 2000 to 2010 was 3.5 million per year. The rate of change for 1900 to 2000 was less than the rate of change for 2000 to 2010.
2. Highest rate of change: Either 1950 to 1960 or 1990 to 2000 could be described as the 10-year period with the highest rate of change, because they seem to have identical rates. Lowest rate of change: 1930 to 1940. Calculate the rate of change for each 10-year period.
3. Answers will vary. Sample answer: The population in 2020 could reach 348,000,000 if the rate of change continues at 3.5 million per year.

Practice 2.4.3: Recognizing Average Rate of Change, p. U2-220

1. ≈ 0.001 gallons per foot
2. ≈ 0.001 gallons per foot
3. ≈ 0.8 euros per U.S. dollar
4. ≈ 0.8 euros per U.S. dollar
5. Yes, a prediction is possible. Sample prediction: The rate of change would be the same as the rate of change in questions 3 and 4 because the function is linear.

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6. ≈ 25 visitors per year
7. ≈ 29 visitors per year
8. ≈ –271 people per year
9. ≈ –167 people per year
10. The rate of change for the interval [2, 9] is steeper than that for the interval [14, 20]. The population is decreasing at a faster rate for the interval [2, 9] than it is for the interval [14, 20].

Progress Assessment, p. U2-224

1. c
2. a
3. a
4. c
5. b
6. b
7. a
8. c
9. b
10. a
11. Answers:
    a. All values are approximate. The function has one *y*-intercept, (0, 125), but does not have an *x*-intercept. The function is increasing and is positive for $x \geq 0$. The relative minimum is 125, the initial population, but there is no relative maximum because the population continues to increase. The function has a horizontal asymptote at $y = 0$.
    b. The domain of this function is all real numbers greater than or equal to 0, or $x \geq 0$. Negative values are not included because you cannot have negative days.
    c. The rate of change varies greatly depending on the interval you choose to observe. The population initially increases at a less rapid rate because the graph is less steep. The interval [0, 6] has a much less rapid increase than the interval [12, 18]. However, each rate of change is positive, showing growth.

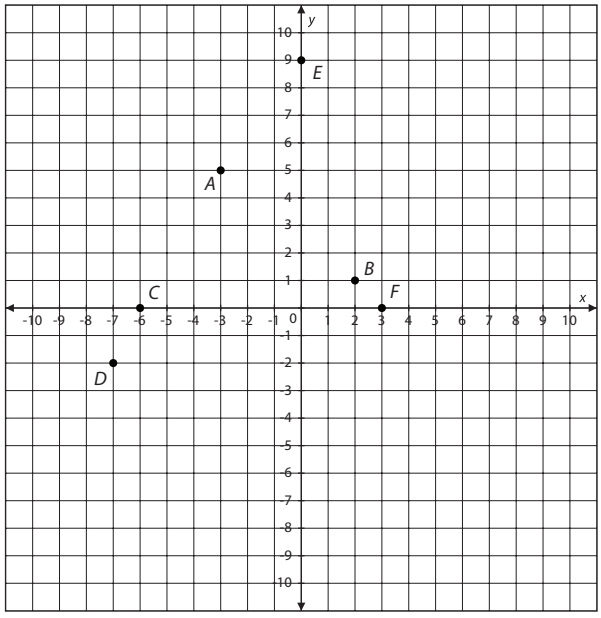
Lesson 5: Analyzing Linear and Exponential Functions

Pre-Assessment, p. U2-232

1. a
2. d
3. c
4. a
5. d

Warm-Up 2.5.1, p. U2-238

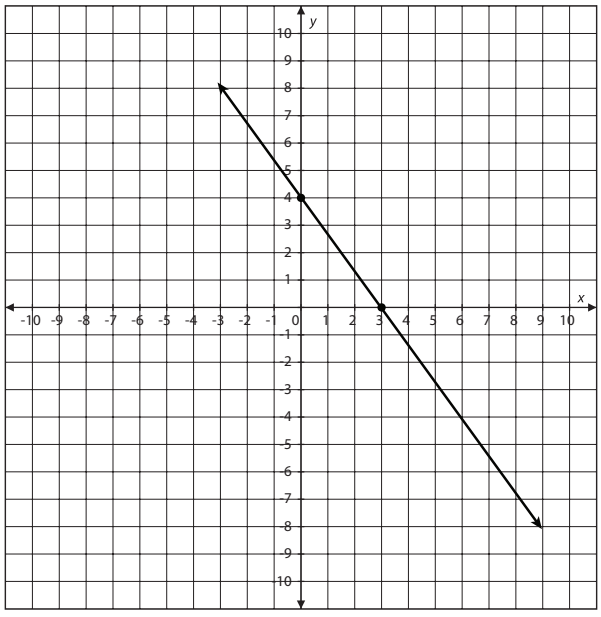
1.



2. Point *A* (–3, 5) is in the second quadrant. Point *B* (2, 1) is in the first quadrant. Point *C* (–6, 0) is on the *x*-axis, so –6 is an *x*-intercept. Point *D* (–7, –2) is in the third quadrant. Point *E* (0, 9) is on the *y*-axis, so 9 is a *y*-intercept. Point *F* (3, 0) is on the *x*-axis, so 3 is an *x*-intercept.

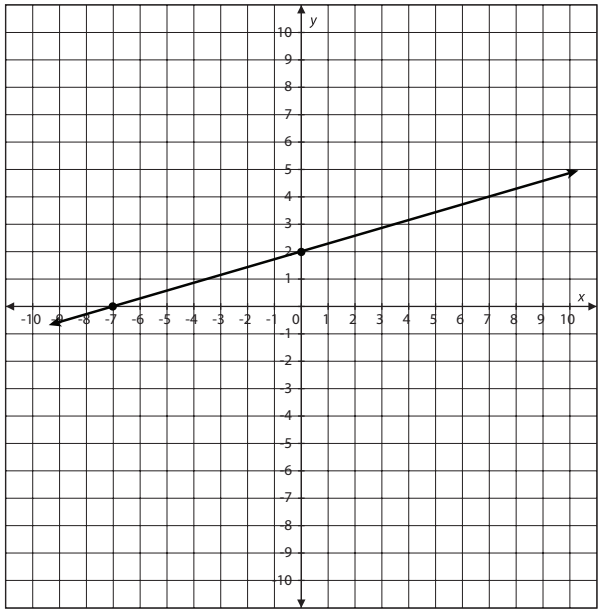
Practice 2.5.1: Graphing Linear Functions, p. U2-254

1. *x*-intercept: 3; *y*-intercept: 4

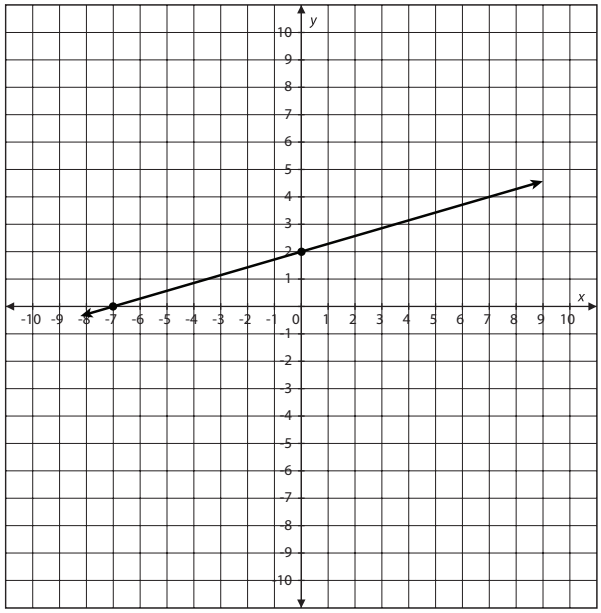


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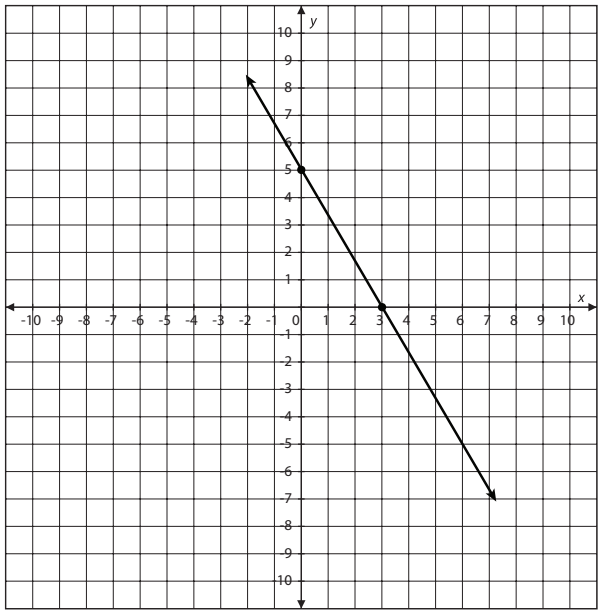
2. *x*-intercept: –7; *y*-intercept: 2



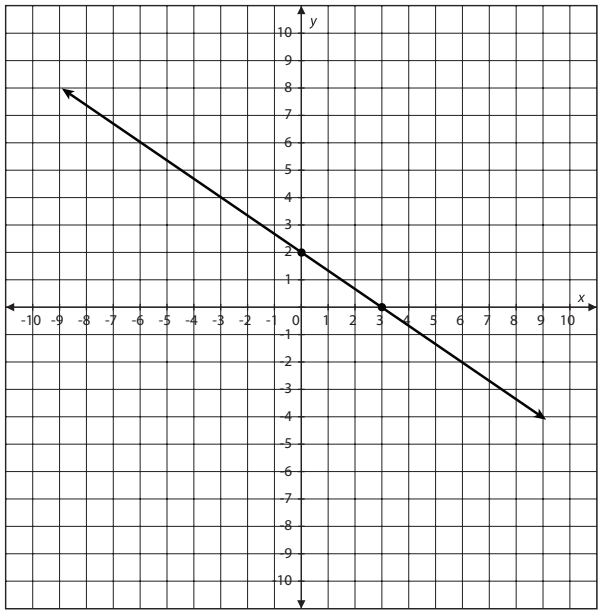
4. *x*-intercept: –7; *y*-intercept: 2



3. *x*-intercept: 3; *y*-intercept: 5

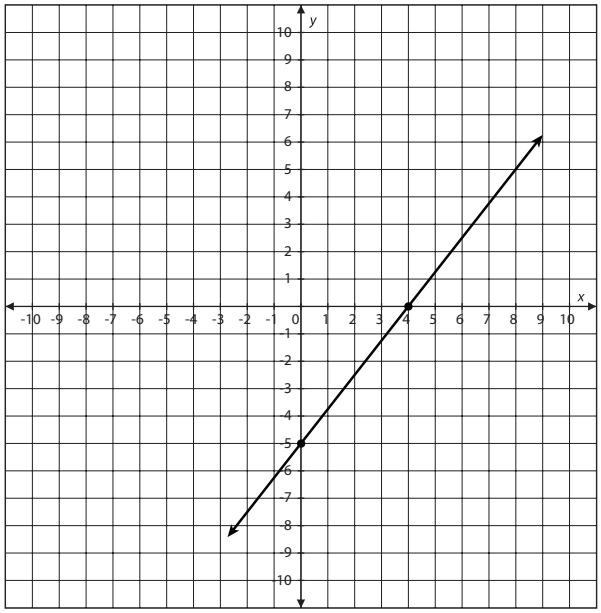


5. *x*-intercept: 3; *y*-intercept: 2

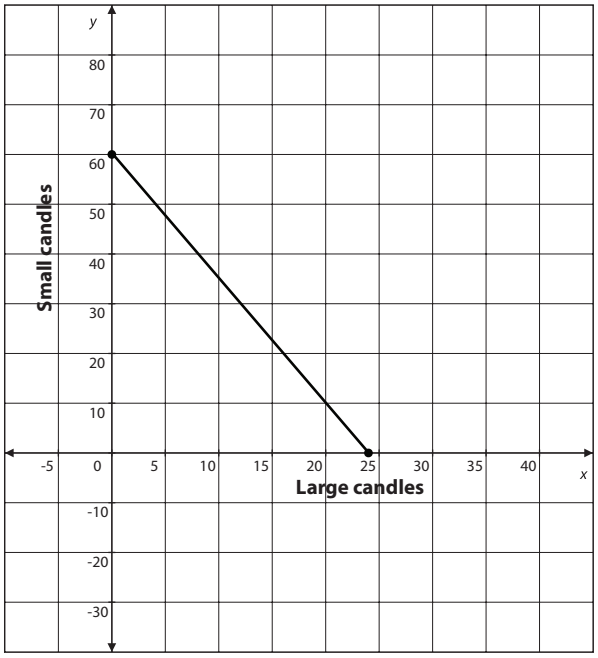


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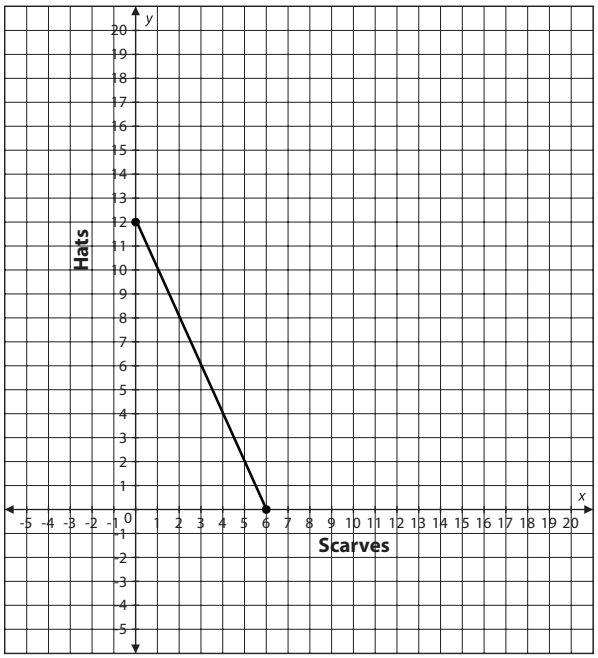
6. *x*-intercept: 4; *y*-intercept: –5



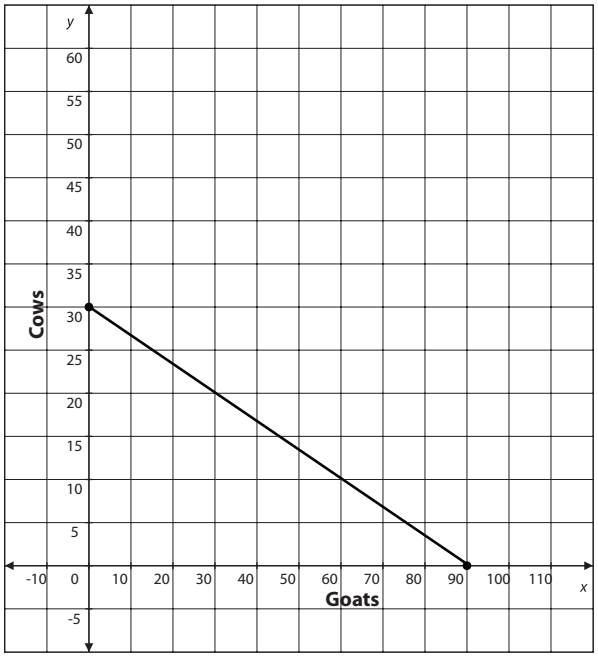
7. 24 large candles, 60 small candles



8. 6 scarves, 12 hats



9. 30 cows, 90 goats



10. Answers will vary; the relationship should be linear.

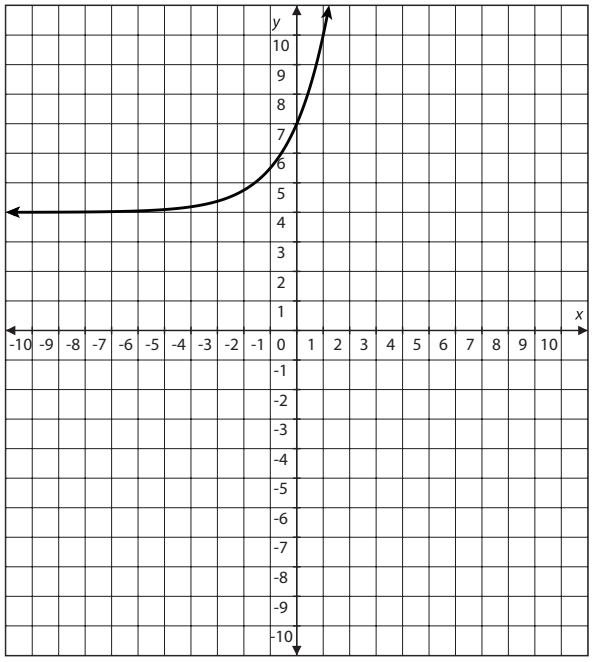
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Warm-Up 2.5.2, p. U2-256

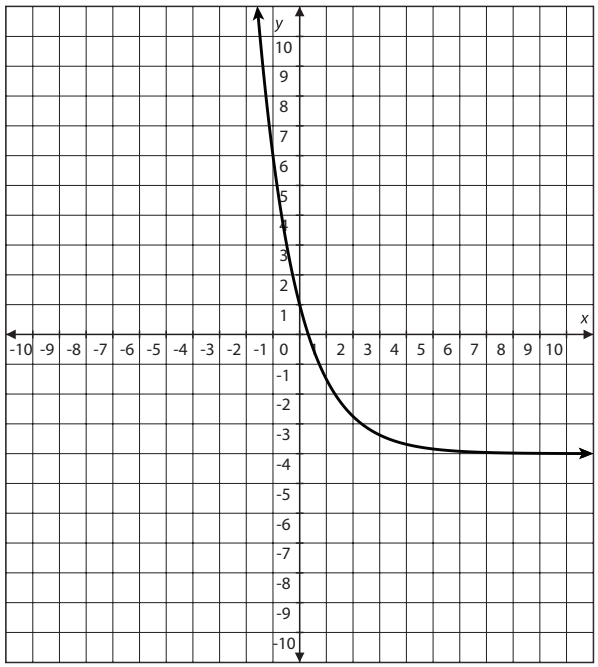
1. 23
2. 5

Practice 2.5.2: Graphing Exponential Functions, p. U2-274

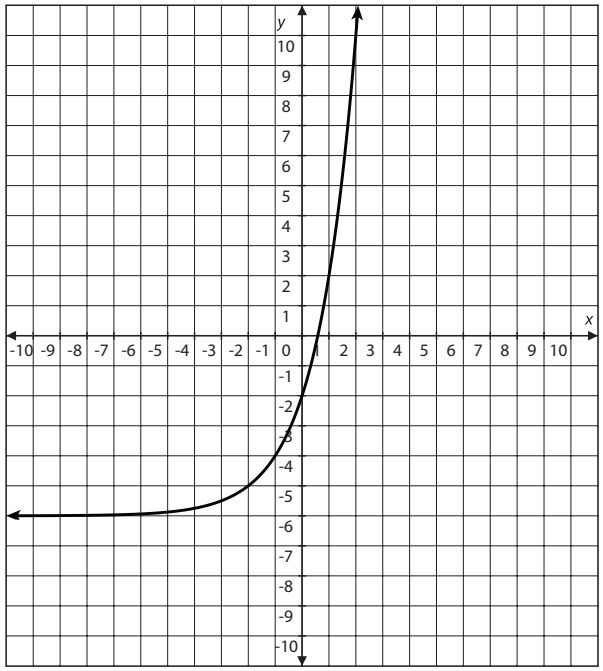
1. *y*-intercept: 7; growth, with a horizontal asymptote of $y = 4$



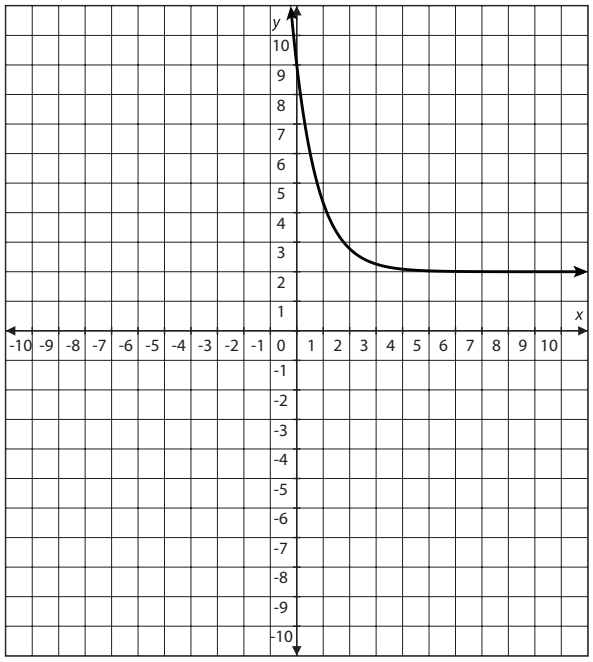
2. *y*-intercept: 1; decay, with a horizontal asymptote of $y = -4$



3. *y*-intercept: –2; growth, with a horizontal asymptote of $y = -6$



4. *y*-intercept: 9; decay, with a horizontal asymptote of $y = 2$



5. *y*-intercept: –1; decay, with a horizontal asymptote of $y = -3$
6. *y*-intercept: 1.25; growth, with a horizontal asymptote of $y = 1$
7. $f(x) = 100\left(\frac{1}{2}\right)^x$; 25 grams
8. $f(x) = 20(3^x)$; 540 organisms
9. $f(x) = 400(1.025^x)$; \$475.47
10. Answers will vary; the scenario should be an example of exponential growth.

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Progress Assessment, p. U2-276

1. d
2. d
3. a
4. a
5. d
6. b
7. b
8. c
9. d
10. c
11. 9,200 grams

Lesson 6: Comparing Functions

Pre-Assessment, p. U2-282

1. c
2. a
3. b
4. c
5. b

Warm-Up 2.6.1, p. U2-287

1. –300 meters per second
2. $x \geq 0$; $y \geq 0$ and $y \leq 2700$

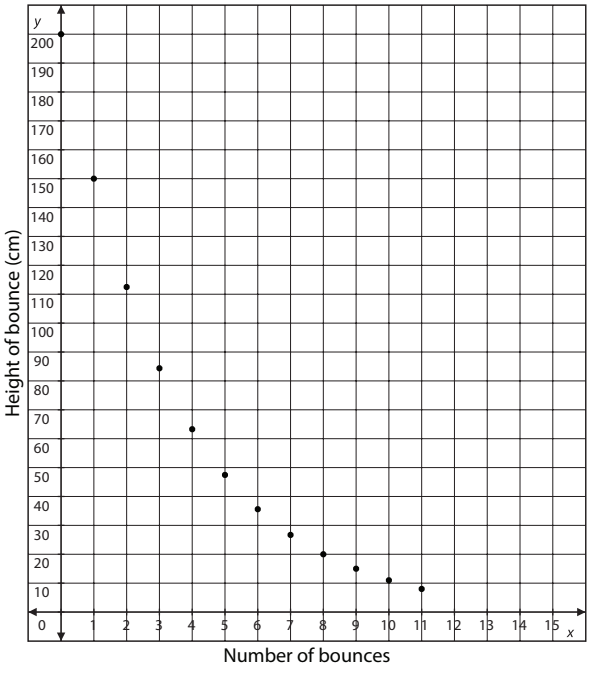
Practice 2.6.1: Comparing Linear Functions, p. U2-302

1. Function A has a rate of change of –4 and a *y*-intercept of –4. Function B has a rate of change of 2 and a *y*-intercept of 3. Function A has a greater rate of change because the absolute value of –4 is greater than the absolute value of 2. Function B has a greater *y*-intercept because 3 is greater than –4.
2. Function A has a rate of change of 1/8 and a *y*-intercept of 2. Function B has a rate of change of –1/5 and a *y*-intercept of –7. Function B has a greater rate of change because the absolute value of –1/5 is greater than the absolute value of 1/8. Function A has a greater *y*-intercept because 2 is greater than –7.
3. Function A has a rate of change of 1/4 and a *y*-intercept of 3. Function B has a rate of change of 0 and a *y*-intercept of 3. Function A has a greater rate of change. Both functions intersect the *y*-axis at (0, 3).
4. Function A has a rate of change of –5 and a *y*-intercept of 0. Function B has a rate of change of 5/4 and a *y*-intercept of 0. Function A has a greater rate of change because the absolute value of –5 is greater than the absolute value of 5/4. Both functions intersect the *y*-axis at (0, 0).
5. Function A has a rate of change of 1.17 and a *y*-intercept of 0. Function B has a rate of change of 0.40 and a *y*-intercept of 0. Function A has a greater rate of change. Both functions intersect the *y*-axis at (0, 0).
6. Function A has a rate of change of 150 and a *y*-intercept of 1,300. Function B has a rate of change of 225 and a *y*-intercept of 950. Function B has a greater rate of change and Function A has a greater *y*-intercept.
7. Function A has a rate of change of 4 and a *y*-intercept of 40. Function B has a rate of change of 7 and a *y*-intercept of 25. Function B has a greater rate of change and Function A has a greater *y*-intercept.
8. Function A has a rate of change of –130 and a *y*-intercept of 1,560. Function B has a rate of change of –130 and a *y*-intercept of 1,600. Both functions have the same rate of change. Function B has a greater *y*-intercept.
9. Function A has a rate of change of –0.25 and a *y*-intercept of 7.5. Function B has a rate of change of –0.25 and a *y*-intercept of 10. Both functions are decreasing at the same rate, meaning that in both scenarios the puppy is being fed the same amount. Function B has a greater *y*-intercept, meaning that the package initially contains more puppy food than the package in Function A.
10. Function A has a rate of change of 20 and a *y*-intercept of 15. Function B has a rate of change of 15 and a *y*-intercept of 10. The rate of change of Function A is greater, meaning that Reggie is bicycling more miles each week than Zac. The *y*-intercept of Function A is also greater, meaning Reggie bicycled more miles the previous week than Zac.

Warm-Up 2.6.2, p. U2-307

1.

Number of bounces	Height of bounce (cm)
0	200
1	150
2	112.5
3	84.38
4	63.28
5	47.46
6	35.60
7	26.70
8	20.02
9	15.02
10	11.26
11	8.45



2. We can see from both the table and the graph that the height of the fifth bounce will be less than 50 centimeters.

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Practice 2.6.2: Comparing Exponential Functions, p. U2-326

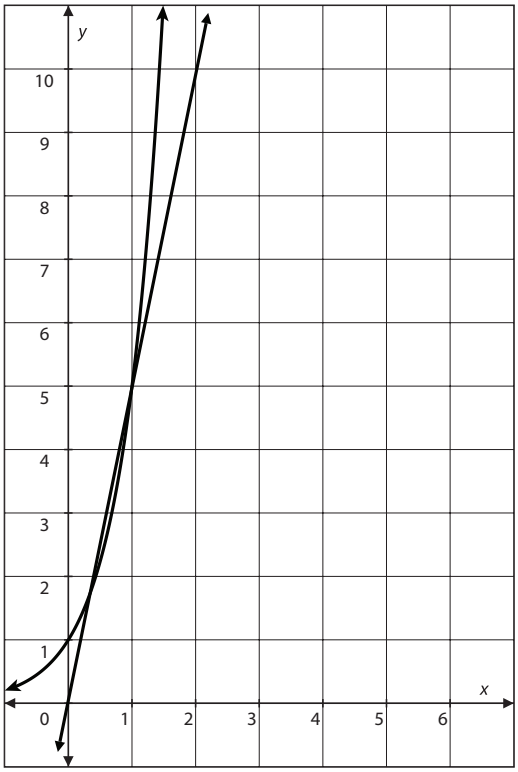
1. Function A has a greater rate of change because the slope for the interval [2, 8] is 89.98 compared to 66.7 for the same interval on Function B. Function B has a lower *y*-intercept of 1,300 compared to the *y*-intercept of 1,400 of Function A.
2. Function B has a greater rate of change of 6.2 compared to the rate of change for Function A of –0.19375. The *y*-intercepts are the same at (0, 1).
3. Function A has a greater rate of change over the interval [2, 8]. The rate of change for Function A is approximately 32.47, compared to 31.57, the rate of change for Function B. The *y*-intercept of Function B is less than Function A. Function B has a *y*-intercept of 350, as seen in the table. The *y*-intercept of Function A is 400, as seen in the equation.
4. Function B has a greater rate of change. The approximate rate of change for the interval [0, 5] is 100, whereas the rate of change of Function A for the same interval is 18.6. The *y*-intercept of Function A is 3, and the *y*-intercept of Function B is 0.
5. Function A has a greater rate of change than Function B. The rate of change for Function A over the interval [0, 10] is –43.313 cubic inches per day, compared to approximately –33 cubic inches per day. The absolute value of –43.313 is greater than the absolute value of –33. Both beach balls have the same *y*-intercept of 800, as stated in the original problem and seen in the graph of Function B. The beach ball in Function A is losing air at a faster rate than Function B.
6. Function B has a greater rate of change with $624.48 per year, compared to $494.62 per year for Function A. Function A has a greater *y*-intercept of $32,000, as stated in the problem. The equation shows that the *y*-intercept of Function B is $30,000.
7. Function A has a greater rate of change with 44.12 students per year, compared to Function B with –27.75 per year. The absolute value of 44.12 is greater than the absolute value of –27.75. Function B has a greater *y*-intercept with 1,900 students, compared to 1,700 for Function A. The enrollment of the students in Function B is greater at the start, but is decreasing compared to the enrollment in Function A, which is increasing.
8. Function A has a rate of change of $2.205 per year compared to approximately $2 per year for Function B. The *y*-intercept of Function A is greater at (0, 52) than the approximate *y*-intercept of (0, 38) of Function B. The stamp in Function A was purchased at a greater price and increases in value at a faster rate than the stamp in Function B.
9. Function A has a greater rate of change, with a decrease of $2,542.42 per year, compared to Function B, which has a decrease of $2,432 per year. The *y*-intercept of Function A is greater at (0, 22,450), as seen in the table, compared to (0, 19,375), as seen in the equation of Function B. The car represented by Function B costs less and depreciates at a lower rate.
10. Function A has a greater rate of change with $45.42 per year compared to the approximate rate of change of Function B of $35 per year. Both functions have a *y*-intercept of (0, 1000), as seen in the equation and on the graph. If the same amount were invested, Function A would increase at a faster rate than Function B.

Warm-Up 2.6.3, p. U2-331

1. $f(x) = 1(2)^{\frac{x}{20}}$
2. approximately 0.12 bacteria per minute
3. 262,144 bacteria

Practice 2.6.3: Comparing Linear to Exponential Functions, p. U2-346

1. $g(x) = 5^x$ will eventually increase at a faster rate than $f(x) = 5x$.



2. $g(x) = 2^x - 4$ will eventually increase at a faster rate than $f(x) = 2x - 4$.

x	$f(x) = 2x - 4$	$g(x) = 2^x - 4$
0	–4	–3
1	–2	–2
2	0	0
3	2	4

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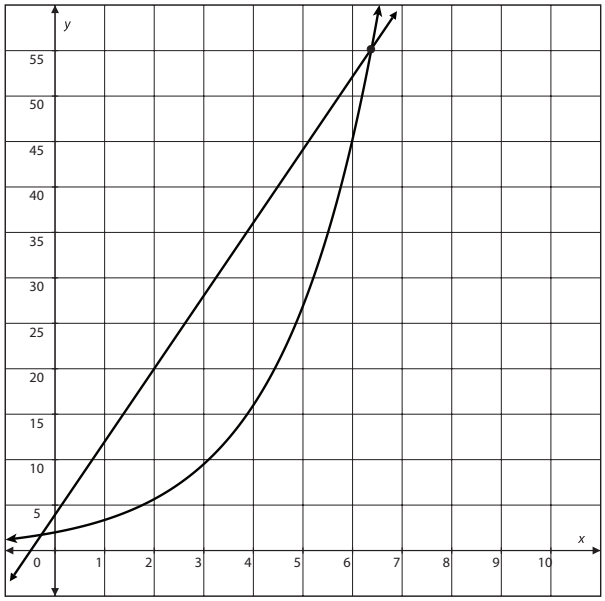
3. $g(x) = 200 - 3.6x$ will eventually decrease at a faster rate than $f(x) = 200(0.97)^x$.



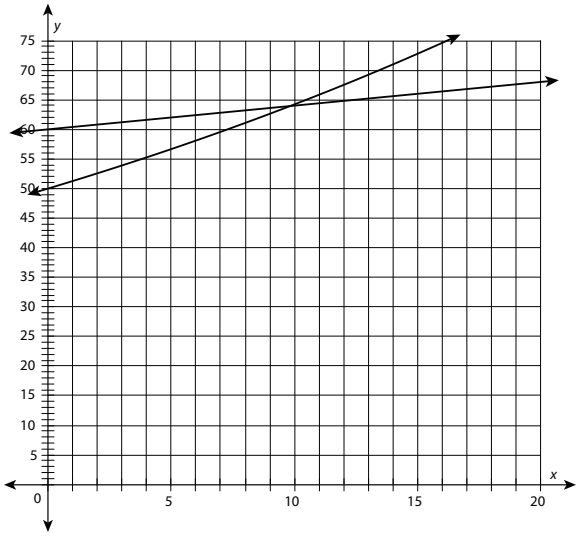
4. $f(x) = 1000(0.85)^x$ will eventually decrease at a faster rate than $g(x) = 1000 - 0.15x$.

x	$g(x) = 1000 - 0.15x$	$f(x) = 1000(0.85)^x$
0	1000	1000
10	998.5	196.87
20	997	38.76
30	995.5	7.63

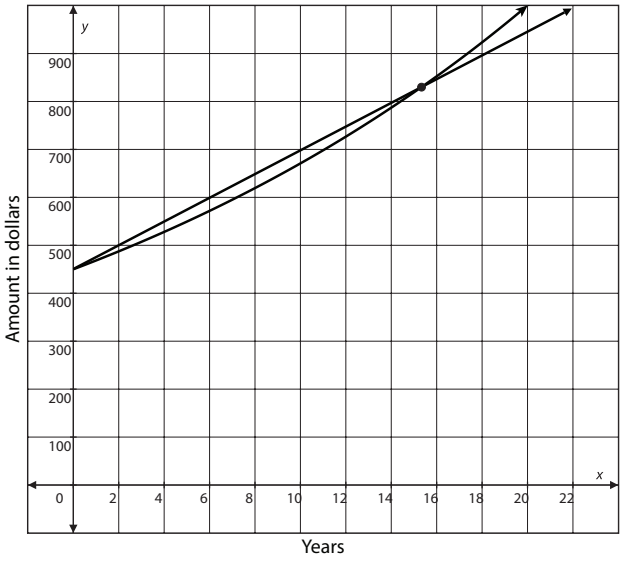
5. The graphs intersect at approximately (6.4, 55); $f(x) > g(x)$ when $x > 6.4$.



6. The graphs intersect at approximately (9.8, 63.9); $f(x) > g(x)$ when $x > 9.8$.



7. $g(x) = 450\left(1 + \frac{0.04}{12}\right)^{12x}$ will eventually increase at a faster rate than $f(x) = 450 + 450(0.055)x$.



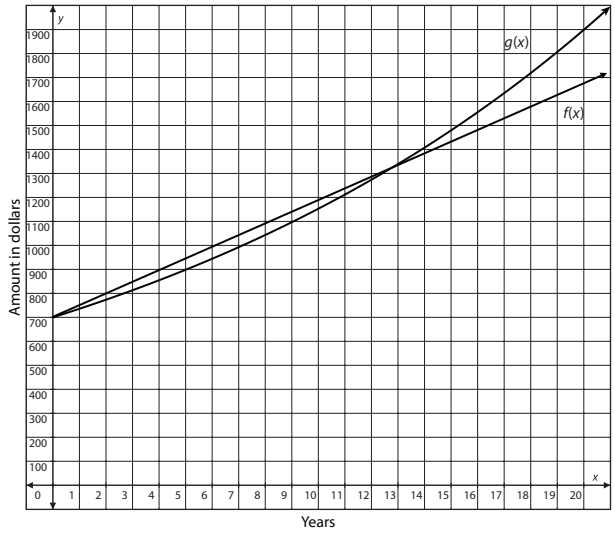
8. At approximately (15, 825), the amount invested will be equal in both savings accounts.
9. Choose $f(x) = 450 + 450(0.055)x$ if investing for less than 10 years. At this rate, you will always have more money than with the option for $g(x)$.
10. Choose $g(x) = 450\left(1 + \frac{0.04}{12}\right)^{12x}$ if investing for more than 20 years. At this rate, you will always have more money than with the option for $f(x)$.

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Progress Assessment, p. U2-347

1. a
2. c
3. a
4. b
5. a
6. a
7. a
8. b
9. b
10. b
11. Answers:

    a. $g(x)$ eventually exceeds $f(x)$.



    b. At approximately (12.5, 1350), the amount invested will be equal in both savings accounts. After that point, the value of $g(x)$ exceeds that of $f(x)$.

    c. Choose $f(x)$ if investing for less than 10 years because $f(x)$ is greater than $g(x)$ during the first 12 years.

    d. Choose $g(x)$ if investing for more than 25 years because $g(x)$ is greater than $f(x)$ after about 12.5 years.

Lesson 7: Building Functions

Pre-Assessment, p. U2-354

1. c
2. b
3. a
4. a
5. d

Warm-Up 2.7.1, p. U2-358

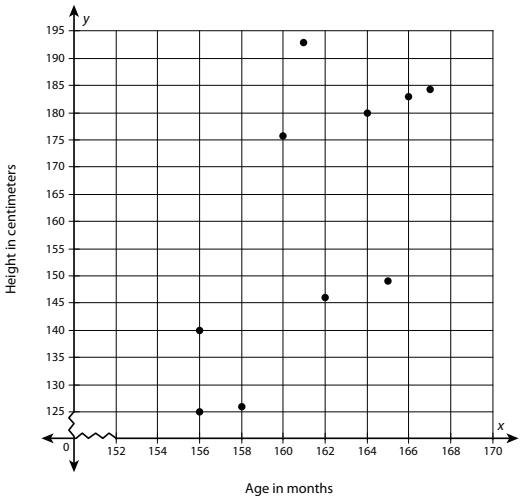
1. $c = 4p$; \$4
2. $d = 55t$; 330 miles
3. $t = 4s$; 120 students
4. $p = 12h$; 36 pages

Practice 2.7.1: Building Functions from Context, p. U2-376

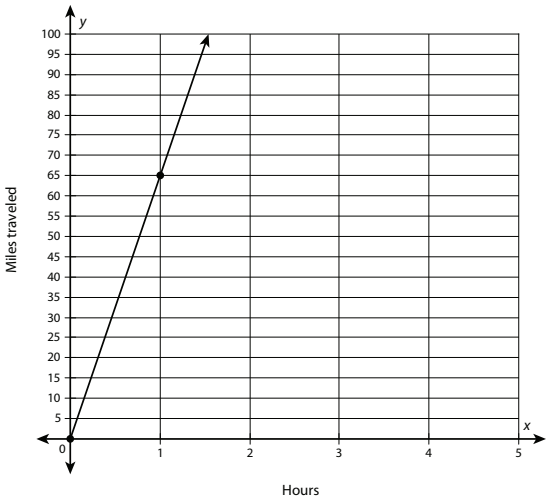
1. $f(x) = 19 + 12x$
2. $f(x) = -5x + 30$
3. $f(x) = 132 - 22x$
4. $f(x) = 12 \cdot (5)^{x-1}$
5. $f(x) = 32 \cdot (0.5)^{x}$
6. $f(x) = 2 + x$
7. $f(x) = 3^{x-1}$
8. $f(x) = 4^{x-1}$
9. $f(x) = 150 + 25(x - 1)$
10. $f(x) = 10{,}000 \cdot (1.10)^{x}$

Warm-Up 2.7.2, p. U2-378

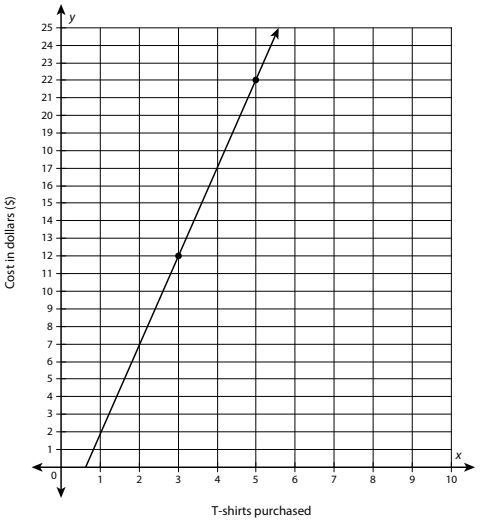
1.



2.



3.



Practice 2.7.2: Constructing Functions from Graphs and Tables, p. U2-394

1. $f(x) = -6x + 4$
2. $f(x) = 2x + 1$
3. $f(x) = -2 \cdot (2)^{x}$
4. $f(x) = 8 \cdot (0.5)^{x}$
5. $f(x) = -0.5x + 10$
6. $f(x) = 10 \cdot (4)^{x}$
7. $f(x) = -1.5x + 16$
8. $f(x) = 8x - 12$
9. $f(x) = -2 \cdot (7)^{x}$
10. $f(x) = -7 \cdot (8)^{x}$

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Progress Assessment, p. U2-396

1. c
2. a
3. b
4. d
5. a
6. b
7. c
8. a
9. a
10. d
11. $f(x) = -13 \bullet (9)^x$

Lesson 8: Operating on Functions and Transformations

Pre-Assessment, p. U2-400

1. b
2. c
3. b
4. c
5. b

Warm-Up 2.8.1, p. U2-405

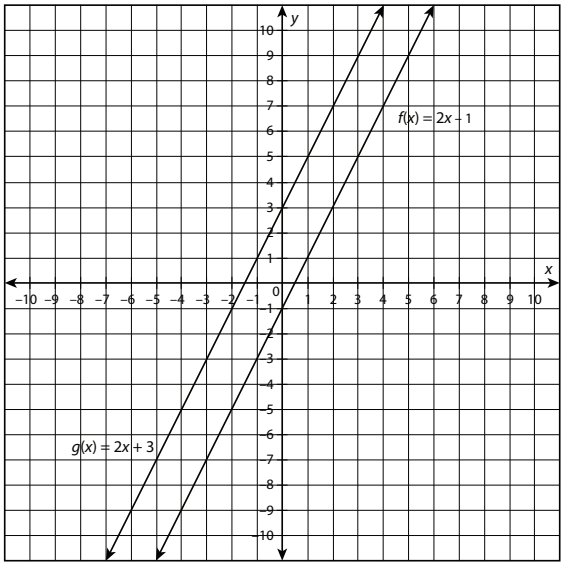
1. 40 pages
2. pages 20 and 21
3. 41
4. $4n + 1$
5. 820
6. $2n(4n + 1)$

Practice 2.8.1: Operating on Functions, p. U2-414

1. $8x + 3$
2. $2x - 7$
3. $4(5^x)$
4. $5^x/4$
5. $7x + 2$
6. $-7x - 6$
7. $-14x - 8$
8. $-\dfrac{2}{7x+4}$
9. $f(x) = 7x + 4$; $f(x) = 7x + 5$; 1 is added to the first function.
10. $f(x) = 2^{\frac{x}{4}}$; $f(x) = 3(2^{\frac{x}{4}})$; the first function is multiplied by 3.

Warm-Up 2.8.2, p. U2-415

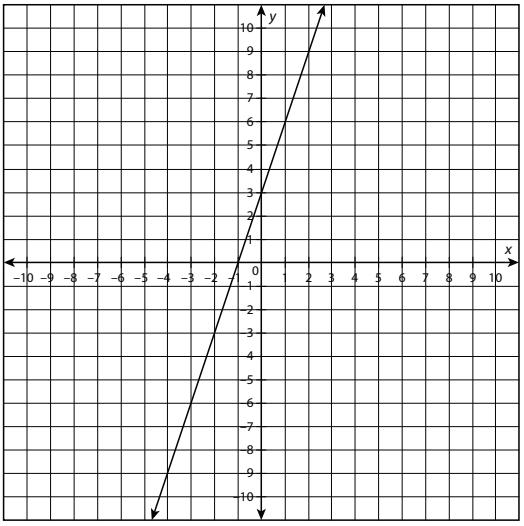
1. Graph:



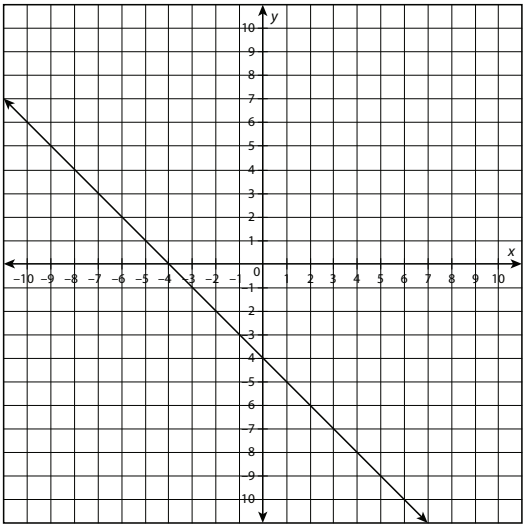
2. The lines have the same slope and are parallel.
3. The graphs have different y-intercepts.
4. $g(x)$ is the graph of $f(x)$ translated up 4 units.

Practice 2.8.2: Transformations of Linear and Exponential Functions, p. U2-426

1.

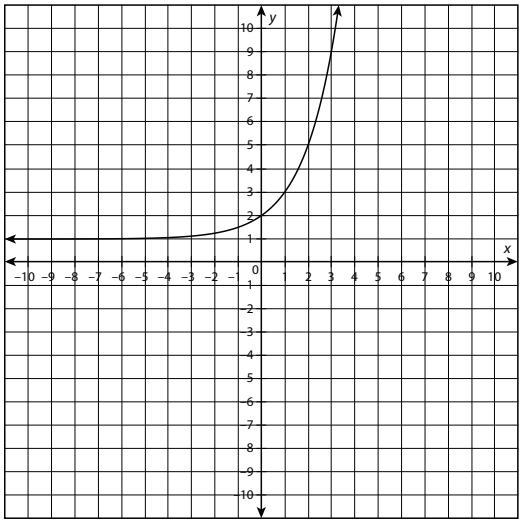


2.

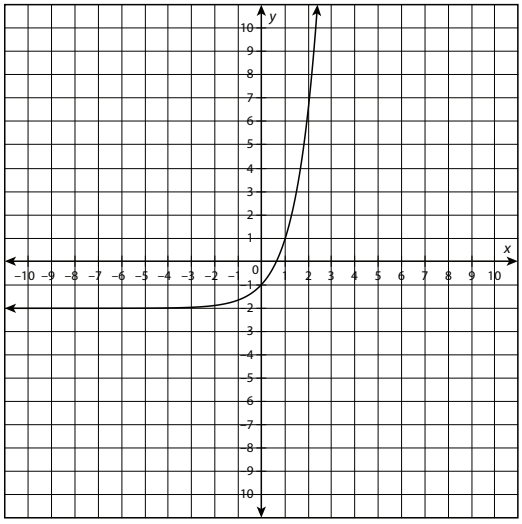


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3.



4.



5. $g(x) = f(x) - 5$
6. $g(x) = f(x) + 6$
7. $g(x) = f(x) + 4$
8. $g(x) = f(x) - 3$
9. –3
10. 4

Progress Assessment, p. U2-429

1. b
2. c
3. a
4. c
5. a
6. c
7. b
8. b
9. d
10. d
11. Aidan's monthly cost: $f(x) = 2.75x + 5$; Raj's monthly cost: $g(x) = f(x) - 1.25$

Lesson 9: Arithmetic and Geometric Sequences

Pre-Assessment, p. U2-434

1. c
2. d
3. d
4. a
5. c

Warm-Up 2.9.1, p. U2-437

1. 9
2. 33
3. –8
4. 6

Practice 2.9.1: Arithmetic Sequences, p. U2-447

1. $a_n = 4n + 23$
2. $a_n = -7n + 11$
3. $a_n = 14n - 115$
4. $a_n = 2n - \frac{3}{2}$
5. 3.2, 5.9, 8.6, 11.3, 14
6. 18, –4, –26, –48, –70
7. $a_n = 10n + 25$
8. $a_n = -15n + 105$, 5 days
9. $a_n = 3n + 9$, 36 miles
10. $a_n = 5n + 65$, 17 weeks

Warm-Up 2.9.2, p. U2-448

1. 81
2. 1
3. 64/81
4. 1/32

Practice 2.9.2: Geometric Sequences, p. U2-459

1. $a_n = 2^{n-1}$
2. $a_n = 10\left(-\frac{1}{5}\right)^{n-1}$
3. $a_n = 5(3)^{n-1}$
4. $a_n = 320\left(-\frac{1}{4}\right)^{n-1}$
5. –1, –3, –9, –27, –81
6. 216, 54, 13.5, 3.375, 0.84375
7. $a_n = 90(1.1)^{n-1}$
8. $623.34
9. $a_n = 4(2)^{n-1}$
10. $8874.11

Progress Assessment, p. U2-460

1. b
2. d
3. b
4. b
5. a
6. a
7. b
8. c
9. b
10. a
11. a. $a_n = 6(0.9)^{n-1}$
    b. 3.9366 ft

Lesson 10: Interpreting Parameters

Pre-Assessment, p. U2-462

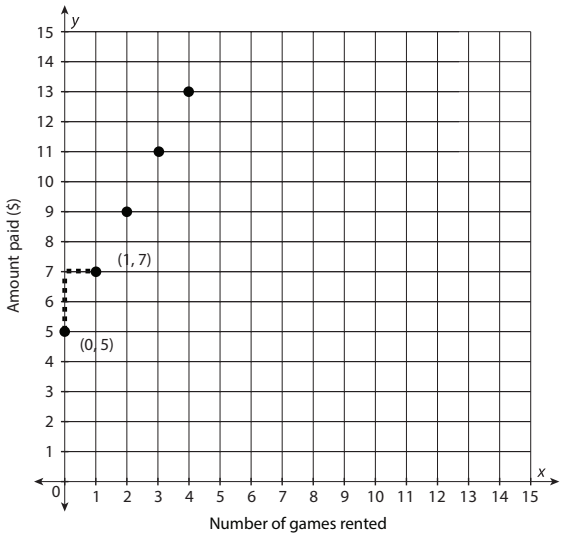
1. b
2. a
3. b
4. c
5. c

Warm-Up 2.10.1, p. U2-465

1. $f(x) = 2x + 5$
2. The domain is the set of non-negative integers.

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3. The *y*-intercept is 5.



4. The *y*-intercept is the membership fee.
5. The slope represents the $2 rental fee per game.

Practice 2.10.1: Interpreting Parameters, p. U2-480

1. 7 and 5
2. 2 and 3
3. –2 and 10
4. 2 and 3
5. 3, 2, and 5
6. $f(x) = 2x + 10$; 2 and 10
7. 1.25 and 5
8. Max picks 18 apples per minute and started with 15 apples in his bag.
9. There were 10 ants in the colony to start with, and the number of ants doubles every 36 hours.
10. $f(x) = 150(2^x) + 200$; 2, $150, and $200

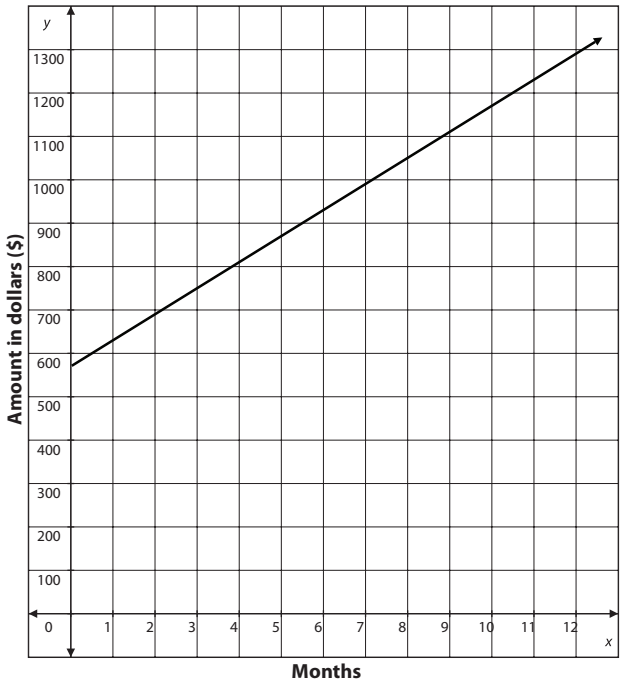
Progress Assessment, p. U2-481

1. d
2. b
3. c
4. b
5. a
6. c
7. d
8. c
9. a
10. a
11. The slope is the hourly rate of $13, and the *y*-intercept is the flat fee. Function: $f(x) = 13x + b$; flat fee: $20

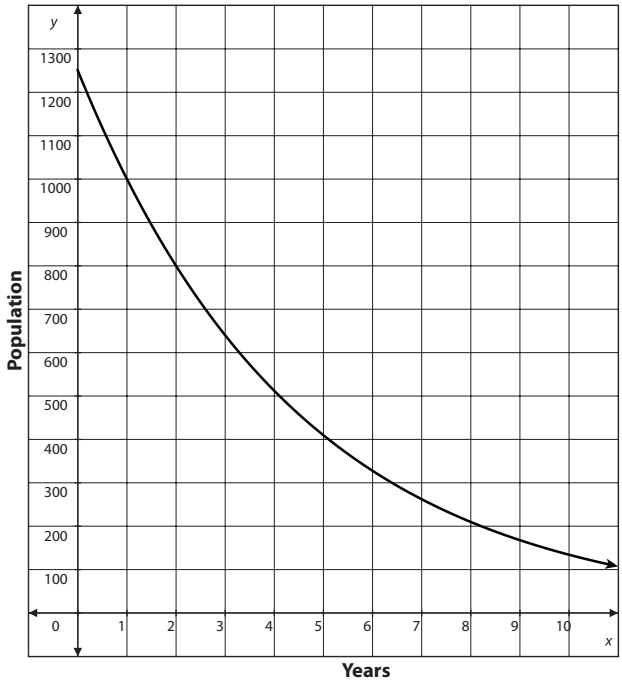
Unit Assessment p. U2-483

1. d
2. c
3. c
4. c
5. a
6. b
7. d
8. a
9. c
10. d
11. b
12. c

13. a. $f(x) = 60x + 575$
    b. Graph:



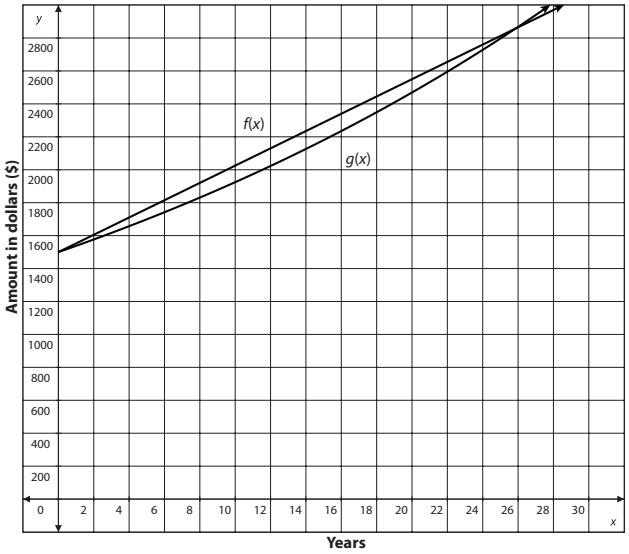
    c. The *y*-intercept is 575, but there is no *x*-intercept because the graph never intersects the *x*-axis. The minimum of this function is $575 and there is no maximum, since the graph continues infinitely. The function is increasing. The rate of change for this function is $60 per month.
14. a. $f(x) = 1250(0.8)^x$
    b. Graph:



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c. The y-intercept is 1,250, but there is no x-intercept because the graph never intersects the x-axis. The maximum of this function is 1,250. The rate of change over the interval [0, 10] is approximately 112 bees per year. The function has an asymptote of $y = 0$.

15. a. The graphs start at the same point, (0, 1500). The linear function $f(x)$ grows more quickly at first, but $g(x)$ passes $f(x)$ and eventually grows faster than $f(x)$. This can be seen in the following graph.



b. Initially, the graphs start at the same point, (0, 1500), but $f(x)$ grows steadily while $g(x)$ starts out growing more slowly. At approximately (26, 2800), the amount invested will be equal in both savings accounts.

c. Choose $g(x)$ if investing at the age of 20. Choose $f(x)$ if investing at the age of 50. In 26 years, the amounts will be equal. After 26 years, $g(x)$ will result in a greater investment than $f(x)$.

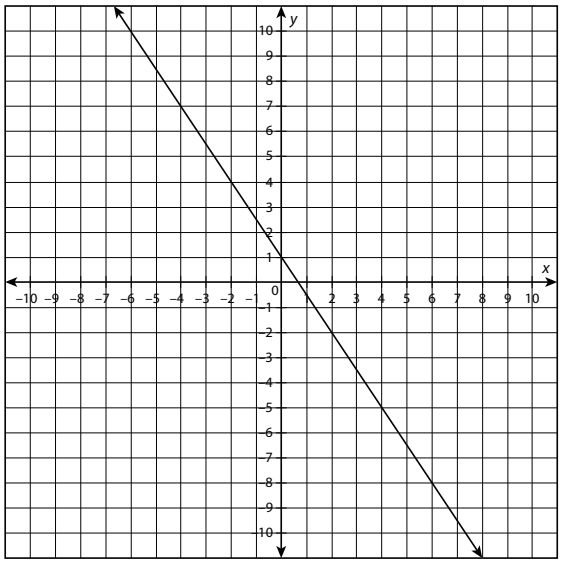
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Student Book Answer Key

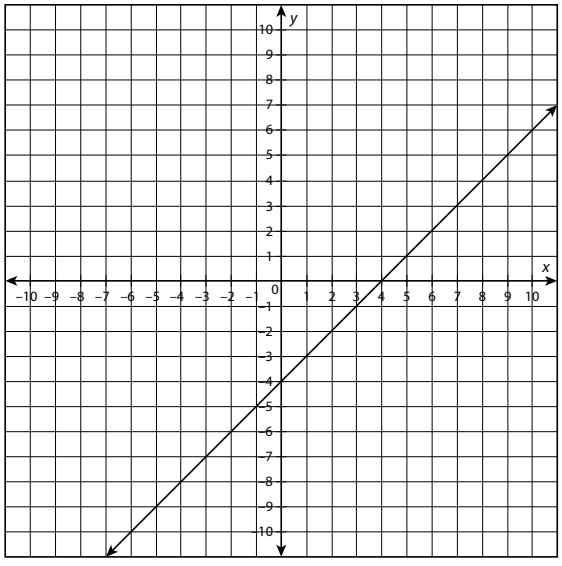
Lesson 1: Graphs As Solution Sets and Function Notation

Practice 2.1.1: Graphing the Set of All Solutions, p. U2-10

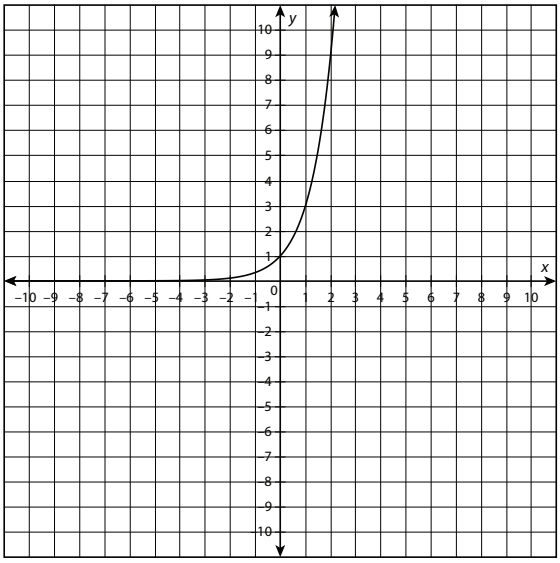
1.



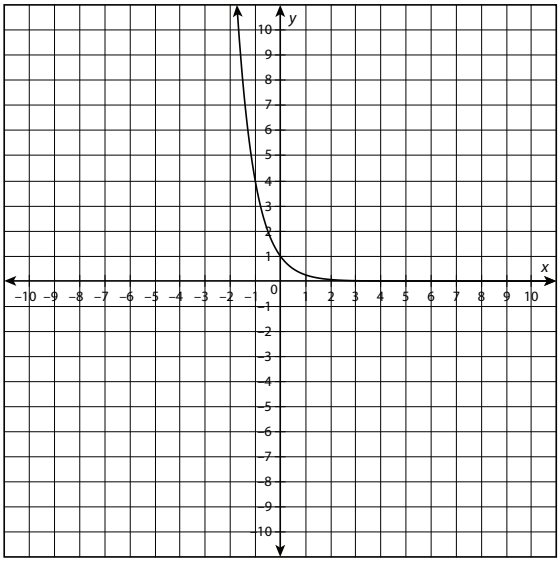
2.



3.



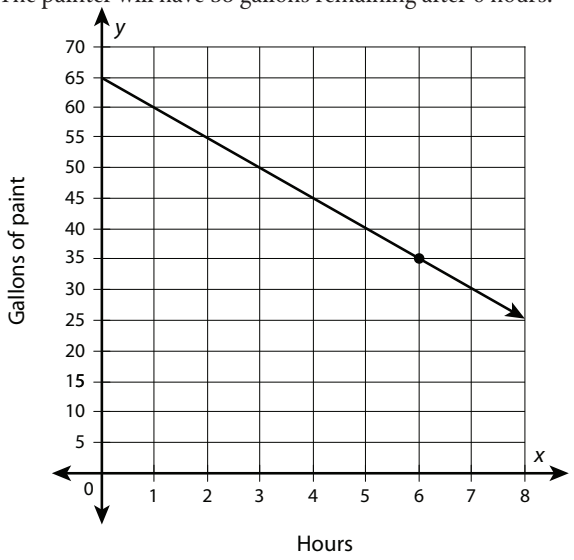
4.



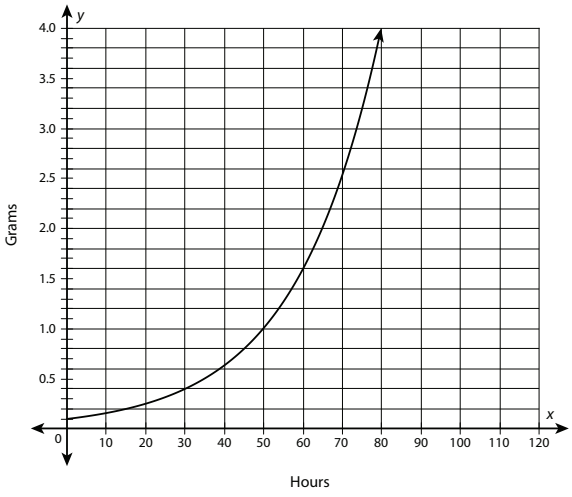
5. {(0, 2); (2, 5); (4, 8)}
6. {(–2, 9); (–1, 3); (0, 1)}

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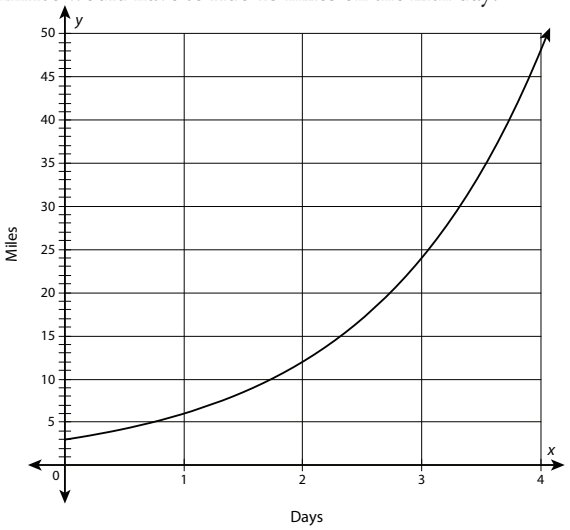
7. The painter will have 35 gallons remaining after 6 hours.



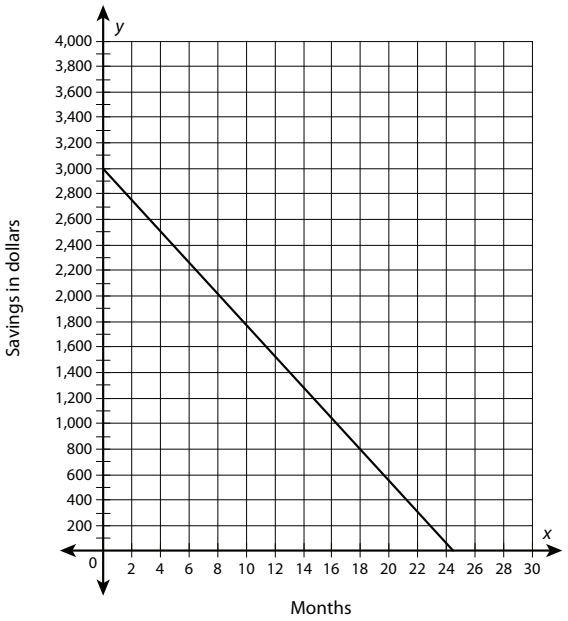
8. There were about 1.6 grams of bacteria after 60 hours.



9. Enrico would have to ride 48 miles on the fifth day.

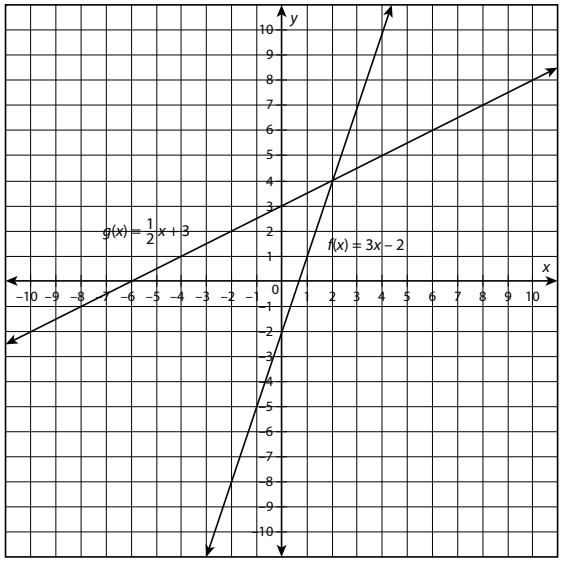


10. It will take just over 24 months for Mr. Samuelson's savings to exceed what he spent on the air conditioning unit.



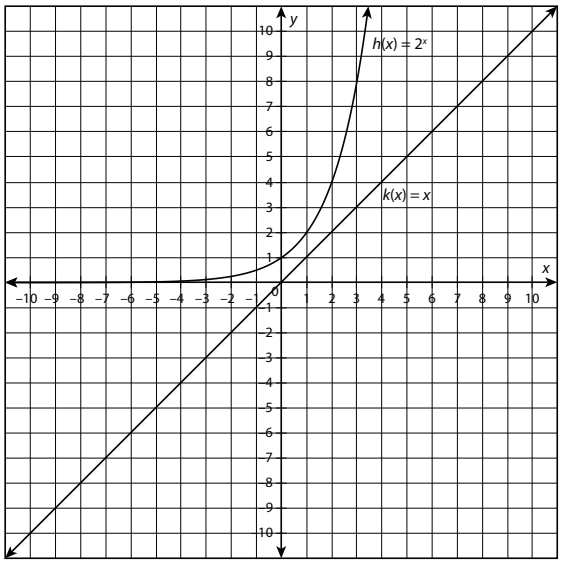
Practice 2.1.2: Intersecting Graphs, p. U2-19

1. If $f(x) = g(x)$ for some value of x, then the point $(x, f(x))$ will be on the curve f and the point $(x, g(x))$ will be on the curve g. Since $f(x) = g(x)$, $(x, f(x))$ and $(x, g(x))$ are the same point.
2. There will be no solutions for x: $f(x) = g(x)$.
3. The graphs will not cross between the values of x in the table.
4. A sign change means that the graphs intersect.
5. The graphs intersect where $x \approx 2$.



6. no solutions

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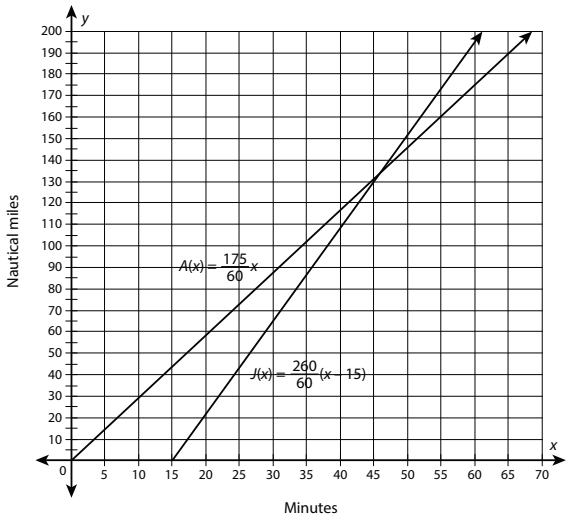
7. $x = 6$

x	$r(x) = x - 6$	$s(x) = \left(\frac{1}{4}\right)^x$	$r(x) - s(x)$
3	–3	0.02	–3.02
4	–2	0.003	–2.003
5	–1	0	–1
6	0	0	0
7	1	0	1
8	2	0	2

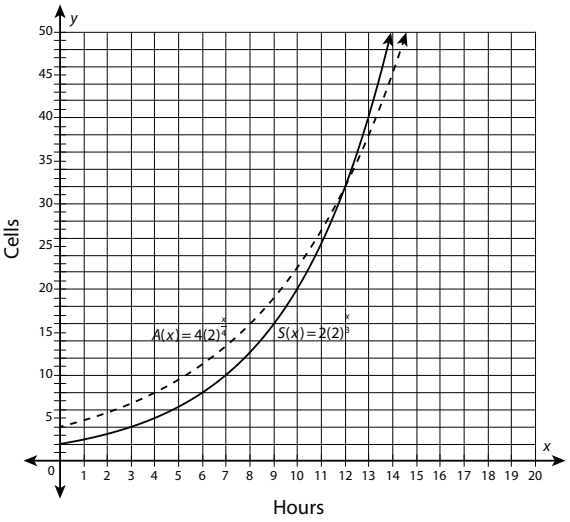
8. $x = -1$

x	$f(x) = x + 7$	$g(x) = \left(\frac{1}{3}\right)^x + 3$	$f(x) - g(x)$
–3	4	30	–26
–2	5	12	–7
–1	6	6	0
0	7	4	3
1	8	$3.3\overline{3}$	$7.6\overline{6}$

9. It will take Josiah about 46 minutes to catch up with Alanna. They will be about 134 nautical miles from the airport.



10. They should see the same number of cells in their cultures at about 12 hours.



Practice 2.1.3: Domain and Range, p. U2-30

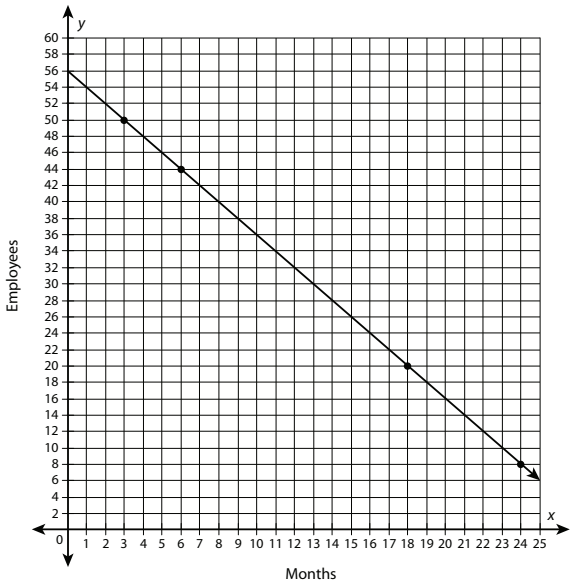
1. No; 10 is mapped to 1 and 4, 20 is mapped to 2 and 5, and 30 is mapped to 3 and 6.
2. Yes; each x is mapped to only one y.
3. Yes; the graph passes the vertical line test.
4. No; the graph fails the vertical line test on the interval $-5 < x < 5$.
5. Yes; each x is paired to only one y.
6. No; $x = 2$ is mapped to $y = 4$, 6, 8, 10, 12, and 14.
7. domain: {all reals}; range: {all reals}
8. domain: {all reals}; range: {$y > 0$}
9. domain: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25};
   range: {–100, –88, –76, –64, –52, –40, –28, –16, –4, 8, 20, 32, 44, 56, 68, 80, 92, 104, 116, 128, 140, 152, 164, 176, 188, 200}
10. domain: {0, 1, 2, 3, 4 }; range: {16, 8, 4, 2, 1}

Practice 2.1.4: Function Notation and Evaluating Functions, p. U2-38

1. {10, 28, 46, 82}

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2. {–3, 1, 5, 9}
3. {3.25, 4, 7, 67}
4. {3, 4, 10, 34}
5. $f(-4) = 7$
6. $f(-2) = 1$
7. Evaluating the function over the domain results in the range of {50, 44, 20, 8}. After 3 months, the store is down to 50 employees. After 6 months, the store is down to 44 employees. After 18 months, the store is down to 20 employees, and after 24 months the store is down to 8 employees. The following are all points on the function of $g(x) = -2x + 56$:
   $g(3) = 50$
   $g(6) = 44$
   $g(18) = 20$
   $g(24) = 8$

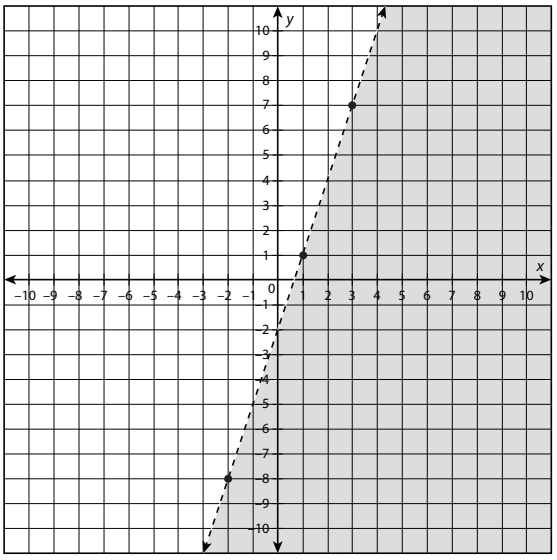


8. Evaluating the function over the domain results in the range {8, 32, 512}. After 2 years, the population doubles to 8. After 6 years, the population grows to 32. After 14 years, the population grows to 512. The following are all points on the function $r(x) = 4(2)^{\frac{x}{2}}$:
   $b(2) = 8$
   $b(6) = 32$
   $b(14) = 512$
9. $d(3) = 300$; $d(5) = 420$; $d(8) = 600$. After 3 hours, Carlos will have driven a total of 300 miles. After 5 hours, he will have driven a total of 420 miles. After 8 hours, he will have driven a total of 600 miles.
10. $p(1) = 5940$; $p(2) = 5880.6$; $p(5) = 5705.9$; $p(10) = 5426.3$. After 1 year, the population will decline to 5,940 people. After 2 years, the population will decline to about 5,881 people. After 5 years, the population will decline to about 5,706 people. After 10 years, the population will decline to about 5,426 people.

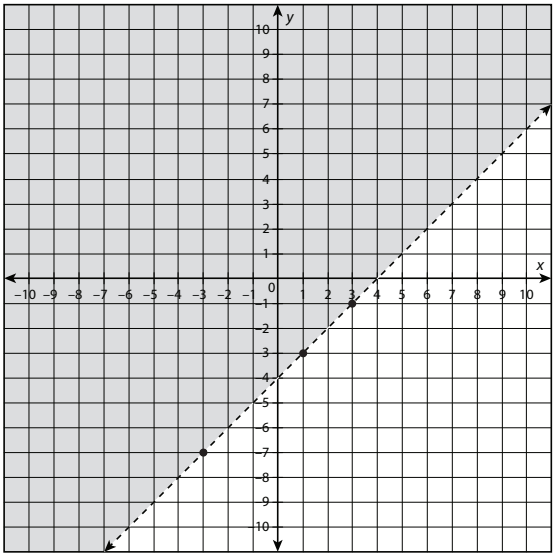
Lesson 2: Solving Linear Inequalities in Two Variables and Systems of Inequalities

Practice 2.2.1: Solving Linear Inequalities in Two Variables, p. U2-56

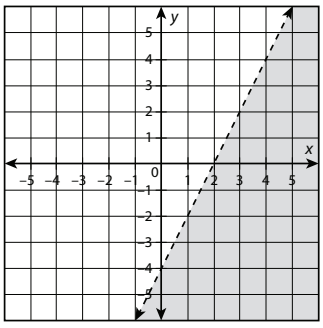
1.



2.

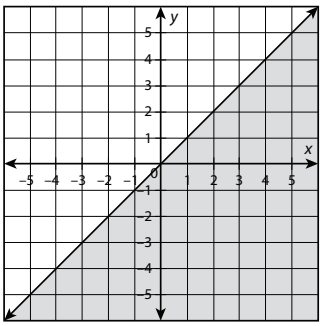


3.

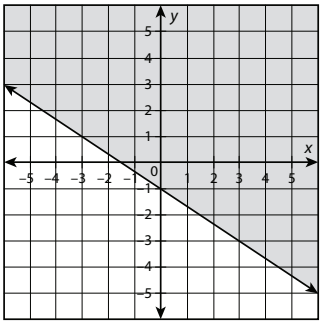


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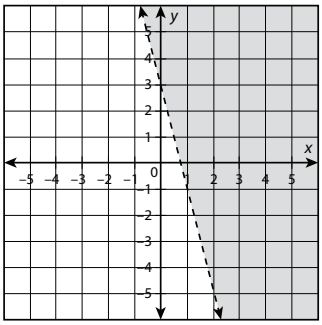
4.



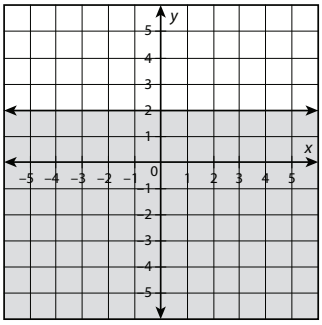
5.



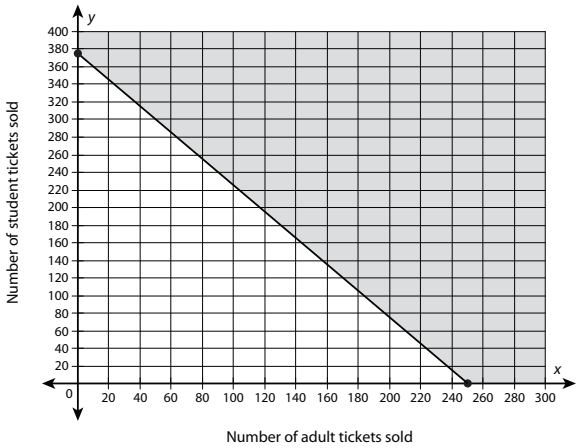
6.



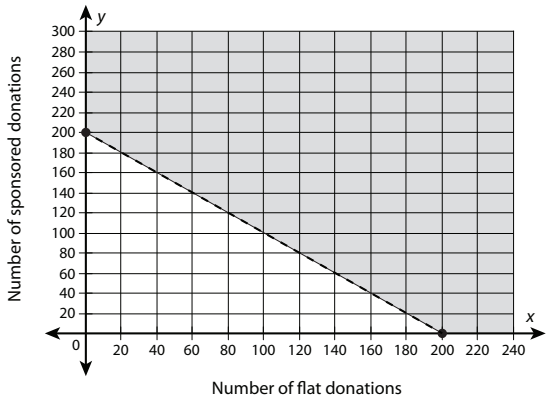
7.



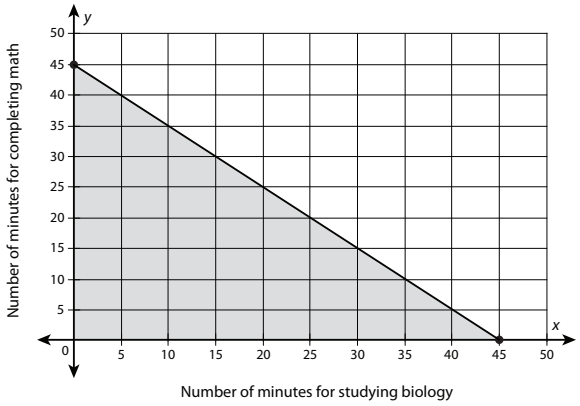
8. $12x + 8y \geq 3000$ (Note that the ideal graph of this situation contains only integer pairs that are in the shaded region.)



9. $x + y > 200$ (Note that the ideal graph of this situation contains only integer pairs that are in the shaded region.)



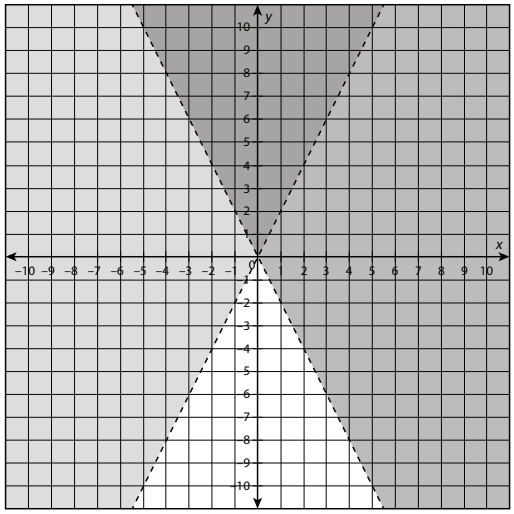
10. $x + y \leq 45$



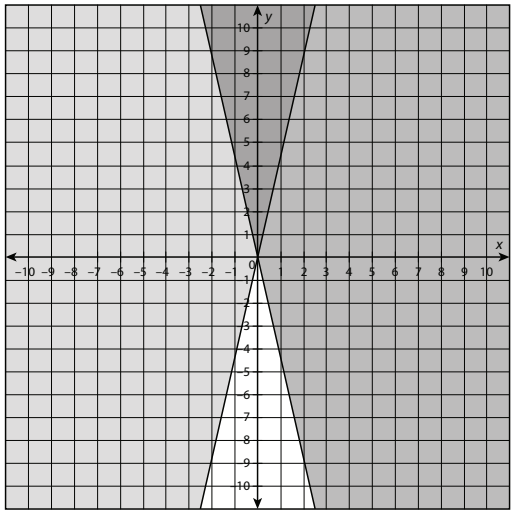
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Practice 2.2.2: Solving Systems of Linear Inequalities, p. U2-69

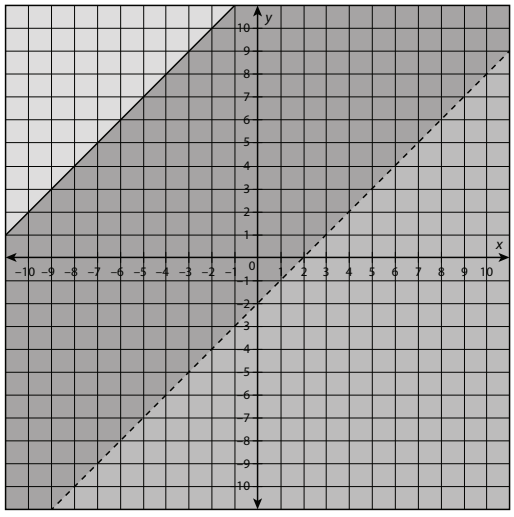
1.



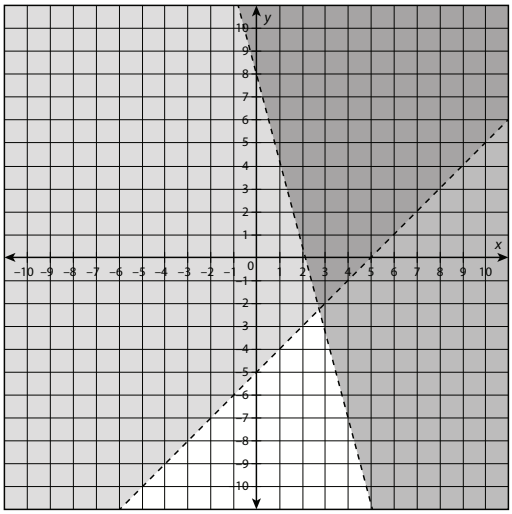
2.



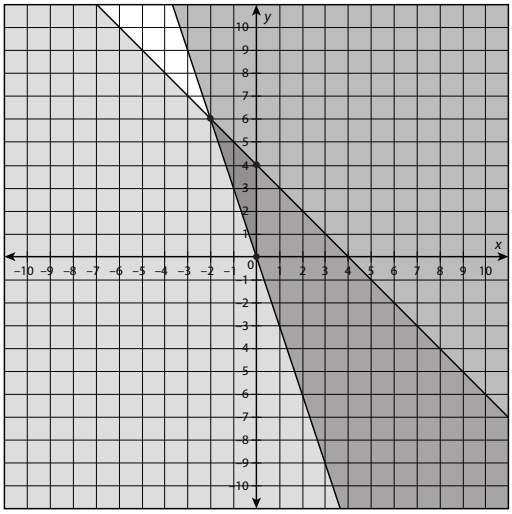
3.



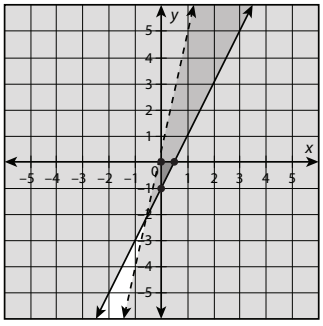
4.



5.

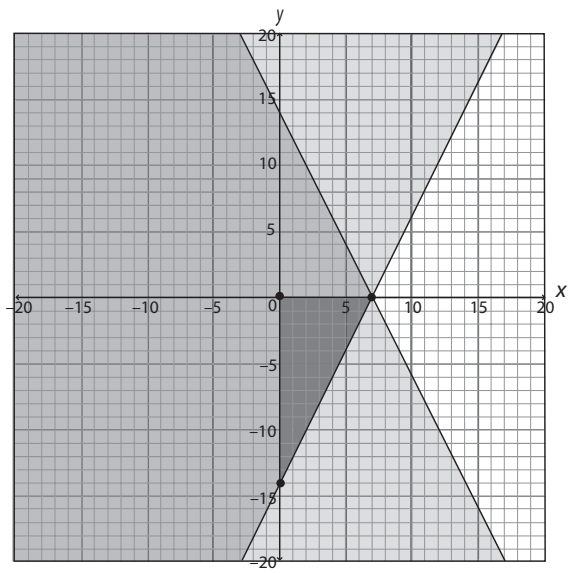


6.



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7.



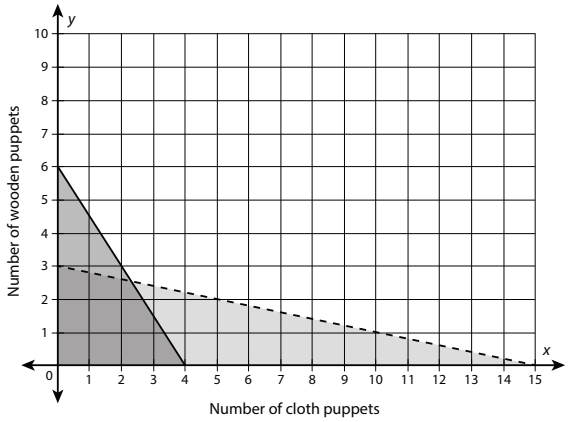
8. a. $\begin{cases} x+\frac{1}{2}y \le 8 \\ 4x+2y \le 16 \\ x \ge 0 \\ y \ge 0 \end{cases}$

b. Graph of the system (note that the ideal graph of this situation contains only integer pairs that are in the darkly shaded region):



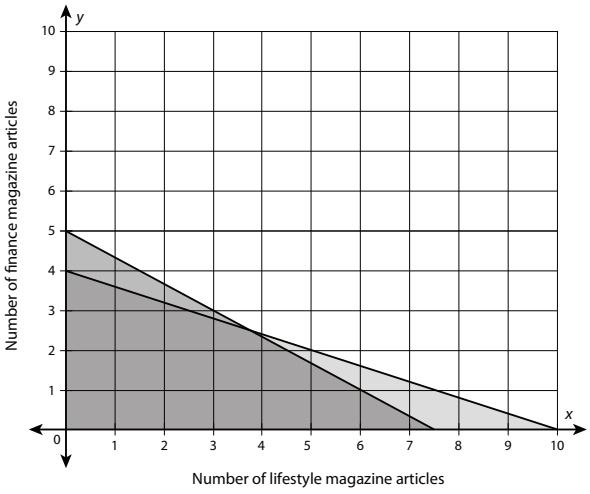
9. a. $\begin{cases} x+5y < 15 \\ 3x+2y \le 12 \\ x \ge 0 \\ y \ge 0 \end{cases}$

b. Graph of the system (note that the ideal graph of this situation contains only integer pairs that are in the darkly shaded region):



10. a. $\begin{cases} 2x+3y \le 15 \\ 2x+5y \le 20 \\ x \ge 0 \\ y \ge 0 \end{cases}$

b. Graph of the system (note that the ideal graph of this situation contains only integer pairs that are in the darkly shaded region):

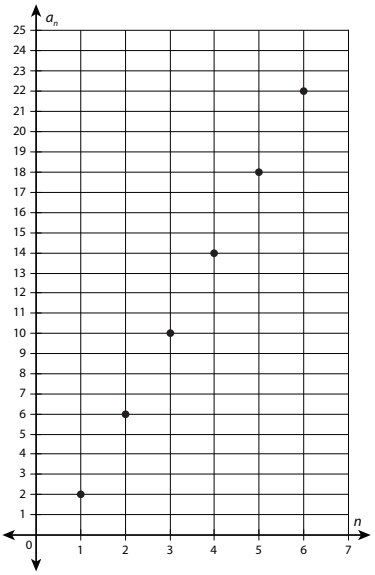


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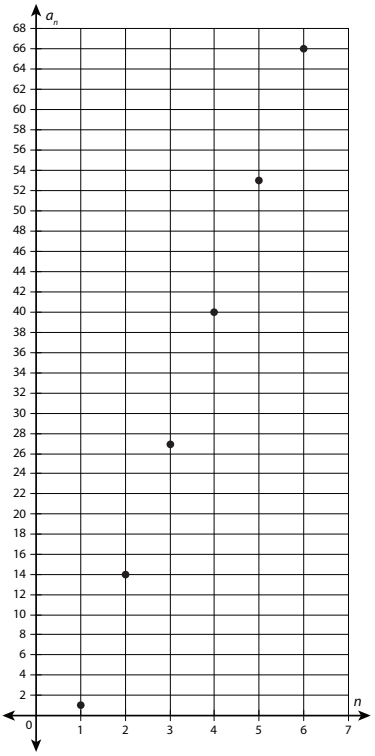
Lesson 3: Sequences As Functions

Practice 2.3.1: Sequences As Functions, p. U2-84

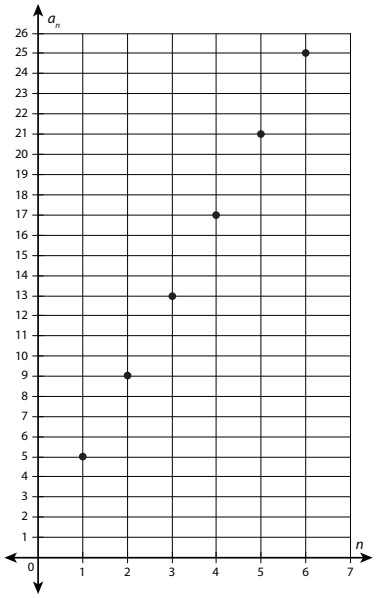
1. 32
2. 22



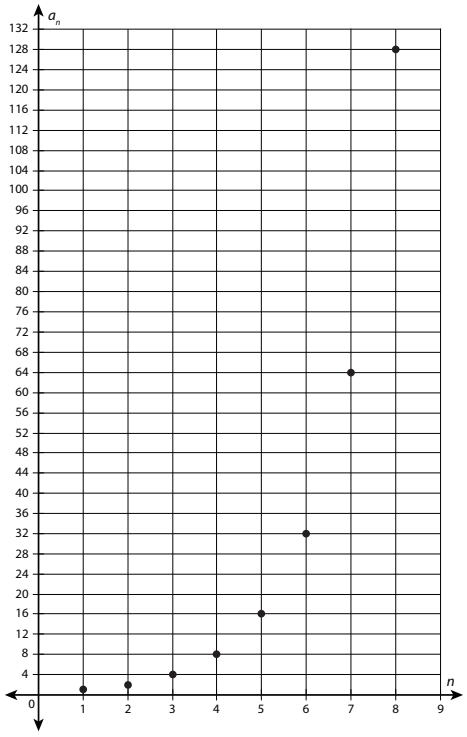
3. 53



4. 55
5. 32
6. –2
7. $a_5 = 21$, $a_6 = 25$



8. $a_5 = 32$, $a_6 = 64$



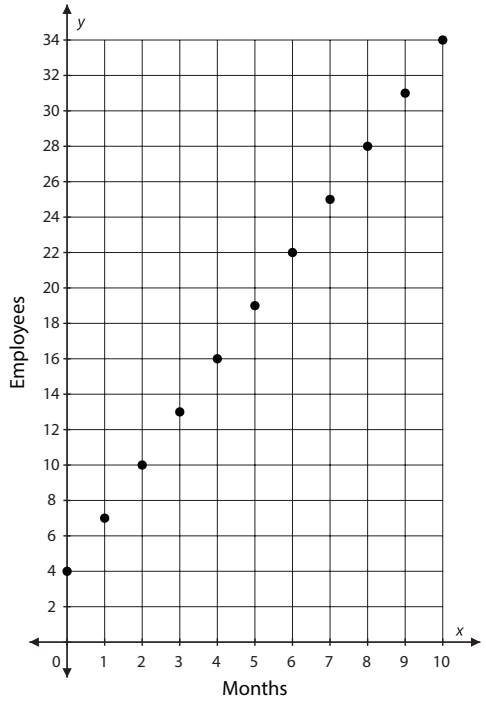
9. 5:53 A.M.
10. The largest container holds 32 gallons; the smallest container holds 2 gallons.

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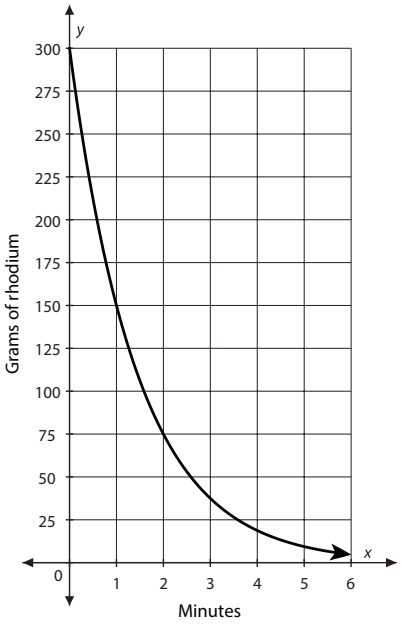
Lesson 4: Interpreting Graphs of Functions

Practice 2.4.1: Identifying Key Features of Linear and Exponential Graphs, p. U2-101

1. domain: whole numbers



2. domain: $x \geq 0$



3. x-intercept: none; y-intercept: (0, 30); the function is decreasing; the function is positive for $0 \leq x \leq 4$; minimum: 18; maximum: 30; asymptote: there is no asymptote; domain: $0 \leq x \leq 4$, where x is a whole number
4. x-intercept: (30, 0); y-intercept: (0, 750); the function is decreasing; the function is positive for $0 \leq x < 30$; minimum: 0; maximum: 750; asymptote: there is no asymptote; domain: $0 \leq x \leq 30$, where x is a whole number
5. x-intercept: (0, 0); y-intercept: (0, 0); the function is increasing; the function is positive for $x \geq 0$; minimum: 0; maximum: there is no maximum; asymptote: there is no asymptote; domain: $x \geq 0$
6. x-intercept: none; y-intercept: (0, 150); the function is increasing; the function is positive for $x \geq 0$; minimum: 150; maximum: there is no maximum; asymptote: there is no asymptote; domain: $x \geq 0$, where x is a whole number
7. x-intercept: none; y-intercept: (0, 32); the function is decreasing; the function is positive for $x \geq 0$; minimum: 1; maximum: 32; asymptote: there is no asymptote; domain: $0 \leq x \leq 5$, where x is a whole number
8. x-intercept: none; y-intercept: (0, 8); the function is increasing; the function is positive for $x \geq 0$; minimum: 8; maximum: there is no maximum; asymptote: $y = 0$; domain: $x \geq 0$
9. x-intercept: none; y-intercept: (0, 1500); the function is increasing; the function is positive for $x \geq 0$; minimum: 1500; maximum: there is no maximum; asymptote: $y = 0$; domain: $x \geq 0$
10. x-intercept: none; y-intercept: (0, 750); the function is increasing; the function is positive for $x \geq 0$; minimum: 750; maximum: there is no maximum; asymptote: $y = 0$; domain: $x \geq 0$

Practice 2.4.2: Average Rate of Change, p. U2-112

1. –0.02
2. 20
3. 30.72
4. –5.47
5. 2.54
6. 0.13
7. 0.13
8. 14
9. 24.40
10. 26.98

Practice 2.4.3: Recognizing Average Rate of Change, p. U2-123

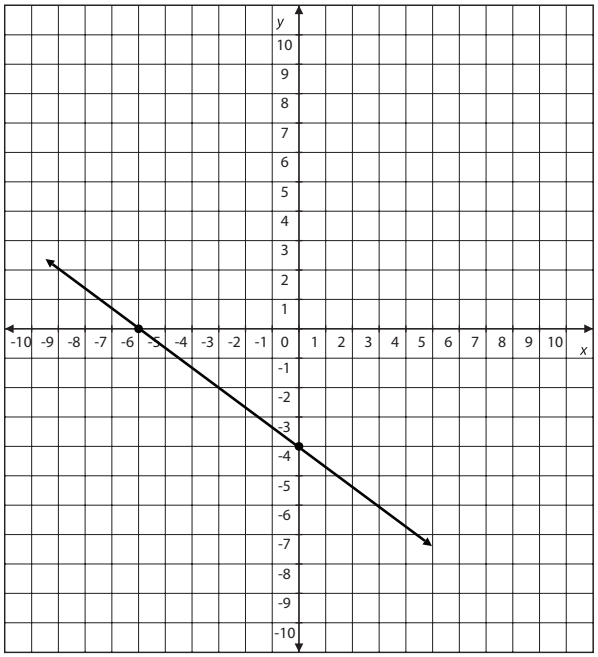
1. ≈ 0.06 gallons per door
2. ≈ 0.06 gallons per door
3. ≈ 0.97 Australian dollars per U.S. dollar
4. ≈ 0.97 Australian dollars per U.S. dollar
5. Yes, a prediction is possible. Sample prediction: The rate of change would be the same as the rate of change in questions 3 and 4 because the function is linear.
6. ≈ 92 campers per year
7. ≈ 150 campers per year
8. ≈ –160 people per year
9. ≈ –110 people per year
10. The rate of change for the interval [1, 6] is steeper than that for the interval [10, 20]. The population is decreasing at a faster rate for the interval [1, 6] than it is for the interval [10, 20].

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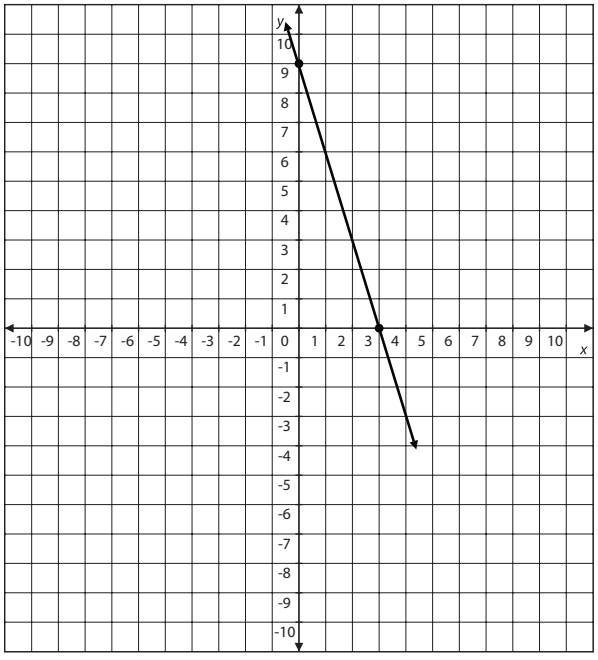
Lesson 5: Analyzing Linear and Exponential Functions

Practice 2.5.1: Graphing Linear Functions, p. U2-138

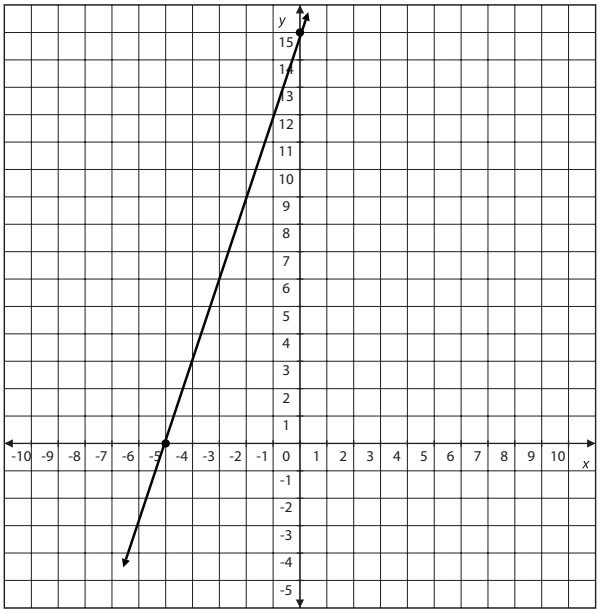
1. *x*-intercept: –6; *y*-intercept: –4



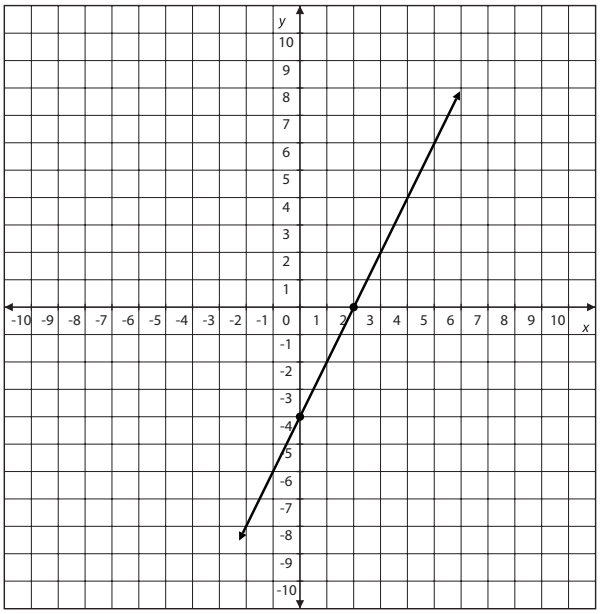
2. *x*-intercept: 3; *y*-intercept: 9



3. *x*-intercept: –5; *y*-intercept: 15

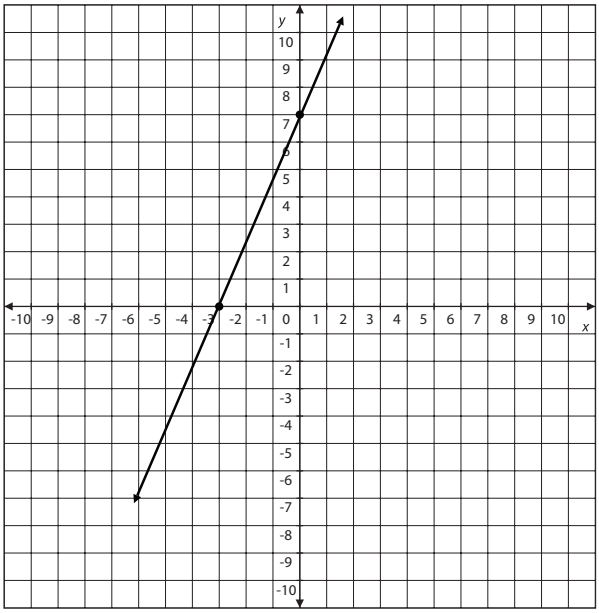


4. *x*-intercept: 2; *y*-intercept: –4

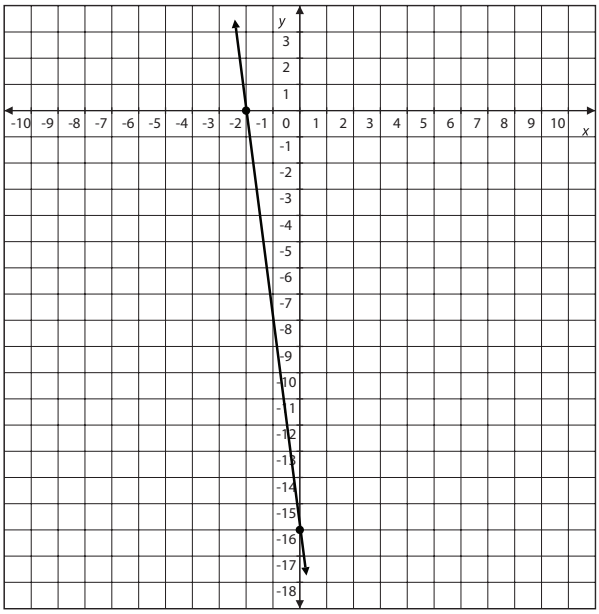


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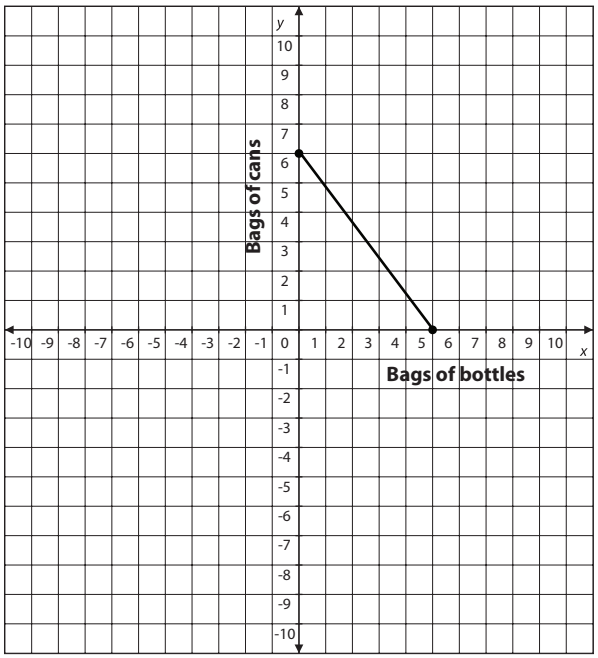
5. *x*-intercept: –3; *y*-intercept: 7



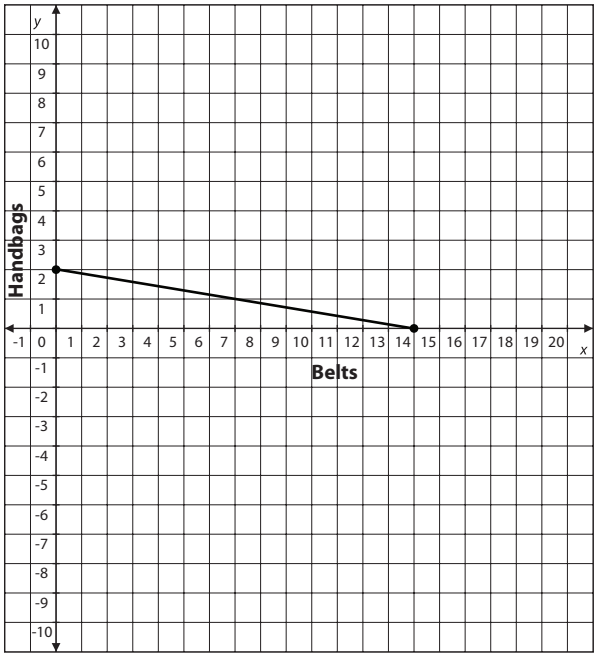
6. *x*-intercept: –2; *y*-intercept: –16



7. 5 bags of bottles, 6 bags of cans

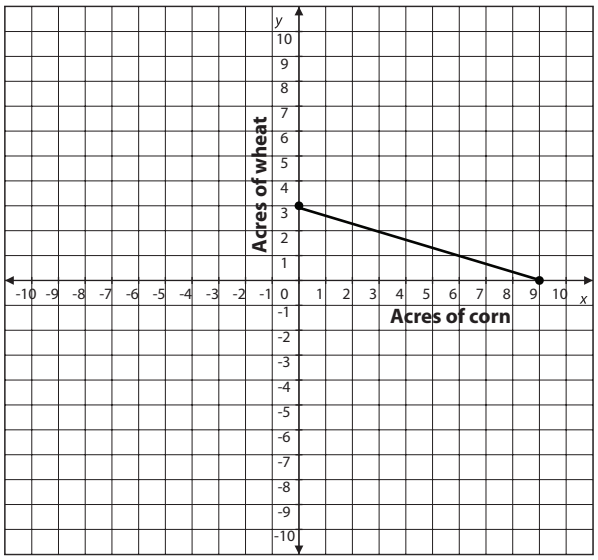


8. 14 belts, 2 handbags



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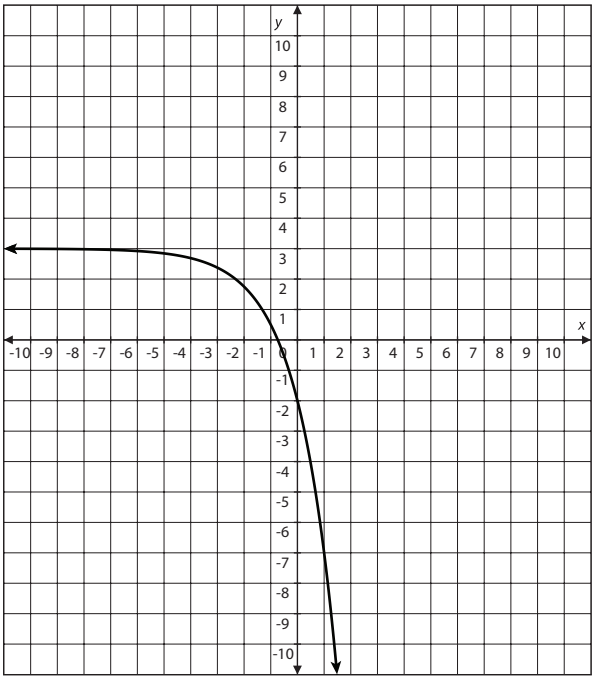
9. 9 acres of corn, 3 acres of wheat



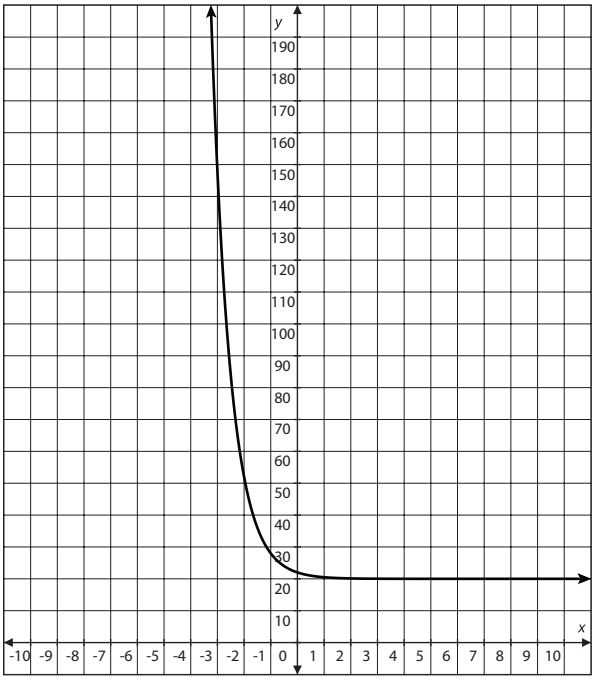
10. Answers will vary; the relationship should be linear.

Practice 2.5.2: Graphing Exponential Functions, p. U2-152

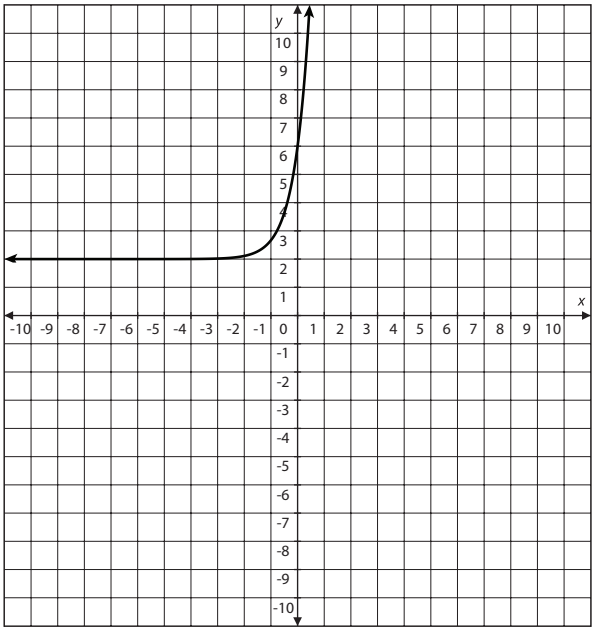
1. *y*-intercept: –2; decay, with a horizontal asymptote of $y = 3$



2. *y*-intercept: 22; decay, with a horizontal asymptote of $y = 20$

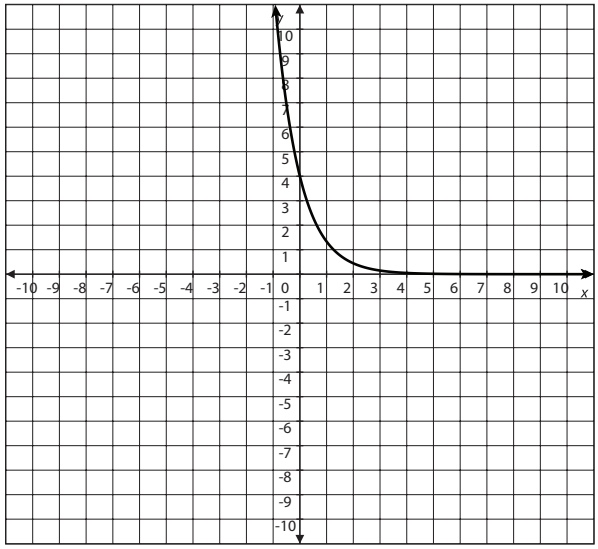


3. *y*-intercept: 6; growth, with a horizontal asymptote of $y = 2$



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4. *y*-intercept: 4; decay, with a horizontal asymptote of $y = 0$



5. *y*-intercept: –7; decay, with a horizontal asymptote of $y = -5$
6. *y*-intercept: 2; growth, with a horizontal asymptote of $y = -8$
7. $f(x) = 55\left(\frac{1}{2}\right)^x$; 6.875 grams
8. $f(x) = 10(4)^x$; 160 organisms
9. $f(x) = 250(1.037)^x$; $278.79
10. Answers will vary; the scenario should be an example of exponential decay.

Lesson 6: Comparing Functions

Practice 2.6.1: Comparing Linear Functions, p. U2-161

1. Function A has a rate of change of 3 and a *y*-intercept of –5. Function B has a rate of change of 2 and a *y*-intercept of 1. Function A has a greater rate of change because 3 is greater than 2. Function B has a greater *y*-intercept because 1 is greater than –5.
2. Function A has a rate of change of –1/7 and a *y*-intercept of –4. Function B has a rate of change of 1/3 and a *y*-intercept of 6. Function B has the greater rate of change because the absolute value of 1/3 is greater than the absolute value of –1/7. Function B also has the greater *y*-intercept.
3. Function A has a rate of change of 2/3 and a *y*-intercept of 9. Function B has a rate of change of 0 and a *y*-intercept of 2. Function A has a greater rate of change and a greater *y*-intercept.
4. Function A has a rate of change of 3 and a *y*-intercept of 0. Function B has a rate of change of –1 and a *y*-intercept of 0. Function A has a greater rate of change because the absolute value of 3 is greater than the absolute value of –1. Both functions share the same *y*-intercept.
5. Function A has a rate of change of 0.23 and a *y*-intercept of 0. Function B has a rate of change of 0.20 and a *y*-intercept of 0. Function A has a greater rate of change. Both functions share the same *y*-intercept.
6. Function A has a rate of change of 175 and a *y*-intercept of 3,400. Function B has a rate of change of 95 and a *y*-intercept of 2,200. Function A represents a magazine with a greater number of initial subscribers. Function A also has a greater rate of change, meaning more subscribers sign up for that magazine each year.
7. Function A has a rate of change of 2.00 and a *y*-intercept of 3.50. Function B has a rate of change of 2.50 and a *y*-intercept of 1. The initial cost to rent a game is higher with Function A. Function B has the higher rate of change, meaning the charge for each extra night is higher than for Function A.
8. Function A has a rate of change of –75 and a *y*-intercept of 1,200. Function B has a rate of change of –75 and a *y*-intercept of 850. Both functions have the same rate of change, meaning that both accounts are being paid down at $75 per month. Function A has a greater *y*-intercept, meaning that the cost of the repairs was greater than the job represented by Function B.
9. Function A has a rate of change of –0.32 and a *y*-intercept of 12.5. Function B has a rate of change of –0.32 and a *y*-intercept of 10. Function A has a greater *y*-intercept, meaning that this container of cat food initially held more food. Functions A and B have the same rate of change, meaning that the cat is being fed the same amount every time in each scenario.
10. Function A has a rate of change of 2 and a *y*-intercept of 8. Function B has a rate of change of 2.5 and a *y*-intercept of 5. Function A has a greater *y*-intercept, meaning that Sophie initially ran more miles last week. Function B has a greater rate of change, meaning that Kaelina plans to run more miles per week than Sophie.

Practice 2.6.2: Comparing Exponential Functions, p. U2-177

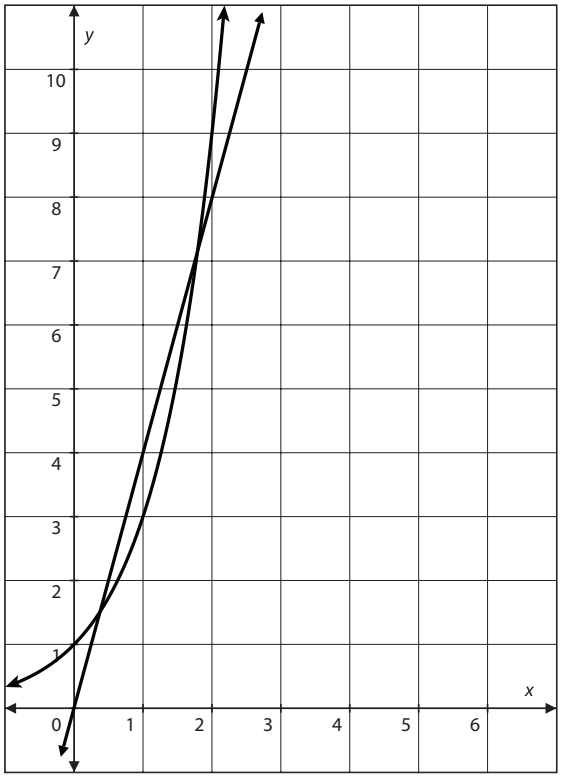
1. Function A has a greater rate of change because the slope of the function over the interval is 33,600, compared to 933.333 for Function B. Function B has a greater *y*-intercept of 50, compared to 25 for Function A.
2. Function A has the greater rate of change, 20, compared to approximately –0.25 for Function B. Both functions have the same *y*-intercept, 1.
3. Function A has the greater rate of change, 17.755, compared to 15.882 for Function B. Function B has the greater *y*-intercept of 300, compared to 250 for Function A.
4. Function A has the greater rate of change, 48.75, compared to 7/3 for Function B. Function A also has the greater *y*-intercept of 4, where Function B has a *y*-intercept of 2.
5. Function A has a greater rate of change and *y*-intercept than Function B. Function A's rate of change is –2,781.40 and its *y*-intercept is at 25,000. Function B's rate of change is –1,638.04 and its *y*-intercept is at 20,000.

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6. Function A has a greater rate of change than Function B, but a lower *y*-intercept. Function A has a rate of change of 1,051.27 and a *y*-intercept of 40,000. Function B has a rate of change of 884.69 and a *y*-intercept of 42,500.
7. Function A has a lower rate of change than Function B, but a greater *y*-intercept. Function A has a rate of change of 13.70 and a *y*-intercept of 930. Function B has a rate of change of 35.4 and a *y*-intercept of 875.
8. Function A has a greater rate of change and *y*-intercept than Function B. Function A has a rate of change of 12.167 and a *y*-intercept of 17.50. Function B has a rate of change of 11.525 and a *y*-intercept of 12.
9. Function A has a greater rate of change and *y*-intercept than Function B. Function A has a rate of change of –4,723.25 and a *y*-intercept of 32,000. Function B has a rate of change of –3,286.25 and a *y*-intercept of 27,500.
10. Function A has a greater rate of change and *y*-intercept than Function B. Function A has a rate of change of 110.11 and a *y*-intercept of 2,500. Function B has an approximate rate of change of 56 and a *y*-intercept of 2,000.

Practice 2.6.3: Comparing Linear to Exponential Functions, p. U2-189

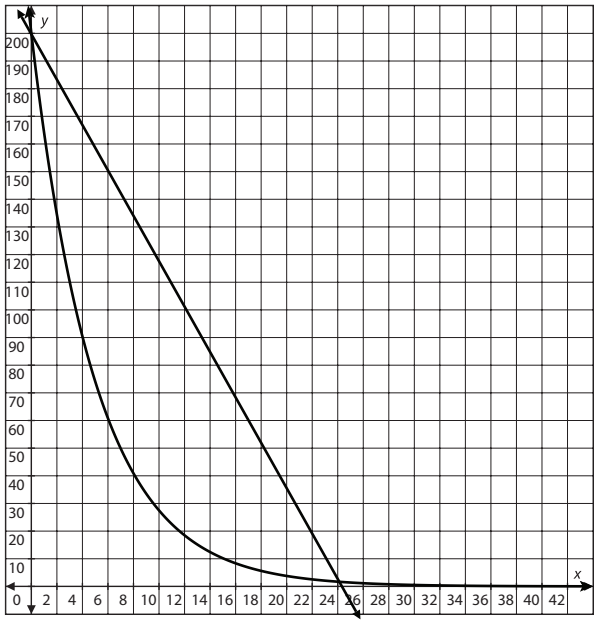
1. $g(x)$ will eventually grow faster than $f(x)$.



2. $g(x)$ grows faster than $f(x)$.

x	$f(x)$	$g(x)$
0	3	3
2	11	19
4	19	259
6	27	4099
8	35	65,539

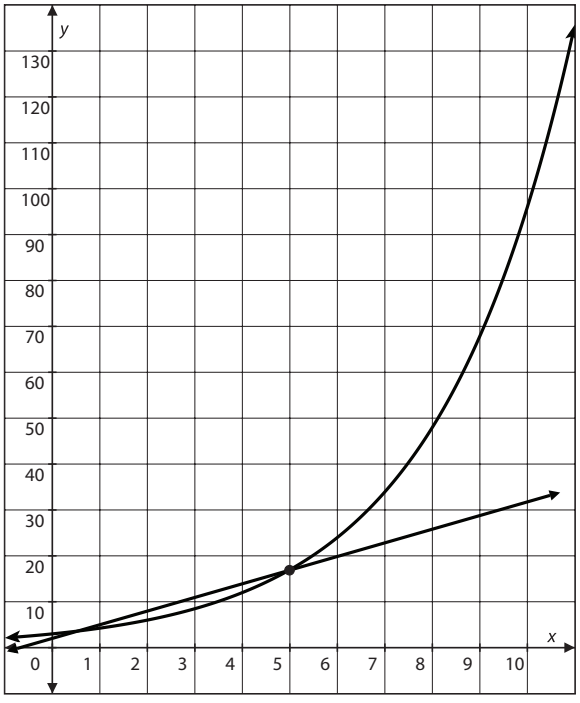
3. $f(x)$ decreases faster than $g(x)$.



4. $f(x)$ decreases faster than $g(x)$.

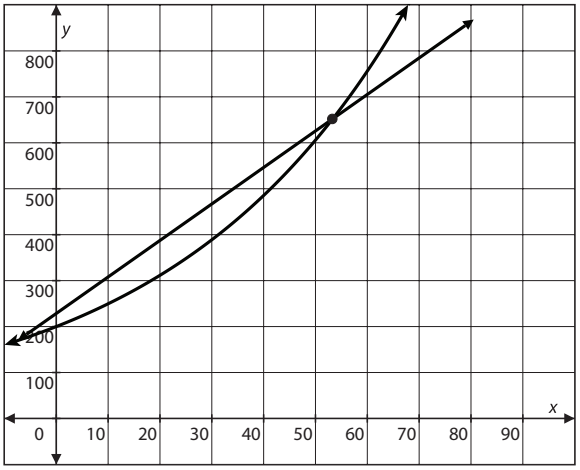
x	$f(x)$	$g(x)$
0	2000	2000
5	474.61	1998.8
10	112.63	1997.5
15	26.727	1996.3
20	6.3424	1995

5. The graphs intersect at approximately (5, 17); $f(x) > g(x)$ when $x > 5$.

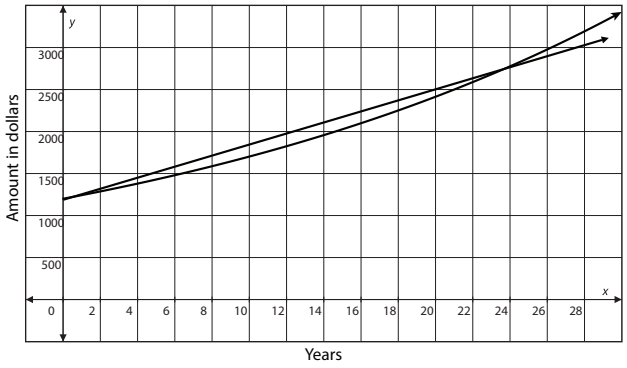


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6. The graphs intersect at approximately (53, 650); $f(x) > g(x)$ when $x > 53$.



7. $g(x)$ eventually increases faster than $f(x)$.



8. At approximately (24, 2800), the amount invested will be equal in both savings accounts. After that point, the value of $g(x)$ exceeds the value of $f(x)$.
9. Choose $f(x)$ if investing for less than 10 years because the value of $f(x)$ is greater than the value of $g(x)$ during the first 24 years.
10. Choose $g(x)$ if investing for more than 25 years because $g(x)$ is greater than $f(x)$ after 24 years.

Lesson 7: Building Functions

Practice 2.7.1: Building Functions from Context, p. U2-202

1. $f(x) = 15x$
2. $f(x) = -18x + 260$
3. $f(x) = 7 + 9x$
4. $f(x) = 6 \cdot (4)^{x-1}$
5. $f(x) = 60 \cdot (0.9)^x$
6. $f(x) = x$
7. $f(x) = 4x - 3$
8. $f(x) = 2^x$ or $f(x) = 2 \cdot (2)^{x-1}$
9. $f(x) = 10 + 0.25x$
10. $f(x) = 15{,}000 \cdot (0.80)^x$

Practice 2.7.2: Constructing Functions from Graphs and Tables, p. U2-212

1. $f(x) = 6 \bullet \left(\frac{1}{3}\right)^x$
2. $f(x) = -0.5x + 9$
3. $f(x) = 5x - 2$
4. $f(x) = 2 \cdot (3)^x$
5. $f(x) = 0.5 \cdot (4)^x$
6. $f(x) = -15x + 82$
7. $f(x) = 22x - 18$
8. $f(x) = 3 \cdot (10)^x$
9. $f(x) = -0.5 \cdot (6)^x$
10. $f(x) = 0.2 \cdot (5)^x$

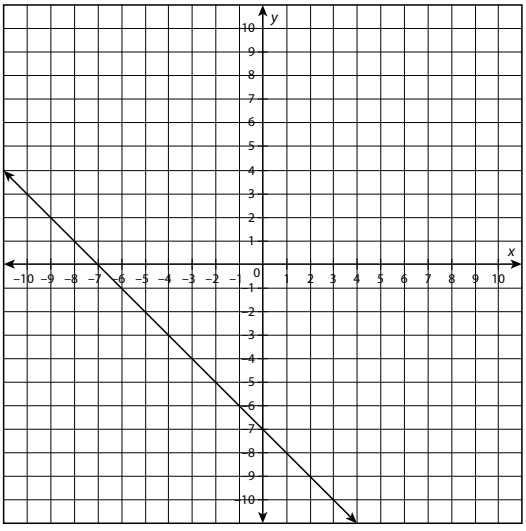
Lesson 8: Operating on Functions and Transformations

Practice 2.8.1: Operating on Functions, p. U2-220

1. $8x - 1$
2. $-2x + 15$
3. $3(2^x)$
4. $2^x/3$
5. $-4x + 19$
6. $4x - 5$
7. $-28x + 84$
8. $7/(-4x + 12)$
9. $f(x) = 100(1.02)^x$; $f(x) = 100(1.02)^x + 200$; 200 is added to the first function.
10. $f(x) = 2.5x + 3.5$; $f(x) = 2.5x + 3.5 + 1.25$; 1.25 is added to the second function.

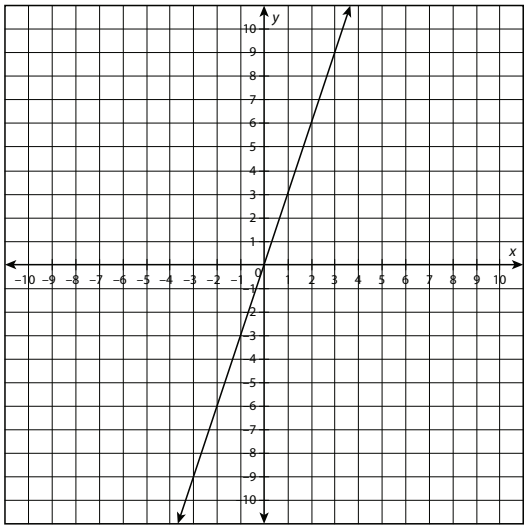
Practice 2.8.2: Transformations of Linear and Exponential Functions, p. U2-226

1.

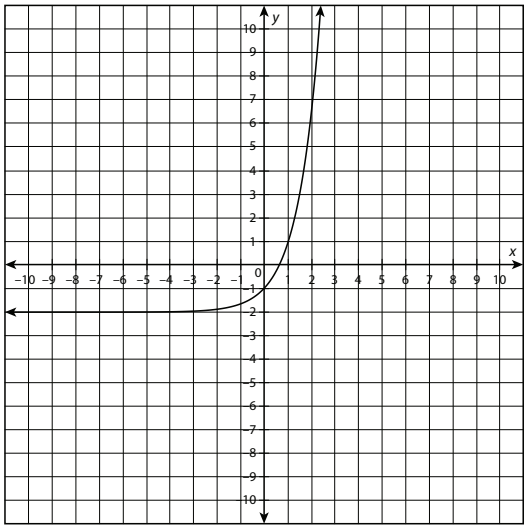


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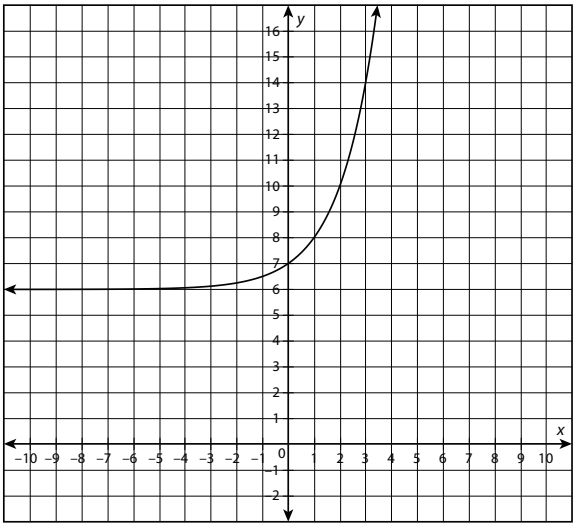
2.



3.



4.



5. $g(x) = f(x) + 13$
6. $g(x) = f(x) + 4$
7. $g(x) = f(x) - 4$
8. $g(x) = f(x) + 6$
9. 13
10. –3

Lesson 9: Arithmetic and Geometric Sequences

Practice 2.9.1: Arithmetic Sequences, p. U2-236

1. $a_n = 1.8n + 2.4$
2. $a_n = -8n + 19$
3. $a_n = 43n - 280$
4. $a_n = n + \frac{2}{3}$
5. 12.3, 12.9, 13.5, 14.1, 14.7
6. 52, 21, –10, –41, –72
7. $a_n = 12n + 3$
8. $a_n = -3n + 53$, 16 days
9. $a_n = \frac{1}{4}n + \frac{1}{4}$, 6 days
10. $a_n = 14n + 78$, \$232

Practice 2.9.2: Geometric Sequences, p. U2-242

1. $a_n = 3^{n-1}$
2. $a_n = \frac{5}{16}(5)^{n-1}$
3. $a_n = 16\left(-\frac{1}{2}\right)^{n-1}$
4. $a_n = 18\left(\frac{1}{2}\right)^{n-1}$
5. $48, -16, \frac{16}{3}, -\frac{16}{9}, \frac{16}{27}$
6. 36, 54, 81, 121.5, 182.25
7. \$461.32
8. $a_n = 12(1.2)^{n-1}$
9. 3 years
10. 9,984 dandelions

Lesson 10: Interpreting Parameters

Practice 2.10.1: Interpreting Parameters, p. U2-253

1. 3 and 12
2. 4 and –8
3. –6 and 13
4. 2 and 5
5. 4, 2, and 9
6. $f(x) = 300(3^x) + 100$; 3, \$300, and \$100
7. 2.5 and 7
8. 3.75 and 12
9. Kendall picks 35 strawberries per minute and started with 20 strawberries in his basket.
10. There were 25 ants to start with, and the population grows at a rate of 3 ants every 4 days.

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Station Activities Set 1: Comparing Linear Models

Instruction

Goal: To provide opportunities for students to develop concepts and skills related to creating and interpreting linear graphs representing real-world situations

Common Core State Standards

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★

A–REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A–REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★

F–IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Student Activities Overview and Answer Key

Station 1

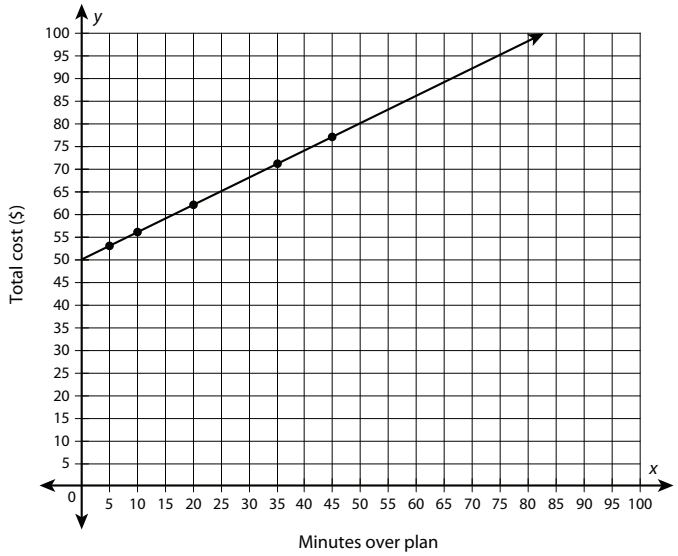
Students will be given a ruler and graph paper. They will work together to graph the linear equation of two cell phone company plans. Then they will use the graph to compare the two cell phone plans.

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Answers

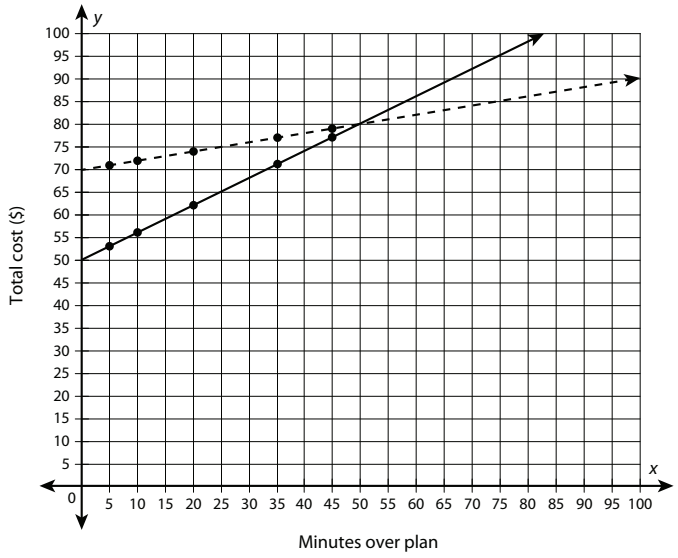
1. $y = 50 + 0.6x$; answers will vary; possible values include:

Minutes (x)	5	10	20	35	45
Cost in \$ (y)	53	56	62	71	77



2. $y = 70 + 0.2x$; answers will vary; possible values include:

Minutes (x)	5	10	20	35	45
Cost in \$ (y)	71	72	74	77	79



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3. Keri should choose 5-Bars Phone's plan because it only costs $68 versus $76.
4. Keri should choose Stellular Phone's plan because it only costs $86 versus $98.
5. At 50 minutes, it doesn't matter which plan Keri chooses because both plans cost $80.
6. The solution would be (50, 80). This can be seen in the graph where the two lines intersect. Substituting in the x-coordinate of 50 gives the same result of 80 for y in both equations.

Station 2

Students will be given a real-world graph of calories burned per mile for runners. They will interpret the graph and explain how to find an equation from the graph.

Answers

1. 60 calories per mile
2. about 69 calories per mile
3. about 81 calories per mile
4. 125 pounds
5. 150 pounds
6. Use two points on the line to find the slope, m. Use a point and the slope in slope-intercept form to find the value of b, then write the equation as $y = mx + b$.

Station 3

Students will be given a graph that represents the temperature change in the United States in January from 1999–2009. They will analyze the temperature increase and decrease and how it relates to slope.

Answers

1. 2005–06
2. The segment between these two years has the steepest positive slope.
3. 2006–07
4. The segment between these two years has the steepest negative slope.
5. 1999–2000, 2000–01, 2002–03, 2003–04, 2006–07, 2007–08
6. 2001–02, 2004–05, 2005–06, 2008–09

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Instruction

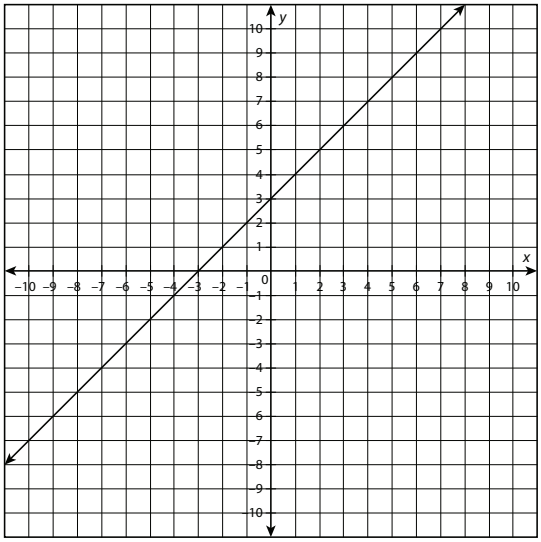
Station 4

Students will be given a linear function and asked to generate a table of values and the graph. Then they will examine the equation, table of values, and graph for defining characteristics of linear functions.

Answers

1. Answers will vary. Sample answer:

x	$f(x)$
–2	1
–1	2
0	3
1	4
2	5

2. x-intercept: –3; y-intercept: 3
3. 

4. Answers will vary. Sample answer: power of 1 on x
5. Answers will vary. Sample answer: There is a constant rate of change in y with respect to x. When x increases by 1, so does y.
6. Answers will vary. Sample answer: straight line

Materials List/Setup

Station 1 graph paper; ruler

Station 2 none

Station 3 none

Station 4 ruler

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. Using a graph, how can you find the x-value given its y-value?
2. Using a graph, how can you find the y-value given its x-value?
3. Using a graph, how can you find the x- and y-intercepts of the graph?
4. How can you use an equation to plot its graph?
5. If two lines are graphed on the same coordinate plane, how can you determine the point of intersection of the two lines?
6. Do graphs of most real-world situations represent a linear equation? Why or why not?
7. How do you determine if a function is linear?
8. What is the general shape of the graph of a linear function?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. On the graph, move your finger across from the y-axis to the line. Move your finger down to the x-axis to find the x-value.
2. On the graph, move your finger from the x-axis up to the line. Move your finger straight across to the y-axis to find the y-value.
3. The x-intercept is where the graph intersects the x-axis. The y-intercept is where the graph intersects the y-axis.
4. Create a table of values that are solutions to the equation. Graph these ordered pairs and draw a line through these points.
5. Look for the intersection point. If the graphs intersect, the x-coordinates will be the same for both equations at this point. Substituting in the x-coordinates will give the same output value for both equations.

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6. No. Linear equations have a consistent slope. In the real world, the rate of increase or decrease is often variable because of many outside factors.
7. A function is linear if the variables are to the power of 1, the variables are not multiplied together, and the variable is not in the denominator.
8. a line

Possible Misunderstandings/Mistakes

- Reversing the *x*-values and the *y*-values when reading the graph
- Incorrectly reading the graph by matching up the wrong *x*- and *y*-values
- Reversing the *x*-values and *y*-values when constructing the graph
- Incorrectly plugging *x*-values into the given equation to find the *y*-values
- Not generating the table of values correctly
- Plotting points incorrectly
- Miscalculating the *x*- and *y*-intercepts

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Station 1

You will be given a ruler and graph paper. Work together to analyze data from the real-world situation described, then, as a group, answer the questions.

> Keri is going to get a new cell phone and she has to choose between two cell phone companies. 5-Bars Phone Company charges $50 per month. The company charges an additional $0.60 per minute if a customer uses more than the monthly number of minutes included in the plan. Stellular Phone Company charges $70 per month. This company charges an additional $0.20 per minute if a customer uses more than the monthly number of minutes included in the plan. Both companies' plans include the same number of minutes each month.

Let x represent the minutes used that exceeded the plan. Let y represent the cost of the plan.

1. Write an equation that represents the monthly cost of 5-Bars Phone Company's plan.

Complete the table by selecting values for x and calculating y.

Minutes (x)					
Cost in $ (y)					

Use your graph paper to graph the ordered pairs. Use your ruler to draw a straight line through the points and complete the graph.

2. Write an equation that represents the monthly cost of Stellular Phone Company's plan.

Complete the table by selecting values for x and calculating y.

Minutes (x)					
Cost in $ (y)					

On the same graph you created for problem 1, plot the ordered pairs. Use your ruler to draw a straight line through the points and complete the graph.

continued

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Use your graph from problems 1 and 2 to answer the following questions.

3. Which plan should Keri choose if she uses 30 minutes of extra time each month? Explain.

4. Which plan should Keri choose if she uses 80 minutes of extra time each month? Explain.

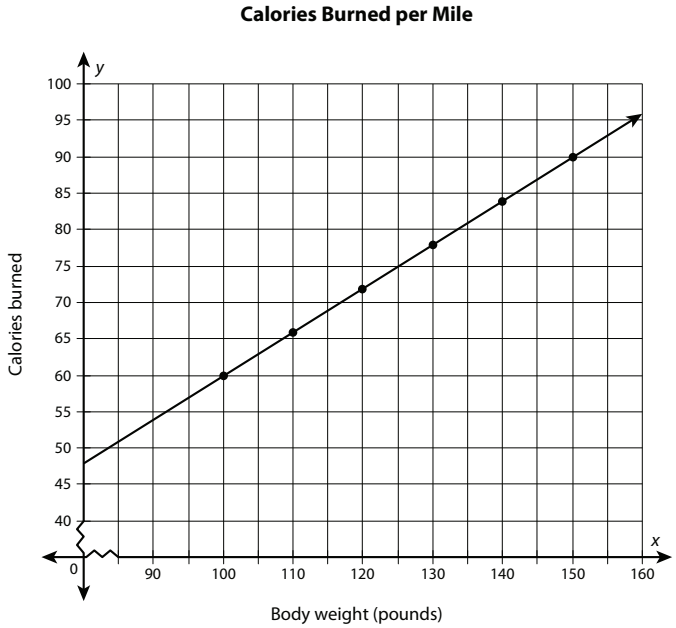
5. At what number of extra minutes per month would it not matter which phone plan Keri chooses since the cost would be the same? Explain.

6. If the equations for the two cell phone plans were solved as a system of equations, what would be the solution? Explain.

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Station 2

The equation $y = 0.6x$ represents the number of calories (y) that a runner burns per mile based on the runner's body weight of x pounds.



For each weight given, use the graph to find the number of calories burned per mile.

1. 100 pounds

2. 115 pounds

3. 135 pounds

For each given number of calories burned per mile, use the graph to find the matching weight of the person.

4. 75 calories burned

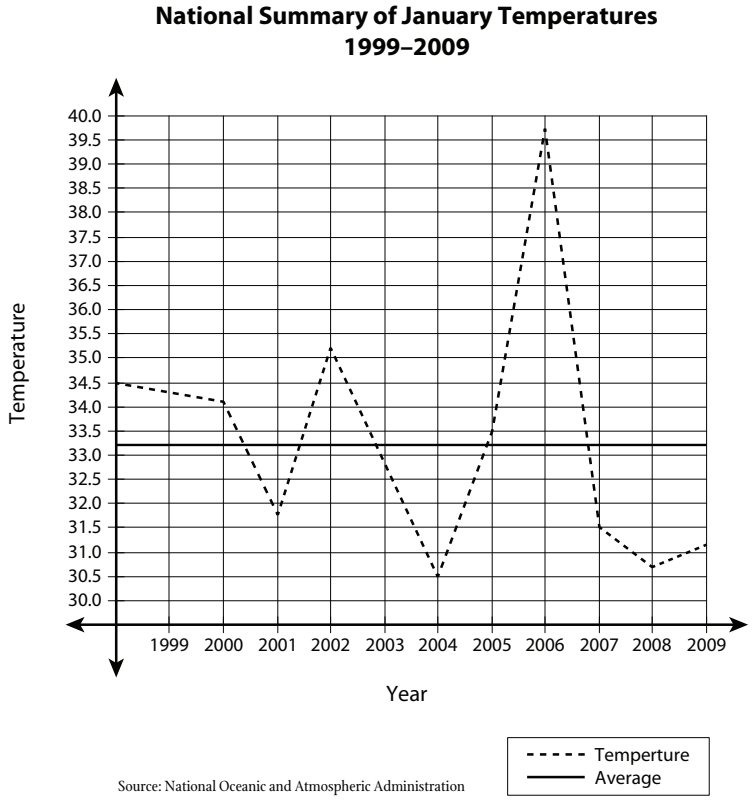
5. 90 calories burned

6. If you didn't know the equation of this graph, how could you use the graph to find the equation of the line? Explain.

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Station 3

NOAA Satellite and Information Service created the following graph, which depicts the U.S. National Summary of the temperature in January from 1999–2009.



Source: National Oceanic and Atmospheric Administration

1. Between which consecutive years did the United States see the greatest increase in average temperature change in January?

2. What strategy did you use to answer problem 1?

continued

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3. Between which consecutive years did the United States see the greatest decrease in average temperature change in January?

4. What strategy did you use to answer problem 3?

5. Between which consecutive years was the temperature change represented as a negative slope? Explain.

6. Between which consecutive years was the temperature change represented as a positive slope? Explain.

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Station 4

You will work with a linear function at this station.

Use the given linear function for the following problems.

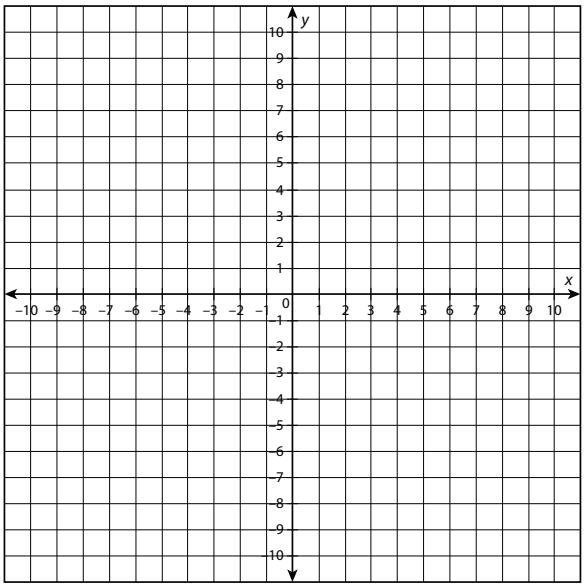
$f(x) = x + 3$

1. Create a table of values for the function.

x	$f(x)$

2. Find the x- and y-intercepts.

3. Graph the function on the coordinate plane.



continued

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4. Looking at the equation, what is a defining characteristic of a linear function?

5. Looking at the table of values, what is a defining characteristic of a linear function's table of values?

6. Looking at the graph, what is a defining characteristic of a linear function's graph?

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Instruction

Goal: To provide opportunities for students to develop concepts and skills related to using function notation, domain, range, relations, and functions

Common Core State Standards

F–IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F–BF.1 Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Student Activities Overview and Answer Key

Station 1

Students will be given eight index cards with functions and function values on them. They will match the functions with the appropriate function values. Then they will evaluate functions.

Answers

1. $f(x) = 2x$ with $f(3) = 6$; $f(x) = -3x + 7$ with $f(3) = -2$; $f(x) = x^2$ with $f(3) = 9$; $f(x) = \frac{2}{3}x$ with $f(3) = 2$
2. $f(x + 3) = x + 8$
3. $f(t - 4) = (t - 4)^2$ or $t^2 - 8t + 16$
4. $f(s + 4) = \frac{1}{5}s + \frac{4}{5}$ or $\frac{(s + 4)}{5}$

Station 2

Students will use a ruler to perform the vertical line test on graphs of relations. They will determine if the relation is a function. They will construct a graph that is a function. Then they will determine if a relation is a function by analyzing coordinate pairs.

Answers

1. Yes; the vertical line test holds.
2. No; the vertical line test does not hold.

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3. Yes; the vertical line test holds; I used the vertical line test, which says if any vertical line passes through a graph at more than one point, then the graph does not represent a function.
4. Answers will vary. Verify that the vertical line test holds.
5. It is not a function because the element 3 in the domain has two assigned elements in the range. (3, 1) and (3, 6)
6. Yes, it is a function.

Station 3

Students will be given a calculator to help them solve a real-world linear function. They will write and solve a linear function based on two data points.

Answers

1. (100, 19), (250, 17)
2. slope = $-\frac{1}{75}$
3. Use the point (100, 19).

   $y - 19 = -\frac{1}{75}(x - 100)$

   $y = -\frac{x}{75} + \frac{61}{3}$ or $f(x) = -\frac{x}{75} + \frac{61}{3}$
4. $f(500) = -\frac{1(500)}{75} + \frac{61}{3} = \$13.67$
5. $f(60) = -\frac{1(60)}{75} + \frac{61}{3} = \$19.53$

Station 4

Students will be given a number cube. They will roll the number cube to populate a relation. They will find the domain and range of the relation and determine if it is a function. Then for given relations, they will determine the domain, range, and whether each relation is a function.

Answers

1. Answers will vary; verify that the domain includes the *x*-values.
2. Answers will vary; verify that the range includes the *y*-values.
3. Answers will vary; a function is a relation in which each *x* input has only one *y* output.
4. Domain: {–1, 2, 3, 4}; range: {2, 5, 10}; yes, it is a function.

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5. Domain: {3, 7, 10}; range: {2, 5, 7}; it is not a function because there are two y-values for $x = 10$: (10, 7) and (10, 5).
6. Domain: {–14, 14, 15, 17}; range: {–9, 8, 17}; yes, it is a function.

Materials List/Setup

Station 1 eight index cards with the following functions and function values written on them:

$f(x) = 2x$; $f(x) = -3t + 7$; $f(x) = x^2$; $f(x) = \frac{2}{3}x$; $f(3) = 6$; $f(3) = 9$; $f(3) = -2$; $f(3) = 2$

Station 2 ruler; graph paper

Station 3 calculator

Station 4 number cube

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. How do you evaluate a function, $f(x)$, when given a value for x?
2. What is the vertical line test for a function?
3. What is the general form of a linear function? How does this relate to a linear equation?
4. How do you find the domain and range of a relation?
5. How can you determine whether a relation is a function?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. Substitute the value of x into the function to solve for $f(x)$.
2. The vertical line test says if any vertical line passes through a graph at more than one point, then the graph does not represent a function.
3. $f(x) = mx + b$, where m and b are real numbers. This is the same as $y = mx + b$.
4. The domain is the x-values. The range is the y-values.
5. For every x input value, there must only be one y output value assigned to it.

Possible Misunderstandings/Mistakes

- Mixing up the domain and range
- Incorrectly thinking that in a function each y-value must have a unique x-value assigned to it
- Not keeping track of variables plugged into a function
- Using a horizontal line test instead of a vertical line test to determine if a relation is a function

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Station 1

You will be given eight index cards with the following functions and function values written on them:

$$f(x) = 2x; f(x) = -3x + 7; f(x) = x^2; \ f(x) = \frac{2}{3}x; f(3) = 6; f(3) = 9; f(3) = -2; f(3) = 2$$

1. Work together to match each function with its corresponding function value. Write your matches in the space provided.

Evaluate each function for the given expression. Show your work.

2. Let $f(x) = x + 5$. What is $f(x + 3)$?

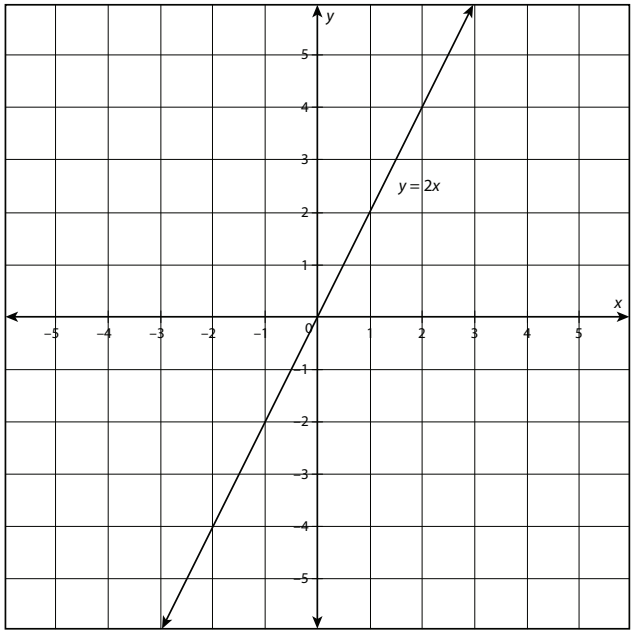
3. Let $f(t) = t^2$. What is $f(t - 4)$?

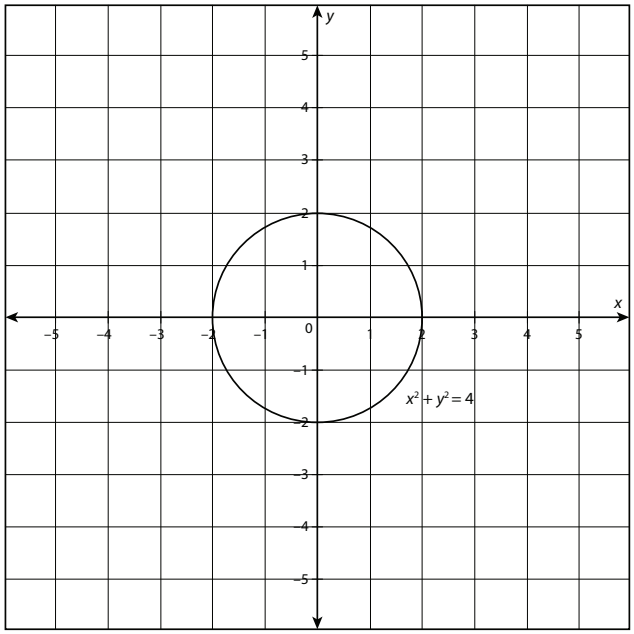
4. Let $f(s) = \frac{1}{5}s$. What is $f(s + 4)$?

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Station 2

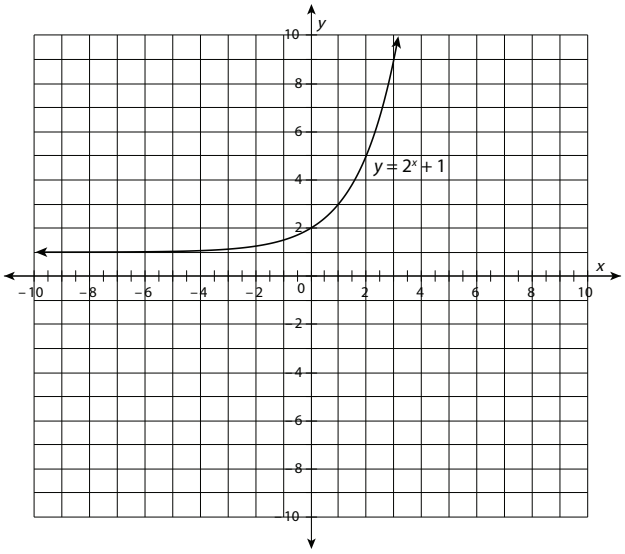
You will be given a ruler and graph paper. As a group, use your ruler to determine whether each of the following relations is a function. Beside each graph, write your answer and reasoning.

1. 

2. 

continued

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3. 

How did you use your ruler to determine whether each relation was a function?

4. Use your ruler and graph paper to sketch a function. Use the vertical line test to verify that it is a function.

Determine whether each of the following relations is a function. Explain your answer.

5. {(2, 5), (3, 1), (1, 4), (3, 6)}

6. {(1, 1), (2, 1), (3, 2)}

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Station 3

A function f is linear if its equation can be written in the form $f(x) = mx + b$, where m and b are real numbers. Use this information and the problem scenario that follows to answer the questions. You may use a calculator.

> The cost of a sweatshirt is linearly related to the number of sweatshirts ordered. If you buy 100 sweatshirts, then the cost per sweatshirt is $19. However, if you buy 250 sweatshirts, then the cost per sweatshirt is only $17.

1. You are given information that determines two points in the function. If x represents the number of sweatshirts and y represents the cost per sweatshirt, write the two ordered pairs represented in the problem scenario above.

2. What is the slope of the function?

3. Find a function which relates the number of sweatshirts and the cost per sweatshirt. Show your work.

4. What would the cost per sweatshirt be for 500 sweatshirts? Explain.

5. What would the cost per sweatshirt be for 60 sweatshirts? Explain.

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Station 4

You will be given a number cube. As a group, roll the number cube and write the result in the first box. Repeat this process until all the boxes contain a number.

$\{(\square, \square), (\square, \square), (\square, \square), (\square, \square)\}$

1. What is the domain of this relation?

2. What is the range of this relation?

3. Is this relation a function? Why or why not?

For problems 4–6, state the domain, range, and whether the relation is a function. Include your reasoning.

4. {(2, 5), (3, 10), (–1, 2), (4, 5)}

5. {(10, 7), (3, 7), (10, 5), (7, 2)}

6. {(–14, 8), (17, 8), (14, –9), (15, 17)}

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Instruction

Goal: To provide opportunities for students to develop concepts and skills related to creating and interpreting exponential graphs representing real-world situations

Common Core State Standards

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F–IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★

d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

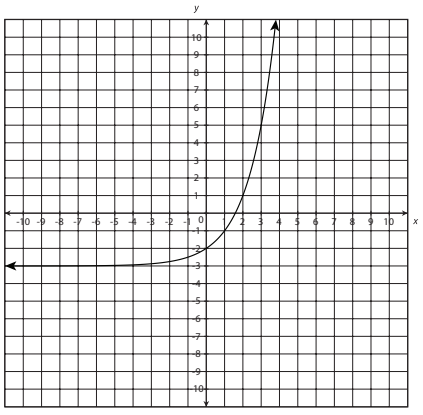
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Student Activities Overview and Answer Key

Station 1

Working with groups, students will determine the *y*-intercepts and solutions to exponential functions using their graphs. Then, students will be given a pair of points and asked to determine the exponential function that passes through those points.

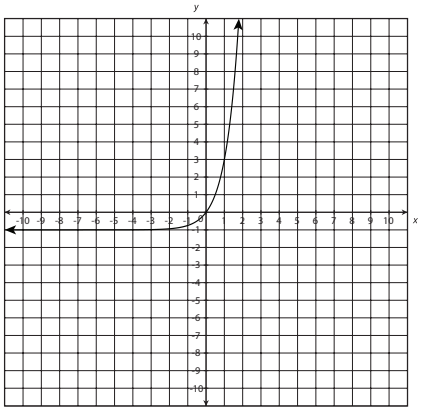
Answers

1. 

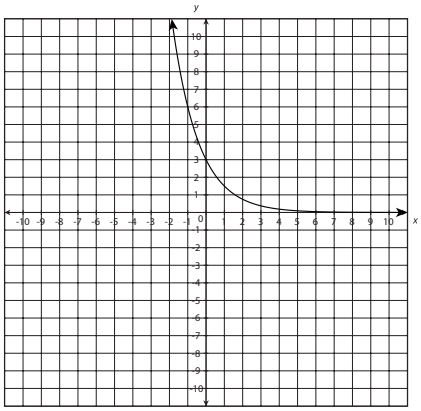
y-intercept: –2

$1 < x < 2$

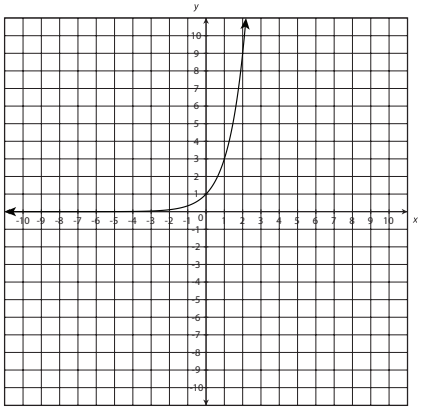
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2. 

x-intercept: 0

3. 

no *x*-intercepts

4. 

$y = 3^x$

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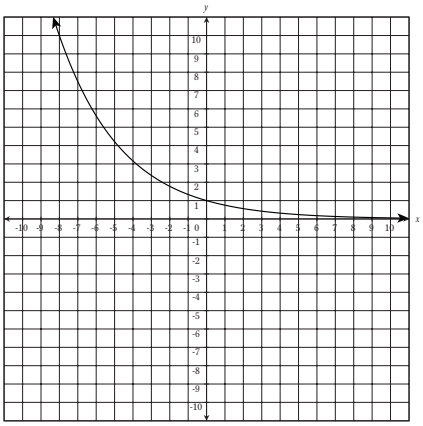
Instruction

Station 2

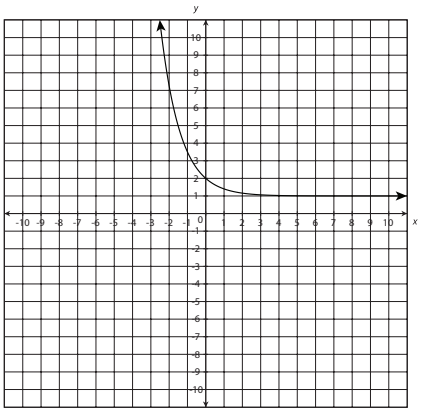
Working with groups, students will use calculators to evaluate and graph exponential functions.

Answers

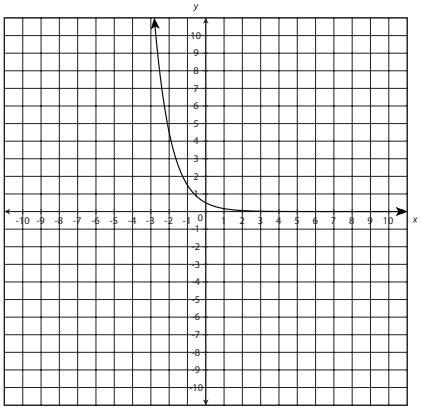
1. $f(4) = 0.316$



2. $f(5) = 1.010$

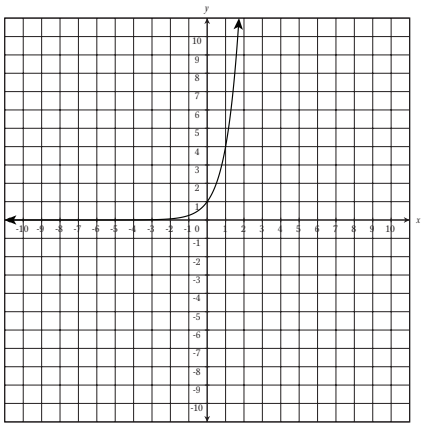


3.

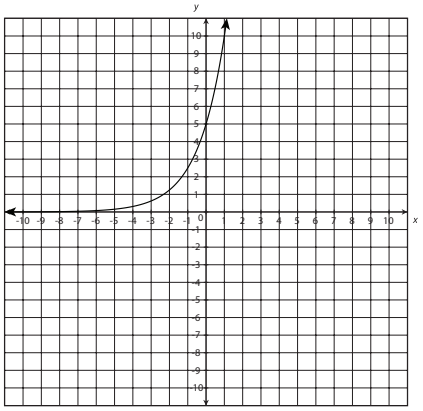


no x-intercepts

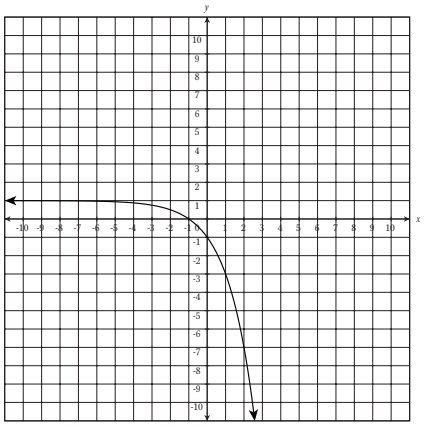
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4. 

no *x*-intercepts

5. 

no *x*-intercepts

6. 

x-intercept: –1

7. The graph must intersect the *x*-axis, so the equation must include an addition or subtraction operation in addition to the exponential operation.

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Instruction

Station 3

Working with groups, students will use exponential functions to calculate compound interest according to the formula $A = P\left(1 + \frac{r}{n}\right)^{tn}$.

Answers

1. $A = 2500\left(1+\frac{0.035}{4}\right)^{4 \cdot 4}$

   $A = \$2873.93$

2. $A = 5000\left(1+\frac{0.027}{12}\right)^{2 \cdot 12}$

   $A = \$5277.10$

3. $A = 500\left(1 + \frac{0.04}{6}\right)^{5 \cdot 6}$

   $A = \$610.30$

4. $A = 6000\left(1+\frac{0.045}{12}\right)^{3 \cdot 12}$

   $A = \$6865.49$

5. $A = 1000\left(1+\frac{0.05}{4}\right)^{2 \cdot 4} = \$1104.49$

   $A = 1000\left(1+\frac{0.046}{12}\right)^{2 \cdot 12} = \$1096.17$

   The account with 5% interest has the better yield since that account will yield approximately $1,104.49 and the account with the 4.6% interest rate will yield approximately $1,W096.17.

Station 4

Students will be given an exponential function and asked to generate a table of values and the graph. Then students will examine the equation, table of values, and graph for defining characteristics of exponential functions.

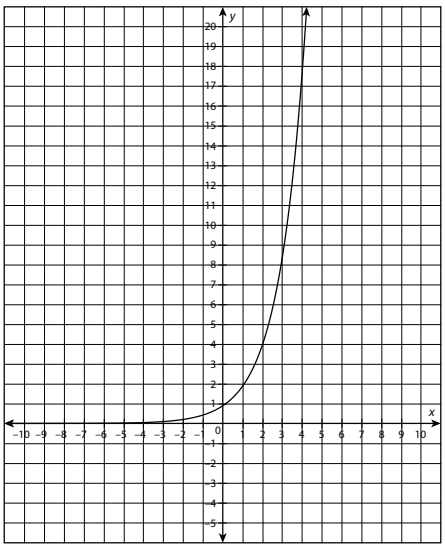
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Instruction

Answers

1. Answers will vary. Sample answer:

x	$f(x)$
–2	1/4
–1	1/2
0	1
1	2
2	4
3	8
4	16

2. x-intercepts: none; y-intercept: 1
3. 

4. Answers will vary. Sample answer: There is a variable in the exponent.
5. Answers will vary. Sample answer: As the x-values increase by 1, the y-values increase by larger and larger amounts.
6. Answers will vary. Sample answer: It rises to the right and levels off toward the left.

Materials List/Setup

Station 1 colored pens or pencils

Station 2 graphing calculator; colored pens or pencils

Station 3 calculator

Station 4 none

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. What is an exponential function?
2. When does a function have an x-intercept?
3. What is compound interest?
4. Why can it be difficult to estimate compound interest?
5. How do you determine if an equation is exponential?
6. What is the general shape of the graph of an exponential function?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. An exponential function is a function in which the variable is in the exponent.
2. A function has an x-intercept when its graph intersects the x-axis.
3. Compound interest is interest that accumulates according to the total (principal plus interest) already in the account, not just according to the principal.
4. The amount on which the percentage is based keeps changing.
5. An exponential equation has a variable in the exponent.
6. The general shape is a curve whose y-value extends toward infinity on one side and approaches an asymptote on the other side.

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Instruction

Possible Misunderstandings/Mistakes

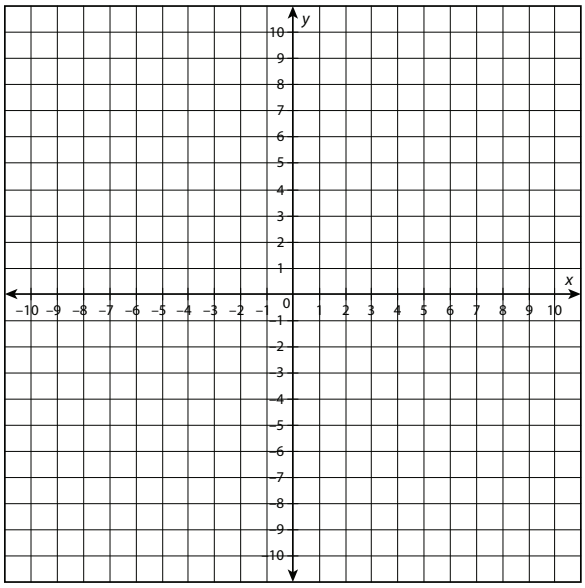
- Incorrectly manipulating numbers, variables, or exponents
- Not understanding the laws of exponents
- Assuming that all functions have zeros
- Incorrectly calculating squares, cubes, etc., of integers between 1 and 10
- Confusing a negative exponent with a fractional exponent
- Incorrectly using the exponent function of a calculator
- Incorrectly applying the formula of compound interest
- Not understanding the relationship between an exponential function and its graph
- Not generating the table of values correctly
- Plotting points incorrectly
- Miscalculating the x- and y-intercepts

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Station 1

Work with your group to answer each question.

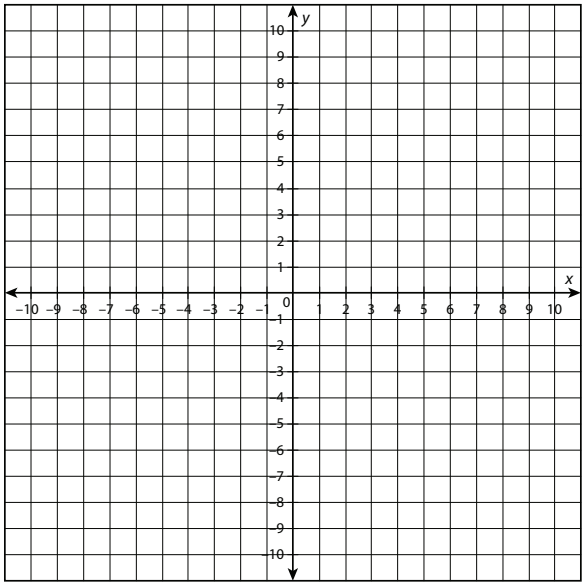
1. Graph $y = 2^x - 3$.



What is the *y*-intercept?

What are the zeros of this function?

2. Graph $y = 4^x - 1$.

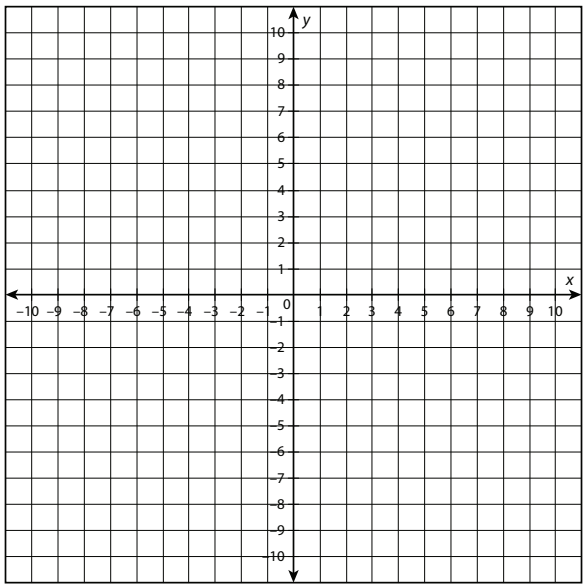


Does this function have an *x*-intercept? If so, what is it?

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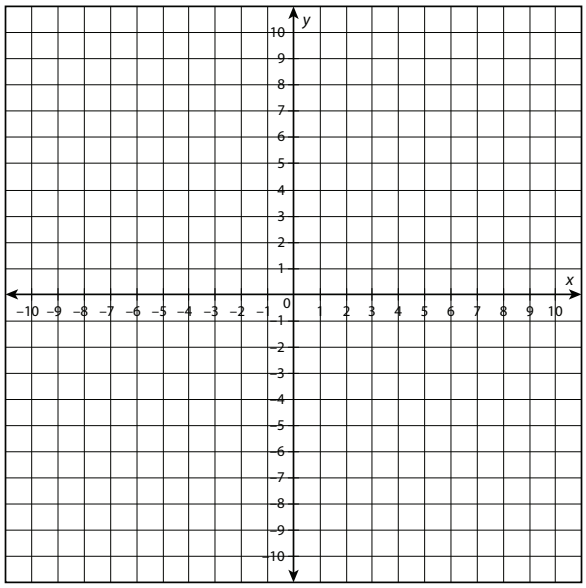
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3. Graph $y = 3\left(\frac{1}{2}\right)^x$.



Does this function have an *x*-intercept? If so, estimate where it is.

4. An exponential function passes through the points (0, 1) and (2, 9). What is the equation of the function? Graph your answer.

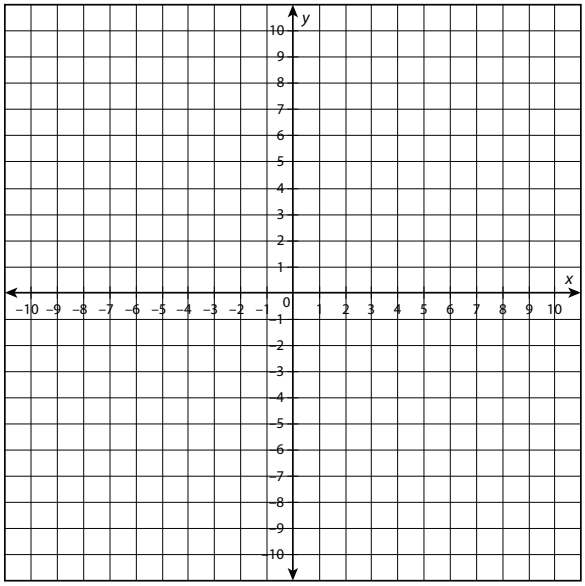


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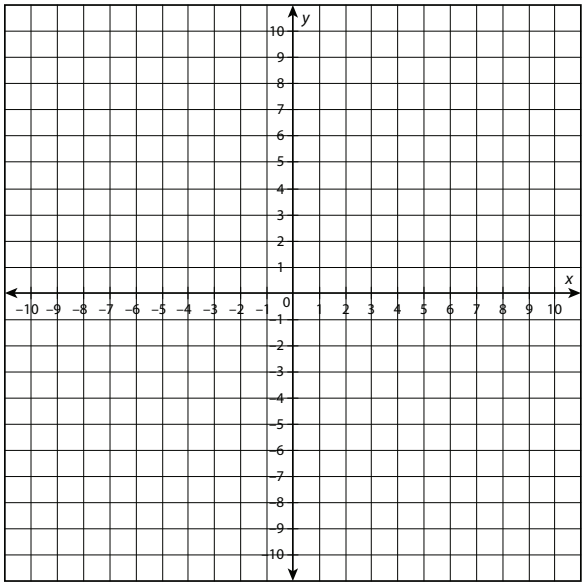
Station 2

Using a calculator, work with your group to graph each function and evaluate the function at the given value.

1. $f(x) = 0.75^x$
   $f(4) =$



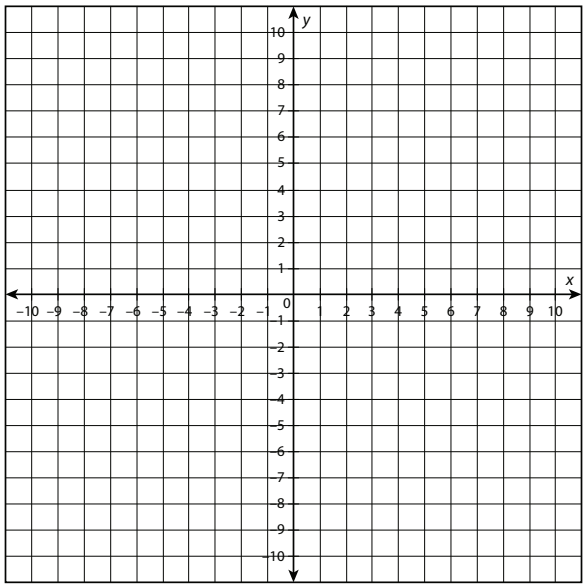
2. $f(x) = 0.4^x + 1$
   $f(5) =$



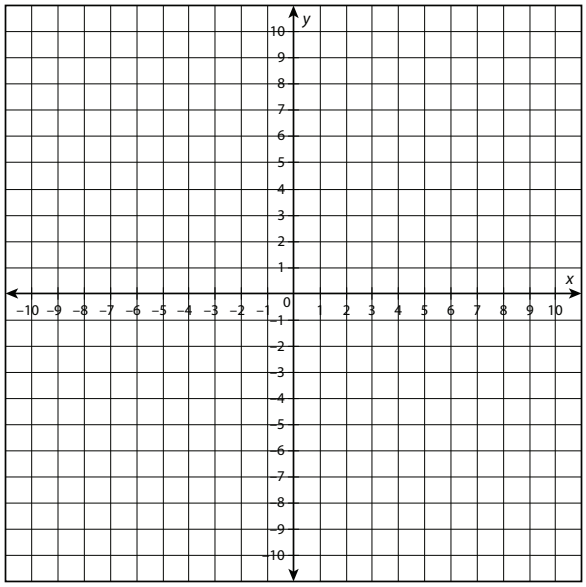
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3. Graph $y = \frac{1}{2}\left(\frac{1}{3}\right)^x$. If there is an x-intercept, what is it?



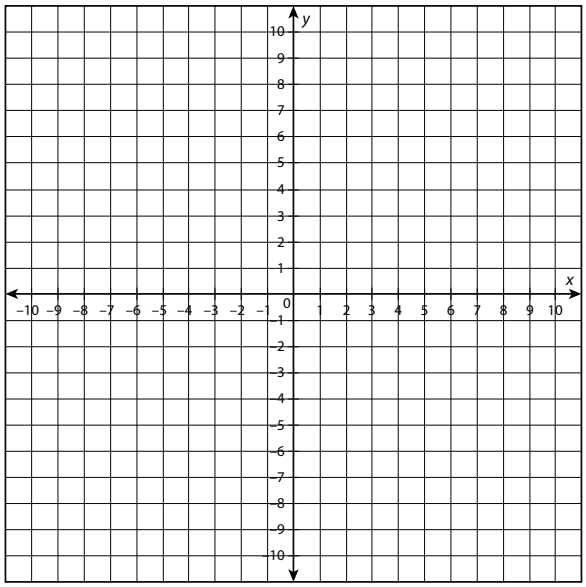
4. Graph $y = 4^x$. If there is an x-intercept, what is it?



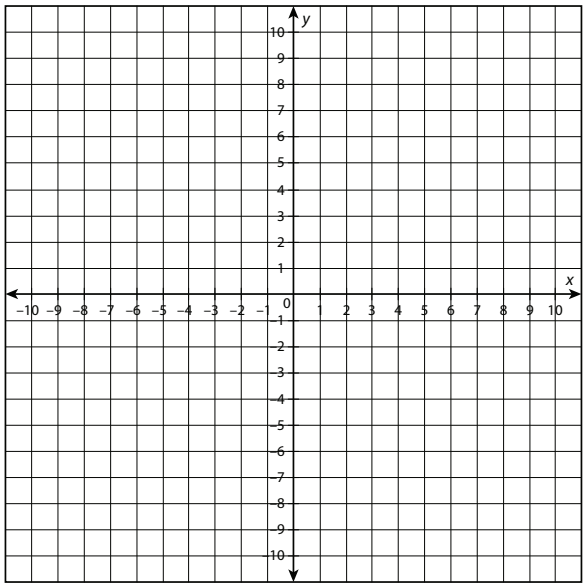
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5. Graph $y = 5(2^x)$. If there is an x-intercept, what is it?



6. Graph $y = -2(2^x) + 1$. If there is an x-intercept, what is it?



7. For an exponential function to have an x-intercept, what must be true of the equation?

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Station 3

The formula for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{tn}$, where A is the final total (principal plus interest), P is the initial amount (principal), r is the interest rate expressed as a decimal, t is the amount of time in years, and n is the number of times the interest compounds per year. Work with your group to set up and then solve each equation. Round answers to the nearest penny.

1. An account with an initial balance of $2,500 has interest of 3.5% that compounds quarterly over four years. What is the balance at the end of the fourth year?

2. An account with an initial balance of $5,000 has interest of 2.7% that compounds monthly over two years. What is the balance at the end of the second year?

3. An account with an initial balance of $500 has interest of 4% that compounds every other month over five years. What is the balance at the end of the fifth year?

4. An account with an initial balance of $6,000 has interest of 4.5% that compounds monthly over three years. What is the balance at the end of the third year?

5. If you have $1,000 to invest for two years, which account has the better yield: an account that compounds quarterly at 5%, or one that compounds monthly at 4.6%?

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Station 4

You will work with an exponential function at this station.

Use the given exponential function for the following problems.

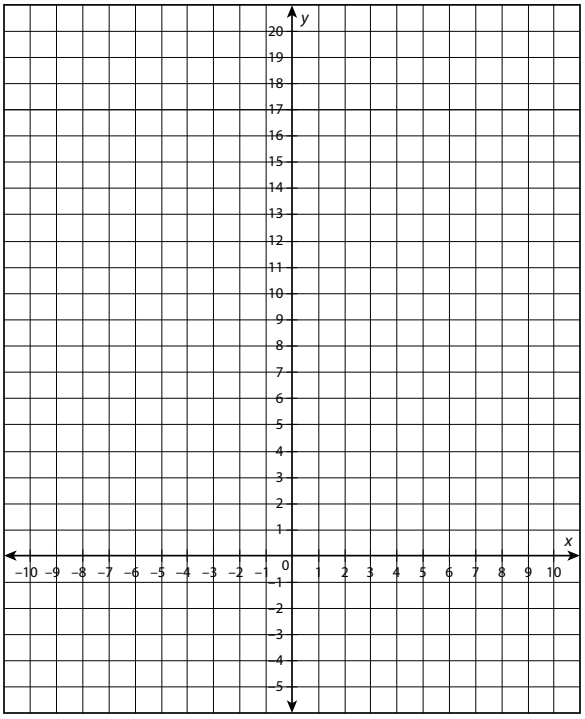
$f(x) = 2^x$

1. Create a table of values for the function.

x	$f(x)$

2. Find the x- and y-intercepts.

3. Graph the function on the coordinate plane.



continued

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4. Looking at the equation, what is a defining characteristic of an exponential function?

5. Looking at the table of values, what is a defining characteristic of an exponential function's table of values?

6. Looking at the graph, what is a defining characteristic of an exponential function's graph?

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Instruction

Goal: To guide students to an understanding of the properties and graphs of exponential functions

Common Core State Standards

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F–IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Student Activities Overview and Answer Key

Station 1

Students will evaluate functions for given values of x in order to explore the properties of exponents.

Answers

1. $f(4) = 81$
2. $f(7) = 128$
   $f\left(\frac{1}{2}\right) = \sqrt{2}$
   $f(0) = 1$
3. $f(10) = 1$
   $f(2) = 1$
4. $f(2) = 100$
   $f(4) = 10,000$
   $f(6) = 1,000,000$
5. $f(1) = 5$
   $f(3) = 125$
   $f(0) = 1$
6. $f(1) = 8$
   $f(2) = 64$

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Instruction

Station 2

Working with groups, students will determine properties of the graphs of exponential functions.

Answers

1. a. 1

   b. $y > 0$ if $b > 0$, and $y < 0$ if $b < 0$

   c. no

   d. $y = 0$

2. a. all real numbers

   b. $y > -1$

   c. There is one zero, at $x = 0$.

   d. $y = -1$

Station 3

Working with groups, students will determine the end behavior of exponential functions. Students will use their observations to determine based on the formula whether a formula represents exponential growth or decay.

Answers

1. a. approaches 0

   b. increases without bound

2. a. increases without bound

   b. approaches 0

3. a. decreases without bound

   b. approaches 0

4. Exponential decay; the function approaches 0 as x becomes infinitely large.

5. Exponential growth; the function approaches infinity or increases without bound as x becomes infinitely large.

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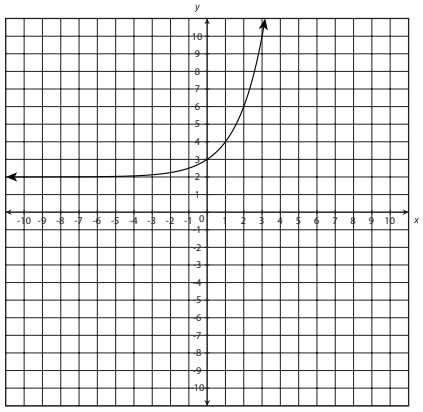
Instruction

Station 4

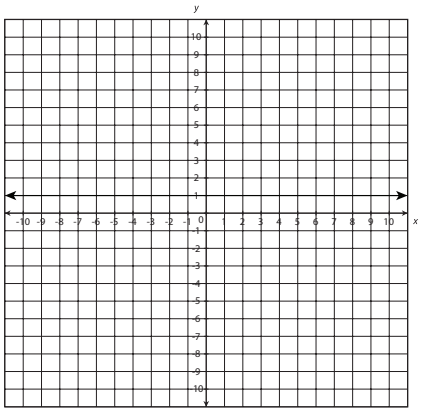
Students will work in pairs to graph exponential functions, checking their work with a graphing calculator.

Answers

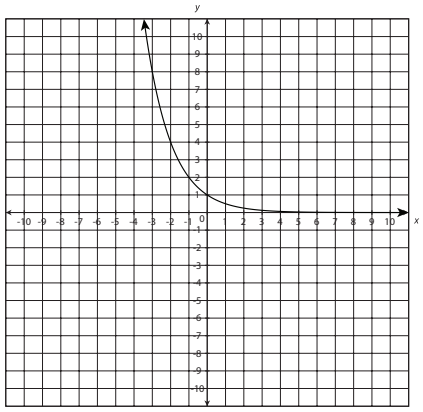
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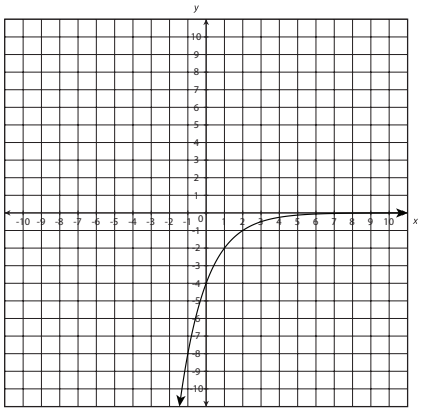
3.



2.



4.



Materials List/Setup

Station 1 graphing calculator

Station 2 graphing calculator

Station 3 graphing calculator

Station 4 graphing calculator; graph paper

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. What is an exponent?
2. What is an exponential function?
3. What are the differences among the graphs of exponential functions when the base is negative, when the base is a fraction, and when the base is a negative fraction?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. An exponent is a number that tells the number of times the base is to be multiplied by itself.
2. An exponential function is a function in which the variable is in the exponent.
3. A base that is negative will cause the graph to be reflected over the x-axis, and a base that is a fraction will cause the graph to be reflected over the y-axis. If the base is a negative fraction, then the graph will be reflected over both the x- and y-axes.

Possible Misunderstandings/Mistakes

- Incorrectly manipulating numbers, variables, or exponents
- Not understanding the laws of exponents
- Assuming that all functions have zeros
- Assuming that an exponential function has a vertical asymptote as it tends toward unbounded growth
- Not understanding the difference between growth and decay
- Incorrectly calculating squares, cubes, etc., of integers between 1 and 10
- Confusing a negative exponent with a fractional exponent

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Station 1

Work with your partner to evaluate each function. Show your work.

1. $f(x) = 3^x$
   $f(4) =$

2. $f(x) = 2^x$
   $f(7) =$
   $f\left(\frac{1}{2}\right) =$
   $f(0) =$

3. $f(x) = 1^x$
   $f(10) =$
   $f(2) =$

4. $f(x) = 10^x$
   $f(2) =$
   $f(4) =$
   $f(6) =$

5. $f(x) = 5^x$
   $f(1) =$
   $f(3) =$
   $f(0) =$

6. $f(x) = 2^{3x}$
   $f(1) =$
   $f(2) =$

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Station 2

Consider the graph of the given function. Work with your group to answer each question. Show all your work.

1. $y = b^x$
   a. What is the y-intercept?
   b. What is the range?
   c. Does the function have any zeros? If so, what are they?
   d. Does the function have any asymptotes? If so, where?

2. $y = 2^x - 1$
   a. What is the domain?
   b. What is the range?
   c. Does the function have any zeros? If so, what are they?
   d. Does the function have any asymptotes? If so, where?

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Station 3

Work with a group to answer each question. Show all your work.

1. $y = 0.5^x$
   a. What is the end behavior as x approaches infinity?
   b. What is the end behavior as x approaches negative infinity?

2. $y = 3^x$
   a. What is the end behavior as x approaches infinity?
   b. What is the end behavior as x approaches negative infinity?

3. $y = -2(2^x)$
   a. What is the end behavior as x approaches infinity?
   b. What is the end behavior as x approaches negative infinity?

4. $y = \left(\frac{1}{4}\right)^x$
   Does this function represent exponential growth or decay? Explain.

5. $y = 4^x$
   Does this function represent exponential growth or decay? Explain.

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Station 4

Work with a partner to graph each function on graph paper. Check your work with a graphing calculator.

1. $f(x) = 2^x + 2$

2. $f(x) = \left(\frac{1}{2}\right)^x$

3. $f(x) = 1^x$

4. $f(x) = -4\left(\frac{1}{2}\right)^x$

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Instruction

Goal: To guide students to an understanding of arithmetic sequences and geometric sequences

Common Core State Standards

F–BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

F–LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

Student Activities Overview and Answer Key

Station 1

Students will find the difference, d, or the nth term, a_n, of various arithmetic sequences.

Answers

1. $d = 4$
2. $d = -3$
3. $a_9 = 16$
4. $a_{10} = 6$
5. $d = 3$
6. $a_{10} = 3x + 36$
7. $a_8 = -5x$
8. Answers will vary. Students should find a_7 of their partners' sequences.

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Instruction

Station 2

Students will find the common ratios, *n*th terms, and formulas of geometric sequences.

Answers

1. $r = 2$

   $a_n = 2^{n-1}$

2. 524,288

3. $r = 3$

   $a_n = 2(3^{n-1})$

4. 39,366

5. $r = 6$

   $a_n = 5\left(6^{n-1}\right)$

6. 233,280

7. $a_1 = 4$

   $r = 2$

8. $a_1 = 10$

   $r = 6$

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Instruction

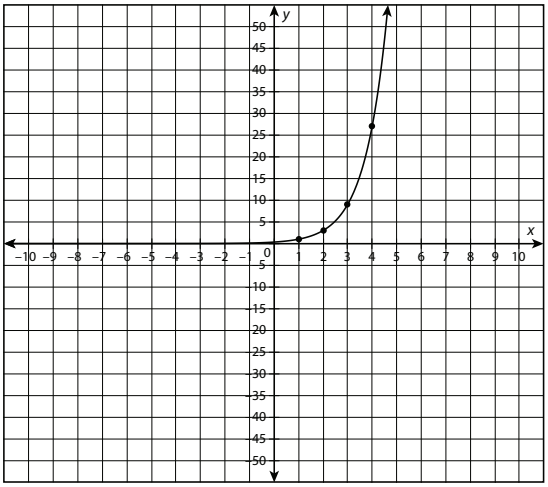
Station 3

Working with groups, students will graph geometric sequences as exponential functions.

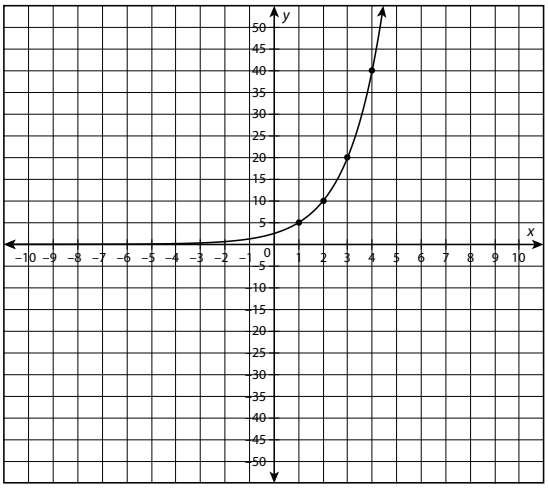
Answers

Note: Graphs of curves added to show shape only.

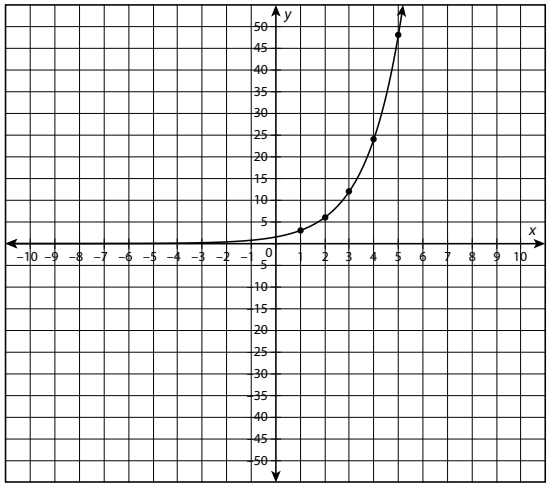
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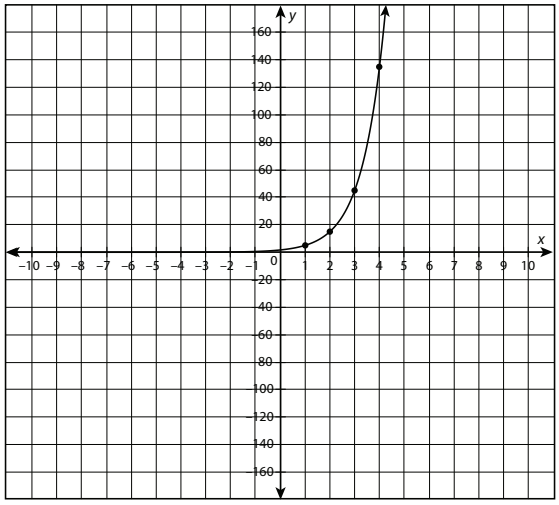
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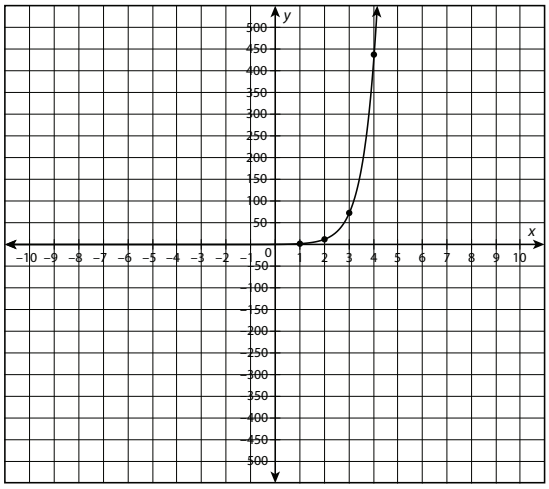
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4.



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5. 

Station 4

Working with groups, students will use exponential equations to answer questions about geometric sequences.

Answers

1. $n = 5$
2. $n = 10$
3. $n = 9$
4. $n = 8$
5. $n = 11$

Materials List/Setup

Station 1 none

Station 2 calculator

Station 3 calculator; colored pens or pencils

Station 4 calculator

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. What is an arithmetic sequence?
2. Is a sequence finite or infinite?
3. What is a geometric sequence?
4. How is a geometric sequence different from an arithmetic sequence?
5. How could a geometric sequence be related to exponential functions?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. An arithmetic sequence is an ordered group of numbers separated by a common difference.
2. A sequence can be bounded (finite) or infinite.
3. A geometric sequence is an ordered set of numbers that increase or decrease by a common ratio, r.
4. An arithmetic sequence is an ordered set of numbers that increase or decrease at a common difference, d. The terms in an arithmetic sequence are defined by addition or subtraction; the terms in a geometric sequence increase or decrease by a common factor.
5. To find the terms of a geometric sequence, we use an exponential calculation.

Possible Misunderstandings/Mistakes

- Not understanding the difference between arithmetical and geometric sequences
- Incorrectly finding the common difference, d
- Making simple arithmetical errors
- Incorrectly manipulating numbers, variables, or exponents

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Instruction

- Not understanding the laws of exponents
- Incorrectly calculating squares, cubes, etc., of integers between 1 and 10
- Incorrectly using the exponent function of a calculator
- Not understanding the relationship between a geometric sequence and an exponential function
- Not understanding the relationship between an exponential function and its graph
- Rounding off numbers too early in calculations
- Confusing geometric and arithmetic sequences
- Confusing sequences with series
- Incorrectly applying the geometric sequence formula

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Station 1

Work with your partner to answer each question. Show your work.

1. Find the difference, d, of the sequence 1, 5, 9, 13, 17, ...

2. Find the difference, d, of the sequence 2, –1, –4, –7, –10, ...

3. Find the ninth term, a_9, of the sequence 0, 2, 4, 6, 8, ...

4. Find the tenth term, a_{10}, of the sequence $\frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

5. Find the common difference of the sequence $a_n = 3n + 2$.

6. Find a_{10} for the sequence $3x, 3x + 4, 3x + 8, 3x + 12, \ldots$

7. Find a_8 for $2x, x, 0, -x, -2x, \ldots$

8. Create a sequence for your partner. Have your partner find a_7.

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Station 2

Work with your partner to answer each question. Use the calculator as you need it.

1. Find r and state the explicit formula of the geometric sequence 1, 2, 4, 8, 16, . . .

2. What is the 20th term of the sequence in problem 1?

3. Find r and state the explicit formula of the geometric sequence 2, 6, 18, 54, 162, . . .

4. What is the 10th term of the sequence in problem 3?

5. Find r and state the explicit formula of the geometric sequence 5, 30, 180, 1080, . . .

6. What is the 7th term of the sequence in problem 5?

7. Find a_1 and r of the geometric sequence with $a_4 = 32$ and $a_9 = 1024$.

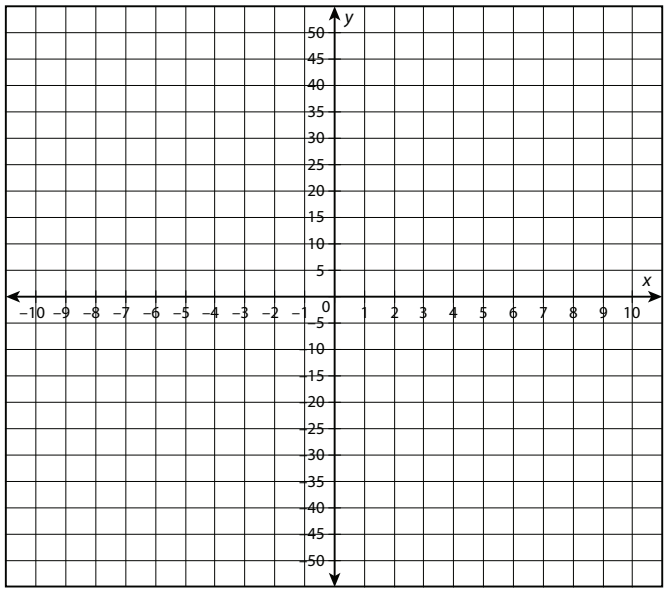
8. Find a_1 and r of the geometric sequence with $a_3 = 360$ and $a_9 = 16{,}796{,}160$.

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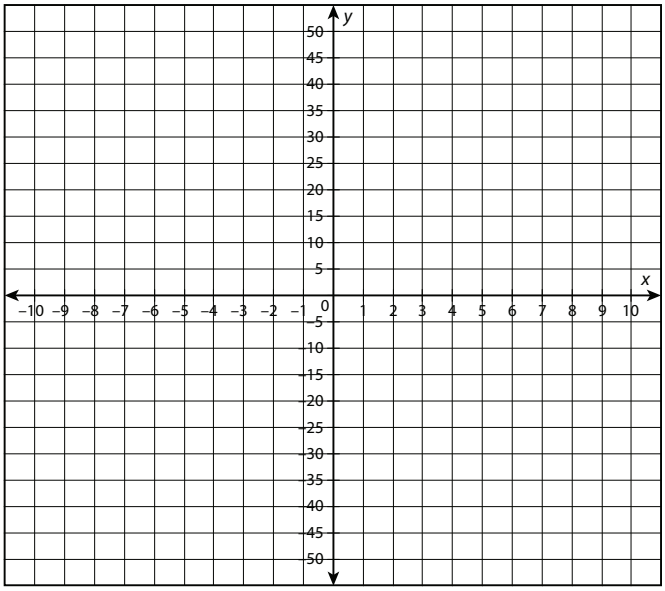
Station 3

Working with your group, graph each geometric sequence as an exponential function.

1. 1, 3, 9, 27, . . .



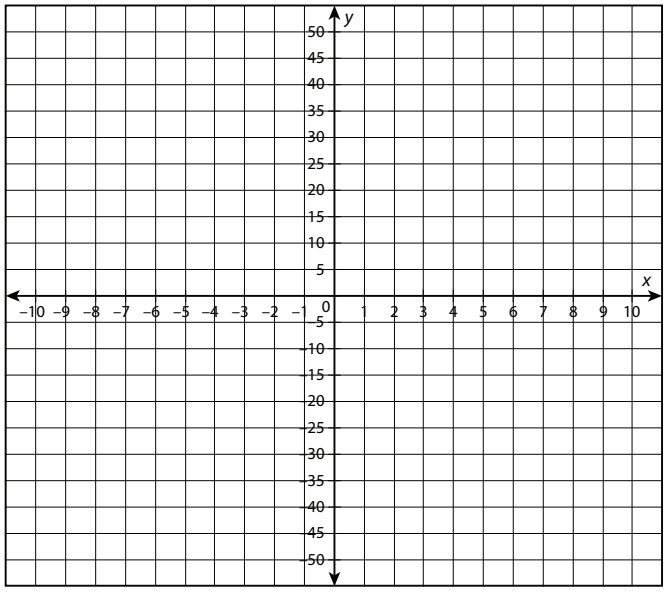
2. 5, 10, 20, 40, . . .



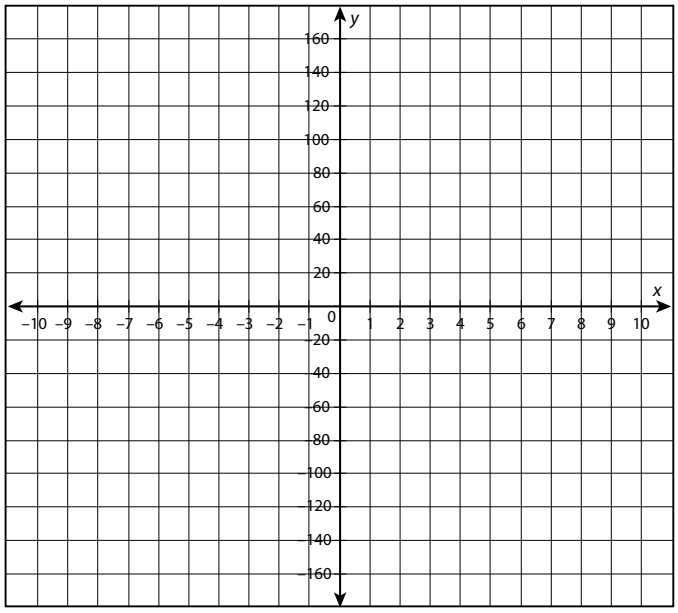
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3. 3, 6, 12, 24, 48, . . .



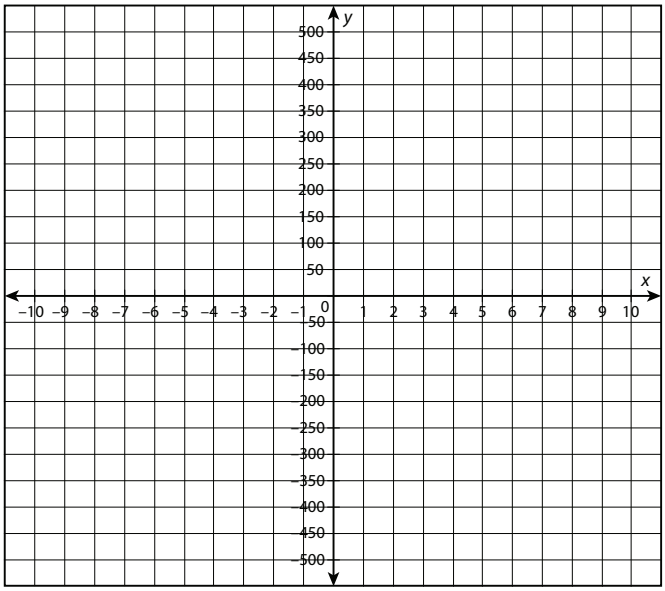
4. 5, 15, 45, 135, . . .



continued

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5. 2, 12, 72, 432, . . .



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Station 4

Work with your group to answer each question. Use a calculator as needed. Show all your work.

1. If some term, a_n, of a geometric sequence is 48, and $a_1 = 3$ and $r = 2$, what is n?

2. If some term, a_n, of a geometric sequence is 1,310,720, and $a_1 = 5$ and $r = 4$, what is n?

3. If some term, a_n, of a geometric sequence is 13,122, and $a_1 = 2$ and $r = 3$, what is n?

4. If some term, a_n, of a geometric sequence is 384, and $a_1 = 3$ and $r = 2$, what is n?

5. If some term, a_n, of a geometric sequence is 181,398,528, and $a_1 = 3$ and $r = 6$, what is n?

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APPENDIX

Problem-Based Task Implementation Guides

Introduction

This addendum contains a collection of Implementation Guides (IGs) designed to support the use of the Problem-Based Tasks found throughout this unit.

Each Implementation Guide includes an overview and suggestions for introducing and facilitating the task. These contain specific recommendations for selected Standards for Mathematical Practice (SMPs) to highlight, guidance on addressing common errors and misconceptions associated with the task, and ideas for monitoring and coaching students as they work through the task. Finally, there is guidance for debriefing the task, with special attention to the SMPs and relevant Key Concepts from the instruction, as well as information on suitable extensions and alternate strategies.

Throughout the Implementation Guides, suggestions are provided in a question/prompt format to support your efforts to elicit student thinking and facilitate classroom discourse.

We developed these Implementation Guides for our Problem-Based Tasks at the request of math educators, and with advice and feedback from mathematics supervisors and teachers. Please let us know if they are helpful. Visit us at www.walch.com, follow us on Twitter (@WalchEd), or e-mail suggestions to customerservice@walch.com.

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Guide

Problem-Based Task: Saving for College

Common Core State Standard

A–REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Task Overview

Focus

How can an equation involving two variables be created to represent a real-world scenario? How can this equation be graphed on the coordinate plane and used to estimate values? In this task, students will create an equation describing Jake's hourly wages and then graph the equation, along with a second given equation, in order to determine his earning potential.

This activity will provide practice with:

- identifying independent and dependent variables in a scenario
- creating a linear equation in two variables
- graphing a linear equation on a coordinate plane
- estimating output values from a graph
- interpreting an ordered pair from a graph
- comparing two linear equations on a graph
- analyzing values in order to determine an income

Introduction

This task should be used to explore or apply the skill of creating and graphing a linear equation and comparing it with another graph of a linear equation in order to draw conclusions involving a real-world scenario. Students should already be familiar with plotting ordered pairs in the coordinate plane and creating equations from context before beginning the task. Graphing may be done using technology, including graphing calculators or online graphing programs.

Begin by reading the problem and clarifying the meaning of the terms *linear equation* and *ordered pair*, as well as the following terms:

commission	the amount of money an employee earns for selling a product, generally based on a percentage of the sales price
earning potential	the top salary a person could possibly earn in a particular job

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Guide

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will first realize that they will need to create a linear equation to model Jake's hourly earnings. Encourage them to identify which variable will be the independent variable and which will be the dependent variable in their equation. Students will then recognize that two graphs will need to be created in order to represent the two equations that model Jake's different earning scenarios. Suggest that students check their estimations algebraically.

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they make sense of the variables given and their relationships in the scenario. Students will reason quantitatively as they substitute values into the equation for Jake's hourly earnings and the given equation for Jake's commission, in order to compare the output values for the two equations.

- **SMP 4:** Model with mathematics.

  The situation presented in this task can be modeled graphically by creating a table of values and plotting the ordered pairs on a coordinate plane for each equation. They will use these models to make estimates and compare Jake's total earnings. This situation is also modeled algebraically, by creating a linear equation to represent Jake's hourly earnings.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly plotting ordered pair solutions on a coordinate plane

  Review the process of plotting ordered pairs on a coordinate plane. Remind students that only the first quadrant is relevant in this scenario because all values are positive.

- misinterpreting the coordinates of an ordered pair

  Remind students that the first coordinate is found by moving right on the x-axis. The second coordinate is found by moving up on the y-axis.

- incorrectly evaluating the equation for different given values

  In the equation for commission, $c = (0.15)(s - 500)$, students may first distribute 0.15 in order to rewrite the equation as $0.15s - 75$, and then substitute the given value of s into the equation. Let students know that this simplified form may be easier to use for the calculations, but that the equation can be used in the original format. Suggest that students use calculators to check their math when evaluating.

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- believing the number of solutions an equation has is limited to points seen on the graph

  Discuss the domain and range of the equations in the scenario. Encourage students to think about why the domain and range are positive values. However, remind them to consider that the amount of sales or the number of hours worked in one week may exceed the dimensions of their graphs, and consequently the domain and range will contain values not shown on the graph.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.
- Before they begin the task, ask students to make a prediction about whether they think, in general, that being paid an hourly wage would yield higher total earnings than being paid only on commission. Then ask students to make predictions about Jake's situation. Encourage and guide a discussion about factors that could affect the outcome of Jake's earnings. (**Sample answer:** Student predictions will vary. Some factors that could affect the outcome are Jake's overall aptitude for sales, the type of market, the quality of the furniture, and the number of hours worked.)
- If students have difficulty beginning the task, ask them to examine the components of the given equation for Jake's commission, $c = (0.15)(s - 500)$, and discuss what each variable represents. Review the meaning of *commission*, and discuss examples of how commission is applied in business. (**Answer:** "The variable s represents the total sales, and c represents the amount of commission. Commission is the amount of money an employee earns by selling a product, generally based on a percentage of the sales price. The more sales a person makes, the higher the amount of commission.")
- If students are unsure of why there are two separate equations for this scenario, or how they relate, ask them to think about some real-world scenarios in which people work on commission. Discuss these examples with students, and draw parallels with the given scenario of Jake working as a furniture salesperson. (**Sample answer:** "A car salesperson who is paid on commission has an extra incentive to work hard at making sales, because his earnings depend on the number and price of cars he sells. If he doesn't sell any cars, he won't be paid. If he sells many cars, he will earn a high commission. However, if he was paid an hourly wage no matter how many cars he sold, he would be paid even if he didn't sell any cars.")

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- If students have difficulty creating the correct linear equation to represent Jake's hourly earnings, encourage them to think about which variable is the independent one and which variable is the dependent one. Ask them to find a pattern by testing a value or by creating a table of values with several ordered pairs. (**Answer:** "The number of hours worked, h, is the independent variable, and the wages, w, is the dependent variable, because the amount of money Jake earns depends on the number of hours he works. For example, if he works 1 hour, he will earn \$12.50. If he works 2 hours, he will earn (2)(12.50), which is \$25.00. An example of a set of ordered pairs that could be used to create a table of values is {(1, 12.5), (2, 25), (10, 125), (20, 250), (30, 375)}.")
- Ask students, "What are the constraints on the domain and range of these equations, based on the context of the scenario?" (**Answer:** "The number of hours worked cannot be negative, so the domain of the equation representing Jake's hourly earnings is $h \geq 0$. Similarly, the amount of sales in the equation for commission also cannot be negative, so $s \geq 0$. As a result, the range of both equations must also be greater than or equal to 0.")
- Ask students if both equations can be graphed on the same coordinate grid, and why. (**Answer:** "No; they must each be on a separate grid because in the equation for Jake's wages, the independent variable stands for the time in hours, and in the equation for his commission, the independent variable stands for the amount of sales in dollars.")
- If students are unsure of how to correctly create a scale for their graphs, encourage them to determine an appropriate domain based on the scenario. For each equation, ask students to identify the largest input value that makes sense in the scenario, and then ask them to substitute this input value into the equation to determine the largest output value, which is the range. (**Answer:** "The largest input value for the equation that represents Jake's hourly earnings is 40, and the corresponding output value is \$500. The largest input value for the equation that represents Jake's commission is undetermined, since it is based on how much he sells, so we can use a reasonable estimate of \$6,000. The corresponding output value is \$825.")
- After students have created the graphs for the two equations, either by hand or with a graphing calculator, ask them which graphing method will provide the best representation of the equations and why. (**Answer:** "Even if I use graph paper and a ruler, a graphing calculator will still provide greater accuracy in graphing, as there is no room for error in regard to plotting the points and drawing the line. However, I must still be careful when inputting their equations into the graphing calculator to avoid errors caused by incorrectly keying in values.")

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- After students have examined their graphs to estimate the amount that Jake will earn based on the given number of hours and amount of sales in the Coaching questions, ask them how they can algebraically check their results and why this will ensure accuracy. (**Answer:** "Because the scales of the graphs contain large intervals, it is difficult to find the exact amount of earnings by finding the corresponding output value from the given input value. The word *estimate* means 'to closely guess or roughly calculate the value of something,' which is what is done when the scale contains large numerical intervals. I can check my estimated results algebraically by evaluating the equations for the specific input values; the output values will be the exact amounts, with no estimating necessary.")
- Ask students to explain why being paid hourly results in a guaranteed wage and being paid on commission does not. (**Answer:** "An hourly wage is only dependent on the number of hours that an employee works. As long as an employee is present at his or her job each day, then he or she earns a wage. However, being paid only on commission depends on many factors, such as the skill of the employee as a salesperson, the type of product being sold, the time of the year, and the economy. Working only on commission does not guarantee a wage.")
- Ask students to compare their answers and show their work to another student. If students' results differ, ask them to discuss and analyze each other's work. Ask students to write down their response to the following question: "What is the maximum amount of earnings that Jake could make—that is, what is Jake's earning potential?" (**Sample answer:** "Jake has a limit on the number of hours he is allowed to work each week, but he does not have a limit on how much furniture he can sell. Therefore, Jake's earning potential is theoretically unlimited if he works on commission, but it is limited if he works for an hourly wage.")

Debriefing the Task

- Ask students to volunteer their initial predictions about the different outcomes of working for an hourly wage compared to working on commission. Ask them to explain their thought processes and reasoning based on the factors involved in working on commission. Ask if their graphical analysis changed their predictions, and why or why not.
- Compare students' strategies and explanations for creating the linear equation and for graphing both equations. Focus on the use of precise mathematical language, including the identification of the variables, as well as how the graphs were created.

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Connecting to Key Concepts

Make explicit connections to key concepts:

- Ordered pairs can be plotted in the coordinate plane.

  In this task, one method for graphing the equations is by creating a table of values to represent Jake's hourly earnings and his commission. These ordered pairs would then be plotted in the coordinate plane.

- An equation whose graph is a straight line is called a linear equation.

  When the two equations are graphed, the points are connected to form a straight line, which identifies each equation as linear. This means that for each equation, the rate of change between any two points is constant.

- The solution to an equation with two variables is the set of ordered pairs (x, y) that satisfies the given equation.

  If the independent variable representing the number of hours Jake works, h, is set at 20, this value is substituted into the equation for his hourly earnings, and the output value, w, is \$250. Therefore, the ordered pair that represents this solution is (20, 250).

Extending the Task

- To extend the task, provide students with a scenario about another person who works in a rival furniture company (Company B). Let this person have an hourly wage close to Jake's hourly wage (for example, \$13.00/hour), and provide a similar equation for this person's commission (e.g., $c = (0.12)(s - 525)$). Ask students to create a table of values for this person's hourly wage, as well as the commission. Ask students to plot the line for this person's hourly wage on the same coordinate plane as used for Jake's hourly wage, and then to plot the line for this person's commission on the same coordinate plane as Jake's commission. Have students compare the graphs visually, and ask them to draw conclusions about whether Jake's company or Company B offers the best wages, based on the same amount of sales or hours worked.

- Another option for extending the task is to ask students to work with a partner to create their own problem scenarios, modeled on the idea of Jake and the furniture company. Ask them to be creative in designing a scenario in which a person works on commission compared to another person who works for only an hourly wage. Ask each pair to list the steps for solving their scenario, as well as detailed explanations and processes for comparing an hourly wage to working on commission.

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Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Sample answer:** "I persevered in solving the problem by following my original solution process of creating a linear equation, creating a table of values for both equations, and then graphing both equations. Then, I used my graphs to estimate my solutions and I checked these solutions algebraically by substituting values into the equations.")
- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Sample answer:** "I reasoned abstractly by choosing an independent variable, h, to represent the number of hours that Jake works and choosing w as the dependent variable so that I could create a linear equation to model Jake's hourly wages.") "Which of your strategies represent quantitative reasoning?" (**Sample answer:** "I used quantitative reasoning when I substituted values into the equation for Jake's hourly earnings and into the given equation for Jake's commission in order to compare the output values for the two equations.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Sample answer:** "I modeled the given scenario graphically by creating a table of values and plotting the ordered pairs on a coordinate plane for each equation. I also modeled the scenario algebraically by creating a linear equation to model Jake's earnings.")

Alternate Strategies or Solutions

- Although the solution method presented in the Coaching questions is based on modeling the equations graphically and estimating Jake's earnings from examining the output values when given specific input values, students may also choose to substitute the input values into the given linear equation for commission and the equation they create for Jake's hourly earnings to find the exact results. Discuss the advantages and disadvantages of estimating values from a graph, including possible errors resulting from the numbers on the scales being large. Explain how substituting values into the linear equations produces exact results.
- Students may choose their own values for the number of hours Jake works, and the amount of sales. They may experiment with these values in order to draw their own conclusions about his earning potential. Encourage them to consider constraints on the input values in the context of the scenario. (For example, is it reasonable for Jake to work 100 hours in one week? Is it reasonable for him to sell $500,000 worth of furniture in one week?)

Technology

Although graphing can be done by hand, students may use graphing technology to draw the graphs and to evaluate the equations for various input values.

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Problem-Based Task: Senior Trip

Common Core State Standard

A–REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

Task Overview

Focus

How can the solution to a system of equations be found? How are functions evaluated for specific domain values? In this task, students will compare two functions that represent different savings methods, and determine which method results in the greater amount saved at the end of 8 years. They will also determine the point of intersection of the two functions.

This activity will provide practice with:

- evaluating functions for given domain values
- graphing functions on the same coordinate plane
- analyzing the output values of functions
- estimating a point of intersection on a graph and from a table of values
- interpreting the point of intersection of two functions on a graph
- creating a table of values to represent input and output values

Introduction

This task should be used to explore or apply the following skills: identifying the solution of a system of equations by graphing; evaluating functions at specific domain values; and interpreting a table of values by identifying and comparing changes in output values of two functions. Graphing may be done using technology, including graphing calculators or online graphing programs.

Begin by reading the problem and clarifying the meaning of the terms *domain*, *function*, *range*, *solution set*, and *system*, as well as the meaning of *interest compounded monthly*:

interest compounded monthly	a percentage of the balance that is added to the account once per month

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will first recognize that both boys have a different function that represents their savings methods. Students will also identify that one method is represented by a linear function, and the other is represented by an exponential function. Students will create a plan for comparing the two functions in order to determine if both boys will have saved enough money for their senior trip at the end of 8 years. Encourage students to discuss comparing the functions graphically and algebraically, and to analyze and interpret the range values that result when the functions are evaluated at specific domain values.

- **SMP 4:** Model with mathematics.

  Students are given two different types of functions representing money-saving methods for two boys. Students will identify the important quantities in the problem and model the functions with a table of values as well as with graphs. They will evaluate both functions and compare the results in order to determine the amount of money each boy will have saved at the end of 8 years. Encourage them to analyze both the table of values and the graphs in order to draw conclusions about the possible solutions of a linear function and an exponential function.

- **SMP 8:** Look for and express regularity in repeated reasoning.

  Students will notice that the same calculations are performed repeatedly in order to achieve the desired results. They will recognize that the number of years is used for the independent variable in both functions, and that the order of operations is used to evaluate the functions correctly. Students will also evaluate the range values and see the reasonableness of the results in regard to the fact that both functions are increasing, and that as more time passes, more money is saved. Remind students to observe the shapes of the graphs of each function, and to compare the shapes with the rate of change for each function from the table of values.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- substituting values incorrectly

  Review the order of operations with students. Remind them to carefully substitute values when evaluating a function, and to check their results with a calculator, if they choose to perform the calculations by hand first.

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- believing that all graphs will cross

  Remind students that there are three possible types of solutions for a system of equations: there may be no solution, one solution, or more than one solution. Graphically, two graphs may not intersect at all, they may intersect at one point, or they may intersect at more than one point.

- incorrectly assuming that one function will yield a greater output value in comparison with a second function based on only a few input values

  Remind students that *all* domain values in the given scenario must be evaluated in order to draw conclusions about which function yields greater output values. For example, in the Coaching Sample Response for part b, we find that Jacob will have saved the most amount of money up through $x = 4$ years. However, in part c, we skip ahead to solving for $x = 6$ years, after which Antonio's savings will be greater than Jacob's. Concluding that Antonio's function will yield higher outputs for all inputs greater than 4 is misleading, because there are many possible values of x between 4 and 6. In fact, Antonio's savings won't surpass Jacob's until x is somewhere between 5.9 and 6.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.
- Before they begin the task, ask students to predict who they think will have saved the most money after 8 years, and whether they think either boy will have saved enough money in time for their trip. Encourage students to base their predictions on the information given in the problem regarding the boys' savings habits, as well as the type of functions used to represent each boy's savings method. (**Answer:** Some students might predict that Jacob will save more money because he continues to work and earn wages each month, whereas Antonio does not work, but has his money invested in a bank account. Some students might predict that the function representing Antonio's savings method is exponential and will therefore yield larger numbers when domain values are evaluated. Encourage students to write their predictions down, and to discuss their predictions with each other before beginning the task.)
- If students have difficulty beginning the task, first ask them to examine the components of each function. Ask:

  "What are the independent and dependent variables in each function? What do these variables represent?" (**Answer:** "In both functions, x is the independent variable, representing the number of years, and y is the dependent variable, representing the amount of money saved.")

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"What types of functions are these? Describe each function's shape and whether the rate of change for each is constant." (**Answer:** "Jacob's function is a linear function, which means the graph will be a straight line, and the rate of change will be constant between any two consecutive ordered pairs. Antonio's function is an exponential function, which means the graph will be a curve, and the rate of change will not be constant between two consecutive ordered pairs.")

- If students are unsure about the meaning of the formula used for the function representing Antonio's savings method, ask them to think about what they already know about savings accounts and interest. Ask:

  "What is the compound interest formula? What do the variables in this formula represent?" (**Answer:** "The formula is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A is the account balance, P is the initial amount or principal, r is the interest rate expressed as a decimal, n is the number of times the investment is compounded in a year, and t is the number of years.")

  "What does each number in Antonio's function represent?" (**Answer:** "In Antonio's function, '1000' represents Antonio's initial deposit of $1,000; '1 +' represents adding 100% of the account balance to the amount of interest earned on that balance each compounding period; '0.18' represents the 18% interest rate; and '12,' which appears in both the denominator of $\frac{0.18}{12}$ and the exponent $12x$, represents the fact that the interest is compounded monthly, or 12 times in 1 year.")

- Ask, "What is the domain of each function and why?" (**Answer:** "The domain is all real numbers greater than or equal to 0, and less than or equal to 8. This can be represented as $0 \le x \le 8$. The domain begins at 0 because that represents when the boys start saving their money. The domain ends at 8 because that represents the total number of years for which they plan to save their money.")
- Although the Coaching questions do not require students to create a table of values for all of the domain values, encourage students to create one in order to compare the results of the domain values of 2, 4, 6, and 8 years for parts a–d. From this table, ask students to note the interval of values in which one boy's savings will have surpassed the other's. (**Answer:** "The table of values shows that Antonio's savings are less than Jacob's when $x = 4$, but greater than Jacob's when $x = 6$. This means that Antonio's savings surpassed Jacob's at some x-value greater than 4, but less than 6. Therefore, the interval of values is $4 < x < 6$.")

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- Students will construct a table of values to approximate when one boy's savings surpasses the other's. If students aren't sure what values of x to use for the table, ask, "What does the intersection of the graphs represent? How can you use this information to determine what values of x to use for your table of values?" (**Answer:** "The point of intersection of the graphs represents the point at which both boys' savings will be equal. One boy's savings will exceed the other's after this point.") Encourage students to construct their table of values using x-values from 5.5 years to 6 years, since the graph shows that the point of intersection is within this interval of time.
- After students have created the table of values for the domain of 5.5 to 6.0 years, ask them why it is important to subtract the output values for each function, and what the results of this represent. (**Answer:** "When the output values are subtracted, a positive difference means that Jacob has more money saved than Antonio. When the difference becomes a negative value, this represents that Jacob now has less money saved than Antonio does. Therefore, the domain value which resulted in the output value that yielded a negative difference represents the point in time at which Antonio's savings surpassed Jacob's savings.")
- Ask students to explain why and/or how it is possible that for the first 5-plus years, Jacob will have saved more money, but then at 5.9 years, Antonio's savings will become greater than Jacob's. (**Answer:** "Since Antonio has his money invested in a bank account that earns interest, the amount he has saved will gradually increase exponentially over time, whereas Jacob's savings will increase linearly. The slope of the line representing Jacob's function is not steep enough to surpass the exponential curve representing Antonio's function.")
- Ask students to compare their answers and show their work to another student. If students' results differ, ask them to discuss and analyze each other's work. Ask students to write down their response to the following questions: "How did you conclude which boy's method will result in the most money saved? How did you determine whether either boy will have enough money saved for their trip at the end of 8 years?" (**Answer:** "By evaluating each boy's function for $x = 8$, I determined that after 8 years, Antonio will have saved \$4,175.80, which is more than Jacob's projected savings of \$3,500. This result also shows that both boys will have accomplished their goal of saving \$3,500 in time for their trip.")
- Ask students, "If the boys continued saving money for another 8 years, would Jacob's savings ever surpass Antonio's in this time?" (**Answer:** "No, because Antonio's total savings will continue to increase exponentially, whereas Jacob's savings will continue to increase linearly; therefore, the graphs will not intersect again.")

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Debriefing the Task

- Ask students to volunteer their initial predictions about who will save more money and whether either boy will have saved enough money in time. Ask students to explain their thought processes and reasoning based on the given information.
- Compare students' strategies and explanations for evaluating the functions, creating a table of values, and graphing the functions. Discuss the analysis used on the table of values from 5.5 to 6.0 years. Focus on the use of precise mathematical language, including the identification of the variables and the solution to the system of equations.

Connecting to Key Concepts

Make explicit connections to key concepts:

- When graphing functions $f(x)$ and $g(x)$, the solution set for $f(x) = g(x)$ is the coordinates of the point(s) of intersection of the graphs of the two functions.

  In this task, students will graph the two functions representing the boys' savings methods on the same coordinate plane. Students will use the graphs to determine that the graphs intersect at one point, when x is approximately equal to 5.9 years.

- To create a table of values for a system of equations, list the inputs for each equation and determine the corresponding outputs in both equations.

  Students will create a table of values for the time interval that contains the point of intersection, and evaluate the functions at specific domain values in order to compare the output values for each function.

- Look for the difference that has the least absolute value, or for a sign change between successive output values; that is, where the values of $f(x) - g(x)$ change from negative to positive or from positive to negative. The solution to the system is found where the sign change occurs and the difference between the two outputs is equal to 0 or has the least absolute value.

  In the table of values, the sign change of the values of the differences between the output values of the two functions occurs between 5.9 and 6.0 years, and the input value that corresponds to the output with the least absolute value is 5.9 years.

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Extending the Task

- To extend the task, provide students with the scenario of a third friend who plans to save for the senior trip using a different savings method that is represented by a new function; for example, a function involving interest compounded quarterly, or a linear function with a steeper slope. Ask students to add an additional column to their table of values to include the results of evaluating this new function. Also, ask them to graph this new function on the same graph as Jacob's and Antonio's functions. Ask students to determine which friend's method will result in the most money saved, and whether this third person will have enough money saved for the trip after 8 years.
- Another option for extending the task is to have students team up with a partner to work through the task again, but this time using different values for the boys' savings goal, time frame, and each boy's initial amount, along with different values for Jacob's monthly contribution and Antonio's interest rate and/or number of compoundings.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I made sense of the problem by creating a table of values for both functions and analyzing the table in order to compare the functions. I also graphed both functions and used the graphs to approximate the solution to the system, which was the point where the graphs intersected.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I modeled the given scenario graphically by creating a table of values for each function and graphing both functions on the same coordinate plane. I also modeled the scenario algebraically by evaluating the functions at specific domain values and analyzing the results in order to determine the solution to the system of equations.")
- **For SMP 8, ASK:** "How did you look for and express regularity in repeated reasoning?" (**Answer:** "I noticed that the same calculations are performed repeatedly in order to achieve the desired results. I recognized that I used the order of operations each time in order to evaluate the functions at specific domain values.")

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Alternate Strategies or Solutions

- Although the solution method presented in the Coaching Sample Responses section is based on only creating a table of values for the time range of interest (5.5 to 6.0 years), students may also create a table of values for the domain values given in parts a–d: 2, 4, 6, and 8 years. It may be helpful for students to create a table to represent these results before they graph both functions. Discuss the advantages and disadvantages of determining a solution for a system of equations both algebraically and graphically.
- Students may choose their own values for the number of years when evaluating the functions. They may experiment with these values in order to draw their own conclusions about when one boy's savings surpasses the other's. Students may also choose domain values that contain more significant digits, such as 5.85 or 5.95, in order to achieve a more accurate approximation. Encourage students to consider and discuss how many significant digits might be necessary in order to obtain an exact result.

Technology

Although graphing can be done by hand, students may use graphing calculators to create the two graphs, or they may use scientific calculators to perform the calculations when substituting values into the equations.

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Problem-Based Task: The Private Library

Common Core State Standard

F–IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

Task Overview

Focus

What is the domain of a function? What is the range of a function? How can the domain and range of functions given in a real-world scenario be determined? In this task, students will analyze given information about Terrence's organizational system for library books, and they will identify the domain and range of three different functions used in the process of sorting the books.

This activity will provide practice with:

- interpreting an organizational system in terms of creating a function
- identifying the domain and range of a function
- applying the order of operations
- analyzing given real-world constraints
- ordering a set of values

Introduction

This task should be used to explore or apply the skill of determining the domain and range of a function, as well as the skill of analyzing the context of a scenario in order to identify real-world constraints on the domain and range. Students should already be familiar with the notation $y = f(x)$ and the concept of the term *function*.

Begin by reading the problem and clarifying the meaning of the terms *domain* and *range*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will first recognize that the functions given in this scenario are not represented with strictly numerical values. They will analyze the verbal descriptions given in the scenario and translate them into functions. Students will examine Terrence's method for sorting the library books, and they will interpret the letters and numbers that represent his sorting system. Students will also analyze the constraints for both the domain and the range for each function based on the context of the given scenario.

- **SMP 6:** Attend to precision.

  Remind students to continuously use correct mathematical terms and definitions clearly and accurately when going through the Coaching questions. Strive toward precise mathematical language, such as by using the terms *function*, *domain*, and *range* correctly. Make sure students use the proper notation when listing the domains and ranges.

- **SMP 7:** Look for and make use of structure.

  In order to identify the range of the year-sorting function, Y, students have to notice the structure of Terrence's formula of subtracting 1800 from the year of publication for each book in the library. They will make the connection between the formula and the function, as they subtract 1800 from each book's year of publication in order to obtain this part of Terrence's coding structure.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- confusing domain and range

  Remind students that the domain is the set of all inputs, or x-values, of a function, and the range is the set of all outputs, or y-values, of a function.

- thinking that if a range value repeats or is the same for a different value in the domain then the relation is not a function (different x-values have the same y-value)

  Remind students that for a relation to be considered a function, each domain value must only be mapped to one range value; in other words, for each x-value there is one and only one y-value. However, two different domain values can be mapped to the same range value, and the relation will still be a function.

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- incorrectly identifying the constraints on the domains and ranges of the functions

  In this problem, the domain values are only limited to the amount (or number) of books in the library. However, the range values are limited to the number of letters in the alphabet, the range of dates from 1800 to 2012, and all possible combinations of these ranges. Remind students to carefully examine all information given in the problem; for example, the year that Terrence was hired. This year affects the range of the year function, *Y*.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.
- Before they begin the task of identifying the domains and ranges of the given functions, ask students to create their own example of a book that could be in Terrence's library, so that they can practice and show understanding of the naming convention that Terrence uses. For example, a student may create a fictional book entitled *Labeling a Book*, with a publication year of 1952. Ask them how the sticker for the book would read. (**Answer:** "Using Terrence's labeling system, since the first letter of the book title is L, the first part of the sticker would have an 'L' on it. Then, since he uses the formula of subtracting 1800 from the year of publication, the number part of the sticker would be 152, since 1952 – 1800 = 152. Therefore, the sticker for *Labeling a Book* would read L152.")
- If students have difficulty beginning the task, first ask them to write the definitions of *domain* and *range* at the top of their papers or in their notes. Encourage students to write the definitions in a way that will help them learn and apply the correct definitions for each term. Note that some students may write the formal definitions of domain and range, while others may write something such as: "domain = *x*-values; range = *y*-values," or "domain = first part; range = second part." While it is important to encourage students to use precise mathematical language, some students may apply the terms better when using their own wording.
- If students are unsure about the domain and range for the title function, *T*, encourage students to think about the domain in terms of the number of books in the library. Then, ask them to think about the range in terms of the constraint that each book's label must use the first letter of the title.

  Ask, "Do we know how many books are in the library? What does this tell us about the domain of *T*?" (**Answer:** "We don't know how many books are in the library. Since there's no specific number of books, the domain has to be 'all possible book titles.' ")

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Ask, "Do we know what letter is the first letter of the title for *every* book in the library? What does this tell us about the range of *T*?" (**Answer:** "We don't know the title for every book in the library, so we can't exclude any of the 26 letters in the alphabet from the range. Therefore, the range of the title function is the letters A–Z.")

- If students are unsure about the domain for the year function, *Y*, encourage them to think about the domain in terms of all of the possible publication dates of books in the library. Ask, "What are the lowest and highest possible publication dates?" (**Answer:** "The earliest publication date in the library is 1800, so 1800 is the lowest possible publication date. Since Terrence was hired in 2012, the highest possible publication date is 2012. Therefore, the domain of the publication dates is all dates from 1800 to 2012.")
- If students are unsure about the range for the year function, *Y*, encourage them to think about the formula that Terrence uses to find the numerical part of the labeling sticker for each book. Ask, "How can the range be determined from the domain?" (**Answer:** "Since the domain of the year-sorting function *Y* is all dates from 1800 to 2012, the range is all possible differences when 1800 is subtracted from a publication date; in other words, the number of years that have passed since the earliest publication date, 1800. Therefore, the range is from 0 to 212.")
- If students are unsure about the domain and range for the sticker function, *S*, remind them that this function is a combination of the other two functions. Ask, "If the sticker function, *S*, is a combination of the functions for *T* and *Y*, what does this tell you about the domain and range of *S*?" (**Answer:** "The domain of *S* is all of the books in the library, because every book has a single sticker. The range of *S*, however, is all possible combinations of the letters A–Z from the range of *T* and the numbers 0 to 212 from the range of *Y*.")
- Ask students, "Why do you think Terrence chose to list the letter before the number in his sticker naming system?" (**Answer:** Students should reason that there are only 26 letters in the alphabet, but there are 212 possible numbers that could represent a book. While it is possible to order the numbers in terms of least to greatest, it is generally more effective and quicker to locate a book labeled alphabetically than numerically when the range of possible numbers is so much larger than the range of possible letters.)

Debriefing the Task

- Compare students' strategies and explanations for determining the domains and ranges for each of the three functions. Discuss any issues that students may have had when they were asked to use the sticker function, *S*, to label the given books.
- Encourage and guide a discussion about whether students felt this task was easier to complete given this real-world scenario involving letters used to create functions, in comparison with a strictly numerical function. Discuss the parallel of the domains and ranges of the functions in this task compared to the domains and ranges of a task involving simple numerical functions, such as $f(x) = x + 3$.

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Guide

Connecting to Key Concepts

Make explicit connections to key concepts:

- The domain is the set of *x*-values that are valid for the function.

  In this task, students are given a real-world scenario in which they are asked to identify the domains of three different functions. In order to accomplish this, they will examine the constraints on the domains for each function.

- The range is the set of *y*-values that are valid for the function.

  After determining the domain values for each function in the problem, students will consider the constraints on the possible output values, or ranges, for each function, based on the domain values.

Extending the Task

- To extend the task, provide students with the scenario of a fourth component and a fourth function that Terrence could use to sort the books in the library. For example, he might also label the books in terms of genre. As a result, a genre-sorting function, *G*, could be created, with a range of the different types of books in the library, such as fiction (f), nonfiction (nf), science fiction (sf), biography (b), history (h), mystery (m), drama (d), and romance (r). The lowercase letter(s) could be located at the end of the sticker label, such that now each book has three different components of classification. Ask students to then create a revised sticker function, and also ask them to find the domain and range of the new sticker function.
- Another option for extending the task is to ask students to work with a partner to create their own functions to represent a sorting system for books in their own library. Encourage them to be creative and experiment with different types of functions that can be used to represent their sorting methods.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I persevered in solving the problem by first analyzing the verbal descriptions given in the scenario and translating them into functions. I interpreted the letters that represented Terrence's sorting system, and I considered the constraints on the domains and ranges as I determined them for each function.")

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- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** "I used correct math terminology in my explanations and I used the proper notation for the domains and ranges. I also used the correct naming system of letters and numbers when asked to do so for the books in the library.")
- **For SMP 7, ASK:** "How did you look for and make use of structure when solving this problem?" (**Answer:** "I noticed the structure of Terrence's formula of subtracting 1800 from the year of publication for each book in the library. I made the connection between the formula and the function, which allowed me to understand how to subtract 1800 from each book's year of publication in order to obtain this part of Terrence's coding structure.")

Alternate Strategies or Solutions

- Although the Coaching questions do not require a table or chart, students may choose to create one in order to list the domain and range for each function. This may help them visually organize the domains and ranges.
- Students may choose to list all of the elements of each domain and range. Let students know that although this approach is correct, it may be too time-consuming. Reinforce that an interval of numbers or letters can be represented by an ellipsis, so that not all elements of a domain and range have to be written. For example, point out that instead of listing every number from 0 to 212, students can use the notation {0, 1, 2, ..., 211, 212} to more efficiently represent all of the elements.

Technology

Students do not need any technology for this task.

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Guide

Problem-Based Task: Party Time

Common Core State Standard

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Task Overview

Focus

How can a function be evaluated given specific domain values? How can the sum of several functions be determined? In this task, students will create a function and evaluate it for various domain values, which represent the number of possible guests at a party, in order to determine the total cost of the party.

This activity will provide practice with:

- identifying the domain of a function
- evaluating a function for given domain values
- applying the order of operations in order to simplify a numerical expression
- interpreting the output values in the context of a scenario
- creating a table of values displaying input and output values

Introduction

This task should be used to explore or apply the skill of evaluating functions for various inputs and interpreting the results in terms of the context of the scenario. Students should already be familiar with substituting values for variables. Scientific calculators may be used for the computations.

Begin by reading the problem and clarifying the meaning of the terms *domain*, *evaluate*, *function*, *function notation*, and *range*.

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Guide

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students can begin to make sense of the problem by discussing the given functions for the number of tables, chairs, and waiters, as well as the venue size to better understand what they represent. Students can then relate these formulas to the function given for the total cost of the party, $P(x)$, in order to make sense of that function.

- **SMP 6:** Attend to precision.

  Strive toward precise mathematical language, such as correctly using the terms *function*, *domain*, and *range*. Make sure students use the proper notation when writing the functions and listing the domains and ranges. Remind students to use the proper units when stating the results of the Coaching questions.

- **SMP 8:** Look for and express regularity in repeated reasoning.

  Students will notice that the same calculations are performed repeatedly in order to achieve the desired results. They will recognize that the number of guests is used for the independent variable in all four functions, and also that the order of operations is used to evaluate the functions correctly.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- thinking function notation means "f times x" instead of "f of x"

  Remind students that $f(x)$ is a way to name a function instead of using y. The x in the parentheses represents the function's independent variable.

- not multiplying each function in this task by its unit rate when evaluating it for a specific domain value

  Remind students that although the functions for each component are stated without the unit cost, each function must be multiplied by its unit cost (or rate). For example, the function for the number of chairs is $c(x) = x + 10$, and chairs cost \$3 each. Therefore, when evaluating the *cost* of the chairs, the expression $x + 10$ must be multiplied by 3.

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Guide

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task:

  Ask students to identify each type of function given. (**Answer:** "The first three functions are linear, and the fourth function is exponential.")

  Ask students to examine the given information, and guide them to notice that the number of guests increases linearly (50, 100, 150, 200). Ask students to make a prediction based on this observation about whether they think the cost of the party will increase linearly with the number of guests. Encourage students to base their predictions on the types of functions given in the problem. Some students might predict that because there is a constant difference between the input values, there will be a constant difference between the output values that represent the cost of the party. Some students might predict that there will be varying differences between the output values, due to the exponential function. Encourage students to write their predictions down, and to discuss their predictions with each other before beginning the task.

- If students have difficulty determining the domain values for each function, ask them to first identify the independent and dependent variables for each function and to define what they represent. (**Answer:** "The independent variable, x, represents the number of guests. The dependent variable is given as $P(x)$, and this represents the total cost of the party.")

  Ask, "Now that you know what the independent variable represents, how can you tell from the given information what the possible domain values are?" (**Answer:** "The problem only gives four possible domain values: 50, 100, 150, and 200. These are the numbers of guests for which we need to evaluate the cost of the event.")

- If students are unsure about how to determine the most that the party can cost, remind them to consider the necessary domain value that should be used to find the "most." Ask, "What is the number of guests that would result in the highest cost for the party?" (**Answer:** "There are four domain values: 50, 100, 150, and 200. The value 200 will be used to determine the most that the party can cost, because it represents the maximum number of guests.")

- Ask students, "How is the cost for tables and chairs for 100 guests calculated? Why do the numbers '50' and '3' need to be part of the calculations?" (**Answer:** "The functions representing the number of tables and the number of chairs will be evaluated at $t(100)$ and $c(100)$. The results will be multiplied by the cost per item, and then added together. The number 50 represents the cost of each table, and the number 3 represents the cost of each chair. Therefore, $t(100)$ must be multiplied by 50, and $c(100)$ must be multiplied by 3.")

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- Ask students, "How can you determine the least amount the party can cost?" (**Answer:** "The party will cost the least amount with the smallest number of guests, which is represented by the domain value of 50. Therefore, substitute 50 into each of the four initial functions, multiply the results by the unit cost for each item, and then add the products to find the total cost of the party for 50 guests.")
- Ask students what they notice about the increase in the total cost of the party when the number of guests rises from 50 guests to 100 guests, from 100 guests to 150 guests, and then from 150 to 200 guests. (**Answer:** "When the number of guests increases from 50 to 100 guests, the cost of the party doubles. When another 50 guests are added, the cost almost doubles again, although not exactly. Finally, when the number of guests increases from 150 to 200, the cost almost doubles again, but also not exactly.") Encourage and guide a discussion about these results, prompting students to consider that although the domain values increase by 50 each time, the range values do not have a constant difference. Discuss possible reasons for this.
- Ask students, "If you created a graph that represented the number of guests and the cost of the party, would the graph be a straight line or a curve?" (**Answer:** "The graph would be a curve, because the output values representing the total cost of the party do not have a constant rate of change.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask students to volunteer their initial predictions about whether they thought the cost of the party would increase linearly. Ask them to explain their thought processes and reasoning based on the given information.
- Compare students' strategies and explanations for evaluating the functions, creating a table of values, and drawing conclusions about the rates of change in the four different functions. Discuss the analysis used on the table of values for the four functions. Focus on the use of precise mathematical language.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Functions can be evaluated for values of variables.

  In this task, students will evaluate four functions at four domain values, which represent different numbers of guests.

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- To evaluate a function such as $f(x)$, substitute values in the domain for all occurrences of x.

  For each of the functions, students will substitute the values 50, 100, 150, and 200 for the variable x, and then they will calculate the results using the order of operations.

Extending the Task

- To extend the task, ask students to calculate the number of guests who could attend the party if the total cost of the party is $14,000. (**Answer:** 120 guests)

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Sample answer:** "I made sense of the problem by evaluating each of the four functions for the domain values of 50, 100, 150, and 200, and then I analyzed the results.")
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Sample answer:** "I used correct math terminology in my explanations, and I used the proper notation for the functions and the domains. I also used the correct units when I stated the results.")
- **For SMP 8, ASK:** "How did you look for and express regularity in repeated reasoning?" (**Sample answer:** "I noticed that the same calculations are performed repeatedly for each function, with the only difference being the domain value each time. I recognized that I used the order of operations each time in order to evaluate the functions at specific domain values.")

Alternate Strategies or Solutions

Students may choose to organize their information in a table. Students may also choose to graph the function $P(x)$ in order to have a visual representation of the behavior of the function representing the total cost of the party. It may be easier for students to recognize that the total cost function is not linear if they see it graphically. Discuss the advantages and disadvantages of analyzing a table of values compared to analyzing a graph. Accept and discuss all paths to the solution.

Technology

Students can use scientific calculators to perform the calculations when evaluating the functions. They may use graphing technology if they choose to create a graph displaying all four functions.

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Problem-Based Task: Cupcakes

Common Core State Standard

A–REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Task Overview

Focus

What inequality can be written to represent a scenario involving two events? How can the inequality be graphed on a coordinate plane to show the solution set? In this activity, students will write and graph an inequality that represents a scenario involving two variables.

This activity will provide practice with:

- writing an inequality involving two variables
- graphing the solution set of an inequality involving two variables
- identifying the correct inequality symbol based on key words given in the problem
- determining the boundary line of a graph of an inequality
- substituting values into an inequality to determine if a statement is true or not
- interpreting the solution of an inequality based on the results of a test point
- analyzing the shaded region based on the context of a scenario

Introduction

This task should be used to explore or apply the skill of solving and graphing linear inequalities containing two variables. Students should have already learned how to graph linear equations containing two variables. Graphing may be verified using technology.

Begin by reading the problem and clarifying the meaning of *slope-intercept form* and *boundary line*, as well as the meaning of *test point*:

test point — a point above or below the graphed line of an inequality that is chosen to substitute into the inequality to determine whether the half plane containing the point is in the solution set of the inequality

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Some students may have difficulty at first with recognizing that two variables are needed to represent the two different types of tasks involved in the scenario. Ask students how the task of frosting the cupcakes and the task of decorating the cupcakes can be represented by the variables x and y, and how the sum of these variables can be used to create an inequality. Also, ask students how they think the solution of the inequality can be represented, and what steps are involved in the process of graphing the inequality and determining its solution set.

- **SMP 2:** Reason abstractly and quantitatively.

  Reasoning abstractly, students will need to determine that two variables are necessary to represent the two types of tasks. Students will need to determine which inequality symbol represents the words "at most." Quantitatively, students will need to analyze the context of the situation to determine where the boundary line will stop on the coordinate plane. Encourage students to make decisions based on the parameters of the scenario; for example, the number of friends cannot be negative, so the graph is limited to the first quadrant.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students may work together and discuss their methods of creating the inequality, as well as how they determined whether the inequality is inclusive or non-inclusive. Students may also discuss the choice of test points for determining the solution of the inequality. For example, ask, "Why is the point (0, 0) a common test point? Is the point (5, 5) another possible test point? Why does one point work when substituted into the inequality, and the other point doesn't?" Encourage students to compare methods for determining test points.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- choosing the incorrect inequality symbol to represent the words "at most"

  Remind students that the phrase "at most 5" indicates a value that cannot be more than 5, but it can be less than or equal to 5.

- using a solid line for a non-inclusive inequality or a dashed line for an inclusive inequality

  Have students review the definitions of *inclusive* and *non-inclusive*. Remind them that inclusive inequalities use a solid line, and the inequality symbol is either ≤ or ≥, and that non-inclusive inequalities use a dashed line, and the inequality symbol is either < or >.

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- shading the region that makes the inequality false

  Remind students that if the test point yields a true statement, then the region containing the test point is the one that is shaded. If the test point makes the inequality false, then the *other* half plane is shaded.

- forgetting to shade the half plane

  Remind students that the half plane either above or below the inequality line has to be shaded, and that the correct half plane to shade is determined using the test point. (If the test point yields a true statement, shade that plane; if it yields a false statement, shade the other plane.)

- not restricting the boundary line to only the first quadrant for real-world problems

  Remind students to consider the context of the problem, in that the number of friends cannot be negative; therefore, the boundary line should only be drawn in the first quadrant, where both the x- and y-coordinates are positive.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

- If students have difficulty choosing the correct inequality symbol, ask:

  "What do the words 'at most' mean?" (**Answer:** "The words 'at most' mean 'no more than' a specific number.")

  "What are the possible values that would meet the requirement of 'at most 5 friends'?" (**Answer:** "There could be 0, 1, 2, 3, 4, or 5 friends as possible values.")

  "Is the number 5 included in these possible values?" (**Answer:** "Yes.")

  "Which inequality symbol would represent this scenario?" (**Answer:** "Since the number 5 is included, the inequality symbol to represent this would be ≤, because the possible values can be less than 5, but also include 5.")

- If students have difficulty graphing the boundary line, ask them how rewriting the inequality in slope-intercept form might make it easier to graph the boundary. (**Answer:** "The equation is easier to graph when it is in slope-intercept form, using the value of m, which is –1, and the value of b, which is 5, because I can start the line at the y-intercept and then use the slope to draw the rest of the line.")

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- If students ask why the boundary line is restricted to the first quadrant of the graph, ask them why it is not necessary to display the entire line on the graph even though the line representing the inequality technically continues infinitely in both directions. (**Answer:** "Since this is a real-world scenario in which the variables represent people, the number of people cannot be negative.") Remind students that the first quadrant contains only positive values of the x- and y-coordinates, so the other quadrants are not necessary in terms of this scenario.

  Ask students if *all* points in the shaded region represent real-world solutions to the problem. (**Answer:** "No; only the coordinate pairs that are whole numbers represent solutions, because it doesn't make sense to have a fraction of a person.")

- If students have trouble choosing a test point, remind them that (0, 0) is commonly used, and ask them why that is. If students decide to use (0, 0), ask them afterward to experiment with additional test points. (**Answer:** "The point (0, 0) is often chosen as a test point because generally it is a convenient point to substitute into an inequality. The math is typically easier and less time-consuming compared to substituting non-zero values.")

- If students are unable to correctly interpret the results of the inequality statement when a test point is substituted, ask them to substitute the same test point into the original inequality, $x + y \leq 5$. Ask them to consider a point that might make sense in the context of the scenario, such as (1, 2). Ask, "What does it mean if $x = 1$ and $y = 2$? Do these values make sense in the context of the problem?" (**Answer:** "If the test point is (1, 2), then it means that 1 friend frosted the cupcakes, and 2 friends decorated the cupcakes. These values do make sense in the context of the problem, because the total number of friends working on the cupcakes is 3, which falls within the parameters of having no more than 5 friends working on the cupcakes.")

Debriefing the Task

- Ask students to volunteer their thought processes for creating the inequality to represent the scenario.
- Discuss the reasons for rewriting the inequality and representing it in slope-intercept form. Explain that knowing the slope and y-intercept can make it easier to graph the inequality.
- Once students have graphed the inequality line, remind them of the definitions of *inclusive* and *non-inclusive*, and ask why the line for this scenario is solid. Then, discuss why it is only necessary to display the portion of the line that is in the first quadrant. Encourage and guide students to consider what the variables stand for when drawing a graph. Reinforce that the number of friends has to be a whole number.
- Ask students to explain how to choose a test point and how to interpret the results of substituting the test point into the inequality. Encourage and guide a discussion about testing other points on both sides of the line for additional reassurance.

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Connecting to Key Concepts

Make explicit connections to key concepts:

- To determine the solution set of an inequality, first graph the boundary line.

  Explain the process involved in graphing the linear inequality in this problem: rewriting the inequality in slope-intercept form, using the slope and *y*-intercept to create the line, and then determining whether the line should be solid or dashed.

- The solution set of the inequality will be all of the points in a half plane, on one side of the boundary line. To find out which side of a boundary line contains the solution, choose a point that is clearly on one side of the line and substitute the coordinates of this test point into the inequality.

  Discuss the process of selecting a test point in order to determine which region should be shaded for the inequality in this task.

- A constraint is a restriction or limitation on any of the variables in an equation or inequality. Generally, linear inequalities in context have one or more constraints such that the variables may not be negative. This means the endpoints of a line segment are its intercepts.

  In this task, the half plane is restricted to the first quadrant in order to exclude negative values of the variables, which wouldn't make sense in the context of the problem.

Extending the Task

- To extend the task, ask students questions that they can answer using their inequality and graph. For example, ask students how the inequality would change if the problem read, "The number of friends who have agreed to help is *at least* 5." Ask students what other parameters would have to be considered to reflect the phrase "at least 5."

- Ask students to work with a partner to create their own real-world scenario in which it is necessary to create and graph a linear inequality, and then determine its solution set. Ask students to consider scenarios in which the variables can be represented by negative numbers, decimals, and/or fractions. Ask each pair of students to present their scenarios to the class.

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Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "First, I determined the process for solving the problem, which was to write the inequality, graph it, and then choose a test point in order to determine which region to shade. Then, I carried out each part of this process, and determined if the inequality, the graph, and the boundary line made sense in the context of the problem. When I didn't know what to do, I asked my classmates.")
- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "I assigned variables and wrote an inequality to represent the situation.") "Which of your strategies represent quantitative reasoning?" (**Answer:** "Choosing a test point and substituting it into the inequality is an example of quantitative reasoning.")
- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "I used the definitions of *inclusive* and *non-inclusive* to determine if the boundary line should be solid or dashed, and I used the results of substituting my test point to make sure that the correct region of the graph was shaded. I compared my methods to those of my classmates to determine which methods were more efficient and precise.")

Alternate Strategies or Solutions

- Instead of rewriting the standard-form inequality $x + y \leq 5$ in slope-intercept form and graphing the slope and y-intercept, students might choose to graph the intercepts of $x + y \leq 5$, substituting 0 for x and solving for y, and vice versa. Students would then graph the points (0, 5) and (5, 0).
- After students use a test point to determine that they should shade the half plane that is below the boundary line, encourage them to verify their results using another test point that is clearly above the line, such as (5, 5). Note that choosing the point (5, 5) would result in a false statement.

Technology

Students can use graphing technology, but it is not required or recommended for this activity.

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Problem-Based Task: Skate Constraints Revisited

Common Core State Standard

A–REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Task Overview

Focus

How can you find all the possible solutions that satisfy a given set of constraints? In this task, students will explore how to find all the possible combinations of figure skates and hockey skates that satisfy the given constraints by graphing the system of inequalities that represents the situation.

This activity will provide practice with:

- writing and graphing inequalities
- writing a system of inequalities
- interpreting constraints
- interpreting a graph of inequalities

Introduction

This task should be used to explore or apply the skill of graphing inequalities. Graphing may be done using technology, such as with graphing calculators or online graphing programs.

Begin by reading the problem and clarifying the meaning of the term *constraint*.

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they develop the system of inequalities that represents this situation and again when they graph their system. Students will reason quantitatively as they interpret the graph in the context of the problem.

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- **SMP 4:** Model with mathematics.

  The situation presented in this task can be modeled algebraically, with a system of inequalities. It can also be modeled graphically and numerically, with a table of values.

- **SMP 5:** Use appropriate tools strategically.

  The system of inequalities in this task is relatively straightforward to graph by hand, since all of the intercepts are integers. However, some students may choose to graph the system on a graphing calculator. Encourage all students to think about the meaning of their graphs, regardless of how the graphs are created. Have students carefully consider a logical scale/window for their graphs. Also, remind students to think about whether the boundary lines should be included, as well as which side of each line should be shaded. If students are not familiar with the shading feature on the graphing calculator, show them how to use it. *Note*: In order to graph the system on a TI-83/84 calculator, the first two inequalities must be solved for y.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- not seeing the solution or identifying where the intersection occurs

  Suggest that students shade their graphs carefully so that the overlapping region is apparent. Since the number of skates produced must be below certain values, the students must shade below the two boundary lines. Likewise, since the number of skates produced must be greater than or equal to 0, students must shade above the x-axis and to the right of the y-axis.

- forgetting to add constraints or boundaries that make sense for the context of the problem

  Many students will forget to include the two real-world constraints, $x \geq 0$ and $y \geq 0$, in the system. Point out that the number of skates made cannot be a negative number.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- If students are struggling with how to start:

  Ask them what kind of situation we are trying to model. (**Answer:** a system of inequalities)

  Ask students how they know the situation can be modeled using inequalities. (**Answer:** "I know this because we have a maximum amount of hours to spend on creating blades and boots, but we could spend anywhere up to and including that maximum, which indicates that this situation is modeled by an inequality.")

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Ask students how they know the situation is modeled by a system of inequalities. (**Answer:** "There are several criteria to be met for two variables. When multiple equations or inequalities work together with the same variables, the result is a system. Because the problem gives a different maximum number of hours for each part, there needs to be one inequality for blades and a second inequality for boots.")

If students try to set up the inequalities so that the information is grouped by skate type instead of by component, ask them to think about the terms in which the criteria for maximum number of hours were given. (**Answer:** "The maximum numbers of hours is given first in terms of creating blades and then in terms of creating boots.")

- If students struggle with writing an inequality representing skate blades, ask:

  "What are the variables in the system? What letters can represent these variables?" (**Answer:** "The variables are the number of figure skates made and the number of hockey skates made. Let x represent figure skates and y represent hockey skates.")

  "How many hours does it take to make a blade for each type of skate?" (**Answer:** "It takes 2 hours to make the blade of a figure skate and 3 hours to make the blade of a hockey skate.")

  "How many hours a week can be spent making blades for both types of skates?" (**Answer:** no more than 40 hours a week)

  "Which inequality symbol should be used?" (**Answer:** "The ≤ symbol, because the amount of hours can be equal to 40, but cannot be greater than 40. Therefore, the total number of hours has to be less than or equal to 40.")

  "Based on this information, what inequality can you write to express the time constraint for making skate blades?" (**Answer:** $2x + 3y \leq 40$)

  Direct students to follow the same process when setting up the inequality for making boots, but provide less coaching. If students get stuck on the second inequality, guide them through reading the problem scenario to determine the variables (x and y), coefficient values (3 and 1), and time limit (≤ 20 hours) for making boots. The resulting inequality is $3x + y \leq 20$.)

- If students forget to specify that the values of both variables cannot be negative, ask them to think about what else must be true about the number of skates made. (**Answer:** "The number of each type of skate made can be equal to 0, but cannot be less than 0. Therefore, the values of the variables cannot be negative.")

  Ask students how to represent these limits with inequalities. (**Answer:** $x \geq 0$ and $y \geq 0$)

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- Students might have all four inequalities written on their papers, but not grouped together as a system. Ask students how to write the four inequalities as a system. (**Answer:** $\begin{cases} 2x+3y \le 40 \\ 3x+y \le 20 \\ x \ge 0 \\ y \ge 0 \end{cases}$)

- Ask students how they can use this system of inequalities to find all the possible combinations of figure skates and hockey skates that can be produced given the constraints. (**Answer:** "We can graph the system of inequalities to see the overlapping region. The overlapping region contains all the solutions that make all of the inequalities true." *Note*: In the context, only integer points are solutions to the system.)

- If students are graphing by hand:

  Make sure students use graph paper and a straightedge to create their graphs so that the result is precise enough to find the solution.

  Make sure students shade the correct side of each line.

  Make sure students consider whether points on the line can be included in the solution set. (**Answer:** They are included.)

  If students are unsure of how to graph the constraints $x \ge 0$ and $y \ge 0$, remind them that the line $x = 0$ is the y-axis, and the line $y = 0$ is the x-axis.

- If students are graphing with a calculator or other technology:

  If students are using a TI-83/84 calculator, ask them what they must do first before they can enter the inequalities into their calculators. (**Answer:** "We must first solve each inequality for y so that it can be entered into the calculator.")

  If necessary, show students how to use their calculator's shading feature.

  Ask, "Is there a constraint that *cannot* be entered on the calculator?" (**Answer:** "Yes; you cannot enter the constraint $x \ge 0$ because $x = 0$ is not a function. We just have to keep in mind that this constraint means that only positive x-values are valid for this scenario.")

- If students graph the system but are unsure of how to use the graph to complete the task:

  Ask students to recall what the variables stand for. (**Answer:** "The variable x is the number of figure skates, and y is the number of hockey skates.")

  Guide students to realize that the number of skates made must be a whole number, so the only possible combinations of skates that are valid for the context are the whole-number coordinate pairs that lie within the shaded region of the graph or on the boundary lines of the region.

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- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Compare strategies used by the groups.
- Ask students how they determined the answers using the graph. (**Sample answer:** "I plotted the integer points that fell within the shaded region or on the boundary of the shaded region, then I wrote down the coordinates of these points.")
- Ask, "Can *any* point in the shaded region of the graph be a solution to the problem?" (**Answer:** "No. Only the integer points represent skates. The task asks for the number of figure skates and hockey skates, which we assume to mean entire skates. An answer such as 2.9 figure skates and 3.4 hockey skates is in the overlapping solution area for the system of inequalities, but (2.9, 3.4) doesn't make sense in the context of the problem because these values are not whole numbers.")

Connecting to Key Concepts

Make explicit connections to key concepts:

- The solution to a system of linear inequalities is the intersection of the half planes of the inequalities. The solution set will be the set of all points that satisfy the inequalities in the system.

  For this task, students will shade the half planes for each of the four inequalities in the system, and find the region where these half planes intersect. Then, they will determine that although every point in the overlapping region is mathematically a solution to the system, they should only consider points whose coordinates are whole numbers due to the context of the problem.

- Remember to add constraints to the system that are not explicitly stated in the problem context but that make sense for the problem.

  For this task, students will include the constraints $x \geq 0$ and $y \geq 0$ because the number of skates cannot be negative. Furthermore, within the context, students will add the constraint that the number of skates must be a whole number.

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Extending the Task

- After students complete the task, ask them to determine how many figure skates and how many hockey skates the company should make each week to maximize income if hockey skates sell for $100 per pair and figure skates sell for $120 per pair. What will the income be? Remind students that the inequalities and graph are based on the number of skates, not the number of pairs of skates. (**Answer:** "The company will maximize its income by making 3 pairs of figure skates and 11 pairs of hockey skates each week. The income will be $1,460.")
- Another option for extending the task is to ask students to find a way to modify the parameters of the problem so that the solution pair (6, 8)—which represents 6 figure skates and 8 hockey skates—is a solution, but (6, 10) is not. (**Sample answer:** "If the first constraint in the problem is altered so that there is a maximum of 39 hours to make the blades, and the second constraint is changed so that it takes 2 hours to make the boot of a figure skate instead of 3 hours, then the system of inequalities becomes:

$$\begin{cases} 2x+3y \le 39 \\ 2x+y \le 20 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

With this new system, it is possible to make 6 figure skates and 8 hockey skates, but not to make 6 figure skates and 10 hockey skates.")

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "When I wrote the system of inequalities to represent the situation, I used abstract reasoning.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I first modeled the scenario with a system of inequalities, and then I modeled the scenario using a graph.")
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** "I graphed the system of inequalities on a graphing calculator to see the overlapping shaded region. I also used the 'calculate' feature of the graphing calculator to test whether certain points were inside or outside of the region.")

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Alternate Strategies or Solutions

Although the task specifies that students should use a graph to find the solutions, students may have made an organized list instead. All paths to the correct solution are valid and should be discussed.

Technology

Students can use graphing technology (including calculators) and/or spreadsheets.

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Problem-Based Task: Which Tree Will It Be?

Common Core State Standard

F–IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n + 1) = f(n) + f(n – 1)$ *for* $n \geq 1$.

Task Overview

Focus

Why is a sequence a function? How can an explicit formula for a sequence be evaluated for specific domain values? In this task, students will evaluate sequences for the growth rates of three different species of trees, and they will analyze their results to draw conclusions about which species of tree should be planted to meet the needs of different groups of people.

This activity will provide practice with:

- identifying variables in an explicit formula
- evaluating functions at given domain values
- applying the order of operations to simplify numerical expressions
- comparing results of different sequences when evaluated with the same domain values
- analyzing different growth rates over time
- drawing conclusions based on the results of evaluating functions with the same domain values

Introduction

This task should be used to explore or apply the following skills: recognizing a sequence as a function; evaluating functions for specific domain values; and comparing the results of different functions in order to draw conclusions about the scenario given in the problem.

Begin by reading the problem and clarifying the meaning of the terms *discrete*, *function*, *natural numbers*, *explicit formula*, and *sequence*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will first recognize that three explicit formulas are given that represent three different species of trees. Students will identify that the goal of the problem is to make conclusions about the growth and sizes of three different species of trees over time, in order to choose one species of tree to meet the needs of three groups of people. Students will recognize that they will reach this goal by evaluating each function for the same domain values and analyzing the results. Students will identify each function as a sequence with an explicit formula, and they will identify each component of the formula for the sequence, specifically that a_1, b_1, and c_1 represent the initial value for each species. Encourage students to discuss their ideas for organizing their calculations in order to successfully analyze their results.

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they make sense of the sequences given, determining that the formulas used to represent the sequences are explicit formulas. Students will also make sense of the variables used in the sequences, and their relationships to the scenario. Specifically, they will recognize a_1, b_1, and c_1 as the initial diameter, or first term of each sequence, and they will recognize n as the number of years. Students will reason quantitatively as they evaluate the functions by substituting values into each sequence formula for the variable n, the number of years. They will apply the order of operations and simplify the remaining numerical expressions, in order to get a resulting value that represents each tree's diameter for the specific number of years of growth.

- **SMP 8:** Look for and express regularity in repeated reasoning.

  Students will notice that the same calculations are performed repeatedly in order to achieve the desired results. They will recognize that the same domain value is used in order to evaluate the sequences for all three species of trees, and then the next value in the list is used for all three species, and so on. They will also note that the order of operations is used to evaluate the sequences correctly. Remind students to observe and apply the order of operations for each individual sequence calculation for each domain value.

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Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- not realizing that a sequence is a function

  Remind students that a function is defined as a relation in which every element of the domain is mapped to exactly one element of the range; that is, for every value of x, there is exactly one value of y. In a sequence, every domain value is individually separate and distinct, so a sequence is a function, specifically a discrete function.

- not understanding that a sequence is graphed without a line connecting the points

  Remind students that the domain of a sequence is the set of natural numbers, which is {1, 2, 3, ...}. Since the domain only includes these numbers, there cannot be fractions, or non-integer numbers, plotted on the graph. Therefore, the graph of the function is a set of coordinates with no line connecting them.

- making a recommendation about which species of tree to choose based only on the results of the trees' growth after 5 years

  Although Species C is clearly the larger tree after 5 years, after 50 years of growth it is surpassed by Species B, and then after 100 years of growth it is surpassed by Species A. Remind students that it is important to check the size of the trees after evaluating the functions at all of the domain values.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

- Before students begin the task, prompt them with the following:

  "Why are the given formulas functions?" (**Answer:** "The formulas are sequences, and sequences are a type of function, namely a discrete function. For each domain value, there is exactly one range value. In other words, each x-value results in one y-value, which is the definition of a function.")

  "Examine the sequence formulas. Can you determine if they are recursive or explicit formulas, and if they are arithmetic or geometric sequences? Explain." (**Answer:** "The given formulas are explicit formulas, because the problem asks for the nth term of a sequence, specifically {5, 10, 25, 50, 100}, and the initial value, a_1, is given. Some students may answer that the sequence is explicit because it matches the general format for an explicit formula. The sequence is geometric because a common ratio is multiplied by the initial term in each sequence. If it were an arithmetic sequence, a common difference would be added instead.")

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- If students have difficulty beginning the task, suggest that they create a table of values to help organize their results. Some students may prefer an organized table as opposed to a list of results for each Coaching question. Ask students how a table of values might be helpful. (**Answer:** A table would allow the diameters for each tree species to be listed vertically for each specific domain value, or the number of years. This type of visual representation might help when comparing the results of the growth of the trees over time.)
- If students have difficulty recognizing which values are substituted for each variable, remind them to make a list of each component of the formula. Ask, "What does each of the variables in each formula represent?" (**Answer:** "The a_1 variable represents the diameter of each tree at its first year, or the initial value to use in the formula. The variable n represents the number of years, which is given in the problem as {5, 10, 25, 50, 100}.")
- After students have evaluated the functions for 5, 10, and 25 years, ask them if they can conclude that Species C will be the larger tree for certain. Ask them if it is necessary to evaluate the functions at 50 and 100. (**Answer:** "After evaluating the three functions at 5, 10, and 25 years, it appears as though Species C will definitely be the largest tree. However, due to the nature of an exponential function, it is possible for the other two species of trees to surpass Species C. Therefore, all of the domain values must be evaluated before a final conclusion can be made." *Note*: Students will see that Species A and Species B do surpass Species C at 50 and 100 years, respectively.)
- After students have completed the evaluation of all of the domain values, ask them how they can explain mathematically which tree species might be the best for all three groups of people involved in the scenario. (**Answer:** Students can subtract the differences in the diameters of the three trees, specifically at 100 years, explaining that the diameter of Species A will be 56.16 centimeters larger than the diameter of Species B (250.4 – 194.24), and 138.44 centimeters larger than the diameter of Species C (250.4 – 111.96).)
- Ask students, "How can the diameters of the trees be converted from centimeters to feet? What are the diameters of the three tree species, to the nearest tenth of a foot, after 100 years of growth?" (**Answer:** There are 30.48 centimeters in 1 foot. Centimeters can be converted to feet by dividing the diameter in centimeters by 30.48. Therefore, after 100 years, Species A will have a diameter of about 8.2 feet, Species B will have a diameter of about 6.4 feet, and Species C will have a diameter of about 3.7 feet.)
- Ask students, "Why is it not practical to be able to choose a tree species that will completely please the city officials, the historians, *and* the community activists?" (**Answer:** "Because all three groups of people want different characteristics of a tree species, it is very difficult to meet all of their needs. For example, it is hard to have a small, medium, and large tree species at the same time. Therefore, one solution is to find a medium-sized tree species, which is a compromise for all three groups.")

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Debriefing the Task

- Encourage and guide a discussion about the growth rates of the different tree species. Ask students to explain their thoughts about why Species C seemed to grow at a faster rate than the other two species for the first 50 years, but then began growing at a slower rate after 50 years.
- Discuss the meaning of the word *reasonable* and how it applies to satisfying the needs of all three groups of people. Discuss the analysis used to make a final decision.
- Compare students' strategies and explanations for evaluating the functions and drawing conclusions about the rates of change in the three different functions in terms of growth rate. Focus on the use of precise mathematical language.

Connecting to Key Concepts

Make explicit connections to key concepts:

- The domain of the function that generates a sequence is the set of all natural numbers.

  In this problem, the domain of the sequences is {5, 10, 25, 50, 100}. These are all natural numbers.

- Sequences are ordered lists that can be expressed as functions.

  In this problem, for each value of the domain, x, there is exactly one value of y. Since each domain value is separate and distinct, these sequences are functions.

- If the sequence is defined explicitly (that is, with an explicit formula), the function is given, and the sequence is used to find the nth term of the sequence.

  For each different species, the initial domain value, a_1, which is the diameter of a first-year sapling, is given. This value is used in the explicit formulas to find the nth term in the sequences.

Extending the Task

- To extend the task, ask students what other factors they might consider when making a decision about choosing a tree species. Ask them to work with a partner and make a list of concerns and needs that each of the three focus groups might have in terms of the size of tree they want. Ask students to volunteer their answers and discuss their results.
- Another option is to provide a fourth tree species, Species D, for which the explicit sequence formula has different values for the constant ratio and the diameter of a first-year sapling (d_1). For example, Species D could have a constant ratio of 1.02, with d_1 equal to 10 centimeters. In this case, the formula would be $d_n = d_1 \bullet 1.02^{n-1}$. Ask students to evaluate this new species for the same domain values as the other three species, and then compare the results for all four species.

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Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I persevered in solving the problem by following my original solution process of evaluating each of the three functions for the domain values of 5, 10, 25, 50, and 100 years, and then I analyzed the results.")
- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "I used abstract reasoning when I identified the given formula as an explicit formula, and when I identified each of the variables in my explicit sequence formula.") "Which of your strategies represent quantitative reasoning?" (**Answer:** "I used quantitative reasoning when I determined which values were substituted for which variables, and when I followed the order of operations to evaluate each function for a specific domain value.")
- **For SMP 8, ASK:** "How did you look for and express regularity in repeated reasoning?" (**Answer:** "I noticed that the same calculations are performed repeatedly for each function, with the only difference each time being the domain value. I recognized that I used the order of operations each time I evaluated the functions at specific domain values.")

Alternate Strategies or Solutions

- Although the Coaching questions do not ask students to create a table of values to display their results, this might help some students in terms of visually understanding and analyzing the results. Encourage students to do this if they have difficulty listing the range values (i.e., the specific diameter measurements) as indicated in the Coaching Sample Responses.
- Students may further investigate the growths of the three tree species by experimenting with additional domain values, such as 60 and 75 years. They may want to predict the year in which Species A and B will become larger than Species C. Encourage and guide a discussion about practical constraints on the domain in the context of the given scenario, as well as choosing an appropriate number of domain values in order to reach more accurate conclusions.
- Students may also choose to use a graphing calculator and graph the three functions on the same graph, so that they can see the curve of each function. From the graph, they can also try to estimate the year in which the graph for Species C intersects the graphs of the other two species.

Technology

Students can use scientific calculators to perform calculations when evaluating the functions. If they choose to graph all three functions, they may use a graphing calculator or other graphing technology.

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Problem-Based Task: Careful Calculations

Common Core State Standards

F–IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F–IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

Task Overview

Focus

How can the relationship between two quantities be represented on a graph? How can the key features of a function be used to sketch its graph? In this task, students will interpret a verbal description of the relationship between time and the value of a car in order to determine which type of graph will represent this relationship, and then they will identify key features of the graph.

This activity will provide practice with:

- determining the type of function used to represent the relationship between two quantities
- interpreting a given verbal description of the relationship between two quantities
- identifying key features of a graph
- explaining the behavior of a graph
- analyzing the domain of a function in the context of a scenario

Introduction

This task should be used to explore or apply the skill of interpreting key features of a graph and the skill of sketching a graph showing key features, given a verbal description of the relationship between the quantities that are being graphed. Students should already be familiar with identifying domains, independent variables, and dependent variables. They should also be familiar with graphing linear and exponential functions from a table of values.

Begin by reading the problem and clarifying the meaning of the terms *asymptote*, *domain*, *exponential function*, *linear function*, *negative function*, *positive function*, *maximum*, and *minimum*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will first recognize the two quantities that are involved in the problem, and they will identify the first quantity (time in years) as the independent variable and the second quantity (the car's value) as the dependent variable. They will then recognize that the quantities will be represented by an exponential function, because the verbal description states that the value of a car decreases exponentially each year. Students will also interpret the given information in terms of what key features will be represented on the graph. Encourage students to discuss their knowledge of the key features of a graph with each other, as well as how the key features will help in sketching a graph and interpreting its behavior.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will analyze the given information about the two quantities in order to construct arguments and draw conclusions about the behavior of the graph in terms of the context of the scenario. They will apply their knowledge of the key features of a graph in order to support their conclusions about which features are present, and which features help with the interpretation of the graph. Encourage students to share their thoughts and ideas with others and to defend their ideas should disagreements arise.

- **SMP 6:** Attend to precision.

  Remind students to continuously use correct mathematical terms and definitions clearly and accurately when working through the Coaching questions. Strive toward precise mathematical language, such as using the correct terms for *domain*, *asymptote*, and *maximum*. Make sure students use the proper notation when listing the domain and the maximum point. Remind students to use proper units (years and dollars) when discussing the quantities.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- believing that exponential functions will eventually touch or intersect an asymptote

  Remind students that the definition of an *asymptote* is a line that a function gets closer and closer to as one of the variables increases or decreases without bound.

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- incorrectly identifying the type of function as either exponential or linear

  Remind students that a linear function has a graph that is a straight line, and an exponential function has a graph that is a curve. Also, remind students to look for key words and phrases that relate to the type of function. For example, "constant rate of change" indicates a linear function, and "exponential growth or decay" indicates an exponential function.

- misidentifying key features on a graph

  Have students make a list of all of the features of a graph of a function, and suggest that they keep this list as a reference when working through problems. Encourage them to draw an example of a graph of a function with all of the key features labeled on the graph.

- incorrectly choosing the domain for a function

  Remind students to carefully consider the context of a scenario and remember that certain quantities cannot be negative, or fractional, in some cases.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the Coaching questions, ask:

  "What part of the given information allows you to determine which type of function is modeled in this scenario?" (**Answer:** "The information states that the value of a car decreases at an exponential rate each year. This determines that the function will be exponential, not linear.")

  "What is necessary in order to create a graph of a linear function? What is necessary in order to create a graph of an exponential function?" (**Answer:** "At least two points are needed to create a graph of a linear function. Since the graph is a straight line, two points will form the straight line. Although two points are a start for an exponential function, more information is helpful in order to create a precise graph of an exponential function; for example, the equation of an asymptote, and knowledge of whether the function is increasing or decreasing.")

- If students have difficulty distinguishing between the independent and dependent variables, remind them to consider which of the quantities given in the problem is dependent on the other. Ask, "Does the number of years depend on the value of the car, or does the value of the car depend on the number of years?" (**Answer:** "The value of the car depends on the number of years that go by, so the value of the car is the dependent variable, y, and the number of years is the independent variable, x.")

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- Ask students, "If we are given that the average car loses nearly half of its original value after 5 years, how can we use this information to determine a point on the graph?" (**Answer:** "If the average price of a new car is $30,750, then after 5 years, its value would be approximately half of the starting value, so 30,750 ÷ 2 = 15,375. Therefore, the point that can be determined from this information is (5, 15,375), which represents a value of $15,375 after 5 years.")
- As students work through the task, encourage them to consider the reasons that the words "average," "approximate," and "nearly half" are used in the given information. Ask, "Why is it not practical to expect exact numbers in this scenario?" (**Answer:** There are many different makes and models of new cars in the United States, with a wide range of prices. Encourage and guide a discussion about how different makes and models will most likely decrease in value at different rates, and also how some cars hold their value better and longer than others.)
- If students have difficulty understanding why there is an asymptote whose equation is $y = 0$, first remind them of the definition of *asymptote*, then ask, "Even if a car has been depreciating in value for more than 20 years, why do you think the car will still be worth something?" (**Answer:** "Quite often people will take very old cars apart and sell their parts, or sell the scrap metal. Therefore, a car still has some value, no matter how much time has passed. This is why the graph approaches the line at $y = 0$ and gets closer and closer to it as x increases, but the graph will never intersect this line or go below it.")
- Ask, "Why does the graph have a maximum point, but no minimum point?" (**Answer:** "There is a maximum point on the graph because the graph starts at the point (0, 30,750). Since the value of a car decreases from this point forward, this point is the highest the graph will ever reach. There is no minimum point because the graph is an exponential function, and there is no restriction on the endpoint of the domain. In other words, the domain is all numbers greater than or equal to 0, so the domain continues indefinitely.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Compare students' strategies and explanations for determining their responses to the Coaching questions. Discuss how they analyzed the original information, and how they interpreted this information in order to identify the key features of the graph.
- If students disagreed about any responses to the Coaching questions, discuss their reasons and explanations. Focus on the use of precise mathematical language as students discuss their explanations.

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Connecting to Key Concepts

Make explicit connections to key concepts:

- A function is positive when its graph is above the x-axis.

  Students are asked to determine the values of x that make the function positive. Since the function's graph never goes below the x-axis, it is positive for all x-values greater than or equal to 0.

- A relative maximum is the greatest value of a function for a particular interval of the function.

  In this task, students are asked to identify the maximum point on the graph. They will identify that the maximum of the graph occurs at the point (0, 30,750).

- The domain of a function is the set of all inputs, or x-values, that are valid for the function.

  Students will identify the domain of the function in the task as all real numbers greater than or equal to 0, since the time is the domain, and time is positive.

- Functions that represent real-world scenarios often include domain restrictions.

  In this task, the amount of time is the domain, and this value will always be positive, so the domain in this scenario is restricted to all real numbers greater than or equal to 0.

- An exponential function in the form $f(x) = a^x$, where $a > 0$ and $a \neq 1$, has an asymptote, which is a line that a function gets closer and closer to as one of the variables increases or decreases without bound.

  Since the function for this task is exponential, it has an asymptote of $y = 0$. The value of the car will never be \$0, although the graph will keep getting closer and closer to $y = 0$.

Extending the Task

- To extend the task, provide students with another scenario involving the average price of a car in Europe (converted to U.S. dollars), with the average European car losing approximately half of its value after 8 years. Ask students to sketch the graphs representing the values of cars in the United States and cars in Europe on the same coordinate plane, and to compare their key features.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Sample answer:** "I persevered in analyzing the given verbal information and identifying the key features of the graph of the scenario. I made sure my answers made sense in the context of the problem and the domain of the scenario.")

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- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Sample answer:** "I analyzed the given information about the two quantities and constructed my arguments based on this information. I applied my knowledge of the key features of a graph in order to support my answers to the questions.")
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Sample answer:** "I used correct math terminology in my explanations and responses and I used the proper notation for the domain. I also used the correct units when I stated the results.")

Alternate Strategies or Solutions

- Students may examine the given description of the graph and use the two identifiable points to determine the equation of the function. With this equation, they may choose to create a table of values, and then sketch the graph by hand. Alternatively, they may graph the equation using graphing calculators or other graphing technology.
- Before beginning the Coaching questions, students may create an organized list of the given information in order to determine which features they already know from the information, and which features they still need to find.

Technology

Technology is not needed for this task, although some students may choose to use graphing calculators.

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Problem-Based Task: Costing Up a Storm

Common Core State Standards

F–IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

F–LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

Task Overview

Focus

How is the rate of change for a function calculated? How can the rate of change determine if a function is linear or exponential? In this task, students will determine the rate of change for two different calculation methods for a depreciated item and analyze the results to determine if the rates of change for each method are constant or not.

This activity will provide practice with:

- analyzing a real-world scenario involving rates of change
- identifying domains of two different functions
- calculating rates of change using the slope formula
- comparing slopes of two different tables of values
- interpreting rates of change within a specified interval

Introduction

This task should be used to explore or apply the skill of calculating rates of change of a function using the slope formula and analyzing the characteristics of the slope in order to draw conclusions about the function. Students should already be familiar with calculating the slope of a line before beginning this task. If necessary, discuss the slope formula and the different types of slope (positive, negative, zero, and undefined).

Begin by reading the problem and clarifying the meaning of the terms *interval*, *rate of change*, and *slope*, along with the meaning of *depreciate*:

depreciate to decrease in value

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 4:** Model with mathematics.

  The situation presented in this task can be modeled algebraically, by calculating the rates of change of Option A and Option B. It can also be modeled graphically, using the given table of values. Some students might try to graph the functions representing both calculation methods instead of calculating the rates of change. Encourage these students to think about how the shapes of the graphs of the functions would help them determine which calculation method has a constant rate of change.

- **SMP 6:** Attend to precision.

  Remind students to continuously use correct mathematical terms and definitions clearly and accurately to explain their reasoning. Strive toward precise mathematical language, such as using the correct terms for *rate of change*, *interval*, *slope*, and the characteristics of slope, such as "steepness." Make sure students use the proper notation for the intervals, values, and variables, as well as the correct monetary unit when explaining the results.

- **SMP 7:** Look for and make use of structure.

  As students work on this problem, they will apply the structure of the slope formula to two different scenarios. They will also use the slope formula to calculate the rate of change between the first and last values in the table, as well as for intermediate values. The structure and meaning of the slope formula will be broadened, and applied to a non-linear function.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly choosing the values of the indicated interval to calculate the rate of change

  Remind students that the domain of both options of calculation methods is {0, 1, 2, 3, 4}. When calculating the rate of change for a specified interval, ask them to double-check that the domain value corresponds to the correct range value given in the table.

- substituting incorrect values into the slope formula when calculating the rate of change

  Have students write the general formula for slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$, at the top of their papers as a reference. Ask them to label each coordinate with x_1, y_1, x_2, or y_2 before entering each value into the slope formula.

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- assuming the rate of change must remain constant over a period of time regardless of the function

  Remind students that in order to determine whether the rate of change is constant over an entire interval, the rate of change must be calculated between all consecutive points. If the rate of the change is the same each time, then it is constant.

- interpreting interval notation as coordinates

  Remind students that parentheses are used for coordinate notation, and they are also used to denote an interval if the endpoints are not included. Brackets are used for interval notation when the endpoints are included. In this task, the endpoints of the intervals are included, so bracket notation is used.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.
- If students have difficulty beginning the task, first ask them to discuss the meaning of *slope* in words, without using the formula. Ask them how the terms *rate of change* and *slope* are related, and how these terms apply to the context of the problem. (**Answer:** "Slope is the ratio of change in the *y*-values to the change in *x*-values. Slope is 'rise over run,' or the steepness of a line. The rate of change is the ratio that describes how much one quantity changes with respect to changes in another quantity. In this problem, one quantity is the number of years that have passed, and the other quantity is the amount of money that the desk is worth with respect to the number of years that have passed.")
- Before they begin the task, ask students if they are able to make a prediction about whether the rates of change are constant for both calculation methods over the 4-year period, based only on the information given in the table of values. Some students might be able to deduce that, although the starting values for the two methods are the same, the ending values are not. Also, some students might be able to estimate the difference between the values in the chart, without doing the exact calculations. Encourage students to write down any predictions, so that they can check them at the end of the task to see if their predictions were correct.
- If students are unsure of how to identify the starting and ending points of the intervals for the two calculation methods, ask them to list the ordered pairs for each method separately. (**Answer:** "For Option A, the list of ordered pairs is: {(0, 487.65), (1, 390.12), (2, 292.59), (3, 195.06), (4, 97.53)}. For Option B, the list of ordered pairs is: {(0, 487.65), (1, 341.36), (2, 238.95), (3, 167.26), (4, 117.08)}.")

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- If students are unsure of how to interpret the negative slope values, ask them to consider instances of money being represented by negative amounts, and ask them to relate these examples to the given scenario. (**Sample answer:** "When dealing with money, a deduction or withdrawal from a checking or savings account is represented by a negative sign. Since the value of the desk depreciates, or decreases, each year, the slope found between intervals will be negative to show this decrease.")
- Ask students which type of function has a constant rate of change; what, if any, specific intervals have to be chosen for this type of function; and how it can be shown algebraically that a function has a constant rate of change. (**Answer:** "A linear function has a constant rate of change, no matter what interval is chosen. The slope between any two points of a linear function will be the same. The slope formula can be used to calculate the rate of change over any interval of a linear function.")
- Ask students to compare their answers and show their work to another student. If students' results differ, ask them to discuss and analyze each other's work. Then, ask them how they can verify their calculations for the rates of change for the two options. Students may choose to verify their calculations by plotting the points for Option A and Option B on the same coordinate plane. Remind them to consider the scales for the x- and y-axes, and also remind them to attend to precision when graphing the points, in order for the points to be plotted as accurately as possible.
- If students incorrectly conclude that both Option A and Option B have a constant rate of change, ask students how to verify their conclusion. Remind students that the rate of change of other intervals must be evaluated in order to conclude whether or not a rate of change is constant. Students will see that Option A has a constant rate of change, because the slope between any two points of the function is the same, but in Option B, the slope is different between different intervals of the function.
- Ask students to identify which types of function Option B can and cannot be based on the results of determining various rates of change in the function. (**Answer:** "Since any interval chosen in Option B results in a different rate of change, Option B cannot represent a linear function. It can, however, represent an exponential function.")
- Ask students if one option is always better than the other option. (**Answer:** "No. For the first 3 years, the desk depreciates less with Option A, and for the fourth year, the desk depreciates less with Option B.")

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Debriefing the Task

- Ask students to volunteer their initial predictions about the outcome of the two options in terms of which has a constant rate of change. Ask them to explain their thought processes and reasoning based on the given table of values. Compare students' strategies and explanations. Focus on the use of precise mathematical language and clarity.
- If students chose to graph the functions, discuss the different types of functions in regard to the change in the shape of the functions' graphs over specific intervals. Ensure that students understand the relationship of the constant rate of change of a linear function to that of the non-constant rate of change of an exponential function.

Connecting to Key Concepts

Make explicit connections to key concepts:

- A rate of change is a ratio describing how one quantity changes with respect to another quantity.

  In this problem, the depreciated value of a desk changes as the number of years passes. The quantities are the value of the desk (in dollars), and the time that has passed (in years).

- Slope is a way to describe the rate of change of a function.

  The slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, is used to calculate the rate of change over a specific interval, [0, 4], in order to determine if the rate of change is constant for both options for depreciation calculation in the task.

- The rate of change between any two points of a linear function will be equal.

  The slope between any two points in the specified interval for Option A is the same. Therefore, it can be concluded that Option A represents a linear function.

- The rate of change between any two points of a non-linear function will not be equal for all points chosen, but will instead be an average for the interval determined by the points.

  For Option B, the slope for the interval [0, 4] is approximately –92.64. However, the slope for the interval [1, 3] is approximately –87.05. Since the slopes are not the same, it can be concluded that Option B represents a non-linear function.

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Extending the Task

- To extend the task, provide students with a third option (Option C) for a depreciation calculation method. Let the table of values for Option C represent a linear function with a slope that is less steep than that of Option A. Ask students to determine the rate of change for Option C over the same interval, [0, 4]. Then, ask them which of the three options is the "better" option in terms of receiving the most value for a depreciated desk.
- Another option for extending the task is to modify the original problem such that the slopes of the two options are less steep, and then have students predict how many years it will take for the desk's value to depreciate to almost $0. For example, keep the initial value of the desk at $487.65, but let the value of the desk after 1 year be $452.35 for Option A, and $438.78 for Option B (adjusting years 2, 3, and 4 accordingly). Encourage and guide a discussion about how to determine at what point the desk will be worth almost nothing, and which option will allow for the desk's value to be retained the longest.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Sample answer:** "I modeled the given scenario algebraically, by calculating the rate of change for both options for calculating depreciation.")
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Sample answer:** "I used correct math terminology in my explanations and I used the proper notation for the intervals, values, and variables, as well as the correct unit of money when explaining the results.")
- **For SMP 7, ASK:** "How did you look for and make use of structure when solving this problem?" (**Sample answer:** "Because I understand the structure and meaning of the slope formula, I was able to apply it in several ways to analyze the rates of change of the two different options. I was able to deduce from the results that one option is linear and the other is not.")

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Alternate Strategies or Solutions

- Although the solution method presented in the Coaching section is based on modeling the scenario algebraically and calculating the rates of change for the two options, students may choose to graph both functions and draw conclusions about the rates of change visually. Discuss the advantages and disadvantages of graphing, including how using the slope formula will provide precise and exact numbers, whereas graphing by hand may result in errors in precision. However, it is important to note that students may use their graphing calculators, which would allow for precise plotting of points.
- Students may also attempt to mentally estimate the change in *y*-values without performing the actual calculations. Encourage students to write down their estimations and make predictions about the results, but remind them to calculate the numbers in order to verify their predictions.

Technology

Students can use scientific calculators for their slope calculations, and they may use graphing calculators if graphing is used as an alternate solution method.

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Problem-Based Task: Dwindling Concentrations

Common Core State Standards

F–IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

F–LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Task Overview

Focus

How is a rate of change between two points on a graph calculated? How can the rate of change for one interval be compared to the rate of change for another interval? In this task, students will calculate the rate of change for different intervals on an exponential graph, and then they will interpret and analyze the results in order to make a conclusion about the amount of medication remaining in the body after a specific interval of time.

This activity will provide practice with:

- estimating *y*-values on a graph
- identifying coordinates to be used in the slope formula
- calculating the rate of change by using the slope formula
- simplifying numerical expressions
- comparing different rates of change
- interpreting a graph in the context of a real-world scenario
- drawing conclusions based on the interpretation of calculations

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Introduction

This task should be used to explore or apply the skill of calculating and interpreting the average rate of change over a specified interval and comparing two rates of change. Students should already be familiar with applying the slope formula, as well as interpreting the value of a slope in terms of direction and steepness. If necessary, review the slope formula and the different types of slope.

Begin by reading the problem and clarifying the meaning of the terms *interval*, *rate of change*, and *slope*.

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will first recognize that the graph is an exponential function that represents the amount of medication that remains in a body after a specified interval of time. They will identify that two different intervals are given, and they will realize that the slope formula will be applied in order to calculate the rates of change for the two intervals. They will also recognize that because of the large scale on the *y*-axis, it will be necessary to estimate, or approximate, the *y*-values on the graph, as opposed to identifying the exact values. Encourage students to discuss their ideas for comparing the rates of change in terms of the amount of time that the medication will stay in the body.

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they make sense of the quantities represented on the graph and their context in the scenario. They will identify that the independent variable is the time in hours that a medication stays in the body, and they will identify that the dependent variable is the amount of medication in milligrams remaining in the body. Students will determine that the graph represents an exponential function, which results in variable rates of change in different intervals on the graph. Students will reason quantitatively as they substitute values into the slope formula in order to determine the rate of change for a specified interval. They will apply the order of operations and simplify the remaining numerical expressions.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will analyze the given graph in order to construct arguments about the behavior of the graph in terms of the context of the scenario. They will perform calculations on specific intervals in order to support their claims about the rate at which the medication leaves the body. Based on their results for the rates of change for the intervals, students will draw conclusions and make reasonable arguments about the need for a second dose of medication after 12 hours.

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Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly choosing the values of the indicated interval to estimate the rate of change

  Remind students that the given intervals represent an interval of domain values, so the corresponding *y*-values will be found for each endpoint of the interval.

- incorrectly estimating the values for the indicated interval

  Remind students that in order to find the coordinates for a point on a graph, first find the *x*-value by moving to the right of 0 on the *x*-axis. Then, move straight up on the *y*-axis until the graph is intersected; this value represents the *y*-value for the corresponding *x*-value.

- substituting incorrect values into the formula for calculating rate of change

  Have students write the slope formula on their papers, and remind them to label their coordinates with x_1, y_1, x_2, and y_2 before substituting the values into the slope formula.

- incorrectly assuming that the graph will eventually go below 0 on the *y*-axis

  Remind students to consider the real-life constraints of the scenario. There will never be a negative amount of medication in the body; therefore, the graph will not extend below 0 on the *y*-axis.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

- Before they begin the task:

  Ask students to identify the type of graph given for the scenario, and ask them to predict just by their visual examination of the graph what happens to the medication in the body in a 12-hour time frame. (**Answer:** "The graph is an exponential function because it is a curve that does not have a constant rate of change; note that the graph decreases rapidly for approximately the first 3 hours, and then it seems to decrease at a slower rate after 3 hours.") Encourage students to connect their observations with the concept of rate of change, and discuss how they will be able to calculate the rate of change for a specific interval of time.

  Ask students why only the first quadrant of the graph is displayed. (**Answer:** "In the context of this real-world scenario, the amount of time, which is in hours, is always positive. The graph shows a specific interval of time: from 0 to 12 hours. Also, the amount of medication is represented by positive quantities, as there cannot be a negative amount of medication in the body.")

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Ask students to identify the domain and range of the function given by the graph, and to explain the choices for the starting points and endpoints of the domain and range. (**Answer:** "The domain of the graph is the amount of time in hours from 0 to 12 hours. This is the interval of interest for this problem, as the theme of the problem is to study the rate at which medicine is eliminated from the body over a 12-hour period. The range is 0 to 750 milligrams. The initial dose of the medicine is 750 milligrams, which represents the endpoint of the range. Eventually the medicine will be eliminated from the body entirely, so 0 is the starting point of the range.")

- As students begin answering the Coaching questions, discuss why it may be necessary to approximate their answers instead of answering precisely. (**Answer:** "On the given graph, the scale on the y-axis ranges from 0 to 750, with intervals of 50. Due to the large scale, it is difficult to identify the exact number of milligrams that the graph represents at any given hour. Therefore, it is necessary to approximate the answer.")
- If students have difficulty interpreting the rates of change in terms of steepness, remind them that the rate of change is the slope, and review the different types of slope. Ask them, "How can you tell from the graph that the rate of change for the interval [1, 3] is steeper than the rate of change for the interval [3, 10]? How does this match with the actual calculations for the rates of change?" (**Answer:** "From the interval [1, 3], the graph decreases sharply, or at a steep slope. From the interval [3, 10], the graph does not decrease as much; that is, the slope is less steep. This matches with the rate of change of –145 from [1, 3], which is steeper than the rate of change of –22 from [3, 10].")
- Ask students, "Although the domain value of 12 hours does not show on the graph, how can you use the graph to predict the amount of medication left in the body after 12 hours?" (**Answer:** "The arrow on the graph indicates that the function continues on; however, it can be seen that the line is getting closer and closer to 0 after 10 hours, so it can be concluded that the amount of medication left in the body after 12 hours will be minimal.")
- Ask students, "Why do you think a new dose of medication would be prescribed after 12 hours, when it is not certain that all of the previous medication has been eliminated from the body?" (**Answer:** "Although the graph does not show where the function will get the closest to 0, it can be approximated from the curve of the graph that after 10 hours, the amount of medication remaining is getting smaller and smaller. Most likely, after this 12-hour mark, there is not enough medication left in the body to be effective.")

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- Encourage and guide a discussion with students about how this problem relates to everyday situations involving dosages of medicine. For example, ask students to think about medicines that they have taken in the past for a cold or flu. Ask them, "How do you think chemists/pharmacists determine how often and how much medicine a person should receive? What factors might contribute to the prescribed dosage?" (**Answer:** "Since different medications are made up of different chemicals and substances, each medication most likely has its own function that represents how it breaks down in the body. For example, some medicines may break down completely after only 4 hours, and some medicines may last in the body for 24 hours. Other factors that might contribute to a dosage are the age and weight of a person. A medication might take longer to break down in an adult than in a small child, due to the differences in weight.")

Debriefing the Task

Compare students' strategies and explanations for calculating the rates of change and for drawing conclusions about the behavior of the graph. Discuss the analysis used on the rates of change for the two intervals, and how this analysis led students to answer the question of why a person would need a new dose of medication after 12 hours.

Connecting to Key Concepts

Make explicit connections to key concepts:

- To determine the rate of change of a function, first identify the coordinates of the endpoints of the interval being observed.

  In this problem, the desired intervals for finding the rate of change are given as [1, 3] and [3, 10]. Students will find the coordinates of these domain values and then use them in the slope formula to determine the rate of change. The coordinates for the interval [1, 3] are (1, 450) and (3, 160), and the coordinates for the interval [3, 10] are (3, 160) and (10, 5).

- Sometimes it is necessary to estimate the values for y.

  Since the scale on the y-axis is so large, the y-values, which represent the amount of medication remaining, have to be estimated.

- The resulting calculation may be an estimation of the rate of change for the interval identified for the given function.

  When the y-values for the specified domain values in the given intervals are approximated, then the calculations used for the slope formula are also approximated.

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Extending the Task

- To extend the task, provide students with the scenario of a second type of medication, with a graph that is also exponential, but that begins with a lower initial dosage. The curve of this second graph could also be less steep than the original graph. This graph could be displayed on the same coordinate axes as the original one, or on a separate graph. Ask students to analyze the rates of change for the same intervals, [1, 3] and [3, 10], and to compare these with the rates of change for the original graph.
- Another option for extending the task is to provide students with a linear function that has a negative slope representing a different medication. Ask students to graph the linear function on the same coordinate axes as the exponential function. For example, provide the function $y = 62.5x + 550$. Discuss how the rate of change is constant for this function, and ask students to compare it to the rates of change for the exponential graph for the specified intervals. Ask students to work together to analyze the differences between the two medications, in terms of the initial doses and how long each medication stays in the body.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Sample answer:** "I persevered in solving the problem by following my original solution process of calculating the rates of change for the specified intervals and interpreting the results. I then examined the behavior of the graph and made conclusions about the amount of medication remaining in the body after 12 hours.")
- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Sample answer:** "I reasoned abstractly by carefully interpreting the variables and quantities represented on the graph.") "Which of your strategies represent quantitative reasoning?" (**Sample answer:** "When I substituted coordinates into the slope formula in order to calculate the rate of change for the specified intervals, I used quantitative reasoning.")
- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Sample answer:** "I constructed viable arguments about the behavior of the graph in terms of the context of the scenario by analyzing the rates of change for two intervals on the graph. I interpreted the results and made conclusions about the necessity for a second dose of medication after 12 hours.")

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Alternate Strategies or Solutions

- Although students are asked to find the rate of change for two specific intervals, they may choose to examine the rate of change for other intervals, such as [0, 5] and [5, 10]. Encourage students to compare the rates of change on these intervals just as they compared the intervals given in the problem. Discuss how analyzing additional intervals may allow for a more accurate interpretation of the behavior of the graph.
- Students may try to determine the function represented by the graph. Encourage them to work together and use their prior knowledge of key features of an exponential function in order to accomplish this goal. Remind them to use correct terminology in their discussions, such as the terms *intercepts* and *zeros*.

Technology

No technology is needed for this task.

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Problem-Based Task: Fund-raising Concert

Common Core State Standard

F–IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Task Overview

Focus

How can the relationship between two quantities be represented with an equation and also on a graph? How can *x*- and *y*-intercepts on a graph help indicate solutions to an equation in a real-world scenario? In this task, students will write an equation representing the relationship between two quantities, and they will graph this equation and find the intercepts in order to determine the number of tickets that need to be sold in order to reach a fund-raising goal.

This activity will provide practice with:

- creating a linear equation representing two quantities
- writing a linear equation in standard form
- evaluating an equation for specific values
- plotting points on a coordinate plane
- identifying *x*- and *y*-intercepts on a graph

Introduction

This task should be used to explore or apply the skill of graphing a linear function and showing key features of the graph, specifically intercepts. Students should already be familiar with graphing points on a coordinate plane, as well as with substituting values into an equation for *x* in order to solve for *y* and vice versa.

Begin by reading the problem and clarifying the meaning of the terms *linear function*, *standard form of a linear equation*, *x-intercept*, and *y-intercept*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will first recognize that two different quantities are being examined and analyzed in this task, and that it will be necessary to solve for these quantities in order to find the solution to the task. They will identify that there are two types of tickets—advance tickets and same-day tickets—and there are different prices for each type of ticket. Students will recognize that it will be necessary to write a linear equation representing the two quantities, and then they will plan to find the *x*- and *y*-intercepts of the equation, which will represent the solutions for each type of ticket. Encourage students to keep in mind that the overall goal is to solve an equation that contains two variables, and that in the context of the scenario, the goal is to raise $6,000. Discuss their ideas and understanding of the process of finding the intercepts of an equation.

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they make sense of the quantities represented in the context of the scenario. They will identify that there are two types of tickets involved in the task. They will assign a variable for each type of ticket, and apply the rate of each ticket when creating the linear equation. They will also reason abstractly by manipulating the variables through substituting 0 for one variable and then solving for the other. Students will also reason quantitatively as they substitute 0 for one variable and solve for the other, as this will result in the *x*- and *y*-intercepts of the equation. They will apply the order of operations and simplify the remaining numerical expressions.

- **SMP 4:** Model with mathematics.

  Students will recognize that the scenario presented can be modeled by the creation of a linear equation. They will then identify that the *x*- and *y*-intercepts for the equation can be found, and these intercepts can be modeled visually on a graph. Encourage and guide students to make the connection between graphing the intercepts and the context of the scenario. Encourage students to analyze their solutions for each Coaching question to ensure that the results are practical and make sense in the context.

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Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly plotting points

  Remind students that the first coordinate, which is the *x*-coordinate, is always plotted first, going left or right on the *x*-axis, and the *y*-coordinate is always plotted second, going up or down on the *y*-axis.

- mistaking the *y*-intercept for the *x*-intercept and vice versa

  Remind students that the *y*-intercept is the point where the graph intersects the *y*-axis, and has the coordinate (0, *y*), and the *x*-intercept is the point where the graph intersects the *x*-axis, and has the coordinate (*x*, 0).

- being unable to identify key features of a linear model

  Have students list the different features of a linear model to use as a reference as they work throughout this task. Ask students to focus specifically on the definitions and examples of *x-intercept* and *y-intercept* so that they can apply their knowledge of intercepts to this task.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task:

  Ask them how they can determine which type of ticket they will need to sell more of by itself in order to reach the goal of $6,000, and why. (**Answer:** "Since the price of an advance ticket is less than the price of a same-day ticket, I will need to sell more advance tickets than same-day tickets.")

  Ask students to predict which key feature(s) of the graph they will use in order to answer the questions presented in the task. Ask, "What part of the linear equation, and its resulting graph, do you think will help you determine how many of each type of ticket will need to be sold in order to reach the fund-raising goal?" (**Answer:** "The points where the *x*- and *y*-intercepts are located will be the correct features.") If students do not make this prediction, encourage them to think about what it means in the context of the problem if only one variable is considered at a time.

- If students have difficulty creating the linear equation with the two variables, ask them to verbalize the relationship between the two variables in the context of the scenario. Ask, "How is the number of advance tickets and same-day tickets related to the goal of raising $6,000?" (**Answer:** "The variable *x* represents one type of ticket, and the variable *y* represents the other type of ticket. Each variable is multiplied by the price for that type of ticket to find the total amount of money raised by selling that type of ticket. The sum of these two products, 20*x* and 30*y*, needs to equal the fund-raising goal of $6,000.")

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- Ask students, "What are some other ways to explain the concept of 'intercepts'?" (**Answer:** "On a graph, an intercept is where two lines meet, or touch; specifically, an intercept is where the graph of the line intersects the *x*- or *y*-axis.")
- Before students begin creating the graph of the linear equation, ask, "Why is it only necessary to display the first quadrant when graphing the equation in terms of the domain?" (**Answer:** "Since this scenario represents a real-life context of raising money and selling tickets, the values for the variables will never be negative. There cannot be a negative number of tickets sold, so the smallest value for the domain is 0. The first quadrant represents positive *x*- and *y*-values.")
- When students are creating the graph, ask, "How did you determine which quantity to label on the *x*-axis and which quantity to label on the *y*-axis?" (**Answer:** "The choice of which quantity to represent on each axis is arbitrary. I chose the *x*-axis to represent the number of advance tickets sold because the problem statement presented that quantity first. Then I used the remaining axis, the *y*-axis, to represent the number of same-day tickets sold. However, using *x* to represent same-day tickets and *y* to represent advance tickets is equally valid.")
- Ask students, "Although the task only focuses on determining the number of each type of ticket sold if that type was the only type sold (which is represented by the intercepts on the graph), what do the other points on the graphed line indicate?" (**Answer:** "The other points indicate which combinations of both types of tickets sold add up to reach the goal of $6,000.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask students to volunteer their initial predictions about which type of features they thought would be used to answer the questions in the task. Ask students to explain their thought processes and reasoning based on the given information.
- Compare students' strategies and explanations for creating the linear equation, and for how the equation was graphed. Discuss the analysis used on the *x*- and *y*-intercepts of the graph in terms of solutions for the variables. Focus on the use of precise mathematical language.

Connecting to Key Concepts

Make explicit connections to key concepts:

- To find the *y*-intercept in function notation, evaluate $f(0)$.

  In this task, students will evaluate the linear equation for $x = 0$, which results in a *y*-intercept of 200.

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- To locate the *y*-intercept of a function, determine the *y*-coordinate of the point where the line intersects the *y*-axis.

  Students will see that when the *y*-intercept, 200, is plotted on the graph, it is the *y*-coordinate of the point at which the line of the equation intersects the *y*-axis.

- To locate the *x*-intercept of a function, determine the *x*-coordinate of the point where the line intersects the *x*-axis.

  Students will substitute 0 for *y* and solve for *x*, and the result will be 300. When they graph the *x*-intercept, 300, on the *x*-axis, they will see that the line of the graph intersects the *x*-axis at this point.

Extending the Task

- To extend the task, present students with an additional question: "How many additional tickets of each type would the club need to sell in order to raise $8,400?" Ask students to work together to create a solution plan for answering this question. Discuss the different solution methods possible. Encourage students to think of how they can use their prior solutions to help answer the new question, rather than solving the problem from scratch.
- Another option for extending the task is to present students with another fund-raising organization that also has a goal of raising $6,000, but with different ticket prices for their advance and same-day tickets. For example, suppose the second club sells tickets in advance for $4 and same-day tickets for $15. Ask students to determine how many of each ticket would need to be sold to reach the goal of $6,000, and ask them to determine which ticket-pricing scenario might result in more money raised: the original scenario in the task or this new scenario. Encourage students to consider the real-world possibility of how lower ticket prices might result in more tickets purchased overall.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Sample answer:** "I made sense of the problem by first recognizing the quantities involved in the scenario, which I used to create a linear equation to represent the scenario. I persevered by solving for the *x*- and *y*-intercepts of the equation and graphing them on a coordinate plane in order to determine how many of each ticket I need to sell.")

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- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Sample answer:** "I reasoned abstractly by carefully interpreting the variables and quantities represented in the scenario.") "Which of your strategies represent quantitative reasoning?" (**Sample answer:** "I used quantitative reasoning when I substituted 0 for each variable in order to find the *x*- and *y*-intercepts of the equation and then applied the order of operations to simplify the remaining numerical expressions.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I used mathematics to model this scenario by first creating a linear equation. Then, I found the *x*- and *y*-intercepts of the equation and graphed them on a coordinate plane to create a visual model of the scenario.")

Alternate Strategies or Solutions

- Instead of graphing the equation, students may first use trial and error in order to find the solutions for the *x*- and *y*-variables. They may experiment with substituting different values for each variable into the equation until they get a true statement. Remind them that this method may be more time-consuming than substituting 0 for one variable and then solving for the other.
- Instead of finding the *x*- and *y*-intercepts first, students may create a table of values. They may graph the values from the table and discover the *x*- and *y*-intercepts from the graph, or they may notice where those intercepts are in the table of values. Point out to students that while this method will allow them to arrive at the same graph of the equation, the task only asks for the required number of tickets of each type alone, and not in combination with the other type of ticket. Therefore, only the intercepts are needed to solve this task. Also, remind students that only two points are necessary to graph a linear equation.

Technology

Although students will be able to create a graph by hand, they may use graphing calculators or other graphing technology to check their results.

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Problem-Based Task: Trout Pond

Common Core State Standard

F–IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Task Overview

Focus

How can the relationship between two quantities be represented on a graph? How can technology be used to model a scenario involving two quantities? In this task, students will interpret a verbal description of the relationship between time and the number of trout in a pond, and they will use graphing technology to visually represent the relationship and determine the number of trout in the pond given a specific number of years.

This activity will provide practice with:

- identifying the correct model for representing a given scenario
- finding a base for an exponential function
- writing an exponential function
- graphing an exponential function using technology
- evaluating a function for specific domain values

Introduction

This task should be used to explore or apply the skill of identifying the type of function necessary for representing a given scenario, based on a verbal description of the relationship between two quantities, and then using technology to graph the two quantities. Students should already be familiar with graphing points on the coordinate plane by hand, as well as the steps involved in graphing using technology. They should also be familiar with the rules of exponents as well as the general shape of an exponential function.

Begin by reading the problem and clarifying the meaning of the term *exponential function*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will first recognize the two quantities that are involved in the problem. They will identify the first quantity, time in years, as the independent variable, and the second quantity, the number of trout in the pond, as the dependent variable. They will then determine that the quantities will be represented by an exponential function, because the verbal description states that the population of trout doubles every year. They will also recognize that graphing technology will be used to represent the exponential model. Encourage students to discuss their knowledge of exponents as well as the steps for using graphing calculators.

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they make sense of the quantities expressed in the given scenario. They will identify that the independent variable is the time in years, and they will identify that the dependent variable is the number of trout in the pond. They will determine that the scenario can be modeled by an exponential function. Students will reason quantitatively as they interpret the results of evaluating the function at specific values for time.

- **SMP 5:** Use appropriate tools strategically.

  Students will use a graphing calculator, or other graphing technology, to visually represent the exponential model they create based on the given scenario. Students will determine the necessary calculator window settings for fully representing the exponential model. They will also use their graphing technology to evaluate the function for specific domain values in order to answer the questions given in the task.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly thinking that the scenario will be modeled linearly as opposed to exponentially

  Remind students to look for key words and phrases that indicate the function will be exponential, such as "increases exponentially" or "doubles." Have them create a sample table of values so they can see that the rate of change between consecutive points is not constant.

- not being able to identify key features of an exponential model

  Remind students that an exponential model has the general form $f(x) = ab^x$, where a is the initial value, b is the base, and x is the time. Have students relate this to the corresponding values for the task, such that a is 275, b is 2, and x is the number of years.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task:

  Ask students to reread the given information. Then ask, "What are the key words and/or phrases that will help you determine which type of model will represent the given scenario?" (**Sample answer:** "The words 'doubles each year' indicates that the scenario will be modeled exponentially, and 'current population' indicates the initial value.")

  Ask, "Why do you think graphing technology will be used in this problem?" (**Answer:** "Based on the context of the scenario, and the fact that the population of trout doubles each year, the number of trout after several years could be a very large number. Graphing by hand might be difficult in terms of being able to get precise results on the scale of the *y*-axis, so graphing technology would be useful.")

- Discuss students' prior knowledge of the nature of different types of functions. Ask, "How can you tell the difference between a linear function and an exponential function?" (**Answer:** "Linear functions are represented by a straight line on a graph, and the rate of change, or slope, between any two consecutive points is constant. In contrast, an exponential function is represented by a curve on a graph, and the rate of change between any two consecutive points is not constant.")

- If students have difficulty determining that the scenario will be modeled by an exponential function, ask them to create a sample table of values, unrelated to the given scenario, based on the concept of "doubling." For example, have them create a list of ordered pairs for the *x*-values of 1, 2, 4, 8, and 16, based on questions such as, "If 1 is doubled, what is the corresponding *y*-value?" and "If 2 is doubled, what is the corresponding *y*-value?" (**Answer:** Ordered pairs: (1, 2), (2, 4), (4, 8), (8, 16), and (16, 32).) Encourage students to analyze the *y*-values and recognize that the rate of change is not constant, because the values are "doubled." Remind students that the word "doubling" indicates an exponential function.

- If students have difficulty relating the word "doubles" to a base of 2, encourage them to think of the mathematical operation that is used when something is doubled. Ask, "What does it mean when something is doubled?" (**Answer:** "When something is doubled, it is multiplied by 2.")

  Ask students, "What would the base be if the number of trout tripled each year? What if it quadrupled each year?" (**Answer:** "The word 'tripled' represents multiplying something by a factor of 3, so the base would be 3. If the number of trout quadrupled each year, then the base would be 4.")

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- As students are preparing to graph the function with their calculators or other technology, ask, "Why is it necessary for the domain to include negative numbers when the window settings for the calculator are being chosen?" (**Answer:** "The starting time for the scenario is 0, which represents 'now.' We need to find the number of trout in the pond 1 year ago. The value that represents this time is –1. Therefore, the minimum window setting for the domain will be at least –2, in order to include the value representing '1 year ago.'")

  Before students graph the function, ask, "Which ordered pair will represent the starting population of the trout?" (**Answer:** "Since the starting population of the trout is given as 275, this amount represents a time of 0. Time represents the independent variable, x, and the number of trout represents the dependent variable, y. Therefore, the corresponding point on the graph would be (0, 275).")

  Ask, "Why is it appropriate to represent the y-values using integers?" (**Answer:** "The number of trout will be a whole number, because there will not be a fractional or a decimal representation of a trout.")

- As students work through the last few Coaching questions and determine the number of trout in the pond in 3 years, ask them, "Although an exact number of trout can be determined using the exponential function, why do you think the answer can be considered an estimation?" (**Answer:** "Given the context, the exponential model that represents the scenario is only an approximate model due to real-world factors and constraints. For example, it is possible that not all of the trout will remain in the pond; some might be eaten by other fish, or die from some other cause. Therefore, the model represents an approximate function to represent the number of trout.")

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Compare students' strategies and explanations for determining their responses to the Coaching questions. Discuss the analysis used on the original information and how this information was interpreted in order to create the exponential model and then graph it using technology.

- If students disagree on the answers to any of the Coaching questions, discuss their reasons and explanations. Focus on the use of precise mathematical language.

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Connecting to Key Concepts

Make explicit connections to key concepts:

- To find the *y*-intercept of an exponential function, evaluate $f(0)$.

  In this task, students will recognize that the *y*-intercept of the function is the point where the *x*-value is 0. This value represents the starting population of the trout, and when 0 is substituted into the function, the resulting *y*-intercept is 275. The coordinates of the *y*-intercept are (0, 275).

- Not all exponential functions have an *x*-intercept.

  In this task, students will recognize that because the number of trout will never be negative in the context of the scenario, the function will never intersect the *x*-axis.

- Graph complex exponential models using technology, as values can become quite large or small very quickly.

  Students will notice that the range of the *y*-values, which represents the number of trout in the pond, will reach numbers that are quite large. Using graphing technology will help students produce a better representation of the function with these larger *y*-values.

Extending the Task

- To extend the task, ask students to create a table of values in order to identify the predicted number of trout in the pond after 4 years, 5 years, and then 10 years. Ask students to work together and analyze the results, in terms of practicality and validity of the number of trout in the pond in 10 years. Encourage and guide a discussion about the considerations that would cause the number of trout in the pond to be less than the predicted numbers that the exponential model yields.

- Another option for extending the task is to create another scenario where students are asked to create a model for the number of trout in a second pond, Pond B. Present students with a starting population of 40 trout, with the population tripling each year. Ask students to compare the graphs of the functions for this scenario and the original scenario. Ask, "In approximately how many years will it take for the trout population of Pond B to surpass the trout population of Pond A? Explain how you found this answer." (**Answer:** "After 4 years, the trout population of Pond B will surpass the trout population of Pond A. I found my answer by first creating an equation to represent the population for Pond B, $f(x) = 40(3)^x$, and graphing the new equation with the equation for Pond A. I then compared the two graphs and looked to see where Pond B had a higher population than Pond A.")

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Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I made sense of the problem by identifying the two quantities involved in the task and then recognizing that an exponential function will model the scenario. Then, I graphed the function using technology and evaluated it for specific domain values.")
- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "I reasoned abstractly by carefully interpreting the variables and quantities represented on the graph, and I determined that the given scenario could be represented with an exponential function.") "Which of your strategies represent quantitative reasoning?" (**Answer:** "When I substituted specific x-values into the function in order to determine the number of trout in the pond, I used quantitative reasoning.")
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** "I used calculators strategically by first determining the window settings to use to best represent the domain and range values, and then I carefully entered the function into my calculator and graphed it.")

Alternate Strategies or Solutions

- Although the Coaching questions specifically ask students to use technology in order to graph the function, some students may prefer creating a table of values and making a graph by hand. Encourage them to be mindful of creating the appropriate scales for the x- and y-axes. Also, remind students that their results may not be as precise with this method compared to graphing with a calculator.
- Some students may use different window settings for their calculator. For example, they might use a wider range of x-values, such as –5 to 10, and a wider range of y-values, such as 0 to 25,000. Remind students of the constraints of the given scenario. Point out that although widening the range of years is acceptable, the graph should focus on the time frame spanning from 1 year ago to 3 years from now.

Technology

Students can use graphing calculators or other graphing technology for this task.

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Problem-Based Task: Supply and Demand

Common Core State Standard

F–IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Task Overview

Focus

How can two functions be compared when one is represented with an equation and one is represented numerically in a table? How are the properties of these functions determined? In this task, students will compare rates of change and the *y*-intercepts of two linear functions, which represent the supply and demand of a product, and they will interpret these properties in order to identify when supply exceeds demand.

This activity will provide practice with:

- identifying a rate of change from the slope-intercept form of a linear equation
- calculating the rate of change of a linear function when given coordinates in a table
- determining the *y*-intercept from a graph, an equation, and a table of values
- graphing linear functions
- interpreting the solution of a system of linear equations
- analyzing information from a graph of two linear equations

Introduction

This task should be used to explore or apply the skill of comparing properties of two functions that are each represented in different formats. After the properties are determined and the functions are graphed, students graphically solve a system of equations to analyze the situation. Students should already be familiar with calculating the slope of a line and graphing linear functions.

Begin by reading the problem and clarifying the meanings of *supply* and *demand*.

demand	the amount of a product that is wanted by customers
supply	the amount of a product that is available

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will first recognize that this task involves analyzing the relationship between supply and demand. They will make sense of the problem by identifying the two different representations of the functions for supply and demand. Perseverance may be necessary if students have difficulty finding a way to compare the two functions in their given formats.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  There are several ways that students can compare the two given functions. Students should justify their reasoning to classmates who approached the problem in a different way. Encourage students to discuss their arguments with each other and explain their reasoning if they do not agree with each other.

- **SMP 4:** Model with mathematics.

  Students may recognize that one way that the scenario can be modeled is by graphing both linear functions on the same coordinate axes. They will also see that although linear functions can be modeled in different formats, the properties of the functions can still be compared and analyzed in these different formats, as well as with a graphic representation.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly determining the rate of change

  Encourage students to write the slope formula on the top of their papers to use as a reference when working through this task. Remind them to carefully label each of the coordinates with x_1, y_1, x_2, and y_2 before substituting them into the formula.

- not comparing the absolute values of the slopes to determine which function is steeper

  Remind students that a positive slope does not necessarily mean that the slope is steeper. What determines the steepness of the slope is the absolute value of the rate of change. The graph of the demand function is steeper than the graph of the supply function, even though the demand function has a negative slope and the supply function has a positive slope.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task:

  Discuss students' prior knowledge of the concept of supply and demand. Ask, "What are some real-world examples of supply and demand?" (**Sample answer:** "When a new product comes out that is considered to be a desirable item, the price is usually high at first, based on the fact that the item is in great demand and the supply is limited. The price usually drops once the demand is not as great. Some examples include newly released phone models and gaming consoles.")

  Discuss the other possible formats in which the two functions can be represented. Point out that the supply function is given as an equation, and the demand function is given as a table of values. Ask, "How else can the functions be represented?" (**Answer:** "Each function can be represented graphically, algebraically, as a table of values, or even as a verbal description. This variety of ways to analyze a function is known as the 'rule of four.' ")

- As students begin to calculate the rates of change for the two functions, ask the following:

  "What do you know about the rate of change for a linear function?" (**Answer:** "A linear function has a constant rate of change. This means that the slope of the line will be the same between any two points on the line.")

  "Which has the steeper slope—the supply function or the demand function? How do you know?" (**Answer:** "The demand function has a steeper slope because the absolute value of –0.6 is 0.6, and this is greater than the absolute value of the supply function, which is 0.3.")

- Ask students, "What do the *y*-intercepts of the functions represent in the context of the scenario?" (**Answer:** "The *y*-intercepts of the functions represent the initial values for each function. One possible interpretation is that before the tablets first came out, when there were 0 tablets available, they could be priced at $100 per tablet according to the supply function, which has a *y*-intercept of 100. For the demand function, the *y*-intercept is 550, which represents the suggested price of $550 for a tablet before any tablets were available.")

- If students have difficulty understanding why the *x*-variable represents the number of tablets and the *y*-variable represents the price per tablet, remind them of the meaning of the terms *independent variable* and *dependent variable*. Ask, "In this scenario, which quantity is dependent on the other?" (**Answer:** "In this scenario, the price of the tablet depends on the number of tablets that are available and in demand. Therefore, the independent variable, *x*, is the number of tablets, and the dependent variable, *y*, is the price per tablet.")

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- Ask students, "What is the point of intersection of the two functions, and what does the point of intersection represent in the context of the scenario?" (**Answer:** "The two lines intersect at the point (500, 250). This represents the fact that when the seller has 500 tablets, each tablet will be priced at $250.")
- If students have difficulty understanding what it means when the supply function exceeds the demand function, encourage them to think about real-life examples of products that have been on the market for a long period of time. Discuss how the price decreases for many products as time goes on, since the current demand for those products is lower than when they first came on the market. Ask students, "What does it mean that the supply function for the tablets exceeded the demand function for the tablets?" (**Answer:** "It means that when the supply function exceeds the demand function, the number of tablets available is greater than the number of tablets demanded by the buyers.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

Compare students' strategies and explanations for calculating the rates of change and the *y*-intercepts for both functions. Discuss the comparisons of these properties for the two functions and what this means in the context of the scenario. Focus on the use of precise mathematical language.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Linear functions can be represented in words or as equations, graphs, or tables.

  In this task, students are presented with a linear function represented by an equation, and also a linear function represented by a table, and they may choose to present both of these functions graphically.

- To compare linear functions, determine the rate of change and intercepts of each function.

  In this task, students will determine the rate of change and the *y*-intercept for both linear functions. Students will compare these properties in order to draw conclusions about the two functions.

- Linear functions are increasing if the rate of change is a positive value.

  Students will determine that the rate of change for the supply function is 0.3; therefore, they can conclude that the supply function in increasing.

- Linear functions are decreasing if the rate of change is a negative value.

  Students will determine that the rate of change for the demand function is –0.6; therefore, they can conclude that the demand function is decreasing.

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Extending the Task

To extend the task, replace one of the linear functions in this problem with an exponential function, and ask students to compare the properties of the functions on specific domain intervals and summarize supply versus demand for this new situation. Discuss how comparing the properties of two linear functions differs from comparing the properties of a linear function and an exponential function.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I made sense of the problem by calculating the rates of change for both linear functions, and then graphing them on the same coordinate plane in order to visually compare their properties. I analyzed the properties of each function in order to draw conclusions about supply and demand.")
- **For SMP 3, ASK:** "How did you construct viable arguments and critique the reasoning of others?" (**Answer:** "I constructed viable arguments by comparing the properties of each function and basing my conclusions about the supply and demand of the tablet on these properties.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I modeled this scenario visually by graphing both functions on the same coordinate plane.")

Alternate Strategies or Solutions

Rather than modeling both functions graphically in order to compare them, students may decide to make a table for the first function, and then compare the two tables. Or, they may decide to write an equation for the second function, compare the two equations, and solve the system of equations algebraically. Point out to students that it is often easier to compare the two functions when they are in the same format.

Technology

Students can use scientific calculators for their calculations of slope, and they can use graphing calculators or other graphing technology when graphing the functions.

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Problem-Based Task: Analyzing Kidney Function

Common Core State Standard

F–IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Task Overview

Focus

How does the half-life of one radioactive material compare with the half-life of another radioactive material? How can it be determined which material decays at a faster rate? In this activity, students will compare two exponential functions by analyzing their equations and their graphs.

This activity will provide practice with:

- writing exponential functions from a context
- graphing exponential functions using graphing technology
- comparing functions by examining their graphs on the same coordinate plane
- analyzing the characteristics of an exponential function, specifically the exponent representing time, or half-life
- evaluating a function for a specific value of time x by determining the corresponding y-coordinate on the graph

Introduction

This task should be used to explore or apply the skills of setting up and graphing exponential functions and comparing two exponential functions. Graphing may be done using technology, including with graphing calculators or online graphing programs.

Begin by reading the problem and clarifying the meaning of *exponential function*. In addition, review the following terms:

decay factor	the factor by which a quantity decreases over time
half-life	the time required for half of the amount of a radioactive material to decay

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Some students may first be inclined to conclude that there is not enough information given in the problem to be able to compare the two functions, because the function for one radioactive material is not given. Ask students how they can use the function given for one material to create the function for the second material.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students may work together and discuss different methods of drawing conclusions about how to determine which material has the faster rate of decay. Other than examining the graph, students may want to create a table of values, or randomly substitute values into each function. Encourage students to compare methods with each other and decide which of these methods is more efficient and accurate.

- **SMP 5:** Use appropriate tools strategically.

  Students will need to graph two exponential functions on the same graph in order to compare them. Students will see the usefulness of graphing technology in order to analyze the rate of decay.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- using the exponential growth model instead of the exponential decay model

  Remind students of the definition of *half-life*, which is the amount of time it takes for half of the amount of a radioactive material to decay. Because this problem represents decay, the exponential decay model must be used.

- choosing the wrong values to substitute into the exponential functions, such as using the initial value in the place of the decay rate, or vice versa

  Have students write the general form of an exponential function, $f(x) = ab^x$, at the top of their papers, along with what the variables a, b, and x represent.

- incorrectly concluding that the graph with the less-steep curve has a faster rate of decay

  Remind students that one way to determine which graph has a faster rate of decay is to substitute a specific value for x into both functions and compare the corresponding outcome; have them analyze which graph gets closer to the x-axis first.

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- incorrectly entering the exponential functions into their graphing calculator

  Remind students to carefully enter each part of the function, and to double-check the function before graphing it.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.
- If students say, "There is no function given for the half-life of indium-113m," explain that since these are both radioactive materials with the same initial value, the function for indium-113m can be created based on the function for technetium-99m.

  If students are struggling with creating this function, ask them to write the function for technetium-99m, and then rewrite it a second time, focusing on which value will be different for indium-113m. (**Answer:** The exponent of the function for indium-113m will have a denominator of 1.7 instead of 6, because the half-life of indium-113m is 1.7 hours.)

  If students don't understand the format of the equation, explain that the standard form of an exponential equation is $f(x) = ab^x$. In the context of this task, a is the initial amount of the radioactive substance, b is the rate of decay, x is the time in hours, and $f(x)$ is the amount of the substance at time x. Because the decay rate of $\frac{1}{2}$ applies to given time periods (the half-lives), we have to divide x by this half-life. If we call the half-life t, then the equation becomes $f(x) = ab^{\frac{x}{t}}$.

- Some students may be able to reason just by looking at the problem that because the half-life of indium-113m is 1.7 hours and the half-life of technetium-99m is 6 hours, the indium-113m is decaying faster. This can be concluded without making a table or a graph. Ask these students to find a way to justify their answer graphically.
- Ask why an exponential function is used to model the scenario instead of a linear function. (**Answer:** "An exponential function is used because the rate at which the radioactive materials decay is not a constant. The substance does not decrease by the same amount for each half-life.")

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- If students are unable to conclude solely from examining the curves of each function's graph which one decays at a faster rate, ask them to choose a value for the time, such as $x = 12$, and substitute this value into each function. Ask students how much of each substance is left after 12 hours. (**Answer:** "After 12 hours, there are approximately 125 mg of technetium-99m left, and there are approximately 7 mg of indium-113m left.") Point out these values on the graph. Encourage students to draw the conclusion that both substances started with the same amount, so the indium-113m must decay faster because there is less of it left after 12 hours.

  Ask students, "What does the difference in the values of the half-life for each material mean in the context of the problem?" (**Answer:** "The indium-113m has a half-life of 1.7 hours, and the technetium-99m has a half-life of 6 hours, so it takes longer for the technetium-99m to decay than it does for the indium-113m to decay.")

  Ask, "How does graphing the functions verify that the indium-113m decays at a faster rate than the technetium-99m?" (**Answer:** "After the same amount of time has passed, there is less indium-113m than technetium-99m, because the *y*-values of the graph for the same *x*-value are lower for indium-113m than for technetium-99m. In other words, the graph for indium-113m is below the graph for technetium-99m.")

Debriefing the Task

- Ask students to volunteer their thought processes for creating an exponential function to model the half-life of indium-113m based on the exponential function already given for the half-life of technetium-99m.

- Discuss each component of the basic formula. Ask students, "Why is the fraction $\frac{1}{2}$ used for the variable *b*?" Help students make the connection between the definition of *half-life* and using the fraction $\frac{1}{2}$ to represent this rate of decay.

- Once students have graphed both functions, ask for their initial reactions. Some students might incorrectly conclude that "faster rate" refers to the graph that is farther to the right. Discuss why this is incorrect, and help students compare the amount of material remaining for both types of material at a specific time, so that they can see that "faster" means "less" in this context.

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Connecting to Key Concepts

Make explicit connections to key concepts:

- Exponential functions are functions that can be written in the form $f(x) = ab^x + k$.

  Ask students to explain the meanings of the following variables in the context of the problem: a is the initial amount of the radioactive substance; b is the rate of decay, or $\frac{1}{2}$; and x is the number of hours that have passed. Note that for each substance, x is divided by the half-life in hours for that substance, resulting in an exponent of $\frac{x}{t}$.

- Exponential functions can be represented in words or as equations, graphs, or tables.

  The task describes the functions using both verbal statements and equations. Students can create a table of values for each function in order to graph them without graphing calculators. Students use graphs of the functions to analyze and compare the decay rates of each material.

Extending the Task

- Provide students with a list of domain values representing the number of hours, such as {2, 4, 6, 8, 10}. Ask them to create a table of values for each function. Then, ask students to determine the rate of change for the interval between several pairs of points. Encourage them to calculate the rate of change for many different intervals in order to determine if a conclusion can be drawn about which function has the faster decay rate without graphing the functions.
- Ask students to work together to determine the number of hours it will take for each material to completely decay (i.e., when the amount of material for each is 0), and to show their calculations.
- Ask students to create a third exponential function that, when graphed, will be in between the graphs for the indium-113m and technetium-99m functions. Ask students to write a paragraph explaining the possible range of half-life values that would result in a function's curve that is between the other two functions.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Sample answer:** "I started by deciding which values to substitute for which variables in the function for technetium-99m to create the function for indium-113m. Once I wrote the exponential equation, I graphed the two equations and analyzed the graphs to see which function was decreasing at a more rapid rate. When I got stuck, I discussed the problem with another student.")

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- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Sample answer:** "Yes. When another student arrived at a different answer from mine, I explained what mistake I thought he made and I was able to convince him that my method was correct.")
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Sample answer:** "After I found the second equation, I graphed the two equations using a graphing calculator. Then, I used the 'calculate' feature to evaluate the remaining amount of each substance at different times to see which function was decreasing the fastest.")

Alternate Strategies or Solutions

- In addition to graphing, students can compare the functions by determining the rate of change between several intervals. Students can compare the rates of change over the same interval for each function and draw conclusions about which has the faster rate of decay.
- If students do not have access to a graphing calculator or other graphing technology, but they do have a scientific calculator, they can create a table of values for each function, with x representing the time in hours and y representing the amount of material remaining. Encourage students to choose values for x that make their calculations easier. Then, students can use the tables to graph the functions by hand. Encourage students to realize that the plotted points are approximations and not exact values. The overall shapes of the curves will still allow students to make comparisons and arrive at the correct conclusion that indium-113m decays at a faster rate.

Technology

Students can use graphing technology. Students can also use scientific calculators in order to create a table of values if they do not have access to graphing technology.

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Problem-Based Task: Future Finances

Common Core State Standard

F–LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★

Task Overview

Focus

How can a linear function and an exponential function be compared? Which type of function will increase faster? In this task, students will graphically compare a linear function and an exponential function, both of which represent payment plans, and they will interpret their comparisons in order to determine which payment plan will yield the greatest income over given periods of time.

This activity will provide practice with:

- identifying the type of function based on verbal descriptions
- writing linear and exponential functions
- graphing linear and exponential functions
- analyzing information from a graph of a linear and an exponential function
- interpreting the solution of a system of equations
- comparing rates of change for linear and exponential functions

Introduction

This task should be used to explore or apply the skill of comparing a linear and an exponential function graphically in order to observe that a quantity increasing exponentially will eventually exceed a quantity increasing linearly. Students should already be familiar with identifying linear and exponential functions, as well as with graphing functions.

Begin by reading the problem and clarifying the meaning of the terms *exponential function*, *growth factor*, *linear function*, and *rate of change*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will construct viable arguments about the amount of income each scenario will generate given specific time intervals in order to draw conclusions about which function will yield more income over time. Encourage students to discuss their arguments with each other and explain their reasoning if they disagree.

- **SMP 4:** Model with mathematics.

  Students will recognize that the scenario presented can be modeled by first creating a linear equation and an exponential equation to model the different pay options, and then by graphing both functions on the same coordinate axes. They will analyze the relationships between the two functions in order to draw conclusions about the different pay options.

- **SMP 6:** Attend to precision.

  Make sure students are using math terms and definitions clearly and accurately to explain their reasoning. Encourage them to analyze properties of both functions carefully when constructing their reasoning. Strive toward precise mathematical language; for example, by correctly using the terms *linear function*, *exponential function*, *rate of change*, and *exponential growth*. Also, remind students to carefully label the axes on their graphs.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly identifying a function as linear when it is exponential and vice versa

  Remind students that they must check several intervals of a function in order to determine if the rate of change is constant or not. If the rate is the same for each interval, then the function is linear, and if the rate is not the same for each interval, then the function is exponential.

- incorrectly determining the rate of change

  Remind students that the rate of change for a linear function is constant, and that this rate is also the slope of the function. The rate of change for an exponential function is not constant, but the average rate of change can be found for specific intervals of the function.

- assuming the rate of change of a function is linear by only referencing one interval

  Once a function has been identified as exponential, or non-linear, then there will be different rates of change for different intervals of the function. Remind students to check several intervals of a non-linear function in order to verify that they do not have the same rate of change.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask them to predict which of the two payment options they think will be the better choice for Cole. Ask, "Based on the verbal description of the scenarios, which option do you think will be better for Cole and why?" (**Answer:** Some students may choose the first option because it presents a higher initial value as compared to the second option, which begins with only \$0.01. Some students may initially recognize that the second option is an exponential situation, and reason that it will turn out to be the better option. Ask students to write down their prediction, along with their reasoning.)
- As students begin the task, ask them how they are able to determine that the first payment option will be represented by a linear function. (**Answer:** "Since Cole will be receiving \$3,495.25 each month, then the rate of change between any two points, where the variable x is the time in months, and y is the amount of money, will always be the same. The slope, or rate of change, will be \$3,495.25.")
- If students have difficulty determining that the second pay option is represented by an exponential function, ask, "What term in the problem indicates that this function will be exponential?" (**Answer:** "The word 'doubles' indicates that this will be an exponential function.")
- If students have difficulty writing the exponential function to represent the second pay option, ask them to identify the values for the variables in the format of an exponential function, $g(x) = ab^x + k$. (**Answer:** "In an exponential function, a represents the initial value, which is 0.01, b represents the base, which in this situation is a growth factor of 2, and x represents the time, which is in months for this problem. There is no value of k for this problem because the y-intercept is 0.")
- Ask students, "In the exponential function, $g(x) = (0.01)(2)^{x-1}$, why is the exponent written as $x - 1$ instead of just x?" (**Answer:** "The exponent is written as $x - 1$ because the amount of pay for a given month is based on doubling the previous month's pay. For example, the pay for the third month ($x = 3$) is based on doubling the pay for month 2.")
- Ask students, "What is the significance of '24 months' for the two functions?" (**Answer:** "At 24 months, the two functions intersect each other. This means that they both yield the exact same income at this point. The ordered pair that represents this intersection, according to the graph, is approximately (24, 84,000). Both payment options will pay about \$84,000 at the end of the 24th month.")

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- Ask students, "What is the exact amount of income, to the nearest dollar, for the point of intersection of the two functions, and how can this amount be determined?" (**Answer:** "The value of 24 can be substituted for x in either the linear function or the exponential function. The resulting income, or output value, is $83,886.")
- Ask students, "Under what circumstances should Cole choose the second payment option?" (**Answer:** "If Cole plans to finish his degree program in less than 24 months, he should choose the first option. If he believes he will finish it in exactly 24 months, he could choose either plan because the total pay would be the same. If he believes it will take him longer than 24 months, he should choose the second option.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask volunteers to discuss their initial predictions about which payment option they thought would be the best one. Encourage students to discuss their reasoning for their predictions, as well as how correct or incorrect they were based on the actual results of the task.
- Compare students' strategies and ways of justifying responses. Focus on the use of precise mathematical language and clarity, specifically when referring to each type of function.

Connecting to Key Concepts

Make explicit connections to key concepts:

- The equation of a linear function can be written in the form $f(x) = mx + b$.

  In this task, students will analyze a verbal description of a payment plan and identify it as a function that can be written linearly; specifically, $f(x) = 3495.25x$.

- The rate of change of linear functions remains constant.

  Students will identify that Cole will receive $3,495.25 each month, which represents the slope, or rate of change, for the linear function representing this pay option. Since Cole will receive the exact same amount each month with this option, the rate of change remains constant.

- Exponential functions are written in the form $g(x) = ab^x + k$.

  In this task, students will analyze a verbal description of a payment plan and identify and write it as an exponential function; specifically, $g(x) = (0.01)(2)^{x-1}$.

- A quantity that increases exponentially will always eventually exceed one that increases linearly.

  Students will examine the graphs of both payment options, and they will conclude that the second payment option, which is represented by an exponential function, exceeds the linear function representing the first payment option after 24 months.

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Extending the Task

To extend the task, provide students with a third payment option for Cole. For example, this third option could give Cole a starting bonus of $2,000, with a payment of $3,495.25 a month. Ask students to create a function for this payment option and then compare it with the two given payment options to determine which of the three is the best payment option.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "I constructed viable arguments by comparing the linear function and the exponential function, and I was able to base my conclusions about which payment option is better for Cole based on the graphical representation of both functions.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I modeled this scenario by first creating functions representing the two different pay options based on verbal descriptions, and then I created a visual model of both functions by graphing them on the same coordinate axes.")
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** "I attended to precision by using correct math terminology when referring to the linear and exponential functions. I also double-checked my calculations when determining how much Cole would earn with each option to make sure the answers made sense.")

Alternate Strategies or Solutions

Although students are not asked to create a table of values for the functions before writing the equations and graphing them, students may choose to create tables so that they can examine the rates of change and identify each function as linear or exponential. Discuss how this method may help students determine the correct type of function visually. A table can also help students compare the amount of money at the end of each month for the two plans.

Technology

Students can use scientific calculators for their calculations when substituting values into the functions, and they can use graphing calculators or other graphing technology to graph the functions or analyze the tables for the functions.

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Problem-Based Task: Texting for the Win

Common Core State Standard

F–BF.1 Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Task Overview

Focus

How can an explicit function be written for a scenario? How can the function be used to calculate specific values for a variable? In this task, students will write an explicit function based on a verbal description of a real-life scenario, and they will use the function to determine the number of text messages sent after given numbers of rounds.

This activity will provide practice with:

- drawing a diagram to model a situation
- analyzing a verbal description of a scenario
- creating a table of values
- identifying a pattern based on information given in a table of values
- writing an explicit function to represent a scenario

Introduction

This task should be used to explore or apply the skill of determining an explicit function or a recursive process. Students should already be familiar with evaluating exponential expressions and creating a table of values representing two quantities.

Begin by reading the problem and clarifying the meaning of *explicit function*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students may begin to make sense of the problem by drawing a diagram to represent the first few rounds of texts sent, or by making a table of values to organize the calculations. Encourage students to discuss their ideas for organizing their calculations in order to successfully analyze their results.

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they represent the quantities in the scenario algebraically. They will identify that the independent variable is the number of rounds of text messages sent, and they will identify that the dependent variable is the total number of text messages sent each round. Students will reason quantitatively as they calculate the number of text messages sent after five rounds.

- **SMP 8:** Look for and express regularity in repeated reasoning.

  Students will notice that the same calculations are performed repeatedly in order to achieve the desired results. They will recognize that the number of text messages sent in each round is a factor of 8. From these repeated calculations, students will be able to generalize the pattern and create an exponential function representing the scenario, in which 8 is the base and the round number is the exponent.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- confusing linear and exponential functions when writing a function rule for a problem situation

  Remind students that a linear function will have a constant rate of change between any two consecutive points, and an exponential function will have a common ratio instead. Remind students to check for a common difference, or a common ratio, between consecutive pairs of points in a table of values in order to identify which type of function will represent the situation.

- believing that $f(x)$ means "f times x;" i.e., not understanding that $f(x)$ represents the dependent variable for the function

  Have students write the definition for function notation on their papers, noting that $f(x)$ can also be written with the variable y, which represents the dependent variable.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task:

  Ask them to predict which type of function will be used to represent the given scenario: linear or exponential. Ask them to write down their predictions and reasoning behind their choice. (**Answer:** Some students may predict that the function will be linear, because they may assume that the number 8 represents the common difference between the output values. Other students may recognize that 8 will be the common ratio, since the number of text messages will increase exponentially each round, and not linearly.)

  Ask, "Based on the given information, what are some ways in which the information can be represented and organized?" (**Answer:** "The information can be represented with a tree diagram, or a similar visual representation. A table of values will be useful for organizing the calculations that need to be performed for each round of text messages sent.")

- As students begin the first part of the task and draw a diagram to represent the situation, ask them about the practicality of drawing a diagram to represent more than two rounds. Ask, "Why does it make sense to draw the diagram so that it shows only two rounds of text messages?" (**Answer:** "Based on only the first two rounds, I see that it will become very tedious and time-consuming to draw a diagram of the number of texts sent for rounds 3, 4, and so on, since the numbers become very large. The number of texts sent each round increases quickly, so the scenario will be better represented exponentially.")

- Ask students, "How can the table of values be used to identify the function for this scenario as exponential?" (**Answer:** "When the output values are found for the input values of 1, 2, 3, 4, and 5, the results show that there is a common ratio of 8 between any two consecutive points. A common ratio indicates an exponential function.")

- If students have difficulty understanding why it is necessary to examine the results of the previous round in order to calculate the number of text messages sent for the current round, ask students, "What is the relationship between each round of messages sent and the previous round?" (**Answer:** "Each round increases by a factor of 8, since each friend sends text messages to 8 friends each round. So each new round is found by multiplying the number of text messages sent in the previous round by 8.")

- Ask students, "How can exponents be used in this problem?" (**Answer:** "Exponents can be used as a way of writing expressions in a more 'compact' way. It is more time-consuming to write out a factor of 8 ten times than it is to represent this value with an exponential expression, 8^{10}.")

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- Ask students, "How can the number of text messages sent in Round 6 be calculated?" (**Answer:** "To determine the number of text messages sent after 6 rounds, substitute 6 for the variable x into the function representing the scenario, which is $t(x) = 8^x$: $t(x) = 8^6 = 262{,}144$. Therefore, the number of text messages sent after 6 rounds is 262,144.")
- Discuss with students the practicality and real-world constraints of this scenario. Ask them, "What are some factors which might impede the success of Lucas's plan to get votes for his friend Isabel?" (**Answer:** "It might be difficult to successfully reach 8 friends quickly enough to get them to vote; it might be even more difficult to get them to send text messages to 8 of their friends; and the friends probably have many friends in common, so the same people may receive the text multiple times. Also, in competitions where voting is conducted in real time via calls, texts, etc., there is often a time limit. So, the reality of reaching the expected number of people based on the function is not likely.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask volunteers to share their initial predictions about the type of function that is used to model the given scenario. Encourage students to discuss their reasoning for their predictions, as well as how correct or incorrect they were based on the actual results of the task.
- Compare students' strategies and explanations for analyzing the given information and creating the equation of the function. Focus on the use of precise mathematical language.

Connecting to Key Concepts

Make explicit connections to key concepts:

- An exponential equation is of the form $y = ab^x + k$.

  In this task, students will identify the independent variable, x, as the number of rounds of text messages sent. They will identify the base to be 8, because this is the factor by which the number of texts increases each round. The dependent variable is $t(x)$, representing the number of text messages sent per round.

- Consecutive dependent terms in an exponential function have a common ratio if the corresponding independent variables are consecutive integers.

  When students create a table of values representing input and output values, they will see that there is a common ratio of 8 between any two consecutive output values, or dependent terms.

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Extending the Task

To extend the task, ask students to consider an alternate scenario in which another person is trying to get votes for another singer on the same show. For example, Mario wants to help his friend Nicholas earn votes, so Mario sends texts to 10 people, and asks each of those 10 people to send texts to 5 more people. Mario also asks the 10 people he texts to ask each of the 5 people they text to in turn text 5 additional people. Ask students to model this scenario and compare it with the original scenario in the task. Ask them to calculate the number of texts sent each round for the new scenario.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I persevered in solving the problem by drawing a diagram and creating a table of values to represent the scenario. Then I analyzed the information and created an explicit function.")
- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "I used abstract reasoning when I identified the variables in this scenario and when I recognized that the scenario would be represented by an explicit function containing an exponential equation.") "Which of your strategies represent quantitative reasoning?" (**Answer:** "I used quantitative reasoning when I calculated the number of text messages that were sent after 1, 2, 3, 4, and 5 rounds.")
- **For SMP 8, ASK:** "How did you look for and express regularity in repeated reasoning?" (**Answer:** "I recognized that the constant 8 is multiplied for each round of text messages and that the number of texts sent each round increases by a factor of 8. From these repeated calculations, I generalized the calculations and created an exponential function to represent the scenario.")

Alternate Strategies or Solutions

- When students are asked to draw a diagram to model the situation at the beginning of the task, some students may choose other types of diagrams instead of the tree diagram presented in the Coaching Sample Responses. Encourage any type of diagram that will clearly and effectively display the information given in the scenario.
- Some students may be able to discern the pattern for the exponents without having to write out the factors. Remind students to check their calculations if they choose to write out all of the factors of 8.

Technology

Students can use scientific calculators in this task.

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Problem-Based Task: Interior Angles in Polygons

Common Core State Standard

F–LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

Task Overview

Focus

How can an equation be written to represent a linear or an exponential function that models a real-life scenario? In this task, students will identify the type of equation that represents the relationship between the number of sides of a polygon and the sum of the measures of the interior angles in a polygon, and they will write a function to represent this scenario.

This activity will provide practice with:

- identifying the type of equation based on a table of values and a graph
- analyzing a table of values
- graphing data using a table of values
- calculating the slope from a table of values
- determining the *y*-intercept of a linear equation
- writing a linear equation in slope-intercept form

Introduction

This task should be used to explore or apply the skill of constructing linear functions given a graph, a verbal description, or a table of values. Students should already be familiar with identifying independent and dependent quantities as well as with graphing points on the coordinate plane.

Begin by reading the problem and clarifying the meaning of the terms *polygon* and *interior angle of a polygon*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will first make sense of the problem by identifying that the goal is to determine if there is a relationship between the number of sides of a polygon and the sum of the interior angles. They will persevere by looking for a way to analyze and make use of the information given in the table.

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they represent the numerical data in the table with an algebraic equation. They will identify that the independent variable is the number of sides of the polygon, and the dependent variable is the sum of the measures of the interior angles. Students will reason quantitatively as they calculate the slope of the equation by determining the ratio of the change of the dependent variable to the independent variable.

- **SMP 4:** Model with mathematics.

  Students will see that the scenario can be modeled using a graph and a linear equation written in slope-intercept form. They will analyze the relationships between the two quantities represented, and they will examine the various models representing this relationship in order to identify the type of equation that represents this relationship.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- looking for a common difference between dependent quantities in a table when the independent values are not 1 unit apart

  Remind students that in order to determine that a table of values represents a linear function, the difference between each consecutive pair of dependent values must be the same; however, this will not be the case if the independent values are not 1 unit apart.

- incorrectly calculating the slope

  Have students write the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, at the top of their papers to use as a reference as they work through the task. Also, remind them to label the coordinates of the two chosen points as x_1, y_1, x_2, and y_2 before substituting the values into the formula.

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- writing a linear equation for an exponential function or writing an exponential equation for a linear function

  Remind students that the graph of an exponential function is a curve, and the graph of a linear function is a straight line.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask them to write their prediction about what type of relationship exists between the number of sides of a polygon and the sum of the interior angles. Prompt students to think about the sum of the interior angles of a triangle and a square and then ask, "Can you identify the relationship between the number of sides and the sum of the interior angles in these two polygons?" (**Answer:** "As the number of sides of a polygon increases by 1, the sum of the interior angles increases by 180°.")
- Ask students, "What do you notice about the sequence of the values of the independent variable in this scenario? What do you notice about the difference between consecutive values of the dependent variable?" (**Answer:** "The values for the independent variable are not all consecutive, since the values for 4 and 8 are not represented.")
- If students choose to graph the points by hand instead of using technology, they may not easily see that the points will connect to form a straight line. In this case, ask, "If you cannot identify from the points on the graph what type of equation is represented, how can you verify that the type of equation representing the scenario is a linear equation?" (**Answer:** "The slope can be calculated between several pairs of points. Since the slope is the same between any two points, the equation is linear.")
- Ask, "In the context of the scenario, why is the value of the slope positive? Explain in terms of increasing the number of sides of a polygon." (**Answer:** "As the number of sides of a polygon increases, the measure of each interior angle also increases. Therefore, a linear equation for this situation will have a positive value for the slope because the values are always increasing.")
- Ask, "What is the domain of this function? Explain." (**Answer:** "The domain of this function is all natural numbers greater than or equal to 3. The least number of sides that a polygon can have is 3. There is no such thing as a two-sided polygon, or a one-sided polygon, and there is no such thing as a half-side or fractional number of sides.")
- Once students determine that the slope is equal to 180, ask, "How can you describe the meaning of the slope in the context of the scenario?" (**Answer:** "Each time the number of sides of a polygon increases by 1, the sum of the interior angles increases by 180°.")

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- If students are having difficulty understanding why the *y*-intercept is a negative value, –360, ask them, "What does the value of the *y*-intercept represent? Why is the *y*-intercept not shown/represented on the graph?" (**Answer:** "The *y*-intercept is the point at which the graph of the line intersects the *y*-axis. In this scenario, since the value of *x* has to be greater than or equal to 3, and the sum of the measures of the interior angles cannot be negative, the *y*-intercept has no meaning.")
- Once students have determined that the equation which represents the sum of the interior angles of any polygon is $f(x) = 180x - 360$, ask, "How can this equation be used to determine the sum of the interior angles of a 12-sided polygon? What is this sum?" (**Answer:** "Substitute 12 for *x* in the equation and simplify: $f(12) = 180(12) - 360 = 1800$. The sum of the interior angles of a 12-sided polygon is 1,800°.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask volunteers to share their initial predictions about the type of relationship that exists between the number of sides of a polygon and the sum of its interior angles, and ask them to share their predictions about the type of equation that represents this relationship. Encourage students to discuss the reasoning behind their predictions, as well as how correct or incorrect they were based on the actual results of the task.
- Compare students' strategies and explanations for analyzing the given information and creating the graph and the linear equation, as well as how they calculated the slope. Focus on the use of precise mathematical language.

Connecting to Key Concepts

Make explicit connections to key concepts:

- The graph of a linear equation is a straight line.

  In this task, students will graph 5 points from a given table of values and see that when the points are connected, the result is a straight line.

- The slope-intercept form of a linear function is $f(x) = mx + b$, where *m* is the slope and *b* is the *y*-intercept.

  Students will find the slope and the *y*-intercept of the linear equation, and then they will write the equation using function notation.

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- The slope of a linear function can be calculated using any two points, (x_1, y_1) and (x_2, y_2); the formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

  Students may find the slope between each consecutive pair of data values in order to show that the rate of change is the same, and then conclude that the function represented in the table of values is linear.

- The *y*-intercept is the *y*-coordinate of the point at which the graph of the equation crosses the *y*-axis. It is the value of *y* when $x = 0$.

  Students will substitute the slope and the coordinates of one of the points in the table into the function $f(x) = mx + b$ and solve for *b* in order to determine the *y*-intercept. The *y*-intercept is –360, which means that the graph of the line intersects the *y*-axis at the point (0, –360).

Extending the Task

- To extend the task, ask students to add the values 4, 8, and 10 to the "Number of sides" column in the table of values, and then to fill in the rest of the table with the sum of the interior angles for these new values. Discuss the results. Also, ask students to write the name of each polygon represented in the chart, using the number of sides.
- Or, ask students to create a table of values representing the sum of the *exterior* angles of polygons with 3, 5, 6, 7, and 9 sides. If students have difficulty remembering how to find the measures of the exterior angles of a polygon, encourage them to sketch an equilateral triangle, a square, a regular pentagon, a regular hexagon, etc., so they can see how an interior angle of a polygon relates to an exterior angle. Ask them to use their data from the task to relate the measure of each interior angle of a polygon to the measure of its exterior angle. Once students have determined that the sum of the exterior angles of a polygon is always 360°, ask them what linear equation would represent the table of values, and ask them for the slope of this line.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I made sense of the problem by reading it through a few times to determine what it is about, and by analyzing the table of values. I persevered by graphing the points to determine whether they lie in a straight line. I determined that the type of equation representing the two quantities was a linear equation.")

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- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "I used abstract reasoning when I identified the variables in this scenario, and again when I recognized that the scenario would be represented by a linear equation based on the graph and finding the slope between each pair of consecutive points.") "Which of your strategies represent quantitative reasoning?" (**Answer:** "I used quantitative reasoning when I calculated the slope of the equation by determining the ratio of the change in the dependent variable to that of the independent variable.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I modeled the scenario by creating a graph from the given table of values, and I also modeled it with an algebraic equation.")

Alternate Strategies or Solutions

Students may choose to add the values of 4 and 8 to the given table of values, so that the values for the independent variable are consecutive. Remind students that when calculating the slope between two consecutive points in the table, it will still be the same, regardless of whether the values for the independent variable are consecutive.

Technology

Students can use scientific calculators or graphing calculators for this task.

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Problem-Based Task: Jai's Jeans

Common Core State Standard

F–BF.1 Write a function that describes a relationship between two quantities.★

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Task Overview

Focus

How can a function rule be written to represent a scenario? How can a function rule be used to calculate specific values for a given variable? In this task, students will first write an expression to represent the sale price for jeans, and then they will convert the expression to a function and evaluate the function in order to determine the total cost for a given number of pairs of jeans.

This activity will provide practice with:

- analyzing a verbal description of a scenario
- creating an expression to represent a scenario
- writing a function rule for a scenario
- simplifying a function by using the order of operations and combining like terms
- evaluating a function for a given value

Introduction

This task should be used to explore or apply the skill of writing a function that describes the relationship between two quantities. Students should already be familiar with simplifying and evaluating expressions.

Begin by reading the problem and clarifying the meaning of the terms *expression*, *function*, *function notation*, and *function rule*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students may make sense of the problem by calculating the sale price of a pair of jeans without using any algebra, and then translating their steps into an algebraic function. Encourage students to discuss the necessary steps for creating and simplifying their function rule before using it to evaluate for a specific value.

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they try to write an algebraic function to represent the scenario. Students will reason quantitatively as they simplify the function by using the order of operations and combining like terms, and again when they evaluate the function for a specific value.

- **SMP 6:** Attend to precision.

  Make sure students use math terms and definitions clearly and accurately to explain their reasoning. Encourage them to carefully construct their expressions, and to remember to rewrite the percent as a decimal. Strive toward precise mathematical language, such as using the terms *expression*, *function rule*, and *function notation* correctly.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- forgetting to rewrite the percentage discount as a decimal

  Remind students that a percent is written in decimal form when it is used in an equation.

- not using the correct function notation

  Remind students that function notation is a way to name a function using $f(x)$ instead of y.

- incorrectly combining like terms

  Remind students that *like terms* are terms that have the same variable(s) and exponent(s); therefore, $50x$ and $-15x$ can be combined for a result of $35x$.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask them, "What are the two unknown quantities in this scenario? Which represents the independent variable and which represents the dependent variable?" (**Answer:** "The two variable quantities in this scenario are the number of pairs of jeans purchased and the total cost of the jeans. The independent variable is the number of pairs of jeans, and the dependent variable is the total cost of the jeans.")
- If students have difficulty writing an expression to represent the sale price for the jeans, encourage them to break the scenario down into parts. Ask:

  "If the jeans were *not* on sale, what expression would represent the price of the jeans?" (**Answer:** "Because the jeans originally cost $50 a pair, the expression that represents x number of pairs of jeans at this price is $50x$.")

  "How can the expression representing the amount of the discount be determined?" (**Answer:** "The discount is found by multiplying the original price by the percent of the discount, written in decimal form: 30% = 0.30. Therefore, the expression representing the discount is $0.30(50x)$.")

  "How can you use these expressions to write the expression that reflects the price *after* the discount?" (**Answer:** "A discount represents an amount that is subtracted from the original price. Therefore, I can represent the discounted price of the jeans by subtracting the expression for the discount amount from the expression for the original price. The resulting expression is $50x - 0.30(50x)$.")
- Once students have written the expression representing the sale price for x pairs of jeans as $50x - 0.30(50x)$, ask, "How can write this as a function?" (**Answer:** "The expression written as a function is $f(x) = 50x - 0.30(50x)$.")

  Ask, "How can the function $f(x) = 50x - 0.30(50x)$ be simplified?" (**Answer:** "The function can be simplified by applying the order of operations. First multiply –0.30 and $50x$ to get a result of $-15x$. Then, since $50x$ and $-15x$ are like terms, they can be combined to get a result of $35x$.")
- Ask students, "How can the total cost of 4 pairs of jeans be determined?" (**Answer:** "To determine the total cost of 4 pairs of jeans, evaluate the simplified function by substituting 4 for x: $f(4) = 35(4) = 140$. The total cost of 4 pairs of jeans is $140.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

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Debriefing the Task

- Ask volunteers to share the steps in their process for writing an expression, converting it to function notation, and then simplifying the function. Discuss the different strategies and processes that students used, and encourage students to justify their processes.
- Compare students' strategies and explanations for analyzing the given information and determining the solution plan for this task. Focus on the use of precise mathematical language, specifically when discussing the use of the terms *expression*, *function rule*, and *function notation*.

Connecting to Key Concepts

Make explicit connections to the following key concept:

- Combine linear and exponential expressions using subtraction.

  In this task, students will write expressions for the price of x pairs of jeans and the discount on x pairs of jeans, and then combine the expressions using subtraction.

Extending the Task

To extend the task, provide students with a second scenario involving a pair of jeans that were originally priced at $60 and are marked with a 35% discount, with the added element of a coupon for $20 off the total purchase. Ask students to write a function rule representing the sale price for x pairs of jeans for this new scenario. Finally, ask them to find the price of 4 pairs of jeans in this scenario, and compare their results with their answers for the original scenario.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I made sense of the problem by first calculating the price of the jeans without using algebra, and then by finding a way to represent what I did algebraically.")
- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "When I identified the variables in this scenario and used them to write a function rule, I used abstract reasoning.") "Which of your strategies represent quantitative reasoning?" (**Answer:** "When I used the function by evaluating it for a specific value, I used quantitative reasoning.")
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** "I attended to precision by using correct math terminology, such as *function rule*, *function notation*, and *expression*, as we discussed the problem.")

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Alternate Strategies or Solutions

- Instead of first creating a function to represent the scenario, students may choose to create a table of values representing the price of a pair of jeans with a 30% discount, and then to create a graph from the table in order to find the total cost of 4 pairs of jeans. Remind students that although this is a viable solution path, they may not be able to accurately determine the y-value for 4 pairs of jeans from the graph. Encourage them to create the expression first in order to be able to precisely determine the y-value.
- Instead of writing an expression, students may choose to calculate the discounted price of the jeans, which is $35, and then they may determine that the price of 4 pairs of jeans is (35)(4), which is $140. Remind them that the problem asked them to create a function rule to represent the scenario for any number of pairs of jeans.

Technology

No technology is needed for this task.

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Problem-Based Task: Gym Fees

Common Core State Standard

F–BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Task Overview

Focus

How can function rules be written to represent a scenario? Which component(s) of a function represent a vertical shift in comparison to another function? In this task, students will write function rules to represent the monthly gym fees for two people, and they will determine the difference in the functions in terms of how the graph of one function is a vertical translation of the other.

This activity will provide practice with:

- identifying slopes and y-intercepts of linear functions from their graphs
- writing a function rule representing a scenario
- comparing the slopes and y-intercepts of two functions
- analyzing differences between the graphs of two functions
- interpreting the differences between two functions in the context of a problem

Introduction

This task should be used to explore or apply the skill of identifying the effect of replacing $f(x)$ with $f(x) + k$ for given values of k, as well as determining the value of k given the graph of a function. Students should already be familiar with identifying slopes and y-intercepts from graphs of linear functions.

Begin by reading the problem and making sure the questions and context are understood.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will examine the graphs of two linear functions in order to determine their relationship to each other. Students will make conjectures about the differences between the functions, and then they will construct viable arguments about the translation that has occurred. Encourage students to discuss their reasoning with each other and justify their thought processes.

- **SMP 4:** Model with mathematics.

  Students will see that the scenario in this problem can be modeled algebraically in the form of two function rules written based on the linear equations shown in the graph. They will analyze the differences in the two functions represented graphically as well as algebraically in order to determine what type of vertical shift occurred from one function to the other.

- **SMP 7:** Look for and make use of structure.

  Mathematically proficient students will notice the connection between the equation of a function that has been translated and the graph of the translation. In this task, students will discover that the graph that represents Justin's gym fees is the same as the graph of Paulo's gym fees, except it has been translated up 3 units. They will recognize that this vertical translation is apparent in the equations for the two functions: that is, if 3 is added to the equation that represents Paulo's fees, the result is the equation that represents Justin's fees.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- mistaking vertical shift for horizontal shift

  Remind students that for a function in the form $f(x) + k$, the value of k represents a vertical shift, moving a graph k units up or down from the original function.

- mistaking a y-intercept for the value of a vertical translation

  Remind students that the y-intercept is the y-coordinate of the point at which a graph intersects the y-axis, and the value of the vertical translation, k, is found by subtracting the y-intercept of a function from the y-intercept of its translation.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask them, "What can you determine about the functions representing the total monthly gym fees for Justin and Paulo strictly from examining the graph?" (**Sample answer:** "The lines are parallel; therefore, the two functions have the same slope. Also, Justin pays higher monthly gym fees than Paolo because the line representing Justin's function is higher than the line representing Paulo's function.")
- As students work to find each line's slope, ask:

  "Why might it be difficult to find the slope of each line from the graph?" (**Answer:** "In order to find the slope of each line from the graph, it is necessary to identify two exact points on the graph for each line. It is important to make sure the points are easily identifiable so that the slope calculations will be accurate.")

  "What is true about parallel lines in terms of their slopes?" (**Answer:** "Parallel lines have the same slope.")
- After students determine the difference in the *y*-intercepts, ask:

  "If Justin paid $68 for a month in gym fees, and Paulo was at the gym the same number of hours as Justin, how much did Paulo pay in gym fees? Explain how this amount can be determined." (**Answer:** "If Justin paid $68, then Paulo paid $65. Since the difference in their fees is $3, Paulo will always pay $3 less than Justin if they are at the gym the same number of hours in a month.")

  "If the difference in the *y*-intercepts was 5 instead of 3, what would this mean in terms of a vertical shift, as well as in the context of the scenario?" (**Answer:** "If the difference in the *y*-intercepts was 5, this would mean that the function rule representing Paulo's total gym fees would have been vertically shifted 5 units from the function rule representing Justin's total gym fees. In the context of the scenario, the difference in their fees would be $5, so Justin would pay $5 more than Paulo for any number of monthly hours of gym use.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask volunteers to share the steps in their process for finding the slope and *y*-intercept of each line, as well as writing the equation for each function. Discuss the different strategies and processes that students used, and encourage students to justify their processes.
- Compare students' strategies and explanations for analyzing the given information and determining a solution plan for this task. Focus on the use of precise mathematical language, specifically regarding the terms *slope*, *y-intercept*, *function rule*, and *vertical shift*.

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Connecting to Key Concepts

Make explicit connections to key concepts:

- Vertical translations can be performed on linear and exponential graphs using $f(x) + k$, where k is the value of the vertical shift.

  In this task, students will compare two linear functions and recognize that one has been vertically shifted in comparison with the other one.

- A vertical shift moves the graph up or down k units.

  Students will see that the graph that represents Justin's gym fees has been shifted up 3 units from the graph that represents Paulo's gym fees.

- Given the graphs of two functions that are vertical translations of each other, the value of the vertical shift, k, can be found by finding the distance between the y-intercepts.

  In this task, students will subtract the values of the y-intercepts in the functions representing Justin's and Paulo's gym fees in order to find the value of the vertical shift, k, which is $8 - 5 = 3$.

Extending the Task

To extend the task, ask students to change the per-hour use fee for one of the men. For example, let Justin's per-hour use fee be \$1.10 instead of \$1.50, so that the function rule that represents his monthly fees is $f(x) = 1.1x + 8$. Ask students to graph this new function rule on the same graph as the function that represents Paulo's monthly fees, $f(x) = 1.5x + 5$. Then, ask students to determine the value of x at which the two graphs will intersect, and what this point of intersection means in the context of the problem.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "I was able to construct a viable argument and defend my solution by demonstrating the steps I took to write the function rules to represent each person's total fees. I showed that the difference in the fees could be found by subtracting one function's y-intercept from the other function's y-intercept.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I used mathematics to model this scenario by writing function rules to represent the graphical representation of the two functions.")

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- **For SMP 7, ASK:** "How did you look for and make use of structure when solving this problem?" (**Answer:** "I noticed that if you shifted the line representing Paulo's monthly fees up 3 units, it would be the same as the line representing Justin's monthly fees, so I looked to see if this was represented in the equations. I saw that it was—if you add 3 to the function representing Justin's cost, the result is the function representing Paulo's cost.")

Alternate Strategies or Solutions

- Instead of calculating the slope by examining the graph, students may choose to first create a table of values that they determine from examining the graph of Justin's and Paulo's total gym fees. Then, they can use the slope formula and two points from each table to calculate the slope of each line.
- Instead of examining the graph to identify the y-intercepts for each function, students may use the slope and a point from each line and solve for the y-intercept by using slope-intercept form, $y = mx + b$.

Technology

No technology is needed for this task.

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Problem-Based Task: New Tablet

Common Core State Standard

F–BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Task Overview

Focus

How can an arithmetic sequence be written to represent a real-world scenario? How can the *n*th term of the sequence be determined? In this task, students will write an arithmetic sequence to represent the scenario of saving money to buy a tablet computer, and they will use the sequence to determine the number of weeks it will take to save enough money for the tablet.

This activity will provide practice with:

- analyzing a verbal description of a scenario
- identifying the common difference of an arithmetic sequence
- writing an explicit formula for an arithmetic sequence
- simplifying an expression
- determining the value of a specific term of an arithmetic sequence

Introduction

This task should be used to explore or apply the skill of writing an explicit formula for an arithmetic sequence and using the formula to model a situation. Students should already be familiar with adding and subtracting signed numbers and identifying patterns.

Begin by reading the problem and clarifying the meaning of the terms *arithmetic sequence*, *common difference*, and *explicit formula*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will make sense of the task by analyzing the scenario and identifying that the goal is to determine the number of weeks it will take to save enough money to buy a tablet computer. Students may persevere by making connections between this problem and other problems they have solved, and by making a list of values for the first few weeks in order to look for a pattern.

- **SMP 6:** Attend to precision.

  Make sure students are using math terms and definitions clearly and accurately to explain their reasoning. Strive toward precise mathematical language, such as using the terms *common difference*, *arithmetic sequence*, and *explicit formula* correctly. Remind students to apply the order of operations and to combine like terms when simplifying the expression for the explicit formula.

- **SMP 8:** Look for and express regularity in repeated reasoning.

  Students will notice that the same calculations are performed repeatedly in order to achieve the desired results. They will recognize that there is a pattern of adding \$10 to the amount saved each week, which represents a common difference. Students will use this common difference to write an arithmetic sequence to represent the scenario.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- not identifying the sequence as an arithmetic sequence

  Remind students that the terms of an arithmetic sequence have a common difference; that is, a specific number is added to each consecutive term in the sequence.

- incorrectly identifying the common difference

  Have students list the total amount of money saved after 1 week, 2 weeks, and 3 weeks. Students should observe that the amount increases by \$10 from week to week; therefore, 10 is the common difference in the sequence.

- incorrectly evaluating the nth term of a sequence when using an explicit formula

  Remind students that in the explicit formula $a_n = a_1 + (n - 1)d$, the common difference d must be distributed to all of the terms inside the parentheses.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask them to predict how many weeks they think it will take to save enough money to buy the tablet, based on the given information. (**Answer:** Students' predictions will vary. Encourage students to write down their reasoning for their predictions.)

  Ask students to identify and list the known information from the verbal description of the scenario. Ask, "Which values are known, and what do each of these values represent in terms of a sequence?" (**Answer:** "One known value is the cost of the tablet, which is \$450, and this is represented by a_n in the sequence formula. Another known amount is the first term of the sequence, a_1, found by adding \$50 and \$10 to get \$60. The third known value is the common difference, d, which is \$10.")

  "How do you know that this scenario will be represented by an arithmetic sequence?" (**Answer:** "The scenario will be represented by an arithmetic sequence because the same amount, \$10, is added each week. Therefore, each consecutive term of the sequence will have the same difference, or common difference, which is the definition of an arithmetic sequence.")

- If students don't understand why the first term is 60 instead of 50, ask, "What does the number 60 represent in the context of the problem, and why is the first term of the sequence *not* 50?" (**Answer:** "The number 60 represents the amount of money that has been saved after 1 week. The initial amount of money was \$50. Then, after the first week, \$10 was saved. Therefore, the first term, a_1, is \$60. The amount of \$50 represents the amount that was present *before* the savings process began; therefore, it is not the first term of the sequence.")

  Ask, "Why is it important to determine the first few terms of the sequence before writing the formula for the sequence?" (**Answer:** "It is important to determine the first few terms of the sequence in order to calculate whether the terms have a common difference. If they have a common difference, then the sequence is an arithmetic sequence.")

- As students are writing the explicit form of the arithmetic sequence, ask:

  "What is a property that is used when simplifying the expression $60 + (n - 1)(10)$?" (**Answer:** "The Distributive Property is used to simplify the expression: 10 is distributed to n and -1 to yield a result of $10n - 10$.")

  "What information can be found by writing a sequence in an explicit form, and how could this be used to complete the task?" (**Answer:** "An explicit form allows you to determine the value for any specific term number in the sequence. In this task, for example, if we wanted to find out how much money will have been saved after 20 weeks, we could substitute 20 for n in the formula $a_n = 10n + 50$: $a_{20} = 10(20) + 50 = 250$. This means that after 20 weeks, a total of \$250 will have been saved.")

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"What is the advantage of writing a formula for an arithmetic sequence?" (**Answer:** "A formula allows you to put in any term number of a sequence and determine the value. Otherwise, it would be necessary to list all of the terms of the sequence in order, which could be lengthy and time-consuming.")

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask volunteers to discuss their initial predictions about how many weeks it would take to save enough money for the tablet. Encourage students to discuss their reasoning, as well as how correct or incorrect they were based on the actual results of the task.
- Compare students' strategies and ways of justifying responses. Focus on the use of precise mathematical language and clarity, specifically when referring to the terms *common difference*, *arithmetic sequence*, and *explicit formula*.

Connecting to Key Concepts

Make explicit connections to key concepts:

- An arithmetic sequence is a list of terms separated by a common difference, the number added to each consecutive term in the sequence.

  In this task, students will find that the given scenario forms an arithmetic sequence in which the common difference is 10.

  An arithmetic sequence is a linear function whose domain is the set of positive consecutive integers and whose range is made up of the terms in the sequence. Students will see that the domain of the sequence is the number of weeks in which money is saved, where the number of weeks is a positive integer. They will also see that the range reflects the total amount of money saved each week.

- The explicit rule for the nth term of an arithmetic sequence is $a_n = a_1 + (n - 1)d$, where a_1 is the first term in the sequence, n is the term number and is a positive integer, d is the common difference, and a_n is the nth term in the sequence.

  In this task, students will write an explicit formula for the arithmetic sequence that represents the scenario, where $a_1 = 60$ and $d = 10$.

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Extending the Task

To extend the task, provide students with the following scenario: "You start with $100 in your savings account, and you plan to save $20 a week for a trip that will cost $3,000." Ask students to write an explicit formula to represent this scenario, and ask them to determine how much money will have been saved after 52 weeks. Then ask how long it will take to save enough money for the trip. (**Answer:** $a_n = 20n + 100$; after 52 weeks, you will have saved $1,140. It will take 145 weeks, or nearly 3 years, to save enough for the trip.)

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I made sense of the problem by recognizing that the given scenario can be represented by an arithmetic sequence. I persevered by comparing this scenario to other linear functions I've studied and trying methods that I used in those instances.")
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** "I attended to precision by using correct math terminology in my explanations and explaining my reasoning in a mathematically correct and precise way.")
- **For SMP 8, ASK:** "How did you look for and express regularity in repeated reasoning?" (**Answer:** "I recognized that there is a pattern of adding 10 in the sequence. I used this common difference to create an explicit formula to represent the scenario.")

Alternate Strategies or Solutions

- Students may choose to create a table of values in order to generalize the pattern of the sequence, with x representing the number of weeks and y representing the total amount of money saved. Discuss with students how creating a table may become time-consuming; they should quickly realize that the table of values will need to contain a large number of values in order to reach the goal of $450. Encourage students to write a formula for the sequence instead.
- Students may choose to create a graph representing the scenario, either by hand or with graphing technology, in order to determine the number of weeks it will take to save a total of $450. Remind students that if they choose to graph points by hand, it is necessary for their points to be plotted precisely and accurately in order to find the exact number of weeks. Again, encourage students to write a formula for the sequence as a way to ensure accuracy when determining their answer.

Technology

No technology is required for this task, though students may wish to use graphing technology.

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Guide

Problem-Based Task: Glass Recycling

Common Core State Standard

F–BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Task Overview

Focus

How can an explicit formula for a geometric sequence be written to represent a real-world scenario? How can the formula be applied to determine values for the *n*th term of the sequence? In this task, students will write an explicit formula for a geometric sequence to represent the number of pounds of glass that a recycling club can expect to collect on a given day, and they will use this formula to find specific values.

This activity will provide practice with:

- analyzing a verbal description of a scenario
- writing equations from a verbal description
- applying properties of equality to isolate a variable
- creating an explicit formula for a geometric sequence
- determining the value of a given term of a geometric sequence

Introduction

This task should be used to explore or apply the skill of writing a geometric sequence with an explicit formula and using the sequence to model a situation. Students should already be familiar with multiplying fractions and applying the order of operations to simplify an expression.

Begin by reading the problem and reviewing the meaning of the terms *common ratio*, *geometric sequence*, and *explicit formula*.

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Guide

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students can make sense of the problem by creating a table of values and thinking about how the values in the table might be generated with a geometric sequence. They could begin by guessing and checking several values to determine the approximate value of the common ratio, and then determine a way to find the exact value algebraically.

- **SMP 6:** Attend to precision.

  Make sure students use math terms and definitions clearly and accurately to explain their reasoning. Strive toward precise mathematical language, such as using the terms *common ratio*, *geometric sequence*, and *explicit formula* correctly. Also, remind students to apply the order of operations and properties of equality when solving equations.

- **SMP 7:** Look for and make use of structure.

  Students will be asked to determine a pattern for the terms of a geometric sequence, and they will find this pattern by calculating the common ratio between consecutive terms. They will use the structure of the explicit formula in order to solve for missing components of the formula.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly evaluating the nth term in a geometric sequence

  Have students write the order of operations at the top of their papers to use as a reference when working through the task. Remind them that the common ratio in the explicit formula of a geometric sequence is raised to the power of the exponent first, and then this result is multiplied by the value of the first term of the sequence.

- incorrectly using the negative value of the square root when calculating the common ratio

  Remind students that in the context of this scenario, the number of pounds of glass increases each day, so the value of the common ratio must be positive. Therefore, the negative value of the square root does not need to be considered.

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Guide

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask:

  "In the given information, it states that the club estimates that the number of pounds of glass collected will form a geometric sequence. What must be determined in order to identify a sequence as geometric?" (**Answer:** "In order to identify if a sequence is geometric, we must determine whether the consecutive terms of the sequence have a common ratio.")

  "Which values are we given in the first two sentences of the problem, and what do each of these values represent in terms of a geometric sequence?" (**Answer:** "The first sentence gives the amount of glass collected on the second day; in other words, the value of the second term in the sequence. So, $a_2 = 39.6$. The second sentence gives the amount of glass collected on the fourth day, which is the value of the fourth term of the sequence. So, $a_4 = 57.024$.")

  "In terms of writing an explicit formula, which values are unknown?" (**Answer:** "We don't know the value of the first term of the sequence, a_1, or the value of the common ratio, r.")

- If students don't understand how the two equations written for the second and fourth days can be used to find the common ratio, encourage them to examine each equation carefully. Ask, "Which part is exactly the same in both equations?" (**Answer:** "Both equations have an unknown quantity of a_1.")

  Remind students that equations can be manipulated so that a common variable is isolated. Ask, "What property of equality can be applied to both equations in order to isolate a_1?" (**Answer:** "The Division Property of Equality can be used to isolate a_1 in both equations.")

- As students begin to solve for the common ratio, r, ask, "When the two expressions for a_1 are set equal to each other, the result is $\frac{57.024}{r^3} = \frac{39.6}{r}$. What steps are necessary to solve for r?" (**Answer:** "First, multiply both sides of the equation by r^3 to eliminate the fractions. Next, eliminate the variable from $57.024r$ by dividing both sides by r. Then, isolate r^2 by dividing both sides of the equation by 39.6. Finally, solve for r by taking the positive square root of 1.44, which is 1.2.")

  When students are performing the final calculation for r, ask, "Although $r = \pm 1.2$, why is it only necessary to consider the positive value of the square root?" (**Answer:** "In the context of the problem, the number of pounds of glass is increasing. Therefore, the common ratio must be a positive value. If the common ratio was a negative number, the sequence would be decreasing.")

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- Ask, "Once the common ratio, *r*, has been calculated, how can the first term, a_1, be determined?" (**Answer:** "The value of *r* can be substituted in either of the two original equations in order to find a_1.")
- Ask, "Once the explicit formula for the geometric sequence has been determined, how can the formula be used to estimate the number of pounds of glass the club can expect to collect on any given day of the collection drive, such as day 10? Explain how to determine this result." (**Answer:** "The explicit formula is $a_n = 33(1.2)^{n-1}$. To find the number of pounds of glass collected on the tenth day, substitute 10 for *n* in the formula: $a_{10} = 33(1.2)^{10-1} \approx 170.3$. Therefore, the club can expect to collect about 170 pounds of glass on the tenth day of the collection drive.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask volunteers to discuss their processes for setting up the equations, as well as their solution plans for finding the missing components of the explicit formula. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Focus on the use of precise mathematical language and clarity, specifically when referring to the terms *common ratio*, *geometric sequence*, and *explicit formula*.

Connecting to Key Concepts

Make explicit connections to key concepts:

- A geometric sequence is a list of terms separated by a common ratio, the number that each consecutive term is multiplied by in a geometric sequence.

  Students will determine that the terms in a sequence are separated by a common ratio, and they will write and manipulate equations in order to find the value of the common ratio.

- A geometric sequence is an exponential function whose domain is the set of positive consecutive integers and whose range is made up of the terms in the sequence.

  Students will see that the domain of the sequence is the number of days when the Recycling Club collects glass, where the number of days is a positive integer. They will also see that the range reflects the amount of glass collected each day for at least 6 days.

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- The explicit formula for the *n*th term of a geometric sequence is $a_n = a_1 \bullet r^{n-1}$, where a_1 is the first term in the sequence, n is the term number and is a positive integer, r is the common ratio, and a_n is the nth term in the sequence.

  In this task, students will write an explicit formula for the geometric sequence that represents the scenario, and they will use the values of $a_1 = 33$ and $r = 1.2$ in the formula.

Extending the Task

To extend the task, provide students with the first 3 terms of a geometric sequence reflecting another school's glass collection efforts. For example, state that a rival school's Recycling Club collected 30, 39, and 50.7 pounds on the first 3 days of their collection drive. Ask students to find the common ratio and then write an explicit formula representing the geometric sequence for this scenario. Then, ask students to calculate the number of pounds of glass this rival school's club will have collected on the sixth day, and compare this result to the amount collected by the original school's Recycling Club.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I persevered by trying different ideas to figure out a way to find the value of *r*. I ended up using the given information in the formula for a geometric sequence to write equations that I could manipulate in order to find the missing values.")
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** "I attended to precision by using correct math terminology in my explanations and explaining my reasoning in a mathematically correct and precise way. I also checked my calculations by using the formula I found to make sure it produced the given values.")
- **For SMP 7, ASK:** "How did you look for and make use of structure when solving this problem?" (**Answer:** "I used the structure of the explicit formula for a geometric sequence to find missing components of the formula. I also recognized the pattern of the common ratio between consecutive terms.")

Alternate Strategies or Solutions

- When solving for the value of a_1 in the explicit formula, students may use the other equation, $a_1 = \frac{57.024}{(1.2)^3}$, in order to isolate a_1. Remind students that they will achieve the same result for a_1 whether they use this equation or $a_1 = \frac{39.6}{1.2}$. Encourage students to use both equations to verify their result for the value of a_1.

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- Some students may determine the value of r by using guess-and-check, which is a valid and useful strategy for this problem. They may recognize that to get from 39.6 to 57.024, they have to multiply by the same number twice. Using a calculator, they can try various values for r, adjusting their guesses each time until they determine that $r = 1.2$.
- Once the first term and the common ratio of the sequence have been determined, students may create a table of values in order to find the values for the other terms in the sequence; i.e., for the third day and the fifth day.

Technology

Students can use scientific calculators for this task.

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Guide

Problem-Based Task: Cell Phone Plans

Common Core State Standard

F–LE.5 Interpret the parameters in a linear or exponential function in terms of a context.★

Task Overview

Focus

How can two cell phone plans be compared algebraically and graphically? In this task, students are asked to decide which of two cell phone plans would best fit their needs. They may do so by modeling the plans with functions, graphing the functions, and interpreting the graphs.

This activity will provide practice with:

- analyzing a situation to determine what type of function would be the best model
- writing and graphing linear functions
- interpreting graphs
- comparing linear functions at various *x*-values

Introduction

This task should be used to explore or apply the concept of linear functions. Graphing may be done using graphing technology.

Begin by reading the problem and clarifying the situation presented in the task. Review the meaning of the term *parameter*.

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Check to make sure that students understand how the pricing on the cell phone plans works. Ask them to determine how much someone would pay under each plan for a given number of minutes.

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- **SMP 4:** Model with mathematics.

  Look for students to recognize that the relationship between the number of minutes used and the monthly fee is linear. Encourage them to identify the slope and *y*-intercept of the lines modeling the plans based on the descriptions. Then, have them write equations to model the situation. Once they've written what they think might be the correct equations, have them test some values to see if they make sense.

- **SMP 5:** Use appropriate tools strategically.

  Graphing calculators can be used to enhance understanding of this task. Once students have identified that the plans can be modeled with linear equations, encourage them to graph the equations using a graphing calculator and then to use the graph to analyze the two plans.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- when reading a word problem, not being able to identify the parameters in the context of the problem

  Remind students that the parameters are the values given in the problem that affect the outcome of the situation. In this problem, the parameters for each phone plan are the monthly fee and the cost per minute.

- mistaking the slope for the *y*-intercept or the *y*-intercept for the slope

  Ask students which parameter they will need to multiply by *x*, the number of minutes used. (**Answer:** the slope) Ask which parameter is not multiplied by the number of minutes used. (**Answer:** the *y*-intercept)

- incorrectly using an exponential function instead of a linear function to model the situation

  Ask students to make a table of values and determine the cost of each plan for 50, 60, and 70 minutes a month. Point out that the rate of change is constant for both plans, which indicates linear relationships.

- forgetting to set the viewing window of the graph on a graphing calculator

  The default setting for most calculator displays is from –10 to 10 on both axes. Make sure students look at the possible range of values for *x*, the number of minutes, and *y*, the total cost, when setting up their viewing windows.

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Guide

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students how they should define their variables. (**Answer:** x represents minutes used, y represents monthly cost)
- Ask students, "How did you determine what types of functions will model the cell phone plans?" (**Answer:** "I calculated the monthly fees at regular intervals of time, and I saw that there is a constant increase in the fee for equal time intervals, so the functions must be linear.")
- Ask, "What parameters did you need to identify to write the functions?" (**Answer:** "I had to identify the slope and the y-intercept.")
- Ask, "How can you be sure that your functions make sense within the context of the problem?" (**Answer:** "I can substitute values for various numbers of minutes used and see if the totals seem reasonable.")
- Ask students how they determined the scale for the graph. (**Answer:** "I made a table of values to see what values need to be included.")
- If students attempt to graph the functions on a graphing calculator but do not see anything, ask, "What is the window for your graph?" (**Answer:** The window default setting on most calculators is from –10 to 10 on each axis with a scale of 1. Encourage students to make table of values to see where the points on the graph lie and to adjust the window so the points will be visible.)
- Ask, "What does it mean if the graph of one line is higher than the other for a given x-value?" (**Answer:** "Whichever graph has a higher y-value for a given x-value represents the plan that costs more for that number of minutes.")
- Ask, "How does looking at the graph help you to determine which plan will be the best choice?" (**Answer:** "The function with lower y-values when $x < 100$ will be the more affordable plan.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

Initiate a discussion around the following questions: "Which cell phone plan would be the best choice? Why?" Ask students to share their reasoning and methods. Compare the strategies used. Ask students to compare the methods and decide which they prefer and why.

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Guide

Connecting to Key Concepts

Make explicit connections to key concepts:

- A linear function may be written in slope-intercept form, $f(x) = mx + b$, where m is the slope of the line and b is the y-intercept.

  The two functions that model the cell phone plans in this task are modeled by linear functions.

- A parameter of a function is a constant that determines the specific graph of the function but not the type of function. It is a value that is built into the function.

  In the function for AT&Me, $f(x) = 0.10x + 25$, the parameters are 0.10 ($0.10 per minute) and 25 (a $25 monthly fee). Note that these values determine the position of the function's graph, but don't affect the nature of the function, which remains linear. The same is true for the parameters of the function for Tracmyphone.

Extending the Task

- Have students observe the behavior of the functions that they wrote over a larger domain; for example, increase the domain to 250 or 300 minutes. Ask them what type of cell phone user might prefer the Tracmyphone plan with this domain. (**Answer:** "Users who use more than 200 minutes would prefer Tracmyphone; the plans are equal in cost when $x = 200$.")

- Encourage students to investigate plans offered by real cell phone carriers. The following lesson plan and worksheet provide information on the monthly cost of buying an iPhone 6 from several different carriers, using 2014 data. Students select a plan and use the given information to write a rule that gives the cost for the phone after any number of months. *Note*: Solutions for the worksheet are only available to YummyMath members, but membership is free.

  YummyMath. "Cell Phone Plans."

  http://www.walch.com/rr/07003

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I made sense of the problem by figuring out how much I would pay with each plan for different numbers of minutes so that I made sure I understood how the pricing works. I didn't let myself get overwhelmed by all the information in the task. I tried out different ideas until I found a model that fit the scenario.")

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- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I found some data points and analyzed their relationship to each other. When I saw that three points lay on the same line, I knew linear functions would model the plans.")
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** "I wrote the equation in a form I could enter into my calculator. I referred to a table of values I made to determine which points were on the graph. I changed the window settings to display the data.")

Alternate Strategies or Solutions

- Students could make a table of values and calculate the monthly fees for various quantities of minutes used.
- Students could use graph paper to construct their own graphs instead of using a graphing calculator.

Technology

Students can use graphing technology.

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Common Core State Standards

Mathematics I

Integrated Pathway



Teacher Resource

Unit 3

WALCH

INTEGRATED MATH

This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

Developers and reviewers include:

Joyce Hale	Valerie Ackley	Doug Kühlmann
Shelly Northrop Sommer	Laura McPartland	Mike May, S.J.
Ruth Estabrook	Cameron Larkins	James Quinlan
Vanessa Sylvester	Jennifer Blair	Peter Tierney-Fife
Zachary Lien	Nancy Pierce	

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Standards Correlations

Each lesson in this Integrated Pathway: Mathematics I program was written specifically to address the Common Core State Standards. Each lesson lists the standards covered in all the sub-lessons, and each sub-lesson lists the standards addressed in that particular section. In this section, you'll find a comprehensive list mapping the sub-lessons to the CCSS.

Guide to Common Core State Standards Annotation

As you use this program, you will come across a symbol included with the Common Core standards for some of the lessons and activities. These symbols are explained below.

Symbol: ⋆

Denotes: Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (⋆).

From http://www.walch.com/CCSS/00003

Symbol: (+)

Denotes: College and Career Readiness Standards

Advanced mathematics standards that are required in higher-level courses such as advanced statistics may also be included in lower-level courses. These additional standards are denoted by (+). According to the Common Core State Standards Initiative, "the evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6–8."

From http://www.walch.com/CCSS/00004

Connections to Future Courses

This section provides a map between topics introduced in each unit of this course and subsequent courses where each topic is revisited and built upon.

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CCSS INTEGRATED PATHWAY: MATHEMATICS I STANDARDS CORRELATIONS

Unit 3: Reasoning with Equations				
Lesson	**Sub-lesson number**	**Title**	**Standard(s)**	**Pages**
Lesson 1	**Solving Equations and Inequalities**			
	3.1.1	Properties of Equality	A–REI.1	U3-3
	3.1.2	Solving Linear Equations	A–REI.3	U3-19
	3.1.3	Solving Linear Inequalities	A–REI.3	U3-34
	3.1.4	Solving Exponential Equations	A–REI.1	U3-46
Lesson 2	**Solving Systems of Equations**			
	3.2.1	Solving Systems of Linear Equations by Substitution and Elimination	A–REI.5 A–REI.6	U3-68
	3.2.2	Solving Systems of Linear Equations by Graphing	A–REI.6	U3-87

TOPICS FROM FUTURE COURSES: CCSS INTEGRATED PATHWAY MATHEMATICS I

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 3	Reasoning with equations (linear equations)	Math II Unit 3	extended to quadratic functions
		Math III Unit 2B	extended to rational and radical functions
	Systems of equations (linear-linear)	Math II Unit 3	includes linear-quadratic systems
		Math III Unit 2A	extended to linear-polynomial, polynomial-polynomial, and absolute value-polynomial systems
		Math III Unit 2B	extended to linear-rational and linear-radical systems
		Math IV	includes systems with vectors and matrices

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Conceptual Activities

Use these interactive open education and/or Desmos resources to build conceptual understanding of mathematical ideas. (*Note*: Activity links will be monitored and repaired or replaced as necessary.)

- Desmos. "Card Sort: Linear Systems."

  http://www.walch.com/ca/01000

  In this activity, students practice what they've learned about solving systems of linear equations. The activity begins with a review of the graphical meaning of a solution to a system. Later, students consider which algebraic method is most efficient for solving a given system. Finally, students practice solving equations using substitution and elimination. Prior to beginning this activity, students should have experience solving systems of linear equations graphically and algebraically.

- Desmos. "The Intersection."

  http://www.walch.com/ca/01011

  Predict the point of intersection for a system of two linear equations: first without a grid, then with one. With the grid in play, participants are able to use the slope of the lines (formally or informally) to improve the accuracy of their predictions.

- Desmos. "Solutions to Systems of Linear Equations."

  http://www.walch.com/ca/01001

  This activity will help students understand what it means for a point to be a solution to a system of equations—both graphically and algebraically.

- Desmos. "Systems of Two Linear Equations."

  http://www.walch.com/ca/01002

  This resource gives a progression of written explanations, equations, and graphs to explain what the algebraic or graphical solution to a system of equations represents.

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Pre-Assessment

Circle the letter of the best answer.

1. Identify the property of equality that justifies the missing step in solving the equation.

Equation	Steps
$23 = 2x - 9$	Original equation
$32 = 2x$	
$16 = x$	Division property of equality
$x = 16$	Symmetric property of equality

a. subtraction property of equality
b. addition property of equality
c. division property of equality
d. multiplication property of equality

2. What is the solution to the equation $3(4x + 5) = 3x - 12$?

a. $x = -3$
b. $x = 3$
c. $x = -\frac{17}{9}$
d. $x = -\frac{1}{3}$

3. What is the solution to the inequality $\frac{4x}{5} + 3 > x + 5$?

a. $x > 10$
b. $x < 10$
c. $x > -10$
d. $x < -10$

4. What is the solution to the inequality $4(3x + 7) \leq 2(4x + 20)$?

a. $x \leq \frac{13}{4}$
b. $x \geq \frac{13}{4}$
c. $x \leq 3$
d. $x \geq 3$

5. What is the solution to the equation $8^x = 512$?

a. $x = \frac{1}{64}$
b. $x = \frac{1}{3}$
c. $x = 64$
d. $x = 3$

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Lesson 1: Solving Equations and Inequalities

Instruction

Common Core State Standards

SMP	
1 ✓	2 ✓
3 ✓	4 ✓
5	6
7 ✓	8 ✓

A–REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A–REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Essential Questions

1. How are the properties of equality used to solve equations?
2. How is solving a literal equation different from solving a linear equation?
3. How is finding the solution to an inequality similar to finding the solution to an equation?
4. What effect does multiplying or dividing an inequality by a negative number have?
5. How is solving an exponential equation different from solving a linear equation?

WORDS TO KNOW

laws of exponents	rules that must be followed when working with exponents
properties of equality	rules that allow you to balance, manipulate, and solve equations
properties of inequality	rules that allow you to balance, manipulate, and solve inequalities

Recommended Resources

- Illuminations. "Pan Balance—Expressions."

  http://walch.com/rr/CAU2L1PanBalance

  Create numeric or algebraic expressions and observe how operations affect equality.

- IXL Learning. "Solve Linear Equations: Mixed Review."

  http://walch.com/rr/CAU2L1EquationsReview

  This site includes various types of linear equations to solve. Explanations are provided for incorrect answers.

- Math Interactives. "Exploring Laws of Exponents—Use It."

  http://walch.com/rr/CAU2L1ExponentDig

  This interactive resource allows users to explore the laws of exponents while completing a virtual paleontological dig.

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Lesson 3.1.1: Properties of Equality

Warm-Up 3.1.1

Sydney subscribes to an e-book service. Her subscription costs a flat fee of $30 for up to 10 e-book downloads each month. For each download over 10, there is an additional charge per download.

1. During the month of September, Sydney downloaded 22 e-books and was charged $75. How much does each additional download cost?

2. In October, Sydney was incorrectly charged $67.50 for 18 e-books. How much should she have been charged?

3. If Sydney received a bill for $101.25, how many e-books did she download?

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Lesson 3.1.1: Properties of Equality

Common Core State Standard

A–REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Warm-Up 3.1.1 Debrief

1. During the month of September, Sydney downloaded 22 e-books and was charged $75. How much does each additional download cost?

   Set up an equation to find the charge for each downloaded e-book over 10.

   $30 + (22 - 10)x = 75$

   Solve the equation.

$30 + (22 - 10)x = 75$	Equation
$30 + 12x = 75$	Subtract 10 from 22.
$12x = 45$	Subtract 30 from both sides.
$x = 3.75$	Divide both sides by 12.

   Each additional download costs $3.75.

2. In October, Sydney was incorrectly charged $67.50 for 18 e-books. How much should she have been charged?

   Set up an equation to find the amount that Sydney should have been charged.

   The cost for 10 e-books is $30. Each e-book over 10 costs an additional $3.75. The total correct cost is $30 + 3.75(x - 10)$.

Total cost $= 30 + 3.75(x - 10)$	Equation
Total cost $= 30 + 3.75[(18) - 10]$	Substitute 18 for x.
Total cost $= 30 + 3.75(8)$	Subtract 10 from 18.
Total cost $= 30 + 30$	Multiply.
Total cost $= 60$	Simplify.

   Sydney should have been charged $60 for 18 e-books.

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3. If Sydney received a bill for $101.25, how many e-books did she download?

   Use the equation for total cost found in problem 2 to determine the number of e-books Sydney downloaded.

Total cost = $30 + 3.75(x - 10)$	Equation
$(101.25) = 30 + 3.75(x - 10)$	Substitute $101.25 for the total cost.
$101.25 = 30 + 3.75x - 37.5$	Distribute 3.75 over $(x - 10)$.
$101.25 = 3.75x - 7.5$	Combine like terms.
$108.75 = 3.75x$	Add 7.5 to both sides of the equation.
$29 = x$	Divide both sides of the equation by 3.75.

   Sydney downloaded 29 e-books.

Connection to the Lesson

- Students will continue to use their knowledge of solving equations, but will be asked to justify the steps used in the process.

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Prerequisite Skills

This lesson requires the use of the following skills:

- applying the order of operations (5.OA.1)
- simplifying expressions (5.OA.1)

Introduction

Equations are mathematical sentences that state two expressions are equal. In order to solve equations in algebra, you must perform operations that maintain equality of the two sides of the equation using the properties of equality. The **properties of equality** are rules that allow you to balance, manipulate, and solve equations.

Key Concepts

- In mathematics, it is important to follow the rules when solving equations, and to be able to justify, or prove, that the steps we are following to solve problems are correct and allowed.
- The following table summarizes some of these rules.

Properties of Equality

Property	In symbols	In words
Reflexive property of equality	$a = a$	A number is equal to itself.
Symmetric property of equality	If $a = b$, then $b = a$.	If numbers are equal, they will still be equal if the order is changed.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.	If numbers are equal to the same number, then they are equal to each other.
Addition property of equality	If $a = b$, then $a + c = b + c$.	Adding the same number to both sides of an equation does not change the equality of the equation.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.	Subtracting the same number from both sides of an equation does not change the equality of the equation.

(*continued*)

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Multiplication property of equality	If $a = b$ and $c \neq 0$, then $a \cdot c = b \cdot c$.	Multiplying both sides of the equation by the same number, other than 0, does not change the equality of the equation.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.	Dividing both sides of the equation by the same number, other than 0, does not change the equality of the equation.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.	If two numbers are equal, then substituting one in for another does not change the equality of the equation.

- You may remember from other classes the properties of operations that explain the effect that the operations of addition, subtraction, multiplication, and division have on equations. The following table describes some of those properties.

Properties of Operations

Property	General rule	Specific example
Commutative property of addition	$a + b = b + a$	$3 + 8 = 8 + 3$
Associative property of addition	$(a + b) + c = a + (b + c)$	$(3 + 8) + 2 = 3 + (8 + 2)$
Commutative property of multiplication	$a \cdot b = b \cdot a$	$3 \cdot 8 = 8 \cdot 3$
Associative property of multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(3 \cdot 8) \cdot 2 = 3 \cdot (8 \cdot 2)$
Distributive property of multiplication over addition	$a \cdot (b + c) = a \cdot b + a \cdot c$	$3 \cdot (8 + 2) = 3 \cdot 8 + 3 \cdot 2$

- When we solve an equation, we are rewriting it into a simpler, equivalent equation that helps us find the unknown value.
- When solving an equation that contains parentheses, apply the properties of operations and perform the operation that's in the parentheses first.
- The properties of equality, as well as the properties of operations, not only justify our reasoning, but also help us to understand our own thinking.

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- When identifying which property is being used, it helps to review each step and make note of what operation was performed, and whether it was done to one side of the equation or both.
- When operations are performed on one side of the equation, such as when an expression is simplified, this signifies the use of the properties of operations.
- When an operation is performed on both sides of the equation, such as subtracting the same value from both sides, this signifies the use of the properties of equality.
- Once you have noted which operations were used, match them to the properties listed in the tables.

Common Errors/Misconceptions

- incorrectly identifying operations
- incorrectly identifying properties
- performing an operation that is not justifiable or does not follow the properties of equality and/or the properties of operations

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Guided Practice 3.1.1

Example 1

Which property of equality is missing in the steps to solve the equation $-7x + 22 = 50$?

Equation	Steps
$-7x + 22 = 50$	Original equation
$-7x = 28$	
$x = -4$	Division property of equality

1. Observe the differences between the original equation and the next equation in the sequence. What has changed?

   Notice that 22 has been subtracted from both expressions, $-7x + 22$ and 50.

2. Refer to the table of Properties of Equality.

   The subtraction property of equality tells us that when we subtract a number from both sides of the equation, the expressions remain equal.

   The missing step is "Subtraction property of equality."

Example 2

Which property of equality is missing in the steps to solve the equation $-3 - \frac{x}{6} = 4$?

Equation	Steps
$-3-\frac{x}{6}=4$	Original equation
$-\frac{x}{6}=7$	Addition property of equality
$-x = 42$	
$x = -42$	Division property of equality

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1. Observe the differences between the original equation and the next equation in the sequence. What has changed?

   Notice that 3 has been added to both expressions, $-3-\frac{x}{6}$ and 4.
   The result of this step is $-\frac{x}{6}=7$.

   In order to move to the next step, the division by 6 has been undone.

   The inverse operation of the division by 6 is the multiplication by 6.
   The result of multiplying $-\frac{x}{6}$ by 6 is $-x$ and the result of multiplying 7 by 6 is 42. This matches the next step in the sequence.

2. Refer to the table of Properties of Equality.

   The multiplication property of equality tells us that when we multiply both sides of the equation by a number, the expressions remain equal.

   The missing step is "Multiplication property of equality."

Example 3

Which property of equality is missing in the steps to solve the equation $76 = 5x - 15 + 2x$?

Equation	Steps
$76 = 5x - 15 + 2x$	Original equation
$76 = 5x + 2x - 15$	Commutative property of addition
$76 = 7x - 15$	Distributive property to combine like terms
$91 = 7x$	Addition property of equality
$13 = x$	Division property of equality
$x = 13$	

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1. Observe the differences between the original equation and the next equation in the sequence. What has changed?

   Notice that the expression $5x - 15 + 2x$ was rearranged using the commutative property of addition. The new expression, $5x + 2x - 15$, although in a different order, has the same meaning.

   The distributive property allows us to combine like terms by thinking of $5x + 2x$ as $x(5 + 2)$. Simplifying this expression to $7x - 15$ has the same meaning as the prior step.

   The addition property of equality allows us to add 15 to both sides of the equation, bringing us one step closer to finding out the value of x.

   To isolate x in the equation $91 = 7x$, we use the division property of equality to divide both sides of the equation by the coefficient 7.

2. Refer to the table of Properties of Equality.

   Notice that the equation now reads $13 = x$. Compare this to the final line, $x = 13$. The symmetric property of equality allows us to write $x = 13$. This more standard way of writing the solution means the same as the previous step.

   The missing step is "Symmetric property of equality."

Example 4

What equation is missing in the steps to solve the equation $5x + 3(x + 4) = 28$?

Equation	Steps
$5x + 3(x + 4) = 28$	Original equation
$5x + 3x + 12 = 28$	Distributive property
$8x + 12 = 28$	Distributive property to combine like terms
	Subtraction property of equality
$x = 2$	Division property of equality

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1. Look at the differences between the original equation and the next equation in the sequence. What has changed?

   Notice that the expression $5x + 3(x + 4)$ was rewritten using the distributive property. The new expression, $5x + 3x + 12$, looks different but has the same meaning.

   The distributive property also allows us to combine like terms by thinking of $5x + 3x$ as $x(5 + 3)$. Simplifying this expression to $8x + 12$ has the same meaning as the prior step.

2. We are told that the subtraction property of equality justifies the next step. This property states that we can subtract the same number from both sides of the equation and not change the equality of the equation. In order to solve for x, we have learned to isolate the variable. We do this by subtracting the constant term from both sides. The constant term is 12, so subtract 12 from both sides.

$$\begin{array}{r} 8x+12=28 \\ \underline{-12\ \ -12} \\ 8x=16 \end{array}$$

   Now we have an equation, $8x = 16$, which may be our missing equation. Let's look at what happens to get to the final statement in the table to see if this equation makes sense.

3. The table lists the division property of equality as the property that leads to $x = 2$. Look at the equation we found: $8x = 16$. In this equation, the coefficient of x is 8. If we divide both sides of the equation by 8, we get the final statement, $x = 2$.

$$\begin{array}{r} 8x=16 \\ \underline{\div 8\ \ \div 8} \\ x=2 \end{array}$$

   The division property of equality justifies the division of the equation $8x = 16$ by the coefficient, 8.

   The missing equation is $8x = 16$.

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Problem-Based Task 3.1.1: Magic Number

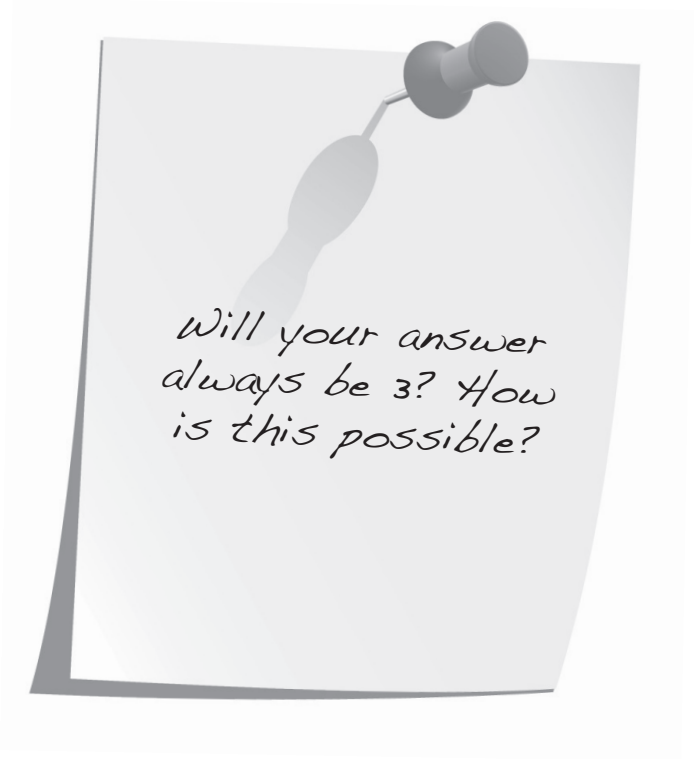
SMP	
1 ✓	2
3 ✓	4
5	6
7 ✓	8

Try this magic number game.

> Think of a number. Then double it. Now add 6. Take half of that number. Finally, subtract the number you started with.

Your answer is 3.

Will your answer always be 3? How is this possible? Use the properties of equality to justify your reasoning.



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Problem-Based Task 3.1.1: Magic Number

Coaching

a. What number did you think of?

b. What expression represents doubling your number?

c. How can you show adding 6 to the expression from part b?

d. How can you show taking half of the expression from part c?

e. How can you show subtracting the number you originally thought of from the expression in part d?

f. How can you show that the result of the operations in parts b through e will always result in 3?

g. How can you modify your equation in part f to work with any number?

h. How can your equation from part g be simplified?

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Instruction

Problem-Based Task 3.1.1: Magic Number

Coaching Sample Responses

a. What number did you think of?

Responses will vary, but could include any number, such as 53.

b. What expression represents doubling your number?

Using the number from part a, $2 \cdot 53$.

c. How can you show adding 6 to the expression from part b?

Using the expression from part b, $2 \cdot 53 + 6$.

d. How can you show taking half of the expression from part c?

Using the expression from part c, $\frac{2 \cdot 53 + 6}{2}$.

e. How can you show subtracting the number you originally thought of from the expression in part d?

Using the expression from part d, $\frac{2 \cdot 53 + 6}{2} - 53$.

f. How can you show that the result of the operations in parts b through e will always result in 3?

Using the expression from part d, $\frac{2 \cdot 53 + 6}{2} - 53 = 3$.

g. How can you modify your equation in part f to work with any number?

Replace the value chosen in part a with the variable x: $\frac{2x + 6}{2} - x = 3$.

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h. How can your equation from part g be simplified?

Begin by rewriting the term $\frac{2x+6}{2}$ as a sum of two fractions.

$$\frac{2x}{2}+\frac{6}{2}-x=3$$

Simplify each fraction.

$$x + 3 - x = 3$$

Use the commutative property of addition to rewrite the expression on the left of the equal sign.

$$x - x + 3 = 3$$

Combine like terms.

$$3 = 3$$

The operations performed on any number will always result in the number 3.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 3.1.1: Properties of Equality

Identify the property of equality that justifies each missing step or equation in each of the following tables.

1.

Equation	Steps
$x - 1.2 = 1.9$	Original equation
$x = 3.1$	

2.

Equation	Steps
$5x = 37$	Original equation
$x = 7.4$	

3.

Equation	Steps
$2x + 3 = 15$	Original equation
$2x = 12$	Subtraction property of equality
$x = 6$	

4.

Equation	Steps
$19 = 2x - 7$	Original equation
$26 = 2x$	
$13 = x$	Division property of equality
$x = 13$	Symmetric property of equality

5.

Equation	Steps
$x + (x - 0.6) = 2$	Original equation
$2x - 0.6 = 2$	Associative property of addition
	Addition property of equality
$x = 1.3$	

continued

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6.

Equation	Steps
$x + (4x + 32) = 12$	Original equation
$5x + 32 = 12$	Associative property of addition
$5x = -20$	
	Division property of equality

7.

Equation	Steps
$4(x - 6) = 40$	Original equation
$x - 6 = 10$	
$x = 16$	

8.

Equation	Steps
$1.4 - 0.3x + 0.7x = 9.4$	Original equation
$1.4 + 0.4x = 9.4$	
$0.4x = 8$	
$x = 20$	

Solve each equation that follows. Justify each step in your process using the properties of equality. Be sure to include the properties of operations, if used.

9. $7x - (4x - 39) = 0$

10. $4(3x + 5) = -4$

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Lesson 3.1.2: Solving Linear Equations

Warm-Up 3.1.2

The cost *C* of mailing a letter first class in the United States is \$0.45 for the first ounce. Each added ounce, or fraction of an ounce, costs an additional \$0.20.

1. Write an equation to represent the total cost, *C*, to mail a first class letter that weighs *n* ounces.

2. How much does it cost to mail a letter that weighs 3.2 ounces?

3. How much does a letter weigh if it costs \$1.85 to mail it?

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Lesson 3.1.2: Solving Linear Equations

Common Core State Standard

A–REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Warm-Up 3.1.2 Debrief

1. Write an equation to represent the total cost, C, to mail a first class letter that weighs n ounces.

   The equation that represents the total cost, C, is $C = 0.45 + 0.20(n - 1)$.

2. How much does it cost to mail a letter that weighs 3.2 ounces?

   The post office charges \$0.20 for each additional ounce or fraction of an ounce; therefore, we must round the weight of the letter up to 4.

   Substitute 4 for the variable n and solve for C.

$C = 0.45 + 0.20(n - 1)$	Equation
$C = 0.45 + 0.20[(4) - 1]$	Substitute 4 for n.
$C = 0.45 + 0.20(3)$	Subtract 1 from 4.
$C = 0.45 + 0.60$	Multiply.
$C = 1.05$	Add.

   The cost of mailing a letter weighing 3.2 ounces is \$1.05.

3. How much does a letter weigh if it costs \$1.85 to mail it?

   Using the same equation as in the answer to problem 2, substitute 1.85 for C and solve for n.

$C = 0.45 + 0.20(n - 1)$	Equation
$(1.85) = 0.45 + 0.20(n - 1)$	Substitute 1.85 for C.
$1.40 = 0.20(n - 1)$	Subtract 0.45 from both sides of the equation.
$1.40 = 0.20n - 0.20$	Distribute 0.20 over $(n - 1)$.
$1.60 = 0.20n$	Add 0.20 to both sides of the equation.
$8 = n$	Divide both sides of the equation by 0.20.

   Since fractions of an ounce are charged at the same rate as full ounces, the weight of a letter that costs \$1.85 to mail is greater than 7 ounces, but no more than 8 ounces.

Connection to the Lesson

- Students will continue to find solutions and apply their knowledge to more complex equations.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- simplifying expressions (5.OA.1)
- using the distributive property (6.EE.3)

Introduction

While it may not be efficient to write out the justification for each step when solving equations, it is important to remember that the properties of equality must always apply in order for an equation to remain balanced.

As equations become more complex, it may be helpful to refer to the properties of equality used in the previous lesson.

Key Concepts

- When solving equations, first take a look at the expressions on either side of the equal sign.
- You may need to simplify one or both sides of the equation before you can solve for the unknown. Sometimes you may need to combine like terms by using the associative, commutative, or distributive properties.
- Pay special attention if the same variable appears on both sides of the equal sign.
- Just like with numbers, variables may be added to or subtracted from both sides of the equation without changing the equality of the statement or the solution to the problem.

Solving Equations with the Variable in Both Sides of the Equation

1. Choose which side of the equation you would like the variable to appear on.
2. Add or subtract the other term containing the variable from both sides of the equation using either the addition or subtraction property of equality.
3. Simplify both sides of the equation.
4. Continue to solve the equation.
5. As with any equation, check that your answer is correct by substituting the value into the original equation to ensure both sides are equal.

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- Some equations have no solution, such as $x = x + 1$. In the case of equations with no solution, after you've completed all of the appropriate steps to solve an equation, the result is something untrue, like $0 = 1$. Such an equation is never true for any value of the variable.
- Some equations have an infinite number of solutions, such as $2x + 4 = 2(x + 2)$. These equations will be true for any value of the variable. This is the case when, following all of the appropriate steps for solving an equation, the result has the same expression on both sides of the equal sign.
- Other equations will only have one solution, where the final step in solving results in the variable being equal to a number, such as $x = 5$.

Common Errors/Misconceptions

- performing the wrong operation when isolating the variable
- incorrectly combining terms

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Instruction

Guided Practice 3.1.2

Example 1

Solve the equation $5x + 9 = 2x - 36$.

1. Move the terms containing the variable to one side of the equation.

   Notice that the same variable, x, is on both sides of the equation: $5x$ is on the left of the equation and $2x$ is on the right. It makes no difference whether you choose to have the variables on the left or on the right; your solution will remain the same. It's common to have the variable on the left, but not necessary.

   It's often easier to move the variable with the smallest coefficient to the opposite side of the equation. Here, $2x$ has a smaller coefficient than $5x$, so let's move $2x$.

   $2x$ is positive, so to get it to the other side of the equal sign you will need to subtract it from both sides of the equation.

   It helps to line up what you are subtracting with the terms that are similar in order to stay organized. In this case, we are subtracting variables from variables.

   $$\begin{array}{r} 5x+9=2x-36 \\ -2x \quad\quad -2x \\ \hline 3x+9= \quad -36 \end{array}$$

   When $2x$ is subtracted, it's important not to forget the remaining terms in the equation. Look out for subtraction signs that now act as negative signs. Here, since 36 was originally being subtracted from $2x$, the subtraction sign left behind makes 36 negative.

   $3x + 9 = -36$

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2. Continue to solve the equation.

To isolate x, subtract 9 from both sides of the equation.

$$\begin{array}{r} 3x+9=-36 \\ \underline{\quad -9 \quad -9} \\ 3x \quad\quad =-45 \end{array}$$

The variable x is multiplied by 3, so divide both sides by 3 because division is the inverse operation of multiplication.

$$\frac{3x}{3}=\frac{-45}{3}$$
$$x=-15$$

3. Verify your solution.

The solution to the equation $5x + 9 = 2x - 36$ is $x = -15$. Substitute -15 for all instances of x in the original equation, and then evaluate both sides.

$5x + 9 = 2x - 36$	Original equation
$5(-15) + 9 = 2(-15) - 36$	Substitute -15 for x.
$-75 + 9 = -30 - 36$	Multiply.
$-66 = -66$	Simplify.

Our check verified that both sides of the equation are equal when x is -15. Therefore, $x = -15$ is correct.

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Example 2

Solve the equation $7x + 4 = -9x$.

1. Move the terms containing the variable to one side of the equation.

   Notice that the variable x is on both sides of the equation. Again, it makes no difference mathematically which term, $7x$ or $-9x$, you choose to move.

   $-9x$ is the only term on the right side of the equation, so it may be easier to move $7x$ from the left side of the equation to the right side.

   Subtract $7x$ from both sides of the equation.

$$\begin{array}{r} 7x+4=-9x \\ \underline{-7x \qquad -7x} \\ 4=-16x \end{array}$$

2. Continue to solve the equation.

   To isolate x, divide both sides of the equation by the coefficient of x, -16.

$$\frac{4}{-16}=\frac{-16x}{-16}$$

$$-\frac{1}{4}=x$$

$$x=-\frac{1}{4}$$

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3. Verify your solution.

The solution to the equation $7x + 4 = -9x$ is $x = -\frac{1}{4}$. Substitute $-\frac{1}{4}$ for all instances of x in the original equation, and then evaluate both sides.

$7x + 4 = -9x$	Original equation
$7\left(-\frac{1}{4}\right) + 4 = -9\left(-\frac{1}{4}\right)$	Substitute $-\frac{1}{4}$ for x.
$\left(-\frac{7}{4}\right) + 4 = \frac{9}{4}$	Multiply.
$\left(-\frac{7}{4}\right) + \left(\frac{16}{4}\right) = \frac{9}{4}$	Convert the whole number 4 to a fraction with the same denominator as the other terms in the equation.
$\frac{9}{4} = \frac{9}{4}$	Add and review the result.

Our check verified that both sides of the equation are equal; therefore, $x = -\frac{1}{4}$ is correct.



Example 3

Solve the equation $2(3x + 1) = 6x + 14$.

1. Simplify each side of the equation.

Notice that the variable x is on both sides of the equation. Also notice the set of parentheses in the expression on the left side of the equation.

Eliminate the parentheses by distributing the 2 over $3x + 1$.

$2(3x + 1) = 6x + 14$	Original equation
$6x + 2 = 6x + 14$	Multiply 2 and $3x + 1$.



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2. Move the terms containing the variable to one side of the equation.

   Again, you need to eliminate one of the terms with the variable x. Subtract $6x$ from both sides of the equation.

$$\begin{array}{r} 6x+2=6x+14 \\ -6x \quad -6x \\ \hline 2=14 \end{array}$$

3. Interpret the result.

   This equation has no solution.

   Subtracting the variables gives us an untrue result: 2 = 14. Therefore, there are no solutions to this equation.

Example 4

Solve the equation $3(4x + 2) = 12x + 6$.

1. Simplify each side of the equation.

   Notice that there are terms containing the variable x on both sides of the equation. Also notice the set of parentheses in the expression on the left side of the equation.

   Eliminate the parentheses by distributing the 3 over $4x + 2$.

$3(4x + 2) = 12x + 6$	Original equation
$12x + 6 = 12x + 6$	Multiply 3 and $4x + 2$.

2. Interpret the result.

   This equation has an infinite number of solutions.

   The expressions on both sides of the equation are the same. This means that any value substituted for x will result in a true statement.

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Instruction

Example 5

Solve the literal equation $A=\frac{1}{2}(b_1+b_2)h$ for b_1.

1. Begin to isolate the specified variable.

   To solve literal equations for a specific variable, we follow the same steps as for solving equations.

   In this equation, we could distribute $\frac{1}{2}$ over b_1+b_2, but this may cause more work for us. Instead, let's eliminate the fraction. Multiply both sides of the equation by the reciprocal of $\frac{1}{2}$, or 2.

   $$A=\frac{1}{2}(b_1+b_2)h$$
   $$2\bullet A=2\bullet\frac{1}{2}(b_1+b_2)h$$
   $$2A=(b_1+b_2)h$$

2. Continue using properties of equality to isolate the specified variable.

   Again, we could distribute h over b_1+b_2, but it's more efficient to divide both sides of the equation by h.

   $$2A=(b_1+b_2)h$$
   $$\frac{2A}{h}=\frac{(b_1+b_2)h}{h}$$
   $$\frac{2A}{h}=b_1+b_2$$

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Instruction

3. Isolate b_1.

   Finally, to solve the equation for b_1, subtract b_2 from both sides of the equation.

$$\frac{2A}{h} = b_1 + b_2$$
$$\underline{-b_2 \qquad -b_2}$$
$$\frac{2A}{h} - b_2 = b_1$$

4. State the results.

   The equation $A = \frac{1}{2}(b_1 + b_2)h$ solved for b_1 is $\frac{2A}{h} - b_2 = b_1$.

   The equation can also be rewritten as $b_1 = \frac{2A}{h} - b_2$ using the symmetric property of equality.

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Problem-Based Task 3.1.2: Guess My Number

SMP	
1 ✓	2
3 ✓	4
5	6
7 ✓	8

Here's another number game to try. See if you can find the number.

2 less than one-third of a number equals 3 more than one-fourth of the number.

What is the number?



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Problem-Based Task 3.1.2: Guess My Number

Coaching

a. How can "2 less than one-third of a number" be written algebraically?

b. How can "3 more than one-fourth of the number" be written algebraically?

c. How can "2 less than one-third of a number equals 3 more than one-fourth of the number" be written algebraically?

d. How can you find the solution to the algebraic sentence?

e. What is the number?

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Instruction

Problem-Based Task 3.1.2: Guess My Number

Coaching Sample Responses

a. How can "2 less than one-third of a number" be written algebraically?

$$\frac{1}{3}x-2$$

b. How can "3 more than one-fourth of the number" be written algebraically?

$$3+\frac{1}{4}x$$

c. How can "2 less than one-third of a number equals 3 more than one-fourth of the number" be written algebraically?

$$\frac{1}{3}x-2=3+\frac{1}{4}x$$

d. How can you find the solution to the algebraic sentence?

Solve the equation for x by first moving the terms with the variable x to one side and then isolating x.

e. What is the number?

$\frac{1}{3}x-2=3+\frac{1}{4}x$	Original equation
$\frac{1}{3}x-\frac{1}{4}x-2=3+\frac{1}{4}x-\frac{1}{4}x$	Subtract $\frac{1}{4}x$ from both sides.
$\frac{1}{12}x-2=3$	Combine like terms.
$\frac{1}{12}x=5$	Add 2 to both sides.
$12\bullet\frac{1}{12}x=5\bullet 12$	Multiply both sides by 12.
$x = 60$	

2 less than one-third of 60 equals 3 more than one-fourth of 60.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 3.1.2: Solving Linear Equations

Solve each equation that follows.

1. $14 - 3y = 4y$

2. $10x - 22 = 29 - 7x$

3. $r - 4 + 6r = 3 + 8r$

4. $-x + 7 - 6x = 19 - 3x$

5. $8(11 - 2b) = -4(4b - 22)$

6. $6(2x - 4) = 3(4x + 8)$

7. $12x - 5 = 8x - x + 50$

8. $5 - 3z - 18 = z - 1 + 8z$

9. Solve the formula $t = \frac{V + K}{g}$ for g.

10. Solve the formula $S = \frac{n}{2}(a + l)$ for l.

SWB p. 23; SRB Practice 3.1.2 p. U3-20

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Lesson 3.1.3: Solving Linear Inequalities

Warm-Up 3.1.3

Luc received $50 from his grandparents for his birthday. He makes $75 each week as a cashier at the supermarket. Since his birthday, he has saved more than enough money to buy a game system that costs $450.

1. How many weeks ago was Luc's birthday?

2. After purchasing the game system, will Luc have any money left over? How do you know?

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Lesson 3.1.3: Solving Linear Inequalities

Common Core State Standard

A–REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Warm-Up 3.1.3 Debrief

1. How many weeks ago was Luc's birthday?

   Set up and solve an inequality to find how many weeks ago Luc had his birthday.

   $50 + 75w > 450$

   $75w > 400$

   $w > 5.\overline{3}$

   Luc is paid for whole weeks of work, not partial weeks, so round up to the nearest whole week.

   Luc has been saving for at least 6 weeks.

2. After purchasing the game system, will Luc have any money left over? How do you know?

   If Luc saved his wages of $75 a week for 6 weeks, he would have saved $450 plus the $50 from his grandparents, for a total of $500.

   The game system costs $450, so Luc would have $50 left after his purchase.

Connection to the Lesson

- Students will continue to find solutions to inequalities and apply their knowledge to more complex inequalities.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- solving equations (8.EE.7b)
- solving simple inequalities (7.EE.4b)
- applying the distributive property (6.EE.3)

Introduction

Solving inequalities is similar to solving equations. To find the solution to an inequality, use methods similar to those used in solving equations. In addition to using properties of equality, we will also use properties of inequality to change inequalities into simpler equivalent inequalities.

Key Concepts

- The **properties of inequality** are the rules that allow you to balance, manipulate, and solve inequalities. The properties are summarized in the following table.

Properties of Inequality

Property	Example
If $a > b$ and $b > c$, then $a > c$.	If $10 > 6$ and $6 > 2$, then $10 > 2$.
If $a > b$, then $b < a$.	If $10 > 6$, then $6 < 10$.
If $a > b$, then $-a < -b$.	If $10 > 6$, then $-10 < -6$.
If $a > b$, then $a \pm c > b \pm c$.	If $10 > 6$, then $10 \pm 2 > 6 \pm 2$.
If $a > b$ and $c > 0$, then $a \bullet c > b \bullet c$.	If $10 > 6$ and $2 > 0$, then $10 \bullet 2 > 6 \bullet 2$.
If $a > b$ and $c < 0$, then $a \bullet c < b \bullet c$.	If $10 > 6$ and $-1 < 0$, then $10 \bullet -1 < 6 \bullet -1$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.	If $10 > 6$ and $2 > 0$, then $10 \div 2 > 6 \div 2$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.	If $10 > 6$ and $-1 < 0$, then $10 \div -1 < 6 \div -1$.

- When solving more complicated inequalities, first clear any parentheses. Do this by either distributing by the multiplier of the quantity in parentheses or dividing both sides of the inequality by it. Then solve the inequality by following the steps learned earlier, as outlined in the table that follows.

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Instruction

Solving Inequalities
1. If a variable appears on both sides of the inequality, choose which side of the inequality you would like the variable to appear on. 2. Add or subtract the other term containing the variable from both sides of the inequality using either the addition or subtraction property of equality. 3. Simplify both sides of the inequality. 4. Continue to solve the inequality by working to isolate the variable. 5. Check that your answer is correct by substituting a value included in your solution statement into the original inequality to ensure a true statement.

- It is important to remember that when solving inequalities, the direction of the inequality symbol ($<$, $>$, $\leq$, or $\geq$) reverses when you divide or multiply by a negative number. Here's an example.
  - If we had the simple statement that $4 < 8$, we know that we can multiply both sides of the inequality by a number, such as 3, and the statement will still be true.

    $4 < 8$ Original inequality

    $4 \cdot 3 < 8 \cdot 3$ Multiply both sides of the inequality by 3.

    $12 < 24$ This is a true statement.

  - We can also divide both sides of the inequality by a number, such as 2.

    $4 < 8$ Original inequality

    $4 \div 2 < 8 \div 2$ Divide both sides of the inequality by 2.

    $2 < 4$ This is a true statement.

  - Notice what happens when we multiply the inequality by –3.

    $4 < 8$ Original inequality

    $4 \cdot -3 < 8 \cdot -3$ Multiply both sides of the inequality by –3.

    $-12 < -24$ This is NOT a true statement.

  - To make this a true statement, change the direction of the inequality symbol.

    $-12 > -24$ This is a true statement.

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Instruction

- The same is true for division. When dividing by a negative number, you must change the direction of the inequality symbol.

$4 < 8$	Original inequality
$4 \div -2 < 8 \div -2$	Divide both sides of the inequality by –2.
$-2 < -4$	This is NOT a true statement. Change the direction of the inequality symbol.
$-2 > -4$	This is a true statement.

Common Errors/Misconceptions

- not understanding why the inequality symbol changes direction when multiplying or dividing by a negative number
- replacing inequality symbols with equal signs
- forgetting to switch the inequality symbol when multiplying or dividing by a negative number
- switching the inequality symbol when subtracting a number

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Guided Practice 3.1.3

Example 1

Solve the inequality $\frac{-3x-4}{7} > 5$.

1. Isolate the variable by eliminating the denominator.

   In this inequality, the denominator means "divide by 7." Eliminate it by performing the inverse operation, multiplication. Multiply both sides of the inequality by 7.

   $$7 \bullet \frac{-3x-4}{7} > 7 \bullet 5$$
   $$-3x-4 > 35$$

2. Eliminate values that are subtracted from the variable term.

   To eliminate the –4, perform the inverse operation of adding 4 to both sides of the inequality.

   $$-3x-4 > 35$$
   $$\underline{\quad +4 \quad +4}$$
   $$-3x \quad > 39$$

3. Eliminate values that are multiplied by the variable.

   To eliminate the –3 that is multiplied by the variable, perform the inverse operation by dividing both sides of the inequality by –3.

   $$\frac{-3x}{-3} > \frac{39}{-3}$$
   $$x < -13$$

   Notice that the direction of the inequality symbol changed because we divided by a negative number.

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Instruction

4. Verify your solution.

The solution to the original inequality, $\frac{-3x-4}{7}>5$, is all numbers less than –13. To check this, choose any number less than –13. Let's try –20. Be sure to substitute the value into the original inequality.

$\frac{-3x-4}{7}>5$	Original inequality
$\frac{-3(-20)-4}{7}>5$	Substitute –20 for x.
$\frac{60-4}{7}>5$	Multiply.
$\frac{56}{7}>5$	Subtract.
$8>5$	Simplify the fraction.

$8 > 5$ is a true statement. Therefore, all numbers less than –13 will result in a true statement.



Example 2

Solve the inequality $5x + 4 \geq 11 - 2x$.

1. Move the terms containing the variable to one side of the inequality.

Notice the variable x is on both sides of the inequality. Begin by choosing which side you want your variable to appear on. Just like with equations, this is a choice, but it is common to choose to have all variables on the left side of the inequality. Add $2x$ to both sides of the inequality.

$$5x+4 \geq 11-2x$$
$$\underline{+2x \qquad\qquad +2x}$$
$$7x+4 \geq 11$$

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Instruction

2. Eliminate values that are added to the variable term.

   Subtract 4 from both sides.

$$\begin{array}{r} 7x+4 \geq 11 \\ -4 \quad -4 \\ \hline 7x \quad \geq 7 \end{array}$$

3. Eliminate values that are multiplied by the variable.

   To eliminate the 7 that is multiplied by the variable, perform the inverse operation by dividing both sides of the inequality by 7.

$$\frac{7x}{7} \geq \frac{7}{7}$$
$$x \geq 1$$

4. Verify your solution.

   The solution to the original inequality, $5x + 4 \geq 11 - 2x$, is all numbers greater than or equal to 1. Choose a number greater than or equal to 1, such as 2, and substitute it for all instances of x in the original inequality.

$5x + 4 \geq 11 - 2x$	Original inequality
$5(2) + 4 \geq 11 - 2(2)$	Substitute 2 for each instance of x.
$10 + 4 \geq 11 - 4$	Multiply.
$14 \geq 7$	Simplify. This is a true statement.

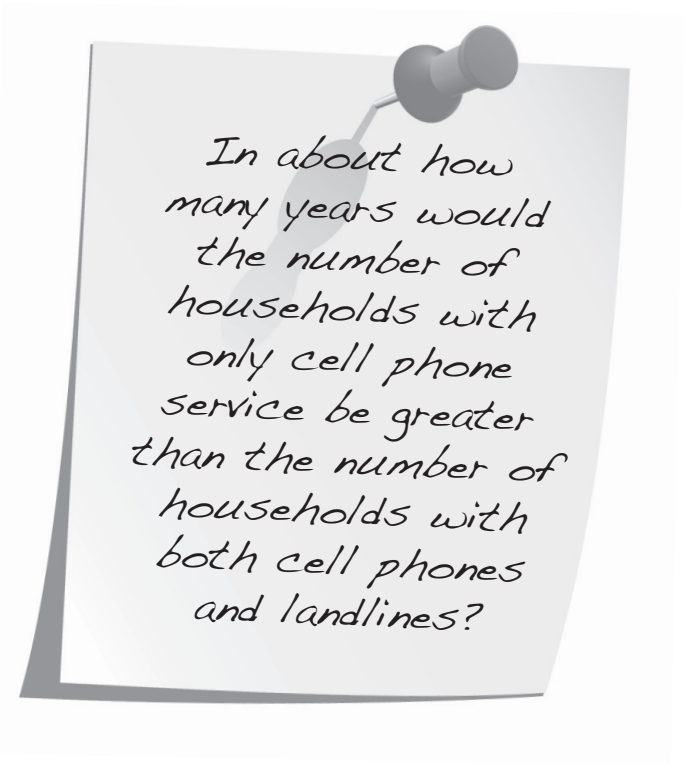
   Our check proved true, so we can be sure that our solution of $x \geq 1$ is accurate.

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Problem-Based Task 3.1.3: Landlines Versus Cell Phones

SMP	
1 ✓	2 ✓
3	4
5	6
7	8 ✓

In 2008, about 66.1 million U.S. households had both landline phones and cell phones. At that time, it was expected that the number of households with both landline and cell phone service would decrease by an average of 5 million households per year. Approximately 27.8 million households had only cell phones, and it was expected that the number of households with only cell phones would increase by an average of 15 million households per year. According to this data, in about how many years would the number of households with only cell phone service be greater than the number of households with both cell phones and landlines?



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Problem-Based Task 3.1.3: Landlines Versus Cell Phones

Coaching

a. In 2008, how many households had both landlines and cell phones?

b. By how much was this number expected to decrease each year?

c. What expression can be written to represent the number of households with both a landline and a cell phone after x years?

d. In 2008, how many households had just a cell phone?

e. By how much was this number expected to increase each year?

f. What expression can be written to represent the number of households with just cell phones after x years?

g. What inequality can be written to show when the number of households with only cell phones would be greater than the number of households with both cell phones and landlines?

h. According to this model, in about how many years would the number of households with only cell phone service be greater than the number of households with both cell phone service and landlines?

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Instruction

Problem-Based Task 3.1.3: Landlines Versus Cell Phones

Coaching Sample Responses

a. In 2008, how many households had both landlines and cell phones?

66.1 million households had both landlines and cell phones.

b. By how much was this number expected to decrease each year?

The number was expected to decrease by 5 million households each year.

c. What expression can be written to represent the number of households with both a landline and a cell phone after x years?

$66.1 - 5x$ represents the number of households (in millions) with both a landline and a cell phone after x years.

d. In 2008, how many households had just a cell phone?

27.8 million households had just cell phones in 2008.

e. By how much was this number expected to increase each year?

The number was expected to increase by 15 million households each year.

f. What expression can be written to represent the number of households with just cell phones after x years?

$27.8 + 15x$ represents the number of households (in millions) with just a cell phone after x years.

g. What inequality can be written to show when the number of households with only cell phones would be greater than the number of households with both cell phones and landlines?

$27.8 + 15x > 66.1 - 5x$

h. According to this model, in about how many years would the number of households with only cell phone service be greater than the number of households with both cell phone service and landlines?

$27.8 + 15x > 66.1 - 5x$	Inequality from the previous step
$27.8 + 20x > 66.1$	Add $5x$ to both sides of the inequality.
$20x > 38.3$	Subtract 27.8 from both sides.
$x > 1.915$	Divide both sides of the inequality by 20.

According to this model, it would take nearly 2 years for the number of households with just cell phones to be greater than the number of households with both cell phones and landlines.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 3.1.3: Solving Linear Inequalities

Find the solution to each inequality.

1. $3x + 1 > 2x + 7$

2. $4 - 3x \geq 16 + x$

3. $2x + 5 \geq 14 + \frac{7}{3}x$

4. $8x < 5(2x + 4)$

5. $5x \leq 10 + 2(3x - 4)$

6. $2(x - 4) \leq 2 + 3(x - 6)$

7. $4 - 5(x - 2) \leq -2(-9 + 2x)$

8. $10x + x + 10 > 8x$

9. $12x + 3x - 1 < 8x + x$

10. $\frac{4x}{5} + 6 < 3x + 4$

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Lesson 3.1.4: Solving Exponential Equations

Warm-Up 3.1.4

Two schools opened in the same year, each with a population of 500 students. Hillsboro High School's population increased at a rate of 160 students per year. The population of Mountain Ridge High School increased by approximately 20% each year.

1. On the tenth anniversary of the schools' opening, which school has the greater population?

2. How many more students does the school with the larger population have?

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Instruction

Lesson 3.1.4: Solving Exponential Equations

Common Core State Standard

A–REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Warm-Up 3.1.4 Debrief

1. On the tenth anniversary of the schools' opening, which school has the greater population?

   Create an expression to represent the population growth for each school.

   Hillsboro High School started with a population of 500 that increased by 160 students each year. This situation can be modeled by the expression $500 + 160n$, where n represents the number of years after the school opened. To find the population after 10 years, substitute 10 for n and evaluate the expression.

   $500 + 160n$

   $= 500 + 160(10)$

   $= 2100$

   Hillsboro High School has 2,100 students after 10 years.

   Mountain Ridge High School also started with a population of 500, but the population increased by 20% each year. This situation can be modeled by the expression $500(1 + 0.20)^n$, where n represents the number of years after the school opened. To find the population after 10 years, substitute 10 for n and evaluate the expression.

   $500(1 + 0.20)^n$

   $= 500(1 + 0.20)^{10}$

   $= 500(1.20)^{10}$

   ≈ 3096

   Mountain Ridge High School has approximately 3,096 students after 10 years.

   After 10 years, Mountain Ridge High School has more students than Hillsboro High School.

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2. How many more students does the school with the larger population have?

   To find the difference in the populations of the two schools, subtract their populations after 10 years.

   $3096 - 2100 = 996$

   After 10 years, Mountain Ridge High School has approximately 996 more students than Hillsboro High School.

Connection to the Lesson

- Students will be required to solve simple exponential equations and must recognize that linear equations are different from exponential equations.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- working with exponents (raising a base to a power) (6.EE.1)
- using properties of exponents (8.EE.1)

Introduction

As we have already seen, exponential equations are equations that have the variable in the exponent. Some exponential equations are complex and some are quite simple. In this lesson, we will focus on solving exponential equations of the form $b^x = c$, where b is the base and x is the exponent.

Key Concepts

- It may help to look at the **laws of exponents**. These laws, sometimes referred to as properties, describe how exponents combine with each other. The following table summarizes these laws.

Laws of Exponents

Law	General rule	Specific example
Product rule for exponents	$b^m \bullet b^n = b^{m+n}$	$4^6 \bullet 4^3 = 4^9$
Power of a power	$(b^m)^n = b^{mn}$	$(4^6)^3 = 4^{18}$
Power of a product	$(bc)^n = b^n c^n$	$(4 \bullet 2)^3 = 4^3 2^3$
Quotient rule for exponents	$\frac{b^m}{b^n} = b^{m-n}$	$\frac{4^6}{4^3} = 4^{6-3} = 4^3$
Exponents of zero	$b^0 = 1$	$4^0 = 1$
Negative exponents	$b^{-n} = \frac{1}{b^n}$ and $\frac{1}{b^{-n}} = b^n$	$4^{-3} = \frac{1}{4^3}$ and $\frac{1}{4^{-3}} = 4^3$

- Keep these laws in mind when solving exponential equations.
- Some exponential equations can be written so that each side of the equation has the same base, such as $a^b = a^c$. In this case, b and c must be equal as long as $a > 0$ and $a \neq 1$.

SRB p. U3-27

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Instruction

- A different method is used when it isn't possible to write each side of the equation using the same base. The method for solving this type of exponential equation will be covered in a later course.
- Follow a few basic guidelines to solve an exponential equation where the bases of both sides of the equation can be written so that they are equal.

Solving Exponential Equations

1. Rewrite the bases as powers of a common base.
2. Substitute the rewritten bases into the original equation.
3. Simplify the exponents.
4. Solve for the variable by setting the exponents equal to each other.

Common Errors/Misconceptions

- attempting to solve an exponential equation as if it is a linear equation
- not finding a common base prior to attempting to solve the equation
- misidentifying the common base

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Instruction

Guided Practice 3.1.4

Example 1

Solve $4^x = 1024$.

1. Find a common base for both sides of the equation, if one exists.

   You may not recognize right away if it is possible to write 1,024 as an exponential expression with a base of 4. Begin by finding values of powers of 4 to see if it is possible.

   $4^1 = 4$

   $4^2 = 16$

   $4^3 = 64$

   $4^4 = 256$

   $4^5 = 1024$

   We now know that it is possible to write 1,024 as a power of 4.

2. Rewrite the equation so that both sides have the same base.

   The common base is 4.

   $4^x = 4^5$

3. Solve for x by setting the exponents equal to each other.

   $x = 5$

   The solution to the equation $4^x = 1024$ is $x = 5$.

4. Check your answer.

   Substitute 5 for the variable x in the original equation.

   $4^x = 1024$

   $4^{(5)} = 1024$

   $1024 = 1024$ This is a true statement.

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Instruction

Example 2

Solve the equation $\left(\frac{1}{2}\right)^x = 16$.

1. Find a common base for both sides of the equation, if one exists.

   Both $\frac{1}{2}$ and 16 can be written as powers of the base 2.

   Referring back to the Laws of Exponents table, we know that $\frac{1}{2}$ is equal to 2 to the power of –1, or 2^{-1}.

   16 is equal to 2 to the power of 4, or 2^4.

2. Rewrite the equation so that both sides have the same base.

   The common base is 2.

   $\left(\frac{1}{2}\right)^x = 16$

   $(2^{-1})^x = 2^4$

   Following the laws of exponents, we know that we can rewrite $(2^{-1})^x$ as 2^{-x}.

   $(2^{-1})^x = 2^4$

   $2^{-x} = 2^4$

3. Now solve for x by setting the exponents equal to each other.

   $-x = 4$

   Divide by –1 to isolate the variable.

   $x = -4$

   The solution to the equation $\left(\frac{1}{2}\right)^x = 16$ is $x = -4$.

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Instruction

4. Check your answer.

   Substitute –4 for the variable x in the original equation.

   $\left(\frac{1}{2}\right)^{x} = 16$

   $\left(\frac{1}{2}\right)^{(-4)} = 16$

   $2^{4} = 16$

   $16 = 16$ This is a true statement.



Example 3

Solve the equation $\frac{1}{49} = 7^{x}$.

1. Find a common base for both sides of the equation, if one exists.

   $\frac{1}{49}$ can be rewritten as $\frac{1}{7^{2}}$.

   Referring back to the Laws of Exponents table, we know that $\frac{1}{7^{2}}$ is equal to 7 to the power of –2, or 7^{-2}.

2. Rewrite the equation so that both sides have the same base.

   The common base is 7.

   $\frac{1}{49} = 7^{x}$

   $\frac{1}{7^{2}} = 7^{x}$

   $7^{-2} = 7^{x}$

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Instruction

3. Solve for x by setting the exponents equal to each other.

   $x = -2$

   The solution to the equation $\frac{1}{49} = 7^x$ is $x = -2$.

4. Check your answer.

   Substitute –2 for the variable x in the original equation.

   $\frac{1}{49} = 7^x$

   $\frac{1}{49} = 7^{(-2)}$

   $\frac{1}{49} = \frac{1}{7^2}$

   $\frac{1}{49} = \frac{1}{49}$ This is a true statement.



Example 4

Solve the equation $117 = 5^x - 8$.

1. Isolate the term that contains the variable.

   Eliminate the subtraction of 8 from the right side of the equation. Do so by adding 8 to both sides of the equation.

   $117 = 5^x - 8$

   $+8 \qquad +8$

   $125 = 5^x$

2. Find a common base for both sides of the equation, if one exists.

   125 is equal to 5 to the power of 3, or 5^3.

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Instruction

3. Rewrite the equation so that both sides have the same base.

   The common base is 5.

   $125 = 5^x$

   $5^3 = 5^x$

4. Solve for x by setting the exponents equal to each other.

   $x = 3$

   The solution to the equation $117 = 5^x - 8$ is $x = 3$.

5. Check your answer.

   Substitute 3 for the variable x in the original equation.

   $117 = 5^x - 8$

   $117 = 5^{(3)} - 8$

   $117 = 125 - 8$

   $117 = 117$ This is a true statement.



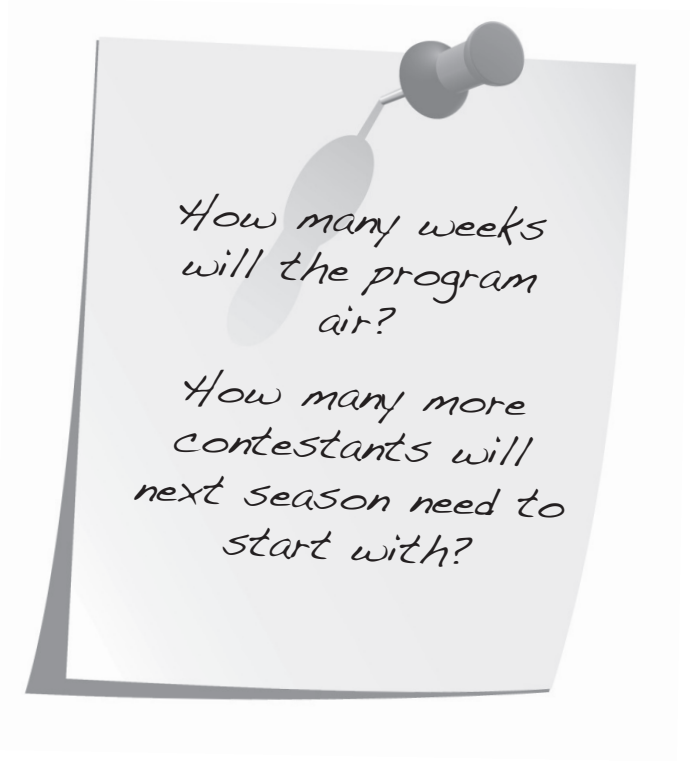
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Problem-Based Task 3.1.4: Singing Celebration

SMP	
1 ✓	2 ✓
3	4 ✓
5	6
7	8

You're a big fan of a new reality series, "Singing Celebration." Contestants from around the country compete to be celebrated as the best singer. Scores are ranked highest to lowest. Those singers falling in the lowest half of the rankings are eliminated. The first season started with 64 contestants. If half of the contestants are eliminated each week, for how many weeks will the program air?

Producers have already decided that next season will be 4 weeks longer than this season. How many more contestants will next season need to start with?



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Problem-Based Task 3.1.4: Singing Celebration

Coaching

a. How many contestants did the show start with in its first season?

b. How many contestants are eliminated each week?

c. What equation can be written to find the number of weeks the program airs?

d. How many weeks will the show's first season air?

e. How many more weeks will the show's second season air?

f. How many weeks will the show's second season air?

g. What equation can be written to find the number of contestants needed for the second season?

h. How many contestants will the second season start out with?

i. How many more contestants will the show's second season have than its first season?

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Instruction

Problem-Based Task 3.1.4: Singing Celebration

Coaching Sample Responses

a. How many contestants did the show start with in its first season?

The show started with 64 contestants in its first season.

b. How many contestants are eliminated each week?

Half the contestants are eliminated each week.

c. What equation can be written to find the number of weeks the program airs?

In order to find the number of weeks the program must air, set up an exponential equation in which the exponent is equal to the unknown number of weeks, x.

$64 = 2^x$

d. How many weeks will the show's first season air?

Solve the exponential equation in part c for x, which represents the number of weeks.

Both 2 and 64 can be rewritten as powers of the base 2.

$2 = 2^1$ and $64 = 2^6$

Substitute these into the original equation and solve for x.

$2^x = 2^6$

$x = 6$

The first season will air for 6 weeks.

e. How many more weeks will the second season air?

The second season will air for 4 more weeks than the first season.

f. How many weeks will the second season air?

The second season will air for 10 weeks (6 + 4).

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g. What equation can be written to find the number of contestants needed for the second season?

The equation $2^{10} = x$ can be used to find the number of contestants needed for the second season.

h. How many contestants will the second season start out with?

Evaluate 2^{10} to determine the number of contestants needed for a 10-week program.

$2^{10} = 1024$

The second season will start out with 1,024 contestants.

i. How many more contestants will the show's second season have than its first season?

Find the difference in the number of contestants between the first season and the second season.

$1024 - 64 = 960$

There will be 960 more participants in the second season than in the first season.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 3.1.4: Solving Exponential Equations

Solve each equation for x.

1. $8 = 2^x$

2. $3^x = 81$

3. $\frac{1}{25} = 5^x$

4. $4^x = \frac{1}{64}$

5. $4 = \left(\frac{1}{2}\right)^x$

6. $\left(\frac{1}{6}\right)^x = 216$

7. $100 = 10^x$

8. $0.0625 = 0.5^x$

9. $223 = 6^x + 7$

10. $4 = 2401^x - 3$

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Progress Assessment

Circle the letter of the best answer.

1. Identify the property of equality that justifies the missing step to solve the given equation.

Equation	Steps
$3x + (x - 8) = 12$	Original equation
$4x - 8 = 12$	Associative property of addition
$4x = 20$	
$x = 5$	Division property of equality

a. subtraction property of equality
b. addition property of equality
c. division property of equality
d. multiplication property of equality

2. Identify the property of equality that justifies the missing step to solve the given equation.

Equation	Steps
$6x + 14 = 12.8$	Original equation
$6x = -1.2$	Subtraction property of equality
$x = -0.2$	

a. subtraction property of equality
b. addition property of equality
c. division property of equality
d. multiplication property of equality

3. What is the solution to the equation $6x - 3 + 3x = 60 + 2x$?

a. $x = 9$
b. $x = \frac{63}{11}$
c. $x = \frac{57}{7}$
d. $x = 63$

4. What is the solution to the equation $2x + 4(3x - 5) - 2 = 98 - 6x$?

a. $x = 15$
b. $x = -20$
c. $x = 7.5$
d. $x = 6$

continued

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Assessment

5. What is the solution to the inequality $12x - 5 \leq 2x + 15$?

   a. $x \leq 2$
   c. $x \leq 1$
   b. $x \geq 2$
   d. $x \geq 1$

6. What is the solution to the inequality $\frac{2x}{5} - 3 < x + 9$?

   a. $x > 20$
   c. $x < 20$
   b. $x > -20$
   d. $x < -20$

7. What is the solution to the inequality $10x - 15 \geq 5(2 + 3x)$?

   a. $x \leq 5$
   c. $x \leq -5$
   b. $x \geq 5$
   d. $x \geq -5$

8. What is the solution to the equation $2^x = 64$?

   a. $x = -6$
   c. $x = 8$
   b. $x = 6$
   d. $x = 32$

9. What is the solution to the equation $3^x = \frac{1}{243}$?

   a. $x = -5$
   c. $x = 5$
   b. $x = 81$
   d. $x = -81$

10. What is the solution to the equation $55 = 4^x - 9$?

    a. $x = 3$
    c. $x = 11.5$
    b. $x = 4$
    d. $x = 16$

Write an equation based on the information given, then solve the equation.

11. As of 2014, the New York Mets had played in 14 less than twice the number of World Series Championships than the Atlanta Braves. The Mets had played in 5 fewer series than the Braves had. How many World Series had the Atlanta Braves played in?

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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. What is the solution to the system $\begin{cases} y=-2x+4 \\ x-5y=2 \end{cases}$?
   a. $\left(\frac{6}{11},\frac{32}{11}\right)$
   b. (2, 0)
   c. There are infinitely many solutions to this system of equations.
   d. There are no solutions to this system of equations.

2. What is the solution to the system $\begin{cases} x+3y=5 \\ 5x-3y=7 \end{cases}$?
   a. (2, 1)
   b. $\left(\frac{1}{2},\frac{3}{2}\right)$
   c. There are infinitely many solutions to this system of equations.
   d. There are no solutions to this system of equations.

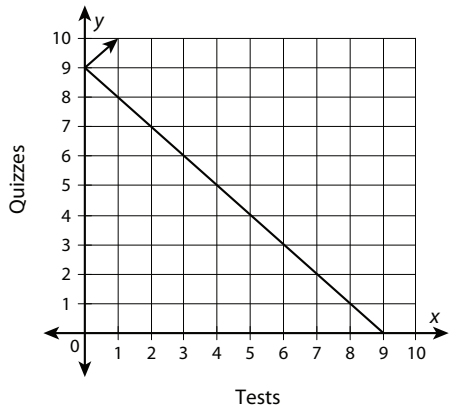
3. What is the solution to the system $\begin{cases} 2x-3y=-28 \\ -6x+9y=84 \end{cases}$?
   a. (0, 0)
   b. $\left(3,\frac{34}{3}\right)$
   c. There are infinitely many solutions to this system of equations.
   d. There are no solutions to this system of equations.

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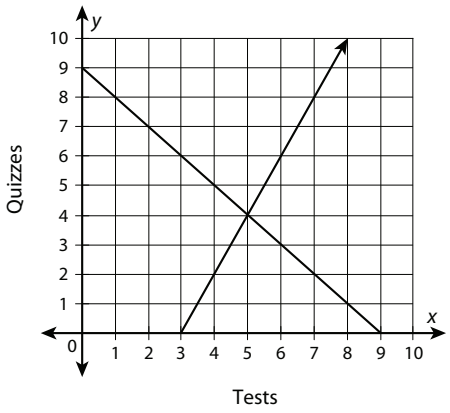
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4. Your science class has 9 assessments. Tests are worth 100 points, and quizzes are worth 50 points. There were 600 points possible in the quarter. Which graph represents the number of tests and quizzes you had last quarter?

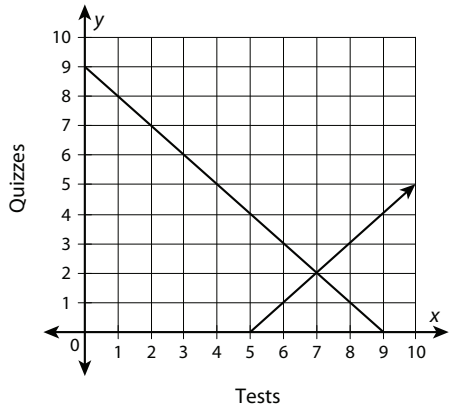
a.



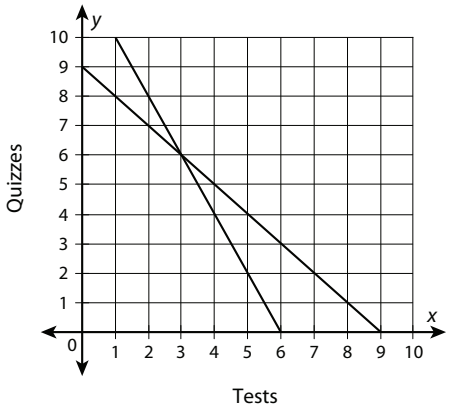
c.



b.



d.

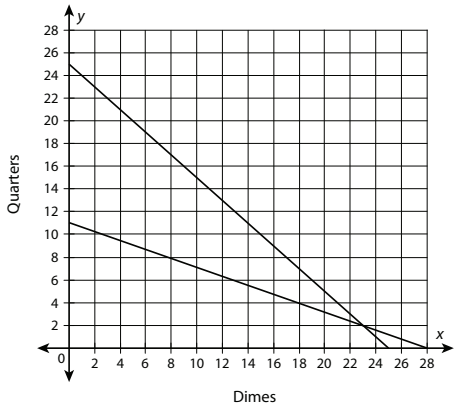


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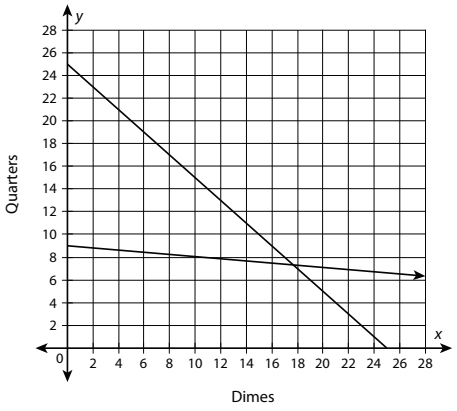
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5. Lucy has a total of 25 dimes and quarters that add up to $2.80. Which graph represents the number of dimes and quarters Lucy has?

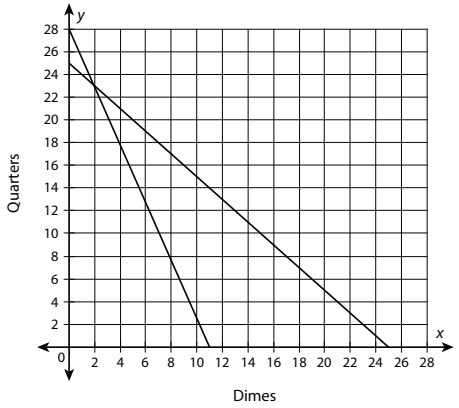
a.



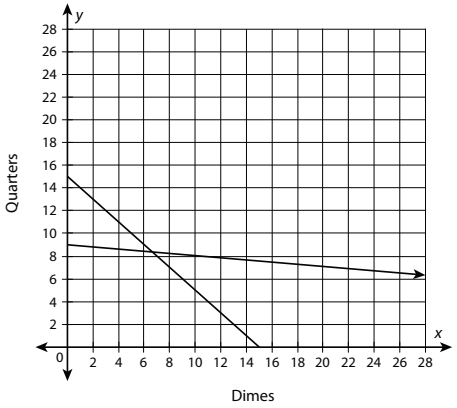
c.



b.



d.



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Lesson 2: Solving Systems of Equations

Instruction

Common Core State Standards

A–REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A–REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

SMP	
1 ✓	2 ✓
3 ✓	4 ✓
5	6
7	8

Essential Questions

1. What does it mean for two systems of linear equations to be equivalent to each other?
2. How many solutions can a system of equations have?
3. Why is knowing the solution to a system of equations helpful in the real world?
4. What are the benefits of having different types of strategies to solve systems of equations related to real-world situations?

WORDS TO KNOW

consistent	a system of equations with at least one ordered pair that satisfies both equations
dependent	a system of equations that has an infinite number of solutions; lines coincide when graphed
elimination method	adding or subtracting the equations in the system together so that one of the variables is eliminated; multiplication might be necessary before adding the equations together
graphing method	solving a system by graphing equations on the same coordinate plane and finding the point of intersection
inconsistent	a system of equations with no solutions; lines are parallel when graphed
independent	a system of equations with exactly one solution
point of intersection	the point at which two lines cross or meet
substitution method	solving one of a pair of equations for one of the variables and substituting that into the other equation
system of equations	a set of equations with the same unknowns

SRB p. U3-35

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Recommended Resources

- Math Warehouse. "Interactive System of Linear Equations."

  http://walch.com/rr/CAU2L2InteractiveGraph

  This site features an interactive graph of a system of equations. Users can move the various points around to observe how the solution changes.

- Purplemath. "Systems of Linear Equations."

  http://walch.com/rr/CAU2L2LinearEquations

  This resource provides a summary of systems of linear equations and methods to solve them.

- Tutorials for *Finite Math.* "Systems of Two Linear Equations in Two Unknowns."

  http://walch.com/rr/CAU2L2EquationTutorial

  This tutorial summarizes systems of linear equations and methods to solve them.

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Lesson 3.2.1: Solving Systems of Linear Equations by Substitution and Elimination

Warm-Up 3.2.1

The Pendleton County School District is trying to decide on a new copier. The purchasing committee has been given quotes on two new machines. One sells for $20,000 and costs $0.02 per copy to operate. The other sells for $17,500, but its operating cost is $0.025 per copy. Your teacher will give you extra credit for writing a mathematically sound recommendation to the school district on which copier to buy based on the total cost during the first year.

1. The district estimates the number of copies made each year is 515,000. Based on this estimation, which machine would you recommend? Justify your choice with clear mathematics.

2. The superintendent of the district has decided that all forms will be distributed electronically. This will decrease the number of copies made in the district by 6%. Based on this information, does this change your recommendation? Justify your choice with clear mathematics.

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Instruction

Lesson 3.2.1: Solving Systems of Linear Equations by Substitution and Elimination

Common Core State Standards

A–REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A–REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Warm-Up 3.2.1 Debrief

1. The district estimates the number of copies made each year is 515,000. Based on this estimation, which machine would you recommend? Justify your choice with clear mathematics.

   The total cost for each copier can be determined using equations. Let C represent the operating cost and p represent the number of pages copied.

   Copier 1: $C = 20{,}000 + 0.02p$

   Copier 2: $C = 17{,}500 + 0.025p$

   Substitute 515,000 for the number of copies, p, to determine the cost for each copier.

   Copier 1: $C = 20{,}000 + 0.02(515{,}000) = 30{,}300$

   Copier 2: $C = 17{,}500 + 0.025(515{,}000) = 30{,}375$

   Copier 1 costs \$30,300 for 515,000 copies.

   Copier 2 costs \$30,375 for 515,000 copies.

   Copier 1 would be the best copier for this district because it is cheaper at the estimated number of pages. The advantage becomes even greater with each year of use.

2. The superintendent of the district has decided that all forms will be distributed electronically. This will decrease the number of copies made in the district by 6%. Based on this information, does this change your recommendation? Justify your choice with clear mathematics.

   It was estimated that the district would make 515,000 copies.

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Calculate the number of copies if the amount is decreased by 6%.

$515{,}000 - 0.06(515{,}000)$

$= 515{,}000(1 - 0.06)$

$= 515{,}000(0.94)$

$= 484{,}100$

If 6% fewer copies were made, the district would make 484,100 copies.

Use the same equations from question 1 to determine the cost for 484,100 copies.

Copier 1: $C = 20{,}000 + 0.02(484{,}100) = 29{,}682$

Copier 2: $C = 17{,}500 + 0.025(484{,}100) = 29{,}602.50$

Copier 1 costs $29,682 for 484,100 copies.

Copier 2 costs $29,602.50 for 484,100 copies.

With the reduced number of copies, Copier 2 would be less expensive during the first year of use. However, after the first year, Copier 1 is less expensive because of the cheaper cost per page.

Connection to the Lesson

- Students will be asked to take a scenario like this a step further and determine at which point the equations are equal.
- Students will be asked to solve a system of equations using various methods and will be asked to justify their thinking.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- graphing equations of lines (A–CED.2★)
- using properties of equality to solve equations (8.EE.7b)

Introduction

Two or more equations that are solved together are called **systems of equations**. The solution to a system of equations is the point or points that make both equations true. Systems of equations can have one solution, no solutions, or an infinite number of solutions. Finding the solution to a system of equations is important to many real-world applications.

Key Concepts

- There are various methods to solving a system of equations. Two methods include the substitution method and the elimination method.

Solving Systems of Linear Equations by Substitution

- The **substitution method** involves solving one of the equations for one of the variables and substituting that into the other equation.

Substitution Method
1. Solve one of the equations for one of the variables in terms of the other variable. 2. Substitute, or replace the resulting expression into the other equation. 3. Solve the equation for the second variable. 4. Substitute the found value into either of the original equations to find the value of the other variable.

- A solution to a system of equations is written as an ordered pair, (x, y). This is the point where the lines would intersect if graphed.
- If the resulting solution is a true statement, such as $9 = 9$, then the system has an infinite number of solutions. The lines would coincide if graphed.

SRB p. U3-37

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- If the result is an untrue statement, such as $4 = 9$, then the system has no solutions. The lines would never intersect, so they would be parallel if graphed.
- Check your answer by substituting the x and y values back into the original equations. If the answer is correct, the equations will result in true statements.

Solving Systems of Linear Equations by Elimination Using Addition or Subtraction

- The **elimination method** involves adding or subtracting the equations in the system, using properties of equality, so that one of the variables is eliminated.

Elimination Method Using Addition or Subtraction
1. Add the two equations if the coefficients of one of the variables are opposites of each other. 2. Subtract the two equations if the coefficients of one of the variables are the same. 3. Solve the resulting equation for the remaining variable. 4. Substitute the found value into either of the original equations to find the value of the other variable.

Solving Systems of Linear Equations by Elimination Using Multiplication

- Use this method when the coefficients of one of the variables are neither opposites nor the same. The multiplication property of equality can be used with one or both equations in order to make one pair of coefficients match.

Elimination Method Using Multiplication
1. Multiply each term of one equation by the same number. It may be necessary to multiply the second equation by a different number in order to have coefficients on one of the variables that are opposites or the same. 2. Add or subtract the two equations to eliminate one of the variables. 3. Solve the resulting equation for the remaining variable. 4. Substitute the found value into either of the original equations to find the value of the other variable.

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Instruction

Common Errors/Misconceptions

- finding the value for only one of the variables of the system
- forgetting to distribute negative signs when substituting expressions for variables
- forgetting to multiply each term by the same number when solving by elimination using multiplication

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Guided Practice 3.2.1

Example 1

Solve the following system by substitution.

$$\begin{cases} x+y=2 \\ x-y=0 \end{cases}$$

1. Solve one of the equations for one of the variables in terms of the other variable.

   It doesn't matter which equation you choose, nor does it matter which variable you solve for.

   Let's solve $x + y = 2$ for the variable y.

   Isolate y by subtracting x from both sides.

$$\begin{array}{rrr} x+y= & 2 \\ -x & & -x \\ \hline y= & 2 & -x \end{array}$$

   Now we know that y is equal to $2 - x$.

2. Substitute $2 - x$ for y in the other equation.

   It helps to place parentheses around the expression you are substituting.

$x - y = 0$	Second equation of the system
$x - (2 - x) = 0$	Substitute $2 - x$ for y.

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Instruction

3. Solve the equation for the remaining variable.

$x - (2 - x) = 0$	Equation from the previous step
$x - 2 + x = 0$	Distribute the negative over $(2 - x)$.
$2x - 2 = 0$	Combine like terms.
$2x = 2$	Add 2 to both sides.
$x = 1$	Divide both sides by 2.

4. Substitute the found value, $x = 1$, into either of the original equations to find the value of the other variable.

$x + y = 2$	First equation of the system
$(1) + y = 2$	Substitute 1 for x.
$1 + y = 2$	Simplify.
$y = 1$	Subtract 1 from both sides.

The solution to the system of equations is (1, 1).
If graphed, the lines would intersect at (1, 1).

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Instruction

Example 2

Solve the following system by elimination.

$$\begin{cases} 2x-3y=-11 \\ x+3y=11 \end{cases}$$

1. Add the two equations if the coefficients of one of the variables are opposites of each other.

   The coefficients 3 and –3 are opposites, so the equations can be added.

   Add downward, combining like terms only.

   $$\begin{array}{rcl} 2x-3y & = & -11 \\ x+3y & = & 11 \\ \hline 3x+0 & = & 0 \end{array}$$

   Simplify.

   $3x = 0$

2. Solve the equation for the second variable.

$3x = 0$	Equation from the previous step
$x = 0$	Divide both sides by 3.

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Instruction

3. Substitute the found value, $x = 0$, into either of the original equations to find the value of the other variable.

$2x - 3y = -11$	First equation of the system
$2(0) - 3y = -11$	Substitute 0 for x.
$-3y = -11$	Simplify.
$y = \frac{11}{3}$	Divide both sides by –3.

The solution to the system of equations is $\left(0, \frac{11}{3}\right)$.

If graphed, the lines would intersect at $\left(0, \frac{11}{3}\right)$.



Example 3

Solve the following system by elimination.

$$\begin{cases} x - 3y = 5 \\ -2x + 6y = 4 \end{cases}$$

1. Multiply each term of one equation by the same number.

   The variable x has a coefficient of 1 in the first equation and a coefficient of –2 in the second equation.

   Multiply the first equation by 2.

$x - 3y = 5$	Original equation
$2(x - 3y) = 2(5)$	Multiply both sides of the equation by 2.
$2x - 6y = 10$	Simplify.



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Instruction

2. Add or subtract the two equations to eliminate one of the variables.

   Replace the first equation in the system with the equivalent equation found in the previous step.

$$\begin{array}{r} 2x-6y=10 \\ +(-2x+6y=4) \\ \hline 0+0=14 \end{array}$$

3. Simplify.

   $0 + 0 = 14$

   $0 = 14$ This is NOT a true statement.

   The system $\begin{cases} x-3y=5 \\ -2x+6y=4 \end{cases}$ does not have a solution. There are no points that will make both equations true, and the graphs of the two equations are parallel lines.

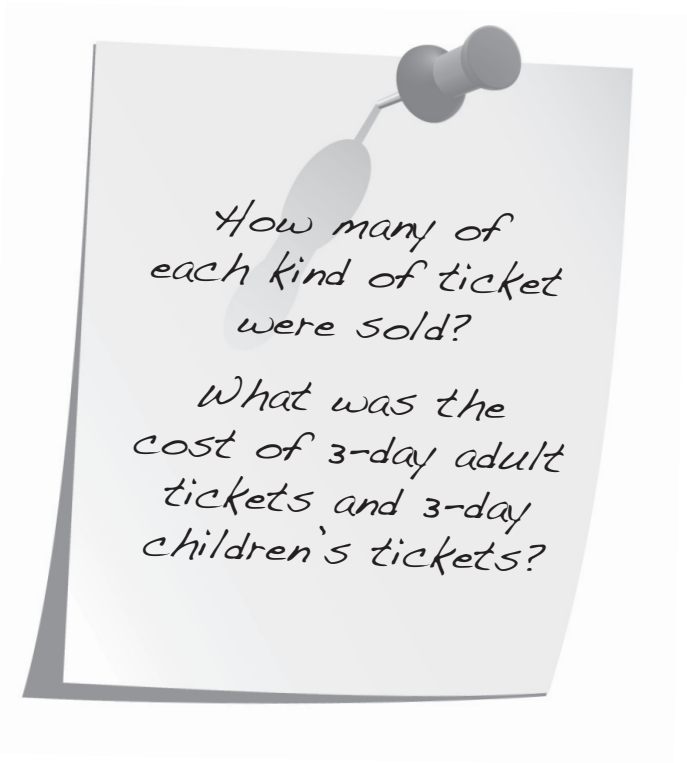
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Problem-Based Task 3.2.1: Ticket Sales

SMP	
1 ✓	2 ✓
3	4 ✓
5	6
7	8

Tickets sales to the local gaming convention, PlayerCon, are on the rise! Sales of 2-day tickets brought in $188,100. A total of 6,600 tickets were sold. Adult 2-day tickets cost $36.00 and children's 2-day tickets cost $26.00. How many of each kind of ticket were sold?

Sales of 3-day tickets to PlayerCon brought in $347,600. The combined cost of one 3-day adult ticket and one 3-day children's ticket is $90. One-third more adult 3-day tickets were sold than adult 2-day tickets. One-fifth more 3-day children's tickets were sold than 2-day children's tickets. What was the cost of 3-day adult tickets and 3-day children's tickets?



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Problem-Based Task 3.2.1: Ticket Sales

Coaching

a. What equation can be written to represent the total number of 2-day tickets sold?

b. What equation can be written to represent the total amount of sales of 2-day tickets?

c. How can you use the equations from parts a and b to find the number of adult 2-day tickets sold and the number of children's 2-day tickets sold?

d. What equation can be written to represent the total price of one 3-day adult ticket and one 3-day children's ticket?

e. How many 3-day adult tickets were sold if there were one-third more 3-day adult tickets sold than 2-day tickets?

f. How many 3-day children's tickets were sold if there were one-fifth more 3-day children's tickets sold than 2-day tickets?

g. What equation can be written to represent the total amount of sales of 3-day tickets?

h. How can you use the equations from parts d and g to find the price of adult 3-day tickets and the price of children's 3-day tickets?

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Instruction

Problem-Based Task 3.2.1: Ticket Sales

Coaching Sample Responses

a. What equation can be written to represent the total number of 2-day tickets sold?

Let a represent adult tickets and c represent children's tickets.

The total number of 2-day tickets sold was 6,600.

Totals are found by adding; the number of adult tickets plus the number of children's tickets is equal to the total number of tickets sold.

$a + c = 6600$

b. What equation can be written to represent the total amount of sales of 2-day tickets?

Adult 2-day tickets cost $36.

Children's 2-day tickets cost $26.

The total amount of sales was $188,100.

The numbers of adult 2-day tickets and children's 2-day tickets sold are unknown.

Use the same variables as in part 1 for adult 2-day tickets, a, and children's 2-day tickets, c.

$36a + 26c = 188{,}100$

c. How can you use the equations from parts a and b to find the number of adult 2-day tickets sold and the number of children's 2-day tickets sold?

Create a system of equations using both equations.

$$\begin{cases} a + c = 6600 \\ 36a + 26c = 188{,}100 \end{cases}$$

Use either substitution or elimination to solve the system. Notice here that substitution might be the most efficient method.

Solve the first equation, $a + c = 6600$, for either a or c. Let's choose a.

$a + c = 6600$

$a = 6600 - c$

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Substitute $6600 - c$ for a in the second equation ($36a + 26c = 188{,}100$) and solve.

$36a + 26c = 188{,}100$

$36(6600 - c) + 26c = 188{,}100$

$237{,}600 - 36c + 26c = 188{,}100$

$237{,}600 - 10c = 188{,}100$

$-10c = -49{,}500$

$c = 4950$

4,950 children's 2-day tickets were sold.

Substitute 4,950 into the original equation to find the number of adult tickets sold.

$a + c = 6600$

$a + (4950) = 6600$

$a = 1650$

1,650 adult 2-day tickets were sold.

d. What equation can be written to represent the total price of one 3-day adult ticket and one 3-day children's ticket?

Let a represent adult tickets and c represent children's tickets.

The total price of one 3-day adult ticket and one 3-day children's ticket is \$90.

Totals are found by adding; the price of 1 adult ticket plus the price of 1 children's ticket is equal to the total price.

$a + c = 90$

e. How many 3-day adult tickets were sold if there were one-third more 3-day adult tickets sold than 2-day tickets?

In part c, we found that 1,650 2-day adult tickets were sold.

One-third more tickets can be found by multiplying 1,650 by $\frac{1}{3}$, then adding that result to 1,650.

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$1650 \bullet \frac{1}{3} = 550$

$1650 + 550 = 2200$

There were 2,200 3-day adult tickets sold.

f. How many 3-day children's tickets were sold if there were one-fifth more 3-day children's tickets sold than 2-day tickets?

In part c, we found that 4,950 children's 2-day tickets were sold.

One-fifth more tickets can be found by multiplying 4,950 by $\frac{1}{5}$, then adding that result to 4,950.

$4950 \bullet \frac{1}{5} = 990$

$4950 + 990 = 5940$

There were 5,940 3-day children's tickets sold.

g. What equation can be written to represent the total amount of sales of 3-day tickets?

2,200 adult 3-day tickets were sold.

5,940 children's 3-day tickets were sold.

The total amount of sales was $347,600.

The prices of adult 3-day tickets and children's 3-day tickets are unknown.

Use the same variables as in part d for adult 3-day tickets, a, and children's 3-day tickets, c.

$2200a + 5940c = 347{,}600$

h. How can you use the equations from parts d and g to find the price of adult 3-day tickets and the price of children's 3-day tickets?

Create a system of equations using both equations.

$$\begin{cases} a + c = 90 \\ 2200a + 5940c = 347{,}600 \end{cases}$$

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Instruction

Use either substitution or elimination to solve the system. Notice that substitution might be the most efficient method here.

Solve the first equation ($a + c = 90$) for either a or c. Let's choose a.

$a + c = 90$

$a = 90 - c$

Substitute $90 - c$ for a in the second equation ($2200a + 5940c = 347{,}600$) and solve.

$2200a + 5940c = 347{,}600$

$2200(90 - c) + 5940c = 347{,}600$

$198{,}000 - 2200c + 5940c = 347{,}600$

$198{,}000 + 3740c = 347{,}600$

$3740c = 149{,}600$

$c = 40$

3-day children's tickets cost $40.

Substitute 40 into the original equation to find the price of 3-day adult tickets.

$a + c = 90$

$a + (40) = 90$

$a = 50$

3-day adult tickets cost $50.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 3.2.1: Solving Systems of Linear Equations by Substitution and Elimination

Find the solution to each of the following systems by using the substitution method.

1. $\begin{cases} y+3=x \\ 3x+4y=16 \end{cases}$

2. Antony works at a local clothing store. He can earn either $600 per month plus a 5% commission or $450 per month and a 7% commission. Write and solve a system of equations that you could use to determine the amount of merchandise that Antony would need to sell in order to earn the same amount of money for each pay scale.

Find the solution to each of the following systems by using the elimination method.

3. $\begin{cases} 5x-2y=-13 \\ 2x+y=11 \end{cases}$

4. At an Italian bistro, the cost of 2 plates of spaghetti and 1 salad is $27.50. The cost for 4 plates of spaghetti and 3 salads is $59.50. Write and solve a system of equations to find the cost of 1 plate of spaghetti and the cost of 1 salad.

Choose an appropriate method to solve each system of equations, then find the solution.

5. $\begin{cases} 3x-3y=-6 \\ 2x-2y=-16 \end{cases}$

6. $\begin{cases} 3x=4y \\ 4x-5y=2 \end{cases}$

continued

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For problems 7–10, write a system of equations, choose an appropriate method to solve each system, and then solve it.

7. There are 120 people attending a wedding reception. There are 16 tables. Some are round and some are square. The round tables seat 8 people and the square tables seat 6 people. How many of each type of table must there be at the reception?

8. Tickets to the theater cost $12 for adults and $9 for children. A group of 15 adults and children are attending the theater. If the total cost was $153, how many adults and how many children went?

9. Simon invests $1,200 into two savings accounts. One account earns 4% annual interest and the other earns 5.9% annual interest. At the end of 1 year, Simon earned $64.15 in interest. How much did he invest at each rate?

10. Carmen walks at a rate of 2 miles per hour and jogs at a rate of 4 miles per hour. She walked and jogged 3.4 miles in 1.2 hours. For how long did Carmen jog and for how long did she walk?

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Lesson 3.2.2: Solving Systems of Linear Equations by Graphing

Warm-Up 3.2.2

Benito is a waiter. He earns a base salary of $1,500 a month, plus 20% of the price of the meals he serves.

1. Write an equation to predict the amount of money Benito will earn if he serves $350 in meals in one month.

2. Benito earned $1,650 in one month. What was the total price of the meals that Benito served?

3. Create a graph to show the possible amount of money Benito could earn each month.

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Lesson 3.2.2: Solving Systems of Linear Equations by Graphing

Common Core State Standard

A–REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Warm-Up 3.2.2 Debrief

1. Write an equation to predict the amount of money Benito will earn if he serves $350 in meals in one month.

   Translate the verbal description of Benito's pay into an algebraic equation.

   Let x represent the total cost of the meals Benito serves in one month.

   Let y represent the amount of money Benito earns.

   Benito earns a base salary of $1,500, plus 20% of the price of the meals.

   20% of the price of the meals is $0.20x$.

   $y = 1500 + 0.20x$

   We are asked to find Benito's earnings if he serves $350 in meals in one month.

   Substitute $350 for x and solve the equation for y, the amount of money Benito will earn.

$y = 1500 + 0.20x$	Equation
$y = 1500 + 0.20(350)$	Substitute 350 for x.
$y = 1500 + 70$	Multiply.
$y = 1570$	Add.

   Benito will earn $1,570 if he serves $350 worth of meals in one month.

2. Benito earned $1,650 in one month. What was the total price of the meals that Benito served?

   Use your equation from part 1 to find the price of the meals Benito served.

   Substitute $1,650 for y, the total amount of money Benito earned.

   $1650 = 1500 + 0.20x$

   Solve for x.

$1650 = 1500 + 0.20x$	Equation
$150 = 0.20x$	Subtract 1,500 from both sides.
$750 = x$	Divide both sides by 0.20.

   Benito served $750 worth of meals.

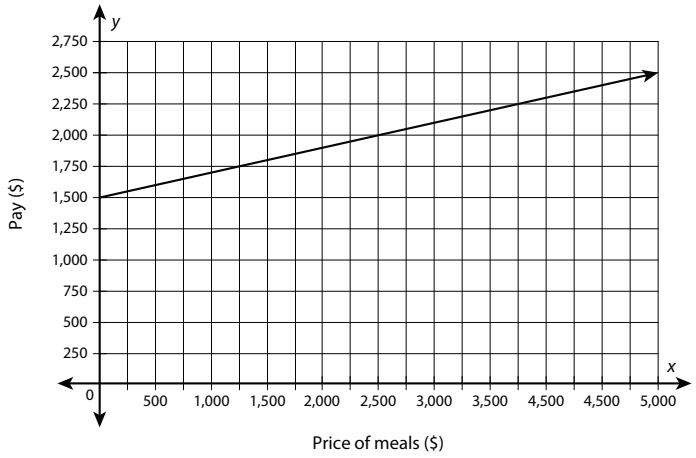
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3. Create a graph to show the possible amount of money Benito could earn each month.

   The equation $y = 1500 + 0.20x$ can be graphed using slope-intercept form.

   The slope of the equation is 0.20.

   The y-intercept is 1,500.



Connection to the Lesson

- Students will be asked to analyze similar situations and go a step further by finding one solution for both scenarios.
- Students will be asked to use graphs to determine solutions to scenarios.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- graphing equations of lines (A–CED.2★)
- analyzing situations involving linear equations (8.F.5)
- creating linear equations to solve problems (A–CED.2★)

Introduction

The solution to a system of equations is the point or points that make both equations true. Systems of equations can have one solution, no solutions, or an infinite number of solutions. On a graph, the solution to a system of equations can be easily seen. The solution to the system is the **point of intersection**, the point at which two lines cross or meet.

Key Concepts

- There are various methods for solving a system of equations. One is the graphing method.

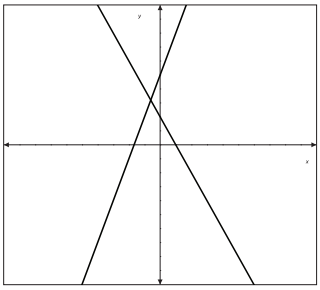
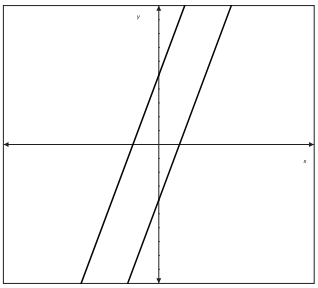
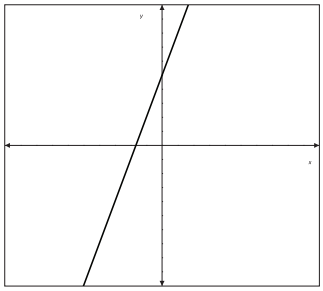
Solving Systems of Linear Equations by Graphing

- The **graphing method**, which involves solving a system of equations by graphing them on the same coordinate plane, allows you to visualize the solution to the system.
- To graph the equations, use a table of values, the slope-intercept form ($y = mx + b$) of the equations, or a graphing calculator.
- Creating a table of values can be time consuming depending on the equations, but will work for all equations.
- Equations not written in slope-intercept form can be rewritten in order to determine the slope and y-intercept.
- Once graphed, you can determine the number of solutions the system has.
- Graphs of systems with one solution have two intersecting lines. The point of intersection is the solution to the system. These systems are considered **consistent**, or having at least one solution, and are also **independent**, meaning there is exactly one solution.
- Graphs of systems with no solution have parallel lines. There are no points that satisfy both of the equations. These systems are referred to as **inconsistent**.

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Instruction

- Systems with an infinite number of solutions are equations of the same line—the lines for the equations in the system overlap. These are referred to as **dependent** and also consistent, having at least one solution.

Intersecting lines	Parallel lines	Same line
		
One solution	No solutions	Infinitely many solutions
Consistent Independent	Inconsistent	Consistent Dependent

- Graphing the system of equations can sometimes be inaccurate, but solving the system algebraically will always give an exact answer.

Graphing Equations Using a TI-83/84:

Step 1: Press [Y=] and key in the first equation using [X, T, θ, *n*] for *x*.

Step 2: Press [ENTER] and key in the second equation.

Step 3: Press [WINDOW] to change the viewing window, if necessary.

Step 4: Enter in appropriate values for Xmin, Xmax, Xscl, Ymin, Ymax, and Yscl, using the arrow keys to navigate.

Step 5: Press [GRAPH].

Step 6: Press [2ND] and [TRACE] to access the Calculate Menu.

Step 7: Choose 5: intersect.

Step 8: Press [ENTER] three times for the point of intersection.

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Graphing Equations Using a TI-Nspire:

Step 1: Press the home key.

Step 2: Arrow over to the graphing icon (the picture of the parabola or the U-shaped curve) and press [enter].

Step 3: At the blinking cursor at the bottom of the screen, enter in the equation and press [enter].

Step 4: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom, and click the center button of the navigation pad.

Step 5: Choose 1: Window settings by pressing the center button.

Step 6: Enter in the appropriate XMin, XMax, YMin, and YMax fields.

Step 7: Leave the XScale and YScale set to auto.

Step 8: Use [tab] to navigate among the fields.

Step 9: Press [tab] to "OK" when done and press [enter].

Step 10: Press [tab] to enter the second equation, then press [enter].

Step 11: Enter the second equation and press [enter].

Step 12: Press [menu] and choose 6: Analyze Graph.

Step 13: Choose 4: Intersection.

Step 14: Select the lower bound.

Step 15: Select the upper bound.

Common Errors/Misconceptions

- graphing equations incorrectly
- misidentifying the point of intersection

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Instruction

Guided Practice 3.2.2

Example 1

Graph the system of equations. Then determine whether the system has one solution, no solution, or infinitely many solutions. If the system has a solution, find it.

$$\begin{cases} 4x-6y=12 \\ y=-x+3 \end{cases}$$

1. Solve each equation for y.

   To simplify graphing, solve the first equation for y.

$4x - 6y = 12$	Original equation
$-6y = 12 - 4x$	Subtract $4x$ from both sides.
$y=-2+\frac{2}{3}x$	Divide both sides by -6.
$y=\frac{2}{3}x-2$	Write the equation in slope-intercept form ($y = mx + b$).

   The second equation ($y = -x + 3$) is already in slope-intercept form.

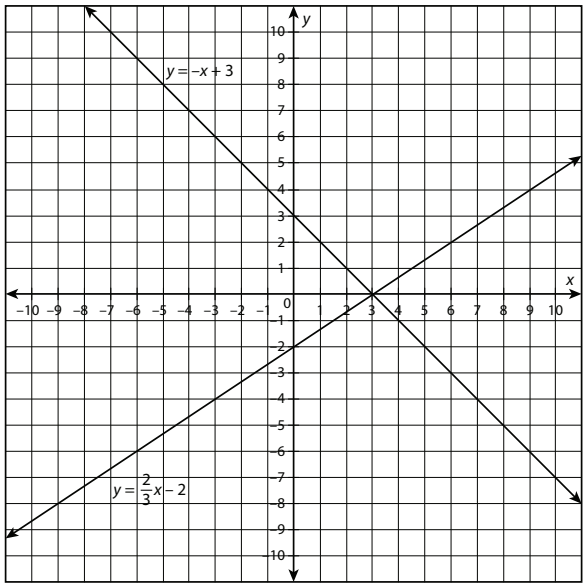
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Instruction

2. Graph both equations using the slope-intercept method.

The y-intercept of $y = \frac{2}{3}x - 2$ is –2. The slope is $\frac{2}{3}$.

The y-intercept of $y = -x + 3$ is 3. The slope is –1.



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Instruction

3. Observe the graph.

The lines intersect at the point (3, 0).

This appears to be the solution to this system of equations.

To check, substitute (3, 0) into both original equations. The results should be true statements.

$4x - 6y = 12$	First equation in the system
$4(3) - 6(0) = 12$	Substitute (3, 0) for x and y.
$12 - 0 = 12$	Simplify.
$12 = 12$	This is a true statement.

$y = -x + 3$	Second equation in the system
$(0) = -(3) + 3$	Substitute (3, 0) for x and y.
$0 = -3 + 3$	Simplify.
$0 = 0$	This is a true statement.

4. State your conclusion.

The system $\begin{cases} 4x - 6y = 12 \\ y = -x + 3 \end{cases}$ has one solution, (3, 0).



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Instruction

Example 2

Graph the system of equations. Then determine whether the system has one solution, no solution, or infinitely many solutions. If the system has a solution, find it.

$$\begin{cases} -8x + 4y = 4 \\ y = 2x + 1 \end{cases}$$

1. Solve each equation for y.

   To simplify graphing, solve the first equation for y.

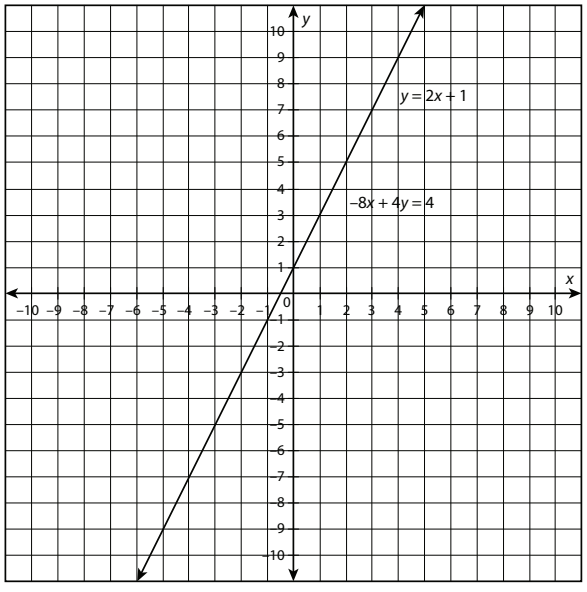
$-8x + 4y = 4$	Original equation
$4y = 4 + 8x$	Add $8x$ to both sides.
$y = 1 + 2x$	Divide both sides by 4.
$y = 2x + 1$	Write the equation in slope-intercept form ($y = mx + b$).

   The second equation ($y = 2x + 1$) is already in slope-intercept form.

2. Graph both equations using the slope-intercept method.

   The y-intercept of both equations is 1.

   The slope of both equations is 2.



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Instruction

3. Observe the graph.

The graphs of $y = 2x + 1$ and $-8x + 4y = 4$ are the same line.

There are infinitely many solutions to this system of equations.

To check, choose any point on the graph of $-8x + 4y = 4$ and substitute it into both original equations. The result should be a true statement. Let's use (2, 5).

$-8x + 4y = 4$	First equation of the system
$-8(2) + 4(5) = 4$	Substitute (2, 5) for x and y.
$-16 + 20 = 4$	Simplify.
$4 = 4$	This is a true statement.

$y = 2x + 1$	Second equation of the system
$(5) = 2(2) + 1$	Substitute (2, 5) for x and y.
$5 = 4 + 1$	Simplify.
$5 = 5$	This is a true statement.

You could choose any other point on the line of $-8x + 4y = 4$ to show it is true for any point, and not just for the point you originally chose. Let's try again with (1, 3).

$-8x + 4y = 4$	First equation of the system
$-8(1) + 4(3) = 4$	Substitute (1, 3) for x and y.
$-8 + 12 = 4$	Simplify.
$4 = 4$	This is a true statement.

$y = 2x + 1$	Second equation of the system
$(3) = 2(1) + 1$	Substitute (1, 3) for x and y.
$3 = 2 + 1$	Simplify.
$3 = 3$	This is a true statement.

4. State your conclusion.

The system $\begin{cases} -8x + 4y = 4 \\ y = 2x + 1 \end{cases}$ has infinitely many solutions.

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Instruction

Example 3

Graph the system of equations. Then determine whether the system has one solution, no solution, or infinitely many solutions. If the system has a solution, find it.

$$\begin{cases} -6x+2y=8 \\ y=3x-5 \end{cases}$$

1. Solve each equation for y.

   To simplify graphing, solve the first equation for y.

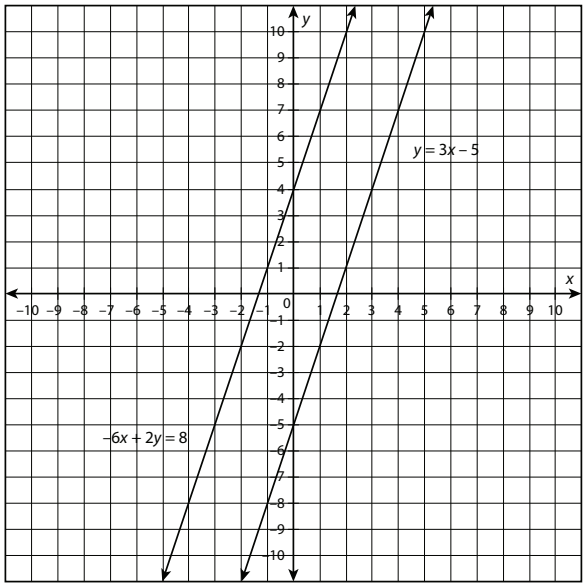
$-6x + 2y = 8$	Original equation
$2y = 8 + 6x$	Add $6x$ to both sides.
$y = 4 + 3x$	Divide both sides by 2.
$y = 3x + 4$	Write in slope-intercept form ($y = mx + b$).

   The second equation ($y = 3x – 5$) is already in slope-intercept form.

2. Graph both equations using the slope-intercept method.

   The y-intercept of $y = 3x + 4$ is 4. The slope is 3.

   The y-intercept of $y = 3x – 5$ is –5. The slope is 3.



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Instruction

3. Observe the graph.

   The graphs of $-6x + 2y = 8$ and $y = 3x - 5$ are parallel lines and never intersect.

   There are no values for x and y that will make both equations true.

4. State your conclusion.

   The system $\begin{cases} -6x+2y=8 \\ y=3x-5 \end{cases}$ has no solutions.



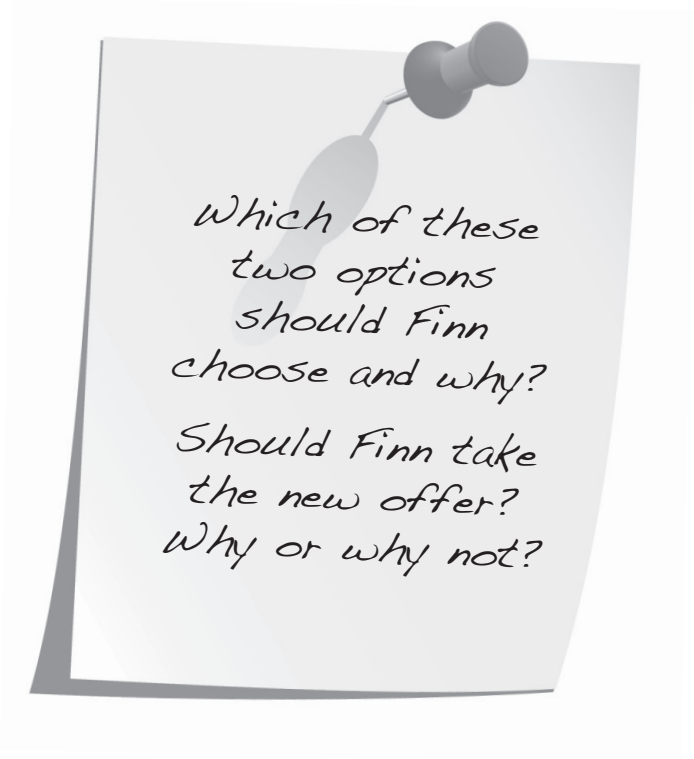
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Problem-Based Task 3.2.2: Salary Scales

SMP	
1	2 ✓
3 ✓	4 ✓
5	6
7	8

Finn has recently been offered a part-time job as a salesperson at a local cell phone store. He has a choice of two different pay scales. The first option is to receive a base salary of $300 a week plus 10% of the price of the merchandise he sells. The second option is a base salary of $220 a week plus 18% of the price of the merchandise he sells. The average phone sells for $200. Which of these two options should Finn choose and why?

Finn's boss has agreed to give him a third salary option. This option includes a base salary of $160 plus 20% of the amount of merchandise he sells. Should Finn take the new offer? Why or why not?



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Problem-Based Task 3.2.2: Salary Scales

Coaching

a. The first salary option is $300 a week plus 10% of the price of the merchandise Finn sells. What equation represents his total salary?

b. The second salary option is $220 a week plus 18% of the price of the merchandise Finn sells. What equation represents his total salary?

c. What system of equations represents the two salary options?

d. How could you compare the salary options for various amounts of sales?

e. Which salary option should Finn choose and why?

f. What equation represents the third option?

g. Is the third option more appealing than the first two options?

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Instruction

Problem-Based Task 3.2.2: Salary Scales

Coaching Sample Responses

a. The first salary option is $300 a week plus 10% of the price of the merchandise Finn sells. What equation represents his total salary?

Let x represent the price of the merchandise Finn sells.

Let y represent Finn's total salary.

$y = 300 + 0.10x$

b. The second salary option is $220 a week plus 18% of the price of the merchandise Finn sells. What equation represents his total salary?

Let x represent the amount of merchandise Finn sells.

Let y represent Finn's total salary.

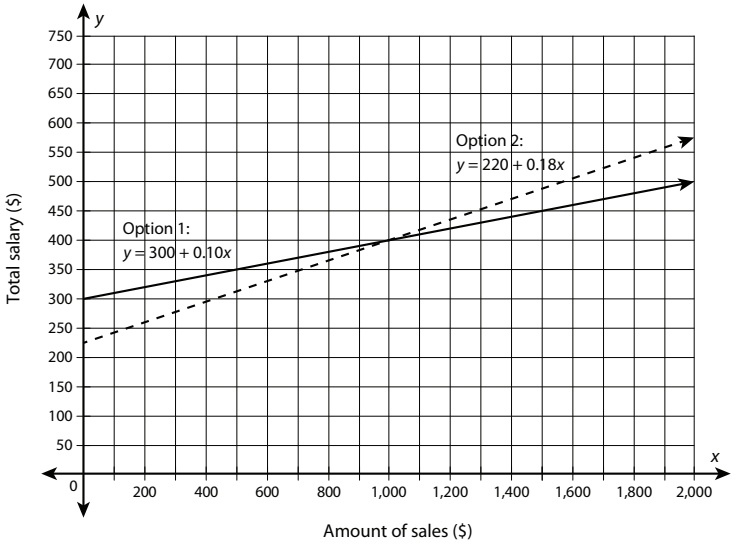
$y = 220 + 0.18x$

c. What system of equations represents the two salary options?

$$\begin{cases} y = 300 + 0.10x \\ y = 220 + 0.18x \end{cases}$$

d. How could you compare the salary options for various amounts of sales?

You can create a graph to compare the values.



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e. Which salary option should Finn choose and why?

The salary options are equal at approximately \$1,000 in sales. If the average phone costs \$200, that would mean Finn would need to sell about 5 phones a week. If he sells less than \$1,000 in merchandise each week, he would probably want to choose Option 1: $y = 300 + 0.10x$. The base salary is higher, but he would earn less on the amount he sells.

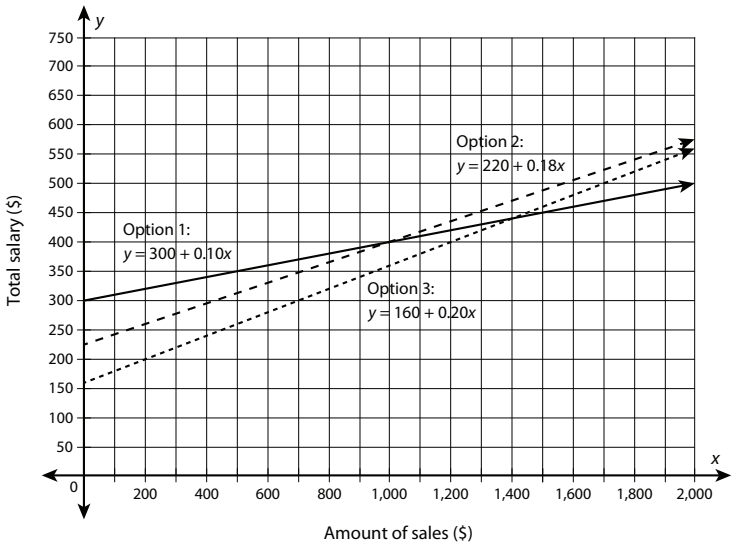
Option 2: $y = 220 + 0.18x$, is a better option if Finn thinks he could sell more than \$1,000 per week. The base salary is less, but the percentage per sale is higher. This can be seen on the graph.

f. What equation represents the third option? Again, let x represent the amount of merchandise Finn sells and y represent his total salary.

$y = 160 + 0.20x$

g. Is the third option more appealing than the first two options?

Graph the equation for Option 3, $y = 160 + 0.20x$, on the same coordinate plane.



From the graph, it's clear that the base salary for Option 3 is the lowest, but the percentage per sale is the highest. If Finn sold \$1,400 worth of merchandise, it wouldn't matter whether he chose Option 1 or 3 since they're equal. However, if he sold more than \$1,000, Option 2 would be the best choice. If the graph were continued, it appears as if Option 3 would eventually be the best choice.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 3.2.2: Solving Systems of Linear Equations by Graphing

Graph each system of equations. Then determine whether the system has one solution, no solution, or infinitely many solutions. If the system has a solution, find it.

1. $\begin{cases} x-2y=8 \\ 3x-2y=12 \end{cases}$

2. $\begin{cases} x+y=7 \\ x-y=1 \end{cases}$

3. $\begin{cases} 2x-y=1 \\ 2y=4x-2 \end{cases}$

4. $\begin{cases} 3x-y=-2 \\ y=3x \end{cases}$

For problems 5–10, write a system of equations, and then use the graphing method to solve each system.

5. You are buying one flower for each member of the swim team to wear at awards night. There are 19 members on the team. You have decided to buy pink roses and yellow roses. The pink roses cost $3, and the yellow roses cost $2. You will spend $50. How many pink roses should you buy? How many yellow roses should you buy?

6. A test worth 100 points has 26 questions. True/false questions are worth 2 points and short answers are worth 5 points. How many 2-point questions are on the test? How many 5-point questions are on the test?

7. The basketball team scored a total of 79 points last game. They made 35 shots, including 2-point shots and 3-point shots. How many 2-point shots did they make? How many 3-point shots did they make?

8. Your class field trip is to a museum that charges $4 for adults and $2 for students. It cost $60 for 27 tickets for your class, the teacher, and chaperones. How many adult tickets were purchased? How many student tickets were purchased?

9. Minh and Mikayla are selling wrapping paper for a school fund-raiser. Customers can buy either packages of folded paper or rolls of paper. Minh sold 6 rolls of paper and 2 packages of folded paper for a total of $44. Mikayla sold 3 rolls of paper and 9 packages of folded paper for a total of $54. How much does each roll cost? How much does each package of folded paper cost?

10. Erin has quarters and nickels for a total of 16 coins. The value of his coins is $2.60. How many quarters does Erin have? How many nickels does Erin have?

SWB p. 63; SRB Practice 3.2.2 p. U3-56

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Progress Assessment

Circle the letter of the best answer.

1. What is the solution to the system $\begin{cases} y = 6x - 3 \\ 6x - y = 72 \end{cases}$?
   a. (3.1, 15.6)
   b. (5.75, 31.5)
   c. There are infinitely many solutions to this system of equations.
   d. There are no solutions to this system of equations.

2. Tickets to the carnival cost $9.00 for adults and $7.50 for children. A group of 11 people went to the carnival and paid $87 for tickets. How many adult tickets were purchased? How many children's tickets were purchased?
   a. 3 adult tickets and 8 children's tickets were purchased.
   b. 8 adult tickets and 3 children's tickets were purchased.
   c. 5 adult tickets and 6 children's tickets were purchased.
   d. The number of each ticket purchased can't be determined from the given information.

3. What is the solution to the system $\begin{cases} 4x - 6y = 42 \\ x + 6y = 48 \end{cases}$?
   a. (5, 18)
   b. (18, 5)
   c. There are infinitely many solutions to this system of equations.
   d. There are no solutions to this system of equations.

4. What is the solution to the system $\begin{cases} 3x + y = 10 \\ -3x + 2y = 65 \end{cases}$?
   a. (–5, 5)
   b. (–5, 25)
   c. There are infinitely many solutions to this system of equations.
   d. There are no solutions to this system of equations.

continued

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5. The sum of two numbers is 32. The difference of those numbers is –2. What are the numbers?

   a. 17 and 19

   b. 15 and –2

   c. 15 and 17

   d. It is not possible to determine the numbers with the given information.

6. What is the solution to the system $\begin{cases} 6x-2y=58 \\ -4x+3y=-42 \end{cases}$?

   a. (9, –2)

   b. (3.75, 16)

   c. There are infinitely many solutions to this system of equations.

   d. There are no solutions to this system of equations.

7. Four times a number plus 6 times another number is 20. The sum of the two numbers is 6. What are the numbers?

   a. Both numbers are 2.

   b. 2 and –8

   c. 8 and –2

   d. It is not possible to determine the numbers with the given information.

8. Samuel invests $2,600 into two savings accounts. One account earns 4% annual interest; the other earns 3.5% annual interest. At the end of 1 year, Samuel has earned $99.50 in interest. How much did he invest at each rate?

   a. $1,200 at 4% and $1,400 at 3.5%

   b. $1,200 at 3.5% and $1,400 at 4%

   c. $900 at 4% and $1,700 at 3.5%

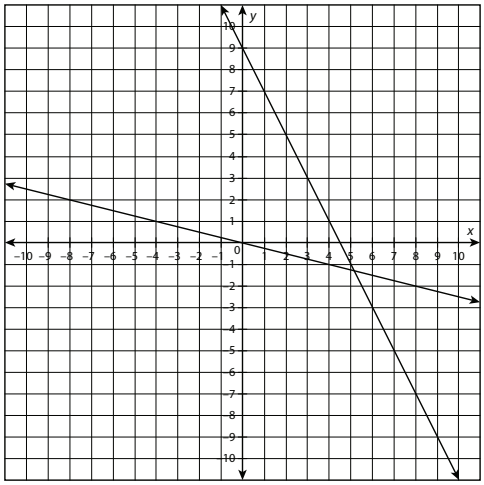
   d. $900 at 3.5% and $1,700 at 4%

continued

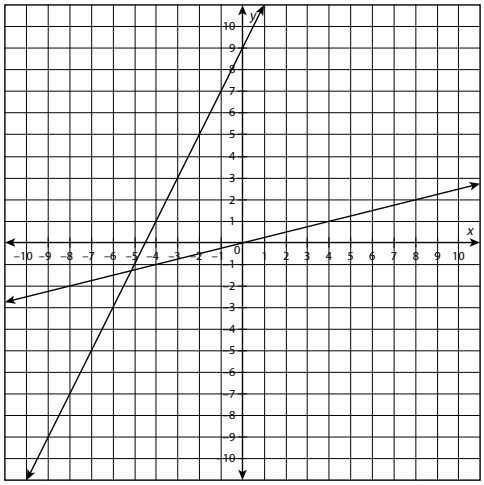
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9. Which graph represents the solution to the system $\begin{cases} 2x+y=9 \\ -x+4y=0 \end{cases}$?

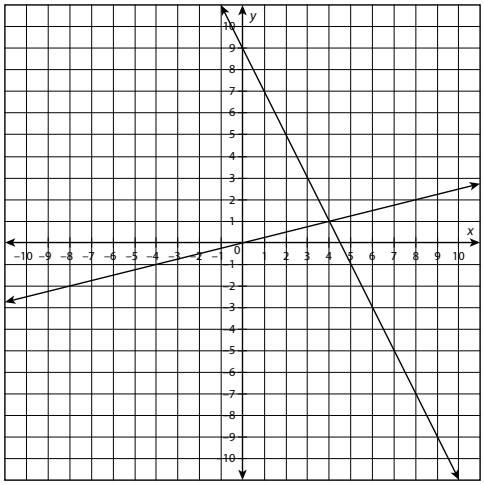
a.



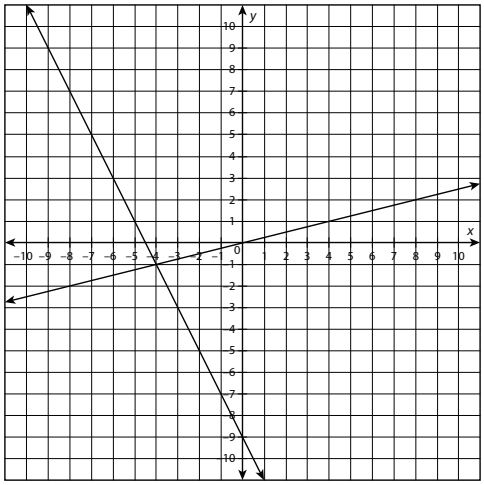
c.



b.



d.

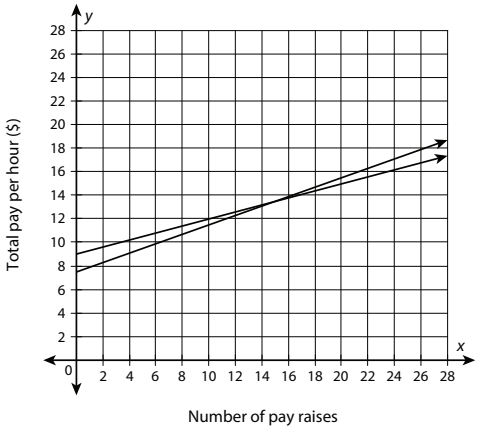


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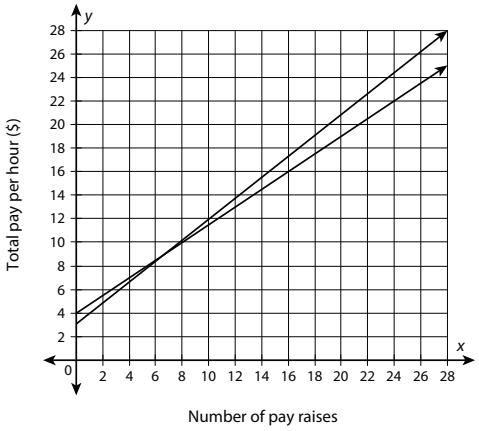
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10. The starting pay at a pizzeria is \$9.00 an hour, with a raise of \$0.30 every 6 months. The starting pay at a burger shop is \$7.50 an hour, with a raise of \$0.40 every 6 months. Which graph shows when the pay at the burger shop will be more than at the pizzeria?

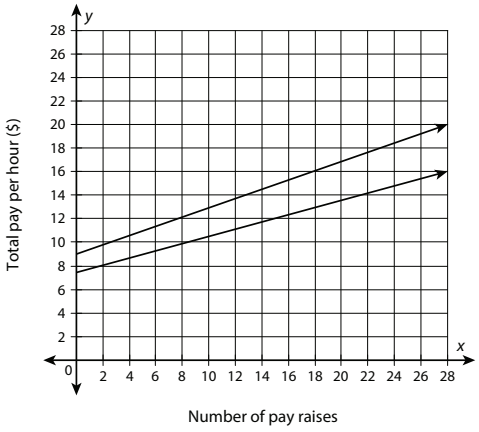
a.



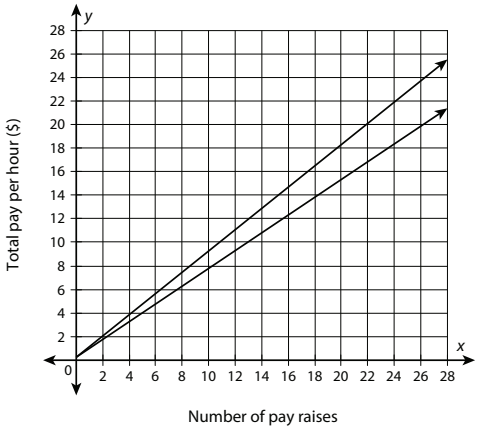
c.



b.



d.



Write a system of equations using the given information, then use the system to solve the problem.

11. A photographer sells large photos for a \$27 profit and small photos for an \$11 profit. This past year, she sold 126 photos and made a profit of \$2,250. How many of each size photo did she sell?

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Unit 3 Assessment

Circle the letter of the best answer.

1. Identify the property of equality that justifies the missing step in solving the equation.

Equation	Steps
$3x - (x + 4) = -6$	Original equation
$3x - x - 4 = -6$	Distributive property
$2x - 4 = -6$	Addition to combine like terms
$2x = -2$	
$x = -1$	Division property of equality

a. subtraction property of equality
b. addition property of equality
c. division property of equality
d. multiplication property of equality

2. What equation is missing in the steps to solve $\frac{1}{2}x + 6 = 2$?

Equation	Steps
$\frac{1}{2}x + 6 = 2$	Original equation
$\frac{1}{2}x = -4$	Subtraction property of equality
	Multiplication property of equality

a. $x = 2$
b. $x = -2$
c. $x = 8$
d. $x = -8$

continued

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3. What is the solution to the equation $4x - 7 - 9x = 13 + 5x$?

   a. $x = -2$

   b. $x = 2$

   c. $x = \frac{1}{2}$

   d. There are no solutions to this equation.

4. What is the solution to the equation $3x + 5(4x - 6) - 8 = 3x - 14$?

   a. $x = 1$

   b. $x = 1.7$

   c. $x = 1.2$

   d. There are no solutions to this equation.

5. What is the solution to the inequality $10x - 7 \geq 3x + 28$?

   a. $x \geq 5$

   b. $x \leq 5$

   c. $x > 5$

   d. $x < 5$

6. What is the solution to the inequality $\frac{4x}{7} - 6 < 2x + 4$?

   a. $x \geq -7$

   b. $x \leq -7$

   c. $x > -7$

   d. $x < -7$

7. What is the solution to the equation $4^x = \frac{1}{64}$?

   a. $x = 3$

   b. $x = -3$

   c. $x = 16$

   d. $x = -16$

continued

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8. What is the solution to the equation $25 = 2^x - 7$?

   a. $x = 12.5$

   b. $x = 9$

   c. $x = -5$

   d. $x = 5$

9. What is the solution to the system $\begin{cases} x + 2y = 15 \\ 2x + y = 9 \end{cases}$?

   a. (1, 7)

   b. (7, 1)

   c. There are infinitely many solutions to this system of equations.

   d. There are no solutions to this system of equations.

10. What is the solution to the system $\begin{cases} 2x + y = 5 \\ 6x + 3y = -9 \end{cases}$?

    a. (5, –9)

    b. (–9, 5)

    c. There are infinitely many solutions to this system of equations.

    d. There are no solutions to this system of equations.

11. What is the solution to the system $\begin{cases} 3x + 5y = 4 \\ -2x + 2y = 8 \end{cases}$?

    a. (2, –2)

    b. (–2, 2)

    c. There are infinitely many solutions to this system of equations.

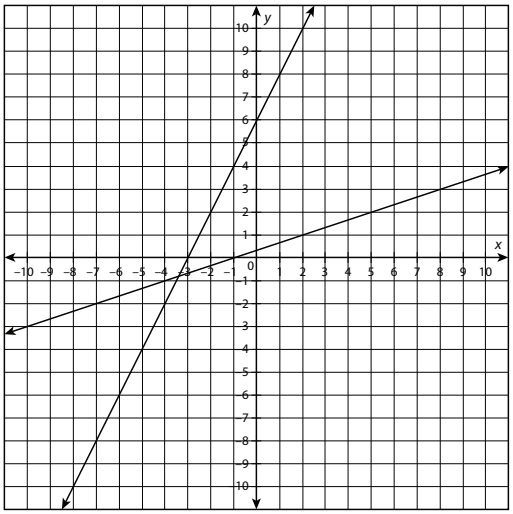
    d. There are no solutions to this system of equations.

continued

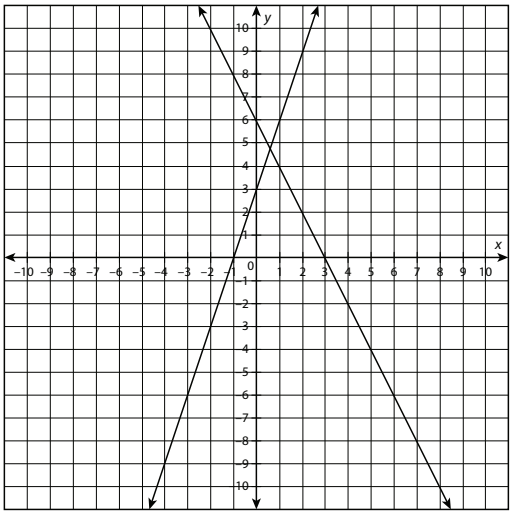
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12. Which graph represents the solution to the system $\begin{cases} 2x+y=6 \\ -x+3y=1 \end{cases}$?

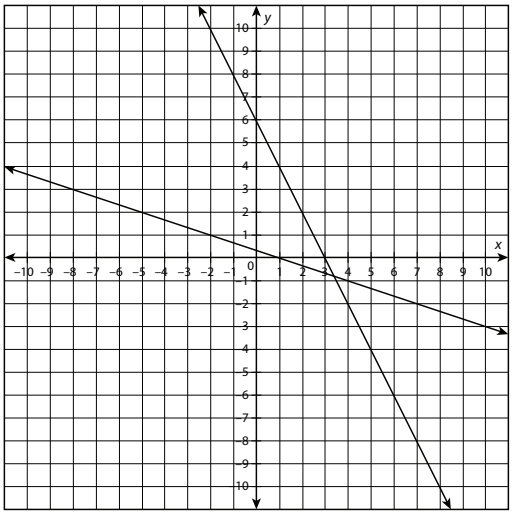
a.



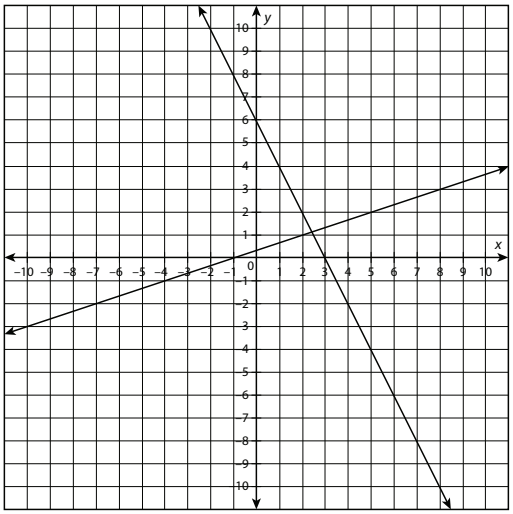
c.



b.



d.



continued

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For problem 13, write equations and/or inequalities, and then use them to solve each part.

13. Jaehee's flight to an all-star track meet was cancelled due to a storm. The airline gave Jaehee a voucher for $10 to spend at any vendor in the airport. Jaehee can only use the voucher at one vendor and, according to the voucher rules, no change will be given. Any money not spent on the voucher is lost.

    a. If a bottle of soda costs $2.50 plus an additional 6% sales tax, write an inequality to show at most how many bottles of soda Jaehee could buy.

    b. At most, how many bottles of soda could Jaehee buy?

    c. How much money would Jaehee lose if she bought the maximum number of sodas?

    d. If a bottle of water costs $2 plus an additional 6% tax, would you recommend Jaehee buy water instead? Justify your answer with mathematics.

For problem 14, write a system of equations for the given information, and then use the system to find the solution or solutions.

14. A math teacher bought 25 new scientific calculators for the classroom, some Acme calculators and some Zenith calculators. Acme calculators cost $15 each. Zenith calculators cost $12 each. The math teacher spent exactly $330. How many of each calculator did she buy?

For problem 15, write a system of equations for the given information, and then graph the system on graph paper to find the solution or solutions.

15. Kirk takes a job earning $9.50 an hour, and will get a $0.25 hourly raise every 3 months. Micah's new job pays $8.50 an hour, with a raise of $0.50 every 3 months. After how many pay raises will Micah earn more per hour than Kirk? How long will this take?

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Answer Key

Lesson 1: Solving Equations and Inequalities

Pre-Assessment, p. U3-1

1. b
2. a
3. d
4. c
5. d

Warm-Up 3.1.1, p. U3-3

1. $3.75
2. $60
3. 29 e-books

Practice 3.1.1: Properties of Equality, p. U3-17

1. addition property of equality
2. division property of equality
3. division property of equality
4. addition property of equality
5. $2x = 2.6$; division property of equality
6. $x = -4$; subtraction property of equality
7. division property of equality; addition property of equality
8. distributive property of multiplication over addition to combine like terms; subtraction property of equality; division property of equality
9.

Equation	Steps
$7x - (4x - 39) = 0$	Original equation
$3x + 39 = 0$	Distributive property of multiplication over addition to combine like terms
$3x = -39$	Subtraction property of equality
$x = -13$	Division property of equality

10.

Equation	Steps
$4(3x + 5) = -4$	Original equation
$3x + 5 = -1$	Division property of equality
$3x = -6$	Subtraction property of equality
$x = -2$	Division property of equality

Warm-Up 3.1.2, p. U3-19

1. $C = 0.45 + 0.20(n - 1)$
2. $1.05
3. more than 7 ounces, but not more than 8 ounces

Practice 3.1.2: Solving Linear Equations, p. U3-33

1. $y = 2$
2. $x = 3$
3. $r = -7$
4. $x = -3$
5. all real solutions
6. no solution
7. $x = 11$
8. $z = -1$
9. $g = (V + K)/t$
10. $l = 2s/n - a$

Warm-Up 3.1.3, p. U3-34

1. at least 6 weeks
2. Yes, Luc will have $50 left. He will have saved a total of $500; the game system costs $450.

Practice 3.1.3: Solving Linear Inequalities, p. U3-45

1. $x > 6$
2. $x \leq -3$
3. $x \leq -27$
4. $x > -10$
5. $x \geq -2$
6. $x \geq 8$
7. $x \geq -4$
8. $x > -10/3$
9. $x < 1/6$
10. $x > 10/11$

Warm-Up 3.1.4, p. U3-46

1. Hillsboro High School: 2,100 students; Mountain Ridge High School: ≈ 3,096 students
2. ≈ 996 students

Practice 3.1.4: Solving Exponential Equations, p. U3-60

1. $x = 3$
2. $x = 4$
3. $x = -2$
4. $x = -3$
5. $x = -2$
6. $x = -3$
7. $x = 2$
8. $x = 4$
9. $x = 3$
10. $x = 1/4$

Progress Assessment, p. U3-61

1. b
2. c
3. a
4. d
5. a
6. b
7. c
8. b
9. a
10. a
11. The equation $2B - 14 = B - 5$ results in the number of series the Braves had played in. The Braves had played in 9 series, so $B = 9$.

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Lesson 2: Solving Systems of Equations

Pre-Assessment, p. U3-63

1. b
2. a
3. c
4. d
5. a

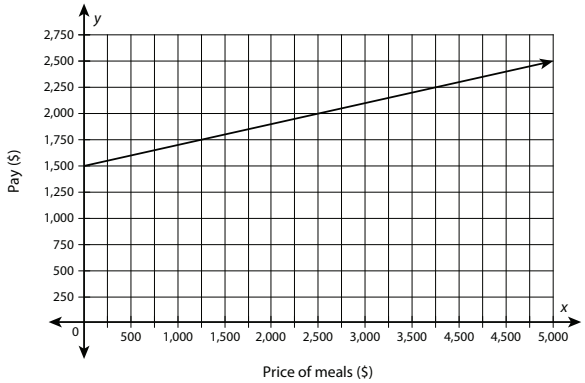
Warm-Up 3.2.1, p. U3-68

1. Copier 1: $30,300; Copier 2: $30,375; Copier 1 is cheaper.
2. Copier 1: $29,682; Copier 2: $29,602.50; Copier 2 is cheaper during the first year. After the first year, Copier 1 is less expensive because of the cheaper cost per page.

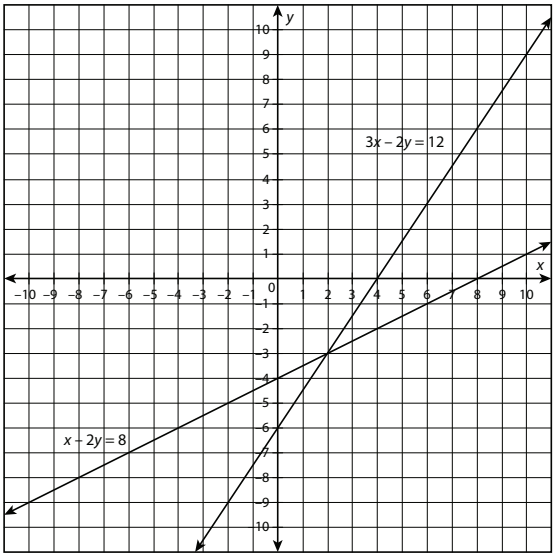
Practice 3.2.1: Solving Systems of Linear Equations by Substitution and Elimination, p. U3-85

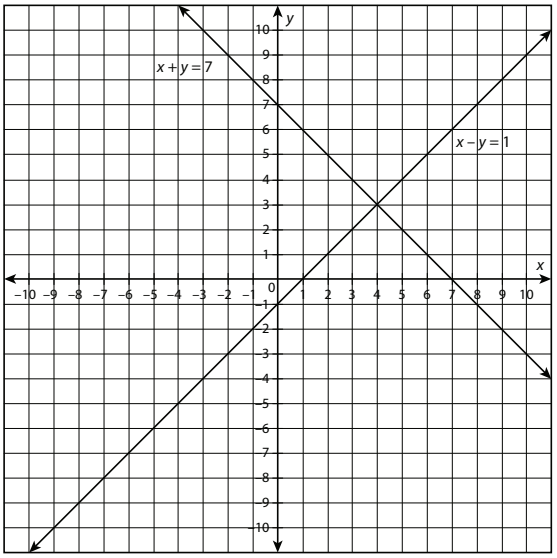
1. (4, 1)
2. $\begin{cases} y = 0.05x + 600 \\ y = 0.07x + 450 \end{cases}$; (7500, 975);
   with $7,500 in sales, Antony will earn $975.
3. (1, 9)
4. $\begin{cases} 2x + y = 27.50 \\ 4x + 3y = 59.5 \end{cases}$; (11.50, 4.50);
   1 plate of spaghetti costs $11.50 and 1 salad costs $4.50.
5. no solution
6. (8, 6)
7. $\begin{cases} x + y = 16 \\ 8x + 6y = 120 \end{cases}$; (12, 4);
   12 round tables and 4 square tables are needed.
8. $\begin{cases} x + y = 15 \\ 12x + 9y = 153 \end{cases}$; (6, 9);
   6 adults and 9 children purchased tickets.
9. $\begin{cases} x + y = 1200 \\ 0.059x + 0.04y = 64.15 \end{cases}$; (850, 350);
   $850 was invested at 5.9% annual interest and $350 was invested at 4% annual interest.
10. $\begin{cases} x + y = 1.2 \\ 2x + 4y = 3.4 \end{cases}$; (0.7, 0.5);
    Carmen walked for 0.7 hours and jogged for 0.5 hours.

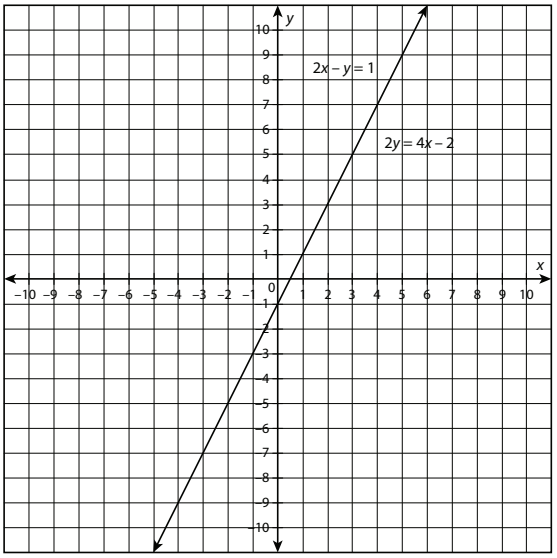
Warm-Up 3.2.2, p. U3-87

1. $y = 1500 + 0.20x$; $1,570
2. $750
3. 

Practice 3.2.2: Solving Systems of Linear Equations by Graphing, p. U3-104

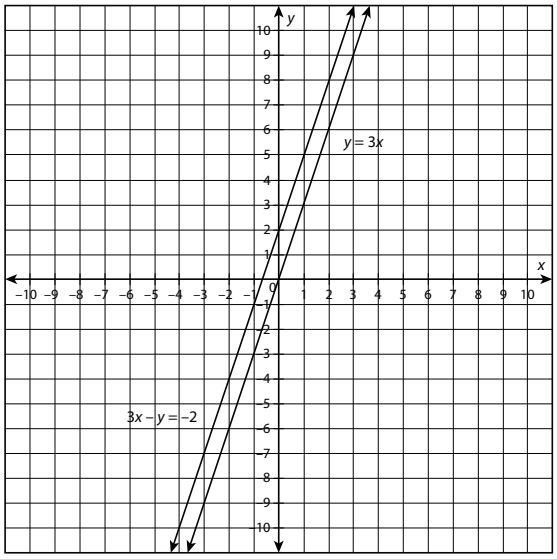
1. one solution: (2, –3)


2. one solution: (4, 3)


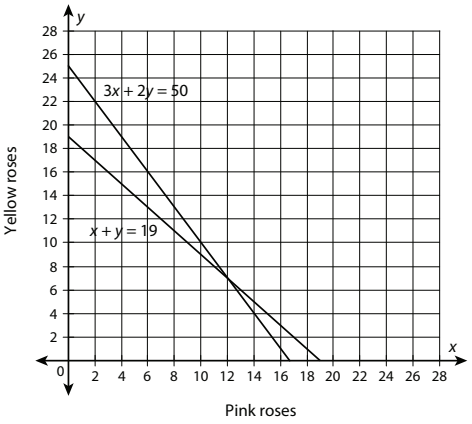
3. infinitely many solutions


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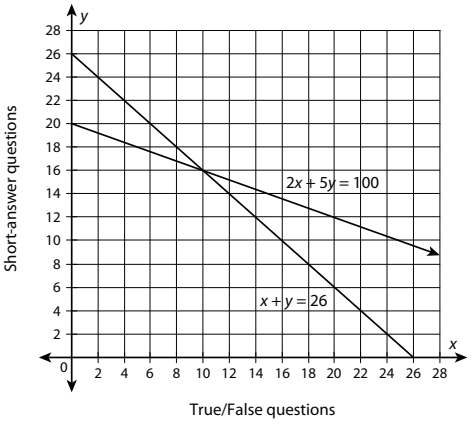
4. no solution



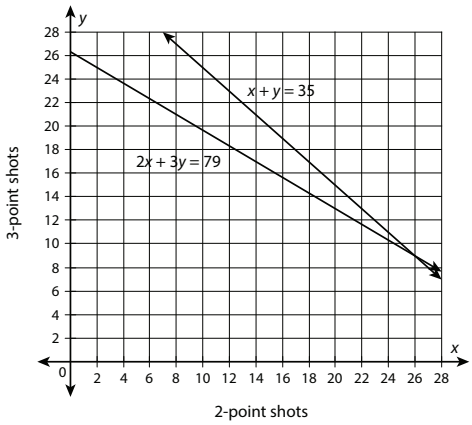
5. $\begin{cases} 3x+2y=50 \\ x+y=19 \end{cases}$; (12, 7); 12 pink roses and 7 yellow roses



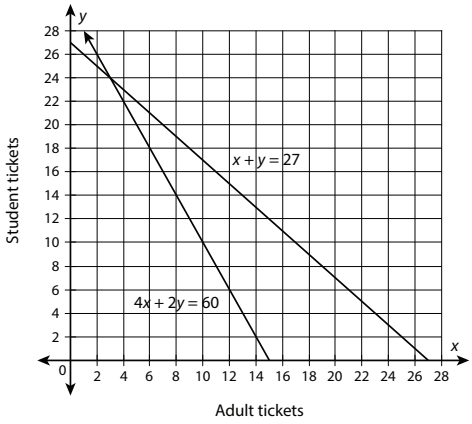
6. $\begin{cases} 2x+5y=100 \\ x+y=26 \end{cases}$; (10, 16); 10 true/false, 16 short answer



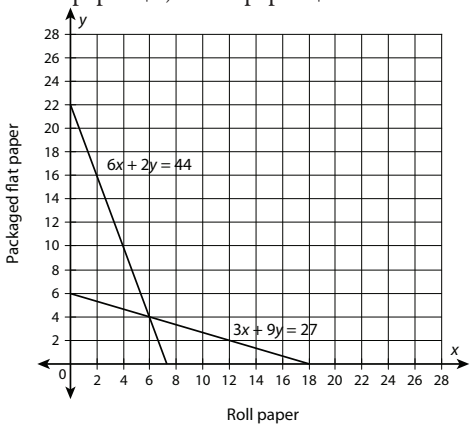
7. $\begin{cases} 2x+3y=79 \\ x+y=35 \end{cases}$; (26, 9); 26 2-point shots, 9 3-point shots



8. $\begin{cases} 4x+2y=60 \\ x+y=27 \end{cases}$; (3, 24); 3 adult tickets, 24 student tickets

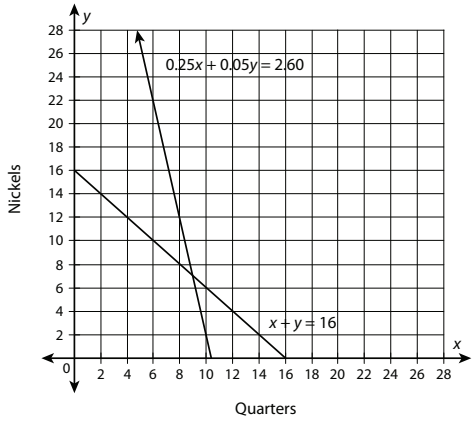


9. $\begin{cases} 6x+2y=44 \\ 3x+9y=54 \end{cases}$; (6, 4);
rolls of paper: \$6; folded paper: \$4



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10. $\begin{cases} 0.25x + 0.05y = 2.60 \\ x + y = 16 \end{cases}$; (9, 7); 9 quarters and 7 nickels



Progress Assessment, p. U3-105

1. d
2. a
3. b
4. b
5. c
6. a
7. c
8. d
9. b
10. a

11. $\begin{cases} 27x + 11y = 2250 \\ x + y = 126 \end{cases}$; (54, 72);
54 large photos and 72 small photos

Unit Assessment
p. U3-109

1. b
2. d
3. a
4. c
5. a
6. c
7. b
8. d
9. a
10. d
11. b
12. d

13. Answers:
    a. $2.50x + 0.06(2.50)x \le 10$ or $(1.06)2.50x \le 10$
    b. Jaehee can buy 3 sodas at most.
    c. \$2.05
    d. Answers will vary. Sample answer: Jaehee should buy bottles of water instead of soda because she could buy 1 more bottle of water than soda and she would lose less money. If Jaehee bought bottles of water, she could buy 4 bottles and she would only lose \$1.52. If she bought bottles of soda, she could only buy 3 bottles and she would lose \$2.05.

Number of bottles that can be purchased

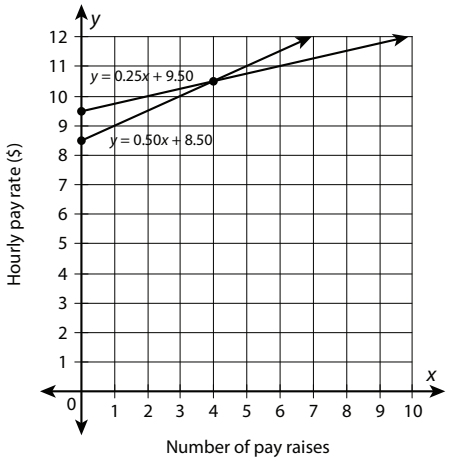
Bottles of soda	Bottles of water
$(1.06)2.50x \le 10$	$(1.06)2x \le 10$
$2.65x \le 10$	$2.12x \le 10$
$x \le 3.77$	$x \le 4.71$

Amount lost (\$)

Bottles of soda	Bottles of water
$10 - [(1.06)2.50x]$	$10 - [(1.06)2x]$
$10 - [(1.06)2.50(3)]$	$10 - [(1.06)2(4)]$
$10 - 7.95$	$10 - 8.48$
2.05	1.52

14. $\begin{cases} x + y = 25 \\ 15x + 12y = 330 \end{cases}$; (10, 15);
10 Acme calculators and 15 Zenith calculators

15. $\begin{cases} y = 0.25x + 9.50 \\ y = 0.50x + 8.50 \end{cases}$



Micah will earn more per hour after 4 pay raises or 1 year of employment.

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Student Book Answer Key

Lesson 1: Solving Equations and Inequalities

Practice 3.1.1: Properties of Equality, p. U3-9

1. subtraction property of equality
2. multiplication property of equality
3. division property of equality
4. addition property of equality
5. addition property of equality; $x = 2$
6. $x/4 = 11$; multiplication property of equality
7. addition property of equality; multiplication property of equality; division property of equality
8. division property of equality; addition property of equality; division property of equality
9.

Equation	Steps
$\frac{4x}{9} = 20$	Original equation
$4x = 180$	Multiplication property of equality
$x = 45$	Division property of equality

10.

Equation	Steps
$13 = \frac{1}{3}x - 5$	Original equation
$18 = \frac{1}{3}x$	Addition property of equality
$54 = x$	Multiplication property of equality
$x = 54$	Symmetric property of equality

Practice 3.1.2: Solving Linear Equations, p. U3-20

1. $x = 4$
2. $x = 4/5$
3. $t = 3$
4. $a = 5$
5. all real numbers
6. no solution
7. $a = 20$
8. $m = 14$
9. $a = 5Kn + k$
10. $c = 2A/h - b$

Practice 3.1.3: Solving Linear Inequalities, p. U3-26

1. $x < -2$
2. $x \leq 8$
3. $x > -3$
4. $y < 2$
5. $x > 2$
6. $x > -24$
7. $x > 3/19$
8. $x \geq -9$
9. $x < 9/8$
10. $x < 1$

Practice 3.1.4: Solving Exponential Equations, p. U3-34

1. $x = 4$
2. $x = 6$
3. $x = -2$
4. $x = -4$
5. $x = -3$
6. $x = -2$
7. $x = 4$
8. $x = 3$
9. $x = 3$
10. $x = 1/3$

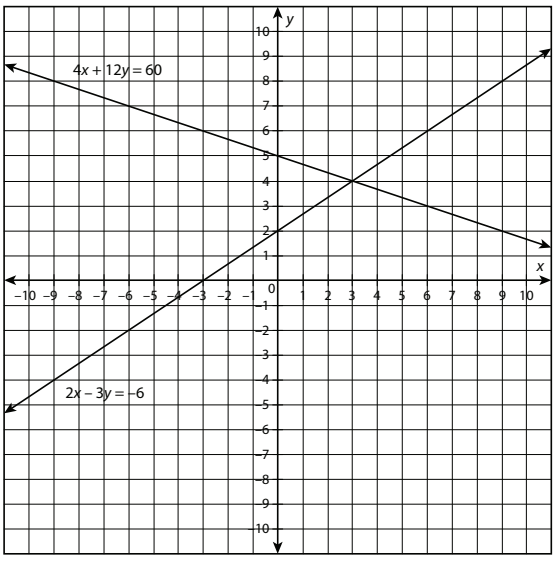
Lesson 2: Solving Systems of Equations

Practice 3.2.1: Solving Systems of Linear Equations by Substitution and Elimination, p. U3-44

1. (–18, 82)
2. $\begin{cases} y = 0.08x \\ y = 0.06x + 8 \end{cases}$; (400, 32);
   at 400 minutes, the cost for each plan will be $32.
3. infinitely many solutions
4. $\begin{cases} 3x + 5y = 36.10 \\ 2x + 3y = 23.65 \end{cases}$; (9.95, 1.25);
   1 pizza costs $9.95 and 1 beverage costs $1.25.
5. (–5, 2)
6. (6, 4)
7. $\begin{cases} x + y = 10 \\ 1.10x + 1.35y = 12 \end{cases}$; (6, 4); 6 muffins and 4 bottles of milk
8. $\begin{cases} x + y = 20 \\ 8x + 5y = 118 \end{cases}$; (6, 14);
   6 adult and 14 child tickets were purchased.
9. $\begin{cases} x + y = 1750 \\ 0.035x + 0.047y = 73.85 \end{cases}$; (700, 1050);
   $700 was invested at 3.5% and $1,050 was invested at 4.7%.
10. $\begin{cases} 7x + 4y = 490 \\ 5x + 2y = 290 \end{cases}$; (30, 70);
    each shirt costs $30 and each pair of shoes costs $70.

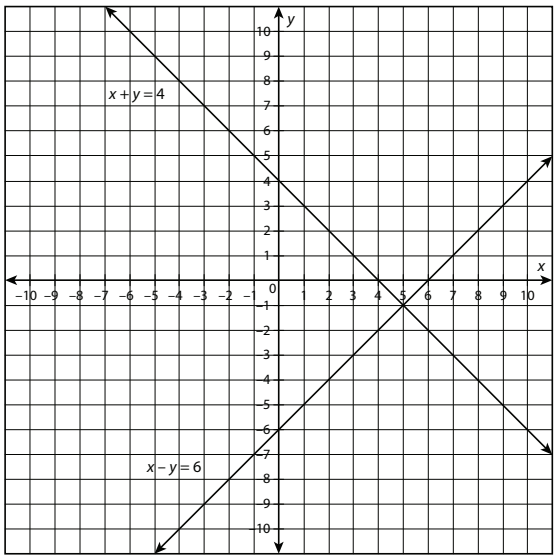
Practice 3.2.2: Solving Systems of Linear Equations by Graphing, p. U3-56

1. one solution: (3, 4)

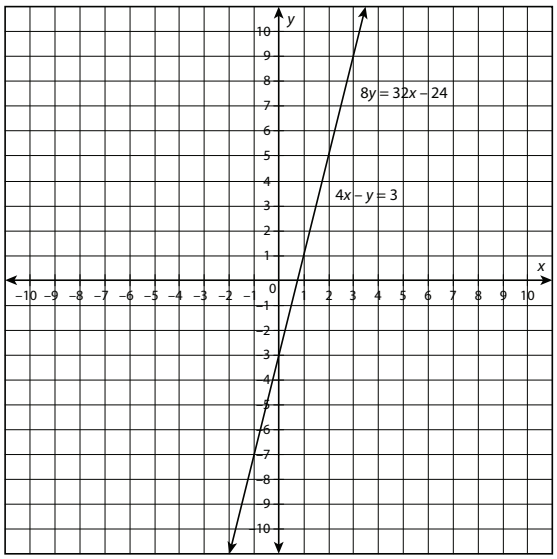


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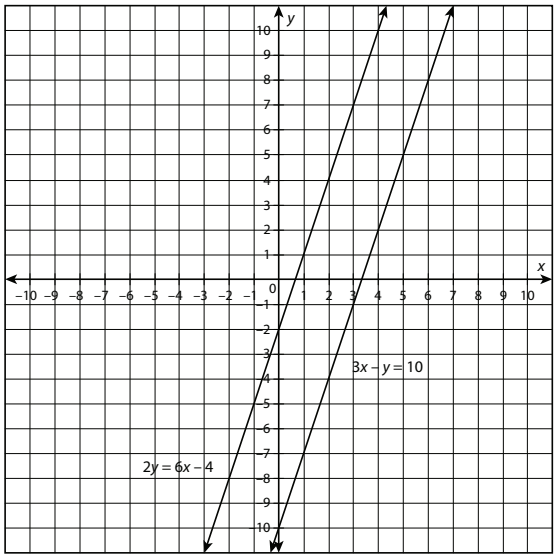
2. one solution: (5, –1)



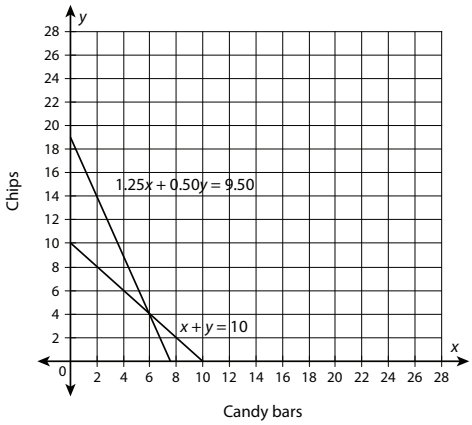
3. infinitely many solutions



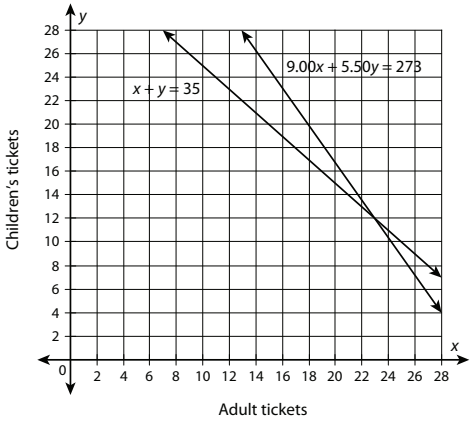
4. no solution



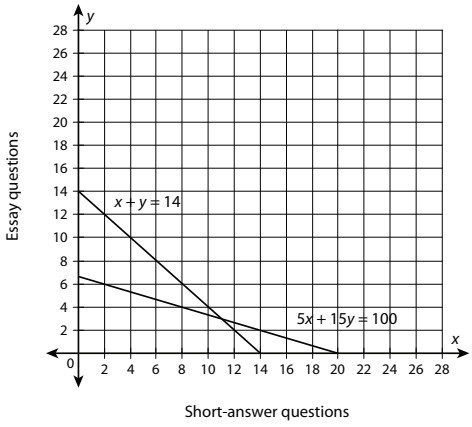
5. $\begin{cases} x+y=10 \\ 1.25x+0.50y=9.50 \end{cases}$; (6, 4); 6 candy bars and 4 chips



6. $\begin{cases} x+y=35 \\ 9.00x+5.50y=273 \end{cases}$; (23, 12);
23 adult tickets and 12 children's tickets



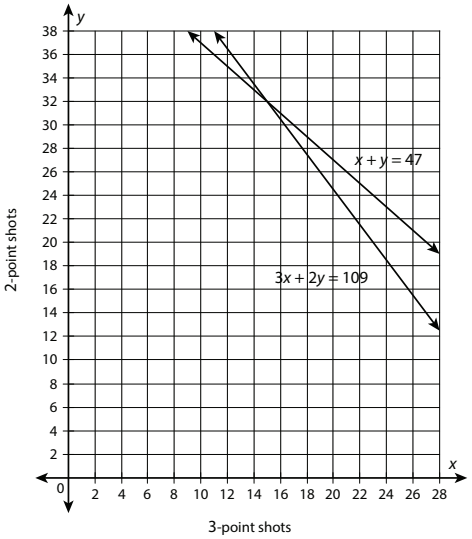
7. $\begin{cases} x+y=14 \\ 5x+15y=100 \end{cases}$; (11, 3);
11 short-answer questions and 3 essay questions



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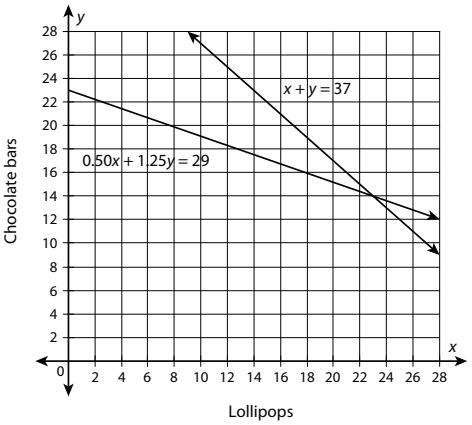
8. $\begin{cases} x+y=47 \\ 3x+2y=109 \end{cases}$; (15, 32);

15 3-point shots and 32 2-point shots

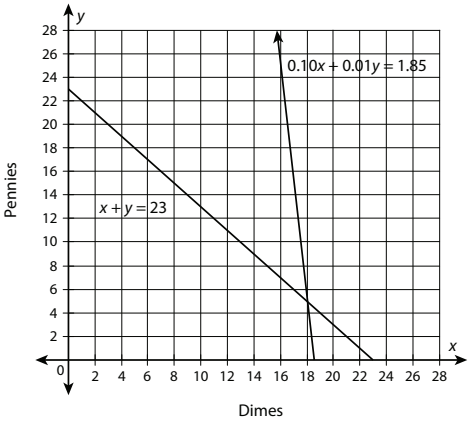


9. $\begin{cases} 0.50x+1.25y=29 \\ x+y=37 \end{cases}$; (23, 14);

Abe sold 23 lollipops and 14 chocolate bars.



10. $\begin{cases} x+y=23 \\ 0.10x+0.01y=1.85 \end{cases}$; (18, 5); 18 dimes and 5 pennies



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Station Activities Set 1: Solving Systems by Substitution and Elimination

Instruction

Goal: To provide opportunities for students to develop concepts and skills related to solving systems of linear equations using substitution and elimination

Common Core State Standard

A–REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Student Activities Overview and Answer Key

Station 1

Students will be given 10 blue, 10 yellow, and 40 red algebra tiles. They will also be given index cards with *x*, *y*, +, –, and = written on them. They will use the algebra tiles and index cards to model a system of linear equations. They will rearrange the tiles and index cards to solve for *x*. Then they will solve for *y* to find the solution to the system of linear equations.

Answers

1. x and y
2. $x = -y + 10$
3. $2(-y + 10) + y = 15$; $y = 5$
4. Substitute the value of y into the first equation; $x + (5) = 10$; $x = 5$
5. $x + y = 10$ and $5 + 5 = 10$; $2x + y = 15$ and $2(5) + 5 = 15$

Station 2

Students will be given a system of linear equations and nine index cards with the following equations written on them:

$y = 1$; $10x = 30$; $y = 2x - 5$; $x = 3$; $2x - 12x + 30 = 0$; $-10x + 30 = 0$;

$2(3) - y = 5$; $2x - 6(2x - 5) = 0$; and $6 - y = 5$

Each index card represents a step in solving a system of linear equations by substitution. Students will work together to arrange the steps in the correct order. Then they will give the strategy they used to arrange the index cards.

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Answers

1. $y = 2x - 5$
2. $2x - 6(2x - 5) = 0$
3. $2x - 12x + 30 = 0$
4. $-10x + 30 = 0$
5. $10x = 30$
6. $x = 3$
7. $2(3) - y = 5$
8. $6 - y = 5$
9. $y = 1$
10. Answers will vary. Possible answer: I looked for one variable written in terms of the other variable. Then I solved by substitution in a step-by-step manner.

Station 3

Students will be given 10 yellow, 10 red, and 20 green algebra tiles. They will also be given index cards with *x*, *y*, –, +, and = written on them. They will use the algebra tiles and index cards to model and solve systems of linear equations by using the elimination method.

Answers

1. yellow algebra tiles
2. red algebra tiles
3. Student drawings should show 7 yellow tiles $(7x) = 14$ green tiles.
4. $x = 2$
5. Substitute *x* into the first equation to solve for *y*.
6. (2, 1)
7. (2, 16)

Station 4

Students will be given eight index cards with the following written on them:

$$5x = 0;\ \begin{cases} 10x + 4y = 16 \\ -10x - 15y = -60 \end{cases};\ (0, 4);\ \begin{cases} 2(5x + 2y) = 2(8) \\ -5(2x + 3y) = -5(12) \end{cases};$$

$$x = 0;\ -11y = -44;\ 5x + 2(4) = 8;\ y = 4$$

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Station Activities Set 1: Solving Systems by Substitution and Elimination

Instruction

Students will work together to arrange the index cards to reflect how to find the solution of the system of linear equations. Then they will explain the strategy they used to find this order.

Answers

1. $\begin{cases} 2(5x + 2y) = 2(8) \\ -5(2x + 3y) = -5(12) \end{cases}$
2. $\begin{cases} 10x + 4y = 16 \\ -10x - 15y = -60 \end{cases}$
3. $-11y = -44$
4. $y = 4$
5. $5x + 2(4) = 8$
6. $5x = 0$
7. $x = 0$
8. $(0, 4)$
9. Answers will vary. Possible answers: We figured out which variable was going to be multiplied and added to cancel it out. Then we figured out the rest of the steps from this information.

Materials List/Setup

Station 1 10 blue, 10 yellow, and 40 red algebra tiles; index cards with *x*, *y*, +, –, and = written on them

Station 2 nine index cards with the following written on them:

$y = 1$; $10x = 30$; $y = 2x - 5$; $x = 3$; $2x - 12x + 30 = 0$; $-10x + 30 = 0$;

$2(3) - y = 5$; $2x - 6(2x - 5) = 0$; and $6 - y = 5$

Station 3 10 yellow, 10 red, and 20 green algebra tiles; index cards with *x*, *y*, –, +, and = written on them

Station 4 eight index cards with the following written on them:

$5x = 0$; $\begin{cases} 10x + 4y = 16 \\ -10x - 15y = -60 \end{cases}$; $(0, 4)$; $\begin{cases} 2(5x + 2y) = 2(8) \\ -5(2x + 3y) = -5(12) \end{cases}$;

$x = 0$; $-11y = -44$; $5x + 2(4) = 8$; $y = 4$

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. How do you solve systems of linear equations using substitution?
2. Why does the "substitution method" have this name?
3. How do you solve a system of linear equations by elimination?
4. What is an example of a system of linear equations that you can solve by elimination?
5. How do you solve a system of linear equations by multiplication and elimination?
6. What is an example of a system of linear equations that you can solve by multiplication and elimination?
7. Does it matter which variable you cancel out through multiplication and/or elimination? Why or why not?
8. What type of relationship must the same variable in each equation have before you can cancel that variable?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. Solve for one variable in the first equation. Substitute this quantity in for that variable in the second equation. Solve for the variable still in the equation. Then substitute your answer for that variable into the original equation to find the value of the other variable.
2. You substitute one variable in terms of the other variable to find x or y.
3. Add the two equations by combining like terms so that one variable is eliminated. Solve for the other variable. Then solve for the original variable.
4. Answers will vary. Possible answer: $\begin{cases} x + 2y = 4 \\ x - 2y = 10 \end{cases}$
5. Multiply the same variable in each equation so they have coefficients of opposite values. Then use addition to cancel out that variable. Solve for the other variable. Then solve for the original variable.

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6. Answers will vary. Possible answer: $\begin{cases} -2x + y = 2 \\ x + 2y = 10 \end{cases}$
7. No, it doesn't matter. You will arrive at the same answer.
8. The same variable must have opposite coefficients in the two equations.

Possible Misunderstandings/Mistakes

- Incorrectly solving for one variable when rewriting an equation in step 1 of the process
- Incorrectly substituting x in the first equation for y in the second equation
- Not double-checking that the values of x and y satisfy both equations
- Not making sure that the same variable has opposite coefficients in both equations before canceling it out
- When using multiplication, forgetting to apply it to all terms in the equation
- Selecting the wrong multipliers to multiply by each equation in order to cancel out a variable
- Trying to solve the system without canceling out a variable through the elimination method

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Station 1

At this station, you will find 10 blue algebra tiles, 10 yellow algebra tiles, and 40 red algebra tiles. The blue tiles represent x, the yellow tiles represent y, and the red tiles represent the constant term. You will also find index cards with x, y, +, −, and = written on them.

As a group, arrange the algebra tiles and index cards to model the following system of linear equations:

$$\begin{cases} x + y = 10 \\ 2x + y = 15 \end{cases}$$

1. What values do you need to find in order to solve this system of linear equations?

2. Rearrange the tiles that represent $x + y = 10$ so that it is solved for x. Write the algebraic equivalent of the rearranged tiles.

3. Replace each x in the tile representation of the second equation, $2x + y = 15$, with the tiles that you found to be equal to x in problem 2. Write the algebraic equivalent of this step.

   Manipulate the tiles in this new equation to solve for y. What is the value of y?

4. How can you use this value of y to find x? Show this step with your tiles, and write the algebraic equivalent.

   What is the value of x?

5. Verify that x and y satisfy both equations. Show your work.

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Station 2

At this station, you will find nine index cards with the following written on them:

$y = 1$; $10x = 30$; $y = 2x - 5$; $x = 3$; $2x - 12x + 30 = 0$; $-10x + 30 = 0$;

$2(3) - y = 5$; $2x - 6(2x - 5) = 0$; and $6 - y = 5$

Each index card represents one step in solving the linear system of equations by substitution. Shuffle the index cards. Work as a group to arrange the index cards in the correct order to solve the following system of linear equations by substitution:

$$\begin{cases} 2x - y = 5 \\ 2x - 6y = 0 \end{cases}$$

List the steps for solving the system.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. What strategy did you use to find the order of the steps?

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Station 3

At this station, you will find 10 yellow, 10 red, and 20 green algebra tiles. The yellow algebra tiles represent x, the red algebra represent y, and the green algebra tiles represent the constant term. You will also find index cards with x, y, –, +, and = written on them.

As a group, arrange the algebra tiles and index cards to model the following system of linear equations by elimination:

$$\begin{cases} 3x + 2y = 8 \\ 4x - 2y = 6 \end{cases}$$

To solve this system of linear equations, you add the two equations.

1. Which algebra tiles will be added together?

2. Which algebra tiles will cancel each other out?

3. Arrange the algebra tiles to model the new equation you found. Draw a picture of this equation in the following space.

4. Move the algebra tiles around to solve for the remaining variable. What is this variable and its solution?

continued

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5. How can you find the value of the other variable?

6. What is the solution to this system of linear equations?

7. Use your algebra tiles to model and solve the following system of linear equations. Write the solution of the system.

$$\begin{cases} -3x + y = 10 \\ 3x + y = 22 \end{cases}$$

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Station 4

At this station, you will find eight index cards with the following written on them:

$$5x = 0;\ \begin{cases} 10x + 4y = 16 \\ -10x - 15y = -60 \end{cases};\ (0, 4);\ \begin{cases} 2(5x + 2y) = 2(8) \\ -5(2x + 3y) = -5(12) \end{cases};$$

$$x = 0;\ -11y = -44;\ 5x + 2(4) = 8;\ y = 4$$

Each index card represents a step in solving a system of linear equations by elimination. Shuffle the index cards. Work as a group to arrange the index cards in the correct order to solve the following system of linear equations by elimination:

$$\begin{cases} 5x + 2y = 8 \\ 2x + 3y = 12 \end{cases}$$

List the steps for solving the system.

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. What strategy did you use to arrange the index cards?

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Instruction

Goal: To provide opportunities for students to develop concepts and skills related to solving linear systems of equations by graphing

Common Core State Standard

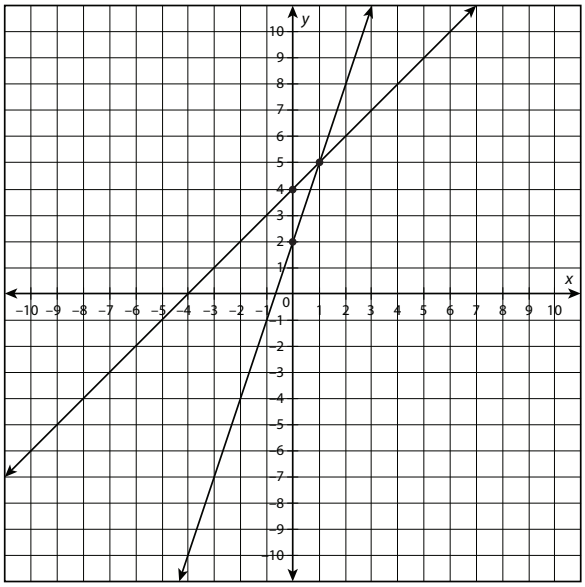
A–REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Student Activities Overview and Answer Key

Station 1

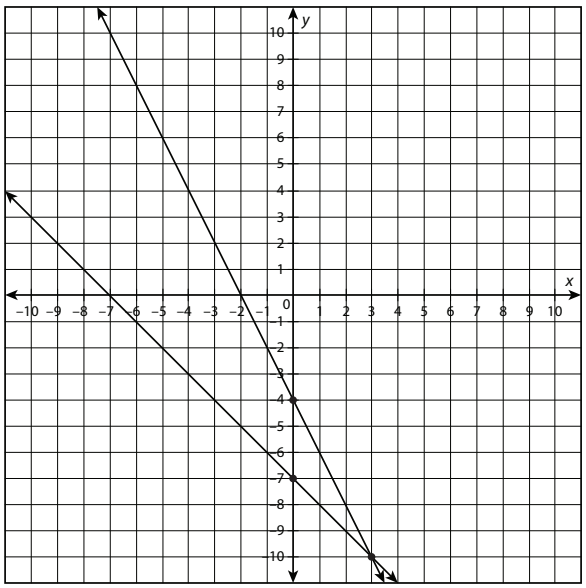
Students will be given graph paper and a ruler. Students will graph a system of linear equations. They will find the point of intersection of the lines and realize that this is the solution to the linear system. They will double-check the point of intersection by substituting it into the original equations.

Answers

1. 

2. Yes, (1, 5)
3. The point of intersection is the solution of the system of linear equations.
4. Substitute (1, 5) into both equations to verify that it satisfies both equations.

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5. 

6. (3, –10)

Station 2

Students will be given four index cards with the following systems of linear equations written on them:

$$\begin{cases} 10x - 2y = 10 \\ 5x - y = 5 \end{cases}; \begin{cases} 5x + y = -3 \\ 7x + y = -5 \end{cases}; \begin{cases} y = 4 \\ -x + y = 10 \end{cases}; \begin{cases} y = 2x + 5 \\ -2x + y = 8 \end{cases}$$

Students will work together to match each system of linear equations with the appropriate graph. They will explain the strategy they used to match the graphs.

Answers

1. $\begin{cases} 5x + y = -3 \\ 7x + y = -5 \end{cases}$
2. $\begin{cases} y = 4 \\ -x + y = 10 \end{cases}$
3. $\begin{cases} 10x - 2y = 10 \\ 5x - y = 5 \end{cases}$
4. $\begin{cases} y = 2x + 5 \\ -2x + y = 8 \end{cases}$
5. Answers will vary. Possible answer: Finding the point of intersection of the lines. This is the solution to the linear system. Substitute this point into the equations to see which systems of linear equations have this solution.

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Instruction

Station 3

Students will be given linear equations and will find points that satisfy the equations. Then they will look at the equations as a linear system. They will find the solution, or no solution, of the linear system. They will describe the strategy they used to determine the solution of the linear system.

Answers

1.

x	*y*
–5	–14
0	–4
5	6
10	16
15	26

2.

x	*y*
–4	37
0	31
8	19
10	16
12	13

3. Yes, (10, 16)
4. They had the same ordered pair of (10, 16), which means the lines intersect at that point.

5.

x	*y*
–2	6
0	10
2	14
4	18

6.

x	*y*
–2	–14
0	–10
2	–6
4	–2

7. The table of values may or may not contain the solution.
8. You can use the slope of each line and *y*-intercepts, or you can graph the equations and see if they intersect.
9. There is no solution. Each equation has the same slope, but different *y*-intercepts.

Station 4

Students will be given a graphing calculator. Students will be shown how to find the solution of a system of linear equations using the graphing calculator. Then they will use the graphing calculator to find the solutions of given systems of linear equations.

Answers

1. (–1.67, –9.67)
2. (–8, –12)
3. (1.29, 3.86)
4. (0.73, 3.27)

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Instruction

Materials List/Setup

Station 1 graph paper; ruler

Station 2 four index cards with the following systems of linear equations written on them:

$\begin{cases} 10x - 2y = 10 \\ 5x - y = 5 \end{cases}$; $\begin{cases} 5x + y = -3 \\ 7x + y = -5 \end{cases}$; $\begin{cases} y = 4 \\ -x + y = 10 \end{cases}$; $\begin{cases} y = 2x + 5 \\ -2x + y = 8 \end{cases}$

Station 3 none

Station 4 graphing calculator

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. How do you solve systems of linear equations by graphing?
2. How do you know if a system of linear equations has infinitely many solutions?
3. How do you know if a system of linear equations has no solutions?
4. How can you use a graphing calculator to graph and solve systems of linear equations?
5. Give an example of a graph of a linear system that is used in the real world.

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. Graph each line. Their point of intersection is the solution to the linear system.
2. Graph each line. If the lines are the same line then they have infinite solutions. In other words, if the lines have the same slope and *y*-intercept, then there are infinite solutions.
3. Graph each line. If the lines are parallel or have no point of intersection, the system has no solutions.
4. Type the equations into Y1 and Y2. Hit "GRAPH," "2ND," "Trace," and select "5: Intersect." Then hit the "ENTER" key three times to find the point of intersection. The point of intersection is the solution to the linear system.
5. Answers will vary. Possible answer: comparing service plans from two different cell phone companies

Possible Misunderstandings/Mistakes

- Incorrectly graphing the equations by not finding the correct slope and intercepts
- Not realizing that equations with the same slope and *y*-intercept are the same line. If they are a system of linear equations then the system has infinite solutions.
- Not realizing that parallel lines have no solutions
- Not realizing the point of intersection of the two lines is the solution to the system of linear equations

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Station 1

At this station, you will find graph paper and a ruler. You can find the solution to a system of linear equations by graphing each equation in the same coordinate plane.

1. As a group, graph each of the following linear equations on the same graph.

   $y = 3x + 2$

   $y = x + 4$

2. Do the lines intersect? If so, find the point at which the two lines intersect.

3. What does this point of intersection tell you about the two equations?

4. How can you double-check your answer to problem 3?

5. As a group, graph the system of linear equations on a new graph.

$$\begin{cases} y = -2x - 4 \\ y = -x - 7 \end{cases}$$

6. Using this graph, what is the solution to the linear system?

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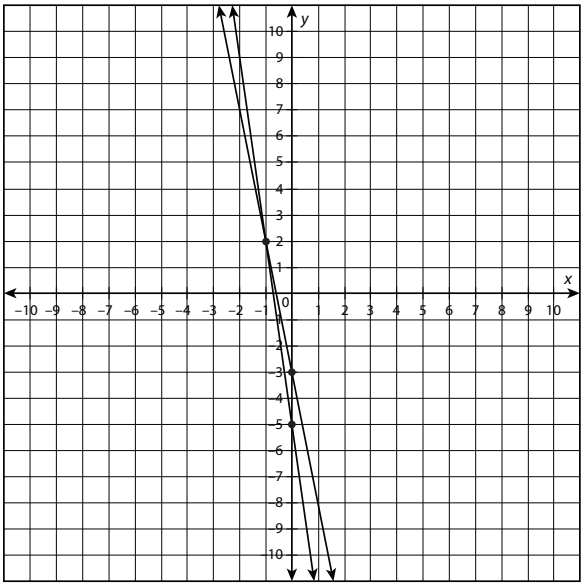
Station 2

At this station, you will find four index cards with the following linear systems of equations written on them:

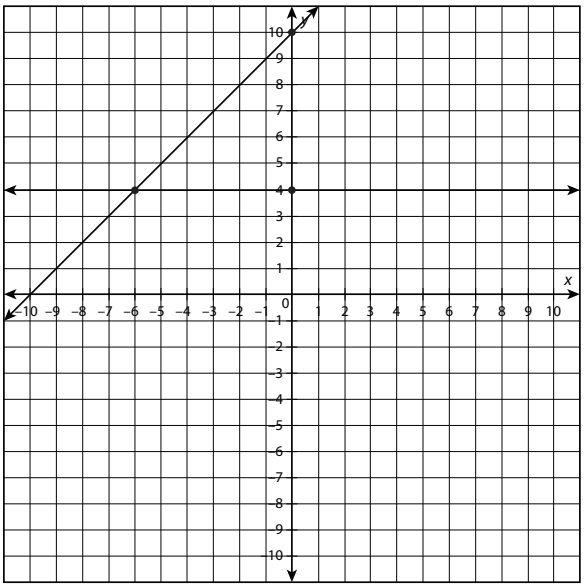
$$\begin{cases} 10x - 2y = 10 \\ 5x - y = 5 \end{cases}; \begin{cases} 5x + y = -3 \\ 7x + y = -5 \end{cases}; \begin{cases} y = 4 \\ -x + y = 10 \end{cases}; \begin{cases} y = 2x + 5 \\ -2x + y = 8 \end{cases}$$

Work together to match each system of linear equations with the appropriate graph for problems 1–4. Write the appropriate system of linear equations beside each graph.

1.



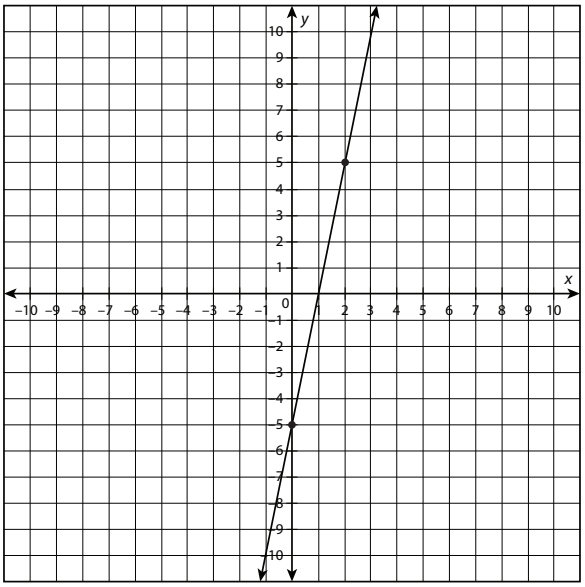
2.



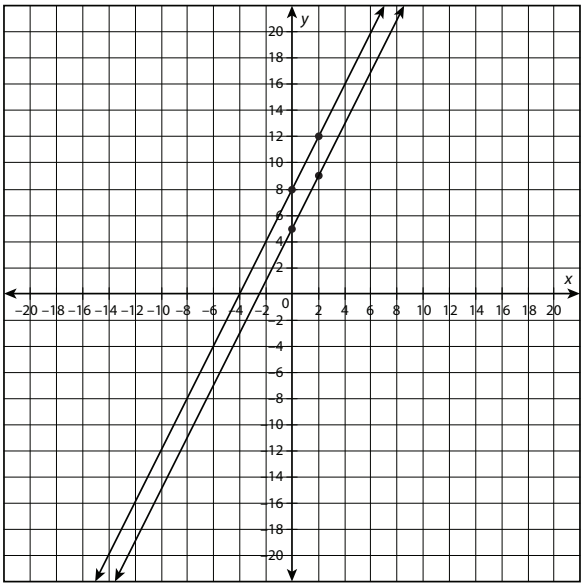
continued

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3.



4.



5. What strategy did you use to match the systems of linear equations with the appropriate graph?

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Station 3

Use what you know about linear equations to solve the following problems.

1. Complete the following table for the linear equation $2x - y = 4$.

x	y
–5	
0	
5	
10	
15	

2. Complete the following table for the linear equation $3x + 2y = 62$.

x	y
–4	
0	
8	
10	
12	

3. If $\begin{cases} 2x - y = 4 \\ 3x + 2y = 62 \end{cases}$ were a set of linear equations, would it have a solution? If so, what would it be?

4. What strategy did you use in problem 3 to find your answer?

5. Complete the following table for the linear equation $2x = y - 10$.

x	y
–2	
0	
2	
4	

continued

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6. Complete the following table for the linear equation $2x = y + 10$.

x	y
–2	
0	
2	
4	

7. If $\begin{cases} 2x = y - 10 \\ 2x = y + 10 \end{cases}$ were a set of linear equations, why would you need more information than just the values in the tables you found for problems 5 and 6 to determine any solutions?

8. What information would you need to determine whether or not a system of linear equations has a solution, infinite solutions, or no solutions?

9. Does $\begin{cases} 2x = y - 10 \\ 2x = y + 10 \end{cases}$ have a solution? Why or why not?

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Station 4

At this station, you will find a TI-83/84 or similar graphing calculator. You can graph systems of linear equations on your calculator to find their point of intersection. Work together to graph systems of linear equations on your calculator.

Given: $\begin{cases} y = 4x - 3 \\ y = x - 8 \end{cases}$

Finding the point of intersection on a TI-83/84:

1. Hit the "Y =" key. Type Y1 = $4x - 3$.
2. Type Y2 = $x - 8$.
3. Hit the "GRAPH" key.
4. Hit "2ND" and then "TRACE". Select "5: Intersect".
5. Hit "ENTER" three times. You will be given the x- and y-values of the intersection.

1. What is the intersection/solution of $\begin{cases} y = 4x - 3 \\ y = x - 8 \end{cases}$?

Using the same steps, find the solution to each of the following systems of linear equations on your graphing calculator.

2. $\begin{cases} y = 3x + 12 \\ y = x - 4 \end{cases}$

   Solution = _____

3. $\begin{cases} y = -4x + 9 \\ y = 3x \end{cases}$

   Solution = _____

4. $\begin{cases} 2x + 2y = 8 \\ y = 10x - 4 \end{cases}$

   Solution = _____

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Instruction

Goal: To provide opportunities for students to develop concepts and skills related to solving systems of linear equations using graphing, substitution, and elimination methods

Common Core State Standards

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

A–CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★

A–REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A–REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A–REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

Student Activities Overview and Answer Key

Station 1

Students will be given a real-world application of systems of linear equations. They will set up the system of linear equations. Then they will find the solution using substitution, graphing, or elimination. They will explain the strategy behind the method they used to find the solution.

Answers

1. $x + y = 100$
2. $10x$
3. $25y$
4. $10x + 25y = 2050$
5. $\begin{cases} x + y = 100 \\ 10x + 25y = 2050 \end{cases}$

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6. Answers will vary. Possible answer:
$$y = 100 - x$$
$$10x + 25(100 - x) = 2050$$
$$x = 30$$
$$y = 70$$

7. Answers will vary. Possible answer: We used the substitution method because it was convenient to solve for y to begin the process.

Station 2

Students will be given six index cards with the following equations written on them:

$14x + y = 233$; $x + y = 233$; $14x + 20y = 3748$; $x + 20y = 2128$;

$x + y = 2128$; $14x + 20y = 233$

Students will work together to solve a real-world application of a system of linear equations. Students will choose which of the two index cards represents the equations in the system. They will solve the system of linear equations using either substitution or elimination. Then they will explain why graphing the system to find the solution is not the best way to solve the problem.

Answers

1. $\begin{cases} x + y = 233 \\ 14x + 20y = 3748 \end{cases}$

2. Number of T-shirts + number of sweatshirts = 233, so $x + y = 233$

   Cost per T-shirt = \$14, so $14x$

   Cost per sweatshirt = \$20, so $20y$

   Total made on T-shirts and sweatshirts = \$3,748, so $14x + 20y = 3748$

3. Answers will vary. Possible answer: Solving by elimination;
   $-14(x + y) = -14(233)$
   $14x + 20y = 3748$
   $y = 81$; $x = 152$

4. 152 T-shirts

5. 81 sweatshirts

6. Graphing by hand wouldn't be a good choice because the numbers are large.

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Instruction

Station 3

Students will be given a graphing calculator. They will set up a system of linear equations for a real-world example. They will use the graphing calculator to find the solution to the real-world example.

Answers

1. y = number of paperbacks
2. $x + y = 225$
3. $7y$
4. $16x$
5. $16x + 7y = 2583$
6. $\begin{cases} x + y = 225 \\ 16x + 7y = 2583 \end{cases}$
7. I typed $Y_1 = -x + 225$ and $Y_2 = \frac{-16}{7}x + 369$. I hit "GRAPH." I hit "2ND," "Trace," and selected "5: Intersect." Then I hit "ENTER" three times. This gave me a solution of $x = 112$ and $y = 113$.
8. 112
9. 113

Station 4

Students will be given a real-world application of systems of linear equations that involves the $d = rt$ formula. They will set up the system of linear equations. Then they will solve the system to find two rates and distances traveled.

Answers

1. Tom's speed
2. $x = y + 15$
3. $2x$
4. $2y$
5. $2x + 2y = 300$
6. $\begin{cases} x = y + 15 \\ 2x + 2y = 300 \end{cases}$

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7. Answers will vary. Possible answer: Substitution method;
   $2(y + 15) + 2y = 300$
   $y = 67.5$
   $x = 82.5$
8. 82.5 mph
9. 67.5 mph
10. Tom = 165 miles and Jeremy = 135 miles

Materials List/Setup

Station 1 none

Station 2 six index cards with the following equations written on them:
$14x + y = 233$; $x + y = 233$; $14x + 20y = 3748$; $x + 20y = 2128$;
$x + y = 2128$; $14x + 20y = 233$

Station 3 graphing calculator

Station 4 none

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. What is the first step you should take when setting up a system of linear equations from a real-world example?
2. How can you find the coefficients of the variables in the equations you use based on a real-world example?
3. To solve for two variables, how many equations do you need in your system of linear equations?
4. Will solving systems of linear equations by substitution, elimination, or graphing give you the same solution? Explain your answer.
5. In what situation would you want to avoid solving a system of linear equations by graphing it by hand?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. Define what the x and y variables represent.
2. Answers will vary. Possible answer: Look for like units such as $.
3. Two equations; the number of equations matches the number of variables.
4. Yes, they will give you the same solution just reached in a different manner.
5. when you have large numbers

Possible Misunderstandings/Mistakes

- Not defining what each variable represents
- Only using one equation instead of two equations to find the solution
- Incorrectly setting up each equation in the system of linear equations
- Not recognizing that similar units (such as $) should be in the same equation
- Incorrectly using substitution, elimination, and graphing methods to find the solution

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Station 1

At this station, you will find a real-world application of systems of linear equations. Work together to answer the questions.

> A science teacher wants to buy 100 new test tubes for the classroom lab. Acme and Zenith make two different types of test tubes. Acme test tubes cost $10 each. Zenith test tubes cost $25 each. The science teacher has $2,050 to spend. How many of each test tube could she buy?

Let x represent the number of test tubes the science teacher buys from Acme test tubes. Let y represent the number of test tubes she buys from Zenith.

1. Write an equation that represents the total number of test tubes the science teacher will buy.

2. Write a term that represents "Acme test tubes cost $10 each."

3. Write a term that represents "Zenith test tubes cost $25 each."

continued

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4. Use the terms you found in problems 2 and 3 to write an equation that represents the amount of money the science teacher has to spend.

5. Write the two equations you found as a system of linear equations.

6. Solve the system of linear equations by substitution, graphing, OR elimination. Show your work.

7. Which method did you use to solve the system of linear equations? Why?

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Station 2

At this station, you will find six index cards with the following equations written on them:

$14x + y = 233$; $x + y = 233$; $14x + 20y = 3748$; $x + 20y = 2128$;

$x + y = 2128$; $14x + 20y = 233$

Shuffle the cards and place them face-down. As a group, read the following real-world problem scenario:

> For a school fund-raiser, students sold T-shirts with the school logo for $14. They sold sweatshirts with the school logo for $20. They sold 233 pieces of clothing and raised $3,748. How many of each type of shirt did they sell at the fund-raiser?

Let x = the number of T-shirts sold. Let y = the number of sweatshirts sold.

Flip over the index cards. Work together to find the two equations that represent the system of equations for this real-world application.

1. Write the system of linear equations.

2. Explain why you chose each equation.

continued

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3. Solve the system of linear equations by either substitution or elimination. Show your work.

4. How many T-shirts were sold?

5. How many sweatshirts were sold?

6. Explain why solving this system of equations by constructing a graph by hand isn't the best way to solve this problem.

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Station 3

At this station, you will find a graphing calculator. Work as a group to set up a system of linear equations for a real-world problem. Then use the graphing calculator to find the solution to the system of linear equations. You might need to use the Zoom feature or set the viewing window to see the intersection point.

> A bookstore sold 225 books that were either hardcover or paperback. The cost of a paperback book is $7. The cost of a hardcover book is $16. The bookstore made $2,583. How many of each type of book did the bookstore sell?

1. If you let x = the number of hardcover books sold, then what can you let y equal?

2. Write an equation that represents "A bookstore sold 225 books that were either hardcover or paperback."

3. Write a term that represents "The cost of a paperback is $7."

4. Write a term that represents "The cost of a hardcover is $16."

5. Write an equation that represents the number of hardcovers and paperbacks sold to make $2,583.

6. Write the system of linear equations you have created.

continued

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7. Use your graphing calculator to find the number of hardcovers and paperbacks sold. Explain how you used your graphing calculator to find this answer.

8. How many hardcovers were sold?

9. How many paperbacks were sold?

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Station 4

Work together to set up a system of linear equations for the problem scenario that follows. Then find the solution for the system.

> Tom and Jeremy are 300 miles apart, driving to meet each other at a certain restaurant. Tom drives 15 miles per hour faster than Jeremy. They arrive at the same point after 2 hours. How fast is each person driving?

1. If you let y = Jeremy's speed, what can you let x equal?

2. Write x in terms of y.

   This is the first equation in your system of equations.

3. The distance formula is *distance* = *rate* • *time*. If the time is 2 hours, how you can you express the distance Tom traveled?

4. How can you express the distance Jeremy traveled?

5. Write an equation that represents "The distance Tom traveled + the distance Jeremy traveled is equal to 300 miles."

   This is the second equation in your system of linear equations.

6. Write the system of linear equations.

continued

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7. Find the solution to the system of linear equations using the substitution or elimination method. Show your work.

8. How fast was Tom driving?

9. How fast was Jeremy driving?

10. How many miles did each person travel?

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APPENDIX

Problem-Based Task Implementation Guides

Introduction

This addendum contains a collection of Implementation Guides (IGs) designed to support the use of the Problem-Based Tasks found throughout this unit.

Each Implementation Guide includes an overview and suggestions for introducing and facilitating the task. These contain specific recommendations for selected Standards for Mathematical Practice (SMPs) to highlight, guidance on addressing common errors and misconceptions associated with the task, and ideas for monitoring and coaching students as they work through the task. Finally, there is guidance for debriefing the task, with special attention to the SMPs and relevant Key Concepts from the instruction, as well as information on suitable extensions and alternate strategies.

Throughout the Implementation Guides, suggestions are provided in a question/prompt format to support your efforts to elicit student thinking and facilitate classroom discourse.

We developed these Implementation Guides for our Problem-Based Tasks at the request of math educators, and with advice and feedback from mathematics supervisors and teachers. Please let us know if they are helpful. Visit us at www.walch.com, follow us on Twitter (@WalchEd), or e-mail suggestions to customerservice@walch.com.

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Guide

Problem-Based Task: Magic Number

Common Core State Standard

A–REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Task Overview

Focus

What are properties of equality and properties of operations? How can these properties be used to justify steps taken in solving equations? In this activity, students will use properties of equality and properties of operations to justify a solution method to an equation. They will form a generalization from simplifying a numerical expression, and they will construct a viable argument as to why the answer to the equation described in the task is always 3.

This activity will provide practice with:

- performing simple calculations with the four operations
- identifying properties to use in the solution process
- justifying steps taken in the solution process with properties
- simplifying numerical expressions and algebraic expressions
- generalizing a numerical expression which will work for any value of x
- interpreting the results of a simplified equation

Introduction

This task should be used to explore or apply the skill of explaining the steps involved in solving an equation by identifying appropriate properties of equality and properties of operations. Students should already be familiar with simplifying numerical expressions using the order of operations, as well as evaluating an algebraic expression for a given value. If necessary, review the order of operations.

Begin by reading the problem and clarifying the meaning of the terms *commutative property of addition*, *expression*, and *order of operations*.

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Guide

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will recognize that the magic number game first requires them to choose any number. Students will most likely choose several different numbers in order to conclude that the result, or "magic number," will always be 3. Once students are convinced, they will plan to determine the pattern or structure of the equation that makes this magic number work. They will identify the properties that are necessary in order to justify the general equation form used for the game. Encourage students to discuss their thoughts and/or theories about why the result is always 3 before they begin examining the structure of the equation.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  The focus of the task is to construct a viable argument as to why the answer to the magic number game is always 3. Students will construct their arguments based on their creation of an equation, which is based on following steps and performing calculations on a numerical expression. They will use properties of equality and properties of operations to justify their steps and explanations. Ask students to put their ideas into writing, and encourage students who disagree with each other on any of the steps in the process to discuss and explain their thinking.

- **SMP 7:** Look for and make use of structure.

  In order to create an equation that will allow any number to work when substituted into the equation, students will identify the structure of the equation, based on the steps used to create it. They should recognize the pattern formed by the operations in the equation, such that any number substituted for the variable would follow the same pattern of operations when the equation is simplified. Make sure students choose several different test numbers so that they can see the process of the pattern.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly identifying operations

  Remind students that when they are identifying which step is being used in a problem, it is helpful to note what operation was performed, and whether it was performed on one side of the equation or both.

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- incorrectly identifying properties

  Have students make a reference sheet of the properties of equality, as well as the properties of operations. Also, ask students to quiz each other in order to help each other learn the properties, so they are able to correctly identify and apply them while solving equations.

- performing an operation that is not justifiable or does not follow the properties of equality and/or the properties of operations

  Remind students that if a step cannot be justified by a valid property of equality, or a valid property of operation, then the step should not be taken. Have students use a list of properties as a reference when working through problems.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task:

  Ask students to write three different numbers to use as their test values for the magic number game. Encourage them to choose three numbers that are not close in value, such as 5, 28, and 102. Ask them to predict whether the numbers they select will have any effect on the outcome of the game, in terms of the size of the numbers. (**Answer:** Some students may predict that the magic number game will only work on relatively small numbers, since the desired result is 3, which is also a small number. Encourage them to discuss their reasoning about why they think the size of the number might make a difference.)

  After students have tested their three chosen numbers, ask them if they think the game will only work if the numbers chosen are positive integers. Ask them to try negative numbers, as well as decimals and/or fractions. Ask, "What is the domain of x-values that will work in the magic number game?" (**Answer:** When students have tested a negative number, as well as a decimal and/or fraction, they should conclude that the result is still 3, and the magic number game works for any rational number. The domain is all rational numbers.)

- If students have difficulty performing the mathematical operations in their heads, encourage them to write the steps down on their papers, and carefully organize the mathematical computations. Remind students that simple mathematical errors can occur when computations are not written down. Ask, "Why do you think it is important to write down all the steps in the calculations?" (**Answer:** Students may conclude that with multiple steps involved in the number game, it is easier to make a careless mathematical error, even with simple numbers, when steps are not written down. Also, if students choose larger numbers to test in the game, the chances for error increase if students attempt to do the calculations mentally.)

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- Ask students, "What is the order of operations, and why is it useful in this task? (**Answer:** "The order of operations is a set of rules that dictates the order in which calculations are performed. Operations within parentheses are performed first, then exponents, then multiplication and division from left to right, and finally addition and subtraction from left to right. Applying the order of operations correctly helps reach the correct answer when evaluating an expression for a given value.")
- Once students have modified the equation they created for part f to work with any number, ask, "What is the next step in proving that the equation works for any number?" (**Answer:** "In order to prove that the equation works for any number, the equation must be simplified.")
- Once students have simplified the equation, ask them, "How does the simplified equation prove that any number will work in the equation?" (**Answer:** "When the equation is simplified, the result is '3 = 3.' Therefore, it does not matter which value is used for x, as the simplification will result in '3 = 3' every time.)
- Ask students, "What does '3 = 3' represent in terms of the type of solution for this equation? What would the results mean if the equation simplified to $x = 3$ instead?" (**Answer:** "When the variables cancel in an equation, and a true statement such as 3 = 3 results, then any number can be substituted into the equation, and the equation will always simplify to a number being equal to itself. If the result of the simplification was $x = 3$, then 3 would be the only solution to the equation, and therefore the only number that would work in the magic number game.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask students to volunteer their initial thoughts about whether only positive integers would work in the magic number game. Ask them to explain their thought processes and reasoning based on the given information, and then discuss how they can conclude that any rational number will work.
- Compare students' strategies and ways of justifying their steps in the process of creating the equation. Focus on the use of precise mathematical language and clarity as they discuss their arguments regarding why the magic number game works for any rational number.

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Guide

Connecting to Key Concepts

Make explicit connections to key concepts:

- When we solve an equation, we are rewriting it into a simpler, equivalent equation that helps us find the unknown value.

  Students will take the equation they write to work with any number, and they will use properties of equality and properties of operations to simplify the equation.

- When operations are performed on one side of the equation, such as when an expression is simplified, this signifies the use of the properties of operations.

  The commutative property of addition is used to rewrite the expression on the left of the equal sign in the equation. This property is only used on the left side of the equation.

- Once you have noted which steps were used, match them to the properties listed in the tables.

  As students work through the steps to simplify the equation they created, they will identify the associative property of addition as the reason for rewriting the expression on the left of the equal sign.

Extending the Task

- To extend the task, ask students to work with a partner to create a different magic number game with different steps of operations. Ask them to write down the steps, and also to work through their equation, identifying the properties of equality and operations that they use as they go. Once they have completed their new game, ask them to share the game with the class. Encourage and guide a discussion about each pair's number game, and encourage students to use the proper terminology when explaining why their magic number game works.

- Another option for extending the task is to present students with the three types of solutions to an equation. Provide them with three examples of equations: an equation like the one in the task, where a true statement results when the solution is simplified; an equation for which there is no solution because the equation simplifies to a false statement; and an equation for which the solution is a specific value of x. Encourage and guide a discussion about the general formats of these three types of equations, focusing on what happens to the variables when the equations are simplified. Help students make generalizations about the three types of solutions.

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Guide

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I persevered by creating an equation based on the steps given for the magic number game. I chose several numbers in order to begin constructing my arguments as to why the game will work for any number. I then examined the steps necessary for creating the equation and analyzed the pattern. Finally, I simplified the equation to prove that any number works in this magic number game.")
- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "Yes. I constructed viable arguments based on examining the steps for creating an equation and then simplifying the resulting equation. I was able to analyze the results and conclude that the game will work for any number, and the result will always be 3.")
- **For SMP 7, ASK:** "How did you look for and make use of structure when solving this problem?" (**Answer:** "I noticed the structure of the equation that was created, using the steps given for the magic number game. I interpreted the pattern of the operations used in the equation.")

Alternate Strategies or Solutions

- When students reach part h in the Coaching questions, which asks them to simplify the equation, instead of rewriting the term $\frac{2x+6}{2}$ as a sum of two fractions, some students may instead multiply the entire equation by 2 in order to eliminate the fraction, or they may first add x to both sides and then continue simplifying. Encourage their alternate choices in terms of simplifying the equation, but remind students to always be careful to apply the correct properties of equality and/or properties of operations.
- Students may desire to try a variety of rational numbers to further prove that the magic number game works for any rational number, before they follow the Coaching questions. They may try to discern the pattern with only the test values, instead of creating an equation. Discuss the factors involved in this method, such that it may be more time-consuming to try many numbers, as opposed to trying to generalize an equation that works for any number.

Technology

Although generally technology is not needed for this task, if students choose large values for their test values, a scientific calculator may be used.

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Problem-Based Task: Guess My Number

Common Core State Standard

A–REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Task Overview

Focus

How can a verbal description be translated into an algebraic equation? What are properties of equality, and how can these properties be used to justify steps taken in solving equations? In this activity, students will write an equation based on a verbal description, and they will use properties of equality to solve the equation.

This activity will provide practice with:

- translating a verbal description into an algebraic equation
- identifying properties to use in the solution process
- justifying steps taken in the solution process with properties
- simplifying numerical expressions
- interpreting the solution to an equation

Introduction

This task should be used to explore or apply the skill of solving a linear equation in one variable, using properties of equality. Students should already be familiar with simplifying numerical expressions using the order of operations, and they should also be familiar with the properties of equality and properties of operations. If necessary, review the properties of equality and properties of operations.

Begin by reading the problem and clarifying the meaning of the terms *algebraic expression*, *like terms*, and *properties of equality*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will recognize that the first step in this task is to translate the given verbal sentence into an algebraic equation. Students will determine which operations are used to represent the words "less than" and "more than," as well as the order in which the constants and the expressions are written in the algebraic equation. Students will also identify the properties that are necessary in order to justify the steps in solving the equation. Encourage students to discuss their thoughts about the process of solving the equation, as well as their choices for operations to represent the words.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will construct viable arguments for their choices for translating the verbal sentence into an algebraic equation. They will use properties of equality and properties of operations to justify their steps and explanations as they go through the process of solving the equation. Ask students to write down their steps as they solve the equation, and encourage students who disagree with each other on any of the steps in the process to discuss and explain their thinking.

- **SMP 7:** Look for and make use of structure.

  In order to create an algebraic equation, students will identify the structure and order of the terms in the equation, based on the precise wording given in the verbal description. Students should recognize that when like terms are on the same side of the equation, they can regroup the terms in order to make the simplification easier. Encourage students to discuss the interpretation of the solution with each other as well as how to verify its accuracy.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- performing the wrong operation when isolating the variable

  Remind students of opposite operations, and that, when solving an equation, the opposite operation must be performed on both sides of the equation in order to "undo" a step.

- incorrectly combining like terms

  Remind students that the definition of *like terms* is "terms that contain the same variables raised to the same power." Also, remind students that if no exponent is shown, then the exponent is understood to be 1.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask them to identify possible terms and phrases to represent the four operations: addition, subtraction, multiplication, and division. Then have students create a chart by labeling four columns with the four operations, and then filling in each column with the terms and phrases. (**Sample answers:** Addition: *sum*, *more than*, *altogether*, *in all*. Subtraction: *difference*, *fewer*, *less than*, *less*. Multiplication: *product*, *times*, *rate*, *of*, *as many as*. Division: *quotient*, *each*, *divided by*.)
- As students begin translating the verbal sentence to an algebraic equation, ask them to explain why "2 less than one-third of a number" should be represented as $\frac{1}{3}x-2$ instead of $2-\frac{1}{3}x$. (**Answer:** "It is important to determine which part is being subtracted from the other part. Since the problem states '2 less than,' the 2 is subtracted from the other term, which is $\frac{1}{3}x$. If the subtraction had been reversed, then the problem would have read, 'one-third of a number less than 2.'")

  Ask, "Does it matter if the left side of the equation is written $2-\frac{1}{3}x$ instead of $\frac{1}{3}x-2$? Explain in terms of properties of operations." (**Answer:** "Yes, the order does matter, because subtraction is not commutative. Only addition and multiplication are commutative.")

  Ask, "Does it matter if the right side of the equation is written as $\frac{1}{4}x+3$ instead of $3+\frac{1}{4}x$? Explain in terms of properties of operations." (**Answer:** "No, the order does not matter, because addition is commutative. Therefore, either order of the terms will not affect the outcome.")

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- As students are working through the process of solving the equation, ask, "Which properties of equality and/or properties of operations did you use when solving the equation? In what order did you use them?" (**Answer:** The subtraction property of equality, the addition property of equality, and the multiplication property of equality were used when solving the equation. Students may also note that the commutative property of addition was used when rearranging the terms on the left side of the equation when $\frac{1}{4}x$ was subtracted from both sides.)

- After students have solved the equation, ask them how they can check that the solution of $x = 60$ is correct. (**Answer:** "I can check by substituting 60 for x in the original equation. When the numerical expressions are simplified, the result is a true statement, 18 = 18.")

  Ask students, "If the variables had canceled out when solving the equation, and only numbers remained, what would that mean about the type of solution for this equation?" (**Answer:** "If the variables in the equation had canceled out, and the result was something that is not true, such as 4 = 7, then the resulting equation would never be true for any value of the variable. Therefore, the equation would have no solution.")

  Ask students, "What if the result of solving the equation was the same value on each side of the equal sign, such as $5x = 5x$? What type of solution would this represent?" (**Answer:** "If the same value had been the result on both sides of the equal sign, then the equation would always be true for any value of the variable. Therefore, the equation would have an infinite number of solutions.")

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

Debriefing the Task

- Ask students to share any difficulties they had in terms of writing and/or solving the equation. Ask them to explain their thought processes and reasoning based on the given information, and then discuss how students can verify that their solution is correct.

- Compare students' strategies in the process of creating the equation. Focus on the use of precise mathematical language and clarity as they discuss the steps and properties they used in solving the equation.

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Connecting to Key Concepts

Make explicit connections to key concepts:

- Pay special attention if the same variable appears on either side of the equal sign.

  In this task, when students translate the given verbal description to an algebraic equation, the variable appears on both sides of the equation.

- Just like with numbers, variables may be added to or subtracted from both sides of the equation without changing the equality of the statement or the solution to the problem.

  Students will find that it is necessary to subtract one of the terms with the variable from both sides of the equation as a step toward isolating the variable.

- Other equations will only have one solution, where the final step in solving results in the variable being equal to a number, such as $x = 5$.

  In this task, students will conclude that there is one and only one solution to the equation, because the last step results in $x = 60$.

Extending the Task

- To extend the task, ask students to work with a partner, and ask each person to create their own verbal description of an algebraic equation. Ask the partners to trade verbal descriptions and then create the corresponding algebraic equation for the description they've been given. Encourage students to discuss their results with each other, as well as the various properties of equalities they used in the process of solving the equation.
- Another option for extending the task is to challenge students to create their own algebraic equation using all four operations, and then create the verbal description for the equation.
- Students can also be challenged to create a verbal description of either an equation that has no solution, or an equation that is true for any value of the variable. Encourage students to generalize the format of all three possible outcomes of an equation: one solution, no solution, and infinitely many solutions.

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Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I made sense of the problem by creating an equation based on the verbal description I was given. Then, I solved the equation using properties of equality.")
- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "I constructed viable arguments by using properties of equality to justify the steps used in solving the equation. I constructed these arguments based on my knowledge of the properties. I was able to analyze the results and check my answer by substituting the solution back into the original equation.")
- **For SMP 7, ASK:** "How did you look for and make use of structure when solving this problem?" (**Answer:** "I noticed the structure and order of the terms used in creating the equation, based on the wording given in the verbal description. I recognized the order of the like terms, which were then simplified.")

Alternate Strategies or Solutions

- When students begin the process of solving the equation they created, some may subtract $\frac{1}{3}x$ from both sides instead of subtracting $\frac{1}{4}x$ from both sides, as shown in the Coaching Sample Responses. Assure students that this method is also correct, and that the result of the equation will still be the same.
- Students may experiment with substituting different numbers into the equation to see if they are able to make both sides of the equation equal. Encourage students that although this method is viable, it will most likely be more time-consuming than taking the steps necessary to solve the equation.

Technology

No technology is needed for this task.

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Problem-Based Task: Landlines Versus Cell Phones

Common Core State Standard

A–REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Task Overview

Focus

How long will it take for one quantity to exceed another? In this task, students will compare two quantities by writing and solving a linear inequality. Students will use the linear inequality to determine the length of time it will take for the number of households with just cell phone service to exceed the number of households with both cell phone and landline service.

This activity will provide practice with:

- writing linear inequalities from a context
- comparing two quantities using a linear inequality
- solving linear inequalities
- interpreting the solution to an inequality

Introduction

This task should be used to explore or apply the concept of creating inequalities from a given context. It is best to implement this task after students have learned how to translate verbal descriptions of inequalities into symbolic representations. Students may graph using technology, including graphing calculators or online graphing programs.

Begin by reading the problem and clarifying the difference between cell phones and landlines. Review the following terms:

exceed	to be greater in number or size
landline	a telephone that physically connects to a network using metal wire or fiber optic cable

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students may be reluctant to use algebraic symbols. Encourage students to process each piece of the task verbally before trying to translate into algebraic symbols. Students must understand how the phrases "*x* years" and "greater than" fit into their linear inequality.

- **SMP 2:** Reason abstractly and quantitatively.

  Students may choose to begin this task quantitatively, by creating a table for both cell phones and landlines. With appropriate guidance, a student's understanding of this process can help lead to the abstract inequality. Remind students that the abstract representation allows them to gather information using a more efficient method. Also, in this situation, a table (using whole numbers) will not allow students to arrive at an exact answer.

- **SMP 8:** Look for and express regularity in repeated reasoning.

  Encourage students to draw parallels between and make generalizations about solving an equation compared to solving an inequality. Also, discuss why it is acceptable to write 15 million, for example, as "15" in the inequality: if every number is written with the same label (millions), it is as though both sides of the inequality were divided by 1 million.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- replacing inequality symbols with equal signs

  Remind students that the words "greater than" require an inequality symbol because it means the two quantities being compared are unequal.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- If students have trouble sorting out which information relates to the two different types of households, have them use highlighters to distinguish between the parts of the task that relate to households with both landline and cell phone service and the parts relating to households with just cell phone service.

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- If students do not know how to set up the expression for households with both landlines and cell phones, ask:

  "Which number is a constant in this situation?" (**Answer:** 66.1 million)

  "What is happening each year to households with both a landline and a cell phone?" (**Answer:** "This number is decreasing by an average of 5 million households per year.")

  "How is the change occurring each year related to the variable, *x*?" (**Answer:** "The decrease of 5 million per year must be multiplied by *x* in order to show the rate of change for each year.")

  "How do the constant and the rate of change relate to each other in this situation?" (**Answer:** "The rate of change must be subtracted from the constant because the number of households with both landlines and cell phones is decreasing each year.")

- If students do not know how to set up the expression for households with just cell phones, ask:

  "Which number is a constant in this situation?" (**Answer:** 27.8 million)

  "What is happening each year to households with just cell phones?" (**Answer:** "This number is increasing by an average of 15 million households per year.")

  "How is the change occurring each year related to the variable, *x*?" (**Answer:** "The increase of 15 million per year must be multiplied by *x* in order to show the rate of change for each year.")

  "How do the constant and the rate of change relate to each other in this situation?" (**Answer:** "The rate of change must be added to the constant because the number of households with just cell phones is increasing each year.")

- Students may be tempted to write the full numbers, such as 15,000,000 for 15 million. Explain that although this would be correct, it is much easier to use the abbreviated version of the numbers. If students don't understand why they are allowed to do this, explain that it is as though both sides of the inequality were divided by 1 million.

- If students are unsure of how to set up the inequality:

  Ask, "What phrase in the problem statement indicates an inequality? What is the inequality symbol for this phrase?" (**Answer:** "The phrase 'greater than' indicates an inequality. The inequality symbol for this phrase is >.")

  Encourage students to initially use words to create the expressions on either side of the inequality symbol. (**Answer:** "Households with just cell phones > Households with both landlines and cell phones")

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- If students start with the correct inequality, but arrive at an incorrect answer, check to make sure that students followed the properties of inequalities throughout the process of solving the inequality. Ask, "Did you have to change the direction of the inequality symbol? Why or why not?" (**Answer:** "No, because when I moved the terms with the variables to the left side of the inequality, the coefficient of x remained positive and the inequality symbol didn't have to change.")
- If students attempted to use a table to solve the problem and could not determine an exact answer, ask:

  "Had the number of households with just cell phones exceeded the number of households with both landlines and cell phones at the 1-year mark?" (**Answer:** "No.")

  "Had the number of households with just cell phones exceeded the number of households with both landlines and cell phones at the 2-year mark?" (**Answer:** "Yes.")

  "Do you need an exact answer for this situation?" (**Answer:** "No. The question asks 'in *about* how many years,' so the answer can be estimated.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

Compare the strategies used by various groups. Ask students if they think one strategy was easier or harder than another, and ask them to explain why they chose one strategy over another.

Connecting to Key Concepts

Make explicit connections to key concepts:

- The properties of inequality are the rules that allow you to balance, manipulate, and solve inequalities.

  Accurately following the properties of inequalities allows one to arrive at the correct solution. Refer students back to the properties of inequality if necessary.
- If a variable appears on both sides of the inequality, choose which side of the inequality you would like the variable to appear on.

  A variable appears on both sides of the inequality in this task. Discuss with students the advantages and disadvantages of moving the variables to the left side of the inequality versus the right side of the inequality. (In this task, it is more advantageous to move the variables to the left side so that the coefficient remains positive.)

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Extending the Task

To extend the task, use current real-life data to give the problem more relevance. For example:

- Have students research the current number of households with both landline and cell phone service versus households with just cell phone service. Use this data to rework the problem using the same rates of change (or updated rates of change, if possible).
- Compare actual data from 2010 to the data given for 2008 to see if the solution for x (2 years) proved to be accurate. Discuss the purpose of mathematical estimates in real-life situations.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I reread the problem to find information and I considered each part of the problem individually before trying to put all the pieces together.")
- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "I used abstract reasoning when I created an inequality from the verbal description, and when I translated the solution back into a verbal description to answer the question posed in the task.") "Did any of your strategies represent quantitative reasoning?" (**Answer:** "I used quantitative reasoning by creating a table of values to represent the number of households with each type of phone service.")
- **For SMP 8, ASK:** "How did you look for and express regularity in repeated reasoning?" (**Answer:** "I know the rules for solving an equation, so I was able to see the parallels and apply the same rules to the inequality. Also, I figured out that I don't have to write the numbers given in millions with all of their zeros, because I know that if every number is written with the same label, 'million,' then the result will be the same.")

Alternate Strategies or Solutions

- Students could create a table of values to determine when one quantity exceeds the other.
- Alternatively, the problem can be solved by graphing on a graphing calculator or using technology. Students can graph the expression for households with landline and cell phone service as Y1 and the expression for households with only cell phone service as Y2.

Technology

Students can use a spreadsheet or a graphing calculator to determine when the number of households with just cell phones will exceed the number of households with both landlines and cell phones.

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Problem-Based Task: Singing Celebration

Common Core State Standard

A–REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Task Overview

Focus

How can an exponential equation be used to model a real-life scenario and make predictions? How can an exponential equation be rewritten so that both sides of the equation have the same base? In this activity, students will analyze a scenario and create an exponential equation to model it.

This activity will provide practice with:

- identifying the base and exponent in an expression
- rewriting numbers in exponential form
- applying the laws of exponents
- solving exponential equations in which the bases of both sides of the equation are equal
- analyzing and making decisions about a real-world scenario
- predicting a future value based on an exponential model

Introduction

This task should be used to explore or apply the laws of exponents and to learn how to solve exponential equations. It should be implemented after students have learned how to distinguish between linear and exponential equations. Students also should have learned the basic parts of a power (the base and the exponent). Students may use a scientific calculator for computation of exponents.

Begin by reading the task and asking students to think about whether this situation should be represented by a linear or an exponential model. Do not tell the students which type of model they should use; allow them to discover this on their own.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Some students may first be inclined to choose a linear equation to model this task. Ask these students to explain why they think a linear equation would be appropriate in this situation. Encourage them to consider whether the number of contestants decreased by the same number of people each week or whether it was multiplied by the same factor each week.

- **SMP 2:** Reason abstractly and quantitatively.

  Students should reason that a linear equation is not possible in this situation because the rate of decline of the contestants each week does not follow a linear model. Some students may make a table to determine the number of contestants each week after half of them are eliminated. Allow and encourage these methods, while guiding students to discover that an exponential equation will allow for the same results they find by using a table of values. Also, guide students to reason that when a larger quantity is used, an equation will provide a much more efficient way of finding a future value.

- **SMP 4:** Model with mathematics.

  Students will apply what they know about creating an equation in order to model a real-life scenario. They can be guided to identify the quantities in the task, and then decide which of the quantities is the independent variable and which is the dependent variable. Encourage students to conclude that in order to make a prediction about a future value, based on a change in the conditions, an equation that models the scenario is useful.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- believing that the situation is modeled with a linear equation instead of an exponential equation

  Point out that the amount of change in the number of contestants from week to week is not constant, so a linear model is not appropriate. Instead, the number of contestants is multiplied by the same value each week, so the correct model is exponential.

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- incorrectly using $\left(\frac{1}{2}\right)^x$ instead of 2^x in the equation

  Point out that the final episode begins with two contestants, so the previous episode began with 4 contestants, the episode before that began with 8 contestants, and so forth. Working backward, the number of contestants is multiplied by 2 each time, so the base is 2.

- believing that the second season will air for 4 weeks, instead of 4 weeks *longer* than the first season

  Have students underline important information in the task. Be sure they include "longer" when underlining. Point out that "4 weeks longer" indicates that the show will air 4 more weeks than the previous season.

- incorrectly applying the laws of exponents when solving the exponential equation

  Refer students to the Laws of Exponents table in the Key Concepts. Identify each law to be applied, and verify accuracy by comparing the result to that of the general rule and the given example.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- If students have trouble deciding on the type of model, ask, "What information in the task can be used to help you decide whether this situation should be modeled with a linear or an exponential model?" (**Answer:** "The number of contestants decreasing by half, or a factor of $\frac{1}{2}$, indicates that the model will not be linear, because the change in the output values is not constant. Therefore, the model should be exponential.")
- If students mistakenly create a linear model, ask them to substitute different values to represent the number of weeks to see if the outputs make sense. (**Answer:** If students use a linear model, the outputs will not be cut in half each week. This should guide students to realize that a linear model does not reflect the situation.)
- To suggest alternative strategies, ask, "In addition to creating an exponential equation to model the situation, what other way can the data be represented in order to help you analyze the scenario?" (**Answer:** "I could create a table of values or a graph, such that the independent variable is the number of weeks and the dependent variable is the number of contestants.")

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- To prompt for conceptual understanding, ask, "Based on the context of the scenario, what do the domain and range represent, and why are the domain and range restricted to positive integers?" (**Answer:** "The domain is the number of weeks, and the range is the number of contestants; both of these values must be positive integers.")
- If students are struggling with rewriting a number with a specific base, provide some concrete examples outside of the task, such as, "How can 27 be rewritten with a base of 3?" Help students work through the example, and then extend this to rewriting 2 and 64 with the same base.

Debriefing the Task

- Ask students to volunteer their thought processes for determining that the task requires an exponential equation instead of a linear equation. Discuss ways in which the data can be represented before the equation is determined. To do this, ask questions such as "After week 1, how many contestants are remaining?" Or, "After week 2, how many contestants are remaining?" Explain how a table of values can be created, which will lead students to see that the change in y-values is *not* the same; therefore, the equation cannot be linear, because the rate of change is not constant.
- Once students have determined the number of weeks during which the first season of the series will air, ask them to explain the steps necessary for determining the number of weeks that the second season will air.
- Discuss why the equation $2^{10} = x$ is chosen for finding the number of contestants that the second season will have.
- Have students share their work with a partner. Direct student pairs to compare their different solution processes and then share their processes with the class. Point out any errors or misconceptions in the process, if necessary.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Some exponential equations can be solved by writing both sides with the same base.

  In this task, both 2 and 64 can be rewritten with a common base of 2: $2^x = 2^6$. Once an exponential equation is in the form where the bases are equal, then the exponents can be set equal, and then the equation can be solved: $2^x = 2^6$; $x = 6$.

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Extending the Task

- To extend the task, ask students to use their equations to solve for the required number of weeks for a different number of contestants, or the number of contestants for a different number of weeks.
- Alternatively, provide students with the following scenario: A competing series follows the same format of eliminating contestants each week and begins its first season with 30 contestants. Instead of eliminating half of the contestants each week, the competing show follows this pattern: 7 contestants are eliminated after the first week, 6 contestants are eliminated the next week, 5 contestants are eliminated the following week, and so on until there are only 2 contestants left for the final show. Which of the two shows will be on the air longer during its first season, and for how much longer?

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I evaluated the situation to decide whether the decrease in the amount of contestants each week was reflected by a linear model or an exponential model. When I realized that the number of contestants was not reduced by a constant each week, I knew that the model had to be exponential.")
- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "I reasoned abstractly when choosing an exponential equation instead of a linear equation, and then writing the equation.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I used algebra to create and solve an exponential equation that modeled the scenario.")

Alternate Strategies or Solutions

- Students may choose to create a table of values for the first season without creating an equation. They could come up with sets of ordered pairs, with x representing the number of weeks and y representing the number of contestants at the beginning of that week's episode, continuing until the y-coordinate is equal to 2. The ordered pairs are: (1, 64), (2, 32), (3, 16), (4, 8), (5, 4), (6, 2). Students can create a graph of these points, which will help them visually determine that the equation that models the scenario cannot be linear, since the graph of the points is a curve.

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- In order to find a solution for the second part of the task, which asks students to determine the number of contestants the next season will start with if it is 4 weeks longer than the first season, students may find the value of 2^{10} by multiplying the base, 2, out 10 times. Encourage students to realize that while this computation is not too time-consuming, a calculator is a more efficient way to evaluate the exponent.

Technology

Students can use scientific calculators and graphing calculators in order to visually analyze the task and to calculate the exponents.

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Problem-Based Task: Ticket Sales

Common Core State Standards

A–REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A–REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Task Overview

Focus

How can the solutions to two different variables be found? How can a system of equations be created and solved? In this task, students will write equations representing different scenarios involving ticket sales at a convention. Given information about different types of tickets (adult and children's tickets, for either 2 or 3 days), students will use their equations to determine how many of each type of 2-day ticket were sold, along with the prices for 3-day adult and children's tickets.

This activity will provide practice with:

- writing equations to represent different quantities
- creating a system of equations
- solving a system of equations using different methods
- applying properties of equality to justify steps taken in the solution process
- evaluating an equation for a given value
- interpreting the results of a solution to a system of equations

Introduction

This task should be used to explore or apply the skill of creating a system of equations in two variables and then using various methods, such as substitution or multiplication, in order to solve the system of equations. Students should already be familiar with using properties of equality to solve equations. If necessary, review the properties of equality.

Begin by reading the problem and clarifying the meaning of the terms *elimination method*, *substitution method*, and *system of equations*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will first recognize that two different quantities are being examined and analyzed in this task, which will result in determining the total number of each type of 2-day ticket sold, and the unknown prices for 3-day tickets. Students will identify that there are two basic types of tickets—adult and child—that are good for either 2 days or 3 days. Furthermore, there are different prices for each type and date range of ticket. Students will recognize that a system of equations will be necessary in order to solve for the number of 2-day adult and children's tickets sold, and a second system will be necessary to find the prices of 3-day adult and children's tickets. Students will also recognize that the systems can be solved by either substitution or elimination. Encourage students to discuss their ideas for implementing the different methods for solving the systems of equations.

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they make sense of the quantities represented in the context of the scenario. They will identify that there are two types of tickets involved in the task, with options for 2-day or 3-day versions of each. Students will assign a variable to each type of ticket, and apply the appropriate rate to each ticket type when creating the systems of equations. They will also reason abstractly by manipulating the variables when solving the systems of equations. Students will reason quantitatively as they substitute the value of one variable back into an equation in order to solve for the other variable. They will apply the order of operations to simplify the remaining numerical expressions. Students will also ensure the proper units are assigned to the solutions; for example, by expressing the cost of each 3-day ticket in terms of dollars.

- **SMP 4:** Model with mathematics.

  Students will recognize that the information presented can be modeled using algebraic systems of equations. Students will identify the important quantities and analyze the relationships between these quantities in order to solve for each variable. Encourage and guide students to make the connection between applying different methods of solving a system of equations and creating a model for this real-world scenario. Also, encourage students to analyze their solutions for each Coaching question to ensure that the results are practical and make sense in the context of the scenario.

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Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- finding the value for only one of the variables of a system

  In this task, students will create two separate systems of equations that each solve for two unknowns: one system to determine how many 2-day adult and children's tickets were sold, and another to find the cost of 3-day adult and children's tickets. In either case, if students find the value of one variable and then neglect to find the value of the other, encourage them to reread the questions posed in the problem scenario so they understand all that they are being asked to find. Remind students that the process for solving a two-variable system of equations involves first solving for one of the variables by either substitution or elimination, then substituting the result into the original equation(s) to solve for the *other* variable.

- forgetting to distribute negative signs when substituting expressions for variables

  Remind students that negative signs must be distributed throughout the expression; have them carefully double-check each step when substituting an expression for a variable.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, encourage them to reread the given information and try to organize it before beginning the Coaching questions. Ask, "What are some ways to better organize the given information before beginning the task?" (**Answer:** Some students may suggest an organizational chart or table that displays the information in rows and/or columns. Encourage students to group the given information by the broadest categories first before getting into the more detailed information they are given, so that the different steps of answering the Coaching questions can be seen more clearly. For example, point out that the first paragraph of the problem scenario focuses on 2-day tickets, and the second focuses on 3-day tickets, so it would be logical to separate the information according to days first, then by the next given piece of information (total sales in dollars), and so on.)

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- As students are writing each system of equations, ask them to predict which solution method—substitution or elimination—might work best for that system, and to justify their predictions. (**Sample answer:** "I think substitution would be the easiest method to use for both systems. The elimination method calls for adding equations if the coefficients of one of the variables are opposites, or subtracting equations if the coefficients are the same. The equations in my systems do not have opposite coefficients for either variable, nor do they have coefficients that are the same for either variable. So, in order to use the elimination method, I would have to first multiply each term in one of the equations of the system by some value so that I could add or subtract the equations.") Encourage students to look at the structure of the equations once they have created them, and to think about how much time each method might require. Also encourage students to discuss their predictions and justifications with each other.

- If students have difficulty writing the equations for the first system, remind them that the two different parts to consider are the total number of tickets sold (6,600) and the total amount of sales in dollars ($188,100).

  Ask, "Does the total number of tickets sold depend on the price of each ticket? What does this tell you about how to write an equation for the total number of tickets sold?" (**Answer:** "The total number of tickets sold does not depend on the price of each ticket; this means I do not need to multiply the variables for adult tickets and children's tickets by the price per ticket to write the equation. Therefore, I can write an equation for the total number of tickets sold by adding the number of adult tickets sold, a, to the number of children's tickets sold, c, and setting this expression equal to the total number of tickets sold, 6,600. The resulting equation is $a + c = 6600$.")

  Ask, "Does the total amount of sales depend on the price of each ticket? What does this tell you about how to write an equation for the total amount of sales?" (**Answer:** "The total amount of sales *does* depend on the price of each ticket, since the price per ticket matters in terms of the amount of money made in all from selling the tickets. This means that I must multiply the variables representing each ticket type (a and c) by the price for that ticket ($36 and $26, respectively). Therefore, the expressions for the amount of money made from each type of ticket are $36a$, representing sales from adult tickets that cost $36 each, and $26c$, representing children's tickets that cost $26 each. The total amount of sales in dollars can be found by adding these two expressions, and setting the result equal to the total amount of sales in dollars, $188,100. The resulting equation is $36a + 26c = 188{,}100$.")

  If students encounter the same difficulties in writing the equations for the second system of equations, encourage them to follow the same process.

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- When students have solved one of the systems, ask them how to determine if their solution is practical and makes sense. Ask, "What parameters can you use to analyze your solution to make sure it is valid? What would be an example of a solution that is *not* valid?" (**Answer:** "Since the task asks for numbers sold and prices for particular tickets, the answers cannot be negative, because a negative number of tickets does not make sense, nor does a negative price per ticket. An example of a solution that would not be valid is $c = -4950$.")
- If students solve for a variable and the result *is* a negative number, ask them, "What should you do if your result is a negative number, since you realize that this type of solution does not make sense?" (**Answer:** "I should first go back and double-check to make sure that all negative signs were distributed properly, if applicable. Then, I should go back through each step in the process of solving the equation, making sure that I applied the correct properties of equality." Remind students that careless errors are often made when attempting to distribute a negative sign, or by neglecting to apply the properties of equality to both sides of the equations.)
- Ask students, "How can you double-check your solutions in a system of equations?" (**Answer:** "Once I have substituted the value for one variable back into the original equation in order to find the value of the second variable, I can check the results by substituting the values for both variables back into *both* of the original equations in the system. If both equations do not result in true statements, then there is an error in one, or both, of the solutions.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask students to volunteer their initial predictions about which of the solution methods they thought would work best in the given scenario. Ask them to explain their thought processes and reasoning based on the given information.
- As a class, compare students' strategies and explanations for creating the systems of equations, solving them, and interpreting their solutions in the real-world context. Discuss the analysis used on the solutions in terms of validity. Focus on the use of precise mathematical language.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Systems of equations can have one solution, no solutions, or an infinite number of solutions.

  In this task, students solve two systems of equations, both of which have one solution.

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- The substitution method involves solving one of the equations in a system of equations for one of the variables and substituting that into the other equation.

  The Coaching Sample Responses for this task reflect using the substitution method for solving each system; however, the elimination method could also be used.

- Check your answer by substituting the *x*- and *y*-values back into the original equations. If the answer is correct, the equations will result in true statements.

  For each system, remind students to check their results for both variables by substituting the values back into both of the original equations. The equations should result in true statements.

Extending the Task

- To extend the task, provide students with information reflecting a 4-day ticket option. Ask students to write and solve a system to determine the total number of adult and children's 4-day tickets sold, given a total number of tickets sold and the total revenue. Or, ask students to write and solve a system for the price per ticket for adult and children's 4-day tickets, given the combined cost of these tickets and a description of how many 4-day adult and children's tickets sold compared to the corresponding 2- or 3-day ticket.

- Another option for extending the task is to ask students to solve each system of equations using the opposite method used initially—i.e., by using the elimination method instead of the substitution method, or vice versa. After students have completed the task using both methods, ask them to write a paragraph explaining the advantages and disadvantages of each solution method.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I persevered in solving the problem by first recognizing the quantities involved in the scenario, then I created systems of equations for each part, and finally I chose a solution method and solved the systems of equations and interpreted the results.")

- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "I reasoned abstractly by carefully interpreting the variables and quantities represented in the scenario.") "Which of your strategies represent quantitative reasoning?" (**Answer:** "When I substituted the value of one variable back into an equation in order to solve for the other variable, and then applied the order of operations to simplify the remaining numerical expressions, I used quantitative reasoning.")

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- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I used mathematics to model this scenario by creating an algebraic system of equations. I analyzed the relationships between the quantities in the scenario, and I also interpreted the solutions to the system in order to ensure the results were practical and made sense in the context of the scenario.")

Alternate Strategies or Solutions

Although the Coaching Sample Responses reflect using the substitution method, some students may choose to use the elimination method. Encourage students to consider the time involved in using the elimination method in this case, as well as the ease of calculations. Discuss how using two different methods can help verify the results and help ensure accuracy.

Technology

Although no technology is needed for this task, students may use scientific calculators in order to check their calculations when simplifying numerical expressions.

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Problem-Based Task: Salary Scales

Common Core State Standard

A–REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Task Overview

Focus

How can a system of equations be created and solved? How can the graphs of two linear equations be analyzed and interpreted in the context of a real-world scenario? In this task, students will create linear equations representing different salary options and then graph the equations in order to determine which option is the most profitable.

This activity will provide practice with:

- writing linear equations to represent different salary options
- graphing a system of equations
- comparing the graphs of a system of equations on the same coordinate plane
- identifying a point of intersection on a graph of two equations
- analyzing the results of a system of equations in the context of the scenario
- drawing conclusions based on the analysis of the graphs of a system of equations

Introduction

This task should be used to explore or apply the skill of creating a system of equations in two variables and then applying the graphing method in order to identify the solution(s) to the system. Students should already be familiar with writing equations in slope-intercept form, as well as with graphing linear equations. If necessary, review slope-intercept form and the process of graphing.

Begin by reading the problem and clarifying the meaning of the terms *graphing method*, *point of intersection*, *slope-intercept form*, and *system of equations*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they make sense of the quantities represented in the context of the scenario. They will identify that the independent variable is the price of merchandise that Finn sells, in dollars, and the dependent variable is his total salary, also in dollars. They will recognize the relationship between the variables, such that his total salary depends on the price of merchandise that he sells. They will also reason abstractly by analyzing the graphs of the equations that represent the different salary options. Students will reason quantitatively as they substitute values for the independent variable, x, back into the equations in order to determine the total salary. They will apply the order of operations and simplify the remaining numerical expressions as they evaluate for a specific value of x. Students will also ensure the proper units (i.e., dollars) are assigned to the solutions.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will analyze the graphs of the linear equations in order to determine which salary option is best. They will construct viable arguments by examining the slope of the linear equations, the point of intersection, and by evaluating the equations at specific values. Based on their analysis of all three equations and graphs for the salary options, they will choose a salary option that they think will provide the most income for Finn.

- **SMP 4:** Model with mathematics.

  Students will recognize that the scenario presented can be modeled by an algebraic system of equations. They will model the equations by graphing them on the same coordinate plane and analyzing the resulting lines, in terms of slope and points of intersection.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- graphing equations incorrectly

  Remind students to double-check each input value as they enter the equations into their graphing calculators. If students are graphing by hand, remind them to check the labeling of the axes on the graph, and to check their calculations for the points to be graphed. Remind students that the points should connect to form a straight line.

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- misidentifying the point of intersection

  Remind students that the point of intersection is the point at which the graphs cross, or meet. On a graphing calculator, if each equation was input correctly, the calculator should display the exact point of intersection. On a graph created by hand, remind students to always look first on the *x*-axis, and then find the corresponding point on the *y*-axis.

- mistakenly thinking that one option is better than the other based only on the first part of the graph

  Remind students that when lines have different slopes and *y*-intercepts, it is important to view the behavior of the lines over time, and not just at the beginning of the graph. Note to students that at the point of intersection of the graphs, the functions will reverse in terms of which one has the larger *y*-values.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task:

  Ask them to predict which pay option they think will make Finn the most money in the long run. Ask students to write their predictions down and explain their reasoning behind their choice. (**Answer:** Some students may choose the first option because it has a higher base salary than the other two options. Some students may choose one of the other options because they both would pay Finn a greater percentage of the price of merchandise he sells.)

  Ask students to identify the components of each pay option in terms of the parts of an equation. Ask them, "Which part of each pay option represents a constant, and which part represents the variable? Explain your answers." (**Answer:** "The base salary for each option represents the constant in each equation, because Finn will receive this amount regardless of how much merchandise he sells. The percentage of the price of the merchandise that Finn sells represents the variable, as this amount will depend on the price of merchandise that he sells, which will vary each week.")

- Once students have written their equations, if they have difficulty understanding the format of the equations, encourage them to think about what the components of an equation in slope-intercept form represent. Ask, "In the equation representing the first pay option, $y = 300 + 0.10x$, what does the 300 mean in the context of graphing, and what does the 0.10 mean in the context of graphing?" (**Answer:** "In the equation, '300' represents the *y*-intercept, or the *y*-coordinate of the point at which the graph intersects the *y*-axis; '0.10' represents the slope of the line.")

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- After students have created the equations and are beginning the graphing process, ask them, "In this context, why is it necessary to restrict the graph to the first quadrant? Explain your answer in terms of domain and range." (**Answer:** "In the context of this real-world scenario, the domain is any value greater than or equal to 0. The domain will never be negative, because Finn will never sell a negative amount of merchandise, and the price of the merchandise will never be a negative number. Since the domain is positive, the range will also be positive. Again, Finn cannot make a negative amount of money. Even if he does not sell any phones, he will still make a base salary. Therefore, only the first quadrant should be shown.")

- Once students have graphed the first two pay options, ask, "What does the point of intersection for the first two pay options represent in the context of the scenario?" (**Answer:** "The point of intersection of the first two pay options is (1,000, 400), which is the solution to the system of equations for the first two pay options. In the context, this point represents that when Finn sells $1,000 worth of merchandise, his total pay is $400 for both pay options.")

  Ask, "Why is it necessary to continue examining the graph of the equations past the point of intersection? What does the rest of the graph tell you about the pay options in terms of which one will be better for Finn?" (**Answer:** "The point of intersection shows that Finn will be paid the same amount regardless of which option he chose only if he sells precisely $1,000 worth of merchandise. If he sells any more or less than $1,000, his pay will not be the same regardless of his choice. I need to look at the rest of the graph to see which option is best for Finn if he sells less than $1,000, and which is best if he sells more than $1,000. The graph tells me that if Finn sells more than $1,000 in merchandise, the second option will pay him more. However, if he sells anything less than $1,000, then the first option will pay him more.")

- Once students have added the third pay option to their graphs, if they have difficulty determining from the graph which of the three options is best, encourage them to expand their domain values on their calculators (or to extend the *x*-axis on their hand-drawn graphs). Ask, "How can increasing the domain values help you to have a better understanding of the third pay option?" (**Answer:** "If I increase the domain, then I will be able to see more of how the graph of the third pay option behaves as the amount of sales increases. If Finn chooses the third option, his total salary will initially be lower than that for the other two options. However, by extending the domain, I will be able to tell whether Finn's total salary for Option 3 will ever be the same as, or more than, what he would earn with either of the first two options.")

- Based on the equation for the third pay option, as well as the lines for the graphs of the third option and the first option, students should note that eventually the lines for the first and third options will cross, and therefore the third option will be better than the first. Ask students, "How can you determine the point of intersection of the first and third options? What will this solution help you conclude?" (**Answer:** The solution to the system of equations for the first and third options can be found by graphing, with a sufficiently large domain, or they can be found using substitution or elimination. Students should conclude that after this point of intersection, the third option would be better than the first for Finn.)

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Ask, "If you compare the graphs and equations for Option 3 and Option 2 in the same manner, what can you conclude from your observations?" (**Answer:** "It looks as though the graph of Option 3 will eventually intersect that of Option 2, but at a much higher x-value. This tells me that Option 3 will eventually be a better choice than Option 2, but Finn will have to sell thousands of dollars more in merchandise before the two options intersect.") Guide students to compare the other options as needed.

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask students to share their initial predictions about which pay option they thought would make Finn the most money in the long run. Ask them to explain their thought processes and reasoning based on the given information. Discuss how and why students' predictions were correct, or not correct.
- Compare students' strategies and explanations for creating the systems of equations, graphing them, and interpreting the interaction of the graphs in the real-world context. Discuss the analysis students used to determine which pay option is best for Finn in the long run. Focus on the use of precise mathematical language.

Connecting to Key Concepts

Make explicit connections to key concepts:

- The graphing method, which involves solving a system of equations by graphing them on the same coordinate plane, allows you to visualize the solution to the system.

  In this task, students use the graphing method to analyze and compare different linear equations. They will see the point of intersection or solution for each two-equation system by graphing the equations on the same coordinate plane.

- To graph the equations, use a table of values, the slope-intercept form ($y = mx + b$) of the equations, or a graphing calculator.

  Students will use the given information to write equations in slope-intercept form. Then they will graph the equations they have written, either by hand or using a graphing calculator.

- Graphs of systems with one solution have two intersecting lines. The point of intersection is the solution to the system.

  Students will conclude from examining the graphs of the first two equations that the lines intersect, and that there is one and only one point of intersection. Therefore, they can conclude that the system has one solution. Upon graphing the third equation on the same coordinate plane, students will see that it also intersects the first two lines, each at a distinct point.

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Extending the Task

- To extend the task, provide students with the scenario of a fourth pay option that has a higher base salary than the other three options, but a lower rate of pay for the merchandise sold. Ask students to first predict if they think this will be a better option than any or all of the other three options. Then, ask students to graph the fourth option and analyze the results in comparison to the other three options. Discuss which factor has the most impact on the total salary: the *y*-intercept (base salary) or the slope (the commission rate).
- Another option for extending the task is to provide students with a list of several scenarios of how much merchandise Finn sells, and ask them to choose the best option for each scenario. For example, tell students, "Finn wants to analyze his pay options based on the following possible sales amounts for merchandise he sells in one week: $300, $500, $1,800, and $3,500. Determine which option is the best pay option for each amount." Encourage students to work together and discuss their results with each other.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** "I reasoned abstractly by identifying the quantities involved in the scenario and creating a system of equations to represent the relationship between the quantities. I also analyzed the graphs of the equations, which represent the different pay options.") "Which of your strategies represent quantitative reasoning?" (**Answer:** "In order to graph the equations by hand, I needed to generate points with which to graph the equations. I used quantitative reasoning when I substituted values for the independent variable, *x*, back into the equations in order to determine the total salary and generate these points. I also applied the order of operations and simplified the remaining numerical expressions when I evaluated the equations for a specific value of *x*.")
- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "Yes; I constructed viable arguments when I analyzed the graphs of the equations in order to determine which salary option is best. I examined the slope of the linear equations and the points of intersection, and I also evaluated the equations at specific values as a basis for constructing my arguments when discussing my conclusions with others.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I used mathematics to model this particular scenario by first creating an algebraic system of equations to represent the quantities and relationships involved in the problem. I also modeled with mathematics by graphing the equations on the same coordinate plane and analyzing the resulting graphs.")

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Alternate Strategies or Solutions

- Although the Coaching Sample Responses reflect using the graphing method to complete the task, some students may choose to use either the substitution method or the elimination method in order to determine the point of intersection for each pair of equations. Encourage students to consider the time involved in using these alternate methods. Point out that these methods will not help students identify the *best* pay option, since these methods only allow them to determine the solution to each two-equation system. Remind students that the substitution and elimination methods do not result in a visual representation of the system of equations, which is necessary in order to compare all three equations and draw the desired conclusions.
- Students may choose values for the price of merchandise sold and substitute these values into the equations they create, in order to determine which option provides more income. Discuss how this option will give students a general idea of which pay option might be best, but remind them that graphing allows for the behavior of the equations to be seen and analyzed over a greater domain. Therefore, students will only be able to make a more precise decision by analyzing the equations on a graph.

Technology

Students can use graphing calculators or other graphing technology for this task.

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Common Core State Standards

Mathematics I

Integrated Pathway



Teacher Resource

Unit 4

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INTEGRATED MATH

This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

Developers and reviewers include:

Joyce Hale	Valerie Ackley	Doug Kühlmann
Shelly Northrop Sommer	Laura McPartland	Mike May, S.J.
Ruth Estabrook	Cameron Larkins	James Quinlan
Vanessa Sylvester	Jennifer Blair	Peter Tierney-Fife
Zachary Lien	Nancy Pierce	

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Standards Correlations

Each lesson in this Integrated Pathway: Mathematics I program was written specifically to address the Common Core State Standards. Each lesson lists the standards covered in all the sub-lessons, and each sub-lesson lists the standards addressed in that particular section. In this section, you'll find a comprehensive list mapping the sub-lessons to the CCSS.

Guide to Common Core State Standards Annotation

As you use this program, you will come across a symbol included with the Common Core standards for some of the lessons and activities. These symbols are explained below.

Symbol: ⋆

Denotes: Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (⋆).

From http://www.walch.com/CCSS/00003

Symbol: (+)

Denotes: College and Career Readiness Standards

Advanced mathematics standards that are required in higher-level courses such as advanced statistics may also be included in lower-level courses. These additional standards are denoted by (+). According to the Common Core State Standards Initiative, "the evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6–8."

From http://www.walch.com/CCSS/00004

Connections to Future Courses

This section provides a map between topics introduced in each unit of this course and subsequent courses where each topic is revisited and built upon.

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CCSS INTEGRATED PATHWAY: MATHEMATICS I STANDARDS CORRELATIONS

Unit 4: Descriptive Statistics				
Lesson	**Sub-lesson number**	**Title**	**Standard(s)**	**Pages**
Lesson 1	**Working with a Single Measurement Variable**			
	4.1.1	Representing Data Sets	S–ID.1★	U4-7
	4.1.2	Comparing Data Sets	S–ID.2★	U4-29
	4.1.3	Interpreting Data Sets	S–ID.3★	U4-50
Lesson 2	**Working with Two Categorical and Quantitative Variables**			
	4.2.1	Summarizing Data Using Two-Way Frequency Tables	S–ID.5★	U4-77
	4.2.2	Solving Problems Given Functions Fitted to Data	S–ID.6a★	U4-95
	4.2.3	Analyzing Residuals	S–ID.6b★	U4-118
	4.2.4	Fitting Linear Functions to Data	S–ID.6c★	U4-141
Lesson 3	**Interpreting Linear Models**			
	4.3.1	Interpreting Slope and *y*-intercept	S–ID.7★	U4-173
	4.3.2	Calculating and Interpreting the Correlation Coefficient	S–ID.8★	U4-194
	4.3.3	Distinguishing Between Correlation and Causation	S–ID.9★	U4-211

TOPICS FROM FUTURE COURSES: CCSS INTEGRATED PATHWAY MATHEMATICS I

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 4	Descriptive statistics	Math III Unit 1	extended to using statistics to draw conclusions

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Conceptual Activities

Use these interactive open education and/or Desmos resources to build conceptual understanding of mathematical ideas. (*Note*: Activity links will be monitored and repaired or replaced as necessary.)

- Desmos. "LEGO Prices."

  http://www.walch.com/ca/01012

  Use the concept of linear regression to predict the cost of a LEGO set with x pieces. (This activity does NOT use the calculator, just the concept. Participants draw the line on the graph, and Desmos calculates the equation.)

- Desmos. "Predicting Movie Ticket Prices."

  http://www.walch.com/ca/01018

  Build a model to describe the relationship between average movie ticket prices and time, then use that model to make predictions about past and future ticket prices. Participants also interpret the parameters of their equation in context.

- Desmos. "Polygraph: Histograms."

  http://www.walch.com/ca/01024

  This activity is designed to spark vocabulary-rich conversations about histograms. Key vocabulary terms that may appear in student questions include *shape*, *center*, *spread*, *roughly symmetric*, *skew right*, *skew left*, *mean*, *median*, *range*, *peak*, *unimodal*, and *bimodal*.

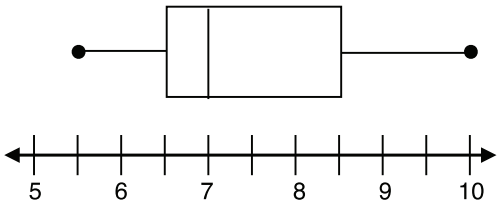
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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. What is the median of the data set used to create the box plot?



a. 5.5
c. 7
b. 6.5
d. 8.5

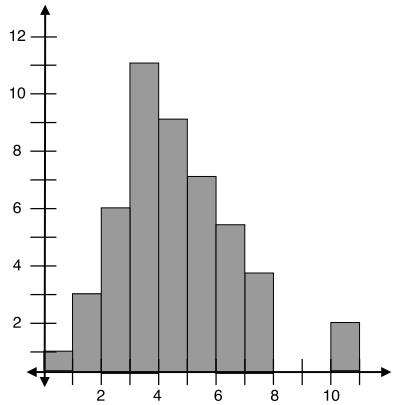
2. What is the interquartile range of the data?

18 21 22 23 26 27 28 29

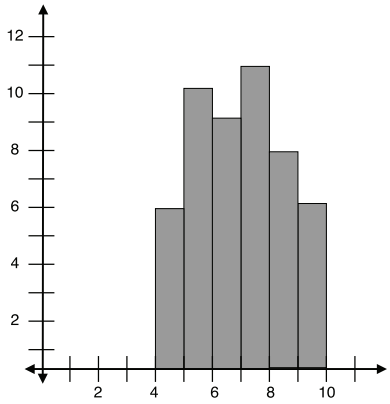
a. 3
c. 7
b. 6
d. 21.5

3. Look at the data presented in the following histograms. Which comparison is true?

Data Set 1



Data Set 2

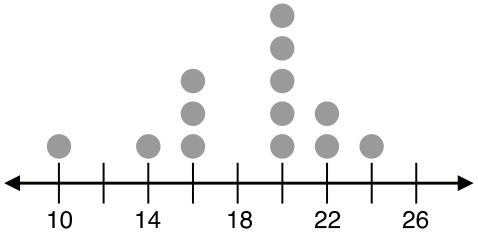


a. Data Set 1: greater mean; Data Set 1: greater variation
b. Data Set 1: greater mean; Data Set 2: greater variation
c. Data Set 2: greater mean; Data Set 2: greater variation
d. Data Set 2: greater mean; Data Set 1: greater variation

continued

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4. Which data set does the dot plot represent?



   a. Data Set 1: 10, 14, 16, 18, 18, 20, 20, 20, 20, 20, 22

   b. Data Set 2: 10, 14, 16, 16, 16, 20, 20, 20, 20, 20, 22, 22, 24

   c. Data Set 3: 10, 14, 14, 16, 16, 16, 18, 18, 18, 18, 18, 18, 22, 22, 24

   d. Data Set 4: 10, 14, 16, 16, 16, 16, 18, 18, 18, 18, 18, 22, 22, 24

5. Which value in the data set appears to be an outlier?

   41 51 27 48 45 46 39 45 38 39

   a. 27

   b. 41

   c. 46

   d. 48

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Lesson 1: Working with a Single Measurement Variable

Instruction

Common Core State Standards

SMP	
1 ✓	2
3 ✓	4 ✓
5	6 ✓
7	8

S–ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).★

S–ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.★

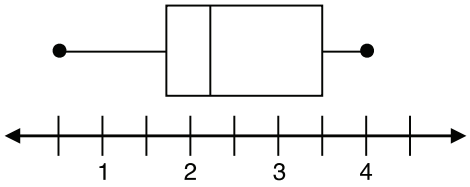
S–ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).★

Essential Questions

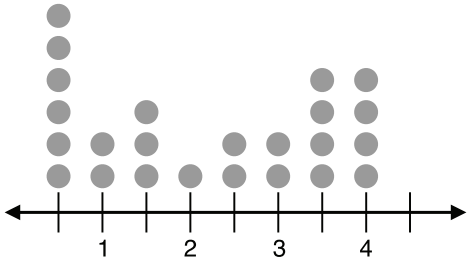
1. What do measures of spread describe about a data set?
2. What can be learned about a data set by looking at a frequency plot?
3. How can two data sets be compared quantitatively?
4. How do outliers influence measures of center?

WORDS TO KNOW

box plot a plot showing the minimum, maximum, first quartile, median, and third quartile of a data set; the middle 50% of the data is indicated by a box. Example:



dot plot a frequency plot that shows the number of times a response occurred in a data set, where each data value is represented by a dot. Example:



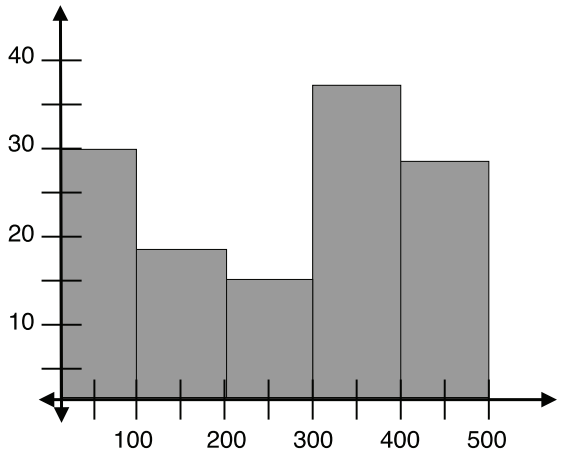
SRB p. U4-1

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Instruction

first quartile the value that identifies the lower 25% of the data; the median of the lower half of the data set; written as Q_1

histogram a frequency plot that shows the number of times a response or range of responses occurred in a data set. Example:



interquartile range the difference between the third and first quartiles; 50% of the data is contained within this range

mean the average value of a data set, found by summing all values and dividing by the number of data points

mean absolute deviation the average distance between each data point and the mean; found by summing the absolute values of the difference between each data point and the mean, then dividing this sum by the total number of data points

measures of center values that describe expected and repeated data values in a data set; the mean and median are two measures of center

measures of spread a measure that describes the variance of data values, and identifies the diversity of values in a data set

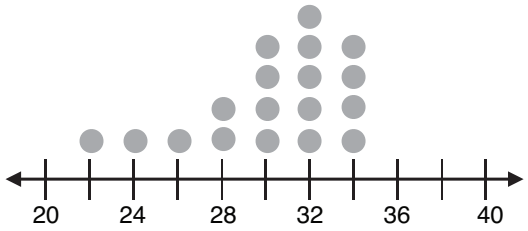
median the middle-most value of a data set; 50% of the data is less than this value, and 50% is greater than it

outlier a data value that is much greater than or much less than the rest of the data in a data set; mathematically, any data less than $Q_1 - 1.5(IQR)$ or greater than $Q_3 + 1.5(IQR)$ is an outlier

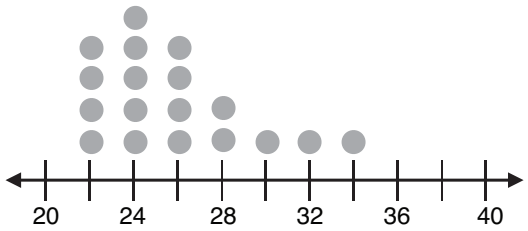
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Instruction

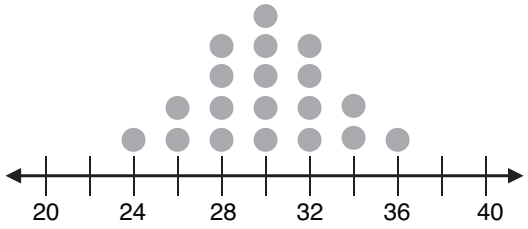
skewed to the left data concentrated on the higher values in the data set, which has a tail to the left. Example:



skewed to the right data concentrated on the lower values in the data set, which has a tail to the right. Example:



symmetric situation in which data is concentrated toward the middle of the range of data; data values are distributed in the same way above and below the middle of the sample. Example:



third quartile value that identifies the upper 25% of the data; the median of the upper half of the data set; 75% of all data is less than this value; written as Q_3

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Recommended Resources

- Braining Camp. "Box Plots—Questions."

  http://walch.com/rr/CAU4L1BoxPlotQuestions

  This site provides an interactive quiz with 10 multiple-choice questions about box plots. The computer scores the results.

- Khan Academy. "Box-and-Whisker Plots."

  http://walch.com/rr/CAU4L1BoxPlotVideo

  This page offers a set of videos that demonstrate how to create a box plot (also known as a box-and-whisker plot) given a data set.

- MathIsFun.com. "Histograms."

  http://walch.com/rr/CAU4L1Histograms

  This site gives an overview of what histograms are and aren't. Users can answer questions about histograms, and the computer scores the results.

- Onlinestatbook.com. "Dot Plots."

  http://walch.com/rr/CAU4L1DotPlots

  This site describes four different types of dot plots and then asks questions about interpreting the dot plots.

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Lesson 4.1.1: Representing Data Sets

Warm-Up 4.1.1

There are 10 ninth-grade classrooms. The number of students in each classroom is in the following table. Use the table to answer the questions about the number of students.

Classroom	Number of students
1	25
2	32
3	30
4	30
5	29
6	35
7	25
8	35
9	29
10	26

1. What is the median number of students in a class?

2. What is the first quartile?

3. What is the third quartile?

4. Which classroom has the least number of students?

5. Which classroom has the greatest number of students?

6. Are there any striking deviations in the data? Explain.

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Instruction

Lesson 4.1.1: Representing Data Sets

Common Core State Standard

S–ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).★

Warm-Up 4.1.1 Debrief

1. What is the median number of students in a class?

   Order the data (the number of students in each classroom) from least to greatest.

Classroom	Number of students
1	25
7	25
10	26
5	29
9	29
3	30
4	30
2	32
6	35
8	35

   The median is the middle-most value of the data set. If there is an odd number of data values, find the middle-most value. If there is an even number of data values, find the average of the two middle-most values. The median number of students is the average of the fifth and sixth values, 29 and 30.

$$\frac{29+30}{2}=29.5$$

2. What is the first quartile?

   The median divides the data into two halves. The lower half of this data set is the first five data values: 25, 25, 26, 29, and 29. The first quartile is the middle-most value of the lower half of the data. The middle-most value of the lower half of the data is the third data value, 26.

3. What is the third quartile?

   The upper half of this data set is the last five data values: 30, 30, 32, 35, and 35. The third quartile is the middle-most value of the upper half of the data. The middle-most value of the upper half of the data is the third value, 32.

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4. Which classroom has the least number of students?

   There are two classrooms with the least number of students. Both classrooms 1 and 7 have 25 students.

5. Which classroom has the greatest number of students?

   There are two classrooms with the greatest number of students. Both classrooms 6 and 8 have 35 students.

6. Are there any striking deviations in the data? Explain.

   Think about the typical number of students in a classroom. This can vary widely from school to school. In this data set though, the interquartile range is 6:

   $Q_3 - Q_1 = 32 - 26 = 6$

   Comparing the interquartile range and the median of 29.5 to the lowest data value, 25, the lowest data value is not far away from the mean in terms of the interquartile range. The lowest data value is not a striking deviation.

   Now compare the interquartile range of 6 and the median of 29.5 to the highest data value, 35. The highest value is not far away from the mean in terms of the interquartile range. The highest data is not a striking deviation.

Connection to the Lesson

- In this lesson, students will plot data on the real number line using dot plots, histograms, and box plots.
- To create box plots from a given data set, students will need to determine the least data value, or minimum; the greatest data value, or maximum; the first quartile; the median; and the third quartile.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- finding the median of a data set (6.SP.5c)
- finding the first and third quartiles of a data set (6.SP.5c)
- plotting values on a real number line (6.NS.6c)

Introduction

Measures of center and variability can be used to describe a data set. The mean and median are two measures of center. The **mean** is the average value of the data, found by finding the sum of all of the values in the data set and dividing by the number of values. The **median** is the middle-most value in a data set. These measures are used to generalize data sets and identify common values. Interquartile range and mean absolute deviation describe variability of the data set. **Interquartile range** is the difference between the third and first quartiles. The **first quartile** is the median of the lower half of the data set. The **third quartile** is the median of the upper half of the data set. The **mean absolute deviation** is the average distance between each data point and the mean. **Measures of spread** describe the variance of data values (how spread out they are), and identify the diversity of values in a data set. Measures of spread are used to help explain whether data values are very similar or very different.

Data can be represented graphically using a number line. Graphs provide a visual representation of data; just by looking at a graph, you can quickly understand the spread and center of a data set. Dot plots and histograms show the frequency of a data value. In a **dot plot**, each data value is represented by a dot. The number of times a value is repeated corresponds to the number of dots above that value. In a **histogram**, the height of a rectangle above a value corresponds to the number of data values with that value. When looking at either a dot plot or histogram, it is easy to see both the most repeated data values and the spread of the data. If a data set is large, a histogram is easier to use because a single dot does not need to be drawn for each data value.

A **box plot** shows the minimum, maximum, first quartile, median, and third quartile of numerical data. The middle 50% of the data is represented with a box. Lines on either side of the box extend to the minimum and maximum data values. A box plot shows the range of data in a data set, and measures of center can be easily seen on a box plot. Box plots can be used to compare the values of multiple data sets.

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Instruction

Key Concepts

- Numerical data can be represented graphically on the real number line.
- Dot plots and histograms show the frequency of each data value in a data set.
- Each data value in a data set is represented by a dot over that value in a dot plot.
- In a histogram, a rectangle is drawn above each value in a data set. The height of each rectangle corresponds to the number of data points with that value.
- A histogram can show the frequency of a range of values.
- The minimum, maximum, first quartile, median, and third quartile of a data set must be calculated before creating a box plot.
- In a box plot, a rectangle is drawn starting at the first quartile and ending at the third quartile. The rectangle shows the middle 50% of the data set. The median is represented in the rectangle by a line. Whiskers are drawn from the rectangle to the minimum and maximum data values.
- A box plot shows more information about the distribution of a data set than a dot plot or histogram.
- A dot plot or histogram provides information about the size of a data set, which cannot be seen in a box plot.

Common Errors/Misconceptions

- confusing the mean and median
- using the mean in a box plot instead of the median
- incorrectly setting up the number line before creating a dot plot or histogram
- forgetting to include data values on a dot plot or histogram because they are not labeled on the number line

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Guided Practice 4.1.1

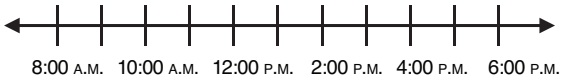
Example 1

A pharmacy records the number of customers each hour that the pharmacy is open. The staff is using the information to determine how many people need to be working at the pharmacy at each time of day. The number of customers is in the table below. Use the table to create a histogram to help the pharmacy staff understand how many customers are in the pharmacy at each time of day.

Time frame	Number of customers
8:00 A.M.–9:00 A.M.	2
9:00 A.M.–10:00 A.M.	0
10:00 A.M.–11:00 A.M.	8
11:00 A.M.–12:00 P.M.	14
12:00 P.M.–1:00 P.M.	23
1:00 P.M.–2:00 P.M.	12
2:00 P.M.–3:00 P.M.	7
3:00 P.M.–4:00 P.M.	3
4:00 P.M.–5:00 P.M.	5

1. Draw a number line on an *x*-axis that corresponds to the range of the data.

   The *x*-axis for this data will show the times the customers were counted. The number line for the pharmacy must include the times from 8:00 A.M. until 5:00 P.M. If using a number line that counts by twos, extend the number line to 6:00 P.M.



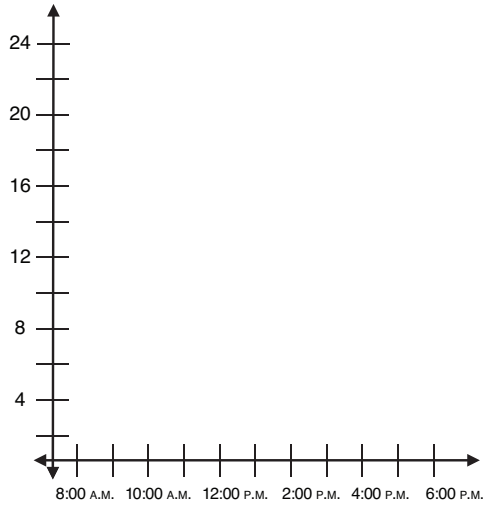
SWB p. 9; SRB p. U4-7

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Instruction

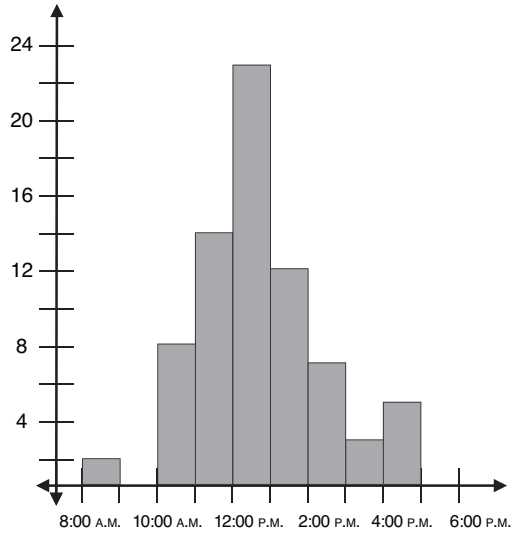
2. Draw a *y*-axis that corresponds to the least and greatest number of times a data value is repeated. The *y*-axis should be to the left of the labeled *x*-axis.

   The number of customers arriving in each time frame ranges from 0 customers to 23 customers. The *y*-axis needs to show values from 0 to 23. If using a number line that counts by twos, extend the number line to 24.



3. Create a rectangle at each value showing the number of data points at each data value.

   The rectangles will each span an hour, and will show the number of customers in that hour. There will be no rectangle from 9:00 A.M. to 10:00 A.M., because there were no customers at that hour.



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Instruction

Example 2

Anna and Ethan watch 20 thirty-minute shows during the month of June. They record the number of commercials that air during each show in the following table. Create a dot plot to display the number of commercials that aired during the 20 shows.

Television show	Number of commercials
A	17
B	17
C	15
D	17
E	14
F	17
G	15
H	19
I	15
J	16
K	12
L	14
M	15
N	17
O	18
P	18
Q	18
R	18
S	13
T	14

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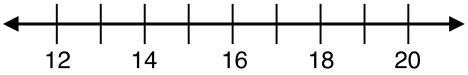
Instruction

1. Arrange the data from least to greatest.

Television show	Number of commercials
K	12
S	13
E	14
L	14
T	14
C	15
G	15
I	15
M	15
J	16
A	17
B	17
D	17
F	17
N	17
O	18
P	18
Q	18
R	18
H	19

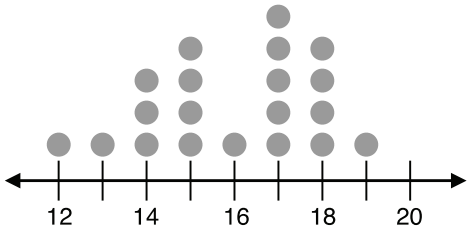
2. Draw a number line on an *x*-axis that corresponds to the range of the data values.

   The *x*-axis for this data will show the number of commercials during a 30-minute television show. The number line must include values from 12 to 19. If counting by twos, extend the number line to 20.



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3. Draw each data value as a dot above the number line. The number of dots above each data value shows the number of times that value occurs in the data set.



Example 3

The website Rate My Phone conducts reviews of smartphones. One aspect of the phones that is tested is battery life. The minutes of battery life for the newest 25 phones is recorded in the following table. Draw a box plot to represent the data.

Smartphone	Minutes of battery life	Smartphone	Minutes of battery life
A	380	N	470
B	530	O	280
C	350	P	300
D	390	Q	440
E	520	R	490
F	520	S	530
G	430	T	340
H	330	U	250
I	550	V	260
J	290	W	730
K	360	X	520
L	550	Y	320
M	370		

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Instruction

1. Order the data from least to greatest. Note the minimum and maximum data values.

Smartphone	Minutes of battery life	Smartphone	Minutes of battery life
U	250	G	430
V	260	Q	440
O	280	N	470
J	290	R	490
P	300	X	510
Y	320	E	520
H	330	F	520
T	340	B	530
C	350	S	530
K	360	I	550
M	370	L	550
A	380	W	730
D	390		

The minimum data value is 250, and the maximum data value is 730.

2. Find the median of the data.

The median is the middle-most data value. There are an odd number of data values, so the median is the 13th data value, 390.

3. Find the first quartile of the data.

The first quartile is the middle-most value of the lower half of the data. There are 12 data values in the lower half of the data, so the first quartile is the average of the sixth and seventh data values (320 and 330).

$$\frac{320+330}{2}=325$$

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Instruction

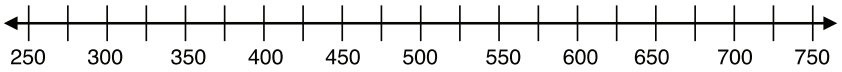
4. Find the third quartile of the data.

   The third quartile is the middle-most value of the upper half of the data. There are 12 data values in the upper half of the data, so the third quartile is the average of the 19th and 20th data values (520 and 520).

$$\frac{520+520}{2}=520$$

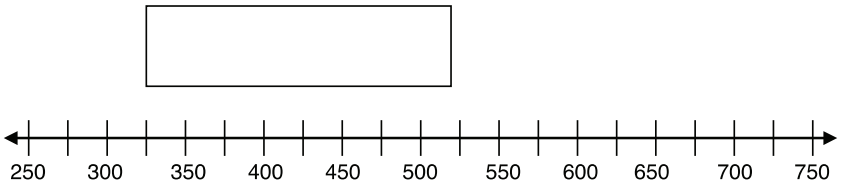
5. Draw a number line that includes the minimum and maximum data values.

   The minimum data value is 250, and the maximum data value is 730. If counting by 50s, extend the number line to 750.



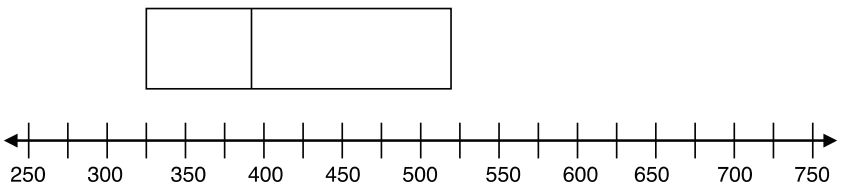
6. Draw a box, beginning at the first quartile and ending at the third quartile.

   The first quartile is 325 and the third quartile is 520.



7. Draw a line in the box at the median.

   The median is 390.

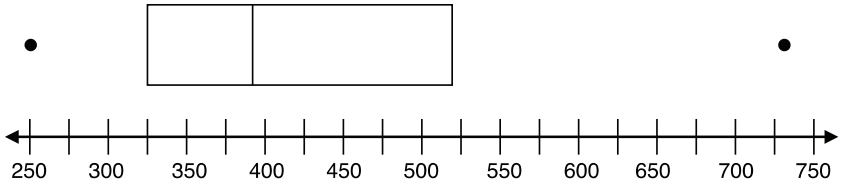


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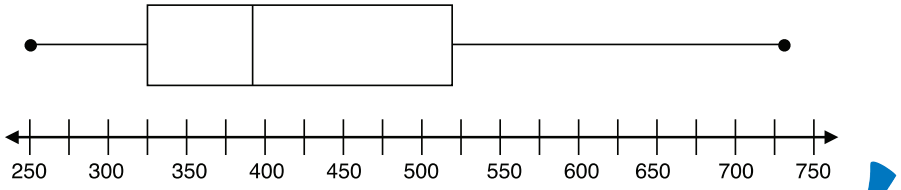
Instruction

8. Draw a point at the minimum and maximum data values.

   The minimum is 250 and the maximum is 730.



9. Connect the minimum and maximum data values to the box.



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Problem-Based Task 4.1.1: Representing Data Sets

SMP
1 ✓ 2
3 ✓ 4
5 6
7 8

Companies conduct research to learn more about how much money customers will pay for new products or services. A new tutoring service, Favorite Tutors, surveys local college students and professors. Look at the following information from three surveys. Help Favorite Tutors determine how to organize the information from the surveys into graphs. Use the survey information to find the following information for the company:

- an expected number of hours per week professors spend helping students with assignments
- the number of students willing to pay different prices per semester
- the number of different classes for which a single student might need assignment help

Survey 1

- People surveyed: Professors
- Question asked: How many hours per week do you spend helping students with assignments?

Professor	Hours per week
A	8
B	12
C	4
D	4
E	2
F	4
G	12
H	3
I	8
J	6

Survey 2

- People surveyed: College students
- Question asked: How much money would you pay per semester for an unlimited tutoring service?

Amount	Number of students
$0–$100	30
$101–$200	18
$201–$300	15
$301–$400	37
$401–$500	29

continued

SWB p. 13

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Survey 3

- People surveyed: College students
- Question asked: This semester, for how many different classes have you visited a professor for assignment help?

Student	Number of classes
a	2
b	1
c	2
d	0
e	4
f	1
g	2
h	4
i	1
j	2
k	0
l	0
m	3



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Problem-Based Task 4.1.1: Representing Data Sets

Coaching

a. Favorite Tutors wants to use data from the first survey to learn about the expected number of hours per week a professor spends helping students with assignments. Which type of graph shows the median value of a data set?

b. Organize the data from the first survey from least to greatest. Note the minimum and maximum value.

c. Find the median, first quartile, and third quartile of the data in Survey 1.

d. Use the minimum, maximum, first quartile, median, and third quartile to create a box plot of the data.

e. Favorite Tutors wants to use data from the second survey to understand the number of students willing to pay different amounts for tutoring each semester. Which type(s) of graph(s) should be used to show the frequency of responses?

f. Look at the data in the table. Were there many responses to this survey, or just a few?

g. Were the responses in this survey a range of values or a single value?

h. Which type of graph best shows the number of students willing to pay different amounts for tutoring?

i. Draw an x-axis that shows the ranges of student responses in Survey 2.

j. Create a y-axis with a maximum value that is greater than or equal to the number of students selecting each dollar amount.

k. Create a histogram showing the amount students are willing to pay for tutoring each semester.

l. Favorite Tutors wants to use data from the third survey to understand the number of different classes for which a student needed assignment help. Which type(s) of graph(s) should be used to show the frequency of responses?

m. Look at the data in the table. Were there many responses to this survey, or just a few?

n. Were the responses in this survey a range of values or a single value?

o. Which type of graph best shows the number of different classes for which a student needed assignment help?

p. Arrange the data from least to greatest.

q. Draw an x-axis that shows the range of responses in Survey 3.

r. Draw a dot above each value to represent a student who gave that response.

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Instruction

Problem-Based Task 4.1.1: Representing Data Sets

Coaching Sample Responses

a. Favorite Tutors wants to use data from the first survey to learn about the expected number of hours per week a professor spends helping students with assignments. Which type of graph shows the median value of a data set?

A box plot shows the median of a data set.

b. Organize the data from the first survey from least to greatest. Note the minimum and maximum value.

Professor	Hours per week
E	2
H	3
C	4
D	4
F	4
J	6
A	8
I	8
B	12
G	12

The minimum number of hours is 2, and the maximum number of hours is 12.

c. Find the median, first quartile, and third quartile of the data in Survey 1.

The middle-most value, the median, is the average of the fifth and sixth data values:

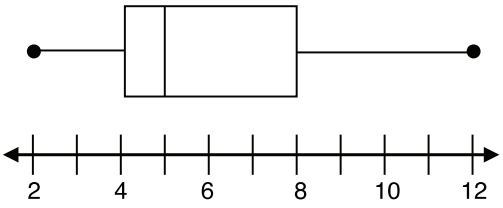
$$\frac{4+6}{2}=5$$

The first quartile is the middle-most value of the lower half of the data, or the third data value: 4. The third quartile is the middle-most value of the upper half of the data, or the eighth data value: 8.

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d. Use the minimum, maximum, first quartile, median, and third quartile to create a box plot of the data.

The *x*-axis should include the range of values: 2 to 12. Draw a vertical line at the first quartile, median, and third quartile. Create a box using the first and third quartile lines. Connect the minimum and maximum to the box by drawing a line from each point to the box.



e. Favorite Tutors wants to use data from the second survey to understand the number of students willing to pay different amounts for tutoring each semester. Which type(s) of graph(s) should be used to show the frequency of responses?

Both dot plots and histograms show frequencies.

f. Look at the data in the table. Were there many responses to this survey, or just a few?

There were many responses; more than 100 students responded to this survey.

g. Were the responses in this survey a range of values or a single value?

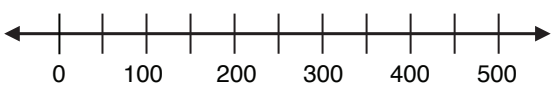
The responses were a range of values. Each student selected a range they were willing to pay, instead of giving a single value.

h. Which type of graph best shows the number of students willing to pay different amounts for tutoring?

A histogram is the best frequency graph to show the number of students willing to pay the different amounts. A histogram is easier to draw when there are many data points, and a histogram can be used for ranges of values.

i. Draw an *x*-axis that shows the ranges of student responses in Survey 2.

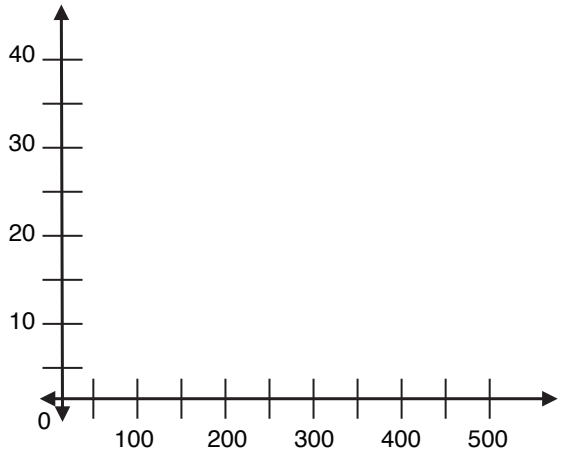
The range of responses is the lowest value, \$0, to the highest value, \$500. The *x*-axis should include all of these values.



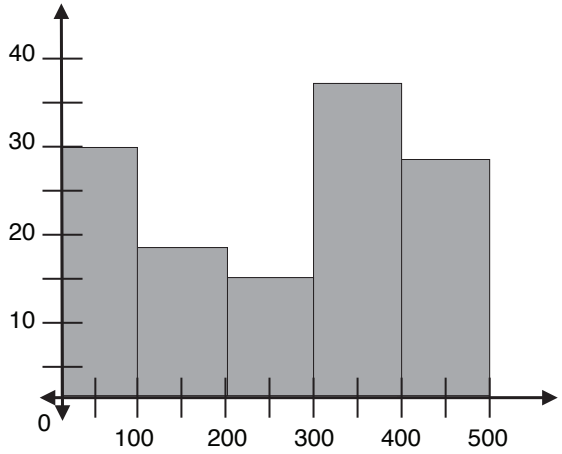
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j. Create a *y*-axis with a maximum value that is greater than or equal to the number of students selecting each dollar amount.

The least number of responses for a dollar amount was 15, and the greatest number of responses for a dollar amount was 37. The *y*-axis should include all of these values.



k. Create a histogram showing the amount students are willing to pay for tutoring each semester.



l. Favorite Tutors wants to use data from the third survey to understand the number of different classes for which a student needed assignment help. Which type(s) of graph(s) should be used to show the frequency of responses?

Both dot plots and histograms show frequencies of responses.

m. Look at the data in the table. Were there many responses to this survey, or just a few?

There were just a few responses; 13 students responded to the survey.

n. Were the responses in this survey a range of values or a single value?

The responses were a single value.

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o. Which type of graph best shows the number of different classes for which a student needed assignment help?

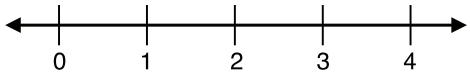
A dot plot would be the best way to show the number of different classes for which a student needed assignment help.

p. Arrange the data from least to greatest.

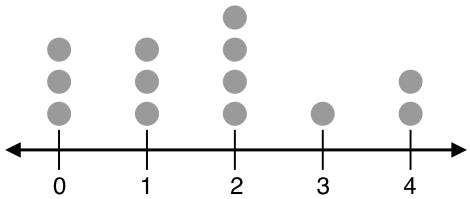
Student	Number of classes
d	0
k	0
l	0
b	1
f	1
i	1
a	2
c	2
g	2
j	2
m	3
e	4
h	4

q. Draw an *x*-axis that shows the range of responses in Survey 3.

The responses ranged from 0 to 4 classes.



r. Draw a dot above each value to represent each student who gave that response.



Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 4.1.1: Representing Data Sets

Kirsten plays softball in the spring. Each game, she records the number of times she reaches first base without being called out. Use the data in the table to solve problems 1–5.

Game	Number of times at first	Game	Number of times at first
1	5	10	0
2	1	11	1
3	2	12	1
4	0	13	0
5	2	14	5
6	2	15	5
7	4	16	4
8	4	17	0
9	0	18	4

1. Create a dot plot showing the number of times Kirsten reached first base in each game.

2. What is the median number of times Kirsten reached first base?

3. Find the minimum, maximum, first quartile, and third quartile of the data set.

4. Create a box plot showing the number of times Kirsten reached first base.

5. Kirsten wants to analyze her performance using this data. She wants to understand the range of her data and the frequency of different results. Which graph, the dot plot or the box plot, will be most useful to Kirsten? Explain.

continued

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Dr. Singh is a veterinarian. He records the weights of each pet. The weights of 10 German shepherds, all 4-year-old males, are in the following table, rounded to the nearest pound. Use this information to solve problems 6–10.

Weight in pounds
80
78
82
84
81
89
83
81
81
82

6. Create a histogram showing the weights of Dr. Singh's German shepherds.

7. What is the median weight of the German shepherds?

8. Find the minimum, maximum, first quartile, and third quartile of the data set.

9. Create a box plot showing the weights of the German shepherds.

10. Dr. Singh wants to analyze the weights of the German shepherds. He wants to understand the center and spread of his data, so that he has a better idea of an expected weight for a 4-year-old male German shepherd. Which graph would be most useful to Dr. Singh? Explain.

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Lesson 4.1.2: Comparing Data Sets

Warm-Up 4.1.2

A group of 10 young mountaineers is climbing Mount Everest. The climbers' ages are listed below. Use this data to answer the questions.

28 23 15 22 21 25 22 18 22 14

1. What is the mean age of the data set?

2. What is the median age of the data set?

3. What is the mean absolute deviation of the data set?

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Instruction

Lesson 4.1.2: Comparing Data Sets

Common Core State Standard

S–ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.★

Warm-Up 4.1.2 Debrief

1. What is the mean age of the data set?

   The mean is the average of the data values.

   $$\frac{28+23+15+22+21+25+22+18+22+14}{10}=\frac{210}{10}=21$$

2. What is the median age of the data set?

   Arrange the data from least to greatest.

   14 15 18 21 22 22 22 23 25 28

   There is an even number of data values. The median is the average of the two middle-most values of the data set.

   $$\frac{22+22}{2}=22$$

3. What is the mean absolute deviation of the data set?

   Find the difference between each data value and the mean (21).

Data value	Absolute difference from mean
14	$\|14-21\|=7$
15	$\|15-21\|=6$
18	$\|18-21\|=3$
21	$\|21-21\|=0$
22	$\|22-21\|=1$
22	$\|22-21\|=1$
22	$\|22-21\|=1$
23	$\|23-21\|=2$
25	$\|25-21\|=4$
28	$\|28-21\|=7$

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Find the sum of the absolute differences and divide by the number of data points (10).

$$\frac{7+6+3+0+1+1+1+2+4+7}{10}=\frac{32}{10}=3.2$$

Connection to the Lesson

- In this lesson, students will compare data sets using measures of center and spread.
- Students will need to be able to compute two measures of center: the mean and median.
- Students will need to compute measures of spread, such as the mean absolute deviation.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- calculating the mean and median of data (6.SP.5c)
- calculating the mean absolute deviation and interquartile range of data (6.SP.5c)
- creating graphical representations of data, such as dot plots, box plots, and histograms (6.SP.4)

Introduction

Data sets can be compared by examining the differences and similarities between measures of center and spread. The mean and median of a data set are measures of center. The mean absolute deviation is a measure of spread that describes the range of data values, with respect to the mean. The mean absolute deviation is the average of the absolute values of the differences between each data value and the mean. The interquartile range is a measure of spread that describes the range of the middle 50% of a data set. The interquartile range is the difference between the third and first quartiles.

The center and spread of a data set can also be seen in the shape of a graphical representation of the data. The range of data values can be seen in the *x*-axis of a graphical representation. Clusters of data values can be seen in graphs that show frequency, such as dot plots and histograms. The interquartile range and median are shown in box plots.

Key Concepts

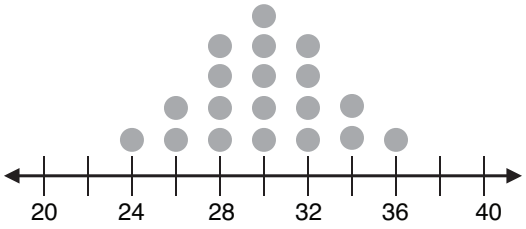
- Measures of center, such as the mean and median, describe the most common values of a data set. The mean is influenced by very small or large data values, whereas the median is not.
- Measures of spread describe the range of data in a set. Interquartile range and mean absolute deviation are measures of spread.
- The interquartile range shows the range of the middle 50% of a data set. It is the difference between the third and first quartiles.
- The mean absolute deviation compares data values to the mean of a data set. If the mean absolute deviation is large, this is a sign that the data points are distributed farther from the mean.
- Two or more data sets can be compared using measures of center and spread. When choosing a measure of center or spread, identify whether there are very large or very small data values that may influence the mean.
- Data can be compared graphically. The shape of a data set can be seen in a frequency plot, such as a dot plot or histogram.

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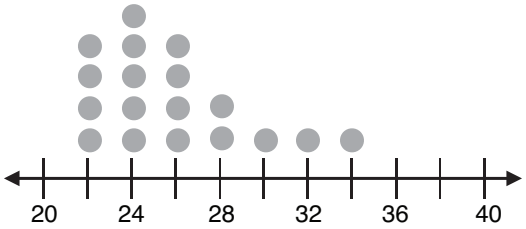
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Instruction

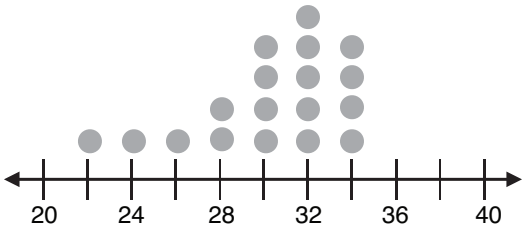
- Data that is **symmetric** is concentrated toward the middle of the range of data. The data is arranged the same way on both sides.



- Data that is **skewed to the right** is concentrated toward the lower range of the data; it has a tail to the right.



- Data that is **skewed to the left** is concentrated toward the upper range of the data; it has a tail to the left.



- Data that is widely or evenly distributed has greater variation, and data that clusters around a set of values has less variation.
- Data can also be compared using a box plot. The width of the box displays the range of the middle 50% of the data; the width increases as variation increases.

Common Errors/Misconceptions

- comparing graphical data that is not drawn using the same scale on the *x*- and/or *y*-axes
- comparing different measures of center or variation
- using an average, such as the mean, to compare data that has very small or very large data values

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Guided Practice 4.1.2

Example 1

Two hockey teams recorded the number of goals scored each game in the following tables. Use the tables to compare the number of goals scored per game for the two teams using both measures of center and spread.

Team 1: Ice Kings	
Game	**Number of goals scored**
1	0
2	0
3	0
4	2
5	1
6	2
7	2
8	1
9	2
10	6
11	5
12	2
13	5
14	2

Team 2: Gliders	
Game	**Number of goals scored**
1	3
2	6
3	0
4	6
5	0
6	1
7	1
8	2
9	0
10	3
11	3
12	2
13	4
14	2

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Instruction

1. Order the data to determine which measure of center to use.

   Order the goals scored for each team from least to greatest.

Team 1: Ice Kings	
Game	**Number of goals scored**
1	0
2	0
3	0
5	1
8	1
4	2
6	2
7	2
9	2
12	2
14	2
11	5
13	5
10	6

Team 2: Gliders	
Game	**Number of goals scored**
3	0
5	0
9	0
6	1
7	1
8	2
12	2
14	2
1	3
10	3
11	3
13	4
2	6
4	6

2. Look at the range of values. Determine if there are any data values in either table that are much larger or much smaller than the rest of the data set. If there are, use the median. If there are not, use either the mean or the median.

   The largest and smallest data values in each data set are not much smaller or much larger than the rest of the data. There aren't any values in the tables that will greatly influence the mean, so either the median or mean would be valid measures of center.

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3. Calculate the chosen measure of center.

   Since it was determined that either measure of center was acceptable, select one. Let's use the mean. To calculate the mean, find the sum of goals scored for each team, and then divide the total goals by the number of games played.

   Ice Kings:

   $$\frac{0+0+0+1+1+2+2+2+2+2+2+5+5+6}{14} \approx 2.14$$

   Gliders:

   $$\frac{0+0+0+1+1+2+2+2+3+3+3+4+6+6}{14} \approx 2.36$$

4. Compare the measures of center for the two teams.

   The two teams have performed similarly over the 14 games. In any given game, you could expect either team to score at least 2 goals.

5. Calculate a measure of spread.

   Since we chose to calculate the mean, now we will use the mean absolute deviation to compare the goals per team. Mean absolute deviation compares the range of values to the mean value. To calculate the mean absolute deviation, first find the absolute value of the difference between each data point and the mean:

   |data point – mean|

   The mean for the Ice Kings is 2.14. The mean for the Gliders is 2.36.

(*continued*)

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Team 1: Ice Kings		
Game	**Number of goals scored**	**Absolute difference**
1	0	$\|0 - 2.14\| = 2.14$
2	0	$\|0 - 2.14\| = 2.14$
3	0	$\|0 - 2.14\| = 2.14$
5	1	$\|1 - 2.14\| = 1.14$
8	1	$\|1 - 2.14\| = 1.14$
4	2	$\|2 - 2.14\| = 0.14$
6	2	$\|2 - 2.14\| = 0.14$
7	2	$\|2 - 2.14\| = 0.14$
9	2	$\|2 - 2.14\| = 0.14$
12	2	$\|2 - 2.14\| = 0.14$
14	2	$\|2 - 2.14\| = 0.14$
11	5	$\|5 - 2.14\| = 2.86$
13	5	$\|5 - 2.14\| = 2.86$
10	6	$\|6 - 2.14\| = 3.86$

Next, sum the absolute differences.

$2.14 + 2.14 + 2.14 + 1.14 + 1.14 + 0.14 + 0.14 + 0.14 + 0.14 + 0.14 + 0.14 + 2.86 + 2.86 + 3.86 = 19.12$

Lastly, divide the sum of the absolute differences by the number of data points (14).

$$\frac{19.12}{14} \approx 1.37$$

The mean absolute deviation for the Ice Kings is 1.37.

(*continued*)

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Team 2: Gliders		
Game	**Number of goals scored**	**Absolute difference**
3	0	$\|0 - 2.36\| = 2.36$
5	0	$\|0 - 2.36\| = 2.36$
9	0	$\|0 - 2.36\| = 2.36$
6	1	$\|1 - 2.36\| = 1.36$
7	1	$\|1 - 2.36\| = 1.36$
8	2	$\|2 - 2.36\| = 0.36$
12	2	$\|2 - 2.36\| = 0.36$
14	2	$\|2 - 2.36\| = 0.36$
1	3	$\|3 - 2.36\| = 0.64$
10	3	$\|3 - 2.36\| = 0.64$
11	3	$\|3 - 2.36\| = 0.64$
13	4	$\|4 - 2.36\| = 1.64$
2	6	$\|6 - 2.36\| = 3.64$
4	6	$\|6 - 2.36\| = 3.64$

Next, sum the absolute differences.

$2.36 + 2.36 + 2.36 + 1.36 + 1.36 + 0.36 + 0.36 + 0.36 + 0.64 + 0.64 + 0.64 + 0.64 + 1.64 + 3.64 + 3.64 = 21.72$

Lastly, divide the sum of the absolute differences by the number of data points (14).

$$\frac{21.72}{14} \approx 1.55$$

The mean absolute deviation for the Gliders is 1.55.

6. Compare the measures of spread and center for the two teams.

The Ice Kings have a slightly lower mean, but the number of goals scored per game also varies less than the Gliders. The Gliders have a higher mean, but with higher variation.

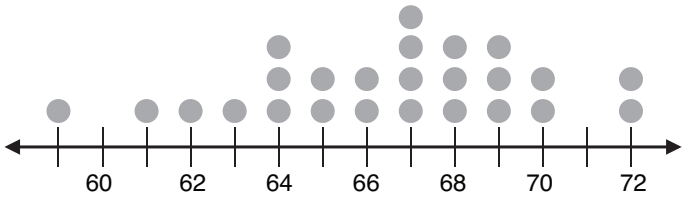
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Instruction

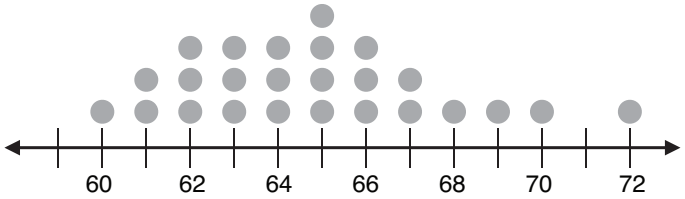
Example 2

Each girl in Mr. Sanson's class and in Mrs. Kwei's class measured her own height. The heights were plotted on the dot plots below. Use the dot plots to compare the heights of the girls in the two classes.

Mr. Sanson's Class



Mrs. Kwei's Class



1. Compare the range of recorded values.

   The overall range of heights of girls in the two classes is similar. The heights in the two classes range from 59 inches to 72 inches, and 60 inches to 72 inches.

2. Compare the middle values of the data sets.

   The girls in Mr. Sanson's class appear to be taller than the girls in Mrs. Kwei's class. By looking at where the dots are clustered, we can estimate that the middle height in Mr. Sanson's class is around 67 inches. The middle height in Mrs. Kwei's class is 65 inches.

3. Compare the variation in the data sets.

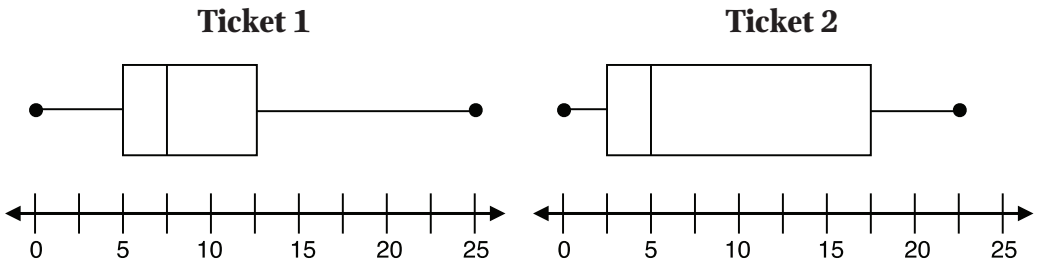
   The variation in the two sets of heights appears to be similar, except Mr. Sanson's data is skewed to the left and Mrs. Kwei's data is skewed to the right. The majority of the heights are within approximately 6 inches in both classes. The majority of the girls in Mr. Sanson's class are between 64 and 70 inches, and the majority of the girls in Mrs. Kwei's class are between 61 and 67 inches.

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Instruction

Example 3

Sam wants to buy a lottery ticket. There are two different tickets that he can buy, and each costs $10. He found a website with information about how much money others have won with their lottery tickets. The information is presented in two box plots, as shown. Use the two box plots to compare the amounts others have won with Ticket 1 and with Ticket 2.



1. Compare the range of data.

   The range of amounts won is similar. With Ticket 1, winnings ranged from $0 to $25, and with Ticket 2, winnings ranged from $0 to $22.50.

2. Compare the center of the data sets.

   The median of the data sets is shown by the vertical line inside the box. The median winnings from Ticket 1 are higher than the median winnings from Ticket 2.

3. Compare the variation of the data.

   The interquartile range, or middle 50% of each data set, is contained within the box in each box plot.

   The IQR of winnings with Ticket 1 is smaller than the IQR of winnings with Ticket 2. The median winnings with Ticket 1 are $7.50, indicating that half of the winnings were less than $7.50 and half were greater. The median winnings with Ticket 2 are $5, indicating that half the winnings were less than $5 and half were greater. Notice that for Ticket 1, 75% of buyers win at least $5, whereas for Ticket 2, only 50% of buyers win at least $5. It is more likely that Sam will win a greater amount with Ticket 1.

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Problem-Based Task 4.1.2: Comparing Data Sets

SMP	
1	2
3 ✓	4 ✓
5	6 ✓
7	8

The number of miles per gallon for any given car can vary. Factors such as speed and weather affect gas mileage. Liam fills the gas tank in his car 15 times and records his gas mileage for each tank. Alyssa also fills her car's gas tank 15 times and records her gas mileage for each tank. The gas mileages are listed in the following tables.

Liam's Gas Mileage	
Tank	**Miles per gallon**
1	21
2	17
3	17
4	18
5	22
6	21
7	18
8	17
9	19
10	21
11	19
12	18
13	21
14	19
15	18

Alyssa's Gas Mileage	
Tank	**Miles per gallon**
1	23
2	24
3	25
4	23
5	23
6	24
7	24
8	25
9	25
10	22
11	23
12	24
13	23
14	22
15	21

Sarah would like to buy a car that needs the least amount of gas. She is thinking of either buying a car like Liam's, or a car like Alyssa's. If Sarah would like a car with the highest gas mileage, which should she buy? Compare the data using measures of center and spread, and include a graph to show the differences between the data.

If Sarah would like a car with the highest gas mileage, which should she buy?

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Problem-Based Task 4.1.2: Comparing Data Sets

Coaching

a. Sort each data set from least to greatest.

b. Determine which measure of center to use to compare the data.

c. Calculate the measure of center for each data set.

d. Determine which measure of spread to use to compare the data.

e. Calculate the measure of spread for each data set.

f. Determine which type of graph would best help Sarah compare the gas mileage of the two cars.

g. Plot the data on two graphs.

h. Which kind of car should Sarah buy? Use the measures of center, the measures of spread, and the graphs to identify the car with the highest gas mileage.

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Problem-Based Task 4.1.2: Comparing Data Sets

Coaching Sample Responses

a. Sort each data set from least to greatest.

Liam's Gas Mileage	
Tank	**Miles per gallon**
2	17
3	17
8	17
4	18
7	18
12	18
15	18
9	19
11	19
14	19
1	21
6	21
10	21
13	21
5	22

Alyssa's Gas Mileage	
Tank	**Miles per gallon**
15	21
10	22
14	22
1	23
4	23
5	23
11	23
13	23
2	24
6	24
7	24
12	24
3	25
8	25
9	25

b. Determine which measure of center to use to compare the data.

If there are data values that are very high or very low in either data set, use the median. The largest and smallest data values in each data set are not much smaller or much larger than the rest of the data. There aren't any values that will greatly influence the mean, so either the median or mean would be a valid measure of center. Use the mean.

c. Calculate the measure of center for each data set.

Sum the data values, and divide by the number of data points.

Mean gas mileage of Liam's car:

$$\frac{17+17+17+18+18+18+18+19+19+19+21+21+21+21+22}{15} \approx 19.07$$

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Mean gas mileage of Alyssa's car:

$$\frac{21+22+22+23+23+23+23+23+24+24+24+24+25+25+25}{15}=23.4$$

d. Determine which measure of spread to use to compare the data.

Since the mean was used as the measure of center, the mean absolute deviation can be used to measure the spread.

e. Calculate the measure of spread for each data set.

To find the mean absolute deviation, first find the absolute value of the difference between each data point and the mean of the data set. Next, sum these differences and then divide by the number of data points in the set.

Liam's Gas Mileage		
Tank	**Miles per gallon**	**Absolute difference from mean**
2	17	$\|17 - 19.07\| = 2.07$
3	17	$\|17 - 19.07\| = 2.07$
8	17	$\|17 - 19.07\| = 2.07$
4	18	$\|18 - 19.07\| = 1.07$
7	18	$\|18 - 19.07\| = 1.07$
12	18	$\|18 - 19.07\| = 1.07$
15	18	$\|18 - 19.07\| = 1.07$
9	19	$\|19 - 19.07\| = 0.07$
11	19	$\|19 - 19.07\| = 0.07$
14	19	$\|19 - 19.07\| = 0.07$
1	21	$\|21 - 19.07\| = 1.93$
6	21	$\|21 - 19.07\| = 1.93$
10	21	$\|21 - 19.07\| = 1.93$
13	21	$\|21 - 19.07\| = 1.93$
5	22	$\|22 - 19.07\| = 2.93$

Mean absolute difference:

$$\frac{2.07+2.07+2.07+\ldots+1.93+1.93+2.93}{15}=\frac{21.35}{15}\approx 1.42$$

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Alyssa's Gas Mileage		
Tank	**Miles per gallon**	**Absolute difference from mean**
15	21	$\|21 - 23.4\| = 2.4$
10	22	$\|22 - 23.4\| = 1.4$
14	22	$\|22 - 23.4\| = 1.4$
1	23	$\|23 - 23.4\| = 0.4$
4	23	$\|23 - 23.4\| = 0.4$
5	23	$\|23 - 23.4\| = 0.4$
11	23	$\|23 - 23.4\| = 0.4$
13	23	$\|23 - 23.4\| = 0.4$
2	24	$\|24 - 23.4\| = 0.6$
6	24	$\|24 - 23.4\| = 0.6$
7	24	$\|24 - 23.4\| = 0.6$
12	24	$\|24 - 23.4\| = 0.6$
3	25	$\|25 - 23.4\| = 1.6$
8	25	$\|25 - 23.4\| = 1.6$
9	25	$\|25 - 23.4\| = 1.6$

Mean absolute difference:

$$\frac{2.4+1.4+1.4+\ldots+1.6+1.6+1.6}{15}=\frac{14.4}{15}=0.96$$

f. Determine which type of graph would best help Sarah compare the gas mileage of the two cars.

To compare the gas mileage of the two cars, Sarah needs to get a sense of the expected mileage. Dot plots and histograms show the frequency of responses, but box plots show spread and center by displaying the median and interquartile range. A box plot would help Sarah compare the gas mileage of the two cars.

g. Plot the data on two graphs.

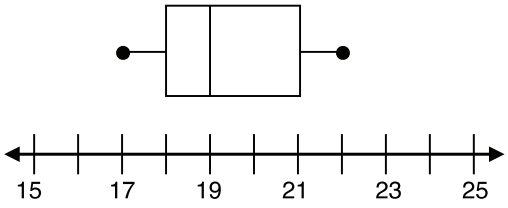
To create a box plot, find the minimum, maximum, first quartile, median, and third quartile of each data set. The middle-most value, the median, is the eighth data value. The first quartile is the middle-most value of the lower half of the data, or the fourth data value. The third quartile is the middle-most value of the upper half of the data, or the 12th data value. The *x*-axis of both graphs should be identical, so that the two graphs can be compared.

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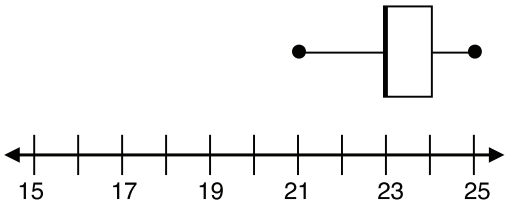
For Liam's data, the minimum is 17, the maximum is 22, the first quartile is 18, the median is 19, and the third quartile is 21.

For Alyssa's data, the minimum is 21, the maximum is 25, the first quartile is 23, the median is 23, and the third quartile is 24.

Liam's graph



Alyssa's graph



h. Which kind of car should Sarah buy? Use the measures of center, the measures of spread, and the graphs to identify the car with the highest gas mileage.

If Sarah is looking for higher gas mileage, Alyssa's car has a higher mean and median gas mileage. The mean absolute deviation and interquartile range of her car's gas mileage is smaller, and has more consistent performance. Liam's car has a gas mileage that is more varied, which can be seen in both the mean absolute deviation calculation and the shape of the box plot. Sarah should buy a car like Alyssa's.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 4.1.2: Comparing Data Sets

Brian recorded the number of commercials played during two types of television shows: an educational documentary and a primetime drama. Both types of shows lasted 1 hour, including commercials. He recorded the number of commercials for 10 episodes of each type of show. Use Brian's tables to solve problems 1–4.

Educational Documentary	
Show	**Number of commercials**
1	26
2	28
3	30
4	18
5	27
6	18
7	20
8	31
9	17
10	17

Primetime Drama	
Show	**Number of commercials**
1	39
2	39
3	35
4	29
5	40
6	27
7	41
8	29
9	32
10	30

1. Determine which measure of center to use to compare the data.

2. Calculate the measure of center for both data sets.

3. Calculate the mean absolute deviation of each data set.

4. Use the measures of center and spread to describe any similarities and differences between the data sets.

continued

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The number of elementary school students who ride the buses for two Springfield schools are listed in the following tables. Use these tables to solve problems 5–8.

Springfield School 1	
Bus	**Number of students**
1	17
2	34
3	41
4	47
5	16
6	36
7	37
8	20

Springfield School 2	
Bus	**Number of students**
1	29
2	34
3	22
4	26
5	26
6	31
7	22
8	35
9	27
10	30

5. Determine the minimum, maximum, first quartile, median, and third quartile of each data set.

6. Create a box plot of each data set.

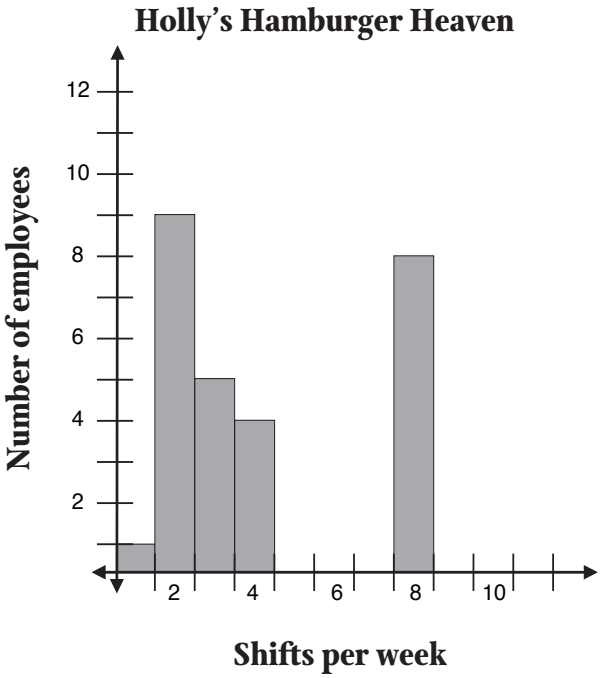
7. Compare the center and spread of the data from each school.

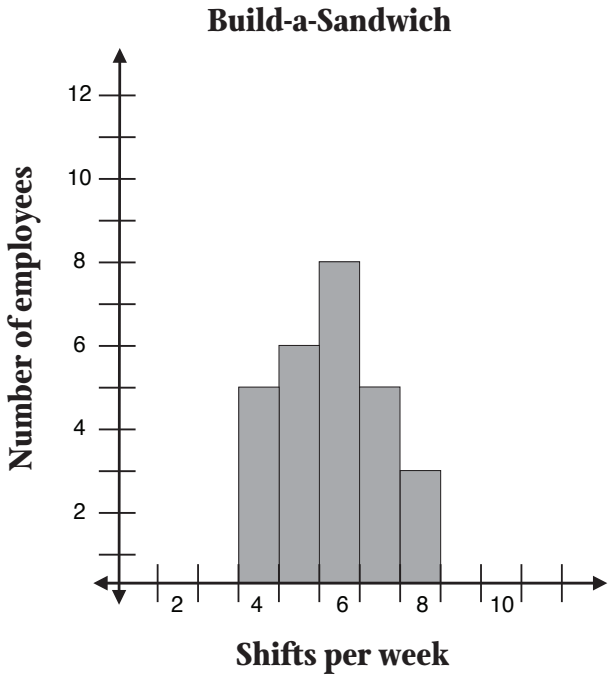
8. Why do you think the data is distributed differently at the two schools?

continued

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Employees at a restaurant sometimes work more than one shift in a day. Managers at two different restaurants, Holly's Hamburger Heaven and Build-a-Sandwich, each note how many shifts their employees worked in a single week. The data is plotted in the following histograms. Use these histograms to solve problems 9 and 10.





9. At which restaurants do employees work, on average, fewer shifts? Explain.

10. Describe the difference in the variation of the number of shifts worked at each restaurant.

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Lesson 4.1.3: Interpreting Data Sets

Warm-Up 4.1.3

A group of friends compares shoe sizes. Their shoe sizes are listed below. Use the data set to complete the problems that follow.

5 6 6 7 9 9 10 10

1. What is the median of the data set?

2. What is the first quartile of the data set?

3. What is the third quartile of the data set?

4. Create a box plot of the data.

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Lesson 4.1.3: Interpreting Data Sets

Common Core State Standard

S–ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).★

Warm-Up 4.1.3 Debrief

1. What is the median of the data set?

   The median is the middle-most value: the average of the fourth and fifth data values.

   $$\frac{7+9}{2}=8$$

2. What is the first quartile of the data set?

   The first quartile is the middle-most value of the lower half of the data set: the average of the second and third data values.

   $$\frac{6+6}{2}=6$$

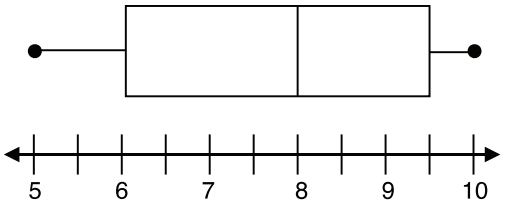
3. What is the third quartile of the data set?

   The third quartile is the middle-most value of the upper half of the data set: the average of the sixth and seventh data values.

   $$\frac{9+10}{2}=9.5$$

4. Create a box plot of the data.

   Use the minimum, maximum, first quartile, median, and third quartile to create a box plot.



Connection to the Lesson

- In this lesson, students will analyze the shape, center, and spread of data sets.
- Students will recall how to calculate measures of center and spread. These calculations are used to describe and compare data sets.
- Students will use graphical representations to compare and describe data sets, and should recall how to create these representations.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- calculating the mean and median of data (6.SP.5c)
- calculating the mean absolute deviation and interquartile range of data (6.SP.5c)
- creating graphical representations of data, such as dot plots, box plots, and histograms (6.SP.4)
- understanding how the shape of data relates to the center and spread (6.SP.5d)

Introduction

Data sets can be compared and interpreted in the context of the problem. Data values that are much greater than or much less than the rest of the data are called **outliers**. Outliers can impact the mean and any calculations that depend on the mean, such as the mean absolute deviation. An outlier that is less than the rest of the data will decrease the mean, and an outlier that is greater than the rest of the data will increase the mean. The median is not influenced by outliers, because it is the middle-most value and not an average. Outliers can be interpreted in the context of the data; an outlier may be the result of poor data collection, or a value that is not typical in the given context.

Key Concepts

- An outlier is a data value that is much greater than or less than the rest of a data set.
- Outliers influence calculations related to center and spread, and can cause misleading conclusions.

Calculating Outliers

1. Determine the first quartile (Q_1) and third quartile (Q_3).
2. Calculate IQR, the interquartile range, which is $Q_3 - Q_1$.
3. Multiply 1.5 by the IQR.
4. Take Q_1 and subtract 1.5(IQR). Any data values less than $Q_1 - 1.5(IQR)$ are outliers.
5. Take Q_3 and add 1.5(IQR). Any data values greater than $Q_3 + 1.5(IQR)$ are outliers.

If outliers are present in the data set, use the median for the measure of center since the outlier will influence the mean and pull it toward the outlier.

SRB p. U4-30

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Instruction

Common Errors/Misconceptions

- incorrectly identifying an outlier without considering the context of the data first
- drawing a conclusion about the center or spread of a data set when calculations have been influenced by an outlier

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Instruction

Guided Practice 4.1.3

Example 1

Each student in Mr. Lamb's class measured the length of his or her pencil. The following data set shows the pencil lengths in centimeters (cm). What is the expected length of any given pencil? Describe the shape, center, and spread of the data.

Student	Pencil length in cm	Student	Pencil length in cm
1	17.8	11	14.0
2	19.0	12	14.3
3	16.7	13	15.1
4	16.5	14	17.3
5	16.1	15	15.6
6	15.6	16	16.1
7	10.2	17	16.2
8	15.7	18	16.2
9	17.9	19	18.6
10	15.7	20	16.0

1. Order the data from least to greatest.

Student	Pencil length in cm	Student	Pencil length in cm
7	10.2	16	16.1
11	14.0	17	16.2
12	14.3	18	16.2
13	15.1	4	16.5
6	15.6	3	16.7
15	15.6	14	17.3
8	15.7	1	17.8
10	15.7	9	17.9
20	16.0	19	18.6
5	16.1	2	19.0

SWB p. 35; SRB p. U4-31

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Instruction

2. Calculate the interquartile range (IQR).

   To find the IQR, first find the median, first quartile (Q_1), and third quartile (Q_3).

   There are 20 data points in the set. First, find the median.

   The median is the average of the 10th and 11th data points.

   $$\frac{16.1+16.1}{2}=16.1$$

   Next find Q_1, the median of the lower half of the data set.

   Q_1 is the average of the fifth and sixth data values.

   $$\frac{15.6+15.6}{2}=15.6$$

   Next find Q_3, the median of the upper half of the data.

   Q_3 is the average of the 15th and 16th data values.

   $$\frac{16.7+17.3}{2}=17$$

   Now find the IQR, which is the difference between Q_3 and Q_1 ($Q_3 - Q_1$).

   $17 - 15.6 = 1.4$

   IQR = 1.4

3. Multiply 1.5 by the IQR.

   $1.5(IQR) = 1.5(1.4) = 2.1$

4. Determine if there are any outliers at the lower end of the data.

   Subtract 1.5(IQR) from Q_1.

   $Q_1 - 1.5(IQR) = 15.6 - 2.1 = 13.5$

   Any data values less than 13.5 are outliers. Examine the data. There is one value in the data set that is less than 13.5. Therefore, there is one outlier: 10.2.

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5. Determine if there are any outliers at the upper end of the data.

   Add 1.5(IQR) to Q_3.

   $Q_3 + 1.5(IQR) = 17 + 2.1 = 19.1$

   Any data values greater than 19.1 are outliers. Examine the data. There are no values in the data set that are greater than 19.1. Therefore, there are no more outliers.

6. List all the outliers.

   All of the outliers are those values that are less than $Q_1 - 1.5(IQR)$ and those values that are greater than $Q_3 + 1.5(IQR)$.

   The only outlier for this data set is 10.2.

7. Calculate a measure of center.

   A measure of center will give us an approximate expected length of any pencil, based on Mr. Lamb's data.

   Because there is an outlier, the mean would be influenced by this value. Instead of using the mean, use the median as a measure of center, because it is not influenced by outliers.

   We already determined that the median is 16.1.

   The expected length of any given pencil is 16.1 cm.

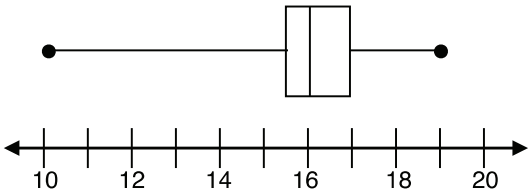
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Instruction

8. Describe the shape, center, and spread in the context of the problem.

   To describe the shape, plot the data using a box plot.

   Create the box plot using the five values found earlier: minimum = 10.2; Q_1 = 15.6; median = 16.1; Q_3 = 17; maximum = 19.0.



   The data is skewed left because the box plot has a long tail on the left side. This is due to the low value of the outlier, 10.2. The spread of the data is large. Notice the width of the box compared to the width of the minimum and maximum values. The center is better described by the median, which is not influenced by outliers. Therefore, the approximate length of a pencil based on Mr. Lamb's data is 16.1 cm, with a large variation in the data.

Example 2

Kayla is trying to estimate the cost of a house painter. She receives the following estimates, in dollars.

1288 1640 1547 1842 1553 1604 2858 1150 1844 1045 1347

She takes the mean of the data and states that the estimated cost of a house painter is $1,610.73. Is her estimate accurate?

1. Order the data from least to greatest.

   1045 1150 1288 1347 1547 1553 1604 1640 1842 1844 2858

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Instruction

2. Calculate the interquartile range (IQR).

   To find the IQR, first find the median, first quartile (Q_1), and third quartile (Q_3).

   There are 11 data points in the set. First, find the median.

   The median is the sixth data point: 1,553.

   Next find Q_1, the median of the lower half of the data set.

   Q_1 is the third data value: 1,288.

   Next find Q_3, the median of the upper half of the data.

   Q_3 is the ninth data value: 1,842.

   Now find the IQR, which is the difference between Q_3 and Q_1 ($Q_3 - Q_1$).

   $1842 - 1288 = 554$

3. Multiply 1.5 by the IQR.

   $1.5(IQR) = 1.5(554) = 831$

4. Determine if there are any outliers at the lower end of the data.

   Subtract 1.5(IQR) from Q_1.

   $Q_1 - 1.5(IQR) = 1288 - 831 = 457$

   Any data values less than 457 are outliers. Examine the data. There are no values in the data set that are less than 457.

5. Determine if there are any outliers at the upper end of the data.

   Add 1.5(IQR) to Q_3.

   $Q_3 + 1.5(IQR) = 1842 + 831 = 2673$

   Any data values greater than 2,673 are outliers. Examine the data. There is one value in the data set that is greater than 2,673. Therefore, there is one outlier: 2,858.

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Instruction

6. List all the outliers.

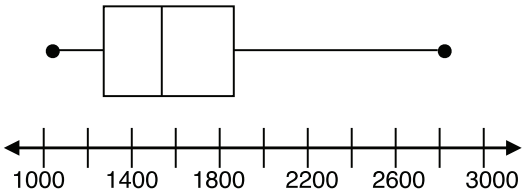
   All of the outliers are those values that are less than $Q_1 - 1.5(IQR)$ and those values that are greater than $Q_3 + 1.5(IQR)$.

   The only outlier for this data set is 2,858.

7. Create a box plot of the data set.

   Use the values we found for minimum, maximum, Q_1, median, and Q_3 to create a box plot.

   The minimum is the lowest value, 1,045; the maximum is the greatest value, 2,858; Q_1 is 1,288; the median is 1,553; and Q_3 is 1,842.



8. Describe how the outlier has influenced the center of the data.

   Compare the calculated mean to the median.

   The outlier is larger than the rest of the data set, and increased the mean. This can also be seen when comparing the mean and median; Kayla's mean is greater than the median.

9. Describe how the outlier has influenced the shape of the data.

   Identify if the data is skewed to the left or right.

   Data represented in the box plot that is skewed to the left has a longer tail on the left side of the box plot.

   Similarly, data that is skewed to the right has a longer tail on the right side of the box plot.

   The data is skewed to the right because of the influence of the large outlier.

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Instruction

10. Describe how the outlier has influenced the spread of the data.

The overall range of the data has been increased by the outlier. Without the outlier, the maximum would be 1,844, making the range (the difference between the maximum and minimum) 799:

$1844 - 1045 = 799$

With the outlier, the range is 1,813:

$2858 - 1045 = 1813$

11. Determine a reasonable estimate for the cost of a house painter.

Either the median or the mean without the outlier could be used as an estimate. The mean without the outlier is the mean of the remaining 10 data points:

$$\frac{1045+1150+1288+1347+1547+1553+1604+1640+1842+1844}{10} = 1486$$

Kayla's estimate is not correct. One of the values is an outlier, which increased her estimate. A more accurate estimate is the mean without the outlier ($1,486) or the median ($1,553).

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Instruction

Example 3

The National Basketball Association has strict regulations about the dimensions of basketballs used during games. The circumference of the basketball must be between 749 mm and 762 mm. A basketball manufacturer is sending a shipment of various brands of basketballs to the NBA. Brandon, who works in the shipping department, notices that the mean circumference of the basketballs is in this range, so he decides to send all the basketballs to the NBA. The NBA gets the shipment, and is not happy. Many of the basketballs in the shipment do not have the right circumference. What could have happened? How could the mean circumference be in this range, but many of the individual basketballs have the wrong circumference?

1. Consider how the mean is calculated. Is there any way the mean could not accurately represent the rest of the data?

   The mean is an average of all the data, but the mean doesn't address the variability. The maximum and minimum sizes of the basketballs must lie within the given range. In addition, if there are very large or very small data values, the mean could be greater than or less than many of the data points.

2. Describe how the mean could misrepresent the data, and what Brandon could do to ensure he doesn't send the wrong size basketballs.

   Brandon should use more measures of center and spread to understand the circumferences of the basketballs being delivered. More specifically, he should focus on variability, and make sure the circumferences lie within the necessary range.

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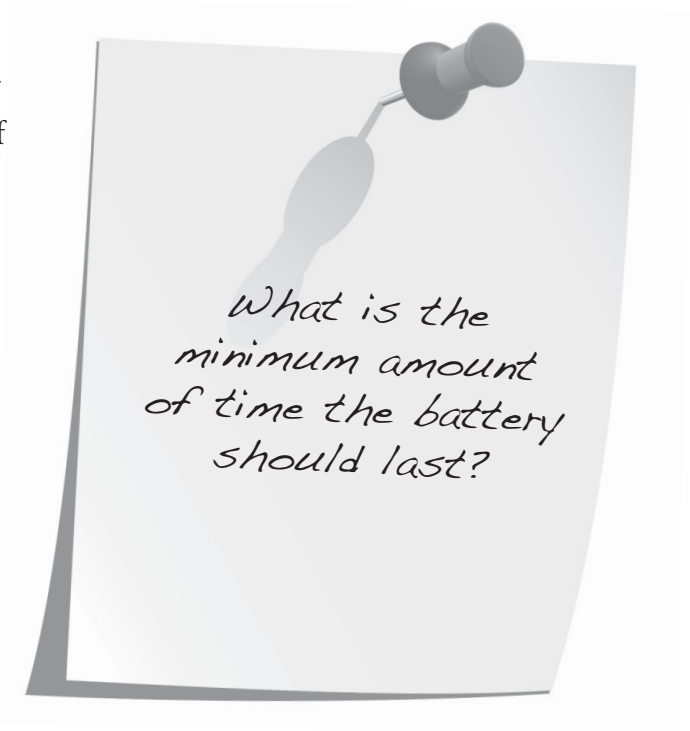
Problem-Based Task 4.1.3: Family Car Trips

SMP	
1	2
3 ✓	4 ✓
5	6
7	8

Engineers with XY Electronics Company are conducting research before designing a new portable DVD player. Because portable DVD players are often used by families traveling by car, the engineers want to understand the average length of a car trip. This way, they can design a battery that lasts long enough for the average car trip. The engineers surveyed 20 families about the length, in hours, of their most recent car trip. The results are in the following table.

Family	Trip length in hours	Family	Trip length in hours
A	1	K	2
B	2	L	3
C	7	M	3
D	3	N	1
E	6	O	4
F	7	P	1
G	18	Q	6
H	5	R	15
I	7	S	8
J	5	T	3

Describe the data, and create a graph to represent the information. The engineers want the battery to last long enough for at least 50% of family car trips. What is the minimum amount of time the battery should last?



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Problem-Based Task 4.1.3: Family Car Trips

Coaching

a. Sort the data from least to greatest.

b. Calculate the interquartile range.

c. Are there any outliers in the data set?

d. How do the outliers influence the center of the data?

e. Determine which measure of center to use to describe the data.

f. Which type of graph will show the average estimated time of a car trip?

g. Create the identified graph.

h. How do the outliers influence the shape and spread of the data?

i. Which measure divides the data in two halves?

j. Given that the engineers want the battery to last long enough for at least 50% of family car trips, what is the minimum amount of time the battery should last?

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Instruction

Problem-Based Task 4.1.3: Family Car Trips

Coaching Sample Responses

a. Sort the data from least to greatest.

Family	Trip length in hours	Family	Trip length in hours
A	1	H	5
N	1	J	5
P	1	E	6
B	2	Q	6
K	2	C	7
D	3	F	7
L	3	I	7
M	3	S	8
T	3	R	15
O	4	G	18

b. Calculate the interquartile range.

To find the interquartile range, or IQR, first find the median, first quartile, and third quartile. The median is the average of the 10th and 11th data values: $\frac{4+5}{2}=4.5$. The first quartile is the middle-most value of the lower half of the data set, the average of the fifth and sixth data values: $\frac{2+3}{2}=2.5$. The third quartile is the middle-most value of the upper half of the data set, the average of the 15th and 16th data values: $\frac{7+7}{2}=7$. The interquartile range is the difference between Q_3 and Q_1:

$7 - 2.5 = 4.5 = \text{IQR}$

c. Are there any outliers in the data set?

Calculate 1.5(IQR): 1.5(4.5) = 6.75. To determine if there are any outliers at the upper end of the data, find Q_3 + 1.5(IQR): 7 + 6.75 = 13.75. Any data values greater than 13.75 are outliers. The two greatest data values, 15 and 18, are outliers.

To determine if there are any outliers at the lower end of the data, find Q_1 – 1.5(IQR): 2.5 – 6.75 = –4.25. Any data values less than –4.25 are outliers. There are no outliers at the lower end of the data.

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d. How do the outliers influence the center of the data?

Both outliers are at the upper end of the data. If the mean of the data set were calculated, including these two values, the mean would be increased by the outliers.

e. Determine which measure of center to use to describe the data.

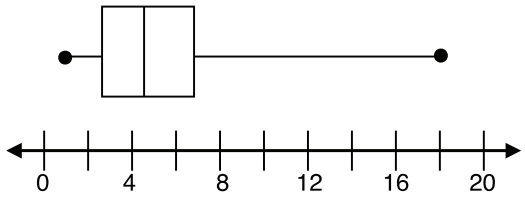
Because the data contains two outliers, either the mean calculated without these values or the median would be a good measure of center. Use the median, 4.5.

f. Which type of graph will show the average estimated time of a car trip?

A box plot would show information about the four quartiles of the data, as well as any very large or very small values in the data set.

g. Create the identified graph.

To create a box plot, find the minimum, maximum, first quartile, median, and third quartile. The minimum is 1, the maximum is 18, the median is 4.5, the first quartile is 2.5, and the third quartile is 7.



h. How do the outliers influence the shape and spread of the data?

Data represented in a box plot that is skewed to the left has a longer tail on the left side of the box plot. Similarly, data that is skewed to the right has a longer tail on the right side of the box plot. The two large outliers have skewed the data to the right. The overall range of the data has also increased to the two large outliers.

i. Which measure divides the data in two halves?

The median divides the data into two halves.

j. Given that the engineers want the battery to last long enough for at least 50% of family car trips, what is the minimum amount of time the battery should last?

If the battery lasts at least as long as the median, 4.5 hours, the DVD player will last long enough for at least 50% of family car trips.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 4.1.3: Interpreting Data Sets

Mrs. Wong is looking for a new babysitter. She asks 10 babysitters the rate they charge per hour. Each babysitter's rate in dollars per hour is listed in the following table. Use the table to solve problems 1–5.

Babysitter	Rate in dollars per hour
1	9.60
2	9.80
3	15.50
4	10.60
5	9.70
6	10.20
7	8.80
8	11.20
9	8.80
10	10.10

1. Are there any outliers in the data set? Explain.

2. Mrs. Wong wants to estimate the hourly rate of any given babysitter. How should she estimate the rate? Why?

3. What is the estimated rate of a babysitter?

4. Create a box plot showing Mrs. Wong's data.

5. How do the outliers influence the shape and spread of the data?

continued

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Brady is keeping track of the calories in each snack he eats. He records the number of calories in each snack he eats in a week in the following table. Use the table to solve problems 6–10.

Snack	Calories	Snack	Calories
1	440	11	120
2	270	12	80
3	360	13	450
4	430	14	470
5	220	15	170
6	180	16	150
7	500	17	370
8	50	18	300
9	140	19	210
10	410	20	100

6. Are there any outliers in the data set? Explain.

7. Brady wants to estimate the number of calories in each snack he eats. Which measure of center should he use? Calculate the measure of center.

8. Brady wants to understand how many calories are in the middle 50% of his snacks. Which graph should he create to show this information? Create the graph.

9. Describe the shape and spread of the data, and how it is influenced by any outliers.

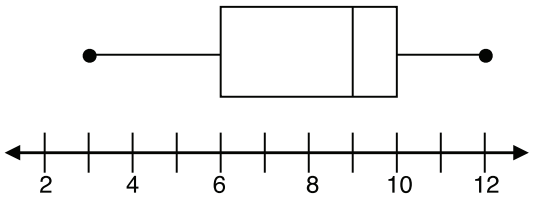
10. Brady's doctor recommends that he eat a couple of healthy snacks each day, and that each snack should be around 200 calories. Using the measure of center and the graph, determine whether Brady is following his doctor's recommendation.

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Progress Assessment

Circle the letter of the best answer.

1. What is the median of the data set used to create the box plot?



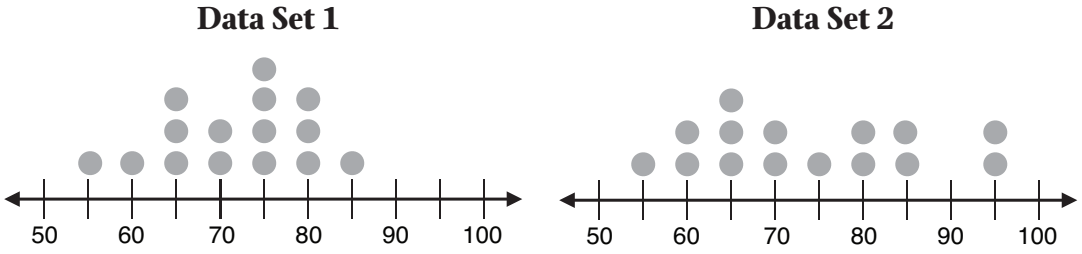
a. 3
b. 9
c. 10
d. 12

2. What is the interquartile range of the data?

39 50 37 32 35 32 39 47 56 41 49 45 53 38 36

a. 13
b. 36
c. 39
d. 49

3. Look at the data presented in the following dot plots. Which comparison is true?

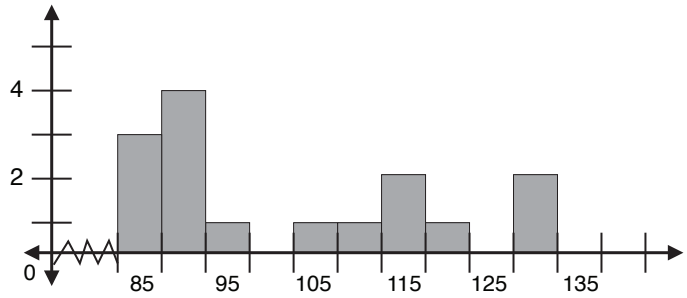


a. Data Set 1: greater mean; Data Set 1: greater variation
b. Data Set 1: greater mean; Data Set 2: greater variation
c. Data Set 2: greater mean; Data Set 2: greater variation
d. Data Set 2: greater mean; Data Set 1: greater variation

continued

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4. Which data set does the histogram represent?



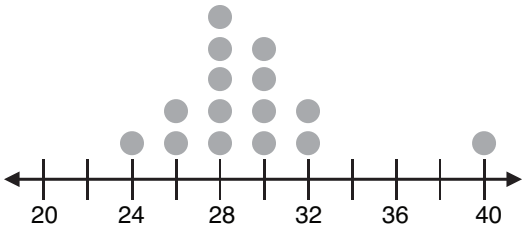
a. Data Set 1: 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 135

b. Data Set 2: 85, 90, 95, 105, 110, 115, 120, 130

c. Data Set 3: 85, 85, 90, 90, 90, 90, 90, 95, 105, 110, 115, 115, 120, 130, 130

d. Data Set 4: 85, 85, 85, 90, 90, 90, 90, 95, 105, 110, 115, 115, 120, 130, 130

5. Which value in the dot plot appears to be an outlier?



a. 24

b. 26

c. 32

d. 40

6. What is the mean absolute deviation of the data?

183 148 169 201 136 150 180 205 129 171
198 160 161 176 132 185 143 116 180 196

a. 21.86

b. 38.5

c. 145.5

d. 165.95

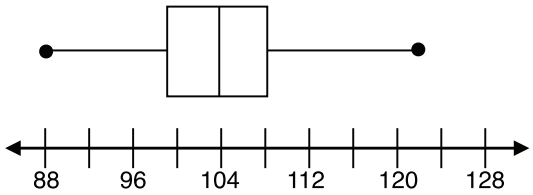
continued

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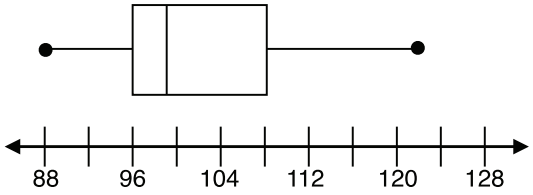
7. Which box plot represents the data?

88 90 96 96 96 99 99 99 103 103 107 108 109 117 122

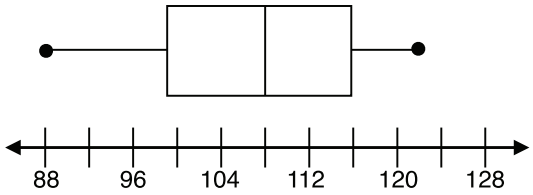
a.



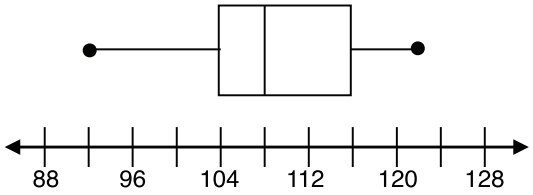
c.



b.



d.



8. Look at the data in the following tables. Which comparison is true?

Data Set 1
75
70
60
81
73
70
75
81
79
78
64
63

Data Set 2
69
74
61
67
69
59
62
76
65
73
71
55

a. Data Set 1: greater mean; Data Set 1: greater absolute deviation

b. Data Set 1: greater mean; Data Set 2: greater absolute deviation

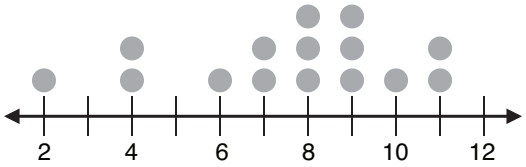
c. Data Set 2: greater mean; Data Set 2: greater absolute deviation

d. Data Set 2: greater mean; Data Set 1: greater absolute deviation

continued

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9. Which data set does the dot plot represent?



a. Data Set 1: 2, 4, 6, 7, 8, 9, 10, 11

b. Data Set 2: 2, 4, 4, 6, 7, 8, 8, 8, 9, 9, 9, 10, 10, 11

c. Data Set 3: 2, 4, 6, 6, 8, 8, 8, 9, 9, 9, 10, 10, 11, 11

d. Data Set 4: 2, 4, 4, 6, 7, 7, 8, 8, 8, 9, 9, 9, 10, 11, 11

10. Which value in the data set is an outlier?

128 133 118 118 122 126 131 119 135 135
116 158 116 119 135 129 122 134 123 133

a. 116

b. 123

c. 135

d. 158

The following tables show a city's daily high temperature in degrees Fahrenheit for two weeks in March. Use the data to solve problem 11.

Week 1	
Day	**Temperature in °F**
Sunday	63
Monday	65
Tuesday	59
Wednesday	61
Thursday	64
Friday	68
Saturday	62

Week 2	
Day	**Temperature in °F**
Sunday	52
Monday	59
Tuesday	63
Wednesday	62
Thursday	60
Friday	61
Saturday	65

11. Using measures of center, shape, and spread, describe the similarities and differences in the temperatures for the two weeks. Calculate at least one measure of center and spread for each week.

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Pre-Assessment

Circle the letter of the best answer.

1. Ruby asks her classmates how many hours they sleep each night during the week, and separates the responses by gender in the following two-way frequency table. What is the joint frequency of males who sleep 8–10 hours?

Gender	Hours of sleep			
	4–6	**6–8**	**8–10**	**10–12**
Male	5	14	8	2
Female	11	10	5	1

a. 5
b. 8
c. 10
d. 14

2. Anna asks her friends which book they prefer in a trilogy. She separates the responses by age. What is the marginal frequency of Book 1?

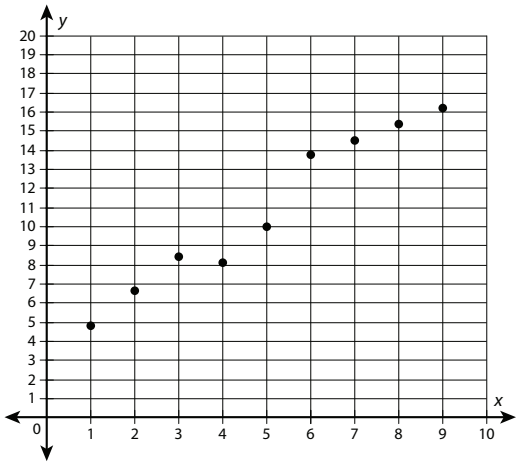
Age	Preferred book		
	Book 1	**Book 2**	**Book 3**
14 years old	8	5	19
15 years old	10	12	7
16 years old	16	0	11

a. 17
b. 29
c. 34
d. 37

continued

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3. Which function could be used to approximate the data in the following scatter plot?



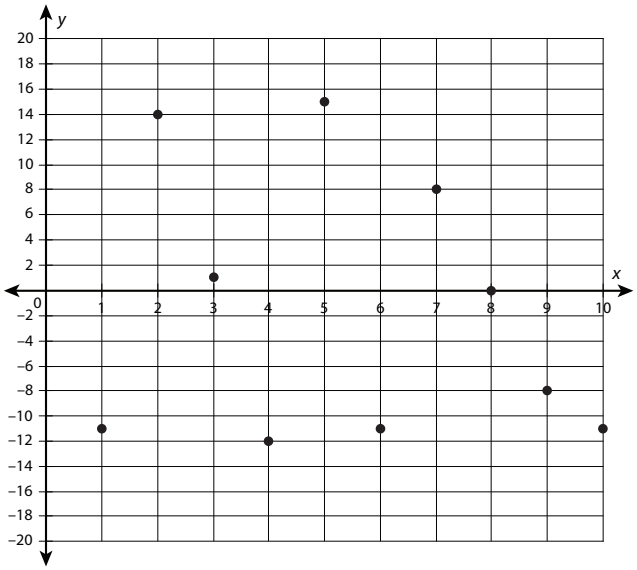
a. $y = -1.5x + 3.4$

b. $y = 1.5x - 3.4$

c. $y = -1.5x - 3.4$

d. $y = 1.5x + 3.4$

4. Sam created a residual plot to analyze a linear function fitted to data. The plot is shown. What does the plot tell him about how well his line fits the data?



a. A linear function is not a good fit for the data.

b. A quadratic function is the best fit for the data.

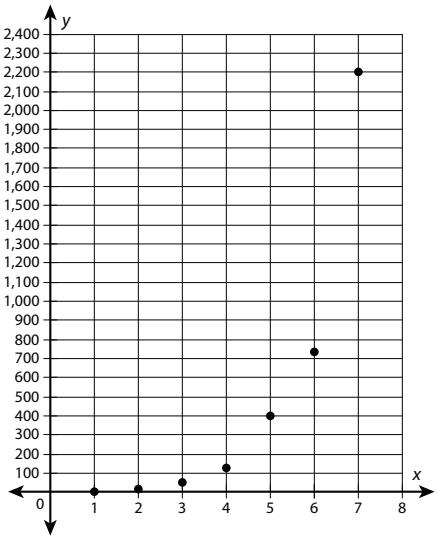
c. An exponential function is the best fit for the data.

d. A linear function is a good fit for the data.

continued

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5. Which function could be used to estimate the data in the following scatter plot?



a. $y = 3x + 35$

b. $y = 3^x + 35$

c. $y = (-3)^x + 35$

d. $y = 3x - 35$

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Lesson 2: Working with Two Categorical and Quantitative Variables

Instruction

Common Core State Standards

SMP	
1 ✓	2
3 ✓	4 ✓
5 ✓	6
7	8

S–ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★

S–ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

b. Informally assess the fit of a function by plotting and analyzing residuals.

c. Fit a linear function for a scatter plot that suggests a linear association.

Essential Questions

1. When is a two-way frequency table a good way to present data?
2. Why is data represented in a scatter plot?
3. What does a residual plot display?
4. How can it be determined graphically that a line is a good estimate for a data set?

WORDS TO KNOW

conditional relative frequency	a joint frequency compared to either the sum of all frequencies or the sum of the frequencies for a category of the given data
function	a relation in which each element in the domain is mapped onto exactly one element in the range; that is, for every value of x, there is exactly one value of y
joint frequency	the number of times a pair of characteristics appear together
marginal frequency	the total number of times a characteristic occurs

SRB p. U4-42

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residual	the vertical distance between an observed data value and an estimated data value on a line of best fit
residual plot	provides a visual representation of the residuals for a set of data; contains the points (x, residual for x)
scatter plot	a graph of data in two variables on a coordinate plane, where each data pair is represented by a point
trend	a pattern of behavior, usually observed over time or over multiple iterations
two-way frequency table	a table that divides responses into categories, showing both a characteristic in the table rows and a characteristic in the table columns; values in cells are a count of the number of times each response was given by a respondent with a certain characteristic

Recommended Resources

- Interactivate. "Finding Residuals."

  http://walch.com/rr/CAU4L2FindingResiduals

  This site provides a discussion about residuals—what they are and how they are calculated—and gives an example.

- Interactivate. "Regression."

  http://walch.com/rr/CAU4L2Regression

  This site allows users to plot points in a scatter plot and then have the computer generate a line of best fit. Users can also fit a line to the data using their own equation.

- MathIsFun.com. "Scatter Plots."

  http://walch.com/rr/CAU4L2ScatterPlots

  This site explains how to create a scatter plot, draw a line of best fit, and analyze correlations. The site ends with a short, interactive quiz.

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Lesson 4.2.1: Summarizing Data Using Two-Way Frequency Tables

Warm-Up 4.2.1

Elizabeth surveys 9th graders, 10th graders, and 11th graders in her school. She asks each student how many hours they spend doing homework each night. She records the responses in the following table.

Grade	Hours spent on homework		
	0–2	**2–4**	**More than 4**
9	38	12	2
10	21	25	9
11	14	18	20

1. How many 9th graders spend 0–2 hours on homework each night?

2. How many 10th graders spend 2–4 hours on homework each night?

3. Which response was the most popular among 11th graders? 0–2 hours, 2–4 hours, or more than 4 hours?

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Instruction

Lesson 4.2.1: Summarizing Data Using Two-Way Frequency Tables

Common Core State Standard

S–ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★

Warm-Up 4.2.1 Debrief

1. How many 9th graders spend 0–2 hours on homework each night?

   Look for the row for grade 9, then the column for 0–2 hours. 38 9th graders spend 0–2 hours on homework each night.

2. How many 10th graders spend 2–4 hours on homework each night?

   Look for the row for grade 10, then the column for 2–4 hours. 25 10th graders spend 2–4 hours on homework each night.

3. Which response was the most popular among 11th graders? 0–2 hours, 2–4 hours, or more than 4 hours?

   Look for the row for grade 11. Find the greatest response, and then identify the heading of the column with the greatest response. The most popular response was "more than 4" hours.

Connection to the Lesson

- In this lesson, students will read and interpret two-way frequency tables.
- This warm-up introduces students to data presented in a two-way frequency table.
- Students will translate from this informal introduction to more formal terminology.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- creating tables with multiple rows and columns (no standard)
- reading tables with multiple rows and columns (no standard)

Introduction

When data compares characteristics of two or more categories, the data can be displayed in a two-way frequency table. In this lesson, we focus on two-way frequency tables that compare survey results. When used to analyze survey results, a **two-way frequency table** is a table of data that separates responses by a characteristic of the respondents.

Type of characteristic	Type of response	
	Response 1	Response 2
Characteristic 1	a	b
Characteristic 2	c	d

Each cell in the table contains a count of the people with a given characteristic who gave each response. For example, in this table, a, b, c, and d would each represent the number of responses given by people with each characteristic. That is, a represents people with Characteristic 1 who gave Response 1, b represents people with Characteristic 1 who gave Response 2, and so on. The sum of all the cells, $a + b + c + d$, is the total number of respondents. Two-way frequency tables help organize information and provide greater insight into features of a population being surveyed. A **trend**, or pattern in the data, can be examined using a two-way frequency table.

A **joint frequency** is the number of times a pair of characteristics appear together. The entries in the cells of a two-way frequency table are joint frequencies. In the sample table, a, b, c, and d are each a joint frequency. A **marginal frequency** is the total number of times a characteristic occurs. This is the sum of either a row or a column in a two-way frequency table. In the sample table, $a + b$ would be the marginal frequency of people with Characteristic 1.

A **conditional relative frequency** allows a comparison to be made between a joint frequency and the total number of responses in a single row, a single column, or the entire table. Relative frequencies are usually expressed as a decimal. They are found by dividing the number of responses by either the total number of people who gave that response, the total number of people with a given characteristic, or the total number of respondents. In the sample table, $\frac{a}{a+b}$ is the relative frequency of Response 1 for people with Characteristic 1.

SRB p. U4-44

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Instruction

Key Concepts

- A two-way frequency table divides survey responses by characteristics of respondents.
- The number of times a response was given by people with a certain characteristic is called a joint frequency.
- A marginal frequency is the total number of times a response is given, or the total number of people with a certain characteristic.
- A conditional relative frequency is usually written as a decimal. It expresses a number of responses divided by the total number of respondents, the total number of people with a given characteristic, or the total number of times a specific response was given.
- Trends, or patterns of responses, can be identified by looking at the frequency of responses.

Common Errors/Misconceptions

- incorrectly locating frequencies in the table
- incorrectly calculating conditional relative frequencies by being inconsistent in the method used (dividing by the number of times a response was given, the number of people with a given characteristic, or the total number of respondents)

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Guided Practice 4.2.1

Example 1

Cameron surveys students in his school who play sports, and asks them which sport they prefer. He records the responses in the following table.

Gender	Preferred sport		
	Tennis	Soccer	Basketball
Male	49	52	16
Female	23	64	33

What is the joint frequency of male students who prefer soccer?

1. Look for the row of male students.

   The characteristic "male" is in the first row of responses.

2. Look for the column with the response "soccer."

   The response "soccer" is in the second column of responses.

3. The frequency for the given characteristic and the given response is the joint frequency.

   The cell in the first characteristic row and the second response column is 52.

   The joint frequency of male students who prefer soccer is 52.

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Instruction

Example 2

Abigail surveys students in different grades, and asks each student which pet they prefer. The responses are in the following table.

Grade	Preferred pet			
	Bird	Cat	Dog	Fish
9	3	49	53	22
10	7	36	64	10

What is the marginal frequency of each type of pet?

1. Sum the responses of people with each characteristic for the first pet type, "bird."

   3 people in grade 9 preferred birds, and 7 people in grade 10 preferred birds.

   3 + 7 = 10 people who preferred birds

2. Sum the responses of people with each characteristic for the second pet type, "cat."

   49 people in grade 9 preferred cats, and 36 people in grade 10 preferred cats.

   49 + 36 = 85 people who preferred cats

3. Sum the responses of people with each characteristic for the third pet type, "dog."

   53 people in grade 9 preferred dogs, and 64 people in grade 10 preferred dogs.

   53 + 64 = 117 people who preferred dogs

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4. Sum the responses of people with each characteristic for the fourth pet type, "fish."

   22 people in grade 9 preferred fish, and 10 people in grade 10 preferred fish.

   22 + 10 = 32 people who preferred fish

5. Organize the marginal frequencies in a two-way frequency table.

   Create a row and include the marginal frequencies of each response under the name of each response.

Grade	Preferred pet			
	Bird	Cat	Dog	Fish
9	3	49	53	22
10	7	36	64	10
Total	10	85	117	32

Example 3

Ms. Scanlon surveys her students about the time they spend studying. She creates a table showing the amount of time students studied and the score each student earned on a recent test.

Time spent studying, in hours	Test score			
	0–25	26–50	51–75	76–100
0–2	2	8	12	2
2–4	0	10	8	24
4–6	1	0	2	9
6+	0	0	1	4

Ms. Scanlon wants to understand the distribution of scores among all the students, and to get a sense of how students are performing and how much students are studying. Find the conditional relative frequencies, expressed as decimals.

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Instruction

1. Find the total number of students represented in the table by summing the joint frequencies.

$$2 + 8 + 12 + 2 + 0 + 10 + 8 + 24 + 1 + 0 + 2 + 9 + 0 + 0 + 1 + 4 = 83$$

2. Divide each joint frequency by the total number of students.

$$\frac{2}{83} \approx 0.024 \quad \frac{8}{83} \approx 0.096 \quad \frac{12}{83} \approx 0.145 \quad \frac{2}{83} \approx 0.024$$
$$\frac{0}{83} \approx 0 \quad \frac{10}{83} \approx 0.120 \quad \frac{8}{83} \approx 0.096 \quad \frac{24}{83} \approx 0.289$$
$$\frac{1}{83} \approx 0.012 \quad \frac{0}{83} \approx 0 \quad \frac{2}{83} \approx 0.024 \quad \frac{9}{83} \approx 0.108$$
$$\frac{0}{83} \approx 0 \quad \frac{0}{83} \approx 0 \quad \frac{1}{83} \approx 0.012 \quad \frac{4}{83} \approx 0.048$$

3. Represent the conditional joint frequencies in a new table.

   Insert each conditional joint frequency in a table set up the same way as the two-way frequency table.

Time spent studying, in hours	Test score			
	0–25	**26–50**	**51–75**	**76–100**
0–2	0.024	0.096	0.145	0.024
2–4	0	0.120	0.096	0.289
4–6	0.012	0	0.024	0.108
6+	0	0	0.012	0.048

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Problem-Based Task 4.2.1: FunZone America Survey

SMP
1 ✓ 2
3 ✓ 4 ✓
5 6
7 8

FunZone America, an amusement park, collects information from park visitors. The park uses this information to determine how to attract certain guests to the park. For example, one summer FunZone America learned that 12- to 17-year-olds were most interested in roller coasters. When FunZone America wanted to try to get more 12- to 17-year-olds to visit the park, the park ran advertisements about roller coasters. The park surveyed recent visitors and recorded the information in the following table. The three main attractions were roller coasters, shows, and the water park.

Visitor	Age	Favorite attraction	Visitor	Age	Favorite attraction
1	27	Roller coasters	26	31	Roller coasters
2	30	Shows	27	12	Roller coasters
3	18	Roller coasters	28	38	Water park
4	35	Shows	29	29	Roller coasters
5	31	Shows	30	28	Shows
6	46	Roller coasters	31	16	Roller coasters
7	25	Water park	32	47	Shows
8	39	Shows	33	37	Shows
9	8	Water park	34	9	Water park
10	14	Water park	35	48	Shows
11	31	Shows	36	22	Water park
12	25	Roller coasters	37	49	Roller coasters
13	35	Shows	38	19	Roller coasters
14	46	Roller coasters	39	53	Shows
15	53	Roller coasters	40	15	Roller coasters
16	27	Shows	41	16	Water park
17	33	Water park	42	14	Shows
18	34	Shows	43	39	Shows
19	5	Shows	44	52	Shows
20	41	Shows	45	20	Shows
21	20	Roller coasters	46	33	Roller coasters
22	24	Water park	47	21	Water park
23	48	Shows	48	53	Water park
24	34	Roller coasters	49	39	Roller coasters
25	14	Shows	50	6	Shows

continued

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Create a two-way frequency table showing the joint frequencies of visitors with the following age ranges: 5–15, 16–25, 26–35, 36–45, 46–55. Include in the table the marginal frequency for the types of attractions and for the ages of the visitors. Are there any trends in the type of attractions preferred by each age group? Use conditional relative frequencies to describe your response.



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Problem-Based Task 4.2.1: FunZone America Survey

Coaching

a. Sort the data first by attraction. Then, within each attraction, sort by age.

b. Count the number of attractions selected for each given age range. For example, to fill in the first joint frequency, count the number of people aged 5 through 15 who selected roller coasters. Fill in a two-way frequency table with this information.

c. Find the marginal frequencies. Sum each row, and sum each column.

d. Which type of conditional relative frequency would show the type of attraction preferred by each age group?

e. Calculate the conditional relative frequencies and put them in a table.

f. Look at the conditional relative frequencies for each age group. Is there an attraction that is preferred by each group?

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Problem-Based Task 4.2.1: FunZone America Survey

Coaching Sample Responses

a. Sort the data first by attraction. Then, within each attraction, sort by age.

Visitor	Age	Favorite attraction
27	12	Roller coasters
40	15	Roller coasters
31	16	Roller coasters
3	18	Roller coasters
38	19	Roller coasters
21	20	Roller coasters
12	25	Roller coasters
1	27	Roller coasters
29	29	Roller coasters
26	31	Roller coasters
46	33	Roller coasters
24	34	Roller coasters
49	39	Roller coasters
6	46	Roller coasters
14	46	Roller coasters
37	49	Roller coasters
15	53	Roller coasters

Visitor	Age	Favorite attraction
9	8	Water park
34	9	Water park
10	14	Water park
41	16	Water park
47	21	Water park
36	22	Water park
22	24	Water park
7	25	Water park
17	33	Water park
28	38	Water park
48	53	Water park

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Visitor	Age	Favorite attraction
19	5	Shows
50	6	Shows
25	14	Shows
42	14	Shows
45	20	Shows
16	27	Shows
30	28	Shows
2	30	Shows
5	31	Shows
11	31	Shows
18	34	Shows
4	35	Shows
13	35	Shows
33	37	Shows
8	39	Shows
43	39	Shows
20	41	Shows
32	47	Shows
23	48	Shows
35	48	Shows
44	52	Shows
39	53	Shows

b. Count the number of attractions selected for each given age range. For example, to fill in the first joint frequency, count the number of people aged 5 through 15 who selected roller coasters. Fill in a two-way frequency table with this information.

Age range	Favorite attraction		
	Roller coasters	Shows	Water park
5–15	2	4	3
16–25	5	1	5
26–35	5	8	1
36–45	1	4	1
46–55	4	5	1

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c. Find the marginal frequencies. Sum each row, and sum each column.

Age range	Favorite attraction			Total
	Roller coasters	Shows	Water park	
5–15	2	4	3	9
16–25	5	1	5	11
26–35	5	8	1	14
36–45	1	4	1	6
46–55	4	5	1	10
Total	17	22	11	

d. Which type of conditional relative frequency would show the type of attraction preferred by each age group?

We need to look at how the values in each row are distributed to understand which attraction is preferred by each age group. The conditional relative frequency that is the joint frequency divided by the number of people in each age group will show the percentage of each age group that preferred each attraction.

e. Calculate the conditional relative frequencies and put them in a table.

Divide each value by the total people in that age group. Make sure the sum of each row is 1.

Age range	Favorite attraction			Total
	Roller coasters	Shows	Water park	
5–15	$\frac{2}{9} \approx 0.22$	$\frac{4}{9} \approx 0.44$	$\frac{3}{9} \approx 0.33$	1.0
16–25	$\frac{5}{11} \approx 0.45$	$\frac{1}{11} \approx 0.09$	$\frac{5}{11} \approx 0.45$	1.0
26–35	$\frac{5}{14} \approx 0.36$	$\frac{8}{14} \approx 0.57$	$\frac{1}{14} \approx 0.07$	1.0
36–45	$\frac{1}{6} \approx 0.17$	$\frac{4}{6} \approx 0.67$	$\frac{1}{6} \approx 0.17$	1.0
46–55	$\frac{4}{10} = 0.4$	$\frac{5}{10} = 0.5$	$\frac{1}{10} = 0.1$	1.0

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f. Look at the conditional relative frequencies for each age group. Is there an attraction that is preferred by each group?

The shows are preferred by ages 5–15, the roller coasters and water parks are equally preferred by ages 16–25, shows are preferred by ages 26–35, shows are preferred by ages 36–45, and shows are preferred by ages 46–55. There are some age groups with a strong preference for one attraction, such as the shows in the 36–45 age group. In others, there is only a slight preference, or there are two equally preferred attractions.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 4.2.1: Summarizing Data Using Two-Way Frequency Tables

Dylan asked his classmates about their favorite school subject, and wanted to see if there was any difference in the classes preferred by boys and girls. His data is recorded in the following table. Use the data for problems 1–5.

Student	Gender	Favorite subject	Student	Gender	Favorite subject
1	Boy	English	21	Boy	Social studies
2	Girl	Math	22	Boy	Science
3	Girl	Math	23	Girl	Social studies
4	Boy	English	24	Girl	Social studies
5	Boy	Science	25	Boy	Social studies
6	Girl	Social studies	26	Boy	English
7	Boy	Math	27	Boy	Science
8	Girl	Math	28	Boy	Science
9	Girl	Social studies	29	Girl	English
10	Girl	Math	30	Boy	English
11	Girl	Math	31	Girl	Science
12	Boy	Science	32	Girl	Math
13	Boy	Social studies	33	Girl	English
14	Girl	Social studies	34	Girl	English
15	Boy	Math	35	Boy	Science
16	Girl	Social studies	36	Girl	Science
17	Boy	English	37	Boy	Social studies
18	Boy	English	38	Boy	English
19	Boy	Science	39	Girl	Math
20	Girl	Science	40	Girl	Math

1. Create a two-way frequency table showing the subjects preferred by students of each gender.
2. Find the marginal frequencies for each gender and for each subject. Include the marginal frequencies in the table.
3. What are the conditional frequencies relative to the total number of people surveyed? Include the values in a table.
4. What are the conditional frequencies relative to the total number of boys and the total number of girls?
5. Describe any trends in the subjects preferred by all students and the subjects preferred by boys versus girls.

continued

SWB p. 51; SRB Practice 4.2.1 p. U4-50

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To better understand which type of cell phones people will purchase, a cell phone company collects information about its customers. Customers could select three of the following ages: under 25, 25–35, and over 35. Each customer indicated whether they used a basic phone or a smartphone. The information is recorded in the following table. Use the data for problems 6–10.

Customer	Age range	Type of cell phone used	Customer	Age range	Type of cell phone used
1	25–35	smartphone	26	over 35	smartphone
2	under 25	smartphone	27	under 25	smartphone
3	under 25	smartphone	28	25–35	smartphone
4	25–35	smartphone	29	25–35	smartphone
5	25–35	smartphone	30	25–35	smartphone
6	under 25	smartphone	31	over 35	basic phone
7	over 35	smartphone	32	25–35	smartphone
8	over 35	basic phone	33	under 25	smartphone
9	25–35	smartphone	34	under 25	basic phone
10	25–35	basic phone	35	over 35	smartphone
11	under 25	smartphone	36	under 25	smartphone
12	over 35	basic phone	37	25–35	basic phone
13	25–35	smartphone	38	25–35	basic phone
14	over 35	smartphone	39	over 35	basic phone
15	under 25	smartphone	40	under 25	smartphone
16	under 25	smartphone	41	over 35	smartphone
17	under 25	basic phone	42	under 25	basic phone
18	under 25	smartphone	43	under 25	smartphone
19	25–35	smartphone	44	25–35	basic phone
20	25–35	smartphone	45	over 35	basic phone
21	25–35	basic phone	46	25–35	smartphone
22	25–35	smartphone	47	over 35	basic phone
23	25–35	smartphone	48	25–35	smartphone
24	under 25	smartphone	49	under 25	smartphone
25	over 35	basic phone	50	over 35	smartphone

continued

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6. Create a two-way frequency table showing the phones used by customers of each age group.

7. Find the marginal frequencies for each age and for each phone type. Include the marginal frequencies in the table.

8. What are the conditional frequencies relative to the types of phones? Include the values in a table.

9. What are the conditional frequencies relative to all customers surveyed?

10. The cell phone company is thinking of creating a new phone. It wants to sell the cell phone type that is most popular to the age group that is most popular. Which type of cell phone should the company make, and to whom should the company sell it?

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Lesson 4.2.2: Solving Problems Given Functions Fitted to Data

Warm-Up 4.2.2

A local dollar store sells goods for around $1, but the name is a little misleading these days with the rising costs of goods. You have kept track of the number of items you've bought and the prices you paid for that number of goods. You recorded your data in the following table. Use the table for problem 1.

Number of goods	Cost in dollars ($)
2	5
4	8
9	14
10	19

1. Plot each point in the table on a coordinate plane.

2. Yasin is a welder. For his job, he requires 1 hour to set up and then 3 hours for each project. The time it takes on his job to complete x projects in one day can be modeled by the function $y = 3x + 1$. Graph the function $y = 3x + 1$.

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Instruction

Lesson 4.2.2: Solving Problems Given Functions Fitted to Data

Common Core State Standard

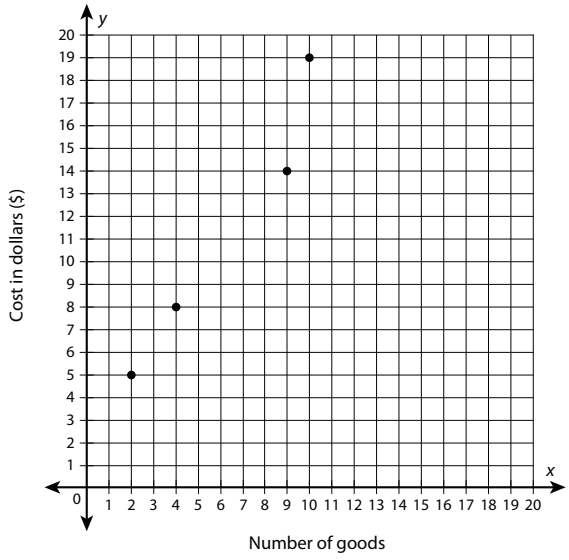
S–ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

Warm-Up 4.2.2 Debrief

1. Plot each point in the table on a coordinate plane.

   To plot each point, find the value of x along the x-axis (the horizontal axis), and then find the value of y along the y-axis (the vertical axis). Let the x-axis represent the number of goods, and the y-axis represent the cost in dollars.



2. Yasin is a welder. For his job, he requires 1 hour to set up and then 3 hours for each project. The time it takes on his job to complete x projects in one day can be modeled by the function $y = 3x + 1$. Graph the function $y = 3x + 1$.

   A function of the form $y = mx + b$ is a linear function, and the graph is a line. To graph a line, find two points on the line. Evaluate the function at two values of x. Two easy values to use are 0 and 1.

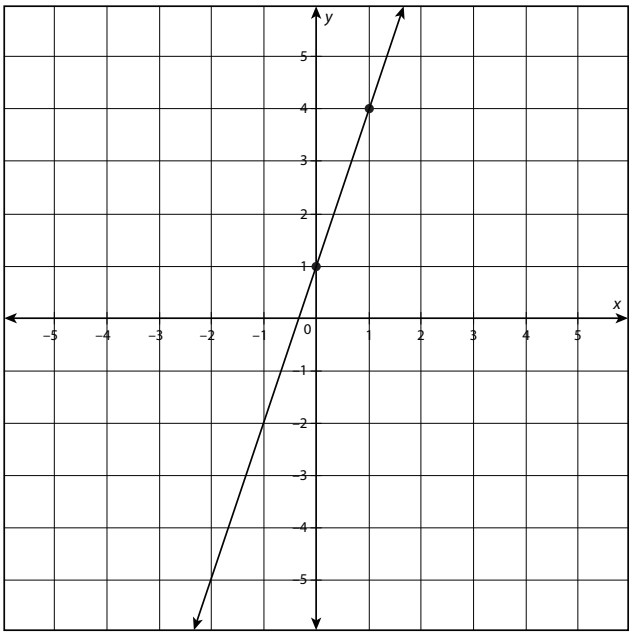
   $y = 3(0) + 1 = 1$

   $y = 3(1) + 1 = 4$

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Two points on the line are (0, 1) and (1, 4).

Graph the two points, and draw a line through the two points.



Connection to the Lesson

- In this lesson, students will examine the relationship between functions and data in a scatter plot.
- This warm-up will help students recall how to create a scatter plot given a data set.
- Students will also need to know how to graph a linear function given an algebraic equation for the function.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- plotting points on the coordinate plane, given data in a table (5.G.1)
- plotting the graph of a linear function from an equation (A–CED.2★)
- plotting the graph of an exponential function from an equation (A–CED.2★)
- evaluating a function at a given input value (F–IF.2)
- solving a function for x given a y-value (A–CED.4★)
- interpreting a function in context using a graph or an equation (8.F.5)

Introduction

Data with two quantitative variables can be represented using a scatter plot. A **scatter plot** is a graph of data in two variables on a coordinate plane, where each data pair is represented by a point. Relationships between the two quantitative variables can be observed on the graph. A **function** is a relation of two variables where each input is assigned to one and only one output. Functions in two variables can be represented algebraically with an equation, numerically with a table, or graphically on the coordinate plane. Graphing a function on the same coordinate plane as a scatter plot for a data set allows us to see if the function is a good estimation of the relationship between the two variables in the data set. The graph and the equation of the function can be used to estimate coordinate pairs that are not included in the data set.

Key Concepts

- Data with two quantitative variables can be shown graphically on a scatter plot.
- To create a scatter plot, plot each pair of data as a point on a coordinate plane.
- To compare a data set and a function, plot the function on the same coordinate plane as the scatter plot of the data set. The graph of the function should approximate the shape of the scatter plot.
- Evaluating a function that has a similar shape as a data set can provide an estimate for data not included in the plotted data set.
- Evaluate a function algebraically for a given value of x or y by substituting the given value for x or y and solving for the remaining variable.

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Instruction

- Evaluate a function graphically for a given value of x or y by finding the point on the graph of the function with the known coordinate, then finding the corresponding x- or y-value of that point.
- To determine whether a data set is best represented by a linear function or an exponential function, plot the data points and determine whether the points follow the shape of a straight line or an exponential curve.
- This determination can also be made by considering the context of the problem. If there is a constant rate of change, a linear function is the best model. However, if the rate of change increases or decreases as the independent variable increases, then an exponential function may be the best model.
- Graph a linear function by plotting at least two points and drawing a line through those points.
- Graph an exponential function by plotting at least five points. Connect the points with a curve.

Common Errors/Misconceptions

- confusing when to evaluate and when to solve a function
- using a linear function to estimate a relationship between two variables when an exponential function is a better fit
- using an exponential function to estimate a relationship between two variables when a linear function is a better fit
- confusing x and y when graphing data points or analyzing a graph

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Guided Practice 4.2.2

Example 1

Andrew wants to estimate his gas mileage, or miles traveled per gallon of gas used. He records the number of gallons of gas he purchased and the total miles he traveled with that gas.

Gallons	Miles
15	313
17	340
18	401
9	205
12	275
17	379
11	230
19	437
16	366
20	416

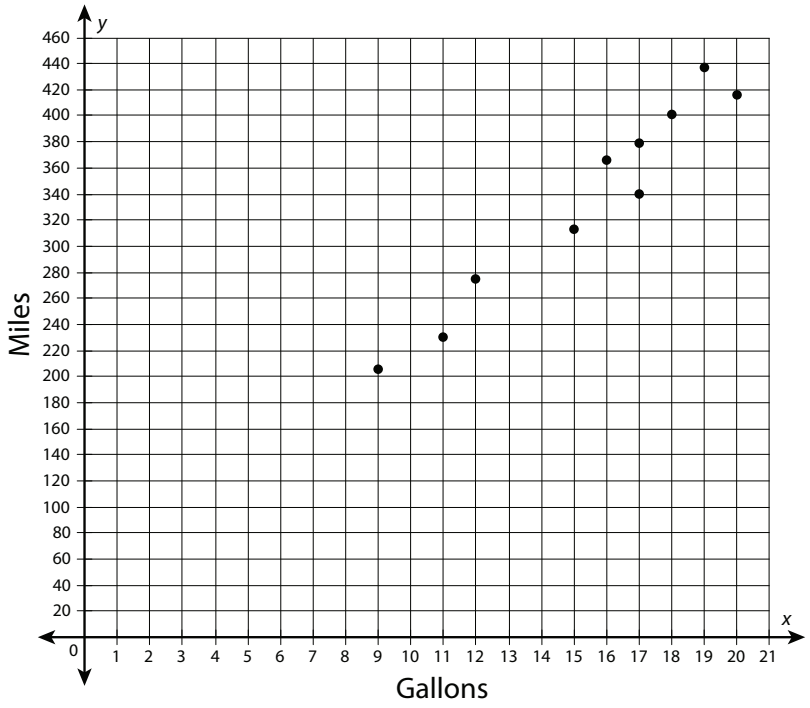
Create a scatter plot showing the relationship between gallons of gas and miles driven. Which is a better estimate for the function that relates gallons to miles: $y = 10x$ or $y = 22x$? How is the equation related to his gas mileage?

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Instruction

1. Plot each point on the coordinate plane.

   Let the *x*-axis represent the number of gallons and the *y*-axis represent the number of miles.



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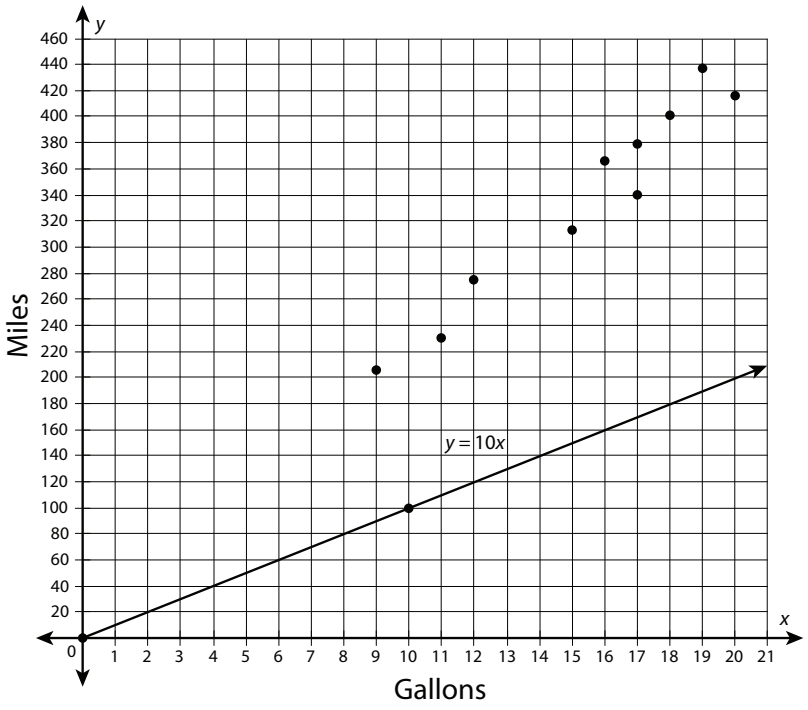
2. Graph the first function on the coordinate plane.

   The function $y = 10x$ is linear, so only two points are needed to draw the line.

   Evaluate the equation at two values of x, such as 0 and 10, and draw a line through these points on the scatter plot.

$y = 10x$	First equation
$y = 10(0) = 0$	Substitute 0 for x.
$y = 10(10) = 100$	Substitute 10 for x.

   Two points on the line are (0, 0) and (10, 100).



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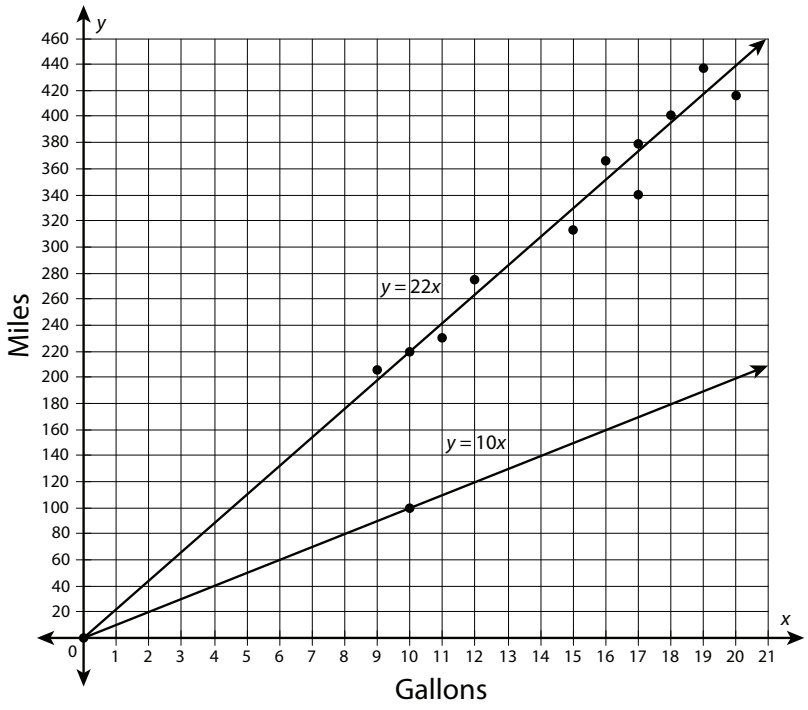
3. Graph the second function on the same coordinate plane.

   The function $y = 22x$ is also linear, so only two points are needed to draw the line.

   Evaluate the equation at two values of x, such as 0 and 10, and draw a line through these points on the scatter plot.

$y = 22x$	Second equation
$y = 22(0) = 0$	Substitute 0 for x.
$y = 22(10) = 220$	Substitute 10 for x.

   Two points on the line are (0, 0) and (10, 220).



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Instruction

4. Identify which function comes closer to the data values.

   The function whose line comes closer to the data values is the better estimate for the data.

   The graph of the function $y = 22x$ goes through the points in the scatter plot, with some points falling above this line and some points falling below it. The function $y = 10x$ is not steep enough to match the data values. Therefore, the function $y = 22x$ is a better estimate of the data.

5. Interpret the equation of the function in the context of the problem, using the units of the x- and y-axes.

   For a linear equation in the form $y = mx + b$, the slope (m) of the equation is the rate of change, or the change in y over the change in x. The y-intercept (b) of the equation is the initial value.

   In this example, y is miles and x is gallons. The slope is $\frac{\text{change in miles}}{\text{change in gallons}}$.

   For the equation $y = 22x$, the slope of 22 is equal to $\frac{22 \text{ miles}}{1 \text{ gallon}}$.

   The gas mileage of Andrew's car is the miles driven per gallon of gas used. The gas mileage is equal to the slope of the line that fits the data.

   Andrew's car has a gas mileage of approximately 22 miles per gallon.

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Instruction

Example 2

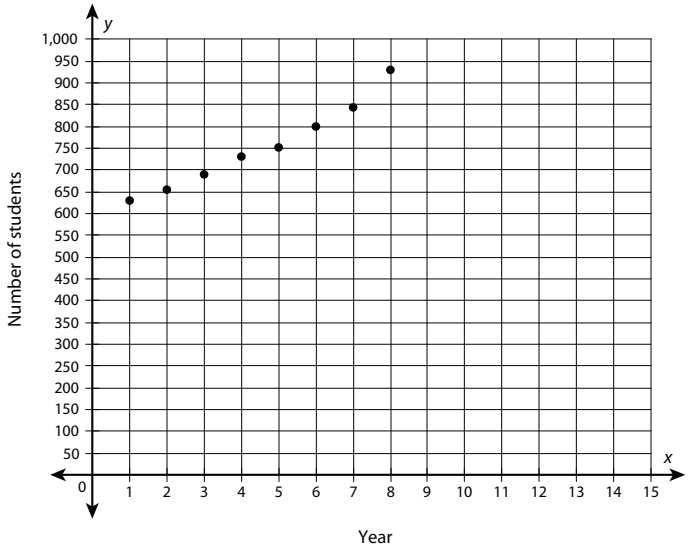
The principal at Park High School records the total number of students each year. The following table shows the number of students for each of the last 8 years.

Year	Number of students
1	630
2	655
3	690
4	731
5	752
6	800
7	844
8	930

Create a scatter plot showing the relationship between the year and the total number of students. Show that the function $y = 600(1.05)^x$ is a good estimate for the relationship between the year and the population. Approximately how many students will attend the high school in year 9?

1. Plot each point on the coordinate plane.

   Let the *x*-axis represent the year number and the *y*-axis represent the number of students.



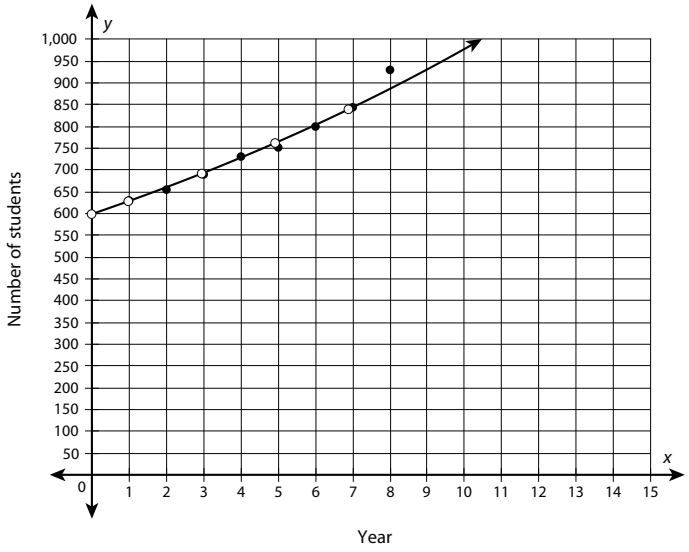
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2. Graph the given function on the coordinate plane.

The function is $y = 600(1.05)^x$. Calculate the value of y for at least five different values of x. Start with $x = 0$. Calculate the value of the function for at least four more x-values that are in the data table.

x	y
0	$600(1.05)^0 = 600$
1	$600(1.05)^1 = 630$
3	$600(1.05)^3 = 694.575$
5	$600(1.05)^5 = 765.769$
7	$600(1.05)^7 = 855.260$

Plot these points on the same coordinate plane. Connect the points with a curve. *Note*: The new points are plotted in white.



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3. Compare the graph of the function to the scatter plot of the data.

   The graph of the function appears to be very close to the points in the scatter plot. Therefore, $y = 600(1.05)^x$ is a good estimate of the data.

4. Use the equation to answer the question.

   Use the equation to estimate the population in year 9.

   Evaluate the equation $y = 600(1.05)^x$ for year 9, when $x = 9$.

   $y = 600(1.05)^9 = 930.797$

   The equation $y = 600(1.05)^x$ is a good estimate of the population. There will be approximately 931 students in the school in year 9.

Example 3

The weights of oranges vary. Maria wants to come up with a way to estimate the number of oranges given a weight. She weighs oranges and records the weights in the following table.

Number of oranges	Weight in pounds
1	0.47
3	1.29
5	2.54
6	2.65
8	4.12
10	5.57
12	7.18
13	8.48
14	7.07

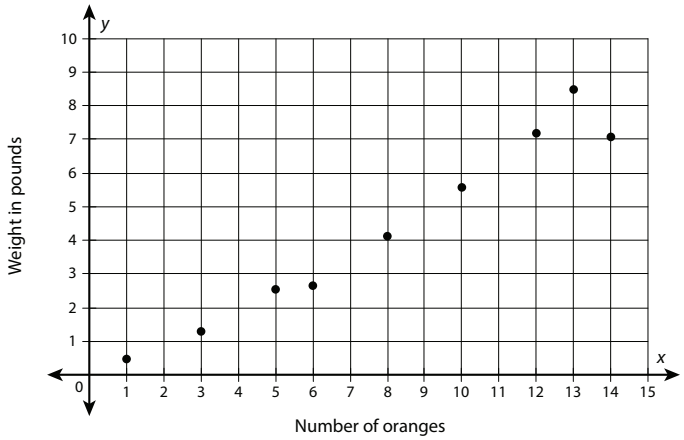
Create a scatter plot showing the relationship between the number of oranges and the weight in pounds. Which is a better estimate for the function that relates the number of oranges to their weight: $y = 0.6x - 0.5$ or $y = 1.2^x - 1$? Maria has a bag of oranges that weighs 2 pounds. Approximately how many oranges are in the bag?

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Instruction

1. Plot each point on the coordinate plane.

   Let the x-axis represent the number of oranges and the y-axis represent the weight in pounds.



2. Determine if this data is best estimated with a linear graph or an exponential graph.

   The graph of the data appears to be linear. This is supported by the fact that the data represents the weight of oranges, so as the number of oranges increases by 1, the weight should increase by approximately the same amount, indicating a constant rate of change.

   Check this conclusion by graphing the two functions.

   Graph the linear equation, $y = 0.6x - 0.5$. Find two points on the line by evaluating the equation at two values of x.

   Two easy values to use are 0 and 1.

$y = 0.6x - 0.5$	Linear equation
$y = 0.6(0) - 0.5 = -0.5$	Substitute 0 for x.
$y = 0.6(1) - 0.5 = 0.1$	Substitute 1 for x.

   Two points on the line are $(0, -0.5)$ and $(1, 0.1)$.

   Graph the two points on the same graph as the scatter plot, and then draw a line through the two points.

(*continued*)

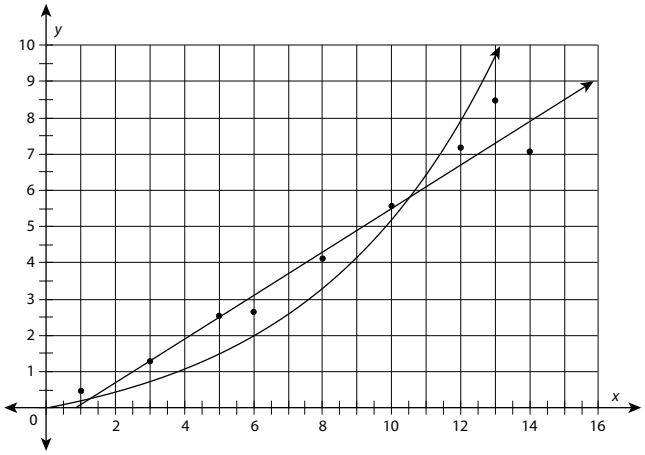
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Instruction

Now, graph the exponential equation, $y = 1.2^x - 1$. Find points on the curve by evaluating the equation at several values of x.

$y = 1.2^x - 1$	Exponential equation
$y = 1.2^0 - 1 = 0$	Substitute 0 for x.
$y = 1.2^3 - 1 \approx 0.7$	Substitute 3 for x.
$y = 1.2^6 - 1 \approx 2.0$	Substitute 6 for x.
$y = 1.2^9 - 1 \approx 4.2$	Substitute 9 for x.
$y = 1.2^{12} - 1 \approx 7.9$	Substitute 12 for x.

Five points on the curve are (0, 0), (3, 0.7), (6, 2.0), (9, 4.2), and (12, 7.9).



3. Look at the relationship between the graphs of the two equations and the graph of the data from the table. Determine which equation's graph most resembles the graph of the data.

   If a linear function is a good approximation for a data set, some of the data values will be above the line and some will be below the line. It appears that the linear equation is a good fit for the data.

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4. Use the equation to estimate the number of oranges weighing 2 pounds.

   In the equation $y = 0.6x - 0.5$, x is the number of oranges and y is the weight of the oranges in pounds. Substitute 2 for y and solve for x to estimate the number of oranges that weigh 2 pounds.

$(2) = 0.6x - 0.5$	Substitute 2 for y.
$2.5 = 0.6x$	Add 0.5 to both sides of the equation.
$4.2 \approx x$	Divide both sides by 0.6.

   Maria can use the equation $y = 0.6x - 0.5$ to estimate how many oranges have a given weight. The weight of 4 oranges is approximately 2 pounds. Therefore, her 2-pound bag probably contains 4 oranges.



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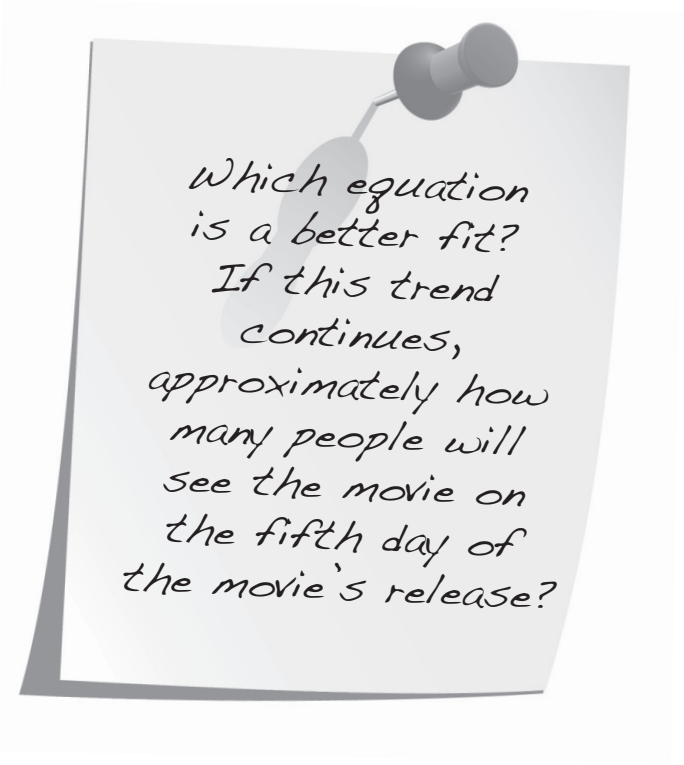
Problem-Based Task 4.2.2: Movie Buzz

SMP	
1	2
3 ✓	4 ✓
5 ✓	6
7	8

Word of mouth can be a great way to increase a movie's popularity. A movie theater released a movie. On the first day, only 5 people saw the movie. They all loved it, and each told at least 5 more people to go see the movie. The second day of the movie's release, many of the people who had been told to see the movie went to the theater. Each day, each person who viewed the movie told approximately 5 other people to go to the theater. The following table shows the number of people who viewed the movie in its first 4 days.

Day	Number of viewers
1	5
2	27
3	124
4	626

Create a scatter plot showing the number of viewers each day the movie played at the theater. Which type of function would best approximate the data? Two theater employees each try to determine a equation to fit the data. One thinks that the equation $y = 5^x$ is a good fit for the data; the other thinks the equation $y = 200x - 200$ is a good fit for the data. Which equation is a better fit? If this trend continues, approximately how many people will see the movie on the fifth day of the movie's release?



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Problem-Based Task 4.2.2: Movie Buzz

Coaching

a. Create a scatter plot of the given data.

b. Look at the shape of the scatter plot. Is the data linear or exponential?

c. Graph the equations $y = 5^x$ and $y = 200x - 200$ on the scatter plot with the data.

d. Which equation is a better fit for this data?

e. Use this equation to estimate the number of people who will see the movie on the fifth day.

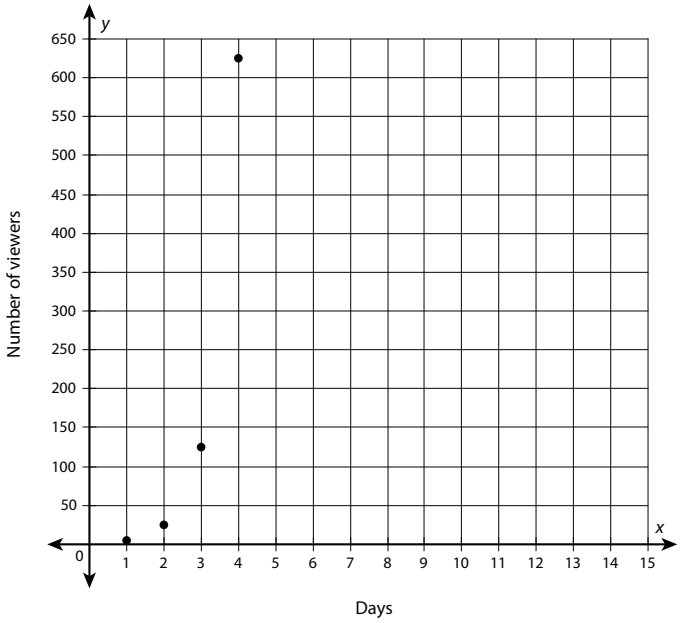
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Instruction

Problem-Based Task 4.2.2: Movie Buzz

Coaching Sample Responses

a. Create a scatter plot of the given data.



b. Look at the shape of the scatter plot. Is the data linear or exponential?

If the data were linear, the slope between each pair of points would be the same. As the x-values increase by 1, the y-values increase by different amounts. The change in y between the first two points is much less than the change in y between the second and third points. The change in y between the second and third points is much less than the change in y between the third and fourth points. The shape of the scatter plot is curved, and it appears to follow the shape of an exponential curve.

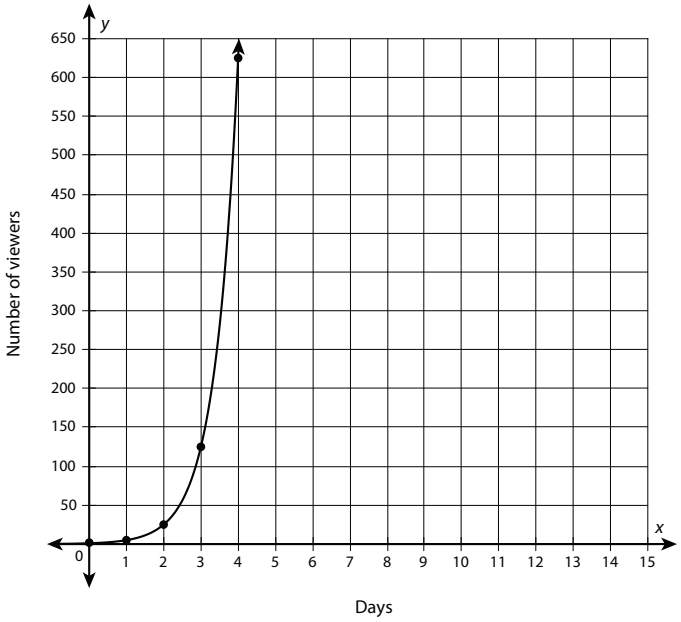
c. Graph the equations $y = 5^x$ and $y = 200x - 200$ on the scatter plot with the data.

To create a graph of the exponential equation $y = 5^x$, evaluate the equation at several values in the domain of the data set; 0 is an easy value with which to start. The original data set contains the values 1, 2, 3, and 4 in the domain; evaluate the equation at these values.

x	$y = 5^x$
0	$5^0 = 1$
1	$5^1 = 5$
2	$5^2 = 25$
3	$5^3 = 125$
4	$5^4 = 625$

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Plot each of these values on the scatter plot, and connect the lines with a curve.

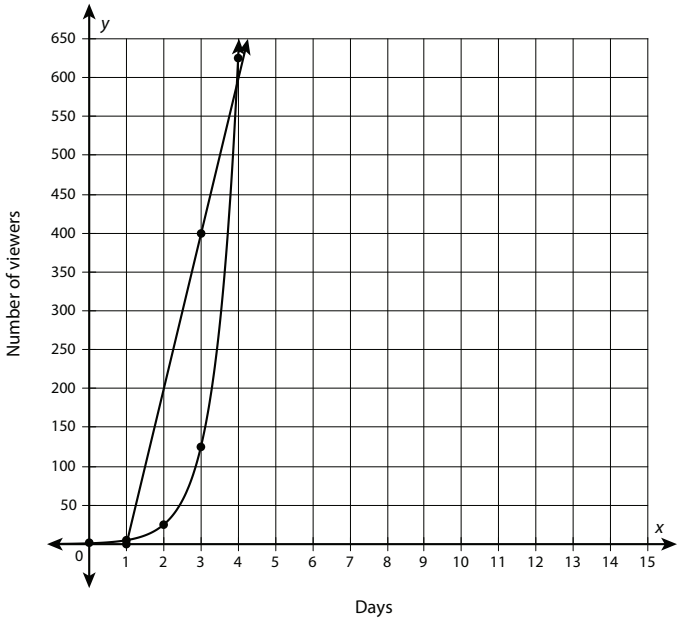


The graph of a linear equation is a line. To plot a linear equation, find at least two points on the line. Two values to evaluate are $x = 1$ and $x = 3$.

$y = 200(1) - 200 = 0$ Substitute 1 for x.

$y = 200(3) - 200 = 400$ Substitute 3 for x.

Plot the points (1, 0) and (3, 400) on the same graph as the scatter plot and draw a straight line through the points.



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d. Which equation is a better fit for this data?

Look at the shape of each graph. The first graph of $y = 5^x$ comes very close to the data set, and matches the shape of the data. This equation is a better fit for the data.

e. Use this equation to estimate the number of people who will see the movie on the fifth day.

The x-values are the number of days, so replace x in the equation $y = 5^x$ with 5. Evaluate the expression to find the value of y.

$y = 5^{(5)} = 3125$

On the fifth day, there will be approximately 3,125 viewers if this pattern continues.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 4.2.2: Solving Problems Given Functions Fitted to Data

Racecar tracks vary in length. A racecar driver records the time it takes him to circle various tracks once at top speed. The distance of the track and his time to circle each track once are listed in the following table. Use the data for problems 1–5.

Time in minutes	Track length in miles
0.42	1.5
0.15	0.53
0.42	1.5
0.43	1.4
0.82	2.5
0.31	1
0.56	2
0.26	0.9
0.15	0.5
0.75	2.7

1. Create a scatter plot of the data set.

2. Would a linear or exponential function be a better estimate for the data? Explain.

3. Which function is a better fit for the data: $y = 2.3x$ or $y = 3.3x$? Use a graph to support your answer.

4. Approximately how long would it take the driver to circle a track that is 1.8 miles long?

5. It takes the driver 0.6 minutes to circle a track. Approximately how long is the track?

continued

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The value of a car is expected to depreciate over time. Bethan buys a car for $20,000. Each year, she determines how much her car is worth. She records the value of her car each year in the following table. Use the data for problems 6–10.

Year	Value in dollars ($)
0	20,000
1	16,000
2	14,500
3	13,200
4	12,000
5	11,000
6	10,000

6. Create a scatter plot showing the value of her car over time.

7. Would a linear or exponential function be a better estimate for the data? Explain.

8. Is $y = 19{,}000(1.10)^x$ or $y = 19{,}000(0.90)^x$ a good estimate for the data? Use your graph to explain why or why not.

9. Bethan wants to sell her car when it's worth approximately $9,000. After how many years should Bethan sell it? Use your graph to explain your answer.

10. How much will her car be worth in 12 years?

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Lesson 4.2.3: Analyzing Residuals

Warm-Up 4.2.3

Felicia is learning to ride a unicycle. She started at her house, which is considered to be at the origin of a coordinate plane. She went 2 blocks east and wasn't able to successfully ride the unicycle. Then she started going north toward her friend's house. After many failed attempts and falling off for 4 blocks, she had success for 2 blocks.

1. Plot the points (2, 4) and (2, 6) on a coordinate plane.

2. Find the distance between the two points. How far did Felicia successfully ride her unicycle?

3. Felicia is going to sell unicycle pins to raise money for medical research. She spent \$2 of her own money to buy the pins and will sell each pin for \$4. Her revenue can be modeled by the function $y = 4x - 2$. Plot the function $y = 4x - 2$ over the domain of all real numbers.

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Instruction

Lesson 4.2.3: Analyzing Residuals

Common Core State Standard

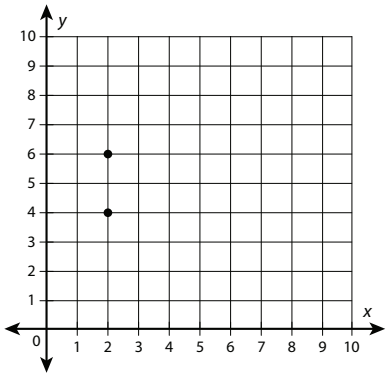
S–ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

b. Informally assess the fit of a function by plotting and analyzing residuals.

Warm-Up 4.2.3 Debrief

1. Plot the points (2, 4) and (2, 6) on a coordinate plane.

   The first number in each ordered pair is the *x*-value, and the second number is the *y*-value.



2. Find the distance between the two points. How far did Felicia successfully ride her unicycle?

   Look at the location of the points on the coordinate plane. The two points have the same *x*-value, so the only distance between the two points is a vertical distance. The distance is the absolute value of the difference between the two *y*-values: $|6 - 4| = 2$. Felicia rode 2 blocks.

3. Felicia is going to sell unicycle pins to raise money for medical research. She spent $2 of her own money to buy the pins and will sell each pin for $4. Her revenue can be modeled by the function $y = 4x - 2$. Plot the function $y = 4x - 2$ over the domain of all real numbers.

   An equation of the form $y = mx + b$ is a linear function. The graph is a line, so only two points are needed to create the graph. Evaluate the function at two values of x to find two points on the line. For example, evaluate the function at $x = 0$ and $x = 1$.

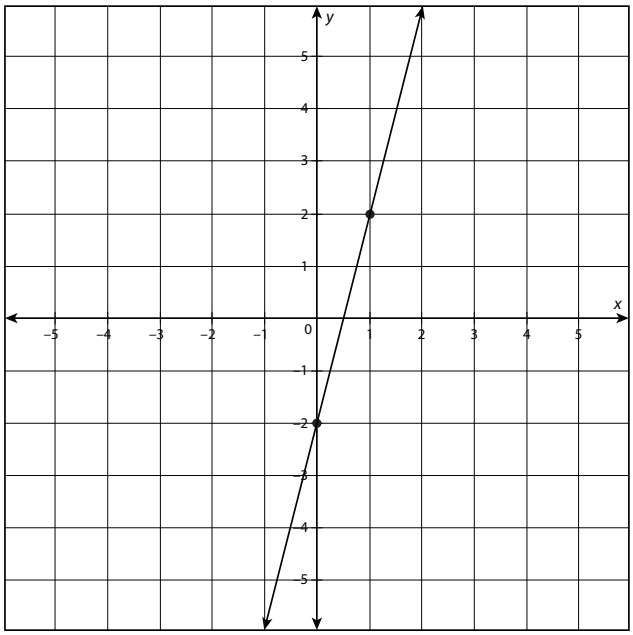
   $y = 4(0) - 2 = -2$ Substitute 0 for x.

   $y = 4(1) - 2 = 2$ Substitute 1 for x.

   Two points on the line are (0, –2) and (1, 2).

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Plot these points, and then draw a line through them.



Connection to the Lesson

- In this lesson, students will analyze the fit of linear functions to a set of data.
- Students will need to be familiar with calculating the vertical distance between two points on the coordinate plane.
- Students will also need to be able to plot points and graphs of linear functions.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- plotting points on the coordinate plane (5.G.1)
- calculating the vertical distance between two points on the coordinate plane (6.NS.8)
- graphing linear functions on the coordinate plane (A–CED.2★)

Introduction

The fit of a linear function to a set of data can be assessed by analyzing residuals. A **residual** is the vertical distance between an observed data value and an estimated data value on a line or curve that models the data. For linear functions, the line that models the data is called the line of best fit. Representing residuals on a residual plot provides a visual representation of the residuals for a set of data. A **residual plot** contains the points (x, residual for x). A residual plot in which the distribution of points appears random, with both positive and negative residual values, indicates that the line is a good fit for the data. If the residual plot follows a pattern, such as a U-shape, the line is likely not a good fit for the data.

Key Concepts

- A residual is the distance between an observed data point and an estimated data value on a line of best fit. For the observed data point (x, y) and the estimated data value on a line of best fit (x, y_0), the residual is $y - y_0$.
- A residual plot is a plot of each x-value and its corresponding residual. For the observed data point (x, y) and the estimated data value on a line of best fit (x, y_0), the point on a residual plot is $(x, y - y_0)$.
- A residual plot with a random pattern indicates that the line of best fit is a good approximation for the data.
- A residual plot with a U-shape, an inverted U-shape, or some other pattern indicates that the line of best fit is not a good approximation for the data.

Common Errors/Misconceptions

- incorrectly finding the residual
- incorrectly plotting points on the residual plot

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Guided Practice 4.2.3

Example 1

Pablo's science class is growing plants. He recorded the height of his plant each day for 10 days. The plant's height, in centimeters, over that time is listed in the following table.

Day	Height in centimeters
1	3
2	5.1
3	7.2
4	8.8
5	10.5
6	12.5
7	14
8	15.9
9	17.3
10	18.9

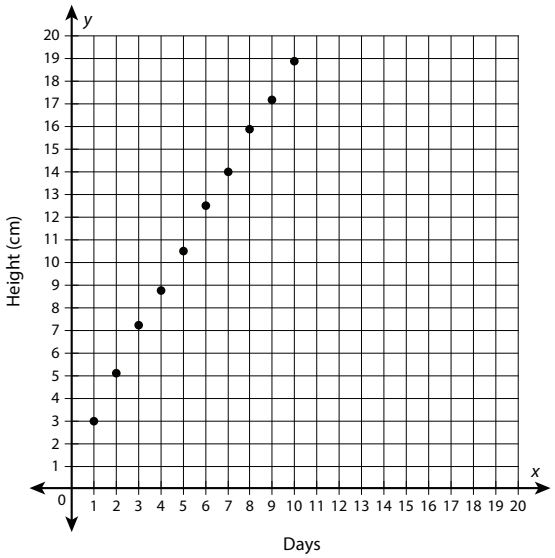
Pablo determines that the function $y = 1.73x + 1.87$ is a good fit for the data. How close is his estimate to the actual data? Approximately how much does the plant grow each day?

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Instruction

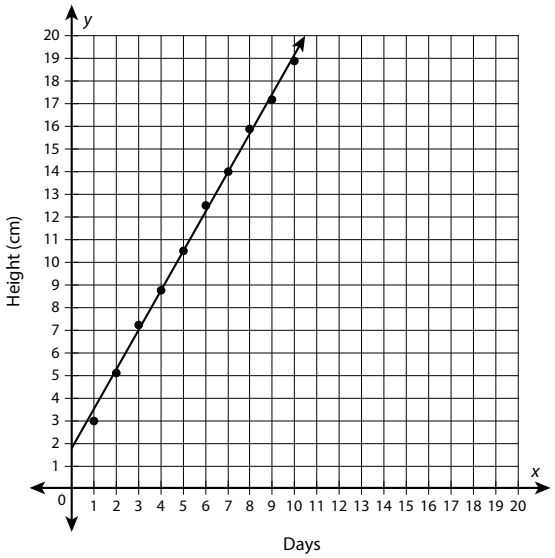
1. Create a scatter plot of the data.

   Let the x-axis represent the number of days and the y-axis represent the height in centimeters.



2. Draw the line of best fit.

   A good line of best fit will have some points below the line and some above the line. Pablo estimated that the graph of $y = 1.73x + 1.87$ is a good line of best fit. Use the graph of this function to initially determine if it is a good fit for the data.



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3. Find the residuals for each data point.

The residual for each data point is the difference between the observed value and the estimated value using a line of best fit. Evaluate the function of the line at each value of x.

x	$y = 1.73x + 1.87$
1	$y = 1.73(1) + 1.87 = 3.6$
2	$y = 1.73(2) + 1.87 = 5.33$
3	$y = 1.73(3) + 1.87 = 7.06$
4	$y = 1.73(4) + 1.87 = 8.79$
5	$y = 1.73(5) + 1.87 = 10.52$
6	$y = 1.73(6) + 1.87 = 12.25$
7	$y = 1.73(7) + 1.87 = 13.98$
8	$y = 1.73(8) + 1.87 = 15.71$
9	$y = 1.73(9) + 1.87 = 17.44$
10	$y = 1.73(10) + 1.87 = 19.17$

Next, find the difference between each observed value and each calculated value for each value of x.

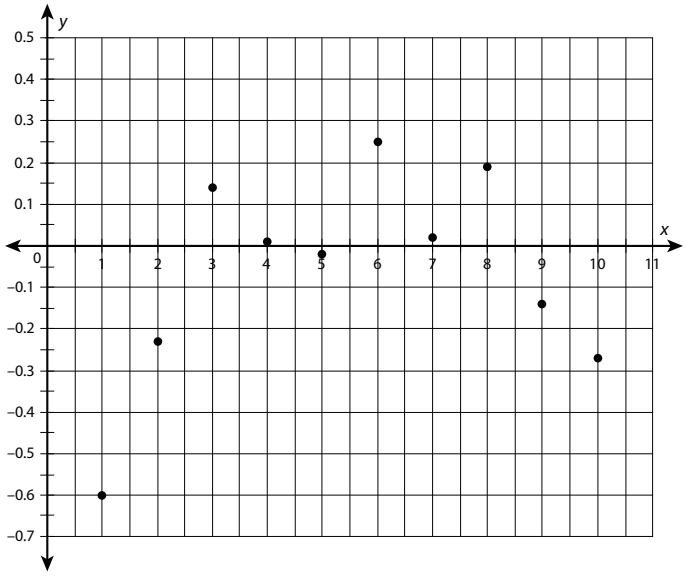
x	Residual
1	$3 - 3.6 = -0.6$
2	$5.1 - 5.33 = -0.23$
3	$7.2 - 7.06 = 0.14$
4	$8.8 - 8.79 = 0.01$
5	$10.5 - 10.52 = -0.02$
6	$12.5 - 12.25 = 0.25$
7	$14 - 13.98 = 0.02$
8	$15.9 - 15.71 = 0.19$
9	$17.3 - 17.44 = -0.14$
10	$18.9 - 19.17 = -0.27$

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Instruction

4. Plot the residuals on a residual plot.

   Plot the points (x, residual for x).



5. Describe the fit of the line based on the shape of the residual plot.

   The plot of the residuals appears to be random, with some negative and some positive values. This indicates that the line is a good line of fit.

6. Use the equation to answer the question.

   Use the equation to estimate the centimeters grown each day.

   The change in the height per day is the centimeters grown each day. In the equation of the line, the slope is the change in height per day. The plant is growing approximately 1.73 centimeters each day.

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Instruction

Example 2

Lindsay created the following table, which shows the population of fruit flies over the last 10 weeks.

Week	Number of flies
1	50
2	78
3	98
4	122
5	153
6	191
7	238
8	298
9	373
10	466

She estimates that the population of fruit flies can be represented by the function $y = 46x - 40$. Using residuals, determine if her representation is a good estimate.

1. Find the estimated population at each x-value.

   Evaluate the function at each value of x.

x	$y = 46x - 40$
1	$46(1) - 40 = 6$
2	$46(2) - 40 = 52$
3	$46(3) - 40 = 98$
4	$46(4) - 40 = 144$
5	$46(5) - 40 = 190$
6	$46(6) - 40 = 236$
7	$46(7) - 40 = 282$
8	$46(8) - 40 = 328$
9	$46(9) - 40 = 374$
10	$46(10) - 40 = 420$

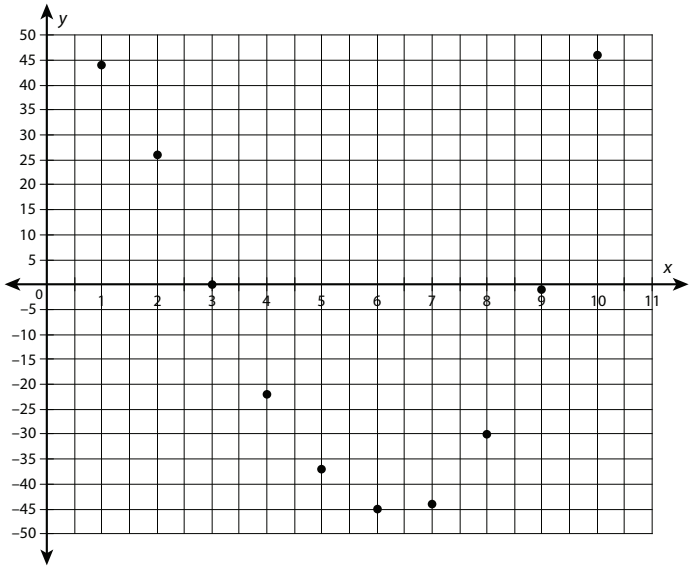
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2. Find the residuals by finding each difference between the observed population and estimated population.

x	Residual
1	50 – 6 = 44
2	78 – 52 = 26
3	98 – 98 = 0
4	122 – 144 = –22
5	153 – 190 = –37
6	191 – 236 = –45
7	238 – 282 = –44
8	298 – 328 = –30
9	373 – 374 = –1
10	466 – 420 = 46

3. Create a residual plot.

Plot the points (x, residual for x).



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4. Analyze the residual plot to determine if the function is a good estimate for the data.

   The residual plot has a U-shape. This indicates that a non-linear estimation would be a better fit for this data set.

   The shape of the residual plot indicates that the function $y = 46x - 40$ is not a good estimate for this data set.



Example 3

Anthony is traveling across the country by car. He keeps track of the hours he has driven and total miles he has traveled in the following table.

Hours	Miles
1	38
3	170
4	234
8	390
11	495
12	528
15	699
17	767
20	857

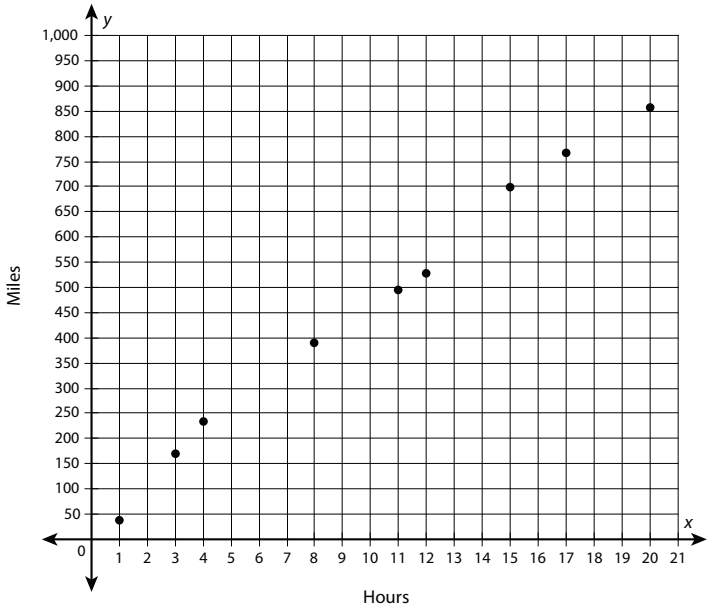
Anthony uses the function $y = 42.64x + 42.12$ to estimate his total miles driven after any number of hours. Use a residual plot to determine how well the line fits the data. Approximately how many miles had Anthony driven after 13 hours?

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Instruction

1. Create a scatter plot of the data set.

   Let the *x*-axis represent the number of hours and the *y*-axis represent the number of miles.



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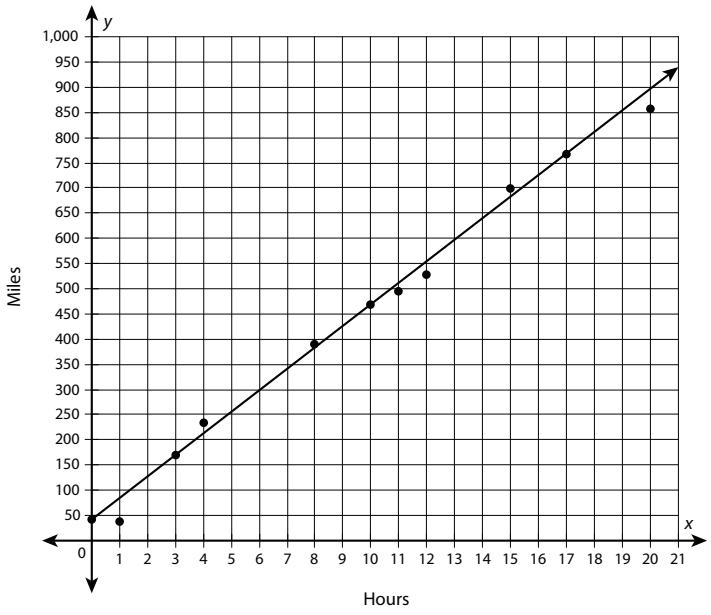
2. Graph the given function.

The function that Anthony used to estimate the total miles driven is $y = 42.64x + 42.12$. To graph a linear equation, find two points on the line. Then draw a straight line through the two points. Two easy values of x to use are 0 and 10.

$y = 42.64(0) + 42.12 = 42.12$ Substitute 0 for x.

$y = 42.64(10) + 42.12 = 468.52$ Substitute 10 for x.

Two points on the line are (0, 42.12) and (10, 468.52).



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3. Evaluate the equation for each value of x.

x	$y = 42.64x + 42.12$
1	42.64(1) + 42.12 = 84.76
3	42.64(3) + 42.12 = 170.04
4	42.64(4) + 42.12 = 212.68
8	42.64(8) + 42.12 = 383.24
11	42.64(11) + 42.12 = 511.16
12	42.64(12) + 42.12 = 553.8
15	42.64(15) + 42.12 = 681.72
17	42.64(17) + 42.12 = 767
20	42.64(20) + 42.12 = 894.92

4. Find the residuals.

   Find the difference between each observed distance and estimated distance.

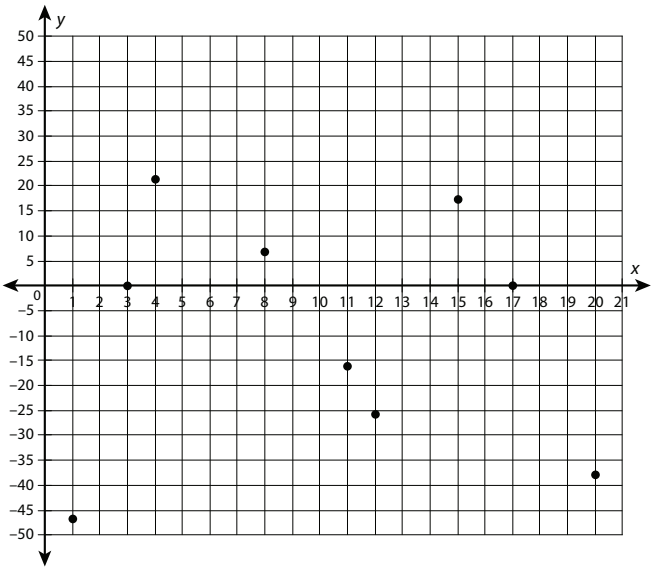
x	Residual
1	38 – 84.76 = –46.76
3	170 – 170.04 = –0.04
4	234 – 212.68 = 21.32
8	390 – 383.24 = 6.76
11	495 – 511.16 = –16.16
12	528 – 553.8 = –25.8
15	699 – 681.72 = 17.28
17	767 – 767 = 0
20	857 – 894.92 = –37.92

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Instruction

5. Create a residual plot.

   Plot the points (x, residual for x).



6. Determine if the linear function is a good estimate for the data.

   The residual plot has a random shape, indicating that the linear function is a good estimate for the data.

7. Use the function to estimate the total miles driven when the time equals 13 hours.

   In the line of best fit, x represents the number of hours driven and y represents the total number of miles driven. Evaluate the function for $x = 13$ to estimate the total miles driven after 3 hours.

   $y = 42.64(13) + 42.12 = 596.44$ Substitute 13 for x.

   After 13 hours, Anthony had driven approximately 596 miles.

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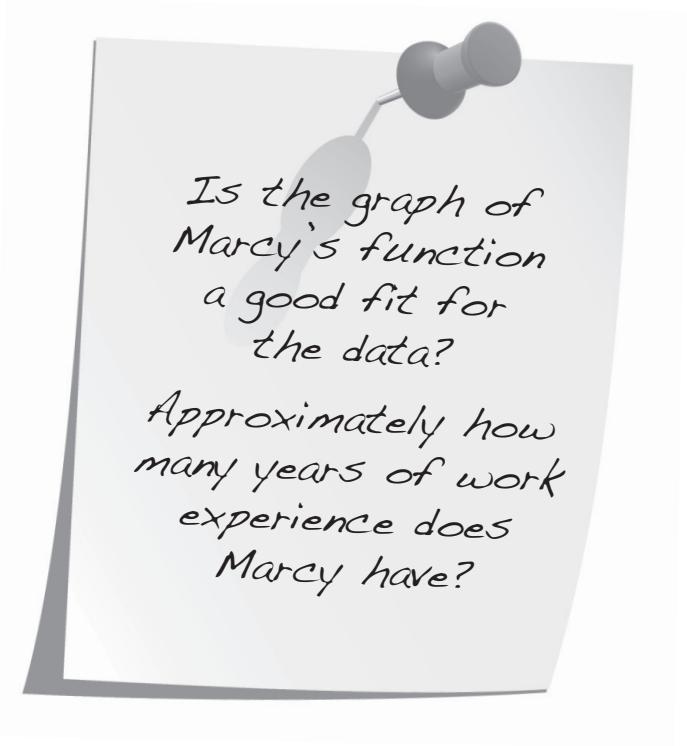
Problem-Based Task 4.2.3: Estimating Salaries

SMP	
1	2
3 ✓	4 ✓
5 ✓	6
7	8

Marcy surveys 10 people who work at a software company. She asks each person how many years they have worked, and what their estimated salary was last year. Her results are in the following table.

Years of experience	Salary in dollars ($)
1	52,810
2	61,615
3	80,831
4	77,201
5	136,566
6	100,707
8	135,460
10	208,889
11	228,831
13	209,726

Marcy believes that the salaries can be estimated using the function $y = 14{,}000x + 34{,}000$. Is the graph of Marcy's function a good fit for the data? Marcy estimates that her salary should be $130,000. Approximately how many years of work experience does Marcy have?



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Problem-Based Task 4.2.3: Estimating Salaries

Coaching

a. Create a scatter plot of the data.

b. Plot the line of best fit on the scatter plot.

c. Does it appear that the line is a good fit for the data?

d. Calculate the estimated salary for each x-value.

e. Find the residual for each x-value, or the difference between the observed and estimated salaries for each value of x.

f. Create a residual plot.

g. Is the line a good fit for the data?

h. If Marcy estimates her salary should be $130,000, how can the years she has worked be approximated using her function?

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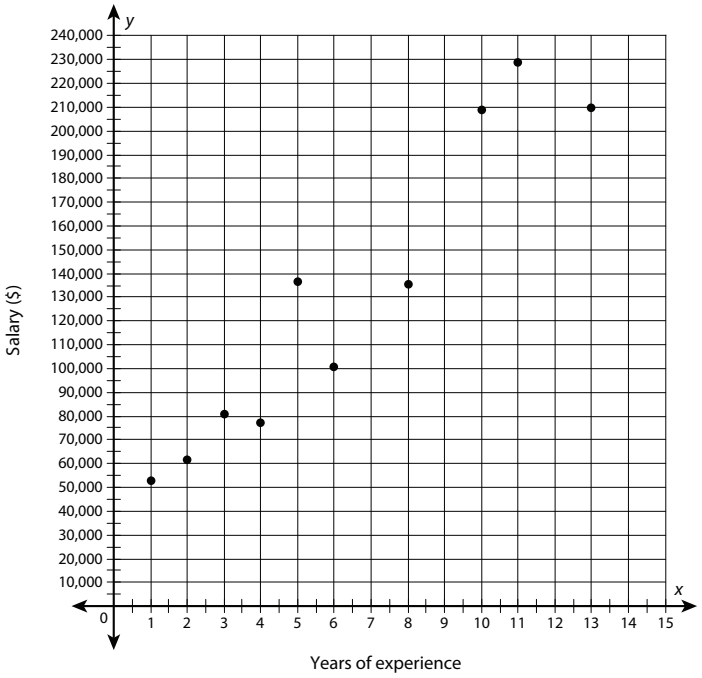
Instruction

Problem-Based Task 4.2.3: Estimating Salaries

Coaching Sample Responses

a. Create a scatter plot of the data.

Let the *x*-axis represent years of experience and the *y*-axis represent salary.



b. Plot the line of best fit on the scatter plot.

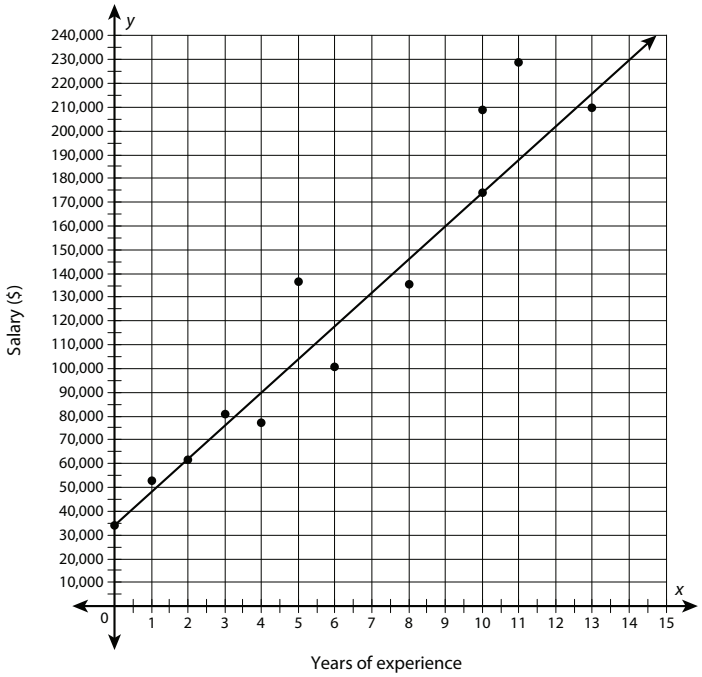
Find the value of two points on the line, and then draw a line through those two points. Two easy values of *x* to use are 0 and 10.

$y = 14{,}000(0) + 34{,}000 = 34{,}000$ Substitute 0 for x.

$y = 14{,}000(10) + 34{,}000 = 174{,}000$ Substitute 10 for x.

The two points are (0, 34,000) and (10, 174,000).

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c. Does it appear that the line is a good fit for the data?

A good line of best fit goes through the middle of the point set, where some of the observed values are above the line and some are below the line. It appears that the line is a good fit for the data.

d. Calculate the estimated salary for each x-value.

Evaluate the function $y = 14{,}000x + 34{,}000$ for each observed x-value.

x	$y = 14{,}000x + 34{,}000$
1	14,000(1) + 34,000 = 48,000
2	14,000(2) + 34,000 = 62,000
3	14,000(3) + 34,000 = 76,000
4	14,000(4) + 34,000 = 90,000
5	14,000(5) + 34,000 = 104,000
6	14,000(6) + 34,000 = 118,000
8	14,000(8) + 34,000 = 146,000
10	14,000(10) + 34,000 = 174,000
11	14,000(11) + 34,000 = 188,000
13	14,000(13) + 34,000 = 216,000

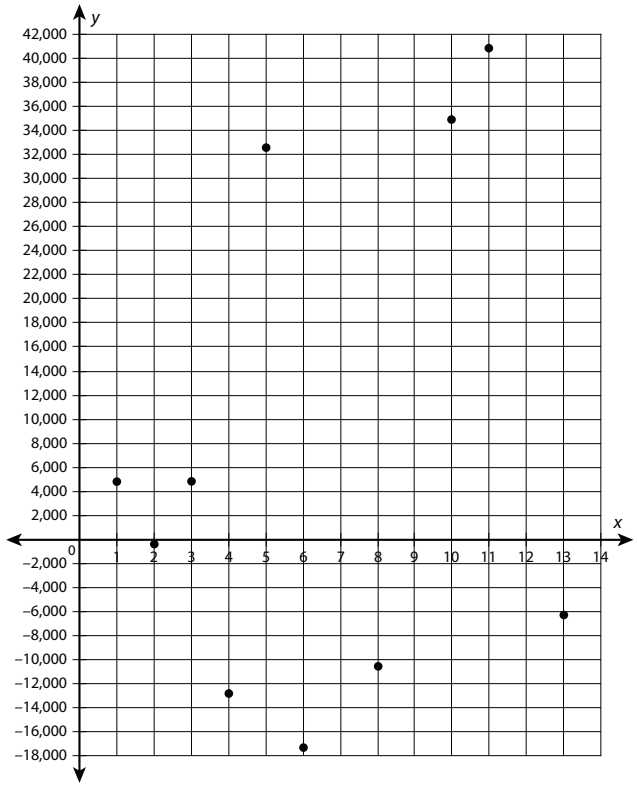
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e. Find the residual for each *x*-value, or the difference between the observed and estimated salaries for each value of *x*.

x	Residual
1	52,810 – 48,000 = 4810
2	61,615 – 62,000 = –385
3	80,813 – 76,000 = 4831
4	77,201 – 90,000 = –12,799
5	136,566 – 104,000 = 32,566
6	100,707 – 118,000 = –17,293
8	135,460 – 146,000 = –10,540
10	208,889 – 174,000 = 34,889
11	228,831 – 188,000 = 40,831
13	209,726 – 216,000 = –6274

f. Create a residual plot.

Let the *x*-axis represent years worked and the *y*-axis represent the residual values. Make sure the *y*-axis includes all values in the range of residuals. The least residual is –17,293, and the greatest residual is 40,831, so the *y*-axis must include both these values.



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g. Is the line a good fit for the data?

The residual plot is random, which indicates that the line is a good fit for the data.

h. If Marcy estimates her salary should be $130,000, how can the years she has worked be approximated using her function?

y is the salary, and x is the years worked. Substitute the salary estimate for y, and solve for x using the line of best fit.

$y = 14{,}000x + 34{,}000$	Function
$130{,}000 = 14{,}000x + 34{,}000$	Substitute values for x and y.
$96{,}000 = 14{,}000x$	Subtract 34,000 from both sides.
$6.86 \approx x$	Divide both sides by 14,000.

Round up to the nearest whole year.

Marcy has approximately 7 years of work experience.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 4.2.3: Analyzing Residuals

To understand the density of a deer population, Lewis counts the deer in different areas of a forest. He records the deer in each portion of the forest in a data table. Use the data for problems 1–4.

Acres of forest	Deer population
5	10
8	0
10	0
14	42
20	100
22	66
30	90
45	180
50	100
58	116

1. Create a scatter plot showing the deer population in each acreage.

2. Lewis states that the population can be estimated using the equation $y = 2x + 22$. Graph the equation on the scatter plot.

3. Does it appear that this line is a good fit for the data? Explain.

4. Use a residual plot to determine if a linear function is a good fit for the data.

continued

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Skylar has a savings account. She records the balance in the account each year. Use the data for problems 5–8.

Years	Account balance in dollars ($)
2	551
3	578
4	608
5	638
6	670
7	704
8	739
9	776

5. Create a scatter plot of the account balances.

6. Skylar estimates that the account balance can be represented by the equation $y = 32x + 483$. Draw the line of the equation on the scatter plot.

7. Does it appear that this line is a good fit for the data? Explain.

8. Use a residual plot to determine if a linear function is a good fit for the data.

Esmeralda is training for a marathon. She records the distance and time of her recent runs in the following table. Use the data for problems 9 and 10.

Distance in miles	Time in minutes
10	120
11	115.5
12.5	121.25
15	168
16.8	169.68
19	163.4
21	224.7
22	228.8
24	230.4

9. Create a scatter plot of the running times.

10. Esmeralda determines that her time can be approximated using the equation $y = 9x + 19$. Use a residual plot to determine if a linear function is a good fit for the data.

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Lesson 4.2.4: Fitting Linear Functions to Data

Warm-Up 4.2.4

The data table shows temperatures in degrees Fahrenheit taken at 7:00 A.M. and noon on 8 different days throughout the year in a small town in Siberia. Use the table to complete problems 1 and 2.

7:00 A.M.	Noon
0	–3
1	–1
2	1
5	7
7	11
10	17
16	29
20	37

1. Plot the points on a scatter plot.

2. Describe the shape of the points.

Use your knowledge of equations to complete problems 3 and 4.

3. Suppose x represents the number of students in a class, and y represents the number of index cards a teacher needs to purchase for the class. Every student needs 8 index cards, and the teacher wants 2 extra cards. This can be modeled by the function $y = 8x + 2$. What is the slope of the line that models this scenario?

4. What is the graph of the equation $y = -x + 1$?

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Instruction

Lesson 4.2.4: Fitting Linear Functions to Data

Common Core State Standard

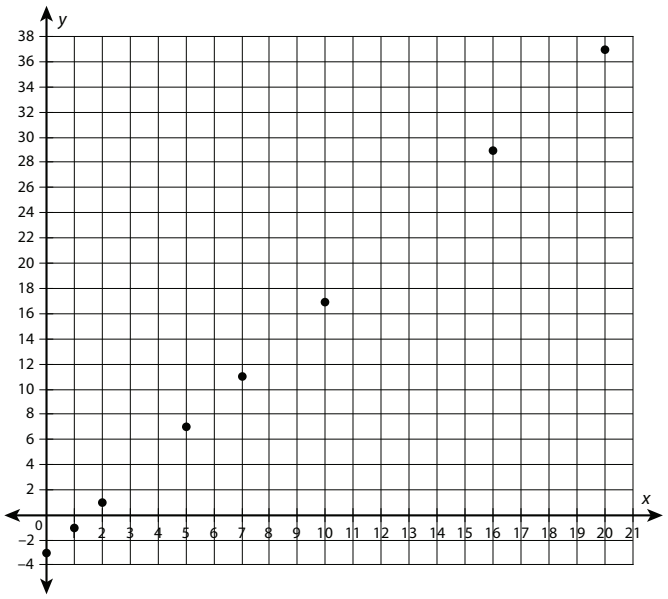
S–ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

c. Fit a linear function for a scatter plot that suggests a linear association.

Warm-Up 4.2.4 Debrief

1. Plot the points on a scatter plot.

   Plot each pair of temperatures in the form (x, y).



2. Describe the shape of the points.

   The points appear to lie on a straight line. The points are linear.

3. Suppose x represents the number of students in a class, and y represents the number of index cards a teacher needs to purchase for the class. Every student needs 8 index cards, and the teacher wants 2 extra cards. This can be modeled by the function $y = 8x + 2$. What is the slope of the line that models this scenario?

   For a line in the form $y = mx + b$, m is the slope and b is the y-intercept. The slope of the line is 8.

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4. What is the graph of the equation $y = -x + 1$?

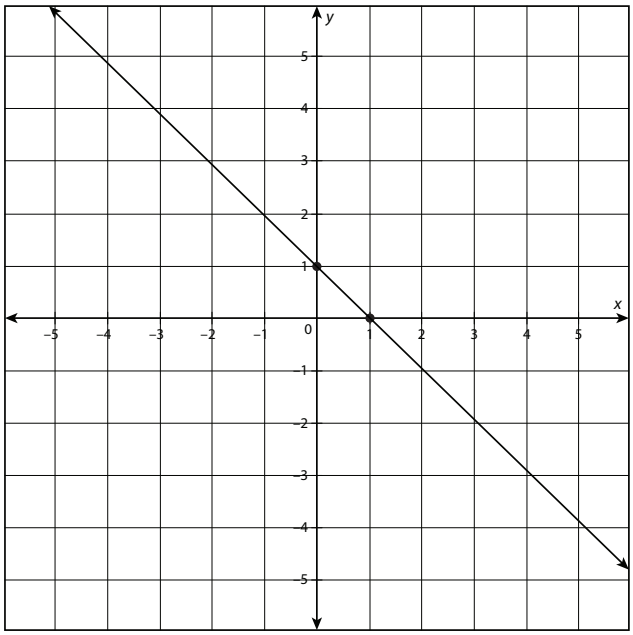
   The equation is in the form $y = mx + b$, so the graph will be a line. To graph a line, find two points on the line. Evaluate the function at two values of x. Easy values of x to use are 0 and 1.

   $y = -(0) + 1 = 1$ Substitute 0 for x.

   $y = -(1) + 1 = 0$ Substitute 1 for x.

   Two points on the line are (0, 1) and (1, 0).

   Graph the two points and draw a line through them.



Connection to the Lesson

- In this lesson, students will recognize that a set of points in a scatter plot can be approximated with a linear function.
- Students will need to understand that the general shape of a linear function is a line.
- Students will make connections between linear graphs and equations, and will need to understand how the slope of a line relates to the equation of the line.
- Students will also need to know how the equation of a line relates to the graph of a line, and will be finding equations of lines based on two points.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- plotting points on a coordinate plane (5.G.1)
- creating the graph of a line given the equation of the line (A–CED.2★)
- finding the equation of a line using two points on the line (8.EE.6)
- relating the slope and *y*-intercept of a line to an equation in slope-intercept form (8.EE.6)
- understanding that the general shape of the graph of a linear function is a line (8.F.3)

Introduction

The relationship between two variables can be approximated using a function. The function's equation can be used to estimate values that are not in the observed data set. To determine which type of equation should be used for a data set, first create a scatter plot of the data. Data that has a linear shape, or can be approximated by a straight line, can be fitted to a linear equation.

To draw a line of best fit, use a ruler to draw a line that passes through the data points as closely as possible, with some points below the line and some points above the line, minimizing the residuals. After creating a graphical representation of a line that fits the data, find the equation of this line using two points on the line. Use the two points to calculate the slope and *y*-intercept of the line.

Key Concepts

- A scatter plot that can be estimated with a linear function will look approximately like a line.
- A line that passes through the points in the scatter plot as closely as possible can be used to find a linear function that fits the data.
- If a line is a good fit for a data set, some of the data points will be above the line and some will be below the line.
- The general equation of a line in slope-intercept form is $y = mx + b$, where m is the slope and b is the *y*-intercept.
- To find the equation of a line with two known points, use the points to calculate the slope and *y*-intercept, then write the equation in slope-intercept form.
- Slope is the change in *y* divided by the change in *x*. To calculate the slope of a line through the points (x_1, y_1) and (x_2, y_2), use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

SRB p. U4-82

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Instruction

- To find the *y*-intercept, or *b* in the equation $y = mx + b$, replace *m* with the calculated slope, and replace *x* and *y* with values of *x* and *y* from a point on the line. Then solve the equation for *b*.
- For example, for a line with a slope of 2 containing the point (1, –3), $m = 2$, $y = -3$, and $x = 1$; $-3 = (2)(1) + b$, so $-5 = b$.

Common Errors/Misconceptions

- thinking that a line is a good estimate for data that is not linear
- drawing a line that is not a good fit for the data, and calculating the equation of this line
- miscalculating the slope of a line using two points on the line
- incorrectly calculating the *y*-intercept when finding the equation of a line given a graph

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Instruction

Guided Practice 4.2.4

Example 1

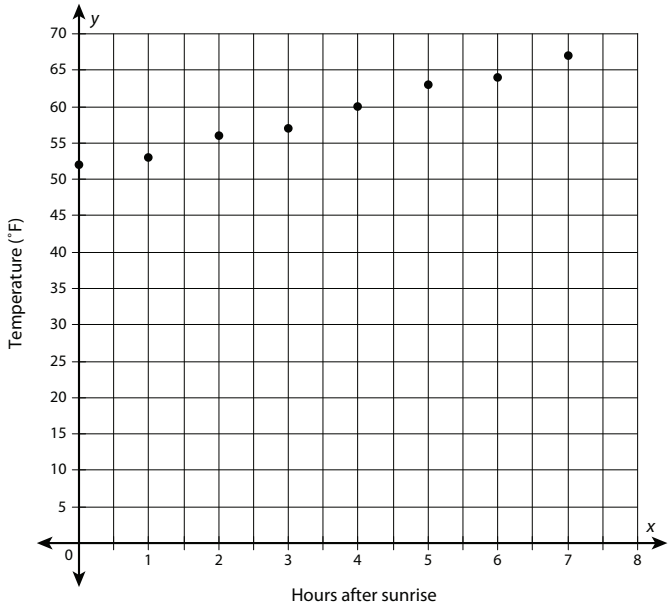
A weather team records the weather each hour after sunrise one morning in May. The hours after sunrise and the temperature in degrees Fahrenheit are in the following table.

Hours after sunrise	Temperature in °F
0	52
1	53
2	56
3	57
4	60
5	63
6	64
7	67

Can the temperature between 0 and 7 hours after sunrise be represented by a linear function? If yes, find the equation of the function.

1. Create a scatter plot of the data.

   Let the *x*-axis represent the hours after sunrise and the *y*-axis represent the temperature in degrees Fahrenheit.



SWB p. 83; SRB p. U4-83

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Instruction

2. Determine if the data can be represented by a linear function.

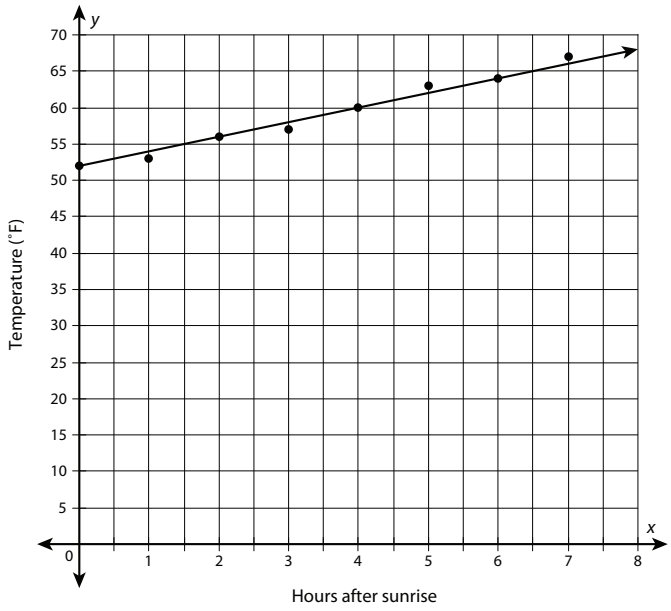
   The graph of a linear equation is a line. If the data looks like it could fit a line, then a linear equation could be used to represent the data.

   The data points appear to form a line, so a linear equation could be used to represent the data set.

3. Draw a line to estimate the data set.

   Draw a line that approximates that data. Some of the data values should be above the line, and some should be below the line.

   Choose two points on the line. It is best to choose points whose coordinates you can determine using the grid lines of the graph, and that are as far apart as possible. Two such points that the line appears to pass through are the points (1.5, 55) and (6.5, 65).



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Instruction

4. Find the equation of the line.

The general equation of a line in slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.

Find the slope, m, of the line through the two chosen points. The slope m is $\frac{\text{change in } y}{\text{change in } x}$. For any two points (x_1, y_1) and (x_2, y_2), $m = \frac{y_2 - y_1}{x_2 - x_1}$.

For the two points (1.5, 55) and (6.5, 65), the slope is $m = \frac{65 - 55}{6.5 - 1.5} = 2$.

Next, find the y-intercept, b. Use the general equation of a line to solve for b. Substitute x and y from a known point on the line, and replace m with the calculated slope.

$y = mx + b$

For the point (1.5, 55): $(55) = 2(1.5) + b$; $b = 52$

Replace m and b with the calculated values in the general equation of a line.

$y = 2x + 52$

The temperature between 0 and 7 hours after sunrise can be approximated with the equation $y = 2x + 52$.



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Instruction

Example 2

To learn more about the performance of an engine, engineers conduct tests and record the time it takes the car to reach certain speeds. A stopped car accelerates to 75 miles per hour. The following table shows the time, in seconds, after the car starts to accelerate and the speed it reaches at each time.

Time in seconds	Speed in miles per hour
0	0
1	2.3
2	6.6
3	13.5
4	22.4
5	32.2
6	44.2
7	57.8
8	74.6

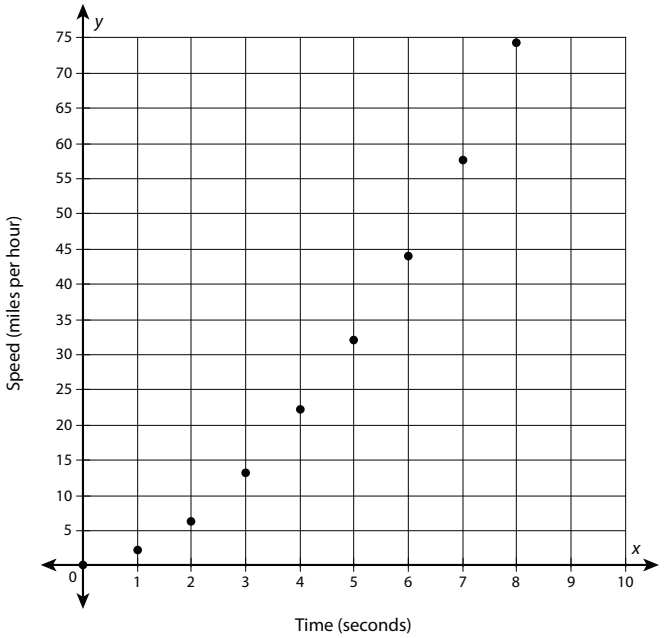
Can the speed between 0 and 8 seconds be represented by a linear function? If yes, find the equation of the function.

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Instruction

1. Create a scatter plot of the data set.

   Let the *x*-axis represent the time, in seconds, and the *y*-axis represent the speed in miles.



2. Determine if the data can be represented by a linear function.

   The *x*-values of each point are increasing by 1 unit. The *y*-values of each point are increasing by greater amounts as *x* gets larger. The *y*-values (speed) of the first two points are close together, but the last two points show a large change in the speed. A curved graph has been created.

   This data should not be approximated using a line, and therefore should not be represented by a linear equation.

   No, the speed should not be represented by a linear equation.

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Instruction

Example 3

Automated tractors can mow lawns without being driven by a person. A company runs trials using fields of different sizes, and records the amount of time it takes the tractor to mow each field. The field sizes are measured in acres.

Acres	Time in hours
5	15
7	10
10	22
17	32.3
18	46.8
20	34
22	39.6
25	75
30	70
40	112

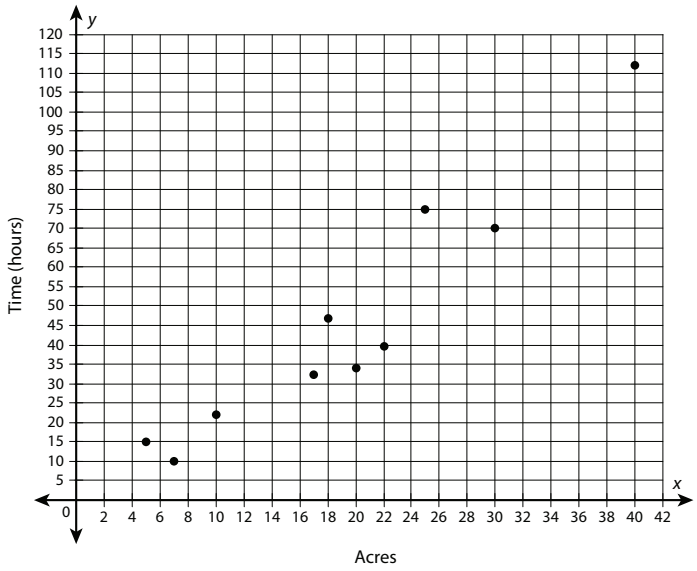
Can the time to mow acres of a field be represented by a linear function? If yes, find the equation of the function.

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Instruction

1. Create a scatter plot of the data.

   Let the *x*-axis represent the number of acres and the *y*-axis represent the time in hours.



2. Determine if the data can be represented by a linear function.

   The graph of a linear equation is a line. If the data looks like it could fit a line, then a linear equation could be used to represent the data.

   The data points appear to form a line, so a linear equation can be used to represent the data set.

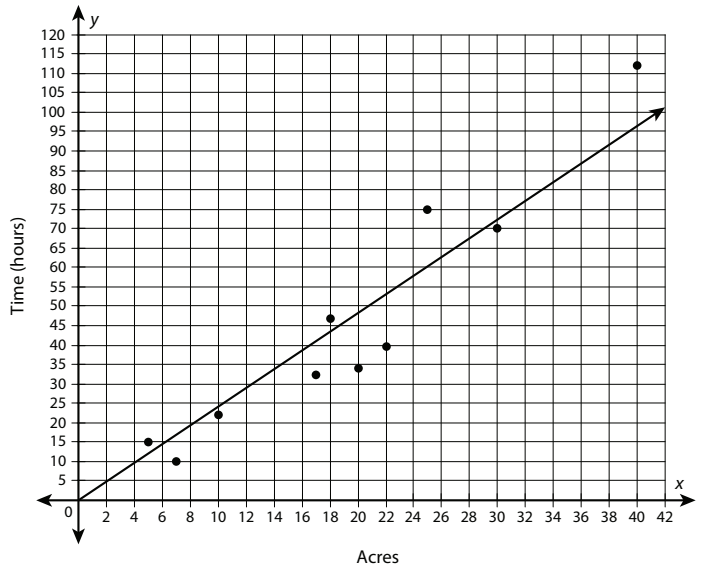
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Instruction

3. Draw a line to estimate the data set.

   Draw a line that estimates the data. Some of the data values should be above the line, and some should be below the line. In addition, because 0 acres are mown in 0 hours, it is a good idea to have the line of best fit pass through the origin.

   A line through (0, 0) and (25, 60) looks like a good fit for the data.



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Instruction

4. Find the equation of the line.

The general equation of a line in slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.

Find the slope, m, of the line through the two chosen points. The slope m is $\frac{\text{change in } y}{\text{change in } x}$. For any two points (x_1, y_1) and (x_2, y_2), $m = \frac{y_2 - y_1}{x_2 - x_1}$.

For the two points (0, 0) and (25, 60), the slope is $m = \frac{60 - 0}{25 - 0} = 2.4$.

Next, find the y-intercept, b. This line passes through the origin, so $b = 0$.

Replace m and b with the calculated values in the general equation of a line.

$y = 2.4x + 0$

The amount of time to mow the acres of a field can be represented using the equation $y = 2.4x$.



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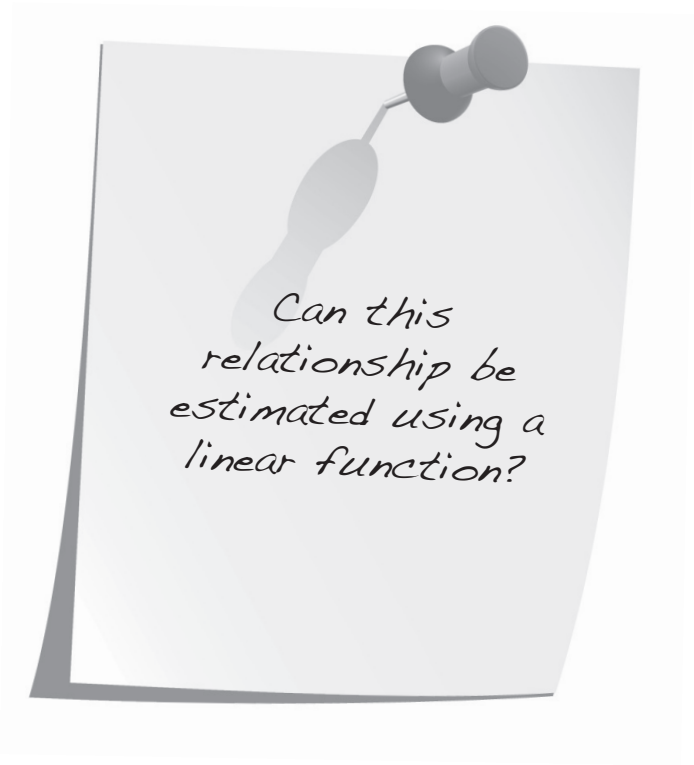
Problem-Based Task 4.2.4: Lion Cub Births

SMP	
1	2
3	4 ✓
5 ✓	6
7	8

A zoologist studies different prides, or groups of lions, living throughout Africa. He records the number of adult females in each pride, and the number of newborn cubs. His results are in the table that follows.

Adult females	Cubs
6	5
13	7
7	6
17	9
14	7
3	1
10	6
7	4
4	3
15	8
8	5
3	0
13	8
12	7
11	7
14	9
6	4

The zoologist would like to use this information to estimate the number of cubs born each year. He would like an equation that relates the number of adult females to the number of newborn cubs. Can this relationship be estimated using a linear function? If yes, find the equation of the function.



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Problem-Based Task 4.2.4: Lion Cub Births

Coaching

a. Create a scatter plot of the data set.

b. What is the shape of the graph?

c. Can this data set be estimated using the graph of a linear function?

d. Draw a line to estimate the data set.

e. Find the equation of the line.

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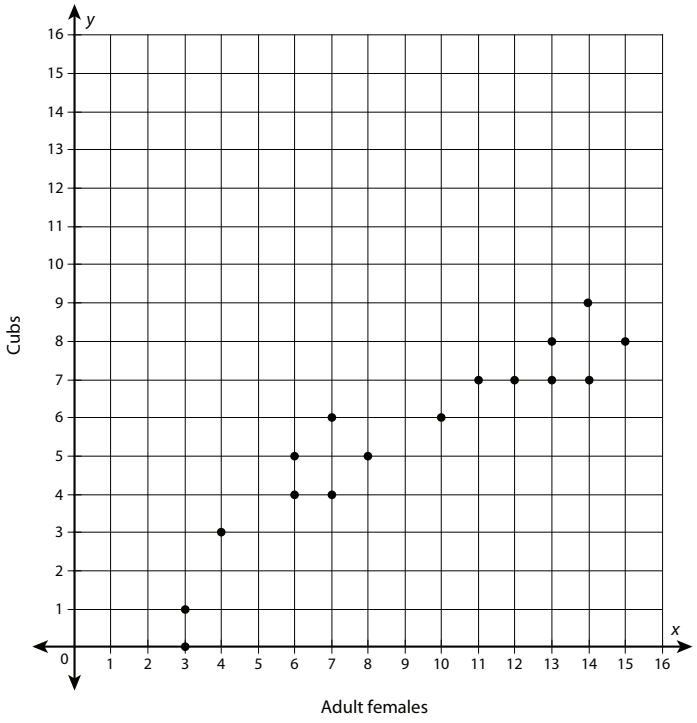
Instruction

Problem-Based Task 4.2.4: Lion Cub Births

Coaching Sample Responses

a. Create a scatter plot of the data set.

Let the *x*-axis represent the number of adult female lions, and the *y*-axis represent the number of cubs.



b. What is the shape of the graph?

The data seems to show that as the *x*-values increase, the *y*-values increase at a steady rate. The data points appear linear.

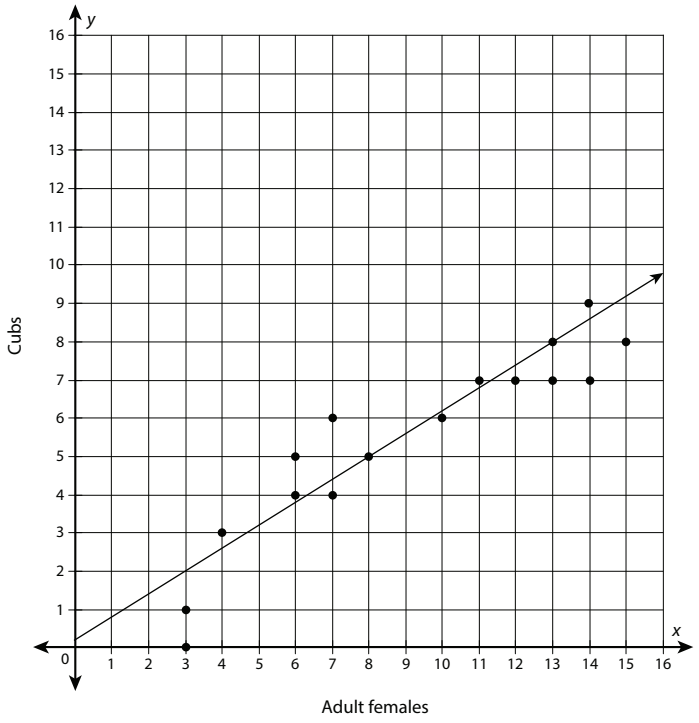
c. Can this data set be estimated using the graph of a linear function?

The graph of a linear function is a line. Since the data appears to follow a line, the data can be approximated using a linear function.

d. Draw a line to estimate the data set.

Draw a line that approximates that data. Some of the data values should be above the line, and some should be below the line. A line through (3, 2) and (13, 8) looks like a good fit for the data.

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e. Find the equation of the line.

The general equation of a line in slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept. Find the slope, m, of the line through the two chosen points.

The slope m is $\frac{\text{change in } y}{\text{change in } x}$. For any two points (x_1, y_1) and (x_2, y_2), $m = \frac{y_2 - y_1}{x_2 - x_1}$.

For the two points (3, 2) and (13, 8), $m = \frac{8-2}{13-3} = 0.6$.

Next, find the y-intercept, b. Use the general equation of a line to solve for b. Substitute x and y from a known point on the line, and replace m with the calculated slope.

$y = mx + b$

For the point (3, 2): $(2) = 0.6(3) + b$; $b = 0.2$

Replace m and b with the calculated values in the general equation of a line.

$y = 0.6x + 0.2$

The equation of the line is $y = 0.6x + 0.2$.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 4.2.4: Fitting Linear Functions to Data

Each of Mrs. Jackson's students records the number of hours he or she studied for a recent quiz. Mrs. Jackson then compared this time to the score earned by each student. Her data is in the following table. Use the data for problems 1–4.

Time studied, in hours	Score earned, out of 100
4.5	90
2.5	69
3	70
5	85
1	43
4.5	85
3.5	73
5	98
5	100
2	46
1.5	56
1.5	48
4	69

1. Create a scatter plot of the data set.

2. Describe the shape of the data.

3. Draw a line to estimate the data set.

4. Find the equation of the line that estimates the relationship between the hours studied and the score earned.

continued

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A clothing store manager conducts research on how many articles of clothing each customer purchases. The manager is trying to understand if there is a relationship between the number of items tried on in the dressing room and the number of items purchased. The data for 10 customers is in the following table. Use the data for problems 5 and 6.

Number of items tried on	Number of items purchased
12	8
14	0
1	4
2	5
13	2
12	11
14	7
1	3
0	4
0	1

5. Create a scatter plot of the data set. Describe the shape of the data.

6. Can the data be represented using a linear equation? If yes, find the equation. If no, explain why not.

continued

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To learn more about the relationship between years of schooling and yearly income, a company surveys 20 people with jobs. Each person identifies the number of years he or she attended school and his or her current yearly income. Use the data for problems 7 and 8.

Years of schooling	Income in dollars ($)
11	16,000
19	91,500
10	20,000
10	19,500
15	49,000
9	10,000
10	13,000
11	18,500
18	81,000
15	49,500
17	78,000
17	69,500
12	26,000
8	5,000
14	51,000
16	69,000
18	88,000
20	91,500
16	72,500
19	77,500

7. Create a scatter plot of the data set, and draw a line to fit the data.

8. Find the equation to estimate the relationship between years of schooling and income.

continued

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The following table shows the cost of lunch at a high school each year for the last 10 years. Use the data for problems 9 and 10.

Year	Cost of lunch, in dollars ($)
1	1.10
2	1.20
3	1.31
4	1.43
5	1.54
6	1.64
7	1.75
8	1.89
9	2.01
10	2.12

9. Create a scatter plot of the data set, and draw a line to fit the data.

10. Find the equation to estimate the relationship between the year and the cost of lunch.

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Progress Assessment

Circle the letter of the best answer.

1. Vince asks baseball players how many years they have played baseball. He separates the responses by school grade and records them in the following table. What is the joint frequency of ninth graders who have played for 4 years?

Grade	Years playing baseball				
	4	**5**	**6**	**7**	**8**
8th grade	9	7	4	1	1
9th grade	6	8	5	3	2
10th grade	2	4	10	8	3
11th grade	1	0	7	14	5
12th grade	0	1	2	9	10

a. 1
b. 2
c. 6
d. 9

2. Grace asks her friends which car they prefer. She separates the responses by gender and lists them in the following table. What is the marginal frequency of males?

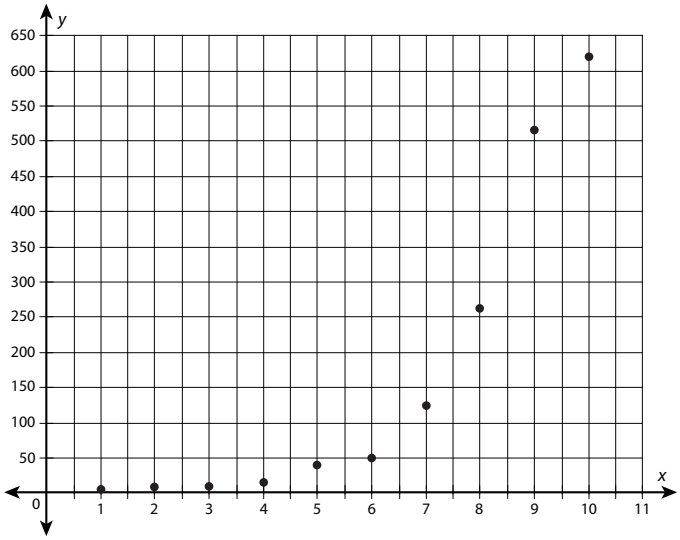
Gender	Preferred car		
	Car 1	**Car 2**	**Car 3**
Male	14	13	10
Female	8	15	12

a. 14
b. 22
c. 35
d. 37

continued

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3. Which equation could be used to approximate the data in the scatter plot?



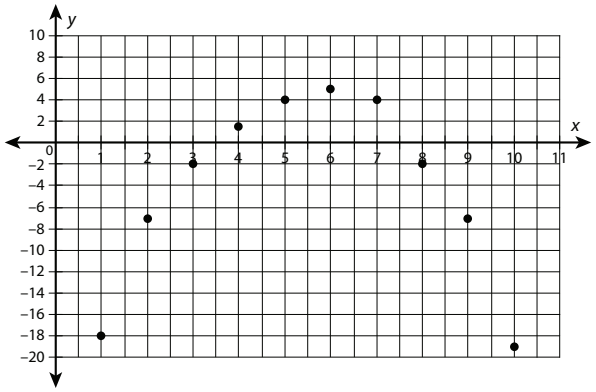
a. $y = 2^x$

b. $y = 2x$

c. $y = \frac{1}{2}x$

d. $y = \left(\frac{1}{2}\right)^x$

4. Malia creates a residual plot to analyze a linear function fitted to data. The plot is shown. What does the plot tell her about her line fitted to the data?



a. A linear function is not a good fit for the data.

b. A linear function is a good fit for the data.

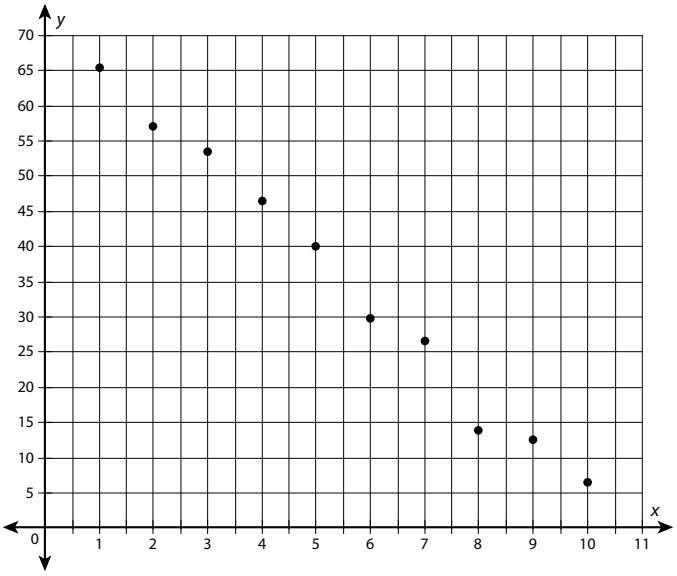
c. A quadratic function is not a good fit for the data.

d. An exponential function is not a good fit for the data.

continued

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5. Which equation could be used to approximate the data in the scatter plot?



a. $y = (-6.8)^x + 72$

b. $y = -6.8x + 72$

c. $y = 6.8^x + 72$

d. $y = 6.8x + 72$

6. Ms. Ward records the field trip preferences of students, separated by age, in the following table. What is the joint frequency of 14-year-olds who prefer the movie theater?

Age	Field trip preference		
	Water park	Movie theater	Sports game
14 years old	8	19	5
15 years old	3	5	20
16 years old	2	15	3

a. 8

b. 5

c. 15

d. 19

continued

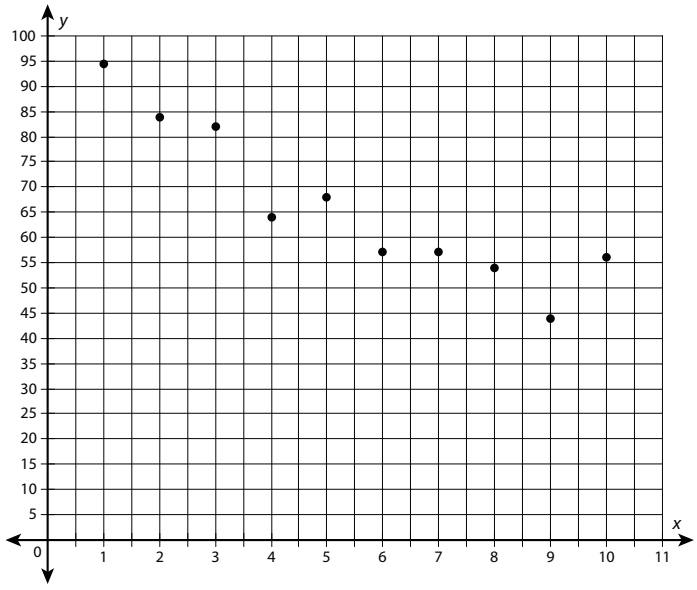
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7. Dylan records whether people prefer to drink coffee, tea, water, or orange juice in the morning. He separates the responses by which of three eastern United States regions they live in: the Northeast, the Mid-Atlantic, or the South. What is the conditional frequency of people from the Northeast who prefer coffee, relative to the total number of respondents?

Region	Preferred morning drink			
	Coffee	Tea	Water	Orange juice
Northeast	28	24	10	2
Mid-Atlantic	20	31	7	9
South	15	29	6	12

a. 0.10
b. 0.15
c. 0.33
d. 0.44

8. Which equation could be used to approximate the data in the scatter plot?

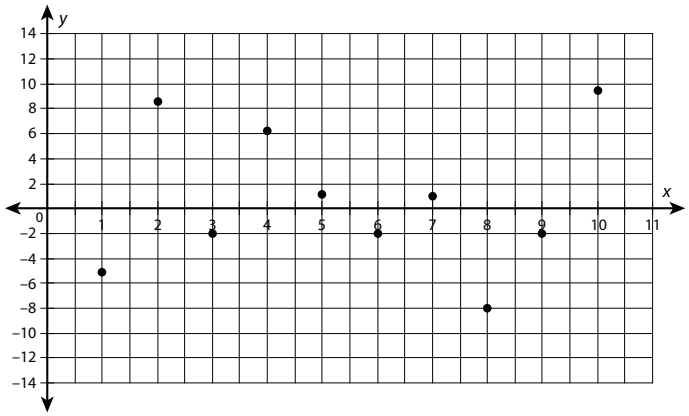


a. $y=(-5)^x+93$
b. $y=5x+93$
c. $y=5^x+93$
d. $y=-5x+93$

continued

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9. Liam creates a residual plot to analyze a linear function fitted to data. The plot is shown. What does the plot tell him about his line fitted to the data?



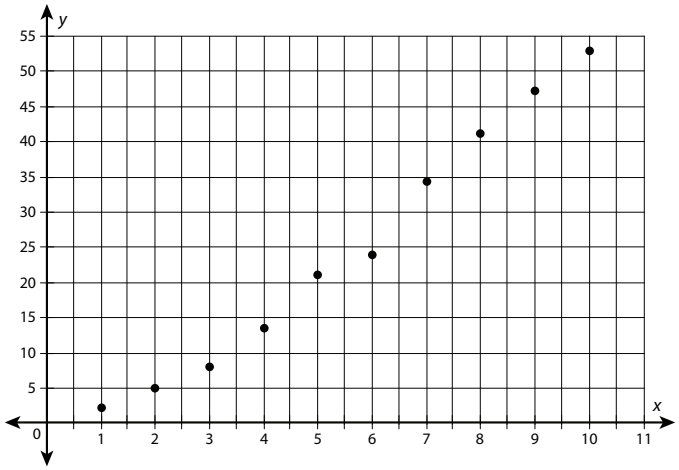
a. A linear function is not a good fit for the data.

b. A linear function is a good fit for the data.

c. A quadratic function is a good fit for the data.

d. An exponential function is a good fit for the data.

10. Which equation could be used to approximate the data in the scatter plot?



a. $y=(-6)^x-9$

b. $y=6^x-9$

c. $y=6x-9$

d. $y=-6x-9$

continued

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Assessment

Read the given information, then follow the directions to complete problem 11.

11. Isaiah uses the equation $y = 12x + 1$ to estimate the data in the set shown in the following table. Create a residual plot for the data set.

x	y
1	12
2	25
3	36
4	47
5	58
6	78
7	83
8	93
9	117
10	113

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Pre-Assessment

Circle the letter of the best answer.

1. Sam tracks the growth of a plant, and records its height in centimeters each week. He determines that the equation $y = 2.3x + 16$ can be used to estimate the plant's height for any week. Which statement is true based on Sam's equation?

   a. The plant grows approximately 16 centimeters each week.

   b. The starting height of the plant is approximately 16 centimeters.

   c. The starting height of the plant is approximately 2.3 centimeters.

   d. The plant did not grow during the time Sam tracked its height.

2. Isabella makes deposits to her savings account each month, and she also earns interest. She records the amount of money in her savings account each month, and finds that the equation $y = 218x + 100$ can be used to estimate the dollars in her savings account for any month. Which statement is true based on Isabella's equation?

   a. She started her account with approximately $218.

   b. The amount of money in her account increases by approximately $100 each month.

   c. The amount of money in her account increases by approximately $218 each month.

   d. Isabella takes approximately $218 out of her account each month.

continued

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3. What is the correlation coefficient, *r*, of the data in the table? Use technology to calculate *r*.

x	*y*
3	35
1	14
6	67
2	27
2	15
2	13
8	35
7	76
4	52
4	51
6	57
2	18

a. 0.598
b. 0.773
c. 7.321
d. 38.33

4. A data set has a correlation coefficient of –0.916. Which statement about the data set is true?
   a. The data has a strong positive linear correlation.
   b. The data has a weak positive linear correlation.
   c. The data has a weak negative linear correlation.
   d. The data has a strong negative linear correlation.

5. Event *x* and event *y* have a strong negative linear correlation. Which statement do you know is true about events *x* and *y*?
   a. If *x* increases, *y* decreases.
   b. If *x* increases, *y* increases.
   c. If *x* increases, it is unknown how *y* will change.
   d. Event *x* is responsible for the change in *y*.

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Lesson 3: Interpreting Linear Models

Instruction

Common Core State Standards

SMP	
1 ✓	2
3 ✓	4 ✓
5 ✓	6
7	8

S–ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★

S–ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★

S–ID.9 Distinguish between correlation and causation.★

Essential Questions

1. How can you use units to help interpret the slope and *y*-intercept of a line fitted to data?
2. What is the relationship between the correlation coefficient and the correlation of two events?
3. What is the difference between correlation and causation?

WORDS TO KNOW

causation a relationship between two events where a change in one event is responsible for a change in the second event

correlation a relationship between two events, where a change in one event is related to a change in the second event. A correlation between two events does not imply that the first event is responsible for the change in the second event; the correlation only shows how likely it is that a change also took place in the second event.

correlation coefficient a quantity that assesses the strength of a linear relationship between two variables, ranging from –1 to 1; a correlation coefficient close to –1 indicates a strong negative correlation, a correlation coefficient close to 1 indicates a strong positive correlation, and a correlation coefficient of 0 indicates a very weak or no linear correlation

linear fit (or linear model) an approximation of data using a linear function

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slope	the measure of the rate of change of one variable with respect to another variable; $\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$; the slope in the equation $y = mx + b$ is m.
***y*-intercept**	the y-coordinate of the point where a line or a curve intersects the y-axis

Recommended Resources

- Illuminations. "Linear Regression I."

  http://walch.com/rr/CAU4L3CorrelationCoefficient

  This site allows users to plot points on a coordinate plane. The applet then generates the line of best fit and displays the correlation coefficient. *Note*: Requires Java.

- Office for Mathematics, Science, and Technology Education, University of Illinois. "Linear Regression."

  http://walch.com/rr/CAU4L3LinearRegression

  This applet generates a theoretical line of best fit for points entered by the user. Users then manipulate a slider to change the slope of another line to try to match the theoretical line of best fit. A thermometer shows if the user's line is a good fit.

- wiseGEEK.com. "What Is the Difference Between Cause and Correlation?"

  http://walch.com/rr/CAU4L3CauseVsCorrelation

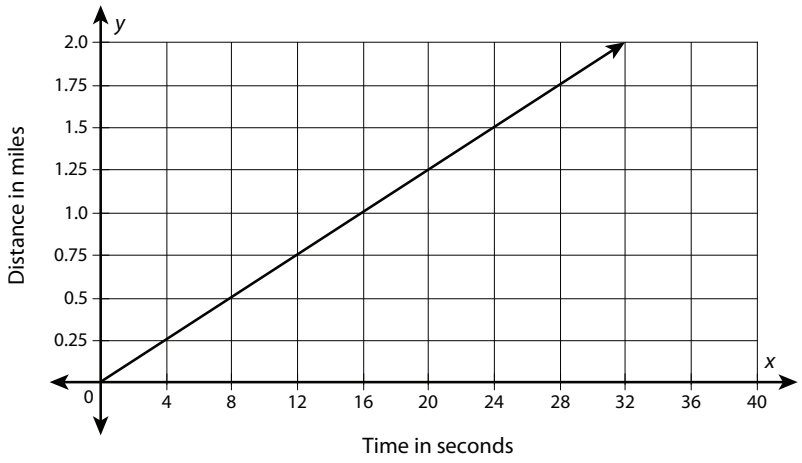
  This site offers an explanation of the difference between causation and correlation.

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Lesson 4.3.1: Interpreting Slope and *y*-intercept

Warm-Up 4.3.1

A top fuel dragster (a car built for drag racing) can travel $\frac{1}{4}$ mile in 4 seconds. The dragster's distance over time is graphed as shown. The graph assumes a constant speed. Use the graph to complete problems 1 and 2.



1. Find the slope and *y*-intercept of the function shown in the graph.

2. Write the algebraic equation of the line.

Use what you know about slope-intercept form to answer questions 3 and 4.

3. What is the slope of a line with the equation $y = -x + 7$?

4. What is the *y*-intercept of a line with the equation $y = 3x - 2$?

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Instruction

Lesson 4.3.1: Interpreting Slope and *y*-intercept

Common Core State Standard

S–ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★

Warm-Up 4.3.1 Debrief

1. Find the slope and *y*-intercept of the function shown in the graph.

   The slope is $m = \frac{\Delta y}{\Delta x}$ or $m = \frac{\text{change in } y}{\text{change in } x}$. To calculate the slope, find any two points on the line.

   The graph shows that (0, 0) and (16, 1) are both points on the line. The formula to find the slope between two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Substitute (0, 0) and (16, 1) into the formula to find the slope.

   $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{16 - 0} = \frac{1}{16}$$

   The slope between the two points (0, 0) and (16, 1) is $\frac{1}{16}$.

   The *y*-intercept is the point at which the graph crosses the *y*-axis. The graph shows that the *y*-intercept is 0, or (0, 0).

2. Write the algebraic equation of the line.

   The equation of a line can be written in the form $y = mx + b$, where m is the slope of the line and b is the *y*-intercept. The equation of the line is $y = \frac{1}{16}x + 0$ or $y = \frac{1}{16}x$.

3. What is the slope of a line with the equation $y = -x + 7$?

   If the equation of a line is in the form $y = mx + b$, m is the slope of the line.

   The slope of the line $y = -x + 7$ is –1.

4. What is the *y*-intercept of a line with the equation $y = 3x - 2$?

   If the equation of a line is in the form $y = mx + b$, then b is the *y*-intercept.

   The *y*-intercept of $y = 3x - 2$ is –2.

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Instruction

Connection to the Lesson

- In this lesson, students will need to know how to determine the slope and *y*-intercept of a linear function using both graphical and algebraic representations.
- This warm-up will remind students how to determine both slope and *y*-intercept using either representation.
- Students will interpret these values in relation to the real-world model the linear function represents.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- creating a scatter plot given data in a table (8.SP.1)
- finding a linear fit given a scatter plot (S–ID.6c★)
- understanding the connection between a graph and an equation of a linear function (8.F.3)
- determining the slope and *y*-intercept of a linear function given a graph or an equation (8.F.4)

Introduction

When linear functions are used to model real-world relationships, the slope and *y*-intercept of the linear function can be interpreted in context. Recall that data in a scatter plot can be approximated using a linear fit, or linear function that models real-world relationships. A **linear fit** is the approximation of data using a linear function.

The **slope** of a linear function is the change in the dependent variable divided by the change in the independent variable. When written as a decimal, the slope describes how much *y* changes when *x* changes by 1. When analyzing the slope in the context of a real-world situation, remember to use the units of *x* and *y* in the calculation of the slope. For example, if the *x*-axis of a graph represents hours and the *y*-axis represents miles traveled, the slope of a linear function graphed on these axes would be $\frac{\text{change in miles}}{\text{change in hours}}$, or the miles traveled each hour.

The ***y*-intercept** of a function is the value of *y* at which the graph of the function intersects the *y*-axis, or the value of *y* when *x* equals 0. When analyzing the *y*-intercept in a real-world context, it is the initial value of whatever quantity is represented by the *y*-axis. For example, if the *x*-axis represents hours and the *y*-axis represents miles traveled, the *y*-intercept would be the miles traveled when the number of hours equals 0. The *y*-intercept in the equation $y = mx + b$ is b.

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Instruction

Key Concepts

- The slope of a line with the equation $y = mx + b$ is m, and the y-intercept of a line with the equation $y = mx + b$ is b.
- The slope of a line is $m = \frac{\text{change in } y}{\text{change in } x}$; the slope between two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- In context, when written as a decimal, the slope describes how much the dependent variable changes each time the independent variable changes by 1 unit.
- In context, the y-intercept is the initial value of the quantity represented by the y-axis, or the quantity of y when $x = 0$.

Common Errors/Misconceptions

- incorrectly calculating the slope
- confusing the y- and x-intercepts, both in context and when calculating using a graph or equation

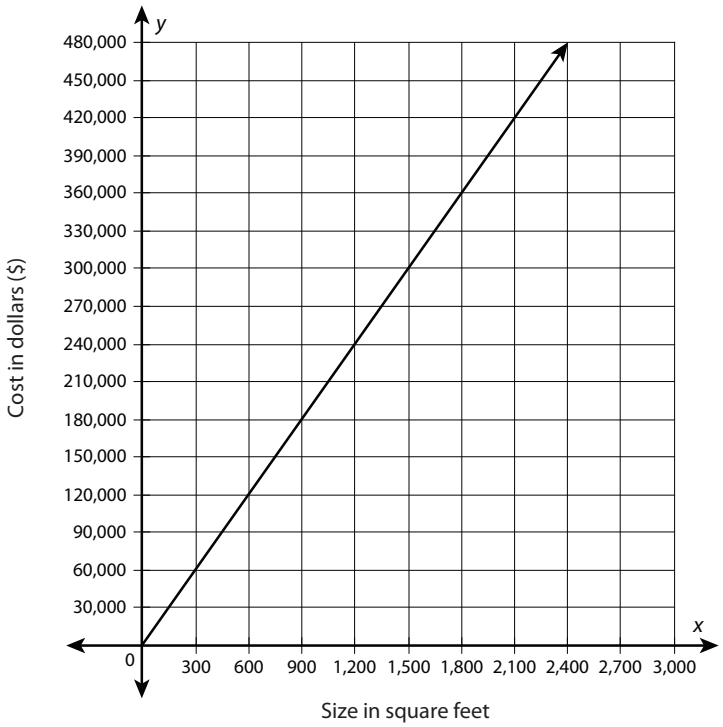
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Instruction

Guided Practice 4.3.1

Example 1

The following graph shows a linear model that approximates the relationship between the size of a home and how much it costs. The *x*-axis represents size in square feet, and the *y*-axis represents cost in dollars. Calculate the slope and the *y*-intercept of the linear model, and describe what these values mean in terms of housing prices.



SWB p. 97; SRB p. U4-100

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Instruction

1. Find the equation of the linear fit.

The general equation of a line in slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.

Find two points on the line using the graph.

The graph contains the points (300, 60,000) and (600, 120,000).

The formula to find the slope between two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Substitute (300, 60,000) and (600, 120,000) into the formula to find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$= \frac{120{,}000 - 60{,}000}{600 - 300}$	Substitute (300, 60,000) and (600, 120,000) for (x_1, y_1) and (x_2, y_2).
$= \frac{60{,}000}{300} = 200$	Divide.

The slope between the two points (300, 60,000) and (600, 120,000) is 200.

Find the y-intercept. Use the equation for slope-intercept form, $y = mx + b$, where b is the y-intercept.

Replace x and y with values from a single point on the line. Let's use (300, 60,000).

Replace m with the slope, 200. Solve for b.

$y = mx + b$	Equation for slope-intercept form
$(60{,}000) = (200)(300) + b$	Substitute values for x, y, and m.
$60{,}000 = 60{,}000 + b$	Multiply.
$0 = b$	Subtract 60,000 from both sides.

The y-intercept of the linear model is 0.

The equation of the line is $y = 200x$.

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Instruction

2. Determine the units of the slope.

   Divide the units on the y-axis by the units on the x-axis: $\frac{\text{dollars}}{\text{square feet}}$.

   The units of the slope are dollars per square foot.

3. Describe what the slope means in context.

   The slope is the change in cost of a home for each square foot of the home. The slope describes how price is affected by the size of the home purchased. A positive slope means the quantity represented by the y-axis increases when the quantity represented by the x-axis also increases.

   The cost of a home increases by $200 for each additional square foot of space in the home.

4. Determine the units of the y-intercept.

   The units of the y-intercept are the units of the y-axis: dollars.

5. Describe what the y-intercept means in context.

   The y-intercept is the value of the equation when $x = 0$, or when the size of the home is 0 square feet. For a home with no area, or for no home, the cost is $0.

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Instruction

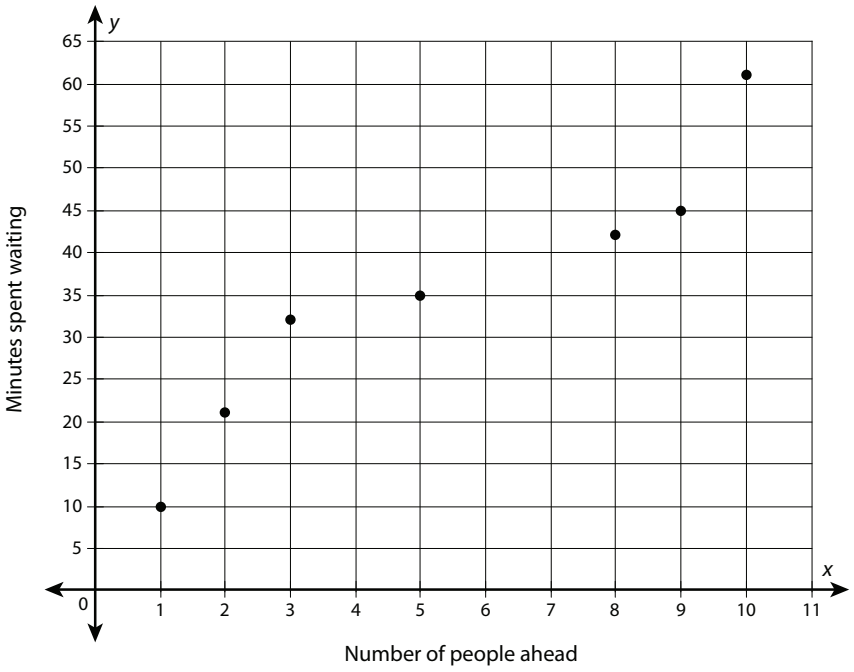
Example 2

A teller at a bank records the amount of time a customer waits in line and the number of people in line ahead of that customer when he or she entered the line. Describe the relationship between waiting time and the people ahead of a customer when the customer enters a line.

People ahead of customer	Minutes waiting
1	10
2	21
3	32
5	35
8	42
9	45
10	61

1. Create a scatter plot of the data.

   Let the *x*-axis represent the number of people ahead of the customer and the *y*-axis represent the minutes spent waiting.



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Instruction

2. Find the equation of a linear model to represent the data.

A line that fits the data well should have approximately the same number of data points both above and below the line. A line through the data points (2, 20) and (8, 45) appears to be a good approximation of the data. Find the slope.

The slope between two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Substitute the points into the formula to find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$= \frac{45 - 20}{8 - 2}$	Substitute (2, 20) and (8, 45) for (x_1, y_1) and (x_2, y_2).
$= \frac{25}{6} \approx 4.2$	Divide.

The slope between the two points (2, 20) and (8, 45) is approximately 4.2.

Find the *y*-intercept. Use the equation for slope-intercept form, $y = mx + b$, where b is the *y*-intercept.

Replace x and y with values from a single point on the line. Let's use (2, 20).

Replace m with the slope, 4.2. Solve for b.

$y = mx + b$	Equation for slope-intercept form
$(20) = (4.2)(2) + b$	Substitute values for x, y, and m.
$20 = 8.4 + b$	Multiply.
$11.6 = b$	Subtract 8.4 from both sides.

The *y*-intercept of the linear model is approximately 11.6.

The equation of the line is $y = 4.2x + 11.6$.

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Instruction

3. Determine the units of the slope.

   Divide the units on the *y*-axis by the units on the *x*-axis:

$$\frac{\text{minutes spent waiting}}{\text{number of people ahead}} = \frac{\text{minutes}}{\text{person}}$$

   The units of the slope are minutes per person.

4. Describe what the slope means in context.

   The slope describes how the waiting time increases for each person in line ahead of the customer. A customer waits approximately 4.2 minutes for each person who is in line ahead of the customer.

5. Determine the units of the *y*-intercept.

   The units of the *y*-intercept are the units of the *y*-axis: minutes.

6. Describe what the *y*-intercept means in context.

   The *y*-intercept is the value of y when $x = 0$, and represents the minutes spent waiting when the number of people ahead of the customer is 0. The *y*-intercept is 11.6.

   In this context, this *y*-intercept doesn't make sense, because a customer who enters the line with no people ahead of him or her would have a wait of 0 minutes, not 11.6 minutes. This discrepancy can be explained by closer analysis of the data. It appears that the first three data points on the graph follow a slope that appears to align with the origin, indicating a 0-minute wait time if there are 0 customers ahead. However, the wait times generally seem to increase at a slower rate if there are more than 3 people in line, so the wait time for each additional person in line is less. This may be explained by the addition of another teller or by the tellers picking up their pace to accommodate the long line of customers.

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Instruction

Example 3

For hair that is 12 inches or longer, a hair salon charges for haircuts based on hair length according to the equation $y = 5x + 35$, where x is the number of inches longer than 12 inches (hair length – 12) and y is the cost in dollars. Describe what the slope and y-intercept mean in context.

1. Determine the units of the slope.

   Divide the units of the dependent variable, y, by the units of the independent variable, x:

   $$\frac{\text{cost in dollars}}{\text{hair length greater than 12 inches}} = \frac{\text{dollars}}{\text{inch}}$$

2. Describe what the slope means in context.

   The units of the slope are dollars per inch. The slope describes how the cost of the haircut increases for each inch of hair length greater than 12 inches. The slope of 5 means that the cost of the haircut increases by \$5 for every inch of hair length over 12 inches.

3. Determine the units of the y-intercept.

   The units of the y-intercept are the units of the dependent variable, y: dollars.

4. Describe what the y-intercept means in context.

   The y-intercept is the value of the equation when $x = 0$, or when hair length is not greater than 12 inches. The y-intercept is the cost of a haircut when a customer's hair is no longer than 12 inches. A haircut is \$35 if a customer's hair isn't longer than 12 inches.

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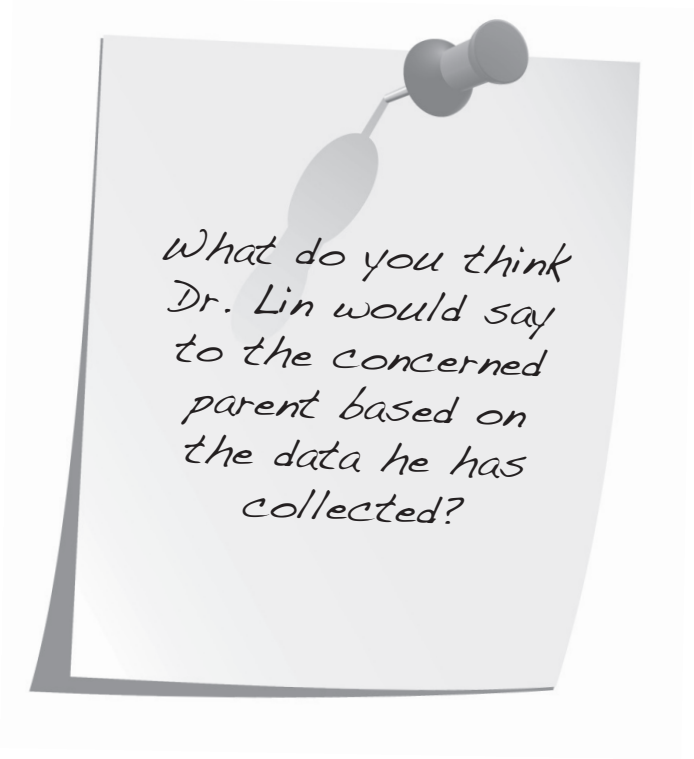
Problem-Based Task 4.3.1: Learning to Speak

SMP
1 ✓ 2
3 ✓ 4 ✓
5 6
7 8

Dr. Lin is a pediatrician. He tracks how a child's vocabulary increases when the child first starts speaking. He records the number of months a child has been speaking, and the number of words spoken each month. His data for three different children is in the following table.

Months speaking	Words spoken
0	5
0	2
0	2
1	32
1	41
1	43
2	82
2	94
2	77
3	96
3	99
3	132
4	170
4	122
4	160

One parent whose child was not involved in the study is concerned that her daughter isn't speaking enough words. When the child had been speaking for 3 months, she spoke 96 words, and now that the child has been speaking for 4 months, she speaks 144 words. What do you think Dr. Lin would say to the concerned parent based on the data he has collected?



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Problem-Based Task 4.3.1: Learning to Speak

Coaching

a. Create a scatter plot of the data.

b. Determine a line of best fit. What are two points on this line?

c. What are the units of the slope?

d. Use the units and the problem statement to describe what the slope means in context.

e. What is the rate of change for the vocabulary of the concerned parent's child?

f. How does the rate of the increase in the child's vocabulary from part e compare to the slope of the linear model?

g. What do you think Dr. Lin would say to the concerned parent based on the data he has collected?

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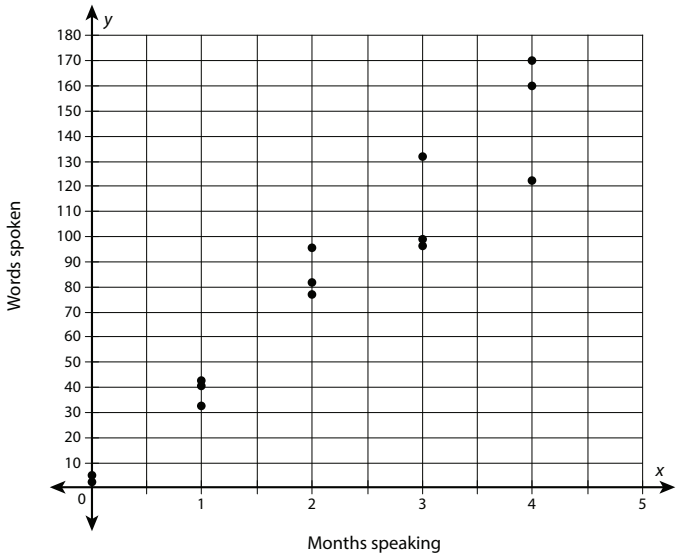
Instruction

Problem-Based Task 4.3.1: Learning to Speak

Coaching Sample Responses

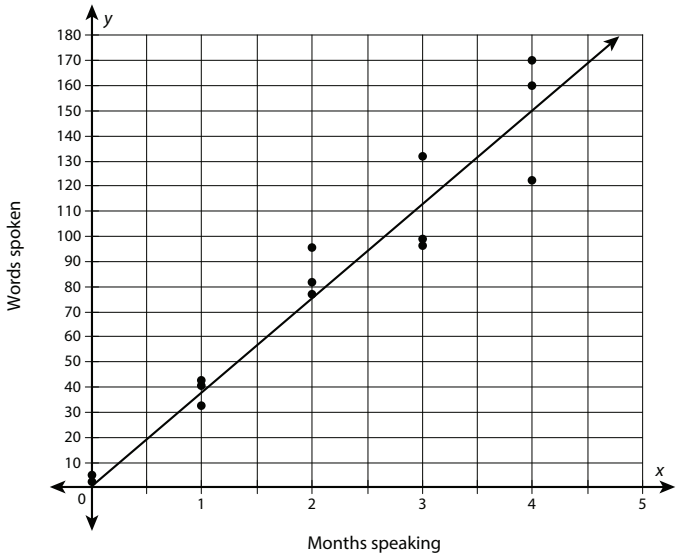
a. Create a scatter plot of the data.

Let the *x*-axis represent the number of months the child has been speaking and the *y*-axis represent the number of words spoken by the child.



b. Determine a line of best fit. What are two points on this line?

A line of best fit should have approximately the same number of data values both above and below the line. A line through the points (0, 0) and (4, 150) appears to be a good fit for the data. Draw the line on the scatter plot to see the fit.



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Instruction

The general equation of a line in slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept. The slope between two points (x_1, y_1) and (x_2, y_2) is found using $m = \frac{y_2 - y_1}{x_2 - x_1}$, so the slope between the two points (0, 0) and (4, 150) is $m = \frac{150 - 0}{4 - 0} = 37.5$.

Use the general equation for slope-intercept form, $y = mx + b$, to calculate the y-intercept, b. Replace x and y with values from a single point on the line, and replace m with the calculated value. Let's use (0, 0).

Solve for b.

$(0) = (37.5)(0) + b$ Substitute values for x, y, and m.

$0 = b$ Simplify as needed.

The equation of the line is $y = 37.5x + 0$, or simply $y = 37.5x$.

c. What are the units of the slope?

Slope is the change in y divided by the change in x. To find the units of the slope, divide the units of the dependent variable, y, by the units of the independent variable, x: $\frac{\text{words}}{\text{month}}$.

The slope is in words per month.

d. Use the units and the problem statement to describe what the slope means in context.

The slope of the equation describes the change in the number of words spoken each month. This is how many additional words a child speaks each month in the first four months after speaking his or her first words. By looking at the slope-intercept form of the equation found in part b, $m = 37.5$; thus, each month a child learns approximately 37 or 38 words.

e. What is the rate of change for the vocabulary of the concerned parent's child?

When the child had been speaking for 3 months, she spoke 96 words. After 4 months of speaking, she spoke 144 words. Substitute (3, 96) and (4, 144) for (x_1, y_1) and (x_2, y_2) into the formula for slope, $\frac{y_2 - y_1}{x_2 - x_1}$.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{144 - 96}{4 - 3} = 48$$

The rate of change for this child's vocabulary is 48 words per month.

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f. How does the rate of the increase in the child's vocabulary from part e compare to the slope of the linear model?

The slope in the linear model is 37.5 words per month. This child's rate is 48 words per month.

g. What do you think Dr. Lin would say to the concerned parent based on the data he has collected?

Even though the child's data points for these two months falls below the linear model, she is still increasing her vocabulary and actually is increasing at a faster rate than the model.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 4.3.1: Interpreting Slope and *y*-intercept

A town tracks the number of new homes being built over 10 years. The data is in the following table. Use the table for problems 1–3.

Year	New homes
1	130
2	233
3	340
4	340
5	709
6	642
7	809
8	1,011
9	1,324
10	1,511

1. Create a scatter plot of the data set.

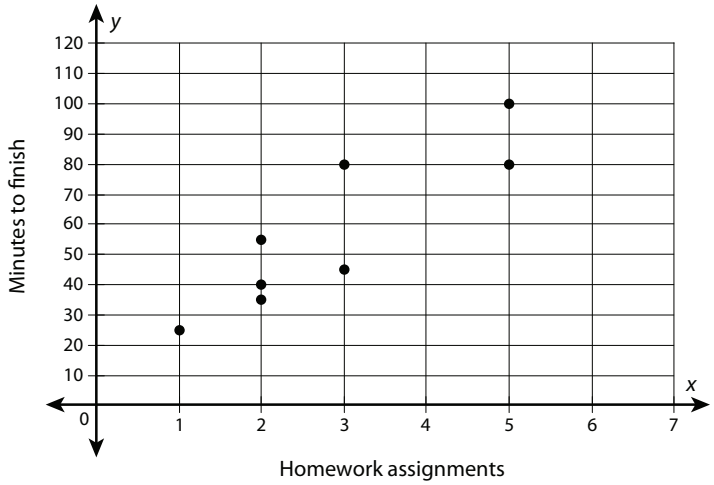
2. Find the equation of a line that fits the data.

3. Interpret the slope and *y*-intercept of the equation in context.

continued

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Madeline records the number of homework assignments she has and the total time it takes her to complete her homework. Her data is in the following scatter plot. Use the scatter plot for problems 4 and 5.



4. Find the equation of a line that fits the data.

5. Interpret the slope and *y*-intercept of the equation in context.

continued

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Will practices his basketball free throws. He records the number of free throws he attempts and the number of free throws he makes in the following table. Use the table for problems 6–8.

Free throws attempted	Free throws made
11	8
27	20
15	9
11	7
30	25
12	10
27	17
17	15
22	15
27	21

6. Create a scatter plot of the data set.

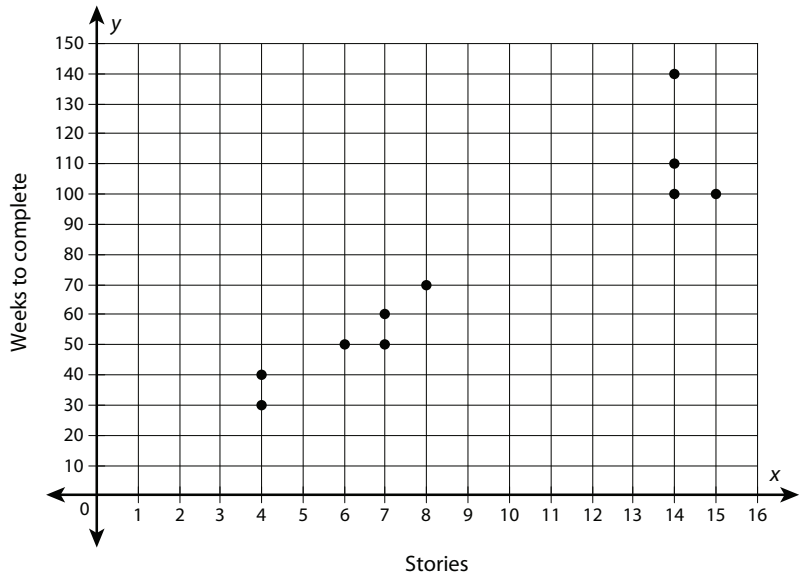
7. Find the equation of a line that fits the data.

8. Interpret the slope and y-intercept of the equation in context.

continued

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A construction company records the number of stories of each building it constructs and the number of weeks it takes to construct the building. The results are in the following scatter plot. Use the scatter plot for problems 9 and 10.



9. Find the equation of a line that fits the data.

10. Interpret the slope and *y*-intercept of the equation in context.

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Lesson 4.3.2: Calculating and Interpreting the Correlation Coefficient

Warm-Up 4.3.2

A new social networking company launched a TV commercial. The company tracked the number of users in thousands who joined the network after each week the commercial aired. Use the table of data to answer the questions that follow.

Number of weeks	New users in thousands
1	6
2	9
3	15
4	19
5	19
6	22
7	32
8	31
9	37
10	45
11	44

1. Create a scatter plot of the data.

2. Does the data appear to have a linear or exponential relationship? Explain.

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Instruction

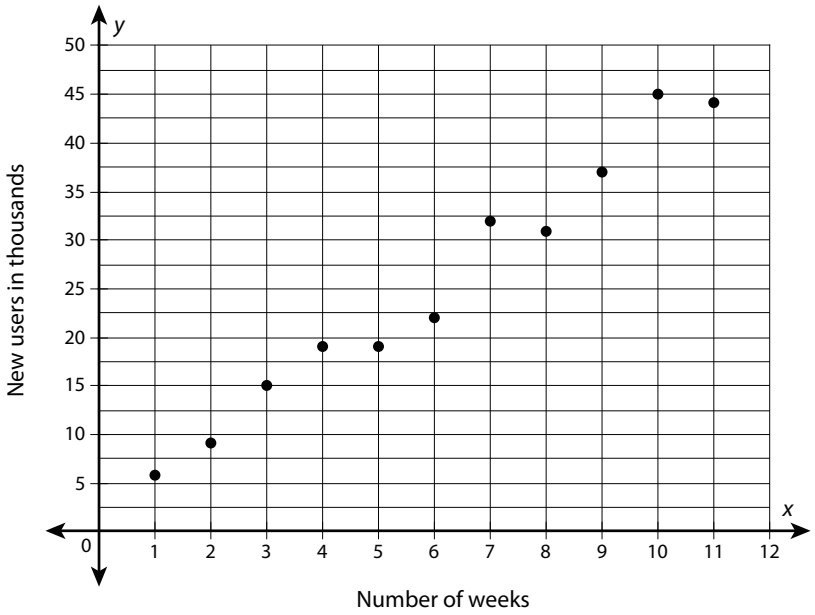
Lesson 4.3.2: Calculating and Interpreting the Correlation Coefficient

Common Core State Standard

S–ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★

Warm-Up 4.3.2 Debrief

1. Create a scatter plot of the data.



2. Does the data appear to have a linear or exponential relationship? Explain.

   The graph of a linear function is a line. The shape of the data in the graph is linear because a line could be drawn on the graph with an approximately equal number of points above and below the graph. The data appears to have a linear relationship.

Connection to the Lesson

- In this lesson, students will need to know how to create scatter plots of data sets.
- This warm-up will remind students how to create scatter plots and how to identify whether the relationship between two variables is linear using a graphical representation.
- Students will examine the strength of linear relationships and be introduced to the concept of the correlation coefficient.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- creating a scatter plot given data in a table (8.SP.1)
- understanding that the general shape of the graph of a linear function is a line (8.F.3)

Introduction

In previous lessons, we have plotted and analyzed data that appears to have a linear relationship. The data points in some data sets were very close to a linear model, while other data sets had points that were farther from the linear model. For variables that seem to have a linear relationship, the strength of that relationship can be analyzed using the correlation coefficient. A **correlation** is a relationship between two events, such as x and y, where a change in one event implies a change in another event. The **correlation coefficient**, r, is a quantity that allows us to determine how strong this relationship is between two events. It is a value that ranges from –1 to 1. A correlation coefficient close to –1 indicates a strong negative correlation, a correlation coefficient close to 1 indicates a strong positive correlation, and a correlation coefficient of 0 indicates a very weak or no linear correlation. A positive correlation indicates that as one variable increases, the other variable also increases. A negative correlation indicates that as one variable increases, the other variable decreases. You will use a calculator to calculate the correlation coefficient. Note that a correlation between two events does not imply that changing one event *causes* a change in the other event—only that a change might have taken place in the other event. This will be explored more later.

Key Concepts

- A correlation is a relationship between two events, where a change in one event implies a change in another event.
- Correlation doesn't mean that a change in the first event *caused* a change in the other event.
- The strength of a linear correlation can be measured using a correlation coefficient.
- Before determining the correlation coefficient, look at the scatter plot of the data and make an initial assessment of the strength of a linear relationship between the two events.
- Use the following steps to determine the correlation coefficient on a graphing calculator.

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On a TI-83/84:

Step 1: Set up the calculator to find correlations. Press [2ND], then [CATALOG] (the "0" key). Scroll down and select DiagnosticOn, then press [ENTER]. (This step only needs to be completed once. The calculator will stay in this mode until changed in this menu.)

Step 2: To calculate the correlation coefficient, first enter the data into a list. Press [2ND], then L1 (the "1" key). Scroll to enter data sets. Press [2ND], then L2 (the "2" key). Enter the second event in L2.

Step 3: Calculate the correlation coefficient. Press [STAT], then select CALC at the top of the screen. Scroll down to 8: LinReg(a+bx), and press [ENTER].

The *r* value (the correlation coefficient) is displayed along with the equation.

On a TI-Nspire:

Step 1: Go to the lists and spreadsheet page. The icon looks like a table.

Step 2: Enter the data into the first column underneath the shaded row, pressing [enter] after each data value.

Step 3: Use the nav pad to arrow up to the first row below the shaded row and then arrow over to the right so that you are in the second column. Enter the data values, pressing [enter] after each data value.

Step 4: Press the [menu] key.

Step 5: Arrow down to 4: Statistics, and press the center click key.

Step 6: Press the center click key again to select 1: Stat Calculations.

Step 7: Choose 3: Linear Regression (mx+b).

Step 8: At the XList field, press [clear] and then type in "a[]". To type "[]", press the [ctrl] key and then the [(] key.

Step 9: Press [tab] to go the YList field and type in "b[]".

Step 10: Press [tab] to go the Results field and check that results are listed in "c[]". If not, change them.

Step 11: Press [tab] to "OK" and press the center click key.

Step 12: Arrow down until you see the "r" and look to the right. The number to the right is the correlation coefficient, *r*.

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Instruction

- A correlation coefficient close to –1 indicates a strong negative correlation.
- A correlation coefficient close to 1 indicates a strong positive correlation.
- A correlation coefficient of 0 indicates a very weak or no linear correlation.
- The correlation coefficient assesses the strength of a linear relationship between two variables.
- The correlation coefficient does not assess causation—that one event causes the other.

Common Errors/Misconceptions

- using the correlation coefficient to analyze data that is not linear
- incorrectly using the correlation coefficient to assess the strength of a relationship

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Guided Practice 4.3.2

Example 1

An education research team is interested in determining if there is a relationship between a student's vocabulary and how frequently the student reads books. The team gives 20 students a 100-question vocabulary test, and asks students to record how many books they read in the past year. The results are in the following table. Is there a linear relationship between the number of books read and test scores? Use the correlation coefficient, *r*, to explain your answer.

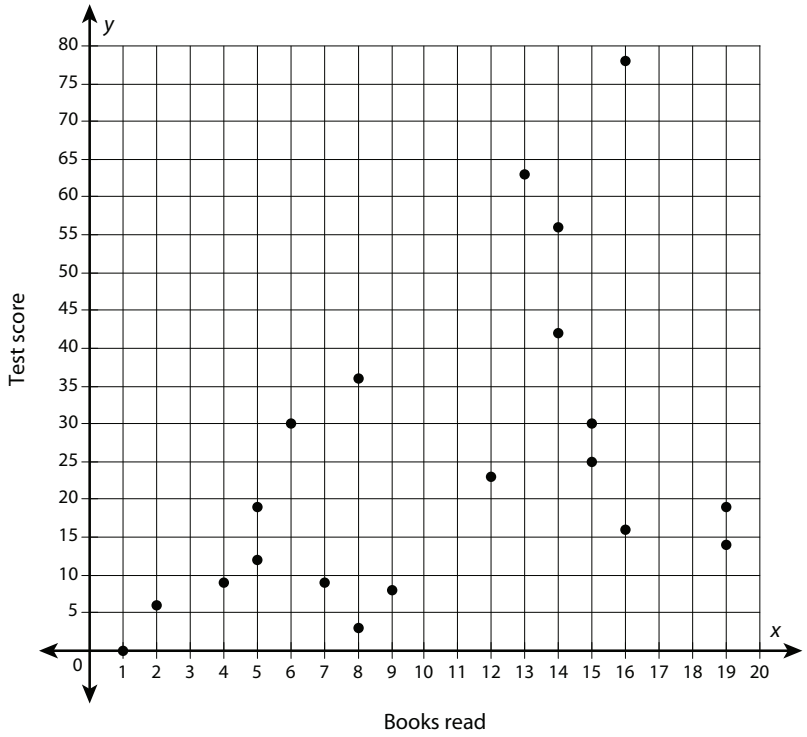
Books read	Test score
12	23
8	3
19	14
9	8
14	56
19	19
15	25
6	30
2	6
14	42
5	12
15	30
8	36
5	19
1	0
13	63
4	9
16	78
16	16
7	9

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Instruction

1. Create a scatter plot of the data.

   Let the *x*-axis represent books read and the *y*-axis represent test scores.



2. Describe the relationship between the data using the graphical representation.

   It appears that the higher scores were from students who read more books, but the data does not appear to lie on a line. There is not a strong linear relationship between the two events.

3. Calculate the correlation coefficient on your graphing calculator. Refer to the steps in the Key Concepts section.

   The correlation coefficient, *r*, is approximately 0.48.

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4. Use the correlation coefficient to describe the strength of the relationship between the data.

   A correlation coefficient close to 1 indicates a strong positive correlation, and a correlation of 0 indicates no correlation. A correlation coefficient of 0.48 is about halfway between 0 and 1, and indicates that there is a weak positive linear relationship between the number of books a student read in the past year and his or her score on the vocabulary test. This means that students who read more books are more likely to score higher on the vocabulary test.



Example 2

A hockey coach wants to determine if players who take many practice shots during practice have a higher shooting percentage. The shooting percentage is calculated by dividing the number of goals scored by the number of shots taken. The coach records the average number of practice shots 20 players take each practice, and compares the number with each player's shooting percentage over the season. Is there a linear relationship between the practice shots and shooting percentage? Create a scatter plot of the data, and use the correlation coefficient, *r*, to explain your answer.

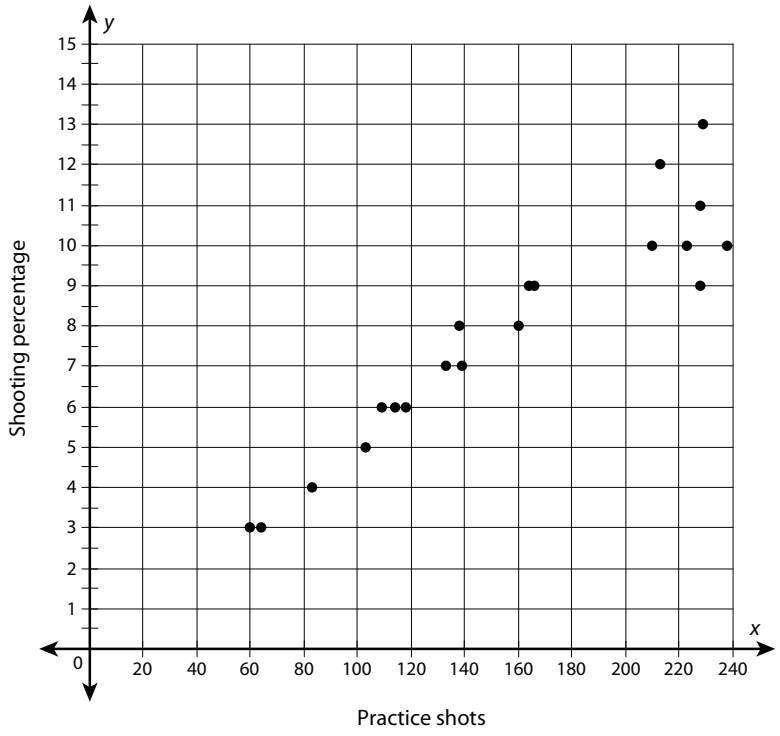
Practice shots	Shooting percentage	Practice shots	Shooting percentage
228	9	223	10
164	9	133	7
64	3	238	10
213	12	228	11
166	9	138	8
60	3	139	7
109	6	118	6
83	4	210	10
229	13	103	5
160	8	114	6

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Instruction

1. Create a scatter plot of the data.

   Let the *x*-axis represent the number of practice shots and the *y*-axis represent the shooting percentage.



2. Describe the relationship between the data using the graphical representation.

   It appears that there is a linear relationship between the number of shots taken during practice and the shooting percentage. As the number of practice shots increases, shooting percentage also increases, and the graph appears to have a linear shape.

3. Calculate the correlation coefficient on your graphing calculator. Follow the steps in the Key Concepts section.

   The correlation coefficient, *r*, is 0.94.

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4. Use the correlation coefficient to describe the strength of the relationship between the data.

   The closer the correlation coefficient is to 1, the stronger the positive correlation. A correlation coefficient of 0.94 is close to 1, and indicates that there is a strong positive linear relationship between the number of shots taken during practice and the shooting percentage. This means that players who took more shots during practice had higher shooting percentages.



Example 3

Caitlyn thinks that there may be a relationship between class size and student performance on standardized tests. She recorded the number of students in 20 classes, along with the average test score in each class, in the following table. Is there a linear relationship between class size and average test score? Use the correlation coefficient, *r*, to explain your answer.

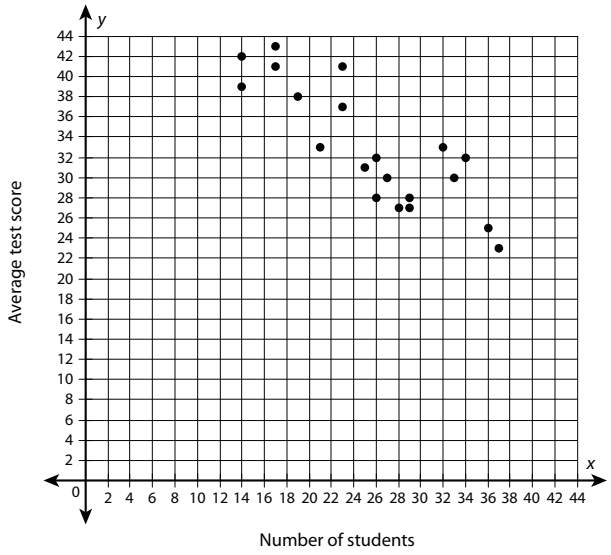
Class size	Average student test score	Class size	Average student test score
26	28	32	33
36	25	27	30
29	27	21	33
26	32	28	27
19	38	23	41
34	32	29	28
17	43	37	23
14	42	14	39
23	37	25	31
17	41	33	30

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Instruction

1. Create a scatter plot of the data.

   Let the *x*-axis represent the number of students in each class and the *y*-axis represent the average test score.



2. Describe the relationship between the two variables using the graphical representation.

   As the class size, *x*, increases, the average test score, *y*, decreases. It appears that there is a linear relationship with a negative slope between the two variables.

3. Calculate the correlation coefficient on your graphing calculator. Follow the steps in the Key Concepts section.

   The correlation coefficient, *r*, is approximately –0.84.

4. Use the correlation coefficient to describe the strength of the relationship between the data.

   A correlation coefficient close to –1 indicates a strong negative correlation. A correlation coefficient of –0.84 is close to –1, and indicates that there is a strong negative correlation between class size and average test score. This means classes with more students are more likely to have lower standardized test scores.

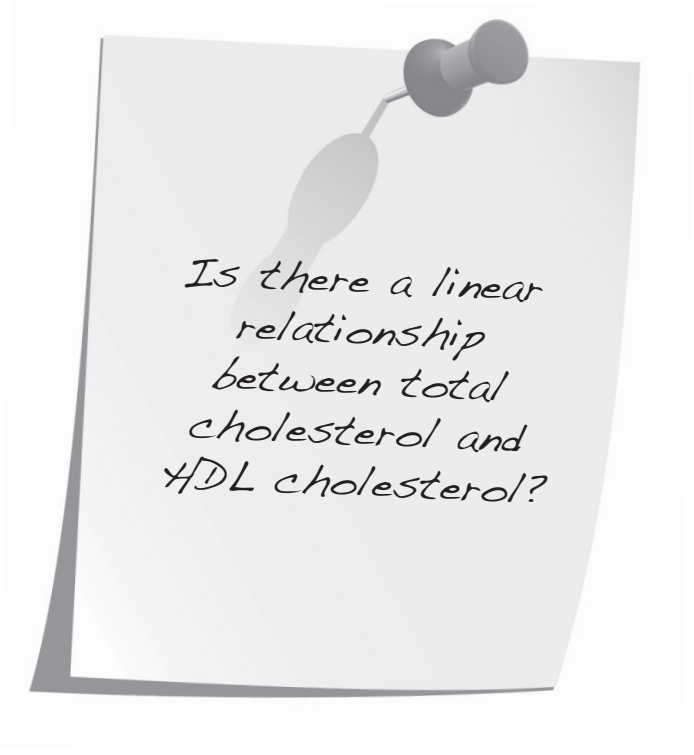
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Problem-Based Task 4.3.2: Good and Bad Cholesterol

SMP	
1	2
3 ✓	4 ✓
5 ✓	6
7	8

Cholesterol is a substance found in human blood. There are two types of cholesterol: HDL (high-density lipoprotein) and LDL (low-density lipoprotein). HDL is known as the "good" type of cholesterol because it lowers the risk of heart disease, and LDL is the type of cholesterol that can lead to heart attacks and strokes. The sum of HDL and LDL cholesterol levels is your total cholesterol: HDL + LDL = total cholesterol. The following table shows the total cholesterol and HDL cholesterol for 20 patients, in milligrams per deciliter (mg/dL). Is there a linear relationship between total cholesterol and HDL cholesterol? Use the correlation coefficient, *r*, to explain your answer.

Total cholesterol (mg/dL)	HDL cholesterol (mg/dL)
251	47
159	30
289	63
198	54
298	75
265	53
258	86
140	45
267	49
262	71
271	50
240	40
218	47
210	57
187	31
256	52
278	79
267	50
186	58
198	38



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Problem-Based Task 4.3.2: Good and Bad Cholesterol

Coaching

a. Create a scatter plot of the data.

b. What do you think is the relationship between total cholesterol and HDL cholesterol? Use the shape of the scatter plot to explain.

c. Calculate the correlation coefficient, *r*, using your graphing calculator.

d. What does the correlation coefficient tell you about the relationship between the two events?

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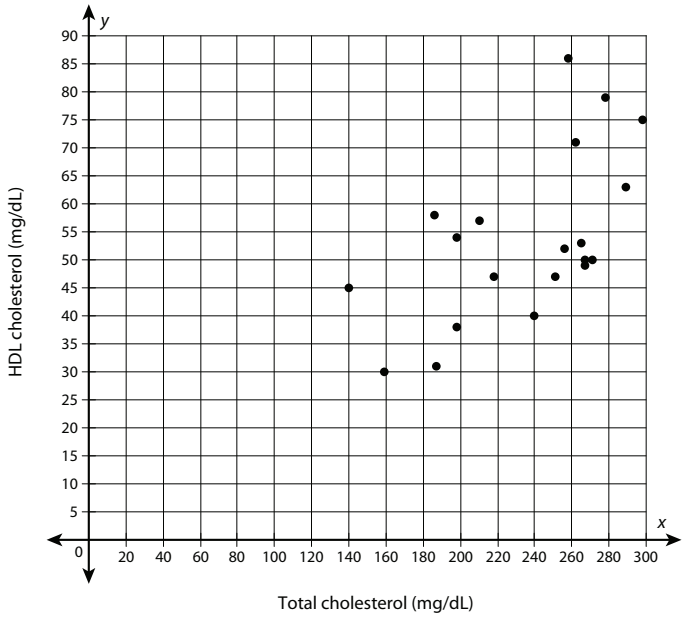
Instruction

Problem-Based Task 4.3.2: Good and Bad Cholesterol

Coaching Sample Responses

a. Create a scatter plot of the data.

Let the *x*-axis represent total cholesterol and the *y*-axis represent HDL cholesterol.



b. What do you think is the relationship between total cholesterol and HDL cholesterol? Use the shape of the scatter plot to explain.

It appears that people with higher values of HDL cholesterol have higher total cholesterol, but the graph does not have a strong linear shape.

c. Calculate the correlation coefficient, *r*, using your graphing calculator.

The correlation coefficient, *r*, is approximately 0.596.

d. What does the correlation coefficient tell you about the relationship between the two events?

A correlation coefficient close to 1 indicates a strong positive linear correlation, and a correlation coefficient of 0 indicates no linear correlation. A correlation coefficient of 0.596 tells us that there is a positive linear correlation between the two values, but it is not very strong.

Recommended Closure Activity

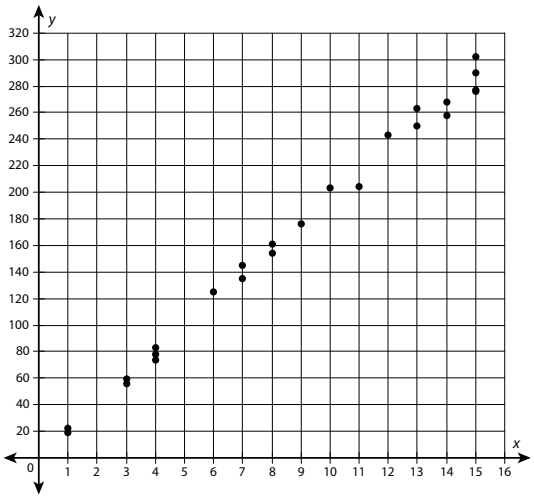
Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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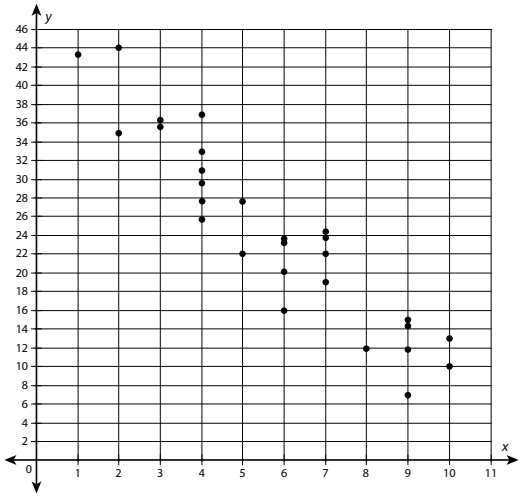
Practice 4.3.2: Calculating and Interpreting the Correlation Coefficient

For each of the following scatter plots, describe the type of linear correlation between the two variables: positive, negative, or no correlation, and identify whether it is strong or weak.

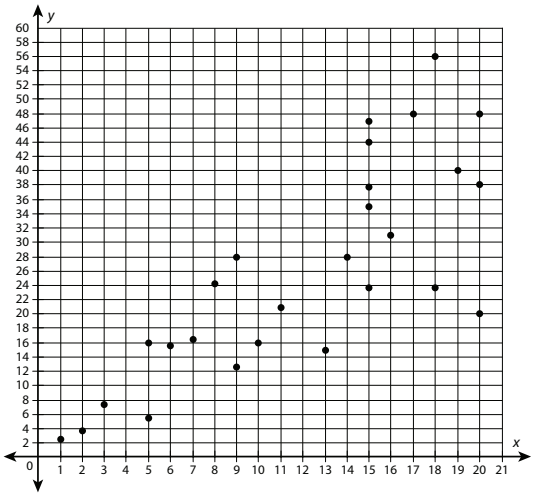
1.



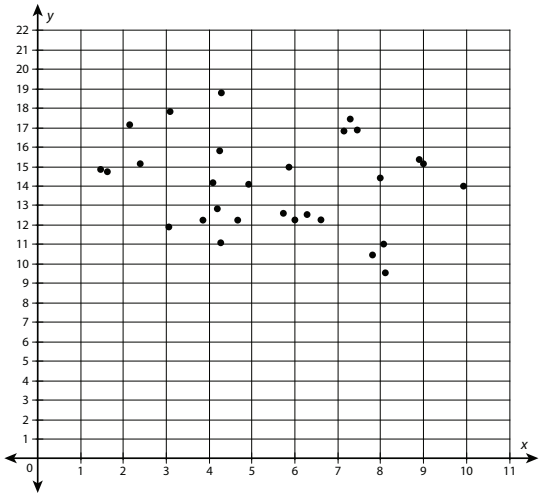
3.



2.



4.



continued

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Warmer weather can be an inspiration to plant gardens and work on landscaping. A plant nursery manager thinks there may be a relationship between weather and plant sales. Each day, the manager records the average temperature in °F and the number of plants sold in a table. Use the table for problems 5–7.

Average temperature (°F)	Plants sold	Average temperature (°F)	Plants sold
52	18	69	119
78	281	64	59
76	101	54	20
67	152	50	4
69	113	57	33
75	120	76	263
56	25	65	58
54	37	76	133
77	157	78	275

5. Create a scatter plot of the data.

6. Use your graph to describe the relationship between temperature and plant sales.

7. Find the correlation coefficient, *r*, of the data. Describe what the correlation coefficient indicates about the relationship between the data.

continued

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A cruise ship captain wants to know if there is a relationship between the number of children on the ship and the average attendance at a nightly pool party. The ship counted anyone under age 17 as a child. The results are in the following table. Use the table for problems 8–10.

Number of children	Average pool party attendance
663	23
454	76
737	23
200	112
101	116
216	139
666	23
415	52
978	61
930	62
850	22
891	63
253	110
795	22
858	64
117	144
842	65
275	136

8. Create a scatter plot of the data.

9. Use your graph to describe the relationship between the number of children and pool party attendance.

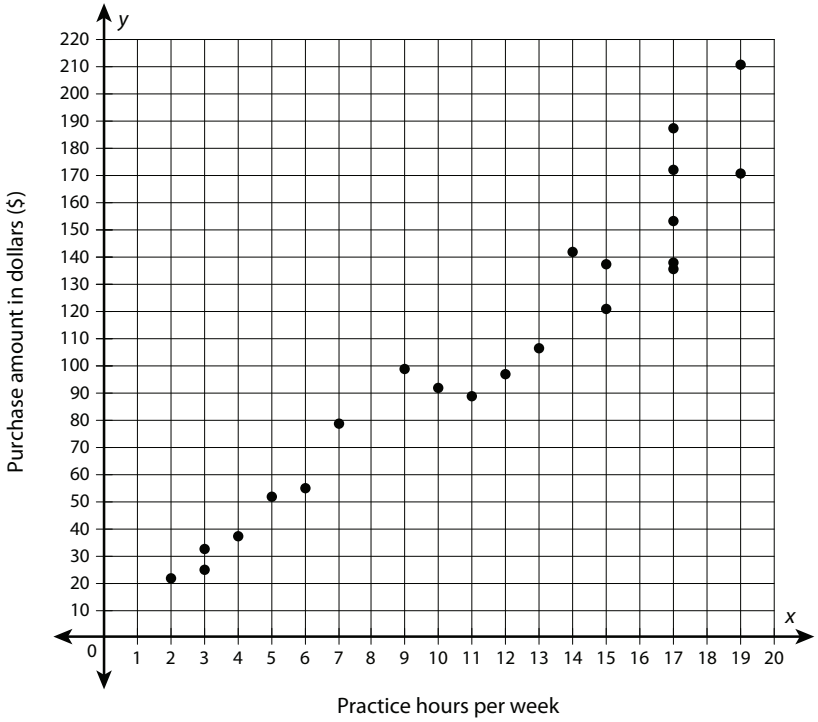
10. Find the correlation coefficient, r, of the data. Describe what the correlation coefficient indicates about the relationship between the data.

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Lesson 4.3.3: Distinguishing Between Correlation and Causation

Warm-Up 4.3.3

The owner of a guitar store asks customers how many hours a week they practice. He also tracks how much the customers spend for a guitar. The relationship is shown in the following scatter plot. Use the scatter plot to answer the question that follows.



1. Is there a linear correlation between x and y? Explain.

continued

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The following data represents the number of minutes guitarists practice per day and the number of mistakes they make during a performance. Use the data in the table to complete the remaining problems.

x	y
4	35
7	26
3	37
6	32
6	29
8	28
9	26
1	40
4	33
3	37
3	38
10	22
3	38

2. Create a scatter plot of the data set.

3. Find the correlation coefficient, r, of the data. Is there a linear relationship between x and y?

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Instruction

Lesson 4.3.3: Distinguishing Between Correlation and Causation

Common Core State Standard

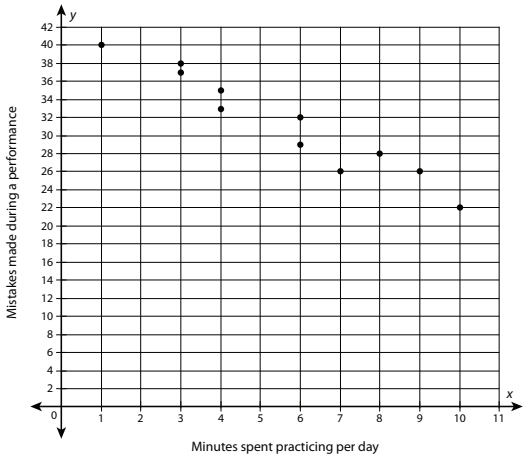
S–ID.9 Distinguish between correlation and causation.★

Warm-Up 4.3.3 Debrief

1. Is there a linear correlation between x and y? Explain.

   The shape of a linear graph is a straight line. The data seems to follow a straight line, so it appears that there is a linear correlation between x and y.

2. Create a scatter plot of the data set.



3. Find the correlation coefficient, r, of the data. Is there a linear relationship between x and y?

   Use a graphing calculator to find r.

   On a TI-83/84:

Step 1: The calculator must first be set up to find correlations. Press [2ND], then [CATALOG] (the "0" key). Scroll down and select DiagnosticOn, then press [ENTER]. (This step only needs to be completed once; the calculator will stay in this mode until changed in this menu.)
Step 2: To calculate the correlation coefficient, first enter the data into a list. Press [2ND], then L1 (the "1" key). Scroll to enter data sets. Press [2ND], then L2 (the "2" key). Enter the second event in L2.
Step 3: Now calculate the correlation coefficient. Press [STAT], then select CALC at the top of the screen. Scroll down to 8: LinReg(a+bx), and press [ENTER].
Step 4: The r value (the correlation coefficient) is displayed along with the equation.

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Instruction

On a TI-Nspire:

Step 1: Go to the lists and spreadsheet page. The icon looks like a table.

Step 2: Enter the data into the first column underneath the shaded row, pressing [enter] after each data value.

Step 3: Use the nav pad to arrow up to the first row below the shaded row and then arrow over to the right so that you are in the second column. Enter the data values, pressing [enter] after each data value.

Step 4: Press the [menu] key.

Step 5: Arrow down to 4: Statistics and press the center click key.

Step 6: Press the center click key again to select 1: Stat Calculations.

Step 7: Choose 3: Linear Regression (mx+b).

Step 8: At the X List field, press [clear] and then type in "a[]". To type "[]", press the [ctrl] key and then the [(] key.

Step 9: Press [tab] to go the Y List field and type in "b[]".

Step 10: Press [tab] to go the Results field and check that results are listed in "c[]". If not, change them.

Step 11: Press tab to "OK" and press the center click key.

Step 12: Arrow down until you see the "r" and look to the right. The number to the right is the correlation coefficient, r.

Using a calculator, $r = -0.97$. The correlation coefficient is very close to –1, so there is a strong negative linear correlation between x and y.

Connection to the Lesson

- In this lesson, students will continue to examine linear correlations between data.
- This warm-up will remind students how to identify linear correlations graphically and by using the correlation coefficient, r.
- Students will differentiate between linear correlations and causations.

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Instruction

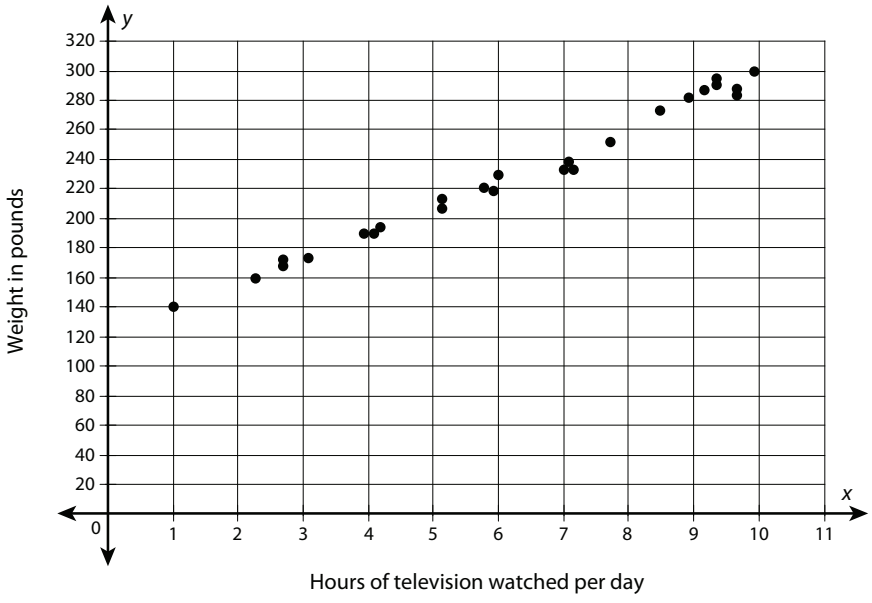
Prerequisite Skills

This lesson requires the use of the following skills:

- creating a scatter plot given data in a table (8.SP.1)
- identifying linear correlations graphically (8.SP.3)
- examining linear correlations using the correlation coefficient, *r* (S–ID.8★)

Introduction

A correlation between two events simply means that there is a consistent relationship between two events, and that a change in one event implies that the other event will change according to the linear relationship. A correlation does not imply **causation**, or that a change in one event causes the change in the second event. Because many factors influence changes in events, it is very difficult to prove causation, sometimes referred to as a causal relationship. For example, researchers gathered the following data comparing the hours of television watched per day by viewers and each viewer's weight in pounds.



The correlation coefficient, *r*, between the two events is 0.994. According to this data, a person's weight could be estimated based on the hours of television he or she watches each day. This is due to the strong correlation between the two events. Can the researchers conclude that watching television is responsible for increased weight, or that watching television causes a person to gain weight?

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Instruction

The correlation does not give any information about *why* weight increases when a person watches more television. If the data is accurate, it is likely that there is a factor not included in the data set that is associated with both events. For example, a person who watches more television may spend less time exercising and more time sitting. Or, a person who watches more television may eat less healthy foods than someone who spends less time watching television.

There may be many contributing factors that explain how this relationship works. The important note is that watching television by itself cannot cause weight gain. It is also possible that a correlation appears to exist because of a small sample size or a poorly selected sample. Knowing the difference between correlation and causation is particularly important when reading advertisements or other persuasive materials that claim to show a causal relationship. Think closely about outside factors that may have influenced both events and led to a strong correlation before believing that one event is responsible for the change in another event.

Key Concepts

- Correlation does not imply causation.
- If a change in one event is responsible for a change in another event, the two events have a causal relationship.
- Outside factors may influence and explain a strong correlation between two events.
- Use a calculator to find the correlation coefficient.

  On a TI-83/84:

  Step 1: Set up the calculator to find correlations. Press [2ND], then [CATALOG] (the "0" key). Scroll down and select DiagnosticOn, then press [ENTER]. (This step only needs to be completed once. The calculator will stay in this mode until changed in this menu.)

  Step 2: To calculate the correlation coefficient, first enter the data into a list. Press [2ND], then L1 (the "1" key). Scroll to enter data sets. Press [2ND], then L2 (the "2" key). Enter the second event in L2.

  Step 3: Calculate the correlation coefficient. Press [STAT], then select CALC at the top of the screen. Scroll down to 8: LinReg(a+bx), and press [ENTER].

  The *r* value (the correlation coefficient) is displayed along with the equation.

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On a TI-Nspire:

Step 1: Go to the lists and spreadsheet page. The icon looks like a table.

Step 2: Enter the data into the first column underneath the shaded row, pressing [enter] after each data value.

Step 3: Use the nav pad to arrow up to the first row below the shaded row and then arrow over to the right so that you are in the second column. Enter the data values, pressing [enter] after each data value.

Step 4: Press the [menu] key.

Step 5: Arrow down to 4: Statistics, and press the center click key.

Step 6: Press the center click key again to select 1: Stat Calculations.

Step 7: Choose 3: Linear Regression (mx+b).

Step 8: At the X List field, press [clear] and then type in "a[]". To type "[]", press the [ctrl] key and then the [(] key.

Step 9: Press [tab] to go the Y List field and type in "b[]".

Step 10: Press [tab] to go the Results field and check that results are listed in "c[]". If not, change them.

Step 11: Press [tab] to "OK" and press the center click key.

Step 12: Arrow down until you see the "r" and look to the right. The number to the right is the correlation coefficient, *r*.

Common Errors/Misconceptions

- assuming that a strong correlation indicates that one event causes another
- failing to consider outside factors that may influence the strength of the correlation between two events

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Instruction

Guided Practice 4.3.3

Example 1

Alex coaches basketball, and wants to know if there is a relationship between height and free throw shooting percentage. Free throw shooting percentage is the number of free throw shots completed divided by the number of free throw shots attempted, multiplied by 100:

$$\frac{\text{free throw shots completed}}{\text{free throw shots attempted}} \cdot 100$$

He takes some notes on the players in his team, and records his results in the following table. What is the correlation between height and free throw shooting percentage? Alex looks at his data and decides that increased height causes a reduced free throw shooting percentage. Is he correct?

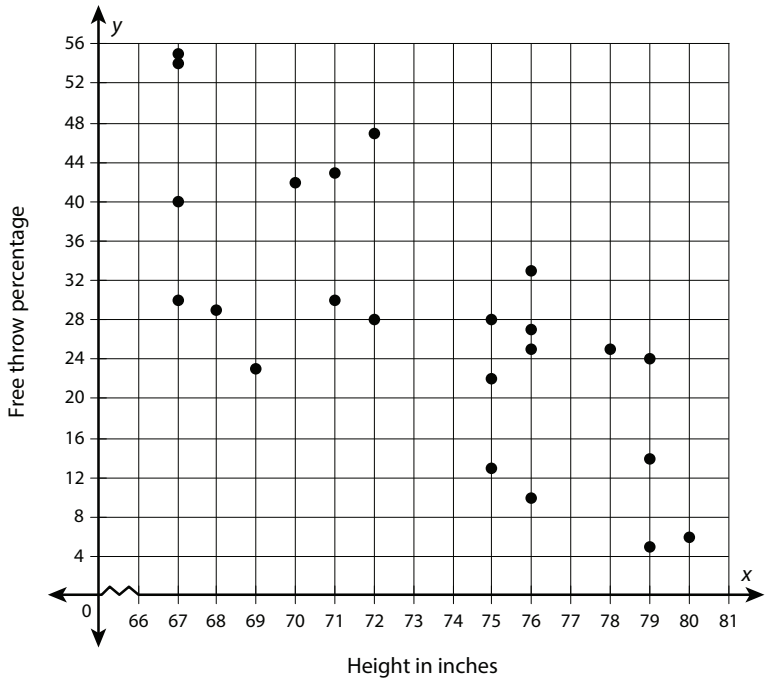
Height in inches	Free throw percentage	Height in inches	Free throw percentage
75	28	72	28
75	22	76	33
67	30	76	25
80	6	67	54
71	43	79	5
67	40	67	55
76	10	78	25
76	25	75	13
70	42	71	30
72	47	68	29
79	24	79	14
69	23	78	25
76	27		

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Instruction

1. Create a scatter plot of the data.

   Let the *x*-axis represent height in inches, and the *y*-axis represent free throw shooting percentage.



2. Analyze the scatter plot, and describe any relationship between the two events.

   As height increases, free throw shooting percentage decreases. It appears that there is a weak negative linear correlation between the two events.

3. Find the correlation coefficient using a graphing calculator. Follow the steps in the Key Concepts section.

   $r = -0.727$

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Instruction

4. Describe the correlation between the two events.

   –0.727 is close to –1. There is a negative linear correlation between the events.

5. Consider the causal relationship between the two events.

   Determine if it is likely that height is responsible for the decrease in free throw shooting percentage.

   Even if there is a correlation between height and free throw percentage, it is not likely that height causes a basketball player to have more difficulty making free throw shots. If two equally skilled players were of different heights, would you expect one of them to make fewer free throws based only on his or her height? What about a very tall player who spends more time practicing free throws than a very short player? What if the sample size is too small to gather data that's true for the larger population? There is most likely not a causal relationship between height and free throw percentage.

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Instruction

Example 2

Mr. Gray's students are interested in learning how studying can improve test performance. Mr. Gray provides students with practice problems related to particular tests. The class records the number of practice problems completed and the score on that related test in the following table. What is the correlation between the number of practice problems completed and the test score? Is there a causal relationship between the number of practice problems completed and the test score?

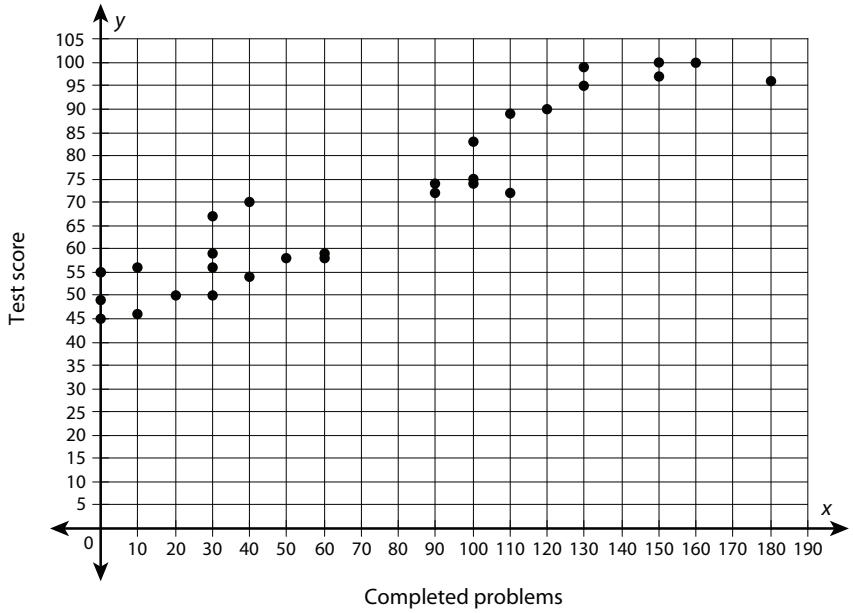
Problems completed	Test score, out of 100 points	Problems completed	Test score, out of 100 points
10	56	100	75
40	70	110	72
100	83	100	74
40	54	120	90
0	45	130	99
50	58	160	100
90	72	0	49
150	97	60	59
30	50	0	55
60	58	180	96
90	74	150	100
110	89	30	67
30	59	30	56
130	95	20	50
10	46		

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Instruction

1. Create a scatter plot of the data.

   Let the *x*-axis represent the number of completed practice problems and the *y*-axis represent the test score.



2. Analyze the scatter plot, and describe any relationship between the two events.

   As the number of completed practice problems increases, the test score also increases. The shape of the graph is approximately a line, and it appears there is a positive linear correlation between the number of practice problems completed and the test score.

3. Find the correlation coefficient using a graphing calculator. Follow the steps in the Key Concepts section.

   $r = 0.942$

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4. Describe the correlation between the two events.

   0.942 is close to 1. There is a strong positive linear correlation between the number of practice problems completed and the test score.

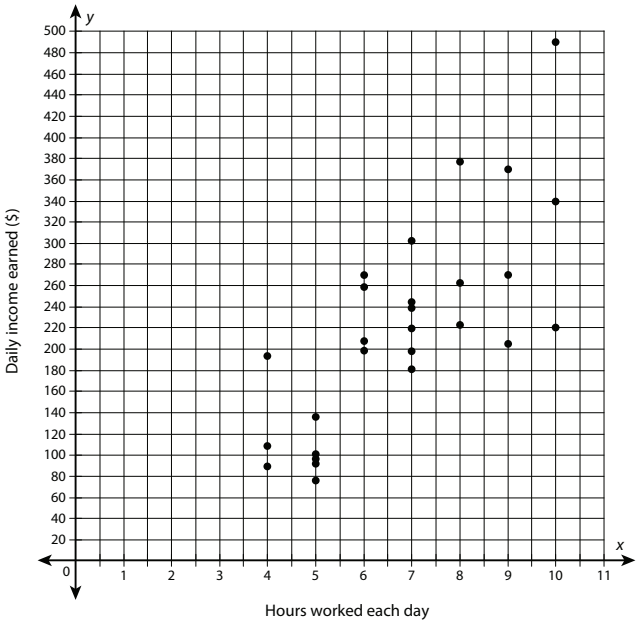
5. Consider the causal relationship between the two events.

   Determine if it is likely that the number of practice problems completed is responsible for increased test scores. Note any other factors that could influence test scores.

   There is a strong correlation between the two events. A score on a test is related to a student's knowledge of the test content, and a student's ability to use content to solve problems. Completing practice problems allows students to develop skills directly related to test performance, and although there are other factors that are related to test performance, such as parental involvement and length of time spent studying, it is likely that there is a causal relationship between the number of practice problems completed and the test score.

Example 3

Nadia is a salesperson at a car dealership. She earns money each time she sells a car. To determine if there is a relationship between the number of hours she works and her income, she records the number of hours worked and the amount of money she earns each day. Her data is in the scatter plot that follows. Is there a causal relationship between the hours Nadia works and her daily income?



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Instruction

1. Analyze the scatter plot, and describe any relationship between the two events.

   As the number of hours increase, the daily income also increases, but there is much variation in the increase of income as hours increase. There appears to be a weak linear correlation between hours and income.

2. Consider the causal relationship between the two events.

   Determine if the number of hours worked is responsible for an increase in income.

   Nadia earns money when she sells a car. The more hours she works, the greater the opportunity she has to sell more cars. However, the only way to earn more money is to either sell more cars or to sell cars that are more expensive. She could work for 10 hours, but if she doesn't sell any cars, or make no effort to sell any cars, then her income will reflect this lack of sales or effort. Working more hours does not cause the increase in income. Selling more cars causes the increase in income. There is likely not a direct causal relationship between hours worked and income earned.

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Problem-Based Task 4.3.3: Good Cholesterol and Exercise

SMP	
1	2
3 ✓	4 ✓
5 ✓	6
7	8

Cholesterol is a substance found in human blood. There are two types of cholesterol: HDL (high-density lipoprotein) and LDL (low-density lipoprotein). HDL is the "good" type of cholesterol that lowers the risk of heart disease. LDL cholesterol can lead to heart attacks and strokes. The sum of HDL and LDL cholesterol is your total cholesterol: HDL + LDL = total cholesterol.

A doctor tested 20 patients' cholesterol levels. She also asked each patient how often he or she exercises each week. The table shows each patient's weekly hours of exercise and HDL cholesterol level, in milligrams per deciliter (mg/dL). The doctor is trying to understand if exercise has an impact on HDL cholesterol. What is the correlation between hours exercised per week and HDL cholesterol levels? Is it likely there is a causal relationship between exercise and HDL cholesterol levels?

Hours of exercise	HDL cholesterol (mg/dL)	Hours of exercise	HDL cholesterol (mg/dL)
2	47	1	50
0	30	7	40
1	63	2	47
1.5	54	5	57
7.5	75	1	31
6	53	4	52
8.5	86	5	79
0.5	45	3.5	50
3	49	2	58
6.5	71	0	38

What is the correlation between hours exercised per week and HDL cholesterol levels?

Is it likely there is a causal relationship between exercise and HDL cholesterol levels?

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Problem-Based Task 4.3.3: Good Cholesterol and Exercise Coaching

a. Create a scatter plot of the data.

b. Describe the shape of the graph.

c. Does it appear that there is a relationship between exercise and HDL cholesterol levels?

d. What is *r*, the correlation coefficient?

e. What does *r* tell us about the relationship between exercise and HDL cholesterol levels?

f. Is it likely that exercising for more hours each week causes higher levels of HDL cholesterol?

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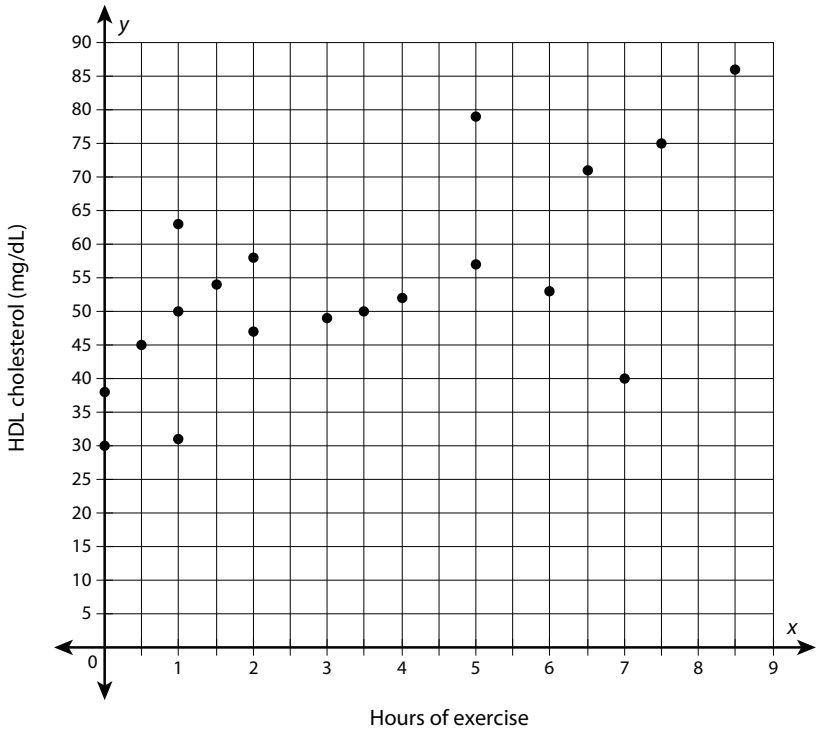
Instruction

Problem-Based Task 4.3.3: Good Cholesterol and Exercise

Coaching Sample Responses

a. Create a scatter plot of the data.

Let the *x*-axis represent the hours of exercise per week, and the *y*-axis represent the HDL cholesterol level in milligrams per deciliter.



b. Describe the shape of the graph.

As the weekly hours of exercise increase, HDL cholesterol generally increases.

c. Does it appear that there is a relationship between exercise and HDL cholesterol levels?

There may be a positive linear relationship between exercise and HDL cholesterol level, based on the shape of the graph.

d. What is *r*, the correlation coefficient?

Using a calculator, $r = 0.672$.

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e. What does *r* tell us about the relationship between exercise and HDL cholesterol levels?

A correlation coefficient close to 1 indicates a strong positive linear relationship, and a correlation coefficient of 0 indicates no correlation. It appears that there is a positive linear correlation between exercise and HDL cholesterol levels.

f. Is it likely that exercising for more hours each week causes higher levels of HDL cholesterol?

The correlation between exercise and HDL cholesterol isn't strong, which is a clue that there isn't a causal relationship between the two events. Given that HDL cholesterol generally increases as exercise increases, exercise may have some effect on HDL cholesterol, but it is difficult to determine given the information collected. Other factors, such as genetics and diet, can also influence HDL levels. These factors are not represented in the graph. People who have healthier diets may also exercise more, which could be responsible for the positive linear trend in the data. It is difficult to identify if there is a causal relationship between exercise and HDL cholesterol level given this data set.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 4.3.3: Distinguishing Between Correlation and Causation

A team of advertisers is trying to measure how effectively the advertising campaigns for several products influence purchases of each product. The team records information about total dollars invested in advertising each product in one large city for one year and the total number sold for each product. The data is in the following table. Use the table for problems 1–4.

Advertising spending ($) per product	Products sold	Advertising spending ($) per product	Products sold
71,000	55,000	31,000	45,000
54,000	125,000	88,000	115,000
73,000	85,000	32,000	80,000
45,000	35,000	80,000	165,000
63,000	150,000	34,000	90,000
55,000	150,000	76,000	50,000
68,000	70,000	38,000	85,000
90,000	110,000	67,000	105,000
87,000	40,000	48,000	125,000
42,000	105,000	18,000	35,000
36,000	65,000	37,000	30,000
24,000	95,000	90,000	55,000
49,000	55,000	26,000	115,000
72,000	90,000	89,000	155,000
87,000	160,000		

1. Create a scatter plot of the data.

2. Describe the shape of the graph.

3. Find the correlation coefficient, *r*, and describe what this indicates about the relationship between the amount of advertising dollars spent and the number of products sold.

4. Is it likely that there is a causal relationship between the amount of advertising dollars spent and the number of products sold?

continued

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A travel agency collects information about its clients. It records a client's age and the number of countries visited by that client. The data is in the following table. Use the table for problems 5–8.

Age of client	Countries visited	Age of client	Countries visited
77	8	47	6
50	7	48	7
26	3	30	4
41	5	26	4
79	13	46	6
36	5	53	7
57	8	77	11
28	4	46	5
73	9	44	7
45	5	47	8
33	4	43	5
46	5	58	7
37	5	52	6
70	10	80	9
28	3		

5. Create a scatter plot of the data.

6. Describe the shape of the graph.

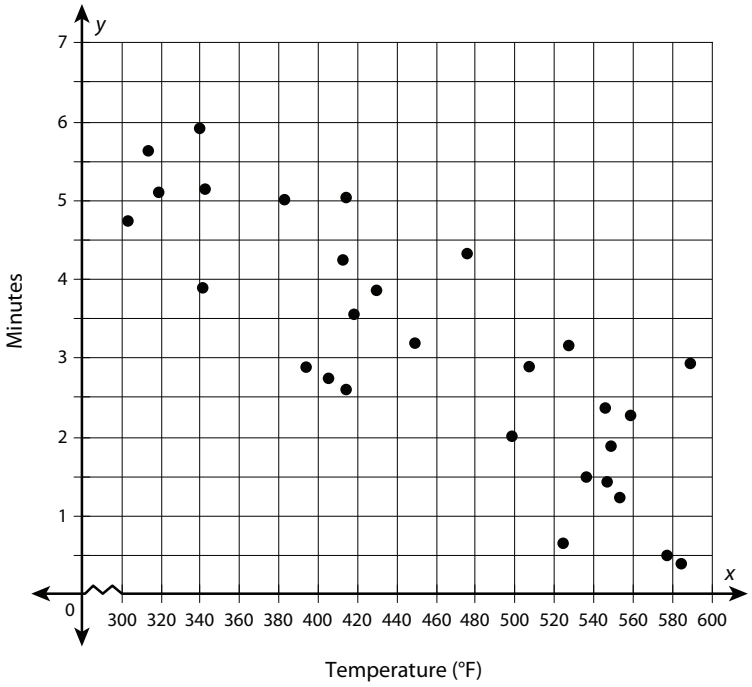
7. Find the correlation coefficient, *r*, and describe what this indicates about the relationship between age and number of countries visited.

8. Is it likely that there is a causal relationship between a client's age and the number of countries visited?

continued

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A science class studies the time it takes a certain amount of water to reach a boil. Each student uses the same shape container for the water and places the container the same distance from a burner. The burners are identical, but can be set to different heat settings. Each student applies a different amount of heat to his or her container of water, and records the temperature applied in degrees Fahrenheit. The students then record the number of minutes it takes their water to reach a boil given the temperature of the applied heat. The results are shown in the following scatter plot. Use the scatter plot for problems 9 and 10.



9. Describe the shape of the graph, and describe any possible correlation between temperature and time.

10. Is it likely that there is a causal relationship between the heat setting in degrees Fahrenheit and the time it takes the water to boil?

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Progress Assessment

Circle the letter of the best answer.

1. Coach Thomas records her athletes' performances at recent softball games. She uses the information to find an equation to represent the number of times a player should reach first base for any number of at bats. Her equation is $y = 0.27x + 0.04$, where x represents the number of times at bat, and y represents the number of times reaching first base. Which statement is true based on the equation?

   a. For each at bat, there is a 27% chance that a player will get on first base.

   b. For each at bat, there is a 4% chance that a player will get on first base.

   c. For each at bat, there is a 2.7% chance that a player will get on first base.

   d. For each at bat, there is a 0.4% chance that a player will get on first base.

2. Ella planted tulips around her house. Each day, she tracks the number of blossoming buds. For the 10 days that she tracks the tulips, she finds that the equation $y = 2x + 3$ estimates the number of blossoming buds on any of the days from 1 to 10. Which statement is true based on this equation?

   a. Approximately 3 buds blossom each day.

   b. Approximately 2 buds blossom each day.

   c. The day Ella started tracking blossoming buds, there were approximately 2 blossoming buds.

   d. Approximately $\frac{2}{3}$ buds blossom each day.

continued

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3. What is the correlation coefficient, *r*, of the data in the table? Use technology to calculate *r*.

x	*y*
20	103
23	116
20	61
21	105
25	127
23	93
26	105
27	108
23	118
26	105
26	81
27	108

a. $r = 0.103$

b. $r = 0.253$

c. $r = 0.321$

d. $r = 2.156$

4. A data set has a correlation coefficient of 0.013. Which statement about the data is true?
   a. The data has little or no linear correlation.
   b. The data has a strong positive linear correlation.
   c. The data has a weak negative linear correlation.
   d. The data has a strong negative linear correlation.

continued

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5. What is the correlation coefficient, *r*, of the data in the table? Use technology to calculate *r*.

x	*y*
68	219
78	192
78	235
60	220
78	240
65	224
74	228
60	212
67	208
68	203
75	191
61	234

a. $r = 0.00337$

b. $r = -0.058$

c. $r = -0.580$

d. $r = -0.134$

6. Anya is studying a textbook. Each day, she works through a few pages of the book. After reading a couple of pages, she began tracking how long it is taking her to work through the textbook, and finds that the equation $y = 14x + 2$ approximates the total pages studied after any number of days. Which statement is true based on this equation?

   a. Anya studies approximately 2 pages each day.

   b. Anya studies approximately 7 pages each day.

   c. Anya studies approximately 14 pages each day.

   d. Anya studies approximately 12 pages each day.

continued

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7. Jordan has a loan at a bank and makes monthly payments. He pays different amounts each month, but he can approximate the total amount he owes the bank using the equation $y = -150x + 1250$, where y is the total amount owed and x is the number of months in the loan repayment. Which statement is true based on the equation?

   a. He started with a total loan of $150.

   b. He started with a total loan of $1,100.

   c. He pays approximately $1,250 each month.

   d. He pays approximately $150 each month.

8. What is the correlation coefficient, r, of the data in the table? Use technology to calculate r.

x	y
15	119
18	73
14	113
14	116
13	135
11	119
13	122
15	92
10	146
12	125
12	125
12	115

   a. $r = -0.846$

   b. $r = 0.716$

   c. $r = -7.462$

   d. $r = 116.67$

9. A data set has a correlation coefficient of –0.892. Which statement about the data set is true?

   a. The data has a strong positive linear correlation.

   b. The data has a weak positive linear correlation.

   c. The data has a strong negative linear correlation.

   d. The data has a weak negative linear correlation.

continued

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10. Which statement describes a causal relationship?

    a. When event *x* increases, event *y* decreases.

    b. Event *x* and event *y* have a linear correlation.

    c. When event *x* increases, event *y* increases.

    d. Event *x* is responsible for a change in event *y*.

Read the scenario and use the table to complete problem 11.

Harris observes small birds that migrate to his yard. He tracks the number of birds by noting how many ounces of birdseed are eaten each day, and records his data in a table.

Day	Ounces eaten
1	4
2	5
3	6
4	12
5	11
6	10
7	18
8	12
9	26
10	22

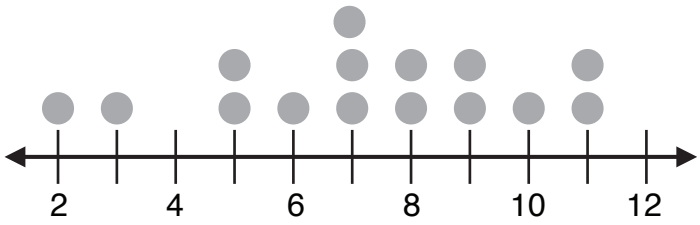
11. Create a scatter plot of the data. Find the correlation coefficient, *r*, and use the shape of the graph and the correlation coefficient to describe the relationship between the day and the bird population.

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Unit 4 Assessment

Circle the letter of the best answer.

1. Which data set is represented by the dot plot?



   a. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
   b. 2, 3, 5, 5, 6, 7, 7, 7, 8, 9, 10, 11, 11
   c. 2, 3, 5, 5, 6, 7, 7, 7, 8, 8, 9, 9, 10, 11, 11
   d. 2, 3, 4, 4, 4, 7, 7, 7, 8, 8, 9, 9, 10, 11, 11

2. Which statement about the following two data sets is true?

   Set A: 15, 18, 19, 19, 20, 24, 24, 25, 25, 25, 28, 28, 29, 30

   Set B: 15, 17, 19, 25, 25, 25, 26, 26, 29

   a. The mean and median are both greater for Set A than for Set B.
   b. The mean and median are both greater for Set B than for Set A.
   c. Set A has a higher mean, but Set B has a higher median.
   d. Set A has a higher median, but Set B has a higher mean.

3. Which points in the data set are outliers?

   73 73 74 75 75 75 77 77 77 77 78 78 89 90

   a. 73, 73
   b. 73, 73, 74
   c. 78, 78, 89, 90
   d. 89, 90

4. What is the interquartile range of the data set?

   16 18 18 19 20 21 23 24 25 26 27 27 27 29 29 29 30

   a. 8.5
   b. 19.5
   c. 24
   d. 28

continued

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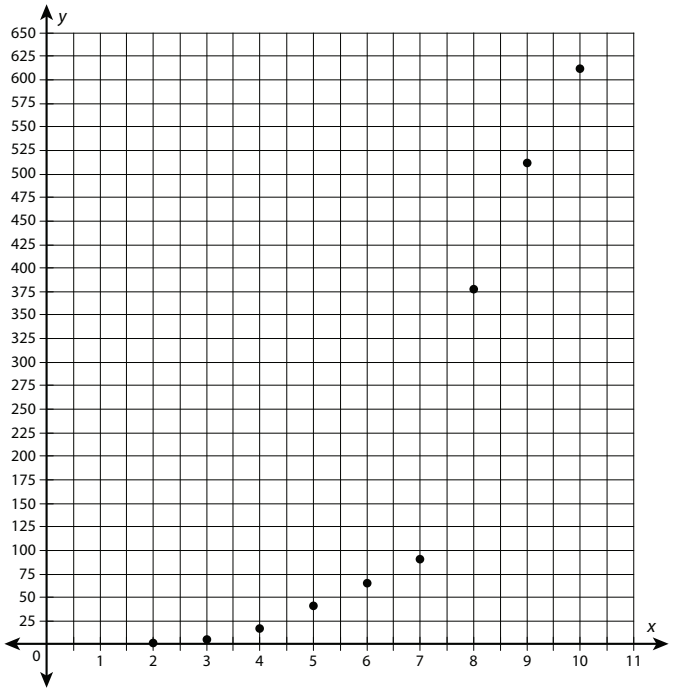
5. Ms. Rosenberg collects information about her students. She records students' favorite movie type in the following table, and separates the responses by age.

Age	Favorite movie genre			
	Comedy	**Romantic comedy**	**Action**	**Thriller**
15 years old	8	14	22	9
16 years old	13	16	18	5

What is the joint frequency of 15-year-olds who prefer action movies?

a. 8
b. 14
c. 18
d. 22

6. Which function is a good fit for the data in the scatter plot?

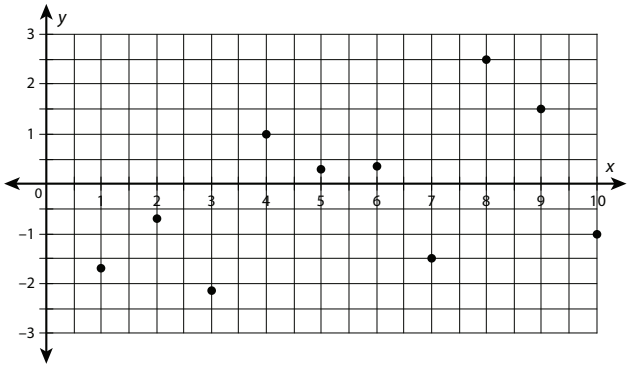


a. $y = 2^x$
b. $y = 77x - 200$
c. $y = 2x - 200$
d. $y = 200x - 77$

continued

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7. A linear function is used to estimate a data set. The residuals of the line fitted to the data are determined and are included in the following residual plot.



What does the residual plot tell you about the line fitted to the data?

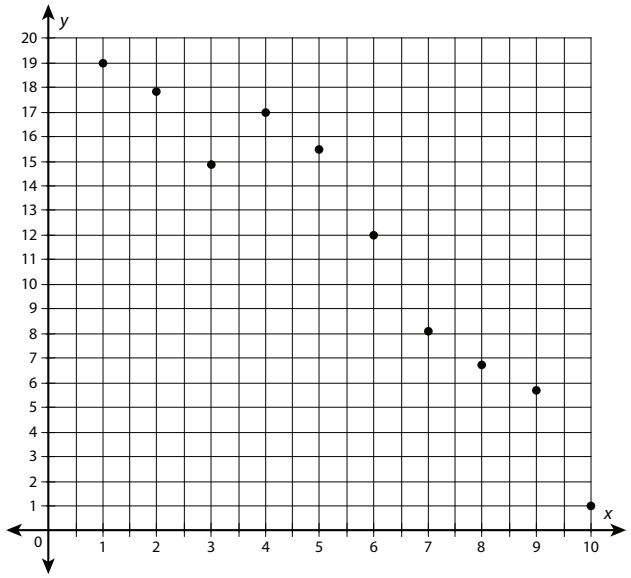
a. A linear function is not a good fit for the data.

b. A quadratic function is the best fit for the data.

c. An exponential function is the best fit for the data.

d. A linear function is a good fit for the data.

8. Which linear function is a good fit for the data in the scatter plot?



a. $y = 1.9x + 22$

b. $y = 1.9x - 22$

c. $y = -1.9x - 22$

d. $y = -1.9x + 22$

continued

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9. A car manufacturer is interested in learning about the amount of money people of different ages spend on a new car. Data regarding customer age and car purchase price are listed in the table.

Customer age	Car price in dollars	Customer age	Car price in dollars
23	25,000	31	30,000
38	35,000	52	44,000
24	25,000	56	47,000
32	31,000	55	47,000
56	47,000	59	50,000
46	40,000	36	34,000
46	41,000	51	44,000
54	46,000	50	43,000
33	31,000		

What is the correlation coefficient, r, of the data in the table? Is this data an example of causation?

a. $r = 0.999$; yes

b. $r = 0.999$; no

c. $r = -0.999$; yes

d. $r = -0.999$; no

10. The events x and y have a correlation coefficient of $r = -0.08$. What is the relationship between x and y?

a. The events have a strong negative linear correlation.

b. The events have a strong positive linear correlation.

c. The events have a weak positive linear correlation.

d. There is little or no linear correlation.

continued

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11. Nolan uses the equation $y = 7.5x$ to estimate the time it will take him to run between 1 and 5 miles, where x is the number of miles and y is the time in minutes. Which statement is true based on the equation?

    a. It takes Nolan approximately 7.5 minutes to run 1 mile.

    b. Nolan runs approximately 7.5 miles in 1 minute.

    c. It takes Nolan approximately 7.5 minutes to run 5 miles.

    d. It takes Nolan approximately 1.5 minutes to run 1 mile.

12. Which linear function is a good fit for the data in the table?

x	*y*
1	6
2	12
3	15
4	24
5	28
6	32
7	35
8	40
9	46
10	52

    a. $y = 4.9x + 2$

    b. $y = 4.9x - 2$

    c. $y = -4.9x + 2$

    d. $y = -4.9x - 2$

continued

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Use your knowledge of data sets to complete the problems that follow.

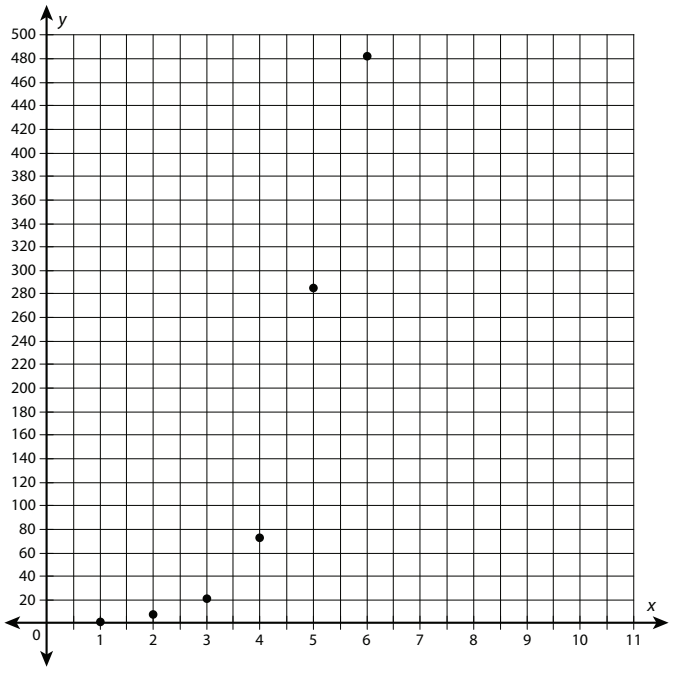
13. Francesca polled her reading group to find out the ages of the group's members (x) and how many books each person has read this year (y). Create a scatter plot for the data set.

x	y
16	30
16	27
18	26
15	30
20	26
16	32
18	29
22	20
19	32
18	31

14. What are the outliers, if any, in the following data set?

93 95 96 98 99 107 113 116 121 121 123 124 125 126 150 163 168

15. What type of function would be a good estimate for the data shown in the scatter plot? Use the shape of the graph to explain your response.



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Answer Key

Lesson 1: Working with a Single Measurement Variable

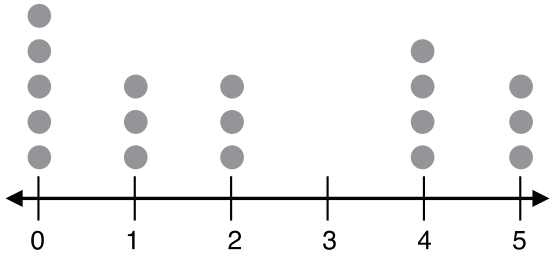
Pre-Assessment, p. U4-1

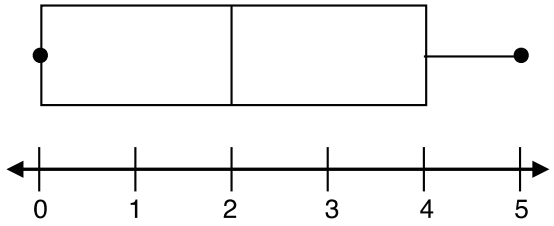
1. c
2. b
3. d
4. b
5. a

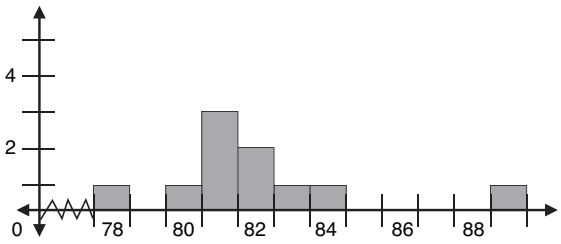
Warm-Up 4.1.1, p. U4-7

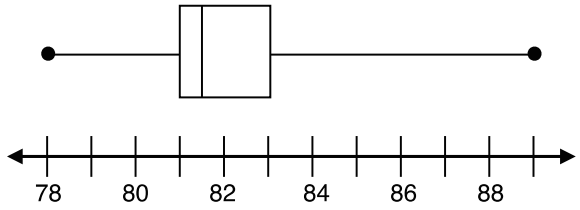
1. 29.5
2. 26
3. 32
4. Classrooms 1 and 7 both have the least number of students.
5. Classrooms 6 and 8 both have the greatest number of students.
6. The interquartile range is 6. Comparing the interquartile range and the median of 29.5 to the lowest data value, 25, the lowest data value is not far away from the mean in terms of the interquartile range. The lowest data value is not a striking deviation. Now compare the median and the interquartile range, 6. The highest data value, 35, is not far away from the mean in terms of the interquartile range. The highest data is not a striking deviation.

Practice 4.1.1: Representing Data Sets, p. U4-27

1. 

2. 2
3. minimum: 0; maximum: 5; first quartile: 0; third quartile: 4
4. 

5. The box plot provides Kirsten with information about the center and spread of her data. The dot plot provides Kirsten with information about the frequency of each response. If Kirsten wants to analyze the frequency of each result, and the range of results, she should use the dot plot.
6. 

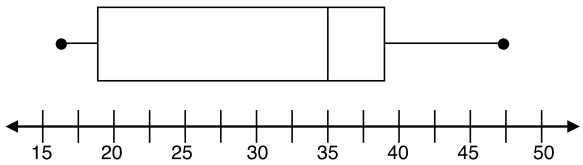
7. 81.5
8. minimum: 78; maximum: 89; first quartile: 81; third quartile: 83
9. 

10. The histogram shows the frequency of each weight. The box plot shows the center and spread of the data. To get a better sense of the expected weight of a 4-year-old German shepherd, Dr. Singh should consult the box plot.

Warm-Up 4.1.2, p. U4-29

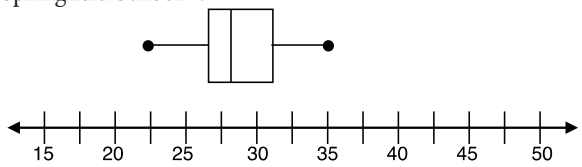
1. 21
2. 22
3. 3.2

Practice 4.1.2: Comparing Data Sets, p. U4-47

1. There are not any data values in either data set that are very small or very large, so either measure of center would be appropriate for this data set. We can use the mean.
2. educational documentary: 23.2; primetime drama: 34.1
3. educational documentary: 5.2; primetime drama: 4.7
4. The mean number of commercials during the primetime dramas is higher than the mean number of commercials during the educational documentaries. The mean absolute deviation of the two data sets is similar, indicating that the variation among the number of commercials during each type of show is similar.
5. Springfield School 1: minimum = 16, maximum = 47, first quartile = 18.5, median = 35, third quartile = 39
Springfield School 2: minimum = 22, maximum = 35, first quartile = 26, median = 28, third quartile = 31
6. Springfield School 1:


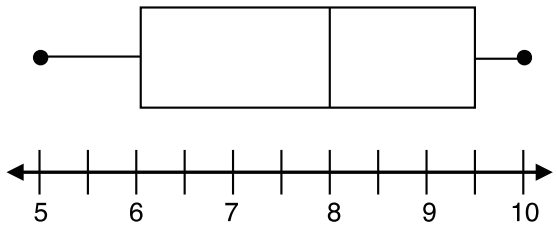
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Springfield School 2:

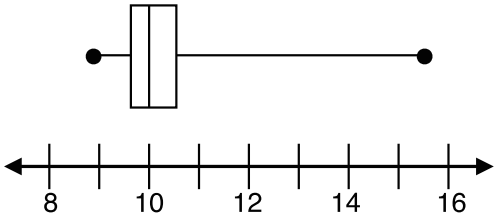


7. The average number of students on Springfield School 1's buses is higher, and the distribution of students on each bus is also higher. There is more variation in the number of students on each of Springfield School 1's buses.
8. The varying distribution of the number of students on the buses could be due to the distribution of the population within each school district. There may be some portions of Springfield School 1's district that only have a few students, whereas other areas of this district have many students. The population in Springfield School 2's district may be more evenly distributed, and each bus route contains a similar number of students. Another possible reason for the variation could be the way the routes were designed. The bus routes in Springfield School 2 may have been more carefully designed to ensure that students were more evenly distributed on the buses.
9. Most employees at Holly's Hamburger Heaven work fewer shifts than the employees at Build-a-Sandwich. This can be seen by examining the distribution of the data in the histograms. Most employees at Holly's Hamburger Heaven worked 4 or fewer shifts per week; however, at Build-a-Sandwich the number of shifts employees worked was clustered around 6 shifts per week.
10. There is a greater variation in the number of shifts worked per employee at Holly's Hamburger Heaven. All of the data for Build-a-Sandwich falls between 4 and 6 shifts per week, whereas the data for Holly's Hamburger Heaven is spread out between 1 and 8.

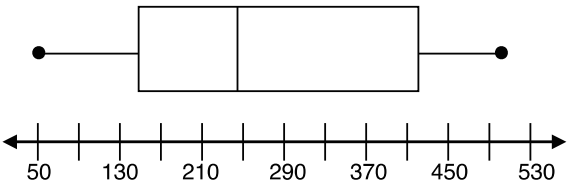
Warm-Up 4.1.3, p. U4-50

1. 8
2. 6
3. 9.5
4. 

Practice 4.1.3: Interpreting Data Sets, p. U4-66

1. Q_1: 9.6, Q_3: 10.6, IQR: 1; 1.5(IQR) = 1.5; Q_1 – 1.5(IQR) = 9.6 – 1.5 = 8.1. There are no data values less than this, so there are no outliers in the lower half of the data. Q_3 + 1.5(IQR) = 10.6 + 1.5 = 12.1. The data value 15.5 is greater than this value, so it is an outlier in the upper half of the data.
2. Mrs. Wong should either use the median or calculate the mean without the outlier. The mean will be increased by the outlier.
3. Median: $9.95; mean without the outlier: $9.87
4. 

5. There is an outlier in the upper half of the data. As a result, the data is skewed to the right. The overall range of the data is increased due to the outlier.
6. Q_1: 145, Q_3: 420, IQR: 275; 1.5(IQR) = 412.5; Q_1 – 1.5(IQR) = 145 – 412.5 = –267.5. There are no data values less than this, so there are no outliers in the lower half of the data. Q_3 + 1.5(IQR) = 420 + 412.5 = 832.5. There are no data values greater than this, so there are no outliers in the upper half of the data.
7. Since there are no outliers, Brady could either use the mean or median to estimate the number of calories in each snack. Mean: 271; median: 245
8. A box plot clearly shows the middle 50% of the data. Minimum: 50; maximum: 500; first quartile: 145; median: 245; third quartile: 420



9. The data is symmetric, because there are no outliers. The tails on either side of the box are both shorter than the box plot. If there were an outlier, one or both tails would be longer.
10. The estimated number of calories in each snack is close to 200 calories, but the box plot shows that the range of the middle 50% of the calories in his snacks is higher than his doctor would likely recommend.

Progress Assessment, p. U4-68

1. b
2. a
3. c
4. d
5. d
6. a
7. c
8. a
9. d
10. d
11. The mean temperature in week 1 is higher, and there is more variation in the temperatures in week 2. Week 1's temperatures are symmetric, and week 2's temperatures are skewed to the left. Week 1 mean: 63.14; week 2 mean: 60.29; week 1 absolute deviation: 2.16; week 2 absolute deviation: 2.82.

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Lesson 2: Working with Two Categorical and Quantitative Variables

Pre-Assessment, p. U4-72

1. b
2. c
3. d
4. d
5. b

Warm-Up 4.2.1, p. U4-77

1. 38 9th graders
2. 25 10th graders
3. more than 4 hours

Practice 4.2.1: Summarizing Data Using Two-Way Frequency Tables, p. U4-92

1. Frequency table:

Gender	English	Math	Science	Social studies
Girl	3	8	3	6
Boy	7	2	7	4

2. Marginal frequencies:

Gender	English	Math	Science	Social studies	Total
Girl	3	8	3	6	20
Boy	7	2	7	4	20
Total	10	10	10	10	40

3. Conditional frequencies relative to the total number of people surveyed:

Gender	English	Math	Science	Social studies
Girl	0.075	0.2	0.075	0.15
Boy	0.175	0.05	0.175	0.1

4. Conditional frequencies relative to gender:

Gender	English	Math	Science	Social studies	Total
Girl	0.15	0.40	0.15	0.30	1.00
Boy	0.35	0.10	0.35	0.20	1.00

5. Using the conditional frequencies relative to the total number of people surveyed, girls' preference for math is the highest value for a subject preference by gender, followed closely by boys' preference of both English and science. Girls preferred math most, and boys equally preferred both English and science.

6. Frequency table:

Age range	Basic phone	Smartphone
Under 25	3	14
25–35	5	15
Over 35	7	6

7. Marginal frequencies:

Age range	Basic phone	Smartphone	Total
Under 25	3	14	17
25–35	5	15	20
Over 35	7	6	13
Total	15	35	50

8. Conditional frequencies relative to type of phone:

Age range	Basic phone	Smartphone
Under 25	0.2	0.4
25–35	0.333	0.429
Over 35	0.467	0.171
Total	1	1

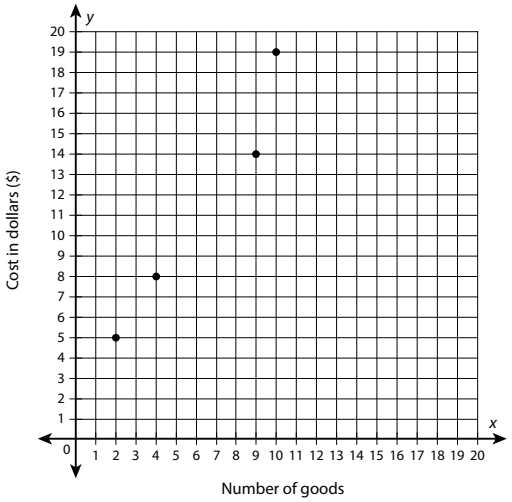
9. Conditional frequencies relative to customers surveyed:

Age range	Basic phone	Smartphone	Total
Under 25	0.06	0.28	0.34
25–35	0.1	0.3	0.4
Over 35	0.14	0.12	0.26
Total	0.3	0.7	1

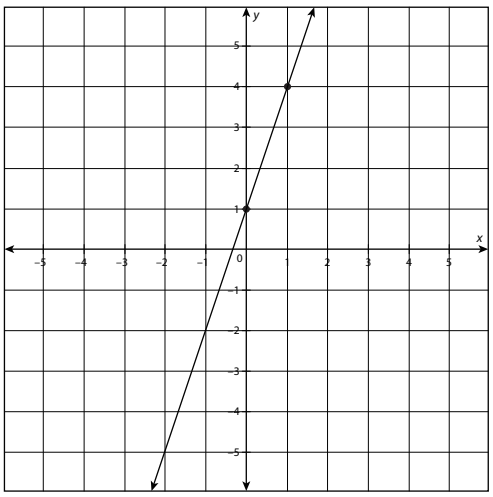
10. Look at the conditional frequencies. Of all people surveyed and all the phones used, the highest conditional frequency is that of customers between 25 and 35 who use smartphones. This is the most popular combination of age and cell phone type, and the company should make smartphones for customers between 25 and 35.

Warm-Up 4.2.2, p. U4-95

1.

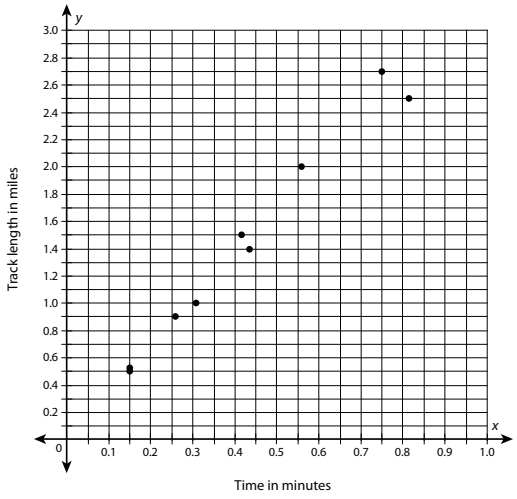


2.

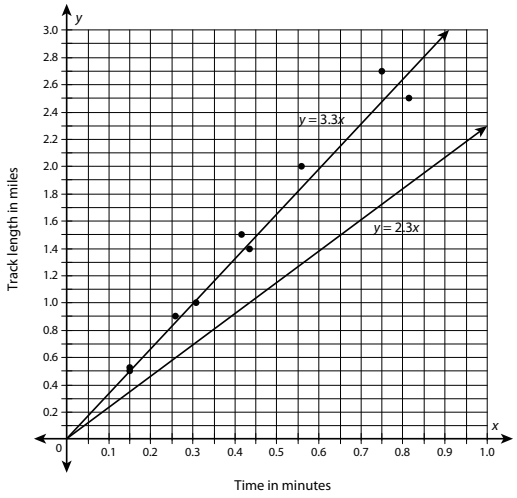


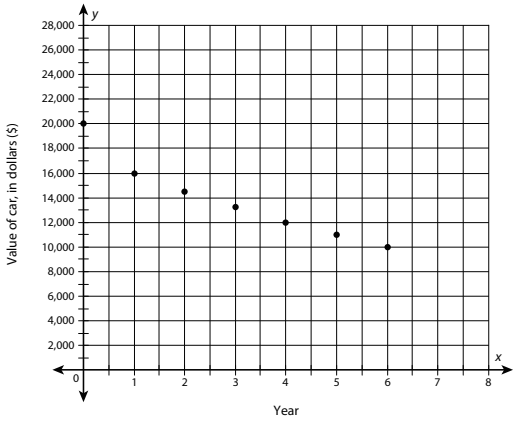
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Practice 4.2.2: Solving Problems Given Functions Fitted to Data, p. U4-116

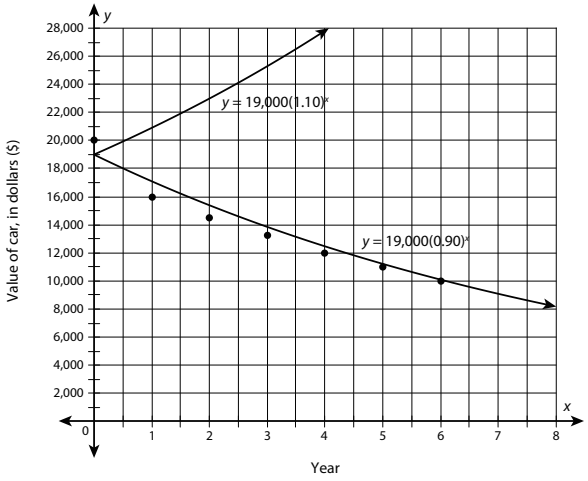
1. 

2. The scatter plot could be estimated using a straight line. A linear function is a better fit for the data.
3. $y = 3.3x$ is a better fit for the data because it more closely follows the trend in the points.

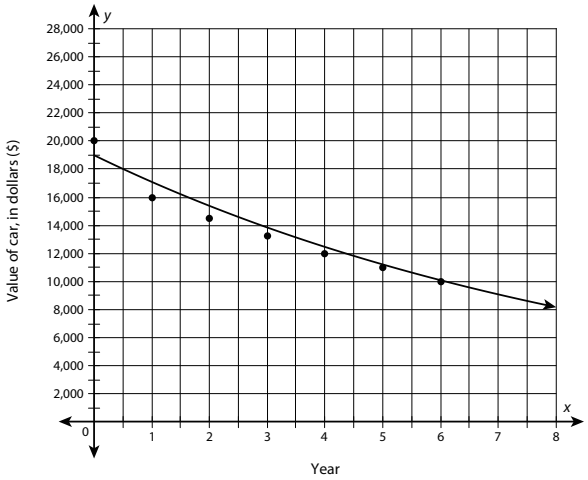


4. 0.545 minutes
5. 1.98 miles
6. 

7. An exponential function; the graph is a curve, and the vertical distance between the points is decreasing as the horizontal distance remains constant.
8. The graph of $y = 19{,}000(1.10)^x$ is not a good estimate for the data because it doesn't approach any of the data points on the scatter plot. However, the graph of $y = 19{,}000(0.90)^x$ is a good estimate because it goes through or closely approaches all of the data points.

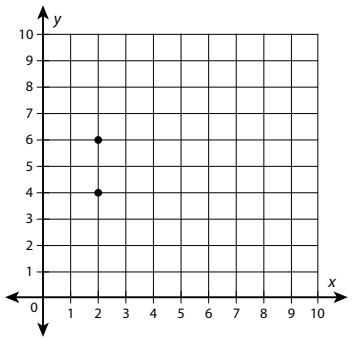


9. 7.1 years, because looking at the trend in the graph, the curve falls through $9,000 at about 7.1 years.



10. approximately $5,366

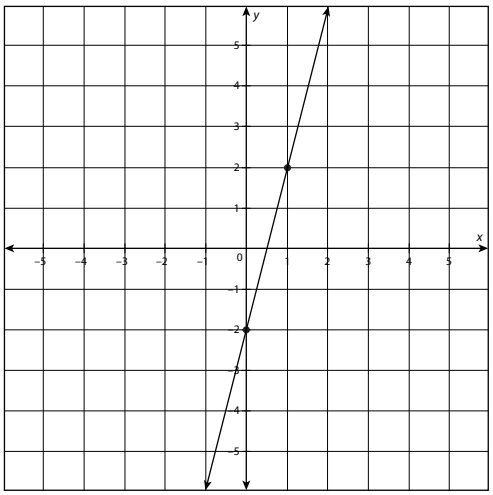
Warm-Up 4.2.3, p. U4-118

1. 

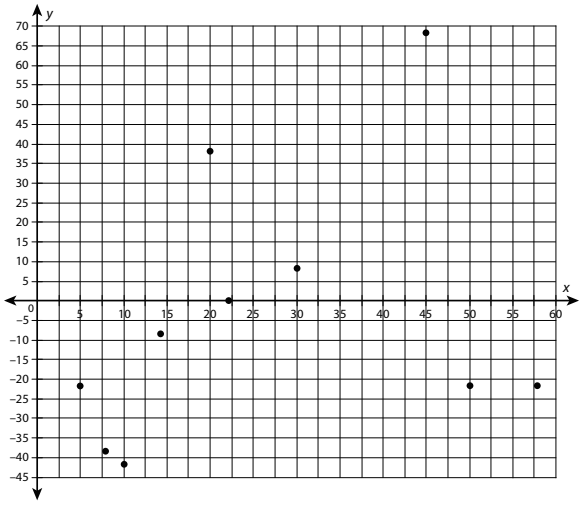
2. The distance is the absolute value of the difference between the two y-values: $|6 - 4| = 2$.

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3.

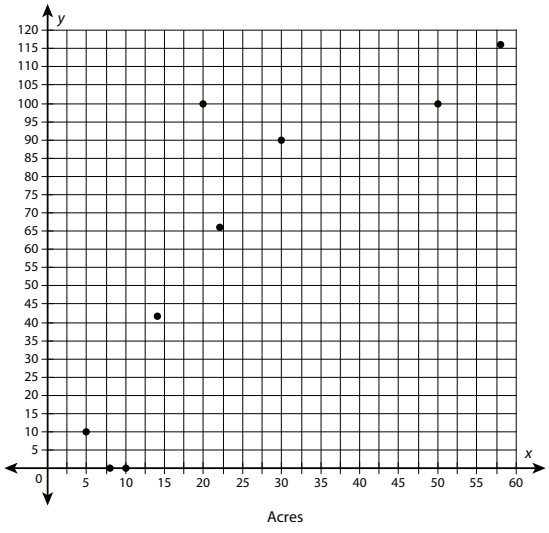


4. The plot appears random, so a linear function is likely a good fit for the data.

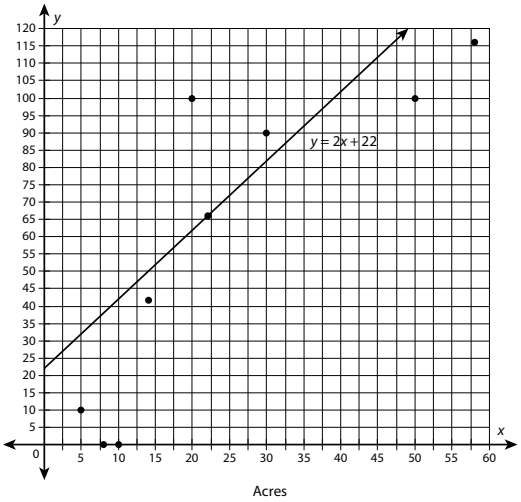


Practice 4.2.3: Analyzing Residuals, p. U4-139

1.

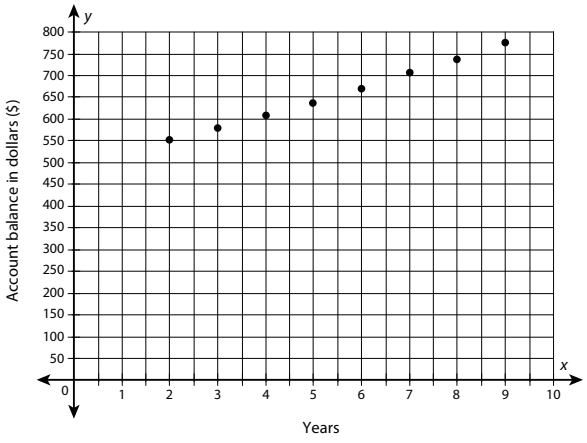


2.

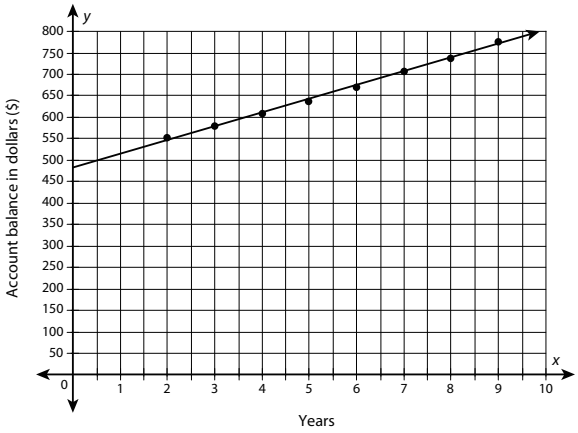


3. Yes; it follows the shape of the data.

5.



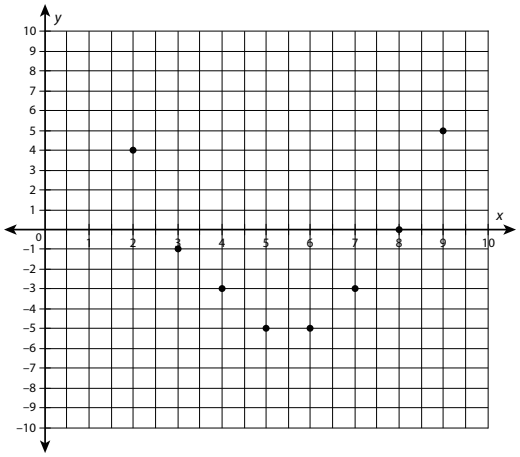
6.



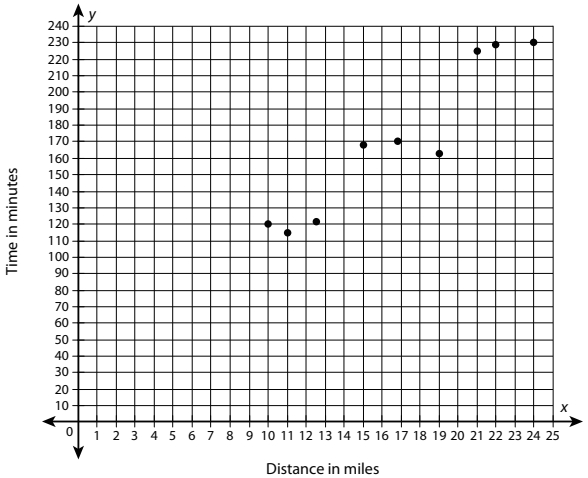
7. Yes; the line appears to follow the shape of the data.

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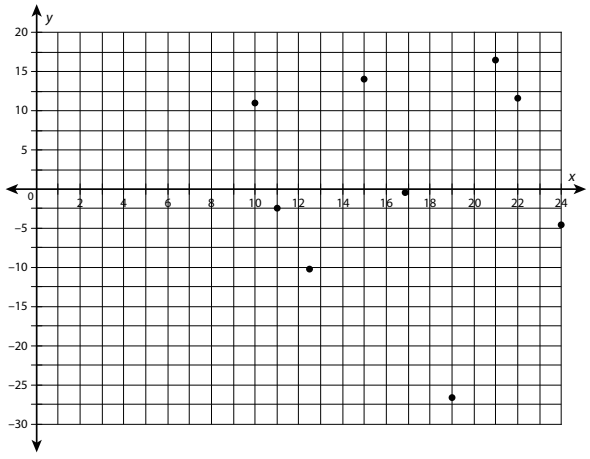
8. Though it appeared that the line fit the data, the U-shape of the residual plot indicates that a linear function is not a good fit for the data.



9.

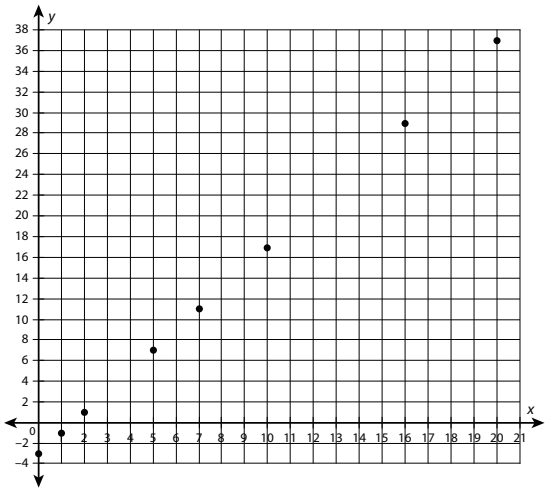


10. The residual plot is random, so a linear function is a good fit.

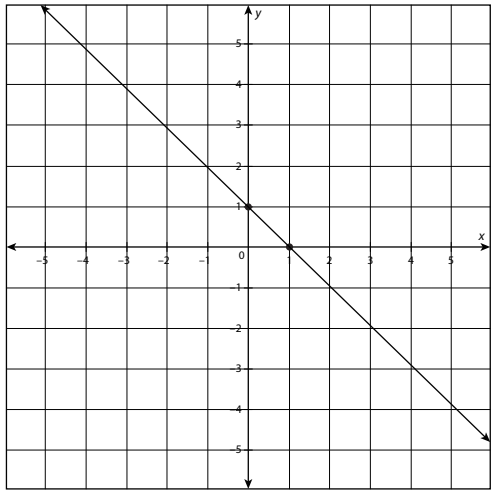


Warm-Up 4.2.4, p. U4-141

1.

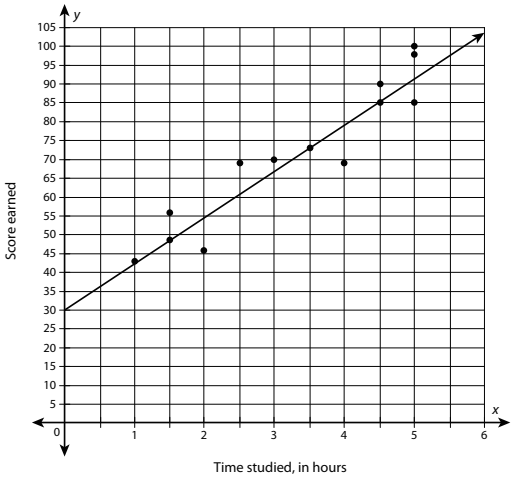


2. The points appear to be on a straight line. The points are linear.
3. The slope of the line is 8.
4.



Practice 4.2.4: Fitting Linear Functions to Data, p. U4-159

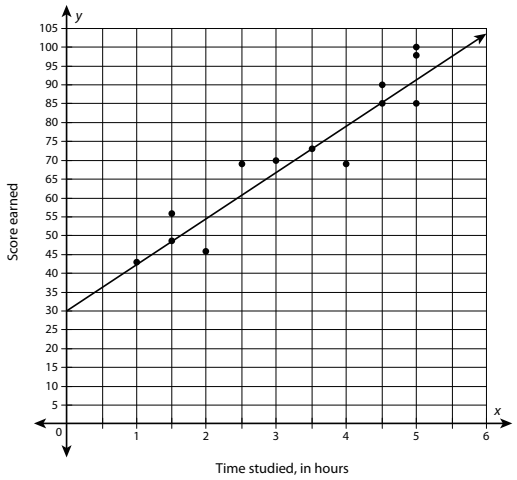
1.



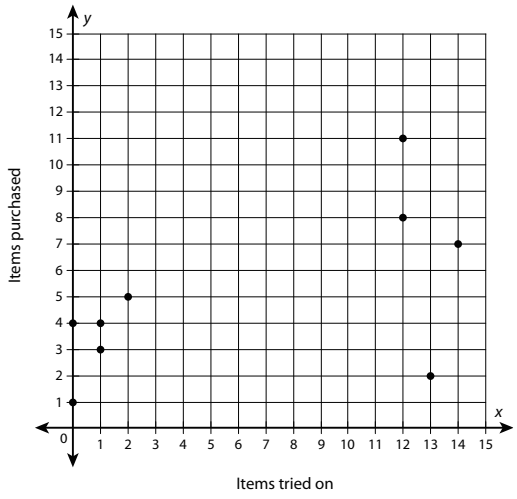
2. The data appears to be in a line.

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3. Answers will vary. One line through (1.5, 48) and (4.5, 85) is $y = 12.3x + 30$.

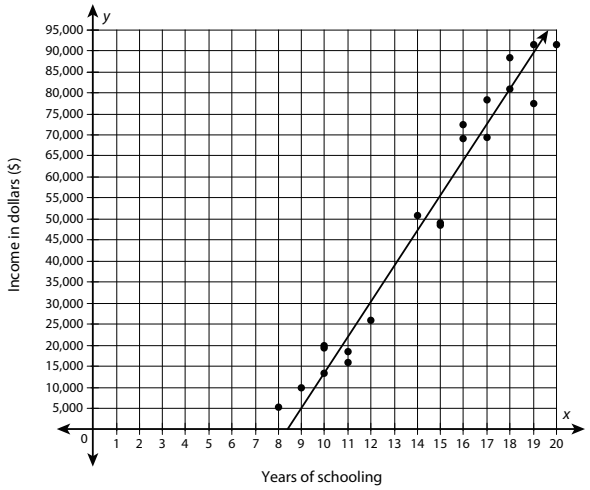


4. Answers will vary depending on points chosen. Sample answer: $y = 12.3x + 30$

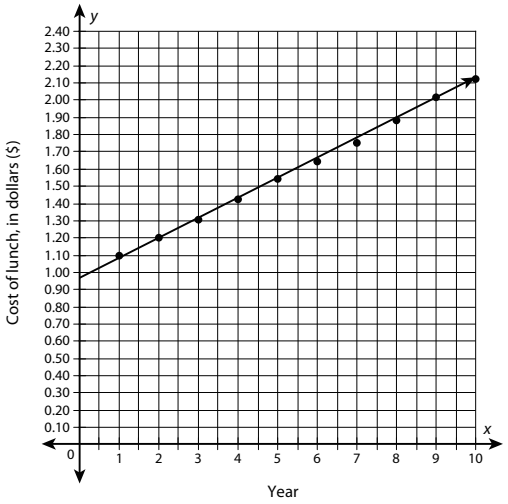
5. 

The data appears to be random; it doesn't follow an exponential or linear shape.

6. No; it does not look like there is a linear relationship between the number of items tried on and the number of items purchased based on the shape of the graph.

7. 

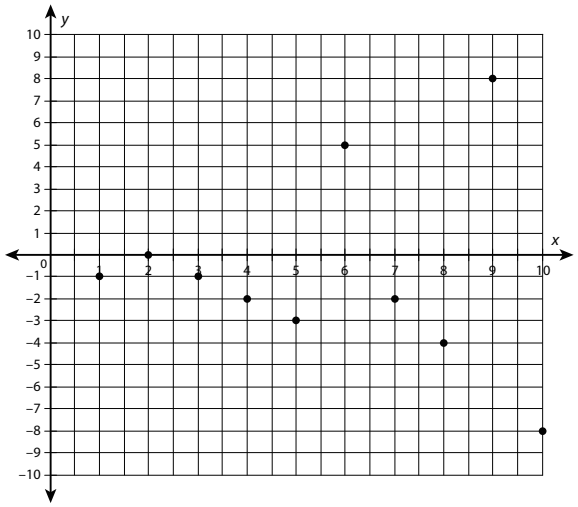
8. Answers will vary depending on points chosen. Sample answer: $y = 8490x - 71{,}848$

9. 

10. Answers will vary depending on points chosen. Sample answer: $y = 0.116x + 0.966$

Progress Assessment, p. U4-163

1. c
2. d
3. a
4. a
5. b
6. d
7. b
8. d
9. b
10. c

11. 

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Lesson 3: Interpreting Linear Models

Pre-Assessment, p. U4-169

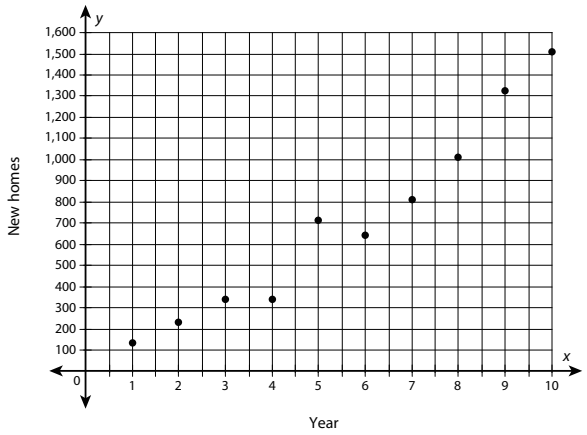
1. b
2. c
3. b
4. d
5. a

Warm-Up 4.3.1, p. U4-173

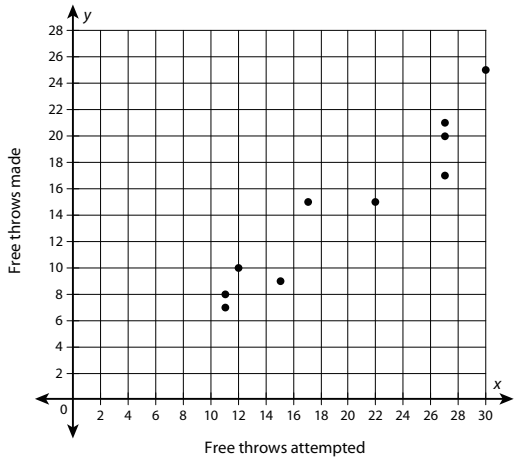
1. Slope = $\frac{1}{16}$; y-intercept = (0, 0)
2. $y = \frac{1}{16}x$
3. –1
4. –2

Practice 4.3.1: Interpreting Slope and y-intercept, p. U4-190

1.



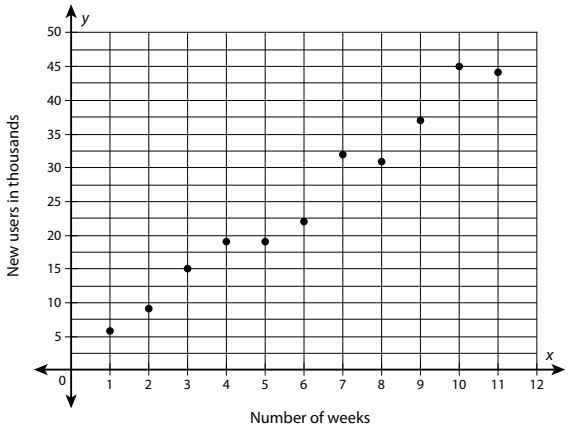
2. Answers may vary; equations should be close to $y = 150x - 120$.
3. The number of homes built each year is increasing by 150. In this case, when $x = 0$, $y = -120$. This value does not make sense in context, since the number of new homes built in a year cannot be negative.
4. Answers may vary; equations should be close to $y = 17.25x + 7.75$.
5. It takes Madeline approximately 17 minutes to complete each homework assignment. In this case, when Madeline has no assignments, her time to complete the assignments would be 0. The y-intercept does not make sense in this context.
6.



7. Answers may vary; equations should be close to $y = 0.77x - 0.7$.
8. Will makes approximately 77% of his free throws. In this context, if Will does not attempt any free throws, he would not make any free throws; when $x = 0$, y should equal 0. The y-intercept is not relevant in this context.
9. Answers may vary; equations should be close to $y = 7.6x + 3.8$.
10. It takes the construction company approximately 7.6 weeks to construct a building for each floor of the building. In this context, if the building doesn't have any floors, the time to construct the building would be 0; when $x = 0$, y should equal 0. The y-intercept is not relevant in this context.

Warm-Up 4.3.2, p. U4-194

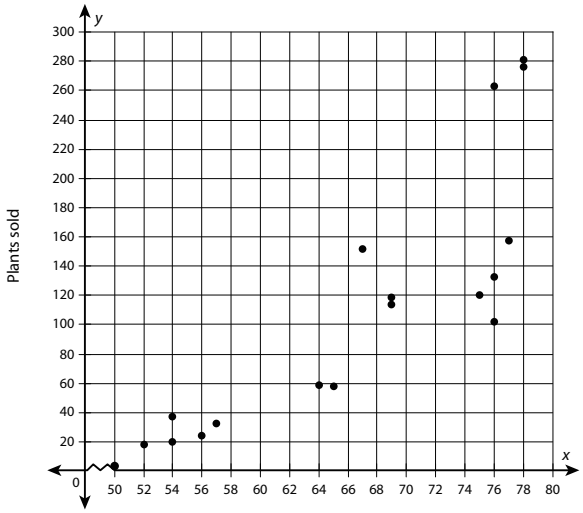
1.



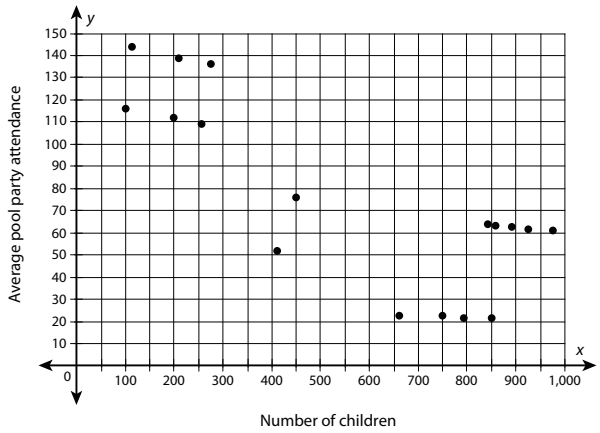
2. The graph of a linear function is a line. The shape of the data in the graph is linear because a line could be drawn on the graph with an approximately equal number of points above and below the line. The data appears to have a linear relationship.

Practice 4.3.2: Calculating and Interpreting the Correlation Coefficient, p. U4-208

1. strong positive linear correlation
2. weak positive linear correlation
3. strong negative linear correlation
4. no correlation, or very weak negative linear correlation
5.

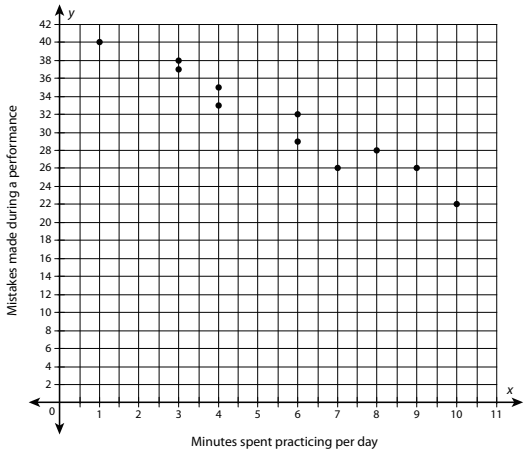


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6. There appears to be a positive linear correlation between temperature and plant sales.
7. $r = 0.848$, which indicates there is a strong positive linear correlation between temperature and plant sales.
8. 

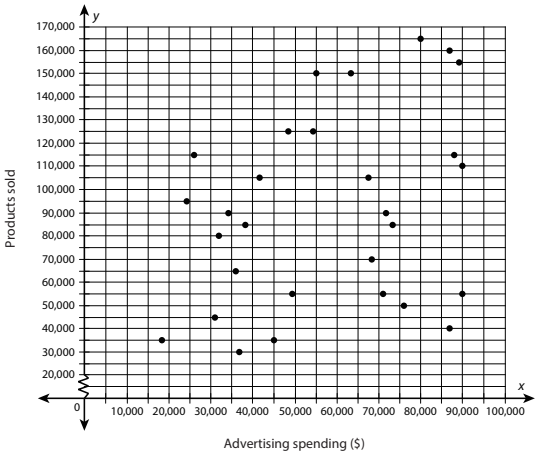
9. Pool party attendance appears to decrease as the number of children increases; there may be a negative linear correlation between the data.
10. $r = -0.779$; there is a negative linear correlation between the data.

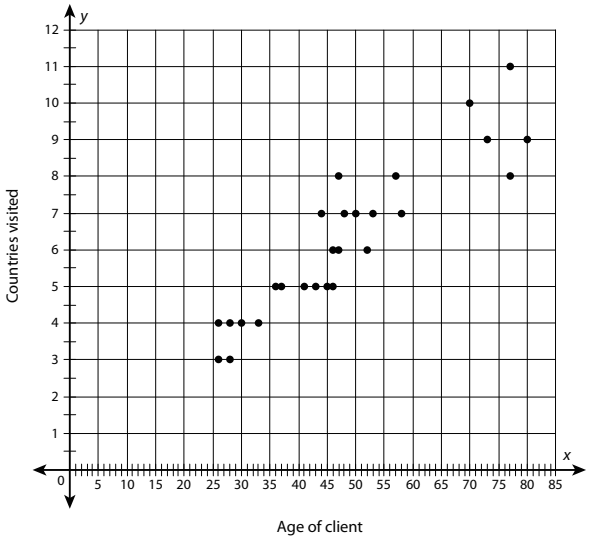
Warm-Up 4.3.3, p. U4-211

1. The shape of a linear graph is a straight line. The data seems to follow a straight line, and it appears that there is a linear correlation between x and y.
2. 

3. Using a calculator, $r = -0.97$. The correlation coefficient is very close to –1, so there is a strong negative linear correlation between x and y.

Practice 4.3.3: Distinguishing Between Correlation and Causation, p. U4-229

1. 

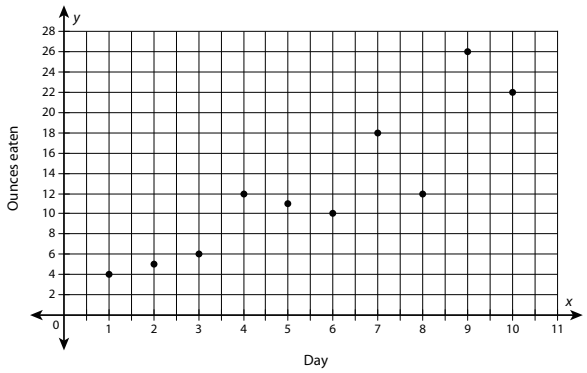
2. The graph appears to be random, with some increase in numbers of products sold as advertising dollars increase.
3. $r = 0.311$; this indicates that there is a weak positive linear correlation between advertising dollars spent and the number of products sold.
4. The correlation between the two values is weak, and it is difficult to determine if there is any causal relationship between advertising dollars spent and the number of products sold.
5. 

6. As age increases, number of countries also increases; the graph follows a linear shape.
7. $r = 0.912$; this indicates that there is a strong positive linear correlation between age and number of countries visited.
8. Even though there is a strong correlation between age and number of countries visited, this does not mean that age is responsible for the number of countries visited. An older person has had more opportunity to travel, but the age of the client does not impact the number of countries the client has chosen to visit. It is not likely that there is a causal relationship between age and number of countries visited.
9. As temperature increases, time decreases. The graph generally has a linear shape with a negative slope.

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10. A few factors influence the time it takes water to boil: volume of water, heat applied to water, distribution of water within its container, and air pressure. The students attempt to keep three of these factors consistent throughout the experiment, by using identical containers, performing the experiment with identical burners, and using the same volume of water. The decrease in time is therefore likely due to the increase in temperature, and it is likely that there is a causal relationship between temperature applied and time to boil.

Progress Assessment, p. U4-232

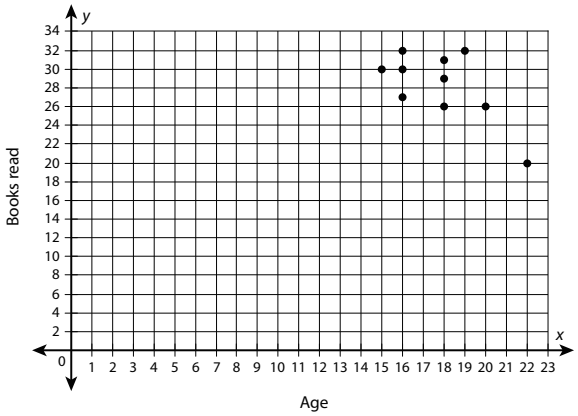
1. a
2. b
3. c
4. a
5. b
6. c
7. d
8. a
9. c
10. d
11. 

$r = 0.892$

There is a strong positive linear correlation between the day and the ounces of birdseed eaten, indicating the population of birds is steadily increasing.

Unit Assessment

p. U4-237

1. c
2. c
3. d
4. a
5. d
6. a
7. d
8. d
9. b
10. d
11. a
12. a
13. 

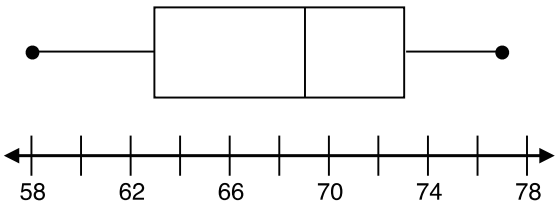
14. Yes; 168
15. Exponential; the graph is curved and there is not a constant slope.

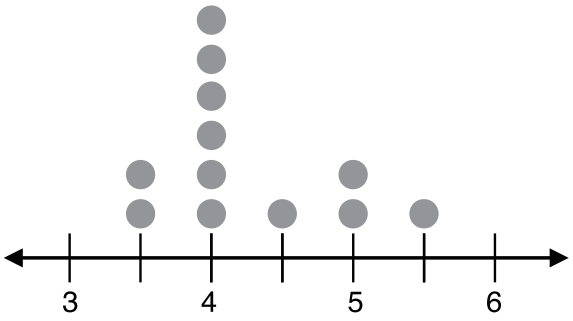
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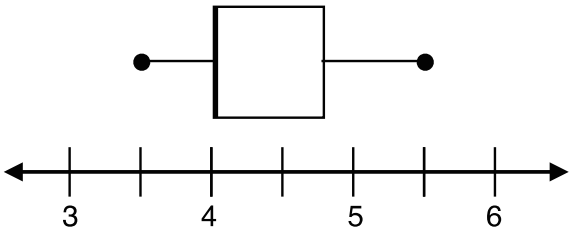
Student Book Answer Key

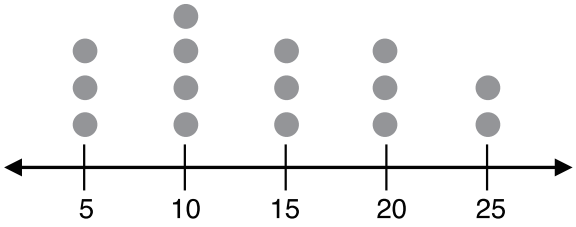
Lesson 1: Working with a Single Measurement Variable

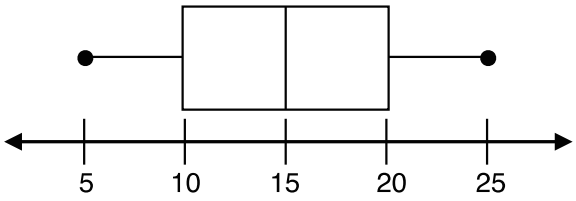
Practice 4.1.1: Representing Data Sets, p. U4-15

1. minimum: 58; maximum: 77; median: 69
2. first quartile: 63; third quartile: 73
3. 

4. There are a couple of different ways to present this information. In a frequency plot, the rainfall in each month would be represented with a dot. The dots could be counted for data values that are greater than 3.5. This information could also be presented with a box plot. Six months is half of the year, so half of the data would need to be greater than 3.5. The median divides the data into two halves. If the median is equal to or greater than 3.5, then in half of the year the rainfall is greater than 3.5 inches a month. (Either graph is acceptable with explanation.)
5. Dot plot:


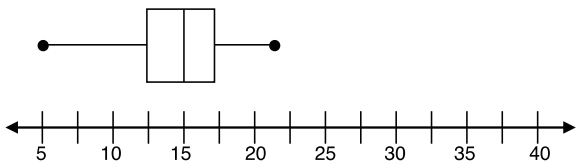
Box plot:


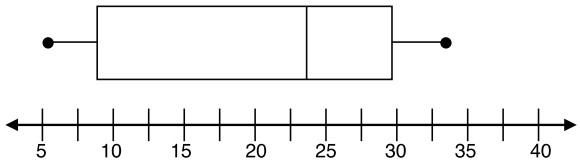
6. A frequency plot will show the toy shop manager the number of times each type of gift card was purchased. Both histograms and dot plots show the frequency of responses. Since there are a small number of data values, and each response is for a single numerical value, a dot plot would be the best type of frequency plot.
7. 

8. A box plot shows the median, which is a measure of center, and interquartile range, which is a measure of spread. A box plot would help the toy store manager understand the center and spread of the data.
9. 

10. The median gift card value was $15. If the new set of data has a median of $20, then more cards must have been sold that were $20 or more.

Practice 4.1.2: Comparing Data Sets, p. U4-27

1. There are not any data values in either data set that are very small or very large, so either measure of center would be appropriate for this data set. We can use the mean.
2. Class 1: 9.4; Class 2: 8.67
3. Class 1: 2.43; Class 2: 3.87
4. The students in the first class had a slightly older average age, but there was less variation in the student responses.
5. Tuesday: minimum = 5.00, maximum = 21.40, first quartile = 12.40, median = 15.00, third quartile = 17.10
Wednesday: minimum = 5.60, maximum = 33.70, first quartile = 8.60, median = 23.50, third quartile: 29.90
6. Tuesday


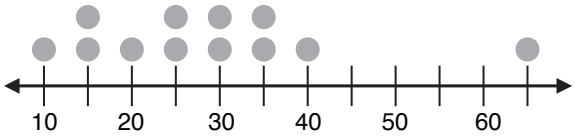
Wednesday


7. Chloe's commissions on Tuesday have less variation, and a lower median than her commissions on Wednesday. Her sales are more consistent Tuesday than on Wednesday.

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8. Though Chloe's commissions have a greater variation on Wednesday, her mean commission is higher than her mean commission on Tuesday. The middle 50% of her commission values are from 8.60 to 29.90, whereas on Tuesday the middle 50% of her commission values are from 12.40 to 17.10. A small portion of the interquartile range for Wednesday is less than the interquartile range for Tuesday, but overall it appears that she is more likely to earn higher commissions on Wednesday.
9. It appears that Florida's school lunch prices have a higher mean than South Carolina's school lunch prices. This is seen in the graph by the clustering of data points around a higher value in Florida's dot plot, from 2.25 to 2.75. South Carolina's school lunch prices cluster around the values 1.75 to 2.25.
10. South Carolina's lunch prices have greater variation. The dots are spread more evenly amongst the range of lunch prices, whereas Florida's dots are concentrated on a few values.

Practice 4.1.3: Interpreting Data Sets, p. U4-39

1. Q_1: 2.5, Q_3: 9.5, IQR: 7; 1.5(IQR) = 10.5; $Q_1 - 1.5(IQR)$ = 2.5 – 10.5 = –8. There are no data values less than this, so there are no outliers in the lower half of the data. $Q_3 + 1.5(IQR)$ = 9.5 + 10.5 = 20. There are no values greater than this value, so there are also no outliers in the upper half of the data.
2. Mr. Wilde should either use the median or the mean.
3. Median: 4.5; mean: 6.05
4. 

5. Q_1: 17.5, Q_3: 35, IQR: 17.5; 1.5(IQR) = 26.25; $Q_1 - 1.5(IQR)$ = 17.4 – 26.25 = –8.75. There are no data values less than this, so there are no outliers in the lower half of the data. $Q_3 + 1.5(IQR)$ = 35 + 26.25 = 61.25. The greatest data value, 65, is greater than this value and is therefore an outlier.
6. Because there is an outlier, Rosa should either use the median or calculate the mean without the outlier. Median: $27.50; mean without the outlier: $25.45
7. Due to the outlier, the data is skewed to the right. The overall range of the data is greater due to the outlier.
8. No; the smallest and greatest values are not much smaller than or much larger than the rest of the data set. $Q_1 - IQR$ = –350 and $Q_3 + 1QR$ = 2450. There are no data values outside of these ranges, and therefore no outliers.
9. Because there are no outliers, Jackson can use either the mean or median to estimate his score. Mean: 1,033.33; median: 1,000.
10. The score on the 16th game, when averaged with the other scores, needs to increase the mean to at least 1033.33 + 100 = 1133.33. Using the histogram, we can see that the scores are:
500 700 700 700 800 900 900 1000
1000 1300 1300 1400 1400 1400 1500
Since the mean score needs to increase, the score needs to be greater than the current mean. Try different score values and calculate the new mean with the 16 scores. A score of 2,700 would increase his mean to 1,137.5.

Lesson 2: Working with Two Categorical and Quantitative Variables

Practice 4.2.1: Summarizing Data Using Two-Way Frequency Tables, p. U4-50

1. Frequency table:

Location	Library	Playground	Recreation center
North	10	8	5
South	9	5	13

2. Marginal frequencies:

Location	Library	Playground	Recreation center	Total
North	10	8	5	23
South	9	5	13	27
Total	19	13	18	50

3. Conditional frequencies, total number of people:

Location	Library	Playground	Recreation center	Total
North	0.2	0.16	0.1	0.46
South	0.18	0.1	0.26	0.54
Total	0.38	0.26	0.36	1

4. Conditional frequencies, total number from each location:

Location	Library	Playground	Recreation center	Total
North	0.43	0.35	0.22	1.00
South	0.33	0.19	0.48	1.00

5. Using the conditional frequencies relative to the total number of people surveyed, the preference for a recreation center by those from the southern part of town has the highest value, followed closely by northern town residents' preference for a library. Northern town residents preferred a library, and southern town residents preferred a recreation center.

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6. Because the northern and southern town residents have distinct preferences, the different types of buildings could be built in the area with the greatest preference frequency. The library could be built in the northern part of town, and the recreation center could be built in the southern part of town. The northern residents preferred the library at a higher rate than the southern residents (0.35 to 0.19, respectively), and the playground could be built in the northern part of town.
7. Marginal frequencies:

Gender	Pizza	Hamburger	Hot dog	Spaghetti	Total
Male	18	17	9	12	56
Female	14	7	13	19	53
Total	32	24	22	31	109

8. Conditional frequencies relative to gender:

Gender	Pizza	Hamburger	Hot dog	Spaghetti	Total
Male	0.32	0.30	0.16	0.21	1.00
Female	0.26	0.13	0.25	0.36	1.00

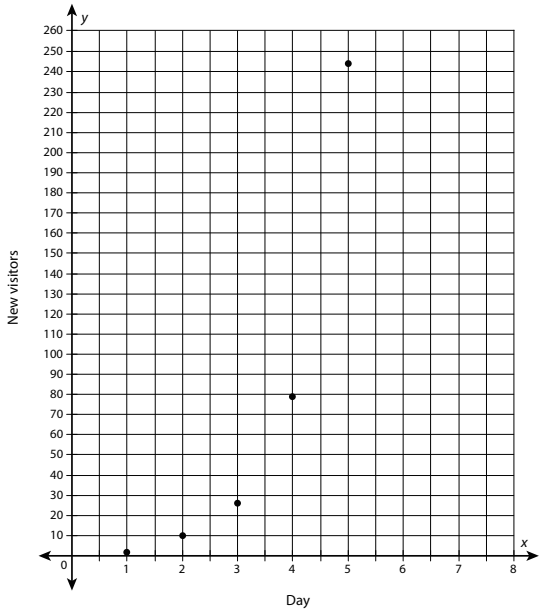
9. Conditional frequencies relative to the total number of people surveyed:

Gender	Pizza	Hamburger	Hot dog	Spaghetti	Total
Male	0.17	0.16	0.08	0.11	0.51
Female	0.13	0.06	0.12	0.17	0.49
Total	0.29	0.22	0.20	0.28	1.00

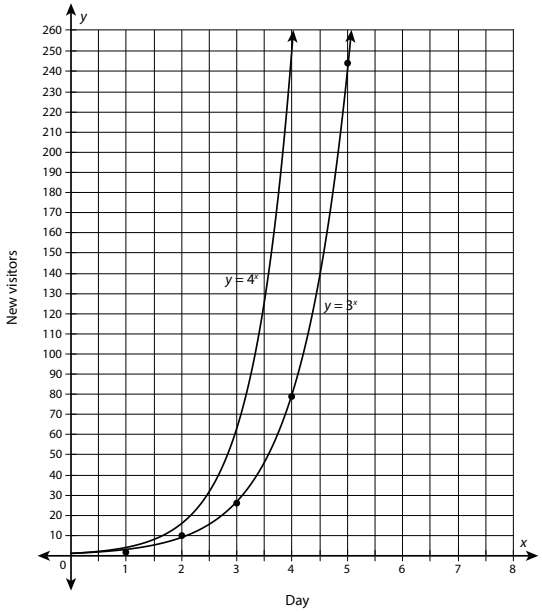
10. The males preferred pizza and the females preferred spaghetti. Overall, these two preferences were nearly equal.

Practice 4.2.2: Solving Problems Given Functions Fitted to Data, p. U4-65

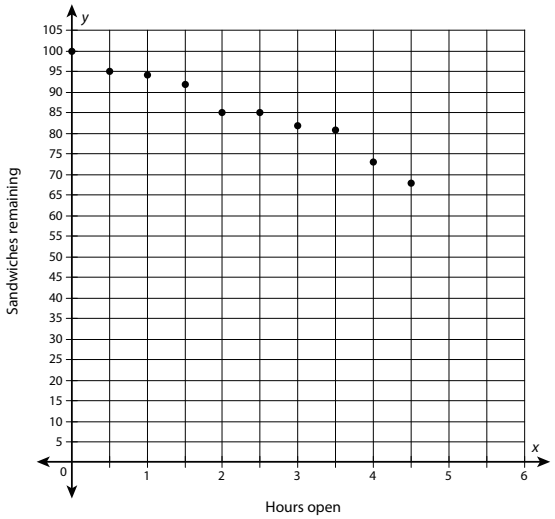
1.



2. An exponential function; the graph is a curve, and the vertical distance between the points is increasing as the horizontal distance remains constant.
3. The function $y = 3^x$ is a better fit because it more closely follows the data trend.



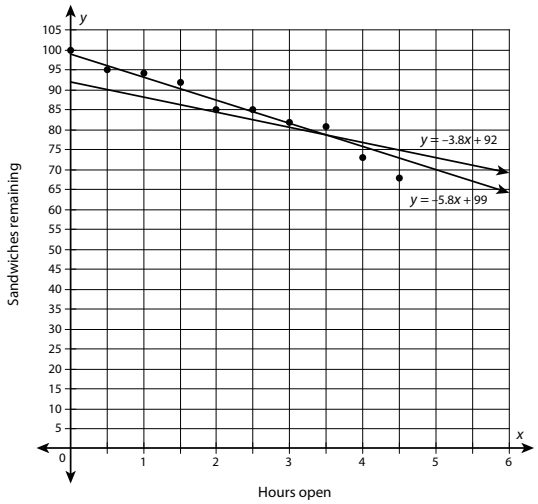
4. on day 6
5. approximately 2,187 visitors
6.



7. The scatter plot could be estimated using a straight line. A linear function is a better fit for the data.

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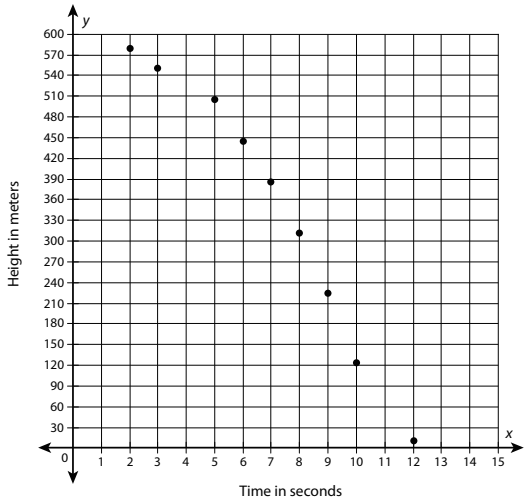
8. $y = -5.8x + 99$, because the curve follows the trend in the data.



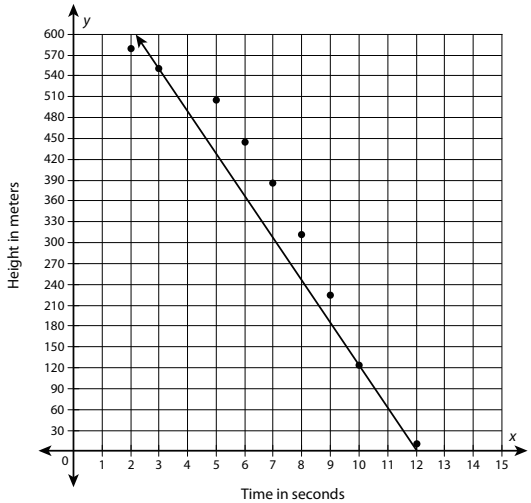
9. approximately 98 sandwiches
10. approximately 17 hours

Practice 4.2.3: Analyzing Residuals, p. U4-79

1.

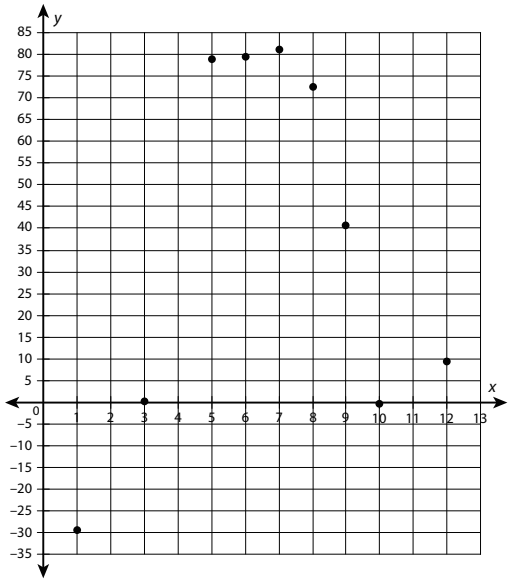


2.

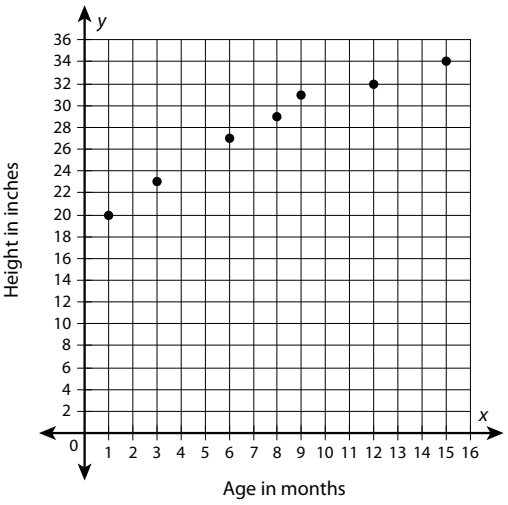


3. It comes close to the data, but the line does not follow the shape of the data.

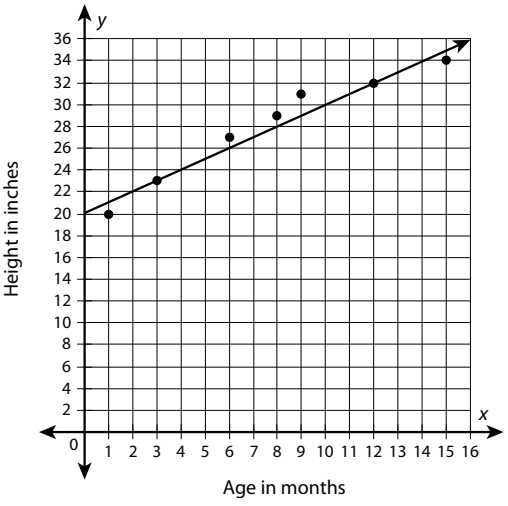
4. The residual plot has a U-shape, indicating a linear function is not the best fit for the data.



5.



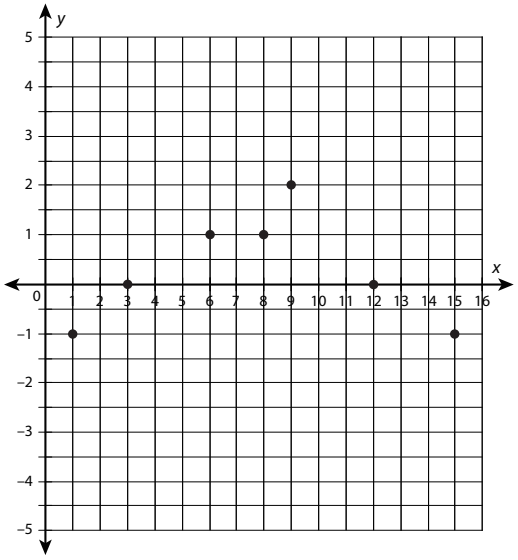
6.



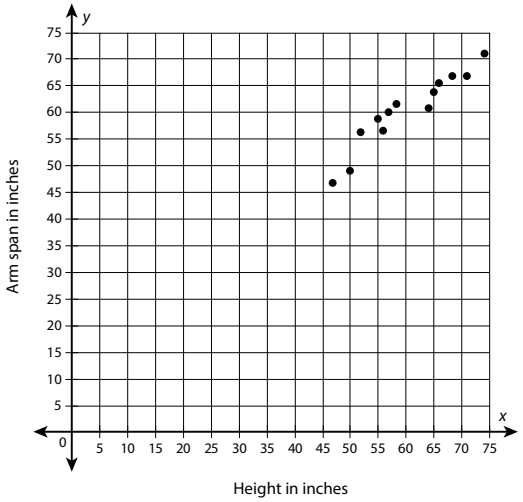
7. Yes; it follows the shape of the data set.

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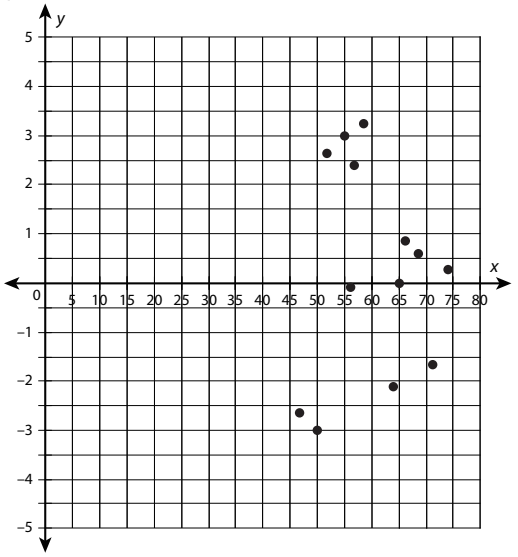
8. When looking at the residual plot, the shape appears to be a U. A linear function is not a good fit for the data.



9.

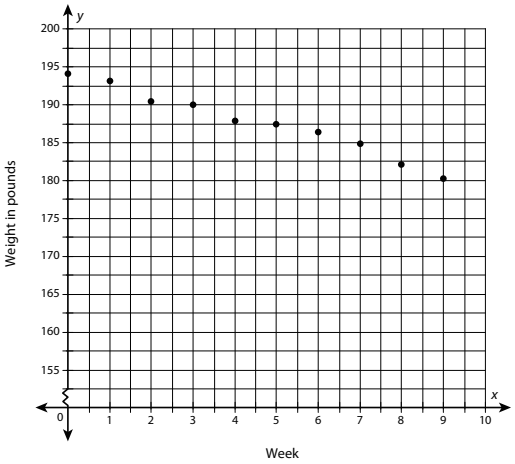


10. The residual plot is random, so a linear function is a good fit.



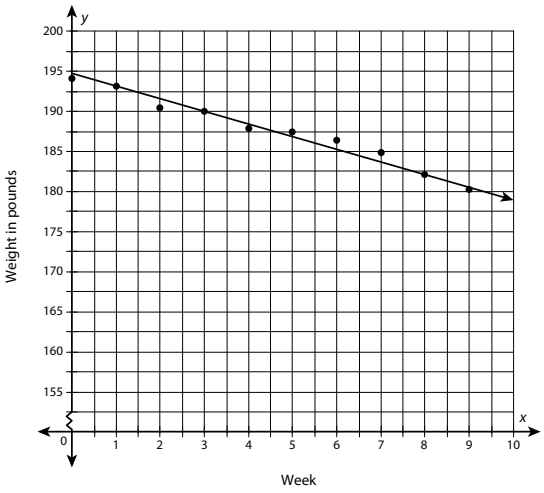
Practice 4.2.4: Fitting Linear Functions to Data, p. U4-92

1.



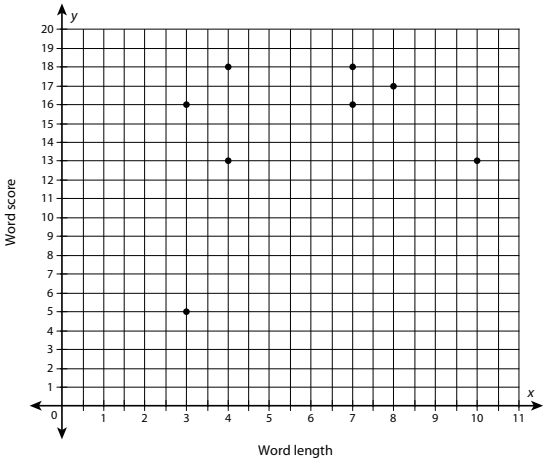
2. The data appears to be linear.

3.



4. Answers will vary based on points chosen. Sample answer: $y = -1.6x + 194.8$

5.

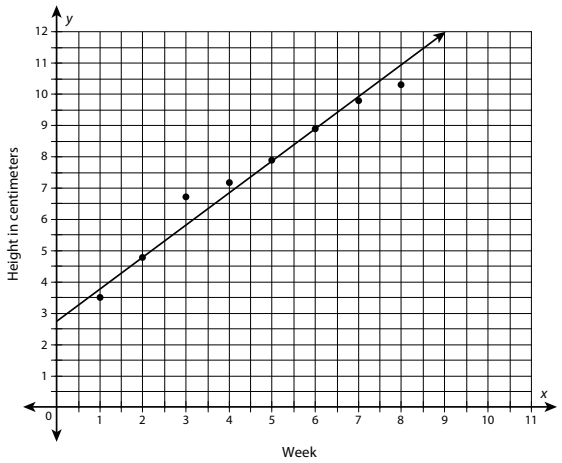


The shape of the data is neither linear nor exponential.

6. No; the data does not appear to be linear based on the randomness of the scatter plot.

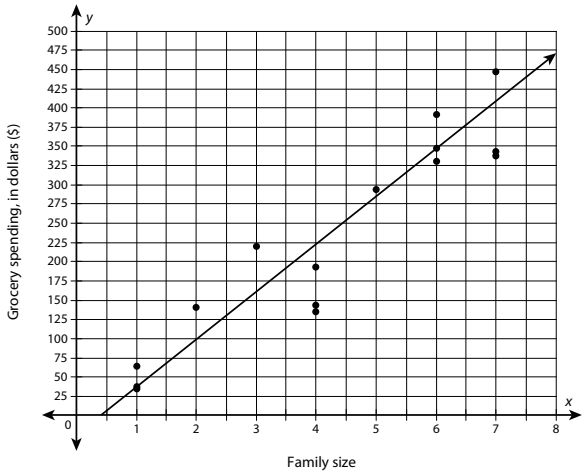
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7.



8. Answers will vary based on points chosen. Sample answer: $y = 1.025x + 2.75$

9.

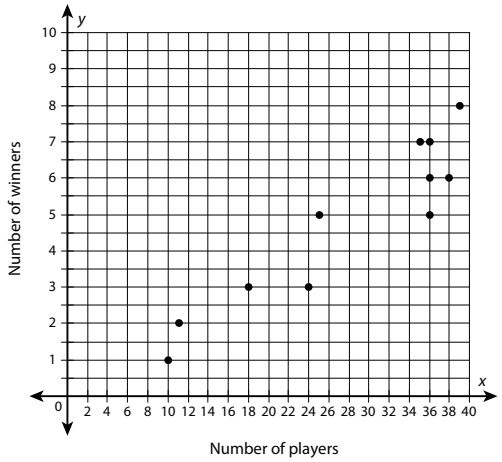


10. Answers will vary based on points chosen. Sample answer: $y = 62.2x - 25.2$

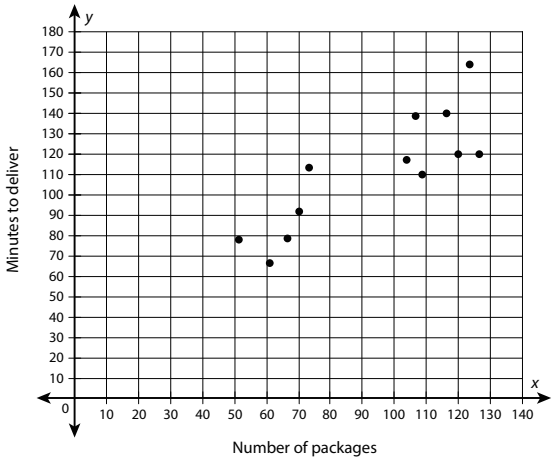
Lesson 3: Interpreting Linear Models

Practice 4.3.1: Interpreting Slope and *y*-intercept, p. U4-107

1.



2. Answers may vary; equations should be close to $y = 0.19x - 0.60$.
3. A player wins the game approximately 19% of the time. In this case, when there are no players, there would be no winners; when $x = 0$, then y should equal 0. The y-intercept is not relevant in this context.
4.



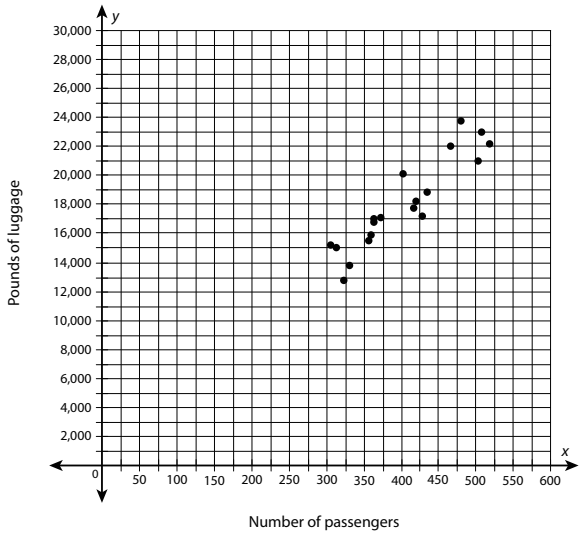
5. Answers may vary; equations should be close to $y = 0.87x + 29.9$.
6. It takes the company approximately 0.87 minutes to deliver each package. In this case, if there are no packages to deliver, it would take no time to deliver the package; when $x = 0$, y should equal 0. The y-intercept is not relevant in this context.
7. Answers may vary; equations should be close to $y = 18.25x - 6.8$.
8. Each cow produces approximately 18 liters of milk each day. In this case, if there are no cows, there is no milk production; when $x = 0$, y should equal 0. The y-intercept is not relevant in this context.
9. Answers may vary; equations should be close to $y = 1.80x + 51.21$.
10. A person's height increases by approximately 1.8 inches for each shoe size. In this case, if a shoe size is 0, there is no height; when $x = 0$, y should equal 0. The y-intercept is not relevant in this context.

Practice 4.3.2: Calculating and Interpreting the Correlation Coefficient, p. U4-120

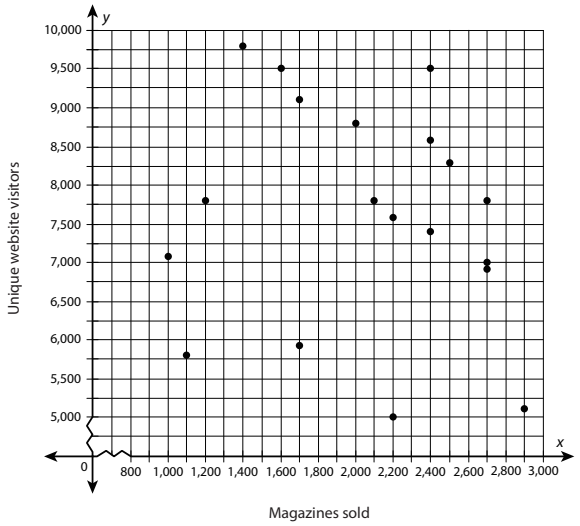
1. weak negative linear correlation
2. strong positive linear correlation
3. no correlation
4. strong negative linear correlation

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5.



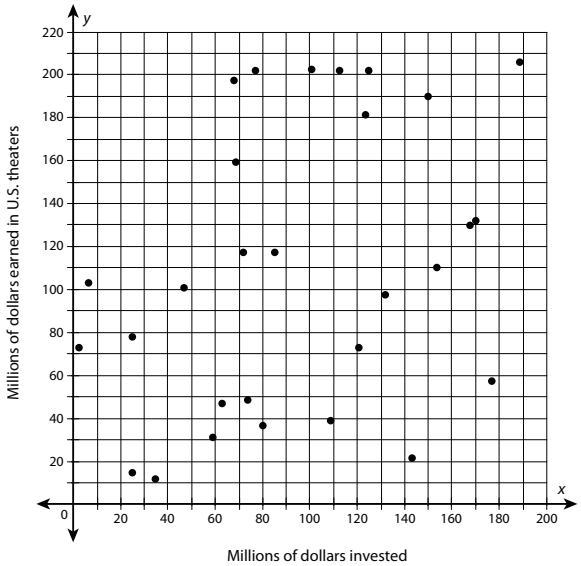
6. There appears to be a positive linear correlation between number of passengers and pounds of luggage.
7. $r = 0.92$; there is a very strong positive linear correlation between the data.
8.



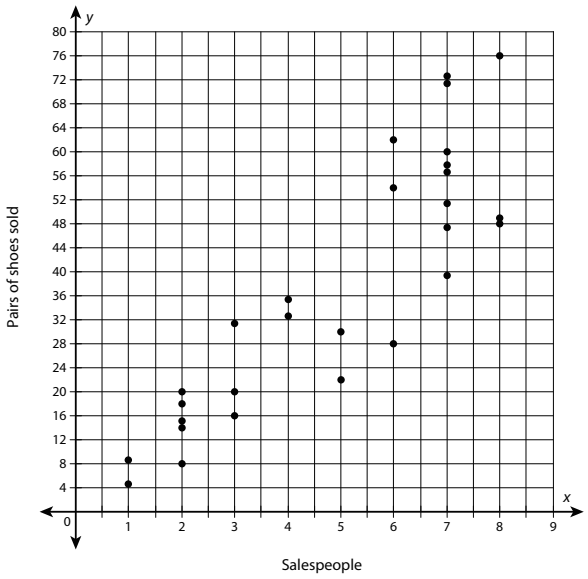
9. There does not appear to be a relationship between the data.
10. $r = –0.168$; there is no relationship between the data, or a very weak negative linear correlation.

Practice 4.3.3: Distinguishing Between Correlation and Causation, p. U4-133

1.



2. The graph appears to be random, with some increase in dollars earned as dollars invested increases.
3. $r = 0.342$; this indicates that there is a weak positive linear correlation between dollars invested and dollars earned by a movie.
4. The correlation between the two values is weak, and it is difficult to determine if there is any causal relationship between dollars invested and dollars earned in U.S. theaters.
5.



6. The pairs of shoes sold increases as the number of salespeople increases, and the graph appears to follow a linear shape.
7. $r = 0.892$; this indicates that there is a strong positive linear correlation between number of salespeople and pairs of shoes sold.

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8. Since customers are dependent on a salesperson to help them get shoes in the correct size, and there is a strong correlation between number of salespeople and pairs of shoes sold, it is likely that there is a causal relationship between the two values. Other factors also influence the sales, but having more salespeople directly impacts the number of people who are able to get shoes in the correct size, leading to greater shoe sales. Note that even if there are a large number of salespeople, sales cannot increase unless there are customers.
9. As the age of cell phone users increases, the number of apps decreases. The graph follows approximately a linear shape with a negative slope.
10. Even though there appears to be a correlation between age and number of apps, it is unlikely that age causes the reduced number of cell phone apps.

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Station Activities Set 1: Displaying and Interpreting Data

Instruction

Goal: To provide opportunities for students to develop concepts and skills related to mean, median, and mode for box plots, including interquartile range; and to use linear regression to analyze data sets

Common Core State Standards

S–ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).★

S–ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.★

S–ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).★

Student Activities Overview and Answer Key

Station 1

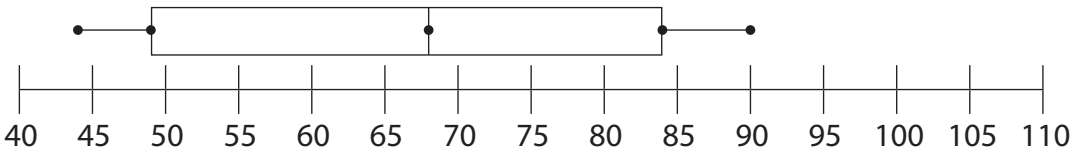
Students will be given nine index cards with the following numbers written on them: 52, 49, 69, 44, 88, 80, 68, 49, 90. Students will also be given graph paper. Students will work together to find the minimum, maximum, mode, median, quartiles, and the interquartile range, and look for outliers. Then on graph paper, they will construct a box plot to represent the data set. They will explain when a box plot would be useful to analyze data.

Answers

1. 44
2. 90
3. 49; mode
4. 68; 68 is the median because there are four numbers below and above it in the data set; 44, 49, 49, 52; $\frac{(49 + 49)}{2} = 49$; 69, 80, 88, 90; $\frac{(80 + 88)}{2} = 84$
5. 49; it is the median of the lower numbers.
6. median: 68
7. 84; it is the median of the upper numbers.
8. 44, because it is the minimum data point; 35; no, because $44 > 49 - 1.5(35)$
9. 90, because it is the maximum data point; no, because $90 < 84 + 1.5(35)$

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10.



11. Answers will vary. Possible answer: Box plots show how the data is dispersed over the data set.

Station 2

Students will be given graph paper and a ruler. Students will create two sets of data. They will find the quartiles, interquartile range, and medians for each data set. Then they will create parallel box plots of the data sets. They will explain when parallel box plots are useful.

Answers

1–5. Answers will vary.

6. Make sure the box plot is graphed correctly.

7–11. Answers will vary.

12. Make sure the box plot is graphed correctly and is on the same graph as the answer for problem 6.

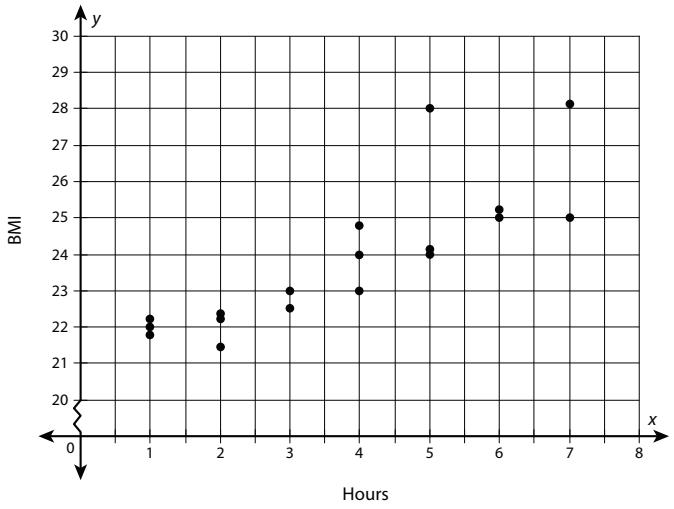
13. Parallel box plots make it easy to compare two sets of data.

Station 3

Working with groups, students will analyze a data set to find a linear relationship between variables. Students will use visual estimates to find the line of best fit.

Answers

1.



2. yes

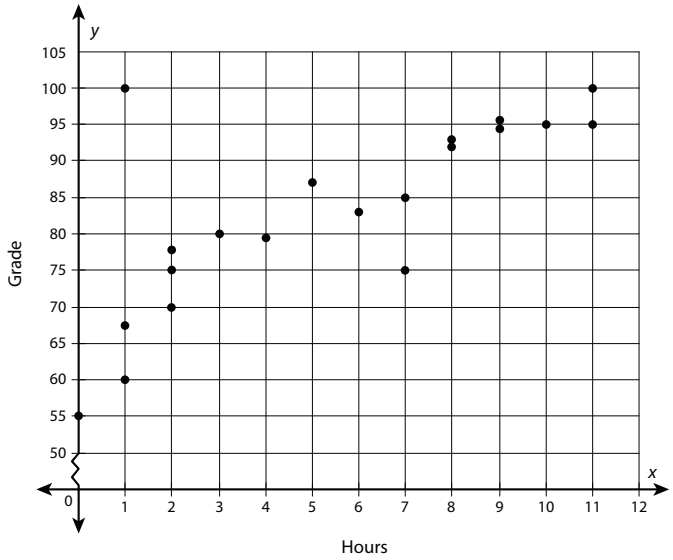
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3. Answers will vary but should be in the form $y = mx + b$. Possible answer: $y = 0.8x + 21$
4. Answers will vary.
5. No. There are too many other factors involved (such as activity level and diet). There seems to be a correlation, but we can't prove causation.
6. Yes; (5, 28) and (7, 28.2)

Station 4

Working with groups, students will analyze a data set to find a linear relationship between variables. Students will use a calculator to create a scatter plot and find an equation for the line of best fit.

Answers

1. 

2. yes
3. Answers will vary. Possible answer: $y = 2.5x + 70$
4. yes; (1, 100)
5. $y = 2.67x + 68.64$
6. Yes. The linear relationship is very close.
7. We can't prove causation from this data. As the outlier shows, some people may not study because they already know the material well. However, the data suggests that the more time spent studying, the higher the test grade will be.

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Instruction

Materials List/Setup

Station 1 graph paper; nine index cards with the following numbers written on them: 52, 49, 69, 44, 88, 80, 68, 49, 90

Station 2 graph paper; ruler

Station 3 calculator; colored pens or pencils; graph paper

Station 4 calculator

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. What is the mode of a data set?
2. How do you find the median of a data set?
3. How do you find the mean of a data set?
4. How do you find the interquartile range in a box plot?
5. How do you determine if there are outliers in a box plot?
6. When are parallel box plots useful?
7. What is a scatter plot?
8. What is a line of best fit?
9. What is correlation?
10. What is causation?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. The mode is the number that occurs most often in the data set.
2. The median is the number that has the same amount of numbers above and below it in the data set.
3. The mean is the sum of the data set divided by the number of numbers in the data set.
4. Interquartile range = $Q_3 - Q_1$
5. An outlier is a number less than $Q_1 - 1.5$(interquartile range) or a number greater than $Q_3 + 1.5$(interquartile range).
6. Parallel box plots let you compare two data sets and compare their dispersion.

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Instruction

7. A scatter plot is a graph of many different pairs of variables as points on a set of coordinate axes.
8. A line of best fit is a line that approximates the points in a scatter plot and gives a pattern to the data.
9. Correlation is a relationship between variables.
10. Causation is a relationship in which one variable causes the other one to behave in a certain way.

Possible Misunderstandings/Mistakes

- Not writing data sets in numerical order before creating stem-and-leaf plots and box plots
- Not identifying outliers in data sets for box plots
- Incorrectly finding Q_1, Q_2, and/or Q_3 in box plots
- Incorrectly graphing points
- Incorrectly finding medians
- Confusing medians with means
- Incorrectly using the calculator's linear regression function
- Confusing correlation with causation
- Incorrectly applying the formula of a line
- Misunderstanding the scenario that provides the data set

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Station 1

At this station, you will find graph paper and nine index cards with the following numbers:

52, 49, 69, 44, 88, 80, 68, 49, 90

As a group, arrange the index cards in numerical order.

1. Which number is the smallest number in the data set?

2. Which number is the largest number in the data set?

3. Which number occurs most often in this data set?

   What is the name for this number?

4. Which number is the median in this data set? Explain your answer.

   Which four numbers are less than the median for this data set?

   What is the median of these four numbers? Show your work.

   Which four numbers are greater than the median for this data set?

   What is the median of these four numbers? Show your work.

continued

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You can create a box plot to represent your data set. A box plot looks like this:



Examine the box plot, and use your observations to complete problems 5–9.

5. Which number from your data set do you think represents the point Q_1? Explain your answer.

6. Which number from your data set do you think represents the point Q_2? Explain your answer.

7. Which number from your data set do you think represents the point Q_3? Explain your answer.

8. Which number from your data set do you think represents the left endpoint of the line? Is this number an outlier? Explain your answer.

   This number may be an outlier.

   Use the interquartile range to see if this minimum number is an outlier.

   Interquartile range = $Q_3 - Q_1$ = _____

   An outlier in this direction will be any point on a number line that is less than $Q_1 - 1.5$(interquartile range).

   Is the minimum number in the data set an outlier? Why or why not?

continued

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9. Which number from your data set do you think represents the right endpoint of the line? Explain your answer.

   This number may be an outlier.

   An outlier in this direction will be any point on a number line that is greater than Q_3 + 1.5(interquartile range).

   Is the maximum number in the data set an outlier? Why or why not?

10. On your graph paper, construct the box plot for your data set.

11. How are box plots useful in analyzing data?

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Station 2

You will be given graph paper and a ruler. You will use these materials, along with data on the birth dates of your group members and your group members' mothers, to answer the questions and construct parallel box plots.

A box plot looks like this:



In the table below, write the number of the day of the month each student in your group was born. For example, if a student was born on February 29, write "29" in the table. Write these numbers in numerical order.

Your birthday (day)

1. What is the median of this data set?

   This will be Q_2 in your box plot.

2. What is the median of the numbers that are *less than* your answer from problem 1?

   This will be Q_1 in your box plot.

continued

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3. What is the median of the numbers that are *greater than* your answer from problem 1?

   This will be Q_3 in your box plot.

4. What is the interquartile range of this data set? Show your work. (*Hint*: interquartile range = $Q_3 - Q_1$)

Find out if there are any outliers in your data set:

A number is an outlier if it is less than $Q_1 - 1.5$(interquartile range).

A number is an outlier if it is greater than $Q_3 + 1.5$(interquartile range).

5. Are there any outliers in this data set? If so, what are they?

6. On your graph paper, graph a box plot that represents the data you gathered on your group members' birthdays.

   Now, you will follow the same process to create a box plot for the birthdays of your group members' mothers.

   In the table below, give the number of the day of the month on which each group member's mother was born. For example, if a group member's mother was born on May 14, write "14" in the table. Write these numbers in numerical order.

Mother's birthday (day)

continued

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7. What is the median of this data set?

   This will be Q_2 in your box plot.

8. What is the median of the numbers that are *less than* your answer from problem 7?

   This will be Q_1 in your box plot.

9. What is the median of the numbers that are *greater than* your answer from problem 7?

   This will be Q_3 in your box plot.

10. What is the interquartile range of this data set? Show your work.

11. Are there any outliers in this data set? If so, what are they?

12. On the same sheet of graph paper you used for problem 6, graph a box plot that represents the data you gathered on your mothers' birthdays.

13. Why is it useful to construct parallel box plots?

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Station 3

Work with your group to answer each question about the data set. Use the calculator to calculate medians, if needed.

A doctor is trying to find out whether there is a correlation between TV viewing and high body mass. She records average daily viewing habits and takes Body Mass Index (BMI) measurements from 18 people.

TV viewing (hours)	BMI
1	22
3	22.5
5	24.1
7	25
4	24.7
7	28.2
5	24
1	22.3
3	23

TV viewing (hours)	BMI
2	22.3
4	23
6	25.2
2	21.5
5	28
6	25
2	22.5
1	21.7
4	24

1. Graph the doctor's results on a scatter plot. Use graph paper.

2. Does there seem to be a linear relationship between the variables?

3. Estimate the equation for the line of best fit. Write your equation in slope-intercept form.

4. Draw your line in a different color on the scatter plot.

5. Can you claim with certainty that increased TV viewing causes higher BMI? Explain.

6. Does the graph have any outliers? If so, what are their coordinates?

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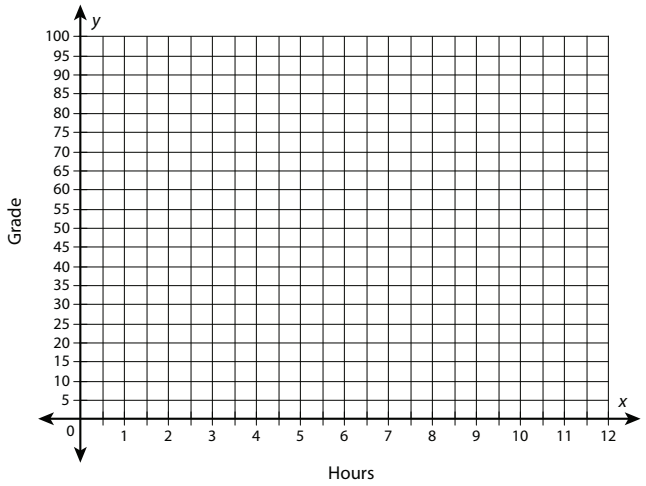
Station 4

Work with your group to answer each question about the data set. Use the calculator to calculate medians and create your graphs.

A class wants to find out if there is a correlation between the number of hours studied and grades on the midterm exam. The 20 students log their hours and their grades, as follows.

Studying (hours)	Grade	Studying (hours)	Grade
10	95	2	78
1	60	2	75
7	75	8	92
11	100	3	80
1	100	0	55
2	70	4	79
9	94	9	96
7	85	6	83
5	87	1	67
8	93	11	95

1. Enter the numbers into your calculator to graph the results on a scatter plot. Sketch your plot.



continued

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2. Does there seem to be a linear relationship between the variables?

3. Estimate the equation for the line of best fit.

4. Are there any outliers? If so, what are their coordinates?

5. Use the calculator to find the equation for the line of best fit.

6. Is there a correlation between the variables? Explain.

7. Is there a causative relationship between the variables? Explain.

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Instruction

Goal: To provide opportunities for students to develop concepts and skills related to creating and analyzing scatter plots and lines of best fit to represent a real-world situation

Common Core State Standards

S–ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

b. Informally assess the fit of a function by plotting and analyzing residuals.

c. Fit a linear function for a scatter plot that suggests a linear association.

S–ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★

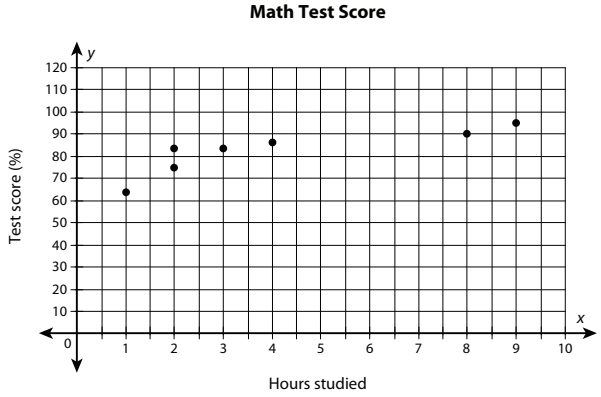
Student Activities Overview and Answer Key

Station 1

Students will be given graph paper and a ruler to help them create a scatter plot. Then they will analyze the scatter plot to determine the correlation between the data and describe the slope.

Answers

1. Graph:



The graph is a scatter plot.

2. The test scores increase as the amount of time she studies for each test increases.

3. Positive correlation; the longer she studied for a test, the higher her test score; positive slope because the line increases from left to right.

4. No, you can't apply this observation to other data sets. She may need to spend more or less time studying in English or history in order to earn the same test scores.

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5. Answers will vary; verify that the scatter plot depicts a positive correlation.
6. Answers will vary; verify that the scatter plot depicts a negative correlation.
7. Answers will vary; verify that the scatter plot depicts no correlation.

Station 2

Students will be given graph paper and a ruler. They will find the line of best fit and describe its slope for the set of data. Then they will create a scatter plot and line of best fit that represents a real-world situation.

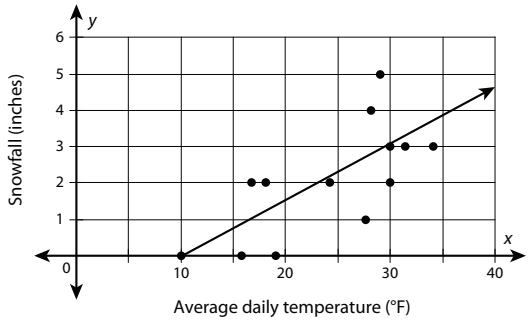
Answers

1. Line D; Line D is the closest to the most number of data points.
2. Answers will vary for line of best fit; check each group's graph paper. The slope is positive.
3. Answers will vary for line of best fit; check each group's graph paper. The slope is negative.
4. Answers will vary for line of best fit; check each group's graph paper. The slope is zero or close to zero.
5. Answers will vary; verify that students created the scatter plot and line of best fit correctly based on their set of data points.

Station 3

Students will be given a ruler and graph paper. They will construct a scatter plot and line of best fit for a real-world situation. They will find the slope of the line and describe how it represents the data set. Then they will write an equation for the line of best fit in slope-intercept form.

Answers

1. 

   (30, 3) and (10, 0) because they are closest to the line of best fit.
2. $m = \frac{3}{20}$; for this data set, a temperature increase of 20° correlates to a snowfall increase of 3 in.
3. $y = \frac{3}{20}x - \frac{3}{2}$

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Instruction

Station 4

Students will be given graph paper, a ruler, and a tape measure. Students will measure their heights using the tape measure and will record this data in a table, along with their shoe sizes. They will create a scatter plot of the data. Then, students will find the equation of the line of best fit, and draw it on their scatter plots using the ruler. They will use this equation to predict points on the line that are inside and outside of the data set. (*Note*: Because men's shoe sizes are different from women's shoe sizes, direct students to create separate tables and scatter plots if necessary.)

Answers

1–7. Answers will vary.

8. Yes, you can predict a person's height based on shoe size. These two values are correlated, since the taller a person is, the larger his or her feet will be.

Materials List/Setup

Station 1 ruler; graph paper

Station 2 ruler; graph paper

Station 3 ruler; graph paper

Station 4 ruler; graph paper; tape measure

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. What is a scatter plot?
2. What is the line of best fit on a scatter plot?
3. How do you find the slope of the line of best fit?
4. How do you find an equation for the line of best fit?
5. On a scatter plot, how can you predict data points that weren't in the original data set?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. A scatter plot is a graph of data points for two variables showing their relationships.
2. The line of best fit is a straight line that best represents the data on the scatter plot.
3. Select two points that appear to be on the line of best fit. Use the points to find the slope.
4. Select two data points that appear to be on the line of best fit. Use the points to find the slope. Use this slope and one of the data points to find the equation of the line of best fit.
5. Find the equation for the line of best fit. Substitute x into this equation to find y and vice versa.

Possible Misunderstandings/Mistakes

- Not understanding that the line of best fit depicts the correlation and slope of the set of data points
- Mistakenly applying the correlation found in the data set to all data sets
- Not understanding the difference between positive, negative, and no correlation
- Using data points that don't best represent the data to find the slope

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Station 1

You will be given a ruler and graph paper. Use them along with the problem scenario and table below to answer the questions.

Theresa wanted to find a relationship between the number of hours she studied before a math test and her test score. She used a table to track the number of hours she studied and the resulting score on her math test.

Hours studied	Test score (%)
2	75
4	88
1	65
2	85
8	91
9	97
3	86

1. Work together to graph these ordered pairs. Do not connect the points.

   What type of graph have you created?

2. What happens to the test scores as Theresa increases the number of hours she studies?

   This relationship is called a correlation between the two variables.

3. Using your graph, what type of correlation is there between the number of hours Theresa studied and her test score? Explain your reasoning.

   Using your graph, does this data set have a positive or negative slope? Explain your reasoning.

continued

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4. Can you apply the correlation you found in this data set to all of Theresa's subjects, such as English and history? Explain your reasoning.

5. On your graph paper, construct a scatter plot that has a positive correlation.

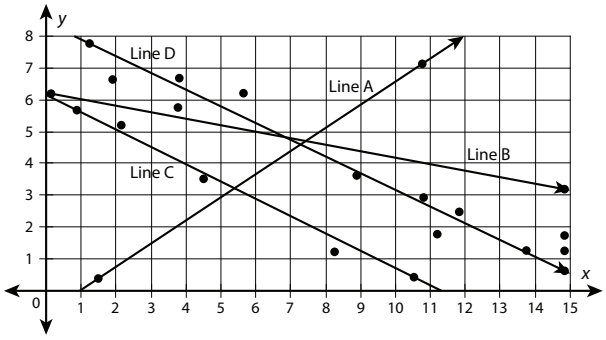
6. On your graph paper, construct a scatter plot that has a negative correlation.

7. On your graph paper, construct a scatter plot that has no correlation.

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Station 2

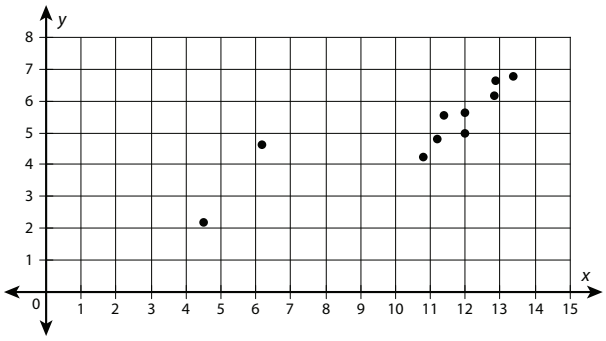
At this station, you will find a ruler and graph paper. The scatter plot below represents a set of data.



1. Does Line A, B, C, or D represent the line of best fit? Explain your answer.

For each of the following scatter plots, draw the line of best fit and describe its slope.

2.

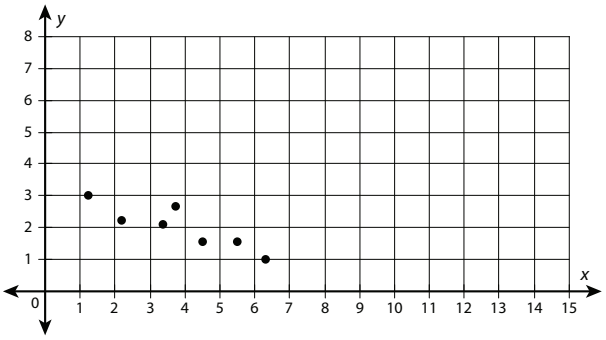


Slope: _____

continued

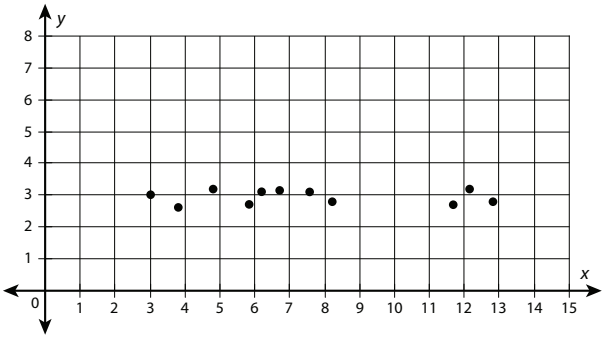
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3.



Slope: _____

4.



Slope: _____

5. In the following table, fill in the numerical value for your birthday (month and day) for each member in your group. For example, a person born February 29 would fill in 2 for the month and 29 for the day. Then use your graph paper to create a scatter plot of this data and find the line of best fit.

Month	Day

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Station 3

At this station, you will find a ruler and graph paper. The data table below represents the average daily temperature versus the daily amount of snowfall for Michigan during a two-week period.

Average daily temperature (°F)	30	31	28	17	34	16	10	19	27	28	24	18	31	30	29
Average daily snowfall (inches)	3	3	4	2	3	0	0	0	1	4	2	2	3	2	5

1. As a group, use your graph paper and ruler to create a scatter plot and line of best fit for this data set.

   Which two data points best represent the data set? Explain your answer.

2. Use these two points to find the slope of the line of best fit. Show your work.

   For this data set, what does the slope represent in terms of temperature and inches of snow?

3. Work together to use these two points and slope to write an equation for the line of best fit. Write your equation in slope-intercept form. Show your work.

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Station 4

At this station, you will find graph paper, a ruler, and a tape measure. As a group, find the shoe size and the height (in inches) of each person in your group, and use this data to create a data table and scatter plot. *Note*: Because men's shoe sizes are different from women's shoe sizes, create two separate tables and scatter plots if necessary.

1. In the following table, fill in the shoe size of each person, and his or her height.

Shoe size	Height (inches)

2. On your graph paper, create a scatter plot of the data points in the table. Let x represent the shoe sizes and y represent the height values.

3. Draw the line of best fit and name two points that are on this line.

4. Find the equation for the line of best fit using these two points. Write the equation in slope-intercept form.

5. How can you predict a new data point using this equation?

6. If $x = 6$, what value do you predict y will be?

7. If $x = 15$, what value do you predict y will be?

8. Can you predict the height of a person based on his or her shoe size? Why or why not?

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APPENDIX

Problem-Based Task Implementation Guides

Introduction

This addendum contains a collection of Implementation Guides (IGs) designed to support the use of the Problem-Based Tasks found throughout this unit.

Each Implementation Guide includes an overview and suggestions for introducing and facilitating the task. These contain specific recommendations for selected Standards for Mathematical Practice (SMPs) to highlight, guidance on addressing common errors and misconceptions associated with the task, and ideas for monitoring and coaching students as they work through the task. Finally, there is guidance for debriefing the task, with special attention to the SMPs and relevant Key Concepts from the instruction, as well as information on suitable extensions and alternate strategies.

Throughout the Implementation Guides, suggestions are provided in a question/prompt format to support your efforts to elicit student thinking and facilitate classroom discourse.

We developed these Implementation Guides for our Problem-Based Tasks at the request of math educators, and with advice and feedback from mathematics supervisors and teachers. Please let us know if they are helpful. Visit us at www.walch.com, follow us on Twitter (@WalchEd), or e-mail suggestions to customerservice@walch.com.

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Guide

Problem-Based Task: Representing Data Sets

Common Core State Standard

S–ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).★

Task Overview

Focus

Is there more than one way to represent numerical data? What makes one type of display better than another? In this task, students will analyze three sets of data and create a graphical display for each set based on the type of data presented and the question being asked.

This activity will provide practice with:

- graphing data using a box plot
- graphing data using a histogram
- graphing data using a dot plot
- analyzing data to determine which type of data display best represents the data

Introduction

This task should be used to explore or apply the concept of using graphical displays to represent data based on the type of data presented. In this task, graphing should be done by hand.

Begin by reading the problem and clarifying the meaning of the terms *minimum*, *maximum*, *first quartile*, *median*, *third quartile*, *box plot*, *dot plot*, *histogram*, *frequency*, and *survey*.

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Guide

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students may make sense of the problem by breaking it down into smaller parts. Encourage students to look at one data set at a time and seek to answer the question that is associated with that particular data set before moving on to the next data set.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will construct viable arguments about which graphical display is the best to use. Encourage students to state why they think their display is the best choice, and have them use the correct mathematical language when supporting their arguments. When students disagree with one another, encourage them to state why they disagree and to provide an alternate solution, and to show why the alternate solution is better.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- using the mean in a box plot instead of the median

  Remind students that box plots are based on finding medians in the data.

- incorrectly setting up the number line before creating a dot plot or histogram

  Have students carefully review the maximum and minimum data values.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask them to identify the first question asked of those participating in the survey. (**Answer:** "Those participating in the survey were asked, 'How many hours per week do you spend helping students with assignments?' ")

  Ask students to analyze the data associated with this survey question, and ask which data display shows a measure of center. (**Answer:** "A box plot shows a measure of center.")

- If students struggle with how to create a box plot, ask, "What information is displayed in a box plot?" (**Answer:** "Box plots show the minimum and maximum values as well as the first, second, and third quartiles of the data.")

  If students struggle with creating a scale for the box plot, ask them to identify the maximum and minimum of the data set. (**Answer:** "The maximum is 12 and the minimum is 2.")

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Ask students what scale they should use in order to show the maximum and minimum values. (**Answer:** "The scale must include both the minimum and maximum values in the data; therefore, the scale must start at 2 or some lesser value, and continue to at least 12.")

- If students struggle with organizing the results of the second survey, ask them to identify the second survey question. (**Answer:** "The second survey question is, 'How much money would you pay per semester for an unlimited tutoring service?' ")

  Ask students how to best display the Survey 2 data graphically. (**Answer:** "Either a dot plot or a histogram would show frequencies, but a histogram would show a price range and would be better than a dot plot for displaying the data, since the data are given in ranges.")

  If students struggle with creating the histogram, ask them to identify which data will be labeled on the *x*-axis and which data will be labeled on the *y*-axis. (**Answer:** "The *x*-axis will be labeled with the price ranges and the *y*-axis will be labeled with the frequencies.")

- If students struggle with organizing the results of the third survey, ask them to identify the third survey question. (**Answer:** "The third survey question is, 'This semester, for how many different classes have you visited a professor for assignment help?' ")

  Ask students how to best display the Survey 3 data graphically. (**Answer:** "Either a dot plot or a histogram would show frequencies, but a dot plot would show all the data. Since there isn't a lot of data to show and the scale is small, a dot plot is feasible.")

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask volunteers to share their methods for organizing each data set and discuss advantages and disadvantages of the varying strategies.
- Discuss the similarities and differences in dot plots and histograms. For example, both dot plots and histograms show frequencies, but dot plots show all of the data points, whereas histograms show frequencies for a range of data.
- Discuss the advantages and disadvantages of dot plots versus histograms. For example, dot plots are preferable if you want to see all of the data values, but sometimes seeing a summary of the data, such as with a histogram, makes the data easier to analyze or describe.
- Compare students' strategies and explanations for analyzing the given information and determining a solution plan for this task. Focus on the use of precise mathematical language, specifically when discussing the use of the terms *median*, *first quartile*, *second quartile*, *third quartile*, *histogram*, *box plot*, and *dot plot*.

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Guide

Connecting to Key Concepts

Make explicit connections to key concepts:

- Dot plots and histograms show the frequency of each data value in a data set. Each data value in a data set is represented by a dot over that value in a dot plot.

  In this task, students will create a dot plot to show the frequency of responses for the data in the third survey question.

- In a histogram, a rectangle is drawn above each value in a data set. The height of each rectangle corresponds to the number of data points with that value.

  In this task, students will create a histogram for the data in the second survey question.

- A histogram can show the frequency of a range of values.

  In this task, the Survey 2 data is given as a range of values, and a histogram is a logical way to display such data.

- The minimum, maximum, first quartile, median, and third quartile of a data set must be calculated before creating a box plot.

  In this task, students will find the minimum, maximum, first quartile, median, and third quartile of the first data set before creating the box plot.

Extending the Task

- To extend the task, have students conduct online research of real-world data that is of interest to them, and create an appropriate graphical representation of the data. Be sure the data is quantitative. Research topics could include the number of home runs hit by a particular baseball team each game, or the monthly rainfall amounts at a favorite vacation destination.
- Another option is to have students design quick surveys to administer during class, and then graph the results. Survey topics could include the number of siblings in one's family, favorite season, the height of girls in the class, or the height of boys in the class. Students could then organize the data and choose the best graphical representation of their results.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I broke the problem down into smaller chunks and approached the problem one question at a time. I only analyzed one set of data at a time.")

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- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "Yes. I explained which graphical displays to use for which data set and explained why we should use them.")

Alternate Strategies or Solutions

- Students could choose to find the average for the first set of data and report that result to describe the data.
- Students could choose to use a box plot instead of a dot plot for the third set of data. However, a box plot would not be as informative because some of the data would get lost in its creation, whereas a dot plot would show every data value.

Technology

Students can use graphing technology to complete the task. They can use computers or tablets and web-based resources to address the extensions.

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Guide

Problem-Based Task: Comparing Data Sets

Common Core State Standard

S–ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.★

Task Overview

Focus

How do you compare two different sets of data? How does comparing data lead to making decisions? In this task, students will analyze two data sets using the measure of center and measure of spread to decide which of two cars would be a better purchase based on gas mileage.

This activity will provide practice with:

- graphing two data sets using box plots
- comparing measures of center and spread
- analyzing measures of center and spread between two data sets
- interpreting graphs
- making a decision based on data

Introduction

This task should be used to explore or apply the concept of using measures of center and spread to compare two or more data sets.

Begin by reading the problem and clarifying the meaning of the terms *measures of center* and *measures of spread*, along with the meaning of *gas mileage*:

gas mileage	the average number of miles a vehicle travels on 1 gallon of gasoline; the ratio of the number of miles traveled to the number of gallons of gasoline used

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Guide

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will construct viable arguments about which car has the better gas mileage given the number of miles per gallon each car traveled per tank of gas over the course of several fill-ups in order to draw conclusions about which car has the better gas mileage. Encourage students to discuss their arguments with each other, and to explain their reasoning if they disagree.

- **SMP 4:** Model with mathematics.

  Students will recognize that the scenario presented can be modeled by first finding the measure of center and the measure of spread of each set of data, and then by graphing both sets of data. They will analyze the data in order to draw conclusions about the gas mileage of each car.

- **SMP 6:** Attend to precision.

  Make sure students are using math terms and definitions clearly and accurately to explain their reasoning. Encourage them to analyze properties of both sets of data carefully when constructing their reasoning. Strive toward precise mathematical language; for example, by correctly using the terms *measure of center*, *measure of spread*, *box plots*, and *histograms*. Also, remind students to carefully label the values on their graphs.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- miscalculating the mean or median

  Students should check to see that they didn't leave out a data point or repeat a data point in their calculations.

- using different scales to compare the box plots for the two data sets

  Require students to use the same scale for both box plots so that the graphs can be compared.

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Guide

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask, "What are you asked to compare? How will you make this comparison? What is the purpose of the graph you're asked to create?" (**Answer:** "I'm being asked to compare the gas mileage for two different cars. I will make this comparison by finding the measures of center and spread. The purpose of the graph is to show the differences between the data.")
- While completing the task, if students have difficulty determining which measure of center to use to compare the data, encourage them to first sort each data set from least to greatest. Then ask:

  "Are there any values in either set that are very high or very low?" (**Answer:** "No; the smallest and largest values in each set are not that much smaller or larger than the rest of the data.")

  "Which measure of center is affected most by much smaller or larger values?" (**Answer:** "The mean is most affected by much smaller or larger values.")

  "Is there a measure of center that would be most appropriate to compare these two sets of data? Why?" (**Answer:** "No; because there aren't any values that are much smaller or larger than the others, the mean and the median are both appropriate measures of center for this scenario.")
- Once students have chosen a measure of center to use, ask, "How is the measure of center calculated?" (**Answer:** "The mean and median are both measures of center. To calculate the mean of a set of data, first find the sum of all the data values and then divide the sum by the number of values in the data set. To find the median, find the middle-most value in each of the ordered sets of data.")

  Ask, "How can the mean be used to determine the measure of spread?" (**Answer:** "The mean can be used to calculate the mean absolute deviation. First, find the absolute value of the difference between each data point and the mean of the data set. Find the sum of these differences, then divide the sum by the number of data points in the set.")

  Ask, "How can the median be used to determine the measure of spread?" (**Answer:** "The median is used to find the interquartile range, which can then be used to find the measure of spread. The median is the first value needed to determine the interquartile range. Once we know the median, we can calculate the first and third quartiles. The interquartile range is the difference between the third and first quartiles.")

  If students have trouble creating the graph, ask, "Which type of graph shows both the measure of center and the measure of spread?" (**Answer:** "A box plot shows both.")

  Ask, "How does this graph separate the data?" (**Answer:** "It separates the data into quarters.")

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Ask, "How could you divide the data into quarters?" (**Answer:** "First find the median, or middle value, of the ordered data set. Then find the median of the lower half of the ordered data set, and then find the median of the upper half.")

Ask students to identify the median, first quartile, and third quartile of Liam's data set. (**Answer:** "The median is 19, the first quartile is 18, and the third quartile is 21.")

Ask students to identify the median, first quartile, and third quartile of Alyssa's data set. (**Answer:** "The median is 23, the first quartile is 23, and the third quartile is 24.")

If students struggle with creating a scale for the box plot, ask students to identify the minimum and maximum of the data sets. (**Answer:** "The minimum of Liam's data set is 17, and the maximum is 22. The minimum of Alyssa's data set is 21, and the maximum is 25.")

Ask students to compare these values and determine the overall minimum and overall maximum of *both* data sets. (**Answer:** "The overall minimum is 17 and the overall maximum is 25.")

Ask students to use this information to identify an appropriate scale to use for both box plots. (**Answer:** "An appropriate scale would start at 15 and end at 25, with increments of 1.")

- If students have difficulty determining which car has the higher gas mileage, ask, "How can the box plots be used to analyze the data and determine which car Sarah should buy if she is looking for higher gas mileage?" (**Answer:** "Set one box plot above the other for comparison, then make observations about the lengths of each box plot.")
- Ask students what observations can be made about the plots. (**Answer:** "Liam's graph is more spread out and is closer to the lower values than Alyssa's.")
- Ask students to interpret the meaning of what they noticed in terms of the context of the situation. (**Answer:** "Alyssa's car has a higher median gas mileage, and her gas mileage is consistently higher. Her interquartile range is smaller than Liam's, meaning that her car had less variation in gas mileage for each fill-up.")
- Ask students to think about the meaning of *interquartile range*, and what this value reveals about the spread of a data set. (**Answer:** "The interquartile range is the middle 50% of the data; the larger it is, the greater the variation about the median. The smaller the IQR, the less the variation about the median.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

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Debriefing the Task

- Ask volunteers to share their reasoning for choosing the measures of center and spread to compare the data sets, and discuss the advantages and disadvantages of using each measure. Discuss the different strategies and processes that students used, and encourage students to justify their processes.
- Focus on the use of precise mathematical language, specifically when discussing the use of the terms *measure of center*, *measure of spread*, *box plots*, and *histograms*.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Two or more data sets can be compared using measures of center and spread. When choosing a measure of center or spread, identify whether there are very large or very small data values that may influence the mean.

  In this task, students determine that neither data set contains very large or very small data values that would influence the mean. For this reason, either the mean or median could be used as the measure of center with which to compare the data. The Coaching Sample Responses reflect using the mean as the measure of center and the mean absolute deviation for the measure of spread. However, the median and interquartile range could also be used.

- Data can also be compared using a box plot. The width of the box displays the range of the middle 50% of the data; the width increases as variation increases.

  In this task, students create box plots to compare the range of the middle 50% of the data for each of two data sets, in order to choose which of two cars is the better purchase based on gas mileage.

Extending the Task

To extend the task, provide students with another data set to add to the comparison. This set could include very large or very small values. Have students discuss why a value might be very high or very low compared with the other values. For example, the driver might have used more gas because the trunk was weighed down, or she might have used less gas because she drove primarily on the highway. Ask students if the measure of center they initially chose would change with the new data set. Have students find the measures of center and spread for the new data set, and compare them with those of the original sets. Finally, have students choose which car has the highest gas mileage.

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Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "I made a case for buying Alyssa's type of car because it got better gas mileage. I was able to back this up by using my graphs to illustrate my point.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I created box plots to model each data set.")
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** "I used mathematical terms in my discussion and arguments and paid attention to the use of decimal points and rounding so that the results made sense in the context of the problem.")

Alternate Strategies or Solutions

The Coaching Sample Responses reflect using the mean and the mean absolute deviation for the measures of center and spread. However, students could also use and discuss the median and interquartile range.

Technology

Students can use computers or tablets to complete the extension, or use an online graphing utility for comparing more than one box plot at a time. Consider the following utility, which allows users to create up to three box plots at once:

- Internet Mathematics Assessment System. "Boxplot Grapher."

  http://www.walch.com/rr/07004

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Problem-Based Task: Family Car Trips

Common Core State Standard

S–ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).★

Task Overview

Focus

Why do we use box plots? How can box plots help us to make decisions? In this task, students will examine the data for 20 families taking car trips and determine how long the battery of a portable DVD player should last in order to make it through 50% of family car trips.

This activity will provide practice with:

- graphing data using box plots
- analyzing data
- interpreting box plots
- making decisions based on data

Introduction

This task should be used to explore or apply the skill of analyzing data using box plots while looking for and calculating outliers.

Begin by reading the problem and clarifying the meaning of *engineer,* as well as discussing what engineers do.

engineer	a designer or creator of machines, structures, systems, or other complex products

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students may work together and discuss their methods for determining which measure of center is more appropriate. Encourage students to calculate both measures, and then calculate and look for any outliers.

- **SMP 4:** Model with mathematics.

  Some students might try finding a measure of center without including a measure of spread. Emphasize that a measure of center must be included with a measure of spread, and that data always needs to be modeled with a display of some type in order to properly analyze the data.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- miscalculating the outliers

  Remind students of the formula for calculating outliers. Have students write the formula at the top of their paper and identify each value before calculating.

- thinking that the mean always represents the cutoff point for 50% of the data

  Remind students that the mean represents the cutoff point for 50% of the data if the data is symmetric, but not if the data is skewed.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask, "Is the mean *always* the center of the data? Why?" (**Answer:** "No, the mean is not always the center of the data, because the mean is pulled toward extreme values or outliers in the data set.")

  Ask students if they think the data might contain any outliers, and to justify their thinking. (**Answer:** Students might point to the values 15 and 18 as possible outliers because they appear to be more extreme than the other values in the set. Emphasize that, until outliers are calculated, any "apparent" outliers are just speculations.)

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- If students struggle to determine if there are outliers in the data set, ask them what information is needed to calculate outliers. (**Answer:** "The values needed to calculate outliers include the first quartile, third quartile, and interquartile range.")

  Ask, "Do you have all the information needed to calculate outliers?" (**Answer:** "No, not yet.")

  Ask, "How can we find the information needed to calculate outliers?" (**Answer:** "Order the data from least to greatest, then find the median of the entire data set. Next, find the median of the lower half of the data, and then find the median of the upper half. Use these values to find the interquartile range, which is the difference between the third quartile and the first quartile.")

  Ask students to identify the cutoff points for outliers for this data set. (**Answer:** "Values greater than 13.75 or less than –4.25 would be outliers of this data set.")

  Once students have determined all the necessary information to find outliers, ask, "Based on your calculations, does the data set contain outliers?" (**Answer:** "The data set contains two outliers at the upper end: 15 and 18.")

- Ask students, "If the engineers want the battery to last long enough for at least 50% of family car trips, where in the data is this percentage represented?" (**Answer:** "The median represents 50% of the data. The median is found at 4.5 hours.")

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask volunteers to share the steps in their process for determining the outliers of the data set, which measure of center to choose, and how long the battery should last.

- Discuss how the median is the cutoff point for 50% of the data, and the first and third quartiles are the cutoff points for 25% and 75% of the data, respectively.

- Compare students' strategies and explanations for analyzing the given information and determining a solution plan for this task. Focus on the use of precise mathematical language, specifically when using the terms *outlier*, *median*, *interquartile range*, *first quartile*, and *third quartile*.

Connecting to Key Concepts

Make explicit connections to key concepts:

- An outlier is a data value that is much greater than or less than the rest of a data set.

  Students will see how outliers influence calculations related to center and spread, and can cause misleading conclusions.

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Extending the Task

- To extend the task, have students calculate the mean, and then compare the mean and median. Ask students to comment on the difference. (**Answer:** "The mean is 5.35, and the median is 4.5. The difference is 0.85; the outliers cause the mean to be higher than the median.")
- Another suggestion is to have students think about data sets for which the median is the better measure of center, rather than the mean. Have students research some data in these areas, calculate the mean and median, and comment on the implications of using the mean instead of the median. Examples include: the selling price of homes in various towns within the state; income levels in various towns; and employment rates in various towns.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "Yes. I used data to back up my decision on which measure of center to use, and to support my answer for the task.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I created a box plot to model the data.")

Alternate Strategies or Solutions

Students could use the mean and mean absolute deviation to calculate the minimum required battery life. They could also use a dot plot to display the data. However, to truly represent 50% of the data, the appropriate measure of center to use is the median, which is best illustrated using a box plot.

Technology

Students can use graphing technology to complete the task. They can use computers or tablets and web-based resources to address the extensions.

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Problem-Based Task: FunZone America Survey

Common Core State Standard

S–ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★

Task Overview

Focus

Are there trends at amusement parks in regard to visitors' preferred activities at different ages? How do you organize data to evaluate trends in categorical data? In this task, students will create and analyze a two-way frequency table by generating conditional relative frequencies.

This activity will provide practice with:

- organizing data in a table
- generating marginal distributions
- creating conditional relative frequencies
- analyzing data
- drawing conclusions about data

Introduction

This task should be used to explore or apply the concept of calculating conditional relative frequencies using a two-way frequency table.

Begin by reading the problem and clarifying the meaning of the terms *two-way frequency table*, *marginal distribution*, *joint frequency*, and *conditional relative frequency*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will persevere to solve the problem by considering if the raw data is enough to evaluate the trends in the data. Encourage students to think about how the number of people in a particular age group in attendance at the park could vary, and how that variation might affect the number of people choosing a particular attraction. Encourage students to think about what the word *relative* means. If students struggle with the formal definitions of the terms *two-way frequency table*, *marginal distribution*, *joint frequency*, and *conditional relative frequency*, encourage them to reason logically about standardizing the data by converting it into percentages.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students should share their observations of the trends they see in the data with one another. Students might want to initially argue for trends in the raw data. Encourage students to think about the validity in using the raw data to make comparisons. Explain to students that they need to find a way to standardize the data so that it can be compared, since each age group has different numbers of people. Encourage students to use data to support their arguments and to require peers to do the same.

- **SMP 4:** Model with mathematics.

  Students might want to draw conclusions based on what they think they know to be true; however, encourage students to model the data in a two-way frequency table for concrete evidence.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- calculating conditional frequencies based on the condition of attraction type rather than on the condition of age

  Remind students that they are looking for trends within the age groups.

- missing or repeating a data value when reorganizing the data

  Have students double-check the numbers of data points associated with each group when they are categorizing the data. Students might find it helpful to put check marks next to each data point as it is categorized.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask, "What are you being asked to find?" (**Answer:** "I am being asked to determine if there any trends in the type of attractions preferred by each age group.")
- Students might be overwhelmed with the amount of data presented. Ask students, "How can the data be organized?" (**Answer:** "There are two ways to organize the data—by type of attraction and by age. I can create a two-way table in which the rows specify the age groups and the columns specify the type of attraction, or vice versa.")
- If students are unsure of how to organize the data, ask:

  "How is the data labeled?" (**Answer:** "It is labeled by visitor number, type of attraction, and age.")

  "Will the visitor numbers matter when determining the ages of people who prefer each type of attraction? Why or why not?" (**Answer:** "The visitor number does not affect the trends. The number is only a way to keep track of the number of people surveyed and does not provide information about age or preferred type of attraction.")

  "What process would you use to organize the data?" (**Answer:** "I would create a frequency chart that shows the types of attractions split up by the given age groups. To populate the chart, I would go through the list of visitors and place a tally mark in the appropriate cell of the table. I would cross out each visitor on the list as I record the data to help keep track, until I've recorded a tally mark for each visitor. Then, I would count the number of tally marks in each cell. I would use the results to create a two-way table where the rows specify the age groups and the columns specify the favorite type of attraction, or vice versa.")
- When students are creating their two-way frequency tables, ask them why it doesn't matter which characteristic (type of attraction or age range) is used for the row headings and which is used for the column headings. (**Answer:** "The table is relative. This means that as long as the joint frequencies reflect the number of visitors in each age group for that particular type of attraction, it doesn't matter whether a characteristic is placed in a row versus a column. It's the joint frequencies that are important.")
- If students struggle with what to do after creating their tables, ask them to review the problem statement and identify the question they're being asked to answer. (**Answer:** "The question posed by the task is, 'Are there any trends in the type of attractions preferred by each age group?' ")

  Ask, "Now that we've filled in a two-way frequency table, can we analyze the data? Why or why not?" (**Answer:** "No, because the data is not standardized. There aren't equal numbers of visitors in each age group; therefore, looking at the raw numbers doesn't give enough information. For example, both the 5–15 age group and the 36–45 age group have 4 people who prefer 'Shows.' However, '4' doesn't represent the same conditional relative frequency with respect to all visitors within both age groups, because there are more people in the 5–15 age group than in the 36–45 age group.")

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- If students struggle with standardizing the data, ask them what other information might be helpful to know about each age group and the number of visitors. (**Answer:** "It would be helpful to know how many visitors there were in each age group. In other words, we need to find the marginal frequency for each age range.")
- Once the marginal frequencies for the age ranges have been calculated, students might see that they now have the information they need to calculate the conditional relative frequencies to use in comparing the data. However, if some students still don't see how to use or standardize the data, ask how percentages could help in comparing the data. (**Answer:** "Each attraction could be expressed in terms of a percentage of the total number of people in each age group who visited the park and preferred that attraction, written as a decimal and then compared.")
- Students might think they are done once they have completed the calculations. Explain that the interpretation is just as important as the calculations, and ask students to think about what the results tell them. (**Answer:** "The data reveals the visitors who prefer each attraction, broken down by age group.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Compare students' methods, and discuss advantages and disadvantages of the varying strategies.
- Discuss why the data must be compared as relative frequencies rather than as just frequencies.
- Discuss the difference between the conditions (age or favorite attraction) in calculating conditional relative frequencies.

Connecting to Key Concepts

Make explicit connections to key concepts:

- A two-way frequency table divides survey responses by characteristics of respondents.

  In this task, students will create a frequency table that divides respondents by age range and their favorite attraction.

- The number of times a response was given by people with a certain characteristic is called a joint frequency.

  Students will determine the joint frequency for each age range and favorite attraction.

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- A marginal frequency is the total number of times a response is given, or the total number of people with a certain characteristic.

  Students will find the marginal frequency for each age range.

- A conditional relative frequency is usually written as a decimal. It expresses a number of responses divided by the total number of respondents, the total number of people with a given characteristic, or the total number of times a specific response was given.

  In order to describe trends, students will calculate the conditional relative frequency of the preferred attractions for each age range.

- Trends, or patterns of responses, can be identified by looking at the frequency of responses.

  Once all calculations are completed, students will use the information to determine trends in the type of attraction preferred by each age group.

Extending the Task

To extend the task, have students look for trends within each attraction, then ask them to interpret the results. For example, ask the following:

- "Which age group had the highest representation of people who prefer roller coasters?" (**Answer:** "The 16–25 group, with a conditional relative frequency of 0.45, or 45%.")
- "Which age group had the lowest representation of people who prefer shows?" (**Answer:** 16–25, with 9%)
- "Which age group had the highest representation of people who prefer the water park?" (**Answer:** 16–25, with 45%)
- "How can you interpret these results? What conclusion can you draw?" (**Answer:** "The age group most likely to prefer the roller coasters was the 16–25 group. This group was also the least likely age group to prefer shows, and the most likely to prefer the water park. From these results, I can conclude that the 16–25 age group is more likely to prefer roller coasters or the water park over shows.")

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "I didn't know how to start or what to do with the data, so I reread the problem, identified what was being asked, and reasoned logically with what I knew about decimals. Then I was able to create the two-way frequency table and use it to find trends in the data.")

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- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "Yes; I drew conclusions and used data to support my answers. I also critiqued the reasoning of others during discussions.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I used a two-way frequency table and calculated conditional relative frequencies to model the data.")

Alternate Strategies or Solutions

Students may attempt to answer the question by reasoning and calculating percentages without actually creating the two-way frequency table. However, this method is not recommended for this task.

Technology

Students can use calculators to find the conditional relative frequencies. Students may also choose to use spreadsheets to create their tables.

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Problem-Based Task: Movie Buzz

Common Core State Standard

S–ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

Task Overview

Focus

How do you evaluate whether a function is a good fit for data? How do you use functions fitted to data to make predictions? In this task, students will compare functions to evaluate their fit with given data. Students will choose the model that better fits the data and use that model to make a prediction.

This activity will provide practice with:

- graphing data
- graphing linear and exponential functions
- comparing functions fitted to data
- analyzing goodness of fit for data
- applying the appropriate function model to make predictions

Introduction

This task should be used to explore or apply the skill of fitting functions to data given two potential models. Graphing may be done using technology, including with graphing calculators or online graphing programs.

Begin by reading the problem and clarifying the meaning of *word of mouth*:

word of mouth a way of passing information by spoken communication

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Guide

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will use mathematical terms and explain their choice of model. Direct students to graph the data and both models on the same coordinate plane and to describe what they see in terms of goodness of fit. Encourage students to use specific language to describe how each model does or does not fit the data. Encourage students to require this of their peers, as well.

- **SMP 4:** Model with mathematics.

  Students will graph the data and the models on the same coordinate plane. Encourage students to see the connection between the algebraic and graphical models.

- **SMP 5:** Use appropriate tools strategically.

  Students using graphing technology may struggle with determining the appropriate scales and/or viewing windows. Encourage students to start by finding the minimum and maximum y-coordinates of the given data, and to adjust their calculator's viewing window settings so that at least these numbers are shown on the y-axis. Remind students that when adjusting their window settings, they will need to extend the y-axis *beyond* the given maximum y-value to leave room for predicting the number of viewers on the fifth day (i.e., when $x = 5$).

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly setting up scales on the axes

  Remind students that the y-axis scale needs to encompass the minimum and the maximum data points from the table, but should also include some extra room for the prediction.

- incorrectly plotting the data points

  Ask students to double-check the placement of their data points to verify they have plotted them correctly.

- incorrectly plotting points between tick marks

  Remind students to estimate where data points are plotted when they are located between two tick marks.

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- thinking that a linear function forms a curve when graphed, or that an exponential function forms a straight line

  Remind students of the differences between linear and exponential functions. Remind students that the x in 5^x is an exponent.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students create their scatter plots, ask them which column in the table represents the independent variable, and which represents the dependent variable. (**Answer:** "The number of viewers depends on how many days have passed. Therefore, the 'Day' column represents the independent variable, x, and the 'Number of viewers' column represents the dependent variable, y.")
- As students are creating their scatter plots, they might struggle with determining an appropriate scale for the y-axis. Ask them to identify the minimum and the maximum y-values given in the table. (**Answer:** "The minimum is 5 and the maximum is 626.")

  Ask students to identify an increment for the scale that would allow them to graph this wide range. (**Answer:** Scales may vary, but increments of 40 or 50 would work.)
- If students are unsure of how to proceed after creating their scatter plots, ask them to reread the problem statement to see what the task is asking them to do. (**Answer:** "We need to determine which of these two functions is a better fit for the data in the table.")

  Ask, "In general, how can you tell if one function is a better fit than another?" (**Answer:** "If one function follows the data more closely than the other function does, then it is a better fit.")

  Ask students what they need to do in order to determine which of the two given functions is the better fit. (**Answer:** "I need to graph the two functions on the same coordinate plane as my scatter plot, so I can observe which function more closely follows the data.")
- Students might see immediately that the exponential function is the better fit, but encourage students to explain their thinking. Ask, "Why do you think the exponential model is the better fit?" (**Answer:** "The exponential model is better because this model more closely follows the data. The linear model misses the path that the data follows.")
- Students might think they are done with the task once they've determined which function fits the data. Ask, "Now that you know which model fits the data, what *else* does the task ask you to find?" (**Answer:** "The task asks us to determine how many people will see the movie on day 5 if this trend continues.")

  Students might argue that they don't have data for that day and can't make a prediction. Ask students what the purpose of fitting a function to data is. (**Answer:** "Fitting a function to data allows us to model the given data and then to make predictions about values that are between or beyond the given data points.")

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Ask students how they will make the prediction using what they have discovered so far. (**Answer:** "I determined that the exponential model, $y = 5^x$, is the best fit for the data. I will substitute $x = 5$ into this equation and solve for y. The result is the approximate number of people that we can predict will see the movie on the fifth day.")

Ask students what their prediction is. (**Answer:** "Evaluating $y = 5^x$ for $x = 5$ gives us $y = 5^{(5)} = 3125$. Therefore, I predict there will be about 3,125 people in the theater to see the movie on the fifth day.")

Ask students if they have any reservations about making predictions beyond the domain of the given data set, and why or why not. (**Answer:** "Predictions made outside the domain of the data set should be made with caution. Factors other than the 'word of mouth' effect on which the exponential model is based could influence the data. For example, beautiful weather on day 5 might entice potential movie viewers to spend the day outdoors, or a sudden storm might prevent people from driving to the theater.")

Ask, "What are some of the implications for making decisions based on predictions from models?" (**Answer:** "Models are just that, models. They do not define or dictate what the data will do. Therefore, using a model to make a decision can be risky. For example, if you are using a model to develop infrastructure in a town based on population growth, but the population then declines or remains steady, money will have been wasted.")

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Compare students' methods, and discuss advantages and disadvantages of the varying strategies.
- Discuss the importance of graphing data before making any decisions based upon it.
- Discuss the pitfalls of predicting beyond the data set.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Data with two quantitative variables can be shown graphically on a scatter plot.

  In this task, students will create a scatter plot to represent the number of viewers at the movie theater each day.

- To create a scatter plot, plot each pair of data as a point on a coordinate plane.

  With the movie theater data, days are represented by the x-coordinates and the number of viewers is represented by the y-coordinates.

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- To compare a data set and a function, plot the function on the same coordinate plane as the scatter plot of the data set. The graph of the function should approximate the shape of the scatter plot.

  In this task, students will create a scatter plot using the given data table. They will then graph both of the given equations on the same coordinate plane to visually determine which function approximately fits the shape of the data.

- Evaluating a function that has a similar shape as a data set can provide an estimate for data not included in the plotted data set.

  Students will determine that the model $y = 5^x$ fits the data, then use this model to predict the number of movie watchers on the fifth day.

Extending the Task

- To extend the task, have students think of a real-world scenario that could be modeled by the other function, $y = 200x - 200$.

- Another suggestion is to provide students with a new data set and two or three possible models. Ask students to follow the same process to determine which of the models is the best model for the data, and to justify their answers. You may also choose to ask students to create a real-world scenario for the data. For example, have students determine whether $y = -0.1x + 250$, $y = -0.25x + 100$, or $y = -0.25x + 75$ best fits the data that follows. (**Answer:** "The function $y = -0.25x + 100$ fits the data best because it more closely follows the path of the given data on a scatter plot.")

x	150	175	200	225	250	275	300
y	60	55	50	45	40	35	30

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "Yes; I used data to back up my reasoning and explanations, and I used data to critique the explanations of others.")

- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I graphed my data points and the two models on the same coordinate plane for comparison and to determine which model better fit the data.")

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- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** "I graphed the data table values and models using a graphing calculator. I adjusted the scales and viewing window so I could tell which model better fit the data. I was able to zoom in and see how closely the $y = 5^x$ model fit, and that the other model, $y = 200x - 200$, did not.")

Alternate Strategies or Solutions

- Students might use the slope-intercept method to graph the linear equation instead of creating a table of values.
- Students might look at the differences in the *y*-coordinates to see if there is a common difference. Upon finding that there is not a common difference, students might deduce that the exponential model is the better fit.
- Students might check to see if there is a common ratio between *y*-coordinates; upon seeing that the ratio is close, they might deduce that the exponential model is the better fit.

Technology

Students can use graphing calculators or any graphing utility to graph the data and the models.

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Guide

Problem-Based Task: Estimating Salaries

Common Core State Standard

S–ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

b. Informally assess the fit of a function by plotting and analyzing residuals.

Task Overview

Focus

How do you determine if a linear model is a good fit for a data set by using residuals? What do we look for in a residual plot to determine goodness of fit? In this task, students will compare data on salaries and years of service to a linear model that is believed to fit the data, and evaluate whether the model is indeed a good fit.

This activity will provide practice with:

- plotting data on the coordinate plane
- graphing a linear model
- calculating, plotting, and analyzing residuals
- interpreting the results of a residual plot

Introduction

This task should be used to explore or apply the concept of interpreting residual plots. Graphing may be done using technology, including with graphing calculators or online graphing programs.

Begin by reading the problem and clarifying the meaning of the terms *residual* and *residual plot*.

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students are asked to determine whether a linear function is a good fit for the given data. Students might try to base their decisions on inspection rather than on using mathematics to justify their answers. Encourage students to use residuals and a residual plot to justify their answers.

- **SMP 4:** Model with mathematics.

  Students will graph the data and the linear model. Help students to see the connection between the data and the linear model by encouraging them to graph both on the same coordinate plane.

- **SMP 5:** Use appropriate tools strategically.

  Students might struggle with how to use graphing calculators effectively. Encourage students to set appropriate viewing windows to be sure they are viewing all of the data. Encourage students to plot the data and the model simultaneously. If desired, have students use the calculator to populate the residuals list and generate the residual plot. Again, encourage students to use the proper viewing window settings. Help students to understand that without the appropriate viewing window, they will not be able to analyze the results effectively.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- mislabeling scales on the axes

  Have students review their axis labels to be sure they include all units, starting with 0, or that they indicated a break in the graph if the axes do not start at 0.

- incorrectly plotting the data

  Remind students to estimate data points that do not fall on tick marks.

- incorrectly plotting the linear model

  Remind students how to graph a linear equation using its parameters; that is, the slope and y-intercept.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- If students are unsure of how to begin, ask them what they should do first in order to determine if the line is a good fit. (**Answer:** "I should make a graphical display of the data.")

  Ask, "What type of graphical display is appropriate for this data? Why?" (**Answer:** "A scatter plot is appropriate because I am interested in the relationship between two quantitative variables: the number of years versus the salary.")

- Before students create their scatter plots, ask them which column in the table represents the independent variable, and which represents the dependent variable. (**Answer:** "Each person's salary depends on their number of years of experience. Therefore, the 'Years of experience' column represents the independent variable, x, and the 'Salary in dollars' column represents the dependent variable, y.")

- If students struggle with determining an appropriate scale for the y-axis while creating the scatter plot, ask them to identify the approximate minimum and maximum y-values in the table. (**Answer:** "The minimum is about \$52,000 and the maximum is about \$210,000.")

  Ask, "By how much do the salaries increase per year?" (**Answer:** "For some years, salaries increase by about \$10,000. For other years, the jump is closer to \$20,000, and for some years the jump is much larger.")

  Ask students to identify an approximate average rate of change. (**Answer:** about \$15,000)

  Ask students to identify a good increment for the y-axis scale using this average. (**Answer:** 15,000)

- Once students have completed their scatter plots, ask them how to evaluate whether the linear model fits the plotted data. (**Answer:** "I need to graph the linear equation on the same coordinate plane as the data.")

- If students struggle with graphing the line, ask them how many points are needed to create a line. (**Answer:** at least two points)

  Ask, "How can you can generate two points on the line of the linear model so that you can graph the line?" (**Answer:** "I can create a table of values by substituting a value for the number of years, x, and then solving for the salary, y.")

  Ask, "Does the linear model *need* to pass through at least one data point? Why or why not?" (**Answer:** "The linear model does not need to pass through at least one data point. The linear model is trying to capture the trend in the data. Therefore, the model does not actually have to pass through a data point, but should be near the points or follow their path.")

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- Once students have graphed the linear model, ask them to make a judgment about whether it is a good fit based on inspection, and to justify their answers. (**Answer:** "The model appears to be a good fit because it looks like the line passes through the data such that it follows the same path. Also, it looks like there are about the same number of data points above and below the line.")

  Ask, "How can you determine whether the model is a good fit using mathematics?" (**Answer:** "I can calculate the residuals and create a residual plot.")

  Ask, "What would you expect to see in a residual plot if the model is a good fit?" (**Answer:** "The residual plot would be random and not follow a particular pattern, with about the same number of points above and below the x-axis.")

  Ask students to identify what a negative residual means. (**Answer:** "A negative residual means that the estimated value was larger than the observed value.")

- Once students create the table of residuals, ask them to analyze it to see if it seems to indicate that the linear model fits the data. (**Answer:** "There appear to be about the same number of positive residuals as there are negative residuals. At first glance, the linear model seems to be a good fit for the data, but I should plot the residuals to see if they follow a pattern.")

- Once the residual values have been plotted, ask students what they should look for and why. (**Answer:** "I should look for any patterns in the residual plot, because this would suggest that the linear model is not a good fit for the data.")

- Ask, "Based on your scatter plot and your residual plot, do you believe the linear model is a good fit for the data? Why?" (**Answer:** "The linear model appears to be a good fit because the model follows the path of the data and the residual plot shows no pattern.")

- Students might forget to answer the second question in the task. Ask students to reread the task and identify what else they need to determine. (**Answer:** "I am asked to determine approximately how many years of work experience Marcy has based on her estimated salary of $130,000.")

  Ask students how they can use the information they already have to answer this question. (**Answer:** "I can substitute Marcy's salary into the linear equation and solve for x, the number of years. The result is approximately 7 years.")

  Ask students if they are comfortable with this estimation and why or why not. (**Answer:** "Yes, because '7 years' lies within the domain of the given data. The prediction is not outside of the given domain. Also, Marcy's salary of $130,000 falls between the salaries of the employees who have 6 and 8 years of experience: $100,707 and $135,460, respectively. Therefore, 7 years is a reasonable number of years of experience for Marcy's salary.")

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

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Debriefing the Task

- Compare students' methods, and discuss advantages and disadvantages of the varying strategies.
- Discuss why, when using a residual plot to evaluate whether a particular linear model is a good fit, the goal is to see a *lack* of a pattern in the residual plot. (**Answer:** A pattern in the residual plot indicates that a linear model is not a good fit for the data, and suggests that another type of model may be more appropriate.)

Connecting to Key Concepts

Make explicit connections to key concepts:

- A residual is the distance between an observed data point and an estimated data value on a line of best fit. For the observed data point (x, y) and the estimated data value on a line of best fit (x, y_0), the residual is $y - y_0$.

  In this task, students will find the residual values for a data set, and use them to determine if the graph of the given function is a good fit for the data.

- A residual plot with a random pattern indicates that the line of best fit is a good approximation for the data.

  In this task, students will observe that the residual plot is random, and therefore the line is a good fit and can be used to make estimations.

Extending the Task

To extend the task, provide students with a different set of data and proposed function for the line of best fit, and have them evaluate the fit of the data by creating a residual plot. Have students justify their conclusions about how well the function fits the data.

For example, students could evaluate how well the function $y = 14{,}000x + 44{,}000$ fits the data in the following table shown on the left. The coordinates of the residuals are shown on the right. (**Answer:** "The linear model is not a good fit because even though there are about the same number of points above and below the line, the residual plot is parabolic, indicating a pattern in the residuals. This indicates that a linear model is not a good fit for this data.")

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Years of experience	Salary in dollars ($)
1	50,000
2	60,000
3	90,000
4	115,000
5	135,000
6	150,000
8	160,000
10	170,000
11	185,000
13	195,000

x	Residual
1	–8,000
2	–12,000
3	4,000
4	15,000
5	21,000
6	22,000
8	4,000
10	–14,000
11	–13,000
13	–31,000

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "Yes; I justified that the linear model was a good fit by using mathematics to support my answer, and critiqued the reasoning of others during discussion.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I graphed the data and the linear model to show the relationship between the data and the proposed model. Then, I created the residual plot to show the difference between the observed values from the table and the estimated values according to the model.")
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** "I used a graphing calculator to graph the scatter plots and the function, and to create a residual plot.")

Alternate Strategies or Solutions

Students could use the graph of the linear model to visually estimate Marcy's years of service instead of calculating the number of years using the equation.

Technology

Students can use calculators, graphing calculators, and/or spreadsheets to plot the data, the linear model, and the residual plot, and to perform calculations.

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Problem-Based Task: Lion Cub Births

Common Core State Standard

S–ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

c. Fit a linear function for a scatter plot that suggests a linear association.

Task Overview

Focus

How do you create a linear model from a data set that appears to be linear? How do you interpret the model in terms of the data set? In this task, students will examine a data set describing the number of lion cubs born in relation to the number of adult female lions within various prides in Africa. Then, they will create a linear function to model the relationship.

This activity will provide practice with:

- plotting data on a coordinate plane
- examining data for a linear relationship
- calculating the line of best fit
- writing a linear function using two points that are on the line

Introduction

This task should be used to explore or apply the skill of creating a line of best fit for data that appears to be linear. Graphing may be done using technology, including with graphing calculators or online graphing programs.

Begin by reading the problem and clarifying the meaning of *pride*:

pride a group of related lions that live together

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 4:** Model with mathematics.

  Students will graph the data in order to get a picture of how the data is trending. Encourage students to spend time visualizing various lines through the data to determine which line captures the essence of the data. Students will then model this line algebraically, by writing its equation using function notation.

- **SMP 5:** Use appropriate tools strategically.

  Students might quickly jump to picking two points with which to write the equation of the line. Encourage students to use a ruler or a straightedge to simulate various models through the data to help them in carefully choosing two points that lie on this line, rather than just guessing or trying to visualize the points in their mind without a tool.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- miscalculating the slope of the line

  Remind students that the slope of a line is calculated by finding the change in y divided by the change in x between two points on the line.

- miscalculating the y-intercept

  Remind students to be careful of signs when writing their equation.

- choosing two points that do not capture the trend in the data

  Remind students that when they connect the two points that they have determined lie on the line of best fit, there should be about the same number of points above the line as there are below the line. Also, make sure students realize that the two points they choose do not have to be two of the given data points.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Students may want to select two data points with which to write an equation of a line without considering the position of the line with respect to the points of a scatter plot. Ask, "What should you do first with data before trying to analyze or summarize it?" (**Answer:** "I should make a graphical display of the data.")

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- Ask, "What type of graphical display is appropriate for this type of data and why?" (**Answer:** "I should use a scatter plot because we are interested in the relationship between one variable and another; that is, the relationship between the number of adult female lions in a pride and the number of cubs in that pride.")
- If students are struggling with which label to place on the x-axis and which label to place on the y-axis, ask, "Which quantity depends on the other? Why?" (**Answer:** "The number of cubs depends on the number of adult females, so the independent variable x represents the number of adult females, and the dependent variable y represents the number of cubs.")
- Students might be tempted to use the first and the last data points from the data set to generate the line of best fit. Ask students to reflect on the terminology *line of best fit*. Ask students what they think this term means. (**Answer:** "This term means 'the line that comes the closest to matching the data set.' ")

  Ask students how they believe they should choose two points with which to write the equation of the line of best fit. (**Answer:** "I should use a ruler or a straightedge to draw a line that passes through the data so that approximately the same number of data points are above and below the line, then choose any two points on this line.")

  Ask students, "Do the two points you choose have to be two of the given data points?" (**Answer:** "No, the chosen points do not have to be from the data set.")

  Ask students to choose two points that when connected capture the data. (**Sample answer:** (3, 2) and (13, 8))
- Ask students to describe how to write the equation of the line that passes through the chosen points. (**Answer:** "Find the slope of the line and then substitute the slope and one of the points into the slope-intercept form of a linear equation, $y = mx + b$. Then, solve for b, the y-intercept. Substitute both the slope and the y-intercept back into the slope-intercept form of the equation to find the specific equation.")
- Ask students to write the equation of the line that models the data. (**Answer:** Equations will vary depending on the two points chosen, but they should be close to $y = 0.6x + 0.2$, the equation given in the Coaching Sample Responses.)
- Ask students how confident they would be in using the equation they've written to predict the number of cubs based on the number of adult females in a pride, and why. (**Sample answer:** "I am fairly confident in my model because the model captures the trend in the data and most of the data points shown are fairly close to the line.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

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Debriefing the Task

Compare methods used by different students, and discuss advantages and disadvantages of the varying strategies. Discuss strategies for finding the line of best fit, discuss methods and procedures for calculating the line of best fit, and discuss interpretations of the line of best fit for this particular task.

Connecting to Key Concepts

Make explicit connections to key concepts:

- A scatter plot that can be estimated with a linear function will look approximately like a line.

  For this task, students will observe that the plotted data points look approximately linear.

- A line that passes through the points in the scatter plot as closely as possible can be used to find a linear function that fits the data.

  Students will use observation to determine the potential location of a line of best fit for the plotted points, then choose two points that lie on this line to find the line's equation.

- If a line is a good fit for a data set, some of the data points will be above the line and some will be below the line.

  Students can place a straightedge on the scatter plot or physically draw a line connecting the two points they chose to verify that the resulting line goes through the middle of the data, with the data basically evenly distributed about the line.

- The general equation of a line in slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.

  In this task, students will find a possible line of best fit modeled by the equation $y = 0.6x + 0.2$.

Extending the Task

The precise line of best fit is called the *least squares regression line* or the *linear regression*. To extend the task, have students find the formal least squares regression line on a graphing calculator and compare these results to the line of best fit that they found. (**Answer:** $y = 0.5461x + 0.4112$)

The following video defines least squares regression lines.

- Central Oregon Community College. "Why a 'Least Squares Regression Line' Is Called That."

  http://www.walch.com/rr/07007

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Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I made a graphical display of the data in the form of a scatter plot, and then I modeled the data algebraically by writing a linear equation describing a line of best fit for the data.")
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** "I used a straightedge to find the position of a line that would best estimate the data.")

Alternate Strategies or Solutions

- Students could use the point-slope form of a linear equation to generate the line of best fit rather than the slope-intercept form.
- Students can plot the data and graph their estimated line of best fit on a graphing calculator to visually confirm that the line is a good fit for the data.

Technology

Students may use graphing calculators.

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Problem-Based Task: Learning to Speak

Common Core State Standard

S–ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★

Task Overview

Focus

How do we interpret the linear models we fit to data? How do we use these models when comparing other data points that were not originally part of the data set? In this task, students will find a line of best fit for a data set, and then analyze new data to see how it compares with the model.

This activity will provide practice with:

- graphing data as a scatter plot
- finding the equation for a line of best fit
- comparing data relative to the line of best fit
- analyzing the differences and similarities between new data points and a model
- interpreting the meaning of the parameters of a model

Introduction

This task should be used to explore or apply the skill of interpreting the slope and intercept of a linear model fitted to data. Graphing may be done using technology, including with graphing calculators or online graphing programs.

Begin by reading the problem and clarifying the meaning of *pediatrician*:

pediatrician a doctor who specializes in treating babies and children

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Encourage students to make sense of the problem by reading it through at least twice, making sure they understand the information given in the table, and devising a plan for comparing the child in question to the three children whose data is presented in the table.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students may disagree about which two points to use in order to write the equation of the line of best fit. Encourage discussion about why one pair of points is better than another, and compare the various lines of best fit on a projector if possible to facilitate this discourse.

- **SMP 4:** Model with mathematics.

  Students will model the data set first with a scatter plot and then with an equation for the line of best fit. Discuss the pros and cons of both the graphical and algebraic model.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- miscalculating the slope of the line

  Remind students that the slope of a line is calculated by finding the change in y divided by the change in x between two points on the line.

- miscalculating the y-intercept

  Remind students to be careful of signs when writing the equation.

- misinterpreting the slope

  Ask students to think about how slope is calculated, and instead of using the terms "change in y over the change in x," ask them to think of the slope in terms of the context: the change in the number of words spoken over the number of months speaking, or more simply, the change in words per month.

- choosing two points that do not capture the trend in the data

  Remind students that when they connect the two points that they have determined are on the line of best fit, there should be about the same number of points above the line as there are below the line. Also, make sure students realize that the two points do not have to be two of the given data points.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- If students have a difficult time getting started, ask:

  "What should you do with the data before you begin to analyze it?" (**Answer:** "I should make a graphical display of the data.")

  "What type of graphical display is appropriate for this data set?" (**Answer:** "A scatter plot is the most appropriate way to display this data because we are comparing two variables that can be interpreted as data points.")

- Students may struggle with which label to place on the x-axis and which label to place on the y-axis. The two quantities are the number of months that a child has been speaking, and the number of words spoken. Ask, "Which of these depends on the other?" (**Answer:** "The number of words spoken depends on the number of months that the child has been speaking, so the number of months speaking is the independent variable, x, and the number of words spoken is the dependent variable, y.")

- After students have created the scatter plot, they may have difficulty deciding how to use it to answer the question posed in the task. Ask students to reread the problem and identify other information they have been given about the child described in the task. (**Answer:** "The additional information is that when the child had been speaking for 3 months, she had a vocabulary of 96 words. After 4 months of speaking, the same child had a vocabulary of 144 words.")

  Ask students, "What are we asked to do with this information?" (**Answer:** "We are asked to put ourselves in the shoes of the pediatrician, and to determine what advice to give to the parent regarding whether her child's vocabulary is growing appropriately.")

  Ask students how they might gather the information necessary to determine the appropriate advice. (**Answer:** "I can find a line of best fit for the original data, and then compare the slope of this line to the slope of the line that connects the two data points for the child in question.")

  Ask why comparing the slopes *may* be a good method of comparison. (**Answer:** "This method compares a rate of change that seems to capture the trend in the data to the rate of change for the child in question.")

  Ask why comparing the slopes *may not* be a good method of comparison. (**Answer:** "The original data only reflects 3 different children who were studied for just 4 months. If more children had been studied, and/or the time frame had been longer, there could have been a different trend in the data.")

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- Ask students how they believe they should choose two points with which to write the equation of the line of best fit. (**Answer:** "I should use a ruler or a straightedge to draw a line through the data so that there are approximately the same number of data points both above and below the line, then choose any two points on this line.")

  Ask students to choose two points that when connected capture the data. (**Sample answer:** (0, 0) and (4, 150))

  Ask students to describe how to write the equation of the line that passes through these two points. (**Answer:** "Find the slope of the line and then substitute the slope and one of the points into the slope-intercept form of a linear equation, $y = mx + b$. Then, solve for b, the y-intercept. Substitute both the slope and the y-intercept back into the slope-intercept form of the equation to find the specific equation.")

  Ask students to write the equation of the line that models the data. (**Answer:** Equations will vary depending on the two points chosen, but should be close to $y = 37.5x$.)

- Ask students to interpret the y-intercept in the context of the problem. (**Answer:** "The y-intercept is 0. This suggests that babies who have been speaking for 0 months start with 0 words in their vocabulary, which is true." *Note*: Depending on the points chosen for the line of best fit, students may find a y-intercept that is something other than 0. In this case, the model would not be accurate for 0 months, but may be a good estimate of the number of words spoken in later months.)

- Ask students to interpret the slope of the line in the context of the problem. (**Answer:** "The children's vocabulary grows by about 37 or 38 new words each month.")

  Ask, "How can this information be used to evaluate the mother's concern for her child?" (**Answer:** "This information can be used to calculate the rate of change of words per month for the child for whom the mother is concerned, and compare it to the rate of change of the linear model.")

  Ask students how they would calculate the rate of change for the child's vocabulary. (**Answer:** "I would write the given information as two data points, (3, 96) and (4, 144), and then calculate the slope between these two points.")

  Ask students to explain how to carry out this calculation. (**Answer:** "Substitute (3, 96) and (4, 144) for (x_1, y_1) and (x_2, y_2) in the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$. The rate of change is 48.")

  Ask students to compare this rate to the slope of the line of best fit. (**Answer:** "The rate of change for the line of best fit is 37.5 words per month, and the rate of change for the child brought into the pediatrician is 48 words per month. The concerned mother's child has a higher rate of word acquisition per month than the children represented in the data table.")

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Ask students to interpret the results based on the rates of change. (**Answer:** "The child is progressing appropriately in acquiring vocabulary because her rate of change is actually higher than the model encompassing all of the children in the study.")

- The mother is concerned that her child isn't speaking enough words. Ask, "How can you alleviate the mother's concerns?" (**Answer:** "After 3 and 4 months of speaking, the child's vocabulary was indeed below the line of best fit of the doctor's data, so the child's vocabulary was slightly behind that of some of her peers. To ease the mother's concerns, I could show her that if her daughter's rate of change continues, she will catch up to her peers soon and eventually exceed her peers in word acquisition.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Compare methods used by the groups and discuss advantages and disadvantages of the varying strategies.
- Discuss the meaning of the slope and the *y*-intercept in the context of the problem.
- Discuss various ways of arguing for and against the mother's concern that the child is behind in vocabulary acquisition.

Connecting to Key Concepts

Make explicit connections to key concepts:

- The slope of a line is $m = \frac{\text{change in } y}{\text{change in } x}$; the slope between two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

  Students will calculate the slope of the line of best fit for a data set to be about 37 or 38 words per month. They will calculate the rate of change of the child's word acquisition to be about 48 words per month.

- In context, when written as a decimal, the slope describes how much the dependent variable changes each time the independent variable changes by 1 unit.

  Students will determine that the slope, or rate of change, for the child's vocabulary was 48 new words in 1 month.

- The *y*-intercept in the equation $y = mx + b$ is b.

  Students will determine that the *y*-intercept of the line of best fit is 0.

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- When analyzing the *y*-intercept in a real-world context, it is the initial value of whatever quantity is represented by the *y*-axis.

  Students will interpret the *y*-intercept of the line of best fit, 0, as meaning that when a child had been speaking for 0 months, he or she can speak 0 words.

Extending the Task

- To extend the task, graph the line that contains the specific child's data points with the line of best fit from the study, and compare the two lines. Analyze when this child will catch up to her peers if we consider her peers to be on the line of best fit. Ask students to show this solution graphically and algebraically, by solving the system of equations. Calculate when the new child will speak more words than her peers.
- Students who carry out the first extension should see that as time goes on, the line of best fit and the line representing the child's vocabulary growth become farther and farther apart. Ask students to propose an interpretation for this. For example, babies grow at different rates. While language emerges at about the same time for most children, the rate at which babies learn language varies a great deal; some children learn very quickly and very early, while others have slower, steadier progress. There is a wider range of typical language acquisition rates as the months progress. It may also be possible that the model in the problem only holds true for the first few months, and then the lines may draw closer together.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** "At first I wasn't sure what to do or how to answer the question, but I saw data and knew that I should try to make a graphical display of the data. Then, I reread the problem and realized that I could make a comparison in the rates of change if I summarized the scatter plot by using a line of best fit.")
- **For SMP 3, ASK:** "How did you construct viable arguments and critique the reasoning of others?" (**Answer:** "I used mathematics to justify my answers, including the use of the scatter plot to see the trend of the data, and the calculations of the slopes to compare the rate of word acquisition of the child to the children in the study. I used data to support my argument.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "First, I modeled the data with a scatter plot, then I found the equation of a line of best fit for the data. I graphed the line of best fit with the data to visualize how the model fit the data. I also plotted the new data with the scatter plot and the model for comparison.")

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Alternate Strategies or Solutions

- Students may have chosen different points when calculating the line of best fit. Accept all reasonable solutions.
- Students could use a graphing calculator to analyze the data and find the line of best fit.

Technology

Students can use graphing calculators to verify solutions graphed by hand or to solve the task outright.

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Problem-Based Task: Good and Bad Cholesterol

Common Core State Standard

S–ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★

Task Overview

Focus

What does a correlation coefficient tell you about a data set? In this task, students will examine the data showing the total cholesterol and HDL cholesterol readings of 20 individuals to see if a relationship exists between these two quantities. Then, they will determine the correlation coefficient and what this value means in the context of this problem.

This activity will provide practice with:

- graphing data on a scatter plot
- examining the relationship between two quantitative variables
- determining the equation of a line of best fit
- analyzing and interpreting the correlation coefficient

Introduction

This task should be used to explore or apply the skill of calculating and interpreting the correlation coefficient. Technology, including graphing calculators or online graphing programs, may be used to create the scatter plot, determine the line of best fit, and calculate the correlation coefficient.

Begin by reading the problem and clarifying the meaning of the terms *HDL* (high-density lipoprotein) *cholesterol*, *LDL* (low-density lipoprotein) *cholesterol*, and *total cholesterol*, as well as the measurement units *milligram* and *deciliter*.

deciliter	a metric measurement of volume; equal to one tenth of a liter
HDL cholesterol	a waxy, fat-like substance found in the bloodstream; referred to as "good" because it reduces the risk of heart disease by removing harmful cholesterol from the bloodstream
LDL cholesterol	a waxy, fat-like substance found in the bloodstream; referred to as "bad" because it increases the risk of heart disease
milligram	a metric measurement of mass; equal to one thousandth of a gram
total cholesterol	the sum of the amounts of all the types of cholesterol found in a person's bloodstream

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students might forget to incorporate the correlation coefficient into their argument. Remind students to use the correlation coefficient as a measure of strength and validity of their model.

- **SMP 4:** Model with mathematics.

  Students should be familiar with creating a scatter plot to model the data, but they may be unsure as to how to proceed after creating the scatter plot. Encourage students to think about how to summarize the data with a linear model by finding the line of best fit.

- **SMP 5:** Use appropriate tools strategically.

  Students might have used technology to generate a line of best fit (least squares regression line) previously, but they may not be accustomed to looking for the correlation coefficient. Some students might want to draw a line of best fit by hand, but this will not help them find the correlation coefficient. Encourage students to use the technology to its maximum capacity: creating the table, creating the scatter plot, finding the equation for the line of best fit, finding the correlation coefficient, and graphing the line of best fit on the same screen as the scatter plot.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- using the correlation coefficient to analyze data that is not linear

  The correlation coefficient measures the strength of the relationship between two variables, but the interpreter must first determine if the relationship is linear based on a residual plot or by inspecting the data.

- incorrectly using the correlation coefficient to assess the strength of a relationship

  Remind students that as the correlation coefficient approaches 0, the strength of the relationship between the two variables is decreasing. As the correlation coefficient approaches 1 or –1, the strength of the relationship is increasing.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students, "When determining the nature of the relationship between two variables that make up a data set, what is the usual first step?" (**Answer:** "The first thing to do with the data is to make a scatter plot.")

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- Students might struggle with how to proceed after creating a scatter plot. Ask them to reread the task and to identify what information they are trying to find. (**Answer:** "I need to determine if there is a linear relationship between total cholesterol and HDL cholesterol.")

  Ask students if they believe there is a linear relationship just by inspection. (**Answer:** "As the x-values increase, the y-values also increase, so there appears to be a weak linear relationship.")

- Ask students to generate the line of best fit (linear regression) using technology. (**Answer:** $y \approx 0.2x + 6.82$)

- Ask students to identify the correlation coefficient. (**Answer:** $r = 0.596$)

  Ask students what this value means. (**Answer:** "By looking at the graph, we identified that relationship looks somewhat linear. The correlation coefficient is measuring the strength of that linear regression model. The value we found for r means that the relationship is positive; since it's a little more than halfway between 0 and 1, this value also means that the strength of the relationship between the two quantities is weak.")

  Ask students what a positive correlation coefficient means in terms of the context. (**Answer:** "As a person's total cholesterol level increases, their HDL cholesterol level also increases.")

- Ask students whether the correlation coefficient indicates causation. (**Answer:** "No, the correlation coefficient does not indicate causation because correlation never indicates causation. Correlation only indicates how likely it is that there is a relationship between the two events.")

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Compare methods used by the groups and discuss advantages and disadvantages of the varying strategies.

- Reiterate that correlation does not imply causation, and discuss why the relationship between total cholesterol and good cholesterol might be of interest.

Connecting to Key Concepts

Make explicit connections to key concepts:

- A correlation is a relationship between two events, where a change in one event implies a change in another event.

  In this task, students will examine whether a change in a person's total cholesterol level indicates a change in the level of HDL or "good" cholesterol.

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- Correlation doesn't mean that a change in the first event caused a change in the other event.

  Remind students that correlation never implies causation. Discuss how, in the context of cholesterol levels, an increase or decrease in total cholesterol does not cause an increase or decrease in HDL cholesterol. Other factors affect HDL cholesterol levels, including diet, exercise, and genetic factors.

- The strength of a linear correlation can be measured using a correlation coefficient.

  In this task, students will calculate the correlation coefficient of data comparing 20 patients' total and HDL cholesterol levels. Students will use this correlation coefficient to determine that the correlation between these two variables is positive, but weak.

- Before determining the correlation coefficient, look at the scatter plot of the data and make an initial assessment of the strength of a linear relationship between the two events.

  Students will create a scatter plot of the given data. They will use it to determine visually that the relationship looks somewhat linear, but that the relationship is not very strong.

Extending the Task

- To extend the task, have students further analyze the data to determine if the linear model is a good fit. For example, students can create a residual plot to assess the goodness of fit for a linear model. Students will see that the residual plot is scattered and random, showing no curvature or pattern. Therefore, a linear model seems to be a good fit.

- Students can look up the meaning of the R^2 value, or the coefficient of determination, that is also given when calculating the least squares regression line on a calculator, and interpret that value in the context of the problem. The R^2 value is a measure of how much the y-variable can be explained by x according to the model. It is another measure of how well the model fits the data. In this case, the R^2 value is about 0.36 or 36%. This means that about 36% of the variation in the HDL cholesterol can be explained by the variation in the total cholesterol. This concept can be further investigated through the following website.

  The Minitab Blog. "Regression Analysis: How Do I Interpret R-squared and Assess the Goodness-of-Fit?"

  http://www.walch.com/rr/07005

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Guide

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "Yes. I used the scatter plot, the regression model, and the correlation coefficient to argue my case for stating that there was a weak linear relationship between total cholesterol and HDL cholesterol.")
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I plotted the data using a scatter plot and I used technology to find the line of best fit.")
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** "I used the calculator to plot the data, calculate the line of best fit, and calculate the correlation coefficient.")

Alternate Strategies or Solutions

Students could use a spreadsheet to plot their data and generate the line of best fit and the correlation coefficient instead of a graphing calculator.

Technology

Students can use graphing calculators, spreadsheets, or online graphing utilities to plot the data and to calculate the line of best fit and the correlation coefficient. Students may use computers or tablets and web-based resources for the extension.

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Guide

Problem-Based Task: Good Cholesterol and Exercise

Common Core State Standard

S–ID.9 Distinguish between correlation and causation.★

Task Overview

Focus

What does a strong correlation between two sets of data mean? Can you determine causation if there is a strong correlation? In this task, students will analyze the relationship between the levels of HDL or "good" cholesterol in the body and the number of hours a person exercises each week.

This activity will provide practice with:

- graphing data in a scatter plot
- determining the equation of a line of best fit
- analyzing a correlation
- interpreting the correlation coefficient
- examining a relationship for causation

Introduction

This task should be used to explore or apply the skill of distinguishing between the concepts of correlation and causation. Graphing may be done using technology, including with graphing calculators or online graphing programs.

Begin by reading the problem and clarifying the meanings of *correlation* and *causation*, as well as the meaning of *HDL* (high-density lipoprotein) *cholesterol*:

HDL cholesterol	a waxy, fat-like substance found in the bloodstream; referred to as "good" because it reduces the risk of heart disease by removing harmful cholesterol from the bloodstream

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Guide

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students might be tempted to equate correlation and causation in their arguments. Encourage students to think about how two variables can be related, without one causing the other to happen. For example, examining the relationship between the number of LCD screens (TVs, computers, tablets, phones) in a household and the number of hours that people in the household exercise might show a negative relationship, but having more screens does not cause people to exercise less. Encourage students to justify their arguments about correlation and causation with examples.

- **SMP 4:** Model with mathematics.

  Students might want to try to answer the question posed in the task without plotting the data and finding the correlation coefficient. Encourage students to model the data with a graphical display, and to then find a measure of the strength of the relationship.

- **SMP 5:** Use appropriate tools strategically.

  Students might struggle with how to use technology efficiently. Encourage students to use the graphing and statistical analysis features available on their calculators or computer programs.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- assuming that a strong correlation indicates that one event causes another

  Discuss examples in which a strong correlation exists without causation. For example, there may be a correlation between the age of a person and the amount of money in his or her bank account, but being older does not "cause" the money to appear.

- failing to consider outside factors that may influence the strength of the correlation between two events

  Although the data suggests that people who exercise more have higher HDL cholesterol, there could be a number of factors that increase HDL cholesterol levels, such as diet.

- incorrectly plotting points, resulting in an incorrect regression line and/or correlation coefficient

  Have students double-check whether they input the data correctly into their graphing calculator or graphing program.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Ask students, "What is the first thing you should do with data in two variables when you are trying to analyze it?" (**Answer:** "I should start by making a scatter plot of the data.")
- Have students enter the data into their calculators and create a scatter plot. Ask them to describe the shape, slope, and approximate strength of the relationship of the data, and to justify their answers. (**Answer:** "The data look roughly linear, with a positive slope and with a moderate correlation. The data looks as if a line could be fit to it such that the data would roughly follow the path of the line. The data have a positive correlation because as the hours of exercise increase, so do the HDL levels. The correlation is moderate because the data is approximately linear, but does deviate from a perfectly linear path.")
- Ask students how they can find the correlation coefficient for this data. (**Answer:** "I can calculate the line of best fit using the linear regression feature of a graphing calculator, and this will give the correlation coefficient along with the equation.")
- Ask students to calculate the linear regression model and the correlation coefficient. (**Answer:** $y = 3.77x + 41.11$; $r = 0.672$)

  Ask students to interpret the correlation coefficient. (**Answer:** "The correlation coefficient is positive, and it is a little more than $\frac{2}{3}$ of the way between 0 and 1. Therefore, it suggests a moderately strong, positive correlation.")

  Ask students, "Does this indicate that increased exercise causes increased HDL levels? Why or why not?" (**Answer:** "Correlation never implies causation. Even if the correlation were closer to 1, the correlation would not indicate that increased exercise *causes* HDL levels to rise.")

  Ask students to explain why correlation does not imply causation. (**Answer:** "Correlation is only a measure of the strength of a linear relationship. It indicates how strong the association is between the variables, but it doesn't explain the factors that lead to the association.")
- Ask students, "What factors might influence HDL levels?" (**Answer:** "Exercise, diet, genetics, environment, and/or medications could be factors.")
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

Compare the students' various methods and conclusions, and discuss advantages and disadvantages of the different strategies. Review the difference between correlation and causation, and discuss factors that might influence HDL levels besides exercise.

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Guide

Connecting to Key Concepts

Make explicit connections to key concepts:

- Correlation does not imply causation.

  In this task, students will see that although there is a correlation between the amount of exercise and the HDL cholesterol level, there is no evidence that the exercise *causes* the HDL level.

- If a change in one event is responsible for a change in another event, the two events have a causal relationship.

  Students will realize that causation can only be demonstrated by conducting a scientific experiment.

- Outside factors may influence and explain a strong correlation between two events.

  Discuss with students how several factors might increase HDL levels, such as exercise, diet, genetics, environment, and/or medications.

- Use a calculator to find the correlation coefficient.

  In this task, students will determine the correlation coefficient using a calculator.

Extending the Task

To extend the task, ask students how they could design an experiment to test whether an increase in exercise causes an increase in HDL levels. For example, have students consider why a control group would probably be used in an experiment, and who would be in the control group. Ask students how to control for other outside factors that might also lead to an increase in HDL levels.

Have students read the tutorial and watch the video about designing experiments found on the following website.

- Stat Trek. "Experimental Design."

  http://www.walch.com/rr/07006

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "Yes. I used the scatter plot and the value of the correlation coefficient to support my reasoning. I used the definitions of *correlation* and *causation* to answer the question posed in the task.")

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- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** "I plotted the data, created a linear model to fit the data, and evaluated the model's strength using the correlation coefficient.")
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** "I used technology to plot the data, create a linear model, and calculate the correlation coefficient in order to answer the question posed in the task.")

Alternate Strategies or Solutions

Students may use a spreadsheet instead of graphing technology to calculate the correlation coefficient.

Technology

Students can use graphing calculators, graphing apps, or spreadsheets to complete the task, as long as the program used is able to generate a correlation coefficient. Students need a computer or tablet and Internet access to complete the extension.

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Common Core State Standards

Mathematics I

Integrated Pathway



Teacher Resource

Unit 5

WALCH

INTEGRATED MATH

This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

Developers and reviewers include:

Joyce Hale	Valerie Ackley	Doug Kühlmann
Shelly Northrop Sommer	Laura McPartland	Mike May, S.J.
Ruth Estabrook	Cameron Larkins	James Quinlan
Vanessa Sylvester	Jennifer Blair	Peter Tierney-Fife
Zachary Lien	Nancy Pierce	

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Standards Correlations

Each lesson in this Integrated Pathway: Mathematics I program was written specifically to address the Common Core State Standards. Each lesson lists the standards covered in all the sub-lessons, and each sub-lesson lists the standards addressed in that particular section. In this section, you'll find a comprehensive list mapping the sub-lessons to the CCSS.

Guide to Common Core State Standards Annotation

As you use this program, you will come across a symbol included with the Common Core standards for some of the lessons and activities. These symbols are explained below.

Symbol: ⋆

Denotes: Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (⋆).

From http://www.walch.com/CCSS/00003

Symbol: (+)

Denotes: College and Career Readiness Standards

Advanced mathematics standards that are required in higher-level courses such as advanced statistics may also be included in lower-level courses. These additional standards are denoted by (+). According to the Common Core State Standards Initiative, "the evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6–8."

From http://www.walch.com/CCSS/00004

Connections to Future Courses

This section provides a map between topics introduced in each unit of this course and subsequent courses where each topic is revisited and built upon.

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CCSS INTEGRATED PATHWAY: MATHEMATICS I STANDARDS CORRELATIONS

Unit 5: Congruence, Proof, and Constructions				
Lesson	**Sub-lesson number**	**Title**	**Standard(s)**	**Pages**
Lesson 1	**Introducing Transformations**			
	5.1.1	Defining Terms	G–CO.1	U5-5
	5.1.2	Transformations As Functions	G–CO.2	U5-18
	5.1.3	Applying Lines of Symmetry	G–CO.3	U5-38
Lesson 2	**Defining and Applying Rotations, Reflections, and Translations**			
	5.2.1	Defining Rotations, Reflections, and Translations	G–CO.4	U5-60
	5.2.2	Applying Rotations, Reflections, and Translations	G–CO.5	U5-72
Lesson 3	**Constructing Lines, Segments, and Angles**			
	5.3.1	Copying Segments and Angles	G–CO.12	U5-94
	5.3.2	Bisecting Segments and Angles	G–CO.12	U5-118
	5.3.3	Constructing Perpendicular and Parallel Lines	G–CO.12	U5-142
Lesson 4	**Constructing Polygons**			
	5.4.1	Constructing Equilateral Triangles Inscribed in Circles	G–CO.13	U5-175
	5.4.2	Constructing Squares Inscribed in Circles	G–CO.13	U5-199
	5.4.3	Constructing Regular Hexagons Inscribed in Circles	G–CO.13	U5-219
Lesson 5	**Exploring Congruence**			
	5.5.1	Describing Rigid Motions and Predicting the Effects	G–CO.6	U5-254
	5.5.2	Defining Congruence in Terms of Rigid Motions	G–CO.6	U5-277
Lesson 6	**Congruent Triangles**			
	5.6.1	Triangle Congruency	G–CO.7	U5-312
	5.6.2	Explaining ASA, SAS, and SSS	G–CO.8	U5-332

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TOPICS FROM FUTURE COURSES: CCSS INTEGRATED PATHWAY MATHEMATICS I

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 5	Transformations and connections between functions and geometric transformations; congruence	Math II Unit 5	includes similarity transformations
		Math II Unit 6	extended to coordinate geometry and proofs
	Perimeter and area	Math II Unit 6	extended to area and volume
		Math III Unit 4B	includes density and cross sections of three-dimensional objects
	Geometric figures (points, lines, and planes)	Math II Unit 5	extended to parallel lines cut by a transversal and other relationships between angle pairs, other special triangles, and parallelograms
		Math II Unit 6	includes circles and special segments
		Math III Unit 4B	extended to three-dimensional objects in modeling and design

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Conceptual Activities

Use these interactive open education and/or Desmos resources to build conceptual understanding of mathematical ideas. (*Note*: Activity links will be monitored and repaired or replaced as necessary.)

- Desmos. "Polygraph: Transformations."

  http://www.walch.com/ca/01025

  This activity is designed to spark vocabulary-rich conversations about transformations. Key vocabulary terms that may appear in student questions include *translation*, *rotation*, *reflection*, *dilation*, *scale factor*, *preimage*, and *image*.

- Desmos. "Symmetry."

  http://www.walch.com/ca/01026

  In this activity, students develop an informal understanding of symmetry of functions. By the end, students should be able to identify the symmetry of a function (reflectional vs. rotational) by considering its graph.

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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. What is the measure of a right angle?
   a. 60°
   b. 90°
   c. 120°
   d. 135°

2. What type of function maps an input onto itself?
   a. opposite function
   b. negative function
   c. identity function
   d. reflective function

3. How many lines of symmetry does a square have?
   a. 1
   b. 5
   c. 2
   d. 4

4. Given $T_{h,k}(x, y) = (x + h, y + k)$ and $P(-4, -3)$, what is $T_{-2,1}(P)$?
   a. $P(-6, -2)$
   b. $P(2, 4)$
   c. $P(-5, -4)$
   d. $P(-3, -5)$

5. What geometric figure has an infinite number of lines of symmetry?
   a. triangle
   b. circle
   c. square
   d. octagon

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Lesson 1: Introducing Transformations

Instruction

Common Core State Standards

SMP	
1 ✓	2
3 ✓	4 ✓
5	6 ✓
7 ✓	8

G–CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G–CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G–CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Essential Questions

1. What does it mean to be parallel? What does it mean to be perpendicular?
2. What is a function? Are transformations functions?
3. What does it mean to be symmetrical?
4. Are there different types of transformations?
5. How do we define transformations?

WORDS TO KNOW

acute angle	an angle measuring less than 90° but greater than 0°
angle	two rays or line segments sharing a common endpoint. Angles can be measured in degrees or radians; written as $\angle A$.
arc length	the distance between the endpoints of an arc; written as $d\left(\overset{\frown}{ABC}\right)$
circle	the set of points on a plane at a fixed distance, or radius, from a given point, the center
circular arc	on a circle, the set of points between the endpoints of two radii
congruent	having the same shape, size, or angle
distance along a line	the linear distance between two points on a given line; written as PQ
image	the new, resulting figure after a transformation

SRB p. U5-1

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Instruction

isometry	a transformation in which the preimage and image are congruent
line	the straight path connecting two points and extending beyond the points in both directions; written as $\overleftrightarrow{PQ}$ or ℓ
line of symmetry	a line separating a figure into two halves that are mirror images
line segment	a part of a line that is between two endpoints and that includes the endpoints; written as $\overline{PQ}$
line symmetry	exists for a figure if for every point on one side of the line of symmetry, there is a corresponding point the same distance from the line of symmetry on the other side
obtuse angle	an angle measuring greater than 90° but less than 180°
one-to-one	a relationship wherein each input is mapped to exactly one output, and each output is associated with exactly one input
parallel lines	lines in a plane that do not share any points and never intersect; written as $\overleftrightarrow{AB} \parallel \overleftrightarrow{PQ}$; line segments and rays can also be parallel
perpendicular lines	two lines that intersect at a right angle (90°); written as $\overleftrightarrow{AB} \perp \overleftrightarrow{PQ}$; line segments and rays can also be perpendicular
point	an exact position or location in a given plane
preimage	the original figure before undergoing a transformation
ray	a part of a line that starts at a point and extends infinitely in one direction; written as $\overrightarrow{PQ}$
reflection	a transformation where a mirror image is created; also called a flip
regular polygon	a closed two-dimensional figure with all sides congruent and all angles congruent
right angle	an angle measuring 90°
rotation	a transformation that turns a figure around a fixed center point; also called a turn
transformation	a change in a geometric figure's position, shape, or size
translation	a transformation that moves each point of a figure the same distance in the same direction; also called a slide

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Instruction

Recommended Resources

- MathIsFun.com. "What is a Function?"

  http://walch.com/rr/CAU5L1Functions

  This site gives a general overview of functions.

- Purplemath. "Function Transformations/Translations."

  http://walch.com/rr/CAU5L1FunctionTrans

  The graphs and explanations at this site provide a thorough look at various function transformations.

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Lesson 5.1.1: Defining Terms

Warm-Up 5.1.1

Marta wants to draw a perfect circle on canvas, but her compass is broken. She decides to take a shoelace from one of her sneakers and a pushpin from her drawing desk. She ties the shoelace around a pencil and then pins the other end of the shoelace into the center of the canvas. Marta then stretches the shoelace out until it's pulled tight, and places the pencil point on the canvas. Keeping the shoelace straight, she moves the pencil around the canvas and begins to draw out a circle.

1. Will Marta's approach to drawing a perfect circle work? Why or why not?

2. How could Marta show that the drawing is or is not a perfect circle?

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Instruction

Lesson 5.1.1: Defining Terms

Common Core State Standard

G–CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Warm-Up 5.1.1 Debrief

1. Will Marta's approach to drawing a perfect circle work? Why or why not?

   Marta's method will yield a perfect circle because she is drawing a path such that every point on it will be the same distance from the center.

2. How could Marta show that the drawing is or is not a perfect circle?

   Marta could measure the distance from one side of her circle through the center point made by the pin to the other side of her circle. This measurement should be the same regardless of where on the circle she begins her measurement.

Connection to the Lesson

- The definition of a circle is explored in a visualization exercise.
- Students will explore the definitions of this and other geometric terms in the lesson.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- understanding the undefined terms in geometry: point, line, and plane (G–CO.1)
- calculating distance on a number line (8.G.8)

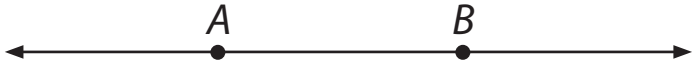
Introduction

Geometric figures can be graphed in the coordinate plane, as well as manipulated. However, before sliding and reflecting figures, the definitions of some important concepts must be discussed.

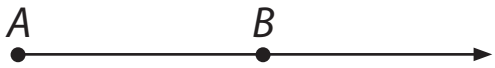
Each of the manipulations that will be discussed will move points along a parallel line, a perpendicular line, or a circular arc. In this lesson, each of these paths and their components will be introduced.

Key Concepts

- A **point** is not something with dimension; a point is a "somewhere." A point is an exact position or location in a given plane. In the coordinate plane, these locations are referred to with an ordered pair (x, y), which tells us where the point is horizontally and vertically. The symbol $A\ (x, y)$ is used to represent point A at the location (x, y).
- A **line** requires two points to be defined. A line is the straight path connecting two points and extending beyond the points in both directions. A line is infinite, without beginning or end. This is shown in the diagram below with the use of arrows. The symbol $\overleftrightarrow{AB}$ is used to represent line AB.



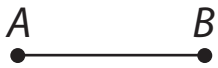
- You can find the linear distance between two points on a given line. **Distance along a line** is written as PQ where P and Q are points on a line.
- Like a line, a **ray** is defined by two points; however, a ray has an endpoint. The symbol $\overrightarrow{AB}$ is used to represent ray AB.



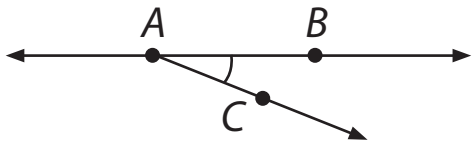
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Instruction

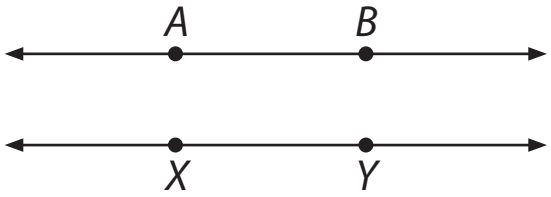
- Similarly, a **line segment** is also defined by two points, but both of those points are endpoints. A line segment can be measured because it has two endpoints and finite length. Line segments are used to form geometric figures. The symbol $\overline{AB}$ is used to represent line segment *AB*.



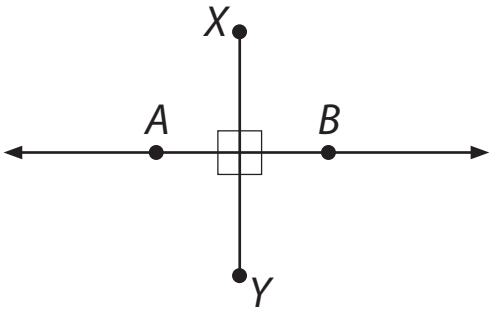
- An **angle** is formed where two line segments or rays share an endpoint, or where a line intersects another line, ray, or line segment. Angles can be measured in degrees or radians. The symbol $\angle A$ is used to represent angle *A*. *A* represents the vertex of the angle. Sometimes it is necessary to use three letters to avoid confusion. In the diagram below, $\angle BAC$ can be used to represent the same angle, $\angle A$. Notice that *A* is the vertex of the angle and it will always be listed in between the points on the rays of the angle.



- An **acute angle** measures less than 90° but greater than 0°. An **obtuse angle** measures greater than 90° but less than 180°. A **right angle** measures exactly 90°.
- Two relationships between lines that will help us define transformations are parallel and perpendicular. **Parallel lines** are two lines that never intersect. The notation for parallel lines is $\parallel$. For the following diagram, $\overline{AB} \parallel \overline{XY}$.



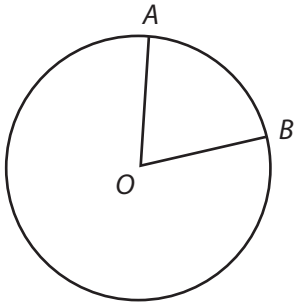
- **Perpendicular lines** meet at a right angle (90°), creating four right angles. The notation for perpendicular lines is $\perp$. For the following diagram, $\overline{AB} \perp \overline{XY}$.



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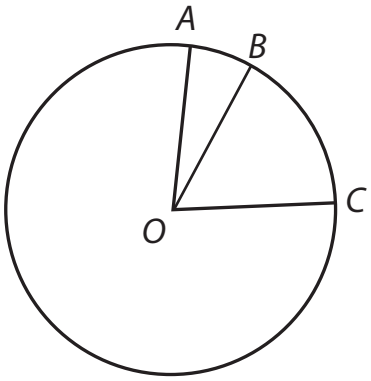
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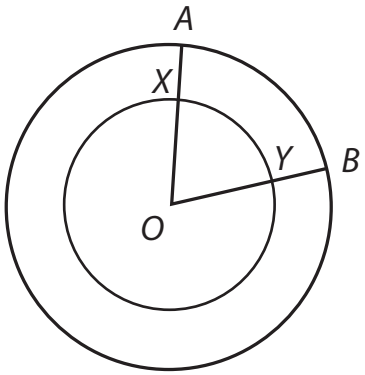
- A **circle** is the set of points on a plane at a fixed distance, or radius, from a given point, the center. Notice that a radius is a line segment. Therefore, if we draw any two radii of a circle, we create an angle where the two radii share a common endpoint, the center of the circle.



- Creating an angle inside a circle allows us to define a **circular arc**, the set of points along the circle between the endpoints of two distinct radii. The **arc length**, or distance along a circular arc, is dependent on the length of the radius and the angle that creates the arc—the greater the radius or angle, the longer the arc.

$\angle AOC > \angle AOB \rightarrow \overset{\frown}{AC} > \overset{\frown}{AB}$ $\qquad$ $\overline{OA} > \overline{OX} \rightarrow \overset{\frown}{AB} > \overset{\frown}{XY}$





Common Errors/Misconceptions

- mislabeling angles or not including enough points to specify an angle
- misusing terms and notations
- finding the length of incorrect arcs

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Instruction

Guided Practice 5.1.1

Example 1

Refer to the figures below. Can a line segment be defined using the points A and B? Can a line segment be defined using the point C? Justify your response to each question.



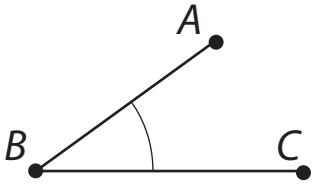


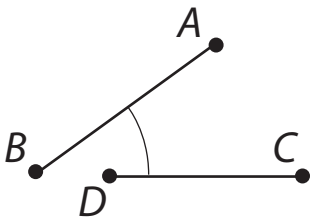
1. The points A and B can be used to define a line segment because A and B are on the same line and are unique points.

2. The point C cannot be used to define a line segment because there is not a second point defined on the line.

Example 2

Refer to the figures below. In the first figure, do the line segments $\overline{AB}$ and $\overline{BC}$ form an angle? In the second figure, do the line segments $\overline{AB}$ and $\overline{CD}$ form an angle? Justify your response to each question.





SWB p. 9; SRB p. U5-7

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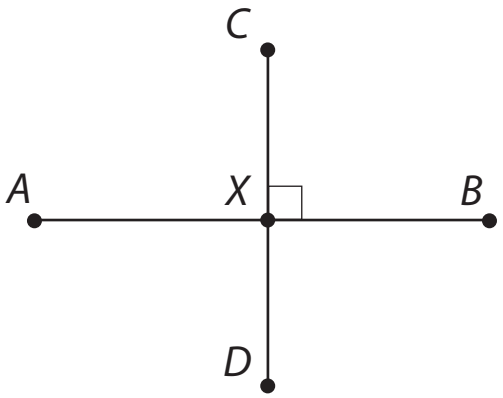
Instruction

1. In the first figure, the line segments $\overline{AB}$ and $\overline{BC}$ meet the angle definition of two lines, rays, or line segments intersecting; the two segments form an angle.

2. In the second figure, the line segments $\overline{AB}$ and $\overline{CD}$ do not intersect, so they do not form an angle.

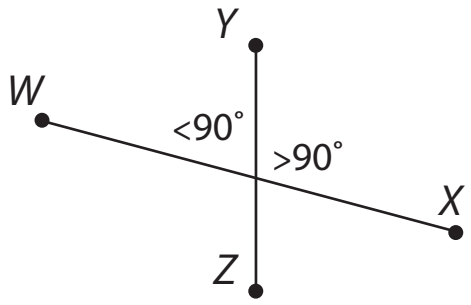
Example 3

By definition, $\overline{AB}$ is perpendicular to $\overline{CD}$ because $m\angle CXB$ is 90°. What are the measures of $\angle AXC$, $\angle AXD$, and $\angle DXB$?



1. The measures of $\angle AXC$, $\angle AXD$, and $\angle DXB$ are all 90°. The importance of the perpendicular relationship is that all four angles created by the intersection are equal.

2. In the figure that follows, we can see the result when the lines are not perpendicular: the angles of intersection are not equal.



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Instruction

Example 4

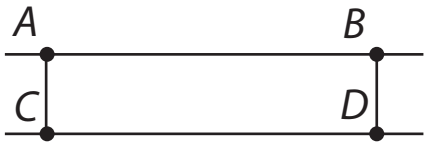
Given the following:

$\overline{AC} \cong \overline{BD}$ $\qquad$ $\overline{WY} < \overline{XZ}$

$\overline{AB} \perp \overline{AC}$ $\qquad$ $\overline{WX} \perp \overline{WY}$

$\overline{AB} \perp \overline{BD}$ $\qquad$ $\overline{WX} \perp \overline{XZ}$

Are $\overline{AB}$ and $\overline{CD}$ parallel? Are $\overline{WX}$ and $\overline{YZ}$ parallel? Explain.





1. $\overline{AC}$ and $\overline{BD}$ intersect $\overline{AB}$ at the same angle and $\overline{AC} \cong \overline{BD}$. $\overline{AB}$ will never intersect $\overline{CD}$. Therefore, $\overline{AB}$ is parallel to $\overline{CD}$.

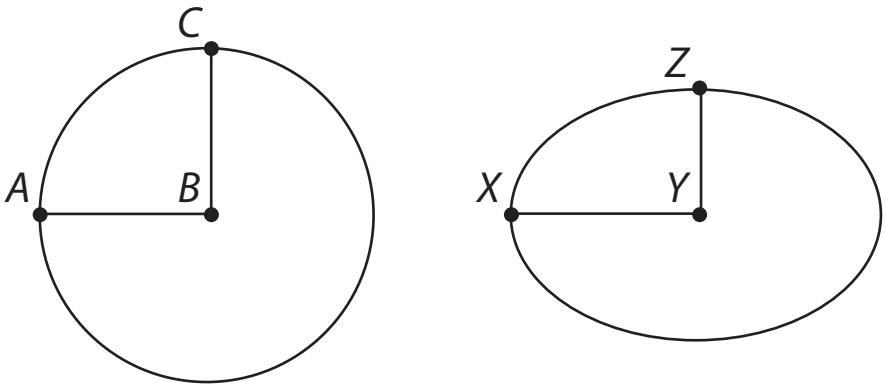
2. $\overline{WY}$ and $\overline{XZ}$ intersect $\overline{WX}$ at the same angle, but $\overline{WY} < \overline{XZ}$. As you move from Z to Y on $\overline{YZ}$, you move closer to, and will eventually intersect, $\overline{WX}$. Therefore, $\overline{WX}$ is not parallel to $\overline{YZ}$.

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Instruction

Example 5

Refer to the figures below. Given $\overline{AB} \cong \overline{BC}$, is the set of points with center B a circle? Given $\overline{XY} > \overline{YZ}$, is the set of points with center Y a circle?



1. The set of points with center B is a circle because all points are equidistant from the center, B.

2. The set of points with center Y is not a circle because the points vary in distance from the center, Y.

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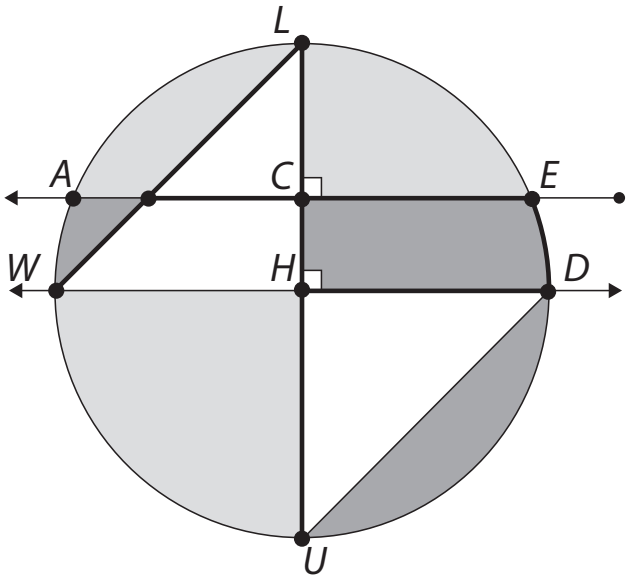
Problem-Based Task 5.1.1: A Mathematical Look

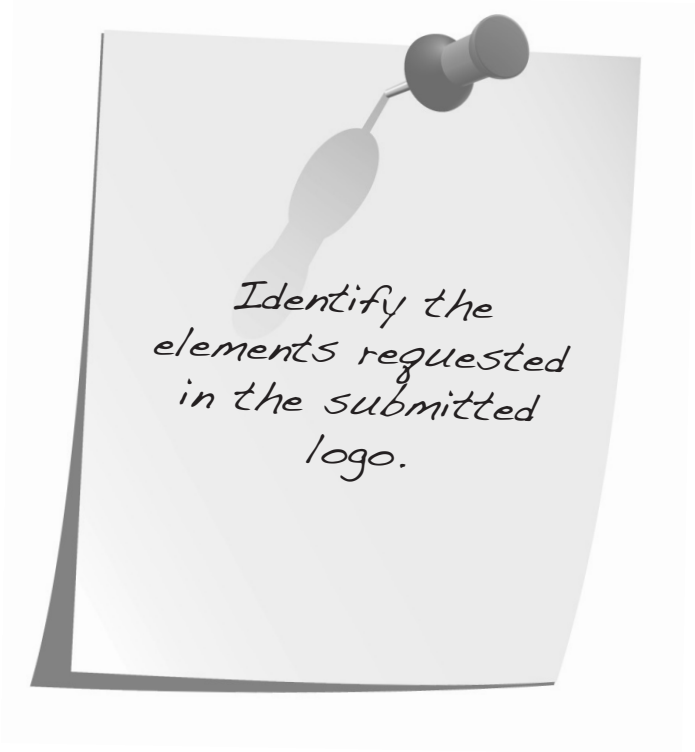
SMP	
1	2
3 ✓	4 ✓
5	6 ✓
7	8

Walch Education contracted Ryan Icons to design a logo for the company. They requested the logo be circular and contain the following elements:

- a line
- a ray
- a line segment
- two pairs of parallel line segments
- a pair of perpendicular line segments

Identify the elements requested in the submitted logo shown below.





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Problem-Based Task 5.1.1: A Mathematical Look

Coaching

a. Identify a line in the submitted logo.

b. Identify a ray in the submitted logo.

c. Identify a line segment in the submitted logo.

d. Identify two pairs of parallel line segments in the submitted logo.

e. Identify one pair of perpendicular lines in the submitted logo.

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Instruction

Problem-Based Task 5.1.1: A Mathematical Look

Coaching Sample Responses

a. Identify a line in the submitted logo.

There is one line in the submitted logo, $\overleftrightarrow{HD}$.

b. Identify a ray in the submitted logo.

One ray in the submitted logo is $\overrightarrow{CA}$.

c. Identify a line segment in the submitted logo.

There are several line segments in the submitted logo, such as $\overline{LC}$ and $\overline{CH}$.

d. Identify two pairs of parallel line segments in the submitted logo.

The two pairs of parallel line segments in the submitted logo are $\overline{AE} \parallel \overline{WD}$ and $\overline{WL} \parallel \overline{UD}$.

e. Identify one pair of perpendicular lines in the submitted logo.

There are two pairs of perpendicular lines in the submitted logo, $\overline{LU} \perp \overline{WD}$ and $\overline{LU} \perp \overline{AE}$.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 5.1.1: Defining Terms

Use what you've learned to answer the questions that follow.

1. What is the part of a line with two endpoints *P* and *Q*? How is it written?

2. What is the definition of a circle?

3. What is a term used to describe two lines that intersect to form a right angle?

4. What is the definition of a point?

5. What type of angle has a measure that is greater than 0° but less than 90°?

6. What term is used to describe two rays or two line segments that share a common endpoint?

7. Two circular arcs, $\overset{\frown}{AB}$ and $\overset{\frown}{XY}$, share the same center, *O*. The point *A* is on $\overline{OX}$ and *B* is on $\overline{OY}$. What can be said about the relation of the lengths of $\overset{\frown}{AB}$ and $\overset{\frown}{XY}$?

8. Two circular arcs, $\overset{\frown}{AB}$ and $\overset{\frown}{AC}$, share the same center, *O*. The point *B* lies on the circle between the points *A* and *C*. What can be said about the relation of the lengths of $\overset{\frown}{AB}$ and $\overset{\frown}{ABC}$?

9. What type of angle has a measure that is greater than 90° but less than 180°?

10. What is the term used to describe the straight path between two points that extends beyond the points in both directions?

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Lesson 5.1.2: Transformations As Functions

Warm-Up 5.1.2

Every weekday morning, cousins Ainsley, Jack, and Caleb are given a different amount of money for lunch by their parents. Ainsley gets $3, Jack receives $4, and Caleb gets $5. On the way to school, they stop by their Grandpa Ray's house to say hello. He always gives them each $2 more for lunch. By the time the cousins reach the bus stop, they each have more money than when they left the house.

Mathematically, one way to describe this story would be to assign each of the cousins a value, and define Grandpa Ray as a function. Example:

> Let Ainsley = $3, Jack = $4, and Caleb = $5. We can then define Grandpa Ray as a function, R, where R "of" x dollars, written $R(x)$, is $x + \$2$. Then we can say $R(\text{Ainsley}) = R(\$3) = \$5$, $R(\text{Jack}) = R(\$4) = \$6$, and $R(\text{Caleb}) = R(\$5) = \$7$.

1. The actual cost of lunch for each cousin is $2. Using the values assigned to the cousins in the example and defining lunch as the function, L, use function notation to symbolize paying for lunch.

2. How does $L(x)$ compare to the original amount each cousin left the house with?

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Lesson 5.1.2: Transformations As Functions

Common Core State Standard

G–CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

Warm-Up 5.1.2 Debrief

1. The actual cost of lunch for each cousin is \$2. Using the values assigned to the cousins in the example and defining lunch as the function, *L,* use function notation to symbolize paying for lunch.

   In the example, after visiting Grandpa Ray, Ainsley = \$5, Jack = \$6, and Caleb = \$7. We are given the cost of lunch to be \$2; therefore, we describe the function for lunch, *L*, as $L(x) = R(x) - 2$.

   $L(\text{Ainsley}) = R(\text{Ainsley}) - 2 = 5 - 2 = \$3$

   $L(\text{Jack}) = R(\text{Jack}) - 2 = 6 - 2 = \$4$

   $L(\text{Caleb}) = R(\text{Caleb}) - 2 = 7 - 2 = \$5$

2. How does $L(x)$ compare to the original amount each cousin left the house with?

   The function $L(x)$ returns the original amount given to each cousin prior to visiting Grandpa Ray.

Connection to the Lesson

- In this lesson, students will learn that functions can describe predictable arithmetic.
- Students will also see that when arithmetic is applied to coordinates, movement can be defined as a function.
- They will see that predictable actions can be described as functions, and functions take in values and produce new values.
- They will understand that when describing functions, letters and symbols can be substituted for elements and actions.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- locating points in the coordinate plane (5.G.1)
- recognizing simple graphs, such as $y = x$ (A–CED.2)

Introduction

The word *transform* means "to change." In geometry, a **transformation** changes the position, shape, or size of a figure on a coordinate plane. The original figure, called a **preimage**, is changed or moved, and the resulting figure is called an **image**. We will be focusing on three different transformations: translations, reflections, and rotations. These transformations are all examples of **isometry**, meaning the new image is congruent to the preimage. Figures are **congruent** if they have the same shape, size, or angle. The new image is simply moving to a new location. In this lesson, we will learn to describe transformations as functions on points in the coordinate plane. Let's first review functions and how they are written.

A function is a relationship between two sets of data, inputs and outputs, where each input has exactly one output. Because of this relationship, functions are defined in terms of their potential inputs and outputs. For example, we can say that a function f takes real numbers as inputs and its outputs are also real numbers. Once we have determined what the potential inputs and outputs are for a given function, the next step is to define the exact relationship between the individual inputs and outputs. To do this, we need to have a name for each output in terms of the input. So, for a function f with input x, the output is called "f of x," written $f(x)$. For example, if we say f takes x as input and the output is $x + 2$, then we write $f(x) = x + 2$.

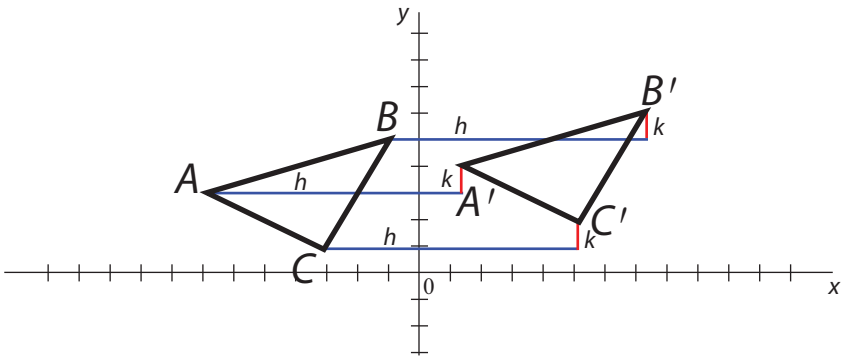
Now that we understand the idea of a function, we can discuss transformations in the coordinate plane as functions. First we need to determine our potential inputs and outputs. In the coordinate plane we define each coordinate pair, or point, in the form (x, y) where x and y are real numbers. Each input for a transformation function f in the coordinate plane will be a real number coordinate pair, (x, y), and each output will be a real number coordinate pair, $f(x, y)$. For example, if f is a function in the coordinate plane such that f of (x, y) is $(x + 1, y + 2)$, we can write this as: $f(x, y) = (x + 1, y + 2)$.

Finally, transformations are generally applied to a set of points such as a line, triangle, square, or other figure. In geometry, these figures are often described by points such as P, rather than coordinates (x, y). The transformation $T(x, y)$ can be written $T(P)$, and the image is called P', or "P prime."

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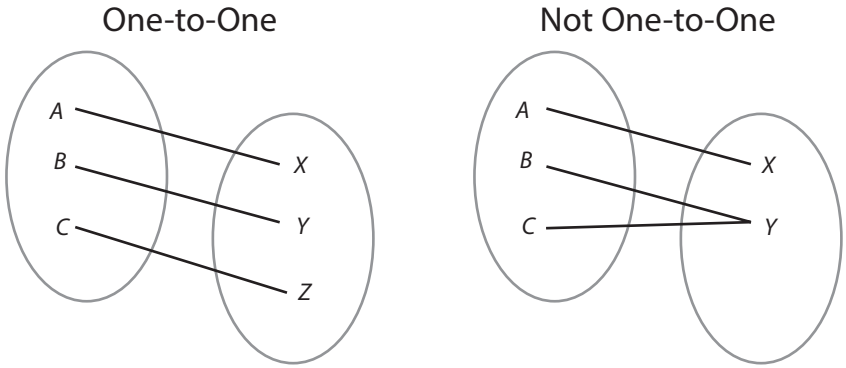
Instruction

When a transformation is applied to a set of points, such as a triangle, then all points in the set are moved according to the transformation. For example, if $T(x, y) = (x + h, y + k)$, then $T(\triangle ABC)$ would be:



Key Concepts

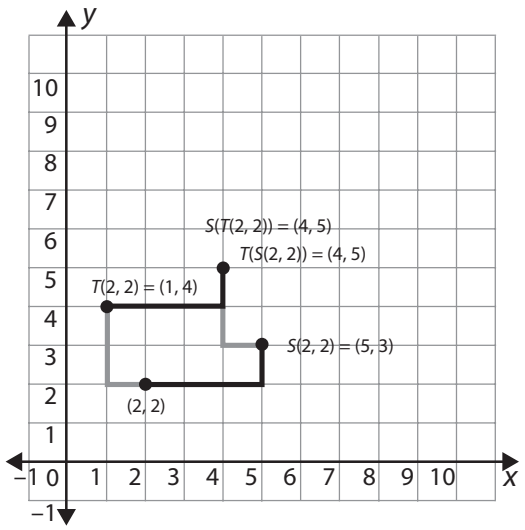
- Transformations are **one-to-one**, which means input is mapped onto exactly one output, and each output is associated with exactly one input.



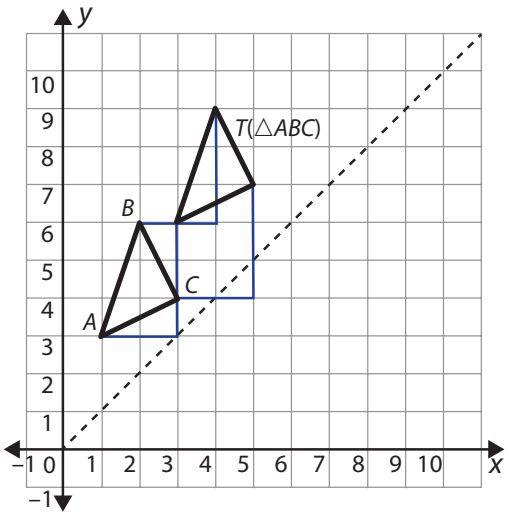
- The simplest transformation is the identity function I where $I(x, y) = (x', y')$, and $(x', y') = (x, y)$.
- Transformations can be combined to form a new transformation that will be a new function.
- For example, if $S(x, y) = (x + 3, y + 1)$ and $T(x, y) = (x - 1, y + 2)$, then $S(T(x, y)) = S((x - 1, y + 2)) = ((x - 1) + 3, (y + 2) + 1) = (x + 2, y + 3)$.
- It is important to understand that the order in which functions are taken may affect the output. In the function above, we see $S(T(x, y)) = (x + 2, y + 3)$. In calculating $T(S(x, y))$, we find that $T(S(x, y)) = (x + 2, y + 3)$ as well. In this case, $T(S(x, y)) = S(T(x, y))$, and in the graph that follows, which applies these transformations to the point (2, 2), we can see why.

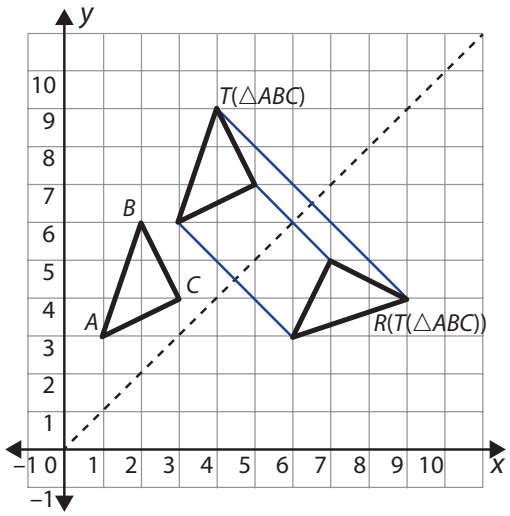
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Instruction



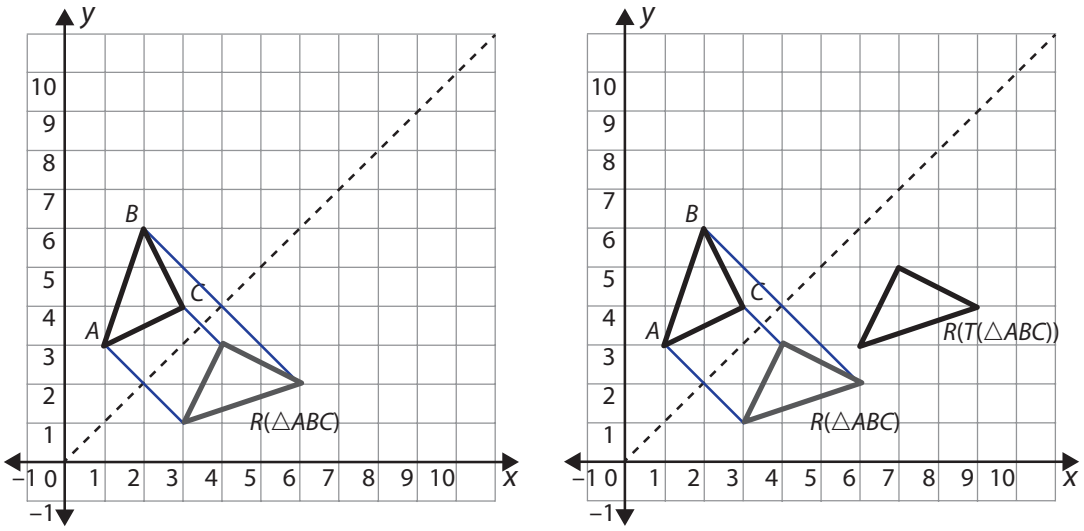
- However, here are two functions where the order in which they are taken changes the outcome: a translation $T_{2,3}(x, y)$, and a reflection through the line $y = x$, $r_{y=x}(x, y)$, on $\triangle ABC$. In the first two graphs we see $r_{y=x}\left(T_{2,3}\left(\triangle ABC\right)\right)$, and in the second two graphs we see $T_{2,3}\left(r_{y=x}\left(\triangle ABC\right)\right)$. Notice the outcome is different depending on the order of the functions.



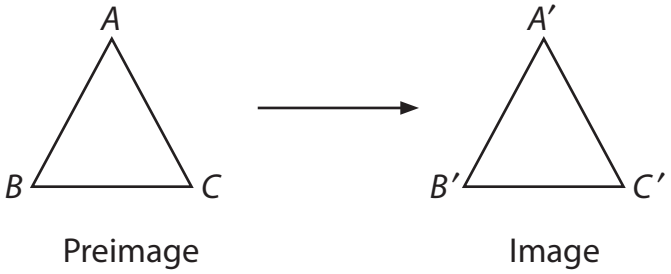


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Instruction



- Because the order in which functions are taken can affect the output, we always take functions in a specific order, working from the inside out. For example, if we are given the set of functions $h(g(f(x)))$, we would take $f(x)$ first and then g and finally h.
- Remember, an isometry is a transformation in which the preimage and the image are congruent. An isometry is also referred to as a "rigid transformation" because the shape still has the same size, area, angles, and line lengths. The previous example is an isometry because the image is congruent to the preimage.

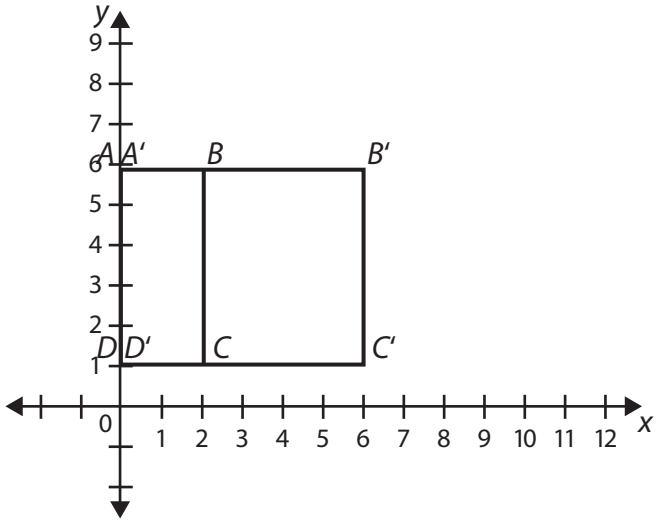


- In this lesson, we will be focusing on three isometric transformations: translations, reflections, and rotations. A **translation**, or slide, is a transformation that moves each point of a figure the same distance in the same direction. A **reflection**, or flip, is a transformation where a mirror image is created. A **rotation**, or turn, is a transformation that turns a figure around a point.

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Instruction

- Some transformations are not isometric. Examples of non-isometric transformations are horizontal or vertical stretches and dilations.
- For example, a horizontal stretch transformation, $T(x, y) = (3x, y)$, applied to rectangle $ABCD$ is one-to-one—every point in rectangle $ABCD$ is mapped to just one point in rectangle $A'B'C'D'$. However, horizontal distance is not preserved.

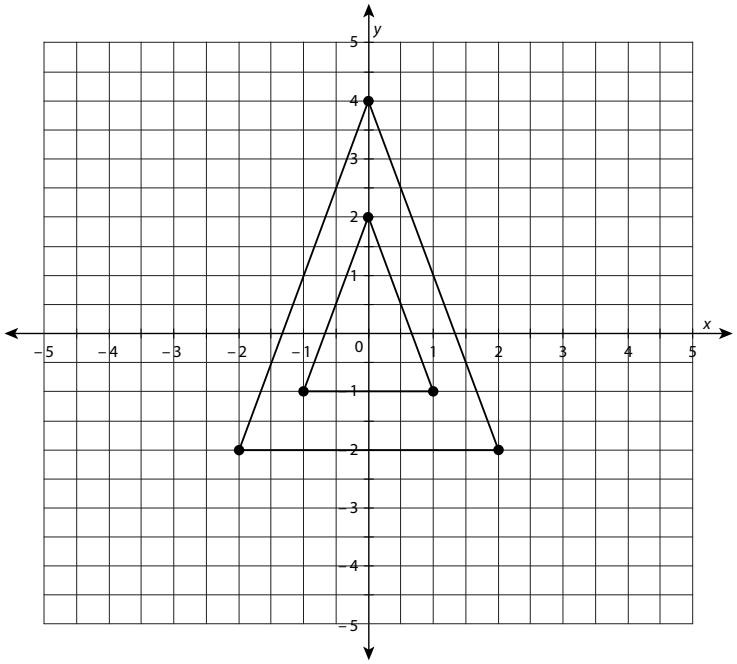


- Note that $\overline{AB} \neq \overline{A'B'}$ and $\overline{CD} \neq \overline{C'D'}$. From the graph, we can see that rectangle $ABCD$ and rectangle $A'B'C'D'$ are not congruent; therefore T is not isometric.
- Another transformation that is not isometric is a dilation. A dilation stretches or contracts both coordinates.

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Instruction

- If we have the dilation $D(x, y) = (2x, 2y)$, we can graph $D(\triangle ABC) = \triangle A'B'C'$, as seen here:



Common Errors/Misconceptions

- taking functions in the wrong order
- not understanding that reflections, rotations, and translations have congruent preimages and images

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Guided Practice 5.1.2

Example 1

Given the point P (5, 3) and $T(x, y) = (x + 2, y + 2)$, what are the coordinates of $T(P)$?

1. Identify the point given.

   We are given P (5, 3).

2. Identify the transformation.

   We are given $T(P) = (x + 2, y + 2)$.

3. Calculate the new coordinates.

   $T(P) = (x + 2, y + 2)$

   $= (5 + 2, 3 + 2)$

   $= (7, 5)$

   $T(P) = (7, 5)$

Example 2

Given $\triangle ABC: A(5,2), B(3,5)$, and $C(2,2)$, and the transformation $T(x, y) = (x, -y)$, what are the coordinates of the vertices of $T(\triangle ABC)$? What kind of transformation is T?

1. Identify the vertices of the triangle.

   We are given the coordinates of the vertices A (5, 2), B (3, 5), and C (2, 2).

2. Identify the transformation.

   We are given $T(x, y) = (x, -y)$.

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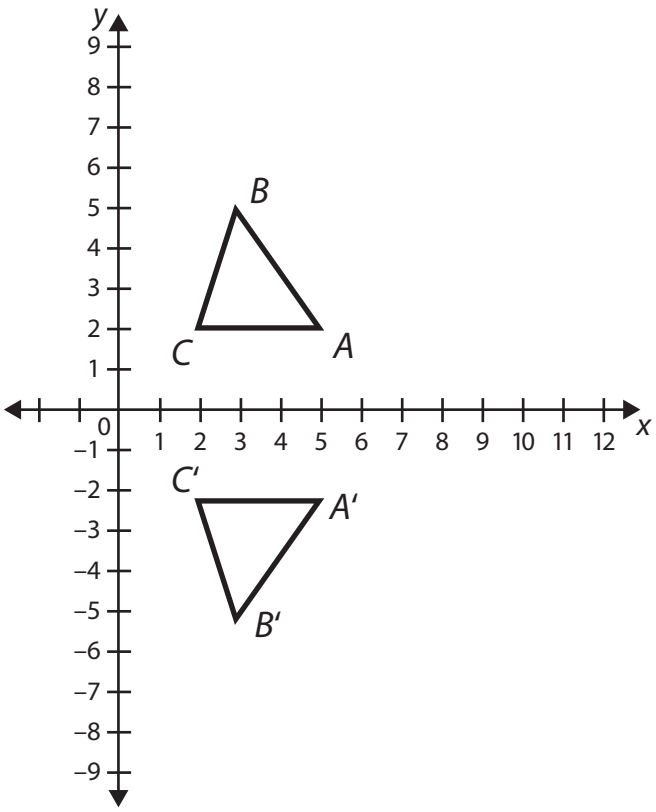
3. Calculate the new coordinates of the triangle.

$T(A)=T(5,2)=(5,-2)=A'$

$T(B)=T(3,5)=(3,-5)=B'$

$T(C)=T(2,2)=(2,-2)=C'$

When $\triangle ABC$ and $\triangle A'B'C'$ are drawn, we can see that the transformation is a reflection through the x-axis.



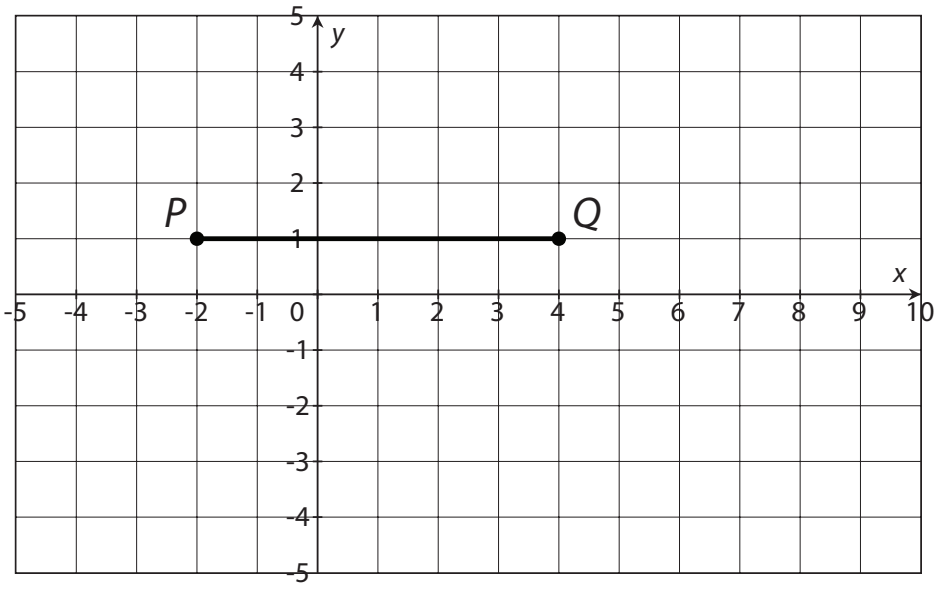
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Example 3

Given the transformation of a translation $T_{5,-3}$, and the points $P(-2, 1)$ and $Q(4, 1)$, show that the transformation of a translation is isometric by calculating the distances, or lengths, of $\overline{PQ}$ and $\overline{P'Q'}$.

1. Plot the points of the preimage.



2. Transform the points.

$$T_{5,-3}(x, y) = (x + 5, y - 3)$$

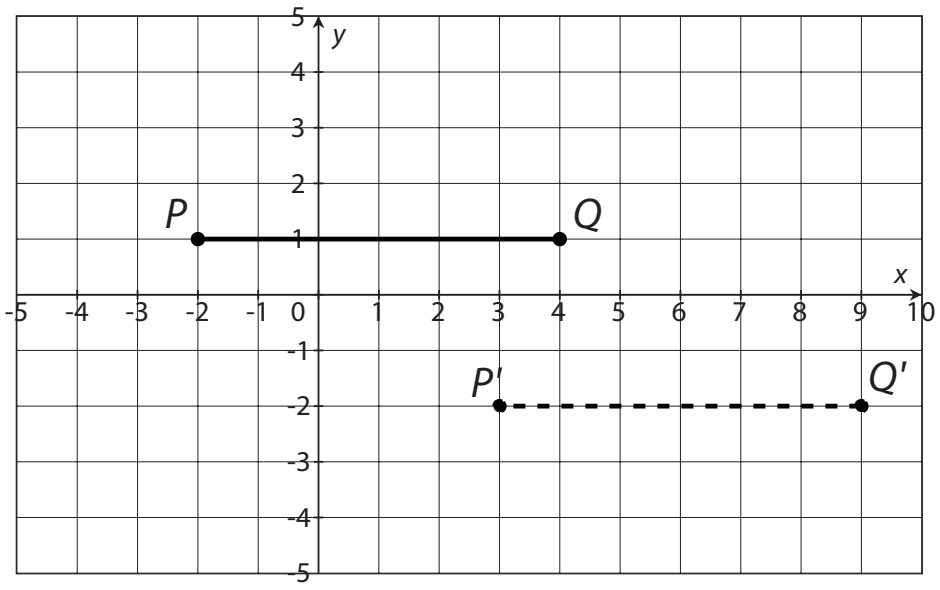
$$T_{5,-3}(P) = (-2+5, 1-3) \Rightarrow P'(3, -2)$$

$$T_{5,-3}(Q) = (4+5, 1-3) \Rightarrow P'(9, -2)$$

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Instruction

3. Plot the image points.



4. Calculate the lengths of the preimage and the image and compare them.

Since the line segments are horizontal, count the number of units the segment spans to determine the length.

$PQ = 6$

$P'Q' = 6$

The lengths of the segments are the same. The translation of the segment is isometric.

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Instruction

Example 4

Given $T_{-6,2}(x, y) = (x - 6, y + 2)$, state the translation that would yield the identity transformation, $I = T_{h,k}(T_{-6,2}(x, y))$.

1. Recall that the identity function brings a preimage back onto itself. Note what changes are made to the coordinates.

   The x-coordinate is being translated 6 units to the left.

   The y-coordinate is being translated 2 units up.

2. Determine what transformation needs to happen to bring the point (x, y) back to its original position.

   The x-coordinate needs to be brought back to the right 6 units.

   The y-coordinate needs to be brought back down 2 units.

3. Write symbolically the translation that will bring the point back to its original position.

   $T_{6,-2}$

4. Put all the pieces together to write the identity transformation.

   $T_{6,-2}(T_{-6,2}(x, y)) = I$

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Problem-Based Task 5.1.2: In the Mail Room

SMP	
1 ✓	2
3	4
5	6 ✓
7 ✓	8

The mail room at a growing retail company stuffs, addresses, weighs, and stamps hundreds of envelopes to be mailed each day. The mail clerks used to do this by hand, but because the company has grown, the process will now be automated. After an open envelope is placed face down on a moving rack, the automated process is as follows:

1. The envelope is stuffed by pushing the open envelope onto the mailer.
2. The envelope is then retracted.
3. The envelope is then moved to where it will be sealed.
4. Once the envelope is sealed it must be flipped face-up to be addressed.
5. The envelope is moved to where it is addressed.
6. The envelope is then rotated 90° and placed on a scale where it is weighed and stamped.
7. Finally, the envelope is pushed off the rack and into a bin to be mailed.

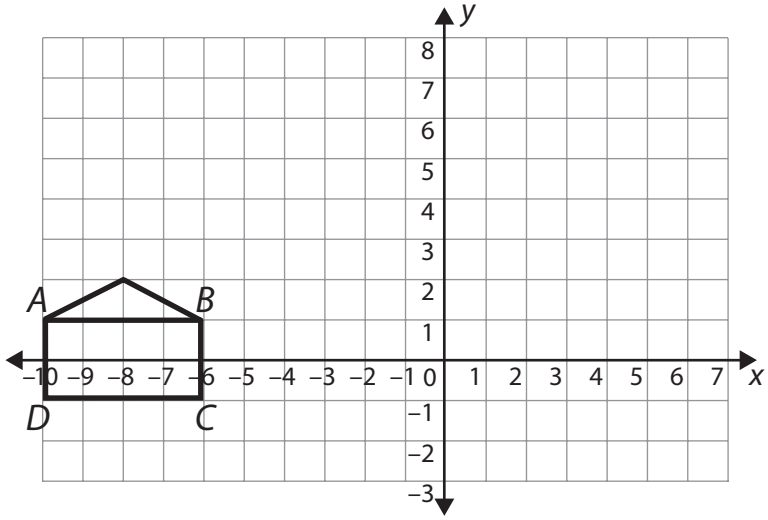
The transformation functions for each step are as follows:

1. $S_1(x, y) = (x, y + 3)$ Stuffing the envelope.
2. $S_2(x, y) = (x, y - 3)$ Retracting the stuffed envelope.
3. $T(x, y) = (x + 4, y)$ Moving the envelope down the rack.
4. $F(x, y) = (x, -y)$ Flipping the envelope over.
5. $T(x, y) = (x + 4, y)$ Moving the envelope down the rack.
6. $R(x, y) = (y, -x)$ Rotating the envelope clockwise 90°.
7. $T(x, y) = (x + 4, y)$ Moving the envelope down the rack.

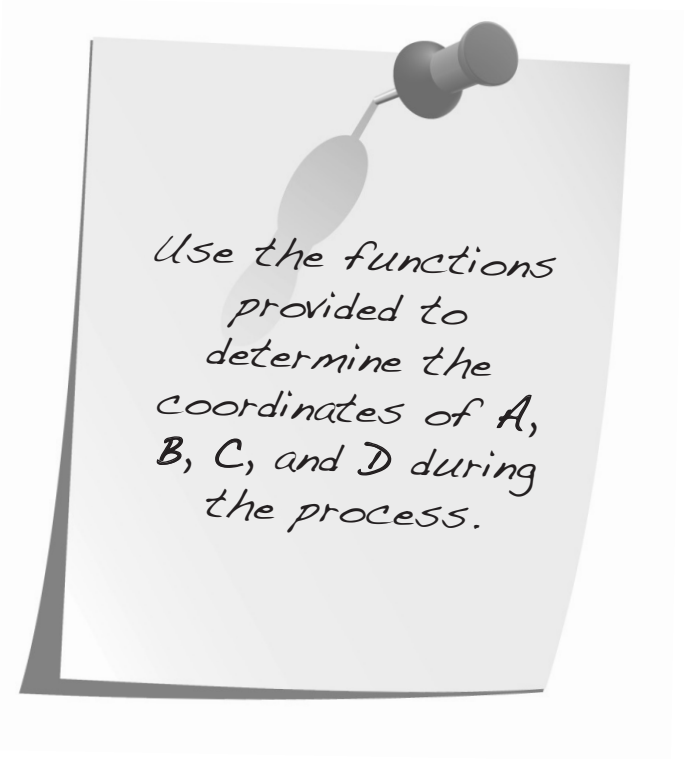
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Each envelope has a width of 4 units and a height of 2 units. Envelopes start at the location shown in the graph below, with the initial coordinates of A (–10, 1), B (–6, 1), C (–6, –1), and D (–10, –1).



Use the functions provided to determine the coordinates of A, B, C, and D during the process.



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Problem-Based Task 5.1.2: In the Mail Room

Coaching

a. What are the coordinates of A, B, C, and D after step 2?

b. What are the coordinates of A, B, C, and D after step 4?

c. What are the coordinates of A, B, C, and D after step 6?

d. What are the final coordinates of A, B, C, and D?

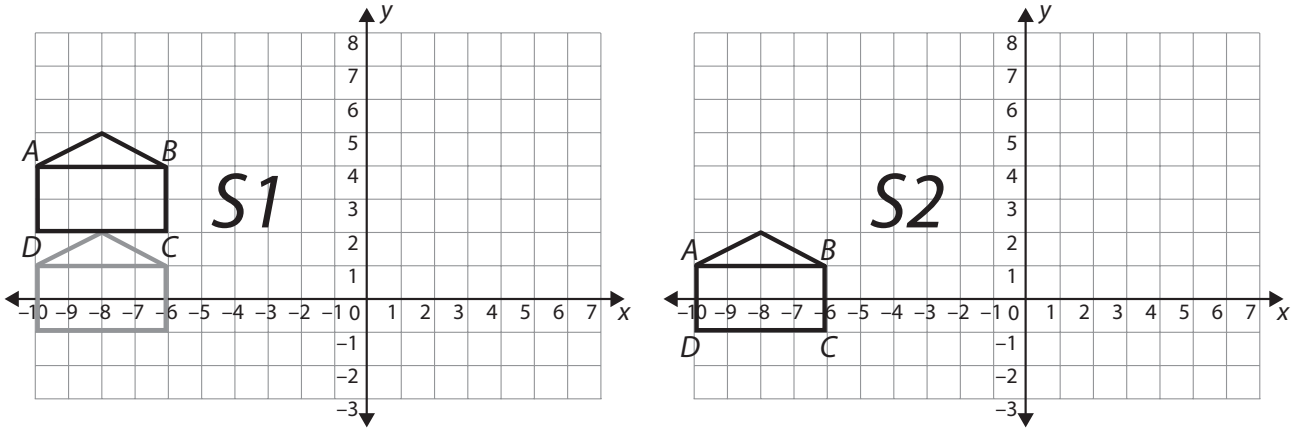
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Instruction

Problem-Based Task 5.1.2: In the Mail Room

Coaching Sample Responses

a. What are the coordinates of *A*, *B*, *C*, and *D* after step 2?



$S_2(S_1(x, y)) = S_2(x, y + 3) = (x, y)$

The original coordinates of the envelope are (x, y).

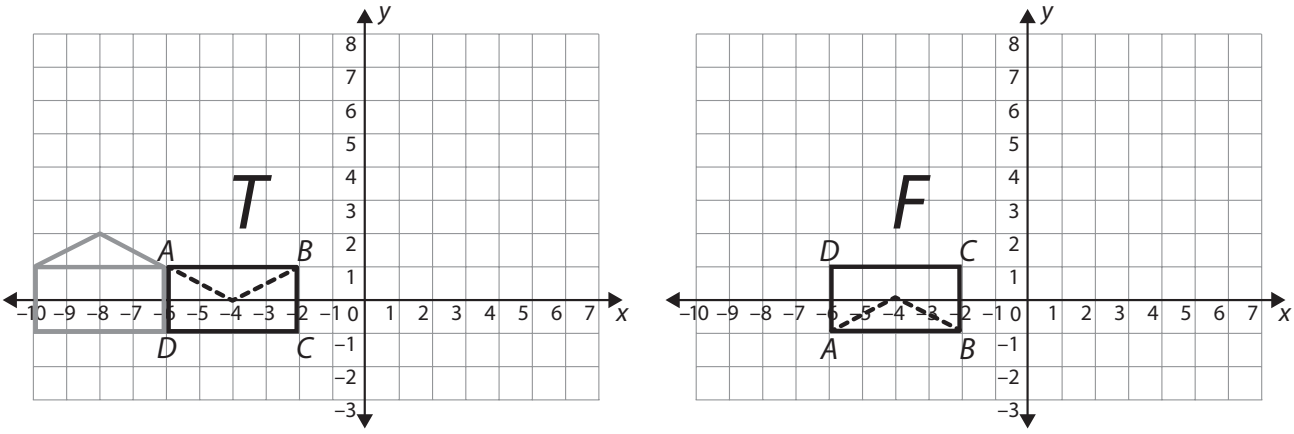
The transformation for step 1 is $(x, y + 3)$. The *x*-value will remain the same, but the *y*-value will be 3 more.

The coordinates after step 1 are *A* (–10, 4), *B* (–6, 4), *C* (–6, 2), and *D* (–10, 2).

The transformation for step 2 is $(x, y – 3)$. Again, the *x*-value will remain the same, but the *y*-value from step 1 will be decreased by 3.

The result is the envelope has been returned to its initial position; the coordinates are *A* (–10, 1), *B* (–6, 1), *C* (–6, –1), and *D* (–10, –1).

b. What are the coordinates of *A*, *B*, *C*, and *D* after step 4?



$F\big(T\big(\text{Step 2}\big)\big) = F\big(T\big(x, y\big)\big) = F\big(x + 4, y\big) = (x + 4, -y)$

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The coordinates after step 2 are *A* (–10, 1), *B* (–6, 1), *C* (–6, –1), and *D* (–10, –1).

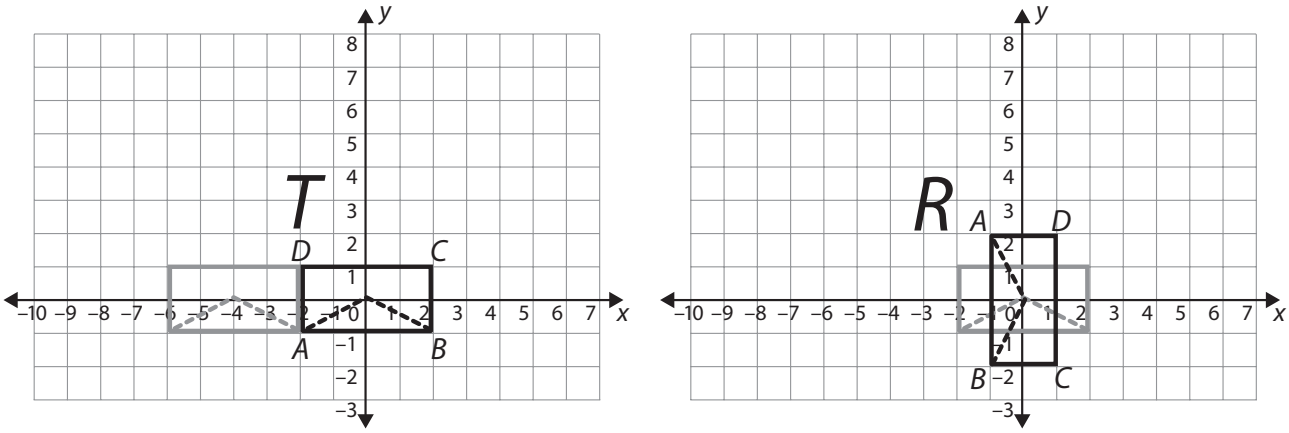
Step 3 moves the envelope down the rack by performing a translation where the *x*-value is increased by 4 and the *y*-value remains the same.

The coordinates after step 3 are *A* (–6, 1), *B* (–2, 1), *C* (–2, –1), and *D* (–6, –1).

Step 4 flips the envelope. This flip causes the *x*-value to remain the same and the *y*-value to become the opposite value.

The coordinates after step 4 are *A* (–6, –1), *B* (–2, –1), *C* (–2, 1), and *D* (–6, 1).

c. What are the coordinates of *A*, *B*, *C*, and *D* after step 6?



$$R\big(T\big(\text{Step 4}\big)\big)=R\big(T\big(x+4,-y\big)\big)=R\big(x+8,-y\big)=\big(-y,-\big(x+8\big)\big)$$

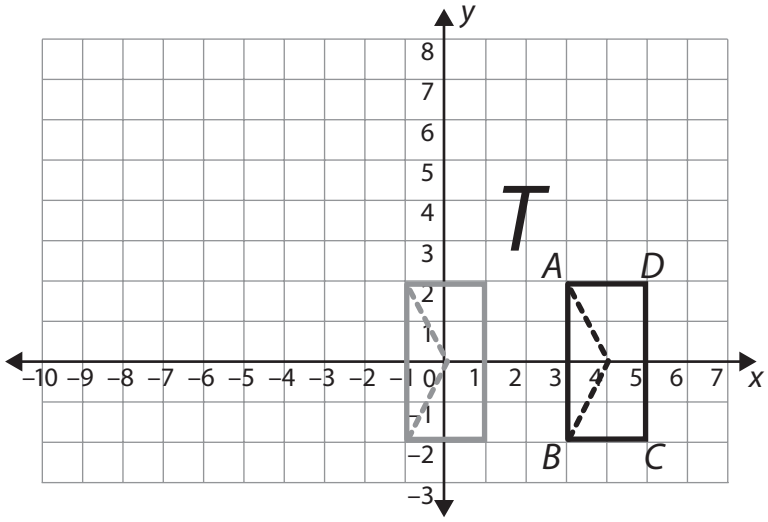
Step 5 moves the envelope down the rack. This translation results in the value of *x* being increased by 4 while the *y*-value remains the same.

The coordinates of the envelope after step 5 are *A* (–2, –1), *B* (2, –1), *C* (2, 1), and *D* (–2, 1).

Step 6 rotates the envelope clockwise 90°. The coordinates after step 6 are *A* (–1, 2), *B* (–1, –2), *C* (1, –2), and *D* (1, 2).

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d. What are the final coordinates of *A*, *B*, *C*, and *D*?



$$T\left(\text{Step } 6\right)=T\left(-y,-\left(x+8\right)\right)=\left(-y+4,-\left(x+8\right)\right)$$

From its position after step 6, the envelope has been translated.

The *x*-value is increased by 4 and the *y*-value remains the same.

The coordinates after the final step are *A* (3, 2), *B* (3, –2), *C* (5, –2), and *D* (5, 2).

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 5.1.2: Transformations As Functions

Use what you know about transformations to answer the questions.

1. When described as functions, can transformations be combined?

2. Given the expression $f(g(k(x)))$, where f, g, and k are functions, what operation should be performed first? Does the order matter? Why or why not?

3. What does it mean for a transformation to be isometric?

4. A figure is transformed by $T_{3,-1}$ and then transformed by $T_{-3,1}$. How does the preimage relate to the final image?

5. If the transformation T is isometric and $d(PQ)=2$, what is $d(T(PQ))$?

6. Given $T_{h,k}(x,y)=(x+h,y+k)$ and the point P (2, 3), what is $T_{5,4}(P)$?

7. Triangle ABC has vertices $A(1, 7)$, $B(3, 3)$, and $C(1, 3)$. What are the vertices of $\triangle A'B'C'$, the image of $\triangle ABC$ after the reflection $r_{y=x}(\triangle ABC)$?

8. Given $R_{90}(x, y) = (-y, x)$ and the point Q (1, 0), what is $R_{90}(Q)$?

9. Find $T(S(x, y))$ if $T(x,y)=(x+2,y+2)$ and $S(x,y)=(x-5,y+1)$. Label your answer P. What values of h and k would prove the equation $T_{h,k}(P) = (x, y)$ true?

10. Given $T_{2,5}(x, y) = (x + 2, y + 5)$, state the translation that would yield the identity transformation, $I = T_{h,k}(T_{2,5}(x, y))$.

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Lesson 5.1.3: Applying Lines of Symmetry

Warm-Up 5.1.3

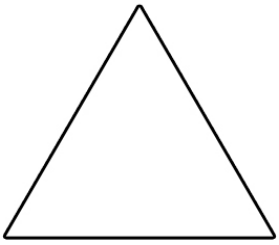
Symmetry is everywhere. We find symmetry in flowers, faces, and buildings. It allows us to predict what something will look like; if we see one half of a basketball court we can assume the other half will match. Because symmetry is common in nature, we see it replicated in art and architecture. Symmetry provides predictability, balance, and strength, which makes it a critical element of mathematics and science. But what defines symmetry? What is found in every symmetrical relationship?

1. In the face below, note the pieces that are symmetrical. Draw a line through the middle of each symmetry.



2. Where would a line of symmetry be on the body of a car?

3. Find where the equilateral triangle below is symmetrical and draw lines bisecting the symmetry. Remember, an equilateral triangle has three sides of equal length. How many lines of symmetry can you draw?



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Instruction

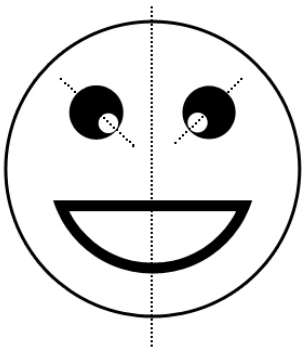
Lesson 5.1.3: Applying Lines of Symmetry

Common Core State Standard

G–CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Warm-Up 5.1.3 Debrief

1. In the face below, note the pieces that are symmetrical. Draw a line through the middle of each symmetry.



The eyes are symmetrical through the center of the whites and also with each other. The mouth is symmetrical. The full shape of the face is symmetrical.

Notice the eyes, mouth, and face share one line of symmetry.

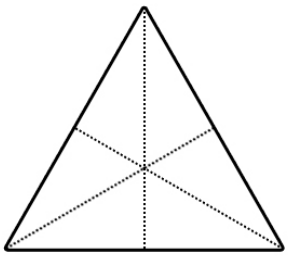
2. Where would a line of symmetry be on the body of a car?

A car body has a line of symmetry running vertically from the front of the car to the rear. Because a car is three-dimensional, this line is a plane of symmetry.

3. Find where the equilateral triangle below is symmetrical and draw lines bisecting the symmetry. Remember, an equilateral triangle has three sides of equal length. How many lines of symmetry can you draw?

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Instruction



There are three lines of symmetry. Notice that each symmetrical item has a line of symmetry where everything on one side is the mirror image of the other side. This is known as line symmetry.

Connection to the Lesson

- Recognizing symmetry in everyday objects helps recognize symmetry in geometric figures.
- Symmetry is defined by lines.

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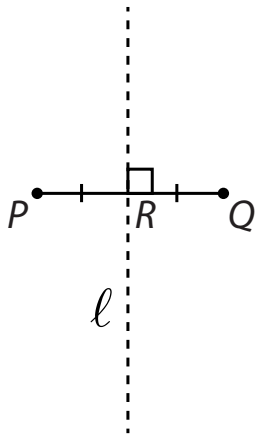
Prerequisite Skills

This lesson requires the use of the following skills:

- recognizing perpendicular lines (4.G.1)
- understanding that the interior angles of a triangle total 180° (7.G.2)

Introduction

A **line of symmetry**, ℓ, is a line separating a figure into two halves that are mirror images. **Line symmetry** exists for a figure if for every point P on one side of the line, there is a corresponding point Q on the other side of the line where ℓ is the perpendicular bisector of $\overline{PQ}$.



From the diagram, we see that ℓ is perpendicular to $\overline{PQ}$. The tick marks on the segment from P to R and from R to Q show us that the lengths are equal; therefore, R is the point that is halfway between $\overline{PQ}$.

Depending on the characteristics of a figure, a figure may contain many lines of symmetry or none at all. In this lesson, we will discuss the rotations and reflections that can be applied to squares, rectangles, parallelograms, trapezoids, and regular polygons that carry the figure onto itself. **Regular polygons** are two-dimensional figures with all sides and all angles congruent.

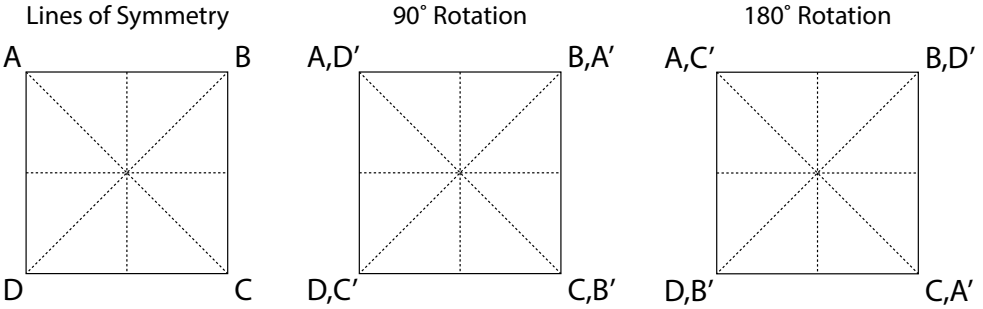
Squares

Because squares have four congruent sides and four congruent angles, squares have four lines of symmetry. If we rotate a square about its center 90°, we find that though the points have moved, the square is still covering the same space.

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Instruction

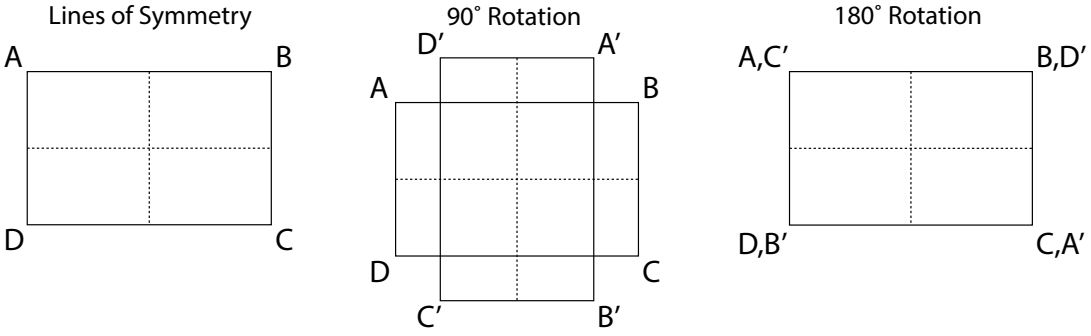
Similarly, we can rotate a square 180˚, 270˚, or any other multiple of 90˚ with the same result.



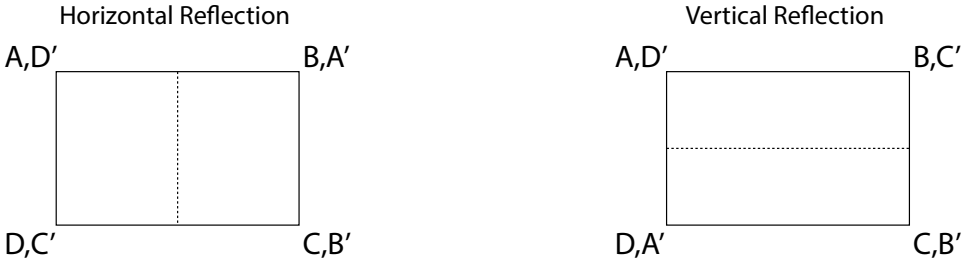
We can also reflect the square through any of the four lines of symmetry and the image will project onto its preimage.

Rectangles

A rectangle has two lines of symmetry: one vertical and one horizontal. Unlike a square, a rectangle does not have diagonal lines of symmetry. If a rectangle is rotated 90˚, the image will not be projected onto its preimage, but if it is rotated 180˚, the image and the preimage are the same.



If a rectangle is reflected through its horizontal or vertical lines of symmetry, the image is projected onto its preimage.

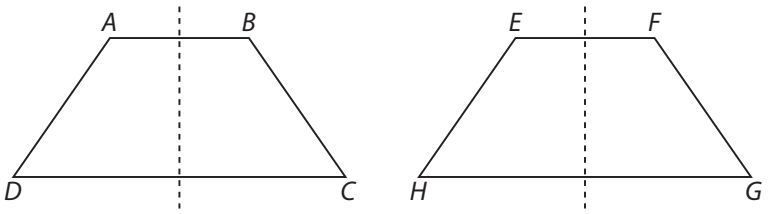


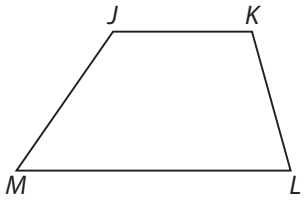
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Instruction

Trapezoids

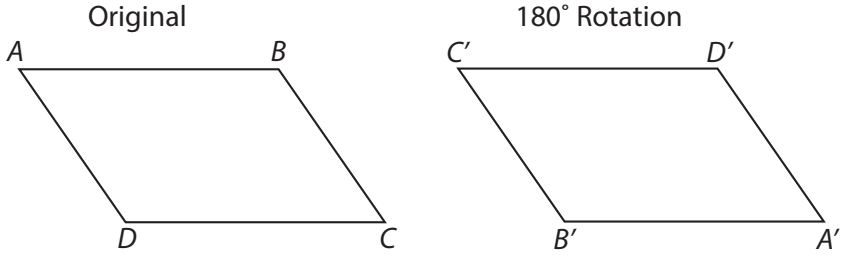
A trapezoid has one line of symmetry, bisecting the parallel sides in half, if and only if the non-parallel sides are of equal length (called an isosceles trapezoid). We can reflect the isosceles trapezoid shown below through the line of symmetry; doing so projects the image onto its preimage. However, notice that in the last trapezoid *JKLM* shown below $\overline{JM}$ is longer than $\overline{KL}$, so there is no symmetry. The only rotation that will carry a trapezoid onto itself is 360˚.





Parallelograms

There are no lines of symmetry in a parallelogram if a 90˚ angle is not present in the figure. Therefore, there is no reflection that will carry a parallelogram onto itself. However, if a parallelogram is rotated 180˚, the image is the same as the preimage.



Key Concepts

- Some figures can be reflected through lines of symmetry onto themselves.
- Lines of symmetry determine the amount of rotation required to carry them onto themselves. A figure with one line of symmetry requires a 360º rotation, a figure with two lines of symmetry requires a 180º rotation, and a figure with four lines of symmetry requires a 90º rotation.

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Instruction

- Not all figures are symmetrical.
- Regular polygons have sides of equal length and angles of equal measure. There are n number of lines of symmetry for a number of sides, n, in a regular polygon.

Common Errors/Misconceptions

- showing a line of symmetry in a parallelogram or rhombus where there isn't one
- missing a line of symmetry

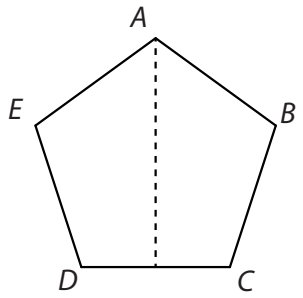
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Guided Practice 5.1.3

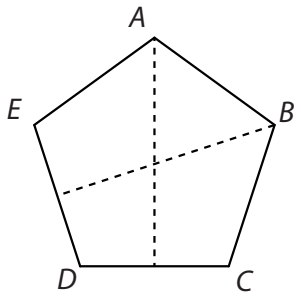
Example 1

Given a regular pentagon *ABCDE*, draw the lines of symmetry.

1. First, draw the pentagon and label the vertices. Note the line of symmetry from *A* to $\overline{DC}$.



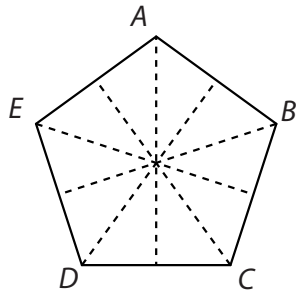
2. Now move to the next vertex, *B*, and extend a line to the midpoint of $\overline{DE}$.



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Instruction

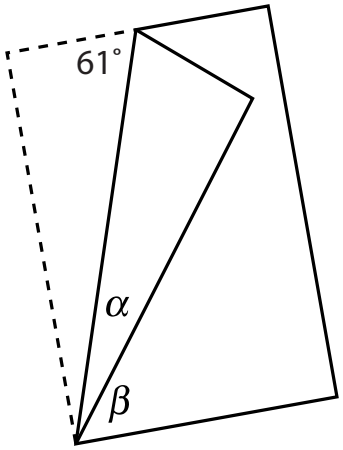
3. Continue around to each vertex, extending a line from the vertex to the midpoint of the opposing line segment.



Note that a regular pentagon has five sides, five vertices, and five lines of symmetry.

Example 2

A piece of rectangular paper is folded in the following way:

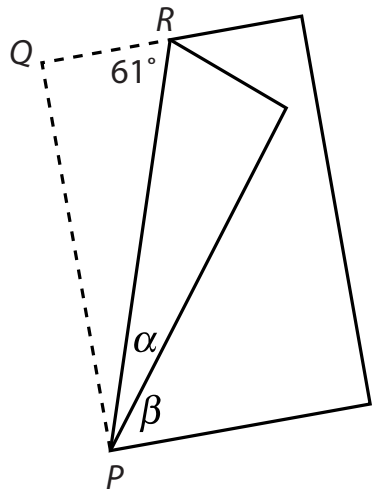


Find the angles alpha, α, and beta, β.

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Instruction

1. First, label the vertices.



2. Find the measure of $\angle QPR$.

   We know the sum of the interior angles of a triangle is 180°. We also know $m\angle Q$ is 90° because we are told the paper is rectangular. Therefore, subtract 61 + 90 from 180 to find the measure of $\angle QPR$.

   $180 - (61 + 90) = 180 - (151) = 29$

3. Now we know $m\angle QPR$ is 29°. Because of the symmetry of the folded paper, we know $m\angle\alpha$ must also be 29°.

4. Finally, we know the measures of $\angle QPR$, $\angle\alpha$, and $\angle\beta$ total 90° because the paper is rectangular, so $m\angle\beta = 90 - 2(29) = 90 - 58 = 32$.

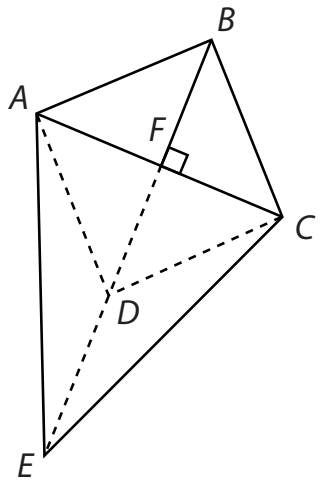
   The measure of $\angle\beta$ is 32°.

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Instruction

Example 3

Given the quadrilateral *ABCE*, the square *ABCD*, and the information that *F* is the same distance from *A* and *C*, show that *ABCE* is symmetrical along $\overline{BE}$.



1. Recall the definition of line symmetry.

   Line symmetry exists for a figure if for every point on one side of the line of symmetry, there is a corresponding point the same distance on the other side of the line of symmetry.

   We are given that *ABCD* is square, so we know $\overline{AB} \cong \overline{BC}$.

   We also know that $\square ABCD$ is symmetrical along $\overline{BD}$.

   We know $\overline{AF} \cong \overline{FC}$.

2. Since $\overline{AB} \cong \overline{BC}$ and $\overline{AF} \cong \overline{FC}$, $\overline{BF}$ is a line of symmetry for $\triangle ABC$ where $\triangle ABF \cong \triangle CBF$.

3. $\triangle AFE$ has the same area as $\triangle CFE$ because they share a base and have equal height. $\overline{AF} \cong \overline{FC}$, so $\triangle AFE \cong \triangle CFE$.

4. We now know $\overline{FE}$ is a line of symmetry for $\triangle ACE$ and $\overline{BF}$ is a line of symmetry for $\triangle ABC$, so $\triangle ABE \cong \triangle CBE$ and quadrilateral *ABCE* is symmetrical along $\overline{BE}$.

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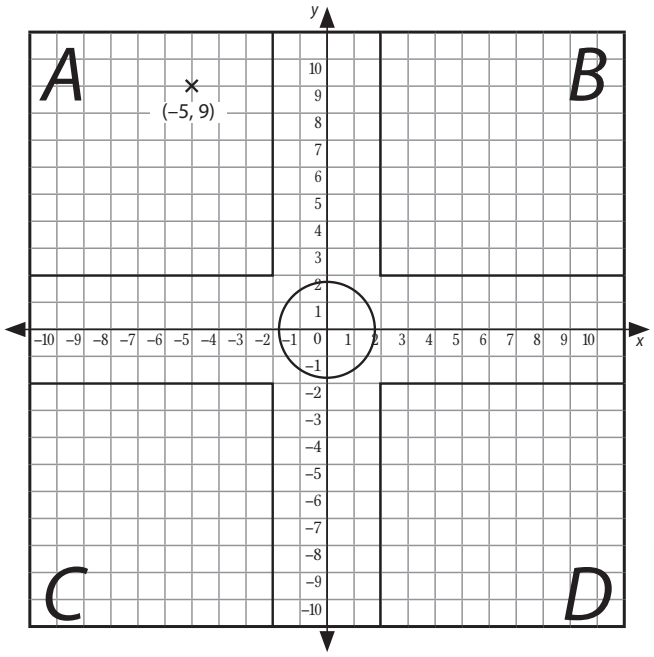
Problem-Based Task 5.1.3: Lining the Plaza

SMP	
1	2
3	4 ✓
5	6 ✓
7	8 ✓

A construction crew is building a park plaza that will include a fountain and eight cherry trees. Two perpendicular walkways will cross through the middle of the plaza, intersecting in the center where the fountain will be placed. The plans call for the trees to be planted in positions that meet the following criteria:

- There are two trees in each quadrant.
- The diagonals of each quadrant are lines of symmetry for the quadrant.
- The walkways are lines of symmetry for the plaza.

The plan for the plaza has been laid onto the coordinate map shown below and the location of the first tree in quadrant A has been marked. Find the locations for the remaining trees.





SWB p. 33

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Problem-Based Task 5.1.3: Lining the Plaza

Coaching

a. What should be the coordinates of the second tree in quadrant A?

b. What should be the coordinates of the two trees in quadrant B?

c. What should be the coordinates of the remaining trees in quadrants C and D?

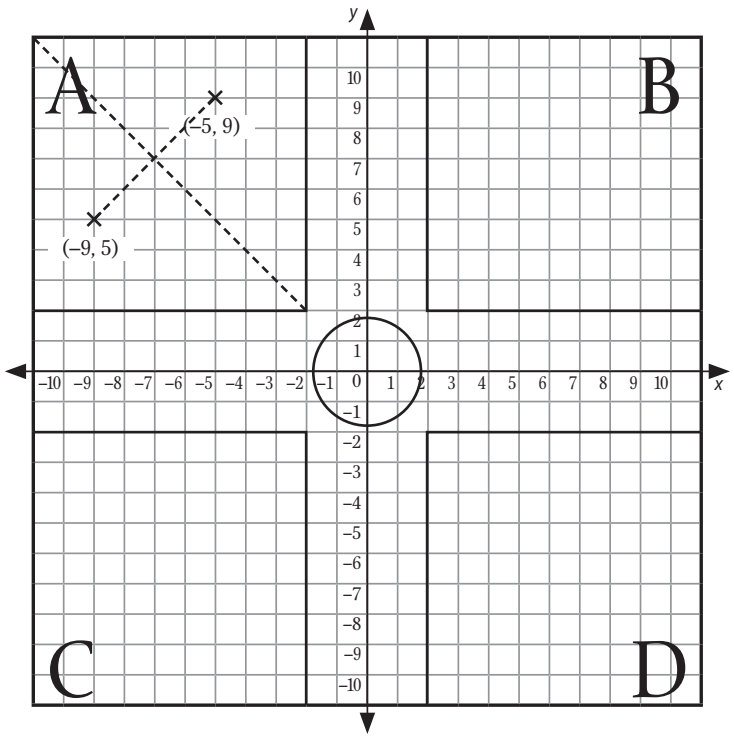
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Instruction

Problem-Based Task 5.1.3: Lining the Plaza

Coaching Sample Responses

a. What should be the coordinates of the second tree in quadrant A?



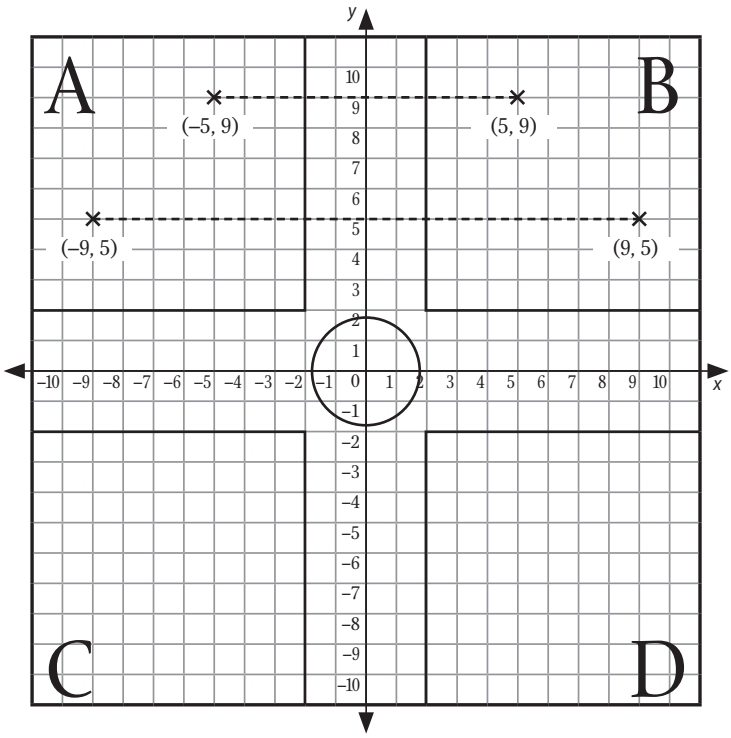
Draw the diagonal across quadrant A from the upper left to lower right corner.

Draw a line perpendicular to the diagonal from the position of the first tree at (–5, 9) to the position of the next tree at (–9, 5). Notice that moving 2 units left and 2 units down from the first tree takes us to the line of symmetry. Moving 2 more units left and 2 more units down takes us to the position of the second tree. This confirms the trees are equidistant to the diagonal, and therefore the positions are symmetrical.

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Instruction

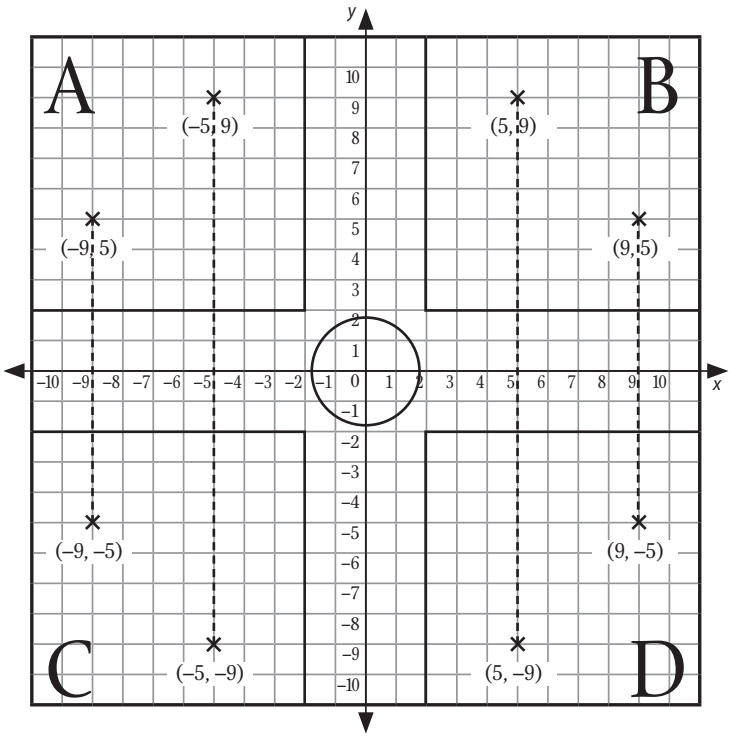
b. What should be the coordinates of the two trees in quadrant B?



Now that the positions of the first two trees have been established, we can reflect their positions across the vertical axis to find the positions of the trees in quadrant B at (5, 9) and (9, 5).

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c. What should be the coordinates of the remaining trees in quadrants C and D?



Reflect the positions of the trees in quadrant A across the horizontal axis to find the positions of the trees in quadrant C at (–9, –5) and (–5, –9). Finally, reflect the positions of the trees in quadrant B across the horizontal axis to find the positions of the trees in quadrant D at (9, –5) and (5, –9).

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 5.1.3: Applying Lines of Symmetry

Use what you've learned about symmetry to answer the questions.

1. What shape has an infinite number of lines of symmetry?

2. How many lines of symmetry does a regular hexagon have?

3. Can a quadrilateral with four equal sides have less than four lines of symmetry?

4. How many lines of symmetry does an isosceles triangle have?

5. How many lines of symmetry does a scalene triangle have?

6. What is the smallest number of degrees needed to rotate a regular pentagon around its center onto itself?

7. What relationship is formed by a line of symmetry and the line between opposite points in the symmetry?

8. How many lines of symmetry are there in one of the five-point stars on the American flag?

9. The face of a cat is symmetrical, with the bridge of the nose falling on the line of symmetry directly between the eyes. If a cat's right eye is 3 inches from the bridge of its nose, how far is the cat's left eye from its right eye?

10. How many different ways can a cube be sliced into 2 equal halves?

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Progress Assessment

Circle the letter of the best answer.

1. A triangle has 2 sides that are perpendicular. What type of triangle is it?
   a. right
   b. isosceles
   c. scalene
   d. obtuse

2. What is the set of points from A to B that extends beyond B in the same direction?
   a. ray
   b. line
   c. line segment
   d. arc

3. How many right angles are in a rectangle?
   a. 1
   b. 2
   c. 3
   d. 4

4. Given an equilateral triangle, if a circular arc is swept from one vertex to another using the third vertex as the center, what would be the measure of the circular arc?
   a. 45°
   b. 30°
   c. 90°
   d. 60°

5. What type of function has exactly one output for each input?
   a. balanced
   b. even
   c. one-to-one
   d. onto

6. If $f(x, y) = (3x, 2y)$, then what is $f(3, 1)$?
   a. (6, 3)
   b. (9, 2)
   c. (0, 1)
   d. (2, 9)

continued

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Assessment

7. If $f(x, y) = (-x, y)$ and $g(x, y) = (x + 1, -y)$, then what is $f(g(2, 7))$?

   a. (–3, –7)
   c. (3, 7)
   b. (7, 3)
   d. (–7, 3)

8. An isometry is a transformation in which the preimage and image _____.

   a. are similar
   c. are mirror images
   b. have equal area
   d. are congruent

9. If $\overline{AB} \perp \overline{XY}$ and $\overline{XY} \perp \overline{KL}$, then

   a. $\overline{AB} \perp \overline{KL}$
   c. $\overline{AB} \parallel \overline{KL}$
   b. $\overline{AY} \perp \overline{BX}$
   d. $\overline{KL} \perp \overline{XY}$

10. How many pairs of parallel lines are there in a regular hexagon?

    a. 1
    c. 3
    b. 2
    d. 4

Use what you have learned about transformations in the coordinate plane to solve.

11. A figure is transformed by $T_{4,-2}$ and then transformed by $T_{4,-2}$. How does the preimage relate to the final image?

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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. How many components does every point in the coordinate plane have?
   a. 1
   b. 2
   c. 3
   d. 4

2. What type of rotation does a rotation transformation require?
   a. point
   b. axis
   c. circumference
   d. line

3. What type of reflection does a reflection transformation require?
   a. line
   b. point
   c. arc
   d. circle

4. Translation transformations move all points:
   a. the same distance but in different directions
   b. a different distance but in the same direction
   c. different distances in different directions
   d. the same distance and direction

5. The image of a translation, reflection, and rotation will be _____ the preimage.
   a. congruent to
   b. a mirror image of
   c. larger than
   d. smaller than

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Lesson 2: Defining and Applying Rotations, Reflections, and Translations

Instruction

Common Core State Standards

SMP	
1	2 ✓
3 ✓	4 ✓
5	6 ✓
7	8

G–CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G–CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Essential Questions

1. What linear relation defines the movement of a translation?
2. What linear relation defines the movement of a reflection?
3. Why do circular arcs define the movement of a rotation rather than a linear point-to-point relationship?

WORDS TO KNOW

clockwise rotating a figure in the direction that the hands on a clock move

counterclockwise rotating a figure in the opposite direction that the hands on a clock move

quadrant the coordinate plane is separated into four sections:

- In Quadrant I, x and y are positive.
- In Quadrant II, x is negative and y is positive.
- In Quadrant III, x and y are negative.
- In Quadrant IV, x is positive and y is negative.

reflection a transformation where a mirror image is created; also called a flip

rotation a transformation that turns a figure around a fixed center point; also called a turn

translation a transformation that moves each point of a figure the same distance in the same direction; also called a slide

SRB p. U5-32

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Recommended Resources

- Interactivate. "3D Transmographer."

  http://walch.com/rr/CAU5L2PolygonsIn3-D

  Create a three-dimensional polygon on the coordinate plane and animate it using options to translate, reflect, or revolve the figure. Drag on the plane itself for a side view. Users may specify angles of rotation, units reflected, and more.

- MathIsFun.com. "Transformations."

  http://walch.com/rr/CAU5L2Transformations

  This site offers an overview of the three main transformations. Click each term to open a page with interactive manipulatives to see the transformations animated. Rotations, translations, and reflections are all covered.

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Lesson 5.2.1: Defining Rotations, Reflections, and Translations

Warm-Up 5.2.1

You are creating a quilting pattern and need to transform a triangle to create your design. Given $\triangle ABC$ with A (5, 8), B (9, 1), and C (–1, 2) as well as the functions $R_{90}(x, y) = (-y, x)$ and $r_{x\text{-axis}}(x, y) = (x, -y)$:

1. What are the coordinates of the vertices after $r_{x\text{-axis}}(\triangle ABC) = \triangle A'B'C'$?

2. What are the coordinates of the vertices after $R_{90}(\triangle A'B'C')$?

3. Are the coordinates of the vertices different if the function R_{90} is taken before $r_{x\text{-axis}}$?

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Instruction

Lesson 5.2.1: Defining Rotations, Reflections, and Translations

Common Core State Standard

G–CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Warm-Up 5.2.1 Debrief

1. What are the coordinates of the vertices after $r_{x\text{-axis}}(\triangle ABC) = \triangle A'B'C'$?

   $A'(5,-8)$, $B'(9,-1)$, and $C'(-1,-2)$

2. What are the coordinates of the vertices after $R_{90}(\triangle A'B'C')$?

   $A''(8,5)$, $B''(1,9)$, and $C''(2,-1)$

3. Are the coordinates of the vertices different if the function R_{90} is taken before $r_{x\text{-axis}}$?

   $R_{90}(A(5,8)\,B(9,1)\,C(-1,2)) \Rightarrow A'(-8,5)\,B'(-1,9)\,C'(-2,-1)$

   $r_{x\text{-axis}}(A'(-8,5)\,B'(-1,9)\,C'(-2,-1)) \Rightarrow A''(-8,-5)\,B''(-1,-9)\,C''(-2,1)$

   Yes, the different order of functions yields a different result.

Connection to the Lesson

- In this lesson, we will define specific reflections and rotations as well as the general form for a translation.
- This lesson will focus on describing and manipulating functions algebraically.
- Students will need to remember the order that functions are taken in given the notation.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

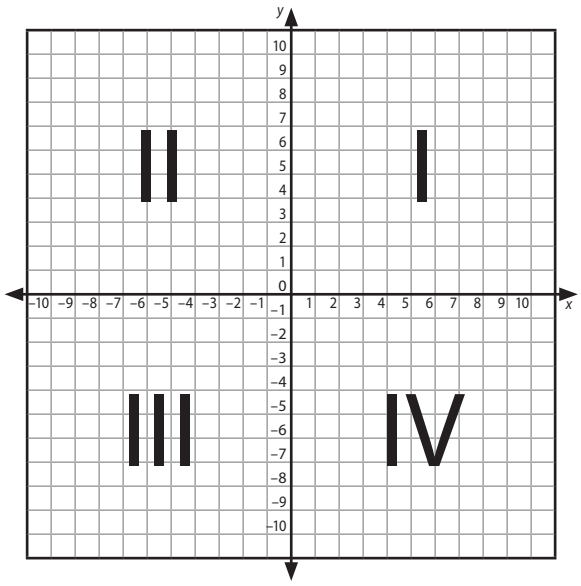
- plotting points in the coordinate plane (5.G.1)
- manipulating functions (F–BF.1b)

Introduction

Now that we have built our understanding of parallel and perpendicular lines, circular arcs, functions, and symmetry, we can expand on the three fundamental transformations from the previous lesson: translations, reflections, and rotations. We will be able to define the movement of each transformation in the coordinate plane with functions that have preimage coordinates as inputs and image coordinates as outputs.

Key Concepts

- The coordinate plane is separated into four **quadrants**, or sections:
  - In Quadrant I, x and y are positive.
  - In Quadrant II, x is negative and y is positive.
  - In Quadrant III, x and y are negative.
  - In Quadrant IV, x is positive and y is negative.

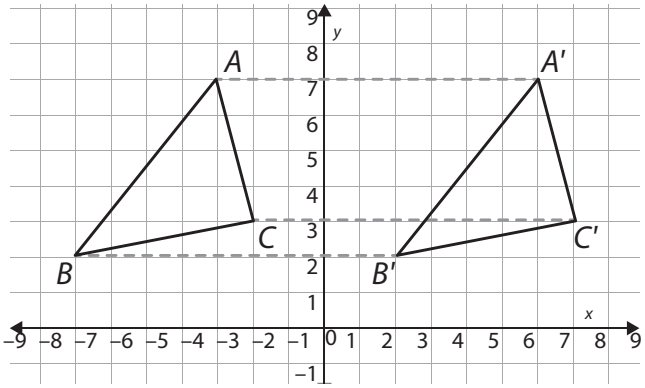


SRB p. U5-34

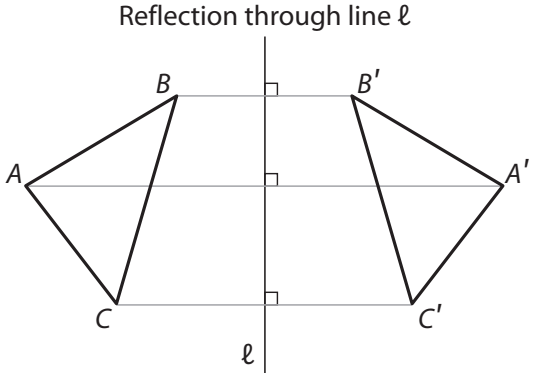
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Instruction

- A **translation** is a transformation that moves each point of a figure the same distance in the same direction. All points in the preimage are moved parallel to a given line. The image has the same orientation as the preimage. Because the orientation does not change, a translation is also called a slide.
- In the translation below, we can see the points A, B, and C are translated along parallel lines to the points A', B', and C'. Each point is carried the same distance and direction, so not only is $\triangle A'B'C'$ congruent to $\triangle ABC$, it also maintains the same orientation.



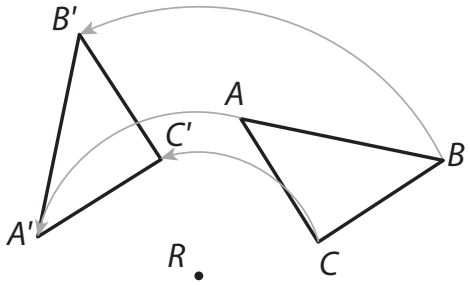
- Translations are described in the coordinate plane by the distance each point is moved with respect to the x-axis and y-axis. If we assign h to be the change in x and k to be the change in y, we can define the translation function T such that $T_{h,k}(x, y) = (x + h, y + k)$.
- A **reflection** is a transformation where a mirror image is created. In a reflection, each point of a figure is moved twice the distance from a line called the line of reflection, along a line that is perpendicular to the line of reflection. Because the result of a reflection is the mirror image of the original figure, a reflection is also called a flip.



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Instruction

- As seen in the previous image, A, B, and C are reflected along lines that are perpendicular to the line of reflection, ℓ. Also note that the line segments $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$ are all of different lengths. Depending on the line of reflection in the coordinate plane, reflections can be difficult to describe as functions. Therefore, in this lesson we will consider the following three reflections:
  - through the x-axis: $r_{x\text{-axis}}(x, y) = (x, -y)$
  - through the y-axis: $r_{y\text{-axis}}(x, y) = (-x, y)$
  - through the line $y = x$: $r_{y=x}(x, y) = (y, x)$
- A **rotation** is a transformation that turns a figure around a fixed center point. Each point in the preimage is moved along a circular arc determined by the center of rotation and the angle of rotation. A rotation is also be called a turn. This transformation can be more complex than a translation or reflection because the image is determined by circular arcs instead of parallel or perpendicular lines. When a rotation is applied to a figure, each point in the figure will move a distance determined by its distance from the point of rotation. A figure may be rotated **clockwise**, in the direction that the hands on a clock move, or **counterclockwise**, in the opposite direction that the hands on a clock move. The following figure shows a 90° counterclockwise rotation around the point R.



- Comparing the arc lengths in the figure, we see that point B moves farther than points A and C. This is because point B is farther from the center of rotation, R.
- Depending on the point and angle of rotation, the function describing a rotation can be complex. Thus, we will consider the following counterclockwise rotations, which can be easily defined:
  - 90° rotation about the origin: $R_{90}(x, y) = (-y, x)$
  - 180° rotation about the origin: $R_{180}(x, y) = (-x, -y)$
  - 270° rotation about the origin: $R_{270}(x, y) = (y, -x)$

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Instruction

Common Errors/Misconceptions

- confusing reflections and rotations (For example, a 90° rotation from Quadrant I to Quadrant II may appear similar to a reflection through the *y*-axis.)
- finding that the final output of combinations of rotations and reflections has the same outcome as a single transformation

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Guided Practice 5.2.1

Example 1

How far and in what direction does the point $P(x, y)$ move when translated by the function $T_{24, 10}$?

1. Each point translated by $T_{24, 10}$ will be moved right 24 units, parallel to the x-axis.

2. The point will then be moved up 10 units, parallel to the y-axis.

3. Therefore, $T_{24, 10}(P) = (x + 24, y + 10)$.

Example 2

Using the definitions described earlier, write the translation $T_{5, 3}$ of the rotation R_{180} in terms of a function F on (x, y).

1. Write the problem symbolically.

   $F = T_{5, 3}(R_{180}(x, y))$

2. Start from the inside and work outward.

   $R_{180}(x, y) = (-x, -y)$

   Therefore, $T_{5, 3}(R_{180}(x, y)) = T_{5, 3}(-x, -y)$.

3. Now translate the point.

   $T_{5, 3}(-x, -y) = (-x + 5, -y + 3)$

4. Write the result of both translations.

   $F = T_{5, 3}(R_{180}(x, y)) = (-x + 5, -y + 3)$

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Instruction

Example 3

Using the definitions described earlier, write the reflection $r_{y=x}$ of the translation $T_{2,3}$ of the reflection $r_{x\text{-axis}}(x, y)$ in terms of a function S on (x, y).

1. Write the problem symbolically.

   $S = r_{y=x}(T_{2,3}(r_{x\text{-axis}}(x, y)))$

2. Start from the inside and work outward. First address the reflection through the x-axis. Solve $r_{x\text{-axis}}(x, y)$ for $(x, -y)$.

   $S = r_{y=x}(T_{2,3}(r_{x\text{-axis}}(x, y))) = r_{y=x}(T_{2,3}(x, -y))$

3. Next, solve for the translation function, $T_{2,3}$, using the input from above, $T_{2,3}(x, -y) = (x + 2, -y + 3)$.

   $S = r_{y=x}(T_{2,3}(x, -y)) = r_{y=x}(x + 2, -y + 3)$

4. Finally, solve the reflection $r_{y=x}$ using the input from above.

   $S = r_{y=x}(x + 2, -y + 3) = (-y + 3, x + 2)$

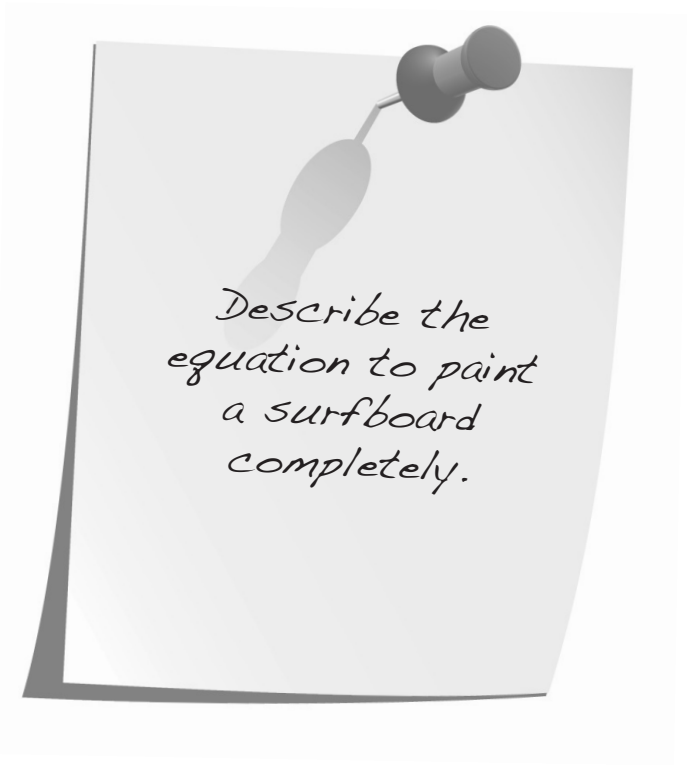


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Problem-Based Task 5.2.1: Painting Surfboards

SMP	
1	2
3 ✓	4 ✓
5	6 ✓
7	8

At a surfboard manufacturer, a conveyor belt moves the board through a paint sprayer. A mechanical arm then rotates the board 180° (turned upside down) and puts it back on the conveyor belt. Then, the surfboard is pushed one more full length through another paint sprayer. Using the variable b for the board, the function C for the motion of the conveyor belt, and the function A for the action of the arm rotating the board 180°, describe the equation to paint a surfboard completely.



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Problem-Based Task 5.2.1: Painting Surfboards

Coaching

a. How many times must the procedure be done to paint both sides of the board?

b. What transformation describes the motion of the conveyor belt?

c. What transformation describes the action of the arm?

d. Using the variable b for the board, the function C for the motion of the conveyor belt, and the function A for the action of the arm rotating the board 180°, what is the equation to paint a surfboard completely?

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Instruction

Problem-Based Task 5.2.1: Painting Surfboards

Coaching Sample Responses

a. How many times must the procedure be done to paint both sides of the board?

The procedure must be completed two times to paint the board completely.

b. What transformation describes the motion of the conveyor belt?

The conveyor belt represents a translation since the board is moved in one direction.

c. What transformation describes the action of the arm?

The action of the arm represents a rotation since the board is rotated 180°.

d. Using the variable b for the board, the function C for the motion of the conveyor belt, and the function A for the action of the arm rotating the board 180°, what is the equation to paint a surfboard completely?

$2(C(b), A(b))$

Note: The comma is used to imply sequence.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 5.2.1: Defining Rotations, Reflections, and Translations

Answer the questions and complete the following problems about transformations.

1. In a rotation of a set of points, what geometric structure defines the motion of points?

2. What linear relationship defines the movement of a translation?

3. What linear relationship defines the movement of a reflection?

4. In a rotation, which point is moved the greatest distance?

5. In a reflection, what happens to the points that lie on the line of reflection?

6. With respect to x and y, how many units does the translation $T_{6, 8}$ move a point $Z(x, y)$?

7. What transformation moves $R_{180}(r_{x\text{-axis}}(P))$ back to P?

8. Find the coordinates of the vertices of $R_{270}(T_{2, 3}(\triangle ABC))$ where A (1, 1), B (2, 8), and C (4, 6).

9. Describe the reflection through the line $y = x$ of the point $T(P)$ for which $T(x - 3, y + 1)$ and P (24, –32).

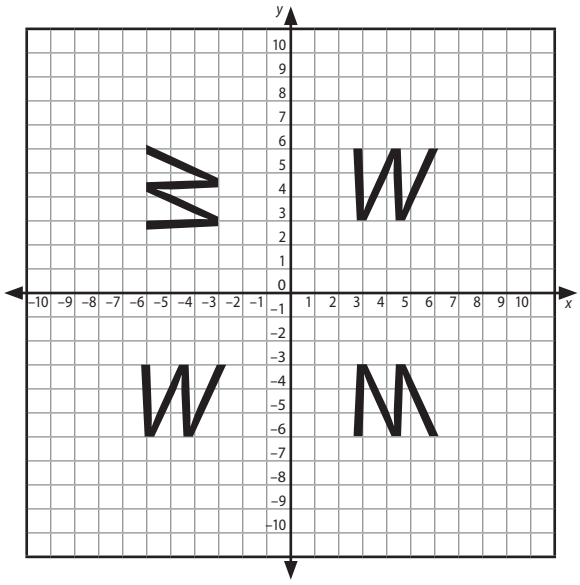
10. In terms of x and y, how does the point V (3, 3) move in the transformation $T_{2, 5}(R_{180}(V))$?

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Lesson 5.2.2: Applying Rotations, Reflections, and Translations

Warm-Up 5.2.2

Willow Springs is a new company that sells yoga mats. Their logo is represented below. Assume the *W* in Quadrant I is the preimage and other *W*'s have been transformed. How have each of the *W*'s been transformed from the preimage?



1. By observation, what transformation occurred on the *W* in Quadrant II?

2. By observation, what transformation occurred on the *W* in Quadrant III?

3. By observation, what transformation occurred on the *W* in Quadrant IV?

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Lesson 5.2.2: Applying Rotations, Reflections, and Translations

Common Core State Standard

G–CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Warm-Up 5.2.2 Debrief

1. By observation, what transformation occurred on the *W* in Quadrant II?

   In Quadrant II, the orientation of the *W* has changed; the *W* is now on its side. This *W* has undergone a rotation about the origin.

2. By observation, what transformation occurred on the *W* in Quadrant III?

   In Quadrant III, the orientation of the *W* is the same as the *W* in Quadrant I. The *W* in Quadrant III appears to have slid to its current position. We can deduce the *W* in Quadrant III has been translated.

3. By observation, what transformation occurred on the *W* in Quadrant IV?

   The *W* in Quadrant IV appears to simply be upside down; however, on closer inspection of the slants of the lines in the *W*, it is clear that this is a mirror image. Therefore, the *W* in Quadrant IV has been reflected through the *x*-axis.

Connection to the Lesson

- Observation of graphs will yield clues to transformations that have occurred.
- Carefully observing graphs as they are drawn can help to identify mistakes quickly.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- understanding the coordinate plane (5.G.1)
- visual recognition of tilt vs. flip (8.G.3)
- understanding the definitions of basic transformations (8.G.3)

Introduction

First we learned that transformations can be functions in the coordinate plane. Then we learned the definitions and properties of three isometric transformations: rotations, reflections, and translations. Now we are able to apply what we have learned to graph geometric figures and images created through transformations.

Key Concepts

- Transformations can be precisely and accurately graphed using the definitions learned.
- Given a set of points and a target, we can determine the transformation(s) necessary to move the given set of points to the target.
- Observing the orientations of the preimage and image is the first tool in determining the transformations required.
- Graphs can be interpreted differently, allowing for several possible transformation functions. While many different transformations will satisfy a particular graph, we will look for the more concise possibilities.
- Formulas can be used to determine translations, reflections, and rotations.
    - Translation: $T_{h,k}(x, y) = (x + h, y + k)$
    - Reflection:
        - through the x-axis: $r_{x\text{-axis}}(x, y) = (x, -y)$
        - through the y-axis: $r_{y\text{-axis}}(x, y) = (-x, y)$
        - through the line $y = x$: $r_{y=x}(x, y) = (y, x)$

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- Rotation:
  - 90° rotation about the origin: $R_{90}(x, y) = (-y, x)$
  - 180° rotation about the origin: $R_{180}(x, y) = (-x, -y)$
  - 270° rotation about the origin: $R_{270}(x, y) = (y, -x)$

Common Errors/Misconceptions

- using the incorrect transformation formula for reflections
- using the incorrect transformation formula for rotations
- translating in the positive directions along the axes when the translations are intended to be in the negative directions
- applying the transformations in the wrong order

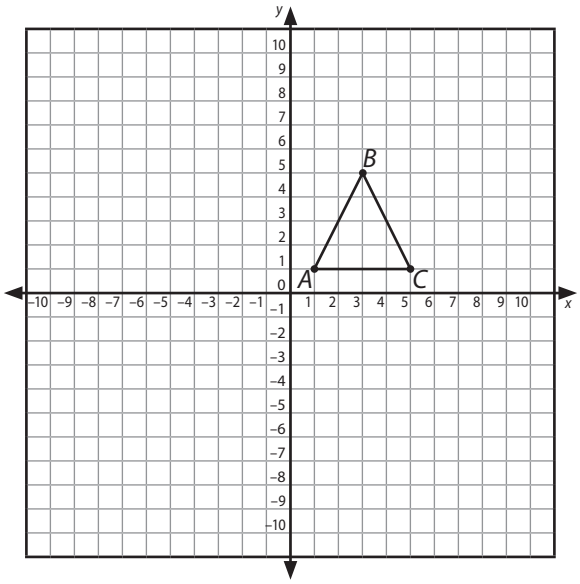
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Guided Practice 5.2.2

Example 1

Use the definitions you have learned to graph the translation $T_{2,3}(\triangle ABC)$ for which $\triangle ABC$ has the vertices A (1, 1), B (3, 5), and C (5, 1).

1. On graph paper, draw the x- and y-axes and graph $\triangle ABC$ with the points A (1, 1), B (3, 5), and C (5, 1).



2. Determine the new points.

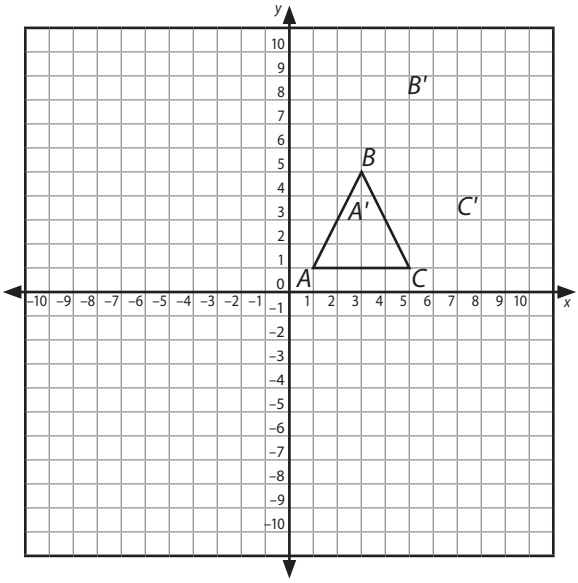
$$A' = T_{2,3}(A) = (1+2, 1+3) = (3, 4)$$

$$T_{2,3}(\triangle ABC) = \triangle A'B'C' \text{ where } B' = T_{2,3}(B) = (3+2, 5+3) = (5, 8)$$

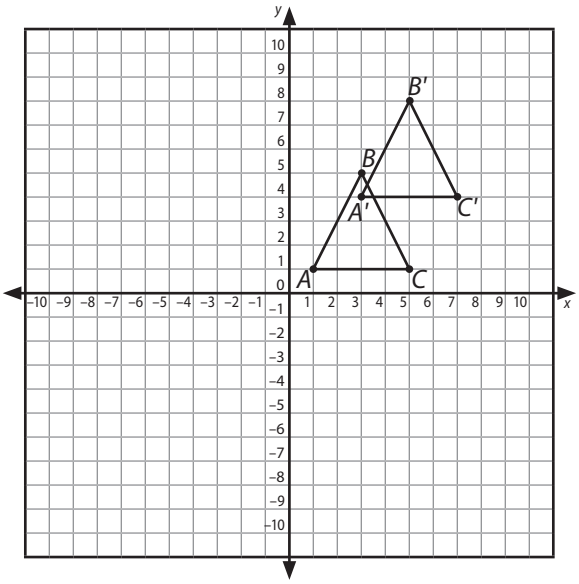
$$C' = T_{2,3}(C) = (5+2, 1+3) = (7, 4)$$

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3. Plot the new points A', B', and C'.



4. Connect the vertices to graph the translation $T_{2, 3}$ of $\triangle ABC$.



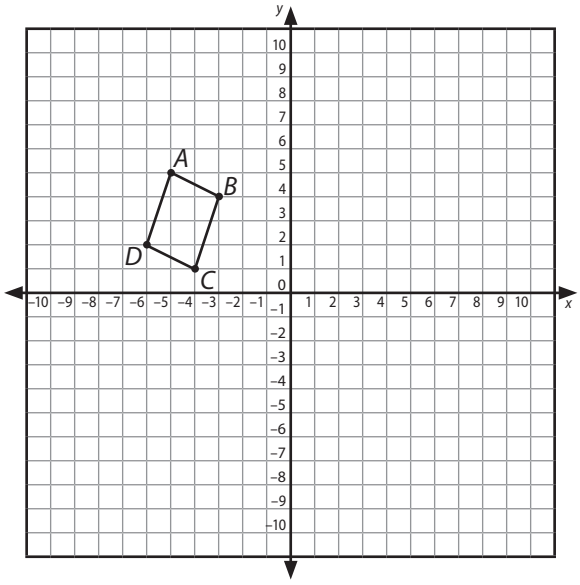
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Instruction

Example 2

Use the definitions you have learned to graph the reflection of parallelogram $ABCD$, or $\square ABCD$, through the y-axis given that $\square ABCD$ has vertices A (–5, 5), B (–3, 4), C (–4, 1), and D (–6, 2).

1. Using graph paper, draw the x- and y-axes and graph $\square ABCD$ with A (–5, 5), B (–3, 4), C (–4, 1), and D (–6, 2).



2. Write the new points.

$r_{y\text{-axis}}(\square ABCD) = \square A'B'C'D'$ where

$A' = r_{y\text{-axis}}(A) = r_{y\text{-axis}}(-5,5) = (-(-5),5) = (5,5)$

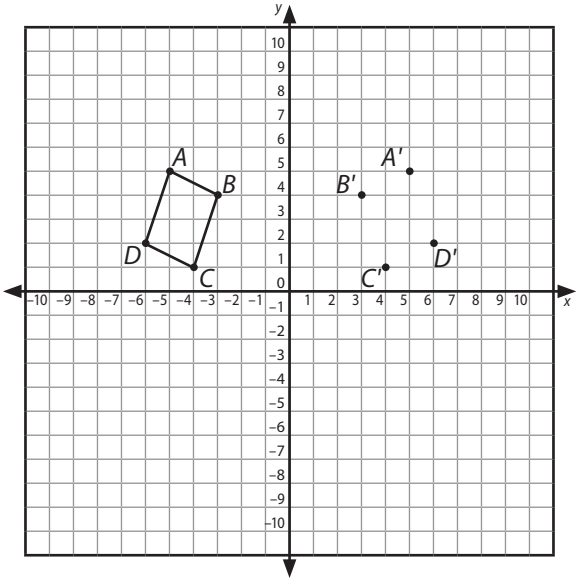
$B' = r_{y\text{-axis}}(B) = r_{y\text{-axis}}(-3,4) = (-(-3),4) = (3,4)$

$C' = r_{y\text{-axis}}(C) = r_{y\text{-axis}}(-4,1) = (-(-4),1) = (4,1)$

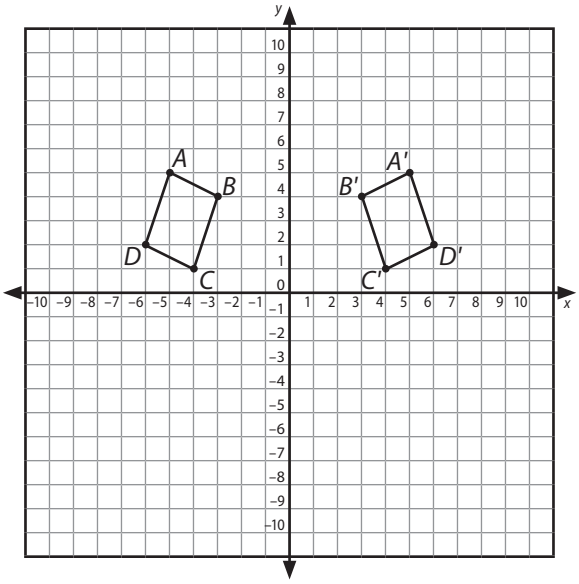
$D' = r_{y\text{-axis}}(D) = r_{y\text{-axis}}(-6,2) = (-(-6),2) = (6,2)$

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3. Plot the new points A', B', C', and D'.



4. Connect the corners of the points to graph the reflection $r_{y\text{-axis}}$ of $\square A'B'C'D'$.



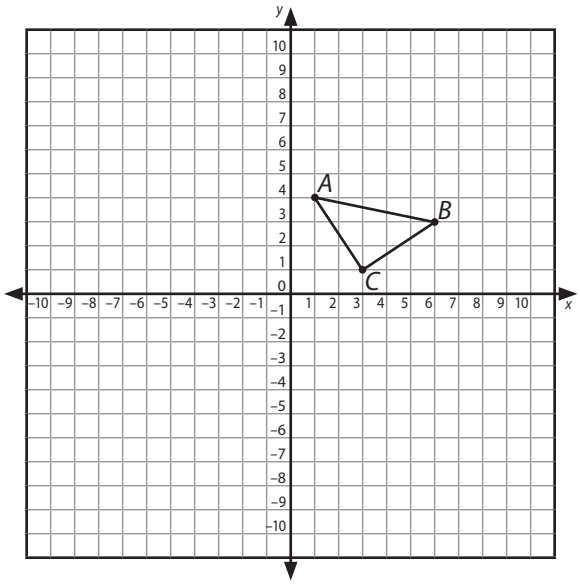
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Instruction

Example 3

Using the definitions you have learned, graph a 90° rotation of $\triangle ABC$ with vertices A (1, 4), B (6, 3), and C (3, 1).

1. Using graph paper, draw the x- and y-axes and graph $\triangle ABC$ with the vertices A (1, 4), B (6, 3), and C (3, 1).



2. Write the new points.

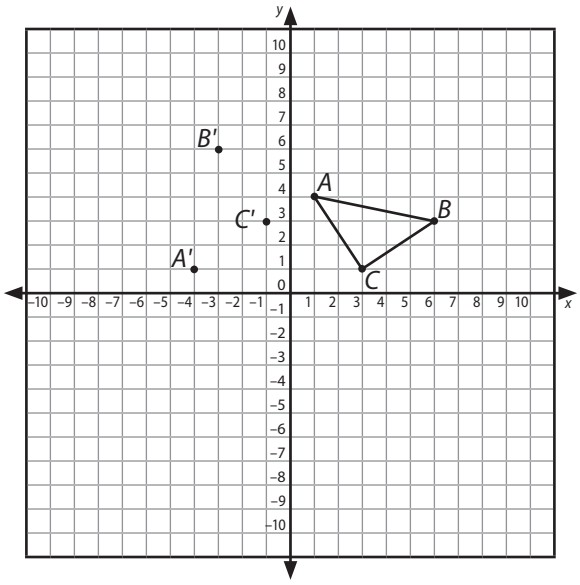
$$A' = R_{90}(A) = R_{90}(1,4) = (-4,1)$$

$$R_{90}(\triangle ABC) = \triangle A'B'C' \text{ where } B' = R_{90}(B) = R_{90}(6,3) = (-3,6)$$

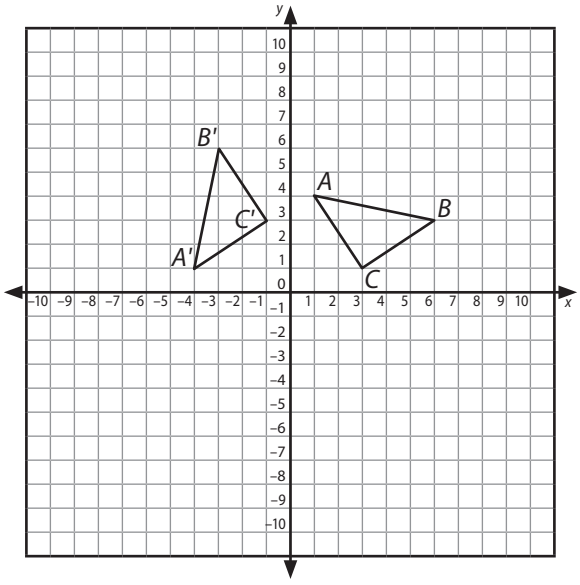
$$C' = R_{90}(C) = R_{90}(3,1) = (-1,3)$$

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3. Plot the new points A', B', and C'.



4. Connect the vertices to graph a 90° rotation of $\triangle ABC$.



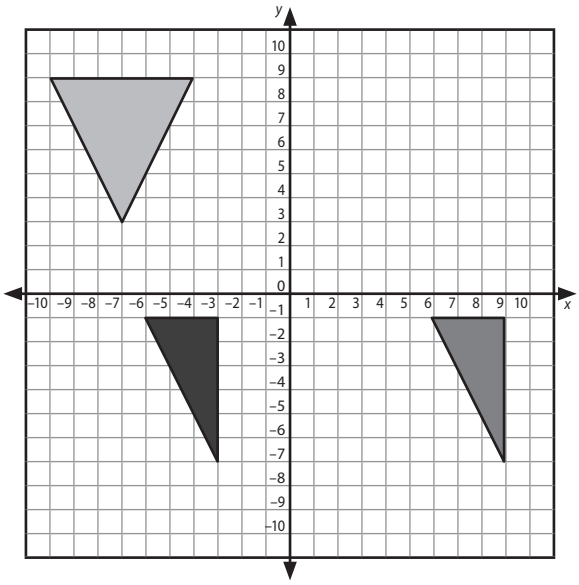
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Problem-Based Task 5.2.2: Animating a Logo

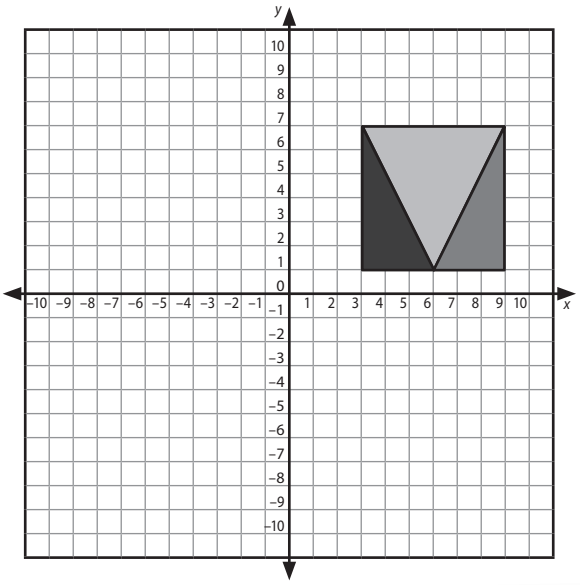
SMP	
1	2 ✓
3	4 ✓
5	6 ✓
7	8

The marketing department for Mitchell Software is designing an animation for their logo. The logo is shown in Quadrant I of Graph B below. The shapes used in the logo can be found in Quadrants II, III, and IV of Graph A. The marketing department will use three unique transformations to bring together the triangles. What transformations must occur in the animation?

Graph A



Graph B





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Problem-Based Task 5.2.2: Animating a Logo

Coaching

Let the triangle from Quadrant II be called II_A and its transformed image be II_B. Let the triangle from Quadrant III be called III_A and its transformed image be III_B. Let the triangle from Quadrant IV be called IV_A and its transformed image be IV_B. Evaluate the triangles one transformed pair at a time.

a. What transformation will take II_A to II_B?

b. What transformation will take III_A to III_B?

c. What transformation will take IV_A to IV_B?

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Problem-Based Task 5.2.2: Animating a Logo

Coaching Sample Responses

a. What transformation will take II_A to II_B?

First look at the orientations of triangles II_A and II_B. The orientations are the same, so the transformation needed a translation.

Determine the change in x from any point P in II_A to P' in II_B. The upper left vertex of II_A is at (–10, 9) and the upper left vertex of II_B is at (3, 7). The change in x is $3 - (-10) = 3 + 10 = 13$.

Now determine the change in y using the same point.

$7 - 9 = -2$

So, the transformation $T_{13, -2} = (x + 13, y - 2)$ will take II_A to II_B.

b. What transformation will take III_A to III_B?

Again, observe the orientation of III_A and III_B. This transformation can be seen as either reflections over both the x- and y-axes, or a rotation of 180°. Let's use a rotation. Taking into account the location of the upper right vertex of III_A at (–3, –1) and the lower right vertex of III_B at (3, 1), we can deduce the point of rotation is the origin.

Thus, $R_{180}(III_A) = (III_B)$.

c. What transformation will take IV_A to IV_B?

Once again, the orientation of IV_A to IV_B will determine the transformation needed. In this case, IV_B is a mirror image of IV_A and our transformation must be a reflection. If we focus on a pair of points, one from IV_A and the other from IV_B, we can determine the line of reflection. The lower left vertex of IV_A is at (6, –1) and the lower left vertex of IV_B is at (6, 1). Since there is no change in x and the y-axis is the midpoint between the two points, the transformation is a reflection through the x-axis.

We now know $r_{x\text{-axis}}(IV_A) = IV_B$.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 5.2.2: Applying Rotations, Reflections, and Translations

Use what you know about transformations to complete each problem.

1. Graph the transformation $T_{2,-3}(T_{2,5}(\triangle ABC))$ where A (1, 2), B (2, 7), and C (4, 1).

2. Graph the transformation $R_{180}(T_{5,1}(\triangle ABC))$ where A (–4, –2), B (–6, –7), and C (–2, –3).

3. In what quadrant is $T_{5,4}(T_{1,1}(P))$ when P (–7, 1)?

4. Given $R_{90}(x, y) = (-y, x)$ and P (5, –1), what is $R_{90}(T_{1,1}(P))$?

5. Given $\square ABCD$ with points A (–2, 2), B (–1, 2), C (–1, 1), and D (–2, 1), what is $R_{180}(\square ABCD)$?

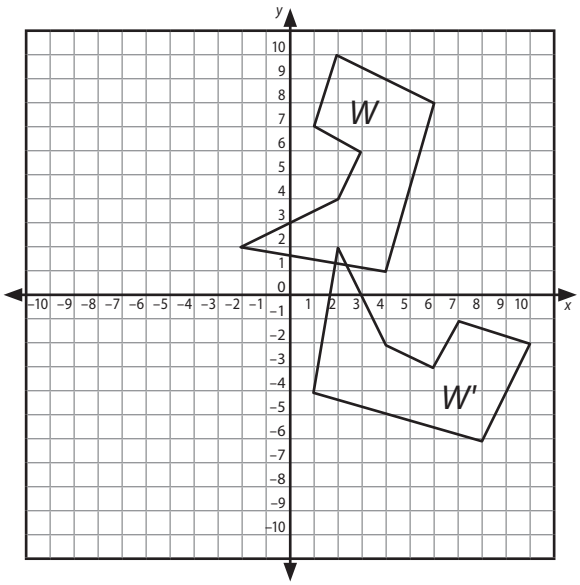
6. Using a rotation R_m, can you create a function $R_m(\square ABCD)$ that is equivalent to $r_{x\text{-axis}}(r_{y\text{-axis}}(\square ABCD))$?

7. A reflection through what line will move P (6, –13) to P' (6, 13)?

8. Graph the transformation $r_{y\text{-axis}}(r_{x\text{-axis}}(\triangle DEF))$ where D (9, 10), E (8, –2), and F (1, 4).

9. Graph the transformation $R_{180}(T_{5,1}(\triangle ABC))$ where A (–1, –1), B (–7, –2), and C (–4, –9).

10. Given the graph below, determine the transformation.



SWB p. 55; SRB Practice 5.2.2 p. U5-47

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Progress Assessment

Circle the letter of the best answer.

1. What geometric form do rotations move points along?
   a. rays
   b. parallel lines
   c. perpendicular lines
   d. circular arcs

2. What type of lines do translations move all points in a set along?
   a. parallel
   b. skew
   c. perpendicular
   d. independent

3. In what transformation are points moved along lines perpendicular to a line of reference?
   a. translation
   b. reflection
   c. rotation of a translation
   d. rotation

4. What coordinates will a point (x, y) rotated 180° about the origin have?
   a. (x, y)
   b. (y, x)
   c. $(x, -x)$
   d. $(-x, -y)$

5. What coordinates will the function $r_{y\text{-axis}}\left(T_{-5,4}\left(x, y\right)\right)$ have?
   a. $(x - 5, y + 4)$
   b. $(x - 5, y - 4)$
   c. $(5 - x, y + 4)$
   d. $(5 + x, y + 4)$

6. The point J (1, 2) undergoes the translation $T_{0,5}$. What are the coordinates of J'?
   a. (0, 5)
   b. (0, 7)
   c. (–1, 7)
   d. (1, 7)

7. The point $S(x, y) = (-x, y)$. What transformation is S?
   a. $T_{-1,0}$
   b. R_{270}
   c. $r_{x\text{-axis}}$
   d. $r_{y\text{-axis}}$

continued

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8. What are the coordinates of $R_{90}\left(R_{270}\left(x, y\right)\right)$ if $(x, y) = (4, 5)$?

   a. (–5, –4)
   c. (5, 4)
   b. (4, 5)
   d. (–5, 4)

9. What translation moves the point Q (–4, –3) to (9, 10)?

   a. $T_{5,7}$
   c. $T_{13,13}$
   b. $T_{-5,7}$
   d. $T_{-13,-13}$

10. If $W'\left(7,8\right)$ and $R_{90}\left(W\right)=W'$, what are the coordinates of W?

    a. (–7, 8)
    c. (8, –7)
    b. (–8, 7)
    d. (7, 8)

Use what you know about transformations and the given information to answer the questions.

11. The vertices of $\triangle ABC$ are A (2, 1), B (–1, 2), and C (–1, –1). Three transformations are performed on this triangle. The first is a transformation of the reflection of $\triangle ABC$ through the x-axis. Then it is translated left 2 units and down 3 units. Finally, the triangle is rotated 90° counterclockwise.

    a. What is the symbolic representation of these transformations?

    b. What are the coordinates of the vertices after the first transformation?

    c. What are the coordinates of the vertices after two transformations?

    d. What are the coordinates of the vertices after all three transformations?

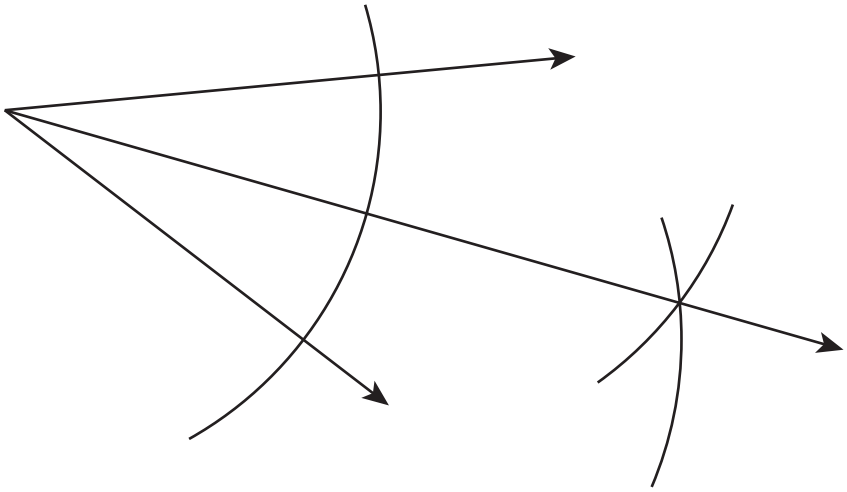
    e. What is the graph of the preimage and image?

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Pre-Assessment

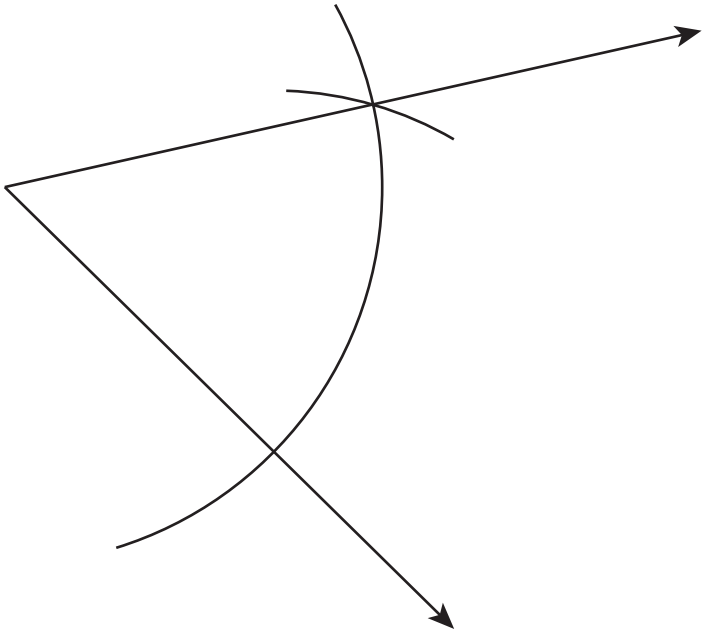
Circle the letter of the best answer.

1. Which geometric construction matches the diagram below?



a. construction of a triangle
b. copying a segment
c. copying an angle
d. construction of an angle bisector

2. Which geometric construction matches the diagram below?

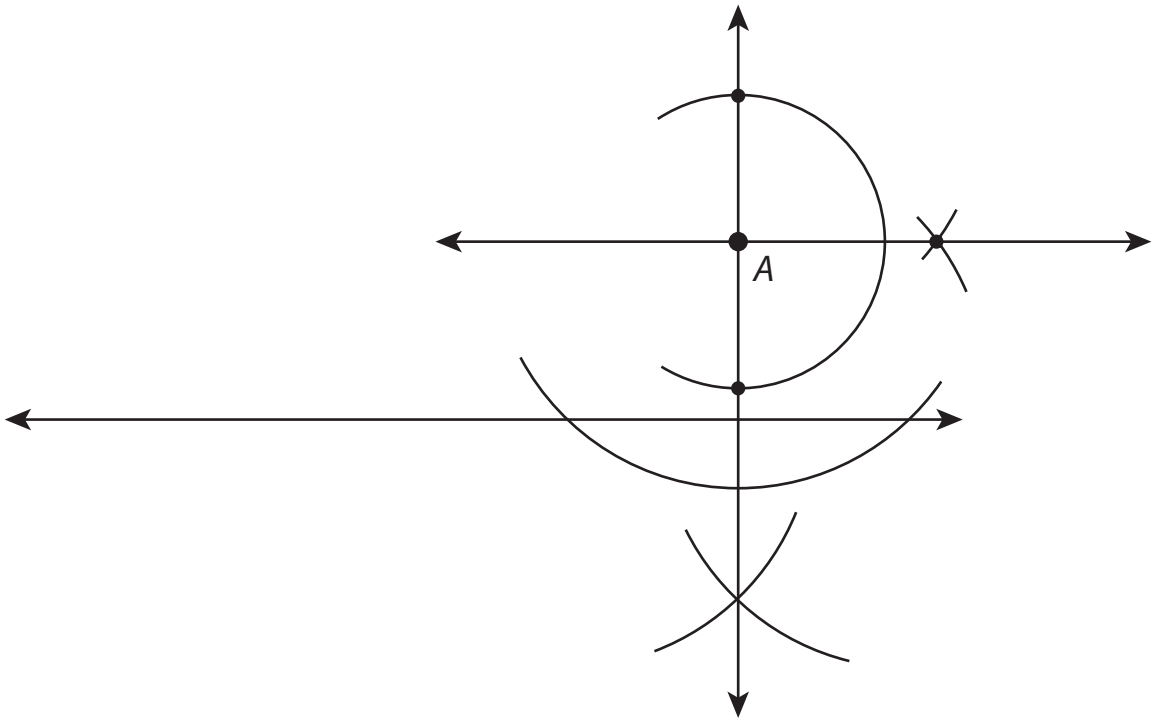


a. construction of a triangle
b. copying a segment
c. copying an angle
d. construction of an angle bisector

continued

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3. Which geometric construction matches the diagram below?



a. copying a segment

b. construction of a perpendicular line through point *A*

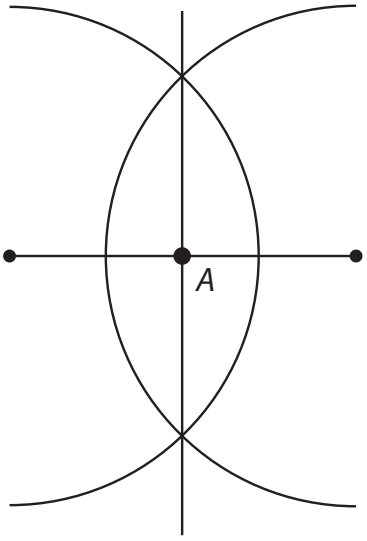
c. construction of a parallel line through point *A*

d. construction of a midpoint

continued

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4. Which geometric construction matches the diagram below?



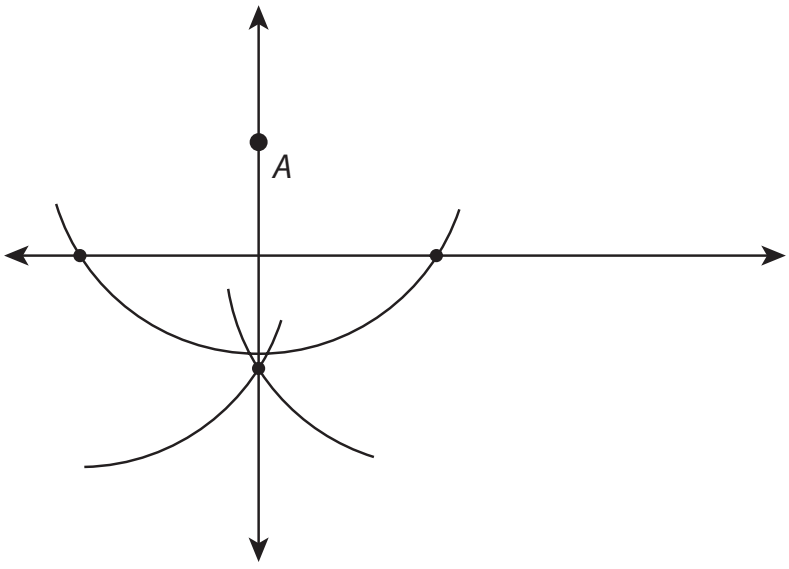
a. copying a segment

b. construction of a perpendicular line through point *A*

c. construction of a parallel line through point *A*

d. construction of a midpoint

5. Which geometric construction matches the diagram below?



a. copying a segment

b. construction of a perpendicular line through point *A*

c. construction of a parallel line through point *A*

d. construction of a midpoint

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Lesson 3: Constructing Lines, Segments, and Angles

Instruction

Common Core State Standard

SMP	
1 ✓	2
3	4
5 ✓	6 ✓
7	8 ✓

G–CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Essential Questions

1. What is the difference between sketching geometric figures, drawing geometric figures, and constructing geometric figures?
2. What tools are used with geometric constructions and why?
3. How can you justify a construction was done correctly?

WORDS TO KNOW

altitude the perpendicular line segment from a vertex of a figure to its opposite side; height

angle two rays or line segments sharing a common endpoint. Angles can be measured in degrees or radians; written as $\angle A$

bisect to cut in half

compass an instrument for creating circles or transferring measurements that consists of two pointed branches joined at the top by a pivot

congruent having the same shape, size, lines, and angles; the symbol for representing congruency between figures is $\cong$.

construct to create a precise geometric representation using a straightedge along with either patty paper (tracing paper), a compass, or a reflecting device

SRB p. U5-48

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Instruction

construction	a precise representation of a figure using a straightedge and a compass, patty paper and a straightedge, or a reflecting device and a straightedge
drawing	a precise representation of a figure, created with measurement tools such as a protractor and a ruler
endpoint	either of two points that mark the ends of a line segment; a point that marks the end of a ray
equidistant	the same distance from a reference point
line	the straight path connecting two points and extending beyond the points in both directions; written as $\overleftrightarrow{PQ}$ or ℓ
line segment	a part of a line that is between two endpoints and that includes the endpoints; written as $\overline{PQ}$
median of a triangle	the segment joining the vertex to the midpoint of the opposite side
midpoint	a point on a line segment that divides the segment into two equal parts
midsegment	a line segment joining the midpoints of two sides of a triangle
parallel lines	lines in a plane that do not share any points and never intersect; written as $\overline{AB} \parallel \overline{PQ}$; line segments and rays can also be parallel
perpendicular bisector	a line that intersects a segment at its midpoint at a right angle
perpendicular lines	two lines that intersect at a right angle (90°); written as $\perp$; line segments and rays can also be perpendicular
ray	a part of a line that starts at a point and extends infinitely in one direction; written as $\overrightarrow{AB}$
sketch	a quickly done representation of a figure; a rough approximation of a figure
straightedge	a bar or strip of wood, plastic, or metal having at least one long edge of reliable straightness

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Instruction

Recommended Resources

- Math Open Reference. "Bisecting an angle."

  http://www.walch.com/rr/00001

  This site provides step-by-step instructions for bisecting an angle with a compass and straightedge.

- Math Open Reference. "Copying a line segment."

  http://www.walch.com/rr/00002

  This site provides step-by-step instructions for copying a line segment with a compass and straightedge.

- Math Open Reference. "Copying an angle."

  http://www.walch.com/rr/00003

  This site provides step-by-step instructions for copying an angle with a compass and straightedge.

- Math Open Reference. "Perpendicular at a point on a line."

  http://www.walch.com/rr/00004

  This site provides step-by-step instructions for constructing a perpendicular line at a point on the given line.

- Math Open Reference. "Perpendicular bisector of a line segment."

  http://www.walch.com/rr/00005

  This site provides step-by-step instructions for constructing a perpendicular bisector of a line segment using a compass and straightedge.

- Math Open Reference. "Perpendicular to a line from an external point."

  http://www.walch.com/rr/00006

  This site provides step-by-step instructions for constructing a perpendicular line from a point not on the given line.

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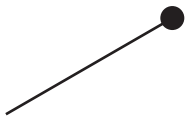
Lesson 5.3.1: Copying Segments and Angles

Warm-Up 5.3.1

Maryellen is caring for a pet goat. To keep the goat from roaming the neighborhood, Maryellen ties the goat to a leash staked in the yard. Maryellen learned quickly that goats enjoy chewing grass, so she must change the location of the stake periodically.

The original location of the stake is shown below on the left with the leash stretched out. Maryellen moved the stake to a new location, shown below on the right. Use the diagrams to solve the problems that follow.

Original stake location **New stake location**





1. Using only a compass, determine if there will be an overlap in the area the goat will chew.

2. Maryellen wants to keep both her goat and her yard healthy. Describe a process for determining where Maryellen should place each new stake in order to prevent the goat from overgrazing on the same patch of grass.

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Instruction

Lesson 5.3.1: Copying Segments and Angles

Common Core State Standard

G–CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Warm-Up 5.3.1 Debrief

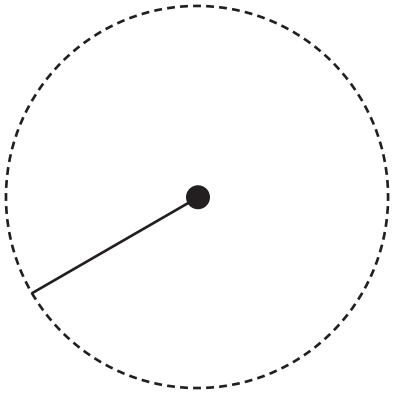
1. Using only a compass, determine if there will be an overlap in the area the goat will chew.

   First, put the sharp point of the compass at the original stake location.

   Open the compass so the pencil point reaches the end of the leash.

   Draw a circle, keeping the sharp point at the original stake location.

Original stake location



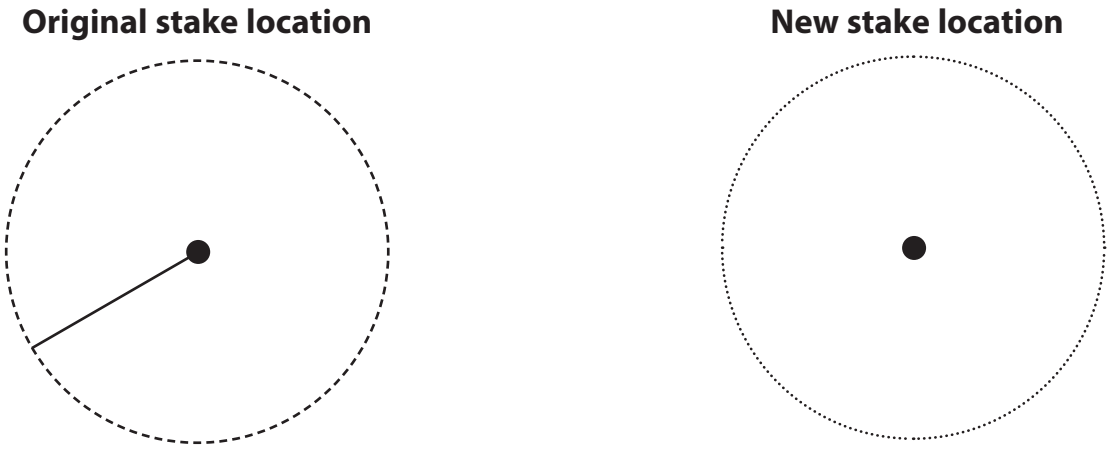
New stake location



   Without adjusting the opening of the compass, place the sharp point of the compass on the new stake location.

   Draw a circle, keeping the sharp point at the new stake location.

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The circles do not overlap; therefore, the areas chewed by the goat will not overlap.

2. Maryellen wants to keep both her goat and her yard healthy. Describe a process for determining where Maryellen should place each new stake in order to prevent the goat from overgrazing on the same patch of grass.

   Using the leash that has already been staked, Maryellen could mark the area the goat can chew by pulling the leash tight and walking the circle.

   Maryellen could then place one end of the leash on the outside edge of the circle, untie the leash from the stake, and stretch the leash out in the opposite direction from where the goat was staked.

   She could then place the stake in the ground.

   The leash acts like a compass for determining the circle.

   Maryellen could continue doing this to minimize the amount of overlap, and ensure that she uses the space in her yard the most effectively.

Connection to the Lesson

- Students will practice using a compass to make circles.
- It is important that students are capable of creating circles effortlessly before proceeding.
- Students will determine lengths with tools other than rulers.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- using a compass (no standard)
- understanding the geometry terms *line*, *segment*, *ray*, and *angle* (G–CO.1)

Introduction

Two basic instruments used in geometry are the straightedge and the compass. A **straightedge** is a bar or strip of wood, plastic, or metal that has at least one long edge of reliable straightness, similar to a ruler, but without any measurement markings. A **compass** is an instrument for creating circles or transferring measurements. It consists of two pointed branches joined at the top by a pivot. It is believed that during early geometry, all geometric figures were created using just a straightedge and a compass. Though technology and computers abound today to help us make sense of geometry problems, the straightedge and compass are still widely used to **construct** figures, or create precise geometric representations. Constructions allow you to draw accurate segments and angles, segment and angle bisectors, and parallel and perpendicular lines.

Key Concepts

- A geometric figure precisely created using only a straightedge and compass is called a **construction**.
- A straightedge can be used with patty paper (tracing paper) or a reflecting device to create precise representations.
- Constructions are different from drawings or sketches.
- A **drawing** is a precise representation of a figure, created with measurement tools such as a protractor and a ruler.
- A **sketch** is a quickly done representation of a figure or a rough approximation of a figure.
- When constructing figures, it is very important not to erase your markings.
- Markings show that your figure was constructed and not measured and drawn.
- An **endpoint** is either of two points that mark the ends of a line segment, or the point that marks the end of a ray.
- A **line segment** is a part of a line that is between two endpoints and that includes the endpoints.

SRB p. U5-51

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Instruction

- An **angle** is formed when two rays or line segments share a common endpoint.
- A constructed figure and the original figure are **congruent**; they have the same shape, size, or angle.
- Follow the steps outlined below and on the next page to copy a segment and an angle.

Copying a Segment Using a Compass and a Straightedge

1. To copy $\overline{AB}$, first make an endpoint on your paper. Label the endpoint *C*.
2. Put the sharp point of your compass on endpoint *A*. Open the compass until the pencil end touches endpoint *B*.
3. Without changing your compass setting, put the sharp point of your compass on endpoint *C*. Make a large arc.
4. Use your straightedge to connect endpoint *C* to any point on your arc.
5. Label the point of intersection of the arc and your segment *D*.

Do not erase any of your markings.

$\overline{AB}$ is congruent to $\overline{CD}$.

Copying a Segment Using Patty Paper and a Straightedge

1. To copy $\overline{AB}$, place your sheet of patty paper over the segment.
2. Mark the endpoints of the segment on the patty paper. Label the endpoints *C* and *D*.
3. Use your straightedge to connect points *C* and *D*.

$\overline{AB}$ is congruent to $\overline{CD}$.

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Instruction

Copying an Angle Using a Compass and a Straightedge

1. To copy $\angle A$, first make a point to represent the vertex A on your paper. Label the vertex E.
2. From point E, draw a ray of any length using the straightedge. This will be one side of the constructed angle.
3. Put the sharp point of the compass on vertex A of the original angle. Set the compass to any width that will cross both sides of the original angle.
4. Draw an arc across both sides of $\angle A$. Label where the arc intersects the angle as points B and C.
5. Without changing the compass setting, put the sharp point of the compass on point E. Draw a large arc that intersects the ray. Label the point of intersection as F.
6. Put the sharp point of the compass on point B of the original angle and set the width of the compass so it touches point C.
7. Without changing the compass setting, put the sharp point of the compass on point F and make an arc that intersects the arc in step 5. Label the point of intersection as D.
8. Use the straightedge to draw a ray from point E to point D.

Do not erase any of your markings.

$\angle A$ is congruent to $\angle E$.

Copying an Angle Using Patty Paper and a Straightedge

1. To copy $\angle A$, place your sheet of patty paper over the angle.
2. Mark the vertex of the angle. Label the vertex E.
3. Use your straightedge to trace each side of $\angle A$.

$\angle A$ is congruent to $\angle E$.

Common Errors/Misconceptions

- inappropriately changing the compass setting
- moving the patty paper before completing the construction
- attempting to measure lengths and angles with rulers and protractors

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Instruction

Guided Practice 5.3.1

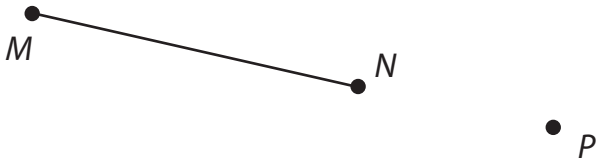
Example 1

Copy the following segment using only a compass and a straightedge.



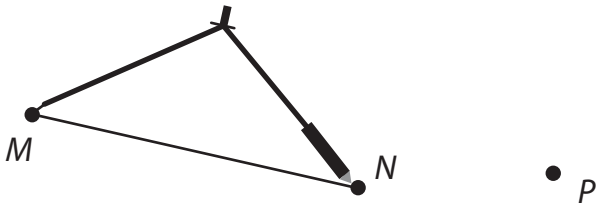
1. Make an endpoint on your paper. Label the endpoint *P*.

Original segment **Construction**



2. Put the sharp point of your compass on endpoint *M*. Open the compass until the pencil end touches endpoint *N*.

Original segment **Construction**

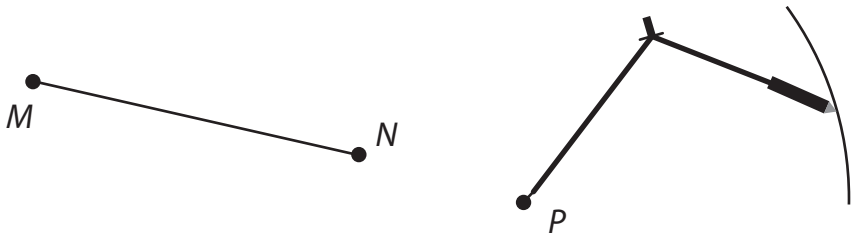


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Instruction

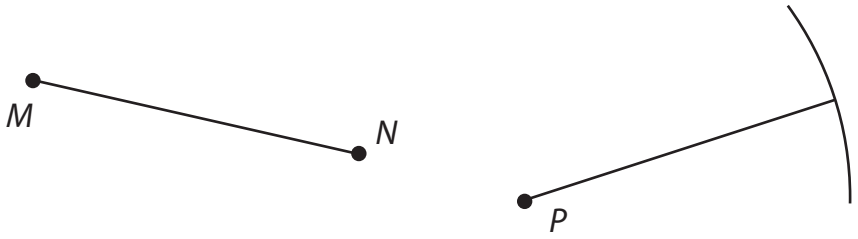
3. Without changing your compass setting, put the sharp point of your compass on endpoint *P*. Make a large arc.

Original segment **Construction**



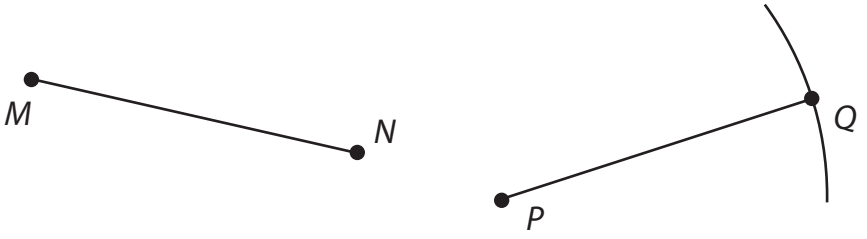
4. Use your straightedge to connect endpoint *P* to any point on your arc.

Original segment **Construction**



5. Label the point of intersection of the arc and your segment *Q*.

Original segment **Construction**



Do not erase any of your markings.

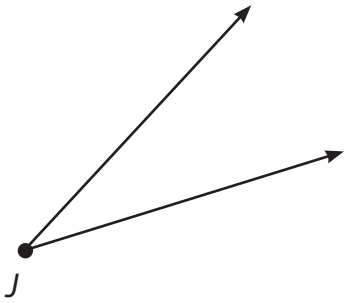
$\overline{MN}$ is congruent to $\overline{PQ}$.

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Instruction

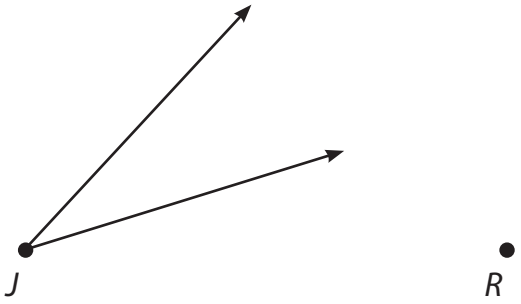
Example 2

Copy the following angle using only a compass and a straightedge.



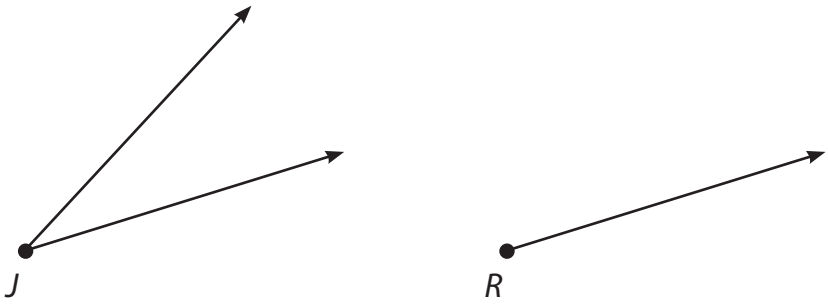
1. Make a point to represent vertex *J*. Label the vertex *R*.

Original angle **Construction**



2. From point *R*, draw a ray of any length. This will be one side of the constructed angle.

Original angle **Construction**

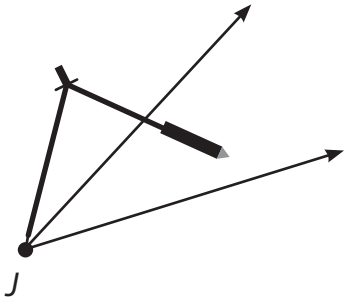


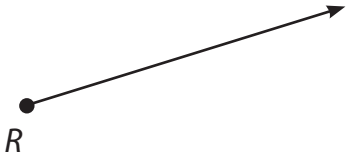
© Walch Education

Instruction

3. Put the sharp point of the compass on vertex J of the original angle. Set the compass to any width that will cross both sides of the original angle.

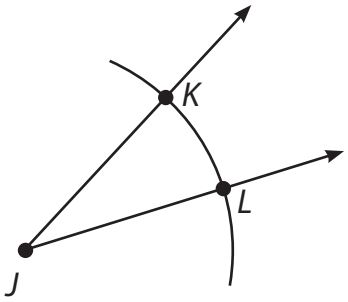
Original angle **Construction**

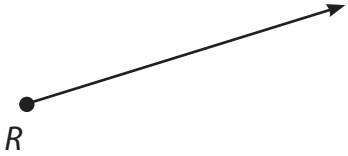




4. Draw an arc across both sides of $\angle J$. Label where the arc intersects the angle as points K and L.

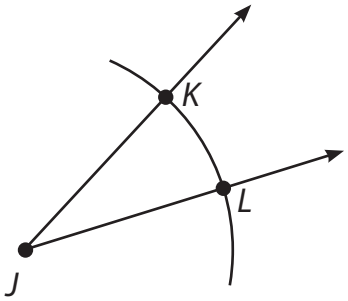
Original angle **Construction**

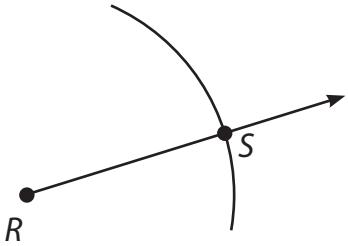




5. Without changing the compass setting, put the sharp point of the compass on point R. Draw a large arc that intersects the ray. Label the point of intersection as S.

Original angle **Construction**



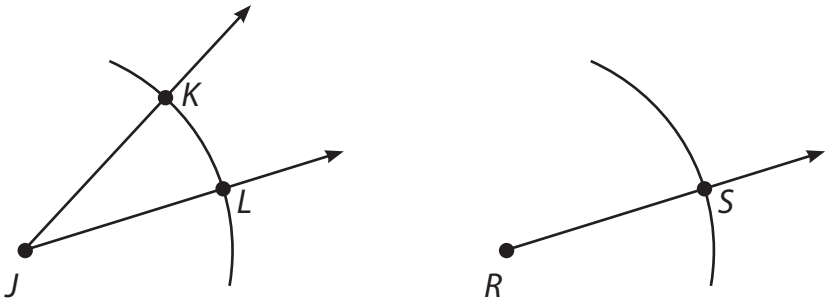


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Instruction

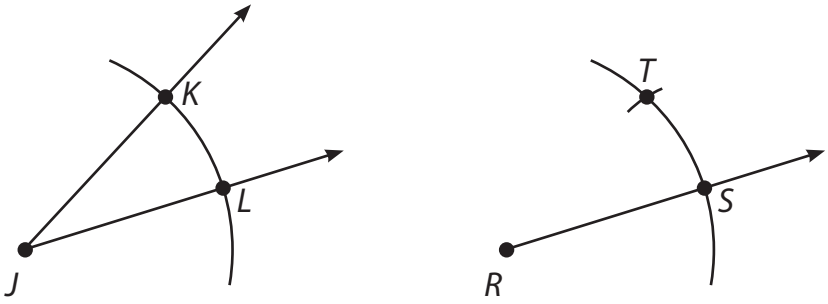
6. Put the sharp point of the compass on point *L* of the original angle and set the width of the compass so it touches point *K*.

Original angle **Construction**



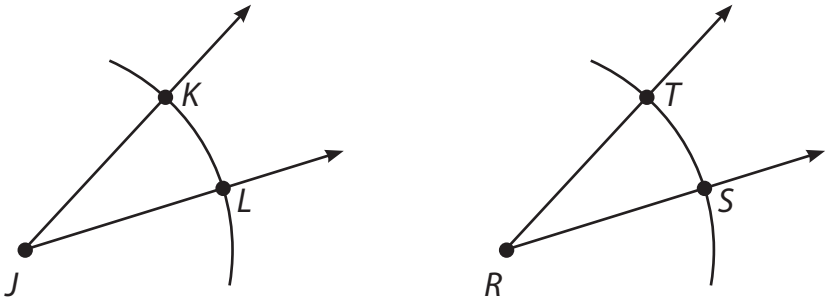
7. Without changing the compass setting, put the sharp point of the compass on point *S* and make an arc that intersects the arc you drew in step 5. Label the point of intersection as *T*.

Original angle **Construction**



8. Use a straightedge to draw a ray from point *R* through point *T*.

Original angle **Construction**



Do not erase any of your markings.

$\angle J$ is congruent to $\angle R$.

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Instruction

Example 3

Use the given line segment to construct a new line segment with length $2AB$.

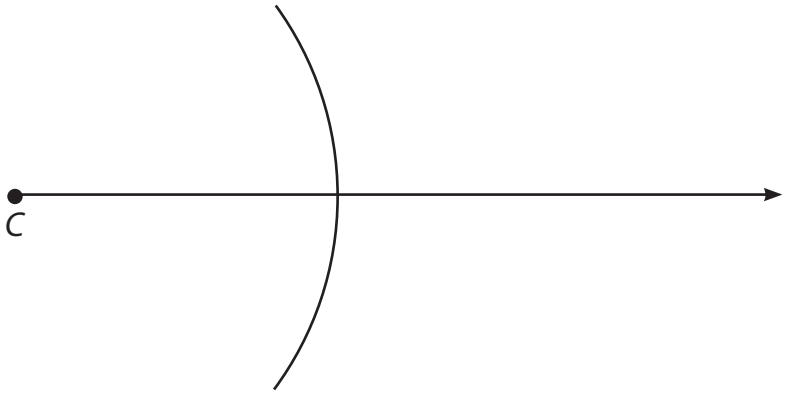


1. Use your straightedge to draw a long ray. Label the endpoint C.

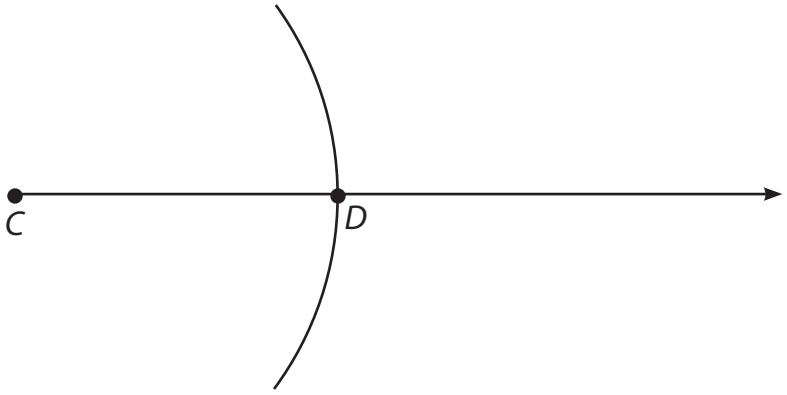


2. Put the sharp point of your compass on endpoint A of the original segment. Open the compass until the pencil end touches B.

3. Without changing your compass setting, put the sharp point of your compass on point C and make an arc that intersects your ray.



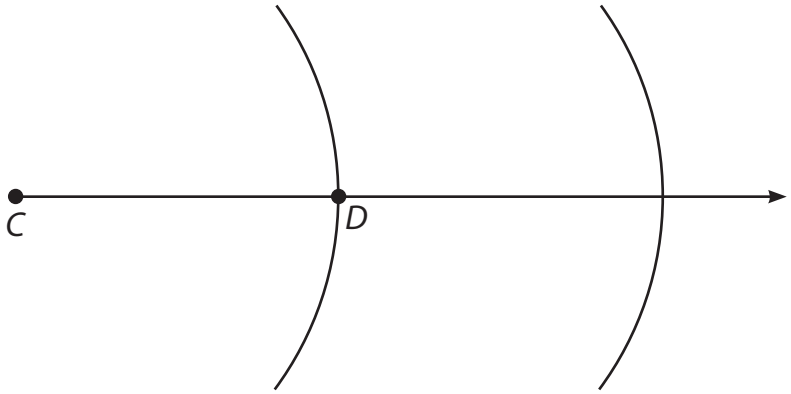
4. Mark the point of intersection as point D.



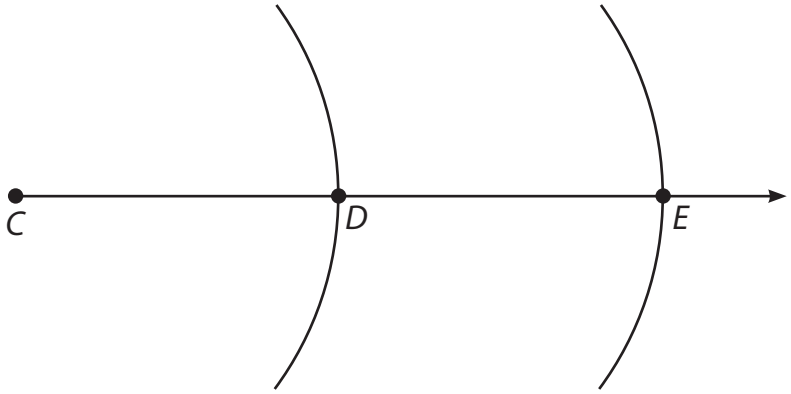
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Instruction

5. Without changing your compass setting, put the sharp point of your compass on point *D* and make an arc that intersects your ray.



6. Mark the point of intersection as point *E*.



Do not erase any of your markings.

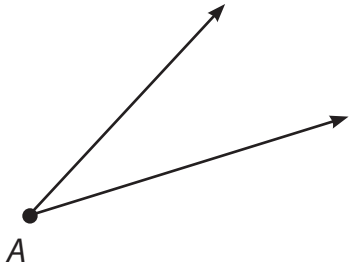
$CE = 2AB$

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Instruction

Example 4

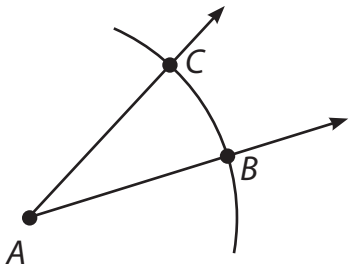
Use the given angle to construct a new angle equal to $\angle A + \angle A$.



1. Follow the steps from Example 2 to copy $\angle A$. Label the vertex of the copied angle G.

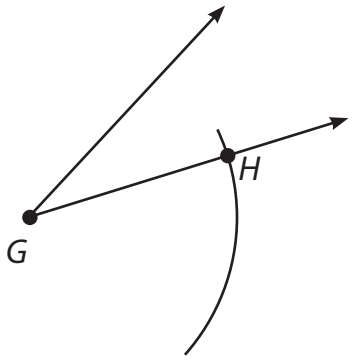
2. Put the sharp point of the compass on vertex A of the original angle. Set the compass to any width that will cross both sides of the original angle.

3. Draw an arc across both sides of $\angle A$. Label where the arc intersects the angle as points B and C.



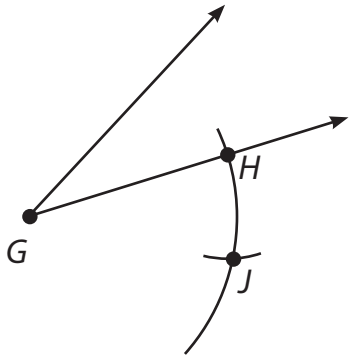
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4. Without changing the compass setting, put the sharp point of the compass on *G*. Draw a large arc that intersects one side of your newly constructed angle. Label the point of intersection *H*.



5. Put the sharp point of the compass on *C* of the original angle and set the width of the compass so it touches *B*.

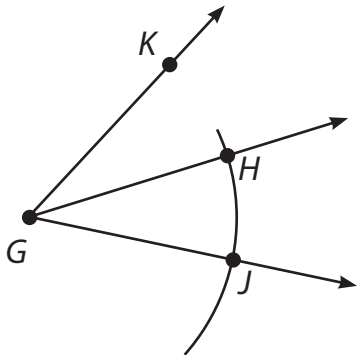
6. Without changing the compass setting, put the sharp point of the compass on point *H* and make an arc that intersects the arc created in step 4. Label the point of intersection as *J*.



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Instruction

7. Draw a ray from point G to point J.



Do not erase any of your markings. Place a point K on the top ray for naming purposes.

$\angle KGJ = \angle A + \angle A$

Example 5

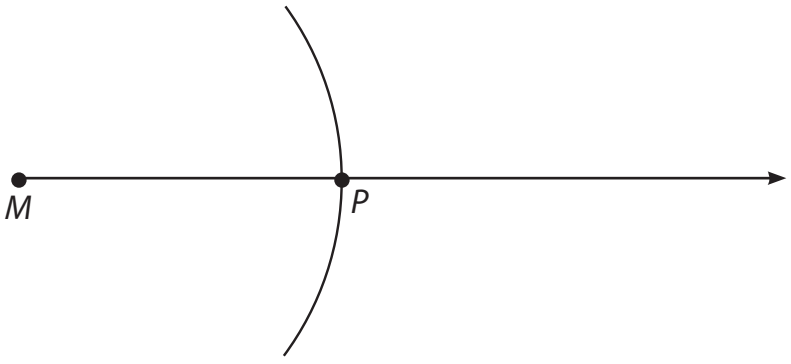
Use the given segments to construct a new segment equal to $AB - CD$.



1. Draw a ray longer than that of $\overline{AB}$. Label the endpoint M.



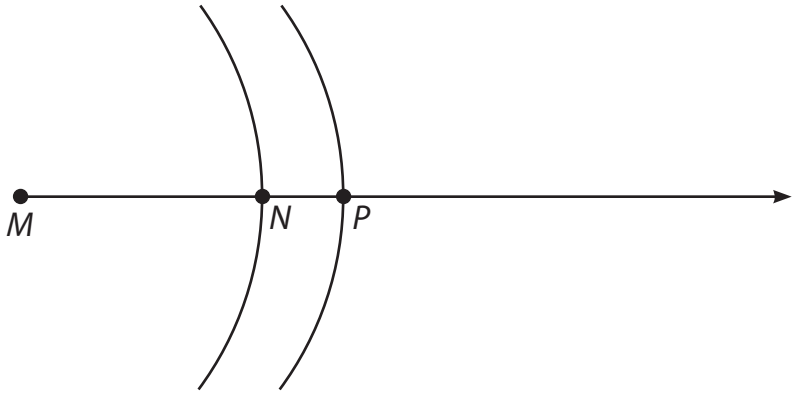
2. Follow the steps from Example 3 to copy $\overline{AB}$ onto the ray. Label the second endpoint P.



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Instruction

3. Put the sharp point of the compass on endpoint M of the ray. Copy segment $\overline{CD}$ onto the same ray. Label the endpoint N.



Do not erase any of your markings.

$NP = AB - CD$

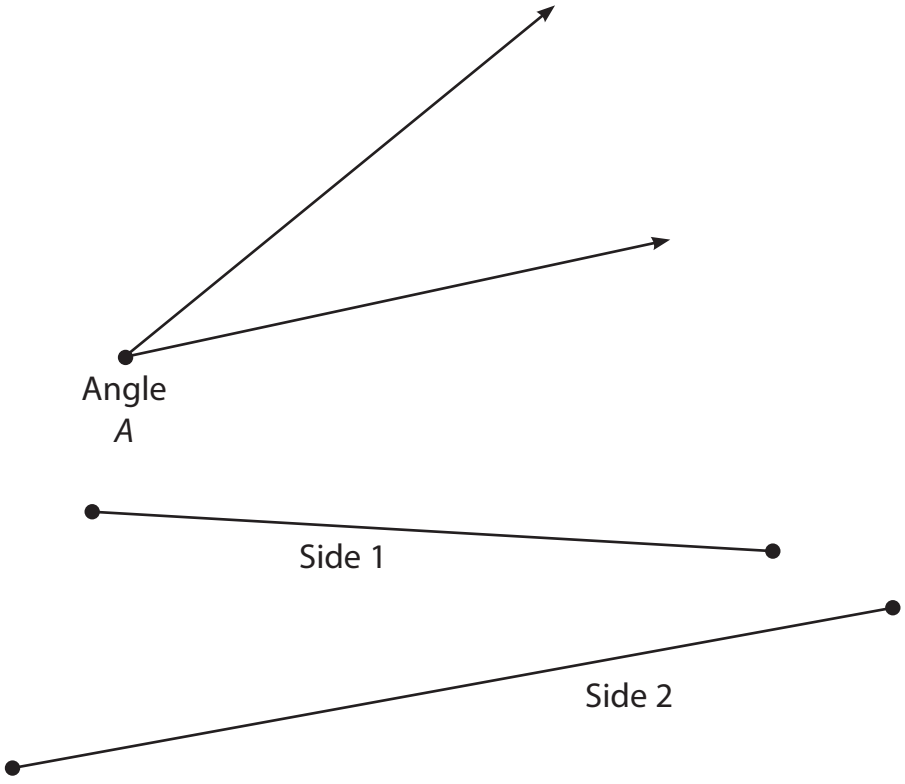
Angles can be subtracted in the same way.

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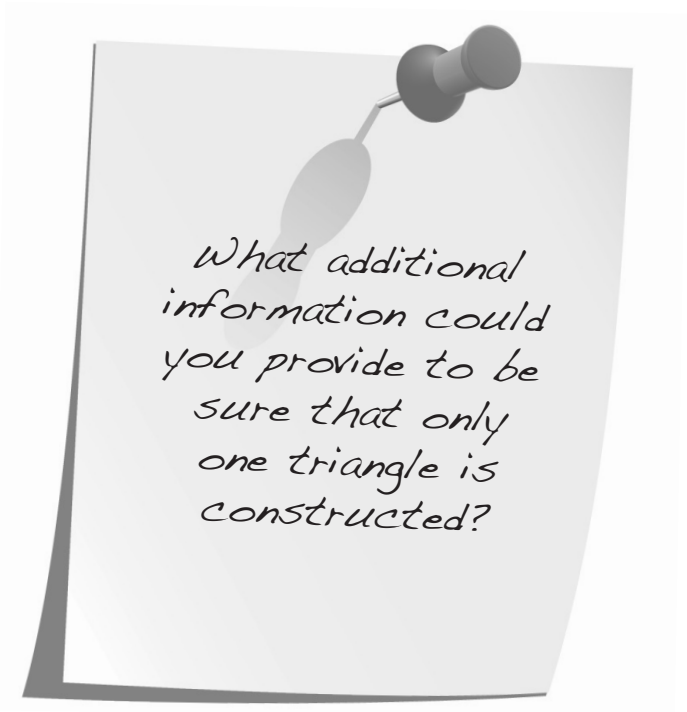
Problem-Based Task 5.3.1: How Many Triangles?

SMP	
1 ✓	2
3	4
5 ✓	6 ✓
7	8

How many non-congruent triangles can be constructed using the two sides and angle given? Use your construction tools to show all possible triangles.



What additional information could you provide to be sure that only one triangle is constructed?



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Problem-Based Task 5.3.1: How Many Triangles?

Coaching

a. A triangle is made up of how many sides?

b. A triangle is made up of how many angles?

c. Are there enough sides and angles to create a triangle?

d. Is it possible to create at least one triangle using the given segments and angle?

e. Using a compass and a straightedge, what is the method for copying an angle?

f. What is the method for copying a segment?

g. Construct and label a triangle using the given segments and angle.

h. Is it possible to rearrange the placement of the sides and create a second triangle that is not congruent to the first? Explain your reasoning.

i. If possible, construct and label a second triangle not congruent to the first using the given segments and angle.

j. Is it possible to rearrange the placement of the sides and create a third triangle that is not congruent to the first or second? Explain your reasoning.

k. If possible, construct and label a third triangle not congruent to the first or second using the given segments and angle.

l. Is it possible to rearrange the placement of the sides and create a fourth triangle that is not congruent to the first, second, or third? Explain your reasoning.

m. If possible, construct and label a fourth triangle not congruent to the first, second, or third using the given segments and angle.

n. How many non-congruent triangles were you able to create using the given segments and angle?

o. What additional information could you provide to be sure that only one triangle could be created?

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Instruction

Problem-Based Task 5.3.1: How Many Triangles?

Coaching Sample Responses

a. A triangle is made up of how many sides?

A triangle is made up of 3 sides.

b. A triangle is made up of how many angles?

A triangle is made up of 3 angles.

c. Are there enough sides and angles to create a triangle?

Two segments are given; each one can be used as a side.

One angle is given.

The remaining side and angles can be created once the given information is used.

d. Is it possible to create at least one triangle using the given segments and angle?

A triangle can be created if the given angle and side lengths are constructed first.

Then the sides can be connected to create the third side and remaining angles.

e. Using a compass and a straightedge, what is the method for copying an angle?

Make a point to represent the vertex. Label the vertex *A*.

From point *A*, draw a ray of any length. This will be one side of the constructed angle.

Put the sharp point of the compass on the vertex of the given angle.

Set the compass to any width that will cross both sides of the given angle.

Draw an arc across both sides of the given angle.

Label where the arc intersects the angle as points *C* and *D*.

Without changing the compass setting, put the sharp point of the compass on point *A*. Draw a large arc that intersects the ray. Label the point of intersection as *S*.

Put the sharp point of the compass on point *C* and set the width of the compass so it touches point *D*.

Without changing the compass setting, put the sharp point of the compass on point *S* and make an arc that intersects the arc.

Label the point of intersection as *T*.

Draw a ray from point *A* to point *T*.

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f. What is the method for copying a segment?

Make an endpoint on your paper. Label the endpoint *P*.

Put the sharp point of your compass on one endpoint of the given segment.

Open the compass until the pencil end touches the second endpoint of the given segment.

Without changing your compass setting, put the sharp point of your compass on endpoint *P*.

Make a large arc.

Label any point on the large arc as *Q*.

Use your straightedge to connect endpoint *P* to point *Q*.

g. Construct and label a triangle using the given segments and angle.

Check students' papers for accuracy.

h. Is it possible to rearrange the placement of the sides and create a second triangle that is not congruent to the first? Explain your reasoning.

Yes, because the placement of the sides is not given in the information provided. Students can rearrange the placement of each segment.

i. If possible, construct and label a second triangle not congruent to the first using the given segments and angle.

Check students' papers for accuracy.

j. Is it possible to rearrange the placement of the sides and create a third triangle that is not congruent to the first or second? Explain your reasoning.

Yes, because the placement of the sides is not given in the information provided. Students can rearrange the placement of each segment.

k. If possible, construct and label a third triangle not congruent to the first or second using the given segments and angle.

Check students' papers for accuracy.

l. Is it possible to rearrange the placement of the sides and create a fourth triangle that is not congruent to the first, second, or third? Explain your reasoning.

No, there are only three possible triangles using the given information: one with the given angle between the two given sides, one with the given angle constructed with Side 1, and one with the given angle constructed with Side 2.

m. If possible, construct and label a fourth triangle not congruent to the first, second, or third using the given segments and angle.

A fourth triangle is not possible.

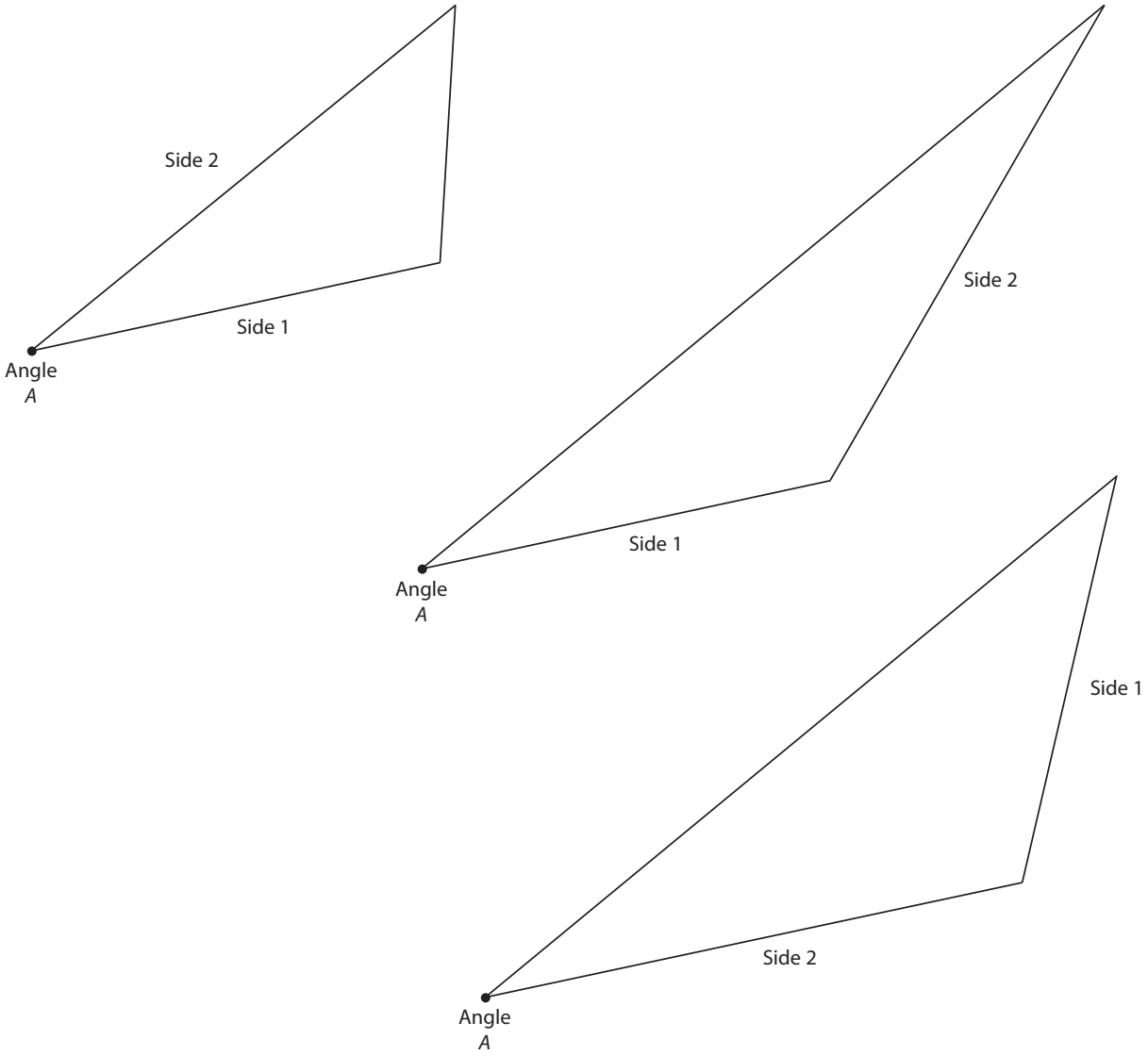
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Instruction

n. How many non-congruent triangles were you able to create using the given segments and angle?

Three non-congruent triangles can be created using the given segments and angle.

The three non-congruent triangles are the following (scaled):



o. What additional information could you provide to be sure that only one triangle could be created?

To be sure that only one triangle can be drawn, more specific information would need to be provided, such as specifying that the given angle must be between the two given sides.

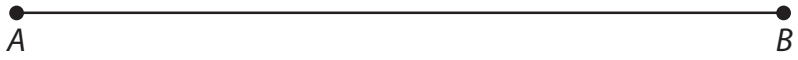
Recommended Closure Activity

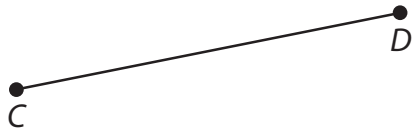
Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 5.3.1: Copying Segments and Angles

Copy the following segments using a straightedge and a compass.

1. 

2. 

3. 

Use the line segments in problems 1–3 to construct the line segments described in problems 4 and 5.

4. $EF - AB$

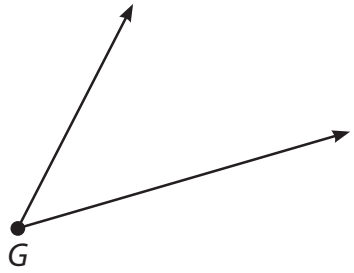
5. $2AB + CD$

continued

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Copy the following angles using a straightedge and a compass.

6.



7.



8.



Use the angles from problems 6–8 to construct the angles described in problems 9 and 10.

9. $\angle J - \angle G$

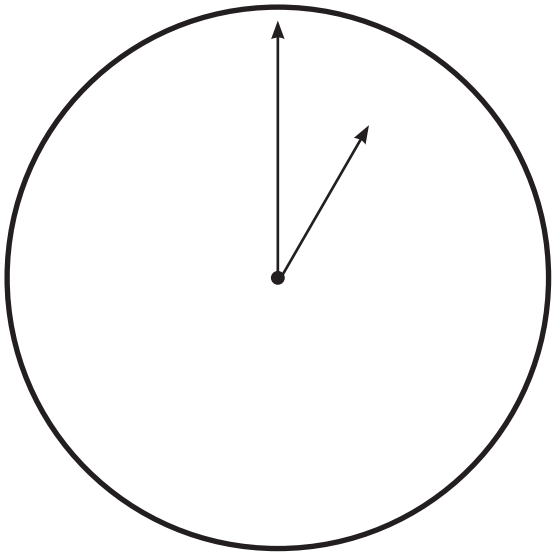
10. $\angle G + \angle H$

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Lesson 5.3.2: Bisecting Segments and Angles

Warm-Up 5.3.2

Henri is designing a set for the upcoming school play. The play opens with the main character in front of the clock tower at 1:00 P.M. Henri easily constructs the hands of the clock for the first scene, but needs help with the placement of the hands for the second scene. The second scene takes place 2 hours later in front of the same tower.



1. Use a compass and a straightedge to construct the hands of the clock for the second scene. Be sure to use construction methods previously learned.

2. If the measure of the angle of the hands for the first scene is 30˚, what is the measure of the angle of the hands for the second scene?

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Instruction

Lesson 5.3.2: Bisecting Segments and Angles

Common Core State Standard

G–CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Warm-Up 5.3.2 Debrief

1. Use a compass and a straightedge to construct the hands of the clock for the second scene. Be sure to use construction methods previously learned.

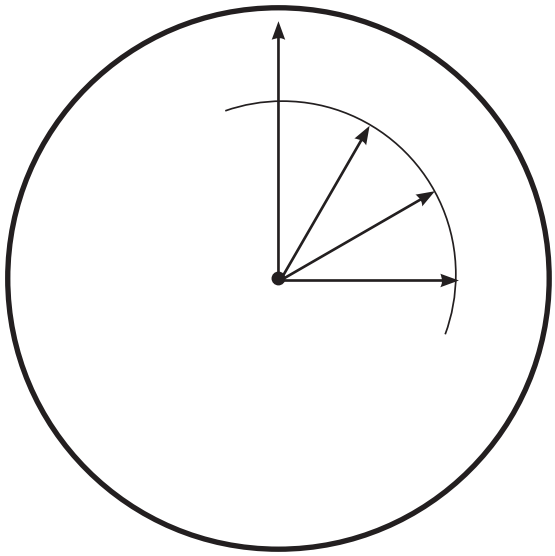
   Use the vertex of the original angle as the vertex for the new angle of the clock hands.

   Copy the original angle, using the lower side of the angle as the starting ray (the hour hand).

   Copy the original angle again to construct the position for the second scene, with the ray representing the hour hand at the 3:00 position.

   The problem scenario specified that the second scene takes place 2 hours later, so the original angle ray representing the minute hand should stay in place, at the 12:00 position. The second angle ray representing the hour hand is at 3:00. Therefore, the angle that represents the time of the second scene is encompassed by the rays pointing at 12:00 and 3:00.

   Check students' papers for accuracy.



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2. If the measure of the angle of the hands for the first scene is 30°, what is the measure of the angle of the hands for the second scene?

   The original position is 30°.

   The second scene is 2 hours later, meaning the angle opened an additional 60° (30 • 2 = 60).

   The resulting angle is 90° (30 + 60 = 90).

Connection to the Lesson

- Students continue to practice copying angles and using construction tools.
- In this lesson, students will use similar construction skills to partition angles.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- using a compass (no standard)
- copying angles and segments (G–CO.12)
- understanding the geometry terms *line*, *segment*, *ray*, and *angle* (G–CO.1)

Introduction

Segments and angles are often described with measurements. Segments have lengths that can be measured with a ruler. Angles have measures that can be determined by a protractor. It is possible to determine the midpoint of a segment. The **midpoint** is a point on the segment that divides it into two equal parts. When drawing the midpoint, you can measure the length of the segment and divide the length in half. When constructing the midpoint, you cannot use a ruler, but you can use a compass and a straightedge (or patty paper and a straightedge) to determine the midpoint of the segment. This procedure is called bisecting a segment. To **bisect** means to cut in half. It is also possible to bisect an angle, or cut an angle in half using the same construction tools. A **midsegment** is created when the midpoints of two sides of a triangle are connected. A triangle has three midsegments.

Key Concepts

Bisecting a Segment

- A segment bisector cuts a segment in half.
- Each half of the segment measures exactly the same length.
- A point, line, ray, or segment can bisect a segment.
- A point on the segment bisector is **equidistant**, or is the same distance, from each endpoint of the segment.

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- The point where the segment is bisected is called the midpoint of the segment.

Bisecting a Segment Using a Compass and a Straightedge

1. To bisect $\overline{AB}$, put the sharp point of your compass on endpoint *A*. Open the compass wider than half the distance of $\overline{AB}$.
2. Make a large arc intersecting $\overline{AB}$.
3. Without changing your compass setting, put the sharp point of the compass on endpoint *B*. Make a second large arc. It is important that the arcs intersect each other in two places.
4. Use your straightedge to connect the points of intersection of the arcs.
5. Label the midpoint of the segment *C*.

Do not erase any of your markings.

$\overline{AC}$ is congruent to $\overline{BC}$.

Bisecting a Segment Using Patty Paper and a Straightedge

1. Use a straightedge to construct $\overline{AB}$ on patty paper.
2. Fold the patty paper so point *A* meets point *B*. Be sure to crease the paper.
3. Unfold the patty paper.
4. Use your straightedge to mark the midpoint of $\overline{AB}$.
5. Label the midpoint of the segment *C*.

$\overline{AC}$ is congruent to $\overline{BC}$.

Bisecting an Angle

- An angle bisector cuts an angle in half.
- Each half of the angle has exactly the same measure.
- A line or ray can bisect an angle.
- A point on the angle bisector is equidistant, or is the same distance, from each side of the angle.

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Instruction

Bisecting an Angle Using a Compass and a Straightedge

1. To bisect $\angle A$, put the sharp point of the compass on the vertex of the angle.
2. Draw a large arc that passes through each side of the angle.
3. Label where the arc intersects the angle as points *B* and *C*.
4. Put the sharp point of the compass on point *B*. Open the compass wider than half the distance from *B* to *C*.
5. Make an arc between the rays of the angle.
6. Without changing the compass setting, put the sharp point of the compass on *C*.
7. Make a second arc. It is important that the arcs intersect each other.
8. Use your straightedge to create a ray connecting the point of intersection of the arcs with the vertex of the angle, *A*.
9. Label a point, *D*, on the new ray.

Do not erase any of your markings.

$\angle CAD$ is congruent to $\angle BAD$.

Bisecting an Angle Using Patty Paper and a Straightedge

1. Use a straightedge to construct $\angle A$ on patty paper.
2. Fold the patty paper so the sides of $\angle A$ line up. Be sure to crease the paper.
3. Unfold the patty paper.
4. Use your straightedge to mark the crease line with a ray.
5. Label a point, *D*, on the ray.

$\angle CAD$ is congruent to $\angle BAD$.

Common Errors/Misconceptions

- inappropriately changing the compass setting
- moving the patty paper before completing the construction
- not creating large enough arcs to find the point of intersection
- attempting to measure lengths and angles with rulers and protractors

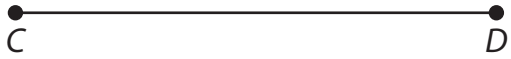
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Instruction

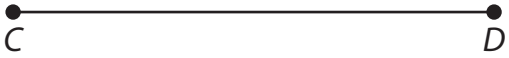
Guided Practice 5.3.2

Example 1

Use a compass and straightedge to find the midpoint of $\overline{CD}$. Label the midpoint of the segment *M*.

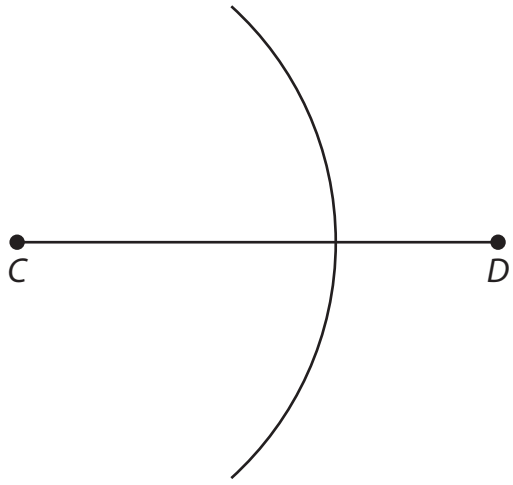


1. Copy the segment and label it $\overline{CD}$.



2. Make a large arc intersecting $\overline{CD}$.

   Put the sharp point of your compass on endpoint *C*. Open the compass wider than half the distance of $\overline{CD}$. Draw the arc.



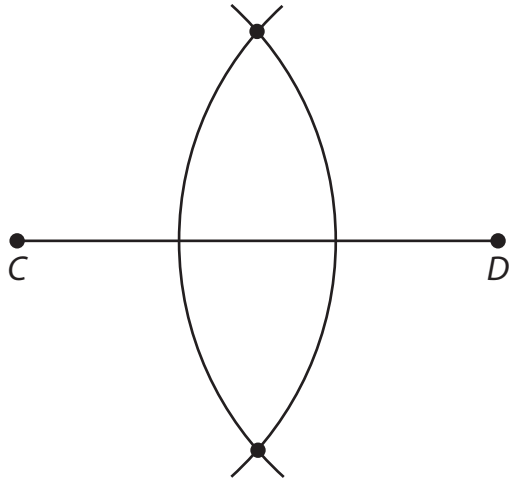
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Instruction

3. Make a second large arc.

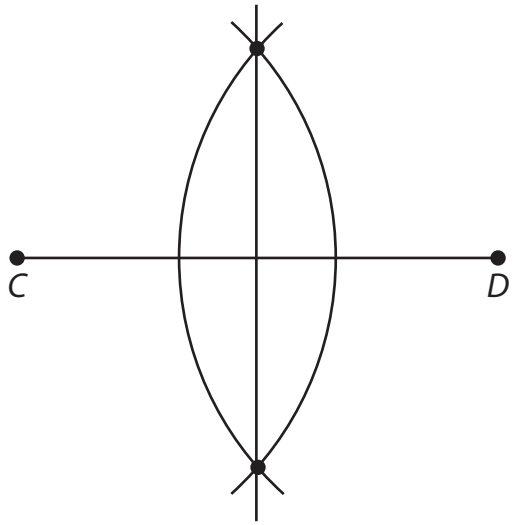
   Without changing your compass setting, put the sharp point of the compass on endpoint *D*, then make the second arc.

   It is important that the arcs intersect each other in two places.



4. Connect the points of intersection of the arcs.

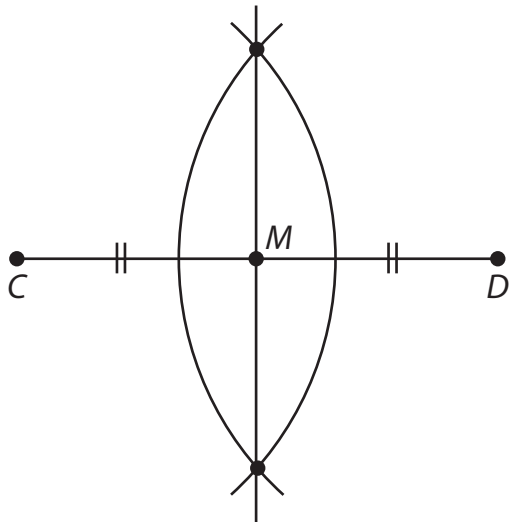
   Use your straightedge to connect the points of intersection.



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Instruction

5. Label the midpoint of the segment M.

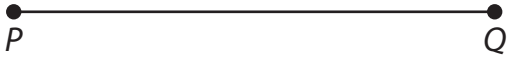


Do not erase any of your markings.

$\overline{CM}$ is congruent to $\overline{MD}$.

Example 2

Construct a segment whose measure is $\frac{1}{4}$ the length of $\overline{PQ}$.



1. Copy the segment and label it $\overline{PQ}$.

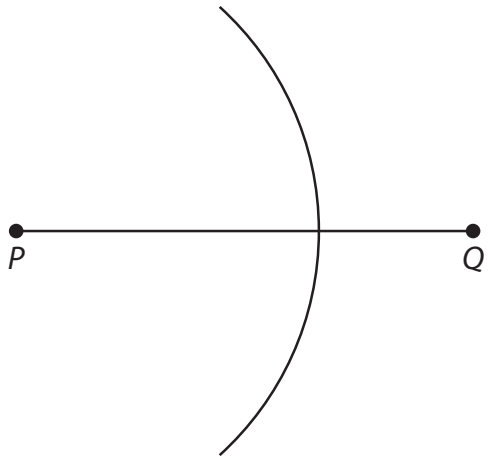


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Instruction

2. Make a large arc intersecting $\overline{PQ}$.

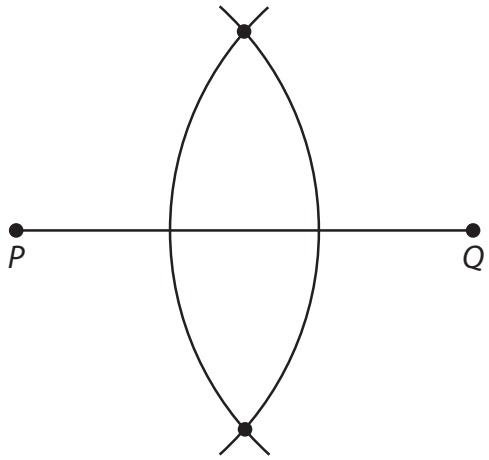
   Put the sharp point of your compass on endpoint *P*. Open the compass wider than half the distance of $\overline{PQ}$. Draw the arc.



3. Make a second large arc.

   Without changing your compass setting, put the sharp point of the compass on endpoint *Q*, then make the second arc.

   It is important that the arcs intersect each other in two places.

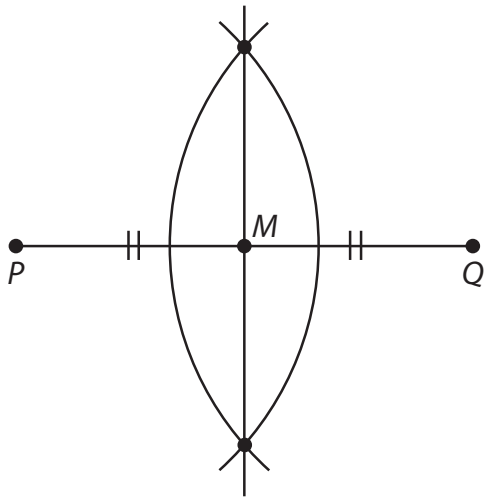


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Instruction

4. Connect the points of intersection of the arcs.

   Use your straightedge to connect the points of intersection. Label the midpoint of the segment M.

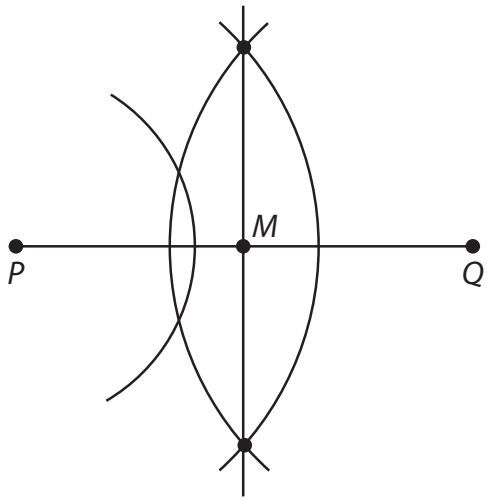


   $\overline{PM}$ is congruent to $\overline{MQ}$.

   $\overline{PM}$ and $\overline{MQ}$ are both $\frac{1}{2}$ the length of $\overline{PQ}$.

5. Find the midpoint of $\overline{PM}$.

   Put the sharp point of your compass on endpoint P and make a large arc intersecting $\overline{PM}$. Open the compass wider than half the distance of $\overline{PM}$. Draw the arc.



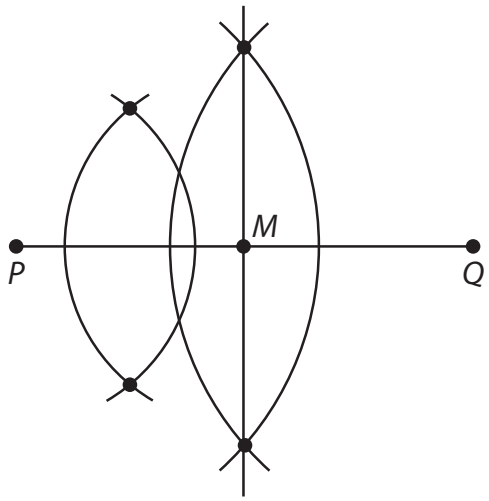
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Instruction

6. Make a second large arc.

   Without changing your compass setting, put the sharp point of the compass on endpoint *M*, and then draw the second arc.

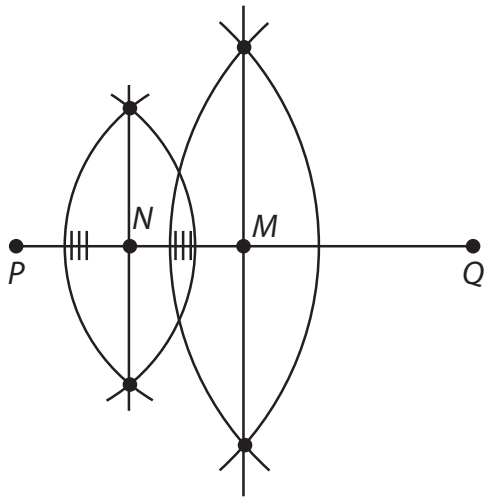
   It is important that the arcs intersect each other in two places.



7. Connect the points of intersection of the arcs.

   Use your straightedge to connect the points of intersection.

   Label the midpoint of $\overline{PM}$ *N*.



   Do not erase any of your markings.

   $\overline{PN}$ is congruent to $\overline{NM}$.

   $\overline{PN}$ is $\frac{1}{4}$ the length of $\overline{PQ}$.

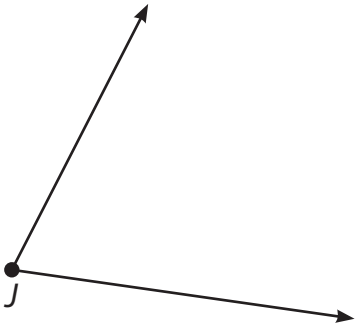
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Instruction

Example 3

Use a compass and a straightedge to bisect an angle.

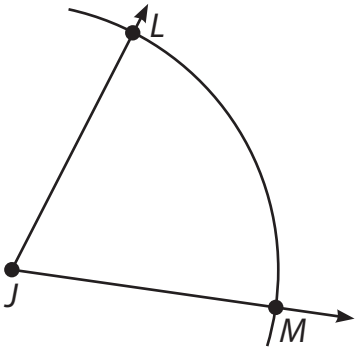
1. Draw an angle and label the vertex *J*.



2. Make a large arc intersecting the sides of $\angle J$.

   Put the sharp point of the compass on the vertex of the angle and swing the compass so that it passes through each side of the angle.

   Label where the arc intersects the angle as points *L* and *M*.



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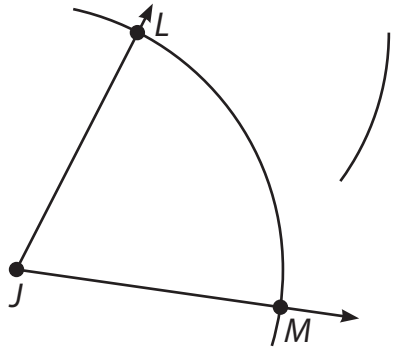
Instruction

3. Find a point that is equidistant from both sides of $\angle J$.

   Put the sharp point of the compass on point *L*.

   Open the compass wider than half the distance from *L* to *M*.

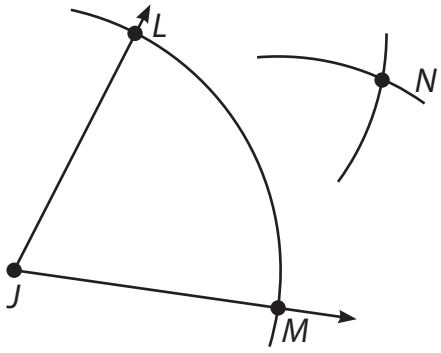
   Make an arc beyond the arc you made for points *L* and *M*.



   Without changing the compass setting, put the sharp point of the compass on point *M*.

   Make a second arc that crosses the arc you just made. It is important that the arcs intersect each other.

   Label the point of intersection *N*.

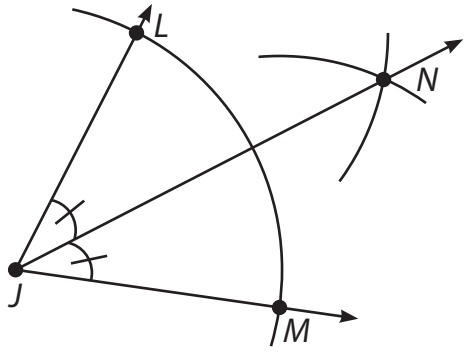


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Instruction

4. Draw the angle bisector.

   Use your straightedge to create a ray connecting point *N* with the vertex of the angle, *J*.

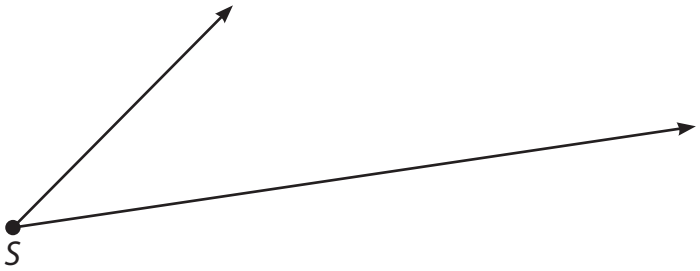


   Do not erase any of your markings.

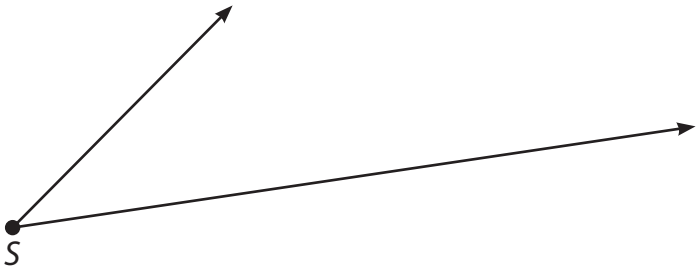
   $\angle LJN$ is congruent to $\angle NJM$.

Example 4

Construct an angle whose measure is $\frac{3}{4}$ the measure of $\angle S$.



1. Copy the angle and label the vertex *S*.



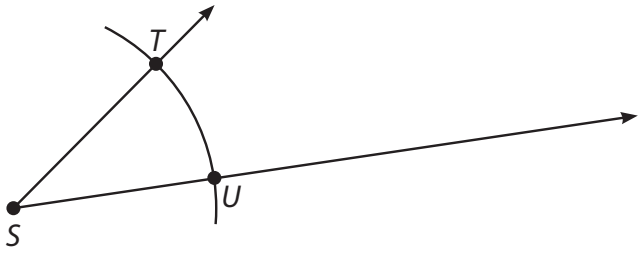
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Instruction

2. Make a large arc intersecting the sides of $\angle S$.

   Put the sharp point of the compass on the vertex of the angle and swing the compass so that it passes through each side of the angle.

   Label where the arc intersects the angle as points T and U.

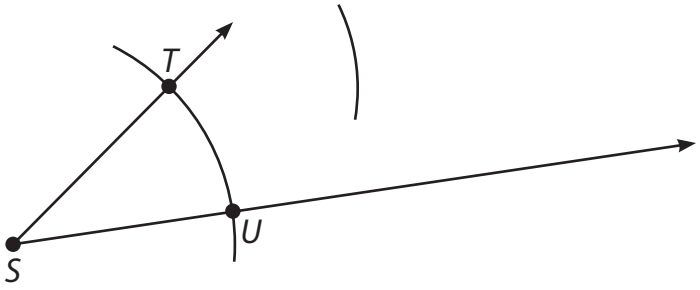


3. Find a point that is equidistant from both sides of $\angle S$.

   Put the sharp point of the compass on point T.

   Open the compass wider than half the distance from T to U.

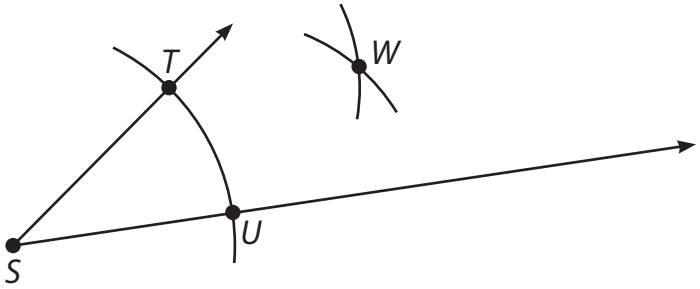
   Make an arc beyond the arc you made for points T and U.



   Without changing the compass setting, put the sharp point of the compass on point U.

   Make a second arc that crosses the arc you just made. It is important that the arcs intersect each other.

   Label the point of intersection W.

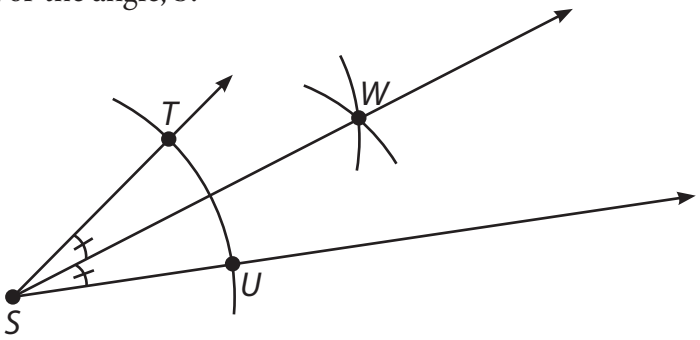


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Instruction

4. Draw the angle bisector.

   Use your straightedge to create a ray connecting point W with the vertex of the angle, S.



   $\angle TSW$ is congruent to $\angle WSU$.

   The measure of $\angle TSW$ is $\frac{1}{2}$ the measure of $\angle S$.

   The measure of $\angle WSU$ is $\frac{1}{2}$ the measure of $\angle S$.

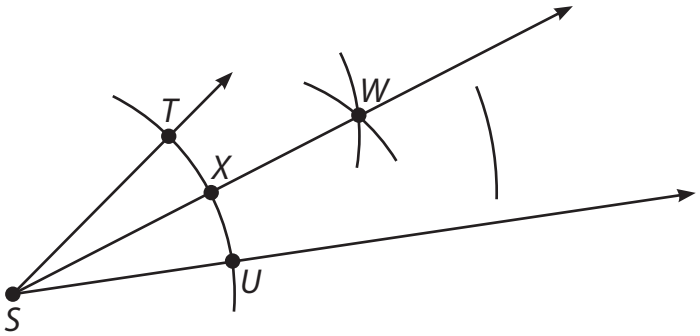
5. Find a point that is equidistant from both sides of $\angle WSU$.

   Label the intersection of the angle bisector and the initial arc as X.

   Put the sharp point of the compass on point X.

   Open the compass wider than half the distance from X to U.

   Make an arc.



(*continued*)

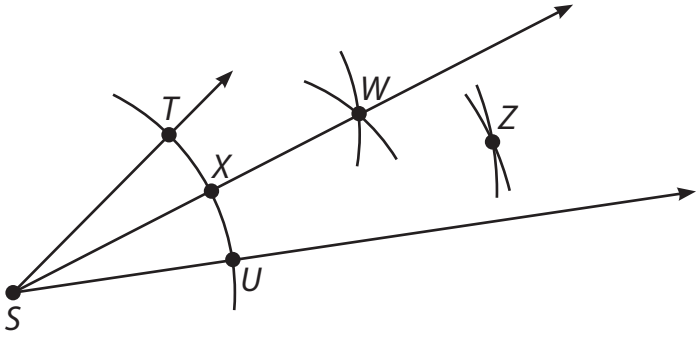
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Instruction

Without changing the compass setting, put the sharp point of the compass on point *U*.

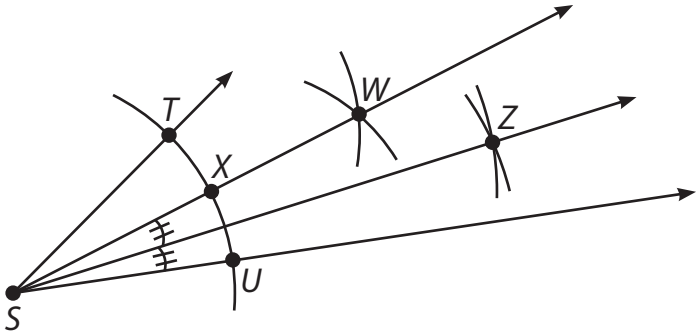
Make a second arc. It is important that the arcs intersect each other.

Label the point of intersection *Z*.



6. Draw the angle bisector.

   Use your straightedge to create a ray connecting point *Z* with the vertex of the original angle, *S*.



Do not erase any of your markings.

$\angle XSZ$ is congruent to $\angle ZSU$.

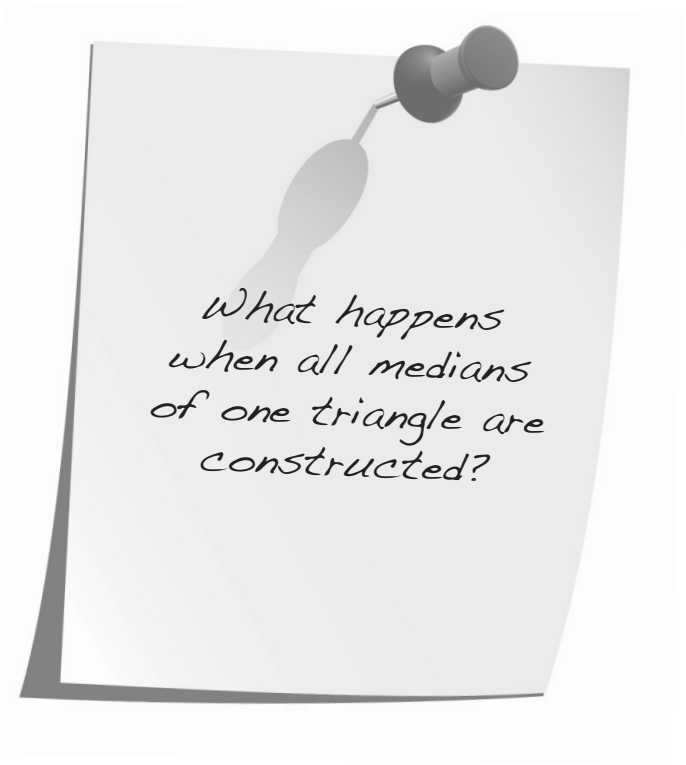
$\angle TSZ$ is $\frac{3}{4}$ the measure of $\angle TSU$.

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Problem-Based Task 5.3.2: Triangle Medians

SMP	
1	2
3	4
5 ✓	6 ✓
7	8 ✓

The median of a triangle is a line segment joining the vertex of a triangle to the midpoint of the opposite side. What happens when all medians of one triangle are constructed?



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Problem-Based Task 5.3.2: Triangle Medians

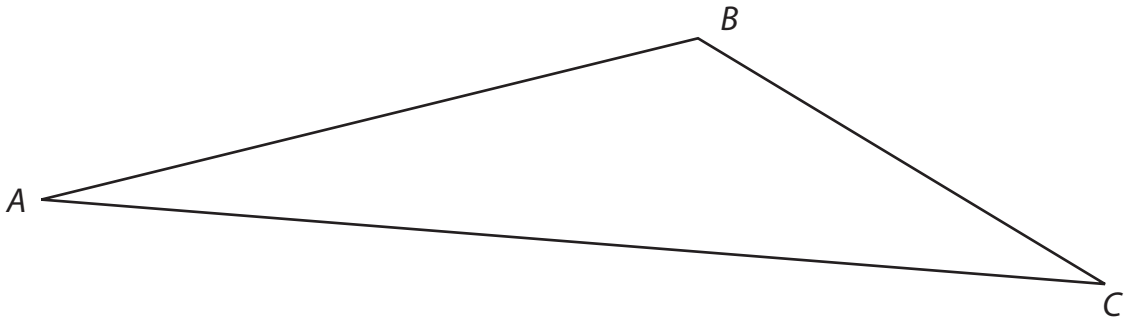
Coaching

a. What is a midpoint?

b. How many midpoints does a triangle have?

c. How many medians does a triangle have?

Use the triangle below for the following constructions.



d. What is the process for constructing the midpoint of $\overline{AB}$?

e. Construct the midpoint of $\overline{AB}$.

f. Construct the midpoint of $\overline{BC}$.

g. Construct the midpoint of $\overline{AC}$.

h. Construct each median.

i. What happens when all medians of the triangle are constructed?

j. Is this true for all triangles?

k. How can you prove your answer to be correct?

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Problem-Based Task 5.3.2: Triangle Medians

Coaching Sample Responses

a. What is a midpoint?

A midpoint is a point on a line segment that divides the segment into two equal parts.

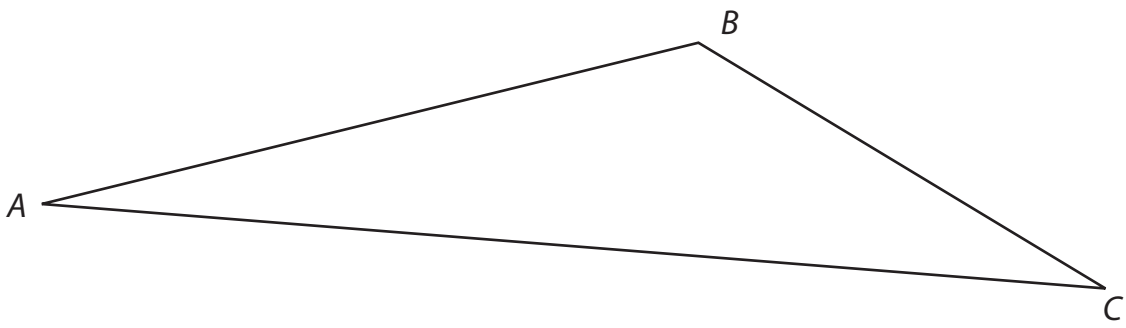
b. How many midpoints does a triangle have?

A triangle has 3 midpoints—one on each side.

c. How many medians does a triangle have?

A triangle has 3 medians—one connecting each vertex to the midpoint of the opposite side.

Use the triangle below for the following constructions.



d. What is the process for constructing the midpoint of $\overline{AB}$?

The construction of the midpoint of $\overline{AB}$ is the same as constructing the segment bisector of $\overline{AB}$.

Open the compass wider than half the distance of $\overline{AB}$, and put the sharp point of your compass on endpoint *A*. Make a large arc intersecting $\overline{AB}$.

Without changing your compass setting, put the sharp point of the compass on endpoint *B* and make a second large arc.

Use a straightedge to connect the points of intersection of the arcs.

Label the midpoint of the segment.

e. Construct the midpoint of $\overline{AB}$.

Check students' papers for accuracy.

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f. Construct the midpoint of $\overline{BC}$.

Check students' papers for accuracy.

g. Construct the midpoint of $\overline{AC}$.

Check students' papers for accuracy.

h. Construct each median.

Check students' papers for accuracy.

i. What happens when all medians of the triangle are constructed?

If constructed correctly, all three medians will intersect at the same point.

j. Is this true for all triangles?

Yes, the three medians of every triangle will always intersect at one point.

k. How can you prove your answer to be correct?

Draw another triangle and construct each median.

Compare results with others.

Recommended Closure Activity

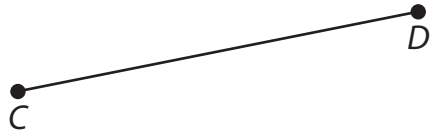
Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 5.3.2: Bisecting Segments and Angles

Use a compass and straightedge to copy each segment, and then construct the bisector of each segment.

1. 

2. 

3. 

Use a compass and straightedge to construct each segment as specified.

4. Construct a segment whose measure is $\frac{1}{4}$ the length of $\overline{AB}$ in problem 1.

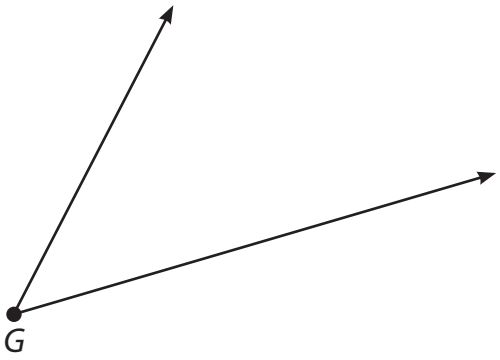
5. Construct a segment whose measure is $\frac{3}{4}$ the length of $\overline{EF}$ in problem 3.

continued

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Use a compass and straightedge to copy each angle, and then construct the bisector of each angle.

6.



7.



8.



Use a compass and straightedge to construct each angle as specified.

9. Construct an angle whose measure is $\frac{1}{4}$ the measure of $\angle G$ in problem 6.

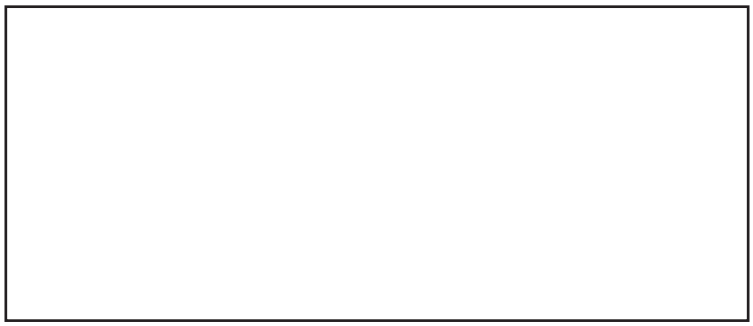
10. Construct an angle whose measure is $\frac{3}{4}$ the measure of $\angle H$ in problem 7.

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Lesson 5.3.3: Constructing Perpendicular and Parallel Lines

Warm-Up 5.3.3

Mikhail would like to create a soccer field in the rectangular field behind his house.



1. Mikhail first needs to create the midline of the soccer field. The midline is the line that is created when the midpoints of the longer sides of the field are connected. Construct the midline of the soccer field.

2. Mikhail must also create the center circle of the field. The center circle is located on the midline of the field and is equidistant from both the longer sides. Construct the center circle of the soccer field.

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Instruction

Lesson 5.3.3: Constructing Perpendicular and Parallel Lines

Common Core State Standard

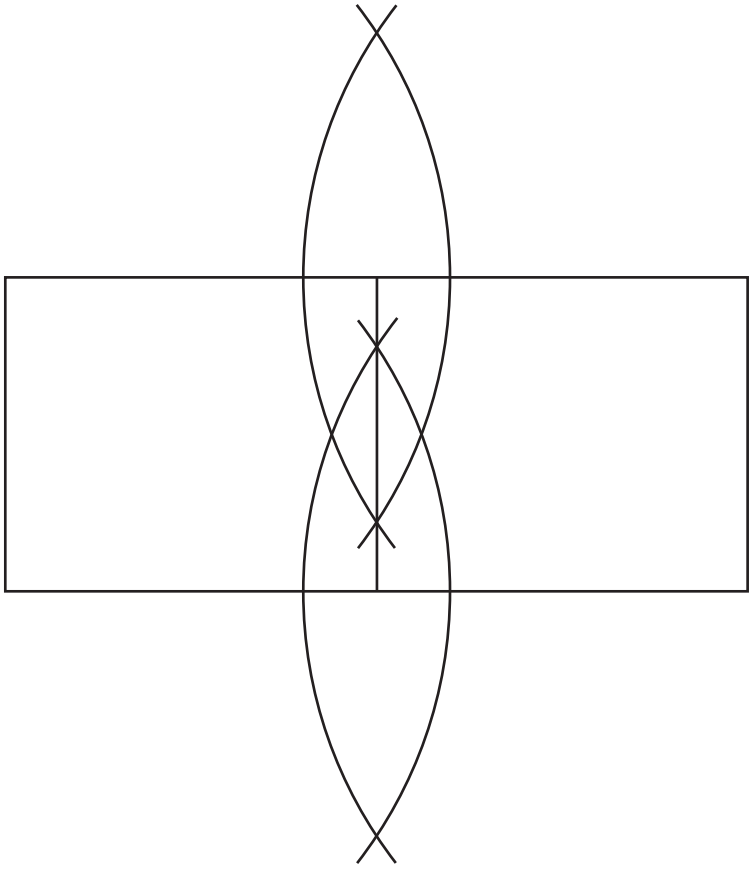
G–CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Warm-Up 5.3.3 Debrief

1. Mikhail first needs to create the midline of the soccer field. The midline is the line that is created when the midpoints of the longer sides of the field are connected. Construct the midline of the soccer field.

   Find the midpoint of each of the longer sides of the field by constructing the segment bisectors.

   Connect the segment bisectors.



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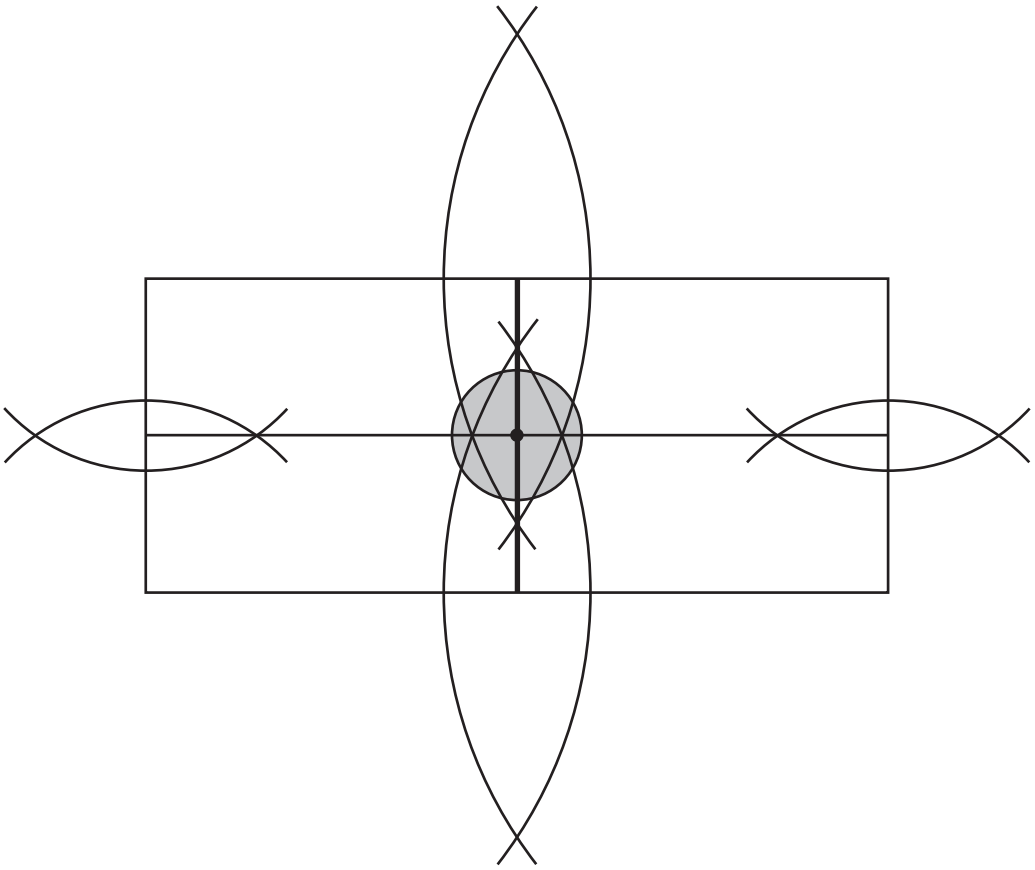
Instruction

2. Mikhail must also create the center circle of the field. The center circle is located on the midline of the field and is equidistant from both the longer sides. Construct the center circle of the soccer field.

   Find the midpoint of each of the shorter sides of the field by constructing the segment bisectors.

   Connect the segment bisectors.

   Construct a circle with the center at the intersection of the midline and the centerline.



Connection to the Lesson

- Students continue to practice constructing segment bisectors.
- In this lesson, students will use similar construction skills to construct perpendicular bisectors and parallel lines.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- using a compass (no standard)
- copying angles and segments (G–CO.12)
- bisecting line segments (G–CO.12)
- understanding the geometry terms *line*, *segment*, *ray*, and *angle* (G–CO.1)

Introduction

Geometry construction tools can be used to create perpendicular and parallel lines. While performing each construction, it is important to remember that the only tools you are allowed to use are a compass and a straightedge, a reflective device and a straightedge, or patty paper and a straightedge. You may be tempted to measure angles or lengths, but in constructions this is not allowed. You can adjust the opening of your compass to verify that lengths are equal.

Key Concepts

Perpendicular Lines and Bisectors

- **Perpendicular lines** are two lines that intersect at a right angle (90˚).
- A perpendicular line can be constructed through the midpoint of a segment. This line is called the **perpendicular bisector** of the line segment.
- It is impossible to create a perpendicular bisector of a line, since a line goes on infinitely in both directions, but similar methods can be used to construct a line perpendicular to a given line.
- It is possible to construct a perpendicular line through a point on the given line as well as through a point not on a given line.

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Instruction

Constructing a Perpendicular Bisector of a Line Segment Using a Compass and a Straightedge

1. To construct a perpendicular bisector of $\overline{AB}$, put the sharp point of your compass on endpoint A. Open the compass wider than half the distance of $\overline{AB}$.
2. Make a large arc intersecting $\overline{AB}$.
3. Without changing your compass setting, put the sharp point of the compass on endpoint B. Make a second large arc. It is important that the arcs intersect each other.
4. Use your straightedge to connect the points of intersection of the arcs.
5. Label the new line m.

Do not erase any of your markings.

Line m is perpendicular to $\overline{AB}$ and it bisects $\overline{AB}$.

Constructing a Perpendicular Bisector of a Line Segment Using Patty Paper and a Straightedge

1. Use a straightedge to construct $\overline{AB}$ on patty paper.
2. Fold the patty paper so point A meets point B. Be sure to crease the paper.
3. Unfold the patty paper.
4. Use your straightedge to mark the creased line.
5. Label the new line m.

Line m is perpendicular to $\overline{AB}$ and it bisects $\overline{AB}$.

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Instruction

Constructing a Perpendicular Line Through a Point on the Given Line Using a Compass and a Straightedge

1. To construct a perpendicular line through the point, *A,* on a line, put the sharp point of your compass on point *A*. The opening of the compass does not matter, but choose a setting that isn't so large or so small that it's difficult to make markings.
2. Make an arc on either side of point *A* on the line. Label the points of intersection *C* and *D*.
3. Place the sharp point of the compass on point *C*. Open the compass so it extends beyond point *A*.
4. Create an arc on either side of the line.
5. Without changing your compass setting, put the sharp point of the compass on endpoint *D*. Make a large arc on either side of the line. It is important that the arcs from step 4 and the new arcs intersect each other.
6. Use your straightedge to connect the points of intersection of the arcs.
7. Label the new line m.

Do not erase any of your markings.

$\overline{CD}$ is perpendicular to line m through point *A*.

Constructing a Perpendicular Line Through a Point on the Given Line Using Patty Paper and a Straightedge

1. Use a straightedge to construct a line, ℓ, on the patty paper. Label a point on the line *A*.
2. Fold the patty paper so the line folds onto itself through point *A*. Be sure to crease the paper.
3. Unfold the patty paper.
4. Use your straightedge to mark the creased line.
5. Label the new line m.

Line ℓ is perpendicular to line m through point *A*.

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Instruction

Constructing a Perpendicular Line Through a Point Not on the Given Line Using a Compass and a Straightedge

1. To construct a perpendicular line through the point, G, not on the given line ℓ, put the sharp point of your compass on point G. Open the compass until it extends farther than the given line.
2. Make a large arc that intersects the given line in two places. Label the points of intersection C and D.
3. Without changing your compass setting, put the sharp point of the compass on point C. Make a second arc below the given line.
4. Without changing your compass setting, put the sharp point of the compass on point D. Make a third arc below the given line. The third arc must intersect the second arc.
5. Label the point of intersection E.
6. Use your straightedge to connect points G and E. Label the new line m.

Do not erase any of your markings.

Line m is perpendicular to line ℓ through point G.

Constructing a Perpendicular Line Through a Point Not on the Given Line Using Patty Paper and a Straightedge

1. Use a straightedge to construct a line, ℓ, on the patty paper. Label a point not on the line, G.
2. Fold the patty paper so the line folds onto itself through point G. Be sure to crease the paper.
3. Unfold the patty paper.
4. Use your straightedge to mark the creased line.
5. Label the new line m.

Line m is perpendicular to line ℓ through point G.

Parallel Lines

- **Parallel lines** are lines in a plane that do not share any points and never intersect.
- Any two points on one parallel line are equidistant from the other line.
- There are many ways to construct parallel lines.
- One method is to construct two lines that are both perpendicular to the same given line.

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Instruction

Constructing a Parallel Line Using a Compass and a Straightedge

1. To construct a line parallel to line ℓ through a point, A, not on the given line ℓ, first construct a line perpendicular to ℓ.
2. Put the sharp point of your compass on point A. Open the compass until it extends farther than line ℓ.
3. Make a large arc that intersects the given line in two places. Label the points of intersection C and D.
4. Without changing your compass setting, put the sharp point of the compass on point C. Make a second arc below the given line.
5. Without changing your compass setting, put the sharp point of the compass on point D. Make a third arc below the given line. The third arc must intersect the second arc.
6. Label the point of intersection E.
7. Use your straightedge to connect points A and E. Label the new line m. Line m is perpendicular to line ℓ.
8. Construct a second line perpendicular to line m.
9. Put the sharp point of your compass on point A.
10. Make a large arc that intersects line m in two places. Label the points of intersection F and G.
11. Without changing your compass setting, put the sharp point of the compass on point F. Make a second arc to the right of line m.
12. Without changing your compass setting, put the sharp point of the compass on point G. Make a third arc to the right of line m. The third arc must intersect the second arc.
13. Label the point of intersection H.
14. Use your straightedge to connect points A and H. Label the new line n.

Do not erase any of your markings.

Line n is perpendicular to line m.

Line ℓ is parallel to line n.

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Instruction

Constructing a Parallel Line Using Patty Paper and a Straightedge

1. Use a straightedge to construct line ℓ on the patty paper. Label a point not on the line A.
2. Fold the patty paper so the line folds onto itself through point A. Be sure to crease the paper.
3. Unfold the patty paper.
4. Fold the new line onto itself through point A.
5. Unfold the patty paper.
6. Use your straightedge to mark the second creased line.
7. Label the new line m.

Line m is parallel to line ℓ.

Common Errors/Misconceptions

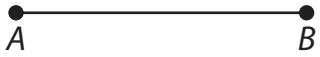
- inappropriately changing the compass setting
- moving the patty paper before completing the construction
- not creating large enough arcs to find the point of intersection
- attempting to measure lengths and angles with rulers and protractors

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Guided Practice 5.3.3

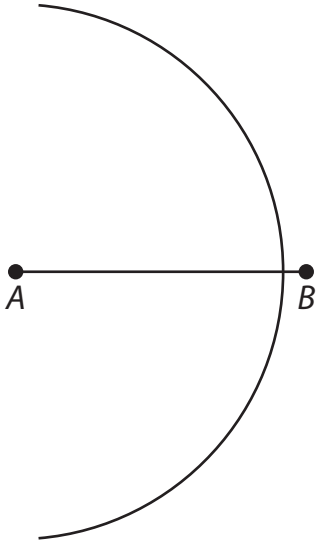
Example 1

Use a compass and a straightedge to construct the perpendicular bisector of $\overline{AB}$.



1. Make a large arc intersecting $\overline{AB}$.

   Put the sharp point of your compass on endpoint A. Open the compass wider than half the distance of $\overline{AB}$. Draw the arc.



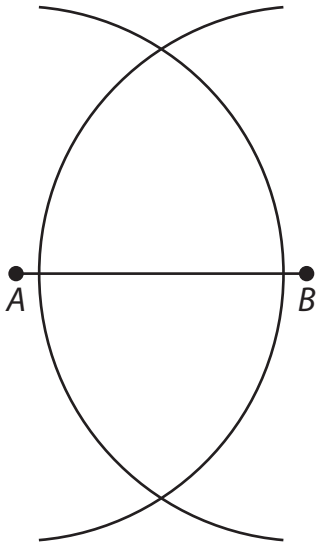
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2. Make a second large arc.

   Without changing your compass setting, put the sharp point of the compass on endpoint *B*.

   Make a second large arc.

   It is important that the arcs intersect each other in two places.



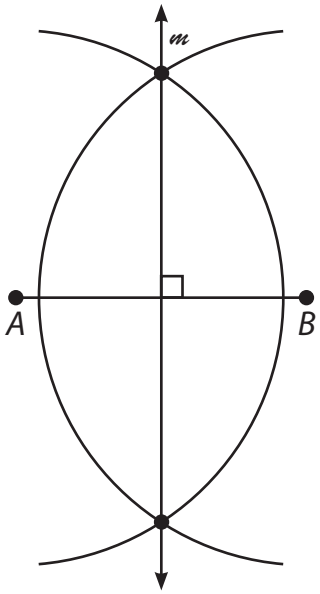
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Instruction

3. Connect the points of intersection of the arcs.

   Use your straightedge to connect the points of intersection of the arcs.

   Label the new line m.



   Do not erase any of your markings.

   Line m is the perpendicular bisector of $\overline{AB}$.

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Instruction

Example 2

Use a compass and a straightedge to construct a line perpendicular to line ℓ through point A.

1. Draw line ℓ with point A on the line.

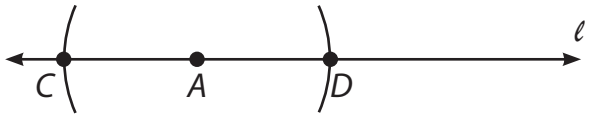


2. Make an arc on either side of point A.

   Put the sharp point of your compass on point A.

   Make arcs on either side of point A through line ℓ.

   Label the points of intersection C and D.

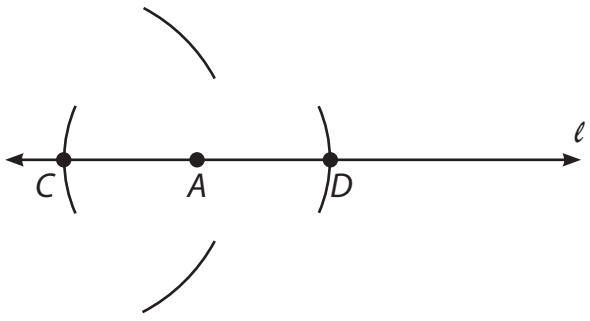


3. Make a set of arcs on either side of line ℓ.

   Place the sharp point of the compass on point C.

   Open the compass so it extends beyond point A.

   Create an arc on either side of the line.



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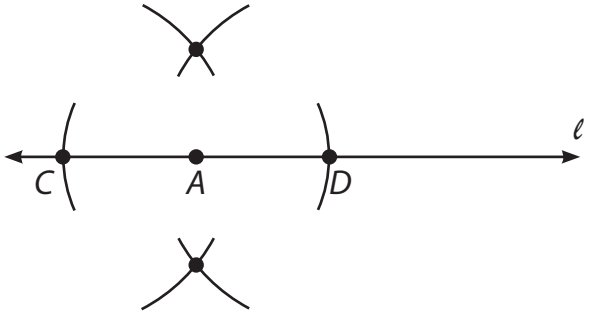
Instruction

4. Make a second set of arcs on either side of line ℓ.

   Without changing your compass setting, put the sharp point of the compass on point D.

   Make an arc on either side of the line.

   It is important that the arcs intersect each other.

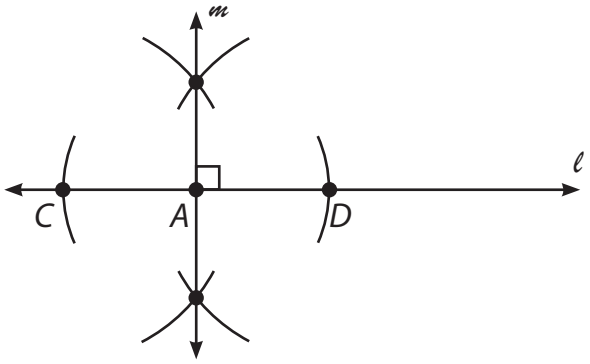


5. Connect the points of intersection.

   Use your straightedge to connect the points of intersection of the arcs.

   This line should also go through point A.

   Label the new line m.



   Do not erase any of your markings.

   Line ℓ is perpendicular to line m through point A.

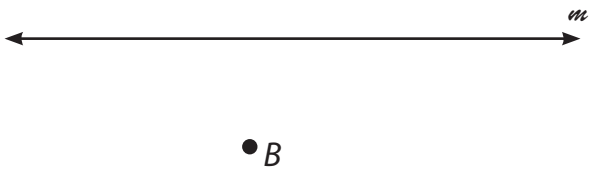
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Instruction

Example 3

Use a compass and a straightedge to construct a line perpendicular to line m through point B that is not on the line.

1. Draw line m with point B not on the line.



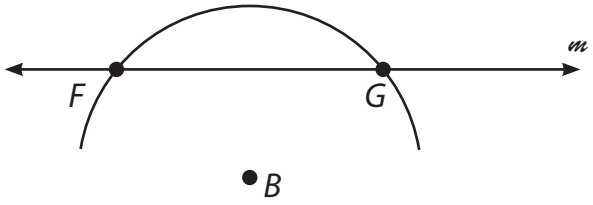
2. Make a large arc that intersects line m.

   Put the sharp point of your compass on point B.

   Open the compass until it extends farther than line m.

   Make a large arc that intersects the given line in two places.

   Label the points of intersection F and G.

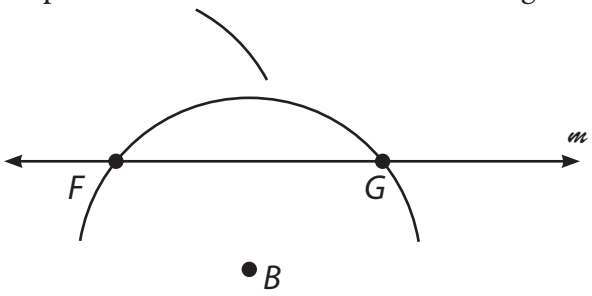


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Instruction

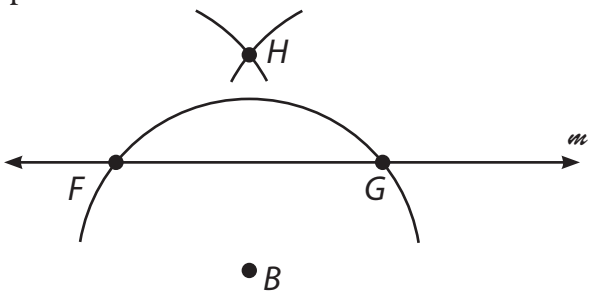
3. Make a set of arcs above line m.

   Without changing your compass setting, put the sharp point of the compass on point F. Make a second arc above the given line.



   Without changing your compass setting, put the sharp point of the compass on point G. Make a third arc above the given line. The third arc must intersect the second arc.

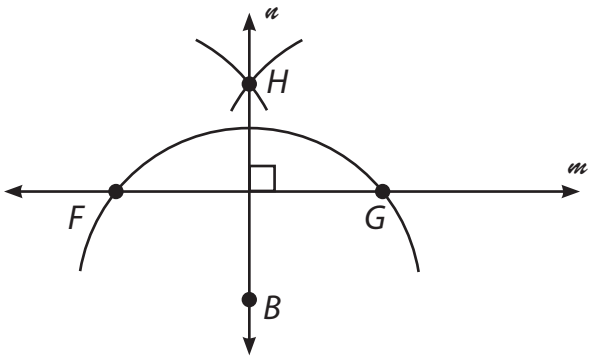
   Label the point of intersection H.



4. Draw the perpendicular line.

   Use your straightedge to connect points B and H.

   Label the new line n.



   Do not erase any of your markings.

   Line n is perpendicular to line m.

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Instruction

Example 4

Use a compass and a straightedge to construct a line parallel to line ℓ through point C that is not on the line.

1. Draw line ℓ with point C not on the line.



2. Construct a line perpendicular to line ℓ through point C.

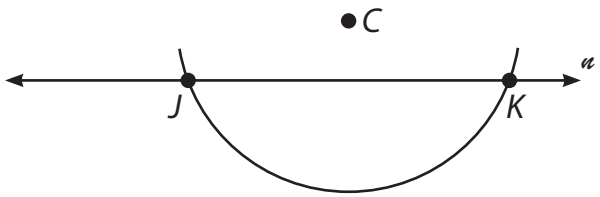
   Make a large arc that intersects line ℓ.

   Put the sharp point of your compass on point C.

   Open the compass until it extends farther than line ℓ.

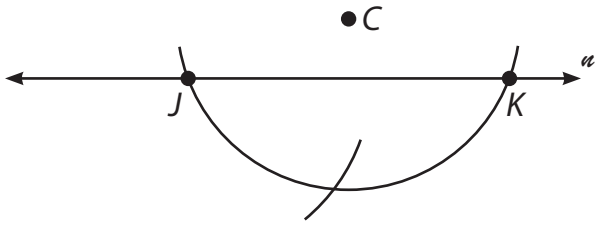
   Make a large arc that intersects the given line in two places.

   Label the points of intersection J and K.

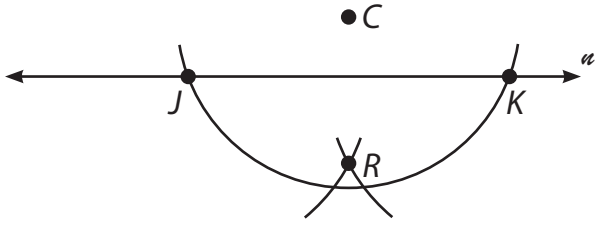


   Make a set of arcs below line ℓ.

   Without changing your compass setting, put the sharp point of the compass on point J. Make a second arc below the given line.



   Without changing your compass setting, put the sharp point of the compass on point K. Make a third arc below the given line.
   Label the point of intersection R.



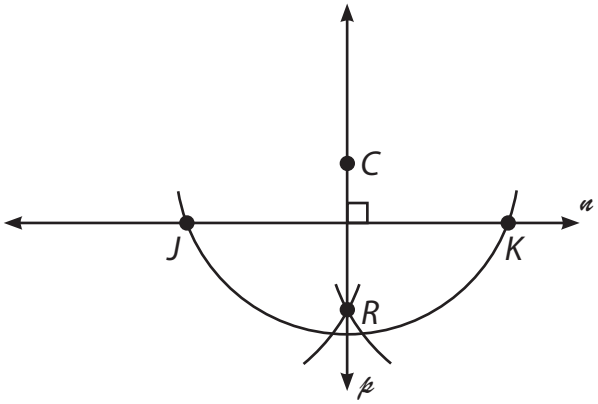
© Walch Education

Instruction

3. Draw the perpendicular line.

   Use your straightedge to connect points *C* and *R*.

   Label the new line ℓ.



   Do not erase any of your markings.

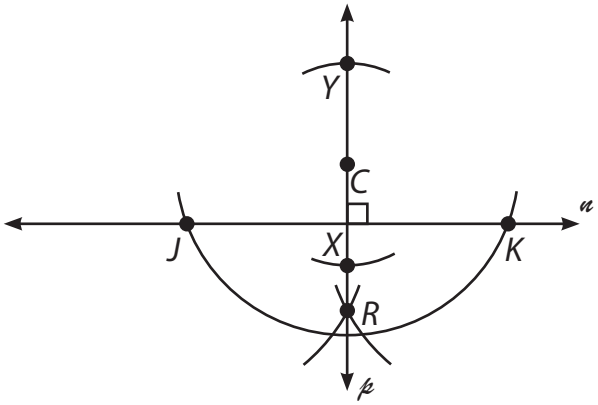
   Line n is perpendicular to line ℓ.

4. Construct a second line perpendicular to line ℓ.

   Put the sharp point of your compass on point *C*.

   Make a large arc that intersects line ℓ on either side of point *C*.

   Label the points of intersection *X* and *Y*.



(*continued*)

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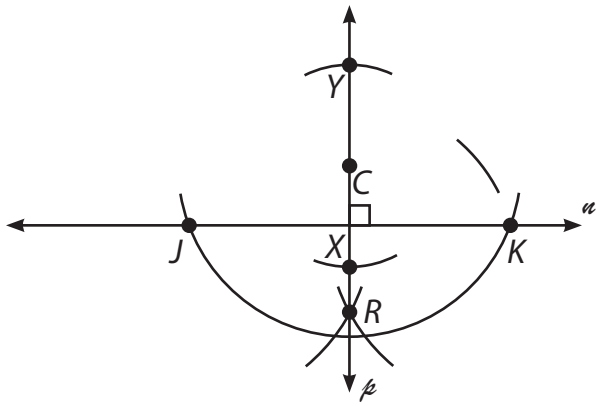
Instruction

Make a set of arcs to the right of line ℓ.

Put the sharp point of your compass on point *X*.

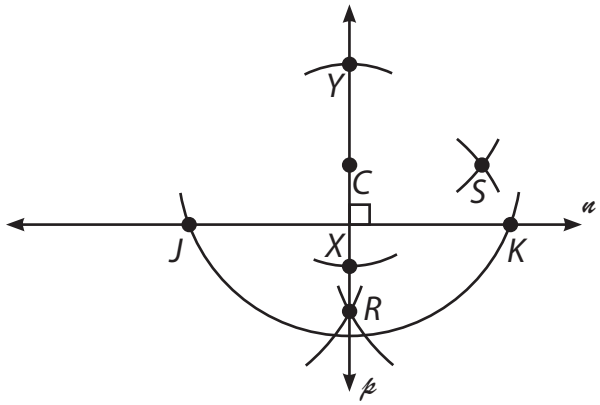
Open the compass so that it extends beyond point *C*.

Make an arc to the right of line ℓ.



Without changing your compass setting, put the sharp point of the compass on point *Y*. Make another arc to the right of line ℓ.

Label the point of intersection *S*.



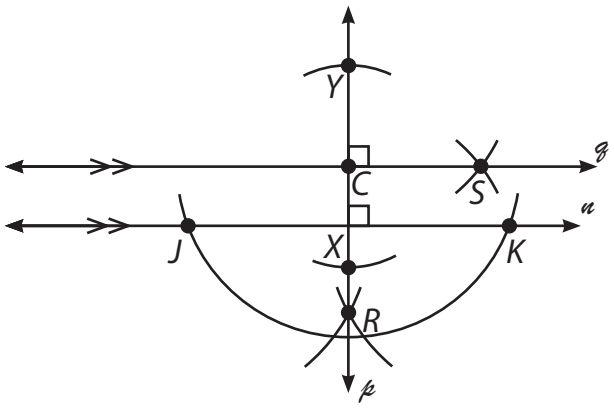
© Walch Education

Instruction

5. Draw the perpendicular line.

   Use your straightedge to connect points *C* and *S*.

   Label the new line g.



   Do not erase any of your markings.

   Line g is perpendicular to line p.

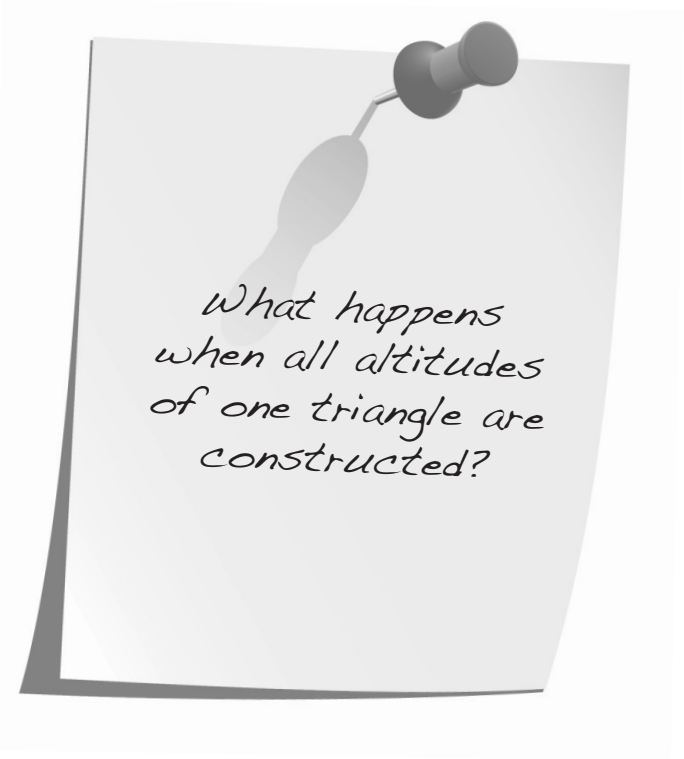
   Line g is parallel to line n.

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Problem-Based Task 5.3.3: Triangle Altitudes

SMP	
1 ✓	2
3	4
5 ✓	6 ✓
7	8

The altitude of a triangle is the perpendicular line from a vertex to its opposite side. The altitude of a triangle is also called the height. What happens when all altitudes of one triangle are constructed?



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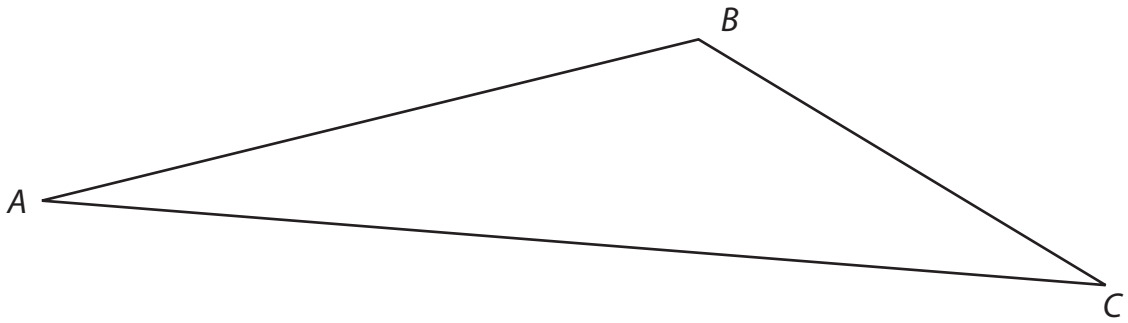
Problem-Based Task 5.3.3: Triangle Altitudes

Coaching

a. What is a perpendicular line?

b. How many altitudes does a triangle have?

Use the triangle for the following constructions.



c. What is the process for constructing the altitude from point B to $\overline{AC}$?

d. Construct the altitude from point B to $\overline{AC}$.

e. Construct the altitude from point C to $\overline{AB}$.

f. Construct the altitude from point A to $\overline{BC}$.

g. What happens when all altitudes of the triangle are constructed?

h. Is this true for all triangles?

i. How can you prove your answer to be correct?

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Instruction

Problem-Based Task 5.3.3: Triangle Altitudes

Coaching Sample Responses

a. What is a perpendicular line?

A perpendicular line determines the shortest distance from a point to a line.

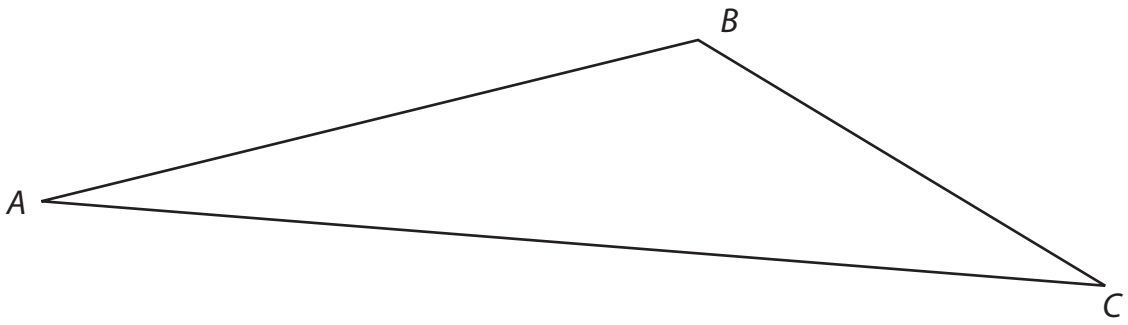
Perpendicular lines create a 90° angle.

b. How many altitudes does a triangle have?

A triangle has 3 altitudes.

Each altitude extends from each vertex to its opposite side.

Use the triangle for the following constructions.



c. What is the process for constructing the altitude from point *B* to $\overline{AC}$?

The construction of the altitude is the same as constructing a perpendicular line segment from point *B* to $\overline{AC}$.

Put the sharp point of the compass on point *B*.

Open the compass until it extends to the other side of $\overline{AC}$.

Make a large arc that intersects $\overline{AC}$ in two places. If the arc does not intersect $\overline{AC}$, extend $\overline{AC}$.

Without changing your compass setting, put the sharp point of the compass on one point of intersection.

Make a second arc below segment $\overline{AC}$.

Without changing your compass setting, put the sharp point of the compass on the second point of intersection.

Make a third arc below segment $\overline{AC}$.

Use a straightedge to connect the intersection of the arcs below $\overline{AC}$ and point *B*.

Do not erase any of your markings.

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d. Construct the altitude from point B to $\overline{AC}$.

Check students' papers for accuracy.

e. Construct the altitude from point C to $\overline{AB}$.

Check students' papers for accuracy. Note that students must extend segment $\overline{AB}$ in order to construct this altitude.

f. Construct the altitude from point A to $\overline{BC}$.

Check students' papers for accuracy. Note that students must extend segment $\overline{BC}$ in order to construct this altitude.

g. What happens when all altitudes of the triangle are constructed?

If constructed correctly, all three altitudes will intersect at the same point.

h. Is this true for all triangles?

The three altitudes of every triangle will always intersect at one point.

Sometimes the altitudes will intersect outside of the triangle.

i. How can you prove your answer to be correct?

Draw another triangle and construct each altitude.

Compare results with others.

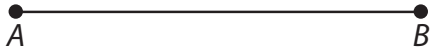
Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 5.3.3: Constructing Perpendicular and Parallel Lines

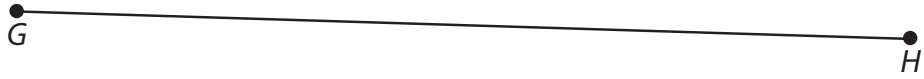
Use a compass and a straightedge to copy each segment, and then construct the perpendicular bisector of each.

1. 

2. 

Use a compass and a straightedge to copy each segment, place a point on the segment, and then construct a perpendicular line through the point.

3. 

4. 

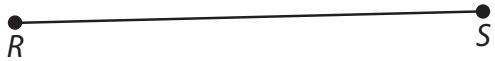
Use a compass and a straightedge to copy each segment, place a point not on the segment, and then construct a perpendicular line through the point.

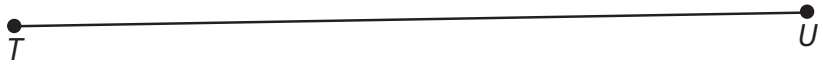
5. 

6. 

Use a compass and a straightedge to copy each segment, place a point not on the segment, and then construct a parallel line through the point.

7. 

8. 

9. 

10. 

SWB p. 85; SRB Practice 5.3.3 p. U5-101

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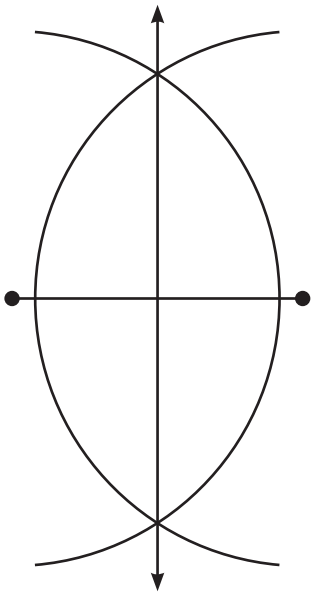
Progress Assessment

Circle the letter of the best answer.

1. What tools are used when making geometric constructions?

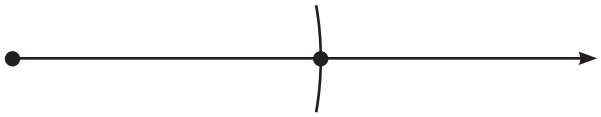
   a. a compass and a ruler
   b. a protractor and a ruler
   c. a compass and a straightedge
   d. a protractor and a straightedge

2. Which geometric construction matches the diagram below?



   a. copying a segment
   b. construction of a segment bisector
   c. copying an angle
   d. construction of an angle bisector

3. Which geometric construction matches the diagram below?

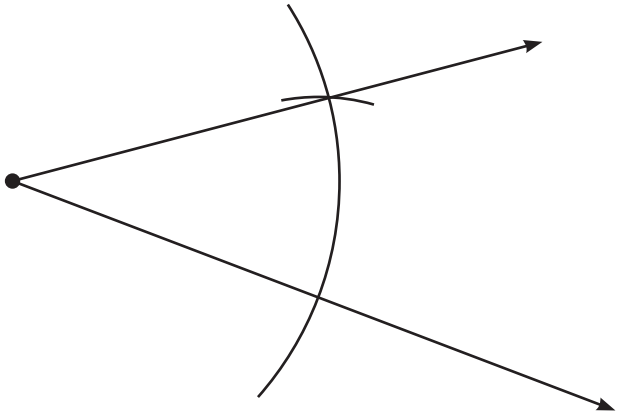


   a. copying a segment
   b. construction of a segment bisector
   c. copying an angle
   d. construction of an angle bisector

continued

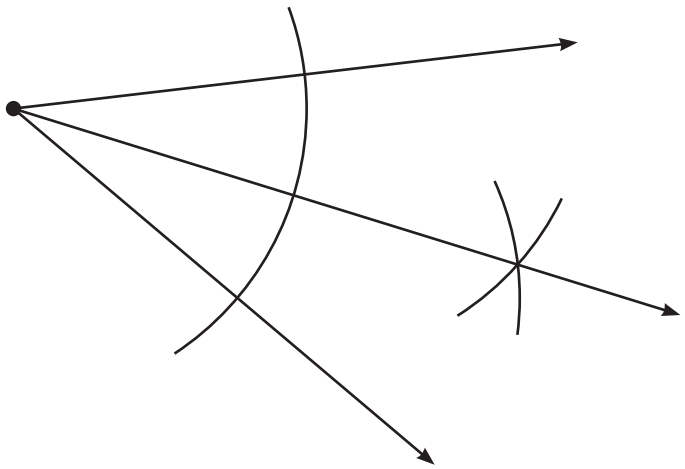
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4. Which geometric construction matches the diagram below?



a. copying a segment
b. construction of a segment bisector
c. copying an angle
d. construction of an angle bisector

5. Which geometric construction matches the diagram below?

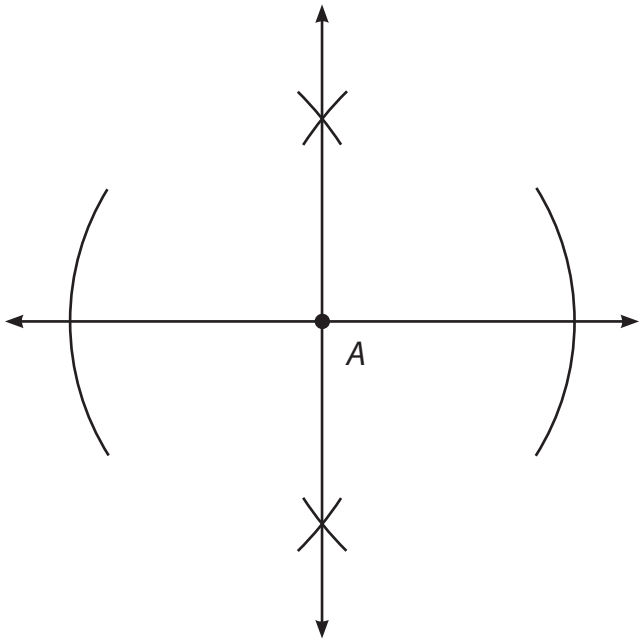


a. copying a segment
b. construction of a segment bisector
c. copying an angle
d. construction of an angle bisector

continued

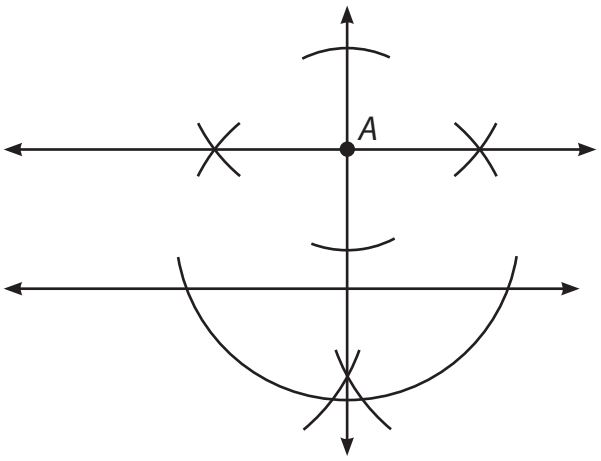
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6. Which geometric construction matches the diagram below?



a. construction of a segment bisector
b. construction of a perpendicular line through point *A*
c. construction of a parallel line through point *A*
d. construction of a median

7. Which geometric construction matches the diagram below?

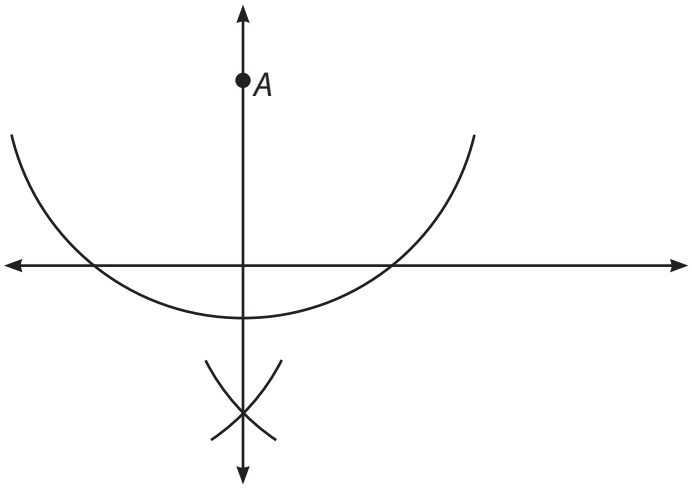


a. construction of a segment bisector
b. construction of a perpendicular line through point *A*
c. construction of a parallel line through point *A*
d. construction of a median

continued

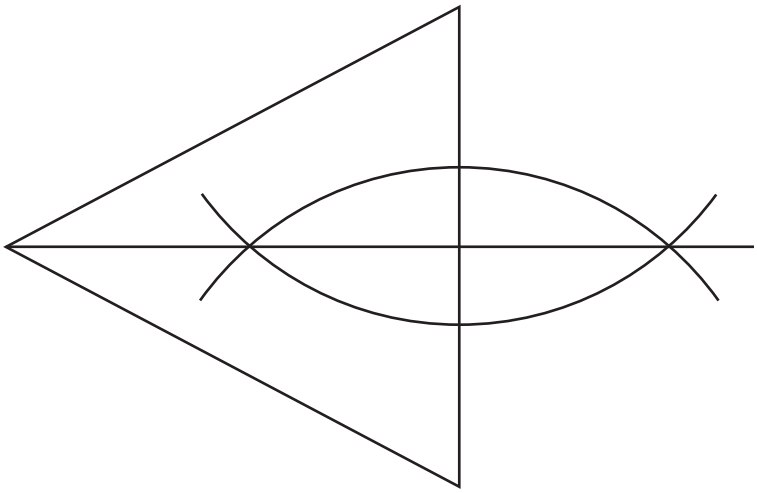
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8. Which geometric construction matches the diagram below?



a. construction of a segment bisector
b. construction of a perpendicular line through point *A*
c. construction of a parallel line through point *A*
d. construction of a median

9. Which geometric construction matches the diagram below?



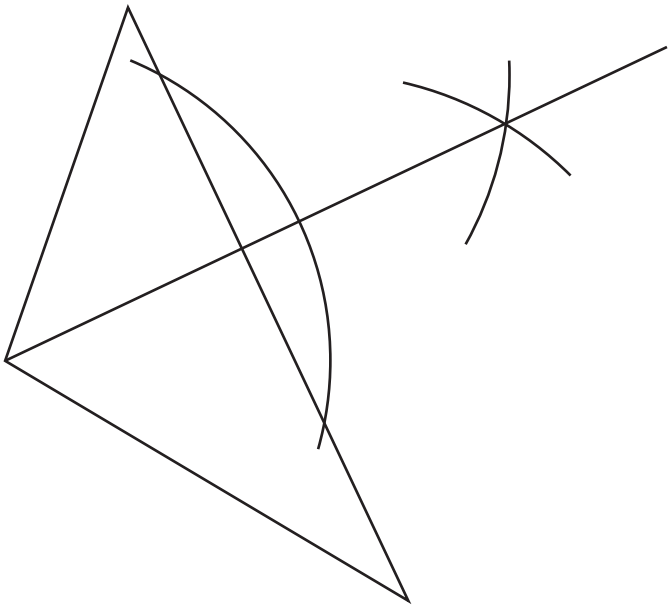
a. construction of a segment bisector
b. construction of an angle bisector
c. construction of an altitude
d. construction of a median

continued

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Assessment

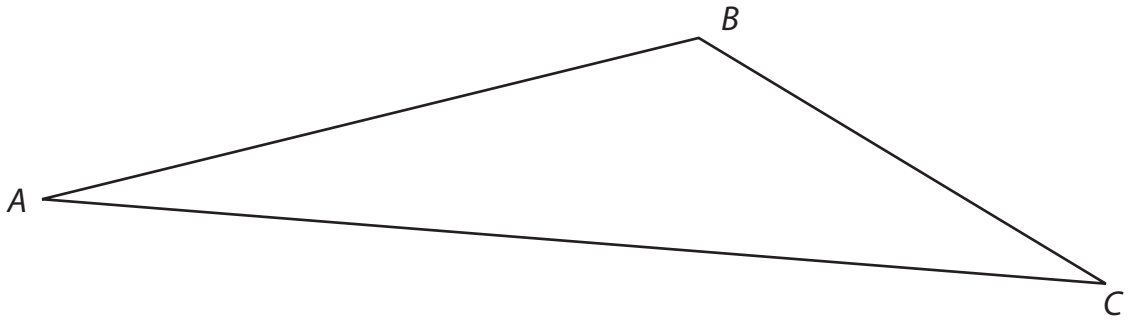
10. Which geometric construction matches the diagram below?



a. construction of a segment bisector
b. construction of an angle bisector
c. construction of an altitude
d. construction of a median

Use what you have learned about constructions to solve the following problem.

11. The midpoint of one side of a triangle is the point that divides that side into two equal parts. The midpoint is found when the side is bisected. When two midpoints of a triangle are connected, a midsegment is created. Every triangle has three midsegments. Construct all midsegments of the triangle shown.



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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. When a figure is inscribed in a circle, how many of the vertices must touch the circle?
   a. Only one of the vertices must touch the circle.
   b. More than one of the vertices must touch the circle.
   c. None of the vertices must touch the circle.
   d. All of the vertices must touch the circle.

2. Which of the following statements is always true?
   a. There is only one way to construct a regular polygon.
   b. The diameter of a given circle determines the size of the polygon inscribed in the circle.
   c. It is not possible to construct regular polygons without using a ruler and a protractor.
   d. Only regular polygons can be inscribed in a circle.

3. What is the first step when constructing an equilateral triangle inscribed in a given circle?
   a. Set the compass equal to the diameter of the circle.
   b. Draw a segment between two points on the circle.
   c. Locate a point on the circle.
   d. Bisect the radius of the circle.

4. What is the first step when constructing a square inscribed in a given circle?
   a. Set the compass equal to the radius of the circle.
   b. Construct the diameter of the circle.
   c. Bisect the diameter of the circle.
   d. Draw a segment between two points on the circle.

5. Think about the process for constructing a regular hexagon in a given circle. Which of the following figures is also inscribed in a circle using the same process?
   a. an equilateral triangle
   b. a scalene triangle
   c. a square
   d. an octagon

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Lesson 4: Constructing Polygons

Instruction

SMP
1 ✓ 2
3 ✓ 4 ✓
5 ✓ 6
7 ✓ 8 ✓

Common Core State Standard

G–CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Essential Questions

1. How can you justify that a construction was done correctly?
2. How can a polygon be constructed given a circle?
3. How are basic constructions used to construct regular polygons?

WORDS TO KNOW

circle	the set of points on a plane at a fixed distance, or radius, from a given point, the center
congruent	having the same shape, size, lines, and angles; the symbol for representing congruency between figures is ≅.
construction	a precise representation of a figure created by using a straightedge and a compass, or a straightedge and patty paper
diameter	a line segment passing through the center of a circle connecting two points on the circle; twice the radius
equilateral triangle	a triangle with all three sides equal in length
inscribe	to draw one figure within another figure so that every vertex of the enclosed figure touches the outside figure
radius	a line segment that extends from the center of a circle to a point on the circle. Its length is half the diameter.
regular hexagon	a six-sided polygon with all sides congruent and all angles measuring 120°
regular polygon	a closed two-dimensional figure with all sides congruent and all angles congruent
square	a regular polygon with four congruent sides and four right angles
triangle	a three-sided polygon with three angles

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Recommended Resources

- DePaul University. "Inscribing Regular Polygons."

  http://www.walch.com/rr/00007

  This site includes overviews and graphics demonstrating how to construct various regular polygons in a given circle.

- Math Open Reference. "Hexagon inscribed in a circle."

  http://www.walch.com/rr/00008

  This site provides animated step-by-step instructions for constructing a regular hexagon inscribed in a given circle.

- Zef Damen. "Equilateral Triangle."

  http://www.walch.com/rr/00009

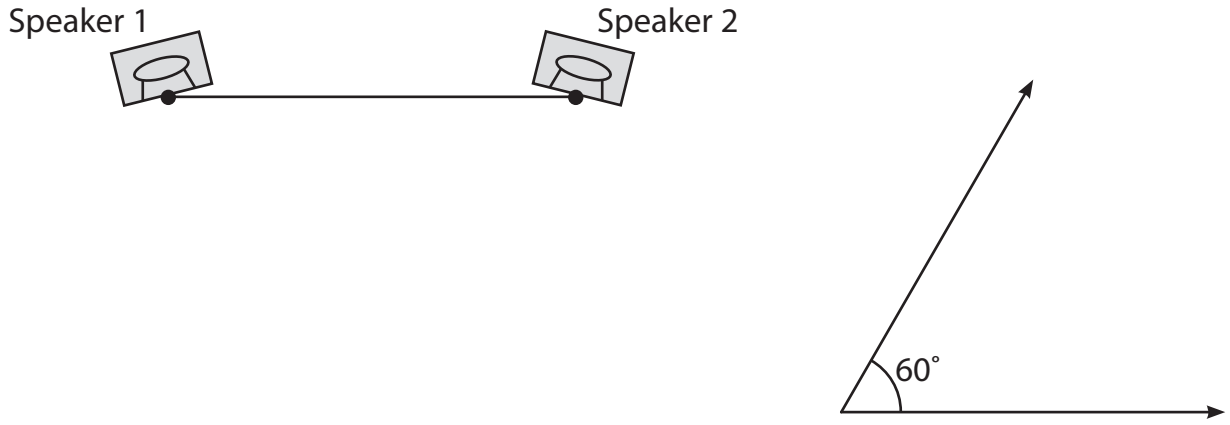
  This site gives step-by-step instructions for constructing an equilateral triangle inscribed in a given circle.

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Lesson 5.4.1: Constructing Equilateral Triangles Inscribed in Circles

Warm-Up 5.4.1

The town of Fairside is planning an outdoor concert in the park. During the planning process, committee members determined there will be two large speakers, but they can't decide where the audience should sit. The best arrangement for two speakers and the center of the audience is a triangle where each angle is 60°. The diagram below depicts the line segment formed by the two speakers; a 60° angle is also shown.



1. Use the 60° angle and the given segment to construct the triangle created by the two speakers and the center of the audience.

2. Triangles are said to be congruent if the angle measures of both triangles are the same and the lengths of the sides are the same. Is it possible to construct a second non-congruent triangle using the given information? Explain your reasoning.

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Instruction

Lesson 5.4.1: Constructing Equilateral Triangles Inscribed in Circles

Common Core State Standard

G–CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Warm-Up 5.4.1 Debrief

1. Use the 60˚ angle and the given segment to construct the triangle created by the two speakers and the center of the audience.

   The speakers represent two of the three vertices of the triangle.

   Use the segment that joins each of the speakers as one side of the triangle.

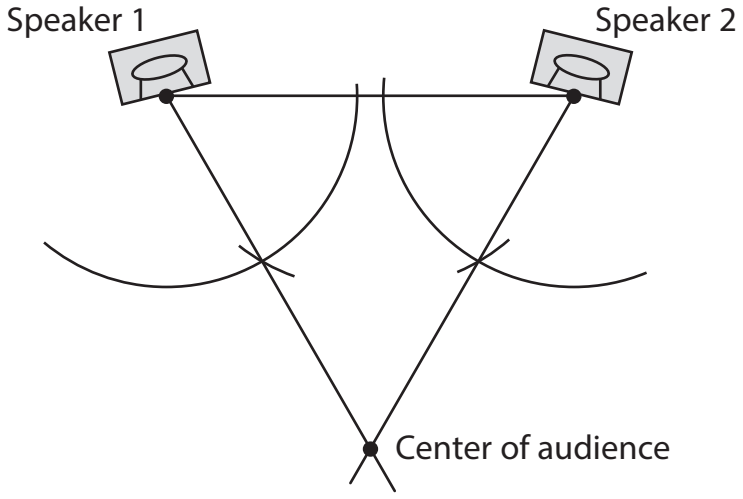
   Copy the given 60˚ angle using construction methods previously learned.

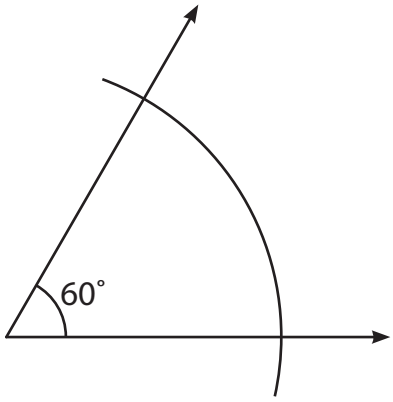
   Use the point on Speaker 1 as the vertex of one of the angles.

   Copy the given 60˚ angle a second time, using the point on Speaker 2 as the vertex of the second angle.

   Extend the sides of the copied angles to find the point of intersection.

   The point of intersection represents the center of the audience.





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Instruction

2. Triangles are said to be congruent if the angle measures of both triangles are the same and the lengths of the sides are the same. Is it possible to construct a second non-congruent triangle using the given information? Explain your reasoning.

   It is not possible to construct a second non-congruent triangle.

   The given information includes two angle measures and a side length between those two angles. It is not possible to construct a different triangle with these measurements.

   If you have two triangles, and any two angles and the included side are equal, then the triangles are congruent.

Connection to the Lesson

- Students will practice geometric constructions by copying angles.
- Students will also learn more about equilateral triangles in this lesson.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- using a compass (no standard)
- copying and bisecting line segments (G–CO.12)
- constructing perpendicular lines (G–CO.12)

Introduction

The ability to copy and bisect angles and segments, as well as construct perpendicular and parallel lines, allows you to construct a variety of geometric figures, including triangles, squares, and hexagons. There are many ways to construct these figures and others. Sometimes the best way to learn how to construct a figure is to try on your own. You will likely discover different ways to construct the same figure and a way that is easiest for you. In this lesson, you will learn two methods for constructing a triangle within a circle.

Key Concepts

Triangles

- A **triangle** is a polygon with three sides and three angles.
- There are many types of triangles that can be constructed.
- Triangles are classified based on their angle measure and the measure of their sides.
- **Equilateral triangles** are triangles with all three sides equal in length.
- The measure of each angle of an equilateral triangle is 60˚.

Circles

- A **circle** is the set of points on a plane at a fixed distance, or radius, from a given point, the center.
- The set of points forms a two-dimensional curve whose central angle measures 360˚.
- Circles are named by their center. For example, if a circle has a center point, *G*, the circle is named circle *G*.
- The **diameter** of a circle is a line segment that goes through the center of a circle and connects two points on the circle. It is twice the radius.

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Instruction

- The **radius** of a circle is a line segment that runs from the center of a circle to a point on the circle.
- The radius of a circle is one-half the length of the diameter.

Inscribing Figures

- To **inscribe** means to draw a figure within another figure so that every vertex of the enclosed figure touches the outside figure.
- A figure inscribed within a circle is a figure drawn within a circle so that every vertex of the figure touches the circle.
- It is possible to inscribe a triangle within a circle. Like with all constructions, the only tools necessary to inscribe a figure are a straightedge and a compass, patty paper and a straightedge, or technology.
- This lesson will focus on constructions with a compass and a straightedge.

Method 1: Constructing an Equilateral Triangle Inscribed in a Circle Using a Compass and a Straightedge

1. To construct an equilateral triangle inscribed in a circle, first mark the location of the center point of the circle. Label the point *X*.
2. Construct a circle with the sharp point of the compass on the center point.
3. Label a point on the circle point *A*.
4. Without changing the compass setting, put the sharp point of the compass on *A* and draw an arc to intersect the circle at two points. Label the points *B* and *C*.
5. Use a straightedge to construct $\overline{BC}$.
6. Put the sharp point of the compass on point *B*. Open the compass until it extends to the length of $\overline{BC}$. Draw another arc that intersects the circle. Label the point *D*.
7. Use a straightedge to construct $\overline{BD}$ and $\overline{CD}$.

Do not erase any of your markings.

Triangle *BCD* is an equilateral triangle inscribed in circle *X*.

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- Once a circle is constructed, it is possible to divide the circle into 6 equal parts.
- Do this by choosing a starting point on the circle and moving the compass around the circle, making marks that are the length of the radius apart from one another.
- Connecting every other point of intersection results in an equilateral triangle.

Method 2: Constructing an Equilateral Triangle Inscribed in a Circle Using a Compass and a Straightedge

1. To construct an equilateral triangle inscribed in a circle, first mark the location of the center point of the circle. Label the point *X*.
2. Construct a circle with the sharp point of the compass on the center point.
3. Label a point on the circle point *A*.
4. Without changing the compass setting, put the sharp point of the compass on *A* and draw an arc to intersect the circle at one point. Label the point of intersection *B*.
5. Put the sharp point of the compass on point *B* and draw an arc to intersect the circle at one point. Label the point of intersection *C*.
6. Continue around the circle, labeling points *D*, *E*, and *F*. Be sure not to change the compass setting.
7. Use a straightedge to connect *A* and *C, C* and *E,* and *E* and *A*.

Do not erase any of your markings.

Triangle *ACE* is an equilateral triangle inscribed in circle *X*.

Common Errors/Misconceptions

- inappropriately changing the compass setting
- attempting to measure lengths and angles with rulers and protractors
- not creating large enough arcs to find the points of intersection

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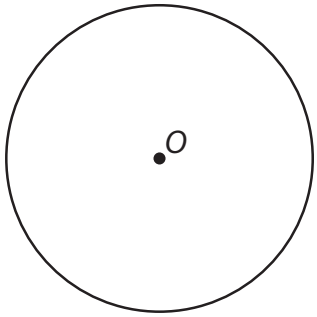
Guided Practice 5.4.1

Example 1

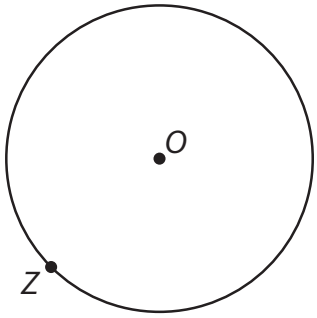
Construct equilateral triangle *ACE* inscribed in circle *O* using Method 1.

1. Construct circle *O*.

   Mark the location of the center point of the circle, and label the point *O*. Construct a circle with the sharp point of the compass on the center point.



2. Label a point on the circle point *Z*.

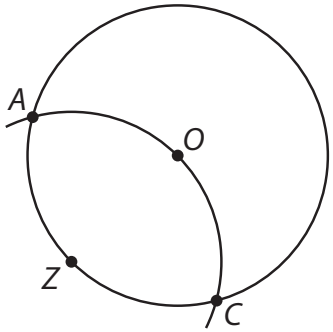


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Instruction

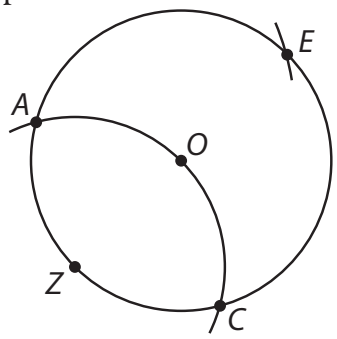
3. Locate vertices A and C of the equilateral triangle.

   Without changing the compass setting, put the sharp point of the compass on Z and draw an arc to intersect the circle at two points. Label the points A and C.



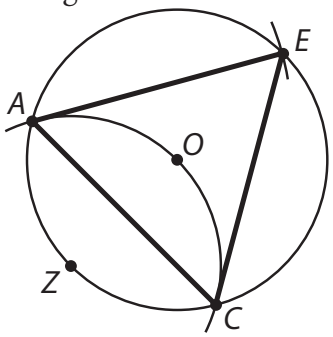
4. Locate the third vertex of the equilateral triangle.

   Put the sharp point of the compass on point A. Open the compass until it extends to the length of $\overline{AC}$. Draw another arc that intersects the circle, and label the point E.



5. Construct the sides of the triangle.

   Use a straightedge to connect A and C, C and E, and A and E. Do not erase any of your markings.



   Triangle ACE is an equilateral triangle inscribed in circle O.

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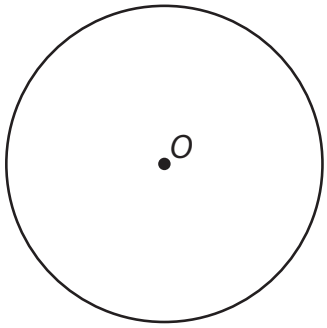
Instruction

Example 2

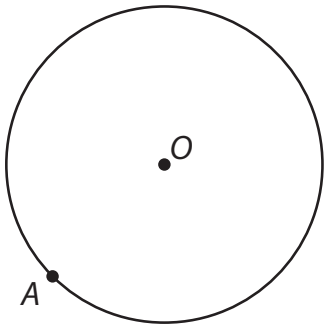
Construct equilateral triangle *ACE* inscribed in circle *O* using Method 2.

1. Construct circle *O*.

   Mark the location of the center point of the circle, and label the point *O*. Construct a circle with the sharp point of the compass on the center point.



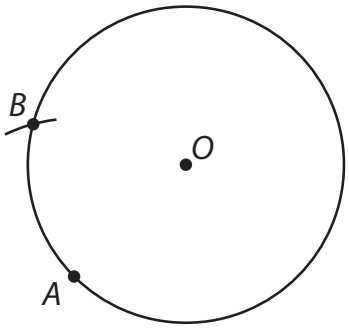
2. Label a point on the circle point *A*.



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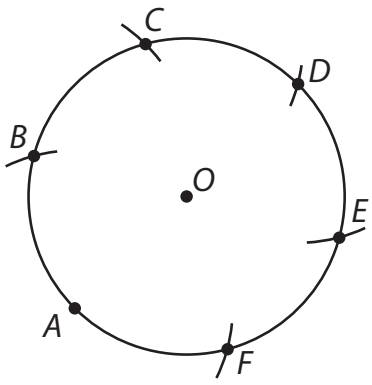
3. Locate vertices *C* and *E*.

   Begin by marking the remaining five equidistant points around the circle. Without changing the compass setting, put the sharp point of the compass on *A*. Draw an arc to intersect the circle at one point. Label the point of intersection *B*.



   Put the sharp point of the compass on point *B*. Without changing the compass setting, draw an arc to intersect the circle at one point. Label the point of intersection *C*.

   Continue around the circle, labeling points *D*, *E*, and *F*. Be sure not to change the compass setting.

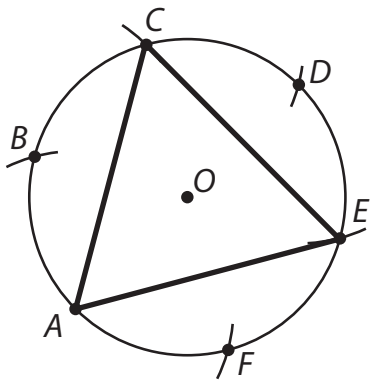


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Instruction

4. Construct the sides of the triangle.

   Use a straightedge to connect *A* and *C*, *C* and *E*, and *E* and *A*. Do not erase any of your markings.



   Triangle *ACE* is an equilateral triangle inscribed in circle *O*.

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Instruction

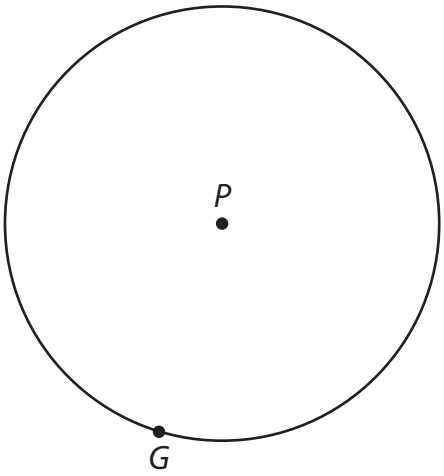
Example 3

Construct equilateral triangle *JKL* inscribed in circle *P* using Method 1. Use the length of $\overline{HP}$ as the radius for circle *P*.



1. Construct circle *P*.

   Mark the location of the center point of the circle, and label the point *P*. Set the opening of the compass equal to the length of $\overline{HP}$. Then, put the sharp point of the compass on point *P* and construct a circle. Label a point on the circle point *G*.

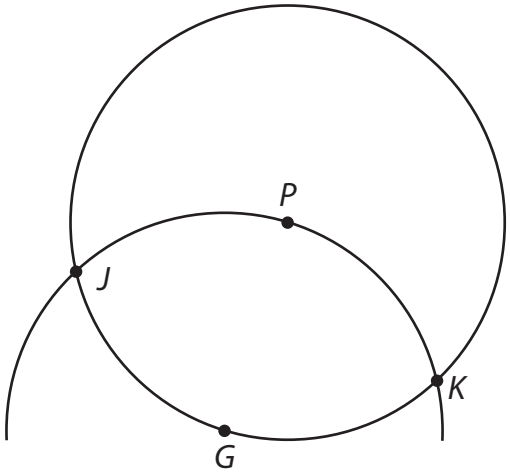


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Instruction

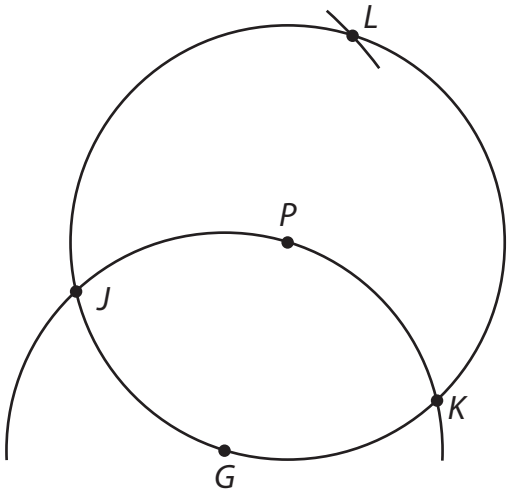
2. Locate vertices J and K of the equilateral triangle.

   Without changing the compass setting, put the sharp point of the compass on G. Draw an arc to intersect the circle at two points. Label the points J and K.



3. Locate the third vertex of the equilateral triangle.

   Put the sharp point of the compass on point J. Open the compass until it extends to the length of $\overline{JK}$. Draw another arc that intersects the circle, and label the point L.

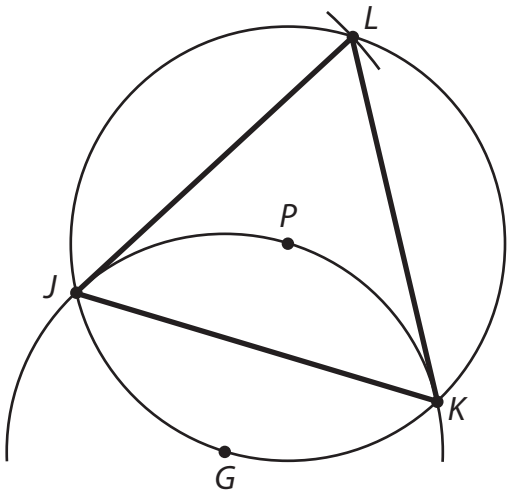


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Instruction

4. Construct the sides of the triangle.

   Use a straightedge to connect *J* and *K*, *K* and *L*, and *L* and *J*. Do not erase any of your markings.



   Triangle *JKL* is an equilateral triangle inscribed in circle *P* with the given radius.

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Instruction

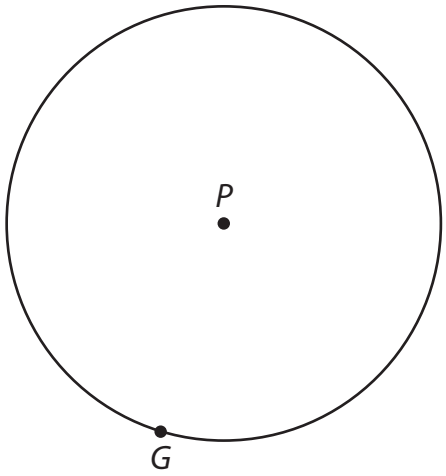
Example 4

Construct equilateral triangle *JLN* inscribed in circle *P* using Method 2. Use the length of $\overline{HP}$ as the radius for circle *P*.



1. Construct circle *P*.

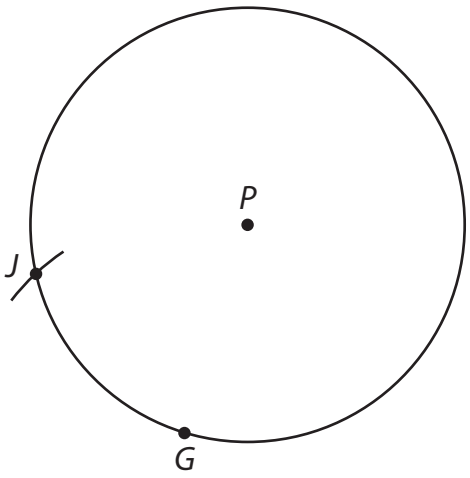
   Mark the location of the center point of the circle, and label the point *P*. Set the opening of the compass equal to the length of $\overline{HP}$. Then, put the sharp point of the compass on point *P* and construct a circle. Label a point on the circle point *G*.



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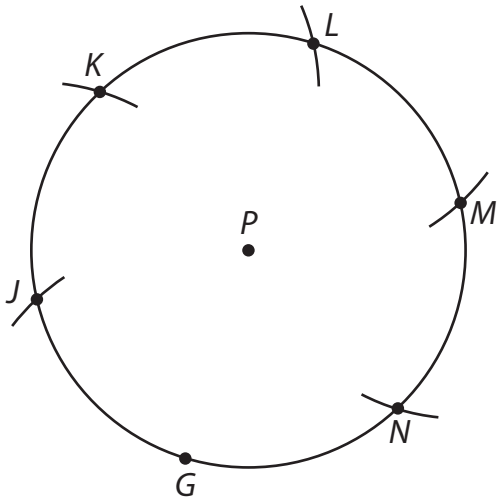
2. Locate vertex *J*.

   Without changing the compass setting, put the sharp point of the compass on *G*. Draw an arc to intersect the circle at one point. Label the point of intersection *J*.



   Put the sharp point of the compass on point *J*. Without changing the compass setting, draw an arc to intersect the circle at one point. Label the point of intersection *K*.

   Continue around the circle, labeling points *L, M,* and *N*. Be sure not to change the compass setting.

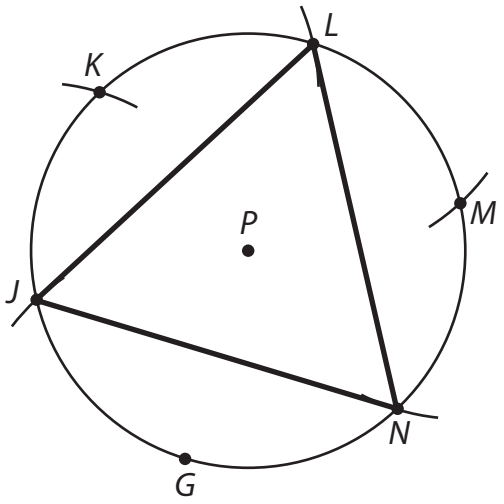


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Instruction

3. Construct the sides of the triangle.

   Use a straightedge to connect *J* and *L*, *L* and *N*, and *J* and *N*. Do not erase any of your markings.



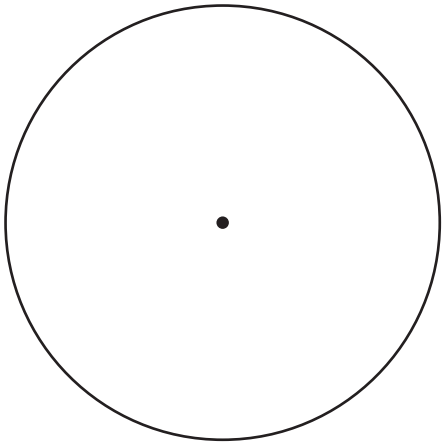
Triangle *JLN* is an equilateral triangle inscribed in circle *P*.

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Problem-Based Task 5.4.1: Vending Machine Placement

SMP	
1 ✓	2
3	4 ✓
5 ✓	6
7	8

As an employee of a skating rink, Jarno was asked to determine the placement of 3 vending machines. Each of the 3 machines needs to be placed along the edge of the circular skating rink. The distance between each machine must be the same. Where should Jarno place each machine? A diagram of the skating rink is provided below.





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Problem-Based Task 5.4.1: Vending Machine Placement

Coaching

a. Suppose the 3 vending machines were placed around the edge of the circular skating rink. If a customer were to skate from the first vending machine to the second, to the third, and back to the first machine, what figure would be created?

b. If the distance between each of the machines is the same, what is true about the length of each of the sides of the figure the customer skated?

c. Which geometric construction can be carried out to identify the locations of the 3 vending machines?

d. What is the procedure for making this geometric construction?

e. Where should Jarno place each machine?

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Instruction

Problem-Based Task 5.4.1: Vending Machine Placement

Coaching Sample Responses

a. Suppose the 3 vending machines were placed around the edge of the circular skating rink. If a customer were to skate from the first vending machine to the second, to the third, and back to the first machine, what figure would be created?

If a customer were to skate from the first vending machine to the second, to the third, and back to the first machine, they would create a triangle.

b. If the distance between each of the machines is the same, what is true about the length of each of the sides of the figure the customer skated?

Each side of the triangle has the same length.

c. Which geometric construction can be carried out to identify the locations of the 3 vending machines?

Constructing an equilateral triangle inscribed in a circle would identify the locations of the 3 vending machines.

d. What is the procedure for making this geometric construction?

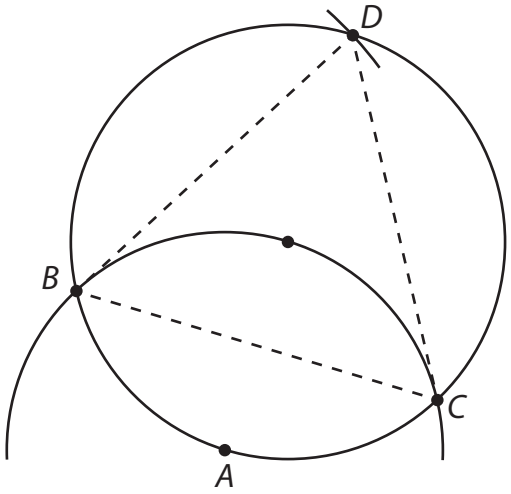
There are two ways of constructing an equilateral triangle inscribed in a circle. Either is correct.

Method 1

- Label a point on the circle point A.
- Put the sharp point of the compass on A and set the compass width equal to the radius of the circle.
- Keeping the sharp point on A, draw an arc to intersect the circle at two points. Label the points B and C.
- Use a straightedge to construct $\overline{BC}$.
- Put the sharp point of the compass on point B. Open the compass until it extends to the length of $\overline{BC}$. Draw another arc that intersects the circle. Label the point D.
- Use a straightedge to construct $\overline{BD}$ and $\overline{CD}$.
- Triangle BCD is an equilateral triangle constructed within the circle. See the following illustration.

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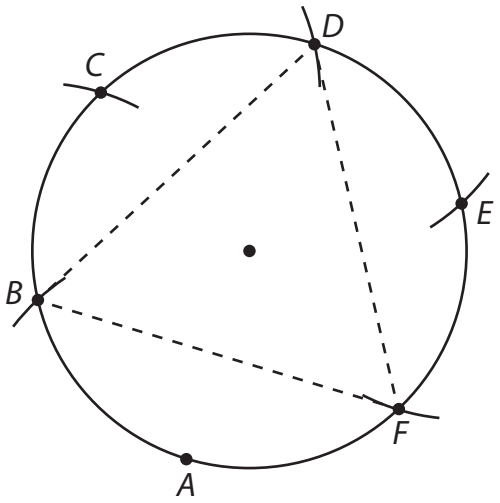
Instruction



Method 2

- Label a point on the circle point *A*.
- Put the sharp point of the compass on *A* and set the compass width equal to the radius of the circle.
- Keeping the sharp point on *A*, draw an arc to intersect the circle at one point. Label the point of intersection *B*.
- Put the sharp point of the compass on point *B*. Without changing the compass setting, draw an arc to intersect the circle at one point. Label the point of intersection *C*.
- Continue around the circle, labeling points *D*, *E*, and *F*. Be sure not to change the compass setting.
- Use a straightedge to connect *B* and *D, D* and *F,* and *F* and *B*.
- Triangle *BDF* is an equilateral triangle constructed within the circle. See the following illustration.

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e. Where should Jarno place each machine?

Jarno should place the vending machines so that the machines are at the vertices of an equilateral triangle. Based on the diagrams, the vending machines should be placed at points *B*, *C*, and *D* for the first method, and at points *B*, *D*, and *F* for the second method.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 5.4.1: Constructing Equilateral Triangles Inscribed in Circles

Use a compass and a straightedge to construct each equilateral triangle using Method 1.

1. Construct equilateral triangle *BCD* inscribed in circle *Z*.

2. Construct equilateral triangle *FGH* inscribed in circle *Y* with radius $\overline{AB}$.



3. Construct equilateral triangle *JKL* inscribed in circle *X* with radius $\overline{CD}$.



4. Construct equilateral triangle *NOP* inscribed in circle *W* with the radius equal to twice $\overline{EF}$.



5. Construct equilateral triangle *RST* inscribed in circle *V* with the radius equal to one-half $\overline{GH}$.



continued

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Use a compass and a straightedge to construct each equilateral triangle using Method 2.

6. Construct equilateral triangle *BCD* inscribed in circle *Z*.

7. Construct equilateral triangle *FGH* inscribed in circle *Y* with radius $\overline{AB}$.



8. Construct equilateral triangle *JKL* inscribed in circle *X* with radius $\overline{CD}$.



9. Construct equilateral triangle *NOP* inscribed in circle *W* with the radius equal to twice $\overline{EF}$.



10. Construct equilateral triangle *RST* inscribed in circle *V* with the radius equal to one-half $\overline{GH}$.

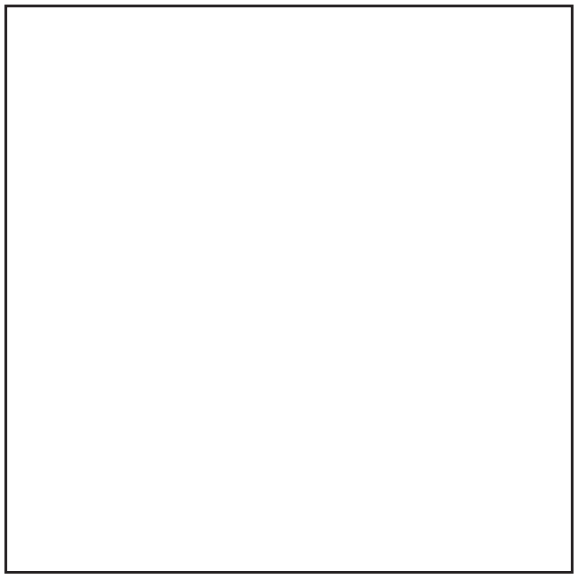


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Lesson 5.4.2: Constructing Squares Inscribed in Circles

Warm-Up 5.4.2

Antonia is making four corner tables, one for each of her three sisters and herself. She has one large square piece of wood that she plans to cut into four tabletops. She begins by marking the needed cuts for the tabletops on the square piece of wood.



1. Each angle of the square piece of wood measures 90°. If Antonia bisects one angle of the square, what is the measure of the two new angles?

2. Bisect one angle of the square. Extend the angle bisector so that it intersects the square in two places. Where does the bisector intersect the square?

3. Bisect the remaining angles of the square. If Antonia cuts along each angle bisector, what figures will she have created?

4. What are the measures of each of the angles of the new figures?

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Instruction

Lesson 5.4.2: Constructing Squares Inscribed in Circles

Common Core State Standard

G–CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Warm-Up 5.4.2 Debrief

1. Each angle of the square piece of wood measures 90˚. If Antonia bisects one angle of the square, what is the measure of the two new angles?

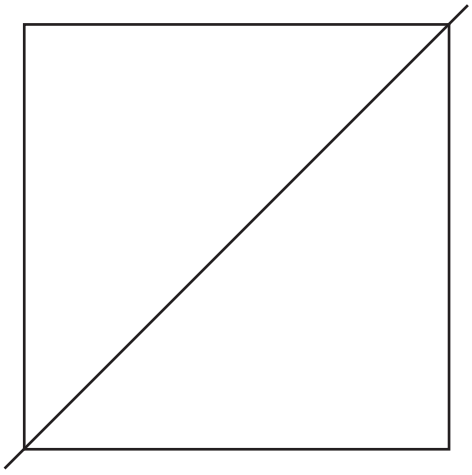
   When an angle is bisected, it is divided into two equal angles.

   $90 \div 2 = 45$

   Each of the new angles measures 45˚.

2. Bisect one angle of the square. Extend the angle bisector so that it intersects the square in two places. Where does the bisector intersect the square?

   The angle bisector intersects the square at the bisected angle and the angle opposite the bisected angle.

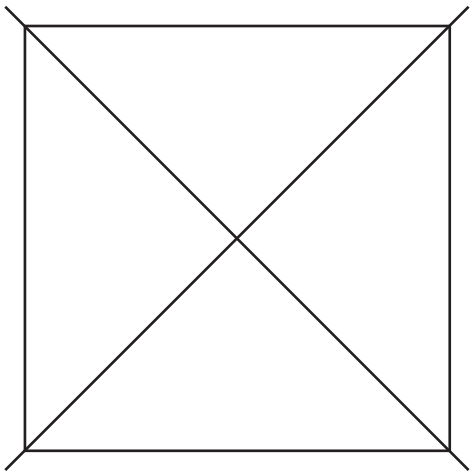


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Instruction

3. Bisect the remaining angles of the square. If Antonia cuts along each angle bisector, what figures will she have created?

   Antonia will have created 4 triangles.



4. What are the measures of each of the angles of the new figures?

   Each triangle will have two 45° angles and one 90° angle.

Connection to the Lesson

- Students will practice geometric constructions by bisecting angles.
- Students will also learn more about squares in this lesson.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- using a compass (no standard)
- copying and bisecting line segments (G–CO.12)
- constructing perpendicular lines (G–CO.12)

Introduction

Triangles are not the only figures that can be inscribed in a circle. It is also possible to inscribe other figures, such as squares. The process for inscribing a square in a circle uses previously learned skills, including constructing perpendicular bisectors.

Key Concepts

- A **square** is a four-sided regular polygon.
- A regular polygon is a polygon that has all sides equal and all angles equal.
- The measure of each of the angles of a square is 90˚.
- Sides that form a 90˚ angle are perpendicular.
- By constructing the perpendicular bisector of a diameter of a circle, you can construct a square inscribed in a circle.

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Instruction

Constructing a Square Inscribed in a Circle Using a Compass and a Straightedge

1. To construct a square inscribed in a circle, first mark the location of the center point of the circle. Label the point *X*.
2. Construct a circle with the sharp point of the compass on the center point.
3. Label a point on the circle point *A*.
4. Use a straightedge to connect point *A* and point *X*. Extend the line through the circle, creating the diameter of the circle. Label the second point of intersection *C*.
5. Construct the perpendicular bisector of $\overline{AC}$ by putting the sharp point of your compass on endpoint *A*. Open the compass wider than half the distance of $\overline{AC}$. Make a large arc intersecting $\overline{AC}$. Without changing your compass setting, put the sharp point of the compass on endpoint *C*. Make a second large arc. Use your straightedge to connect the points of intersection of the arcs.
6. Extend the bisector so it intersects the circle in two places. Label the points of intersection *B* and *D*.
7. Use a straightedge to connect points *A* and *B*, *B* and *C*, *C* and *D*, and *A* and *D*.

Do not erase any of your markings.

Quadrilateral *ABCD* is a square inscribed in circle *X*.

Common Errors/Misconceptions

- inappropriately changing the compass setting
- attempting to measure lengths and angles with rulers and protractors
- not creating large enough arcs to find the points of intersection
- not extending segments long enough to find the vertices of the square

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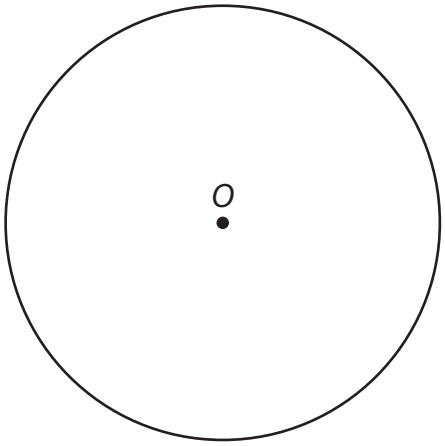
Guided Practice 5.4.2

Example 1

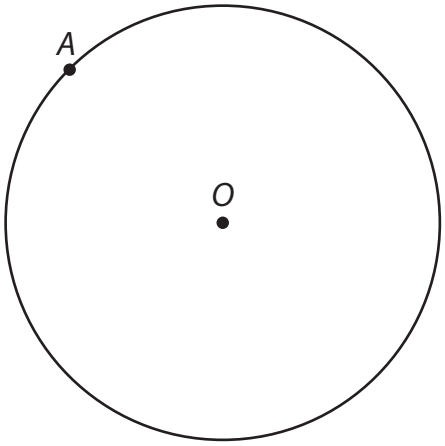
Construct square *ABCD* inscribed in circle *O*.

1. Construct circle *O*.

   Mark the location of the center point of the circle, and label the point *O*. Construct a circle with the sharp point of the compass on the center point.



2. Label a point on the circle point *A*.

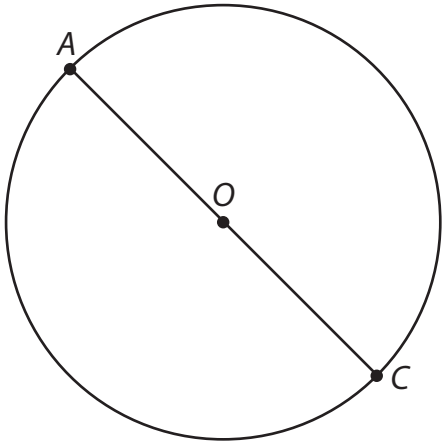


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Instruction

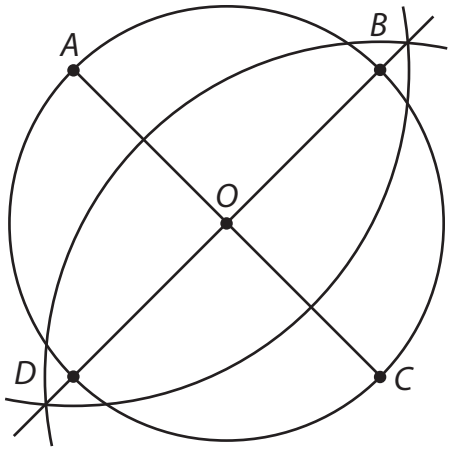
3. Construct the diameter of the circle.

   Use a straightedge to connect point A and point O. Extend the line through the circle, creating the diameter of the circle. Label the second point of intersection C.



4. Construct the perpendicular bisector of $\overline{AC}$.

   Extend the bisector so it intersects the circle in two places. Label the points of intersection B and D.

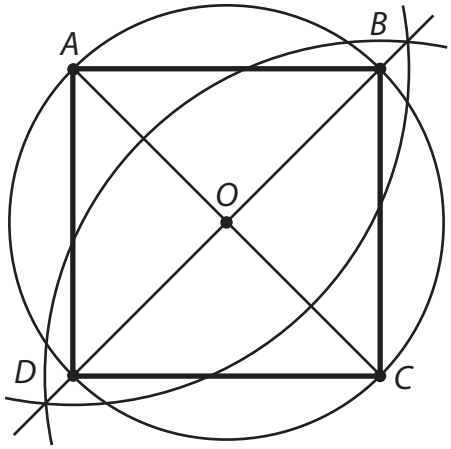


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Instruction

5. Construct the sides of the square.

   Use a straightedge to connect points *A* and *B*, *B* and *C*, *C* and *D*, and *A* and *D*. Do not erase any of your markings.



Quadrilateral *ABCD* is a square inscribed in circle *O*.

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Instruction

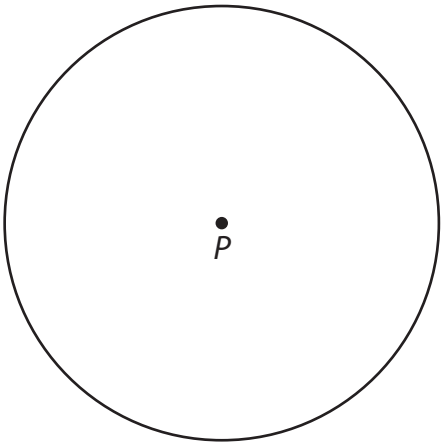
Example 2

Construct square *EFGH* inscribed in circle *P* with the radius equal to the length of $\overline{EP}$.

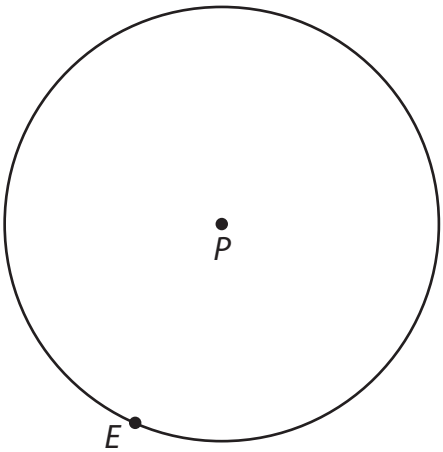


1. Construct circle *P*.

   Mark the location of the center point of the circle, and label the point *P*. Set the opening of the compass equal to the length of $\overline{EP}$. Construct a circle with the sharp point of the compass on the center point, *P*.



2. Label a point on the circle point *E*.

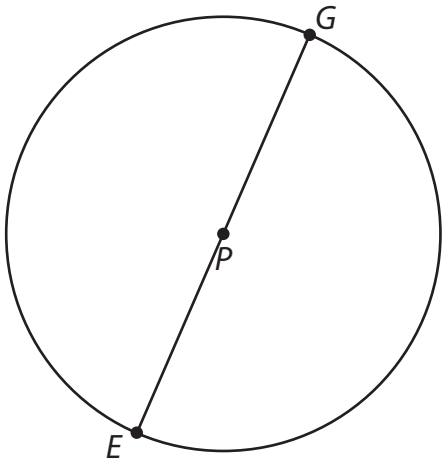


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Instruction

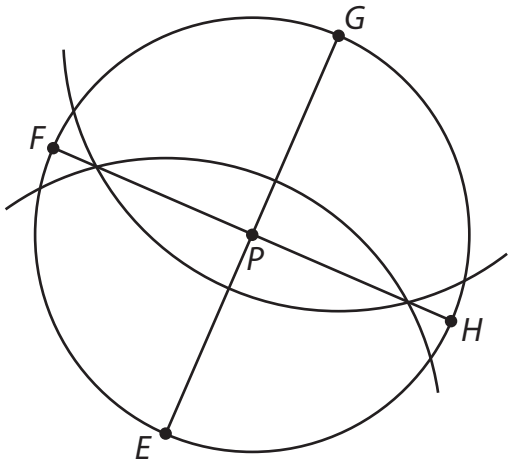
3. Construct the diameter of the circle.

   Use a straightedge to connect point E and point P. Extend the line through the circle, creating the diameter of the circle. Label the second point of intersection G.



4. Construct the perpendicular bisector of $\overline{EG}$.

   Extend the bisector so it intersects the circle in two places. Label the points of intersection F and H.

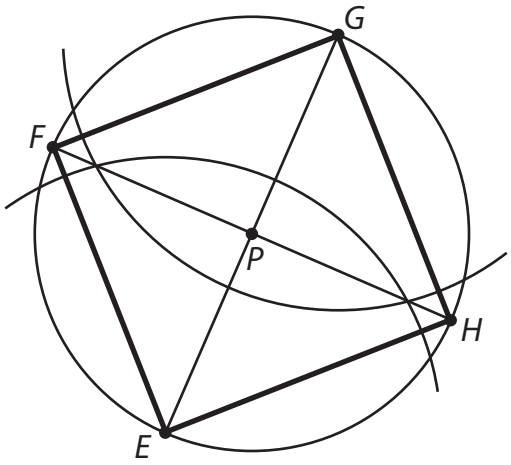


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Instruction

5. Construct the sides of the square.

   Use a straightedge to connect points *E* and *F*, *F* and *G*, *G* and *H*, and *H* and *E*. Do not erase any of your markings.



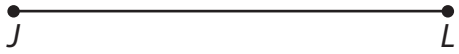
Quadrilateral *EFGH* is a square inscribed in circle *P*.

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Instruction

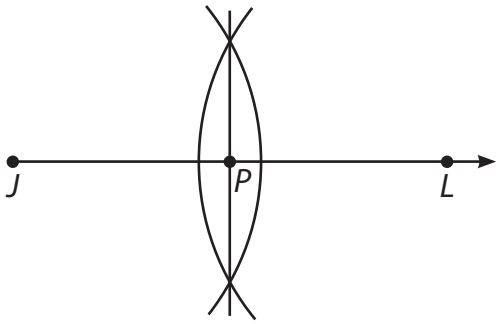
Example 3

Construct square *JKLM* inscribed in circle *Q* with the radius equal to one-half the length of $\overline{JL}$.

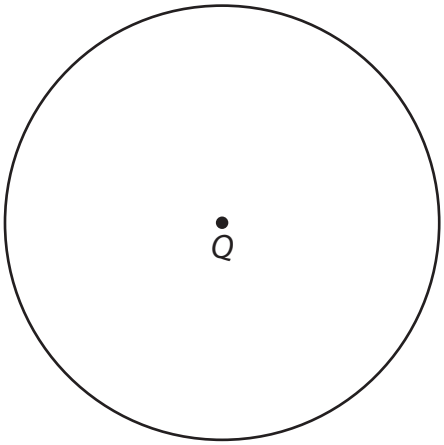


1. Construct circle *Q*.

   Mark the location of the center point of the circle, and label the point *Q*. Bisect the length of $\overline{JL}$. Label the midpoint of the segment as point *P*.



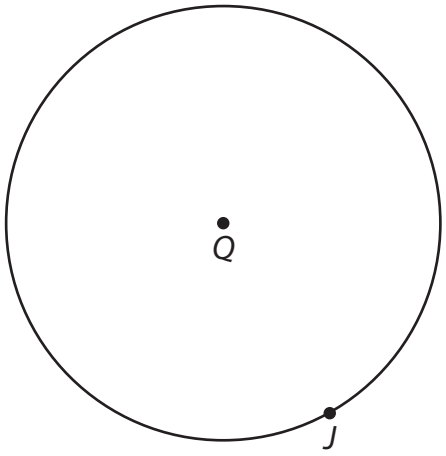
   Next, set the opening of the compass equal to the length of $\overline{JP}$. Construct a circle with the sharp point of the compass on the center point, *Q*.



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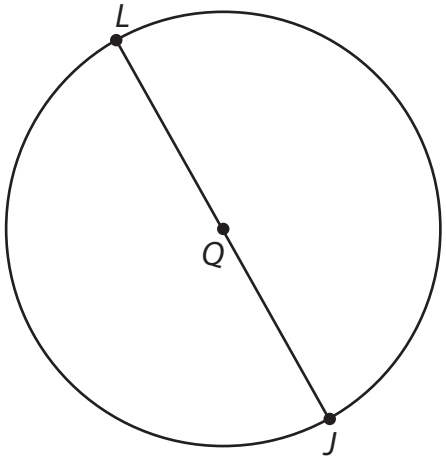
Instruction

2. Label a point on the circle point *J*.



3. Construct the diameter of the circle.

   Use a straightedge to connect point *J* and point *Q*. Extend the line through the circle, creating the diameter of the circle. Label the second point of intersection *L*.

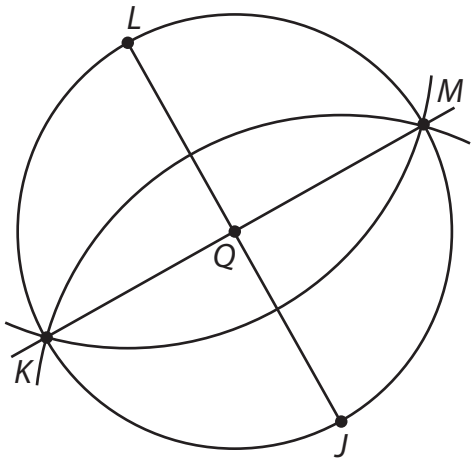


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Instruction

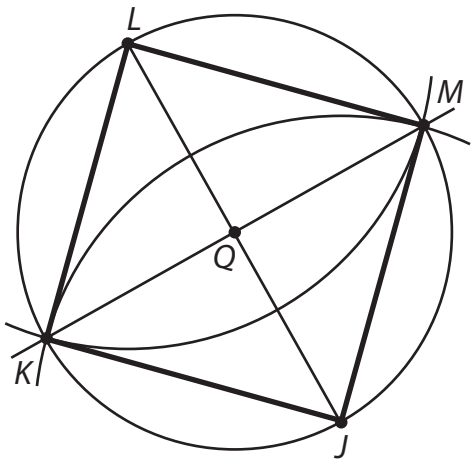
4. Construct the perpendicular bisector of $\overline{JL}$.

   Extend the bisector so it intersects the circle in two places. Label the points of intersection *K* and *M*.



5. Construct the sides of the square.

   Use a straightedge to connect points *J* and *K*, *K* and *L*, *L* and *M*, and *M* and *J*. Do not erase any of your markings.



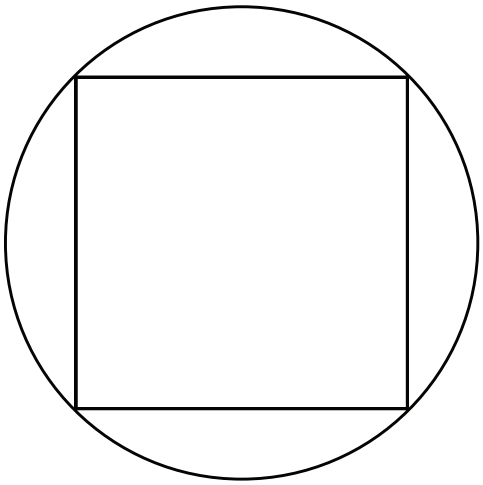
   Quadrilateral *JKLM* is a square inscribed in circle *Q*.

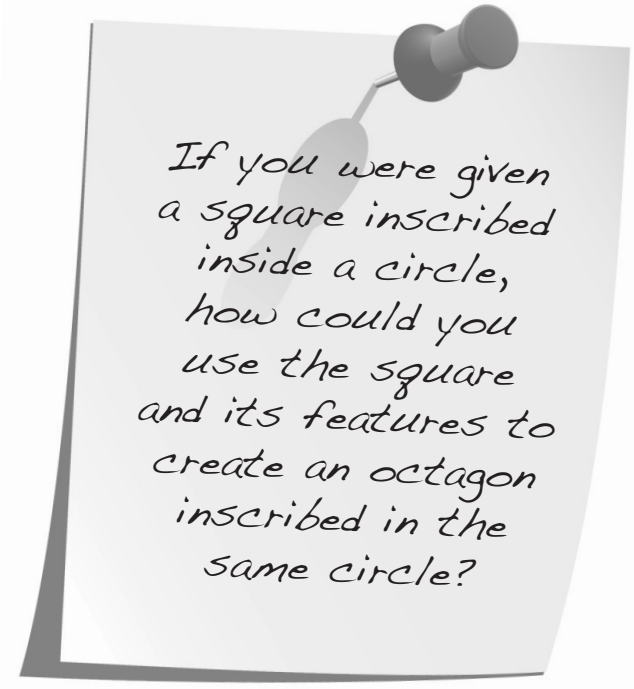
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Problem-Based Task 5.4.2: Constructing a Regular Octagon

SMP	
1	2
3 ✓	4
5 ✓	6
7 ✓	8

A regular octagon is a polygon with eight sides that are equal in length and eight angles that are equal in measure. If you were given a square inscribed inside a circle, how could you use the square and its features to create an octagon inscribed in the same circle?





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Problem-Based Task 5.4.2: Constructing a Regular Octagon

Coaching

Note: There are many ways to construct the regular octagon. This is only an example of one way it can be done.

a. How could you construct the diagonals of a square? Draw the diagonals.

b. Draw a point at the intersection of the diagonals.

c. Which basic construction method could be used to draw a line perpendicular to one side of the square going through the intersection of the diagonals? Using the intersection of the diagonals and the corners of the square, construct this perpendicular line for two adjacent sides of the square long enough to span the entire width of the circle. This will require two separate constructions.

d. In how many places do these lines intersect the circle? Draw a point at each intersection.

e. Do you have enough points to draw a regular octagon? If so, draw the octagon, which should be inscribed in the same circle and use the corners of the original square.

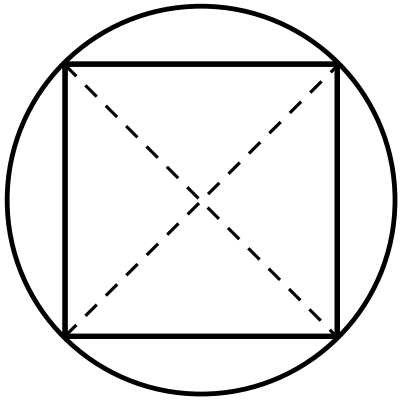
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Instruction

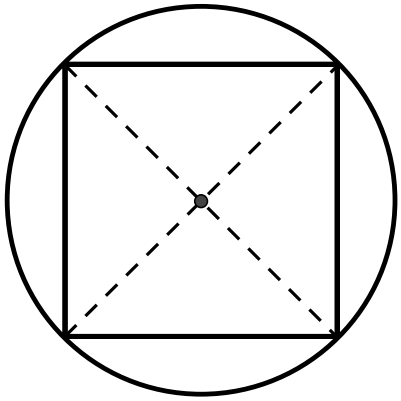
Problem-Based Task 5.4.2: Constructing a Regular Octagon

Coaching Sample Responses

a. How could you construct the diagonals of a square? Draw the diagonals.



b. Draw a point at the intersection of the diagonals.

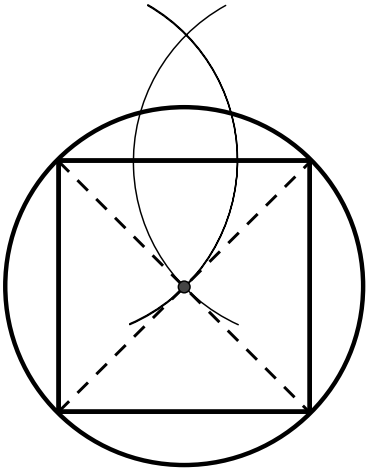


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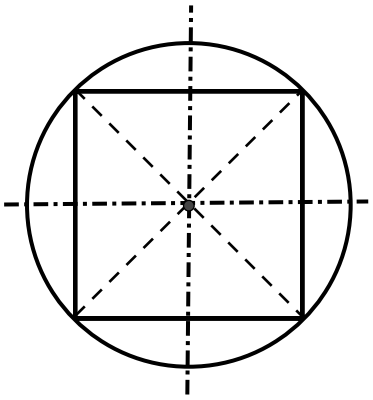
Instruction

c. Which basic construction method could be used to draw a line perpendicular to one side of the square going through the intersection of the diagonals? Using the intersection of the diagonals and the corners of the square, construct this perpendicular line for two adjacent sides of the square long enough to span the entire width of the circle. This will require two separate constructions.

Using a compass, place the compass point on a corner of the square and the pencil on the diagonal intersection. Draw an arc larger than a semi-circle that encompasses both the intersection and the side of the circle. Repeat from the other endpoint of the square's side so that the two arcs intersect in two places.



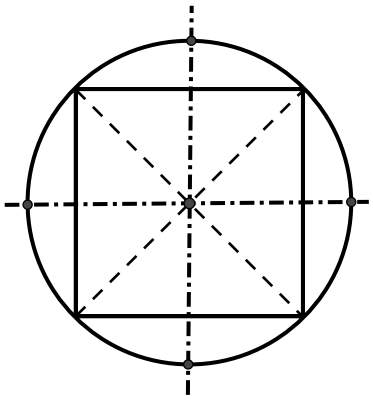
Draw a line through the points where the arcs intersect. Repeat with an adjacent side of the square. You should now have two lines that each intersect the circle in two places.



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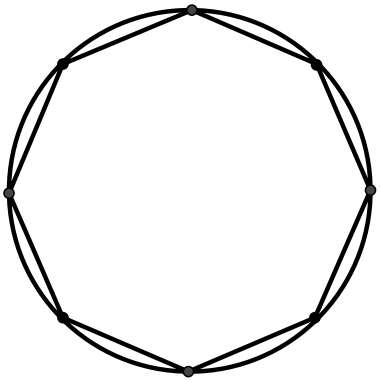
d. In how many places do these lines intersect the circle? Draw a point at each intersection.

There are four places where the new lines intersect the circle.



e. Do you have enough points to draw a regular octagon? If so, draw the octagon, which should be inscribed in the same circle and use the corners of the original square.

Connecting the four new points with the four corners of the square either going clockwise or counter-clockwise will create the regular octagon.



Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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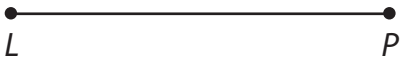
Practice 5.4.2: Constructing Squares Inscribed in Circles

Use a compass and a straightedge to construct each square inscribed in a circle.

1. Construct square *ABCD* inscribed in circle *E.*

2. Construct square *FGHJ* inscribed in circle *K*.

3. Construct square *LMNO* inscribed in circle *P* with radius $\overline{LP}$.



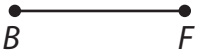
4. Construct square *QRST* inscribed in circle *U* with radius $\overline{QU}$.



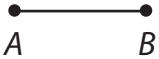
5. Construct square *VWXY* inscribed in circle *Z* with radius $\overline{VZ}$.



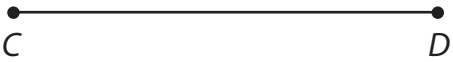
6. Construct square *BCDE* inscribed in circle *F* with radius $\overline{BF}$.



7. Construct square *GHJK* inscribed in circle *L* with the radius equal to twice $\overline{AB}$.



8. Construct square *MNOP* inscribed in circle *Q* with the radius equal to one-half $\overline{CD}$.



9. Construct square *RSTU* inscribed in circle *V* with the diameter equal to $\overline{RT}$.



10. Construct square *WXYZ* inscribed in circle *A* with the diameter equal to $\overline{WY}$.



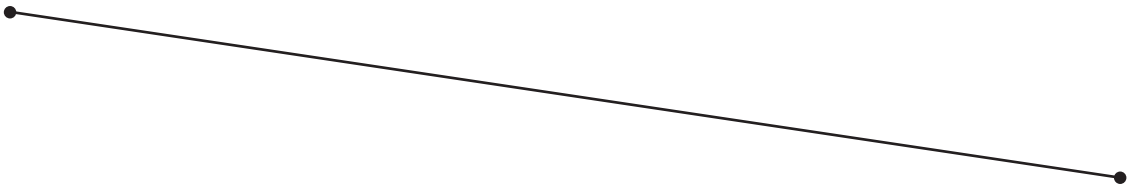
SWB p. 105; SRB Practice 5.4.2 p. U5-130

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Lesson 5.4.3: Constructing Regular Hexagons Inscribed in Circles

Warm-Up 5.4.3

The developers of a community garden would like to construct planting beds using only donated materials. One planting bed will be square. Developers would like to create the square planting bed using only one plank of wood, represented by the line below. Use the given segment to construct a design for the square planting bed.



1. Use a compass and a straightedge to determine the length of each side of the planting bed.

2. Describe the process for determining the length of each side of the planting bed.

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Instruction

Lesson 5.4.3: Constructing Regular Hexagons Inscribed in Circles

Common Core State Standard

G–CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Warm-Up 5.4.3 Debrief

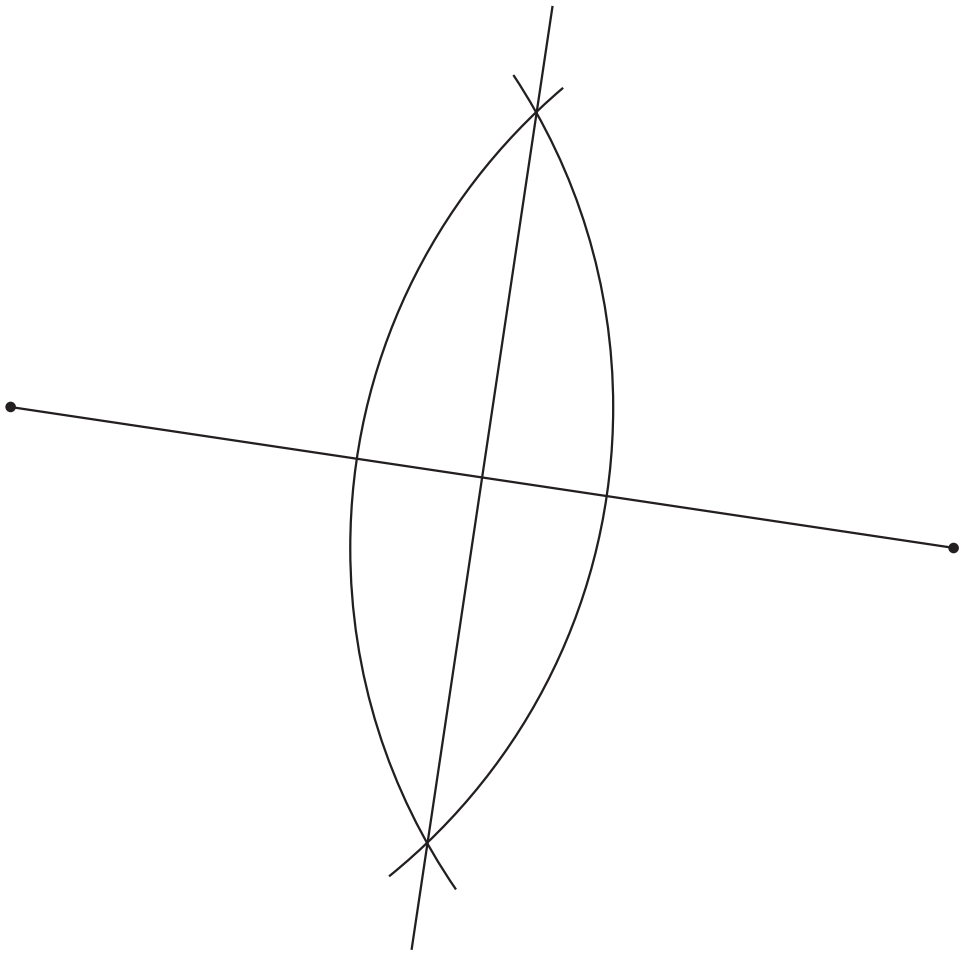
1. Use a compass and a straightedge to determine the length of each side of the planting bed.

   Bisect the original segment, and then bisect each half.

   The length of each side of the square planting bed will be $\frac{1}{4}$ the length of the original plank of wood.

2. Describe the process for determining the length of each side of the planting bed.

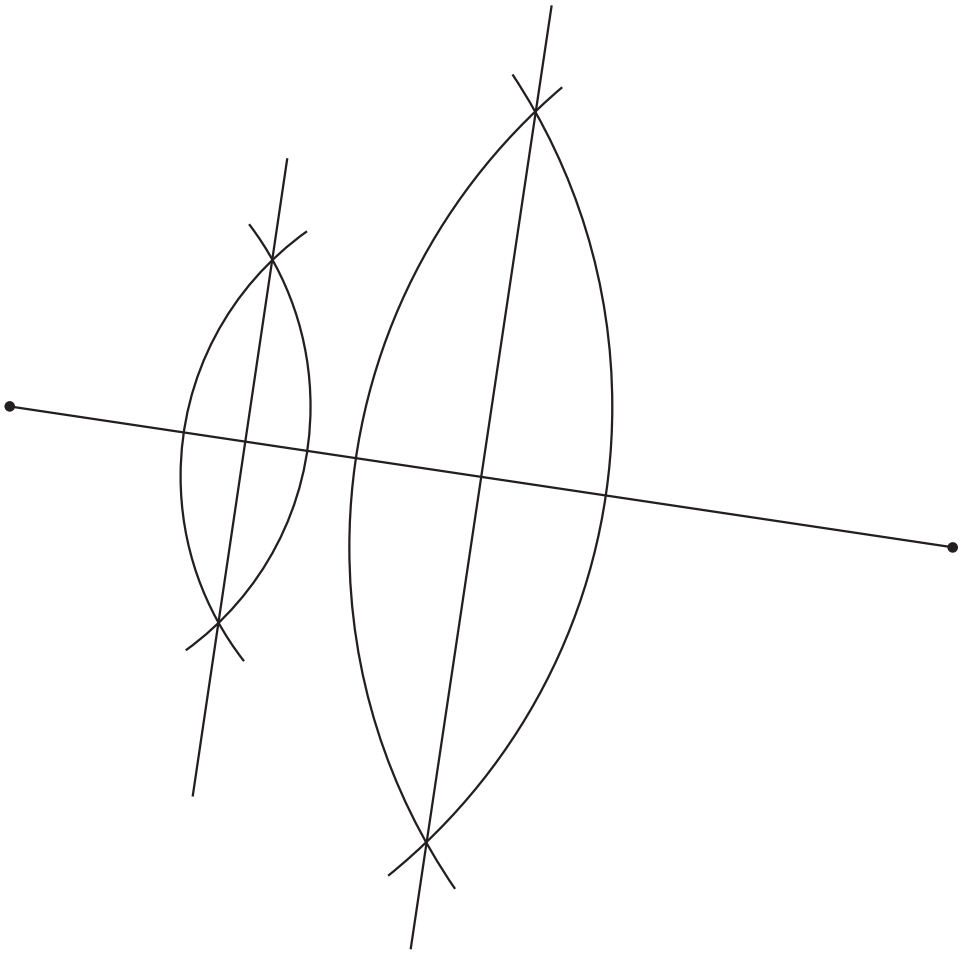
   To determine the length of each side of the square planting bed, first find the midpoint of the plank of wood by bisecting the plank. This construction will divide the plank into two equal pieces.



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Instruction

Next, find the midpoint of one of the halves by bisecting it.



The result is a piece of the plank that is $\frac{1}{4}$ the original length of the plank of wood.



Connection to the Lesson

- Students will practice geometric constructions and apply these methods in the construction of regular hexagons inscribed in a circle.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- using a compass (no standard)
- bisecting line segments (G–CO.12)

Introduction

As we have seen, construction methods can be used to inscribe equilateral triangles and squares in circles. Another figure that can be inscribed in a circle is a hexagon. Hexagons are polygons with six sides.

Key Concepts

- **Regular hexagons** have six equal sides and six angles, each measuring 120˚.
- The process for inscribing a regular hexagon in a circle is similar to that of inscribing an equilateral triangle in a circle.

Method 1: Constructing a Regular Hexagon Inscribed in a Circle Using a Compass and a Straightedge

1. To construct a regular hexagon inscribed in a circle, first mark the location of the center point of the circle. Label the point *X*.
2. Construct a circle with the sharp point of the compass on the center point.
3. Label a point on the circle point *A*.
4. Use a straightedge to connect point *A* and point *X*. Extend the line through the circle, creating the diameter of the circle. Label the second point of intersection *D*.
5. Without changing the compass setting, put the sharp point of the compass on *A*. Draw an arc to intersect the circle at two points. Label the points *B* and *F*.
6. Put the sharp point of the compass on *D*. Without changing the compass setting, draw an arc to intersect the circle at two points. Label the points *C* and *E*.
7. Use a straightedge to connect points *A* and *B*, *B* and *C*, *C* and *D*, *D* and *E*, *E* and *F*, and *F* and *A*.

Do not erase any of your markings.

Hexagon *ABCDEF* is regular and is inscribed in circle *X*.

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Instruction

- In a second method, once a circle is constructed, it is possible to divide the circle into six equal parts.
- Do this by choosing a starting point on the circle and moving the compass around the circle, making marks equal to the length of the radius.
- Connecting consecutive points of intersection results in a regular hexagon.

Method 2: Constructing a Regular Hexagon Inscribed in a Circle Using a Compass and a Straightedge

1. To construct a regular hexagon inscribed in a circle, first mark the location of the center point of the circle. Label the point *X*.
2. Construct a circle with the sharp point of the compass on the center point.
3. Label a point on the circle point *A*.
4. Without changing the compass setting, put the sharp point of the compass on *A*. Draw an arc to intersect the circle at one point. Label the point of intersection *B*.
5. Put the sharp point of the compass on point *B*. Without changing the compass setting, draw an arc to intersect the circle at one point. Label the point of intersection *C*.
6. Continue around the circle, labeling points *D*, *E*, and *F*. Be sure not to change the compass setting.
7. Use a straightedge to connect points *A* and *B*, *B* and *C*, *C* and *D*, *D* and *E*, *E* and *F*, and *F* and *A*.

Do not erase any of your markings.

Hexagon *ABCDEF* is regular and is inscribed in circle *X*.

Common Errors/Misconceptions

- inappropriately changing the compass setting
- attempting to measure lengths and angles with rulers and protractors
- not creating large enough arcs to find the points of intersection
- not extending segments long enough to find the vertices of the hexagon

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Instruction

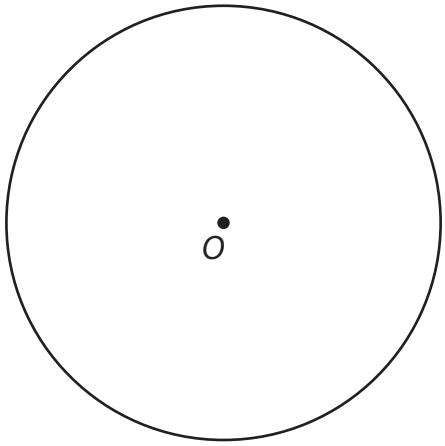
Guided Practice 5.4.3

Example 1

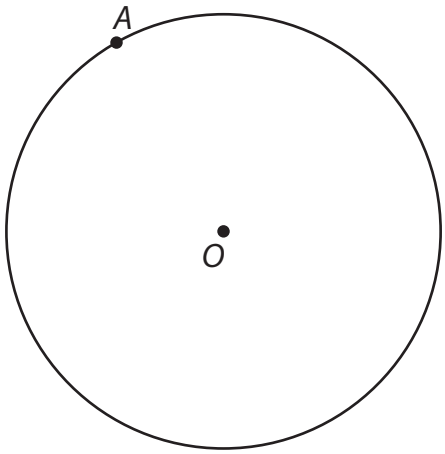
Construct regular hexagon *ABCDEF* inscribed in circle *O* using Method 1.

1. Construct circle *O*.

   Mark the location of the center point of the circle, and label the point *O*. Construct a circle with the sharp point of the compass on the center point.



2. Label a point on the circle point *A*.



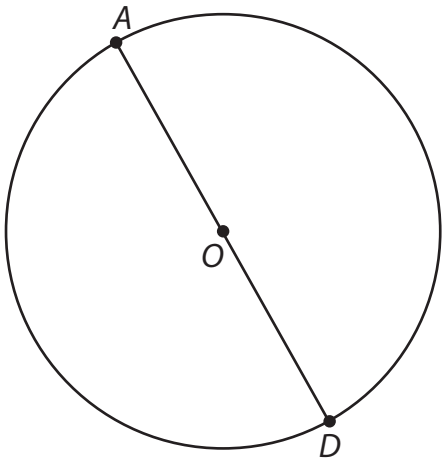
SWB p. 111; SRB p. U5-133

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Instruction

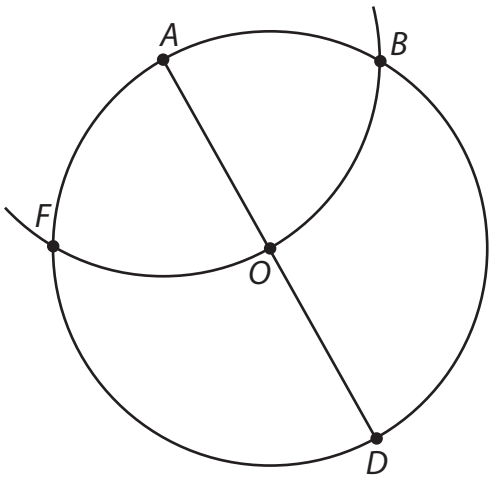
3. Construct the diameter of the circle.

   Use a straightedge to connect point *A* and the center point, *O*. Extend the line through the circle, creating the diameter of the circle. Label the second point of intersection *D*.



4. Locate two vertices on either side of point *A*.

   Without changing the compass setting, put the sharp point of the compass on point *A*. Draw an arc to intersect the circle at two points. Label the points *B* and *F*.

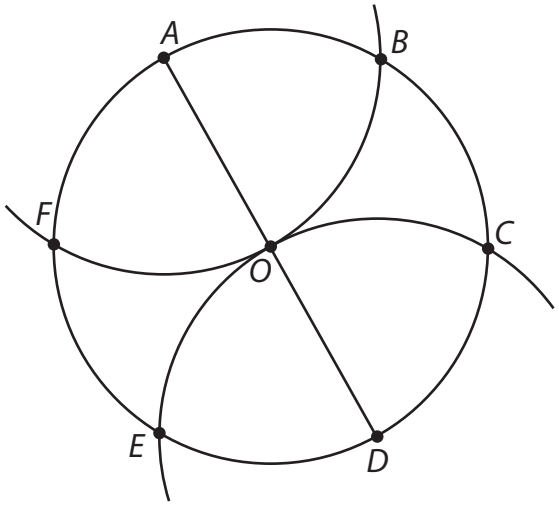


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Instruction

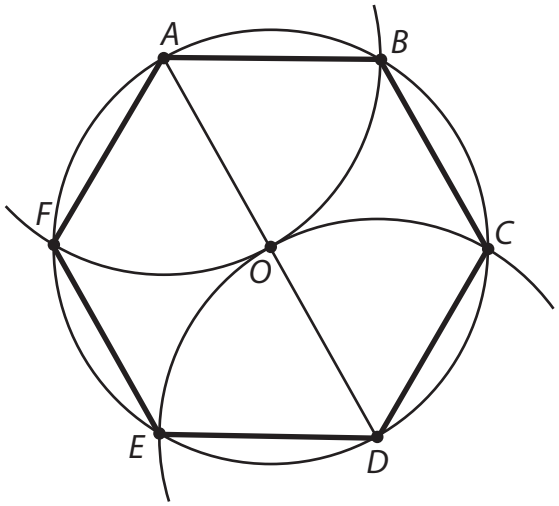
5. Locate two vertices on either side of point *D*.

   Without changing the compass setting, put the sharp point of the compass on point *D*. Draw an arc to intersect the circle at two points. Label the points *C* and *E*.



6. Construct the sides of the hexagon.

   Use a straightedge to connect *A* and *B*, *B* and *C*, *C* and *D*, *D* and *E*, *E* and *F*, and *F* and *A*. Do not erase any of your markings.



Hexagon *ABCDEF* is a regular hexagon inscribed in circle *O*.

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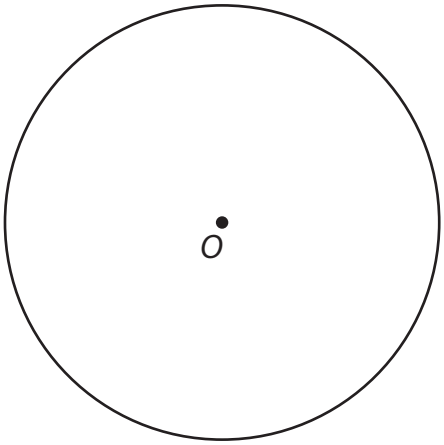
Instruction

Example 2

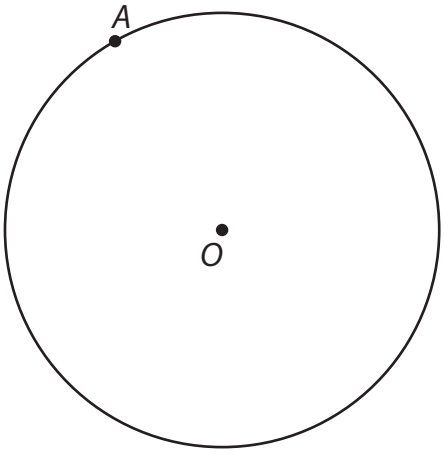
Construct regular hexagon *ABCDEF* inscribed in circle *O* using Method 2.

1. Construct circle *O*.

   Mark the location of the center point of the circle, and label the point *O*. Construct a circle with the sharp point of the compass on the center point.



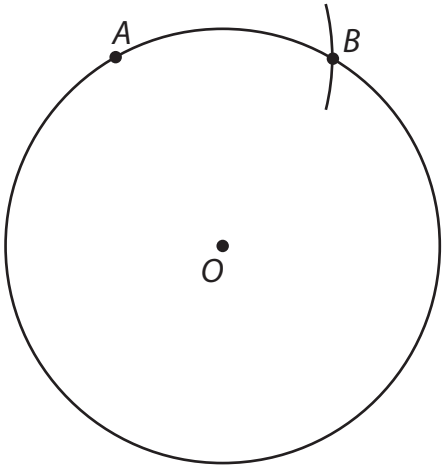
2. Label a point on the circle point *A*.



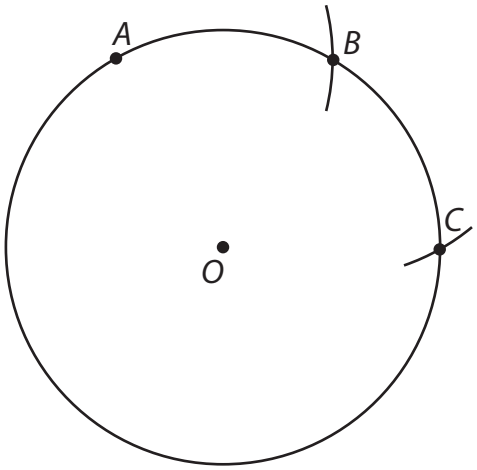
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3. Locate the remaining vertices.

   Without changing the compass setting, put the sharp point of the compass on *A*. Draw an arc to intersect the circle at one point. Label the point of intersection *B*.



   Put the sharp point of the compass on point *B*. Without changing the compass setting, draw an arc to intersect the circle at one point. Label the point of intersection *C*.

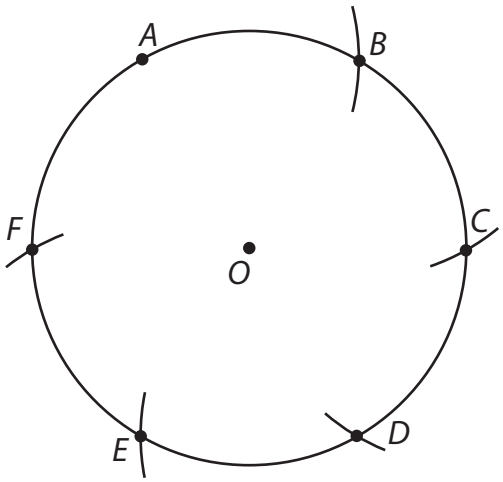


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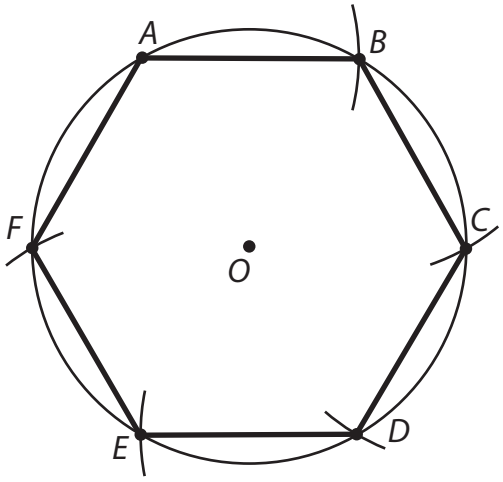
Instruction

Continue around the circle, labeling points *D*, *E*, and *F*. Be sure not to change the compass setting.



4. Construct the sides of the hexagon.

   Use a straightedge to connect *A* and *B*, *B* and *C*, *C* and *D*, *D* and *E*, *E* and *F*, and *F* and *A*. Do not erase any of your markings.



Hexagon *ABCDEF* is a regular hexagon inscribed in circle *O*.

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Instruction

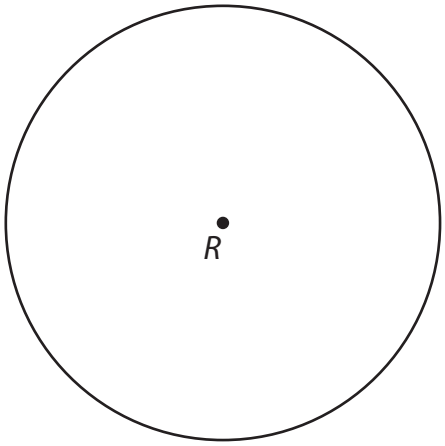
Example 3

Construct regular hexagon *LMNOPQ* inscribed in circle *R* using Method 1. Use the length of $\overline{RL}$ as the radius for circle *R*.

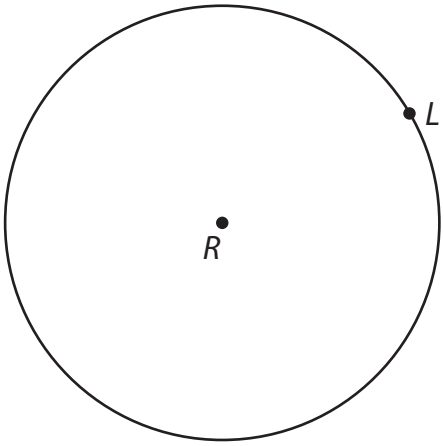


1. Construct circle *R*.

   Mark the location of the center point of the circle, and label the point *R*. Set the opening of the compass equal to the length of $\overline{RL}$. Put the sharp point of the circle on *R* and construct a circle.



2. Label a point on the circle point *L*.

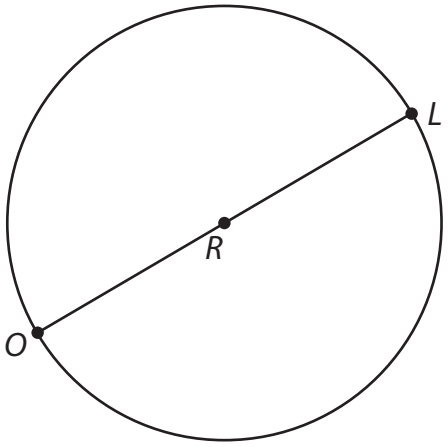


© Walch Education

Instruction

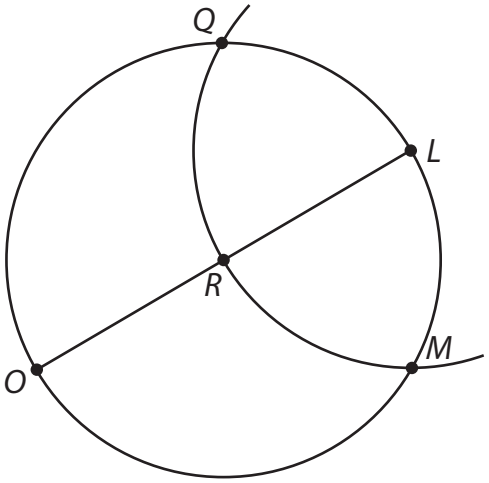
3. Construct the diameter of the circle.

   Use a straightedge to connect point *L* and the center point, *R*. Extend the line through the circle, creating the diameter of the circle. Label the second point of intersection *O*.



4. Locate two vertices on either side of point *L*.

   Without changing the compass setting, put the sharp point of the compass on point *L*. Draw an arc to intersect the circle at two points. Label the points *M* and *Q*.

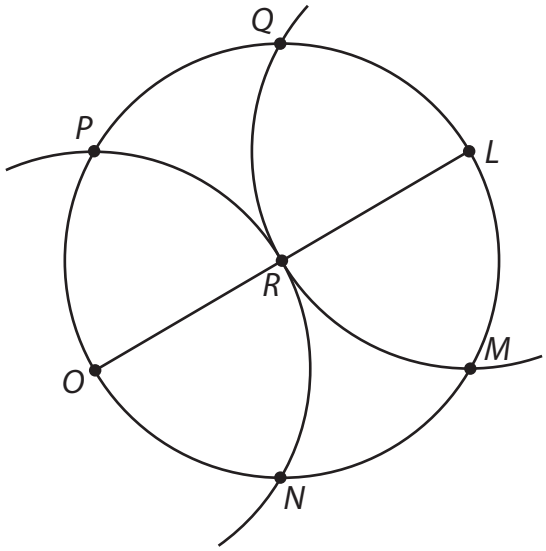


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Instruction

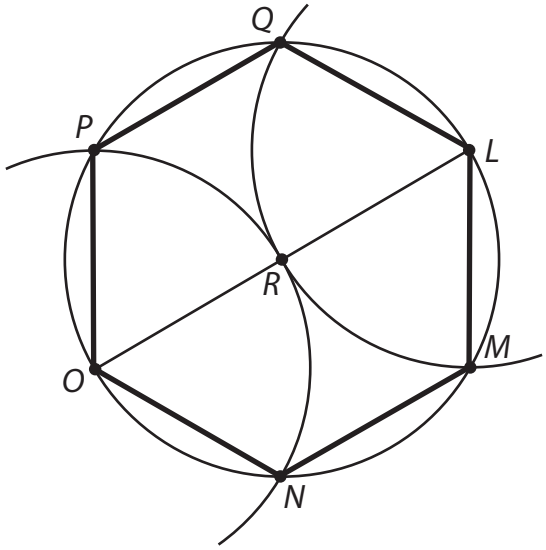
5. Locate two vertices on either side of point *O*.

   Without changing the compass setting, put the sharp point of the compass on point *O*. Draw an arc to intersect the circle at two points. Label the points *P* and *N*.



6. Construct the sides of the hexagon.

   Use a straightedge to connect *L* and *M*, *M* and *N*, *N* and *O*, *O* and *P*, *P* and *Q*, and *Q* and *L*. Do not erase any of your markings.



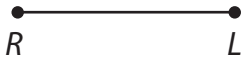
   Hexagon *LMNOPQ* is a regular hexagon inscribed in circle *R*.

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Instruction

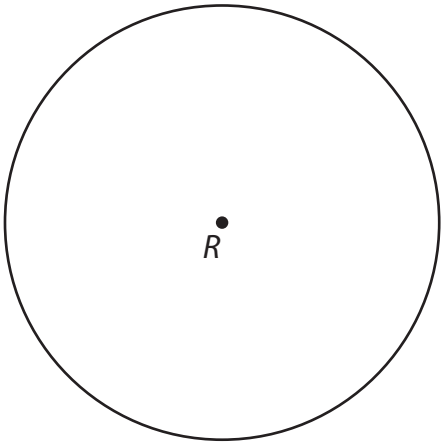
Example 4

Construct regular hexagon *LMNOPQ* inscribed in circle *R* using Method 2. Use the length of $\overline{RL}$ as the radius for circle *R*.

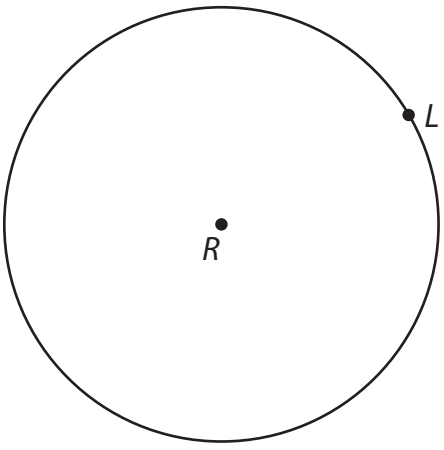


1. Construct circle *R*.

   Mark the location of the center point of the circle, and label the point *R*. Set the opening of the compass equal to the length of $\overline{RL}$. Put the sharp point of the circle on *R* and construct a circle.



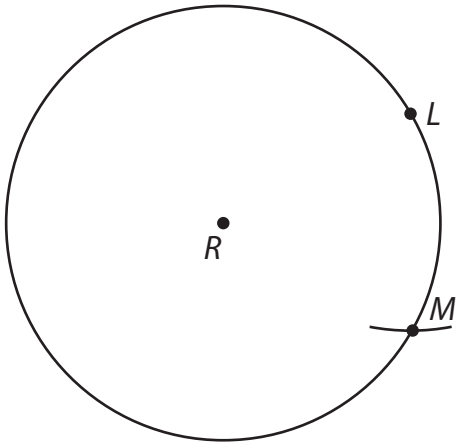
2. Label a point on the circle point *L*.



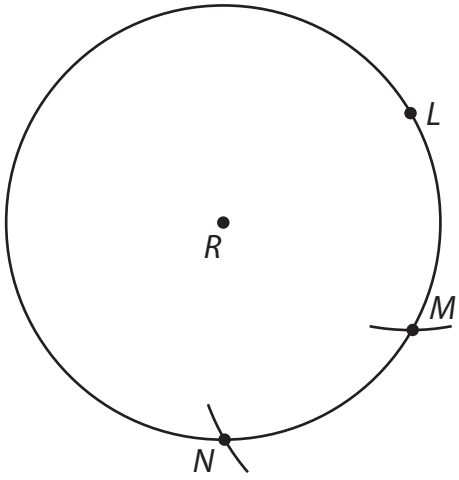
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3. Locate the remaining vertices.

   Without changing the compass setting, put the sharp point of the compass on *L*. Draw an arc to intersect the circle at one point. Label the point of intersection *M*.



   Put the sharp point of the compass on point *M*. Without changing the compass setting, draw an arc to intersect the circle at one point. Label the point of intersection *N*.

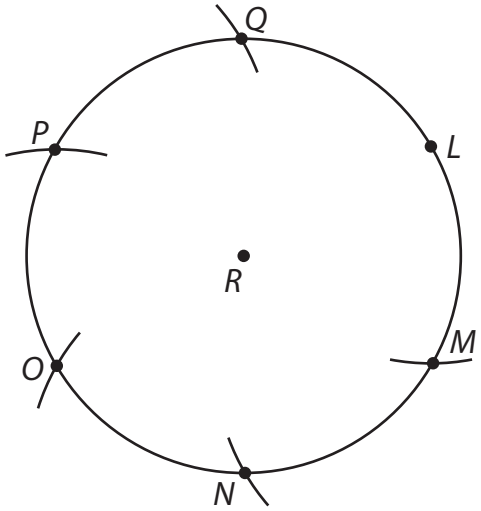


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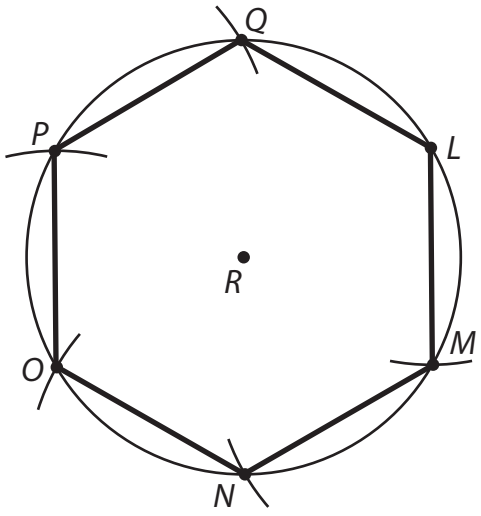
Instruction

Continue around the circle, labeling points *O*, *P*, and *Q*. Be sure not to change the compass setting.



4. Construct the sides of the hexagon.

   Use a straightedge to connect *L* and *M*, *M* and *N*, *N* and *O*, *O* and *P*, *P* and *Q*, and *Q* and *L*. Do not erase any of your markings.



Hexagon *LMNOPQ* is a regular hexagon inscribed in circle *R*.

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Problem-Based Task 5.4.3: Constructing a Regular Dodecagon

SMP	
1 ✓	2
3	4
5 ✓	6
7	8 ✓

A regular dodecagon is a polygon with 12 sides that are equal in length and 12 angles that each measure 150°. How could you construct a regular dodecagon?



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Problem-Based Task 5.4.3: Constructing a Regular Dodecagon

Coaching

a. How many sides does a regular hexagon have?

b. How does the number of sides of a regular hexagon compare to the number of sides of a regular dodecagon?

c. What is the process for constructing a regular hexagon?

d. Which basic construction method could you use to construct the 12 sides of the regular dodecagon from the sides of the regular hexagon inscribed in the circle?

e. Use your method to construct a regular dodecagon inscribed in a circle.

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Instruction

Problem-Based Task 5.4.3: Constructing a Regular Dodecagon

Coaching Sample Responses

a. How many sides does a regular hexagon have?

A regular hexagon has six sides.

b. How does the number of sides of a regular hexagon compare to the number of sides of a regular dodecagon?

A regular hexagon has six sides, whereas a regular dodecagon has 12 sides.

The number of sides of a regular hexagon is half the number of sides of a regular dodecagon.

c. What is the process for constructing a regular hexagon?

There are two methods for constructing a regular hexagon. Either method is correct.

Method 1

- Mark the location of the center point of the circle. Label the point *X*.
- Construct a circle with the sharp point of the compass on the center point.
- Label a point on the circle point *A*.
- Use a straightedge to connect point *A* and point *X*. Extend the line through the circle, creating the diameter of the circle. Label the second point of intersection *D*.
- Without changing the compass setting, put the sharp point of the compass on *A*. Draw an arc to intersect the circle at two points. Label the points *B* and *F*.
- Put the sharp point of the compass on *D*. Without changing the compass setting, draw an arc to intersect the circle at two points. Label the points *C* and *E*.
- Use a straightedge to connect points *A* and *B*, *B* and *C*, *C* and *D*, *D* and *E*, *E* and *F*, and *F* and *A*.
- Hexagon *ABCDEF* is a regular hexagon inscribed in circle *X*.

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Method 2

- Mark the location of the center point of the circle. Label the point *X*.
- Construct a circle with the sharp point of the compass on the center point.
- Label a point on the circle point *A*.
- Without changing the compass setting, put the sharp point of the compass on *A*. Draw an arc to intersect the circle at one point. Label the point of intersection *B*.
- Put the sharp point of the compass on point *B*. Without changing the compass setting, draw an arc to intersect the circle at one point. Label the point of intersection *C*.
- Continue around the circle, labeling points *D*, *E*, and *F*. Be sure not to change the compass setting.
- Use a straightedge to connect points *A* and *B*, *B* and *C*, *C* and *D*, *D* and *E*, *E* and *F*, and *F* and *A*.
- Hexagon *ABCDEF* is a regular hexagon inscribed in circle *X*.

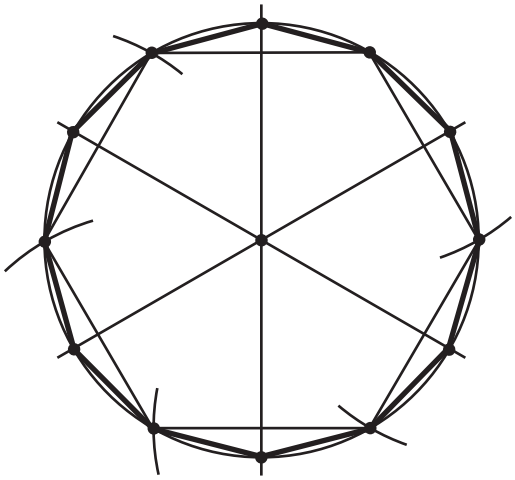
d. Which basic construction method could you use to construct the 12 sides of the regular dodecagon from the sides of the regular hexagon inscribed in the circle?

Bisect each of the sides of the regular hexagon inscribed in the circle. Connect the midpoint of each side of the regular hexagon with the center point of the circle. Extend the segments to the circle and mark the intersection points. Connect the intersection points and the vertices of the regular hexagon to construct a regular dodecagon.

Another possible method to construct a dodecagon is to first bisect the length of the radius. Then set the compass to the distance of half the radius and mark 12 points along the circle to represent each of the vertices of the dodecagon.

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e. Use your method to construct a regular dodecagon inscribed in a circle.



Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

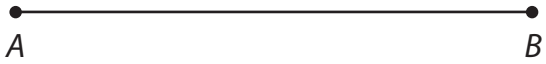
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Practice 5.4.3: Constructing Regular Hexagons Inscribed in Circles

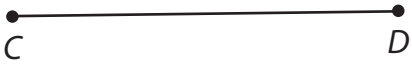
Use a compass and a straightedge to construct each regular hexagon using Method 1.

1. Construct regular hexagon *BCDEFG* inscribed in circle *Z*.

2. Construct regular hexagon *HJKLMN* inscribed in circle *Y* with radius $\overline{AB}$.



3. Construct regular hexagon *PQRSTU* inscribed in circle *X* with radius $\overline{CD}$.



4. Construct regular hexagon *DEFGHJ* inscribed in circle *W* with the radius equal to twice $\overline{EF}$.



5. Construct regular hexagon *RSTUVW* inscribed in circle *A* with the radius equal to one-half $\overline{GH}$.



continued

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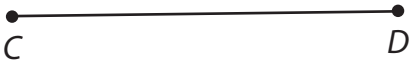
Use a compass and a straightedge to construct each regular hexagon using Method 2.

6. Construct regular hexagon *BCDEFG* inscribed in circle *Z*.

7. Construct regular hexagon *HJKLMN* inscribed in circle *Y* with radius $\overline{AB}$.



8. Construct regular hexagon *PQRSTU* inscribed in circle *X* with radius $\overline{CD}$.



9. Construct regular hexagon *DEFGHJ* inscribed in circle *W* with the radius equal to twice $\overline{EF}$.



10. Construct regular hexagon *RSTUVW* inscribed in circle *A* with the radius equal to one-half $\overline{GH}$.



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Progress Assessment

Circle the letter of the best answer.

1. If a regular hexagon is inscribed in a circle, how many of the vertices must touch the circle?
   a. All of the vertices must touch the circle.
   b. None of the vertices must touch the circle.
   c. At least one vertex must touch the circle.
   d. Only half of the vertices must touch the circle.

2. Which of the following statements is always true?
   a. There is only one way to construct an equilateral triangle inscribed in a circle.
   b. The diameter of a given circle determines the size of the equilateral triangle inscribed in the circle.
   c. It is not possible to construct an equilateral triangle without using a ruler and a protractor.
   d. Equilateral triangles cannot be constructed.

3. What is the first step when constructing an equilateral triangle inscribed in a given circle?
   a. Locate any point on the circle.
   b. Draw a segment between any two points on the circle.
   c. Construct the diameter of the circle.
   d. Bisect the diameter of the circle.

4. After locating a diameter of a given circle, what is the next step in constructing a square inscribed in the circle?
   a. Draw a segment between two points on the circle.
   b. Place the sharp point of the compass on one endpoint of the diameter and swing an arc the same length as the radius of the circle.
   c. Place the sharp point of the compass on one endpoint of the diameter and swing an arc larger than the radius of the circle.
   d. Place the sharp point of the compass on one endpoint of the diameter and swing an arc of random length through two points on the circle.

continued

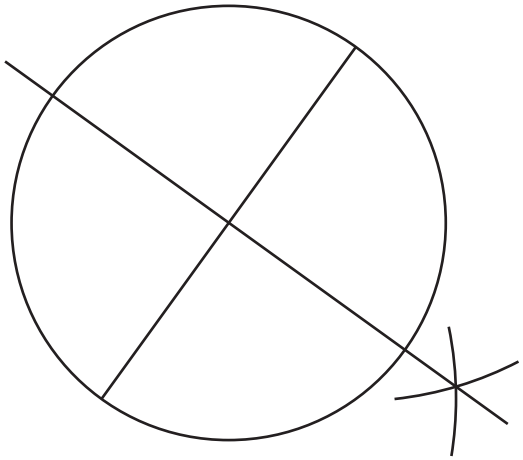
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Assessment

5. The same process used for constructing a regular hexagon inscribed in a given circle can be used to construct which of the following figures inscribed in a given circle?

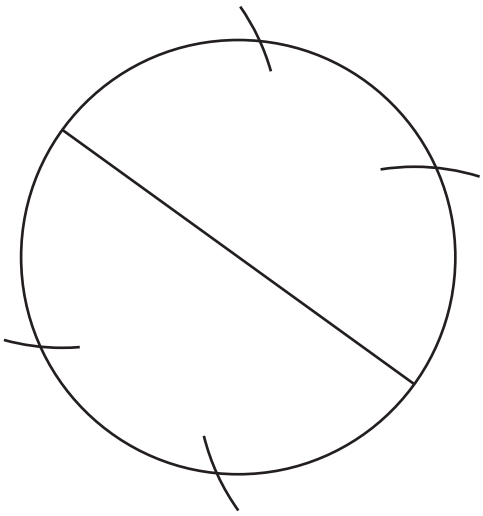
   a. an isosceles triangle
   b. a square
   c. a pentagon
   d. an equilateral triangle

6. The following construction is incomplete. Based on the markings shown, what figure will be constructed?



   a. an equilateral triangle
   b. a square
   c. a hexagon
   d. a dodecagon

7. The following construction is incomplete. Based on the markings shown, what figure will be constructed?

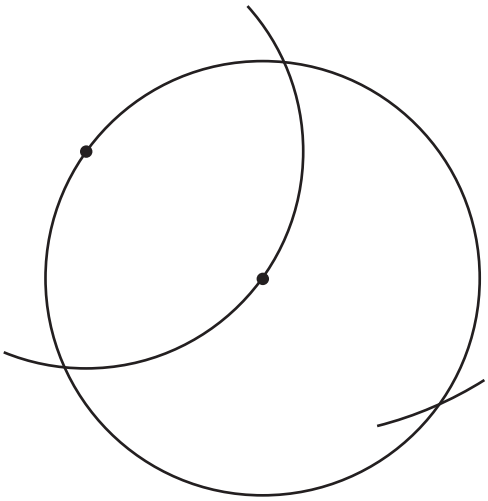


   a. an equilateral triangle
   b. a square
   c. a hexagon
   d. a dodecagon

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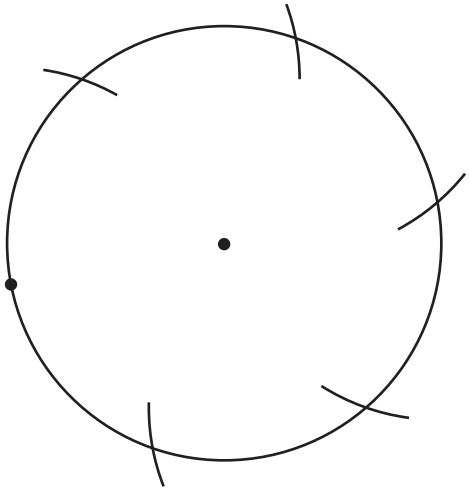
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8. The following construction is incomplete. Based on the markings shown, what figure will be constructed?



a. an equilateral triangle
b. a square
c. a hexagon
d. a dodecagon

9. The following construction is incomplete. Based on the markings shown, what figures will be constructed?



a. an equilateral triangle and a square
b. a square and a hexagon
c. an equilateral triangle and a hexagon
d. a square and an octagon

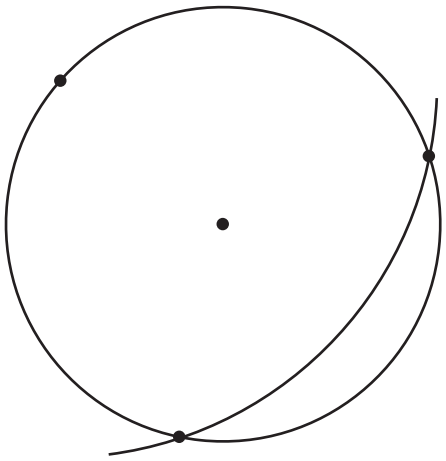
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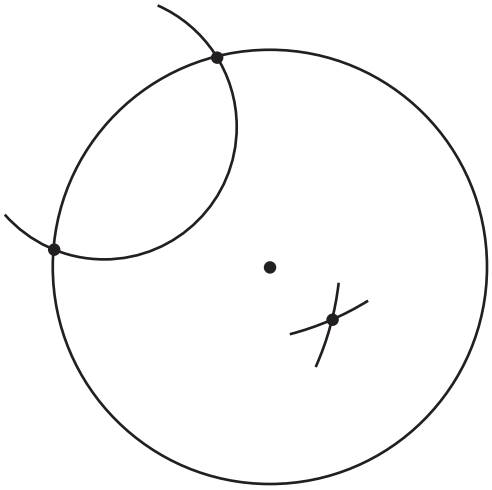
Assessment

10. Which incomplete construction shows the location of the third vertex of an equilateral triangle inscribed in a circle?

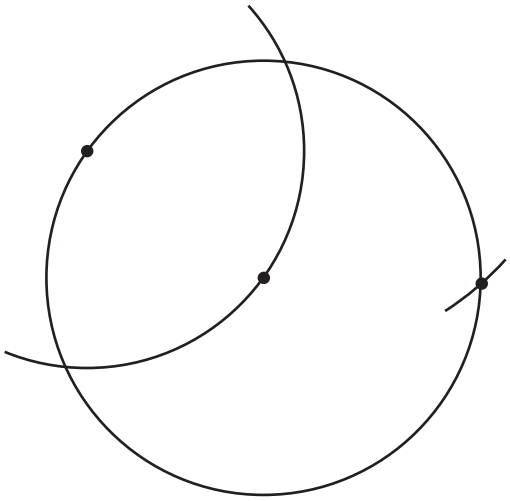
a.



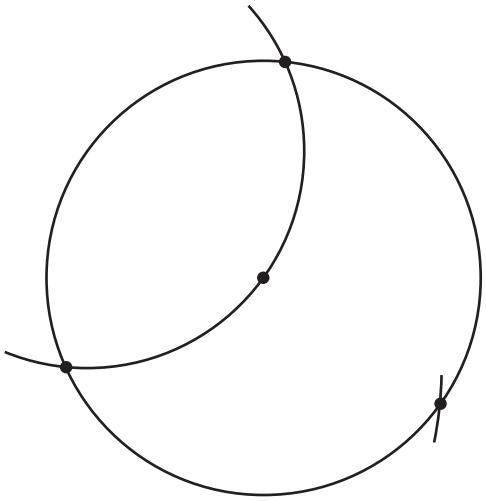
c.



b.



d.



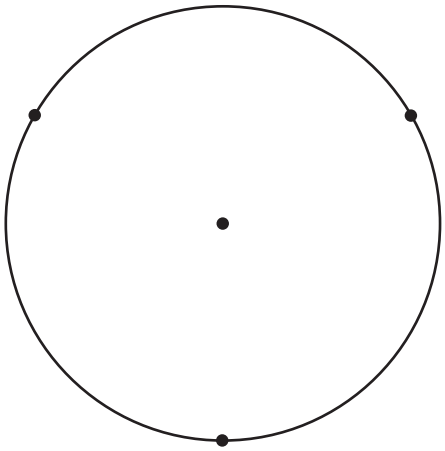
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Assessment

Use what you have learned about constructions to solve the following problem.

11. Jarno determined the placement of 3 vending machines around a circular skating rink. The distance between each machine was the same. As a result of the success of the machines, Jarno was asked to place 3 recycling bins around the skating rink, one between each of the machines. The distance between each vending machine and recycling bin must be the same.



a. Use a compass and a straightedge to determine where Jarno should place the recycling bins.

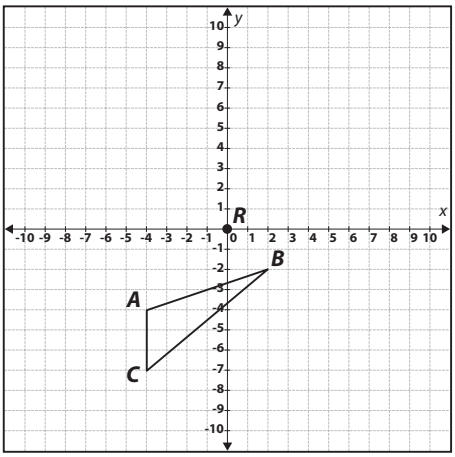
b. Describe the procedure for determining the location of the recycling bins.

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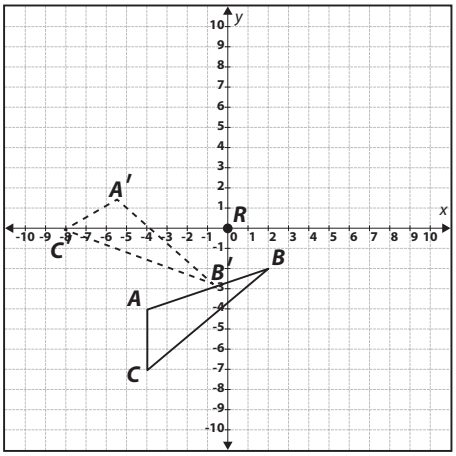
Pre-Assessment

Circle the letter of the best answer.

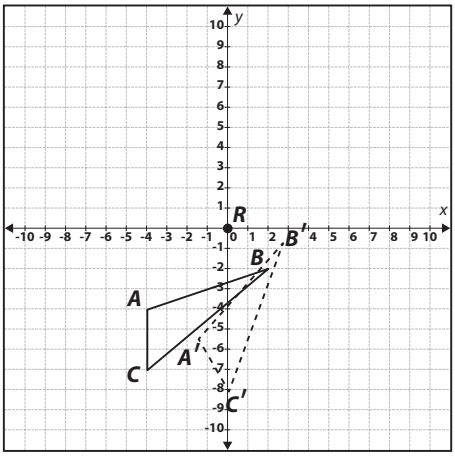
1. Which illustration predicts the effect(s) of rotating the given triangle 60° counterclockwise about *R*?



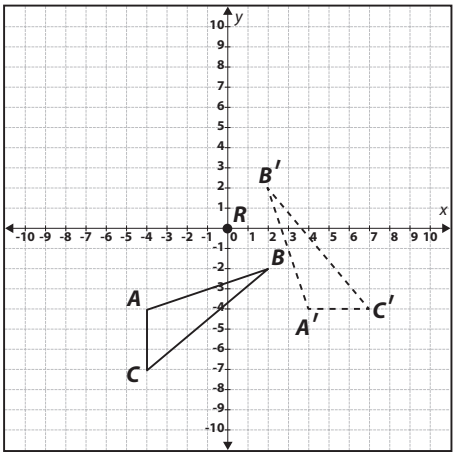
a.



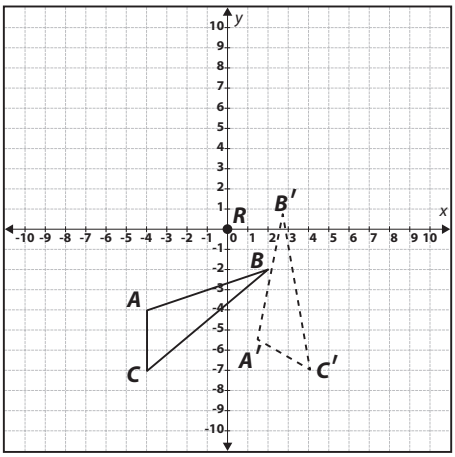
c.



b.



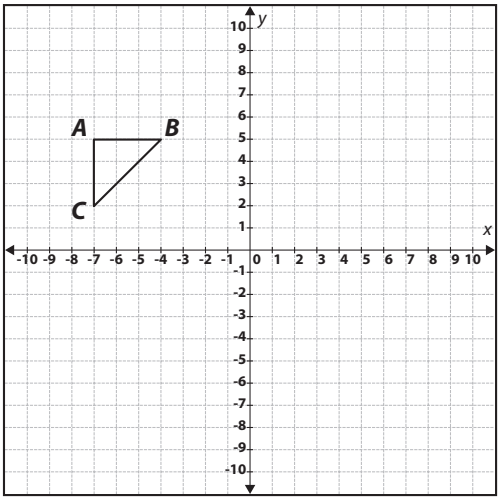
d.



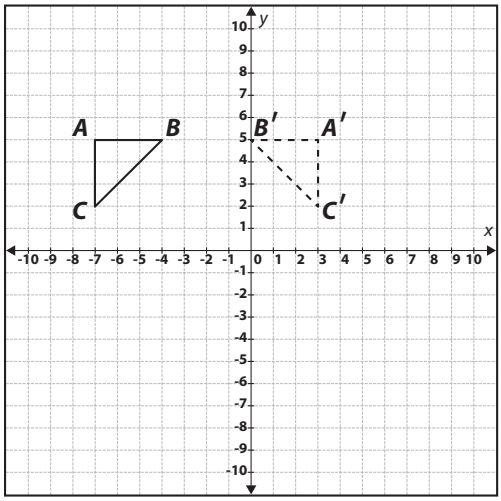
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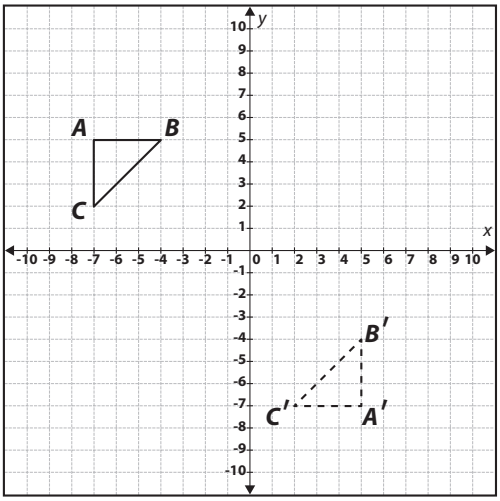
2. Which illustration predicts the effect(s) of reflecting the given triangle over $x = -2$?



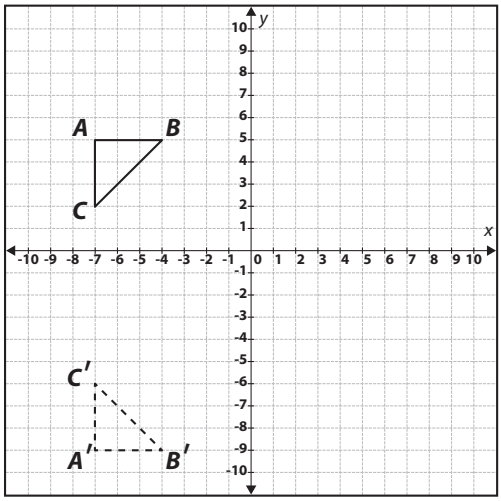
a.



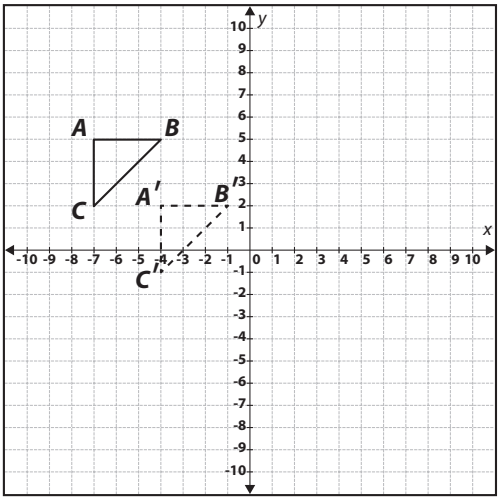
c.



b.



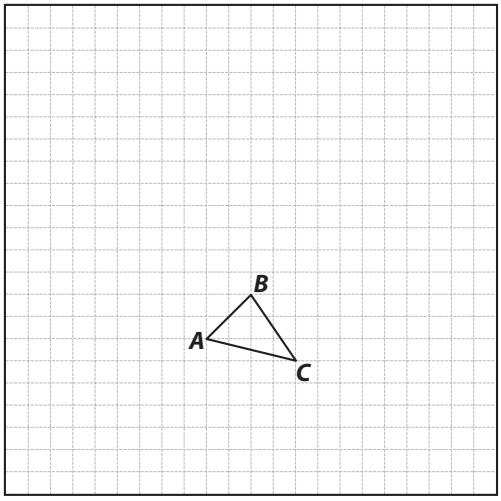
d.



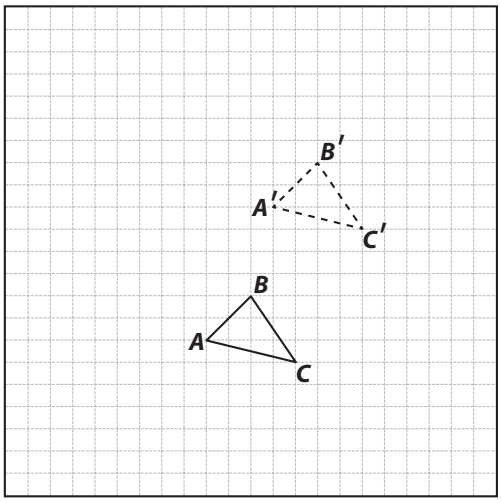
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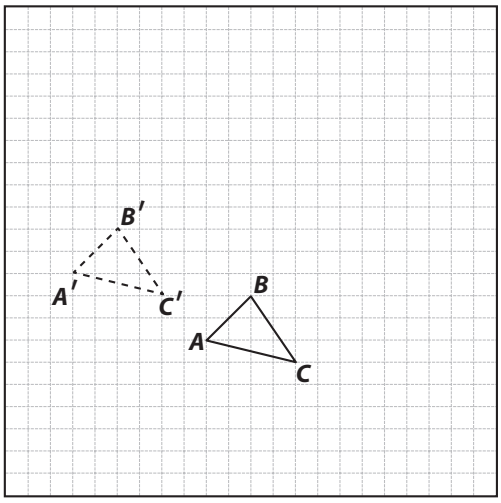
3. Which illustration predicts the effect(s) of translating the given triangle to the left 3 units and up 6 units?



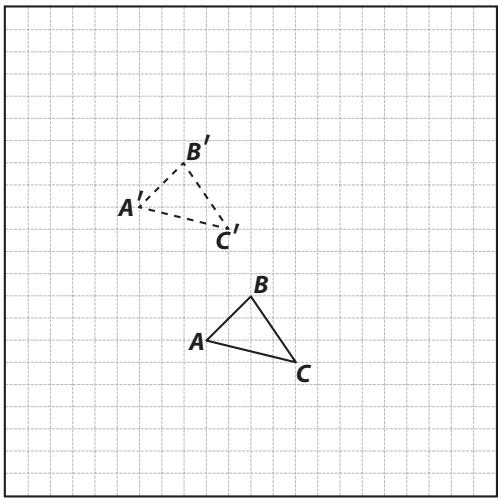
a.



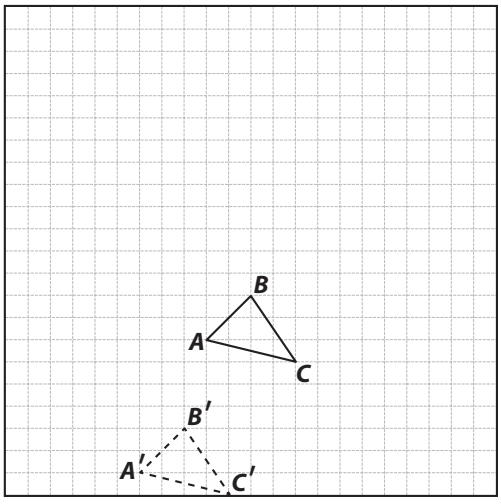
c.



b.



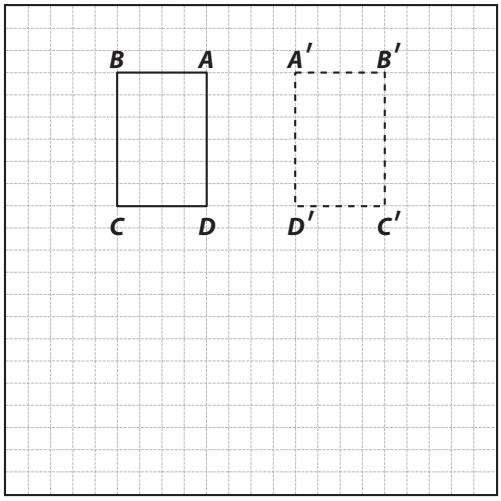
d.



continued

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4. Are the two figures below congruent? Why or why not?



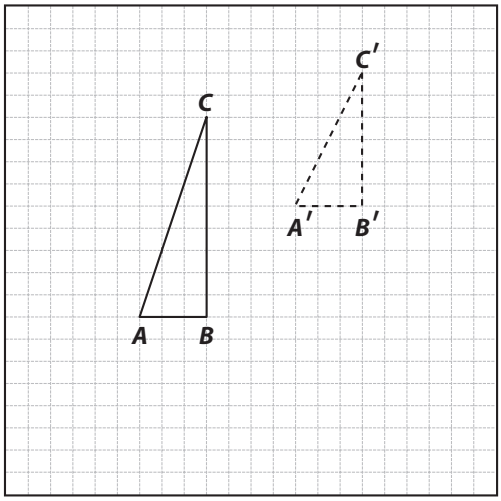
a. Not congruent; $\square ABCD$ has undergone a rigid motion of a reflection.

b. Congruent; $\square ABCD$ has undergone a non-rigid motion of a rotation.

c. Not congruent; $\square ABCD$ has undergone a rigid motion of a rotation.

d. Congruent; $\square ABCD$ has undergone a rigid motion of a reflection.

5. Are the two figures below congruent? Why or why not?



a. Not congruent; $\triangle ABC$ has undergone a non-rigid motion of a vertical stretch.

b. Congruent; $\triangle ABC$ has undergone a non-rigid motion of a dilation.

c. Not congruent; $\triangle ABC$ has undergone a non-rigid motion of a vertical compression.

d. Congruent; $\triangle ABC$ has undergone a rigid motion of a reflection.

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Lesson 5: Exploring Congruence

Instruction

Common Core State Standard

G–CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

SMP	
1	2 ✓
3 ✓	4 ✓
5	6 ✓
7 ✓	8

Essential Questions

1. What are the differences between rigid and non-rigid motions?
2. How do you identify a transformation as a rigid motion?
3. How do you identify a transformation as a non-rigid motion?

WORDS TO KNOW

angle of rotation the angle through which a figure is rotated about a center point; the measure of the angle created by connecting the center of rotation to a point on the preimage and to the corresponding point on the image

clockwise rotating a figure in the direction that the hands on a clock move

compression a transformation in which a figure becomes smaller; compressions may be horizontal (affecting only horizontal lengths), vertical (affecting only vertical lengths), or both

congruency transformation a transformation in which a geometric figure moves but keeps the same size and shape

congruent having the same shape, size, lines, and angles; the symbol for representing congruency between figures is $\cong$

corresponding angles angles of two figures that lie in the same position relative to the figure. In transformations, the corresponding angles are the preimage and image angles, so $\angle A$ and $\angle A'$ are corresponding angles and so on.

corresponding sides sides of two figures that lie in the same position relative to the figure. In transformations, the corresponding sides are the preimage and image sides, so $\overline{AB}$ and $\overline{A'B'}$ are corresponding sides and so on.

counterclockwise rotating a figure in the opposite direction that the hands on a clock move

dilation a transformation in which a figure is either enlarged or reduced by a scale factor in relation to a center point

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equidistant	the same distance from a reference point
image	the new, resulting figure after a transformation
isometry	a transformation in which the preimage and image are congruent
line of reflection	the perpendicular bisector of the segments that connect the corresponding vertices of the preimage and the image
non-rigid motion	a transformation done to a figure that changes the figure's shape and/or size
point of rotation	the fixed location that an object is turned around; the point can lie on, inside, or outside the figure
preimage	the original figure before undergoing a transformation
rigid motion	a transformation done to a figure that maintains the figure's shape and size or its segment lengths and angle measures
scale factor	a multiple of the lengths of the sides from one figure to the transformed figure. If the scale factor is larger than 1, then the figure is enlarged. If the scale factor is between 0 and 1, then the figure is reduced.

Recommended Resources

- MathIsFun.com. "Rotation."

  http://www.walch.com/rr/00010

  This website explains what a rotation is, and then gives the opportunity to experiment with rotating different shapes about a point of rotation and an angle. There are links at the bottom of the page for translations and reflections with similar applets.

- Math Open Reference. "Dilation—of a polygon."

  http://www.walch.com/rr/00011

  This site gives a description of a dilation and provides an applet with a slider, allowing users to explore dilating a rectangle with different scale factors. The website goes on to explain how to create a dilation of a polygon.

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Lesson 5.5.1: Describing Rigid Motions and Predicting the Effects

Warm-Up 5.5.1

Before the digital age, printing presses were used to create text products such as newspapers, brochures, and any other mass-produced, printed material. Printing presses used printing blocks that were the reflection of the image to be printed. Some antique collectors seek out hand-carved printing blocks.

An antique poster of the printed letter "L" was created using a printing block. The "L" has the coordinates *A* (2, 5), *B* (3, 5), *C* (3, 2), *D* (5, 2), *E* (5, 1), and *F* (2, 1). Use this information to solve the following problems.

1. What are the coordinates of the printing block through $r_{x\text{-axis}}$?

2. Graph the preimage and the image.

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Instruction

Lesson 5.5.1: Describing Rigid Motions and Predicting the Effects

Common Core State Standard

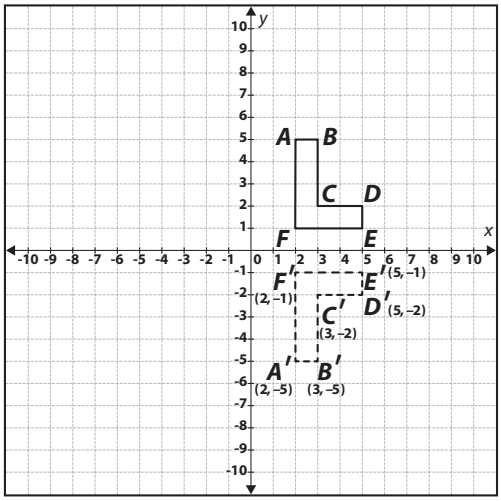
G–CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

Warm-Up 5.5.1 Debrief

1. What are the coordinates of the printing block through $r_{x\text{-axis}}$?

$r_{x\text{-axis}}(x, y) = (x, -y)$	
$r_{x\text{-axis}}(A\ (2, 5)) = A'\ (2, -5)$	$r_{x\text{-axis}}(D\ (5, 2)) = D'\ (5, -2)$
$r_{x\text{-axis}}(B\ (3, 5)) = B'\ (3, -5)$	$r_{x\text{-axis}}(E\ (5, 1)) = E'\ (5, -1)$
$r_{x\text{-axis}}(C\ (3, 2)) = C'\ (3, -2)$	$r_{x\text{-axis}}(F\ (2, 1)) = F'\ (2, -1)$

2. Graph the preimage and the image.



Connection to the Lesson

- Students will be performing reflections in the lesson, but will use lines of reflection other than the *x*-axis, *y*-axis, and the line $y = x$.
- Students will use geometric constructions to generate rigid motions rather than using functions.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- constructing perpendicular bisectors (G–CO.12)
- copying a segment (G–CO.12)
- copying an angle (G–CO.12)

Introduction

Think about trying to move a drop of water across a flat surface. If you try to push the water droplet, it will smear, stretch, and transfer onto your finger. The water droplet, a liquid, is not rigid. Now think about moving a block of wood across the same flat surface. A block of wood is solid or rigid, meaning it maintains its shape and size when you move it. You can push the block and it will keep the same size and shape as it moves. In this lesson, we will examine **rigid motions**, which are transformations done to a figure that maintain the figure's shape and size or its segment lengths and angle measures.

Key Concepts

- Rigid motions are transformations that don't affect an object's shape and size. This means that corresponding sides and corresponding angle measures are preserved.
- When angle measures and sides are preserved, they are congruent, which means they have the same shape and size.
- The congruency symbol ($\cong$) is used to show that two figures are congruent.
- The figure before the transformation is called the preimage.
- The figure after the transformation is the image.
- **Corresponding sides** are the sides of two figures that lie in the same position relative to the figure. In transformations, the corresponding sides are the preimage and image sides, so $\overline{AB}$ and $\overline{A'B'}$ are corresponding sides and so on.
- **Corresponding angles** are the angles of two figures that lie in the same position relative to the figure. In transformations, the corresponding angles are the preimage and image angles, so $\angle A$ and $\angle A'$ are corresponding angles and so on.
- Transformations that are rigid motions are translations, reflections, and rotations.
- Transformations that are not rigid motions are dilations, vertical stretches or compressions, and horizontal stretches or compressions.

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Instruction

Translations

- A translation is sometimes called a slide.
- In a translation, the figure is moved horizontally and/or vertically.
- The orientation of the figure remains the same.
- Connecting the corresponding vertices of the preimage and image will result in a set of parallel line segments.

Translating a Figure Given the Horizontal and Vertical Shift

1. Place your pencil on a vertex and count over horizontally the number of units the figure is to be translated.
2. Without lifting your pencil, count vertically the number of units the figure is to be translated.
3. Mark the image vertex on the coordinate plane.
4. Repeat this process for all vertices of the figure.
5. Connect the image vertices.

Reflections

- A reflection creates a mirror image of the original figure over a reflection line.
- A reflection line can pass through the figure, be on the figure, or be outside the figure.
- Reflections are sometimes called flips.
- The orientation of the figure is changed in a reflection.
- In a reflection, the corresponding vertices of the preimage and image are equidistant from the line of reflection, meaning the distance from each vertex to the line of reflection is the same.
- The **line of reflection** is the perpendicular bisector of the segments that connect the corresponding vertices of the preimage and the image.

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Reflecting a Figure over a Given Reflection Line
1. Draw the reflection line on the same coordinate plane as the figure. 2. If the reflection line is vertical, count the number of horizontal units one vertex is from the line and count the same number of units on the opposite side of the line. Place the image vertex there. Repeat this process for all vertices. 3. If the reflection line is horizontal, count the number of vertical units one vertex is from the line and count the same number of units on the opposite side of the line. Place the image vertex there. Repeat this process for all vertices. 4. If the reflection line is diagonal, draw lines from each vertex that are perpendicular to the reflection line extending beyond the line of reflection. Copy each segment from the vertex to the line of reflection onto the perpendicular line on the other side of the reflection line and mark the image vertices. 5. Connect the image vertices.

Rotations

- A rotation moves all points of a figure along a circular arc about a point. Rotations are sometimes called turns.
- In a rotation, the orientation is changed.
- The **point of rotation** can lie on, inside, or outside the figure, and is the fixed location that the object is turned around. This is sometimes called the center of rotation.
- The **angle of rotation** is the angle through which a figure is rotated about a center point. It is the measure of the angle created by connecting the center of rotation to a point on the preimage and to the corresponding point on the image.
- Rotating a figure clockwise moves the figure in a circular arc about the point of rotation in the same direction that the hands move on a clock.
- Rotating a figure counterclockwise moves the figure in a circular arc about the point of rotation in the opposite direction that the hands move on a clock.

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Instruction

Rotating a Figure Given a Point and Angle of Rotation

1. Draw a line from one vertex to the point of rotation.
2. Measure the angle of rotation using a protractor.
3. Draw a ray from the point of rotation extending outward that creates the angle of rotation.
4. Copy the segment connecting the point of rotation to the vertex (created in step 1) onto the ray created in step 3.
5. Mark the endpoint of the copied segment that is not the point of rotation with the letter of the corresponding vertex, followed by a prime mark (′). This is the first vertex of the rotated figure.
6. Repeat the process for each vertex of the figure.
7. Connect the vertices that have prime marks. This is the rotated figure.

Common Errors/Misconceptions

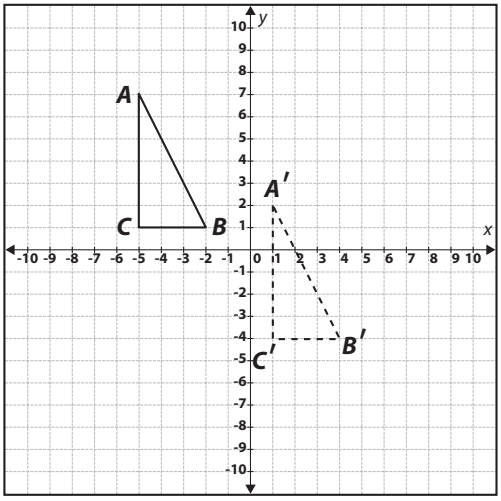
- creating the angle of rotation in a clockwise direction instead of a counterclockwise direction and vice versa
- reflecting a figure about a line other than the one given
- mistaking a rotation for a reflection
- misidentifying a translation as a reflection or a rotation

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Guided Practice 5.5.1

Example 1

Describe the transformation that has taken place in the diagram below.



1. Examine the orientation of the figures to determine if the orientation has changed or stayed the same. Look at the sides of the triangle.

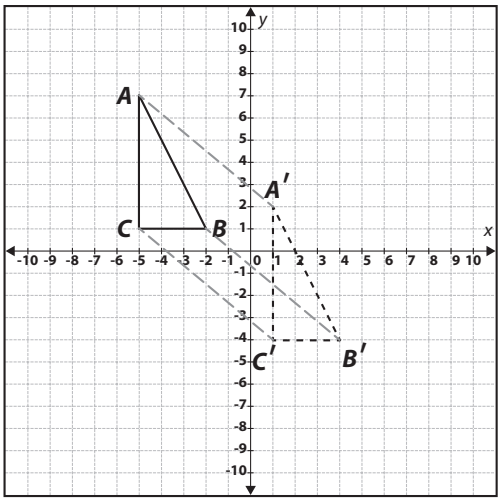
Side length	Preimage orientation	Image orientation
Shortest	Bottom of triangle and horizontal	Bottom of triangle and horizontal
Longest	Right side of triangle going from top left to bottom right	Right side of triangle going from top left to bottom right
Intermediate	Left side of triangle and vertical	Left side of triangle and vertical

The orientation of the triangles has remained the same.

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Instruction

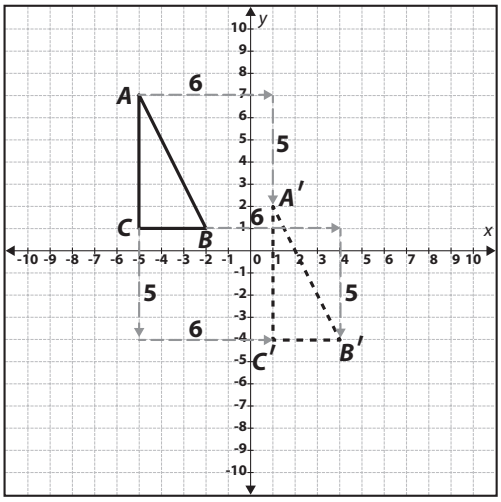
2. Connect the corresponding vertices with line segments.



The segments connecting the corresponding vertices appear to be parallel.

3. Analyze the change in position.

Check the horizontal distance of vertex A. To go from A to A' horizontally, the vertex was shifted to the right 6 units. Vertically, vertex A was shifted down 5 units. Check the remaining two vertices. Each vertex slid 6 units to the right and 5 units down.

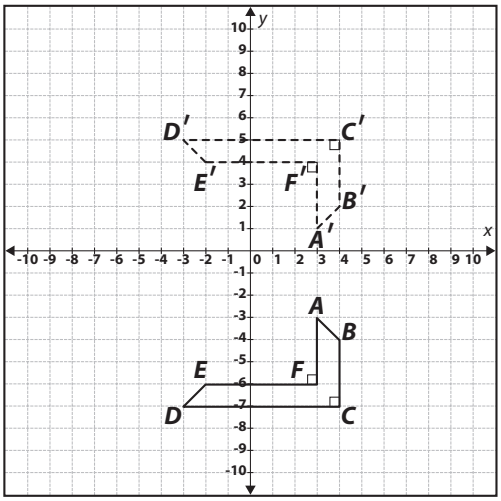


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Instruction

Example 2

Describe the transformation that has taken place in the diagram below.



1. Examine the orientation of the figures to determine if the orientation has changed or stayed the same. Look at the sides of the figures and pick a reference point. A good reference point is the outer right angle of the figure. From this point, examine the position of the "arms" of the figure.

Arm	Preimage orientation	Image orientation
Shorter	Pointing upward from the corner of the figure with a negative slope at the end of the arm	Pointing downward from the corner of the figure with a positive slope at the end of the arm
Longer	Pointing to the left from the corner of the figure with a positive slope at the end of the arm	Pointing to the left from the corner of the figure with a negative slope at the end of the arm

(*continued*)

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Instruction

The orientation of the figures has changed. In the preimage, the outer right angle is in the bottom right-hand corner of the figure, with the shorter arm extending upward. In the image, the outer right angle is on the top right-hand side of the figure, with the shorter arm extending down.

Also, compare the slopes of the segments at the end of the longer arm. The slope of the segment at the end of the arm is positive in the preimage, but in the image the slope of the corresponding arm is negative. A similar reversal has occurred with the segment at the end of the shorter arm. In the preimage, the segment at the end of the shorter arm is negative, while in the image the slope is positive.

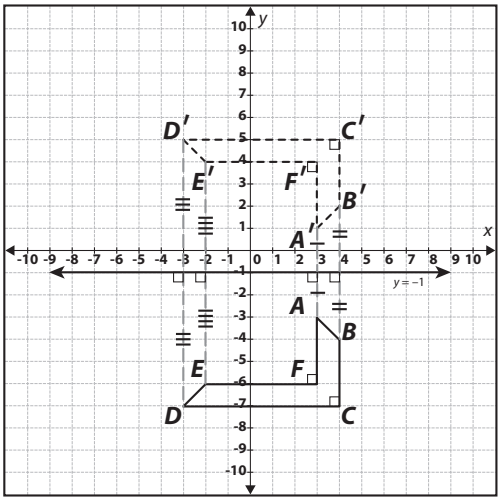
2. Determine the transformation that has taken place.

   Since the orientation has changed, the transformation is either a reflection or a rotation. Since the orientation of the image is the mirror image of the preimage, the transformation is a reflection. The figure has been flipped over a line.

3. Determine the line of reflection.

   Connect some of the corresponding vertices of the figure. Choose one of the segments you created and construct the perpendicular bisector of the segment. Verify that this is the perpendicular bisector for all segments joining the corresponding vertices. This is the line of reflection.

   The line of reflection for this figure is $y = -1$.

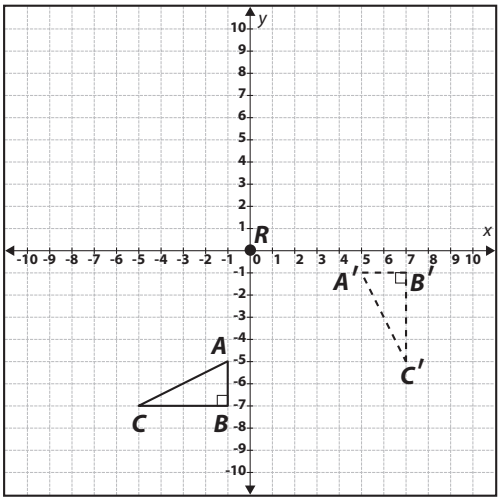


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Instruction

Example 3

Describe the transformation that has taken place in the diagram below.



1. Examine the orientation of the figures to determine if the orientation has changed or stayed the same.

   Look at the sides of the triangle.

Side length	Preimage orientation	Image orientation
Shortest	Right side of triangle and vertical	Top of triangle and horizontal
Longest	Diagonal from bottom left to top right of triangle	Diagonal from top left to bottom right of triangle
Intermediate	Bottom side of triangle and horizontal	Right side of triangle and vertical

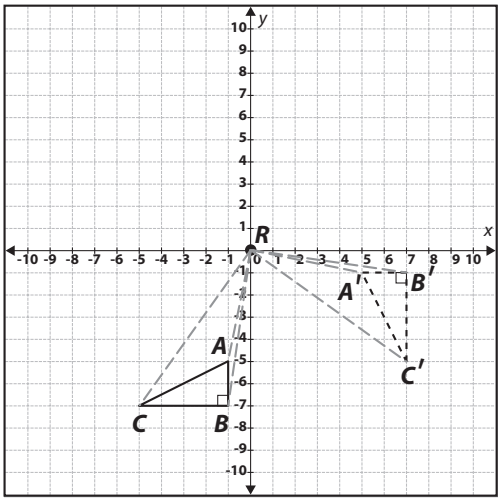
   The orientation of the triangles has changed.

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Instruction

2. Determine the transformation that has taken place.

   Since the orientation has changed, the transformation is either a reflection or a rotation. Since the orientation of the image is NOT the mirror image of the preimage, the transformation is a rotation. The figure has been turned about a point. All angles that are made up of the preimage vertex to the reflection point to the corresponding image vertex are congruent. This means that $\angle ARA' \cong \angle BRB' \cong \angle CRC'$.

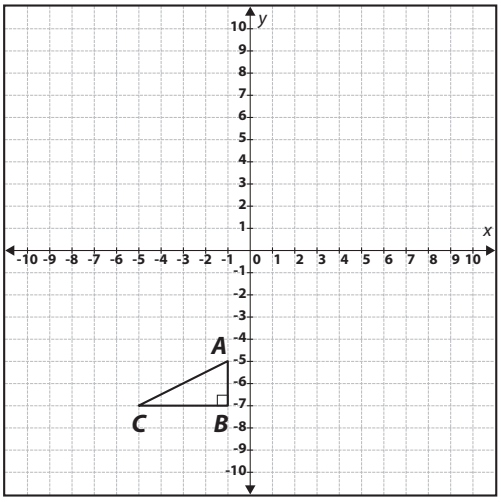


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Instruction

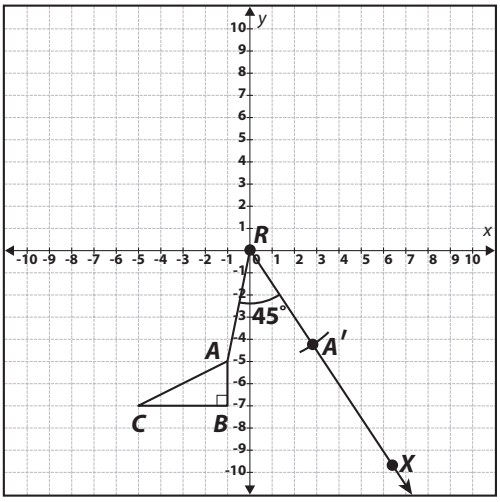
Example 4

Rotate the given figure 45° counterclockwise about the origin.



1. Create the angle of rotation for the first vertex.

   Connect vertex A and the origin with a line segment. Label the point of rotation R. Then, use a protractor to measure a 45° angle. Use the segment from vertex A to the point of rotation R as one side of the angle. Mark a point X at 45°. Draw a ray extending out from point R, connecting R and X. Copy $\overline{AR}$ onto $\overrightarrow{RX}$. Label the endpoint that leads away from the origin A'.

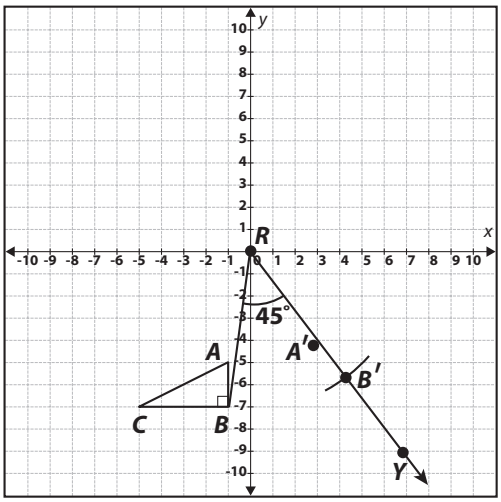


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Instruction

2. Create the angle of rotation for the second vertex.

   Connect vertex B and the origin with a line segment. The point of rotation is still R. Then, use a protractor to measure a 45° angle. Use the segment from vertex B to the point of rotation R as one side of the angle. Mark a point Y at 45°. Draw a ray extending out from point R, connecting R and Y. Copy $\overline{BR}$ onto $\overrightarrow{RY}$. Label the endpoint that leads away from the origin B'.

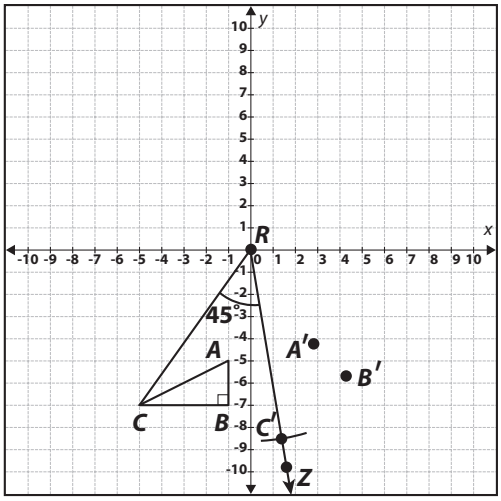


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Instruction

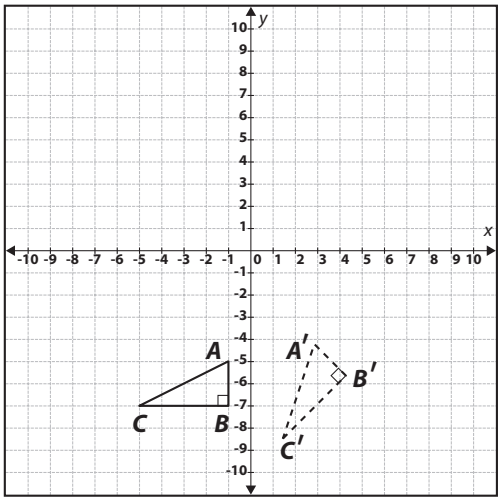
3. Create the angle of rotation for the third vertex.

   Connect vertex C and the origin with a line segment. The point of rotation is still R. Then, use a protractor to measure a 45° angle. Use the segment from vertex C to the point of rotation R as one side of the angle. Mark a point Z at 45°. Draw a ray extending out from point R, connecting R and Z and continuing outward from Z. Copy $\overline{CR}$ onto $\overrightarrow{RZ}$. Label the endpoint that leads away from the origin C'.



4. Connect the rotated points.

   The connected points A', B', and C' form the rotated figure.



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Problem-Based Task 5.5.1: Artifacts Recovered and Restored

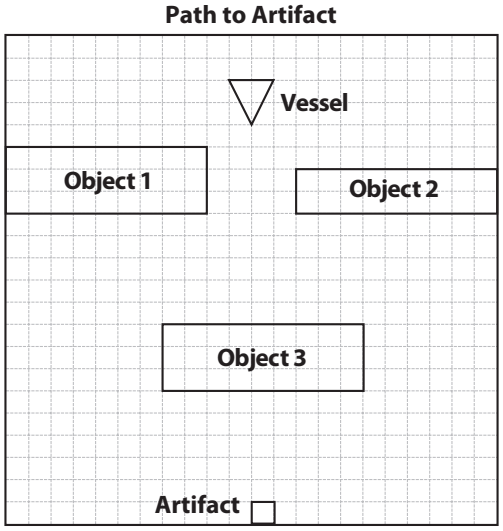
SMP	
1	2 ✓
3	4 ✓
5	6 ✓
7	8

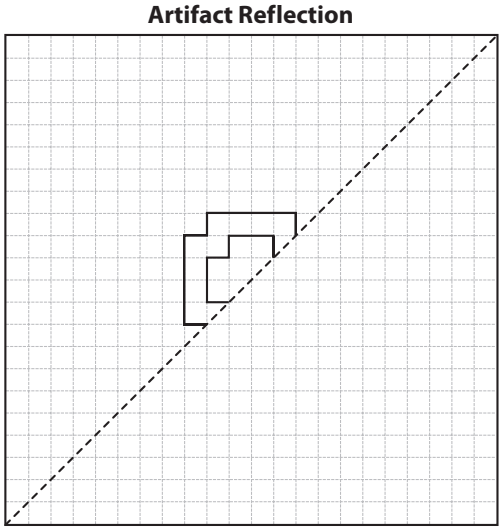
The *Titanic* sank on the morning of April 15, 1912, after hitting an iceberg. In recent years, many artifacts from the ship have been recovered and some have been restored. Your job is to coordinate the recovery and restoration of a particular artifact from the *Titanic*. Your search team will use an unmanned vessel to retrieve the artifact so artists can restore it.

Your team controls the unmanned vessel remotely so it can navigate around objects. The vessel can be translated and rotated, but it cannot touch any of the objects—it must be at least 1 unit away from each item. The point of rotation is at the center of the vessel. The vessel is roughly the shape of a triangle and always points forward when moving. The vessel is currently positioned forward toward Object 3.

To restore the damaged artifact to its original condition, artists will use reflections.

What rigid motions can be used for the unmanned vessel to recover the artifact? What will the artifact look like after it has been restored? Use the following diagrams to answer the questions. The diagram on the left represents a map of the sea floor, including the vessel, the artifact, and objects to avoid. The diagram on the right shows what you expect to recover of the artifact.





What rigid motions must be used for the unmanned vessel to recover the artifact?

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Problem-Based Task 5.5.1: Artifacts Recovered and Restored

Coaching

a. What rigid motion would move the vessel between Objects 1 and 2?

b. Several rigid motions need to be performed to turn and move the vessel to navigate around Object 3. What are they?

c. What rigid motion(s) can be used to navigate toward the artifact?

d. What rigid motion needs to be performed to restore the artifact?

e. What does the artifact look like after this rigid motion is performed?

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Instruction

Problem-Based Task 5.5.1: Artifacts Recovered and Restored

Coaching Sample Responses

a. What rigid motion would move the vessel between Objects 1 and 2?

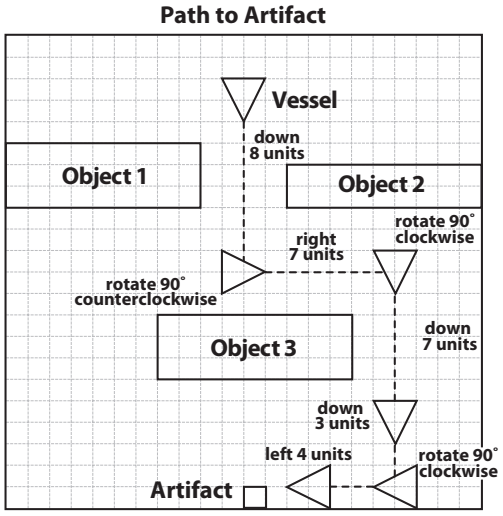
Possible answer: translating the vessel 8 units down

b. Several rigid motions need to be performed to turn and move the vessel to navigate around Object 3. What are they?

Possible answer: First, rotate the vessel 90° counterclockwise. Then translate the vessel 7 units to the right. Rotate the vessel again, but this time rotate it 90° clockwise. Translate the vessel 7 units down.

c. What rigid motion(s) can be used to navigate toward the artifact?

Depending on where the vessel was left in the last series of motions, answers may vary. From where the vessel stands at this point, the vessel needs to be translated 3 units down, then rotated clockwise 90°, then translated 4 units to the left.



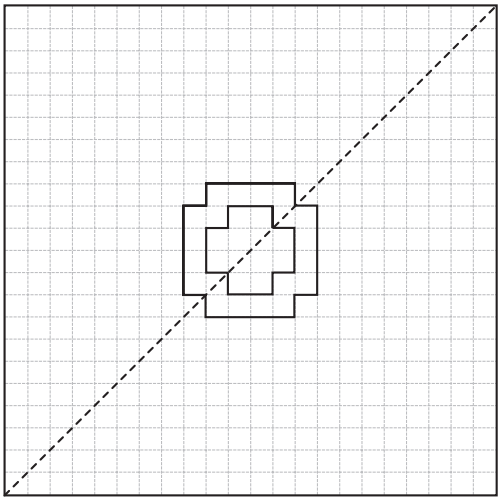
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d. What rigid motion needs to be performed to restore the artifact?

According to the task, artists use reflections to restore missing pieces of artifacts.

e. What does the artifact look like after this rigid motion is performed?

Artifact Reflection



Recommended Closure Activity

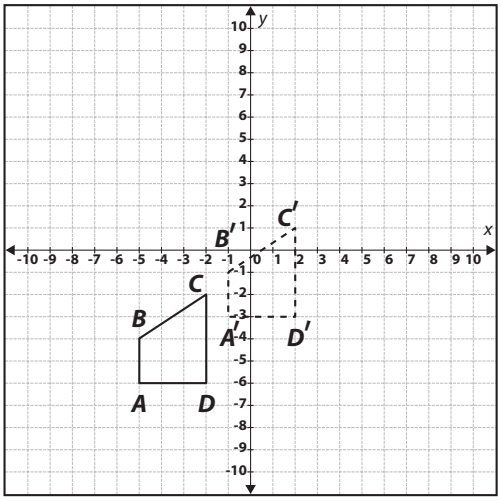
Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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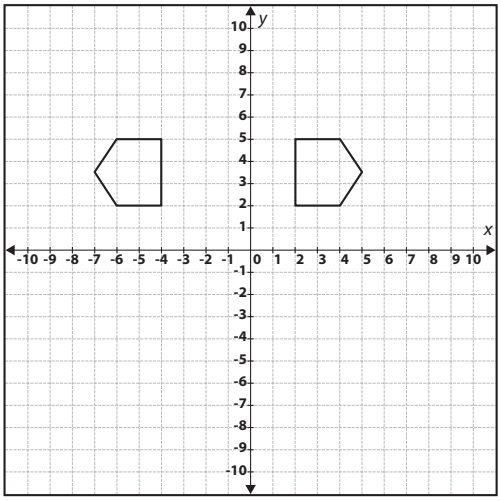
Practice 5.5.1: Describing Rigid Motions and Predicting the Effects

For problems 1–3, describe the rigid motion used to transform each figure. If the transformation is a translation, state the units and direction(s) the figure was transformed. If the transformation is a reflection, state the line of reflection. Write a statement justifying your answer.

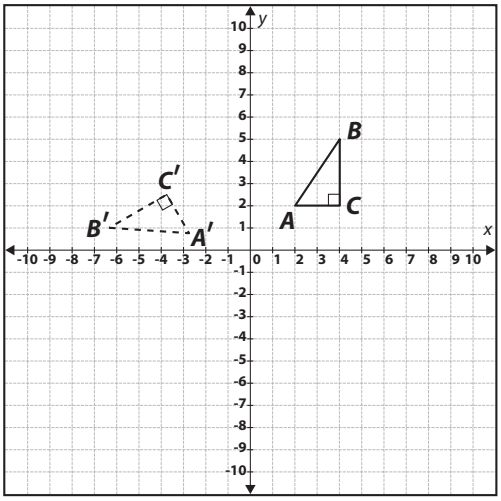
1.



2.



3.



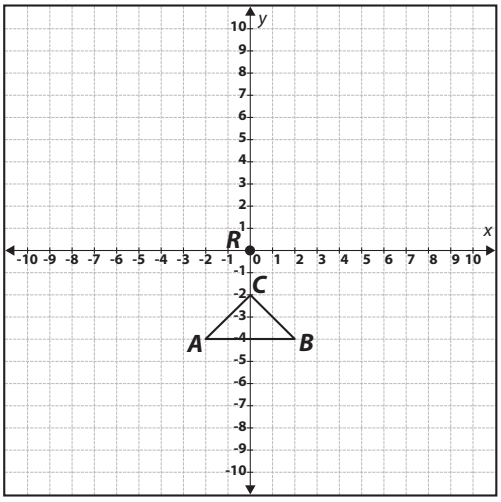
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SWB p. 127; SRB Practice 5.5.1 p. U5-163

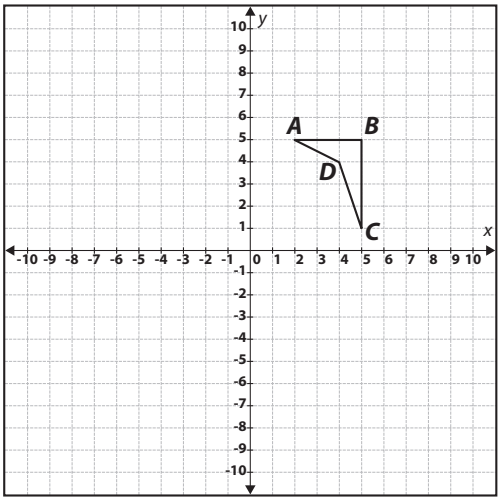
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For problems 4–10, use the given rigid motion to determine the new position of the given figure.

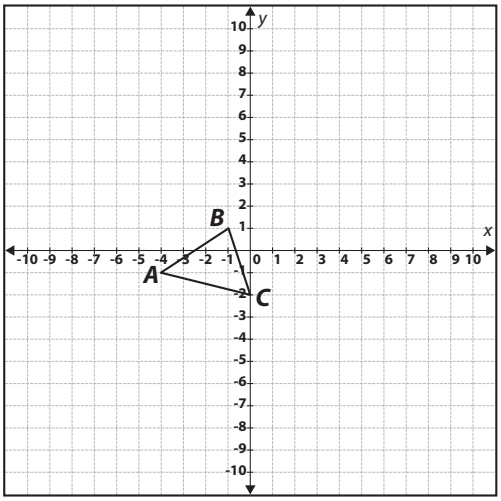
4. Rotate the given triangle 120° counterclockwise about the origin.



5. Translate the given quadrilateral to the left 6 units and down 5 units.



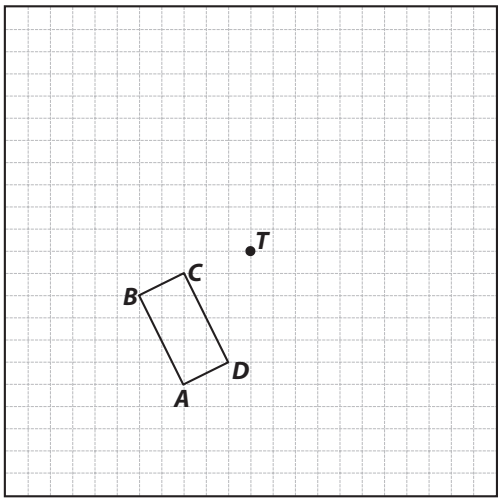
6. Reflect the given triangle over the line $x = 3$.



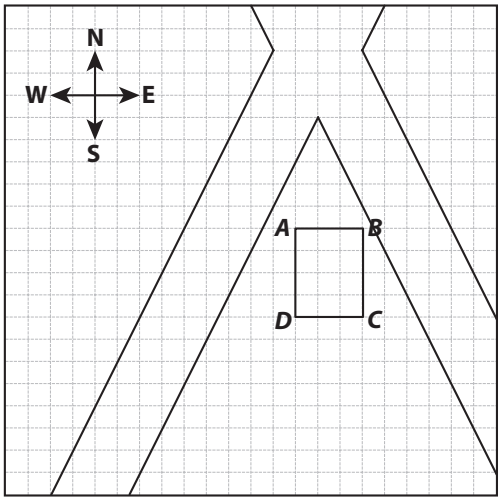
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7. An interior designer wants to rotate the couch in a family room 60° about the center of the coffee table, *T*. What will be the final position of the couch?



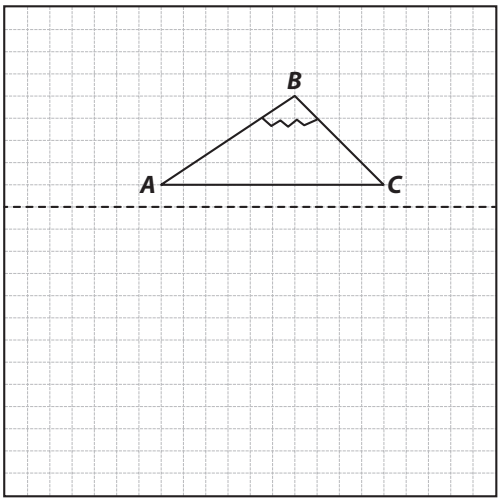
8. Due to the widening of a road, a house needs to be moved west 8 feet and south 32 feet. The house is shown as $\square ABCD$ in the diagram below. Each square on the grid represents 8 feet. What will be the final position of the house?



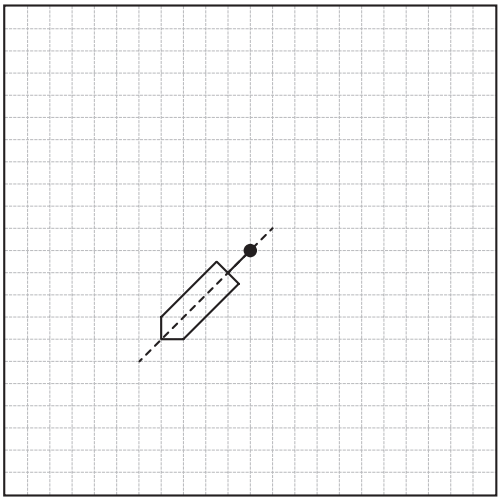
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9. Artists often paint the reflection of a mountain peak in a nearby body of water. Reflect the mountain over the body of water whose edge is represented by the dashed line.



10. A ceiling fan has 4 paddles that are separated by a constant angle of rotation. Each paddle has a line of symmetry, as shown in the diagram below. Describe a series of rigid motions that can be performed with the figure below to generate a ceiling fan with 4 paddles. Keep in mind that there are 360° degrees of rotation to bring a figure back onto itself. Draw the figure of the completed fan.

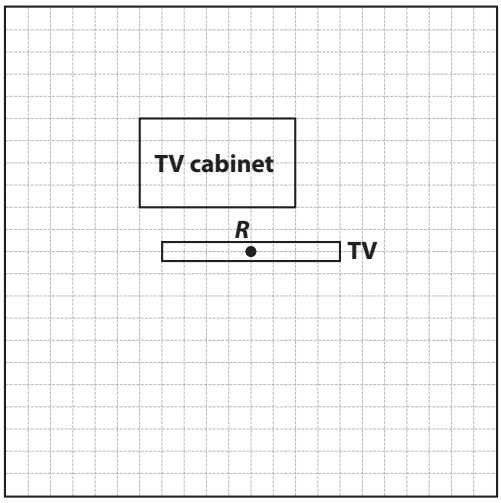


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Lesson 5.5.2: Defining Congruence in Terms of Rigid Motions

Warm-Up 5.5.2

August's grandparents bought her family a new flat-screen TV as a housewarming present. However, the new TV is too wide to fit into the piece of furniture that was holding the old TV. August proposed that they rotate the TV 30° counterclockwise about point *R* to slide the TV into the cabinet. She drew a scaled diagram that shows the top view of the TV and TV cabinet. Use the diagram to solve the problems that follow.



1. Rotate the TV counterclockwise 30° about point *R*.

2. Will the rotated TV fit into the cabinet? Justify your answer.

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Instruction

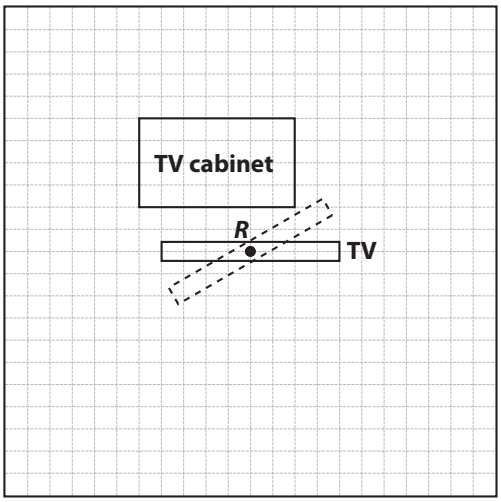
Lesson 5.5.2: Defining Congruence in Terms of Rigid Motions

Common Core State Standard

G–CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

Warm-Up 5.5.2 Debrief

1. Rotate the TV counterclockwise 30° about point *R*.

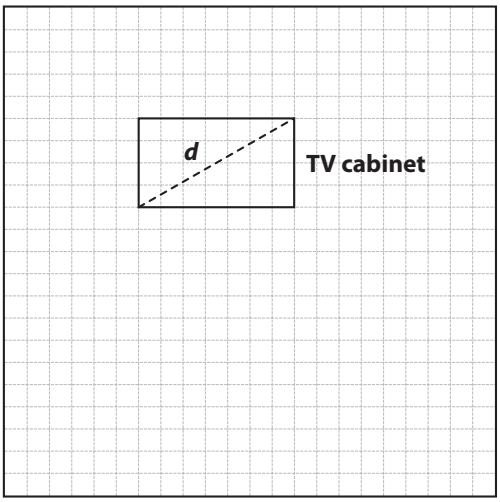


2. Will the rotated TV fit into the cabinet? Justify your answer.

   Since the drawing is a scaled representation of the actual figures, use the units provided on the grid and draw the diagonal of the TV cabinet. Use the Pythagorean Theorem to calculate the length of the diagonal.

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Instruction



$4^2 + 7^2 = d^2$, for which d is the diagonal

$16 + 49 = d^2$

$65 = d^2$

$\sqrt{65} = \sqrt{d^2}$

Since distance can only be positive, $d = \sqrt{65} \approx 8.06$ units.

Count the units to determine the length of the TV.

The TV is 8 units. Therefore, the TV should fit into the cabinet on the diagonal. However, the fit will be tight.

Connection to the Lesson

- Students will be introduced to the terms and concepts of transformations as rigid and non-rigid motions.
- Students will use the Pythagorean Theorem to determine if rigid motions have occurred.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- recognizing rotations, reflections, and translations (8.G.3)
- setting up ratios (6.RP.1)
- using the Pythagorean Theorem (8.G.7)

Introduction

Rigid motions can also be called congruency transformations. A **congruency transformation** moves a geometric figure but keeps the same size and shape. Preimages and images that are congruent are also said to be isometries. If a figure has undergone a rigid motion or a set of rigid motions, the preimage and image are congruent. When two figures are congruent, they have the same shape and size. Remember that rigid motions are translations, reflections, and rotations. Non-rigid motions are dilations, stretches, and compressions. **Non-rigid motions** are transformations done to a figure that change the figure's shape and/or size.

Key Concepts

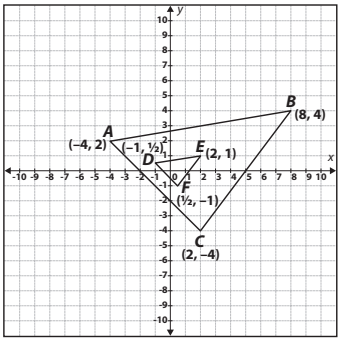
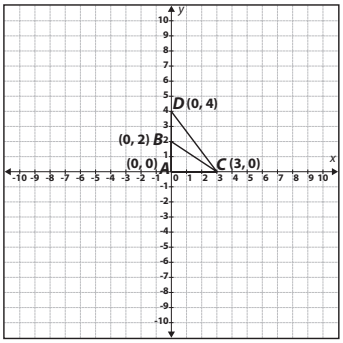
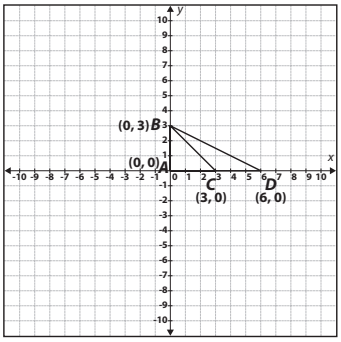
- To decide if two figures are congruent, determine if the original figure has undergone a rigid motion or set of rigid motions.
- If the figure has undergone only rigid motions (translations, reflections, or rotations), then the figures are congruent.
- If the figure has undergone any non-rigid motions (dilations, stretches, or compressions), then the figures are not congruent. A **dilation** uses a center point and a scale factor to either enlarge or reduce the figure. A dilation in which the figure becomes smaller can also be called a **compression**.
- A **scale factor** is a multiple of the lengths of the sides from one figure to the dilated figure. The scale factor remains constant in a dilation.
- If the scale factor is larger than 1, then the figure is enlarged.
- If the scale factor is between 0 and 1, then the figure is reduced.
- To calculate the scale factor, divide the length of the sides of the image by the lengths of the sides of the preimage.

SRB p. U5-168

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Instruction

- A vertical stretch or compression preserves the horizontal distance of a figure, but changes the vertical distance.
- A horizontal stretch or compression preserves the vertical distance of a figure, but changes the horizontal distance.
- To verify if a figure has undergone a non-rigid motion, compare the lengths of the sides of the figure. If the sides remain congruent, only rigid motions have been performed.
- If the side lengths of a figure have changed, non-rigid motions have occurred.

Non-Rigid Motions		
Dilations	**Vertical transformations**	**Horizontal transformations**
Enlargement/reduction	Stretch/compression	Stretch/compression
		
Compare $\triangle ABC$ with $\triangle DEF$. The size of each side changes by a constant scale factor. The angle measures have stayed the same.	Compare $\triangle ABC$ with $\triangle ADC$. The vertical distance changes by a scale factor. The horizontal distance remains the same. Two of the angles have changed measures.	Compare $\triangle ABC$ with $\triangle ABD$. The horizontal distance changes by a scale factor. The vertical distance remains the same. Two of the angles have changed measures.

Common Errors/Misconceptions

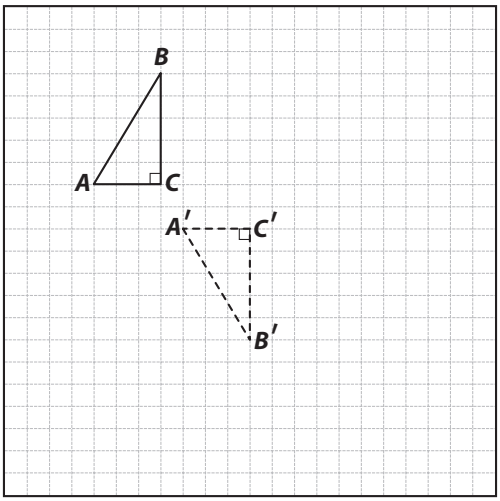
- mistaking a non-rigid motion for a rigid motion and vice versa
- not recognizing that rigid motions preserve shape and size
- not recognizing that it takes only one non-rigid motion to render two figures not congruent

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Guided Practice 5.5.2

Example 1

Determine if the two figures below are congruent by identifying the transformations that have taken place.



1. Determine the lengths of the sides.

   For the horizontal and vertical legs, count the number of units for the length. For the hypotenuse, use the Pythagorean Theorem, $a^2 + b^2 = c^2$, for which a and b are the legs and c is the hypotenuse.

$AC = 3$	$A'C' = 3$
$CB = 5$	$C'B' = 5$
$AC^2 + CB^2 = AB^2$	$A'C'^2 + C'B'^2 = A'B'^2$
$3^2 + 5^2 = AB^2$	$3^2 + 5^2 = A'B'^2$
$34 = AB^2$	$34 = A'B'^2$
$\sqrt{34} = \sqrt{AB^2}$	$\sqrt{34} = \sqrt{A'B'^2}$
$AB = \sqrt{34}$	$A'B' = \sqrt{34}$

   The sides in the first triangle are congruent to the sides of the second triangle. *Note*: When taking the square root of both sides of the equation, disregard the negative value since the value is a distance and distance can only be positive.

SWB p. 135; SRB p. U5-170

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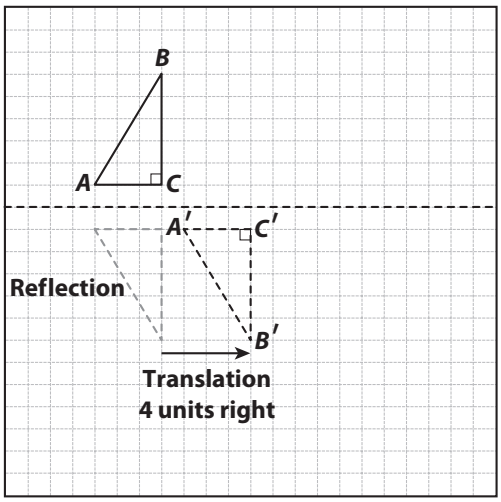
Instruction

2. Identify the transformations that have occurred.

   The orientation has changed, indicating a rotation or a reflection.

   The second triangle is a mirror image of the first, but translated to the right 4 units.

   The triangle has undergone rigid motions: reflection and translation.



3. State the conclusion.

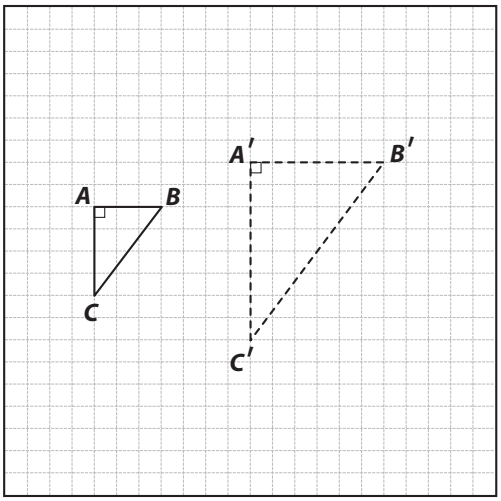
   The triangle has undergone two rigid motions: reflection and translation. Rigid motions preserve size and shape. The triangles are congruent.

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Instruction

Example 2

Determine if the two figures below are congruent by identifying the transformations that have taken place.



1. Determine the lengths of the sides.

   For the horizontal and vertical legs, count the number of units for the length. For the hypotenuse, use the Pythagorean Theorem, $a^2 + b^2 = c^2$, for which a and b are the legs and c is the hypotenuse.

$AB = 3$	$A'B' = 6$
$AC = 4$	$A'C' = 8$
$AB + AC^2 = CB^2$	$A'B'^2 + A'C'^2 = C'B'^2$
$3^2 + 4^2 = CB^2$	$6^2 + 8^2 = C'B'^2$
$25 = CB^2$	$100 = C'B'^2$
$\sqrt{25} = \sqrt{CB^2}$	$\sqrt{100} = \sqrt{C'B'^2}$
$CB = \sqrt{25}$	$C'B' = \sqrt{100}$
$CB = 5$	$C'B' = 10$

   The sides in the first triangle are not congruent to the sides of the second triangle. They are not the same size.

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Instruction

2. Identify the transformations that have occurred.

   The orientation has stayed the same, indicating translation, dilation, stretching, or compression. The vertical and horizontal distances have changed. This could indicate a dilation.

3. Calculate the scale factor of the changes in the side lengths.

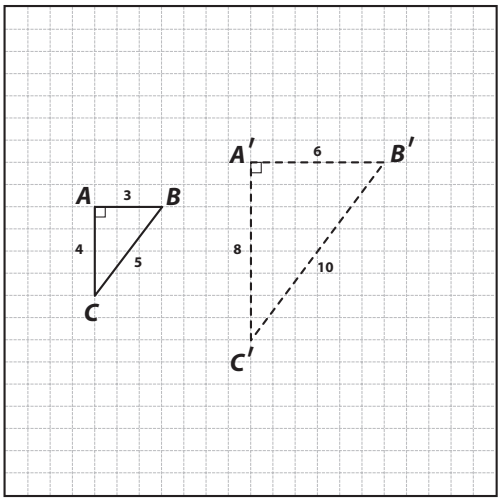
   Divide the image side lengths by the preimage side lengths.

   $$\frac{A'B'}{AB} = \frac{6}{3} = 2$$

   $$\frac{A'C'}{AC} = \frac{8}{4} = 2$$

   $$\frac{C'B'}{CB} = \frac{10}{5} = 2$$

   The scale factor is constant between each pair of sides in the preimage and image. The scale factor is 2, indicating a dilation. Since the scale factor is greater than 1, this is an enlargement.



4. State the conclusion.

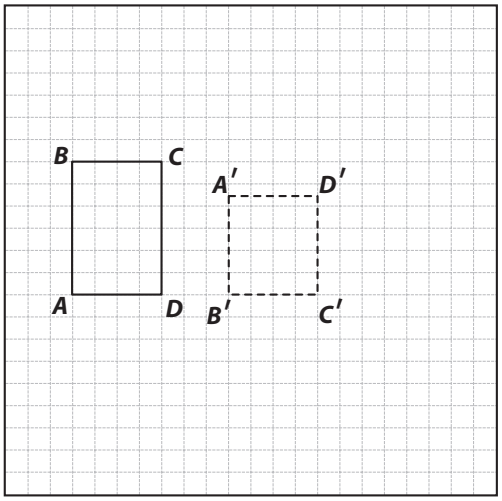
   The triangle has undergone at least one non-rigid motion: a dilation. Specifically, the dilation is an enlargement with a scale factor of 2. The triangles are not congruent because dilation does not preserve the size of the original triangle.

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Instruction

Example 3

Determine if the two figures below are congruent by identifying the transformations that have taken place.



1. Determine the lengths of the sides.

   For the horizontal and vertical sides, count the number of units for the length.

   $AB = 6$ $A'B' = 4.5$
   $BC = 4$ $B'C' = 4$
   $CD = 6$ $C'D' = 4.5$
   $DA = 4$ $D'A' = 4$

   Two of the sides in the first rectangle are not congruent to two of the sides of the second rectangle. Two sides are congruent in the first and second rectangles.

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Instruction

2. Identify the transformations that have occurred.

   The orientation has changed, and two side lengths have changed. The change in side length indicates at least one non-rigid motion has occurred. Since not all pairs of sides have changed in length, the non-rigid motion must be a horizontal or vertical stretch or compression.

   The image has been reflected since $\overline{BC}$ lies at the top of the preimage and $\overline{B'C'}$ lies at the bottom of the image. Reflections are rigid motions. However, one non-rigid motion makes the figures not congruent. A non-rigid motion has occurred since not all the sides in the image are congruent to the sides in the preimage.

   The vertical lengths have changed, while the horizontal lengths have remained the same. This means the transformation must be a vertical transformation.

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Instruction

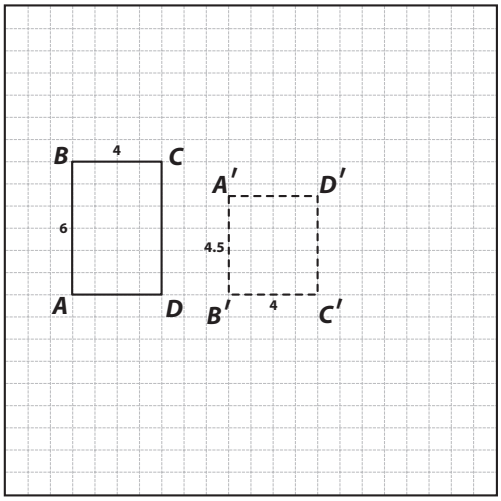
3. Calculate the scale factor of the change in the vertical sides.

   Divide the image side lengths by the preimage side lengths.

$$\frac{A'B'}{AB}=\frac{4.5}{6}=0.75$$

$$\frac{C'D'}{CD}=\frac{4.5}{6}=0.75$$

   The vertical sides have a scale factor of 0.75. The scale factor is between 0 and 1, indicating compression. Since only the vertical sides changed, this is a vertical compression.



4. State the conclusion.

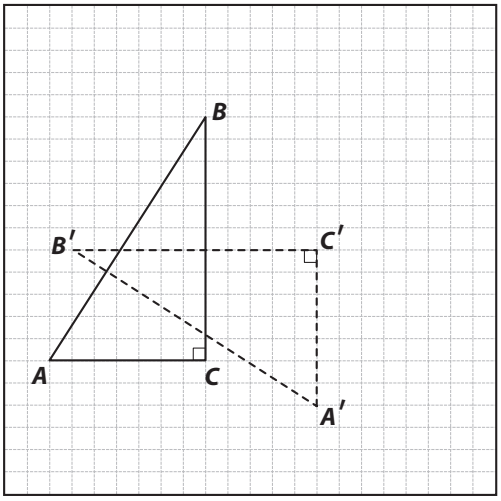
   The vertical sides of the rectangle have undergone at least one non-rigid transformation of a vertical compression. The vertical sides have been reduced by a scale factor of 0.75. Since a non-rigid motion occurred, the figures are not congruent.

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Instruction

Example 4

Determine if the two figures below are congruent by identifying the transformations that have taken place.



1. Determine the lengths of the sides.

   For the horizontal and vertical sides, count the number of units for the length. For the hypotenuse, use the Pythagorean Theorem, $a^2 + b^2 = c^2$, for which a and b are the legs and c is the hypotenuse.

$BC = 11$	$B'C' = 11$
$CA = 7$	$C'A' = 7$
$BC^2 + CA^2 = AB^2$	$B'C'^2 + C'A'^2 = A'B'^2$
$11^2 + 7^2 = AB^2$	$11^2 + 7^2 = A'B'^2$
$170 = AB^2$	$170 = A'B'^2$
$\sqrt{170} = \sqrt{AB^2}$	$\sqrt{170} = \sqrt{A'B'^2}$
$AB = \sqrt{170}$	$A'B' = \sqrt{170}$

   The sides of the first triangle are congruent to the sides of the second triangle.

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Instruction

2. Identify the transformations that have occurred.

   The orientation has changed and all side lengths have stayed the same. This indicates a reflection or a rotation. The preimage and image are not mirror images of each other. Therefore, the transformation that occurred is a rotation.

3. State the conclusion.

   Rotations are rigid motions and rigid motions preserve size and shape. The two figures are congruent.

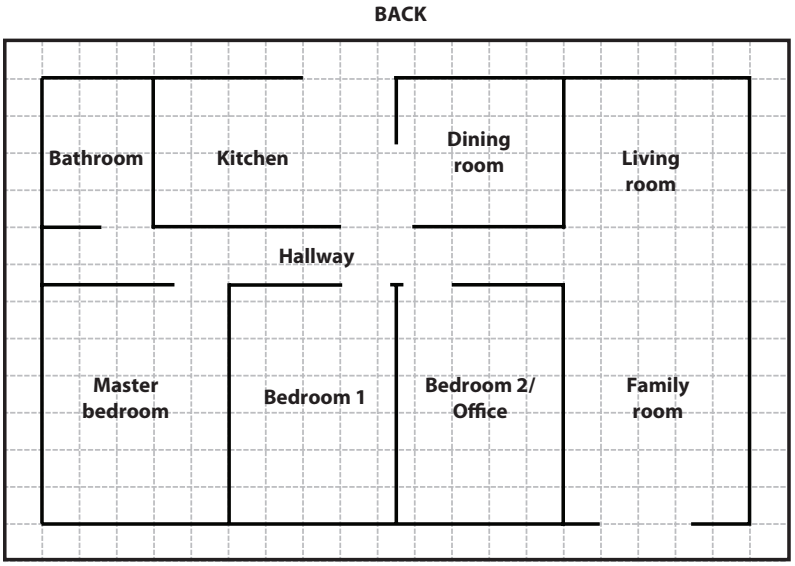


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Problem-Based Task 5.5.2: Architectural Planning

SMP	
1	2
3 ✓	4 ✓
5	6
7 ✓	8

Architects use scale drawings to map out floor plans of houses. In the floor plan below, $\frac{1}{4}$ inch equals 24 inches in the actual house. An architect is creating a housing development of 10 houses. She plans to use one basic floor plan and then use rigid motions to adjust the rooms to create a different look for the 10 houses. Using the floor plan provided, create at least two different floor plans using only rigid motions. Describe the non-rigid motion used to build an actual house based on the floor plan.



Create at least two different floor plans using only rigid motions. Describe the non-rigid motion used to build an actual house based on the floor plan.

SWB p. 139

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Problem-Based Task 5.5.2: Architectural Planning

Coaching

a. What transformation can be used to reverse the front rooms with the back rooms of the house?

b. How do you apply this transformation?

c. What does the result look like? Draw it.

d. Are the floor plans congruent? Explain.

e. What transformation can be used to reverse the rooms on the left with the rooms on the right?

f. How do you apply this transformation?

g. What does the result look like? Draw it.

h. Are the floor plans congruent? Explain.

i. What non-rigid motion is used to scale a figure?

j. What is the scale factor from the floor plan to the actual house?

k. Are the house and the floor plan congruent? Explain.

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Instruction

Problem-Based Task 5.5.2: Architectural Planning

Coaching Sample Responses

a. What transformation can be used to reverse the front rooms with the back rooms of the house?

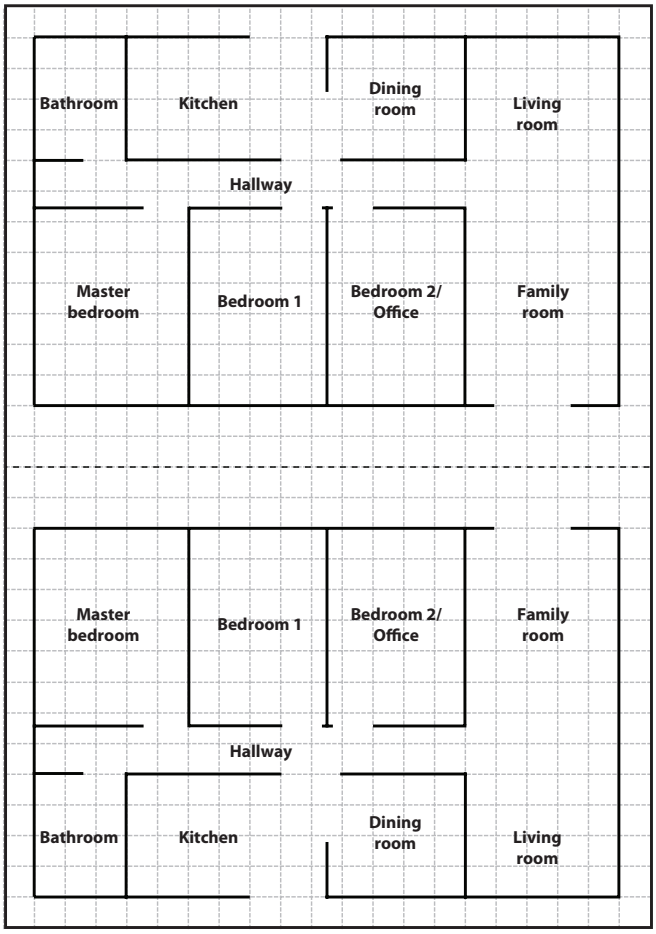
A reflection can be used to create a mirror image, which would switch the front rooms with the back rooms.

b. How do you apply this transformation?

Draw a horizontal line at the front of the house plan and reflect the floor plan over that line. Alternatively, you could draw a horizontal line at the back of the house.

c. What does the result look like? Draw it.

Answers may vary depending on where you draw the line of reflection. Pictured below is the floor plan reflected over a horizontal line at the front of the house.



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Instruction

d. Are the floor plans congruent? Explain.

A reflection is a rigid motion and rigid motions preserve shape and size. The floor plans are congruent.

e. What transformation can be used to reverse the rooms on the left with the rooms on the right?

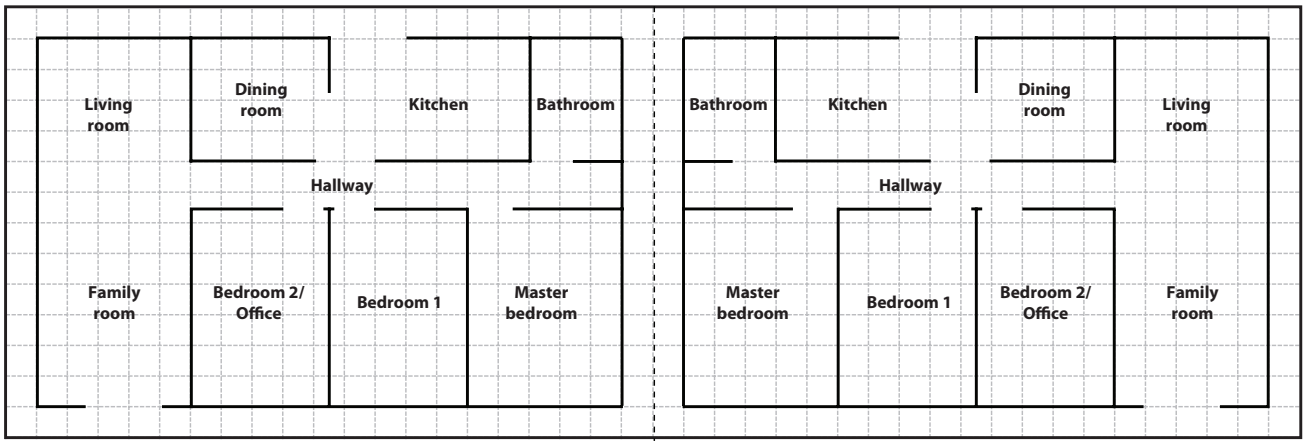
A reflection can be used to create a mirror image, which would switch the rooms on the left with the rooms on the right.

f. How do you apply this transformation?

Draw a vertical line to the left of the house plan and reflect the floor plan over that line. Alternatively, draw a vertical line to the right of the house.

g. What does the result look like? Draw it.

Answers may vary depending on where you draw the line of reflection. Pictured below is the floor plan reflected over a vertical line to the left of the plan.



h. Are the floor plans congruent? Explain.

A reflection is a rigid motion and rigid motions preserve shape and size. The floor plans are congruent.

i. What non-rigid motion is used to scale a figure?

A dilation scales a figure in both directions. If the scale factor is between 0 and 1, then the dilation is a reduction. If the scale factor is greater than 1, then the dilation is an enlargement.

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j. What is the scale factor from the floor plan to the actual house?

$$\frac{\text{house in inches}}{\text{floor plan in inches}} = \frac{24}{\frac{1}{4}} = 24 \bullet \frac{4}{1} = 96$$

The scale factor is 96.

k. Are the house and the floor plan congruent? Explain.

No, they are not congruent, because the sizes are not the same. A dilation has occurred. Specifically, an enlargement occurs from the floor plan to the house, with a scale factor of 96.

Recommended Closure Activity

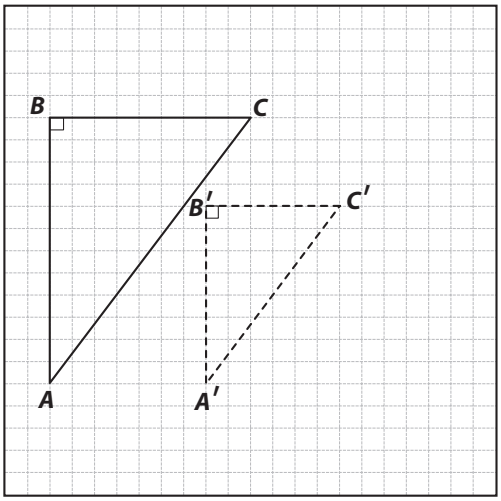
Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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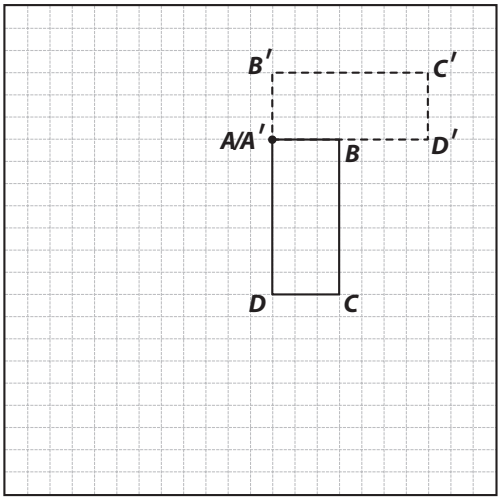
Practice 5.5.2: Defining Congruence in Terms of Rigid Motions

For problems 1–6, determine if the two given figures are congruent by identifying the transformation(s) that occurred. State whether each transformation is rigid or non-rigid.

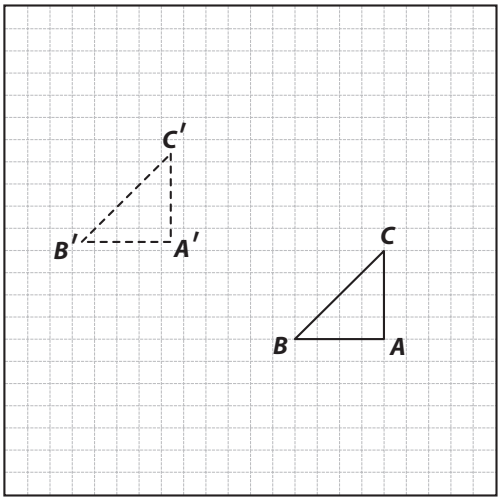
1.



2.



3.

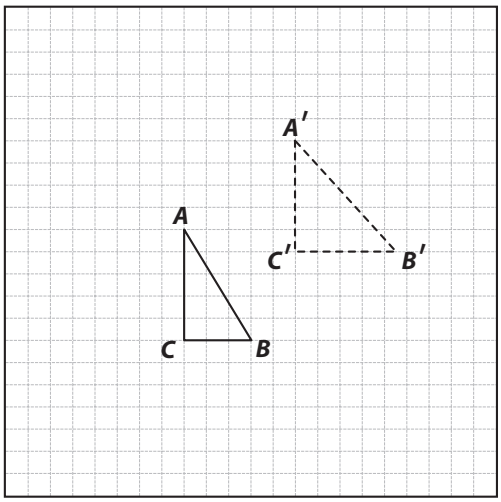


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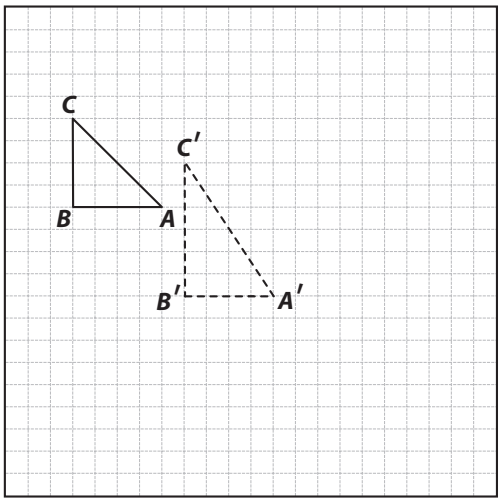
SWB p. 141; SRB Practice 5.5.2 p. U5-179

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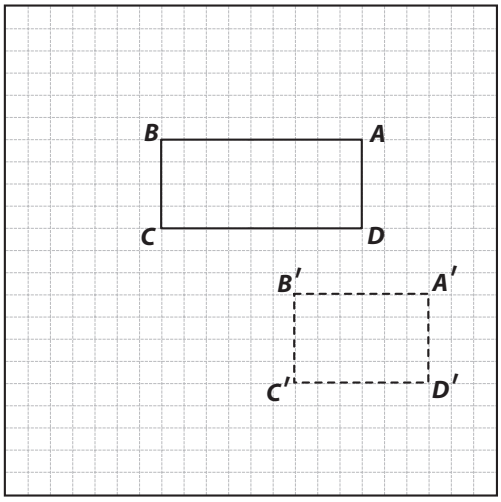
4.



5.



6.

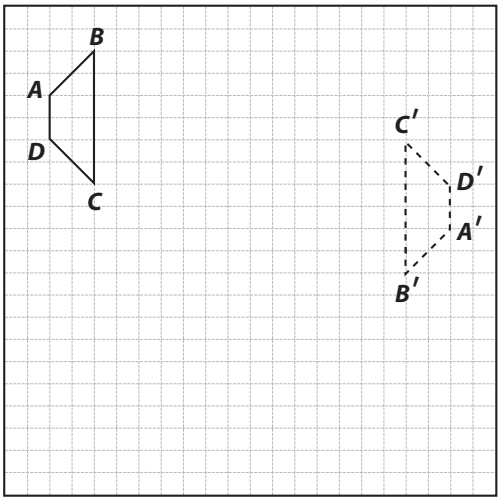


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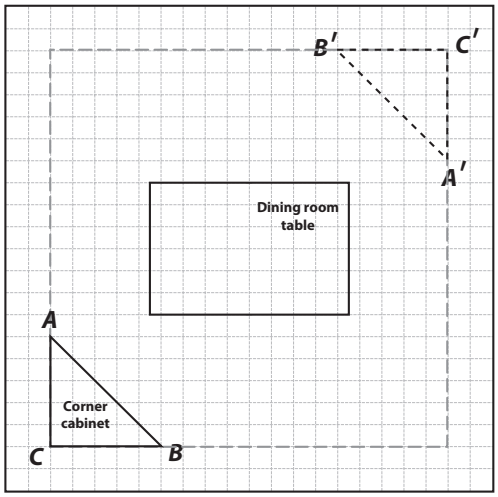
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Use the given information to solve problems 7–10.

7. Xander is rearranging the setup in the school gym for a large presentation. He has to move the speaker to make room for chairs. The diagram of how the speaker is transformed is pictured below. Describe the transformations that have taken place and determine whether the figures are congruent in terms of rigid and non-rigid motions.



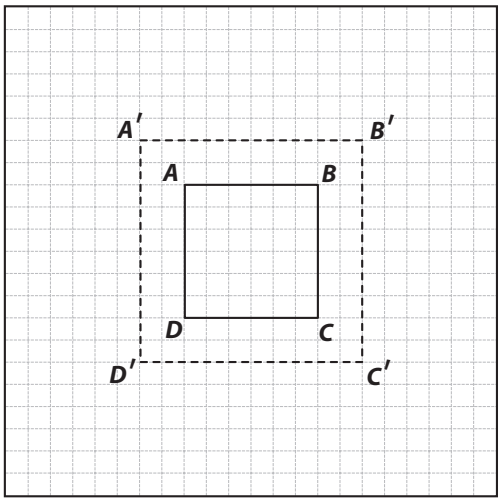
8. A corner cabinet sits in the dining room as pictured below, where the gray line represents the walls of the room. The new location of the corner cabinet is seen as the triangle labeled $A'B'C'$ in the diagram. Describe the transformations that have taken place and determine whether the figures representing the corner cabinet are congruent in terms of rigid and non-rigid motions.



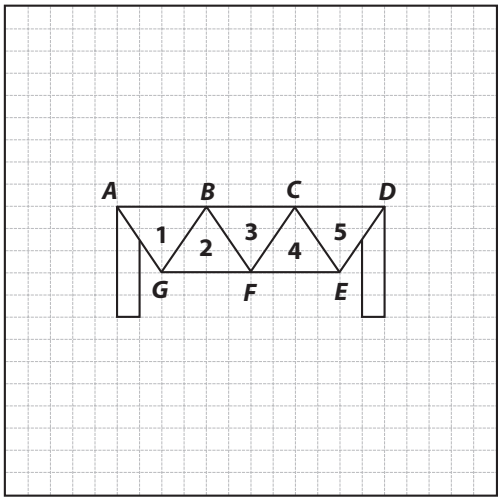
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9. The picture frame pictured below can be thought of as a square inside another square. Describe the transformations that have taken place and determine whether the squares are congruent in terms of rigid and non-rigid motions.



10. A truss is a structure used in building bridges. The bridge truss pictured below is made up of 5 triangles. Describe the transformations that have taken place and determine whether the triangles are congruent in terms of rigid and non-rigid motions.

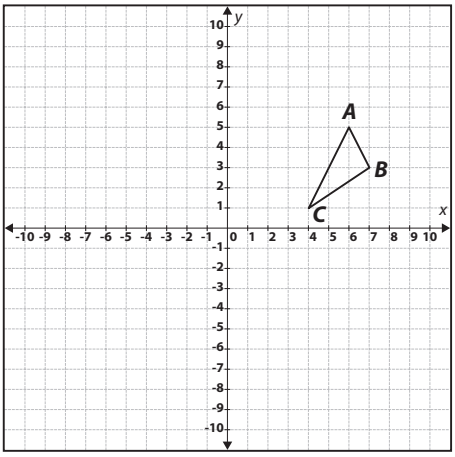


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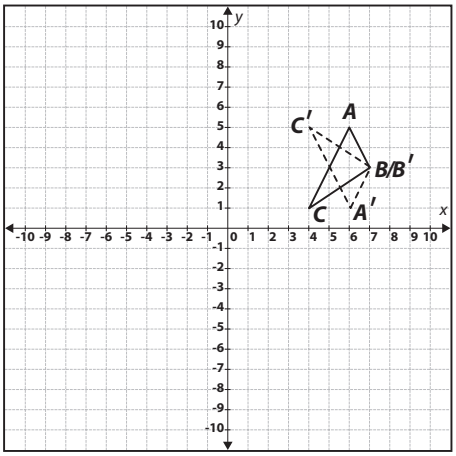
Progress Assessment

Circle the letter of the best answer.

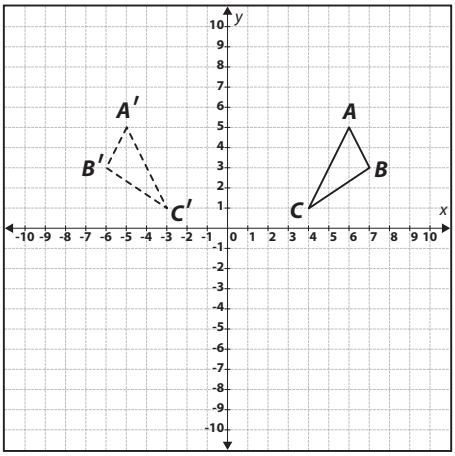
1. Which illustration predicts the effect(s) of reflecting the given triangle over the line $x = 3$?



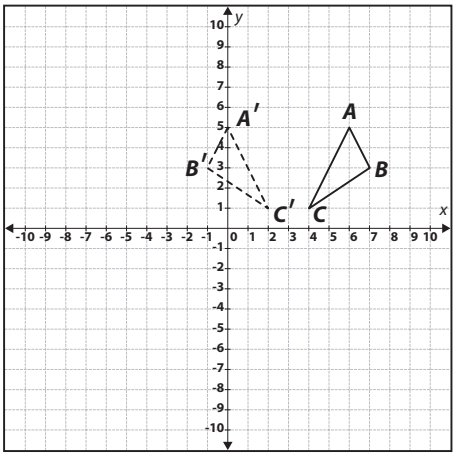
a.



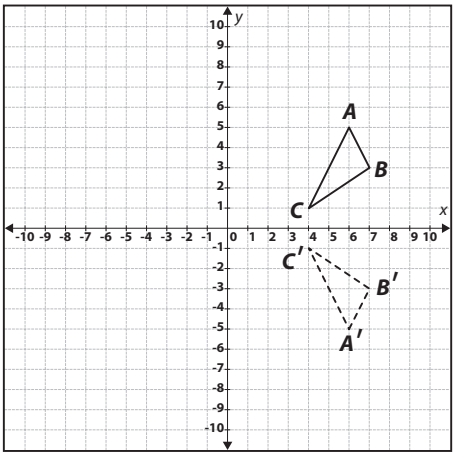
c.



b.



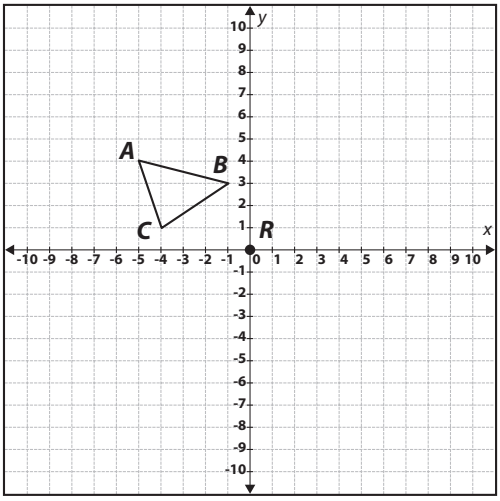
d.



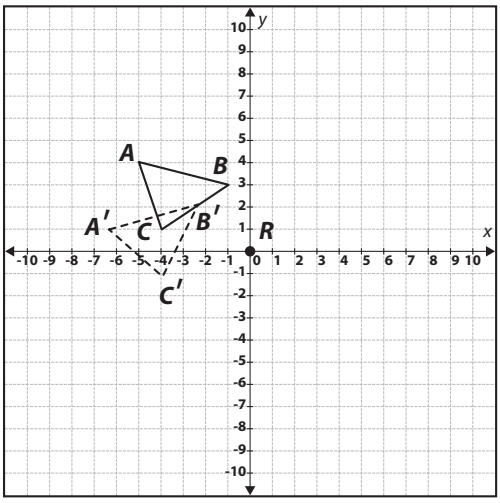
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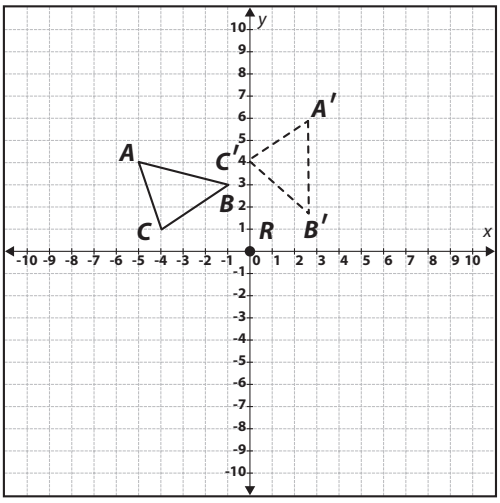
2. Which illustration predicts the effect(s) of rotating the given triangle 75° counterclockwise about *R*?



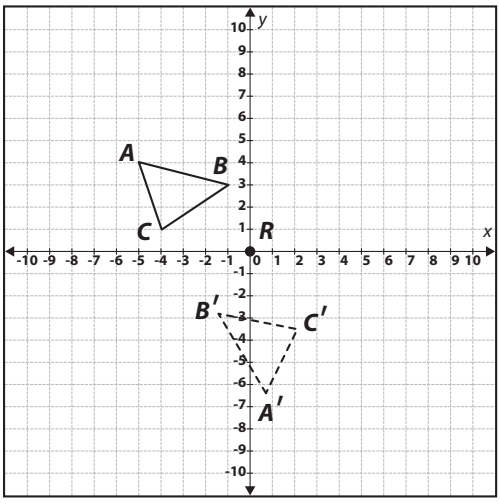
a.



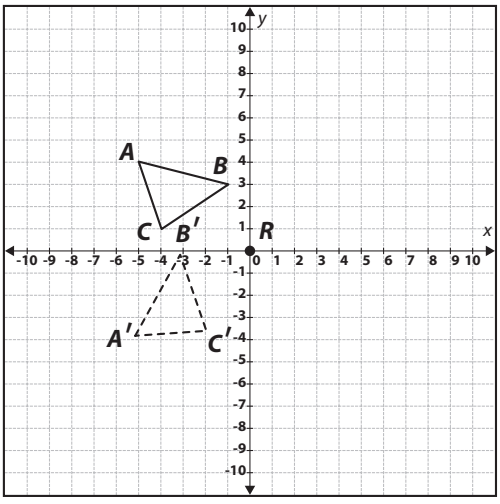
c.



b.



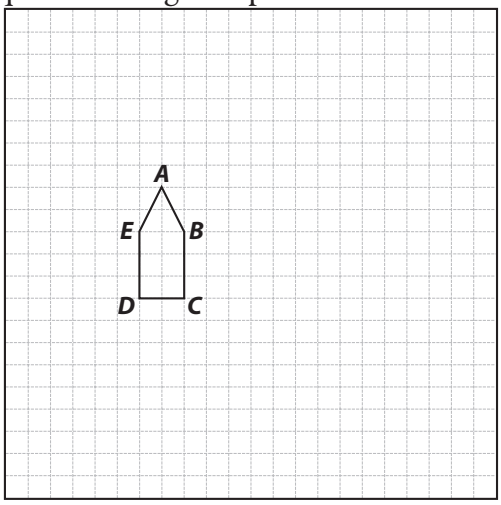
d.



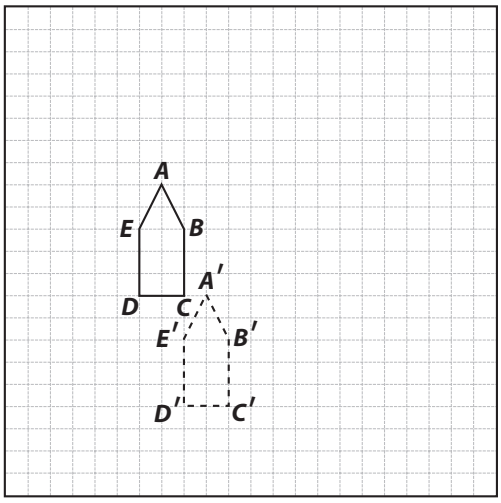
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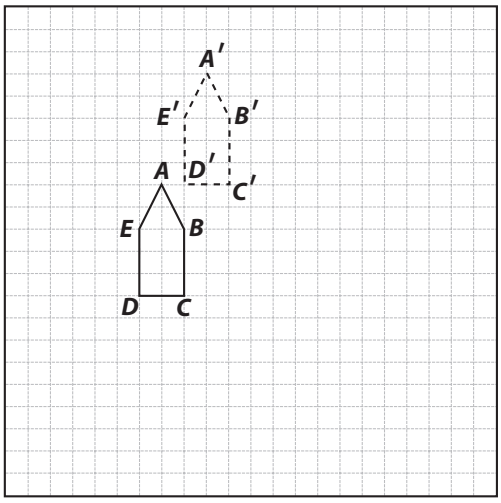
3. Which illustration predicts the effect(s) of translating the given pentagon 2 units to the right and 5 units down? Each square on the grid represents 1 unit.



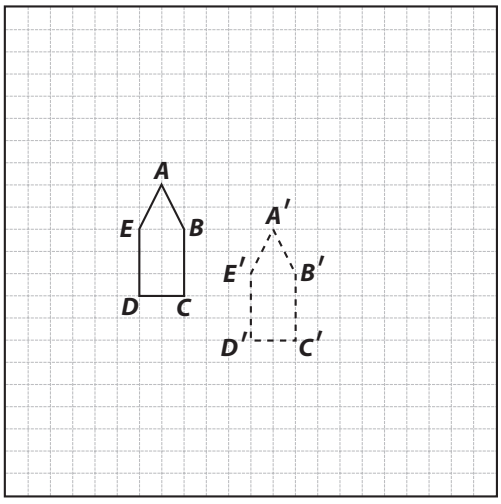
a.



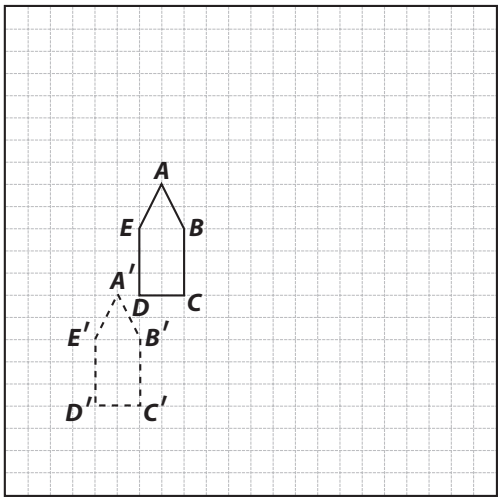
c.



b.



d.

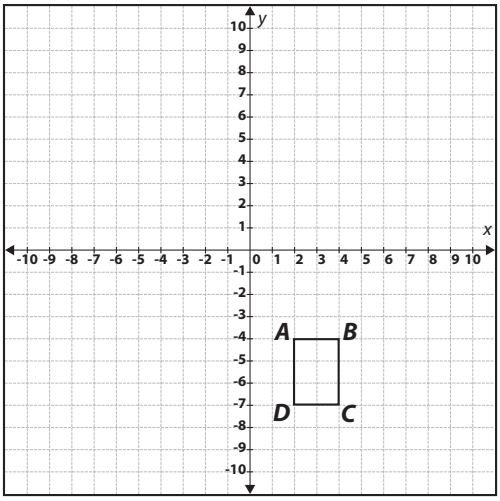


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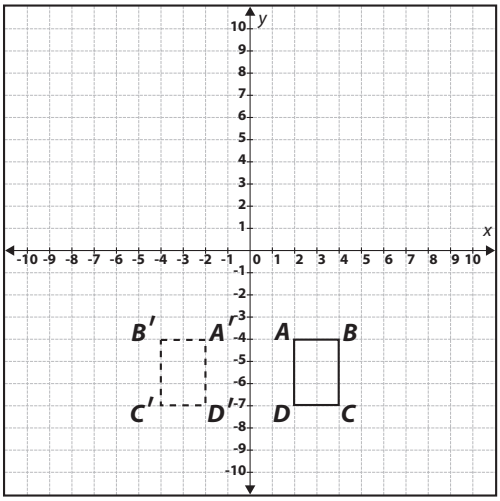
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Assessment

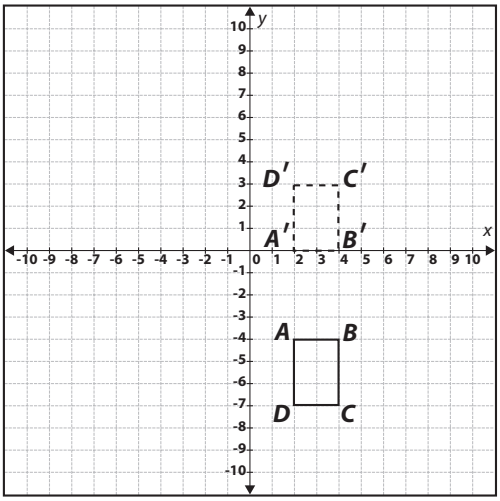
4. Which illustration predicts the effect(s) of reflecting the given rectangle over the line $y = -2$?



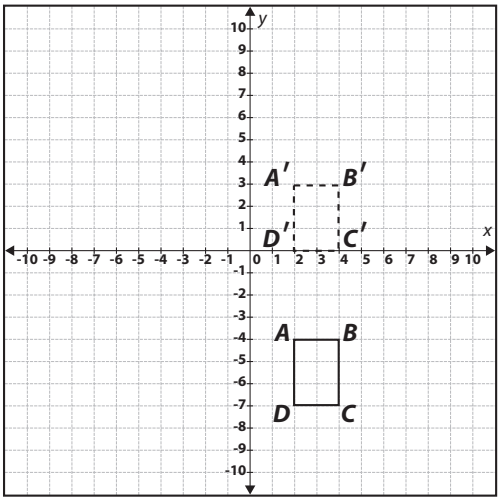
a.



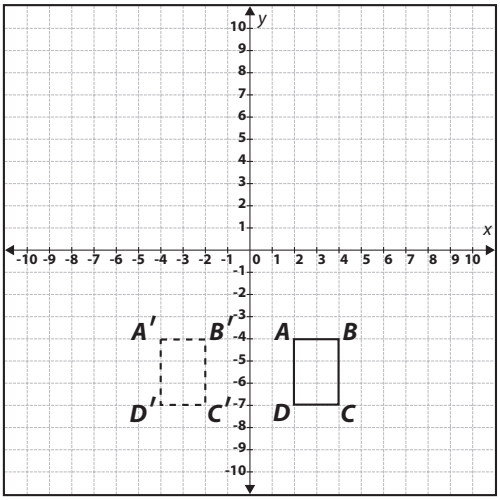
c.



b.



d.

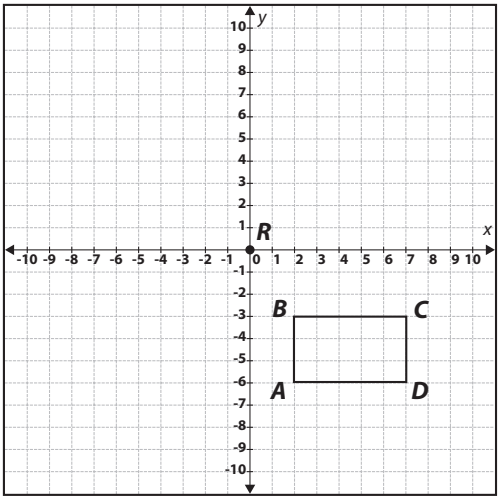


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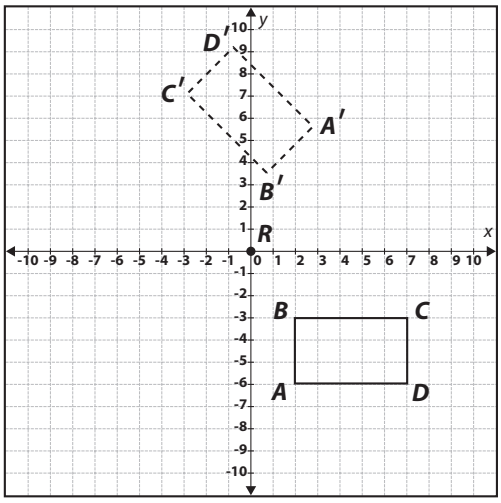
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Assessment

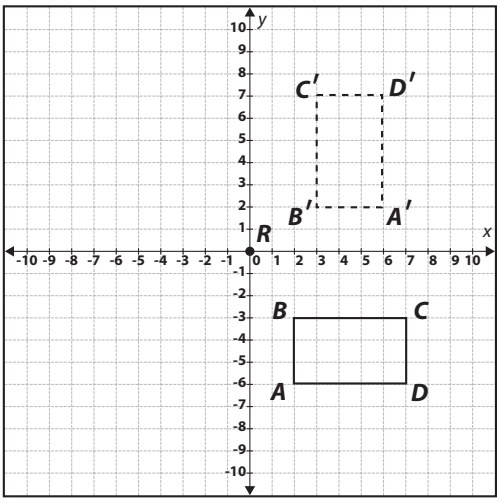
5. Which illustration predicts the effect(s) of rotating the given rectangle 135° counterclockwise about *R*?



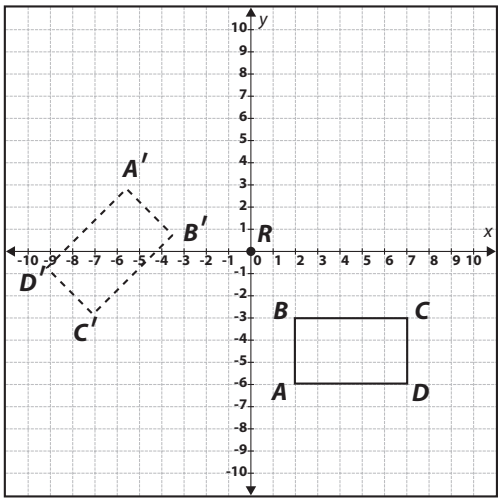
a.



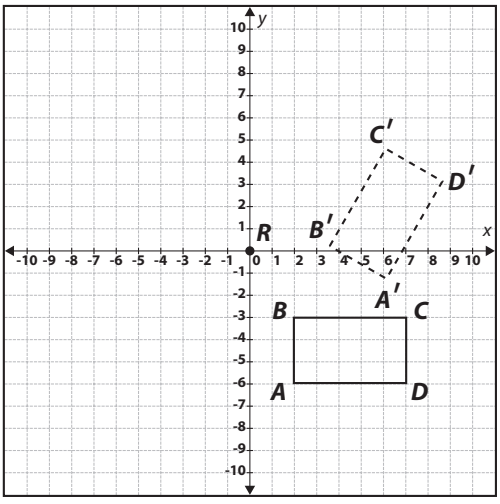
c.



b.



d.



continued

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6. Which of the following are rigid motions?

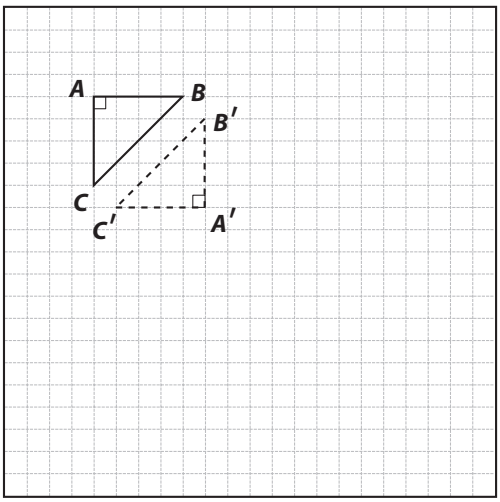
   a. translation, dilation, and rotation

   b. reflection, rotation, and translation

   c. compression, rotation, and reflection

   d. dilation, stretches, and compressions

7. Are the two figures congruent? Why or why not?



   a. Not congruent; $\triangle ABC$ has undergone a rigid motion of a rotation.

   b. Congruent; $\triangle ABC$ has undergone a rigid motion of a reflection.

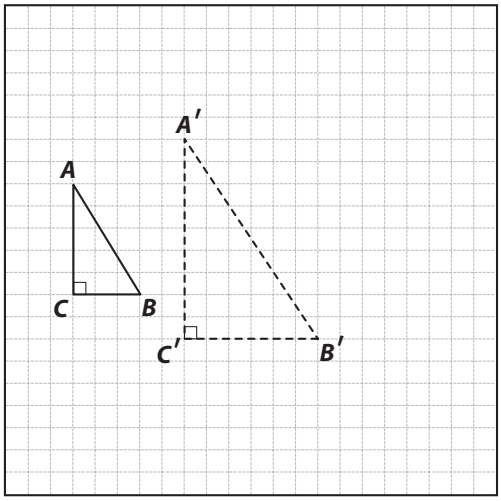
   c. Not congruent; $\triangle ABC$ has undergone a non-rigid motion of a reflection.

   d. Congruent; $\triangle ABC$ has undergone a non-rigid motion of a rotation.

continued

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8. Are the two figures congruent? Why or why not?



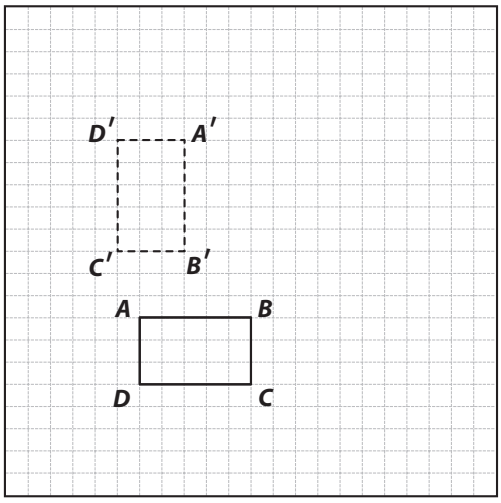
a. Not congruent; $\triangle ABC$ has undergone a rigid motion of a horizontal stretch.

b. Congruent; $\triangle ABC$ has undergone a rigid motion of a vertical stretch.

c. Not congruent; $\triangle ABC$ has undergone a non-rigid motion of a dilation.

d. Congruent; $\triangle ABC$ has undergone a non-rigid motion of a dilation.

9. Are the two figures congruent? Why or why not?



a. Not congruent; $\square ABCD$ has undergone a rigid motion of a reflection.

b. Congruent; $\square ABCD$ has undergone a rigid motion of a rotation.

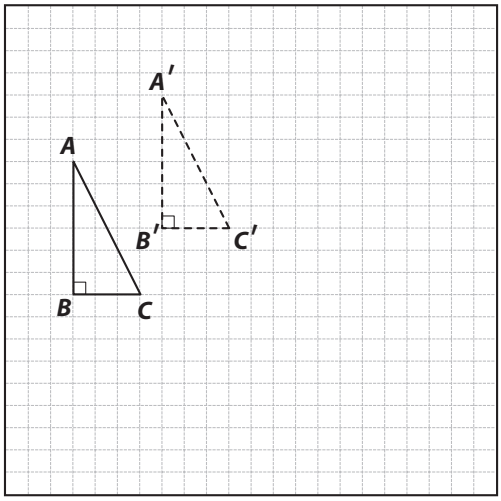
c. Not congruent; $\square ABCD$ has undergone a non-rigid motion of a translation.

d. Congruent; $\square ABCD$ has undergone a non-rigid motion of a dilation.

continued

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10. Are the two figures congruent? Why or why not?



a. Congruent; $\triangle ABC$ has undergone a rigid motion of a rotation.

b. Not congruent; $\triangle ABC$ has undergone a rigid motion of a translation.

c. Congruent; $\triangle ABC$ has undergone a rigid motion of a translation.

d. Not congruent; $\triangle ABC$ has undergone a non-rigid motion of a vertical compression.

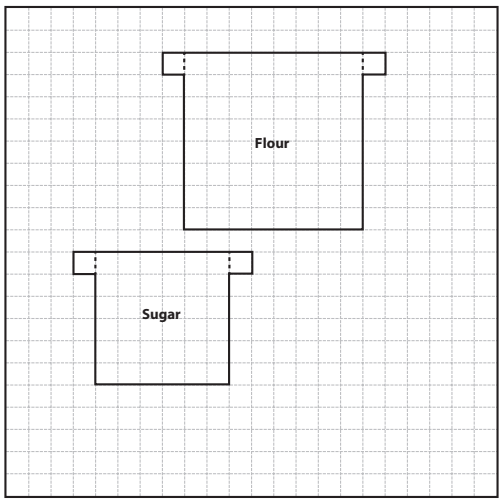
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Assessment

Read the scenario below and answer the questions that follow.

11. Jaisha is designing custom labels for containers of flour and sugar in her kitchen. She sketches the side view of each container to make sure her labels will fit. The lids are not shown, but the handles are. View each figure as a composite figure, meaning that each figure is made up of smaller shapes. Each figure contains a rectangle and two squares.



a. What rigid motions can you identify in Jaisha's sketches, either within a sketch or between the sketches? Explain.

b. What non-rigid motions can you identify in her sketches, either within a sketch or between the sketches? Explain.

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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. $\triangle UVW$ and $\triangle XYZ$ are congruent triangles. Which statement is known to be true?
   a. $\angle U \cong \angle V$
   b. $\angle W \cong \angle X$
   c. $\angle V \cong \angle X$
   d. $\angle V \cong \angle Y$

2. $\triangle ABC$ and $\triangle DEF$ are congruent triangles. Which statement is known to be true?
   a. $\overline{AB} \cong \overline{BC}$
   b. $\overline{AB} \cong \overline{EF}$
   c. $\overline{AC} \cong \overline{DF}$
   d. $\overline{AB} \cong \overline{DF}$

3. A triangle congruent to $\triangle ABC$ is to be constructed. Only three components are measured. Which three components, if constructed in the order listed, will always create a congruent triangle?
   a. side-side-angle
   b. angle-angle-angle
   c. angle-side-angle
   d. Only the three side lengths can be used to create a congruent triangle.

4. Which set of equivalent measures does not indicate that two triangles must be congruent?
   a. angle-angle-angle
   b. angle-side-angle
   c. side-angle-side
   d. angle-angle-side

5. For $\triangle ABC$ and $\triangle DEF$, the following is given: $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$. By which triangle congruence statement can it be concluded that the triangles are congruent?
   a. SSS
   b. SAS
   c. ASA
   d. It cannot be determined if the triangles are congruent.

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Lesson 6: Congruent Triangles

Instruction

Common Core State Standards

G–CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G–CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

SMP	
1 ✓	2
3 ✓	4 ✓
5	6
7 ✓	8

Essential Questions

1. Why is it important to know how to mark congruence on a diagram?
2. What does it mean if two triangles are congruent?
3. If two triangles have two sides and one angle that are equivalent, can congruence be determined?
4. How many equivalent measures are needed to determine if triangles are congruent?

WORDS TO KNOW

angle-side-angle (ASA)	if two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent
congruent angles	two angles that have the same measure
congruent sides	two sides that have the same length
congruent triangles	triangles having the same angle measures and side lengths
corresponding angles	a pair of angles in a similar position
Corresponding Parts of Congruent Triangles are Congruent (CPCTC)	if two or more triangles are proven congruent, then all of their corresponding parts are congruent as well
corresponding sides	the sides of two figures that lie in the same position relative to the figures
included angle	the angle between two sides
included side	the side between two angles of a triangle
postulate	a true statement that does not require a proof

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rigid motion	a transformation done to an object that maintains the object's shape and size or its segment lengths and angle measures
side-angle-side (SAS)	if two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent
side-side-side (SSS)	if three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent

Recommended Resources

- Illuminations. "Congruence Theorems."

  http://www.walch.com/rr/00013

  This site allows users to investigate congruence postulates by manipulating parts of a triangle.

- Math Open Reference. "Congruent Triangles."

  http://www.walch.com/rr/00014

  This site provides an explanation of congruent triangles, as well as interactive graphics that demonstrate how congruent triangles remain congruent when different transformations are applied.

- Math Warehouse. "Corresponding Sides and Angles: Identify Corresponding Parts."

  http://www.walch.com/rr/00015

  This site explains how to identify corresponding parts of a triangle, and provides interactive questions and answers.

- National Library of Virtual Manipulatives. "Congruent Triangles."

  http://www.walch.com/rr/00016

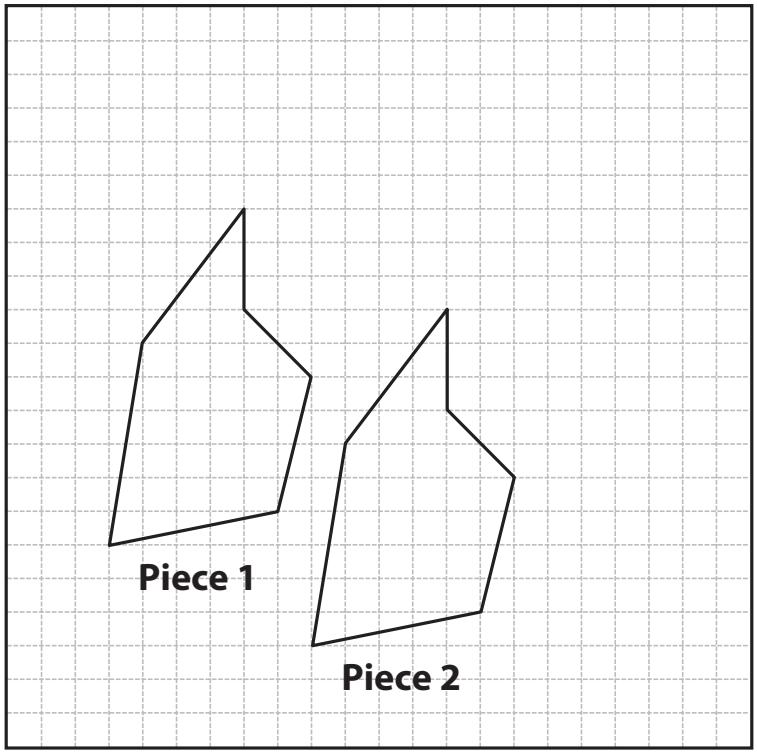
  This site allows users to construct congruent triangles according to each of the congruence postulates. *Note*: Requires Java.

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Lesson 5.6.1: Triangle Congruency

Warm-Up 5.6.1

Cutting mats used in crafting are similar to coordinate planes, except that the axes are labeled with inches or centimeters, rather than positive and negative numbers. Juliet is in the middle of a sewing project and has laid out two congruent pieces of fabric on her cutting mat.

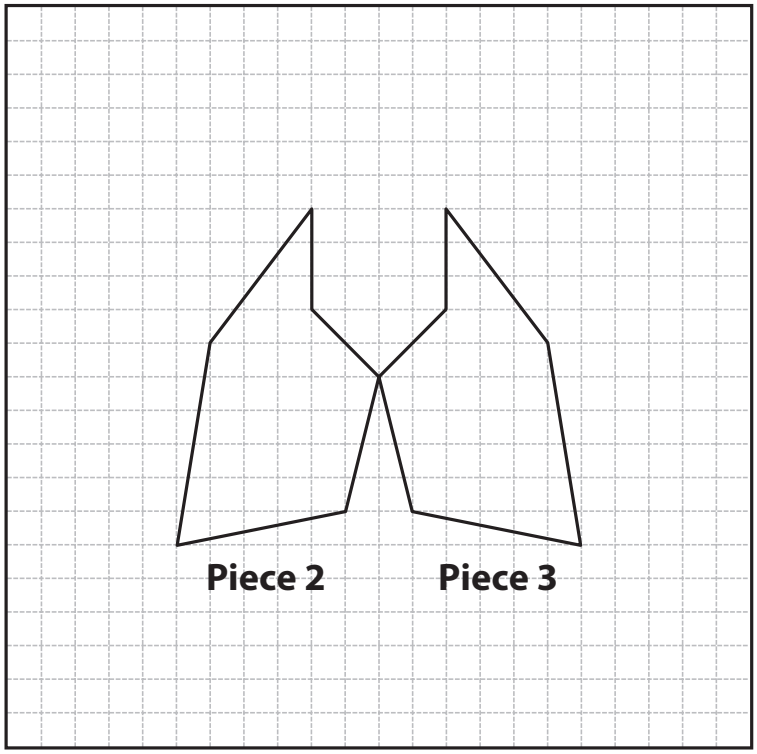


1. What is the series of transformations that has taken place between Piece 1 and Piece 2?

continued

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Juliet takes Piece 1 off the mat. She places a third piece of fabric on the cutting mat.



2. What is the series of transformations that has taken place between Piece 2 and Piece 3?

3. Is Piece 3 congruent to Piece 1? Explain your reasoning.

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Instruction

Lesson 5.6.1: Triangle Congruency

Common Core State Standard

G–CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Warm-Up 5.6.1 Debrief

1. What is the series of transformations that has taken place between Piece 1 and Piece 2?

   Piece 1 was slid, or translated, to the right 6 units.

   Piece 1 was also translated down 3 units.

   The position of Piece 2 is the result of a horizontal translation of 6 units to the right and a vertical translation of 3 units down.

2. What is the series of transformations that has taken place between Piece 2 and Piece 3?

   Piece 3 is the mirror image of Piece 2.

   The position of Piece 3 is the reflection of Piece 2 over a vertical line.

3. Is Piece 3 congruent to Piece 1? Explain your reasoning.

   Piece 3 is congruent to Piece 1.

   A translation is a congruency transformation, so Piece 1 is congruent to Piece 2. A reflection is also a congruency transformation, so Piece 3 is congruent to Piece 2. A series of congruency transformations results in a congruent figure, so Piece 3 is congruent to Piece 1.

Connection to the Lesson

- Students will use the definition of congruence in terms of rigid motions to identify congruent parts of triangles.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- recognizing transformations performed as a combination of translations, reflections, rotations, dilations, contractions, or stretches (G–CO.2)
- understanding that rigid motions maintain shape and size of angles and segments (G–CO.7)

Introduction

If a rigid motion or a series of rigid motions, including translations, rotations, or reflections, is performed on a triangle, then the transformed triangle is congruent to the original. When two triangles are congruent, the corresponding angles have the same measures and the corresponding sides have the same lengths. It is possible to determine whether triangles are congruent based on the angle measures and lengths of the sides of the triangles.

Key Concepts

- To determine whether two triangles are congruent, you must observe the angle measures and side lengths of the triangles.
- When a triangle is transformed by a series of rigid motions, the angles are images of each other and are called corresponding angles.
- Corresponding angles are a pair of angles in a similar position.
- If two triangles are congruent, then any pair of corresponding angles is also congruent.
- When a triangle is transformed by a series of rigid motions, the sides are also images of each other and are called corresponding sides.
- Corresponding sides are the sides of two figures that lie in the same position relative to the figure.
- If two triangles are congruent, then any pair of corresponding sides is also congruent.
- Congruent triangles have three pairs of corresponding angles and three pairs of corresponding sides, for a total of six pairs of corresponding parts.
- If two or more triangles are proven congruent, then all of their corresponding parts are congruent as well. This postulate is known as **Corresponding Parts of Congruent Triangles are Congruent (CPCTC)**. A **postulate** is a true statement that does not require a proof.

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Instruction

- The corresponding angles and sides can be determined by the order of the letters.
- If $\triangle ABC$ is congruent to $\triangle DEF$, the angles of the two triangles correspond in the same order as they are named.
- Use the symbol $\rightarrow$ to show that two parts are corresponding.

  Angle $A \rightarrow$ Angle D; they are equivalent.

  Angle $B \rightarrow$ Angle E; they are equivalent.

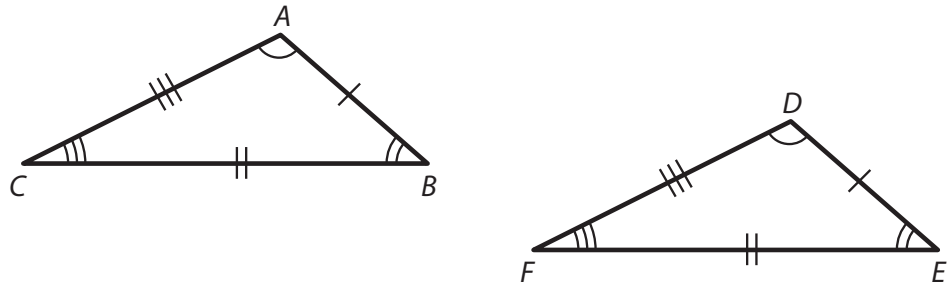
  Angle $C \rightarrow$ Angle F; they are equivalent.
- The corresponding angles are used to name the corresponding sides.

  $\overline{AB} \rightarrow \overline{DE}$; they are equivalent.

  $\overline{BC} \rightarrow \overline{EF}$; they are equivalent.

  $\overline{AC} \rightarrow \overline{DF}$; they are equivalent.
- Observe the diagrams of $\triangle ABC$ and $\triangle DEF$.

$\triangle ABC \cong \triangle DEF$



$\angle A \cong \angle D$ $\quad\quad$ $\overline{AB} \cong \overline{DE}$

$\angle B \cong \angle E$ $\quad\quad$ $\overline{BC} \cong \overline{EF}$

$\angle C \cong \angle F$ $\quad\quad$ $\overline{AC} \cong \overline{DF}$

- By observing the angles and sides of two triangles, it is possible to determine if the triangles are congruent.
- Two triangles are congruent if the corresponding angles are congruent and corresponding sides are congruent.
- Notice the number of tick marks on each side of the triangles in the diagram.

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Instruction

- The tick marks show the sides that are congruent.
- Compare the number of tick marks on the sides of $\triangle ABC$ to the tick marks on the sides of $\triangle DEF$.
- Match the number of tick marks on one side of one triangle to the side with the same number of tick marks on the second triangle.

  $\overline{AB}$ and $\overline{DE}$ each have one tick mark, so the two sides are congruent.

  $\overline{BC}$ and $\overline{EF}$ each have two tick marks, so the two sides are congruent.

  $\overline{AC}$ and $\overline{DF}$ each have three tick marks, so the two sides are congruent.
- The arcs on the angles show the angles that are congruent.
- Compare the number of arcs on the angles of $\triangle ABC$ to the number of arcs on the angles of $\triangle DEF$.
- Match the arcs on one angle of one triangle to the angle with the same number of arcs on the second triangle.

  $\angle A$ and $\angle D$ each have one arc, so the two angles are congruent.

  $\angle B$ and $\angle E$ each have two arcs, so the two angles are congruent.

  $\angle C$ and $\angle F$ each have three arcs, so the two angles are congruent.
- If the sides and angles are not labeled as congruent, you can use a ruler and protractor or construction methods to measure each of the angles and sides.

Common Errors/Misconceptions

- incorrectly identifying corresponding parts of triangles
- assuming corresponding parts indicate congruent parts
- assuming alphabetical order indicates congruence

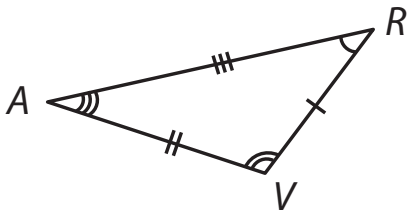
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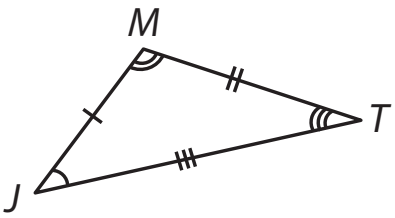
Instruction

Guided Practice 5.6.1

Example 1

Use corresponding parts to identify the congruent triangles.





1. Match the number of tick marks to identify the corresponding congruent sides.

   $\overline{RV}$ and $\overline{JM}$ each have one tick mark; therefore, they are corresponding and congruent.

   $\overline{VA}$ and $\overline{MT}$ each have two tick marks; therefore, they are corresponding and congruent.

   $\overline{RA}$ and $\overline{JT}$ each have three tick marks; therefore, they are corresponding and congruent.

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Instruction

2. Match the number of arcs to identify the corresponding congruent angles.

   $\angle R$ and $\angle J$ each have one arc; therefore, the two angles are corresponding and congruent.

   $\angle V$ and $\angle M$ each have two arcs; therefore, the two angles are corresponding and congruent.

   $\angle A$ and $\angle T$ each have three arcs; therefore, the two angles are corresponding and congruent.

3. Order the congruent angles to name the congruent triangles.

   $\triangle RVA$ is congruent to $\triangle JMT$, or $\triangle RVA \cong \triangle JMT$.

   It is also possible to identify the congruent triangles as $\triangle VAR \cong \triangle MTJ$, or even $\triangle ARV \cong \triangle TJM$; whatever order chosen, it is important that the order in which the vertices are listed in the first triangle matches the congruency of the vertices in the second triangle.

   For instance, it is not appropriate to say that $\triangle RVA$ is congruent to $\triangle MJT$ because $\angle R$ is not congruent to $\angle M$.

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Instruction

Example 2

$\triangle BDF \cong \triangle HJL$

Name the corresponding angles and sides of the congruent triangles.

1. Identify the congruent angles.

   The names of the triangles indicate the angles that are corresponding and congruent. Begin with the first letter of each name.

   Identify the first set of congruent angles.

   $\angle B$ is congruent to $\angle H$.

   Identify the second set of congruent angles.

   $\angle D$ is congruent to $\angle J$.

   Identify the third set of congruent angles.

   $\angle F$ is congruent to $\angle L$.

2. Identify the congruent sides.

   The names of the triangles indicate the sides that are corresponding and congruent. Begin with the first two letters of each name.

   Identify the first set of congruent sides.

   $\overline{BD}$ is congruent to $\overline{HJ}$.

   Identify the second set of congruent sides.

   $\overline{DF}$ is congruent to $\overline{JL}$.

   Identify the third set of congruent sides.

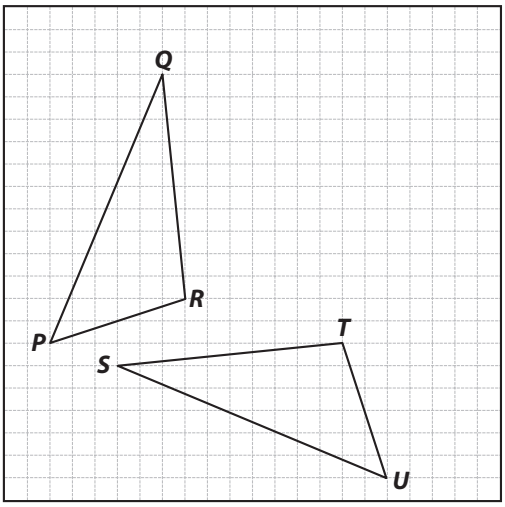
   $\overline{BF}$ is congruent to $\overline{HL}$.

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Instruction

Example 3

Use construction tools to determine if the triangles are congruent. If they are, name the congruent triangles and corresponding angles and sides.



1. Use a compass to compare the length of each side.

   Begin with the shortest sides, $\overline{PR}$ and $\overline{UT}$.

   Set the sharp point of the compass on point *P* and extend the pencil of the compass to point *R*.

   Without changing the compass setting, set the sharp point of the compass on point *U* and extend the pencil of the compass to point *T*.

   The compass lengths match, so the length of $\overline{UT}$ is equal to $\overline{PR}$; therefore, the two sides are congruent.

   Compare the longest sides, $\overline{PQ}$ and $\overline{US}$.

   Set the sharp point of the compass on point *P* and extend the pencil of the compass to point *Q*.

   Without changing the compass setting, set the sharp point of the compass on point *U* and extend the pencil of the compass to point *S*.

   The compass lengths match, so the length of $\overline{US}$ is equal to $\overline{PQ}$; therefore, the two sides are congruent.

(*continued*)

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Instruction

Compare the last pair of sides, $\overline{QR}$ and $\overline{ST}$.

Set the sharp point of the compass on point Q and extend the pencil of the compass to point R.

Without changing the compass setting, set the sharp point of the compass on point S and extend the pencil of the compass to point T.

The compass lengths match, so the length of $\overline{ST}$ is equal to $\overline{QR}$; therefore, the two sides are congruent.

2. Use a compass to compare the measure of each angle.

Begin with the largest angles, $\angle R$ and $\angle T$.

Set the sharp point of the compass on point R and create a large arc through both sides of $\angle R$.

Without adjusting the compass setting, set the sharp point on point T and create a large arc through both sides of $\angle T$.

Set the sharp point of the compass on one point of intersection and open it so it touches the second point of intersection.

Use this setting to compare the distance between the two points of intersection on the second triangle.

The measure of $\angle R$ is equal to the measure of $\angle T$; therefore, the two angles are congruent.

The measure of $\angle Q$ is equal to the measure of $\angle S$; therefore, the two angles are congruent.

The measure of $\angle P$ is equal to the measure of $\angle U$; therefore, the two angles are congruent.

3. Summarize your findings.

The corresponding and congruent sides include:

$\overline{PR} \cong \overline{UT}$ $\overline{PQ} \cong \overline{US}$ $\overline{QR} \cong \overline{ST}$

The corresponding and congruent angles include:

$\angle R \cong \angle T$ $\angle Q \cong \angle S$ $\angle P \cong \angle U$

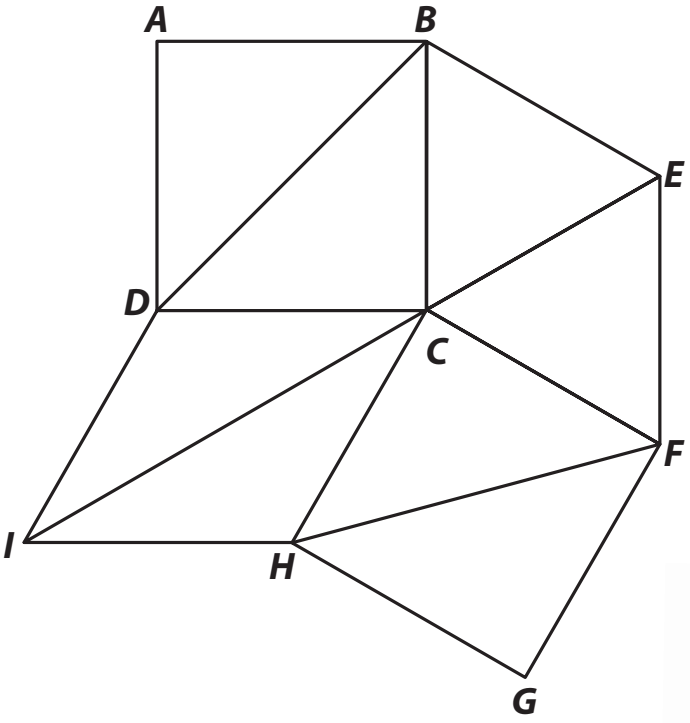
Therefore, $\triangle RQP \cong \triangle TSU$.

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Problem-Based Task 5.6.1: Stained Glass Pattern, Part I

SMP	
1 ✓	2
3 ✓	4
5	6
7 ✓	8

Mary creates stained glass art. She is in the planning stages of creating a new piece and has found a pattern she really likes. Mary studies the pattern to determine which triangles in the pattern are congruent, so that she can cut the correct size pieces of glass. Pictured below is a portion of the pattern. Use the pattern and the information that follows to determine which triangles are congruent. How could Mary use this information to help plan her project?



- Quadrilaterals *ABCD* and *CFGH* are squares. Each diagonal of a square bisects an opposite pair of angles.
- Quadrilaterals *BEFC* and *DCHI* are rhombuses. The diagonals of a rhombus bisect the opposite pairs of angles. Remember that opposite pairs of angles are congruent.
- $\overline{EC} \cong \overline{BC}$



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Problem-Based Task 5.6.1: Stained Glass Pattern, Part I

Coaching

a. What are the properties of the sides and angles of a square?

b. What are the properties of the sides and angles of a rhombus?

c. Which sides in the pattern are congruent?

d. How do you indicate on a diagram that sides are congruent?

e. Mark the congruent sides on the diagram.

f. Which angles are congruent?

g. How do you indicate on a diagram that angles are congruent?

h. Mark congruent angles on the diagram.

i. Name one pair of congruent triangles. Justify your answer by indicating which corresponding parts are congruent.

j. Name a second pair of congruent triangles. Justify your answer by indicating which corresponding parts are congruent.

k. Name a third pair of congruent triangles. Justify your answer by indicating which corresponding parts are congruent.

l. Name a fourth pair of congruent triangles. Justify your answer by indicating which corresponding parts are congruent.

m. Are any pairs of triangles that you identified congruent? If so, name them.

n. How can Mary use this information to plan her project?

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Instruction

Problem-Based Task 5.6.1: Stained Glass Pattern, Part I

Coaching Sample Responses

a. What are the properties of the sides and angles of a square?

All sides are congruent and all angles are congruent.

b. What are the properties of the sides and angles of a rhombus?

All sides are congruent and opposite angles are congruent.

c. Which sides in the pattern are congruent and why?

$\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$ because $\square ABCD$ is a square.

$\overline{BC} \cong \overline{BE} \cong \overline{EF} \cong \overline{FC}$ because $\square BEFC$ is a rhombus.

$\overline{FC} \cong \overline{FG} \cong \overline{GH} \cong \overline{HC}$ because $\square CFGH$ is a square.

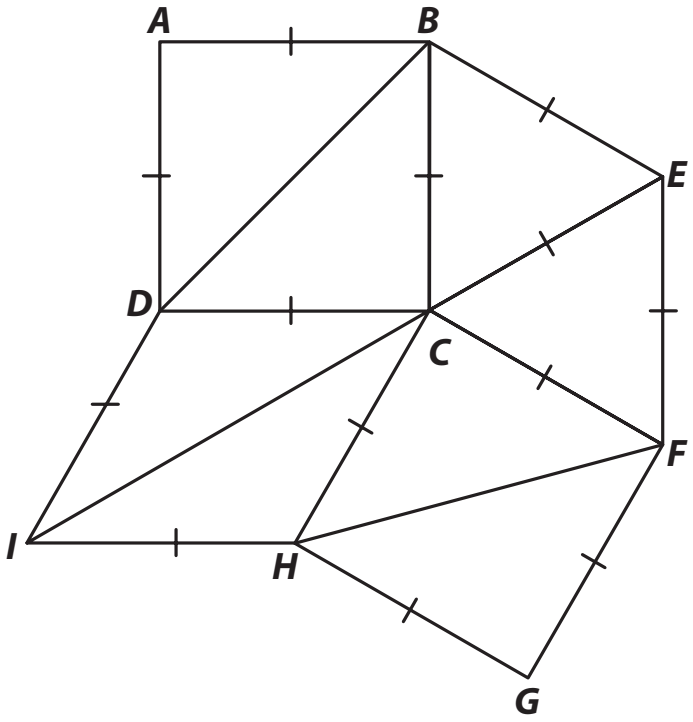
$\overline{HC} \cong \overline{HI} \cong \overline{ID} \cong \overline{DC}$ because $\square DCHI$ is a rhombus.

$\overline{EC} \cong \overline{BC}$ because this was given.

d. How do you indicate on a diagram that sides are congruent?

Use the same number of tick marks on the sides that are congruent.

e. Mark the congruent sides on the diagram.



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f. Which angles are congruent and why?

$\angle DAB \cong \angle HGF \cong \angle DCB \cong \angle HCF$ because these are the vertices of the squares; the measure of the vertex of a square is always 90˚.

$\angle ABD \cong \angle DBC \cong \angle ADB \cong \angle BDC$ because the diagonal of a square bisects the opposite pairs of angles.

$\angle CHF \cong \angle GHF \cong \angle CFH \cong \angle GFH$ because the diagonal of a square bisects the opposite pairs of angles.

$\angle CBE \cong \angle EFC$ because these are the opposite vertices of a rhombus; opposite vertices of a rhombus are congruent.

$\angle BCE \cong \angle ECF \cong \angle BEC \cong \angle FEC$ because the diagonal of a rhombus bisects the opposite pairs of angles.

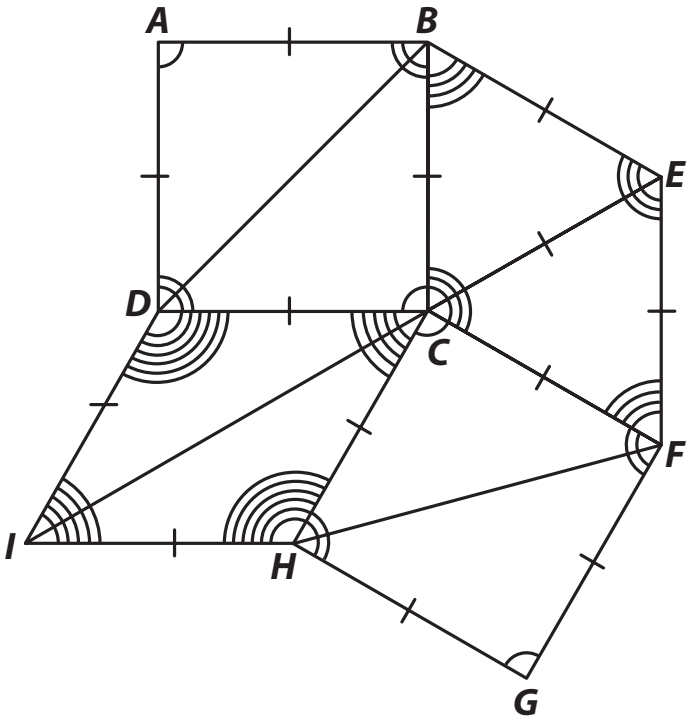
$\angle IDC \cong \angle IHC$ because these are the opposite vertices of a rhombus; opposite vertices of a rhombus are congruent.

$\angle DIC \cong \angle HIC \cong \angle DCI \cong \angle HCI$ because the diagonal of a rhombus bisects the opposite pairs of angles.

g. How do you indicate on a diagram that angles are congruent?

Use the same number of arcs on the angles that are congruent.

h. Mark congruent angles on the diagram.



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Instruction

i. Name one pair of congruent triangles. Justify your answer by indicating which corresponding parts are congruent.

$\triangle ABD \cong \triangle CBD$

$\angle DAB \cong \angle DCB$

$\angle ADB \cong \angle CDB$

$\angle ABD \cong \angle CBD$

$\overline{AB} \cong \overline{CB}$

$\overline{AD} \cong \overline{CD}$

$\overline{BD} \cong \overline{BD}$

j. Name a second pair of congruent triangles. Justify your answer by indicating which corresponding parts are congruent.

$\triangle HCF \cong \triangle HGF$

$\angle HCF \cong \angle HGF$

$\angle CHF \cong \angle GHF$

$\angle CFH \cong \angle GFH$

$\overline{HC} \cong \overline{HG}$

$\overline{CF} \cong \overline{GF}$

$\overline{HF} \cong \overline{HF}$

k. Name a third pair of congruent triangles. Justify your answer by indicating which corresponding parts are congruent.

$\triangle CBE \cong \triangle CFE$

$\angle CBE \cong \angle CFE$

$\angle BEC \cong \angle FEC$

$\angle ECB \cong \angle ECF$

$\overline{CB} \cong \overline{CF}$

$\overline{BE} \cong \overline{FE}$

$\overline{CE} \cong \overline{CE}$

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l. Name a fourth pair of congruent triangles. Justify your answer by indicating which corresponding parts are congruent.

$\triangle IDC \cong \triangle IHC$

$\angle IDC \cong \angle IHC$

$\angle DIC \cong \angle HIC$

$\angle DCI \cong \angle HCI$

$\overline{ID} \cong \overline{IH}$

$\overline{DC} \cong \overline{HC}$

$\overline{IC} \cong \overline{IC}$

m. Are any pairs of triangles that you identified congruent? If so, name them.

Yes, $\triangle ABD \cong \triangle CBD \cong \triangle HCF \cong \triangle HGF$.

n. How can Mary use this information to plan her project?

Mary can use the congruent triangles to create a template for her needed cuts.

She can then verify that the triangles she has cut will fit her pattern if the cut pieces are congruent.

Recommended Closure Activity

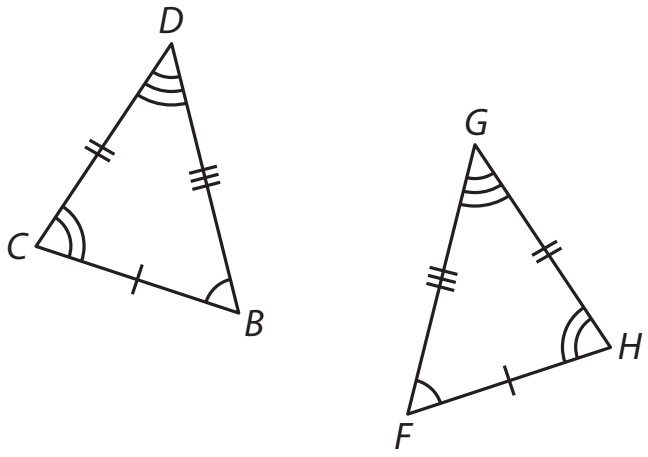
Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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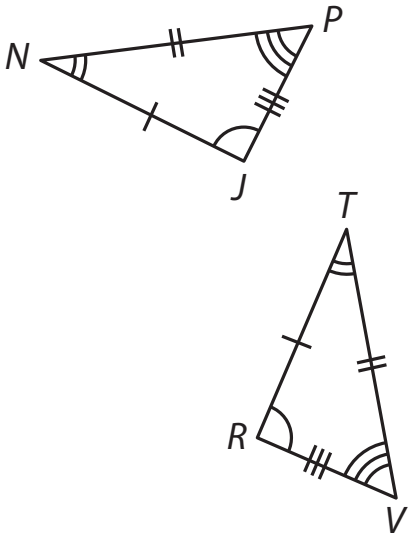
Practice 5.6.1: Triangle Congruency

Use the diagrams to correctly name each set of congruent triangles according to their corresponding parts.

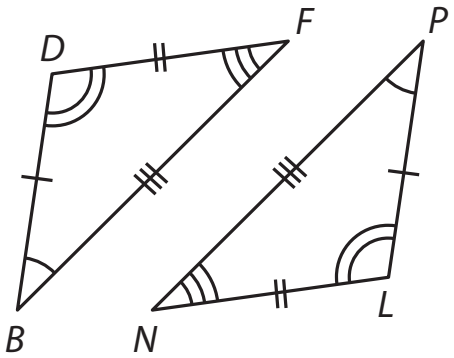
1.



2.



3.



continued

SWB p. 153; SRB Practice 5.6.1 p. U5-194

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Name the corresponding angles and sides for each pair of congruent triangles.

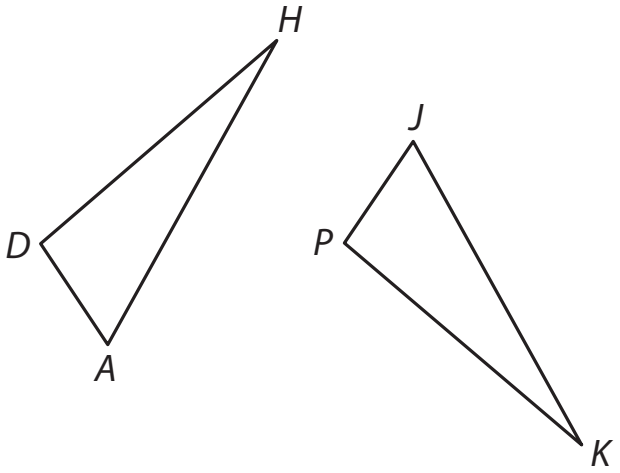
4. $\triangle QRS \cong \triangle WXY$

5. $\triangle AFH \cong \triangle CGJ$

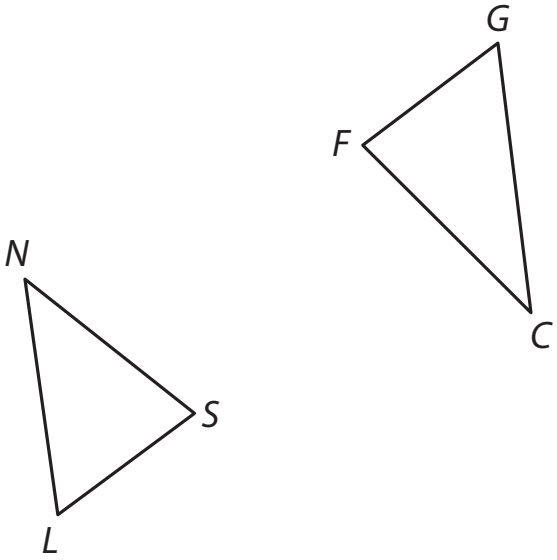
6. $\triangle LPQ \cong \triangle HJK$

Use a ruler and a protractor or construction tools to determine if the triangles are congruent. If they are, name the congruent triangles and their corresponding angles and sides.

7. An architect has two versions of a blueprint. Both blueprints contain a ramp. Are the ramps congruent?



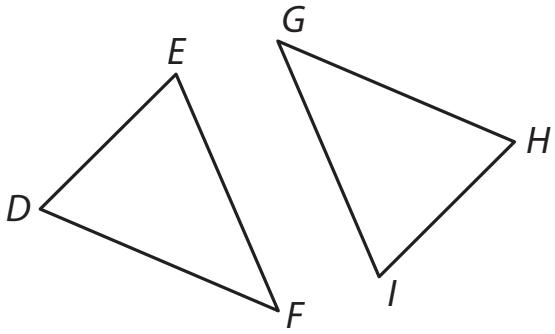
8. A recent delivery to a construction site included several trusses for a new roof structure. Two of the trusses are shown below. Are the trusses congruent?



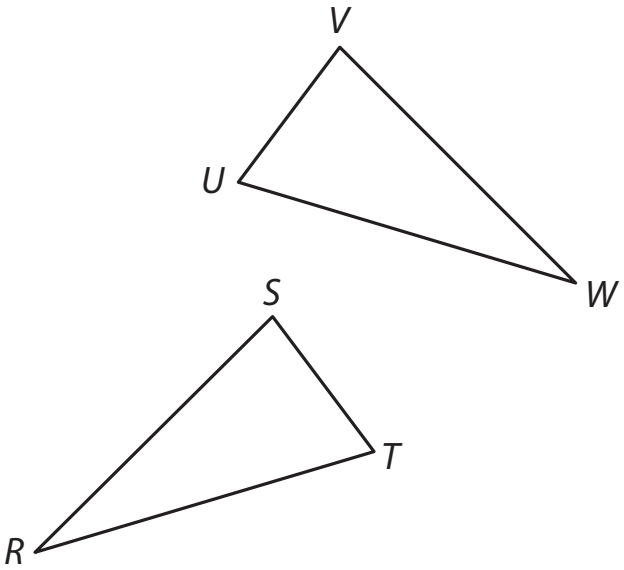
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9. Dave is making a blanket out of his old band shirts and has pre-cut several pieces of fabric. Are the T-shirt pieces congruent?



10. A tile installer wants to replace a broken tile with a scrap piece he has from a recent job. Is the scrap piece of tile congruent to the tile that needs replacement?

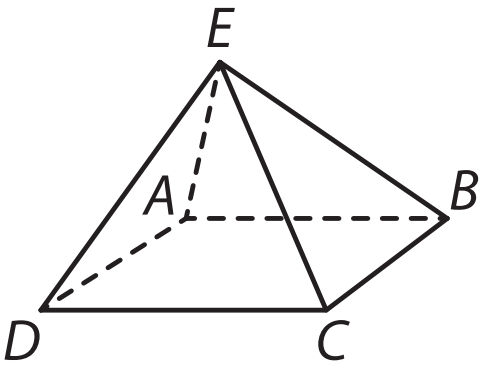


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Lesson 5.6.2: Explaining ASA, SAS, and SSS

Warm-Up 5.6.2

The Great Pyramid of Giza is the oldest and largest of the three pyramids near Giza, Egypt. The pyramid has a square base and four congruent triangular faces.



1. Is it possible to determine if the sides of the triangular faces are congruent? Explain your reasoning.

2. Is it possible to determine if the angles of the triangular faces are congruent? Explain your reasoning.

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Instruction

Lesson 5.6.2: Explaining ASA, SAS, and SSS

Common Core State Standard

G–CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Warm-Up 5.6.2 Debrief

1. Is it possible to determine if the sides of the triangular faces are congruent? Explain your reasoning.

   Yes, it is possible to determine if the sides of the triangular faces are congruent.

   It is known that the base of the pyramid is a square; therefore, $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$.

   Also, it is given that the triangular faces are congruent; therefore, the corresponding parts of the triangles are also congruent, so $\overline{AE} \cong \overline{BE} \cong \overline{CE} \cong \overline{DE}$.

2. Is it possible to determine if the angles of the triangular faces are congruent? Explain your reasoning.

   Yes, it is possible to determine if the angles of the triangular faces are congruent.

   The corresponding angles of the congruent triangles are congruent; therefore, $\angle EBA \cong \angle EAD \cong \angle EDC \cong \angle ECB$. Also, $\angle EAB \cong \angle EBC \cong \angle ECD \cong \angle EDA$.

Connection to the Lesson

- Students will use corresponding parts to determine if triangles are congruent.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- understanding that rigid motions maintain the shape and size of angles and segments, and that rigid motions include the transformations of reflections, rotations, and translations (G–CO.7)
- ability to identify corresponding pairs of sides and angles (no standard)

Introduction

When a series of rigid motions is performed on a triangle, the result is a congruent triangle. When triangles are congruent, the corresponding parts of the triangles are also congruent. It is also true that if the corresponding parts of two triangles are congruent, then the triangles are congruent. It is possible to determine if triangles are congruent by measuring and comparing each angle and side, but this can take time. There is a set of congruence criteria that lets us determine whether triangles are congruent with less information.

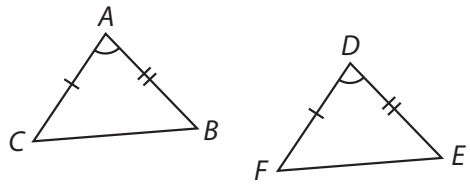
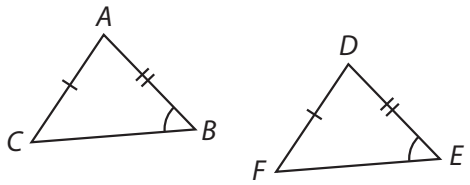
Key Concepts

- The criteria for triangle congruence, known as triangle congruence statements, provide the least amount of information needed to determine if two triangles are congruent.
- Each congruence statement refers to the corresponding parts of the triangles.
- By looking at the information about each triangle, you can determine whether the triangles are congruent.
- The **side-side-side (SSS)** congruence statement states that if three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent.
- If it is known that the corresponding sides are congruent, it is understood that the corresponding angles are also congruent.
- The **side-angle-side (SAS)** congruence statement states that if two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

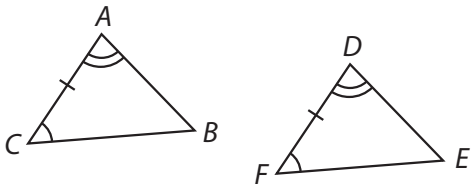
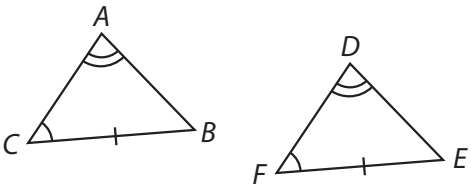
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Instruction

- The **included angle** is the angle that is between the two congruent sides.

Included angle	Non-included angle
	
$\angle A$ is included between $\overline{CA}$ and $\overline{AB}$. $\angle D$ is included between $\overline{FD}$ and $\overline{DE}$.	$\angle B$ is NOT included between $\overline{CA}$ and $\overline{AB}$. $\angle E$ is NOT included between $\overline{FD}$ and $\overline{DE}$.

- The **angle-side-angle** congruence statement, or **ASA**, states that if two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.
- The **included side** is the side that is between the two congruent angles.

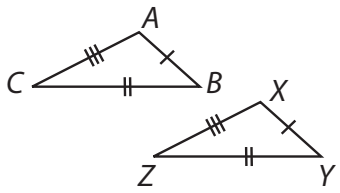
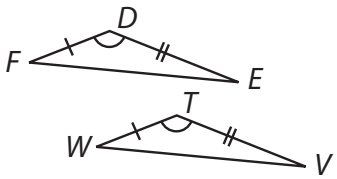
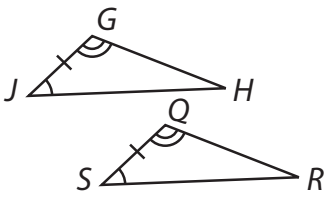
Included side	Non-included side
	
$\overline{AC}$ is included between $\angle C$ and $\angle A$. $\overline{FD}$ is included between $\angle F$ and $\angle D$.	$\overline{CB}$ is NOT included between $\angle C$ and $\angle A$. $\overline{FE}$ is NOT included between $\angle F$ and $\angle D$.

- A fourth congruence statement, angle-angle-side (AAS), states that if two angles and a non-included side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the triangles are congruent.

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Instruction

- This lesson will focus on the first three congruence statements: SSS, SAS, and ASA.
- The following diagram compares these three congruence statements.

Side-Side-Side (SSS)	Side-Angle-Side (SAS)	Angle-Side-Angle (ASA)
		
$\triangle ABC \cong \triangle XYZ$	$\triangle DEF \cong \triangle TVW$	$\triangle GHJ \cong \triangle QRS$

Common Errors/Misconceptions

- misidentifying included sides and angles, resulting in the wrong congruence statement
- misreading congruency symbols of triangles
- changing the order of named triangles, causing parts to be incorrectly interpreted as congruent

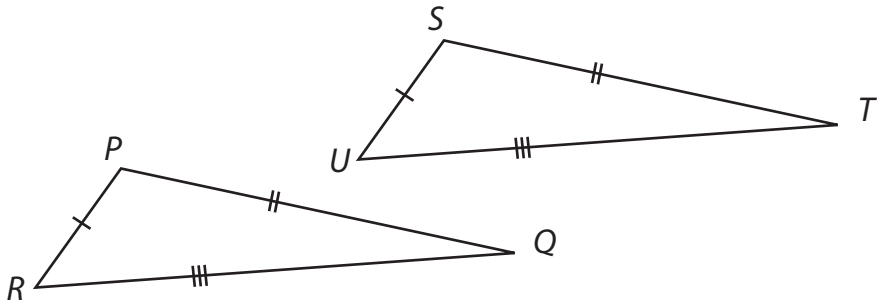
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Instruction

Guided Practice 5.6.2

Example 1

Determine which congruence statement, if any, can be used to show that $\triangle PQR$ and $\triangle STU$ are congruent.



1. Determine which components of the triangles are congruent.

   According to the diagram, $\overline{RP} \cong \overline{US}$, $\overline{PQ} \cong \overline{ST}$, and $\overline{TU} \cong \overline{QR}$.

   Corresponding side lengths of the two triangles are identified as congruent.

2. Determine if this information is enough to state that all six corresponding parts of the two triangles are congruent.

   It is given that all side lengths of the two triangles are congruent; therefore, all their angles are also congruent.

   Because all six corresponding parts of the two triangles are congruent, then the two triangles are congruent.

3. Summarize your findings.

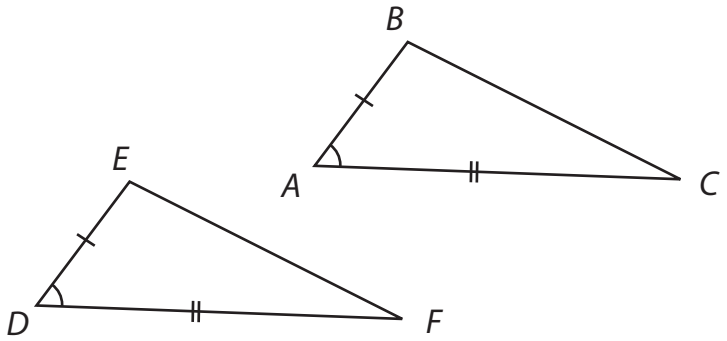
   $\triangle PQR \cong \triangle STU$ because of the congruence statement side-side-side (SSS).

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Example 2

Determine which congruence statement, if any, can be used to show that $\triangle ABC$ and $\triangle DEF$ are congruent.



1. Determine which components of the triangles are congruent.

   According to the diagram, $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle A \cong \angle D$.

   Two corresponding side lengths of the two triangles and one corresponding angle are identified as congruent.

2. Determine if this information is enough to state that all six corresponding parts of the two triangles are congruent.

   Notice that the congruent angles are included angles, meaning the angles are between the sides that are marked as congruent.

   It is given that two sides and the included angle are congruent, so the two triangles are congruent.

3. Summarize your findings.

   $\triangle ABC \cong \triangle DEF$ because of the congruence statement side-angle-side (SAS).

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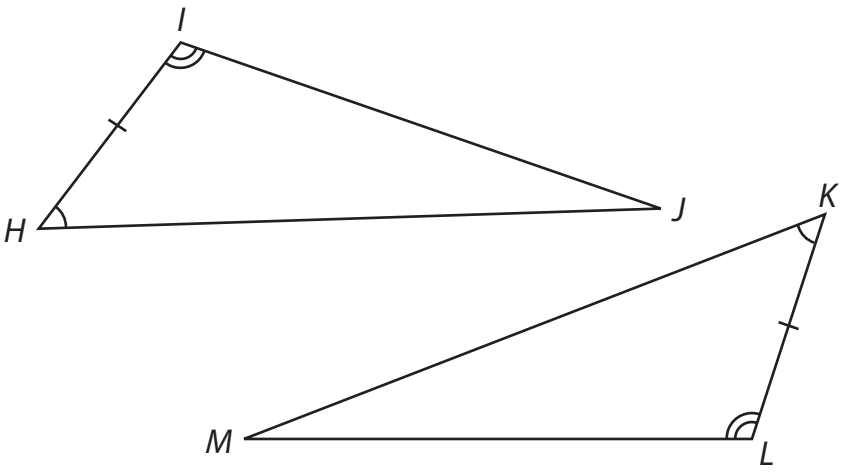
Example 3

Determine which congruence statement, if any, can be used to show that $\triangle HIJ$ and $\triangle KLM$ are congruent if $\overline{HI} \cong \overline{KL}$, $\angle H \cong \angle K$, and $\angle I \cong \angle L$.

1. Determine which components of the triangles are congruent.

   One corresponding side length of the two triangles and two corresponding angles are identified as congruent.

   It is often helpful to draw a diagram of the triangles with the given information to see where the congruent side is in relation to the congruent angles.



2. Determine if this information is enough to state that all six corresponding parts of the two triangles are congruent.

   Notice that the congruent sides are included sides, meaning the sides are between the angles that are marked as congruent.

   It is given that the two angles and the included side are equivalent, so the two triangles are congruent.

3. Summarize your findings.

   $\triangle HIJ \cong \triangle KLM$ because of the congruence statement angle-side-angle (ASA).

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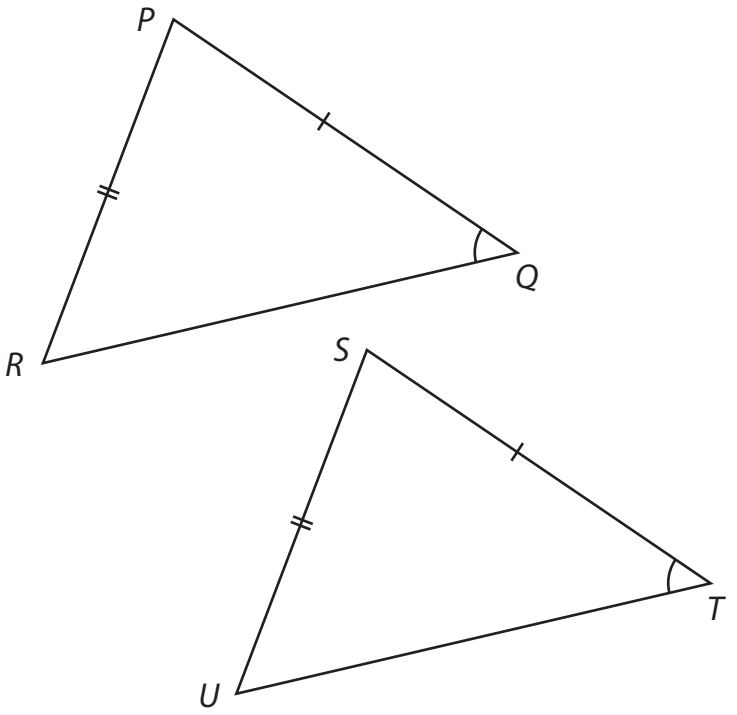
Example 4

Determine which congruence statement, if any, can be used to show that $\triangle PQR$ and $\triangle STU$ are congruent if $\overline{PQ} \cong \overline{ST}$, $\overline{PR} \cong \overline{SU}$, and $\angle Q \cong \angle T$.

1. Determine which components of the triangles are equivalent.

   Two corresponding side lengths of the two triangles and one corresponding angle are identified as congruent.

   Draw a diagram of the triangles with the given information to see where the congruent sides are in relation to the congruent angle.



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Instruction

2. Determine if this information is enough to state that all six corresponding parts of the two triangles are congruent.

   Notice that the congruent angles are not included angles, meaning the angles are not between the sides that are marked as congruent.

   There is no congruence statement that allows us to state that the two triangles are congruent based on the given information.

3. Summarize your findings.

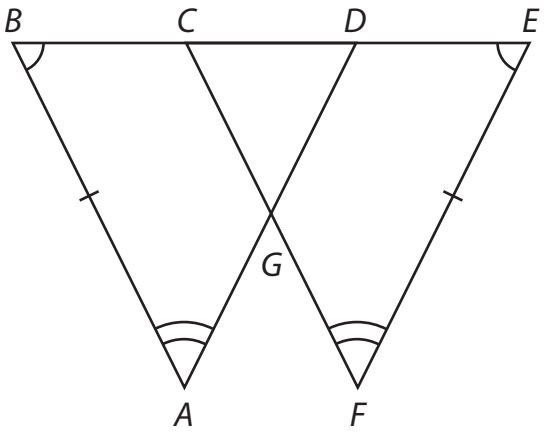
   It cannot be determined whether $\triangle PQR$ and $\triangle STU$ are congruent.

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Instruction

Example 5

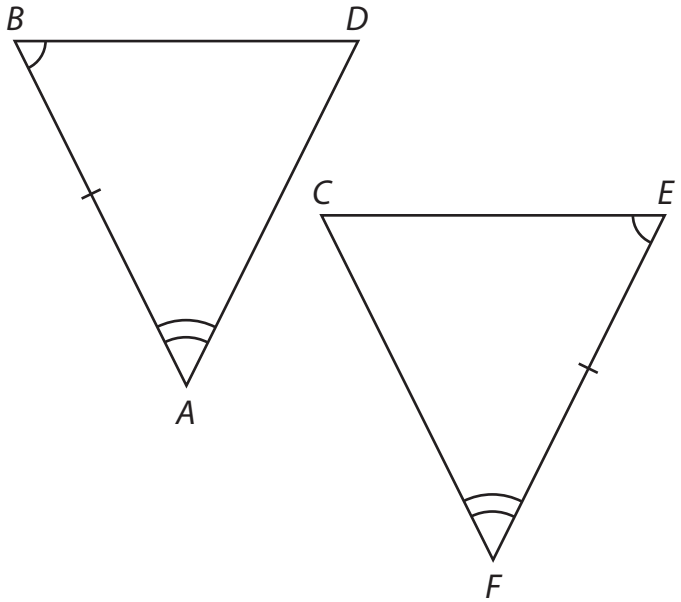
Determine which congruence statement, if any, can be used to show that $\triangle ABD$ and $\triangle FEC$ are congruent.



1. Determine which components of the triangles are congruent.

   Notice that the triangles overlap.

   If you have trouble seeing the two triangles, redraw each triangle.



   According to the diagram, $\overline{AB} \cong \overline{FE}$, $\angle A \cong \angle F$, and $\angle B \cong \angle E$.

   One corresponding side length of the two triangles and two corresponding angles are identified as congruent.

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2. Determine if this information is enough to state that all six corresponding parts of the two triangles are congruent.

   Notice that the congruent sides are included sides, meaning the sides are between the angles that are marked as congruent.

   It is given that two angles and the included side are equivalent, so the two triangles are congruent.

3. Summarize your findings.

   $\triangle ABD \cong \triangle FEC$ because of the congruence statement angle-side-angle (ASA).

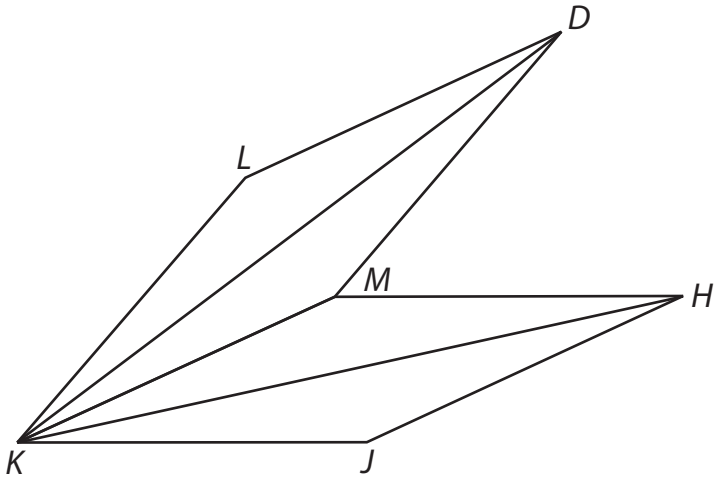


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Problem-Based Task 5.6.2: Stained Glass Pattern, Part II

SMP	
1 ✓	2
3 ✓	4 ✓
5	6
7	8

Mary is making a new stained glass art piece, as shown in the diagram below. As she studied the pattern, she recognized two rhombuses, each with a diagonal. She concluded that all the triangles in the pattern are congruent. Can Mary confidently state that all the triangles are congruent without using measuring tools? Use what you know about congruent triangles to prove or disprove her theory, and explain your reasoning. Remember that rhombuses have opposite angles that are congruent. Also, the diagonal of a rhombus bisects opposite angles.





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Problem-Based Task 5.6.2: Stained Glass Pattern, Part II

Coaching

a. Identify the two rhombuses in the new pattern.

b. In general, what are the properties of the sides of rhombuses?

c. In general, what are the properties of the angles of rhombuses?

d. What segment is shown to be the diagonal of $\square KLDM$?

e. The diagonal of a rhombus bisects opposite angles. Which angles are bisected by $\overline{KD}$?

f. Name the angles created by $\overline{KD}$ and determine if they are congruent to one another.

g. Is it possible to determine if $\triangle DLK \cong \triangle DMK$?

h. What segment is shown to be the diagonal of $\square KMHJ$?

i. The diagonal of a rhombus bisects opposite angles. Which angles are bisected by $\overline{KH}$?

j. Name the angles created by $\overline{KH}$ and determine if they are congruent to one another.

k. Is it possible to determine if $\triangle HMK \cong \triangle HJK$?

l. Mark congruent sides and angles on the diagram.

m. Is it possible to determine if $\triangle DLK \cong \triangle HMK$?

n. Is it possible to determine if $\triangle DLK \cong \triangle HJK$?

o. Can Mary confidently state that all triangles are congruent without using measuring tools?

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Instruction

Problem-Based Task 5.6.2: Stained Glass Pattern, Part II

Coaching Sample Responses

a. Identify the two rhombuses in the new pattern.

One rhombus is $\square KLDM$ and the second rhombus is $\square KMHJ$.

b. In general, what are the properties of the sides of rhombuses?

All sides of a rhombus are congruent.

c. In general, what are the properties of the angles of rhombuses?

Opposite angles of a rhombus are congruent.

d. What segment is shown to be the diagonal of $\square KLDM$?

$\overline{KD}$ is the diagonal of $\square KLDM$.

e. The diagonal of a rhombus bisects opposite angles. Which angles are bisected by $\overline{KD}$?

$\overline{KD}$ bisects $\angle LDM$ and $\angle MKL$.

f. Name the angles created by $\overline{KD}$ and determine if they are congruent to one another.

The angles created by $\overline{KD}$ are $\angle LDK$, $\angle MDK$, $\angle LKD$, and $\angle MKD$.

$\angle LDK \cong \angle MDK \cong \angle LKD \cong \angle MKD$

Opposite angles of the rhombus are congruent, and the diagonal bisects them (cuts them in half). Therefore, each angle created by $\overline{KD}$ is of the same measure. They are congruent.

g. Is it possible to determine if $\triangle DLK \cong \triangle DMK$?

Yes, there is enough information to determine that $\triangle DLK \cong \triangle DMK$.

$\overline{KL} \cong \overline{LD} \cong \overline{DM} \cong \overline{MK}$

$\angle LDK \cong \angle MDK \cong \angle LKD \cong \angle MKD$

$\overline{KD} \cong \overline{KD}$

It follows that $\triangle DLK \cong \triangle DMK$ because of side-angle-side (SAS) or side-side-side (SSS).

h. What segment is shown to be the diagonal of $\square KMHJ$?

$\overline{KH}$ is the diagonal of $\square KMHJ$.

i. The diagonal of a rhombus bisects opposite angles. Which angles are bisected by $\overline{KH}$?

$\overline{KH}$ bisects $\angle MHJ$ and $\angle JKM$.

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j. Name the angles created by $\overline{KH}$ and determine if they are congruent to one another.

The angles created by $\overline{KH}$ are $\angle MHK$, $\angle JHK$, $\angle MKH$, and $\angle JKH$.

$\angle MHK \cong \angle JHK \cong \angle MKH \cong JKH$

Opposite angles of the rhombus are congruent, and the diagonal bisects them, or cuts them in half. Therefore, each angle created by $\overline{KH}$ is of the same measure. They are congruent.

k. Is it possible to determine if $\triangle HMK \cong \triangle HJK$?

There is enough information to determine that $\triangle HMK \cong \triangle HJK$.

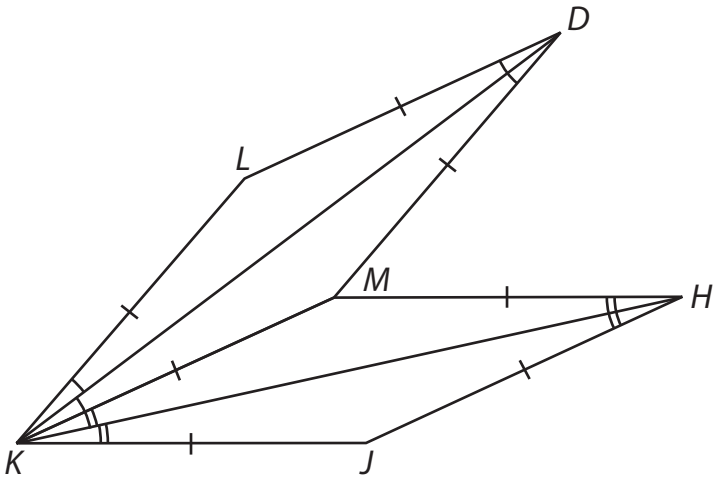
$\overline{JH} \cong \overline{HM} \cong \overline{MK} \cong \overline{KJ}$

$\angle MHK \cong \angle JHK \cong \angle MKH \cong JKH$

$\overline{KH} \cong \overline{KH}$

It follows that $\triangle HMK \cong \triangle HJK$ because of side-angle-side (SAS) or side-side-side (SSS).

l. Mark congruent sides and angles on the diagram.



m. Is it possible to determine if $\triangle DLK \cong \triangle HMK$?

It is not possible to make this determination because there is not enough information given.

The triangles only have two sets of corresponding congruent sides. It is not known whether any angles are congruent, or if the remaining two corresponding sides are congruent.

There is no congruence statement that allows us to state that the two triangles are congruent based on the given information.

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n. Is it possible to determine if $\triangle DLK \cong \triangle HJK$?

It is not possible to make this determination because there is not enough information given.

The triangles only have two sets of corresponding congruent sides. It is not known whether any angles are congruent, or if the remaining two corresponding sides are congruent.

There is no congruence statement that allows us to state that the two triangles are congruent based on the given information.

o. Can Mary confidently state that all triangles are congruent without using measuring tools?

Mary cannot confidently state that all triangles are congruent without using measuring tools because not enough information is known.

Recommended Closure Activity

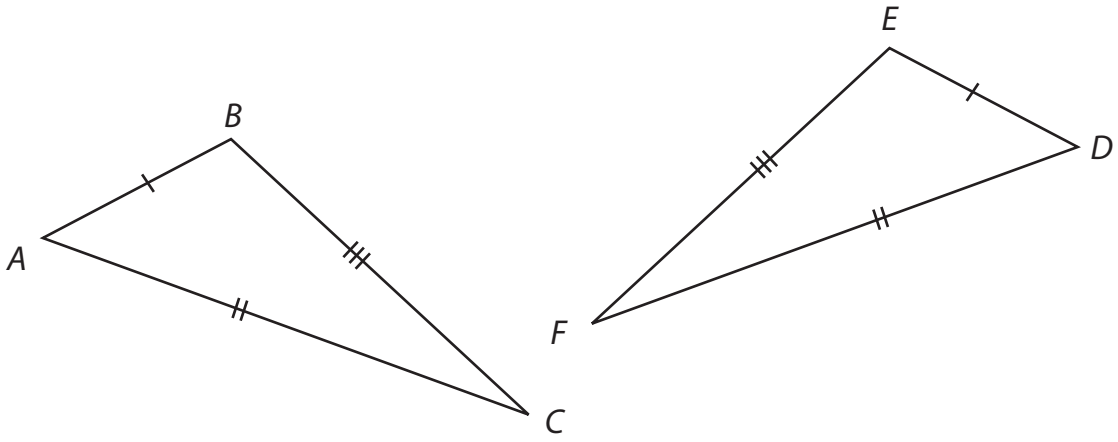
Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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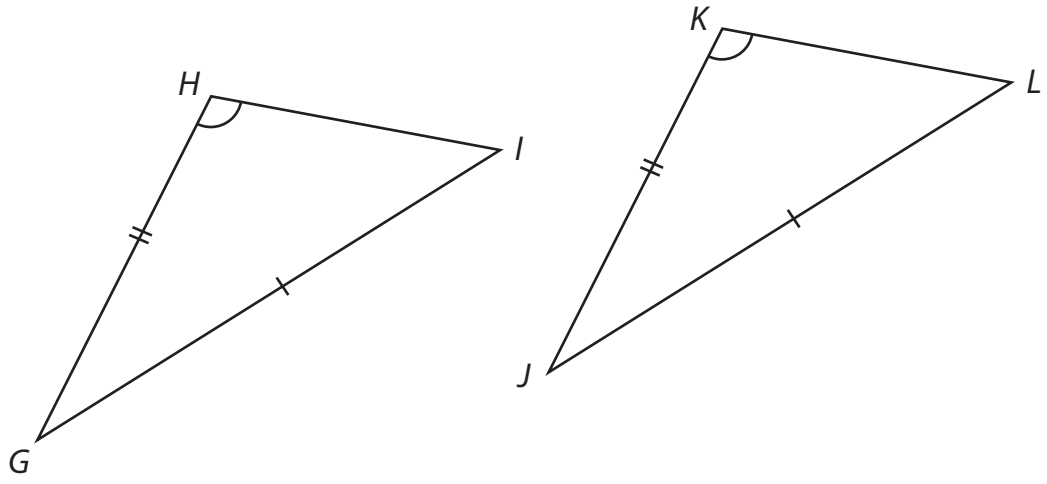
Practice 5.6.2: Explaining ASA, SAS, and SSS

For each diagram, determine which congruence statement can be used to show that the triangles are congruent. If it is not possible to prove triangle congruence, explain why not.

1.



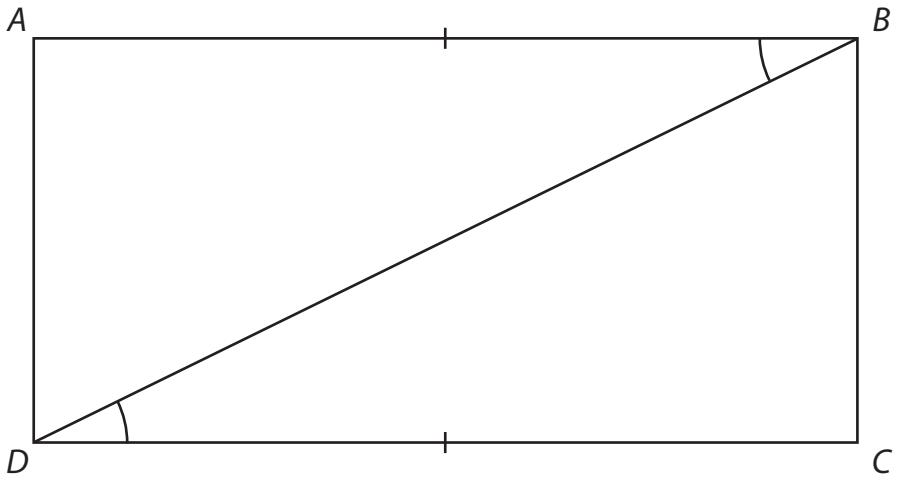
2.



continued

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3. Based on the information in the diagram, is $\triangle ABD$ congruent to $\triangle CDB$?



Use the given information to determine which congruence statement can be used to show that the triangles are congruent. If it is not possible to prove triangle congruence, explain why not.

4. $\triangle STU$ and $\triangle VWX$: $\angle S \cong \angle V$, $\angle T \cong \angle W$, and $\overline{ST} \cong \overline{VW}$

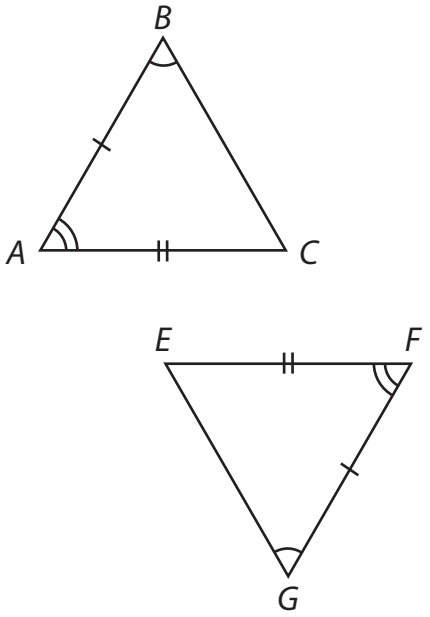
5. $\triangle MNO$ and $\triangle PQR$: $\angle O \cong \angle R$, $\overline{MO} \cong \overline{PR}$, and $\overline{NO} \cong \overline{QR}$

6. $\triangle GHI$ and $\triangle JKL$: $\angle G \cong \angle H$, $\overline{HI} \cong \overline{KL}$, $\angle J \cong \angle K$

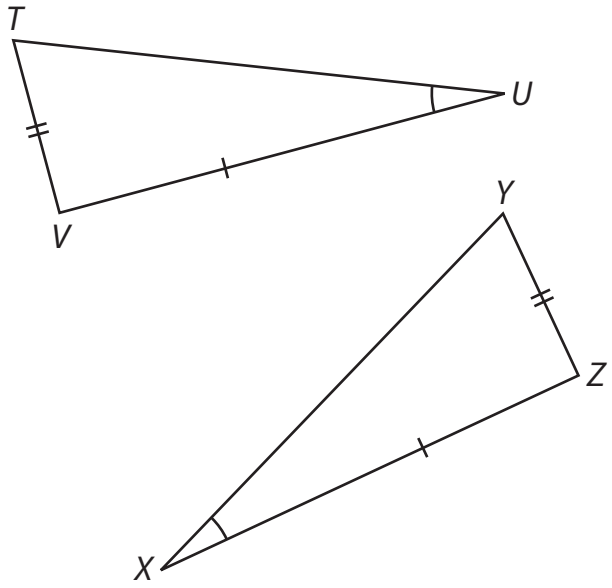
continued

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7. Jessalyn found two vintage road signs at a thrift store. She is re-decorating her room and congruency is important for her decor. Based on the information about each sign shown in the diagram below, determine if the triangles are congruent. If so, name the congruent triangles and identify the congruence statement used.



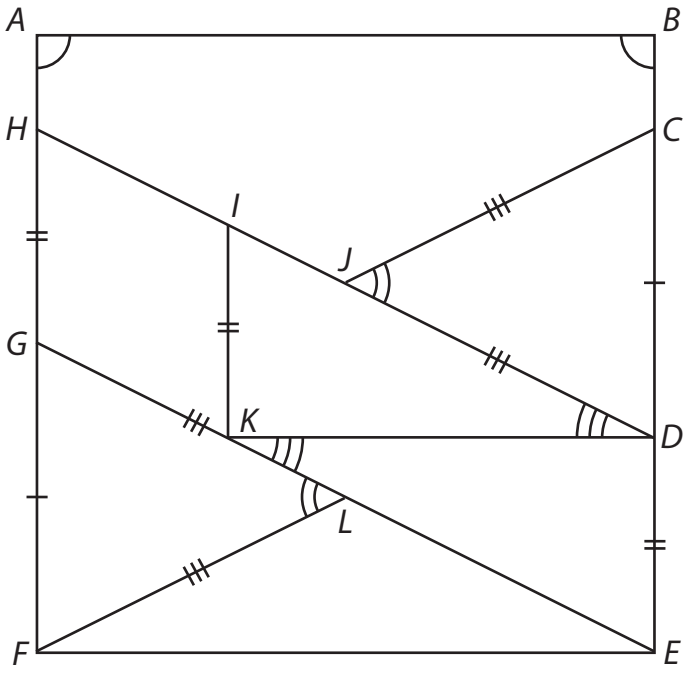
8. Isaac needs two congruent sails for his sailboat. The boat supply shop has only two sails in stock. Based on the information about each sail, determine if the sails are congruent. If so, name the congruent triangles and identify the congruence statement used.



continued

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The diagram below represents a plot of land in the town of Willow Woods. Use this diagram to solve problems 9 and 10.



9. The Kim family owns the plot of land marked by $\triangle IKD$ and the Reed family owns the plot of land marked $\triangle KDE$. Are the plots of land congruent? Explain your reasoning.

10. The Larsen family owns the plot of land marked by $\triangle FGL$ and the Rodriguez family owns the plot of land marked by $\triangle CDJ$. Are the two plots of land congruent? Explain your reasoning.

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Progress Assessment

Circle the letter of the best answer.

1. $\triangle ABC$ and $\triangle XYZ$ are congruent triangles. Which statement is known to be true?
   a. $\angle A \cong \angle B$
   b. $\angle X \cong \angle Y$
   c. $\angle B \cong \angle Y$
   d. $\angle A \cong \angle Y$

2. $\triangle DEF$ and $\triangle TUV$ are congruent triangles. Which statement is known to be true?
   a. $\overline{DE} \cong \overline{TU}$
   b. $\overline{DF} \cong \overline{TU}$
   c. $\overline{DF} \cong \overline{UV}$
   d. $\overline{DE} \cong \overline{TV}$

3. If there are two triangles for which $\overline{FH} \cong \overline{MP}$, $\overline{HJ} \cong \overline{PR}$, and $\overline{FJ} \cong \overline{MR}$, which statement is known to be true?
   a. $\triangle FHJ \cong \triangle RMP$
   b. $\triangle FHJ \cong \triangle MPR$
   c. $\triangle FHJ \cong \triangle RPM$
   d. $\triangle FJH \cong \triangle MPR$

4. $\triangle DEF$ and $\triangle GHI$ are congruent triangles where $\angle F \cong \angle I$. Which pairs of congruent components also indicate that the two triangles are congruent?
   a. $\angle D \cong \angle G$ and $\overline{DF} \cong \overline{GI}$
   b. $\overline{FE} \cong \overline{HI}$ and $\overline{DE} \cong \overline{GH}$
   c. $\angle D \cong \angle G$ and $\overline{DF} \cong \overline{HI}$
   d. $\angle D \cong \angle G$ and $\angle E \cong \angle H$

5. Which set of equivalent measures does not make it possible to determine if any two given triangles are congruent?
   a. angle-side-angle
   b. side-angle-side
   c. angle-angle-side
   d. angle-angle-angle

6. A triangle congruent to $\triangle DEF$ is to be constructed. Only three components are measured. Which three components, if constructed in the order listed, will always create a congruent triangle?
   a. angle-angle-angle
   b. side-angle-side
   c. side-side-angle
   d. Only the three side lengths can be used to create a congruent triangle.

continued

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7. For $\triangle ABC$ and $\triangle DEF$, the following is given: $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$. By which triangle congruence statement can it be concluded that the triangles are congruent?

   a. SSS

   b. SAS

   c. ASA

   d. It cannot be determined if the triangles are congruent.

8. For $\triangle ABC$ and $\triangle DEF$, the following is given: $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\overline{AB} \cong \overline{DE}$. By which triangle congruence statement can it be concluded that the triangles are congruent?

   a. SSS

   b. SAS

   c. ASA

   d. It cannot be determined if the triangles are congruent.

9. For $\triangle ABC$ and $\triangle DEF$, the following is given: $\angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$, and $\overline{BC} \cong \overline{EF}$. By which triangle congruence statement can it be concluded that the triangles are congruent?

   a. SSS

   b. SAS

   c. ASA

   d. It cannot be determined if the triangles are congruent.

10. For $\triangle ABC$ and $\triangle DEF$, the following is given: $\angle C \cong \angle F$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$. By which triangle congruence statement can it be concluded that the triangles are congruent?

    a. SSS

    b. SAS

    c. ASA

    d. It cannot be determined if the triangles are congruent.

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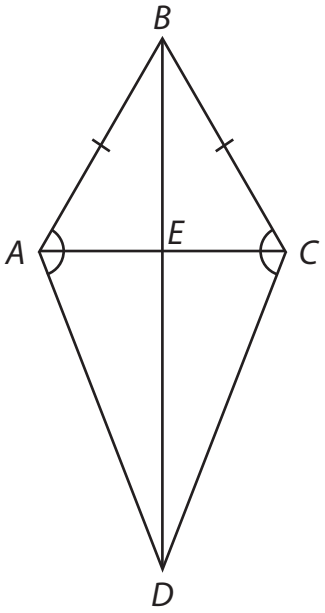
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Assessment

Use what you have learned about triangle congruence to solve the following problem.

11. Angelo is creating a kite and wants to determine congruent parts.

    a. Use the information in the diagram to list congruent parts of the kite.

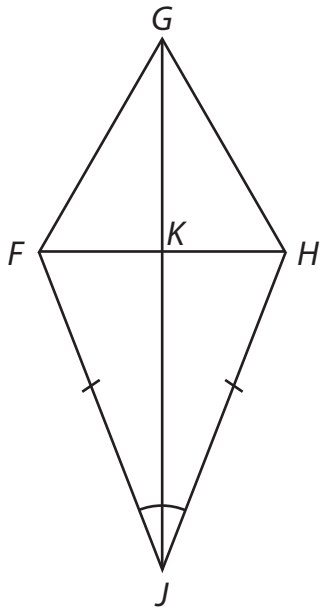


    b. Based on the information provided, is $\triangle ABD \cong \triangle CBD$? Explain your answer.

continued

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c. Angelo is creating a second, smaller kite for his brother. Use the information in the diagram to list congruent parts of the kite.



d. Based on the information provided, is $\triangle FKJ \cong \triangle HKJ$? Explain your answer.

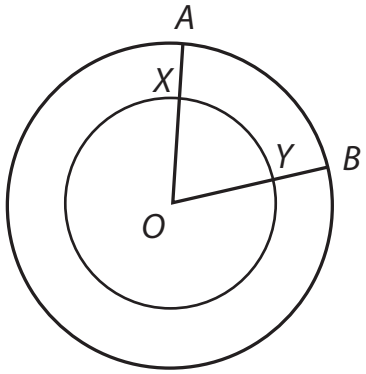
e. List all the different pieces of information needed to prove congruency for each triangle congruence theorem.

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Unit 5 Assessment

Circle the letter of the best answer.

1. Given the diagram below, which of the following is a true statement?



a. $\overline{OX} \cong \overline{OB}$

b. $\overset{\frown}{XY} \cong \overset{\frown}{AB}$

c. $\overline{AX} \| \overline{BY}$

d. $\overset{\frown}{AB} > \overset{\frown}{XY}$

2. If $f(x, y) = (-y, -x)$ and $g(x, y) = (6, -3)$, then what is $f(g(x, y))$?

a. (–6, –3)

b. (–3, 6)

c. (–6, 3)

d. (3, –6)

3. The point A (–5, 2) has undergone the transformation $T_{-1, 2}$. Which point is the preimage of A?

a. (–6, 4)

b. (–6, 0)

c. (–4, 0)

d. (–4, 4)

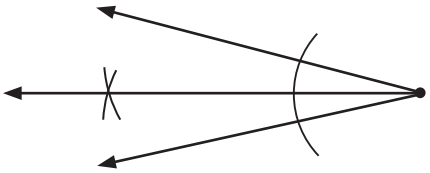
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4. Given A (–5, 7) and $T(x + 3, y – 8)$, state A'' after a reflection over the x-axis of the point $T(A)$.

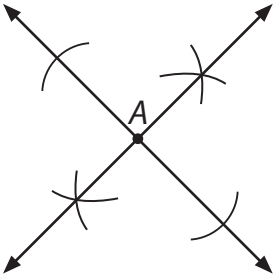
   a. A'' (–2, 1)
   b. A'' (2, –1)
   c. A'' (2, 1)
   d. A'' (–2, –1)

5. Which geometric construction matches the diagram below?



   a. construction of a segment bisector
   b. construction of an angle bisector
   c. copying a segment
   d. copying an angle

6. Which geometric construction matches the diagram below?



   a. construction of a median
   b. construction of a parallel line through point A
   c. construction of a segment bisector
   d. construction of a perpendicular line through point A

continued

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7. Think about the process for constructing an equilateral triangle inscribed in a given circle. Which of the following figures is also inscribed in a circle using the same process?

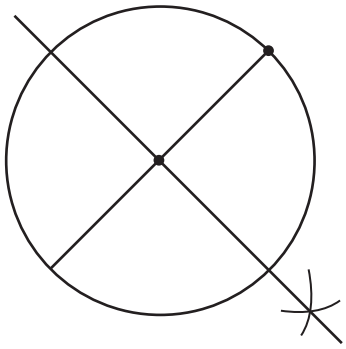
   a. a square

   b. a pentagon

   c. a hexagon

   d. a scalene triangle

8. The following construction is incomplete. Based on the markings shown, what figure will be constructed?



   a. an equilateral triangle

   b. a square

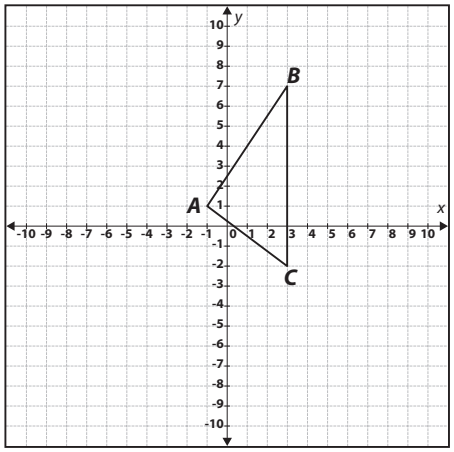
   c. a hexagon

   d. a dodecagon

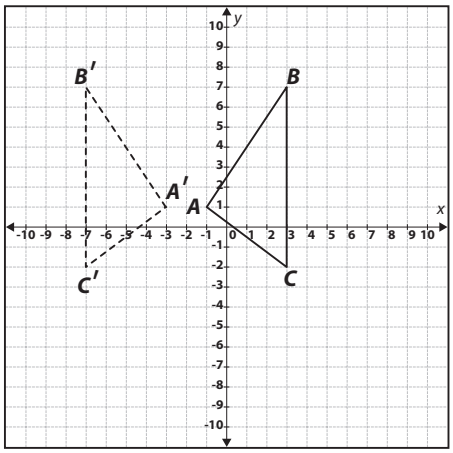
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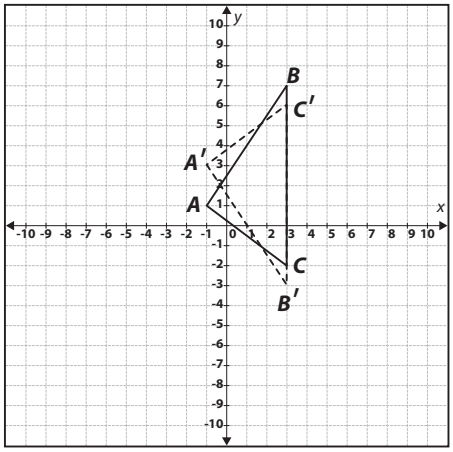
9. Which illustration shows the effect(s) of reflecting the given triangle over the line $x = -2$?



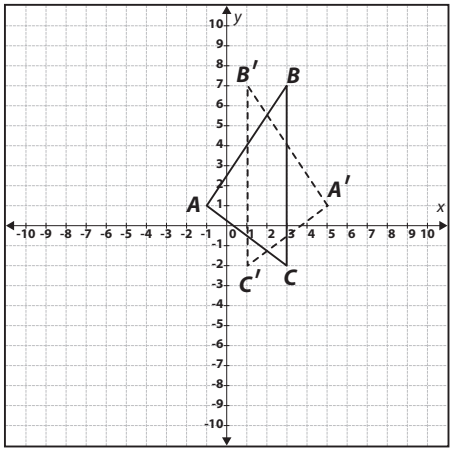
a.



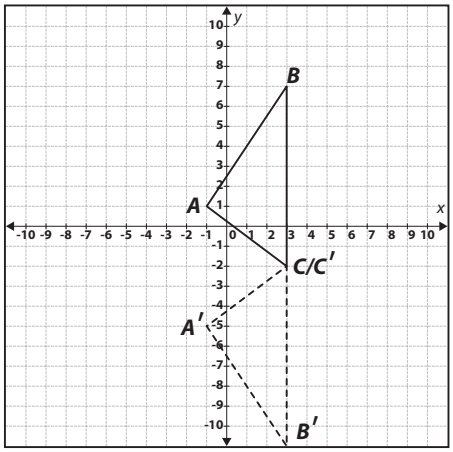
c.



b.



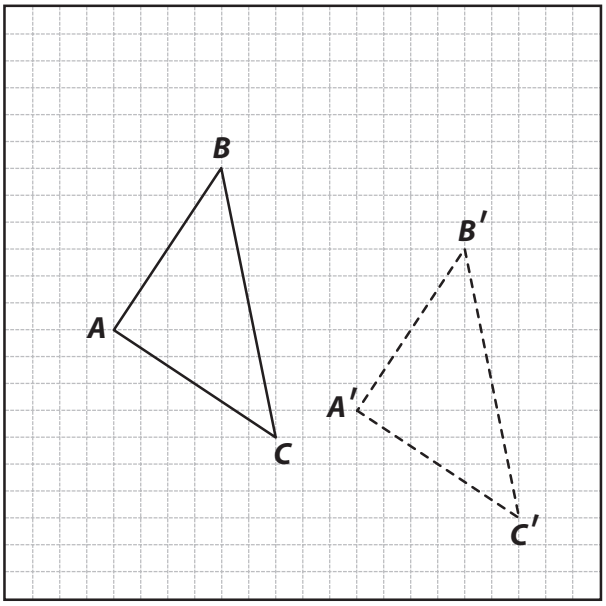
d.



continued

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10. Are the two figures congruent? Why or why not?



a. Congruent; $\triangle ABC$ has undergone a rigid motion of dilation.

b. Congruent; $\triangle ABC$ has undergone a rigid motion of a translation.

c. Not congruent; $\triangle ABC$ has undergone a non-rigid motion of dilation.

d. Not congruent; $\triangle ABC$ has undergone a non-rigid motion of a translation.

11. Which of the following figures has exactly two lines of symmetry?

a. a square

b. a trapezoid

c. a rectangle

d. a circle

12. For $\triangle EFG$ and $\triangle MNP$, it is known that $\overline{EF} \cong \overline{MN}$, $\angle G \cong \angle P$, and $\overline{FG} \cong \overline{NP}$. Determine if the triangles are congruent, and if so, by which type of congruency.

a. SSS

b. SAS

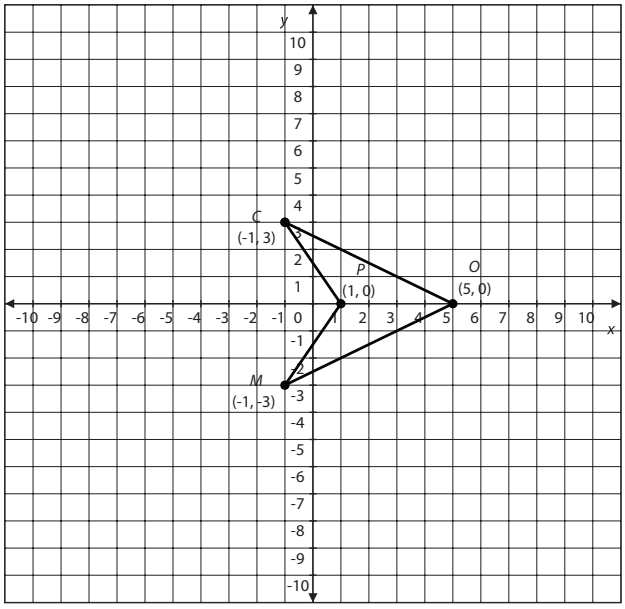
c. ASA

d. It cannot be determined if the triangles are congruent.

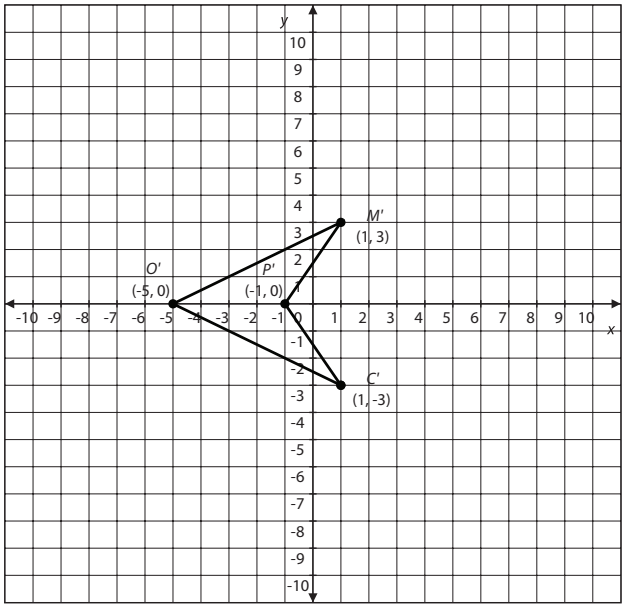
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13. Your computer science teacher has assigned you a video project. The introduction must include an animation of an arrowhead, as shown below. An animation is a series of transformations, called frames. The first three frames given to you by your teacher are:



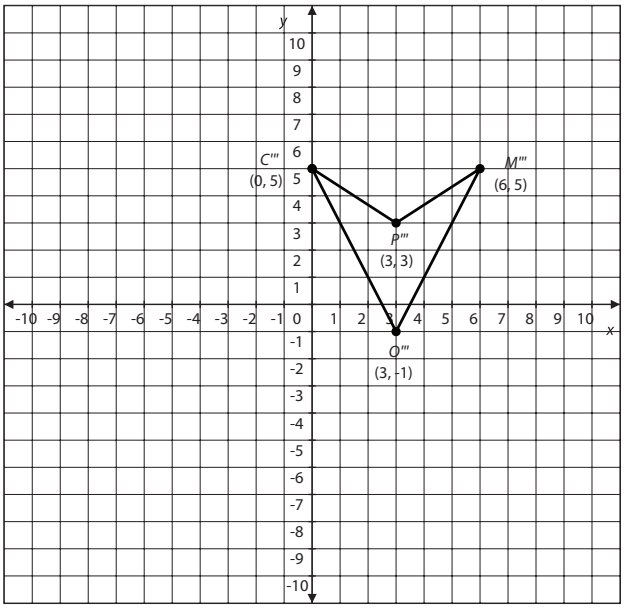
Frame 1



Frame 2

continued

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Frame 3

a. Looking at Frame 1, state the angles within the arrowhead that are acute.

b. What transformation occurred from the preimage in Frame 1 to the image in Frame 2?

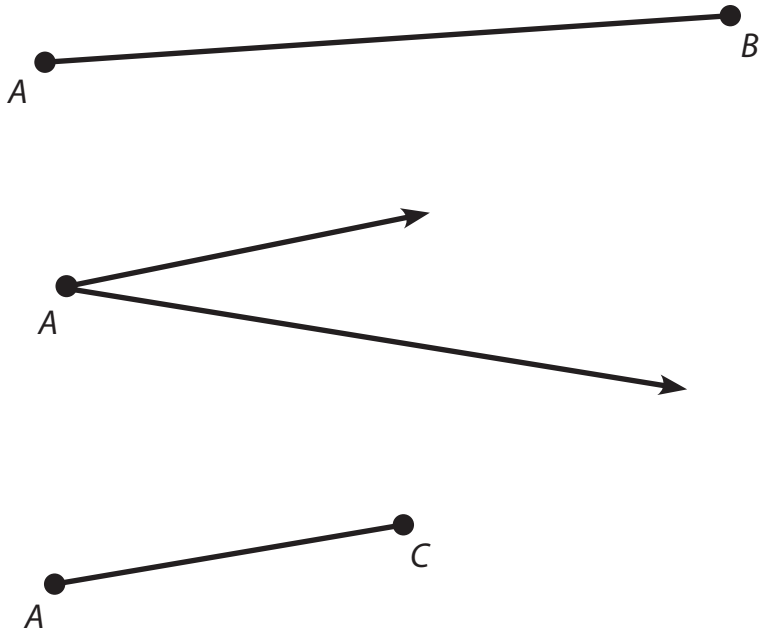
c. Frame 3 is Frame 2 under $T_{3,4}(r_{y=x})$. Frame 3 is the graph of $C'''O'''M'''P'''$. What are the coordinates and graph of $C''O''M''P''$?

d. Are the graphs in Frames 1 and 3 isometric? Explain.

continued

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14. Use the parts of a triangle given below to solve the problems that follow.



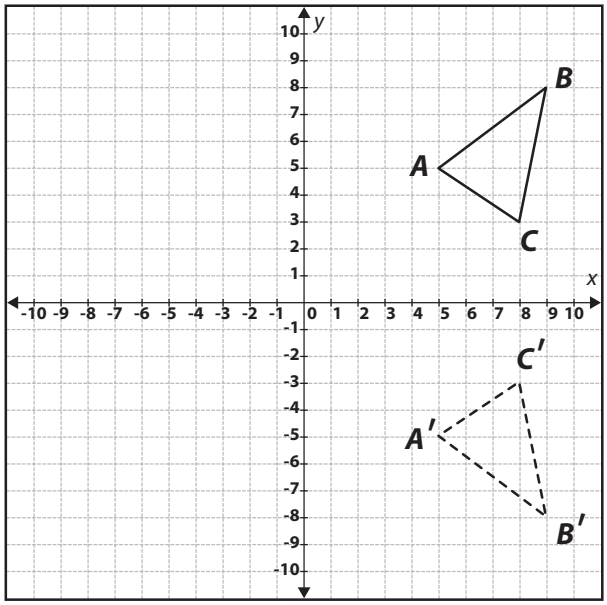
a. Use construction tools to construct $\triangle ABC$ using the given parts.

b. Is it possible to construct a second non-congruent triangle using the given parts? If it is not possible, explain why not. If it is possible, construct a second non-congruent triangle.

continued

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15. $\triangle ABC$ is reflected over the line $y = 0$. Use the diagram to solve the problems that follow.



a. Reflect $\triangle A'B'C'$ over the line $x = 0$.

b. Is it possible to perform one rigid motion to the preimage $\triangle ABC$ rather than a series of reflections to create the same image that you created in question a? Explain your reasoning.

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Answer Key

Lesson 1: Introducing Transformations

Pre-Assessment, p. U5-1

1. b
2. c
3. d
4. a
5. b

Warm-Up 5.1.1, p. U5-5

1. Marta's method will yield a perfect circle because she is drawing a path such that every point on it will be the same distance from the center.
2. Marta could measure the distance from one side of her drawing through the center point made by the pin to the other side of her drawing. This measurement should be the same as any distance from one side of her drawing through the center point to the other side of the drawing.

Practice 5.1.1: Defining Terms, p. U5-17

1. segment; $\overline{PQ}$
2. a set of points on a plane at a certain distance, or radius, from a single point
3. perpendicular
4. A point is an exact position or location in a given plane.
5. acute
6. angle
7. $\overset{\frown}{AB}$ cannot be longer than $\overset{\frown}{XY}$.
8. $\overset{\frown}{AB}$ cannot be longer than $\overset{\frown}{ABC}$.
9. obtuse
10. line

Warm-Up 5.1.2, p. U5-18

1. $L(x) = R(x) - 2$, so L(Ainsley) = R(Ainsley) – 2 = 5 – 2 = \$3; L(Jack) = R(Jack) – 2 = 6 – 2 = \$4; L(Caleb) = R(Caleb) – 2 = 7 – 2 = \$5.
2. The function $L(x)$ returns the original amount given to each cousin prior to visiting Grandpa Ray.

Practice 5.1.2: Transformations As Functions, p. U5-37

1. Yes, transformations can be combined.
2. $k(x)$ comes first, followed by g, then f. Yes, the order matters because if you apply the functions in different orders you get different results.
3. Isometric means equal measure. When a transformation is isometric it means that the distances between any two points in the transformed set of points are equal.
4. The resulting image is the preimage, yielding the identity transformation I, where the position of all points is preserved.
5. Because the transformation is isometric all distances are preserved, so $d\left(T\left(\overset{\frown}{PQ}\right)\right) = 2$.
6. $T_{5,4}(P) = T_{5,4}(2, 3) = (2 + 5, 4 + 3) = (7, 7)$
7. $\Delta A'B'C'$ has vertices A' (7, 1), B' (3, 3), and C' (3, 1).
8. $R_{90}(Q) = R_{90}(1, 0) = (0, 1)$
9. $P(x - 3, y + 3)$; $h = 3$, $k = -3$
10. $T_{-2,-5}$

Warm-Up 5.1.3, p. U5-38

1. The eyes are symmetrical through the center of the whites and also with each other; the mouth is symmetrical; and the full shape of the face is symmetrical. The eyes, mouth, and face share one line of symmetry.
2. A car body has a line of symmetry running vertically from the front of the car to the rear. Because a car is three-dimensional, this line is a plane of symmetry.
3. There are three lines of symmetry.

Practice 5.1.3: Applying Lines of Symmetry, p. U5-54

1. a circle
2. 6
3. Yes; a rhombus without a right angle has less than four lines of symmetry.
4. An isosceles triangle has one line of symmetry bisecting the angle created by the two congruent sides.
5. A scalene triangle has no lines of symmetry.
6. 360/5 = 72; 72°
7. The lines are perpendicular.
8. 5 lines of symmetry
9. 6 inches
10. Nine, because there are 9 symmetry planes.

Progress Assessment, p. U5-55

1. a
2. a
3. d
4. d
5. c
6. b
7. a
8. d
9. c
10. c
11. The resulting image is the preimage, yielding the identity transformation I, where the position of all points is preserved.

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Lesson 2: Defining and Applying Rotations, Reflections, and Translations

Pre-Assessment, p. U5-57

1. b
2. a
3. a
4. d
5. a

Warm-Up 5.2.1, p. 60

1. $A'(5,-8), B'(9,-1),$ and $C'(-1,-2)$
2. $A''(8,5), B''(1,9),$ and $C''(2,-1)$
3. Yes, the different order of functions yields a different result.

Practice 5.2.1: Defining Rotations, Reflections, and Translations, p. U5-71

1. circular arcs
2. Points are translated the same distance along parallel lines.
3. Points are translated twice the distance the point is from a given line (the line of rotation) along perpendicular lines.
4. The point farthest from the point of rotation is moved the greatest distance.
5. Points on the line of reflection are not moved in a reflection.
6. $Z(x, y)$, so $T_{6,8}(Z) = T_{6,8}(x, y) = (x+6, y+8)$
7. $r_{x\text{-axis}}(R_{180}(x, y))$
8. $A'(4,-3), B'(11,-4),$ and $C'(9,-6)$
9. $P' = (-31, 21)$
10. $T_{2,5}(R_{180}(V)) = T_{2,5}(R_{180}(3,3)) = T_{2,5}(-3,-3) = (-1,2)$; therefore, V (3, 3) has moved left 4 units along the x-axis and down 1 unit along the y-axis.

Warm-Up 5.2.2, p. U5-72

1. rotation
2. translation
3. reflection

Practice 5.2.2: Applying Rotations, Reflections, and Translations, p. U5-85

For problems 1, 2, 8, and 9, check students' graphs for the coordinates and labels listed below.

1. $\triangle A'B'C'$: $A'(5,4), B'(6,9),$ and $C'(8,3)$
2. $\triangle A'B'C'$: $A(-1,1), B'(1,6),$ and $C'(-3,2)$
3. Quadrant II
4. $R_{90}(T_{1,1}(P)) = R_{90}(T_{1,1}(5,-1)) = R_{90}(5+1,-1+1) = R_{90}(6,0) = (0,6)$
5. $R_{180}(A) = R_{180}(-2,2) = (2,-2)$

   $R_{180}(B) = R_{180}(-1,2) = (1,-2)$

   $R_{180}(C) = R_{180}(-1,1) = (1,-1)$

   $R_{180}(D) = R_{180}(-2,1) = (2,-1)$
6. $R_m = R_{180}$
7. the x-axis
8. $D'(-9,-10), E'(-8,2),$ and $F'(-1,-4)$
9. $A'(-4, 0), B'(2, 1),$ and $C'(-1, 8)$
10. $R_{270}(W)$

Progress Assessment, p. U5-86

1. d
2. a
3. b
4. d
5. c
6. d
7. d
8. b
9. c
10. c
11. Answers:
    a. $R_{90}(T_{-2,-3}(r_{x\text{-axis}}(\triangle ABC)))$

    b. $A'(2,-1), B'(-1,-2),$ and $C'(-1,1)$

    c. $A''(0,-4), B''(-3,-5),$ and $C''(-3,-2)$

    d. $A'''(4,0), B'''(5,-3),$ and $C'''(2,-3)$

    e. Preimage $A(2,1), B(-1,2),$ and $C(-1,-1)$ and image $A'''(4,0), B'''(5,-3),$ and $C'''(2,-3)$; check student graphs for accuracy.

Lesson 3: Constructing Lines, Segments, and Angles

Pre-Assessment, p. U5-88

1. d
2. c
3. c
4. d
5. b

Warm-Up 5.3.1, p. U5-94

1. The areas do not overlap.
2. Maryellen could use the leash as a compass to determine where to place the stake to minimize overlap.

Practice 5.3.1: Copying Segments and Angles, p. U5-116

1–10. Check students' work for accuracy.

Warm-Up 5.3.2, p. U5-118

1. Check students' work for accuracy.
2. 90°

Practice 5.3.2: Bisecting Segments and Angles, p. U5-140

1–10. Check students' work for accuracy.

Warm-Up 5.3.3, p. U5-142

1. Check students' work for accuracy.
2. Check students' work for accuracy.

Practice 5.3.3: Constructing Perpendicular and Parallel Lines, p. U5-166

1–10. Check students' work for accuracy.

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Progress Assessment, p. U5-167

1. c
2. b
3. a
4. c
5. d
6. b
7. c
8. b
9. d
10. c
11. Check students' work for accuracy.

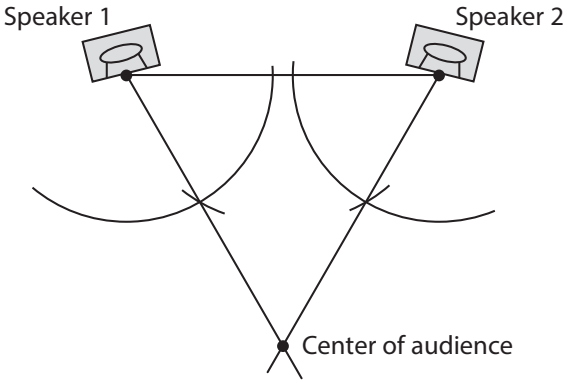
Lesson 4: Constructing Polygons

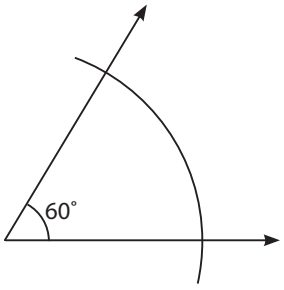
Pre-Assessment, p. U5-172

1. d
2. b
3. c
4. b
5. a

Warm-Up 5.4.1, p. U5-175

1. Copy the given 60° angle to construct a triangle. Extend the sides of the copied angles to determine the center of the audience.





2. No, it is not possible to construct a second non-congruent triangle using the given information. The given information includes two angle measures and a side length between those two angles. If you have two triangles and any two angles and the included side are equal, then the triangles are congruent.

Practice 5.4.1: Constructing Equilateral Triangles Inscribed in Circles, p. U5-197

1–3. Check students' work for accuracy. Be sure each of the vertices lies on the circle.
4. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is twice the length of the given segment.
5. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to half the length of the given segment.

6–8. Check students' work for accuracy. Be sure each of the vertices lies on the circle.
9. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is twice the length of the given segment.
10. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to half the length of the given segment.

Warm-Up 5.4.2, p. U5-199

1. Each of the new angles measures 45°.
2. The angle bisector intersects the square at the bisected angle and the angle opposite the bisected angle.
3. Antonia will have created 4 triangles.
4. Each triangle will have two 45° angles and one 90° angle.

Practice 5.4.2: Constructing Squares Inscribed in Circles, p. U5-218

1–2. Check students' work for accuracy. Be sure each of the vertices lies on the circle.
3–6. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to the length of the given segment.
7. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to twice the length of the given segment.
8. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to half the length of the given segment.

9–10. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to half the length of the given segment.

Warm-Up 5.4.3, p. U5-219

1. The length of each side of the square planting bed will be $\frac{1}{4}$ the length of the original plank of wood.
2. To determine the length of each side of the square planting bed, first find the midpoint of the plank of wood by bisecting the plank. This construction will divide the plank into two equal pieces. Next, find the midpoint of one of the halves by bisecting it. The result is a piece of the plank that is $\frac{1}{4}$ the original length of the plank of wood.

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Practice 5.4.3: Constructing Regular Hexagons Inscribed in Circles, p. U5-241

1. Check students' work for accuracy. Be sure each of the vertices lies on the circle.

2–3. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to the length of the given segment.

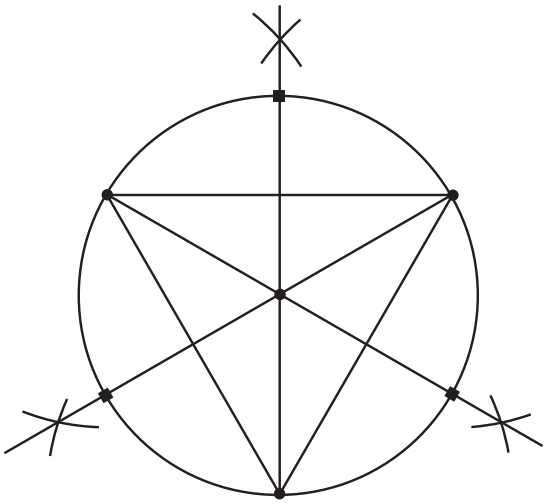
4. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to twice the length of the given segment.
5. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to half the length of the given segment.
6. Check students' work for accuracy. Be sure each of the vertices lies on the circle.

7–8. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to the length of the given segment.

9. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to twice the length of the given segment.
10. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to half the length of the given segment.

Progress Assessment, p. U5-243

1. a
2. b
3. a
4. c
5. d
6. b
7. c
8. a
9. c
10. d
11. a. Check students' work for accuracy.



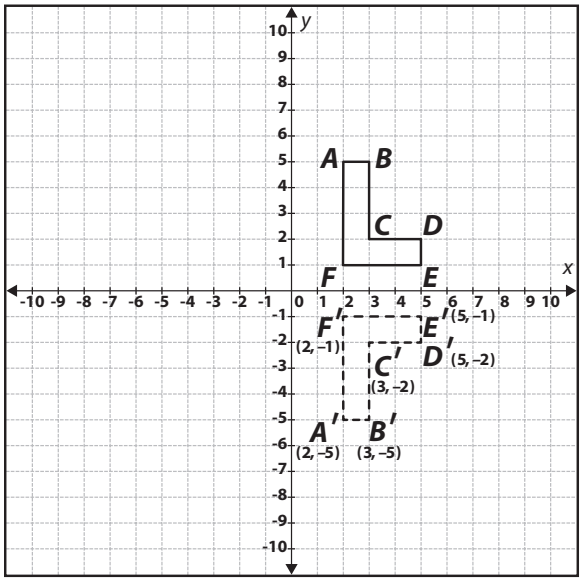
b. Find the perpendicular bisector of each side of the equilateral triangle created when lines are drawn to connect the vending machines. Extend the bisectors to locate intersection points with the circle. The points of intersection represent the placement of the recycling bins.

Lesson 5: Exploring Congruence

Pre-Assessment, p. U5-248

1. d
2. a
3. b
4. d
5. c

Warm-Up 5.5.1, p. U5-254

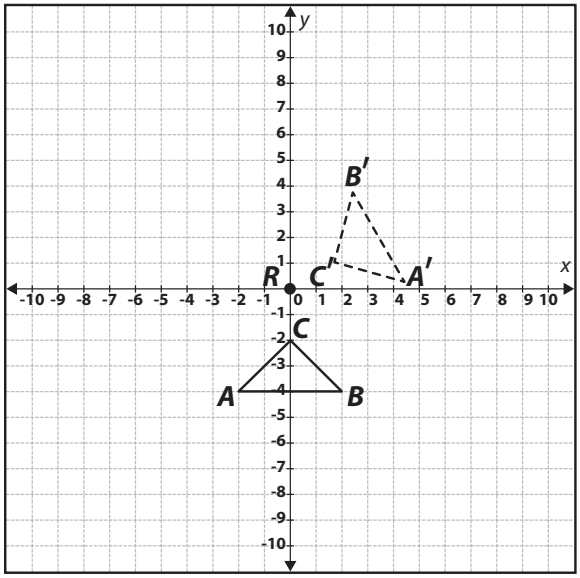
1. The coordinates:
   $r_{x\text{-axis}}(A\ (2, 5)) = A'\ (2, -5)$
   $r_{x\text{-axis}}(B\ (3, 5)) = B'\ (3, -5)$
   $r_{x\text{-axis}}(C\ (3, 2)) = C'\ (3, -2)$
   $r_{x\text{-axis}}(D\ (5, 2)) = D'\ (5, -2)$
   $r_{x\text{-axis}}(E\ (5, 1)) = E'\ (5, -1)$
   $r_{x\text{-axis}}(F\ (2, 1)) = F'\ (2, -1)$
2. 

Practice 5.5.1: Describing Rigid Motions and Predicting the Effects, p. U5-273

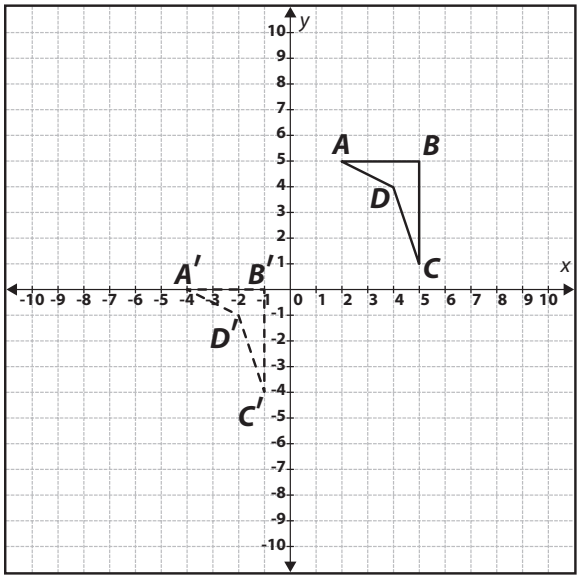
1. Translation; right 4 units and up 3 units; the orientation stayed the same.
2. Reflection; the line of reflection is $x = -1$. The orientation changed, and the preimage and the image are mirror reflections of each other. The line of reflection is the perpendicular bisector of the segments connecting the vertices of the preimage and image.
3. Rotation; the orientation changed, but the figures are not mirror images of each other.

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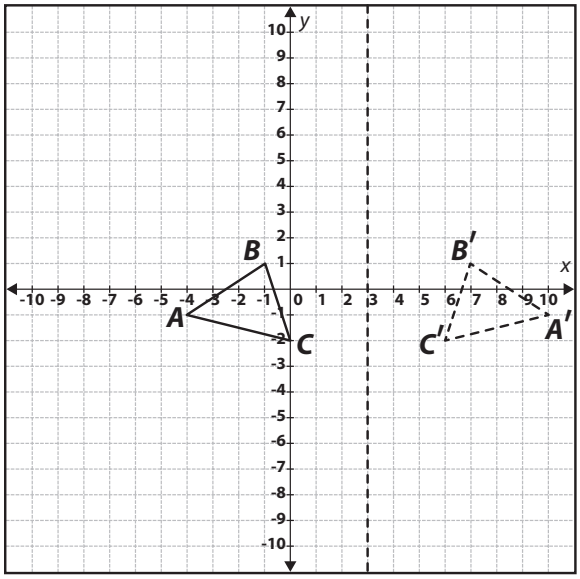
4.



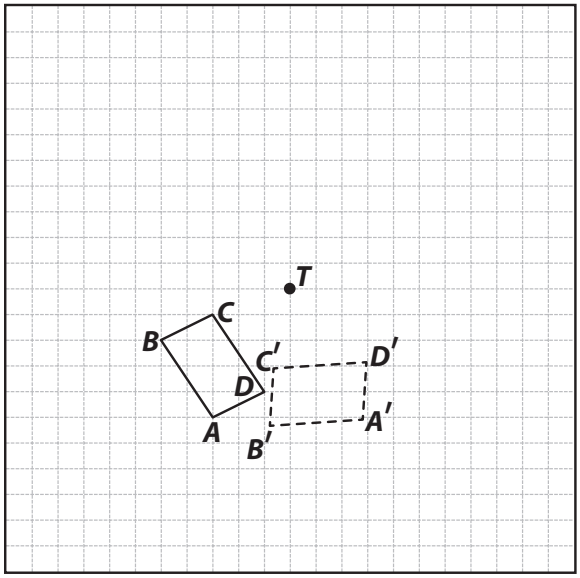
5.



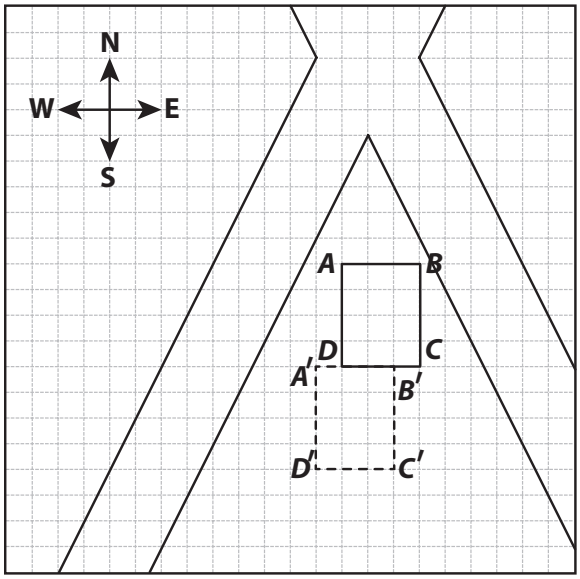
6.



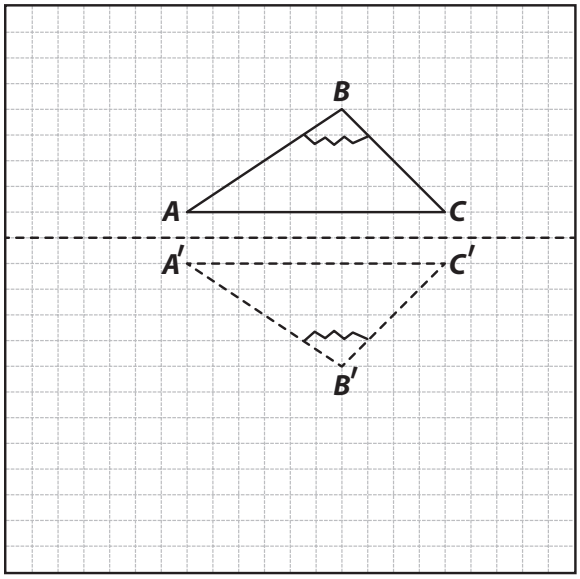
7.



8.

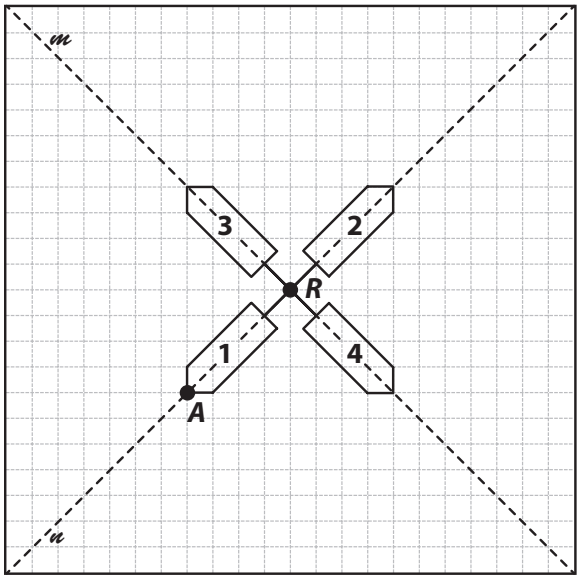


9.



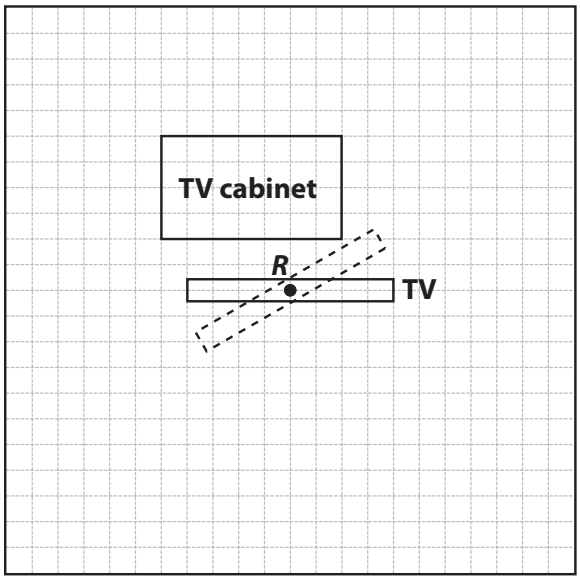
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10. Answers may vary. Method 1: Rotate the paddle 3 times using a 90° angle of rotation and point *R*. Method 2: Reflect the paddle over the line *m* to create paddle 2. Rotate paddle 2 90° counterclockwise about point *R* to create paddle 3. Reflect paddle 3 over line *n* to create paddle 4.



Warm-Up 5.5.2, p. U5-277

1.



2. Since the drawing is a scaled representation of the cabinet and TV, use the units provided on the grid and draw the diagonal. Use the Pythagorean Theorem to calculate $d = \sqrt{65} \approx 8.06$, which is the length of the diagonal of the cabinet. The TV is 8 units. Therefore, the TV should fit, but it will be a tight fit.

Practice 5.5.2: Defining Congruence in Terms of Rigid Motions, p. U5-296

1. Not congruent; a dilation occurred with a scale factor of 2/3, indicating a reduction. Dilations are non-rigid motions.
2. Congruent; a rotation occurred. Rotations are rigid motions.
3. Congruent; a translation occurred by moving the figure 9.5 units to the left and 4.5 units up. Translations are rigid motions.
4. Not congruent; a horizontal stretch has occurred with a scale factor of 1.5. Stretches are non-rigid motions.
5. Not congruent; a vertical stretch has occurred with a scale factor of 1.5. Stretches are non-rigid motions.
6. Not congruent; a horizontal compression has occurred with a scale factor of 2/3. Compressions are non-rigid motions.
7. The speaker has been rotated and translated. Since these are both rigid motions, the figures are congruent.
8. The corner cabinet has been rotated. Since rotations are rigid motions, the figures are congruent.
9. The inner square has been dilated by a scale factor of 5/3. Since dilations are non-rigid motions, the squares are not congruent.
10. Answers may vary. Sample answer: The triangles are congruent. Triangle 1 can be reflected over a vertical line passing through the top right vertex to create triangle 3. Triangle 3 can be reflected over another vertical line passing through the top right vertex to create triangle 5. Triangle 1 can be reflected over a horizontal line passing through the center of the triangle and then translated 2 units to the right to create triangle 2. Triangle 2 can be reflected over a vertical line passing through the bottom right vertex to create triangle 4. Since all the transformations described are rigid motions, the triangles are congruent.

Progress Assessment, p. U5-300

1. b
2. d
3. a
4. c
5. a
6. b
7. b
8. c
9. b
10. c

11. a. Answers may vary. Sample answer: Looking at the sugar and flour containers, the left handle can be reflected over the vertical line passing through the center of each container to create the right handle. The handles are all congruent and can be created from rigid motions.
    b. The rectangles of the containers have undergone a non-rigid motion of a dilation with a scale factor of 4/3. The flour container is larger than the sugar container. This means that the flour and sugar containers are not congruent.

Lesson 6: Congruent Triangles

Pre-Assessment, p. U5-309

1. d
2. c
3. c
4. a
5. d

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Warm-Up 5.6.1, p. U5-312

1. The position of Piece 2 is the result of a horizontal translation of 6 units to the right and a vertical translation of 3 units down.
2. The position of Piece 3 is the reflection of Piece 2 over a vertical line.
3. A series of congruency transformations results in a congruent figure, so Piece 3 is congruent to Piece 1.

Practice 5.6.1: Triangle Congruency, p. U5-329

1. Sample answer: $\triangle FHG \cong \triangle BCD$
2. Sample answer: $\triangle JNP \cong \triangle RTV$
3. Sample answer: $\triangle BDF \cong \triangle PLN$
4. $\angle Q \cong \angle W$, $\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\overline{QR} \cong \overline{WX}$, $\overline{RS} \cong \overline{XY}$, $\overline{QS} \cong \overline{WY}$
5. $\angle A \cong \angle C$, $\angle F \cong \angle G$, $\angle H \cong \angle J$, $\overline{AF} \cong \overline{CG}$, $\overline{FH} \cong \overline{GJ}$, $\overline{AH} \cong \overline{CJ}$
6. $\angle L \cong \angle H$, $\angle P \cong \angle J$, $\angle Q \cong \angle K$, $\overline{LP} \cong \overline{HJ}$, $\overline{PQ} \cong \overline{JK}$, $\overline{LQ} \cong \overline{HK}$
7. Yes, the triangles are congruent; $\triangle ADH \cong \triangle JPK$.
8. No, the triangles are not congruent.
9. Yes, the triangles are congruent; $\triangle EDF \cong \triangle IHG$.
10. Yes, the triangles are congruent; $\triangle RST \cong \triangle WVU$.

Warm-Up 5.6.2, p. U5-332

1. The base of the pyramid is a square; therefore, $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$; the triangles are congruent, so the corresponding parts of the triangles are also congruent: $\overline{AE} \cong \overline{BE} \cong \overline{CE} \cong \overline{DE}$.
2. The corresponding angles of the congruent triangles are congruent; therefore, $\angle EBA \cong \angle EAD \cong \angle EDC \cong \angle ECB$ and $\angle EAB \cong \angle EBC \cong \angle ECD \cong \angle EDA$.

Practice 5.6.2: Explaining ASA, SAS, and SSS, p. U5-349

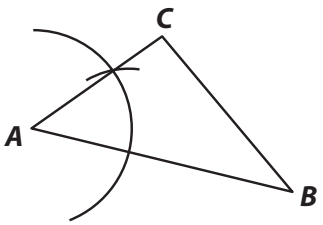
1. SSS
2. Congruency cannot be determined; the identified congruent parts form SSA, which is not a triangle congruence statement.
3. SAS
4. ASA
5. SAS
6. Congruency cannot be determined; there is not enough information about both of the triangles to determine if the triangles are congruent.
7. $\triangle ABC \cong \triangle FGE$; SAS or ASA
8. There is not enough information to determine if the sails are congruent. The information provided follows SSA, which is not a triangle congruence statement.
9. Congruency cannot be determined; there is not enough information about both of the triangles to determine if the triangles are congruent.
10. The plots of land are congruent by SAS or SSS.

Progress Assessment, p. U5-353

1. c
2. a
3. b
4. a
5. d
6. b
7. a
8. c
9. d
10. b
11. a. $\overline{AB} \cong \overline{BC}$, $\angle BAD \cong \angle BCD$
    b. There is not enough information to determine if the triangles are congruent. The information provided follows SSA, which is not a triangle congruence statement.
    c. $\overline{FJ} \cong \overline{HJ}$, $\angle FJK \cong \angle HJK$
    d. $\triangle FKJ \cong \triangle HKJ$ by SAS
    e. The minimum amount of information needed to prove that two triangles are congruent is two angles and the included side, or two sides and the included angle, or three sides. (Students may also recall that given two angles and a side not included, congruency may be proven by AAS.)

Unit Assessment p. U5-357

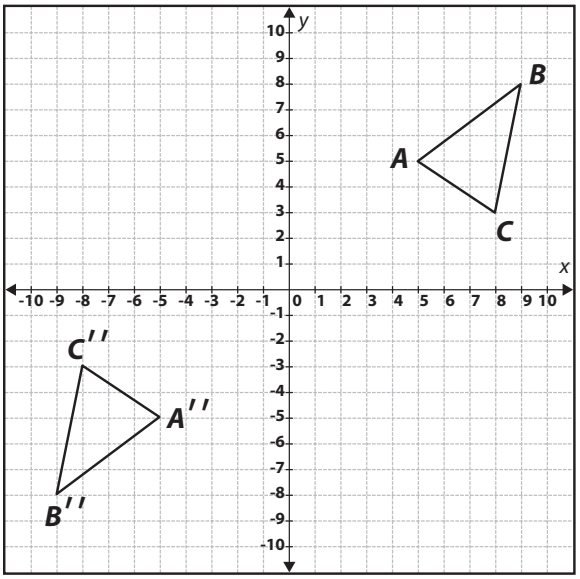
1. d
2. d
3. c
4. a
5. b
6. d
7. c
8. b
9. a
10. b
11. c
12. d
13. a. $\angle C$, $\angle O$, and $\angle M$
    b. Frame 2 is Frame 1 under R_{180} or, written symbolically, Frame 2 = R_{180}(Frame 1).
    c. $C''(-3, 1)$, $O''(0, -5)$, $M''(3, 1)$, and $P''(0, -1)$; check student graphs for accuracy.
    d. Yes, they are isometric. The transformations applied from Frame 1 to 2 to 3 are all isometric transformations and the figure has remained the same shape and size.
14. a.



    b. No, it is not possible to construct a second non-congruent triangle because the parts of the triangle given are two sides and the included angle. The triangle congruence statement side-angle-side ensures that two triangles are congruent given this information.

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15. a.



b. It is possible to rotate the preimage 180° about the origin to arrive at the same image as reflecting over $x = 0$ and $y = 0$.

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Student Book Answer Key

Lesson 1: Introducing Transformations

Practice 5.1.1: Defining Terms, p. U5-11

1. circle
2. obtuse
3. segment AB
4. acute
5. point
6. $\overset{\frown}{CD}$ cannot be longer than $\overset{\frown}{FG}$.
7. 90°
8. two rays or line segments that share a common endpoint
9. the set of points between two points and the infinite number of points that continue beyond them
10. $\overset{\frown}{MN}$ cannot be longer than $\overset{\frown}{MNP}$.

Practice 5.1.2: Transformations As Functions, p. U5-22

1. translation, reflection, and rotation
2. $h(x)$; start on the inside and work outward.
3. that the image is congruent to the preimage
4. You create an identity.
5. 4
6. (–5, –2)
7. $\Delta A'B'C'$ has vertices A' (2, 1), B' (6, –1), and C' (5, 3).
8. (–2, 5)
9. $P(x - 3, y + 2)$; $h = 3$, $k = -2$
10. $T_{-1, 3}$

Practice 5.1.3: Applying Lines of Symmetry, p. U5-31

1. an infinite number
2. 8
3. when a right angle is not present
4. isosceles
5. 3
6. 360/6 = 60; 60°
7. 360/5 = 72; 72°
8. 2
9. 8 inches
10. 5

Lesson 2: Defining and Applying Rotations, Reflections, and Translations

Practice 5.2.1: Defining Rotations, Reflections, and Translations, p. U5-39

1. rotation
2. translation
3. reflection
4. The point that moves the farthest would be the point that is initially the farthest from the center of rotation.
5. $A'(1, 1)$
6. The point would move 4 units to the right and 3 units down.
7. $B''(-2, -5)$
8. $A''(-3, 1)$, $B''(-8, 6)$, and $C''(-5, 9)$
9. $P''(-9, 3)$
10. 5 units to the left

Practice 5.2.2: Applying Rotations, Reflections, and Translations, p. U5-47

For problems 1, 2, 8, and 9, check students' graphs for the coordinates and labels listed below.

1. $A''(9, 2)$, $B''(14, 3)$, and $C''(8, 5)$
2. $D''(-4, -3)$, $E''(-5, -7)$, and $F''(-7, -4)$
3. Quadrant I
4. P'' (–1, 11)
5. $A'(-2, -2)$, $B'(-5, -2)$, $C'(-5, 1)$, and $D'(-2, 1)$
6. R_{270}
7. $r_{y\text{-axis}}$
8. $A''(-2, -2)$, $B''(-4, -4)$, and $C''(-3, 3)$
9. $D''(2, 4)$, $E''(1, 8)$, and $F''(-2, 2)$
10. R_{270}

Lesson 3: Constructing Lines, Segments, and Angles

Practice 5.3.1: Copying Segments and Angles, p. U5-65

1–10. Check students' work for accuracy.

Practice 5.3.2: Bisecting Segments and Angles, p. U5-82

1–10. Check students' work for accuracy.

Practice 5.3.3: Constructing Perpendicular and Parallel Lines, p. U5-101

1–10. Check students' work for accuracy.

Lesson 4: Constructing Polygons

Practice 5.4.1: Constructing Equilateral Triangles Inscribed in Circles, p. U5-118

1–3. Check students' work for accuracy. Be sure each of the vertices lies on the circle.

4. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is twice the length of the given segment.
5. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to half the length of the given segment.

6–8. Check students' work for accuracy. Be sure each of the vertices lies on the circle.

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9. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is twice the length of the given segment.
10. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to half the length of the given segment.

Practice 5.4.2: Constructing Squares Inscribed in Circles, p. U5-130

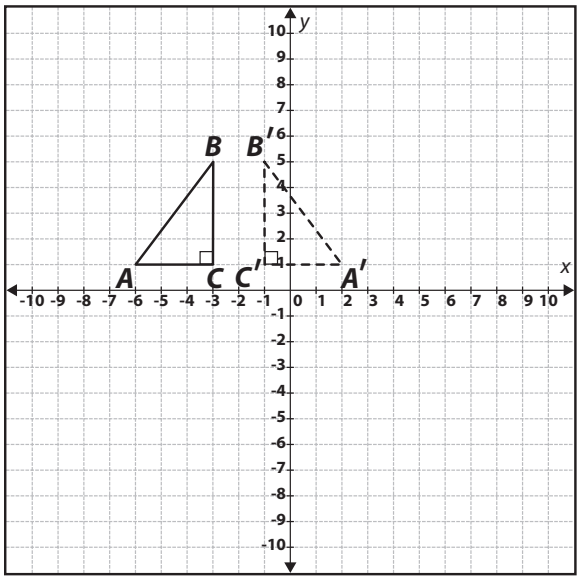
1–2. Check students' work for accuracy. Be sure each of the vertices lies on the circle.
3–6. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to the length of the given segment.
7. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to twice the length of the given segment.
8. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to half the length of the given segment.
9–10. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to half the length of the given segment.

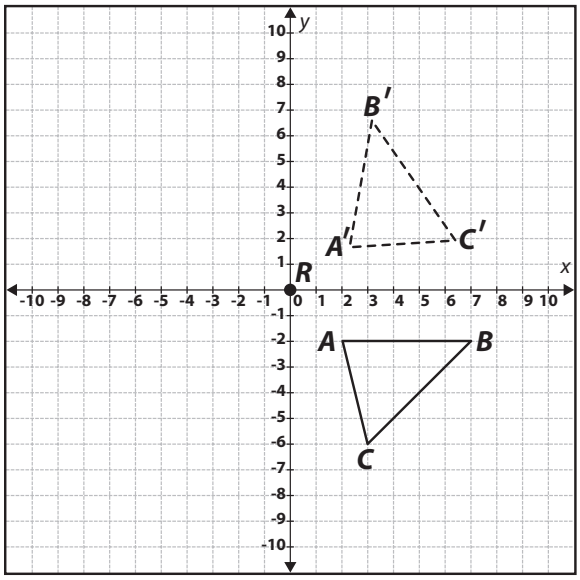
Practice 5.4.3: Constructing Regular Hexagons Inscribed in Circles, p. U5-145

1. Check students' work for accuracy. Be sure each of the vertices lies on the circle.
2–3. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to the length of the given segment.
4. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to twice the length of the given segment.
5. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to half the length of the given segment.
6. Check students' work for accuracy. Be sure each of the vertices lies on the circle.
7–8. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to the length of the given segment.
9. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to twice the length of the given segment.
10. Check students' work for accuracy. Be sure each of the vertices lies on the circle and the radius of the circle is equal to half the length of the given segment.

Lesson 5: Exploring Congruence

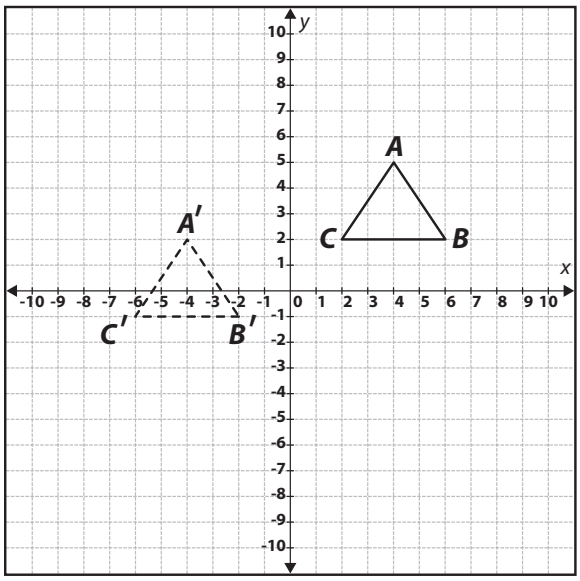
Practice 5.5.1: Describing Rigid Motions and Predicting the Effects, p. U5-163

1. Rotation; the orientation changed, but the images are not mirror reflections of each other.
2. Translation; 5 units left and 3 units up; the orientation stayed the same.
3. Reflection; the line of reflection is $y = -3$; the orientation changed and the preimage and image are mirror reflections of each other; the line of reflection is the perpendicular bisector of the segments connecting the vertices of the preimage and image.
4. 

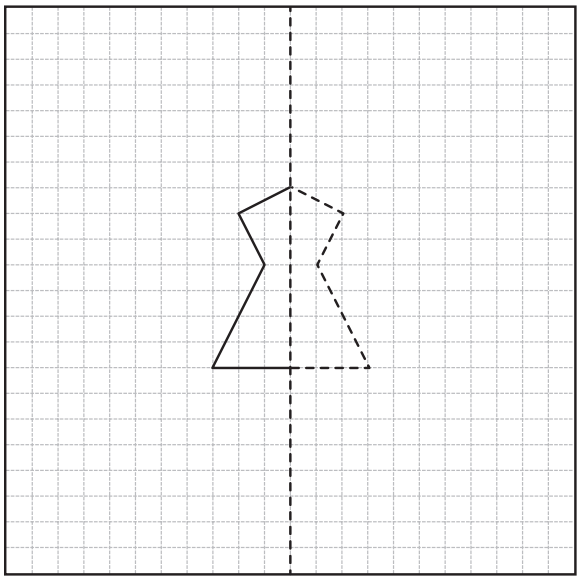
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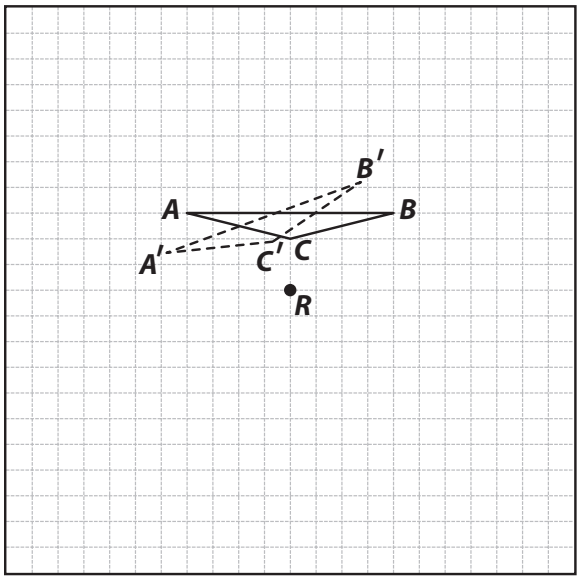
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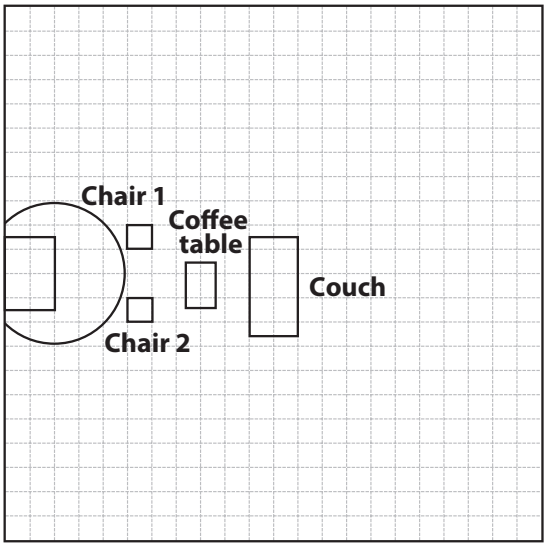
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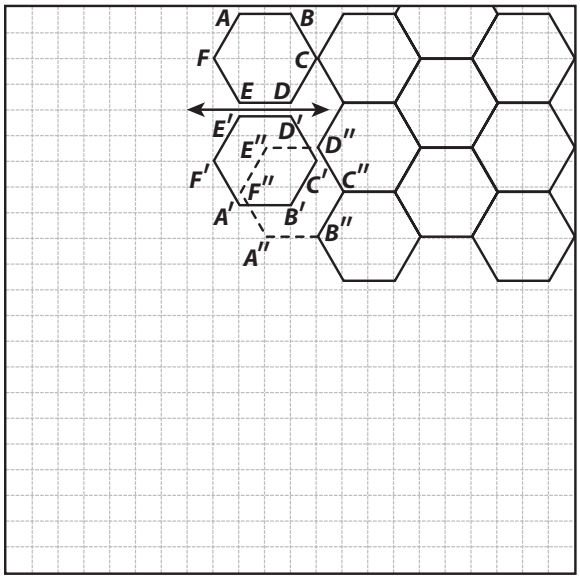
8.



9. Answers may vary. Sample answer: Translate both chairs 2 units to the right.



10. Answers may vary. Sample answer: First, reflect the hexagon over a horizontal line just below the solar panel. Then translate the reflected hexagon about 1 unit to the right and 1 unit down.



Practice 5.5.2: Defining Congruence in Terms of Rigid Motions, p. U5-179

1. Congruent; a translation occurred 7 units to the left and 3 units up. Translations are rigid motions.
2. Congruent; a rotation has occurred. Rotations are rigid motions.
3. Not congruent; a vertical compression has occurred with a scale factor of 1/2. Compressions are non-rigid motions.
4. Not congruent; a horizontal compression has occurred with a scale factor of 2/3. Compressions are non-rigid motions.
5. Congruent; a reflection has occurred. Reflections are rigid motions.

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6. Not congruent; a vertical stretch has occurred with a scale factor of 5/3. Stretches are non-rigid motions.
7. The outer triangle has been dilated by a scale factor of 2/3. Since dilations are non-rigid motions, the triangles are not congruent.
8. The target has undergone a rotation. Since rotations are rigid motions, the targets are congruent.
9. The art is a reflection. Since reflections are rigid motions, the A+ on top is congruent to the A+ on the bottom.
10. Answers may vary. Sample answer: The windowpanes are congruent. Pane 1 can be translated to the right 2 units to create pane 2. Pane 1 can be translated 4 units to the right to create pane 3. Panes 1, 2, and 3 can be reflected over the horizontal line passing through the bottom of the panes to create panes 4, 5, and 6. Since all the transformations described are rigid motions, the panes are congruent.

Lesson 6: Congruent Triangles

Practice 5.6.1: Triangle Congruency, p. U5-194

1. Sample answer: $\triangle NLM \cong \triangle HIJ$
2. Sample answer: $\triangle PQR \cong \triangle TSV$
3. Sample answer: $\triangle USW \cong \triangle LNQ$
4. $\angle H \cong \angle M$, $\angle I \cong \angle N$, $\angle J \cong \angle P$, $\overline{HI} \cong \overline{MN}$, $\overline{IJ} \cong \overline{NP}$, $\overline{HJ} \cong \overline{MP}$
5. $\angle B \cong \angle H$, $\angle D \cong \angle J$, $\angle E \cong \angle L$, $\overline{BD} \cong \overline{HJ}$, $\overline{DE} \cong \overline{JL}$, $\overline{BE} \cong \overline{HL}$
6. $\angle N \cong \angle T$, $\angle P \cong \angle V$, $\angle R \cong \angle X$, $\overline{NP} \cong \overline{TV}$, $\overline{PR} \cong \overline{VX}$, $\overline{NR} \cong \overline{TX}$
7. Yes, the triangles are congruent; $\triangle DGA \cong \triangle LJH$.
8. Yes, the triangles are congruent; $\triangle MNP \cong \triangle RTS$.
9. Yes, the triangles are congruent; $\triangle BCD \cong \triangle FGE$.
10. No, the triangles are not congruent.

Practice 5.6.2: Explaining ASA, SAS, and SSS, p. U5-207

1. SAS
2. ASA
3. Congruency cannot be determined; the identified congruent parts form SSA, which is not a triangle congruence statement.
4. ASA
5. Congruency cannot be determined; the identified congruent parts form SSA, which is not a triangle congruence statement.
6. SSS
7. $\triangle DEF \cong \triangle TVS$; SAS
8. There is not enough information to determine if the pieces of wood are congruent. The information provided follows AAA, which is not a triangle congruence statement.
9. The pieces of fabric are congruent; SAS
10. The pieces of fabric are congruent; SAS or SSS

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Instruction

Goal: To provide opportunities for students to develop concepts and skills related to using coordinate geometry in order to prove that reflections and rotations yield congruent polygons

Common Core State Standards

G–CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G–CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G–CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G–CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Student Activities Overview and Answer Key

Station 1

Students will be given white paper, graph paper, scissors, tape, three toothpicks, and a ruler. Students will construct triangle, pentagon, and trapezoid "lollipops." They will perform reflections of these polygons. Then they will determine if the polygons are congruent.

Answers

1. Answers will vary.
2. Answers will vary.
3. They are congruent because they are the same triangle; symmetric; the line of symmetry is the x-axis.
4. reflection
5. x-axis
6. Answers will vary; yes, they are congruent because the shape and dimensions didn't change; yes, they are symmetric; the line of symmetry is the y-axis.
7. Answers will vary; yes, they are congruent because the shape and dimensions didn't change; yes, they are symmetric; the line of symmetry is $y = x$.
8. Reflected figures are congruent because the shape and dimensions of each figure do not change. Reflected figures are symmetric because a reflection creates a mirror image of the original figure.

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Instruction

Station 2

Students will be given white paper, graph paper, a cork board, scissors, push pins, a protractor, and a ruler. Students will work as a group to construct triangles, rectangles, and squares. They will perform rotations and determine congruence and rotational symmetry.

Answers

1. Answers will vary.
2. Answers will vary; yes, because the triangles have the same dimensions and shape.
3. No, no, no, yes; 3; 120°, 240°, 360°; yes, because the triangle matches itself 3 times.
4. 2; 180°, 360°; yes; 2, because the rectangle matches itself 2 times.
5. Yes, because the square matches itself 4 times.

Station 3

Students will be given graph paper and a ruler. Students will construct and reflect an irregular hexagon. They will find the line of symmetry and determine if reflected figures are congruent.

Answers

1. (0, 3), (–1, 1), (–5, 1), (–6, 3), (–5, 5), (–1, 5); yes
2. (0, –3), (1, –1), (5, –1), (6, –3), (5, –5), (1, –5); yes
3. No; (0, –3), (–1, –1), (–5, –1), (–6, –3), (–5, –5), (–1, –5); $y = 1$

Station 4

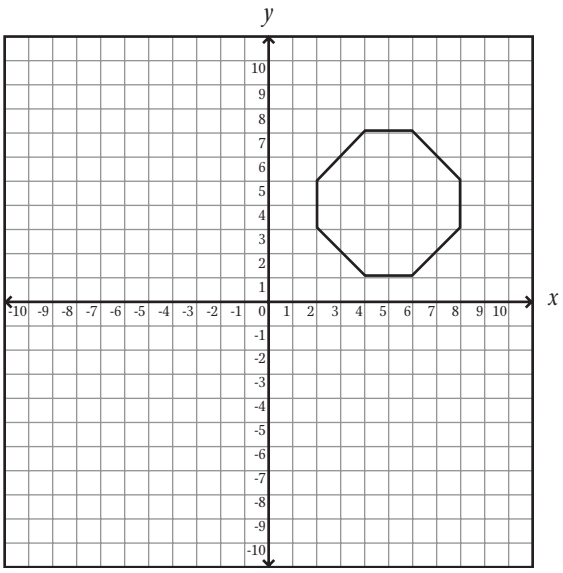
Students will be given graph paper, a protractor, and a ruler. They will construct regular and irregular polygons. They will prove that rotational symmetry yields congruent figures. They will prove that the order of rotational symmetry for regular polygons equals the number of sides of the polygon.

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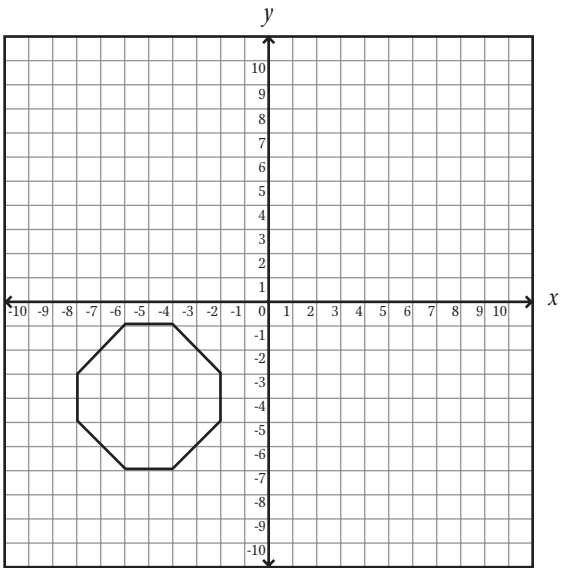
Instruction

Answers

1. Answers will vary. Sample answer:



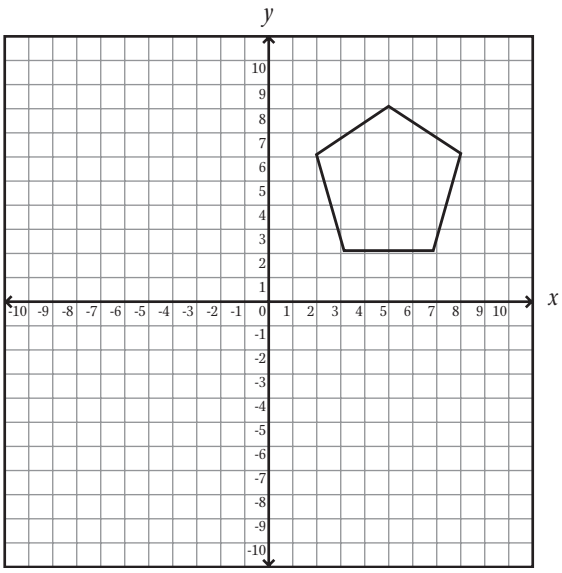
2. Answers will vary. Sample answer:



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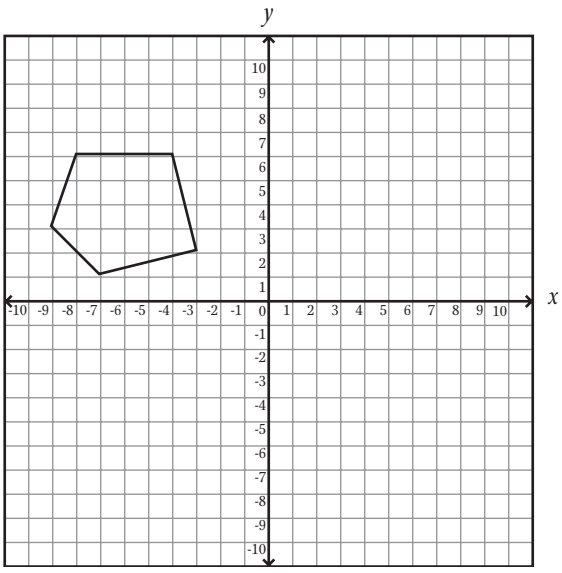
3. Yes, because you are only rotating the polygon.

4. Answers will vary. Sample graph:



The pentagon matches in rotational symmetry 5 times.

5. Answers will vary. Sample graph:



The pentagon matches in rotational symmetry 1 time.

6. For regular polygons, the number of matches during rotation is equal to the number of sides of the polygon. For irregular polygons, there is only one match.

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Instruction

Station 5

Students will be given graph paper and a ruler. They will construct and perform translations on irregular and regular polygons. They will determine if translations yield congruent figures.

Answers

1. Scalene triangle; no, the sides aren't equal.
2. (1, 2), (5, 4), (1, 5); yes
3. Yes, because all sides and angles are equal.
4. (–3, –9), (1, –9), (–3, –5), (1, –5); yes
5. Yes, because the dimensions and shapes of the figures do not change when undergoing translations.

Materials List/Setup

Station 1 white paper; graph paper; scissors; tape; three toothpicks; ruler

Station 2 white paper; graph paper; cork board; scissors; push pins; protractor; ruler

Station 3 graph paper; ruler

Station 4 graph paper; protractor; ruler

Station 5 graph paper; ruler

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. Are reflected polygons congruent?
2. What is rotational symmetry?
3. Are rotated polygons congruent?
4. How can you use coordinate geometry to prove that reflections yield congruent polygons?
5. How can you use coordinate geometry to prove rotational symmetry?
6. How can you use coordinate geometry to prove that translations yield congruent polygons?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. Yes, reflected polygons are congruent.
2. A figure has rotational symmetry if it looks the same after a certain amount of rotation.
3. Yes, rotated polygons are congruent.
4. Reflect the polygon and verify that its size and shape remain the same.
5. Rotate the polygon and see if it matches itself more than once during a 360° rotation with the center of the polygon as the point of rotation.
6. Translate a polygon and verify that its size and shape remain the same.

Possible Misunderstandings/Mistakes

- Not reflecting the polygon across the correct line of symmetry
- Not keeping the size and shape of the polygon the same when performing reflections
- Not recognizing how to find the "match" or order in rotational symmetry
- Not understanding the difference between regular and irregular polygons
- Not keeping the same shape and size when performing translations

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Station 1

At this station, you will find white paper, graph paper, scissors, tape, three toothpicks, and a ruler. Work as a group to construct the polygons and answer the questions.

On your graph paper, draw an x- and y-axis with the origin through the center of the paper.

On the white paper, draw an isosceles triangle with side lengths of 2 inches, 2 inches, and 1 inch. Cut the triangle out of the paper.

Tape the toothpick to the base of the triangle to create a triangle "lollipop."

1. Place the free end of the toothpick on the point (4, 0). Place the triangle so it is in the first quadrant.

   What are the vertices of the triangle?

2. Now place the free end of the toothpick on the point (4, 0) again, but place the triangle in the fourth quadrant.

   What are the vertices of this triangle?

3. Are the two triangles congruent? Explain your answer.

   Are the two triangles symmetric? What is the line of symmetry?

4. What transformation did you perform between the first and second triangle?

continued

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5. Which axis or point did you perform this transformation across?

6. Work as a group to construct a pentagon "lollipop." Reflect the pentagon across the y-axis.
   What are the vertices of the first pentagon?

   What are the vertices of the second pentagon?

   Are the two pentagons congruent? Explain your answer.

   Are the two pentagons symmetric? What is the line of symmetry?

7. Work as a group to construct a trapezoid. Reflect the trapezoid about the line $y = x$.
   What are the vertices of the first trapezoid?

   What are the vertices of the second trapezoid?

continued

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Are the trapezoids congruent? Explain your answer.

Are the trapezoids symmetric? What is the line of symmetry?

8. Based on your observations in problems 1–7, do reflections create congruent figures? Do reflections create symmetric figures? Explain your answer to each question.

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Station 2

At this station, you will find white paper, graph paper, a cork board, scissors, push pins, a protractor, and a ruler. Work as a group to construct the polygons and perform the transformations.

On your graph paper, draw an x- and y-axis with the origin through the center of the paper.

1. On the white paper, draw an equilateral triangle with side lengths of 1 inch. Cut the triangle out of the paper.

   Place one vertex of the triangle at the point (5, 5). Use the push pin and cork board to secure the vertex at this point.

   What are the vertices of the triangle?

2. Rotate the triangle 90° using the push pin as the point of rotation.

   What are the vertices of the triangle?

   Are the two triangles congruent? Explain your answer.

   Remove the push pin.

3. Poke the push pin through the center of the triangle. Then push the pin into the coordinate (5, 5).

   Rotate the triangle 90° using the push pin as the point of rotation.

   Does the triangle look the same as it did before you rotated it?

   Rotate the triangle another 90° using the push pin as the point of rotation.

   Does the triangle look the same as it did before you rotated it?

continued

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Rotate the triangle 90° using the push pin as the point of rotation.

Does the triangle look the same as it did before you rotated it?

Rotate the triangle another 90° using the push pin as the point of rotation.

Does the triangle look the same as it did before you rotated it?

How many times would the triangle match itself during a 360° rotation?

At what degrees of rotation would the triangle match itself?

Does the equilateral triangle have rotational symmetry? Explain your answer.

4. Repeat problem 3 for a rectangle.

   How many times did the rectangle match itself during the 360° rotation?

   At what degrees of rotation did the rectangle match itself?

   Does the rectangle have rotational symmetry? Explain your answer.

5. Does a square have rotational symmetry? Explain your answer.

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Station 3

At this station, you will find graph paper and a ruler. Work as a group to construct the polygons and answer the questions.

On your graph paper, draw an *x*- and *y*-axis with the origin through the center of the paper.

On your graph paper, construct an irregular hexagon that has vertices (1, 1), (5, 1), (6, 3), (5, 5), (1, 5), and (0, 3).

1. Reflect the figure across the *y*-axis.

   What are the vertices of this figure?

   Are the two figures congruent?

2. Reflect the original figure across the *x*-axis.

   What are the vertices of this figure?

   Are the three figures congruent?

3. Katy says that the vertices of the figure from problem 1 reflected across the *x*-axis are (1, 1), (5, 1), (6, –3), (5, –5), (1, –5), and (0, –3). Is she correct?

   If not, what are the correct vertices?

   If Katy didn't reflect the figure across the *x*-axis, what line of symmetry did she use?

   Perform this reflection on your graph paper to justify your answer.

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Station 4

At this station, you will find graph paper, a protractor, and a ruler. Work together to construct the polygons and answer the questions.

On your graph paper, draw an *x*- and *y*-axis with the origin through the center of the paper.

1. On your graph paper, construct an octagon in the first quadrant.

   What are the vertices of the octagon?

   How many units on the graph paper can you shade in the octagon?

2. Rotate the octagon 180° about the origin.

   What are the vertices of this octagon?

   How many units on the graph paper can you shade in this octagon?

3. Should the number of units on the graph paper shaded in for each octagon be equal? Why or why not?

4. Construct a regular pentagon with side lengths of 5 units. (Each interior angle is 108°.)

   What are the vertices of this regular pentagon?

   Rotate the pentagon about its center.

   How many times does the pentagon match in rotational symmetry?

continued

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5. Construct an irregular pentagon in the second quadrant.

   What are the vertices of this irregular pentagon?

   Rotate the pentagon about its center.

   How many times does the pentagon match in rotational symmetry?

6. Based on your observations in problems 4 and 5, what can you say about rotational symmetry for regular polygons?

   What can you say about rotational symmetry for irregular polygons?

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Station 5

At this station, you will find graph paper and a ruler. Work as a group to construct the polygons and answer the questions.

On your graph paper, draw an x- and y-axis with the origin through the center of the paper.

1. On your graph paper, construct a triangle with vertices (–5, 2), (–1, 4), and (–5, 5).

   What type of triangle have you created?

   Is this triangle a regular polygon? Why or why not?

2. Translate the triangle 6 units to the right.

   What are the vertices of this triangle?

   Are the two triangles congruent?

3. On a new graph, construct a square with vertices (1, 1), (5, 1), (1, 5), and (5, 5).

   Is this square a regular polygon? Why or why not?

4. Translate the square 10 units down and 4 units to the left.

   What are the vertices of this square?

   Are the two squares congruent?

5. Based on your observations in problems 1–4, do translations yield congruent figures? Justify your answer.

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Instruction

Goal: To provide opportunities for students to develop concepts and skills related to corresponding parts, transformations, and principles of congruent triangles

Common Core State Standards

G–CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G–CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G–CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

G–CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G–CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G–CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Student Activities Overview and Answer Key

Station 1

Students will use a ruler and protractor to prove that two triangles and corresponding parts are congruent. Then they will use the properties of triangles to prove that two triangles and corresponding parts are congruent.

Answers

1. Measure lengths of corresponding sides and see if they are equal.
2. Measure corresponding angles and see if they are equal.

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3.

$\overline{DB} \perp \overline{AC}$	Given
$\angle DBA$ and $\angle DBC$ are right angles.	Definition of perpendicular lines
$\triangle DBA$ and $\triangle DBC$ are right triangles.	A right triangle has a right angle.
$\overline{DB} \cong \overline{DB}$	Reflexive property of congruence
$\triangle DBA \cong \triangle DBC$	Hypotenuse leg postulate
$\overline{AB} \cong \overline{AC}$	CPCTC (Corresponding Parts of Congruent Triangles are Congruent)

4. Answers will vary. Possible answer: designing a skyscraper
5. Yes, corresponding parts of congruent triangles are congruent.

Station 2

Students will be given graph paper, a ruler, push pins, and rubber bands. Students will construct triangles and perform translations and dilations. They will show how to translate congruent triangles. They will explain why dilations of triangles do not yield congruent triangles.

Answers

1. Triangle with vertices (–7, –7), (7, –7), and (7, 9)
2. Triangle with vertices (–8, –2), (6, –2), and (6, 14); yes; n/a
3. Triangles with vertices (–14, 4), (7, 4), and (7, 20); no, sides are different lengths; answers may vary. Sample answer: (–7, 4), (7, 4), and (7, 20)
4. Triangle with vertices (0, –5), (8, –5), and (0, 5)
5. (0, –10), (16, –10), (0, 10)
6. No, because dilation changed the size of the triangle.

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Instruction

Station 3

Students will be given graph paper, a ruler, and a cardboard triangle. Students will perform reflections and identify congruent triangles that have been reflected. Then they will use the cardboard triangle to perform a rotation and explain how it relates to congruent triangles.

Answers

1. (–4, 2), (–7, 2), (–7, 7); (–4, –2), (–7, –2); (–7, –7). Yes, because the size and shape remained the same.
2. *A* and *B*, because corresponding sides and angles are congruent.
3. Yes, because the size and shape remained the same.
4. They have the same size and shape.

Station 4

Students will be given four index cards with the following written on them: SSS; SAS; ASA; AAS. Students will work together to match the index cards to real-world examples of SSS, SAS, ASA, and AAS. Then they will explain how SSS, SAS, ASA, and AAS relate to congruent triangles.

Answers

1. ASA
2. AAS
3. SSS
4. SAS
5. Answers will vary.
6. side-side-side; side-angle-side; angle-side-angle; angle-angle-side. These are ways to prove two triangles are congruent.

Materials List/Setup

Station 1 ruler; protractor

Station 2 graph paper; ruler; push pins; rubber bands

Station 3 graph paper; ruler; cardboard triangle created from a triangle with vertices (4, 4), (10, 4), and (6, 12) in the coordinate plane

Station 4 four index cards with the following written on them: SSS; SAS; ASA; AAS

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. If two triangles are congruent, then what parts of the triangles are congruent?
2. Does a translation create a congruent triangle? Why or why not?
3. Does dilation create a congruent triangle? Why or why not?
4. Does a rotation create a congruent triangle? Why or why not?
5. Does a reflection create a congruent triangle? Why or why not?
6. What are four ways to prove two triangles are congruent if you know only three pieces of information about the triangles?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. Corresponding parts of congruent triangles are congruent.
2. Yes, because the size and shape of the triangle remain the same.
3. No, because the size does not remain the same.
4. Yes, because the size and shape of the triangle remain the same.
5. Yes, because the size and shape of the triangle remain the same.
6. SSS, ASA, AAS, SAS

Possible Misunderstandings/Mistakes

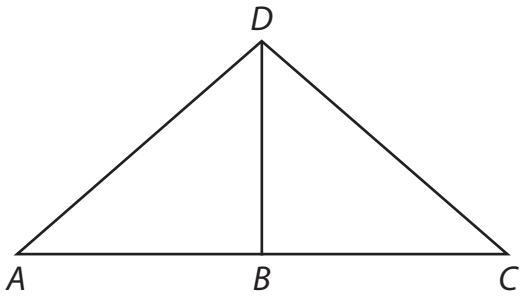
- Not reflecting, rotating, or translating a triangle correctly to create a congruent triangle
- Not realizing that dilation changes the size of the triangle
- Not identifying the relationship between the locations of the sides and angles in triangles when selecting ASA, AAS, or SAS to prove two triangles are congruent
- Incorrectly identifying corresponding parts of triangles

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Station 1

At this station, you will find a ruler and a protractor. Work as a group to answer the questions.

Amir is building a doghouse. He sketched this diagram of the roof:



It is important that the roof is symmetrical. Therefore, $\triangle DBA \cong \triangle DBC$.

1. How can you use your ruler to show that $\triangle DBA \cong \triangle DBC$ and $\overline{AB} \cong \overline{BC}$?

2. How can you use your protractor to show that $\triangle DBA \cong \triangle DBC$ and $\overline{AB} \cong \overline{BC}$?

3. What if you didn't have a ruler or protractor? How could you use the angle and side properties of triangles to prove that $\triangle DBA \cong \triangle DBC$ and $\overline{AB} \cong \overline{BC}$?

   For example, if it is given that $\overline{DC} \cong \overline{DA}$ and $\overline{DB} \perp \overline{AC}$, how can you prove that $\triangle DBA \cong \triangle DBC$ and $\overline{AB} \cong \overline{BC}$?

continued

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4. Provide an example of a real-world situation in which proving two triangles are congruent using the method in problem 3 is much more feasible than actually measuring the two triangles. Explain your reasoning.

5. If you know that two triangles are congruent, are all corresponding sides and angles congruent? Why or why not?

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Station 2

At this station, you will find graph paper, a ruler, push pins, cardboard, and rubber bands. Mount the graph paper on cardboard. Create an *x*- and *y*-axis on your graph paper.

1. As a group, construct a triangle on your graph paper by placing push pins at the points (–7, –7), (7, –7), and (7, 9). Place a rubber band around pairs of push pins to create a triangle.

2. Construct a second triangle on your graph paper by placing push pins at the points (–8, –2), (6, –2), and (6, 14). Place a rubber band around pairs of push pins to create a triangle.

   Is this triangle congruent to the triangle in problem 1? Why or why not?

   If the two triangles are not congruent, how can you modify the second triangle so it is congruent to the first triangle?

3. Construct a third triangle on your graph paper by placing push pins at the points (–14, 4), (7, 4), and (7, 20). Place a rubber band around pairs of push pins to create a triangle.

   Is this triangle congruent to the triangle in problem 1? Why or why not?

continued

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If the two triangles are not congruent, how can you modify the third triangle so it is congruent to the first triangle?

Remove all the push pins and rubber bands from the graph.

4. On your graph paper, construct a triangle by placing push pins at the points (0, –5), (8, –5), and (0, 5). Place one rubber band around all of the push pins to create a triangle.

5. Construct a second triangle by dilating the first triangle to twice its size about the origin, (0, 0).

   At what points did you place the push pins for this new triangle?

6. Are the triangles you created in problems 4 and 5 congruent? Why or why not?

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Station 3

At this station, you will find graph paper, a ruler, and a cardboard triangle.

Create an *x*- and *y*-axis on your graph paper and work together to answer the questions.

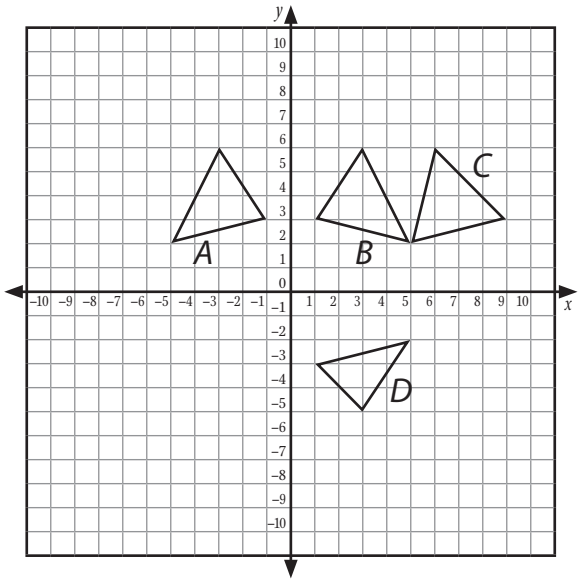
1. On your graph paper, construct a triangle that has vertices (4, 2), (7, 2), and (7, 7).

   Reflect this triangle across the *y*-axis. What are the vertices of this new triangle?

   Reflect this second triangle across the *x*-axis. What are the vertices of this new triangle?

   Are the three triangles congruent? Why or why not?

Use the following diagram to answer question 2.



2. Which triangles are congruent? Explain your answer.

continued

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3. On a new graph, place the cardboard triangle so its vertices are at (4, 4), (10, 4), and (6, 12). Trace the cardboard triangle.

   Rotate the cardboard triangle 90° about the point (4, 4). Trace the cardboard triangle. Are the triangles congruent?

4. Based on your observations in problems 1–3, when properly reflecting or rotating triangles, they are congruent if they:

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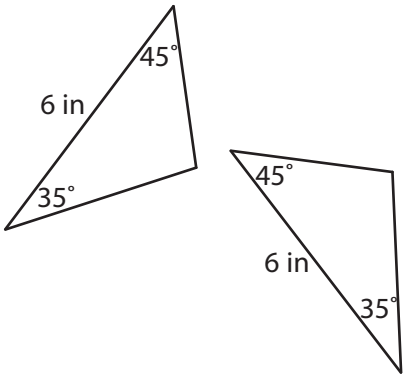
Station 4

At this station, you will find four index cards with the following written on them:

SSS; SAS; ASA; AAS

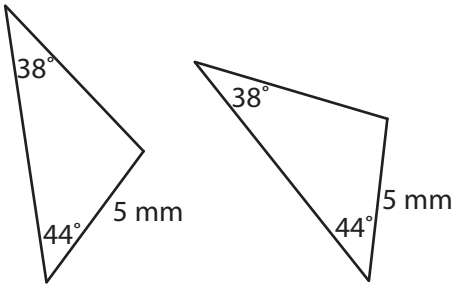
Work as a group to match each index card to the following real-world situations described.

1. A stained glass window contains two triangles.



Matching index card: _____

2. Anna creates earrings in the shape of scalene triangles.

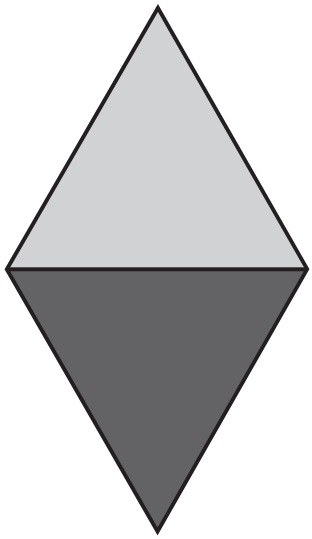


Matching index card: _____

continued

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3. Peter designed a logo for his design company. Each triangle has side lengths of 3 inches, 3 inches, and 3 inches.



Matching index card: _____

4. Janice designed two new buttons for her website. One button was a purple isosceles triangle with a 30° angle between the congruent sides. The other button was a green isosceles triangle with a 30° angle between the congruent sides.

Matching index card: _____

5. What strategy did you use to match the cards?

6. Based on your observations in problems 1–4, what do you think each index card stands for in regard to congruent triangles?

SSS = _____

SAS = _____

ASA = _____

AAS = _____

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APPENDIX

Problem-Based Task Implementation Guides

Introduction

This addendum contains a collection of Implementation Guides (IGs) designed to support the use of the Problem-Based Tasks found throughout this unit.

Each Implementation Guide includes an overview and suggestions for introducing and facilitating the task. These contain specific recommendations for selected Standards for Mathematical Practice (SMPs) to highlight, guidance on addressing common errors and misconceptions associated with the task, and ideas for monitoring and coaching students as they work through the task. Finally, there is guidance for debriefing the task, with special attention to the SMPs and relevant Key Concepts from the instruction, as well as information on suitable extensions and alternate strategies.

Throughout the Implementation Guides, suggestions are provided in a question/prompt format to support your efforts to elicit student thinking and facilitate classroom discourse.

We developed these Implementation Guides for our Problem-Based Tasks at the request of math educators, and with advice and feedback from mathematics supervisors and teachers. Please let us know if they are helpful. Visit us at www.walch.com, follow us on Twitter (@WalchEd), or e-mail suggestions to customerservice@walch.com.

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Guide

Problem-Based Task: A Mathematical Look

Common Core State Standard

G–CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Task Overview

Focus

How are geometric definitions used to define a figure? How can a line, a line segment, a ray, and parallel and perpendicular lines be identified and labeled in a circle? In this lesson, students will identify and label basic geometric concepts that have been drawn into a circular company logo.

This activity will provide practice with:

- analyzing a circular drawing
- identifying a line, a ray, and a line segment in a figure
- applying the definitions of parallel and perpendicular lines to identify pairs of these lines in a figure
- writing lines, rays, line segments, parallel lines, and perpendicular lines using correct notation

Introduction

This task should be used to explore the skill of applying precise definitions of line segment, ray, perpendicular line, and parallel line based on the undefined notions of point, line, and distance along a line. Students should already be familiar with the undefined notions in Geometry of point, line, and plane.

Begin by reading the problem and clarifying the meanings of line, line segment, parallel lines, perpendicular lines, and ray.

line	the set of points between two points P and Q in a plane and the infinite number of points that continue beyond those points; written as $\overleftrightarrow{PQ}$
line segment	a line with two endpoints; written as $\overline{PQ}$
parallel lines	lines in a plane that either do not share any points and never intersect or share all points; written as $\overleftrightarrow{AB} \parallel \overleftrightarrow{PQ}$
perpendicular lines	two lines that intersect at a right angle (90°); written as $\overleftrightarrow{AB} \perp \overleftrightarrow{PQ}$
ray	a line with only one endpoint; written as $\overrightarrow{PQ}$

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will use the given logo diagram to identify lines, rays, line segments, and parallel and perpendicular lines. Students may have difficulty understanding the differences between each of these terms. Encourage them to discuss the definitions of these terms with each other, and to justify their thinking as they work to identify each feature on the diagram.

- **SMP 4:** Model with mathematics.

  Students will recognize that this scenario can be modeled with a diagram. They will use the diagram to identify lines, rays, line segments, and parallel and perpendicular lines.

- **SMP 6:** Attend to precision.

  Students will use the definitions of line, ray, line segment, parallel lines, and perpendicular lines to identify these terms in the circular diagram. They will use exact and precise notation to label and write each term correctly.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- not labeling the elements/parts correctly

  Have students make a list of the symbols for a line, a line segment, a ray, parallel lines, and perpendicular lines to use as a reference when working through this task. Remind them to be careful with the arrows when labeling.

- misusing terms and notations

  Have students make a list of definitions, along with a diagram, for each of the terms to use as a reference when working through the task.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, encourage them to think about their knowledge of circles. Ask them, "What do you know about the parts of a circle?" (**Answer**: A circle has a center, which is equidistant from any point on the circle. A circle also has a radius, which is a line segment with endpoints of the center and any point on the circle. It also has a diameter, which is a line segment going through the center with endpoints on the circle.)

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- As students begin identifying the elements in the circle, ask, "What do you know about a line in comparison to a line segment?" (**Answer**: A line goes on forever in both directions. A line segment has distinct endpoints; in other words, a line segment has a distinct starting and ending point.)
- Ask students, "What is the difference between a ray and a line? Explain a ray in terms of the context of 'the rays of a sun.'" (**Answer**: A ray has one endpoint, and a line has no endpoints. The sun's rays have one endpoint, which is the sun itself, and the rays extend indefinitely.)
- If students are having difficulty naming the line in the figure, encourage them to carefully examine the notation in the figure. Ask, "Since a line goes indefinitely in both directions, which parts in the circle have arrows on both sides?" (**Answer:** There are arrows drawn with the letters *W*, *H*, and *D*.)
- Ask students, "What are three different ways to name the line drawn in the logo?" (**Answer:** The line can be represented as $\overleftrightarrow{WD}$, $\overleftrightarrow{WH}$, or $\overleftrightarrow{HD}$.)
- As students are identifying the ray in the logo, ask, "What is another way to name the ray besides $\overleftarrow{AC}$?" (**Answer:** Another way to name the same ray is $\overrightarrow{CA}$.)
- As students are identifying the parallel lines in the logo, ask, "What is the difference between parallel lines and perpendicular lines?" (**Answer:** Parallel lines are lines that do not intersect each other, and perpendicular lines intersect each other at right angles.)
- Ask students, "In the logo, line segments $\overline{LW}$ and $\overline{AC}$ intersect each other. How do you know that these segments are NOT perpendicular to each other?" (**Answer:** Although the segments intersect each other, they do not intersect each other at right angles, because there is not a right angle symbol at their point of intersection.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

Debriefing the Task

- Ask for volunteers to discuss their reasoning when identifying the requested elements of the logo. Discuss the distinguishing notation for each element. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Focus on the use of precise mathematical language and clarity, specifically when referring to each element of the logo.

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Connecting to Key Concepts

Make explicit connections to key concepts:

- A line requires two points to be defined. A line is the set of points between two reference points and the infinite number of points that continue beyond those two points in either direction.

  In this task, students will identify the line in the logo and name it using two points. They will identify the line by the arrows representing that the line will continue beyond the points in either direction.

- Like a line, a ray is defined by two points; however, a ray has one (and only one) endpoint.

  Students will identify that the ray $\overleftarrow{AC}$, or $\overrightarrow{CA}$, has one endpoint, which is *C*.

- A line segment is defined by two points, and both of those points are endpoints.

  Students will identify the line segments in the logo by naming them with two endpoints.

- Parallel lines are two lines that have unique points and never cross.

  Students will identify the parallel lines in the logo and note that they are parallel because they do not intersect each other in the figure.

- Perpendicular lines meet at a right angle (90°), creating four right angles.

  In this task, students will identify that the lines that intersect each other and are noted with a right angle symbol are perpendicular lines.

Extending the Task

- Ask students to label each radius and diameter in the circle, with the assumption that *H* is the center point of the circle. The radii of the circle are $\overline{HL}$, $\overline{HD}$, $\overline{HU}$, and $\overline{HW}$. There are two diameters of the circle, which can be represented by $\overline{WD}$ and $\overline{LU}$.

- Provide students with a compass and a ruler. Ask them to design their own circular logo that contains the same elements as the logo in the task. Ask them to be creative and see how they can design a different logo and still ensure that they have drawn lines, line segments, rays, and parallel and perpendicular lines, with specific points labeled. Ask students to volunteer their logos, and have the class identify the different elements.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 3, ASK:** "Did you construct viable arguments and critique the reasoning of others?" (**Answer:** I helped other students understand the difference between a line, a ray, and a line segment.)

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- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** I modeled this scenario by analyzing the circular diagram that represents the company logo.)
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** I made sure I used proper terminology and notation when I identified the elements of the logo and labeled them.)

Alternate Strategies or Solutions

- Students may choose to redraw the given logo and color code the elements to help them visually distinguish them. Remind students to carefully label the parts as well as their corresponding symbols and/or notation.
- As students identify the elements and record their responses, some of them may reverse the two endpoints when naming a line or a segment. Remind students that it is all right to reverse the order when naming a line or a line segment, but that the order of the letters DOES make a difference when naming a ray. For example, $\overrightarrow{AC}$ is NOT the same as $\overrightarrow{CA}$. When labeling a ray, the endpoint of the ray does not have the arrow on top of it.

Technology

No technology is needed for this task.

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Problem-Based Task: In the Mail Room

Common Core State Standard

G–CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

Task Overview

Focus

How can an object be transformed without losing its shape and size? How can functions be used to represent the transformation of an object? In this lesson, students will follow a series of transformations and determine the final coordinates of an envelope.

This activity will provide practice with:

- analyzing a transformation
- identifying a transformation of a figure on a coordinate plane
- calculating new coordinates of a figure
- determining the location of a figure on the coordinate plane

Introduction

This task should be used to explore or apply the skill of representing transformations in a plane and describing them as functions that take points in a plane as inputs, and give other points as outputs. Students should already be familiar with locating points in a coordinate plane and recognizing the graph of $y = x$.

Begin by reading the problem and clarifying the meanings of isometry, reflection, rotation, transformation, and translation.

isometry	a transformation in which the preimage and image are congruent
reflection	a transformation in which a mirror image is created; also called a flip
rotation	a transformation that turns a figure around a point; also called a turn
transformation	a change in a geometric figure's position, shape, or size
translation	a transformation that moves each point of a figure the same distance in the same direction; also called a slide

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will recognize that the goal of the task is to follow a series of transformations to determine the final coordinates of an envelope. They will perform the transformations, making note of the new coordinates after each step.

- **SMP 6:** Attend to precision.

  Students will apply the definitions of the transformations to calculate the new coordinates after each transformation. Encourage students to use the correct terminology as they work through the transformations.

- **SMP 7:** Look for and make use of structure.

  Students will look at the structure of the function notation that leads to the transformations. They will also follow and apply the pattern used to move from one transformation to the other, following the same process of noting the new coordinates each time.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- misusing terms and notations

  Have students create a list of the different types of transformations applicable to this task to use as a reference. Have them write the notations for the transformations next to each one.

- incorrectly plotting coordinates

  Remind students that the x-coordinate is found first by moving left or right on the x-axis, and the corresponding y-coordinate is found by moving up or down on the y-axis.

- performing the transformations in the incorrect order

  Have students draw the new figure in a new coordinate plane each time a transformation is performed. Remind them to carefully follow the order of the given transformations.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask them to carefully read through the steps of the automated process for an envelope to identify unfamiliar words. Ask, "In step 2, what does the word 'retracted' mean, and how do you think this definition applies to the envelope?" (**Answer:** "Retracted" means to pull back, bring back, or withdraw. In step 2, when the envelope is retracted, it is brought back to its original position as it was in step 1.)
- Before students begin the task, ask, "What is another word for 'reflection'? What is another word for 'rotation'? What is another word for 'translation'?" (**Answer:** A reflection is also known as a "flip"; a rotation is also known as a "turn"; and a translation is also known as a "slide".)
- Before students begin the task, ask, "As you perform the transformations on the envelope, if the *x*-coordinate changes, will the envelope move up/down, or left/right? What direction will the envelope move if the *y*-coordinate changes?" (**Answer:** If the *x*-coordinate changes, the envelope will move left or right. If the *y*-coordinate changes, the envelope will move up or down.)
- As students perform the first translation, ask, "In your own words, what does it mean if the envelope is transformed by the function $S_1(x, y + 3)$, and what type of transformation is this?" (**Answer:** The *x*-coordinates will stay the same, but 3 is added to each *y*-coordinate, so the envelope will move straight up by 3 units. This is a translation, as the figure slides up.)
- Ask students, "How many degrees would a figure need to be rotated if it is transformed back to its original position?" (**Answer:** A figure would need to be rotated 360° to get it back to its original position.)
- Ask students, "Based on the definition of a one-to-one function, how can you prove that each transformation is one-to-one?" (**Answer:** A one-to-one function means that each point is mapped to exactly one unique point. In each transformation of the envelope, each *x*-coordinate is mapped to exactly one unique point each time.)
- Ask students, "What are the possible combinations of transformations that would be performed on the envelope from its original position if the transformation function was (–*x*, –*y*)? What would the new coordinates of the envelope be?" (**Answer:** The new coordinates of the envelope would be *A*(10, –1), *B*(6, –1), *C*(6, 1), and *D*(10, 1). The original envelope could have been translated to the right and reflected over the *x*-axis, or it could have been rotated 180° about the *x*-axis.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

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Debriefing the Task

- Ask for volunteers to discuss their strategies and processes for performing each transformation on the envelope. Discuss the type of transformation for each step. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of the types of transformations.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Transformations are one-to-one, which means each point in the set of points will be mapped to exactly one other point, and no other point will be mapped to that point.

  In this task, students will observe that as they perform each transformation, each x-coordinate is mapped to exactly one other y-coordinate, which makes each transformation one-to-one.

- An isometry is a transformation in which the preimage and the image are congruent.

  Students will see that each transformation preserves the size and shape of the original envelope.

- A translation, or slide, is a transformation that moves each point of a figure the same distance in the same direction. A reflection, or flip, is a transformation in which a mirror image is created. A rotation, or turn, is a transformation that turns a figure around a point.

  As students are performing the transformations on the envelope, they will perform translations, reflections, and rotations.

Extending the Task

- Provide students with an isosceles triangle with vertices $A(1, 1)$, $B(3, 6)$, and $C(5, 1)$. Ask them to determine the new coordinates of the triangle after three separate transformations have been performed: 1) reflection over the x-axis; 2) reflection over the y-axis; and 3) translated 4 units to the left. Then ask them to write the general transformation function for each transformation.
- Ask students to work together to create their own figure that will undergo a series of transformations. Encourage them to consider other figures besides a rectangle, such as a triangle or a trapezoid. Ask them to create a series of transformations similar to the original task, then ask them to determine the final coordinates for their figure. Ask for volunteers to share their figure and transformations.

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Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** I analyzed the given steps, then I persevered by following the steps for each transformation and finding the final coordinates for the envelope.)
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** I attended to precision by using correct math terminology in my explanations and by explaining my reasoning in a mathematically correct and precise way.)
- **For SMP 7, ASK**: "How did you look for, and make use of, structure when solving this problem?" (**Answer:** I looked at the structure of the function notation that led to the transformations. I also followed and applied the pattern used to move from one transformation to the other, and I followed the same process of noting the new coordinates each time.)

Alternate Strategies or Solutions

Students may choose to draw the envelope after each transformation on the same coordinate plane in a different color each time, or they may choose to use a new coordinate plane for each transformation. With either method, encourage them to carefully label each coordinate and label them with the proper vertices.

Technology

No technology is needed for this task.

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Problem-Based Task: Lining the Plaza

Common Core State Standard

G–CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Task Overview

Focus

How are lines of symmetry found in polygons? How can the coordinates of figures in different quadrants be found by applying lines of symmetry? In this lesson, students will find the coordinates of the locations of trees in each quadrant of a coordinate plane in a scenario in which the coordinate plane represents the construction plans for a park plaza.

This activity will provide practice with:

- drawing diagonals and perpendicular lines
- identifying lines of symmetry
- calculating coordinates
- plotting points in the coordinate plane
- reflecting a figure across the horizontal axis

Introduction

This task should be used to explore or apply the skill of describing reflections of a figure and determining coordinates based on reflections and lines of symmetry. Students should already be familiar with recognizing perpendicular lines and plotting points in the coordinate plane.

Begin by reading the problem and clarifying the meanings of line of symmetry, line symmetry, perpendicular lines, and reflection.

line of symmetry	a line separating a figure into two halves that are mirror images; written as l
line symmetry	exists for a figure if, for every point on one side of the line of symmetry, there is a corresponding point on the opposite side of the line that is the same distance from the line
perpendicular lines	two lines that intersect at a right angle (90°); written as $\overline{AB} \perp \overline{PQ}$
reflection	a transformation in which a mirror image is created; also called a flip

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 4:** Model with mathematics.

  Students will recognize that this scenario can be modeled by a diagram drawn in the coordinate plane that represents the location of an object in the plan for a park plaza. They will use coordinates to locate the trees in each quadrant of the plaza.

- **SMP 6:** Attend to precision.

  Students will apply the definitions of lines of symmetry, reflections, and perpendicular lines to calculate the coordinates, or positions, of the trees that will be planted in each quadrant. Encourage students to use the correct terminology as they work through the calculations of the coordinates.

- **SMP 8:** Look for and express regularity in repeated reasoning.

  Students will use repeated reasoning as they perform the processes of transformations, specifically reflections, and calculate the coordinates for the locations of the trees on the coordinate plane. They will accomplish this by first finding the location of one tree and a line of symmetry, then repeating this process in the other quadrants to find the locations of the other trees.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- miscalculating the location of a coordinate in a quadrant

  Remind students that once a diagonal is drawn in the first quadrant, the coordinates in the other quadrants can be found by reflecting their positions across either the horizontal or vertical axis.

- incorrectly labeling the sign of a coordinate

  Have students label the signs for each quadrant as they work through the task. Quadrant A will have signs of (–, +), Quadrant B will have signs of (+, +), Quadrant C will have signs of (–, –), and Quadrant D will have signs of (+, –).

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask, "What is the definition of a diagonal of a quadrilateral?" (**Answer:** A diagonal of a quadrilateral is a line segment drawn from one vertex of a quadrilateral to the opposite vertex.)
- Before students begin the task, ask, "What quadrants in the coordinate plane correspond to the quadrants listed in the drawing of the plaza?" (**Answer:** Quadrant A corresponds to the second quadrant in the coordinate plane, Quadrant B corresponds to the first quadrant, Quadrant C corresponds to the third quadrant, and Quadrant D corresponds to the fourth quadrant.)
- If students are having difficulty finding the coordinate of the second tree in Quadrant A, remind them that the diagonal in the quadrant is the line of symmetry, and that a point on the other side of the diagonal must be equidistant from the diagonal as the first tree. Ask students, "How many units down and to the left from the line of symmetry should you move to find the location of the second tree in Quadrant A?" (**Answer:** Move two units down and two units to the left.)
- Ask students, "When a point is reflected over the x-axis, which coordinate will have its sign changed to the opposite? Give an example." (**Answer:** When a point is reflected over the x-axis, the sign of the y-coordinate will change. For example, if the original point (2, 5) is reflected over the x-axis, the new coordinate will be (2, –5).)
- Ask students, "When a point is reflected over the y-axis, which coordinate will have its sign changed to the opposite? Give an example." (**Answer:** When a point is reflected over the y-axis, the sign of the x-coordinate will change. For example, if the original point (3, 6) is reflected over the y-axis, the new coordinate will be (–3, 6).)
- Ask students, "Without using a ruler, what would be one way to verify that the coordinates representing the trees in a quadrant were equidistant from the line of symmetry?" (**Answer:** The distance formula could be used to verify that the coordinates of each tree were equidistant from the line of symmetry.)
- Ask students, "If the coordinates representing the location of the trees were connected by line segments, what type of polygon would result? Would this be a regular polygon? Why or why not?" (**Answer:** If the coordinates were connected, the resulting polygon would be an octagon. This would not be a regular octagon, because the sides are not all the same length.)
- Ask students, "What is the equation of the line that represents the line of symmetry going through Quadrants A and D? Quadrants B and C?" (**Answer:** The equation of the line of symmetry going through Quadrants A and D is $y = -x$, and the equation of the line of symmetry going through Quadrants B and C is $y = x$.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

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Debriefing the Task

- Ask for volunteers to discuss their strategies and processes for finding the coordinates of each tree. Discuss how applying a reflection determines the new coordinates. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Focus on the use of precise mathematical language and clarity, specifically when referring to lines of symmetry and reflections involved in each quadrant.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Figures can be reflected through lines of symmetry onto themselves.

  In this task, students will determine how to find the coordinates of a point reflected over the x-axis and the y-axis, and a line of symmetry created by a diagonal in a quadrant in the coordinate plane.

- Line symmetry exists for a figure if, for every point P on one side of the line, there is a corresponding point Q where l is the perpendicular bisector of $\overline{PQ}$.

  To find the location of the second tree in Quadrant A, students will draw a line perpendicular to the diagonal from the position of the first tree. They will see that the location of the trees is equidistant from the diagonal, so the line is a perpendicular bisector.

Extending the Task

- Ask students to work together to create another type of landscaping figure that could be included in the park plaza. Ask them to plot the coordinates of the figure in one quadrant, then ask them to determine the coordinates of the same figure in the other three quadrants so that the figures are all equidistant from the center of the park plaza (the origin).
- Provide students with a blank coordinate plane, and ask to draw an isosceles trapezoid in the first quadrant. Then ask them to transform the trapezoid by reflecting it over the x-axis and the y-axis. Ask them to write down the coordinates for all three trapezoids.

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Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** I used mathematics to model this scenario using a diagram drawn in the coordinate plane that represents the location of an object in the plan for a park plaza.)
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** I attended to precision by using correct math terminology in my explanations and by explaining my reasoning in a mathematically correct and precise way.)
- **For SMP 8, ASK:** "How did you look for and express regularity in repeated reasoning?" (**Answer:** I expressed regularity in repeated reasoning by performing a series of reflections from one quadrant to another to determine the coordinates that represented the location of the trees in each quadrant.)

Alternate Strategies or Solutions

- Students may choose to use the distance formula to determine the coordinates of the second tree in Quadrant A. Once the diagonal and the perpendicular line to the diagonal are drawn, they can use the point of intersection of the diagonal and the perpendicular line as (x_1, y_1), and use the coordinates of the first tree, (–5, 9), as (x_2, y_2). Then they can determine the coordinates of the second tree by finding the corresponding point that is the same distance from the diagonal as the first tree.
- Students may choose to draw the lines of symmetry represented by the equations $y = x$ and $y = –x$. This may help them recognize the symmetry with respect to the origin, as well as the symmetry in each quadrant.

Technology

No technology is needed for this task.

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Problem-Based Task: Painting Surfboards

Common Core State Standard

G–CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Task Overview

Focus

How can an equation be written to describe a series of transformations performed on an object? What is the difference between a translation and a rotation? In this lesson, students will analyze a series of transformations performed on a surfboard as it goes through a conveyor belt, and will describe an equation to represent the transformations.

This activity will provide practice with:

- interpreting verbal information
- identifying a transformation
- applying function notation to a transformation
- writing an equation to describe a series of transformations

Introduction

This task should be used to explore or apply the skill of describing transformations in the form of an equation. Students should already be familiar with plotting points in the coordinate plane and manipulating functions.

Begin by reading the problem and clarifying the meaning of rotation, transformation, and translation.

rotation	an isometry in which all points in the preimage are moved along circular arcs determined by the center of rotation and the angle of rotation; also called a turn
transformation	a change in a geometric figure's position, shape, or size
translation	an isometry in which all points in the preimage are moved parallel to a given line; also called a slide

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will apply definitions of translations and rotations in transforming the surfboard, and will use these results to construct viable arguments about the equation created to describe the transformations.

- **SMP 4:** Model with mathematics.

  Students will recognize that this scenario can be modeled with an algebraic equation using function notation representing a series of transformations performed on a surfboard as it passes through a conveyor belt.

- **SMP 6:** Attend to precision.

  Students will apply the definitions of the transformations to describe the transformations in terms of an equation. Encourage students to use the correct terminology as they work through the transformations and writing the equation.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- confusing reflections and rotations

  Remind students that reflections are transformations that are mirror images of the original image, and that rotations are transformations in which the original image is turned about a set point at a certain angle measure.

- misidentifying the first transformation in the task as a reflection

  Remind students that a translation is also known as a slide, and that a translation does not change the orientation of the original image, as it is moved parallel to the original. Since the surfboard is first moved in one direction, the transformation is a translation.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask, "What is another word for reflection? What is another word for rotation? What is another word for translation?" (**Answer:** A reflection is also known as a "flip"; a rotation is also known as a "turn"; and a translation is also known as a "slide".)
- Before students begin the task, ask, "Why is the beginning movement of the surfboard on the conveyor belt considered a translation?" (**Answer:** If an object is on a conveyor belt, the object is moving forward in one direction. The orientation of the object has not been changed. A translation is a transformation that moves an object parallel in one direction.)
- Before students begin the task, ask, "Although a surfboard is more shaped like an oval than anything else, what other type of shape can you identify the surfboard as to sketch it on a coordinate plane?" (**Answer:** A surfboard could have the shape of a rectangle so it would be easier to sketch on a coordinate plane.)
- Ask students, "After the surfboard has gone through the conveyor belt one full cycle, what must be done to paint the rest of the board?" (**Answer:** The board must be turned upside down, or flipped over, then placed on the conveyor belt for another full cycle for the entire board to be painted.)
- As students begin the task, ask, "How could the movement of the surfboard be modeled on the coordinate plane in terms of the first transformation?" (**Answer:** The surfboard could be drawn as a rectangle in Quadrant II, for example, and the movement of the conveyor belt, which is a translation, could be represented by drawing a translated rectangle in Quadrant I. Remind students to make sure the rectangle has been translated such that it has moved directly over, and not up or down.)
- Ask students, "What information are you given in the problem that indicates the second transformation of the surfboard is a rotation?" (**Answer:** The information states that a mechanical arm rotates the surfboard 90°, so the angle measure indicates that the surfboard has been turned 90°. The type of transformation that involves a rotation about a set point and a specific angle measure is a rotation.)
- Ask students, "When the equation of $2(C(b), A(b))$ is written to describe the transformations involved in painting a surfboard completely, what does "2" represent?" (**Answer:** The "2" represents the number of times the surfboard must go through the given transformations. Since the process only paints one side of the surfboard at a time, it must go through the process twice to be fully painted.)

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- Ask students, "In the equation $2(C(b), A(b))$, what transformations do $C(b)$ and $A(b)$ represent?" (**Answer:** The $C(b)$ represents the motion of the conveyor belt as it moves the surfboard along, which is a translation. The $A(b)$ represents the rotating of the surfboard, which is a rotation.)
- Ask students, "If a reflection had been performed on the surfboard during the painting process, what would this cause the surfboard to do in terms of motion?" (**Answer:** If the surfboard were to be flipped over to its other side during the process, this would be a reflection.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

Debriefing the Task

- Ask for volunteers to discuss their strategies and processes for identifying the types of transformations performed on the surfboard. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of the types of transformations.

Connecting to Key Concepts

Make explicit connections to key concepts:

- A translation is an isometry in which all points in the preimage are moved parallel to a given line.

  In this task, students will conclude that the movement of the surfboard on the conveyor belt represents a translation, because the surfboard is moved parallel on the conveyor belt.

- A rotation is an isometry in which all points in the preimage are moved along circular arcs determined by the center of rotation and the angle of rotation.

  Students will recognize that the type of transformation performed by the mechanical arm in the painting process is a rotation.

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Extending the Task

- Ask students to write an additional transformation into the painting process for the surfboard. Ask them to create the scenario in which another type of mechanical equipment flips the board over at some point in the process. Then ask students to add this type of transformation into the original scenario and write an equation that describes all three transformations. Discuss how this new transformation would eliminate the "2" that is in the equation for the current scenario.
- Ask students to work together to create their own real-life scenario in which an object undergoes a series of transformations. Encourage them to consider objects that are represented by known polygons. Ask them to include at least three transformations, then ask them to write an equation describing their transformations. Ask for volunteers to share.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 3, ASK:** "Did you construct viable arguments and critique the reasoning of others?" (**Answer:** I constructed viable arguments by using the results of the transformations of the surfboard to create an equation describing the transformations.)
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** I used mathematics to model this scenario with the use of an algebraic equation using function notation that represents a series of transformations performed on a surfboard as it passes through a conveyor belt.)
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** I attended to precision by using correct math terminology in my explanations and by explaining my reasoning in a mathematically correct and precise way.)

Alternate Strategies or Solutions

- Although this scenario is not presented using a coordinate plane, students may choose to sketch a graph of an object that represents the surfboard. They may then draw a new image each time a transformation is performed on the surfboard. Encourage students to carefully plot the coordinates for the new image each time, based on their knowledge of the effects of the signs of the coordinates for each type of transformation.
- Instead of writing the final equation for the transformations as $2(C(b), A(b))$, students may choose to write it instead as $(C(b)), A(b)) + (C(b)), A(b))$. Encourage them to examine this equation and determine how to simplify it.

Technology

No technology is needed for this task.

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Problem-Based Task: Animating a Logo

Common Core State Standard

G–CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Task Overview

Focus

How can a series of transformations be performed on shapes to create a composite shape? How can these transformations be represented symbolically? In this lesson, students will determine three unique transformations performed on triangles to create an animated logo for a software company.

This activity will provide practice with:

- locating triangles in the four quadrants
- applying definitions of transformations
- calculating the change in *x*- and *y*-coordinates for a translated triangle
- determining the angle of rotation for a triangle
- identifying the line of reflection for a triangle

Introduction

This task should be used to explore or apply the skill of specifying a sequence of transformations that will carry a given figure onto another. Students should already be familiar with the definitions of basic transformations and the locations of the quadrants in the coordinate plane.

Begin by reading the problem and clarifying the meanings of reflection, rotation, transformation, and translation.

reflection	an isometry in which a figure is moved along a line perpendicular to a given line called the line of reflection
rotation	an isometry in which all points in the preimage are moved along circular arcs determined by the center of rotation and the angle of rotation; also called a turn
transformation	a change in a geometric figure's position, shape, or size
translation	an isometry in which all points in the preimage are moved parallel to a given line; also called a slide

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they make sense of the types of transformations that occur with each triangle to create the final composite logo. They will represent each transformation symbolically using the correct transformation function notation. They will reason quantitatively as they calculate the change in the x- and y-coordinates for the translation.

- **SMP 4:** Model with mathematics.

  Students will recognize that this scenario can be modeled with the use of a coordinate plane that isolates each triangle in a different quadrant so the transformations can be identified. They will use the coordinate plane to locate the coordinates of the transformed triangles.

- **SMP 6:** Attend to precision.

  Students will apply definitions of the transformations to identify each type of transformation for each triangle in the logo. Encourage students to use the correct terminology as they work through the transformations, and to write the correct transformation formula notation.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- using the incorrect transformation formula for reflections

  Have students create a list of the transformation formulas for reflections about the x-axis, the y-axis, and the line $y = x$. Remind them to refer to these formulas and check the signs of the x- and y-coordinates depending on which line the transformation is being reflected.

- using the incorrect transformation formula for rotations

  Have students create a list of the transformation formulas for rotations of 90°, 180°, and 270°. Remind them to refer to these formulas and check the signs of the x- and y-coordinates, depending on which angle of rotation is being used in the transformation.

- translating in the positive directions along the axes when the translations are intended to be in the negative directions

  Remind students that if a translated figure is being translated in a positive direction along the axes, the value of the coordinates in the translation function will be positive, and if it is being translated in a negative direction, the value of the coordinates in the translation function will be negative.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask them to examine the triangle in Quadrant II of Graph A and compare it to the corresponding triangle in Graph B. Ask them, "Without using the specific name of the transformation, how can you compare the two triangles in terms of the movement from one to the other?" (**Answer**: The triangle on Graph B has been moved right to the first quadrant, and the entire triangle has been shifted down a little.)
- Before students begin the task, ask them to examine the triangle in Quadrant III of Graph A and compare it to the corresponding triangle in Graph B. Ask them, "Without using the specific name of the transformation, how can you compare the two triangles in terms of the movement from one to the other?" (**Answer**: The triangle on Graph B has been moved to the first quadrant. It has been flipped up and over; in other words, it has been turned counter-clockwise to a different orientation.)
- Before students begin the task, ask them to examine the triangle in Quadrant IV of Graph A and compare it to the corresponding triangle in Graph B. Ask them, "Without using the specific name of the transformation, how can you compare the two triangles in terms of the movement from one to the other?" (**Answer**: The triangle on Graph B has been flipped up from the fourth quadrant to the first quadrant.)
- Before students begin the task, ask, "What is another word for reflection? What is another word for rotation? What is another word for translation?" (**Answer**: A reflection is also known as a "flip," a rotation is also known as a "turn," and a translation is also known as a "slide.")
- As students are determining the changes in the *x*- and *y*-coordinates for the translation, ask, "If you chose to use the upper right vertex of II_A, what would the coordinates be for II_A and II_B? Show how the transformation is still $T_{13,-2} = (x+13, y-2)$ with these different coordinates." (**Answer:** The coordinates for the upper right vertex of II_A are $(-4,9)$, and the coordinates for II_B are (9, 7). The change in *x* is 9 – (–4) = 13. The change in *y* is 7 – 9 = –2. Therefore, the transformation is still $T_{13,-2} = (x+13, y-2)$.)
- As students are identifying the rotation of 180° that takes III_A to III_B, ask them to consider other transformations for the triangle. Ask, "Other than a rotation, what other transformation or series of transformations would take III_A to III_B?" (**Answer:** The triangle in Quadrant III could have first been reflected over the *x*-axis, which would move it to the second quadrant. Then, the triangle could have been reflected over the *y*-axis, which would move it to its final position shown in Graph B.)

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- Ask students, "Based on the coordinates for the vertices of the triangle from Quadrant II known as II_A, what type of triangle do you think this is, and why? How could this be proven?" (**Answer:** The triangle II_A is an isosceles triangle. A perpendicular bisector could be drawn from the vertex to the base, as the bisected segments are congruent. The distance formula could be used to show that the legs of the triangle are congruent.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

Debriefing the Task

- Ask for volunteers to discuss their strategies and processes for identifying the types of transformations performed on the three triangles. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of the types of transformations and the formula notation for each transformation.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Given a set of points and a target, we can determine the transformation(s) necessary to move the given set of points to the target.

  In this task, students are given triangles graphed in the coordinate plane, and will be able to use the coordinates of the vertices of the triangles to determine the new coordinates based on the type of transformation.

- Observing the orientations of the preimage and image is the first tool in determining the transformations required.

  Before students specifically identify each transformation, they will observe the orientation of each triangle to determine if each triangle was flipped, turned, or moved over.

- Formulas can be used to determine translations, reflections, and rotations.

  Students will apply the formulas for all three types of transformations in this task, based on the original orientation of each triangle. They will use the formulas to write the proper transformation formula using correct notation.

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Extending the Task

- Ask students to determine the final coordinates of the vertices of the logo in Graph B if the composite figure/logo was translated by the function $T_{-5,-1}$. Have them sketch the figure in a new coordinate plane. Ask them, "Which quadrant, or quadrants, will the final translated figure be located?" Discuss the process of determining the new coordinates for all four vertices.
- Ask students to work together to create their own company logo composed of a series of figures being transformed. Encourage them to be creative and have a combination of polygons, such as triangles, squares, and rectangles, in their logo. Ask them to include at least three different transformations, and to represent each transformation function with the appropriate notation. Ask for volunteers to share their logos and discuss their transformations.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** I used abstract reasoning as I made sense of the types of transformations that occurred with each triangle to create the final composite triangle logo. I also represented each transformation symbolically using the correct transformation function notation.) "Which of your strategies represent quantitative reasoning?" (**Answer:** I reasoned quantitatively when I calculated the change in the *x*- and *y*-coordinates for the translation.)
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** I used mathematics to model this scenario with the use of the coordinate plane to help me determine which type of transformation was performed on each triangle.)
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** I attended to precision by using correct math terminology in my explanations and by explaining my reasoning in a mathematically correct and precise way.)

Alternate Strategies or Solutions

- Students may perform the transformations in a different order than what is in the task. Remind them to examine the orientation of each triangle independent of the others.
- When determining the change in the *x*- and *y*-coordinates in the translation, students may choose other vertices to use in the calculation. Ensure them that any of the four vertices will yield the same result for the transformation formula, but remind them to make sure they use the correct pair of corresponding vertices when choosing the pair of coordinates.

Technology

No technology is needed for this task.

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Problem-Based Task: How Many Triangles?

Common Core State Standard

G–CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Task Overview

Focus

Given specific segments and an angle, how many non-congruent triangles can be constructed with a straightedge and a compass? How can constructions be verified for accuracy? In this lesson, students will perform triangle constructions with two given segments and an angle, and will determine the number of possible non-congruent triangles that can be created.

This activity will provide practice with:

- identifying the number of sides and angles in a triangle
- copying an angle and a segment using a compass and a straightedge
- constructing a triangle using a compass and a straightedge
- labeling the endpoints of segments, rays, and the vertices of a triangle
- analyzing triangle constructions to determine congruency

Introduction

This task should be used to explore or apply the skill of making formal geometric constructions with a variety of tools and methods, specifically a compass and a straightedge. Students should already be familiar with the geometric terms line, ray, segment, and angle, and should be familiar with using a compass.

Begin by reading the problem and clarifying the meanings of the following terms:

angle	two rays or line segments sharing a common endpoint; the symbol used is >
compass	an instrument for creating circles or transferring measurements; it consists of two pointed branches joined at the top by a pivot
construct	to create a precise geometric representation using a straightedge along with either patty paper (tracing paper), a compass, or a reflecting device
segment	a part of a line noted by two endpoints
straightedge	a tool having at least one long edge of reliable straightness

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will recognize that the goal of the problem is to construct as many non-congruent triangles as possible using the given segment lengths and angle. They will copy the segments and the angle, using a compass and a straightedge, and orient them in such a way that a triangle is constructed.

- **SMP 5:** Use appropriate tools strategically.

  Students will use a compass and a straightedge for this task. They will follow a series of steps to copy an angle and two segments, then they will construct non-congruent triangles using the angle and the segments.

- **SMP 6:** Attend to precision.

  Make sure students are using math terms and definitions clearly and accurately when constructing their triangles. Encourage them to take their time and very carefully go through each step in the process of copying an angle and segments, as well as when they use the compass and straightedge to perform the constructions. Also remind students to carefully label the endpoints and/or vertex of the segments, rays, angles, and triangles.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- inappropriately changing the compass setting

  Remind students to make sure the compass setting is secure each time they use it for a construction, or when copying an angle or segment. Remind them that the constructions will not be accurate if the compass setting is changed, unless the instructions specifically indicate a change.

- attempting to measure lengths and angles with rulers and protractors

  Remind students that in this task, as well as with most basic geometry constructions, the only tools used are a compass and a straightedge, and that these tools will allow for accurate constructions of segments and angles.

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- constructing fewer than four non-congruent triangles

  Remind students to double-check that all possible orientations of the sides and angles have been drawn. Encourage them to construct the triangles with the angle in the same position each time so they can see how many different combinations of the other sides are possible using the given segment lengths.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask, "How many possible non-congruent triangles do you predict can be drawn with the given angle and segments? Explain your reasoning for your prediction." (**Answer:** Students' predictions will vary. Some may predict that only one triangle may be drawn because they may not see the different possibilities of the orientation of the two given segments. Encourage students to write down their reasoning in detail.)
- Before students begin the task, discuss their prior knowledge about the lengths of the sides that form triangles. Ask, "What do you know about the possible lengths for a triangle in terms of which combination of lengths do and do not make a triangle? What is the name of this theorem?" (**Answer:** The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side.)
- Before students begin the task, ask, "What do you know about the sum of the interior angles of a triangle? What is the name of this theorem?" (**Answer:** The Triangle Sum Theorem states that the sum of the interior angles of a triangle is 180°.)
- Before students begin the task, ask, "Instead of a compass and a straightedge, what other construction tools could you use to perform this task?" (**Answer:** A protractor could be used to measure the given angle, and a ruler could be used to measure the lengths of the given segments.)
- If students are having difficulty with construction of the triangles, encourage them to review the steps necessary for copying an angle and a segment. Ask, "When copying a segment or an angle, why is it important to make sure the compass setting is not changed?" (**Answer:** If the compass setting is changed at an inappropriate time, the construction of the angle or segment will not be accurate.)

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- As students are determining the triangle combinations, ask, "Why are there only two possible non-congruent triangles that can be constructed when Side 1 and Side 2 share the vertex *A*?" (**Answer:** If both Side 1 and Side 2 share vertex *A*, there are only two possible ways the sides can be constructed. Side 1 can be on the upper part of the angle and Side 2 on the lower part; the opposite orientation can also happen.)
- Ask students, "In order to draw a congruent triangle, what additional information needs to be provided?" (**Answer:** Based on the given angle and segments, if only one congruent triangle is drawn, either the length of the third segment or another angle measure would need to be provided.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

Debriefing the Task

- Ask for volunteers to discuss their initial predictions about how many possible non-congruent triangles could be drawn with the given angle and segments. Encourage students to discuss their reasoning for their predictions, as well as how correct or incorrect they were based on the actual results of the task. Also encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Focus on the use of precise mathematical language and clarity, specifically when referring to the basic geometric terms used when performing the constructions.

Connecting to Key Concepts

Make explicit connections to key concepts:

- A geometric figure precisely created using only a straightedge and compass is called a construction.

  In this task, students will create triangle constructions using only a straightedge and a compass.
- An endpoint is either of two points that mark the ends of a line, or the point that marks the end of a ray.

  As students are copying the segments needed for their triangle constructions, they will label and determine the endpoints used in the construction process.
- A line segment is part of a line noted by two endpoints.

  Students will define their line segments by two endpoints as they copy the segments, and when they are constructing triangles using segments.

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- An angle and a segment can be copied using a compass and a straightedge.

  In this task, students will follow a series of steps and use a compass and a straightedge to copy a given angle and given segments that will be used in the construction of triangles.

Extending the Task

- Ask students to examine the four non-congruent triangles and, without using a protractor or ruler, guess what type of triangle each is in terms of angles. Ask them to use a protractor to measure the three angles to verify that the angles sum to 180°, then to determine if each triangle is acute, right, or obtuse. Ask them to determine, without using a ruler, if each triangle is equilateral, isosceles, or scalene. Discuss how the type of triangle in terms of the sides can be determined by the angle measures, and vice versa.
- Ask students to work with a partner, and have each person draw a specific acute angle with a protractor, then two specific segment lengths with a ruler. Ask the partners to switch papers, and have each person go through the steps to copy the segments and angles using a compass and a straightedge, then to construct as many non-congruent triangles as possible. Ask for volunteers to share their results.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** I persevered by recognizing that the goal of the problem was to construct as many non-congruent triangles as possible using the given segment lengths and angle. I copied the segments and the angle using a compass and a straightedge, then I constructed four non-congruent triangles.)
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** I used a compass and a straightedge strategically by carefully following a series of steps for copying a given angle and two segments, then I used the compass and straightedge again to construct triangles.)
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** I attended to precision by using correct math terminology in my geometric constructions and by explaining my reasoning and steps in a mathematically correct and precise way.)

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Alternate Strategies or Solutions

Two of the three triangles are drawn with Side 1 and Side 2 NOT as the sides for Angle A. With these two, students may have different triangles than the given triangles drawn, as the angle connecting Side 1 and Side 2 may be drawn as a right angle, an acute angle, or an obtuse angle (as shown in the task). Encourage students to construct as many different triangles as possible for these two triangles.

Technology

No technology is needed for this task.

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Problem-Based Task: Triangle Medians

Common Core State Standard

G–CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Task Overview

Focus

What is a median of a triangle? What is the result of constructing all three medians in a triangle? In this lesson, students will use a compass and a straightedge to construct the three medians in a triangle and determine the result of the construction of the three medians.

This activity will provide practice with:

- defining a midpoint of a segment and a median of a triangle
- constructing the midpoint of a segment using a compass and a straightedge
- constructing the median of a triangle
- interpreting the results of constructing the three medians in a triangle
- verifying the intersection of the medians of a triangle

Introduction

This task should be used to explore or apply the skill of making formal geometric constructions with a variety of tools and methods, specifically a compass and a straightedge. Students should already be familiar with the geometric terms "line," "ray," "segment," and "angle," and should be familiar with using a compass and a straightedge.

Begin by reading the problem and clarifying the meaning of construction, median, and midpoint.

construction	a precise representation of a figure using a straightedge and a compass, patty paper and a straightedge, or a reflecting device and a straightedge
median	the segment joining the vertex to the midpoint of the opposite side
midpoint	a point on a line segment that divides the segment into two equal parts

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 5:** Use appropriate tools strategically.

  Students will use a compass and a straightedge for this task. They will follow a series of steps with the compass and the straightedge to construct the midpoints of the triangle, then they will use the straightedge to construct the medians.

- **SMP 6:** Attend to precision.

  Make sure students are using math terms and definitions clearly and accurately when performing their constructions of the midpoints. Encourage them to take their time and very carefully go through each step of the construction process. Remind students to carefully label the midpoints of the segments.

- **SMP 8**: Look for and express regularity in repeated reasoning.

  Students will use repeated reasoning by repeating the process used to construct the three medians of a triangle. As a result of the constructions, they will draw a conclusion about the intersection of the three medians.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- inappropriately changing the compass setting

  Remind students to make sure the compass setting is secure each time they use it for a construction. Remind them that the constructions will not be accurate if the compass setting is moved or changed, unless the instructions specifically indicate a change.

- not creating large enough arcs to find the point of intersection

  Remind students that if they do not set their compasses with a large enough arc, the arcs may not intersect each other, which is necessary for the construction to be accurate. Assure students that in terms of constructions with a compass, it is better to overestimate than underestimate the arc size.

- attempting to measure lengths and angles with rulers and protractors

  Remind students that in this task, as well as with most basic geometry constructions, the only tools used are a compass and a straightedge, and that these tools will allow for accurate constructions of segments and angles.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask them, "What do you think will happen when all three medians of a triangle are constructed? Explain your reasoning." (**Answer:** Students' answers will vary. Some students may predict the three medians will intersect in some fashion. Or they may predict they that only two of the medians will intersect. Encourage students to write down their reasoning in detail.)
- Before students begin the task, ask them, "In your own words, what is the difference between the words 'midpoint' and 'median'? What do both words mean, in general?" (**Answer:** The midpoint of a segment is the point that divides the segment into two equal parts. The median is a segment drawn from the midpoint to the vertex of a triangle. In general, both words mean "middle.")
- Before students begin the task, ask them, "What are other words or phrases that have the same meaning when discussing the midpoint of a segment?" (**Answer:** Other words or phrases that also indicate the meaning of a midpoint are "bisect," "equidistant," and "congruent parts.")
- If students are having difficulty with their constructions of the midpoints, encourage them to review the steps necessary for constructing a midpoint. Ask them, "When performing the construction of a midpoint, why is it important to make sure the compass setting is wide enough so the arcs are drawn large enough?" (**Answer:** If the arcs are not drawn large enough, the two arcs will not intersect when performing the construction of the midpoint. As a result, the construction will not be accurate.)
- As students are beginning to construct the medians from the midpoints, ask them, "If all three medians are very close to intersecting at one point, but do not quite do this, where do you think the error comes from?" (**Answer:** If the medians almost intersect but do not exactly intersect, it is likely that one or more of the medians was not drawn accurately from the midpoint to the vertex.)
- Ask students, "Which method do you think is more accurate for constructing midpoints and medians – using a compass and a straightedge, or using a ruler and a protractor? Why?" (**Answer:** The construction method with a compass and a straightedge will most likely be more accurate than using a ruler and a protractor, because error in measurement using a ruler, as well as error in using the protractor, is possible. These types of errors will lead to inaccurate measurements of the midpoint.)
- Ask students, "How can you verify that the medians of any type of triangle will always intersect at exactly one point?" (**Answer:** The triangle given in the task is an obtuse triangle. If a right triangle or an acute triangle is drawn, the medians of these triangles also intersect at exactly one point. Encourage students to draw several triangles to verify this.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

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Debriefing the Task

- Ask for volunteers to discuss their initial predictions about what will happen when all medians of a triangle are constructed. Encourage students to discuss their reasoning for their predictions, as well as how correct or incorrect they were based on the actual results of the task. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Focus on the use of precise mathematical language and clarity, specifically when referring to the basic geometric terms used when performing the constructions.

Connecting to Key Concepts

Make explicit connections to key concepts:

- When constructing the midpoint of a triangle, a compass and a straightedge (or patty paper and a straightedge) can be used to determine the midpoint of the segment.

  In this task, students will use a compass and a straightedge to construct the midpoint on all three sides of a triangle.

- The point where the segment is bisected is called the midpoint of the segment.

  Students will see that when they follow the steps for constructing the midpoint, they are bisecting the side of the triangle. The point that represents this is called the midpoint.

- The median of a triangle is the segment joining the vertex to the midpoint of the opposite side.

  In this task, after students have constructed the midpoints of the sides of the triangle, they will construct the medians for each side by drawing a segment from the midpoint to the opposite vertex.

Extending the Task

- Ask students to use a ruler and a protractor to draw an equilateral triangle. Then ask them to use their compass and a straightedge to construct the midpoints of each triangle, then the medians. Ask students, "What do you notice about the lengths of the medians of an equilateral triangle? What do you notice about the internal triangles that are created by the medians?"
- Ask students to work with a partner and have each draw either an isosceles triangle or a scalene triangle. Then ask the partners to switch papers, and have each go through the steps to construct the midpoints using a compass and a straightedge, then the medians. Ask students to draw conclusions about the lengths of the medians for these types of triangles.

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Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** I used a compass and a straightedge strategically by carefully following a series of steps for constructing the midpoints of the sides of a triangle, then I used the straightedge to construct the medians.)
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** I attended to precision by using correct math terminology in my geometric constructions, and by explaining my reasoning and steps in a mathematically correct and precise way.)
- **For SMP 8, ASK:** "How did you look for and express regularity in repeated reasoning?" (**Answer:** I used repeated reasoning to construct the medians of a triangle. I evaluated these results to determine the intersection of the medians.)

Alternate Strategies or Solutions

Instead of using a compass and a straightedge to construct the midpoints, students may choose to find the midpoint of each side of the triangle by measuring each segment with a ruler and finding the halfway point. Discuss with students that although this method is technically correct, it will be difficult to get as precise and accurate a measurement.

Technology

No technology is needed for this task.

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Problem-Based Task: Triangle Altitudes

Common Core State Standard

G–CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Task Overview

Focus

What is an altitude of a triangle? What is the result of constructing all three altitudes in a triangle? In this lesson, students will use a compass and a straightedge to construct the three altitudes in a triangle and determine the result of the construction of the three altitudes.

This activity will provide practice with:

- defining the altitude of a triangle
- constructing the altitude of a triangle using a compass and a straightedge
- interpreting the results of constructing the three altitudes in a triangle
- verifying the intersection of the altitudes of a triangle

Introduction

This task should be used to explore or apply the skill of making formal geometric constructions with a variety of tools and methods, specifically a compass and a straightedge. Students should already be familiar with the geometric terms "line," "ray," "segment," and "angle," and should be familiar with using a compass and a straightedge.

Begin by reading the problem and clarifying the meaning of altitude, construction, and perpendicular lines.

altitude	the perpendicular line from a vertex of a figure to its opposite side; height
construction	a precise representation of a figure using a straightedge and a compass, patty paper and a straightedge, or a reflecting device and a straightedge
perpendicular lines	two lines that intersect at a right angle (90°)

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will recognize that the goal of the problem is to construct the altitudes of a triangle by constructing a perpendicular line from each vertex to the opposite side, using a compass and a straightedge. They will then determine the outcome of the construction of all three altitudes.

- **SMP 5:** Use appropriate tools strategically.

  Students will use a compass and a straightedge for this task. They will follow a series of steps with the compass and straightedge to construct a perpendicular line, which is the altitude of a triangle.

- **SMP 6:** Attend to precision.

  Make sure students are using math terms and definitions clearly and accurately when performing their constructions of the altitudes. Encourage them to take their time and very carefully go through each step of the construction process.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- inappropriately changing the compass setting

  Remind students to make sure their compass setting is secure each time they use it for a construction. Remind them the constructions will not be accurate if the compass setting is moved or changed, unless the instructions specifically indicate a change.

- not creating large enough arcs to find the point of intersection

  Remind students that if they do not set their compasses with a large enough arc, the arcs may not intersect each other, which is necessary for the construction to be accurate. Assure students that in terms of constructions with a compass, it is better to overestimate than underestimate the arc size.

- attempting to measure lengths and angles with rulers and protractors

  Remind students that in this task, as well as with most basic geometry constructions, the only tools that are used are a compass and a straightedge, and that these tools will allow for accurate constructions of segments and angles.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask, "What are some examples in which you hear the word 'altitude' in real-life scenarios? What is the word commonly associated with altitude?" (**Answer:** One common example is the altitude at which an airplane is flying. Other common examples are discussing the altitude above sea level or the altitude of mountains. The word "height" is most frequently associated with altitude.)
- Before students begin the task, ask, "What do you think will happen when all three altitudes of a triangle are constructed? Explain your reasoning." (**Answer:** Students' answers will vary. Some may predict that the three altitudes will intersect in some fashion, or that only two of the altitudes will intersect. Encourage students to write down their reasoning in detail.)
- If students are having difficulty with their constructions of the altitudes, encourage them to go through the steps necessary for constructing a perpendicular line. Ask them, "When performing the construction of a perpendicular line, why is it important to make sure the compass setting is wide enough so the arcs are drawn large enough?" (**Answer:** If the arcs are not drawn large enough, the two arcs will not intersect when performing the construction of the perpendicular line. As a result, the construction will not be accurate.)
- Once students have constructed the three altitudes for the triangle, ask, "What can you conclude about the location of the altitudes drawn from vertex A, vertex B, and vertex C?" (**Answer:** The altitude drawn from vertex B is located inside the triangle, but the altitudes drawn from vertices A and C are located outside the triangle.)
- Ask students, "How could you verify the construction of the altitudes of a triangle with other tools besides a compass and a straightedge?" (**Answer:** To make sure the altitude is constructed properly, a protractor could be used to check that the angle of the perpendicular line is exactly 90°.)
- Ask students, "Where do you think the three altitudes of an acute triangle will be located? How can you verify this?" (**Answer:** The three altitudes of an acute triangle will all be located inside the triangle. This can be verified by constructing the altitudes in an acute triangle.)
- Ask students, "What do you think is unique about the intersection of the medians of an equilateral triangle and the intersection of the altitudes of an equilateral triangle?" (**Answer:** In an equilateral triangle, the medians and the altitudes will intersect at the same point.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

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Debriefing the Task

- Ask for volunteers to discuss their initial predictions about what will happen when all altitudes of a triangle are constructed. Encourage students to discuss their reasoning for their predictions, as well as how correct or incorrect they were based on the actual results of the task. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Focus on the use of precise mathematical language and clarity, specifically when referring to the basic geometric terms used when performing the constructions.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Perpendicular lines are two lines that intersect at a right angle (90°).

  In this task, students will construct a perpendicular line from the vertex of a triangle to the opposite side.
- It is possible to construct a perpendicular line through a point on a given line as well as through a point not on a given line.

  Students will discover that to construct a perpendicular line from vertices A and C to the opposite side of the triangle, the line will be drawn to a point that is not on the given segment.

Extending the Task

- Ask students to use a ruler and a protractor to draw an equilateral triangle. Then ask them to use a compass and a straightedge to construct the altitudes of the triangle. Ask, "What do you notice about the intersection of the altitudes of an equilateral triangle?" Then ask them to construct the medians of the triangle and compare the intersection of the altitudes to the intersection of the medians.
- Ask students to work with a partner and have each draw an isosceles triangle or a scalene triangle. Then ask the partners to switch papers, and have each go through the steps to construct the altitudes using a compass and a straightedge. Ask students to draw conclusions about the intersections of the altitudes for different types of triangles.

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Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** I persevered by recognizing that the goal of the problem was to construct the altitudes of a triangle using a compass and a straightedge. After constructing the altitudes, I observed that they intersected at a point outside the triangle.)
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** I used a compass and a straightedge strategically by carefully following a series of steps for constructing the altitudes of a triangle.)
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** I attended to precision by using correct math terminology in my geometric constructions, and by explaining my reasoning and steps in a mathematically correct and precise way.)

Alternate Strategies or Solutions

Instead of using a compass and a straightedge to construct the altitudes, students may choose to draw the perpendicular line by using a protractor to draw a 90° angle on the opposite side of the vertex. Discuss with students that although this method is technically correct, it will be difficult to get as precise and accurate a 90° angle as with a compass and a straightedge.

Technology

No technology is needed for this task.

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Problem-Based Task: Vending Machine Placement

Common Core State Standard

G–CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Task Overview

Focus

How can an equilateral triangle be constructed inside a circle using a compass and a straightedge? How can the vertices of an inscribed triangle represent points on a circle in a real-world scenario? In this lesson, students will construct an equilateral triangle inscribed in a circle to determine the location of three vending machines in a skating rink that are represented by the vertices of the triangle.

This activity will provide practice with:

- analyzing written information
- determining the type of inscribed polygon to model the scenario
- identifying the properties of an equilateral triangle
- constructing an equilateral triangle using a compass and a straightedge
- labeling the vertices of a triangle

Introduction

This task should be used to explore or apply the skill of constructing an equilateral triangle inscribed in a circle. Students should already be familiar with using a compass and a straightedge to construct arcs.

Begin by reading the problem clarifying the meaning of construction, equilateral triangle, and inscribe.

construction	a precise representation of a figure using a straightedge and a compass, patty paper and a straightedge, or a reflecting device and a straightedge
equilateral triangle	a triangle with all three sides equal in length
inscribe	to draw one figure within another figure so every vertex of the enclosed figure touches the outside figure

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will recognize that the goal of the problem is to determine the locations of three vertices on a circle that will represent the locations of vending machines in the context of the scenario. They will construct an equilateral triangle inscribed inside the circular-shaped skating rink.

- **SMP 4:** Model with mathematics.

  Students will use geometry to solve a design problem. More specifically, they will apply their knowledge of equilateral triangles as well as their knowledge of constructing a triangle using a compass and a straightedge. They will analyze the results of the construction to determine which segments can be drawn that result in an equilateral triangle.

- **SMP 5:** Use appropriate tools strategically.

  Students will use a compass and a straightedge for this task. They will follow a series of steps with the compass and straightedge to make equidistant arcs on the circle that will represent the vertices of the equilateral triangle.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- inappropriately changing the compass setting

  Remind students to ensure their compass setting is secure each time they use it for a construction. Remind them that the constructions will not be accurate if the compass setting is moved or changed, unless the instructions specifically indicate a change.

- attempting to measure lengths and angles with rulers and protractors

  Remind students that in this task, as well as with most basic geometry constructions, the only tools used are a compass and a straightedge, and that these tools will allow for accurate constructions of segments and angles.

- not creating large enough arcs to find the points of intersection

  Remind students that if they do not set their compasses with a large enough arc size, the arcs may not intersect each other, which is necessary for the construction to be accurate. Assure students that in terms of constructions with a compass, it is better to overestimate than underestimate the arc size.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask them, "In terms of side lengths, what are the three types of triangles?" (**Answer:** An equilateral triangle, which has all three sides equal in length; an isosceles triangle, which has at least two sides equal in length; and a scalene triangle, which does not have any sides equal in length.)
- Ask students, "The problem states that 'the distance between each machine must be the same.' What is a word that represents distance being the same between objects?" (**Answer:** The word "equidistant" means equal distance between objects.)
- Before students begin the task, ask them, "What information is given in the problem that leads to the conclusion an equilateral triangle is the type of polygon that will be constructed inscribed in the circle?" (**Answer:** The problem states that the distance must be the same between the three vending machines on the edge of the rink. Since the side lengths represent the distance between machines, an equilateral triangle will be constructed, because all three sides of an equilateral triangle are the same length.)
- If students are having difficulty with their construction of the equilateral triangle, remind them to pay close attention to placing the sharp point of the compass directly on the center of the circle, and placing the other end of the compass directly on the outer edge of the circle. Ask them, "When performing a construction using a compass, why is it important to make sure the compass setting does not change?" (**Answer:** A construction will not be accurate if the compass setting is moved or changed, unless the instructions specifically indicate a change.)
- Ask students, "How could you verify the construction of the equilateral triangle with other tools besides a compass and a straightedge?" (**Answer:** To ensure the triangle is constructed properly, a ruler could be used to measure the side lengths of the triangle to verify they are all the same length.)
- Ask students, "Why would an isosceles triangle not work for the construction of a polygon, based on the information given in the problem?" (**Answer:** Since the vending machines are to be placed equal distances apart, if an isosceles triangle was constructed, only two sides would have the same length, and the third would have a different length. This would not meet the requirements for the vending machines to be equal distances apart.)
- Ask students, "If Jarno wanted to place four vending machines on the edge of the skating rink equidistant from each other, what polygon would need to be constructed inscribed in the circle?" (**Answer:** The four-sided polygon that has four sides of the same length is a square; therefore, a square would need to be constructed inscribed in the circle.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

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Debriefing the Task

- Ask for volunteers to discuss their methods for determining which polygon to construct, as well as the steps for the construction process. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses, specifically when performing the construction of the equilateral triangle inscribed in the circle. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of the construction.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Equilateral triangles are triangles with all three sides equal in length.

  In this task, students will determine that an equilateral triangle will be constructed inscribed in a circle, because the vertices of the triangle represent the location of vending machines spaced equal distances apart from each other.

- To "inscribe" means to draw a figure within another figure so every vertex of the enclosed figure touches the outside figure.

  Students will inscribe an equilateral triangle inside a circle. They will see that the three vertices of the triangle touch the circle.

- It is possible to inscribe a triangle within a circle. Like with all constructions, the only tools used to inscribe a figure are a straightedge and a compass, patty paper and a straightedge, reflective tools and a straightedge, or technology.

  In this task, students will use a compass and a straightedge to construct an equilateral triangle inscribed in a circle.

Extending the Task

- To extend the task, ask students to use a compass and a straightedge to construct an equilateral triangle. Then ask them to use the compass to draw a circle around the triangle so the circle touches all three vertices of the triangle. Ask them, "How do you determine how to set the compass width so the correct size circle is drawn?" Encourage and guide a discussion about the different methods students can use to find the correct radius of the circle.
- Another option for extending the task is to ask students to use a ruler and draw an isosceles triangle and a scalene triangle. Ask them to use a compass to try to draw a circle around each triangle. Ask for volunteers to share their results. Ask them, "What specific polygons can be inscribed in a circle, and why?"

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Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** I recognized the goal of the problem was to determine the locations of three vertices on a circle that represent the locations of vending machines in the context of the scenario. Then I constructed an equilateral triangle inscribed inside the circular-shaped skating rink.)
- **For SMP 4, ASK:** "How did you model mathematics appropriately?" (**Answer:** I applied my knowledge of geometry by constructing an equilateral triangle inscribed in a circle to determine the equidistant locations of three vending machines in a skating rink.)
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** I used a compass and a straightedge strategically by carefully following a series of steps for constructing an equilateral triangle.)

Alternate Strategies or Solutions

Instead of using a compass and a straightedge to construct the equilateral triangle, students may choose to draw it by first choosing a point on the triangle as one of the vertices. Then they may use a ruler and trial and error to determine the location of the other two vertices. Remind them that although this method may work, it may be time-consuming, whereas the construction of the triangle with a compass and a straightedge will be more precise, accurate, and efficient.

Technology

No technology is needed for this task.

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Guide

Problem-Based Task: Constructing a Regular Octagon

Common Core State Standard

G–CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Task Overview

Focus

What are the properties of a regular polygon? How can a regular octagon be constructed within a circle? In this lesson, students will use a compass and straightedge to construct a regular octagon inside a circle using an inscribed square that already exists.

This activity will provide practice with:

- analyzing diagonals of a square
- identifying the correct construction method of a line perpendicular to another line through a given point
- constructing perpendicular bisectors using a compass and a straightedge
- determining the sides of a regular octagon by connecting points of intersection of perpendicular bisectors on the circle and existing corners of the square
- drawing sides of a regular octagon

Introduction

This task should be used to explore or apply the skill of constructing regular polygons inscribed in other regular polygons. Students should already be familiar with using a compass and straightedge to perform basic constructions such as diagonals, perpendicular bisectors, and midpoints.

Begin by reading the problem and clarifying the meaning of construction and regular polygon.

construction	a precise representation of a figure using a straightedge and compass, patty paper and a straightedge, or a reflecting device and straightedge
regular polygon	a two-dimensional figure with all sides and angles congruent

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Guide

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will work through a series of scaffolded steps and complete a logical progression of constructions to determine the correct location of four of an octagon's vertices on a circle. They will use their constructions to justify the solution to the problem of constructing a regular octagon within a circle.

- **SMP 5:** Use appropriate tools strategically.

  Students will use a compass and straightedge for this task. They will use these tools to construct the diagonals of a square, as well as perpendicular lines through a given point, to achieve the goal of constructing a regular octagon inside the circle.

- **SMP 7:** Look for and make use of structure.

  Students will discover the pattern of their constructions using diagonals and perpendicular lines. They will use the structure of the progressive constructions to draw conclusions about the inscribed octagon and its sides and vertices.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- inappropriately changing the compass setting

  Remind students to make sure the compass setting is secure each time they use it for a construction. Remind them that their constructions will not be accurate if the compass setting is changed, unless the instructions specifically indicate a change.

- attempting to measure lengths and angles with rulers and protractors

  Remind students that in this task, as with most basic geometry constructions, the only tools used are a compass and straightedge. Remind them that these tools will allow for accurate constructions of segments and angles.

- not creating large enough arcs to find the points of intersection

  Remind students that if they do not set the compass with a large enough arc size, the arcs may not intersect, which is necessary for the construction to be accurate. Assure students that in terms of constructions with a compass, it is better to overestimate than underestimate the arc size.

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- not extending segments long enough to intersect the circle

  Remind students that the segments must be long enough for the intersection points on the circle to be identified. Assure students that it is better to overestimate the length of a segment and draw it too long than to underestimate it and draw it too short.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask, "What is the definition of a regular polygon? For example, is a square a regular polygon? Is a rectangle a regular polygon? Explain." (**Answer:** A regular polygon is a two-dimensional figure with all sides and angles congruent. A square is a regular polygon, because all four sides are congruent and all four angles are congruent. A rectangle is not a regular polygon, because only opposite pairs of sides are congruent, and not all four sides are congruent.)
- Once students have identified the definition of a regular polygon, ask, "Is a triangle always a regular polygon? Explain." (**Answer:** A triangle is a regular polygon if it is an equilateral triangle. An equilateral triangle has all three sides and all three angles congruent. However, an isosceles triangle has only two sides congruent, and a scalene triangle does not have any congruent sides. Therefore, an isosceles triangle and a scalene triangle are not regular polygons.)
- As students begin to work on the basic constructions on the square, ask, "When each corner of a square is connected to the opposite corner, what is the measure of the segment created? Also, what types of triangles are the four triangles created inside the square by the intersections of the diagonals?" (**Answer:** Because each diagonal of a square splits the square into two right triangles, each diagonal will be the hypotenuse of that triangle. We can use the Pythagorean Theorem to find the length of each diagonal, which would be $s\sqrt{2}$, where s is the side length of the square. In a square, the diagonals are perpendicular to each other, so each triangle created will be a right triangle.)
- After students have performed the constructions and connected the vertices to construct the regular octagon, ask, "Besides a compass and straightedge, what other tools could you use to verify that the octagon is a regular octagon?" (**Answer:** A ruler could be used to measure the sides of the octagon to verify they are all the same length. A protractor could be used to measure the interior angles of the octagon to verify they are all the same measure.)
- After students have connected the vertices to construct the regular octagon, ask, "The segments that represent the diagonals and the perpendicular bisectors of the square create eight triangles within the octagon. What do you think is true about the eight triangles?" (**Answer:** Because the octagon is a regular octagon with all eight sides congruent, the eight triangles created by the segments are congruent triangles.)

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- Ask students, "What type of triangles do the eight triangles within the octagon appear to be? How could you verify this?" (**Answer:** The eight triangles within the octagon appear to be isosceles triangles, because the segments connecting the center of the octagon to each vertex are congruent. This can be verified by using a ruler to measure the length of each side of a triangle.)
- Ask students, "Suppose you want to determine how much each interior angle of a regular octagon measures, given the fact that the sum of all the interior angles of an octagon is 1,080°. What is the measure of each interior angle? Explain." (**Answer:** In a regular octagon, each side length is the same, and each interior angle measures the same. Because the total of the interior angles is 1,080°, divide this by the number of sides of an octagon (which is 8): 1080 ÷ 8 = 135. Therefore, each interior angle of a regular octagon measures 135°.)
- Ask students, "Do you think a square could be inscribed in the octagon? If yes, how could the square be constructed using only a straightedge?" (**Answer:** Yes, a square could be constructed inside the octagon, because all the sides of the octagon are the same length. Connecting every other vertex of the octagon would create a square.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

Debriefing the Task

- Ask for volunteers to discuss their methods for constructing the diagonals and the perpendicular bisectors of the square. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task, or if they came up with a different method of creating the octagon.
- Compare students' strategies and ways of justifying responses, specifically when explaining why the basic constructions performed on the square were necessary for construction of the regular octagon. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of the constructions and the term "regular polygon."

Connecting to Key Concepts

Make explicit connections to key concepts:

- A regular polygon is a polygon that has all sides equal and all angles equal.

  In this task, students will recognize that a square is a regular polygon because it has four equal sides and four equal angles. They will use this square to construct another regular polygon, specifically a regular octagon.

- Constructions can be performed with the use of a compass and straightedge.

  Students will perform several constructions on a square with the use of a compass and straightedge, specifically constructions of diagonals and perpendicular bisectors.

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Extending the Task

- Once students have completed construction of the regular octagon, ask, "What other regular polygons do you think you can inscribe in the regular octagon? How is it possible to construct another regular octagon within the one already drawn?" Encourage and guide a discussion about constructing a square inscribed in the octagon by using a straightedge to connect every other vertex of the octagon. Ask students to work with a partner to determine how to construct another octagon inside the current octagon. Encourage them to think about how to use the midpoints of each side of the octagon as vertices. Have them perform the construction, and ask for volunteers to share their results.
- Ask students to redo the task by first drawing a circle, using a compass. Then have them construct the square inscribed in the circle by following the steps listed in the lesson. Once they have constructed the square, ask them to erase their markings and go through the original task again.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 3, ASK:** "How did you construct viable arguments?" (**Answer:** I worked through a series of scaffolded steps and completed constructions to determine the correct location of four of the vertices of the octagon on the square. The outcomes of each construction allowed me to explain the location of the vertices of the octagon.)
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** I used a compass and straightedge strategically by carefully following a series of steps in the basic constructions of diagonals and perpendicular bisectors.)
- **For SMP 7, ASK:** "How did you look for, and make use of, structure?" (**Answer:** I found the pattern of the various constructions, then used the structure of the constructions to determine the vertices of the octagon.)

Alternate Strategies or Solutions

Instead of using a compass and straightedge to perform the basic constructions necessary for constructing the regular octagon inside the circle, students may choose to use a ruler and protractor. They may locate four of the octagon's vertices by finding the midpoints of the sides of the square, then drawing the perpendicular bisectors by drawing lines through points on the opposite sides of the square. Alternatively, if they are aware that each interior angle of a regular octagon measures 135°, they may use a protractor to determine the location of each of the remaining four vertices of the octagon by drawing 135° angles. Remind them that although this method may work, it may be time-consuming, whereas using a compass and straightedge to perform the basic constructions will be more precise, accurate, and efficient.

Technology

No technology is needed for this task.

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Guide

Problem-Based Task: Constructing a Regular Dodecagon

Common Core State Standard

G–CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Task Overview

Focus

What is the relationship between a regular hexagon and a regular dodecagon? How can a regular dodecagon be constructed inside a circle? In this lesson, students will use a compass and a straightedge to construct a regular dodecagon inside a circle.

This activity will provide practice with:

- analyzing the relationship between a regular hexagon and a regular dodecagon
- constructing a regular hexagon inscribed in a circle
- identifying a basic construction method for a regular dodecagon
- constructing a regular dodecagon based on the construction of a regular hexagon
- drawing the sides of a regular dodecagon

Introduction

This task should be used to explore or apply the skill of constructing regular polygons inscribed in a circle. Students should already be familiar with using a compass and a straightedge to perform basic constructions, as well as constructing a circle with a given radius.

Begin by reading the problem and clarifying the meanings of construction, regular hexagon, and regular polygon.

construction	a precise representation of a figure using a straightedge and a compass, patty paper and a straightedge, or a reflecting device and a straightedge
regular hexagon	a polygon with six equal sides and six equal angles, each angle measuring 120°
regular polygon	a two-dimensional figure with all sides and angles congruent

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will recognize that the goal of the problem is to construct a regular dodecagon inscribed in a circle. They will construct this dodecagon based on constructing a regular hexagon and applying the results of this construction.

- **SMP 5:** Use appropriate tools strategically.

  Students will use a compass and a straightedge for this task. They will use these tools to construct arcs and midpoints to achieve the goal of constructing a regular dodecagon inside a circle.

- **SMP 8:** Look for and express regularity in repeated reasoning.

  Students will complete the process of constructing a regular hexagon inside a circle by repeatedly drawing arcs and connecting points on the circle. Then they will use the repeated process of bisecting the sides of the hexagon to create the remaining six vertices of the dodecagon.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- inappropriately changing the compass setting

  Remind students to make sure the compass setting is the same each time they use it for a construction. Remind them that the constructions will not be accurate if the compass setting is changed, unless the instructions specifically indicate a change.

- attempting to measure lengths and angles with rulers and protractors

  Remind students that in this task, as well as with most basic geometry constructions, the only tools used are a compass and straightedge, and that these tools will allow for accurate constructions of segments and angles.

- not creating large enough arcs to find the points of intersection

  Remind students that if they do not set their compasses with a large enough arc size, the arcs may not intersect, which is necessary for the construction to be accurate. Assure students that in terms of constructions with a compass, it is better to overestimate the arc size than underestimate it.

- not extending segments long enough to find the vertices of the hexagon

  Remind students that the segments must be long enough so the vertices of a hexagon can be identified. Assure students it is better to overestimate the length of a segment and draw it too long than underestimate it and draw it too short.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask, "What is the definition of a regular polygon? Explain how you could verify that a polygon is a regular polygon." (**Answer:** A regular polygon is a two-dimensional figure with all sides and angles congruent. To verify that a polygon is a regular polygon, the lengths of the sides could be measured with a ruler, and the measures of the angles could be measured with a protractor.)
- Before students begin the task, ask, "What is the relationship between a regular hexagon and a regular dodecagon in terms of number of sides?" (**Answer:** A regular hexagon has six sides, and a regular dodecagon has 12 sides. Therefore, a regular dodecagon has twice the number of sides as a regular hexagon.)
- Ask students, "If a regular dodecagon has twice as many sides as a regular hexagon, what does this mean in terms of the lengths of the sides and the number of vertices of a regular dodecagon?" (**Answer:** Since a regular dodecagon has 12 sides and a regular hexagon has six sides, each side of a regular dodecagon will be half the length of a regular hexagon side. A regular hexagon has six vertices, and a regular dodecagon has 12 vertices.)
- If students are having difficulty understanding the purpose of first constructing a regular hexagon inscribed in a circle, ask, "After a regular hexagon is constructed inside a circle, how do you think the relationship between the length of the sides of a regular dodecagon compared to the length of the sides of a regular hexagon can help in constructing a regular dodecagon?" (**Answer:** Since the length of a side of a regular dodecagon is exactly half the length of a side of a regular hexagon, each side of the hexagon can be bisected using basic construction with a compass, specifically making arcs on the midpoints. The midpoints will represent vertices on the regular dodecagon.)
- If students are having difficulty with the construction of the regular dodecagon, remind them there is more than one method that can be used. Ask, "Instead of bisecting the sides of the regular hexagon to find the midpoints, what other construction method can you use to construct the regular dodecagon?" (**Answer:** The radius of the hexagon can be bisected, and once this bisected length is found, the compass can be set to this length. Then vertices can be marked along the edge of the circle, which would represent the vertices of the regular dodecagon.)
- After students perform the constructions and connect the vertices to construct the regular dodecagon, ask, "Besides a compass and a straightedge, what other tools could you use to verify that the dodecagon is a regular dodecagon?" (**Answer:** A ruler could be used to measure the sides of the dodecagon to verify they are all the same length. A protractor could be used to measure the interior angles of the dodecagon to verify they are all the same measure.)

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- Ask students, "Do you think the same constructions that are used for the regular hexagon and the regular dodecagon would also work for regular polygons that have an odd number of sides, such as a triangle, a pentagon, or a heptagon (a 7-sided figure)? What do you think is the component that relates the number of sides to the ability of being able to inscribe a regular polygon in a circle?" (**Answer:** Since a circle has a total of 360°, any polygon whose number of sides is a factor of 360 can be inscribed in a circle. For example, a triangle can be inscribed in a circle because 3 is a factor of 360, and a pentagon can also be inscribed in a circle because 5 is a factor of 360. However, 7 is NOT a factor of 360, so a regular heptagon cannot be inscribed inside a circle using basic construction methods.)
- Ask students, "What shape do you think a regular polygon will get increasingly closer to looking like as the number of sides increases?" (**Answer:** As the number of sides of a regular polygon increases, the polygon will look more and more like a circle.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

Debriefing the Task

- Ask for volunteers to discuss their methods for constructing the regular hexagon and the regular dodecagon. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses, specifically when explaining which construction method they chose for each regular polygon. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of the constructions and the term "regular polygon."

Connecting to Key Concepts

Make explicit connections to key concepts:

- Regular hexagons have six equal sides and six angles, each measuring 120°.

  In this task, students will construct a regular hexagon and recognize that it has six congruent sides.

- Once a circle is constructed, it is possible to divide the circle into six equal parts.

  Students will construct a circle, then use a compass and a straightedge to construct a regular hexagon by dividing the circle into six equal arcs, which will represent the vertices of the hexagon.

- Connecting every point of intersection made from creating equally spaced arcs on a circle results in a regular hexagon.

  Students will create equally spaced arcs on a circle using their compasses, and will connect the points of intersection created from the arcs and the circle to create a regular hexagon.

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Extending the Task

- Ask students to work together and try to construct a regular pentagon inscribed in a circle, then construct a regular decagon inscribed in a circle. Ask them to write a paragraph explaining their construction methods, as well as the reasoning behind their choices. Encourage and guide a discussion about how the number of sides of each regular polygon helps determine some of the steps of the construction processes.
- Ask students, "How can you construct a regular 24-sided figure inscribed in a circle, based on the construction of a regular dodecagon?" Ask them to explain the process of bisecting the sides of a regular dodecagon and using those midpoints as vertices for the regular 24-sided polygon. Ask them to construct a 24-sided polygon, and ask for volunteers to share their results.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** I recognized that the goal of the problem was to construct a regular dodecagon inscribed in a circle. I used basic constructions to construct a regular hexagon, then I used the results of this to construct the regular dodecagon.)
- **For SMP 5, ASK:** "How did you use appropriate tools strategically?" (**Answer:** I used a compass and a straightedge strategically by carefully following a series of steps in my constructions of a regular hexagon, then a regular dodecagon.)
- **For SMP 8, ASK:** "How did you look for and express regularity in repeated reasoning?" (**Answer:** I expressed regularity in repeated reasoning by repeating the process for constructing the vertices of a regular hexagon, and then by extending this process to bisect the sides of the hexagon to create the vertices of the dodecagon.)

Alternate Strategies or Solutions

- Students may choose to use one or the other basic construction methods presented in the task to construct the regular hexagon as well as the regular dodecagon. Encourage them to use the method with which they are most comfortable, but also encourage them to try the other method as a way to verify the accuracy of their constructions.
- Once students have completed the construction for a regular hexagon, they may choose to measure a side length of the hexagon and determine the midpoint numerically. They may designate the midpoint as a vertex of the regular dodecagon. Then they may repeat this procedure for the other five sides of the hexagon until they have 12 vertices. Remind students that although this method is technically correct, using a compass and a straightedge instead will provide more precise and accurate results.

Technology

No technology is needed for this task.

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Guide

Problem-Based Task: Artifacts Recovered and Restored

Common Core State Standard

G–CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

Task Overview

Focus

How can a series of transformations that are rigid motions be performed on an object to move it to a new location? How can these rigid motions be described? In this lesson, students will use a coordinate plane and determine a series of rigid motions that will move an object (a vessel) around other objects to get the vessel to a specific location.

This activity will provide practice with:

- interpreting verbal information
- applying definitions of transformations
- identifying rigid motions in the coordinate plane
- calculating the number of units in a rigid motion
- determining the necessary rigid motion needed to complete a transformation

Introduction

This task should be used to explore or apply the skill of using geometric descriptions of rigid motions to transform figures and predict the effect of a given rigid motion on a given figure. Students should already be familiar with basic transformations and with plotting points in the coordinate plane.

Begin by reading the problem and clarifying the meanings of reflection, rotation, rigid motion, transformation, and translation.

reflection	an isometry in which a figure is moved along a line perpendicular to a given line called the line of reflection
rotation	an isometry in which all points in the preimage are moved along circular arcs determined by the center of rotation and the angle of rotation; also called a turn
rigid motion	a transformation to a figure that maintains the figure's shape and size or its segment lengths and angle measures
transformation	a change in a geometric figure's position, shape, or size
translation	an isometry in which all points in the preimage are moved parallel to a given line; also called a slide

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they make sense of the types of rigid motions necessary to transform an object to a specific location. They will reason quantitatively as they calculate the number of units the object is either translated, reflected, or rotated a specific number of degrees.

- **SMP 4:** Model with mathematics.

  Students will recognize that this scenario can be modeled with the use of a coordinate plane that represents the movement of a vessel maneuvering around other objects on the coordinate plane. They will use the coordinate plane to plot each rigid motion as it occurs.

- **SMP 6:** Attend to precision.

  Students will apply the definitions of the types of rigid motions to move the object to a specific location in the coordinate plane. Encourage students to use the correct terminology as they work through the rigid motions and identify the location in the coordinate plane after each one.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- creating the angle of rotation in a clockwise direction instead of a counterclockwise direction, and vice versa

  Have students make a chart representing the motion of the hands of a clock as it moves clockwise and counterclockwise to use as a reference when working through this task. Remind them to draw an arrow on the object that is being rotated to see which direction the object has rotated.

- reflecting a figure about a line other than the one given

  Remind students that three of the most common lines of reflection are over the x-axis, over the y-axis, and the line $y = x$. Have students draw a line on the artifact in the "Artifact Reflection" coordinate plane that will create a border for the artifact. Encourage them to observe that the line drawn is the line $y = x$.

- mistaking a rotation for a reflection

  Have students create a list of definitions for reflections, rotations, and translations to use as a reference when working through the task. Remind them that a reflection is also known as a "flip," a rotation is also known as a "turn," and a translation is also known as a "slide."

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- misidentifying a translation as a reflection or rotation

  Have students create a list of definitions for reflections, rotations, and translations to use as a reference when working through the task. Remind them to carefully consider the location of the preimage and the image when identifying the type of rigid motion that occurred.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask, "As you move the vessel through the coordinate plane, what are the restrictions given on the movement of the vessel?" (**Answer:** As the vessel is moved, it cannot touch any of the objects, and it must be at least one unit away from each object on the coordinate plane. The vessel can also be translated or rotated.)
- Before students begin the task, ask, "What is another word for reflection? What is another word for rotation? What is another word for translation?" (**Answer:** A reflection is also known as a "flip"; a rotation is also known as a "turn"; and a translation is also known as a "slide.")
- As students begin the task, ask them to first carefully examine the given diagram, "Path to Artifact." Ask them, "What is the only option for the first rigid motion of the vessel, and why?" (**Answer:** The only choice for the first rigid motion is a translation that will move the vessel in a downward direction. If a rotation had been chosen, this would have oriented the vessel in the wrong direction.)
- After the first rigid motion of translating the vessel down eight units has been performed, instead of then rotating the vessel 90° counterclockwise, ask students, "What other rotation could be performed on the vessel instead of a 90° counterclockwise rotation? Where would the vessel be pointing?" (**Answer:** The vessel could have been rotated 90° in a clockwise direction. Then the vessel would be pointing to the left.)
- Ask students, "After the first rigid motion is performed, if the vessel had been rotated 90° in a clockwise direction, what is the least number of units it would need to be translated to the left? How does this translation to the left compare to the given translation to the right of seven units?" (**Answer:** The least number of units the vessel would be translated to the left is six units. This translation is one unit less than if the vessel is translated seven units to the right.)
- If students choose to move the vessel to the left instead of the right as is done in the task, ask, "Is the amount and are the types of rigid motions the same if the vessel is moved left or right? If not, what is the difference in terms of units or movements?" (**Answer:** The number and types of rigid movements will be the same, but translating the vessel to the left instead of the right will be one less unit of a translation.)

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- Ask students, "How can you determine which line of reflection will be used for the artifact based on the 'Artifact Reflection' diagram?" (**Answer:** Based on the lines drawn for the artifact, the way to complete the diagram would be to reflect the artifact over the line $y = x$. This would allow for the part of the artifact that is drawn to be flipped over on this diagonal line.)
- Ask students, "How could you verify that the line of reflection is $y = x$ using slope?" (**Answer:** When the line $y = x$ is drawn, it can be seen that the slope of the line is 1.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

Debriefing the Task

- Ask for volunteers to discuss their strategies and processes for identifying the types of rigid motions performed on the vessel. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of the types of rigid motions.

Connecting to Key Concepts

Make explicit connections to key concepts:

- Rigid motions are transformations that don't affect an object's shape and size.

  In this task, students will see that a vessel is moved in a series of rigid transformations, such as translations and rotations. However, the shape and size of the vessel do not change.

- Transformations that are rigid motions are translations, reflections, and rotations.

  Students will perform translations, rotations, and a reflection to maneuver a vessel, represented on a coordinate plane, through several objects.

- In a translation, the figure is moved horizontally and/or vertically.

  Students will translate the vessel by moving it horizontally to the left and right and vertically down on the coordinate plane.

- A rotation moves all points of a figure along a circular arc about a point.

  Students will see that the vessel is rotated counterclockwise 90° and then clockwise 90° about its center point.

- A reflection creates a mirror image of the original figure over a reflection line.

  In this task, students will see that when the artifact is reflected over the line $y = x$, a mirror image is created.

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Extending the Task

- Ask students to use a straightedge and draw the *x*- and *y*-axes on the coordinate plane. Ask them to go through the series of transformations for the vessel, but this time using coordinates and transformation function notation to describe each transformation. Finally, ask students to determine the final coordinates of the vertices of the vessel as it intersects the upper right corner of the square that represents the artifact.
- Ask students to work together to create their own scenario in which an object has to be maneuvered through a series of objects/obstacles in a coordinate grid. Encourage them to create a type of maze and/or game that requires a person to determine the least number of rigid motions that need to occur to get an object to the "finish line." Ask them to include at least four different transformations, then represent each transformation function with the appropriate notation. Ask for volunteers to share their scenarios.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** I used abstract reasoning as I made sense of the types of rigid motions necessary to transform an object to a specific location.) "Which of your strategies represent quantitative reasoning?" (**Answer:** I reasoned quantitatively when I calculated the number of units the object was either translated or reflected, or when the object was rotated 90°.)
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** I recognized that this scenario could be modeled with the use of a coordinate plane representing the movement of a vessel maneuvering around other objects located on the coordinate plane. I used the coordinate plane to plot each rigid motion.)
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** I attended to precision by using correct math terminology in my explanations and by explaining my reasoning in a mathematically correct and precise way.)

Alternate Strategies or Solutions

- After the first translation of 6 units down is performed, students may choose to perform the next few transformations in a different order than what is presented in the task. Remind them to carefully examine the orientation of the vessel after each transformation, making sure they abide by the restrictions for the movement of the vessel given in the original problem.
- Students may choose to redraw the "Path to Artifact" diagram on a coordinate plane that contains the *x*- and *y*-axes. Being able to identify the coordinates of the vessel after each transformation may help students better identify the type of transformation.

Technology

No technology is needed for this task.

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Problem-Based Task: Architectural Planning

Common Core State Standard

G–CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

Task Overview

Focus

How can rigid motions be applied to a real-world scenario to create congruent figures? How can a non-rigid motion be described? In this lesson, students will perform rigid motions on a floor plan to create different floor plans, and will determine the scale factor from the floor plan to the actual house.

This activity will provide practice with:

- interpreting verbal information
- identifying possible transformations
- applying definitions of transformations
- drawing transformations in the context of a scenario
- calculating a scale factor
- analyzing the results of a non-rigid motion

Introduction

This task should be used to explore or apply the skill of using geometric descriptions of rigid motions to transform figures, as well as applying the definition of congruence in terms of rigid motions to decide if figures are congruent. Students should already be familiar with recognizing translations, reflections, and rotations, as well as setting up ratios.

Begin by reading the problem and clarifying the meanings of the following terms:

congruency transformation	a transformation in which a geometric figure moves but keeps the same size and shape
dilation	a transformation in which a figure is either enlarged or reduced by a scale factor in relation to a center point
non-rigid motion	a transformation to a figure that changes the figure's shape and/or size
rigid motion	a transformation to a figure that maintains the figure's shape and size or its segment lengths and angle measures
scale factor	a multiple of the lengths of the sides from one figure to the transformed figure

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will use the definitions of rigid motions to construct viable arguments when creating alternate floor plans to ensure congruency of the original floor plan to the alternate ones. Encourage students to discuss their arguments with each other and explain their reasoning if they do not agree with each other.

- **SMP 4:** Model with mathematics.

  Students will recognize that this scenario can be modeled with the use of a coordinate plane that represents the floor plan of the house, and will use the coordinate plane to draw transformations of the floor plan. They will also recognize that the scale factor can be determined by an algebraic representation, specifically by solving a proportion.

- **SMP 7:** Look for and make use of structure.

  Students will apply the structure of rigid motions by determining the new location of a room on the floor plan using the definitions of a reflection. They will analyze these results and draw conclusions on the congruency of the alternate floor plans in comparison with the original.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- mistaking a non-rigid motion for a rigid motion, and vice versa

  Remind students that if a rigid motion is performed, the figure keeps its size and shape; if a non-rigid motion is performed, the size and/or shape of the figure is changed.

- not recognizing that rigid motions preserve shape and size

  Once students perform a rigid motion, remind them to check the lengths of the sides of the preimage and the image to make sure the corresponding sides and angles are congruent.

- not recognizing that it takes only one non-rigid motion to render two figures not congruent

  Remind students that a non-rigid motion changes a figure's shape and/or size; therefore, once a non-rigid motion has been performed on a figure, the two figures cannot be congruent.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask, "What are the possible rigid motions that can be performed on the floor plan?" (**Answer:** To preserve the size and shape of the original floor plan, the types of rigid motions that can be performed are translations, rotations, and reflections.)
- Before students begin the task, ask, "What is another word for reflection? What is another word for rotation? What is another word for translation?" (**Answer:** A reflection is also known as a "flip"; a rotation is also known as a "turn"; and a translation is also known as a "slide.")
- As students begin the task, ask, "Why is a coordinate plane helpful when working through this task?" (**Answer:** Since the floor plan is in the shape of a rectangle, it can be drawn on a coordinate plane. When the rigid motions are performed, the lengths of the sides of the floor plan can be verified by counting units on the x- and y-axes to measure the lengths.)
- As students are creating the different transformations, ask, "What other type of reflection of the floor plan will also create a different floor plan?" (**Answer:** The floor plan can also be reflected over a vertical line at the right side of the house.)
- If students are having difficulty determining other types of rigid motions, ask, "How can the floor plan be rotated? What angles of rotation would work?" (**Answer:** The floor plan could be rotated by a factor of 90°, so the floor plan could be rotated 90°, 180°, or 270°.)
- Ask students, "What would happen if the floor plan were rotated 360° in either a clockwise or counterclockwise direction?" (**Answer:** If the floor plan were rotated in either direction, it would come back to the original floor plan, so no change would occur.)
- As students are preparing to calculate the scale factor from the floor plan to the actual house, ask, "How can you determine the scale factor based on the information given in the problem?" (**Answer:** The problem stated that every $\frac{1}{4}$ inch represents 24 inches in the actual house. Therefore, a proportion can be set up representing the ratio of the house in inches to the floor plan in inches. When the proportion is solved, the result is $24 \cdot 4 = 96$ inches.)
- Ask students, "What is the difference between a rigid motion, such as a reflection, and a non-rigid motion, such as a dilation?" (**Answer:** When a rigid motion is performed on a figure, the size and shape are preserved, and the two figures are congruent. When a non-rigid motion is performed on a figure, the resulting figure is either enlarged or reduced from the original by a scale factor. Therefore, the figures are not congruent, but they are proportional.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

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Debriefing the Task

- Ask for volunteers to discuss their strategies and processes for determining the types of rigid motions performed on the floor plan, as well as their calculation of the scale factor. Encourage them to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of the types of rigid motions and non-rigid motions.

Connecting to Key Concepts

Make explicit connections to key concepts:

- If a figure has undergone only rigid motions (translations, reflections, or rotations), the new figure and the original figure are congruent.

  In this task, students will perform reflections on a floor plan, which are rigid motions, to produce congruent floor plans.

- If a figure has undergone any non-rigid motions (dilations, stretches, or compressions), the new figure and the original figure are not congruent.

  Students will determine that the house and the floor plan are not congruent, as the floor plan is a reduction of the actual size of the house.

- A dilation uses a center point and a scale factor to either enlarge or reduce the figure.

  Students will determine that the non-rigid motion used to scale the house is a dilation.

- A scale factor is a multiple of the lengths of the sides from one figure to the dilated figure.

  Students will determine that the scale factor from the floor plan to the actual house can be calculated by multiplying the lengths of the sides of the floor plan by 96.

- If the scale factor is larger than 1, the figure is enlarged.

  In this task, students will calculate that the scale factor from the floor plan to the actual house is 96, and since this scale factor is larger than 1, the figure is enlarged.

Extending the Task

- Ask students to work together to determine what other rigid motions besides reflections can be used to create other floor plans. Ask them to draw the original floor plan and their new floor plans on the coordinate plane. Encourage them to consider how rotating the floor plan a specific number of degrees would result in the same floor plan as a rotation. Ask for volunteers to share their floor plans and discuss the rigid motions that were used.

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- Ask students to work together to create their own floor plan for a house, different from the one in the task. Ask them to specifically draw the dimensions of each room of their house, then draw their floor plan on a coordinate plane. Encourage them to consider drawing rooms that have the same dimensions, then determine which rigid motions could be performed on the congruent rooms within the floor plan. Ask for volunteers to share their floor plans as well as the rigid motions they used on the rooms.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 3, ASK:** "How did you construct viable arguments, and did you critique the reasoning of others?" (**Answer:** I constructed viable arguments by using the definitions of rigid motions when creating alternate floor plans to ensure congruency of the original floor plan to the alternate ones. I also discussed my results with others, using examples of different floor plans that satisfied the parameters of the problem.)
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** I recognized that this scenario could be modeled with the use of a coordinate plane that represents the floor plan of a house. I used the coordinate plane to draw a new floor plan based on a rigid motion. I also modeled the scenario by using an algebraic equation to determine the scale factor of a dilation.)
- **For SMP 7, ASK:** "How did you make use of structure?" (**Answer:** I made use of structure by applying the definitions of rigid motions, specifically reflections, to determine new locations of rooms on a floor plan while ensuring congruency to the original floor plan.)

Alternate Strategies or Solutions

- Students may choose to reflect the floor plan on a horizontal line at the back of the floor plan instead of at the front as the task shows. Students may also choose to rotate the floor plan 180° in a clockwise direction to create a different floor plan. Remind students that the floor plan can be rotated or reflected in any direction, since this will maintain the size and shape of the original floor plan.
- Students may choose to reorganize the rooms within the floor plan. Encourage them to note the dimensions of each room, being careful not to change the dimensions. Also remind them that the overall size of the floor plan must be preserved.

Technology

No technology is needed for this task.

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Guide

Problem-Based Task: Stained Glass Pattern, Part I

Common Core State Standard

G–CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Task Overview

Focus

How can diagonals of quadrilaterals create congruent triangles? How can corresponding parts of triangles be used to prove congruency? In this lesson, students will identify congruent sides and angles of congruent triangles formed by squares and rhombuses in a pattern of stained glass art.

This activity will provide practice with:

- identifying properties of congruent quadrilaterals
- analyzing a pattern of triangles in the context of a scenario
- determining congruent sides and angles of quadrilaterals and triangles
- drawing congruency markings on sides and angles
- justifying congruent triangles based on corresponding parts

Introduction

This task should be used to apply the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Students should already be familiar with recognizing transformations of figures, as well as understanding that rigid motions preserve the size and shape of figures.

Begin by reading the problem and clarifying the meaning of congruent triangles, corresponding angles, and corresponding sides.

congruent triangles	triangles that have the same angle measures and side lengths
corresponding angles	a pair of angles in a similar position
corresponding sides	the sides of two figures that lie in the same position relative to the figures

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will recognize that the goal of the task is to identify congruent triangles in a stained glass art pattern. They will determine these congruent triangles based on applying their knowledge about the properties of a square and a rhombus. They will also label and identify the corresponding parts of the congruent triangles.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will construct viable arguments about which pairs of triangles in the stained glass pattern are congruent based on the identification of congruent corresponding parts of the triangles.

- **SMP 7:** Look for and make use of structure.

  Students will look for patterns in identifying congruent triangles. Specifically, they will determine the corresponding parts of the triangles and conclude that they are congruent based on the properties of the sides and angles of the figures. They will recognize that the same shapes are used to create congruent triangles, and therefore the corresponding sides and angles can be identified as congruent.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly identifying corresponding parts of triangles

  Remind students that corresponding angles are angles that are in a similar position in two triangles, and that corresponding sides are sides that lie in the same position relative to the figures.

- assuming that corresponding parts indicate congruent parts

  Remind students that corresponding sides and angles of two triangles must be proven to be congruent by given information, by tick marks for the sides or arc marks for the angles, or by applying properties of shapes to prove congruence.

- assuming that alphabetical order indicates congruence

  Remind students that triangles can be named using any combination and order of letters, so it is important to look at the order of the letters in relation to the corresponding parts of the triangles.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask, "What are the properties of a square in regards to sides and angles? What are the properties of a rhombus in regards to sides and angles?" (**Answer:** A square is a quadrilateral with four congruent sides and four right angles. A rhombus is a quadrilateral with four congruent sides and congruent pairs of opposite angles.)
- Before students begin the task, ask, "What do you know about the diagonals of a square and rhombus? What is unique about the diagonals of a square?" (**Answer:** The diagonals of a square and a rhombus bisect the opposite pairs of angles. In a square, the diagonals bisect the opposite pairs of angles and create 45° angles, because each corner of a square is exactly 90°.)
- Ask students, "What must be proven to conclude that any two figures are congruent?" (**Answer:** To conclude that any two figures are congruent, all of the corresponding sides must be the same lengths, and all of the corresponding angles must have the same measures.)
- Ask students, "How do you symbolically label congruent parts on figures? Explain." (**Answer:** Congruent sides are noted by tick marks. One mark on a side of a triangle will indicate congruency to the corresponding side of another triangle with one tick mark. Two tick marks will indicate congruence to the corresponding side that also has two tick marks, etc. Angle measures are noted as congruent by drawing arcs on the angles, as an arc on one triangle matches an arc on the angle of another triangle, two arcs match with two arcs, etc.)
- As students are beginning the task and analyzing the given information, ask them, "It is given that segment *EC* is congruent to segment *BC*. With this information, what are the possible types of triangles that triangle *BEC* could be? Explain." (**Answer:** Since it is given that two sides of triangle *BEC* are congruent, this indicates that the triangle is either an isosceles triangle, which has at least two sides congruent, or an equilateral triangle, which has all three sides congruent.)
- Ask students, "How does a diagonal of a square automatically create two congruent triangles? Explain how the corresponding parts of these two triangles are congruent." (**Answer:** A diagonal of a square divides a square into two congruent triangles, because all six corresponding parts are congruent. The angles that are congruent are the two right angles in the opposite corners, and since a diagonal bisects a 90° angle, the resulting angles are each 45° in both triangles. Since all four sides of a square are congruent, the two legs of the triangle are already congruent, and since the triangles share the third side, which is the diagonal, that side is also congruent for both triangles.)
- If students are having difficulty identifying a pair of congruent triangles, ask them, "How many corresponding parts are there between two triangles?" (**Answer:** Since there are three sides of a triangle and three angles in a triangle, there is a total of six corresponding parts between two triangles.)

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- Once the pairs of congruent triangles have been identified in the pattern, ask students, "How can Mary apply congruent triangles to her project? How will this knowledge help her in terms of creating her stained glass art?" (**Answer:** Mary can use the congruent triangles in her pattern when she is creating a template to use for future patterns. Since she knows that certain pairs of triangles are congruent, she can make cuts to fit her pattern using the congruent triangles.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask for volunteers to discuss their strategies and processes for identifying the congruent parts of the triangles, as well as determining which triangles in the pattern are congruent. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses, specifically when identifying pairs of congruent triangles. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of corresponding parts of triangles.

Connecting to Key Concepts

Make explicit connections to key concepts:

- To determine whether two triangles are congruent, you must observe the angle measures and side lengths of the triangles.

  In this task, students will apply the properties of a square and a rhombus to identify angle measures and side lengths of a number of triangles.

- Congruent triangles have three pairs of corresponding angles and three pairs of corresponding sides, for a total of six pairs of corresponding parts.

  Students will identify and list three pairs of corresponding angles and three pairs of corresponding sides in each pair of triangles that they identify as congruent.

- Two triangles are congruent if the corresponding angles are congruent and corresponding sides are congruent.

  Students will identify and label all corresponding angles and sides between a pair of triangles and determine they are congruent to each other, so they will conclude that the pair of triangles is congruent.

- Tick marks show the sides that are congruent, and arcs on the angles show the angles that are congruent in a pair of triangles.

  In this task, students will draw the tick marks on the sides and the arcs on the angles of the parts of the triangles that are congruent.

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Extending the Task

- Ask students to use a ruler to measure the side lengths of the borders of Mary's pattern. Ask them to extend her pattern by drawing squares, rhombuses, and/or triangles that will continue her pattern of congruent figures. Remind students that the figures should not overlap. Encourage them to see how many figures they can draw. Ask for volunteers to share their new designs and discuss their reasoning in their choices for their types of figures.
- Provide students with a blank coordinate plane and a ruler. Ask them to create their own patterns of polygons similar to what was given in the task. Ask them to use a combination of shapes, and encourage them to be creative in creating a pattern in which congruent figures can be identified. Remind them to carefully measure their shapes using the graph as a means of ensuring congruent sides. Ask for volunteers to share their shapes and discuss the congruency in their patterns.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** I persevered by recognizing that the goal of the task was to identify congruent triangles in a stained glass art pattern. I determined these congruent triangles based on applying my knowledge about the properties of a square and a rhombus. I also labeled and identified the corresponding parts of the congruent triangles.)
- **For SMP 3, ASK:** "Did you construct viable arguments and critique the reasoning of others?" (**Answer:** I constructed viable arguments about which pairs of triangles in the stained glass pattern are congruent based on the identification of congruent corresponding parts of the triangles.)
- **For SMP 7, ASK:** "How did you look for, and make use of, structure?" (**Answer:** I looked for patterns when I identified congruent triangles. I determined the corresponding parts of the triangles and concluded that they are congruent based on the properties of the sides and angles of the figures. I recognized that the same shapes are used to create congruent triangles, and that therefore the corresponding sides and angles can be identified as congruent.)

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Alternate Strategies or Solutions

- Students may choose to redraw the given pattern to break apart the triangles. It may be helpful to reorient the triangles so students can more easily identify the corresponding parts. Encourage students to be careful when labeling the vertices of each triangle with the correct letter to maintain the correspondence between a pair of triangles.
- Students may choose to use a ruler and a protractor to measure each of the angles and sides. Remind them that it is possible to have measurement errors using these techniques. They may also choose to use a compass and a straightedge to measure the sides and angles, using construction methods. Note that construction methods will generally result in more precise and accurate measurements, as opposed to using a ruler and a protractor.

Technology

No technology is needed for this task.

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Problem-Based Task: Stained Glass Pattern, Part II

Common Core State Standard

G–CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Task Overview

Focus

What information is needed to prove that two triangles are congruent? Can triangles created by the diagonal of a rhombus be proven congruent? In this lesson, students will identify corresponding congruent sides and angles in triangles displayed in a pattern represented as a stained glass art piece.

This activity will provide practice with:

- identifying properties of rhombuses
- determining congruent sides and angles of rhombuses and triangles
- drawing congruency markings on sides and angles
- analyzing corresponding sides and angles on two triangles
- justifying congruent triangles based on corresponding parts
- identifying triangle congruency statements; SAS and SSS

Introduction

This task should be used to explore or apply the skill of explaining how the criteria for triangle congruence (ASA, SAS, and SSS) follow the definition of congruence in terms of rigid motions. Students should already be familiar with identifying corresponding sides and angles among triangles.

Begin by reading the problem and clarifying the meaning of congruent triangles, corresponding angles, corresponding sides, side-angle-side (SAS), and side-side-side (SSS).

congruent triangles	triangles having the same angle measures and side lengths
corresponding angles	a pair of angles in a similar position
corresponding sides	the sides of two figures that lie in the same position relative to the figures
side-angle-side (SAS)	if two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, the two triangles are congruent
side-side-side (SSS)	if three sides of one triangle are congruent to three sides of another triangle, the two triangles are congruent

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will recognize that the goal of the task is to determine if there is enough evidence to conclude all of the triangles in the stained glass pattern are congruent. They will identify corresponding parts of the triangles, and use triangle congruency statements to prove triangles congruent.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will construct viable arguments about which pairs of triangles in the stained glass pattern are congruent based on the identification of congruent corresponding parts of the triangles.

- **SMP 4:** Model with mathematics.

  Students will recognize that this scenario can be modeled by applying the definitions and properties of a rhombus. They will also apply triangle congruency statements to draw conclusions about the art piece.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- misidentifying included sides and angles, resulting in the wrong congruence statement

  Remind students that an included side must be directly between two angles, that an included angle must be directly between two sides, and that the triangle congruency statements can only hold true if the included sides and angles are in the correct locations.

- misreading congruency symbols of triangles

  Remind students that tick marks are used to indicate congruent sides, and that the number of tick marks on a side of a triangle must match the number of tick marks on the corresponding side of another triangle to state that the sides are congruent. Remind them that arcs are used to indicate congruent angles, and that the symbol $\cong$ indicates congruency.

- changing the order of named triangles, causing parts to be incorrectly interpreted as congruent

  Remind students that the order in which triangles are named is important, and that they must match the order of the corresponding sides of triangles when they are being compared and analyzed for congruency.

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Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask, "What are the properties of a rhombus in regards to sides and angles?" (**Answer:** A rhombus is a quadrilateral with four congruent sides and congruent pairs of opposite angles.)
- Before students begin the task, ask, "What do you know about the diagonals of a rhombus?" (**Answer:** The diagonals of a rhombus bisect the opposite pairs of angles.)
- If students are having difficulty identifying the angles created by the diagonal $\overline{KD}$, remind them that the diagonals of a rhombus bisect the opposite pairs of angles. Ask, "What does it mean when a diagonal bisects an angle? What types of angles are created?" (**Answer:** When a diagonal bisects an angle, it cuts the angle in half; therefore, two congruent angles are created.)
- Ask students, "Why do you think there is enough information to conclude that two triangles are congruent based only on the fact that all three corresponding sides are congruent?" (**Answer:** If all three corresponding sides of two triangles are congruent, the angles that join the three corresponding sides must also be congruent.)
- Ask students, "If two pairs of corresponding angles in two triangles are congruent, why can you conclude that the third pair of corresponding angles must also be congruent?" (**Answer:** Since the sum of the interior angles of any triangle is 180°, if two pairs of corresponding angles are congruent, the third pair of corresponding angles must have the same measure, which is found by subtracting the sum of the other two angles from 180°.)
- Ask students, "Why is there NOT enough information to conclude that two triangles are congruent based only on the fact that two pairs of corresponding sides are congruent? Describe an example to illustrate this." (**Answer:** Although two pairs of corresponding sides are congruent, that does not necessarily mean the third pair of corresponding sides is congruent. For example, suppose two triangles had two pairs of corresponding sides of 6 inches each. The included angle between the two congruent sides in the first triangle could be 40°. In the second triangle, although two sides of the triangle are also 6 inches each, the included angle between these sides could be 100°. The third sides for each of these triangles would have to be different lengths to join the two congruent sides with the 40° and 100° angles.)
- In the case of Mary's stained glass pattern, although there are congruent sides and angles in the diagram of her pattern, ask, "Why is not possible for Mary to confidently conclude that the triangles in each of the rhombuses are congruent to each other?" (**Answer:** Mary does not have enough information to prove that the triangles in the different rhombuses are congruent to each other. They only have two sets of corresponding congruent sides, and there is not a triangle congruency statement involving only two corresponding congruent sides.)

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- Ask students, "What additional information does Mary need to prove that triangles *DLK* and *HMK*, and triangles *DLK* and *HJK,* are congruent to each other?" (**Answer:** Mary would need to prove that the third pair of corresponding sides for each pair of triangles is congruent, or that two pairs of corresponding angles are congruent in each pair of triangles if one of these angles is located between the sides or a congruent side is located between the two congruent angles.)
- Ask students, "What tools could Mary use to identify additional congruent corresponding sides and angles?" (**Answer:** Mary could use a ruler to measure side lengths, and a protractor to measure angles.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow them to clarify these points for each other.

Debriefing the Task

- Ask for volunteers to discuss their strategies and processes for identifying the congruent parts of the triangles, as well as determining which triangle congruency statements can be used to determine congruent triangles. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses, specifically when identifying corresponding parts of congruent triangles. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of corresponding parts of triangles and triangle congruency statements.

Connecting to Key Concepts

Make explicit connections to key concepts:

- The criteria for triangle congruence, known as triangle congruence statements, provide the least amount of information needed to determine if two triangles are congruent.

  In this task, students will determine whether there are enough congruent corresponding sides and angles in triangles to conclude that triangles are congruent using the triangle congruency statements.

- The side-side-side (SSS) congruence statement states that if three sides of one triangle are congruent to three sides of another triangle, the two triangles are congruent.

  In the individual rhombuses in the stained glass pattern, students will find three pairs of corresponding sides that are congruent in the triangles of each rhombus that were created by the diagonal of the rhombus. They will conclude that triangles are congruent by the SSS congruence statement.

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- The side-angle-side (SAS) congruence statement states that if two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, the two triangles are congruent.

  In the individual rhombuses in the stained glass pattern, students will find at least two pairs of corresponding sides that are congruent in the triangles of each rhombus that were created by the diagonal of the rhombus. They will also be able to identify the included angle between those sides as congruent, since the opposite angles of a rhombus are congruent. They will conclude that triangles are congruent by the SAS congruence statement.

Extending the Task

- Ask students to use a ruler to measure the side lengths of the border sides of Mary's pattern. Ask them to extend her pattern by drawing additional rhombuses that will continue her pattern. Remind students that the figures should not overlap. Encourage them to see how many rhombuses they can draw, and ask them to identify congruent triangles based on the properties of a rhombus, specifically the diagonal, and to label the appropriate triangle congruence statement. Ask for volunteers to share their new designs and discuss the processes used to identify congruent triangles.
- Provide students with a blank coordinate plane and a ruler. Ask them to create their own pattern using different types of triangles, specifically isosceles and equilateral triangles. Ask them to identify and label congruent sides and angles, and to label the vertices of each angle in their patterns. Encourage them to be creative. Ask for volunteers to share their shapes and discuss how they determined which pairs of corresponding sides and angles in their triangles are congruent, as well as which triangle congruence statements they used.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** I persevered by recognizing that the goal of the task was to determine if there is enough evidence to conclude all of the triangles in the stained glass pattern are congruent. I identified corresponding parts of the triangles, and used triangle congruency statements to prove triangles congruent.)
- **For SMP 3, ASK:** "Did you construct viable arguments, and did you critique the reasoning of others?" (**Answer:** I constructed viable arguments about which pairs of triangles in the stained glass pattern were congruent based on the identification of congruent corresponding parts of the triangles.)

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- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** I recognized that this scenario could be modeled by applying definitions and properties of a rhombus, as well as by applying triangle congruency statements to justify the conclusion that more information is needed to classify the triangles as congruent.)

Alternate Strategies or Solutions

- Students may choose to redraw the given pattern to break apart the two original rhombuses and then break apart the resulting triangles as well. It may be helpful to reorient the triangles so they can identify the corresponding parts easier. Encourage students to be careful when labeling the vertices of each triangle with the correct letter to maintain the correspondence between a pair of triangles.
- Students may choose to use a ruler and a protractor to measure each of the angles and the sides. Remind them that it is possible to have measurement errors using these techniques. They may also choose to use a compass and a straightedge to measure the sides and angles, using construction methods. Note that construction methods will generally result in more precise and accurate measurements, as opposed to using a ruler and a protractor.

Technology

No technology is needed for this task.

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Common Core State Standards

Mathematics I

Integrated Pathway



Teacher Resource

Unit 6

WALCH

INTEGRATED MATH

This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

Developers and reviewers include:

Joyce Hale	Valerie Ackley	Doug Kühlmann
Shelly Northrop Sommer	Laura McPartland	Mike May, S.J.
Ruth Estabrook	Cameron Larkins	James Quinlan
Vanessa Sylvester	Jennifer Blair	Peter Tierney-Fife
Zachary Lien	Nancy Pierce	

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Standards Correlations

Each lesson in this Integrated Pathway: Mathematics I program was written specifically to address the Common Core State Standards. Each lesson lists the standards covered in all the sub-lessons, and each sub-lesson lists the standards addressed in that particular section. In this section, you'll find a comprehensive list mapping the sub-lessons to the CCSS.

Guide to Common Core State Standards Annotation

As you use this program, you will come across a symbol included with the Common Core standards for some of the lessons and activities. These symbols are explained below.

Symbol: ⋆

Denotes: Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (⋆).

From http://www.walch.com/CCSS/00003

Symbol: (+)

Denotes: College and Career Readiness Standards

Advanced mathematics standards that are required in higher-level courses such as advanced statistics may also be included in lower-level courses. These additional standards are denoted by (+). According to the Common Core State Standards Initiative, "the evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6–8."

From http://www.walch.com/CCSS/00004

Connections to Future Courses

This section provides a map between topics introduced in each unit of this course and subsequent courses where each topic is revisited and built upon.

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CCSS INTEGRATED PATHWAY: MATHEMATICS I STANDARDS CORRELATIONS

Unit 6: Connecting Algebra and Geometry Through Coordinates				
Lesson	**Sub-lesson number**	**Title**	**Standard(s)**	**Pages**
Lesson 1	**Slope and Distance**			
	6.1.1	Using Coordinates to Prove Geometric Theorems with Slope and Distance	G–GPE.4 G–GPE.5	U6-4
	6.1.2	Working with Parallel and Perpendicular Lines	G–GPE.5	U6-32
Lesson 2	**Lines and Line Segments**			
	6.2.1	Calculating Perimeter and Area	G–GPE.7★	U6-58

TOPICS FROM FUTURE COURSES: CCSS INTEGRATED PATHWAY MATHEMATICS I

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 6	Connecting algebra and geometry	Math II Unit 2	extended to transformations of quadratic functions
		Math III Unit 3	extended to proofs of the laws of sines and cosines
		Math III Units 3 and 4A	includes transformations of families of functions, evaluating trigonometric functions, and graphing trigonometric functions

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Conceptual Activities

Use this interactive open education resource to build conceptual understanding of mathematical ideas. (*Note*: The link will be monitored and repaired or replaced as necessary.)

- Illustrative Mathematics. "A Midpoint Miracle."

  http://www.walch.com/ca/10003

  Students will prove a theorem about quadrilaterals that is somewhat difficult to prove with a straightedge and ruler but relatively easy to prove using coordinates. This task requires that students be comfortable using the formula for the midpoint of a line segment and the parallel line criterion.

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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. What is the distance between the points (7, 4) and (–3, 5)?

   a. $\sqrt{117}$ units
   b. $\sqrt{101}$ units
   c. 10 units
   d. $\sqrt{65}$ units

2. The line through the points (–3, –1) and (0, 5) is parallel to the line through which of the following points?

   a. (1, –3) and (2, –1)
   b. (1, –3) and (3, 3)
   c. (1, –3) and (3, –2)
   d. (1, –3) and (3, –3)

3. Which equation is parallel to the line $y=-\frac{1}{2}x+2$ and passes through the point (10, 4)?

   a. $y = 2x + 16$
   b. $y=\frac{1}{2}x-6$
   c. $y=-\frac{1}{2}x+12$
   d. $y=-\frac{1}{2}x+9$

4. Which equation is perpendicular to the line $y=\frac{1}{3}x-9$ and passes through the point (–6, 2)?

   a. $y=\frac{1}{3}x+4$
   b. $y = –3x – 16$
   c. $y=-\frac{1}{3}x+4$
   d. $y = –3x$

5. Lee's house is located at the point (1, 0). The equation of the road is $y = 2x + 1$. Lee is building a driveway from his house to the road. What is the approximate length of the shortest driveway he can build? (Each unit represents 100 yards.)

   a. 123 yards
   b. 144 yards
   c. 134 yards
   d. 190 yards

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Lesson 1: Slope and Distance

Instruction

Common Core State Standards

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 6
7 ✓ 8

G–GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*

G–G.E.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Essential Questions

1. How are the distance formula and the Pythagorean Theorem related?
2. How can algebra be used to explore properties of geometric shapes in the plane?
3. What are real-world applications of slope and the distance formula?
4. Is there a relationship between the slopes of parallel lines?
5. Is there a relationship between the slopes of perpendicular lines?

WORDS TO KNOW

congruent	having the same shape, size, lines, and angles; the symbol for congruent is $\cong$.
distance formula	a formula that states the distance between points (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
parallel lines	lines in a plane that do not share any points and never intersect; written as $\overleftrightarrow{AB} \parallel \overleftrightarrow{PQ}$
parallelogram	a special type of quadrilateral with two pairs of opposite sides that are parallel
perpendicular lines	two lines that intersect at a right angle (90˚); written as $\perp$; line segments and rays can also be perpendicular
quadrilateral	a polygon with four sides
rectangle	a special parallelogram with four right angles
rhombus	a special parallelogram with four sides congruent

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Instruction

slope measure of the rate of change of one variable with respect to another variable; $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$

square a parallelogram with four congruent sides and four right angles

Recommended Resources

- Math Open Reference. "Distance from a point to a line."

  http://walch.com/rr/CAU6L1ShortestDistance

  This site shows how to find the shortest distance from a point to a line, and provides an animation that allows you to manipulate both a point and a line on a coordinate plane to see how the perpendicular distance changes.

- Math Warehouse. "Parallel and Perpendicular Lines Applet."

  http://walch.com/rr/CAU6L1LineRelationships

  Use this animation to determine the relationship between lines.

- Purplemath. "Straight-Line Equations: Parallel and Perpendicular Lines."

  http://walch.com/rr/CAU6L1Straight-LineEquations

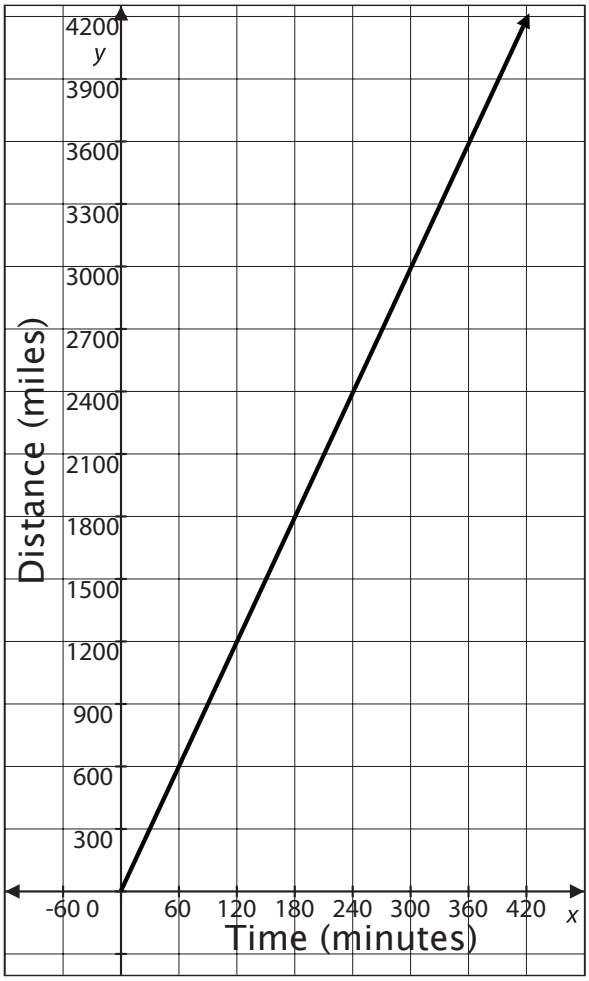
  This site summarizes how to write equations that are parallel and perpendicular to a given point.

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Lesson 6.1.1: Using Coordinates to Prove Geometric Theorems with Slope and Distance

Warm-Up 6.1.1

The graph below shows the average number of miles an airplane travels over the course of its flight.



1. What is the average rate of change per hour?

2. How many miles would the airplane travel in 9 hours?

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Instruction

Lesson 6.1.1: Using Coordinates to Prove Geometric Theorems with Slope and Distance

Common Core State Standards

G–GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*

G–GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Warm-Up 6.1.1 Debrief

1. What is the average rate of change per hour?

   To calculate the average rate of change per hour, first locate any two points on the graph. Let's use (60, 600) and (180, 1800).

   Rate of change can be found using the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

   Let $(x_1, y_1) = (60, 600)$ and $(x_2, y_2) = (180, 1800)$.

   $$\text{Rate of change} = \frac{y_2 - y_1}{x_2 - x_1}$$ Original formula

   $$= \frac{1800 - 600}{180 - 60}$$ Substitute values for x_1, x_2, y_1, and y_2.

   $$= \frac{1200}{120}$$ Simplify.

   $$= 10$$

   The airplane is traveling 10 miles per minute.

   The problem asks for the rate of change per hour. Convert minutes to hours.

   $$\frac{10 \text{ miles}}{1 \text{ minute}} \bullet \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{600 \text{ miles}}{1 \text{ hour}}$$

   The airplane travels an average of 600 miles per hour.

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2. How many miles would the airplane travel in 9 hours?

   Use the information found in question 1 to determine the number of miles the airplane would travel.

$$\frac{600 \text{ miles}}{1 \text{ hour}} \bullet \frac{9 \text{ hours}}{1} = \frac{5400 \text{ miles}}{1}$$

   The airplane would travel an average of 5,400 miles in 9 hours.

Connection to the Lesson

- In this lesson, students will expand upon their understanding of slope.
- Students will need to be comfortable with calculating slopes of given lines as they prove simple geometric theorems algebraically.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- calculating slope (8.EE.5)
- writing equations of lines (A–CED.2)
- using the Pythagorean Theorem (8.G.7)

Introduction

It is not uncommon for people to think of geometric figures, such as triangles and quadrilaterals, to be separate from algebra; however, we can understand and prove many geometric concepts by using algebra. In this lesson, you will see how the distance formula originated with the Pythagorean Theorem, as well as how distance between points and the slope of lines can help us to determine specific geometric shapes.

Key Concepts

Calculating the Distance Between Two Points

- To find the distance between two points on a coordinate plane, you have used the Pythagorean Theorem.
  - After creating a right triangle using each point as the endpoints of the hypotenuse, you calculated the vertical height (a) and the horizontal height (b).
  - These lengths were then substituted into the Pythagorean Theorem ($a^2 + b^2 = c^2$) and solved for c.
  - The result was the distance between the two points.
- This is similar to the **distance formula**, which states the distance between points (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- Using the Pythagorean Theorem:
  - Find the length of a: $|y_2 - y_1|$.
  - Find the length of b: $|x_2 - x_1|$.
  - Substitute these values into the Pythagorean Theorem.

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$$c^2 = a^2 + b^2$$
$$c^2 = |y_2 - y_1|^2 + |x_2 - x_1|^2$$
$$c = \sqrt{|y_2 - y_1|^2 + |x_2 - x_1|^2}$$

- Using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- We will see in the Guided Practice an example that proves the calculations will result in the same distance.

Calculating Slope

- To find the **slope**, or steepness of a line, calculate the change in *y* divided by the change in *x* using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Parallel and Perpendicular Lines

- **Parallel lines** are lines in a plane that do not share any points and never intersect.
  - To prove that two lines are parallel, you can show that the slopes of both lines are equal.
- **Perpendicular lines** are lines that intersect at a right angle (90˚). The slopes of perpendicular lines are always opposite reciprocals.
  - To prove that two lines are perpendicular, you can show that the slopes of both lines are opposite reciprocals.
  - When the slopes of perpendicular lines are multiplied, the result will always be –1.
  - Horizontal and vertical lines are always perpendicular to each other.

Common Errors/Misconceptions

- incorrectly using the *x*- and *y*-coordinates in the distance formula
- subtracting negative coordinates incorrectly
- incorrectly calculating the slope of a line
- incorrectly determining the slope of a line that is perpendicular to a given line
- assuming lines are parallel or perpendicular based on appearance only
- making determinations about the type of polygon without making all the necessary calculations

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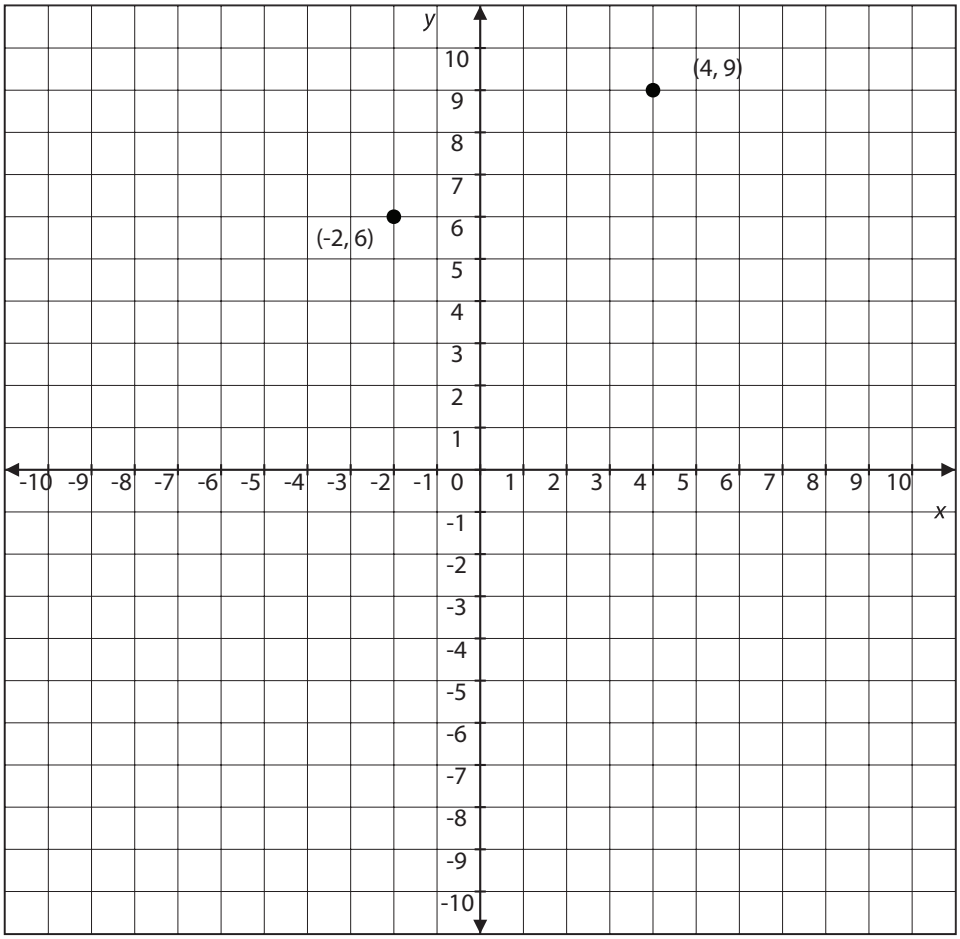
Instruction

Guided Practice 6.1.1

Example 1

Calculate the distance between the points (4, 9) and (–2, 6) using both the Pythagorean Theorem and the distance formula.

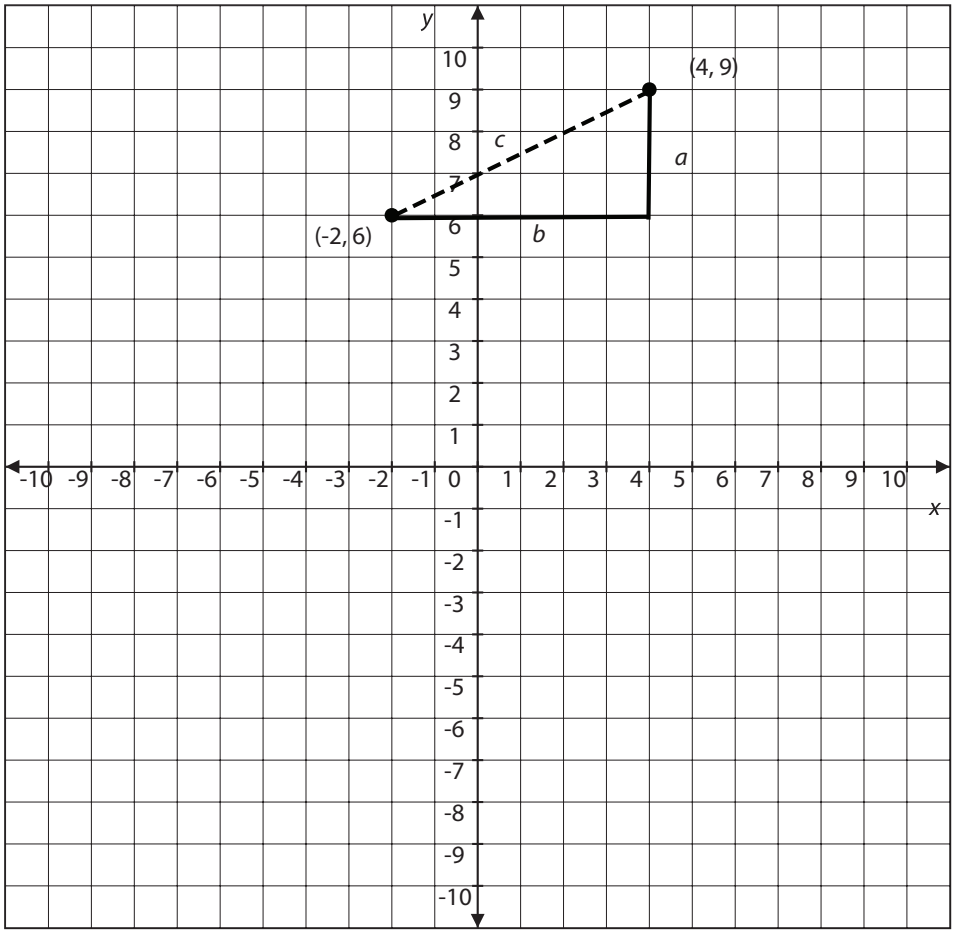
1. To use the Pythagorean Theorem, first plot the points on a coordinate system.



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Instruction

2. Draw lines to form a right triangle, using each point as an endpoint of the hypotenuse.



3. Calculate the length of the vertical side, a, of the right triangle.

Let $(x_1, y_1) = (4, 9)$ and $(x_2, y_2) = (-2, 6)$.

$|y_2 - y_1| = |6 - 9| = |-3| = 3$

The length of side a is 3 units.

4. Calculate the length of the horizontal side, b, of the right triangle.

$|x_2 - x_1| = |-2 - 4| = |-6| = 6$

The length of side b is 6 units.

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Instruction

5. Use the Pythagorean Theorem to calculate the length of the hypotenuse, c.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$3^2 + 6^2 = c^2$	Substitute values for a and b.
$9 + 36 = c^2$	Simplify each term.
$45 = c^2$	Simplify.
$\sqrt{45} = \sqrt{c^2}$	Take the square root of both sides of the equation.
$c = \sqrt{45} \approx 6.7$	

The distance between the points (4, 9) and (–2, 6) is $\sqrt{45}$, or approximately 6.7 units.

6. Now use the distance formula to calculate the distance between the same points, (4, 9) and (–2, 6).

Let $(x_1, y_1) = (4, 9)$ and $(x_2, y_2) = (–2, 6)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
$= \sqrt{(-2-4)^2 + (6-9)^2}$	Substitute (4, 9) and (–2, 6).
$= \sqrt{(-6)^2 + (-3)^2}$	Simplify.
$= \sqrt{36+9}$	Evaluate squares.
$= \sqrt{45}$	Simplify.

The distance between the points (4, 9) and (–2, 6) is $\sqrt{45}$ units, or approximately 6.7 units.

Both calculations will produce the same results each time.

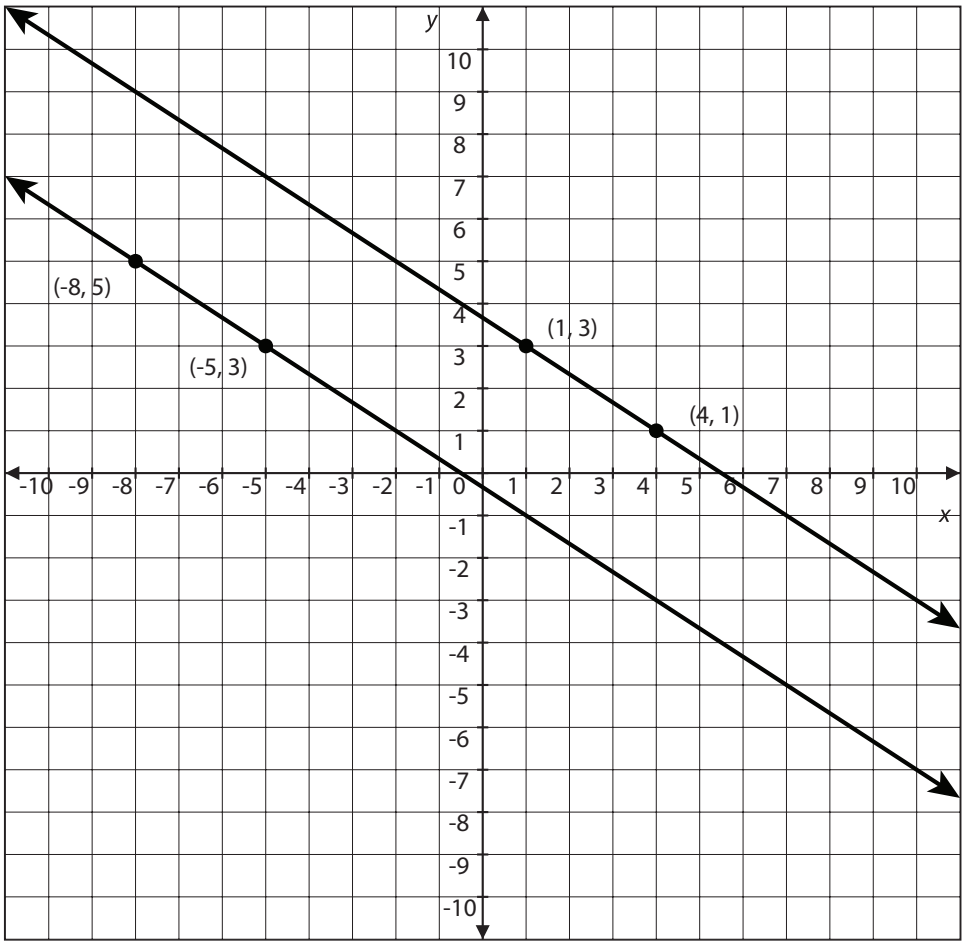
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Instruction

Example 2

Determine if the line through the points (–8, 5) and (–5, 3) is parallel to the line through the points (1, 3) and (4, 1).

1. Plot the lines on a coordinate plane.



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Instruction

2. Calculate the slope of the line through the points (–8, 5) and (–5, 3).

Let $(x_1, y_1) = (-8, 5)$ and $(x_2, y_2) = (-5, 3)$.

Substitute these values into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Original formula

$$= \frac{(3) - (5)}{(-5) - (-8)}$$ Substitute (x_1, y_1) and (x_2, y_2).

$$= -\frac{2}{3}$$ Simplify.

The slope of the line through the points (–8, 5) and (–5, 3) is $-\frac{2}{3}$.

3. Calculate the slope of the line through the points (1, 3) and (4, 1).

Let $(x_1, y_1) = (1, 3)$ and $(x_2, y_2) = (4, 1)$.

Substitute these values into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Original formula

$$= \frac{(1) - (3)}{(4) - (1)}$$ Substitute (x_1, y_1) and (x_2, y_2).

$$= -\frac{2}{3}$$ Simplify.

The slope of the line through the points (1, 3) and (4, 1) is $-\frac{2}{3}$.

4. Determine if the lines are parallel.

Parallel lines have equal slopes.

The slope of each line is $-\frac{2}{3}$; therefore, the lines are parallel.



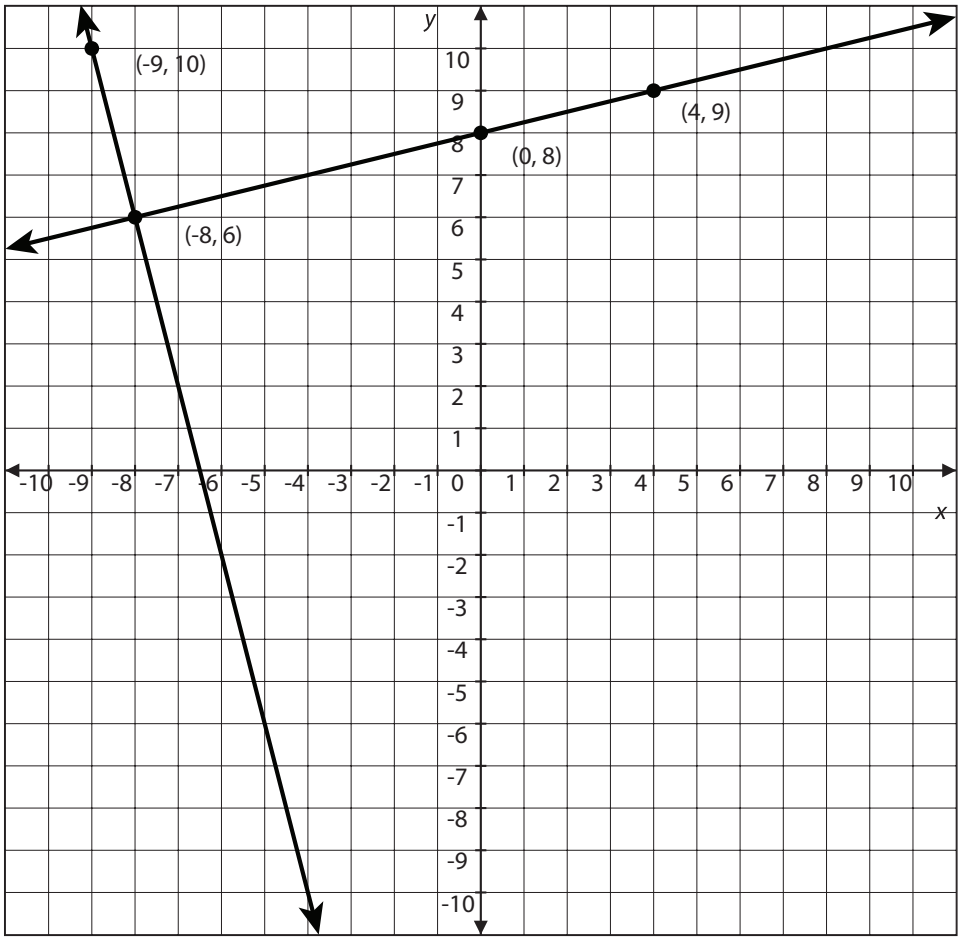
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Instruction

Example 3

Determine if the line through the points (0, 8) and (4, 9) is perpendicular to the line through the points (–9, 10) and (–8, 6).

1. Plot the lines on a coordinate plane.



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Instruction

2. Calculate the slope of the line through the points (0, 8) and (4, 9).

Let $(x_1, y_1) = (0, 8)$ and $(x_2, y_2) = (4, 9)$.

Substitute these values into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Original formula

$$= \frac{(9) - (8)}{(4) - (0)}$$ Substitute (x_1, y_1) and (x_2, y_2).

$$= \frac{1}{4}$$ Simplify.

The slope of the line through the points (0, 8) and (4, 9) is $\frac{1}{4}$.

3. Calculate the slope of the line through the points (–9, 10) and (–8, 6).

Let $(x_1, y_1) = (-9, 10)$ and $(x_2, y_2) = (-8, 6)$.

Substitute these values into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Original formula

$$= \frac{(6) - (10)}{(-8) - (-9)}$$ Substitute (x_1, y_1) and (x_2, y_2).

$$= -\frac{4}{1} = -4$$ Simplify.

The slope of the line through the points (–9, 10) and (–8, 6) is –4.

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Instruction

4. Determine if the lines are perpendicular.

Perpendicular lines have slopes that are opposite reciprocals.

The slope of the first line is $\frac{1}{4}$.

The reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$ or 4.

The opposite of 4 is –4.

The slope of the second line is –4.

The product of the two slopes is –1.

The slopes of the lines are opposite reciprocals; therefore, the lines are perpendicular.



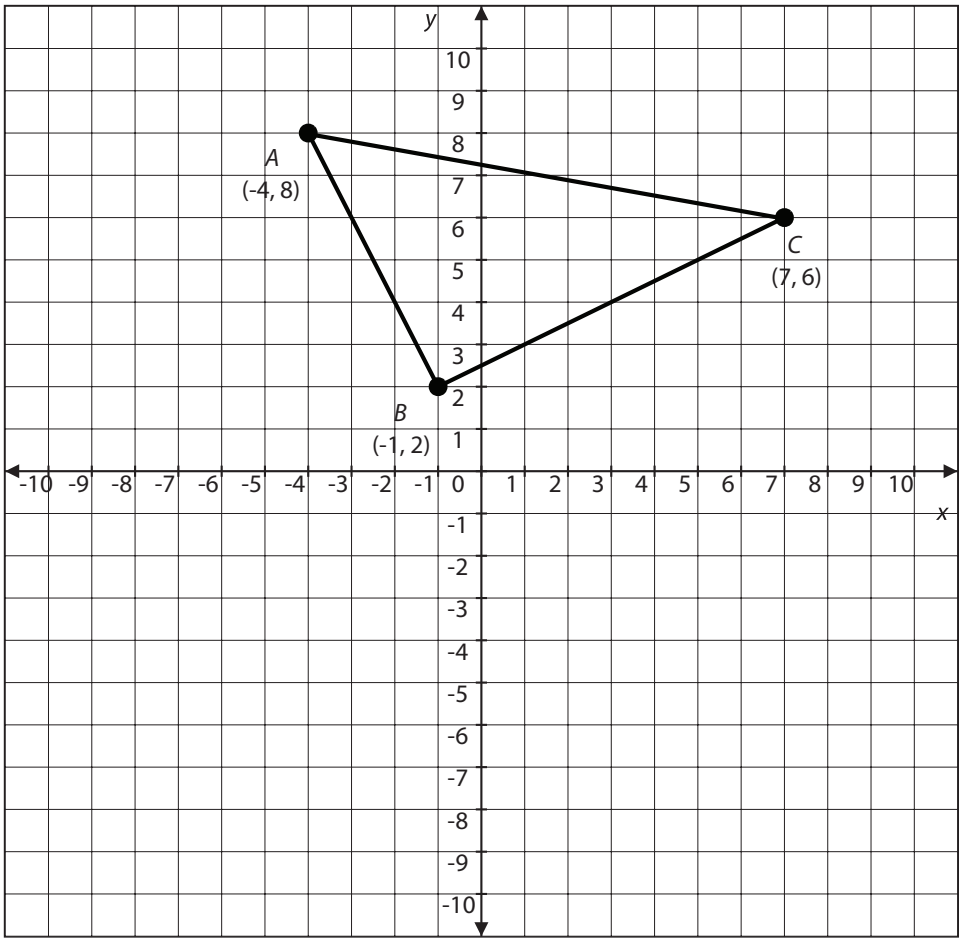
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Instruction

Example 4

A right triangle is defined as a triangle with 2 sides that are perpendicular. Triangle *ABC* has vertices *A* (–4, 8), *B* (–1, 2), and *C* (7, 6). Determine if this triangle is a right triangle. When disproving a conjecture, you only need to show one condition is not met.

1. Plot the triangle on a coordinate plane.



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Instruction

2. Calculate the slope of each side using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

$$\text{slope of } \overline{AB} = \frac{(2)-(8)}{(-1)-(-4)} = \frac{-6}{3} = -2$$

$$\text{slope of } \overline{BC} = \frac{(6)-(2)}{(7)-(-1)} = \frac{4}{8} = \frac{1}{2}$$

$$\text{slope of } \overline{AC} = \frac{(6)-(8)}{(7)-(-4)} = \frac{-2}{11} = -\frac{2}{11}$$

3. Observe the slopes of each side.

The slope of $\overline{AB}$ is –2 and the slope of $\overline{BC}$ is $\frac{1}{2}$.

These slopes are opposite reciprocals of each other, so we know that these two sides are perpendicular.

4. Make connections.

Right triangles have two sides that are perpendicular.

Triangle ABC has two sides that are perpendicular; therefore, it is a right triangle.



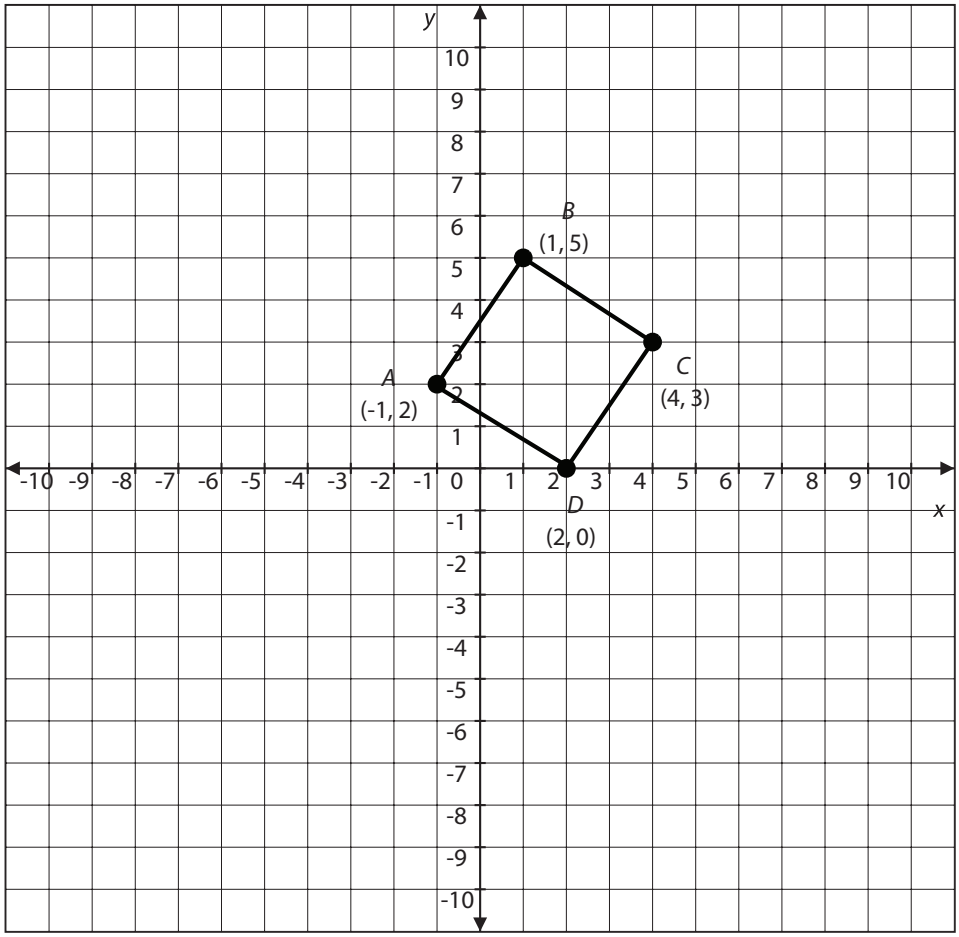
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Instruction

Example 5

A square is a quadrilateral with two pairs of parallel opposite sides, consecutive sides that are perpendicular, and all sides congruent, meaning they have the same length. Quadrilateral *ABCD* has vertices *A* (–1, 2), *B* (1, 5), *C* (4, 3), and *D* (2, 0). Determine if this quadrilateral is a square.

1. Plot the quadrilateral on a coordinate plane.



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Instruction

2. First show the figure has two pairs of parallel opposite sides.

Calculate the slope of each side using the slope formula, $m=\frac{y_2-y_1}{x_2-x_1}$.

slope of $\overline{AB}=\frac{(5)-(2)}{(1)-(-1)}=\frac{3}{2}$

slope of $\overline{BC}=\frac{(3)-(5)}{(4)-(1)}=\frac{-2}{3}=-\frac{2}{3}$

slope of $\overline{CD}=\frac{(0)-(3)}{(2)-(4)}=\frac{-3}{-2}=\frac{3}{2}$

slope of $\overline{AD}=\frac{(0)-(2)}{(2)-(-1)}=\frac{-2}{3}=-\frac{2}{3}$

3. Observe the slopes of each side.

The side opposite $\overline{AB}$ is $\overline{CD}$. The slopes of these sides are the same.

The side opposite $\overline{BC}$ is $\overline{AD}$. The slopes of these sides are the same.

The quadrilateral has two pairs of parallel opposite sides.

$\overline{AB}$ and $\overline{BC}$ are consecutive sides. The slopes of the sides are opposite reciprocals.

$\overline{BC}$ and $\overline{CD}$ are consecutive sides. The slopes of the sides are opposite reciprocals.

$\overline{CD}$ and $\overline{AD}$ are consecutive sides. The slopes of the sides are opposite reciprocals.

$\overline{AB}$ and $\overline{AD}$ are consecutive sides. The slopes of the sides are opposite reciprocals.

Consecutive sides are perpendicular.

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Instruction

4. Show that the quadrilateral has four congruent sides.

Find the length of each side using the distance formula, $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

length of $\overline{AB}=\sqrt{(1-(-1))^2+(5-2)^2}=\sqrt{(2)^2+(3)^2}=\sqrt{4+9}=\sqrt{13}$

length of $\overline{BC}=\sqrt{(4-1)^2+(3-5)^2}=\sqrt{(3)^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$

length of $\overline{CD}=\sqrt{(2-4)^2+(0-3)^2}=\sqrt{(-2)^2+(-3)^2}=\sqrt{4+9}=\sqrt{13}$

length of $\overline{AD}=\sqrt{(2-(-1))^2+(0-2)^2}=\sqrt{(3)^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$

The lengths of all 4 sides are congruent.

5. Make connections.

A square is a quadrilateral with two pairs of parallel opposite sides, consecutive sides that are perpendicular, and all sides congruent.

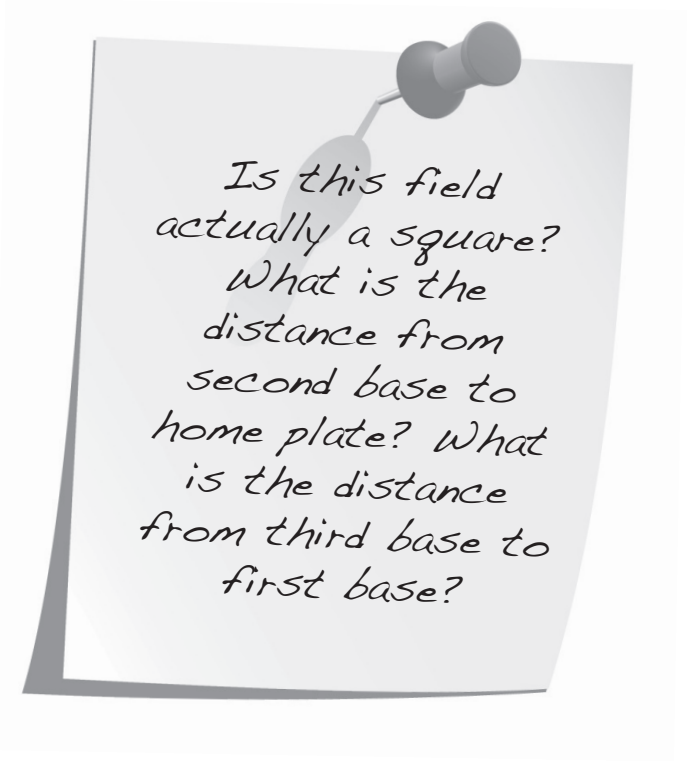
Quadrilateral *ABCD* has two pairs of parallel opposite sides, the consecutive sides are perpendicular, and all the sides are congruent. It is a square.

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Problem-Based Task 6.1.1: Field of Dreams

SMP	
1	2 ✓
3	4 ✓
5	6
7 ✓	8

A square is a quadrilateral with two pairs of parallel opposite sides, consecutive sides that are perpendicular, and all sides congruent. The bases and home plate of a baseball diamond form a square. The local recreation department has created a map of its newest baseball field. First base is represented by the point (7, –3), second base is represented by (4, 6), third base is represented by (–5, 3), and home plate is represented by (–2, –6). Is this field actually a square? What is the distance from second base to home plate? What is the distance from third base to first base?



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Problem-Based Task 6.1.1: Field of Dreams

Coaching

a. What does the map of the field look like?

b. Which sides of the baseball field are opposites?

c. What is the slope of the line connecting first base and second base?

d. What is the slope of the line connecting home plate and third base?

e. What is the slope of the line connecting home plate and first base?

f. What is the slope of the line connecting third base and second base?

g. Are opposite sides parallel?

h. Which sides of the baseball field are consecutive?

i. How is the slope of the line connecting first and second base related to the line connecting first base and home plate?

j. How is the slope of the line connecting second base and third base related to the line connecting third base and home?

continued

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k. How is the slope of the line connecting third base and home plate related to the line connecting home plate and first base?

l. How is the slope of the line connecting first base and second base related to the line connecting second base and third base?

m. What is the formula used for finding the distance between two points?

n. What is the distance between first base and second base?

o. What is the distance between second base and third base?

p. What is the distance between third base and home plate?

q. What is the distance between home plate and first base?

r. Is the baseball diamond a square?

s. What is the distance between second base and home plate?

t. What is the distance between third base and first base?

u. What do you notice about these two distances?

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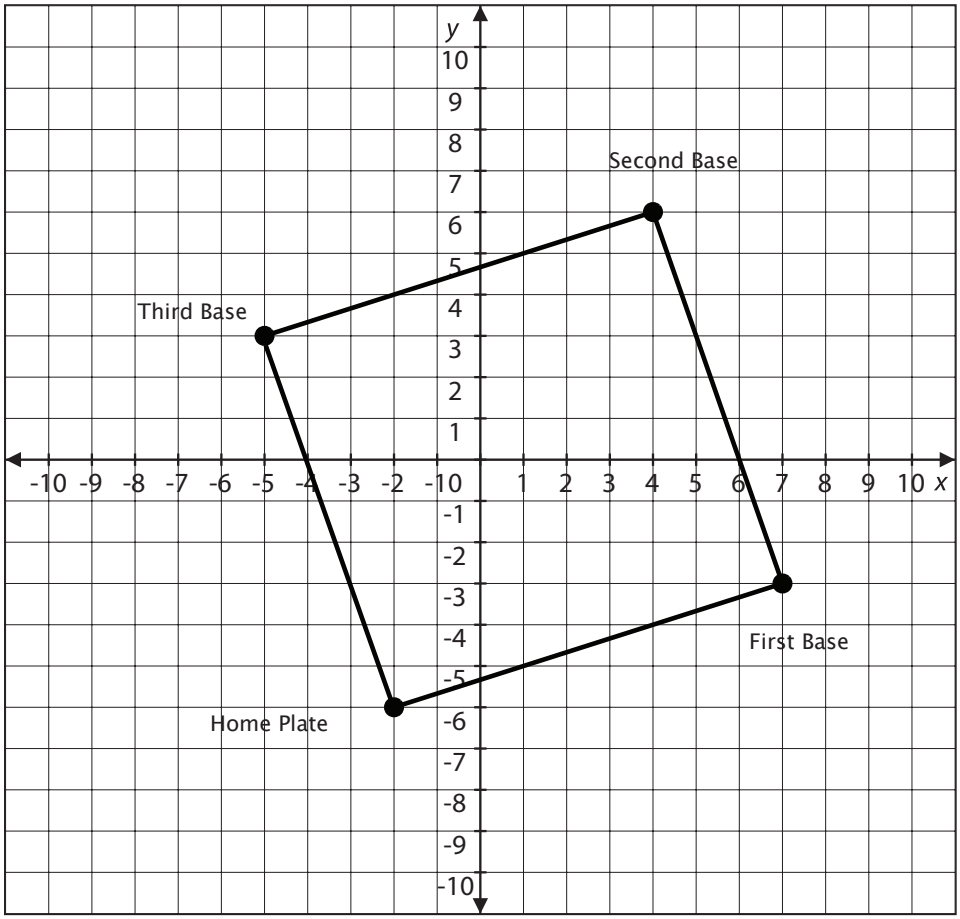
Problem-Based Task 6.1.1: Field of Dreams

Coaching Sample Responses

a. What does the map of the field look like?

Plot each point on a coordinate plane.

Label each of the bases and home plate.



b. Which sides of the baseball field are opposites?

The line connecting first base and second base is opposite the line connecting home plate and third base.

The line connecting second base and third base is opposite the line connecting home plate and first base.

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c. What is the slope of the line connecting first base and second base?

The slope of the line can be found using the slope formula.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{(-3)-(6)}{(7)-(4)}$ Substitute (4, 6) and (7, –3).

$= \frac{-9}{3} = -3$ Simplify.

The slope of the line connecting first base and home plate is –3.

d. What is the slope of the line connecting home plate and third base?

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{(3)-(-6)}{(-5)-(-2)}$ Substitute (–2, –6) and (–5, 3).

$= \frac{9}{-3} = -3$ Simplify.

The slope of the line connecting home plate and third base is –3.

e. What is the slope of the line connecting home plate and first base?

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{(-3)-(-6)}{(7)-(-2)}$ Substitute (–2, –6) and (7, –3).

$= \frac{3}{9} = \frac{1}{3}$ Simplify.

The slope of the line connecting home plate and first base is $\frac{1}{3}$.

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Instruction

f. What is the slope of the line connecting third base and second base?

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{(6) - (3)}{(4) - (-5)}$ Substitute (–5, 3) and (4, 6).

$= \frac{3}{9} = \frac{1}{3}$ Simplify.

The slope of the line connecting third base and second base is $\frac{1}{3}$.

g. Are opposite sides parallel?

Opposite sides have the same slope.

Opposite sides are parallel.

h. Which sides of the baseball field are consecutive?

The line connecting first base and second base is consecutive to the line connecting second base and third base.

The line connecting second base and third base is consecutive to the line connecting third base and home plate.

The line connecting third base and home plate is consecutive to the line connecting home plate and first base.

The line connecting home plate and first base is consecutive to the line connecting first base and second base.

i. How is the slope of the line connecting first base and second base related to the line connecting first base and home plate?

The slopes are opposite reciprocals; therefore, they are perpendicular.

j. How is the slope of the line connecting second base and third base related to the line connecting third base and home?

The slopes are opposite reciprocals; therefore, they are perpendicular.

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Instruction

k. How is the slope of the line connecting third base and home plate related to the line connecting home plate and first base?

The slopes are opposite reciprocals; therefore, they are perpendicular.

l. How is the slope of the line connecting first base and second base related to the line connecting second base and third base?

The slopes are opposite reciprocals; therefore, they are perpendicular.

m. What is the formula used for finding the distance between two points?

The formula for finding the distance between two points is the distance formula.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

n. What is the distance between first base and second base?

Use the distance formula to calculate the distance between each base.

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)}$	Distance formula
$=\sqrt{((4)-(7))^2+((6)-(-3))^2}$	Substitute (7, –3) and (4, 6).
$=\sqrt{(-3)^2+(9)^2}$	Simplify.
$=\sqrt{9+81}$	Evaluate squares.
$=\sqrt{90}$	Simplify.

The distance between first base and second base is $\sqrt{90}$ units.

o. What is the distance between second base and third base?

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	Distance formula
$=\sqrt{((-5)-(4))^2+((3)-(6))^2}$	Substitute (4, 6) and (–5, 3).
$=\sqrt{(-9)^2+(-3)^2}$	Simplify.
$=\sqrt{81+9}$	Evaluate squares.
$=\sqrt{90}$	Simplify.

The distance between second base and third base is $\sqrt{90}$ units.

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p. What is the distance between third base and home plate?

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{((-2) - (-5))^2 + ((-6) - (3))^2}$ Substitute (–5, 3) and (–2, –6).

$= \sqrt{(3)^2 + (-9)^2}$ Simplify.

$= \sqrt{9 + 81}$ Evaluate squares.

$= \sqrt{90}$ Simplify.

The distance between third base and home plate is $\sqrt{90}$ units.

q. What is the distance between home plate and first base?

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{((7) - (-2))^2 + ((-3) - (-6))^2}$ Substitute (–2, –6) and (7, –3).

$= \sqrt{(9)^2 + (3)^2}$ Simplify.

$= \sqrt{81 + 9}$ Evaluate squares.

$= \sqrt{90}$ Simplify.

The distance between home plate and first base is $\sqrt{90}$ units.

r. Is the baseball diamond a square? How do you know?

The baseball diamond is a square because all sides have the same length of $\sqrt{90}$ units.

Opposite sides have the same slope, so they are parallel.

Consecutive sides have opposite reciprocal slopes, so they are perpendicular.

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s. What is the distance between second base and home plate?

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{((-2) - (4))^2 + ((-6) - (6))^2}$ Substitute (4, 6) and (–2, –6).

$= \sqrt{(-6)^2 + (-12)^2}$ Simplify.

$= \sqrt{36 + 144}$ Evaluate squares.

$= \sqrt{180}$ Simplify.

The distance between second base and home plate is $\sqrt{180}$ units.

t. What is the distance between third base and first base?

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{((7) - (-5))^2 + ((-3) - (3))^2}$ Substitute (–5, 3) and (7, –3).

$= \sqrt{(12)^2 + (-6)^2}$ Simplify.

$= \sqrt{144 + 36}$ Evaluate squares.

$= \sqrt{180}$ Simplify.

The distance between third base and first base is $\sqrt{180}$ units.

u. What do you notice about these two distances?

The distance from first base to third base is equal to the distance from second base to home plate.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 6.1.1: Using Coordinates to Prove Geometric Theorems with Slope and Distance

Use the distance formula to calculate the distance between the points indicated.

1. (–1, 1) and (4, 11)

2. (–3, –2) and (6, –1)

A right triangle has a 90° angle made up of two perpendicular sides. Graph each triangle and then determine if each one is a right triangle. Use the slope formula and/or distance formula to justify your answer.

3. *A* (–6, 2), *B* (–2, –2), and *C* (2, 1)

4. *A* (–3, 2), *B* (1, 4), and *C* (3, 0)

A parallelogram is a quadrilateral with opposite sides parallel. Graph each quadrilateral and then determine if each one is a parallelogram. Use the slope formula and/or distance formula to justify your answer.

5. *A* (–2, –1), *B* (–1, 3), *C* (4, 3), and *D* (3, –1)

6. *A* (–2, 2), *B* (1, 8), *C* (4, 5), and *D* (3, –2)

A rectangle is a parallelogram with opposite sides that are congruent and consecutive sides that are perpendicular. Graph each quadrilateral and then determine if each one is a rectangle. Use the slope formula and/or distance formula to justify your answer.

7. *A* (–2, –1), *B* (–3, 1), *C* (1, 3), and *D* (2, 1)

8. *A* (1, –3), *B* (0, 0), *C* (4, 4), and *D* (5, 1)

A square is a parallelogram with four congruent sides and four right angles. Graph each quadrilateral and then determine if each one is a square. Use the slope formula and/or distance formula to justify your answer.

9. *A* (–2, 1), *B* (1, 3), *C* (4, 1), and *D* (1, –1)

10. *A* (–1, 4), *B* (0, 6), *C* (2, 5), and *D* (1, 3)

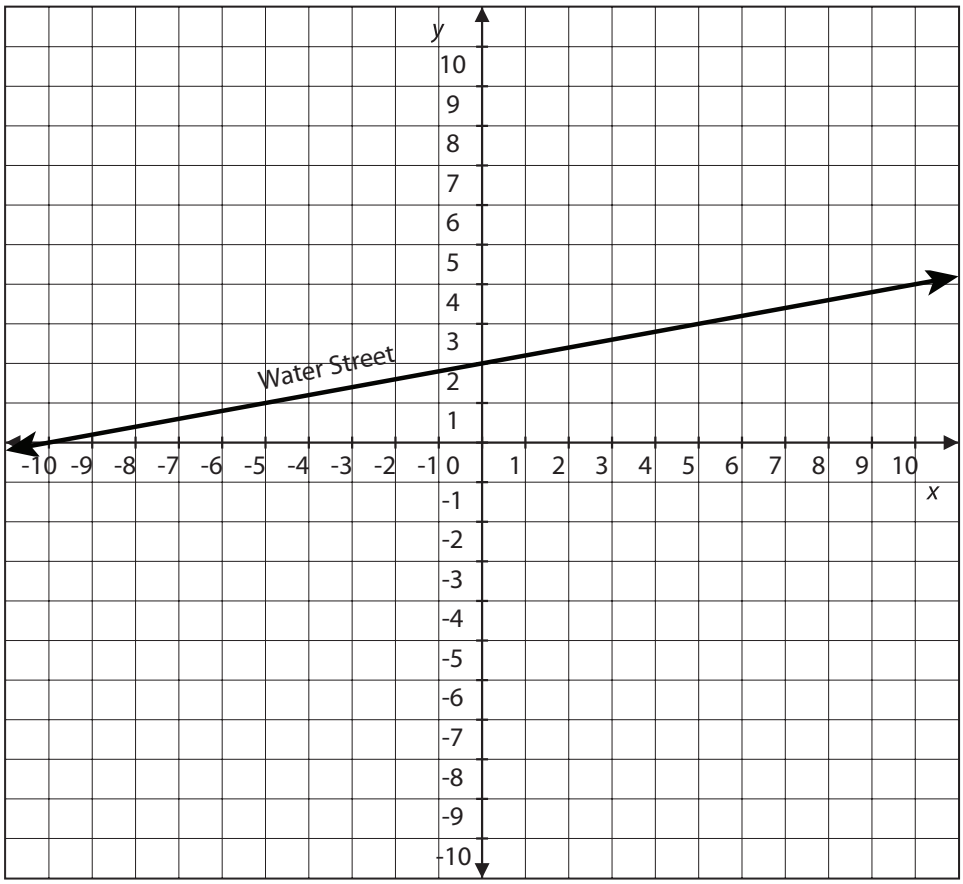
SWB p. 13; SRB Practice 6.1.1 p. U6-18

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Lesson 6.1.2: Working with Parallel and Perpendicular Lines

Warm-Up 6.1.2

Civil engineers are planning a new shopping center downtown. They would like the entrance to the center to run perpendicular to the street represented by the equation $y=\frac{1}{5}x+2$.



1. Write an equation that could represent the entrance to the shopping center. Explain your reasoning.

2. A second entrance will run parallel to the first entrance. Write an equation that could represent the second entrance to the shopping center.

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Lesson 6.1.2: Working with Parallel and Perpendicular Lines

Common Core State Standard

G–GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Warm-Up 6.1.2 Debrief

1. Write an equation that could represent the entrance to the shopping center. Explain your reasoning.

   The equation that represents Water Street is $y = \frac{1}{5}x + 2$. The entrance will be perpendicular to Water Street.

   The slopes of perpendicular lines are opposite reciprocals.

   The slope of the equation representing Water Street is $\frac{1}{5}$.

   The reciprocal of $\frac{1}{5}$ is $\frac{5}{1}$ or 5.

   The opposite of 5 is –5.

   The location of the entrance was not specified, so any equation with a slope of –5 will be perpendicular to the equation $y = \frac{1}{5}x + 2$.

   Possible answers: $y = -5x + 2$ or $y = -5x - 8$

2. A second entrance will run parallel to the first entrance. Write an equation that could represent the second entrance to the shopping center.

   The slopes of parallel lines are equal.

   The slope of the equation representing the first entrance is –5.

   The location of the second entrance was not specified, so any equation with a slope of –5, other than the original equation, will be parallel to the first entrance.

   Possible answers: $y = -5x + 7$ or $y = -5x - 3$

Connection to the Lesson

- Students will extend their understanding of parallel and perpendicular lines in the upcoming lesson.
- Students will be asked to take a scenario like this a step further and find a line parallel and/or perpendicular to a given line through a specified point.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- writing linear equations (A–CED.2)
- graphing linear equations (A–CED.2)

Introduction

The slopes of parallel lines are always equal, whereas the slopes of perpendicular lines are always opposite reciprocals. It is important to be able to determine whether lines are parallel or perpendicular, but the creation of parallel and perpendicular lines is also important. In this lesson, you will write the equations of lines that are parallel and perpendicular to a given line through a given point.

Key Concepts

- You can write the equation of a line through a given point that is parallel or perpendicular to a given line if you know the slope of the given line.
- Writing the given equation in slope-intercept form allows you to quickly identify the slope, m, of the equation.
- If the given equation is not in slope-intercept form, take a few moments to rewrite it.

Writing the Equation of a Line Parallel to a Given Line Through a Given Point

1. Rewrite the given equation in slope-intercept form if necessary.
2. Identify the slope of the given line.
3. Write the general point-slope form of a linear equation: $y - y_1 = m(x - x_1)$.
4. Substitute the slope of the given line for m in the general equation.
5. Substitute x and y from the given point into the general equation for x_1 and y_1.
6. Simplify the equation.
7. Rewrite the equation in slope-intercept form if necessary.

- Writing the equation of a line perpendicular to a given line through a given point is similar to writing equations of parallel lines.
- The slopes of perpendicular lines are opposite reciprocals.

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Instruction

Writing the Equation of a Line Perpendicular to a Given Line Through a Given Point

1. Rewrite the given equation in slope-intercept form if necessary.
2. Identify the slope of the given line.
3. Find the opposite reciprocal of the slope of the given line.
4. Write the general point-slope form of a linear equation: $y - y_1 = m(x - x_1)$.
5. Substitute the opposite reciprocal of the given line for m in the general equation.
6. Substitute x and y from the given point into the general equation for x_1 and y_1.
7. Simplify the equation.
8. Rewrite the equation in slope-intercept form if necessary.

- The shortest distance between two points is a line.
- The shortest distance between a given point and a given line is the line segment that is perpendicular to the given line through the given point.

Finding the Shortest Distance Between a Given Point and a Given Line

1. Follow the steps outlined previously to find the equation of the line that is perpendicular to the given line through the given point.
2. Find the intersection between the two lines by substituting the value of y from the second equation in for y in the first equation.
3. Solve for x.
4. Substitute the x-value into the equation of the given line to find the y-value.
5. Find the distance between the given point and the point of intersection of the given line and the perpendicular line using the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Common Errors/Misconceptions

- attempting to identify the slope of the given line without transforming the equation into slope-intercept form
- incorrectly identifying the slope of the given line
- incorrectly finding the slope of the line parallel to the given line
- incorrectly identifying the slope of the line perpendicular to the given line
- improperly substituting the x- and y-values into the general point-slope equation

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Guided Practice 6.1.2

Example 1

Write the slope-intercept form of an equation for the line that passes through the point (5, –2) and is parallel to the graph of $8x - 2y = 6$.

1. Rewrite the given equation in slope-intercept form.

$8x - 2y = 6$	Given equation
$-2y = 6 - 8x$	Subtract $8x$ from both sides.
$y = -3 + 4x$	Divide both sides by –2.
$y = 4x - 3$	Write the equation in slope-intercept form.

2. Identify the slope of the given line.

   The slope of the line $y = 4x - 3$ is 4.

3. Substitute the slope of the given line for m in the point-slope form of a linear equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - y_1 = 4(x - x_1)$	Substitute 4 for m.

4. Substitute x and y from the given point into the equation for x_1 and y_1.

$y - y_1 = 4(x - x_1)$	Equation
$y - (-2) = 4(x - 5)$	The given point is (5, –2). Substitute 5 for x_1 and –2 for y_1.

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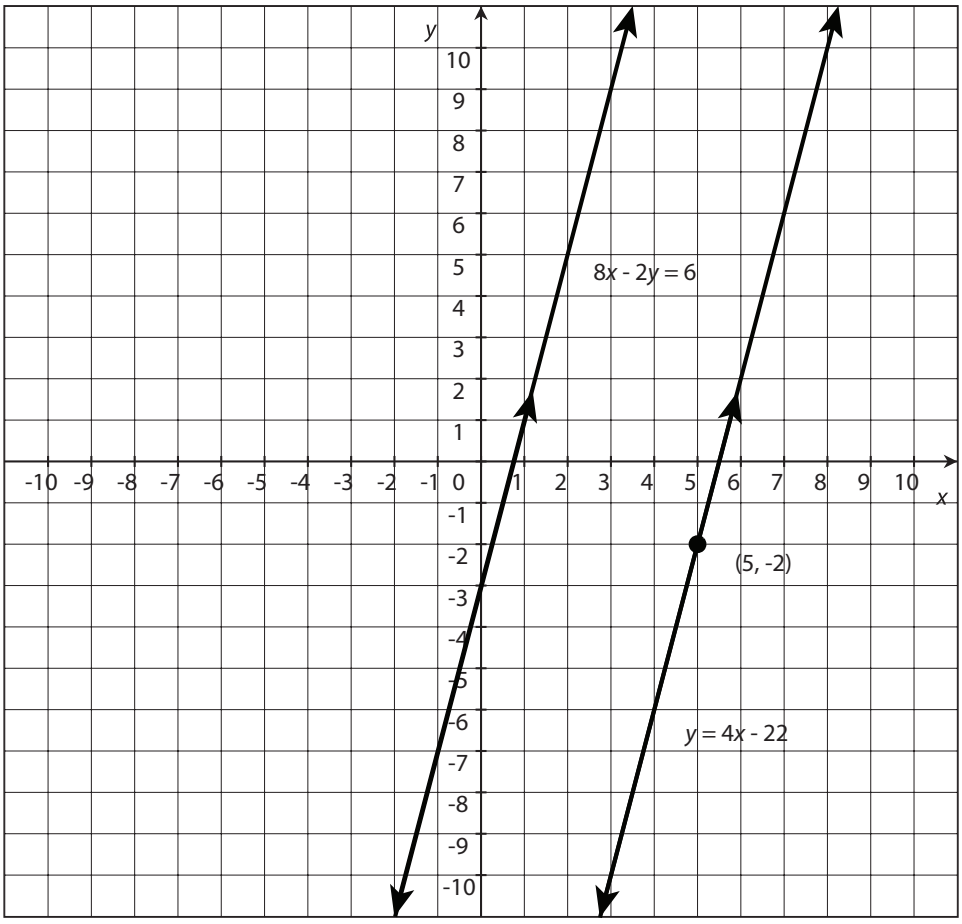
Instruction

5. Simplify the equation.

$y - (-2) = 4(x - 5)$	Equation with substituted values for x_1 and y_1
$y - (-2) = 4x - 20$	Distribute 4 over $(x - 5)$.
$y + 2 = 4x - 20$	Simplify.
$y = 4x - 22$	Subtract 2 from both sides.

The equation of the line through the point (5, –2) that is parallel to the equation $8x - 2y = 6$ is $y = 4x - 22$.

This can be seen on the following graph.



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Instruction

Example 2

Write the slope-intercept form of an equation for the line that passes through the point (–1, 6) and is perpendicular to the graph of $-10x + 5y = 20$.

1. Rewrite the given equation in slope-intercept form.

$-10x + 5y = 20$	Given equation
$5y = 20 + 10x$	Add $10x$ to both sides.
$y = 4 + 2x$	Divide both sides by 5.
$y = 2x + 4$	Write the equation in slope-intercept form.

2. Identify the slope of the given line.

   The slope of the line $y = 2x + 4$ is 2.

3. Find the opposite reciprocal of the slope of the given line.

   The slope of the given line is 2.

   The opposite of 2 is –2.

   The reciprocal of –2 is $-\frac{1}{2}$.

4. Substitute the opposite reciprocal for m in the point-slope form of a linear equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - y_1 = -\frac{1}{2}(x - x_1)$	Substitute $-\frac{1}{2}$ for m.

5. Substitute x and y from the given point into the equation for x_1 and y_1.

$y - y_1 = -\frac{1}{2}(x - x_1)$	Equation
$y - 6 = -\frac{1}{2}(x - (-1))$	The given point is (–1, 6). Substitute –1 for x_1 and 6 for y_1.

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Instruction

6. Simplify the equation.

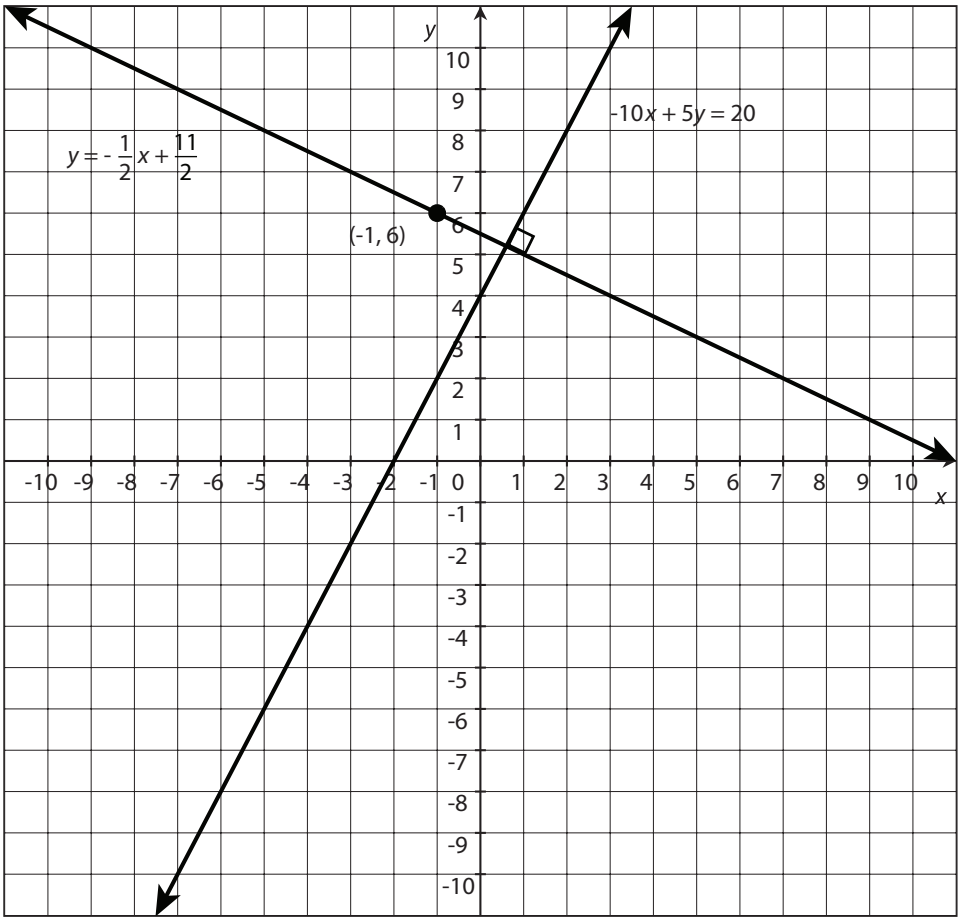
$y-6=-\frac{1}{2}(x-(-1))$ Equation with substituted values for x_1 and y_1

$y-6=-\frac{1}{2}x-\frac{1}{2}$ Distribute $-\frac{1}{2}$ over $(x-(-1))$.

$y=-\frac{1}{2}x+\frac{11}{2}$ Add 6 to both sides.

The equation of the line through the point (–1, 6) that is perpendicular to the graph of $-10x + 5y = 20$ is $y=-\frac{1}{2}x+\frac{11}{2}$.

This can be seen on the following graph.



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Instruction

Example 3

Find the point on the line $y = 4x + 1$ that is closest to the point (–2, 8).

1. Find the line perpendicular to the given line, $y = 4x + 1$, that passes through the point (–2, 8).

2. Identify the slope of the given line.

   The slope of the line $y = 4x + 1$ is 4.

3. Find the opposite reciprocal of the slope of the given line.

   The opposite of 4 is –4.

   The reciprocal of –4 is $-\frac{1}{4}$.

4. Substitute the opposite reciprocal for m in the point-slope form of a linear equation.

   $y - y_1 = m(x - x_1)$ Point-slope form

   $y - y_1 = -\frac{1}{4}(x - x_1)$ Substitute $-\frac{1}{4}$ for m.

5. Substitute x and y from the given point into the equation for x_1 and y_1.

   $y - y_1 = -\frac{1}{4}(x - x_1)$ Equation

   $y - 8 = -\frac{1}{4}(x - (-2))$ The given point is (–2, 8). Substitute –2 for x_1 and 8 for y_1.

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Instruction

6. Simplify the equation.

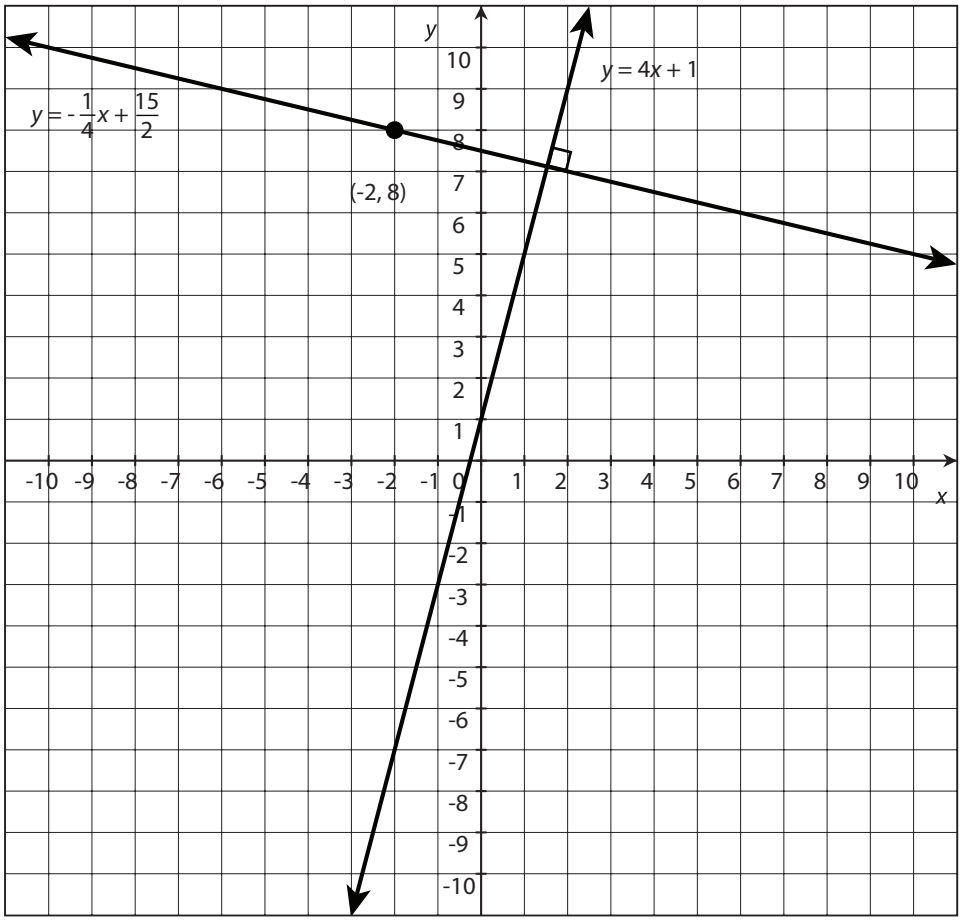
$y-8=-\frac{1}{4}(x-(-2))$ Equation with substituted values for x_1 and y_1

$y-8=-\frac{1}{4}x-\frac{1}{2}$ Distribute $-\frac{1}{4}$ over $(x-(-2))$.

$y=-\frac{1}{4}x+\frac{15}{2}$ Add 8 to both sides.

The equation of the line through the point (–2, 8) that is perpendicular to the graph of $y = 4x + 1$ is $y=-\frac{1}{4}x+\frac{15}{2}$.

This can be seen on the following graph.



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Instruction

7. Find the intersection between the two lines by substituting the value of y in the second equation for y in the first equation, then solve for x.

$4x+1=-\frac{1}{4}x+\frac{15}{2}$	Substitute $4x + 1$ in for y in the first equation.
$4x=-\frac{1}{4}x+\frac{13}{2}$	Subtract 1 from both sides.
$\frac{17}{4}x=\frac{13}{2}$	Add $\frac{1}{4}x$ to both sides.
$x=\frac{26}{17}$	Divide both sides by $\frac{17}{4}$.

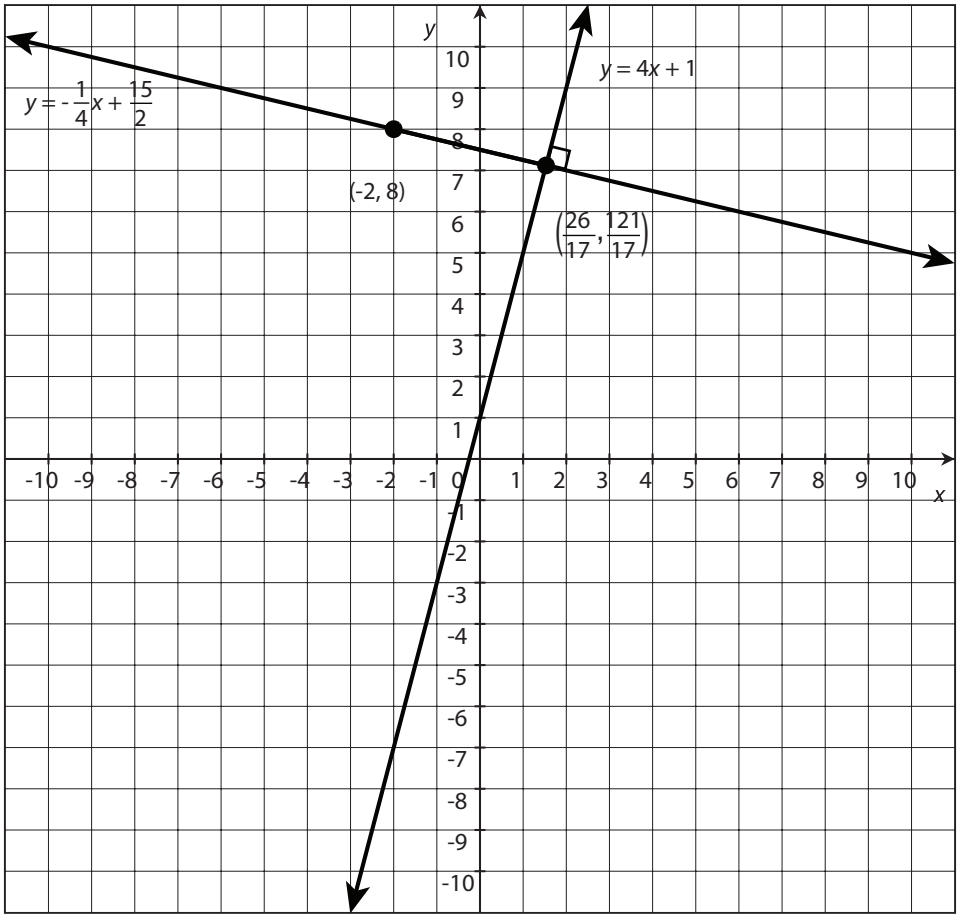
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Instruction

8. Substitute the value of x back into the given equation to find the value of y.

$y = 4x + 1$	Given equation
$y = 4\left(\frac{26}{17}\right) + 1$	Substitute $\frac{26}{17}$ for x.
$y = \frac{121}{17}$	Simplify.

The point on the line closest to (–2, 8) is the point $\left(\frac{26}{17}, \frac{121}{17}\right)$.



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9. Calculate the distance between the two points using the distance formula.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{\left(\frac{26}{17} - (-2)\right)^2 + \left(\frac{121}{17} - 8\right)^2}$ Substitute values for (x_1, y_1) and (x_2, y_2) using the points $(-2, 8)$ and $\left(\frac{26}{17}, \frac{121}{17}\right)$.

$= \sqrt{\left(\frac{60}{17}\right)^2 + \left(\frac{-15}{17}\right)^2}$ Simplify.

$= \sqrt{\left(\frac{3600}{289}\right) + \left(\frac{225}{289}\right)}$ Evaluate squares.

$= \sqrt{\frac{3825}{289}}$ Simplify.

$= \sqrt{\frac{225}{17}}$ Reduce to lowest terms.

The distance between the point of intersection and the given point is $\sqrt{\frac{225}{17}}$ units or approximately 3.6 units.

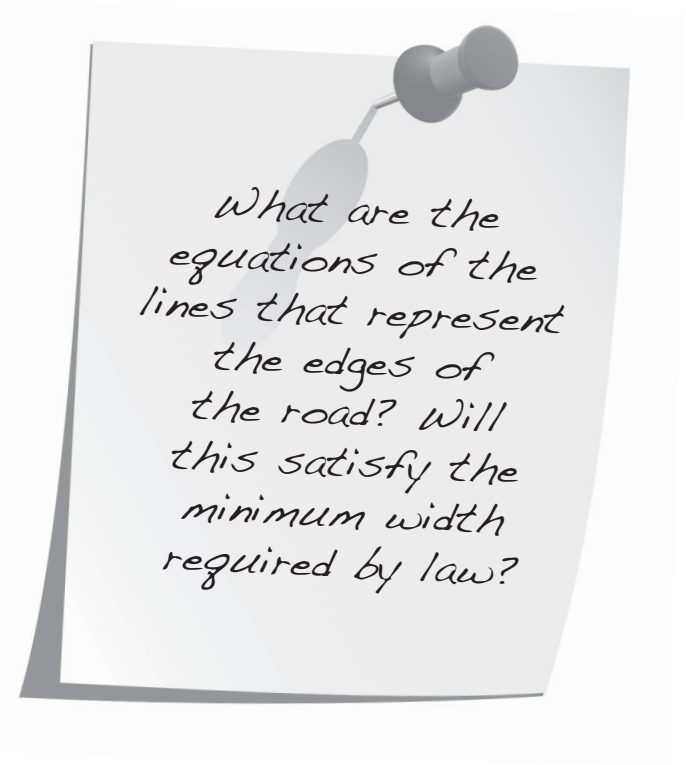


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Problem-Based Task 6.1.2: Pave the Way

SMP	
1 ✓	2 ✓
3 ✓	4
5	6
7	8

The civil engineers are busy again. This time they are expanding one of the well-traveled roads in your town. The center of the road is represented by the equation $y=\frac{3}{5}x+5$. The law says a two-lane road must be at least 24 feet wide. Engineers have mapped out the edges of the road and have one edge running through the point (16, 1) and the other edge running through the point (4, 21), which is directly across the street from the first point. What are the equations of the lines that represent the edges of the road? Will this satisfy the minimum width required by law?



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Problem-Based Task 6.1.2: Pave the Way

Coaching

a. What is the equation of the line that represents the center of the road?

b. What is the slope of the equation that represents the center of the road?

c. What is the slope of the lines that represent each edge of the road? How do you know?

d. Which point represents the edge of one side of the road?

e. What is the equation that represents this edge of the road?

f. Which point represents the other edge of the road?

g. What is the equation that represents this edge of the road?

h. What is the minimum distance for a two-lane road?

i. What formula is used to calculate the distance between points?

j. What are the two points that are directly across the road from each other?

k. What is the distance between these two points?

l. Does the placement of the points meet the minimum distance requirement?

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Instruction

Problem-Based Task 6.1.2: Pave the Way

Coaching Sample Responses

a. What is the equation of the line that represents the center of the road?

The equation of the center of the road is $y=\frac{3}{5}x+5$ as stated in the problem.

b. What is the slope of the equation that represents the center of the road?

The slope of the equation representing the center is $\frac{3}{5}$.

c. What is the slope of the lines that represent each edge of the road? How do you know?

The slope of each edge is $\frac{3}{5}$. The edges of the road are parallel to the center of the road. Lines that are parallel to one another have the same slope.

d. Which point represents the edge of one side of the road?

One edge of the road is marked by the point (16, 1).

e. What is the equation that represents this edge of the road?

Substitute the slope of the given line for m in the point-slope form of a linear equation.

$y-y_1=m(x-x_1)$	Point-slope form
$y-y_1=\frac{3}{5}(x-x_1)$	Substitute $\frac{3}{5}$ for m.

Substitute x and y from the given point into the equation for x_1 and y_1.

$y-y_1=\frac{3}{5}(x-x_1)$	Equation
$y-1=\frac{3}{5}(x-16)$	The given point is (16, 1). Substitute 16 for x_1 and 1 for y_1.

Simplify the equation.

$y-1=\frac{3}{5}(x-16)$	Equation with substituted values for x_1 and y_1
$y-1=\frac{3}{5}x-\frac{48}{5}$	Distribute $\frac{3}{5}$ over $(x-16)$.

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$y = \frac{3}{5}x - \frac{43}{5}$ Add 1 to both sides.

The equation of the line through the point (16, 1) that is parallel to the equation $y = \frac{3}{5}x + 5$ is $y = \frac{3}{5}x - \frac{43}{5}$.

f. Which point represents the other edge of the road?

The other edge of the road is marked by the point (4, 21).

g. What is the equation that represents this edge of the road?

Substitute the slope of the given line for m in the point-slope form of a linear equation.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - y_1 = \frac{3}{5}(x - x_1)$ Substitute $\frac{3}{5}$ for m.

Substitute x and y from the given point into the equation for x_1 and y_1.

$y - y_1 = \frac{3}{5}(x - x_1)$ Equation

$y - 21 = \frac{3}{5}(x - 4)$ The given point is (4, 21). Substitute 4 for x_1 and 21 for y_1.

Simplify the equation.

$y - 21 = \frac{3}{5}(x - 4)$ Equation with substituted values for x_1 and y_1

$y - 21 = \frac{3}{5}x - \frac{12}{5}$ Distribute $\frac{3}{5}$ over $(x - 4)$.

$y = \frac{3}{5}x + \frac{93}{5}$ Add 21 to both sides.

The equation of the line through the point (4, 21) that is parallel to the equation $y = \frac{3}{5}x + 5$ is $y = \frac{3}{5}x + \frac{93}{5}$.

h. What is the minimum distance for a two-lane road?

The minimum distance for a two-lane road is 24 feet.

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Instruction

i. What formula is used to calculate the distance between points?

The distance formula, $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, is used to calculate the distance between two points.

j. What are the two points that are directly across the road from each other?

The two points are (16, 1) and (4, 21).

k. What is the distance between these two points?

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	Distance formula
$=\sqrt{(4-16)^2+(21-1)^2}$	Substitute the points (16, 1) and (4, 21).
$=\sqrt{(-12)^2+(20)^2}$	Simplify.
$=\sqrt{144+400}$	Evaluate squares.
$=\sqrt{544}$	Simplify.

The distance between the points (6, 1) and (4, 21) is $\sqrt{544}$ feet, which is approximately 23 feet.

l. Does the placement of the points meet the minimum distance requirement?

The minimum distance requirement is 24 feet.

The placement of the points does not meet the minimum distance requirement. The distance is too short.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 6.1.2: Working with Parallel and Perpendicular Lines

Write the slope-intercept form of an equation for the line that passes through the given point and is parallel to the graph of the given equation.

1. (3, 1) and $4x + 2y = 10$

2. (–5, 6) and $12x + 9y = 3$

Write the slope-intercept form of an equation for the line that passes through the given point and is perpendicular to the graph of the given equation.

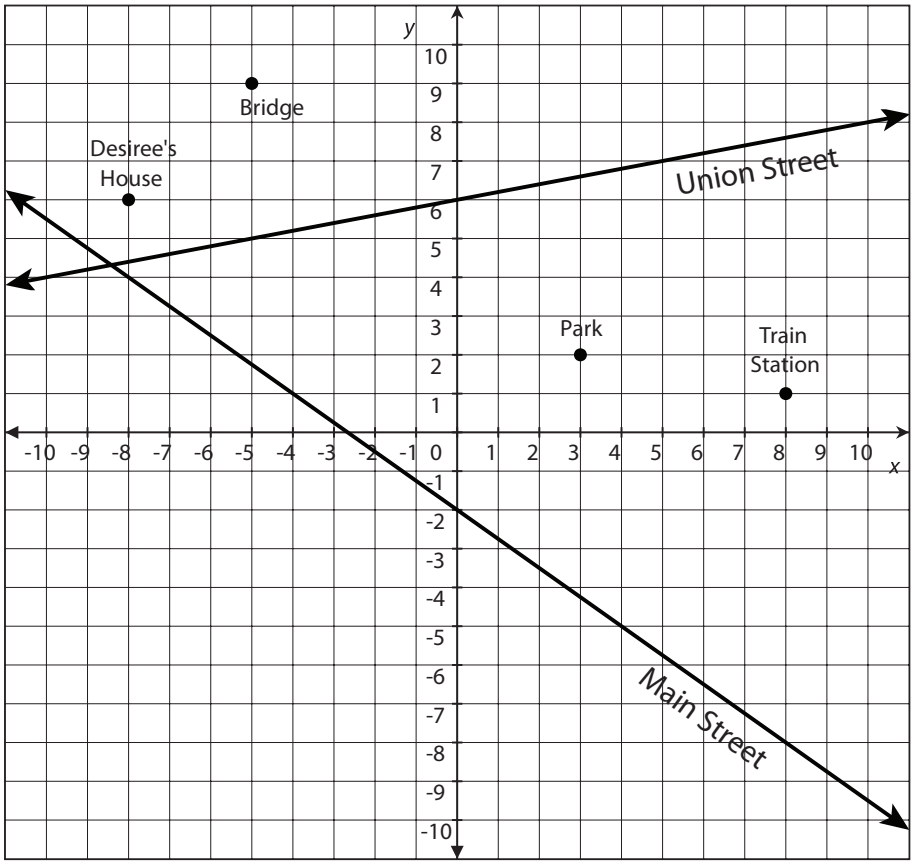
3. (–4, –3) and $8x + 2y = 14$

4. (–6, 5) and $2x – 2y = 10$

Calculate the shortest distance from the given point to the line indicated.

5. (2, 1) to $x – y = –1$

For questions 6–10, refer to the following map. Each unit represents 100 yards.



continued

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6. Main Street is given by the equation $y = -\frac{3}{4}x - 2$. The train station is located at the point (8, 1). What is the equation of the line that represents the train tracks that run parallel to Main Street through the point (8, 1)?

7. Desiree's house is located at the point (–8, 6). Her driveway is perpendicular to Union Street, which is represented by the equation $y = \frac{1}{5}x + 6$. What is the equation of the line that represents Desiree's driveway?

8. Little River runs parallel to Union Street. There is a bridge located at the point (–5, 9). What is the equation of the line that represents Little River?

9. The center of the park is located at the point (3, 2). A walking trail starting at this point runs perpendicular to Main Street. What is the equation of the line that represents the walking trail?

10. What is the shortest distance from the train station to Union Street?

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Progress Assessment

Circle the letter of the best answer.

1. What is the distance between the points (–6, 5) and (1, 1)?

   a. $\sqrt{65}$ units
   b. $\sqrt{33}$ units
   c. $\sqrt{61}$ units
   d. $\sqrt{21}$ units

2. What is the distance between the points (5, 7) and (–1, 11)?

   a. $\sqrt{308}$ units
   b. $\sqrt{20}$ units
   c. $\sqrt{340}$ units
   d. $\sqrt{52}$ units

3. The town of Spring Mountain is located on a map at the point (–4, 8). Winter Valley is located on the same map at the point (2, –7). What is the approximate distance between the towns if each unit represents 10 miles?

   a. 137 miles
   b. 162 miles
   c. 22 miles
   d. 17 miles

4. The line through the points (4, –3) and (–2, 6) is parallel to the line through which of the following points?

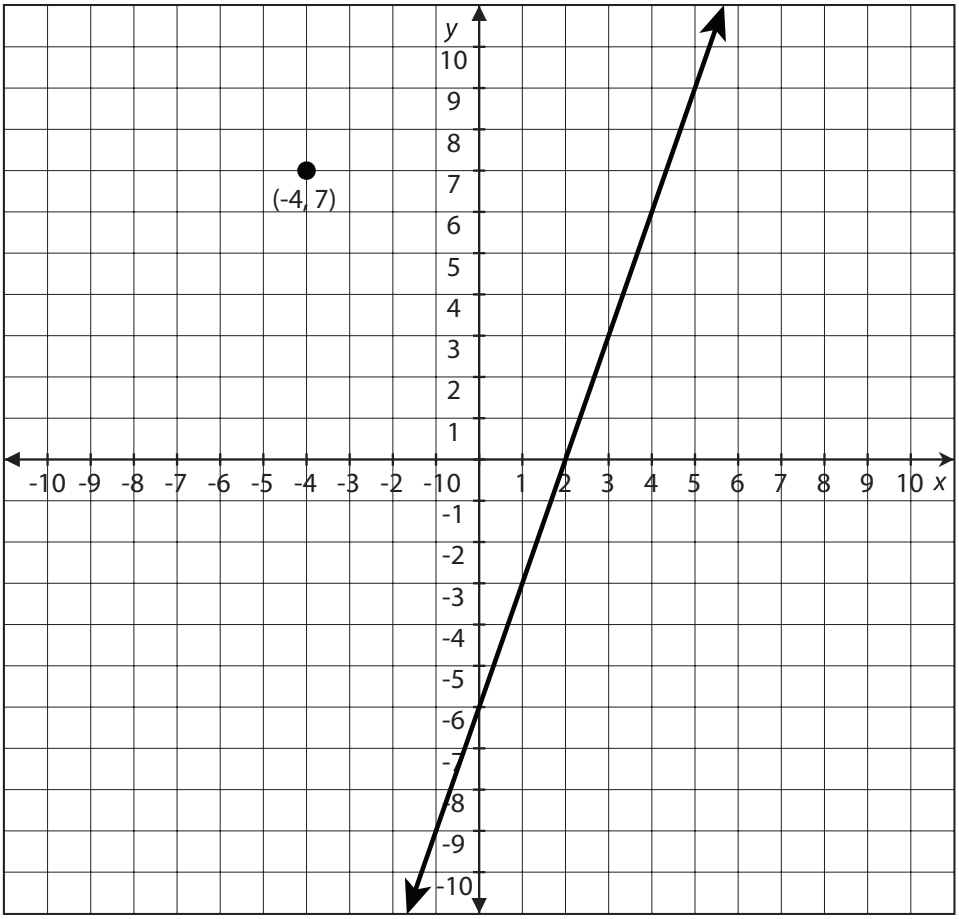
   a. (–1, 2) and (2, 4)
   b. (–1, 2) and (3, –4)
   c. (–1, 2) and (2, –4)
   d. (–1, 2) and (1, 1)

continued

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Assessment

5. Which equation represents a line that is parallel to the graphed line and passes through the point (–4, 7)?



a. $y=-\frac{1}{3}x+\frac{25}{3}$

b. $y = 3x + 17$

c. $y = 3x + 19$

d. $y=\frac{1}{3}x+\frac{25}{3}$

6. A train track runs parallel to a road represented by the equation $y = 4x + 17$. A train station along the track is located at the point (6, –1). What is the equation of the line that represents the train track?

a. $y = 4x – 25$

b. $y=-\frac{1}{4}x-25$

c. $y = 4x + 25$

d. $y=-\frac{1}{4}x+25$

continued

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Assessment

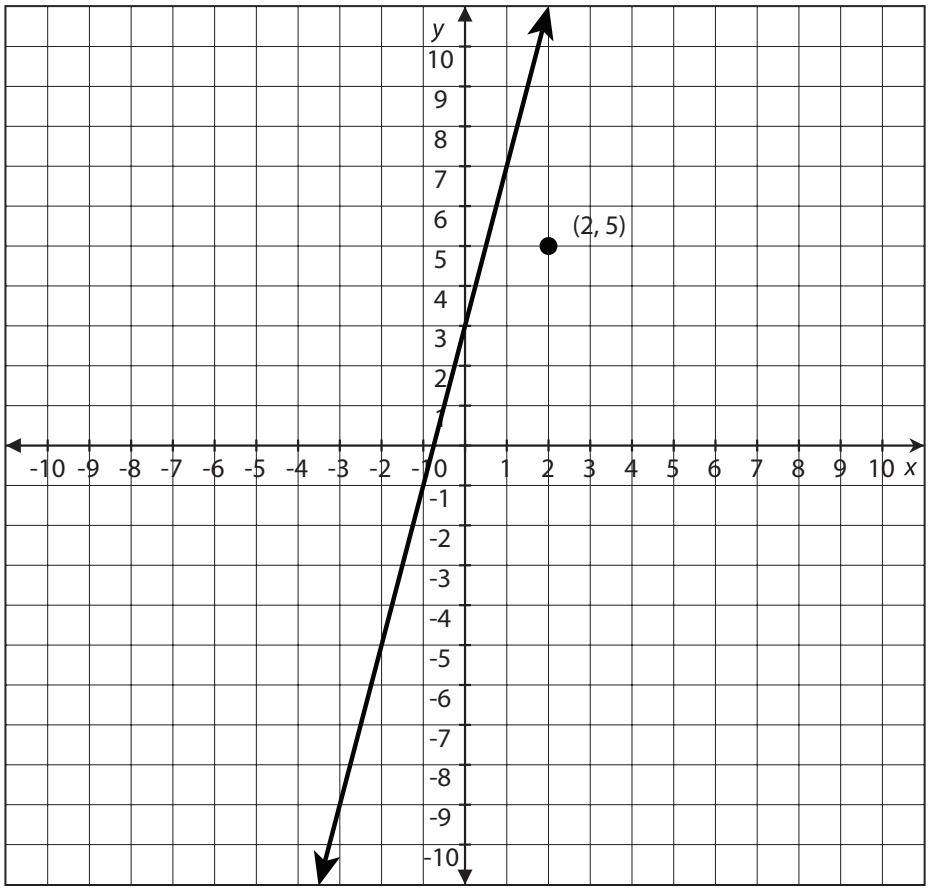
7. Quadrilateral *ABCD* is a parallelogram because the opposite sides are parallel. The locations of three of the vertices are (–3, 2), (0, 8), and (10, 3). What is the location of the fourth vertex?

   a. (–3, 7)
   b. (5, –2)
   c. (8, –1)
   d. (7, –3)

8. The line through the points (6, 3) and (4, –5) is perpendicular to the line through which of the following points?

   a. (1, –4) and (2, 1)
   b. (0, 2) and (8, 0)
   c. (–4, 1) and (1, –4)
   d. (3, 4) and (8, 1)

9. Which equation represents a line that is perpendicular to the graphed line and passes through the point (2, 5)?



   a. $y = \frac{1}{4}x + \frac{9}{2}$
   b. $y = 4x - 3$
   c. $y = -\frac{1}{4}x + \frac{11}{2}$
   d. $y = -4x + 13$

continued

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Assessment

10. Triangle ABC is a right triangle. Sides $\overline{AB}$ and $\overline{BC}$ form the right angle. The equation of the line representing side $\overline{AB}$ is $3x + 6y = 12$. If C is located at the point (9, 5), what is the equation of the line representing side $\overline{BC}$?

a. $y = 2x - 13$

b. $y = 2x + 13$

c. $y = -\frac{1}{2}x - \frac{19}{2}$

d. $y = -\frac{1}{2}x - \frac{9}{2}$

Read the scenario, write an equation to model the walkway, and then solve the problem.

11. Mariah's house is located at the point (–1, 2). The equation of the road is $y + 4x = 2$. Mariah is building a walkway from her house to the road. What is the approximate length of the shortest walkway she can build? Each unit represents 100 yards.

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Assessment

Pre-Assessment

Circle the letter of the best answer.

1. What is the perimeter of quadrilateral *ABCD* with vertices *A* (2, 0), *B* (5, 3), *C* (8, 0), and *D* (5, –3)?
   a. $2\sqrt{12}$ units
   b. 24 units
   c. 36 units
   d. $12\sqrt{2}$ units

2. What is the perimeter of triangle *ABC* with vertices *A* (–7, –5), *B* (–3, –2), and *C* (–3, –8)?
   a. 16 units
   b. 27 units
   c. 5 units
   d. 11 units

3. What is the perimeter of quadrilateral *ABCD* with vertices *A* (2, 4), *B* (6, 7), *C* (10, 4), and *D* (10, –2)?
   a. 25 units
   b. 26 units
   c. 50 units
   d. 1,500 units

4. What is the area of triangle *ABC* with vertices *A* (–6, 4), *B* (–1, 8), and *C* (2, 2)?
   a. 354.2 units^2
   b. 21 units^2
   c. 21.4 units^2
   d. 61.7 units^2

5. What is the area of rectangle *ABCD* with vertices *A* (–6, 2), *B* (–1, 4), *C* (3, –6), and *D* (–2, –8)?
   a. 16.2 units^2
   b. 32.3 units^2
   c. 58 units^2
   d. 29 units^2

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Lesson 2: Lines and Line Segments

Instruction

Common Core State Standard

G–GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

SMP	
1	2 ✓
3	4 ✓
5	6 ✓
7	8

Essential Questions

1. What is the difference between the perimeter and area of a two-dimensional figure?
2. How can the distance formula be used to find the area and perimeter of figures on a coordinate plane?

WORDS TO KNOW

area	the amount of space inside the boundary of a two-dimensional figure
distance formula	formula that states the distance between (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
perimeter	the distance around a two-dimensional figure
polygon	two-dimensional figure with at least three sides

Recommended Resources

- Math Open Reference. "Distance between two points (given their coordinates)."

  http://walch.com/rr/00277

  Users can adjust a line segment by dragging the endpoints around a coordinate plane to see how the distance between the endpoints changes. The equation is displayed off to the side of the plane and changes as the points are dragged.

- Math Open Reference. "Rectangle Area and Perimeter (Coordinate Geometry)."

  http://walch.com/rr/CAU6L2RectanglePerimeter

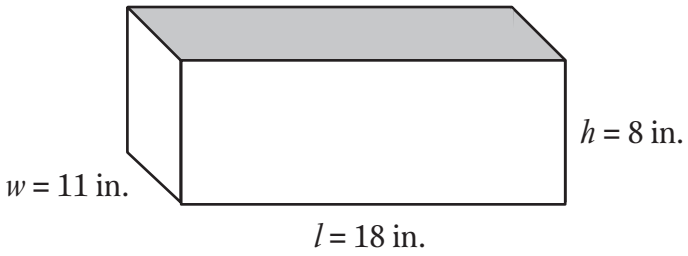
  Drag the vertices of a rectangle around different points on a coordinate plane and watch as the coordinates, area, and perimeter change.

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Lesson 6.2.1: Calculating Perimeter and Area

Warm-Up 6.2.1

Mikko wants to make an open-top plywood box to fit on a certain shelf in his closet. He has drawn the sketch below of the box that he needs.



1. How much plywood would Mikko need to create this box?

2. Mikko's dad has brought him a piece of plywood that measures 2 feet by 3 feet. Is this enough plywood for the project? Explain why or why not.

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Instruction

Lesson 6.2.1: Calculating Perimeter and Area

Common Core State Standard

G–GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Warm-Up 6.2.1 Debrief

1. How much plywood would Mikko need to create this box?

   To find out the amount of plywood Mikko would need, first calculate the area of each of the rectangular sides.

   The formula for calculating the area of a rectangle is *Area* = (*base*)(*height*).

   - The left side of the box has a base that is 11 inches and a height that is 8 inches.

Area = (*base*)(*height*)	Area formula
= (11)(8)	Substitute values for base and height.
= 88 inches^2	

     The right side of the box has the same dimensions and therefore has the same area, 88 inches^2.

   - The front of the box has a base of 18 inches and a height of 8 inches.

Area = (*base*)(*height*)	Area formula
= (18)(8)	Substitute values for base and height.
= 144 inches^2	

     The back of the box has the same dimensions and therefore has the same area, 144 inches^2.

   - The bottom of the box has a base of 18 inches and a height of 11 inches.

Area = (*base*)(*height*)	Area formula
= (18)(11)	Substitute values for base and height.
= 198 inches^2	

   To find the total amount of plywood needed, add the areas of each of the sides.

   Total = area of left side + area of right side + area of front + area of back + area of bottom

   = 88 + 88 + 144 + 144 + 198

   = 662 inches^2

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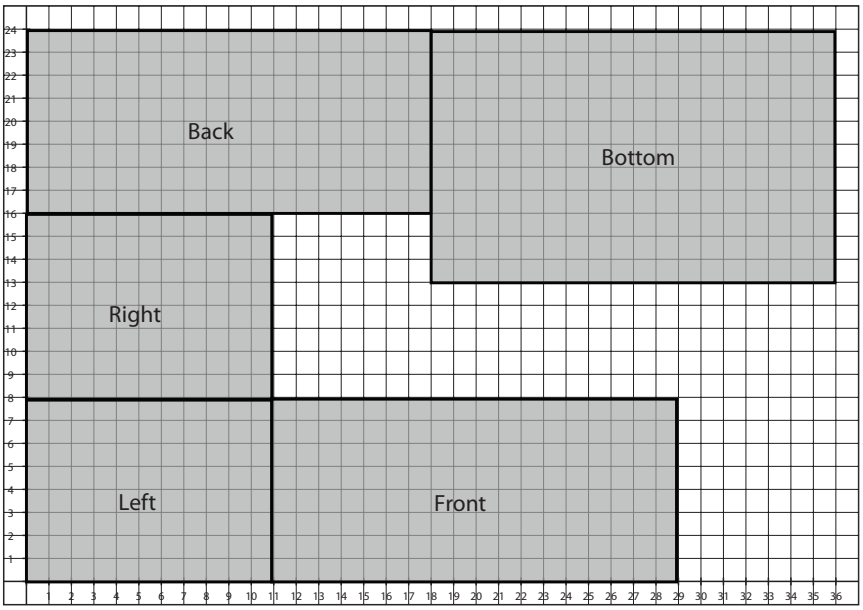
2. Mikko's dad has brought him a piece of plywood that measures 2 feet by 3 feet. Is this enough plywood for the project? Explain why or why not.

   The piece of plywood measures 2 feet by 3 feet or 24 inches by 36 inches.

   $(24)(36) = 864 \text{ inches}^2$

   The total area of the piece of plywood is 864 inches^2, which is more than the 662 inches^2 that Mikko needs for his box.

   One way to cut the pieces is shown below.



Connection to the Lesson

- Students will extend their understanding of area formulas for a rectangle to using coordinates to compute the area of rectangles on the coordinate plane.

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Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- finding the distance between two points on the coordinate plane using the distance formula (8.G.8)
- calculating the perimeter of polygons (no standard)
- calculating the areas of triangles and rectangles (6.G.1)

Introduction

The distance formula can be used to find solutions to many real-world problems. In the previous lesson, the distance formula was used to find the distance between two given points. In this lesson, the distance formula will be applied to perimeter and area problems.

A **polygon** is a two-dimensional figure with three or more sides. Sometimes we need to calculate the **perimeter** or distance around a polygon, as well as find the **area** or the amount of space inside the boundary of a polygon. The distance formula is a valuable tool for both of these calculations.

Key Concepts

- Situations where you would need to calculate perimeter include finding the amount of linear feet needed to fence a yard or a garden, determining the amount of trim needed for a room, or finding the amount of concrete needed to create a barrier around a statue.
- The perimeter is the sum of the lengths of all the sides of a polygon.
- The final answer must include the appropriate label for length (units, feet, inches, meters, centimeters, etc.).
- Sometimes the answer is not a whole number. If it is not, you can approximate the value.

Calculating the Perimeter of a Polygon in the Coordinate Plane

1. Calculate the length of each side of the polygon using the distance formula: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.
2. Add all the lengths of the polygon's sides to find the perimeter.
3. Approximate the value if necessary.

Be sure to include the appropriate label in your answer.

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Instruction

- Calculating area is necessary when finding the amount of carpeting needed for a room in your home, or to determine how much paint to buy to paint a room.
- The area of a triangle is found using the formula $Area = \frac{1}{2}(base)(height)$.
- The height of a triangle is the perpendicular distance from a vertex to the opposite side.
- It may be necessary to determine the equation of the line that represents the height of the triangle before calculating the area. For an example of this, see Example 3 in the Guided Practice.
- Determining the lengths of the base and the height is necessary if these lengths are not stated in the problem.
- The final answer must include the appropriate label for area (units^2, feet^2, inches^2, meters^2, centimeters^2, etc.).

Calculating the Area of a Triangle in the Coordinate Plane

1. Find the equation of the line that represents the base of the triangle.
2. Find the equation of the line that represents the height of the triangle.
3. Find the point of intersection of the line representing the height of the triangle and the line representing the base of the triangle.
4. Calculate the length of the base of the triangle using the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
5. Calculate the length of the height of the triangle using the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
6. Calculate the area using the formula $Area = \frac{1}{2}(base)(height)$.

Be sure to include the appropriate label in your answer.

- By definition, rectangles have adjacent sides that are perpendicular.
- The area of a rectangle is found using the formula *Area* = (*base*)(*height*).
- The lengths of the base and height can be found using the distance formula.
- Sometimes the answer is not a whole number. If it is not, you can approximate the value.

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Instruction

Calculating the Area of a Rectangle in the Coordinate Plane

1. Calculate the length of the base of the rectangle using the distance formula: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.
2. Calculate the length of the height of the rectangle using the distance formula: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.
3. Calculate the area using the formula *Area* = (*base*)(*height*).

Be sure to include the appropriate label in your answer.

Common Errors/Misconceptions

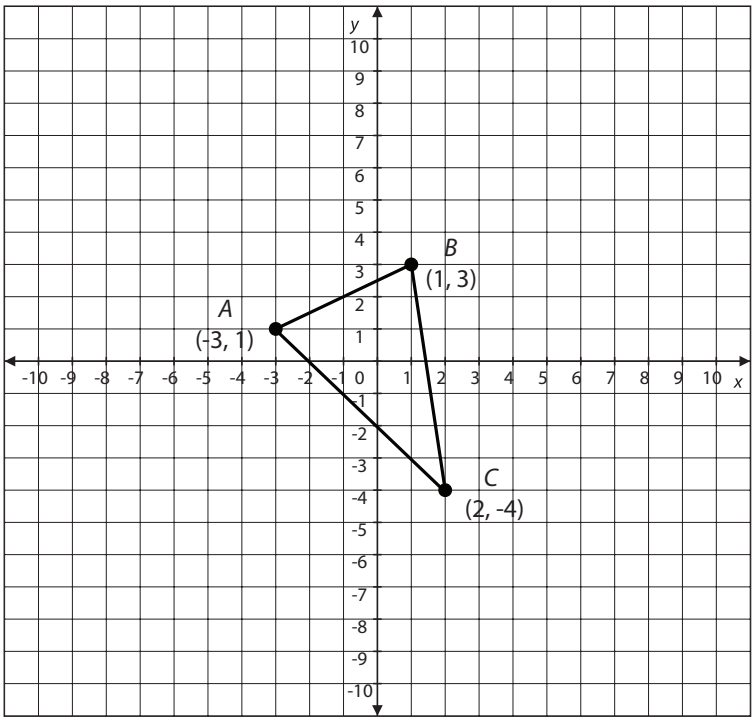
- adding *x*-values and *y*-values rather than subtracting them when using the distance formula
- incorrectly finding the height of a triangle

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Guided Practice 6.2.1

Example 1

Triangle *ABC* has vertices *A* (–3, 1), *B* (1, 3), and *C* (2, –4). Calculate the perimeter of the triangle.



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Instruction

1. Calculate the length of each side of the triangle. Use the distance formula.

Calculate the length of $\overline{AB}$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{((1) - (-3))^2 + ((3) - (1))^2}$ Substitute (–3, 1) and (1, 3).

$= \sqrt{(4)^2 + (2)^2}$ Simplify as needed.

$= \sqrt{16 + 4}$

$= \sqrt{20}$

The length of $\overline{AB}$ is $\sqrt{20}$ units.

Calculate the length of $\overline{BC}$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{((2) - (1))^2 + ((-4) - (3))^2}$ Substitute (1, 3) and (2, –4).

$= \sqrt{(1)^2 + (-7)^2}$ Simplify as needed.

$= \sqrt{1 + 49}$

$= \sqrt{50}$

The length of $\overline{BC}$ is $\sqrt{50}$ units.

Calculate the length of $\overline{CA}$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{((-3) - (2))^2 + ((1) - (-4))^2}$ Substitute (2, –4) and (–3, 1).

$= \sqrt{(-5)^2 + (5)^2}$ Simplify as needed.

$= \sqrt{25 + 25}$

$= \sqrt{50}$

The length of $\overline{CA}$ is $\sqrt{50}$ units.

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Instruction

2. Calculate the perimeter of triangle *ABC*.

Find the sum of the sides of the triangle.

perimeter = $AB + BC + CA$

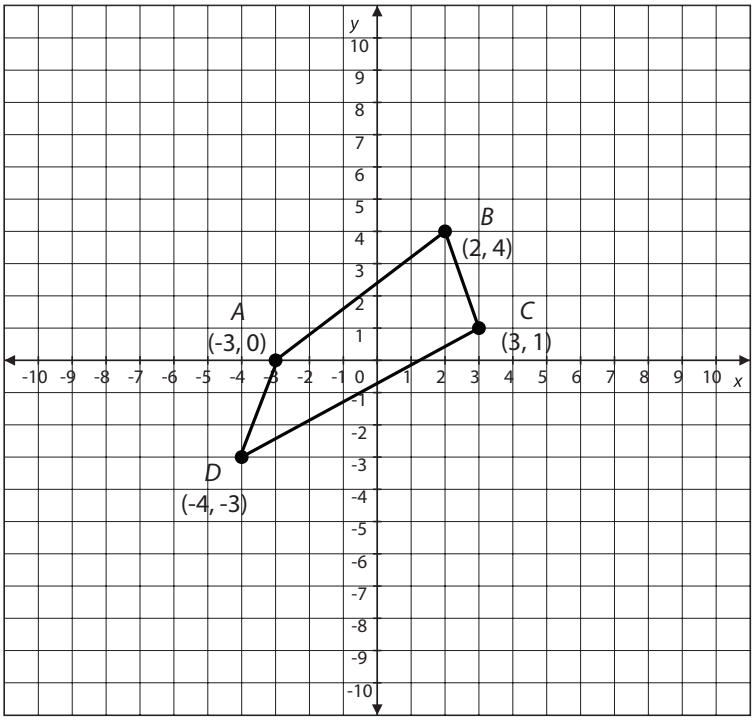
$= \sqrt{20} + \sqrt{50} + \sqrt{50}$

≈ 18.6

The perimeter of triangle *ABC* is approximately 18.6 units.

Example 2

Quadrilateral *ABCD* has vertices *A* (–3, 0), *B* (2, 4), *C* (3, 1), and *D* (–4, –3). Calculate the perimeter of the quadrilateral.



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Instruction

1. Calculate the length of each side of the quadrilateral using the distance formula.

Calculate the length of $\overline{AB}$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{((2) - (-3))^2 + ((4) - (0))^2}$ Substitute (–3, 0) and (2, 4).

$= \sqrt{(5)^2 + (4)^2}$ Simplify as needed.

$= \sqrt{25 + 16}$

$= \sqrt{41}$

The length of $\overline{AB}$ is $\sqrt{41}$ units.

Calculate the length of $\overline{BC}$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{((3) - (2))^2 + ((1) - (4))^2}$ Substitute (2, 4) and (3, 1).

$= \sqrt{(1)^2 + (-3)^2}$ Simplify as needed.

$= \sqrt{1 + 9}$

$= \sqrt{10}$

The length of $\overline{BC}$ is $\sqrt{10}$ units.

Calculate the length of $\overline{CD}$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{((-4) - (3))^2 + ((-3) - (1))^2}$ Substitute (3, 1) and (–4, –3).

$= \sqrt{(-7)^2 + (-4)^2}$ Simplify as needed.

$= \sqrt{49 + 16}$

$= \sqrt{65}$

The length of $\overline{CD}$ is $\sqrt{65}$ units.

(*continued*)

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Instruction

Calculate the length of $\overline{DA}$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{((-3) - (-4))^2 + ((0) - (-3))^2}$ Substitute (–4, –3) and (–3, 0).

$= \sqrt{(1)^2 + (3)^2}$ Simplify as needed.

$= \sqrt{1 + 9}$

$= \sqrt{10}$

The length of $\overline{DA}$ is $\sqrt{10}$ units.

2. Calculate the perimeter of quadrilateral *ABCD*.

Find the sum of the sides of the quadrilateral.

perimeter = $AB + BC + CD + DA$

$= \sqrt{41} + \sqrt{10} + \sqrt{65} + \sqrt{10}$

$= 2\sqrt{10} + \sqrt{41} + \sqrt{65}$

≈ 20.8

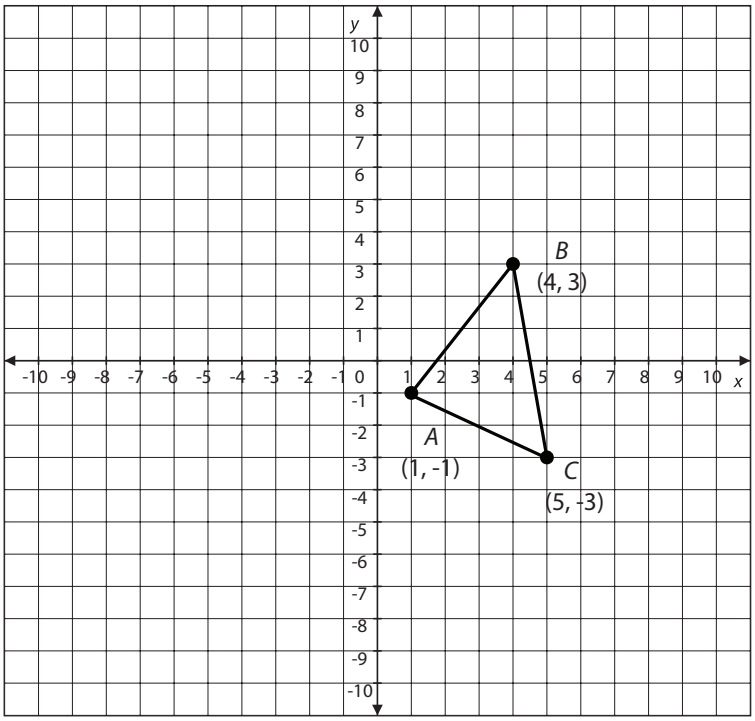
The perimeter of quadrilateral *ABCD* is $2\sqrt{10} + \sqrt{41} + \sqrt{65} \approx 20.8$ units.

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Instruction

Example 3

Triangle *ABC* has vertices *A* (1, –1), *B* (4, 3), and *C* (5, –3). Calculate the area of triangle *ABC*.



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Instruction

1. Find the equation of the line that represents the base of the triangle.

   Let $\overline{AC}$ be the base.

   Calculate the slope of the equation that represents side $\overline{AC}$.

   $m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

   $= \frac{(-3)-(-1)}{(5)-(1)}$ Substitute (1, –1) and (5, –3).

   $= \frac{-2}{4}$ Simplify as needed.

   $= -\frac{1}{2}$

   The slope of the equation that represents side $\overline{AC}$ is $-\frac{1}{2}$.

   Write the equation of the line that represents side $\overline{AC}$.

   $y - y_1 = m(x - x_1)$ Point-slope form

   $y - y_1 = -\frac{1}{2}(x - x_1)$ Substitute $-\frac{1}{2}$ for m.

   $y - (-1) = -\frac{1}{2}(x - (1))$ Substitute (1, –1) for (x_1, y_1).

   $y + 1 = -\frac{1}{2}(x - 1)$ Simplify.

   $y + 1 = -\frac{1}{2}x + \frac{1}{2}$ Distribute $-\frac{1}{2}$ over $(x - 1)$.

   $y = -\frac{1}{2}x - \frac{1}{2}$ Subtract 1 from both sides.

   The equation of the line that represents the base of the triangle is $y = -\frac{1}{2}x - \frac{1}{2}$.

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Instruction

2. Find the equation of the line that represents the height of the triangle.

The equation of the line that represents the height is perpendicular to the base; therefore, the slope of this line is the opposite reciprocal of the base.

The slope of the line representing the height is 2.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - y_1 = 2(x - x_1)$	Substitute 2 for m.
$y - (3) = 2(x - (4))$	Substitute (4, 3) for (x_1, y_1).
$y - 3 = 2(x - 4)$	Simplify.
$y - 3 = 2x - 8$	Distribute 2 over $(x - 4)$.
$y = 2x - 5$	Add 3 to both sides.

The equation of the line that represents the height of the triangle is $y = 2x - 5$.

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Instruction

3. Find the point of intersection of the line representing the height and the line representing the base of the triangle.

Substitute the value of y from the line representing the base into the equation of the line representing the height to determine the point of intersection.

$-\frac{1}{2}x-\frac{1}{2}=2x-5$	Set the expressions equal to each other.
$-\frac{1}{2}x=2x-\frac{9}{2}$	Add $\frac{1}{2}$ to both sides.
$-\frac{5}{2}x=-\frac{9}{2}$	Subtract $2x$ from both sides.
$x=\frac{9}{5}$	Divide both sides by $-\frac{5}{2}$.

The point of intersection has an x-value of $\frac{9}{5}$.

Substitute $\frac{9}{5}$ into either equation to find the y-value.

$y = 2x - 5$	Equation of the line representing height
$y=2\left(\frac{9}{5}\right)-5$	Substitute $\frac{9}{5}$ for x.
$y=\frac{18}{5}-5$	Simplify.
$y=-\frac{7}{5}$	Solve for y.

The point of intersection has a y-value of $-\frac{7}{5}$.

The point of intersection is $\left(\frac{9}{5},-\frac{7}{5}\right)$.

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Instruction

4. Calculate the length of the base, $\overline{AC}$, of the triangle.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{((5) - (1))^2 + ((-3) - (-1))^2}$ Substitute (1, –1) and (5, –3).

$= \sqrt{(4)^2 + (-2)^2}$ Simplify as needed.

$= \sqrt{16 + 4}$

$= \sqrt{20}$

The length of $\overline{AC}$ is $\sqrt{20}$ units.

5. Calculate the length of the height from point *B* to the point of intersection.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{\left(\left(\frac{9}{5}\right) - (4)\right)^2 + \left(\left(-\frac{7}{5}\right) - (3)\right)^2}$ Substitute (4, 3) and $\left(\frac{9}{5}, -\frac{7}{5}\right)$.

$= \sqrt{\left(-\frac{11}{5}\right)^2 + \left(-\frac{22}{5}\right)^2}$ Simplify as needed.

$= \sqrt{\frac{121}{25} + \frac{484}{25}}$

$= \sqrt{\frac{605}{25}}$

The length of the height is $\sqrt{\frac{605}{25}}$ units.

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Instruction

6. Calculate the area of triangle ABC.

$Area = \frac{1}{2}(base)(height)$ — Area formula for triangles

$= \frac{1}{2}\left(\sqrt{20}\right)\left(\sqrt{\frac{605}{25}}\right)$ — Substitute the lengths of the height and the base of the triangle.

$= 11$ — Evaluate with a calculator.

The area of triangle ABC is 11 square units.

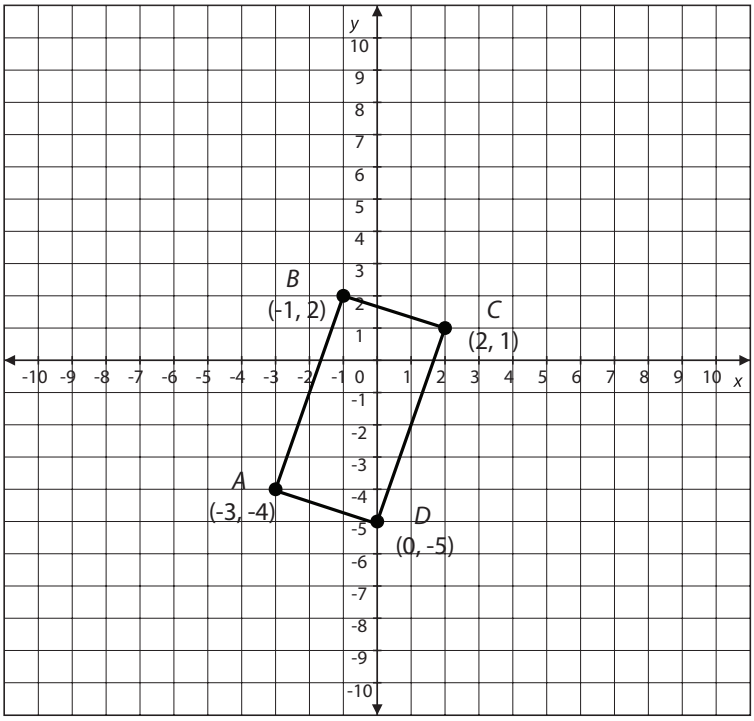


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Instruction

Example 4

Rectangle *ABCD* has vertices *A* (–3, –4), *B* (–1, 2), *C* (2, 1), and *D* (0, –5). Calculate the area of the rectangle.



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Instruction

1. Calculate the length of the base of the rectangle. Use the distance formula.

   Calculate the length of $\overline{AB}$.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$ Distance formula

$$=\sqrt{((-1)-(-3))^2+((2)-(-4))^2}$$ Substitute (–3, –4) and (–1, 2).

$$=\sqrt{(2)^2+(6)^2}$$ Simplify as needed.

$$=\sqrt{4+36}$$

$$=\sqrt{40}$$

   The length of $\overline{AB}$ is $\sqrt{40}$ units.

2. Calculate the length of the height of rectangle *ABCD* using the distance formula.

   Calculate the length of $\overline{BC}$.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$ Distance formula

$$=\sqrt{((2)-(-1))^2+((1)-(2))^2}$$ Substitute (–1, 2) and (2, 1).

$$=\sqrt{(3)^2+(-1)^2}$$ Simplify as needed.

$$=\sqrt{9+1}$$

$$=\sqrt{10}$$

   The length of $\overline{BC}$ is $\sqrt{10}$ units.

3. Calculate the area of rectangle *ABCD*.

$$Area = base \bullet height$$ Area formula for a rectangle

$$=\left(\sqrt{40}\right)\left(\sqrt{10}\right)$$ Substitute the lengths of the height and base of the triangle.

$$=\sqrt{400}$$ Simplify.

$$= 20$$

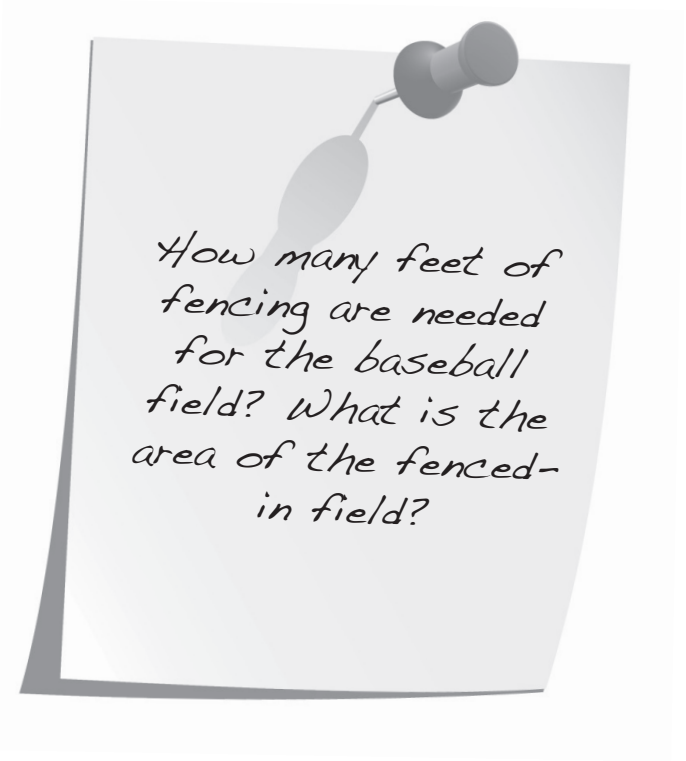
   The area of rectangle *ABCD* is 20 square units.

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Problem-Based Task 6.2.1: Building Fences

SMP	
1	2 ✓
3	4 ✓
5	6 ✓
7	8

The local recreation department has created a map of its newest baseball field. The department is planning to install a rectangular fence around the field. The corners of the field are represented on the map by the points A (–5, 10), B (10, 15), C (14, 3), and D (–1, –2). How many feet of fencing are needed for the baseball field? What is the area of the fenced-in field? Each unit on the map represents 10 feet.



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Problem-Based Task 6.2.1: Building Fences

Coaching

a. What information is needed to calculate the amount of fencing required?

b. What is the distance from corner *A* to corner *B*?

c. What is the distance from corner *B* to corner *C*?

d. What is the distance from corner *C* to corner *D*?

e. What is the distance from corner *D* to corner *A*?

f. How do you calculate the total distance around the field?

g. How many feet of fencing are needed?

h. What information is needed to calculate the area of the fenced-in field?

i. What formula is used to calculate the area of the fenced-in field?

j. What is the value for each variable of the formula?

k. What is the area of the fenced-in field?

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Instruction

Problem-Based Task 6.2.1: Building Fences

Coaching Sample Responses

a. What information is needed to calculate the amount of fencing required?

The length of each side of the fenced-in field is needed to determine the amount of fencing required.

b. What is the distance from corner *A* to corner *B*?

Use the distance formula and the points *A* (–5, 10) and *B* (10, 15).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{((10) - (-5))^2 + ((15) - (10))^2}$ Substitute (–5, 10) and (10, 15).

$= \sqrt{(15)^2 + (5)^2}$ Simplify as needed.

$= \sqrt{225 + 25}$

$= \sqrt{250}$

The distance from corner *A* to corner *B* is $\sqrt{250}$ units.

c. What is the distance from corner *B* to corner *C*?

Use the distance formula and the points *B* (10, 15) and *C* (14, 3).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{((14) - (10))^2 + ((3) - (15))^2}$ Substitute (10, 15) and (14, 3).

$= \sqrt{(4)^2 + (-12)^2}$ Simplify as needed.

$= \sqrt{16 + 144}$

$= \sqrt{160}$

The distance from corner *B* to corner *C* is $\sqrt{160}$ units.

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d. What is the distance from corner *C* to corner *D*?

The length from corner *C* to corner *D* is the same as the length from corner *A* to corner *B*; therefore, the distance is $\sqrt{250}$ units.

e. What is the distance from corner *D* to corner *A*?

The length from corner *D* to corner *A* is the same as the length from corner *B* to corner *C*; therefore, the distance is $\sqrt{160}$ units.

f. How do you calculate the total distance around the field?

The total distance around the field is found by calculating the perimeter of the fenced-in field.

$$\text{Perimeter} = \sqrt{250} + \sqrt{250} + \sqrt{160} + \sqrt{160}$$

$$= 2\sqrt{250} + 2\sqrt{160}$$

The total distance around the field is approximately 56.92 units.

g. How many feet of fencing are needed?

Each unit represents 10 feet. If the distance around the field is approximately 56.92 units, then the length is approximately 569.2 feet. It would be best to plan on 570 feet of fencing.

h. What information is needed to calculate the area of the fenced-in field?

The length of the base and the height of the rectangular field are needed to calculate the area of the fenced-in field.

i. What formula is used to calculate the area of the fenced-in field?

The formula *Area* = (*base*)(*height*) is needed to calculate the area of the fenced-in field.

j. What is the value for each variable of the formula?

The length of the base is $\sqrt{250}$ units.

The length of the height is $\sqrt{160}$ units.

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k. What is the area of the fenced-in field?

$Area = (base)(height)$	Area formula for a rectangle
$=\left(\sqrt{250}\right)\left(\sqrt{160}\right)$	Substitute values for base and height.
$= 200$	Simplify.

The area of the fenced-in field is 200 units^2.

Each unit represents 10 feet, so each square unit represents 100 feet^2.

The fenced-in field is equal to 20,000 feet^2.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

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Practice 6.2.1: Calculating Perimeter and Area

Calculate the perimeter of each of the polygons below.

1. Triangle *ABC* has vertices *A* (–2, 1), *B* (–3, 5), and *C* (3, 6).

2. Trapezoid *ABCD* has vertices *A* (–3, –2), *B* (3, 4), *C* (3, –1), and *D* (1, –3).

3. Quadrilateral *ABCD* has vertices *A* (–4, 0), *B* (–2, 3), *C* (2, 3), and *D* (2, 0).

4. Square *ABCD* has vertices *A* (–2, 2), *B* (0, 4), *C* (2, 2), and *D* (0, 0).

5. Parallelogram *ABCD* has vertices *A* (–5, 4), *B* (–1, 6), *C* (5, 2), and *D* (1, 0).

Calculate the area of each of the polygons below.

6. Rectangle *ABCD* has vertices *A* (–5, 2), *B* (–5, 4), *C* (4, 4), and *D* (4, 2).

7. Rectangle *ABCD* has vertices *A* (–4, –4), *B* (0, 2), *C* (9, –4), and *D* (5, –10).

8. Right triangle *ABC* has vertices *A* (–4, 2), *B* (–4, 6), and *C* (9, 6).

9. Triangle *ABC* has vertices *A* (–2, 5), *B* (3, 1), and *C* (3, 5).

10. Triangle *ABC* has vertices *A* (3, 5), *B* (7, 8), and *C* (5, –3).

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Progress Assessment

Circle the letter of the best answer.

1. What is the approximate perimeter of triangle *ABC* with vertices *A* (–2, 1), *B* (0, 3), and *C* (1, –1)?

   a. 42.0 units
   b. 10.6 units
   c. 5.1 units
   d. 7.9 units

2. What is the perimeter of triangle *ABC* with vertices *A* (–2, –3), *B* (3, 9), and *C* (3, –3)?

   a. 30 units
   b. 60 units
   c. 290 units
   d. 580 units

3. What is the approximate perimeter of quadrilateral *ABCD* with vertices *A* (1, –1), *B* (3, 3), *C* (9, 0), and *D* (7, –4)?

   a. 12.2 units
   b. 30.0 units
   c. 22.4 units
   d. 90 units

4. What is the approximate perimeter of quadrilateral *ABCD* with vertices *A* (–5, 2), *B* (1, 5), *C* (6, 2), and *D* (–4, –3)?

   a. 12.5 units
   b. 39.1 units
   c. 44.0 units
   d. 28.8 units

5. What is the approximate perimeter of quadrilateral *ABCD* with vertices *A* (–3, 1), *B* (1, 4), *C* (1, –2), and *D* (–3, –2)?

   a. 18 units
   b. 22 units
   c. 23 units
   d. 24 units

continued

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Assessment

6. What is the area of rectangle *ABCD* with vertices *A* (–5, 7), *B* (4, 10), *C* (6, 4), and *D* (–3, 1)?

   a. 60 units^2
   c. 240 units^2
   b. 31.2 units^2
   d. 15.8 units^2

7. What is the area of quadrilateral *ABCD* with vertices *A* (–6, 3), *B* (–4, 10), *C* (3, 8), and *D* (1, 1)?

   a. $\sqrt{53}$ units^2
   c. 53 units^2
   b. $\sqrt{848}$ units^2
   d. 26.5 units^2

8. What is the area of quadrilateral *ABCD* with vertices *A* (–7, 4), *B* (1, 7), *C* (7, –9), and *D* (–1, –12)?

   a. 73 units^2
   c. 12.2 units^2
   b. 146 units^2
   d. 8.5 units^2

9. What is the area of triangle *ABC* with vertices *A* (1, 1), *B* (5, 7), and *C* (9, 4)?

   a. 50 units^2
   c. 5 units^2
   b. 10 units^2
   d. 18 units^2

10. What is the area of the triangle with vertices *A* (–2, 6), *B* (3, 1), and *C* (2, –4)?

    a. 22.9 units^2
    c. 388.3 units^2
    b. 15 units^2
    d. 30 units^2

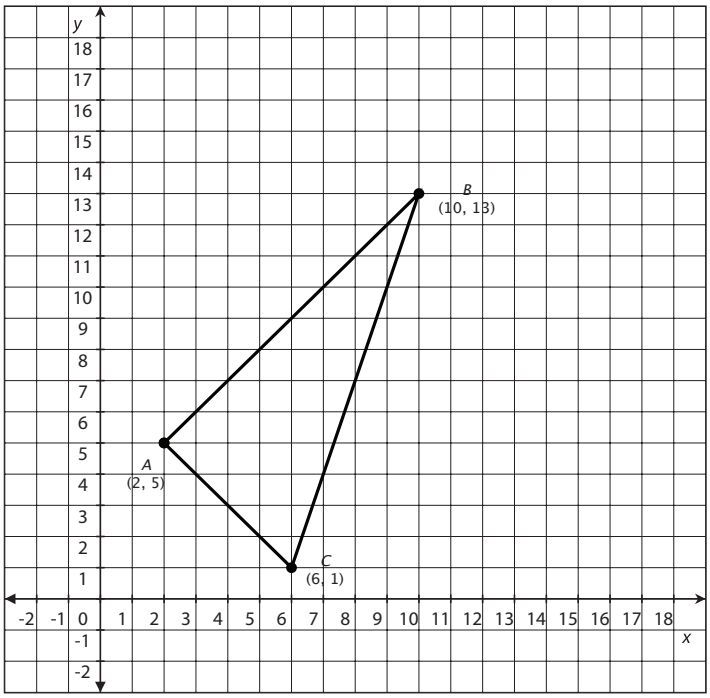
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Assessment

Read the scenario and use the graph to answer each part.

11. A landscaper plotted the design of a triangular garden on a coordinate plane with each box representing 1 foot.



a. The edging for the garden comes in 6-foot rolls. What is the least number of rolls of edging the landscaper needs to go around the perimeter of the garden? Explain your reasoning.

b. The landscaper also needs to calculate the amount of ground cloth needed. What is the area of the garden?

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Unit 6 Assessment

Circle the letter of the best answer.

1. What is the distance between the points (–2, 4) and (10, –1)?

   a. $\sqrt{89}$ units
   c. 169 units
   b. 13 units
   d. $\sqrt{119}$ units

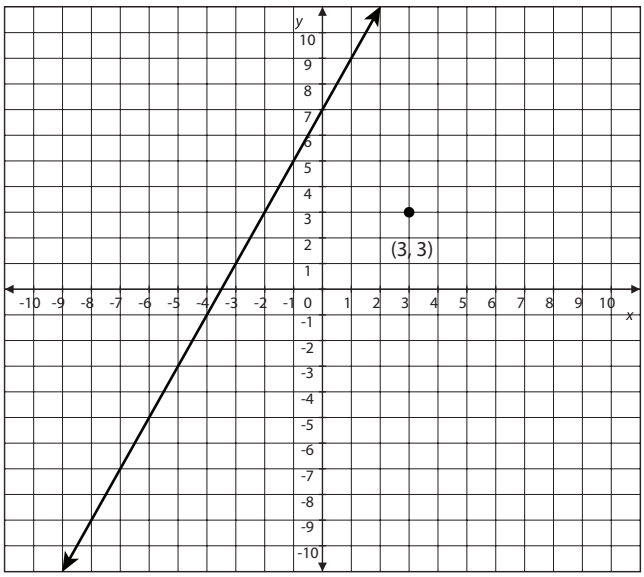
2. What is the distance between the points (–3, –5), and (1, 6)?

   a. $\sqrt{137}$ units
   c. $\sqrt{5}$ units
   b. $\sqrt{117}$ units
   d. $\sqrt{105}$ units

3. The town of Ocean Hills is located on a map at the point (9, –8). Sandy Shores is located on the same map at the point (1, 7). What is the approximate distance between the towns if each unit represents 10 miles?

   a. 127 miles
   c. 170 miles
   b. 81 miles
   d. 2,890 miles

4. Which equation represents a line that is perpendicular to the graph and passes through (3, 3)?



   a. $y=-\frac{1}{2}x+\frac{9}{2}$
   c. $y = 2x - 3$
   b. $y=\frac{1}{2}x+\frac{9}{2}$
   d. $y = 2x + 9$

continued

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5. The line through the points (–3, 8) and (2, –7) is parallel to the line through which of the following points?

   a. (2, 3) and (5, 4)
   b. (2, 3) and (–4, 1)
   c. (2, 3) and (3, 6)
   d. (2, 3) and (4, –3)

6. Triangle *ABC* is a right triangle. Sides $\overline{AB}$ and $\overline{BC}$ form the right angle. The equation of the line representing side $\overline{AB}$ is $4x + 8y = 16$. If *C* is located at the point (1, –3), what is the equation of the line representing side $\overline{BC}$?

   a. $y = -\frac{1}{2}x - \frac{5}{2}$
   b. $y = 2x - 5$
   c. $y = \frac{1}{2}x - \frac{7}{2}$
   d. $y = 2x + 5$

7. What is the approximate perimeter of quadrilateral *ABCD* with vertices *A* (–5, –1), *B* (–4, 1), *C* (2, –2), and *D* (1, –4)?

   a. 17.9 units
   b. 35.8 units
   c. 6.0 units
   d. 4.2 units

8. What is the approximate perimeter of triangle *ABC* with vertices *A* (–2, 1), *B* (4, 5), and *C* (2, –5)?

   a. 30.6 units
   b. 36.8 units
   c. 9.9 units
   d. 24.6 units

9. What is the perimeter of triangle *ABC* with vertices *A* (3, 6), *B* (7, 3), and *C* (3, 3)?

   a. 6 units
   b. 12 units
   c. 18 units
   d. 24 units

10. What is the area of triangle *ABC* with vertices *A* (–4, 7), *B* (0, 9), and *C* (–4, 5)?

    a. 2 units^2
    b. 12.1 units^2
    c. 4 units^2
    d. 16 units^2

continued

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11. What is the area of triangle ABC with vertices A (–2, 1), B (–2, 3), and C (2, 1)?

    a. 4 units^2
    b. 8 units^2
    c. 10.5 units^2
    d. 21 units^2

12. What is the area of rectangle $ABCD$ with vertices A (–6, 3), B (2, 5), C (3, 1), and D (–5, –1)?

    a. 21.6 units^2
    b. 43.2 units^2
    c. 17 units^2
    d. 34 units^2

Read the scenario, and then solve the problem.

13. A right triangle has exactly one 90° angle. The equation of the line that represents $\overline{AB}$ of triangle ABC is $y = x + 3$. Vertex A is located at the point (–3, 0) and vertex B is located at the point (3, 6). Is a right triangle formed if vertex C is located at the point (–1, –6)? If a right triangle is formed, what is the equation of the line that represents $\overline{AC}$?

Read the scenario, and then complete each part.

14. Segment $\overline{AC}$ has endpoints A (–4, 5) and C (4, –11). A perpendicular bisector of $\overline{AC}$ passes through the point (8, 1). Point B is the point of intersection of the perpendicular bisector and $\overline{AC}$.

    a. What is the equation of the perpendicular bisector?

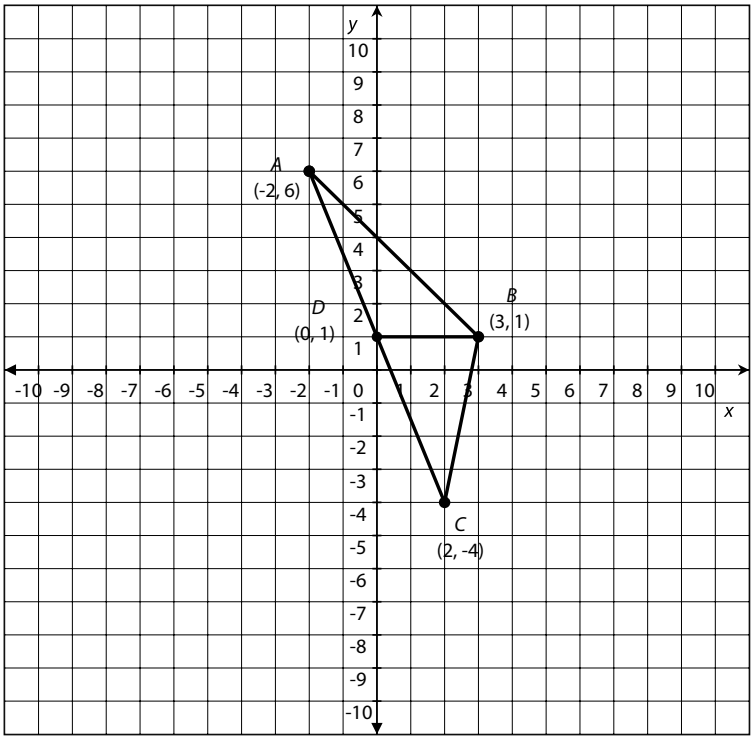
    b. What are the coordinates of point B? Explain how you found these coordinates.

    c. Using the distance formula, verify that $AB = BC$.

continued

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Use the graph below and the given information to complete problem 15.



15. The median of a triangle is the line segment from one vertex to the midpoint of the opposite side. The midpoint is the point that is half the distance between the two endpoints of a segment.

    a. Verify that $\overline{BD}$ is the median of the triangle.

    b. What is the area of triangle *ABD*?

    c. What is the area of triangle *ABC*?

    d. If the area of triangle *CBD* is 7.5 units^2, how does the area of the smaller triangles compare to the area of the original triangle?

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Answer Key

Lesson 1: Slope and Distance

Pre-Assessment, p. U6-1

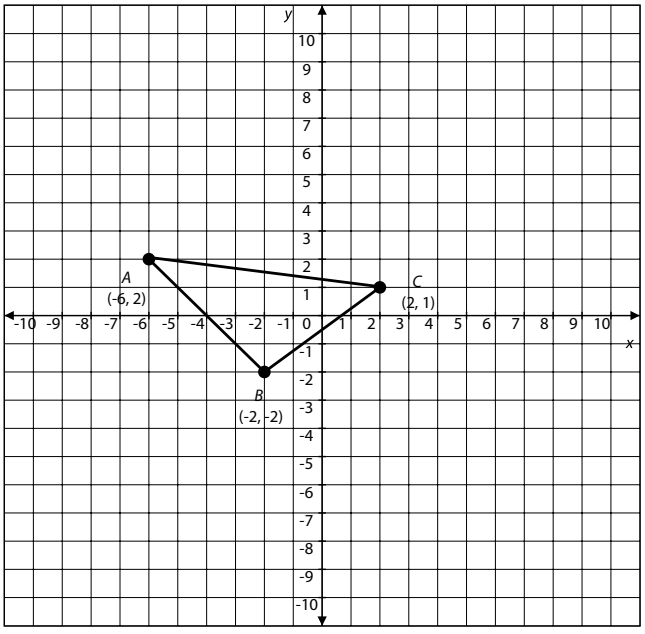
1. b
2. a
3. d
4. b
5. c

Warm-Up 6.1.1, p. U6-4

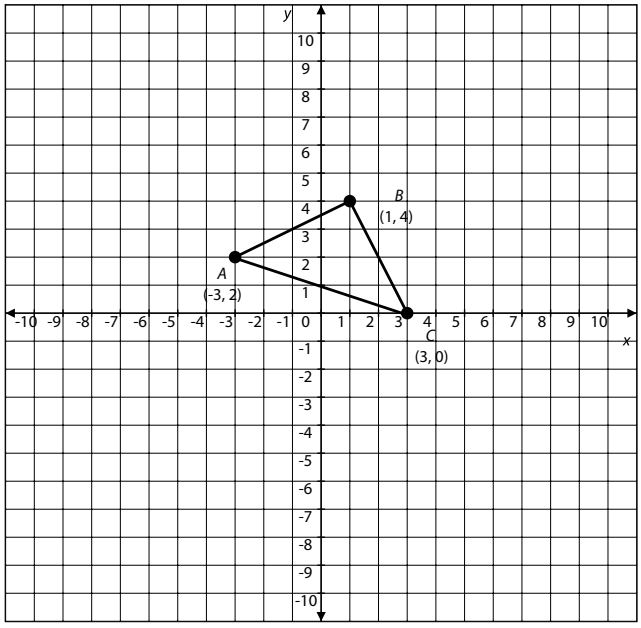
1. 600 miles per hour
2. 5,400 miles

Practice 6.1.1: Using Coordinates to Prove Geometric Theorems with Slope and Distance, p. U6-31

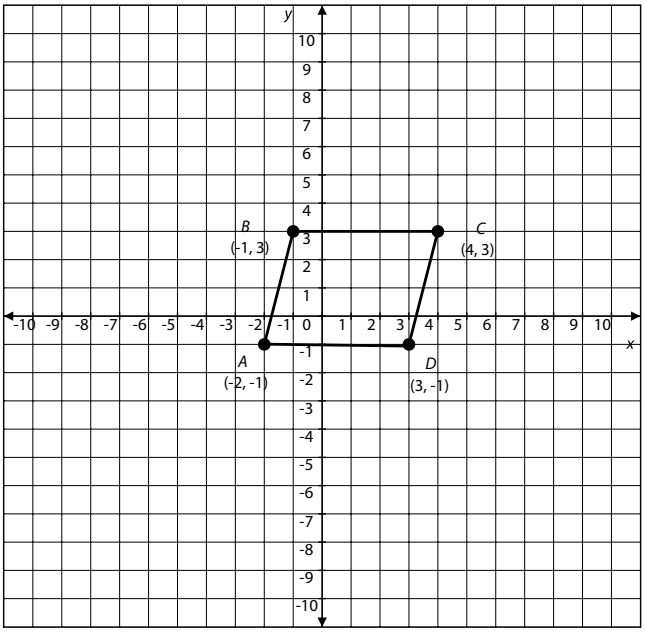
1. $\sqrt{125} \approx 11.2$ units
2. $\sqrt{82} \approx 9.1$ units
3. Triangle ABC is not a right triangle. The slope of $\overline{AB} = -1$, the slope of $\overline{BC} = \frac{3}{4}$, and the slope of $\overline{AC} = -\frac{1}{8}$. None of the slopes are opposite reciprocals of each other; therefore, there are no perpendicular sides. The distance between the points A and C is not equal to the sum of the squares of the lengths of the remaining sides.



4. Triangle ABC is a right triangle. The slope of $\overline{AB} = \frac{1}{2}$, the slope of $\overline{BC} = -2$, and the slope of $\overline{AC} = -\frac{1}{3}$. The slopes of $\overline{AB}$ and $\overline{BC}$ are opposite reciprocals of each other; therefore, they are perpendicular and create a right triangle. The distance between the points A and C is equal to the sum of the squares of the lengths of the remaining sides.

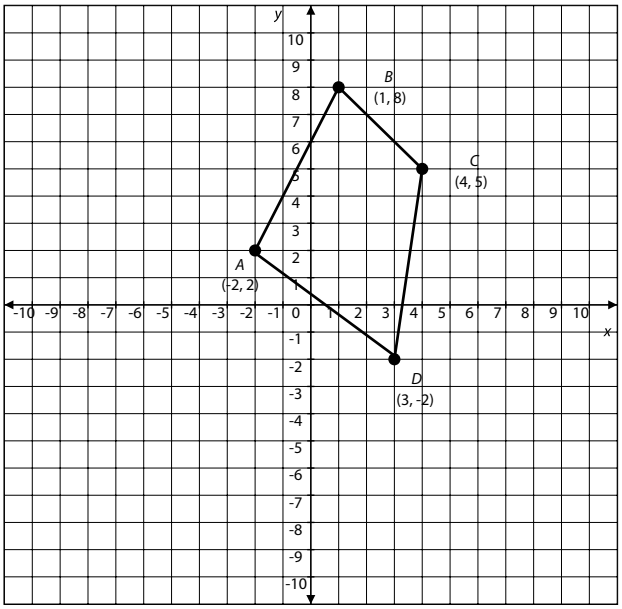


5. Quadrilateral $ABCD$ is a parallelogram. The slopes of $\overline{AB}$ and $\overline{CD}$ are both 4; thus, the segments are parallel. $\overline{BC}$ and $\overline{AD}$ both have a slope of 0; therefore, they are parallel.

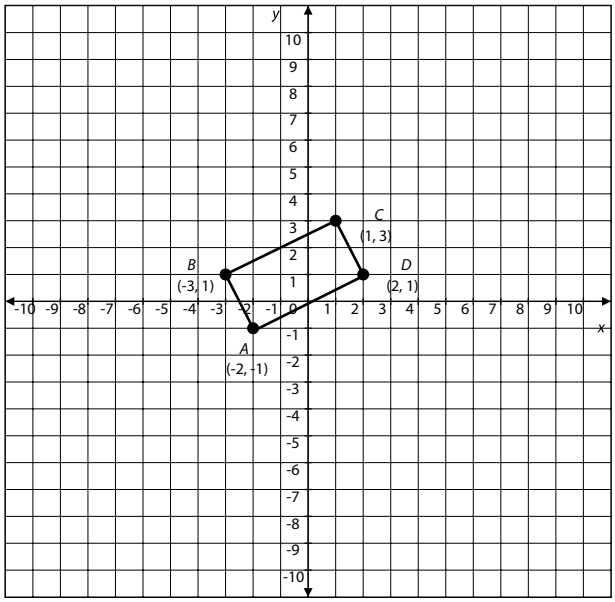


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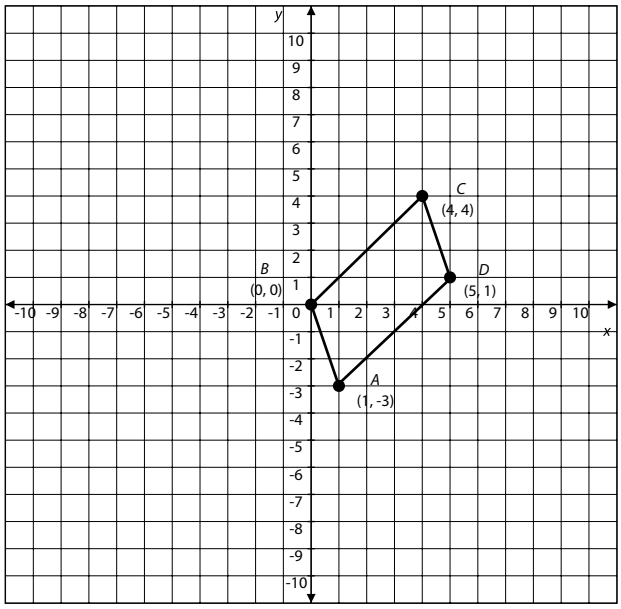
6. Quadrilateral *ABCD* is not a parallelogram. The slope of $\overline{AB}$ = 2, the slope of $\overline{CD}$ = 7, the slope of $\overline{BC}$ = –1, and the slope of $\overline{AD} = -\frac{4}{5}$. Opposite sides do not have equal slopes and are therefore not parallel.



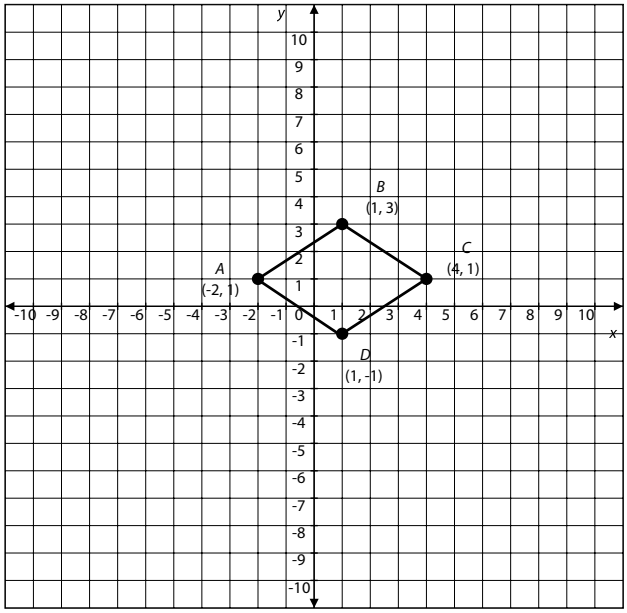
7. Quadrilateral *ABCD* is a rectangle. $\overline{AB}$ and $\overline{CD}$ are opposite sides and both have a slope of –2. $\overline{BC}$ and $\overline{AD}$ are opposite sides and both have a slope of $\frac{1}{2}$. Opposite sides are equal and are therefore parallel. The slope of side $\overline{AB}$ is the opposite reciprocal of the slope of sides $\overline{BC}$ and $\overline{AD}$; therefore, they are perpendicular. Side $\overline{CD}$ is also the opposite reciprocal of the slope of sides $\overline{BC}$ and $\overline{AD}$; therefore, $\overline{CD}$ is perpendicular to $\overline{BC}$ and $\overline{AD}$. Both $\overline{AB}$ and $\overline{CD}$ are $\sqrt{5}$ units long, and both $\overline{BC}$ and $\overline{AD}$ are $\sqrt{20}$ units long; therefore, opposite sides are congruent.



8. Quadrilateral *ABCD* is not a rectangle. $\overline{AB}$ and $\overline{CD}$ are opposite sides and both have a slope of –3. $\overline{BC}$ and $\overline{AD}$ are opposite sides and both have a slope of 1. Opposite sides are equal and are therefore parallel, but the quadrilateral does not have 90° angles. The slope of side $\overline{AB}$ is not the opposite reciprocal of the slope of sides $\overline{BC}$ and $\overline{AD}$; therefore, they are not perpendicular. The slope of side $\overline{CD}$ is not the opposite reciprocal of the slope of sides $\overline{BC}$ and $\overline{AD}$; therefore, $\overline{CD}$ is not perpendicular to $\overline{BC}$ and $\overline{AD}$.

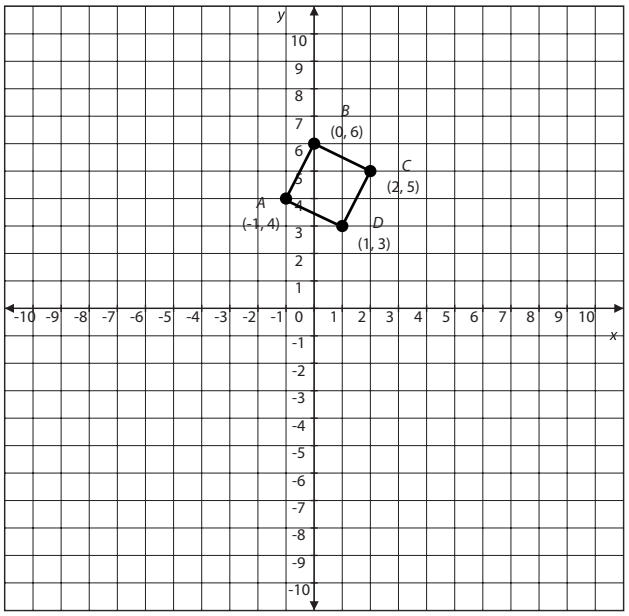


9. Quadrilateral *ABCD* is not a square. The length of each side is $\sqrt{13}$ and opposite sides do have the same slope. Sides $\overline{AB}$ and $\overline{DC}$ are opposite and both have a slope of $\frac{2}{3}$, and sides $\overline{AD}$ and $\overline{BC}$ are opposite and both have a slope of $-\frac{2}{3}$, but $\frac{2}{3}$ and $-\frac{2}{3}$ are opposites, but not reciprocals; therefore, the sides are not perpendicular and the angles are not 90°.



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10. Quadrilateral *ABCD* is a square. The length of each side is $\sqrt{5}$ and opposite sides do have the same slope, so they are parallel. Sides $\overline{AB}$ and $\overline{DC}$ are opposite and both have a slope of 2; sides $\overline{AD}$ and $\overline{BC}$ are opposite and both have a slope of $-\frac{1}{2}$. 2 and $-\frac{1}{2}$ are opposite reciprocals; therefore, the sides are perpendicular and the angles are 90°.



Warm-Up 6.1.2, p. U6-32

1. Answers may vary. Each equation should have a slope of –5. One possible answer is $y = –5x + 2$. *y*-intercepts must be different for lines to be parallel and not coinciding.
2. Answers may vary. Each equation should have a slope of –5. One possible answer is $y = –5x + 7$. *y*-intercepts must be different for lines to be parallel and not coinciding.

Practice 6.1.2: Working with Parallel and Perpendicular Lines, p. U6-50

1. $y = –2x + 7$
2. $y=\frac{-4}{3}x-\frac{2}{3}$
3. $y=\frac{1}{4}x-2$
4. $y = –x – 1$
5. $\sqrt{2} \approx 1.4$ units
6. $y=-\frac{3}{4}x+7$
7. $y = –5x – 34$
8. $y=\frac{1}{5}x+10$
9. $y=\frac{4}{3}x-2$
10. The shortest distance is the line that is perpendicular from the train station to Union Street. This distance is approximately 647 yards.

Progress Assessment, p. U6-52

1. a
2. d
3. b
4. a
5. c
6. a
7. d
8. b
9. c
10. a
11. $y=\frac{1}{4}x+\frac{9}{4}$; 97 yards

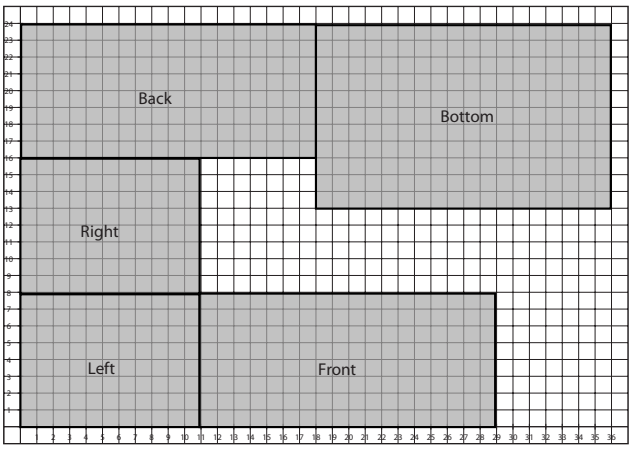
Lesson 2: Lines and Line Segments

Pre-Assessment, p. U6-56

1. d
2. a
3. b
4. b
5. c

Warm-Up 6.2.1, p. U6-58

1. 662 inches^2
2. Yes, there will be enough plywood. One way to cut the pieces is:



Practice 6.2.1: Calculating Perimeter and Area, p. U6-82

1. $\sqrt{50}+\sqrt{17}+\sqrt{37} \approx 17.3$ units
2. $5+\sqrt{128}+\sqrt{17} \approx 20.4$ units
3. $13+\sqrt{13} \approx 16.6$ units
4. $\sqrt{128} \approx 11.3$ units
5. $\sqrt{80}+\sqrt{208} \approx 23.4$ units
6. 18 units^2
7. 78 units^2
8. 26 units^2
9. 10 units^2
10. 19 units^2

Progress Assessment, p. U6-83

1. b
2. a
3. c
4. d
5. a
6. a
7. c
8. b
9. d
10. b
11. a. The perimeter of the garden is approximately 30 feet. The landscaper will need at least 5 rolls of edging for the garden.
    b. The area of the garden is 32 ft^2.

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Unit Assessment

p. U6-86

1. b
2. a
3. c
4. a
5. d
6. b
7. a
8. d
9. b
10. c
11. a
12. d
13. No, a right triangle is not formed because the slopes of the sides are not opposite reciprocals indicating perpendicular lines and a right angle.
14. Answers:
    a. $y = (1/2)x - 3$
    b. B (0, –3); answers may vary. Sample answer: The point of intersection is the point that satisfies both equations or makes both equations true.
    c. $AB = 8.9$ units and $BC = 8.9$ units
15. Answers:
    a. $AD = \sqrt{29}$ units^2 = $DC = \sqrt{29}$ units^2; $\overline{BD}$ intersects side $\overline{AC}$ so that $\overline{AD} \cong \overline{DC}$.
    b. The area of triangle ABD is 7.5 units^2.
    c. The area of triangle ABC is 15 units^2.
    d. The sum of the area of the two smaller triangles is equal to the area of the original triangle.

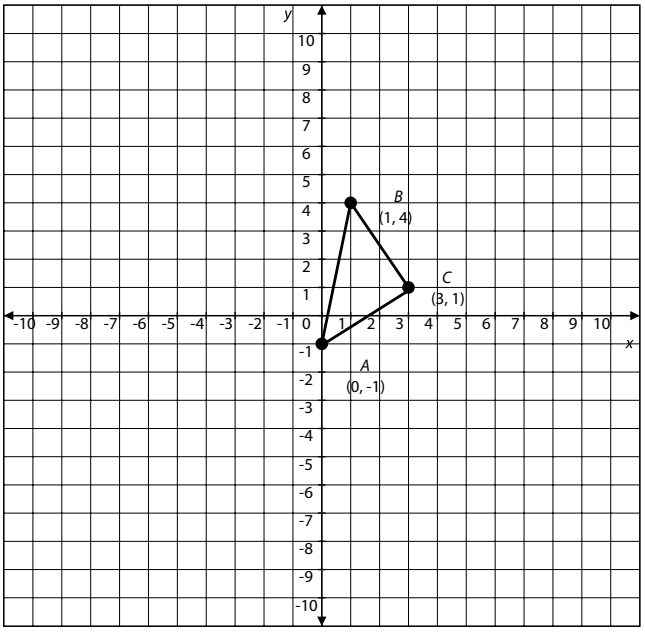
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Student Book Answer Key

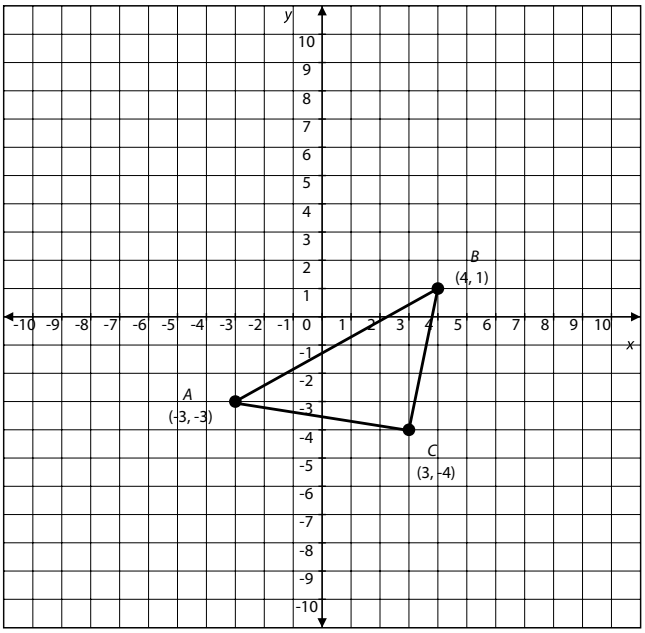
Lesson 1: Slope and Distance

Practice 6.1.1: Using Coordinates to Prove Geometric Theorems with Slope and Distance, p. U6-18

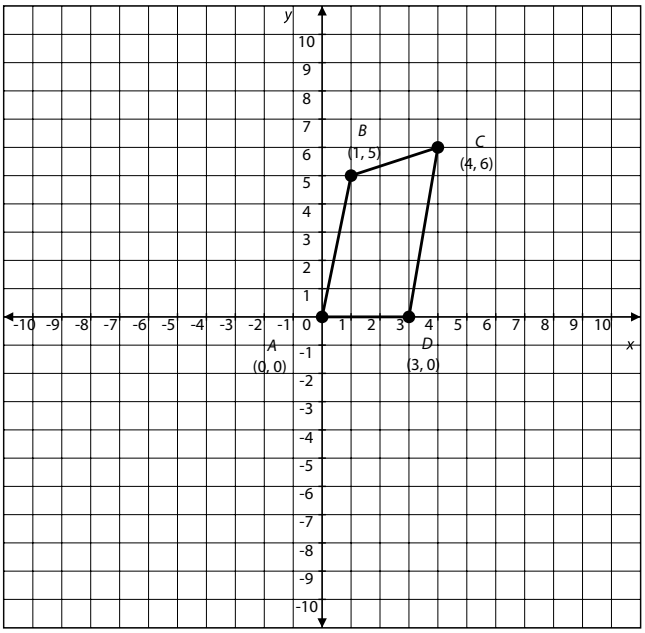
1. $\sqrt{20} \approx 4.5$ units
2. $\sqrt{50} \approx 7.1$ units
3. Triangle *ABC* is a right triangle. The slope of $\overline{AB} = 5$, the slope of $\overline{BC} = -\frac{3}{2}$, and the slope of $\overline{AC} = \frac{2}{3}$. The slopes of $\overline{BC}$ and $\overline{AC}$ are opposite reciprocals of each other; therefore, they are perpendicular and create a right triangle. The distance between points *A* and *B* is equal to the square of the lengths of the remaining sides.



4. Triangle *ABC* is not a right triangle. The slope of $\overline{AB} = \frac{4}{7}$, the slope of $\overline{BC} = 5$, and the slope of $\overline{AC} = -\frac{1}{6}$. None of the slopes are opposite reciprocals of each other; therefore, there are no perpendicular sides. The distance between the points *A* and *B* is not equal to the square of the lengths of the remaining sides.

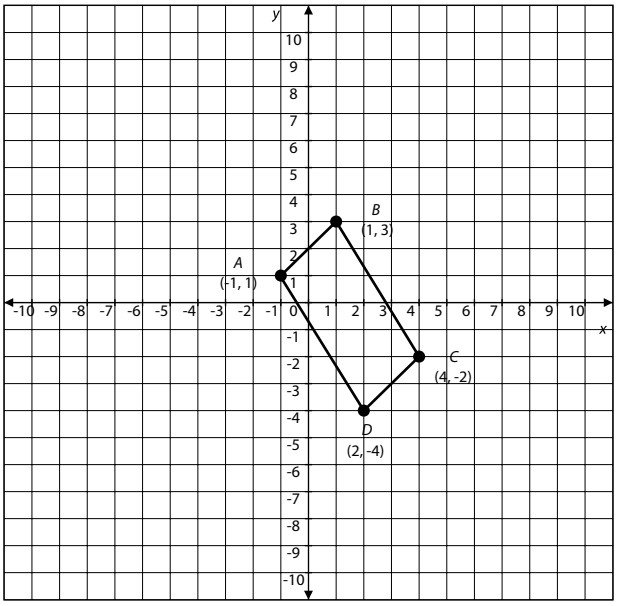


5. Quadrilateral *ABCD* is not a parallelogram. The slope of $\overline{AB} = 5$, the slope of $\overline{CD} = 6$, the slope of $\overline{BC} = \frac{1}{3}$, and the slope of $\overline{AD} = 0$. Opposite sides do not have equal slopes and are therefore not parallel.

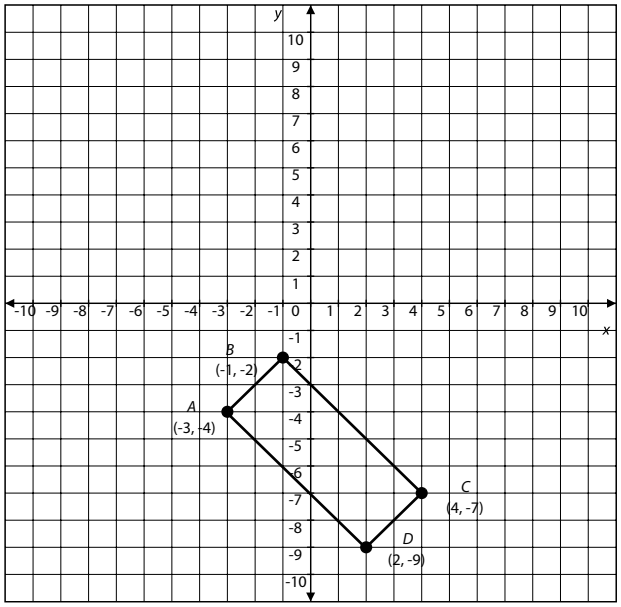


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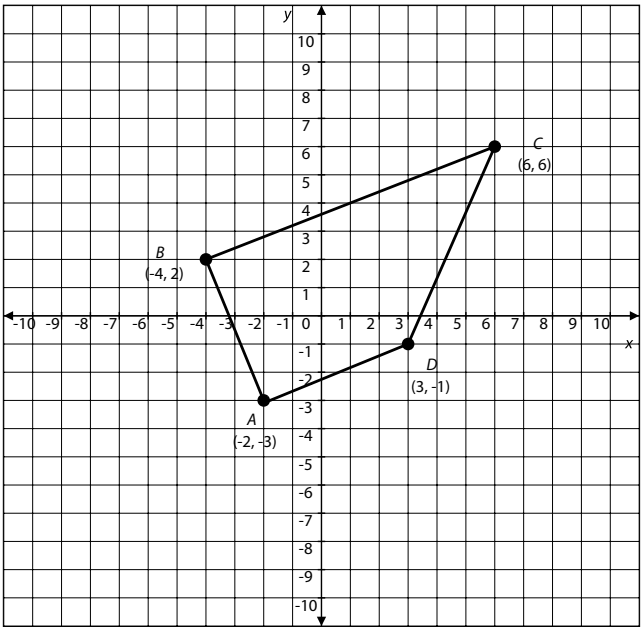
6. Quadrilateral *ABCD* is a parallelogram. Both $\overline{AB}$ and $\overline{CD}$ have a slope of 1, and both $\overline{BC}$ and $\overline{AD}$ have a slope of $-\frac{5}{3}$. The slopes of the opposite sides are equal; therefore, they are parallel.



7. Quadrilateral *ABCD* is a rectangle. $\overline{AB}$ and $\overline{CD}$ are opposite sides and both have a slope of 1. $\overline{BC}$ and $\overline{AD}$ are opposite sides and both have a slope of –1. The slopes of the opposite sides are equal and are therefore parallel. The slope of $\overline{AB}$ is the opposite reciprocal of the slope of sides $\overline{BC}$ and $\overline{AD}$; therefore, they are perpendicular. $\overline{CD}$ is also the opposite reciprocal of the slope of $\overline{BC}$ and $\overline{AD}$; therefore, $\overline{CD}$ is perpendicular to $\overline{BC}$ and $\overline{AD}$. $\overline{AB}$ and $\overline{CD}$ are both $\sqrt{8}$ units long, and $\overline{BC}$ and $\overline{AD}$ are both $\sqrt{50}$ units long; therefore, opposite sides are congruent.

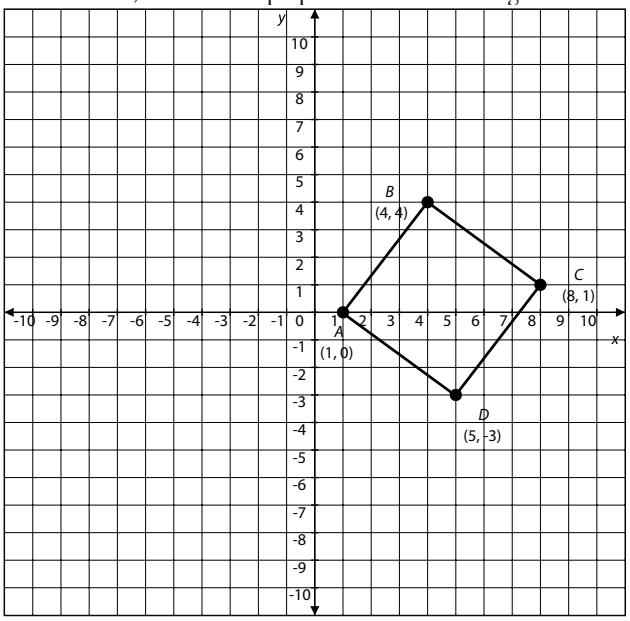


8. Quadrilateral *ABCD* is not a rectangle. $\overline{AB}$ and $\overline{CD}$ are opposite sides. The slope of $\overline{AB}=-\frac{5}{2}$. The slope of $\overline{CD}=\frac{7}{3}$. The slopes are not equal; therefore, they are not parallel. $\overline{BC}$ and $\overline{AD}$ are opposite sides and both have a slope of $\frac{2}{5}$. Opposite sides are equal and are therefore parallel, but not all of the angles of the quadrilateral are 90°. The slope of $\overline{AB}$ is the opposite reciprocal of the slope of $\overline{BC}$ and $\overline{AD}$; therefore, they are perpendicular. However, $\overline{CD}$ is not the opposite reciprocal of the slope of $\overline{BC}$ and $\overline{AD}$; therefore, $\overline{CD}$ is not perpendicular to $\overline{BC}$ and $\overline{AD}$. The length of $\overline{AB}=\sqrt{29}$ units and $\overline{CD}=\sqrt{58}$ units. These opposite sides are not congruent. The length of $\overline{BC}=\sqrt{116}$ units and $\overline{AD}=\sqrt{29}$ units. These opposite sides are also not congruent.

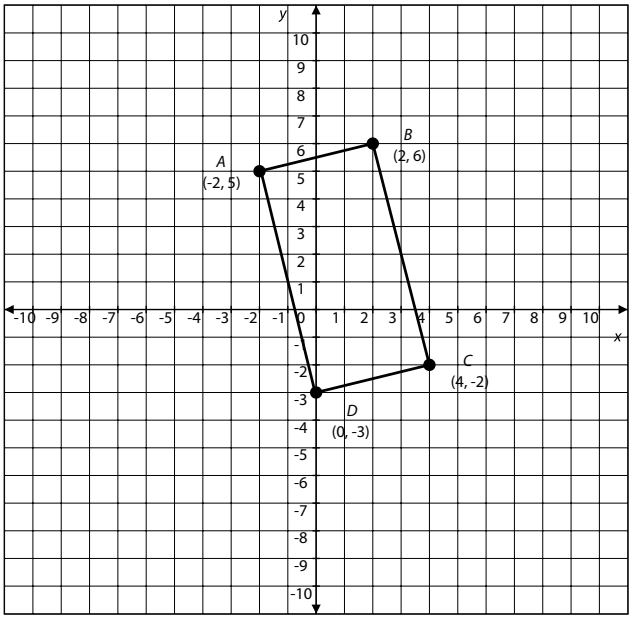


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9. Quadrilateral *ABCD* is a square. The length of each side is 5 units and opposite sides do have the same slope, so they are parallel. $\overline{AB}$ and $\overline{DC}$ are opposite and both have a slope of $\frac{4}{3}$, and $\overline{AD}$ and $\overline{BC}$ are opposite and both have a slope of $-\frac{3}{4}$. $\frac{4}{3}$ and $-\frac{3}{4}$ are opposite reciprocals; therefore, the sides are perpendicular and the angles are 90°.



10. Quadrilateral *ABCD* is not a square. The length of $\overline{AB}$ and $\overline{DC}$ is $\sqrt{17}$ units, and the length of $\overline{AD}$ and $\overline{BC}$ is $\sqrt{68}$ units; therefore, all sides are not congruent. Opposite sides do have the same slope. $\overline{AB}$ and $\overline{DC}$ are opposite sides and both have a slope of $\frac{1}{4}$, and $\overline{AD}$ and $\overline{BC}$ are opposite sides and both have a slope of –4, so the angles are 90° and are perpendicular.



Practice 6.1.2: Working with Parallel and Perpendicular Lines, p. U6-30

1. $y = 3x - 8$
2. $y = -\frac{1}{4}x$
3. $y = \frac{1}{3}x + 7$
4. $y = -2x + 2$
5. $\sqrt{\frac{80}{25}}$ or ≈ 1.79 units
6. $y = \frac{1}{2}x - \frac{7}{2}$
7. $y = \frac{1}{3}x + \frac{7}{3}$
8. $y = -3x - 16$
9. $y = -2x - 3$
10. The shortest distance is the line that is perpendicular from the grocery store to Maple Street. This distance is $\sqrt{\frac{81}{10}} \approx 2.85$ units or 285 yards.

Lesson 2: Lines and Line Segments

Practice 6.2.1: Calculating Perimeter and Area, p. U6-47

1. $\sqrt{18} + \sqrt{116} \approx 15.0$ units
2. $\sqrt{500} \approx 22.4$ units
3. $\sqrt{208} \approx 14.4$ units
4. $19 + \sqrt{17} \approx 23.1$ units
5. $\sqrt{612} \approx 24.7$ units
6. 36 units^2
7. 20 units^2
8. 16 units^2
9. 10.5 units^2
10. 6 units^2

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Station Activities Set 1: Parallel Lines, Slopes, and Equations

Instruction

Goal: To provide opportunities for students to develop concepts and skills related to using coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines

Common Core State Standards

G–GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*

G–GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Student Activities Overview and Answer Key

Station 1

At this station, students explore parallel lines. They determine whether or not a given set of lines is parallel, and explain how they know the answer. Then they apply that by drawing their own parallel lines.

Answers

1. yes
2. They have the same slope; they are always the same distance apart.
3. 5 units
4. Check graphs for a line with a slope of –1.
5. It has the same slope/is always the same distance away.

Station 2

Students will be given graph paper, a ruler, a red marker, and a blue marker. Students will use the graph paper and ruler to construct straight lines. Then they will use the red and blue markers to depict the rise and run of the slope of the lines. They will determine whether two lines are parallel by constructing the lines, or given two points on the line, or given the slope of the lines.

Answers

1. $m = 2/3$
2. $m = 2/3$
3. $m = 4/5$
4. Line 1 and Line 2, because they have the same slope of 2/3.

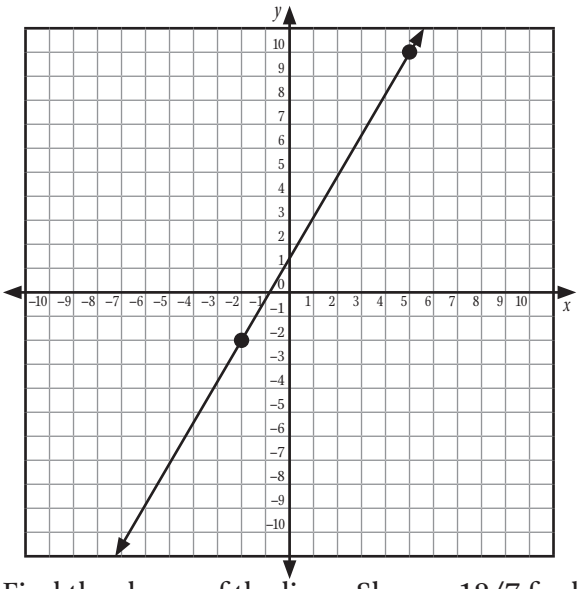
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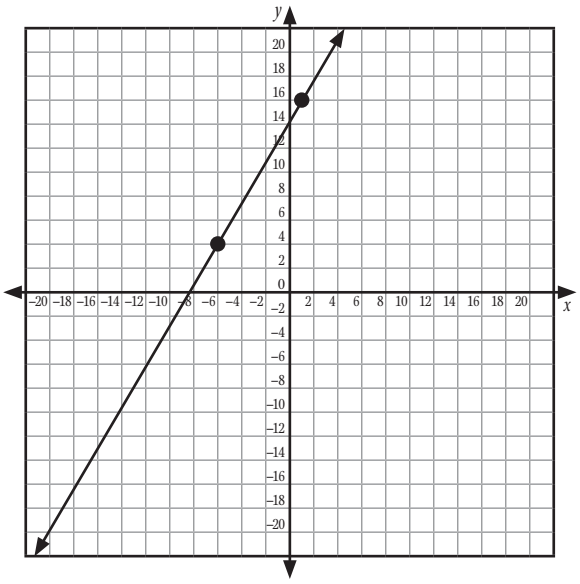
5. No; the slopes are not equal.
6. Yes; the slopes are equal.
7. Yes; the slopes are equal.
8. No; the slopes are not equal.

Station 3

Students will be given uncooked spaghetti noodles, graph paper, and a ruler. Students will use the graph paper and ruler to construct straight lines. They will use the spaghetti noodles to model parallel lines. They will find the slope of the lines to determine if the lines are parallel. Then they will construct a line perpendicular to the two lines to determine if the lines are parallel.

Answers

1. 

2. 

3. Find the slopes of the lines. Slope = 12/7 for both lines, so, yes, the lines are parallel.
4. Slope = –7/12. Yes, because the slope is the opposite reciprocal. Yes, because the slope is the opposite reciprocal.
5. 90°

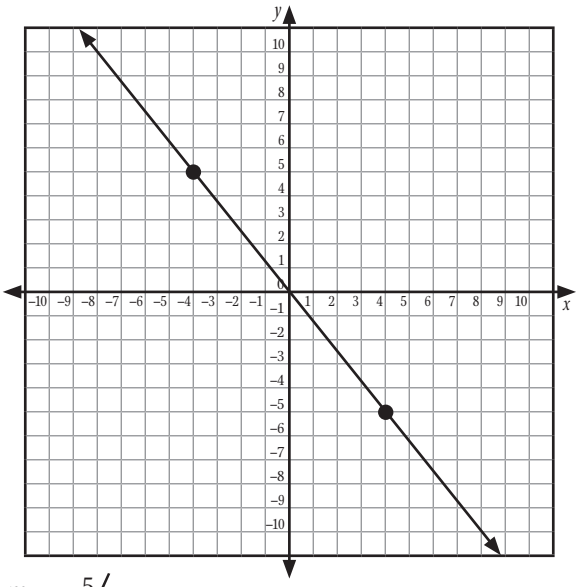
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Instruction

Station 4

Students will be given graph paper and a ruler. Students will construct a straight line and find its slope. Then they will use the slope and the point-slope form of a linear equation to construct two lines that are parallel to the original line. They will write the equations for these parallel lines.

Answers

1. 

2. $m = -\frac{5}{4}$
3. $y = -\frac{5}{4}x$
4. $m = -\frac{5}{4}$
5. Create a line with the same slope. Student graphs will vary but should be parallel.
6. Answers will vary. Possible answer: $y = -\frac{5}{4}x + 10$
7. Answers will vary.
8. Answers will vary. Possible answer: $y = -\frac{5}{4}x - 8$

Materials List/Setup

Station 1 ruler for each group member

Station 2 graph paper; ruler; red marker; blue marker

Station 3 uncooked spaghetti noodles; graph paper; ruler

Station 4 graph paper; ruler

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. If two lines have the same slope, are they parallel or perpendicular?
2. If two lines have slopes that are opposite reciprocals, are they parallel or perpendicular?
3. How can you prove that two lines are parallel?
4. How can you write equations of lines parallel to a given line?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. parallel
2. perpendicular
3. Demonstrate that they have the same slope.
4. Find the slope of the line. Use the point-slope form to find equations of parallel lines.

Possible Misunderstandings/Mistakes

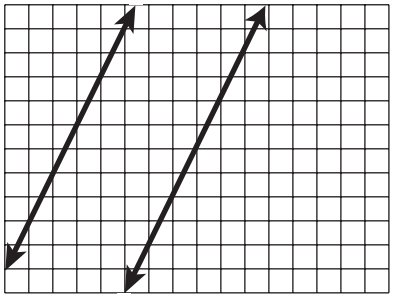
- Not realizing that parallel lines have equal slope
- Not realizing that perpendicular lines have slopes that are opposite reciprocals of each other
- Not recognizing that the equations of parallel lines must have the same slope

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Station 1

At this station, you will find rulers. Use these to help you determine whether or not the following lines are parallel.

Look at the graph below.

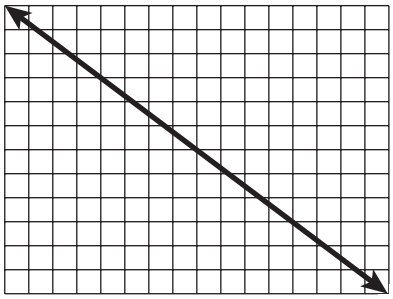


1. Are these lines parallel?

2. Explain two ways you can tell lines are parallel.

3. What is the shortest distance between these two lines?

4. Draw a line that is parallel to the given line below.



5. How do you know your line is parallel?

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Station 2

At this station, you will find graph paper, a ruler, a red marker, and a blue marker. As a group, construct an *x*- and *y*-axis on your graph paper.

1. On your graph paper, construct a straight line that passes through (–2, –5) and (7, 1).

   What is the slope of this line?

   Show how you found this slope by using the red and blue markers to represent rise and run on your graph.

2. Construct a second line that passes through (0, 4) and (18, 16).

   What is the slope of this line?

   Show how you found this slope by using the red and blue markers to represent rise and run on your graph.

3. Construct a third line that passes through (0, 0) and (10, 8).

   What is the slope of this line?

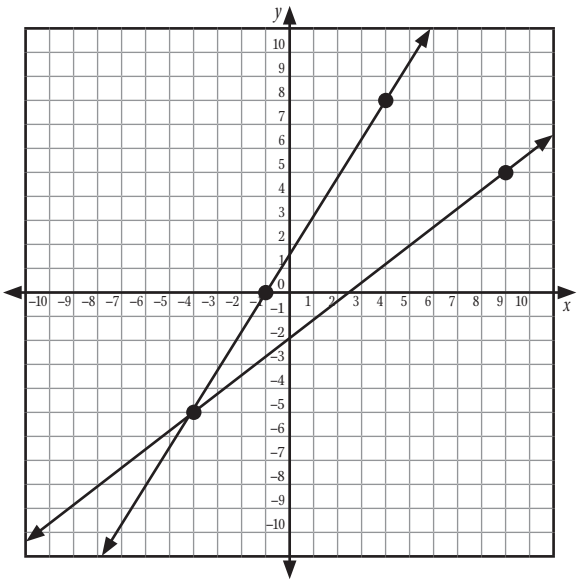
   Show how you found this slope by using the red and blue markers to represent rise and run on your graph.

4. Of the three straight lines you created, which two lines do you think are parallel? Explain your answer.

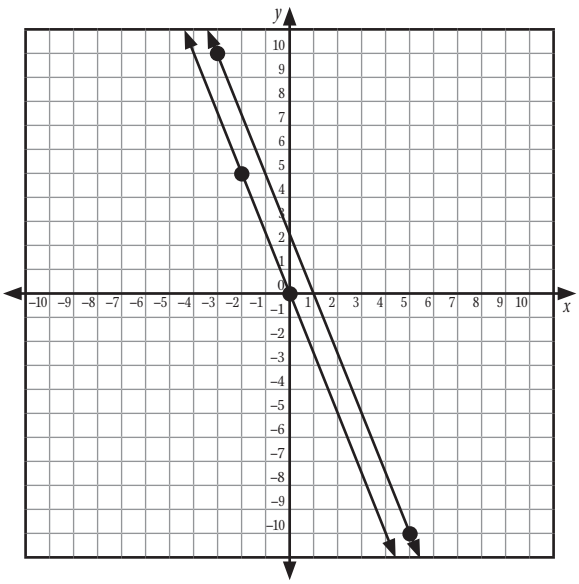
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For problems 5–8, determine whether the lines are parallel. Answer *yes* or *no*. Justify your answers.

5. 

Are the lines parallel?

6. 

Are the lines parallel?

7. One line has a slope of $\frac{2}{3}$ and another line has a slope of $\frac{16}{24}$.
Are the lines parallel?

8. One line has a slope of $-\frac{1}{2}$ and another line has a slope of $\frac{1}{2}$.
Are the lines parallel?

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Station 3

At this station, you will find spaghetti noodles, graph paper, and a ruler. Work as a group to construct the lines and answer the questions.

1. On your graph paper, construct a straight line that passes through (–2, –2) and (5, 10). Place a spaghetti noodle on this line.

2. On the same graph, construct a straight line that passes through (–6, 4) and (1, 16). Place a spaghetti noodle on this line.

By looking at the spaghetti noodles, the two lines may appear to be parallel, but are they?

3. How can you use the points on each line to determine if the two lines are parallel? Show your work. Are the lines parallel? Why or why not?

4. On the same graph, construct a straight line that passes through (5, 10) and (17, 3). Place a spaghetti noodle on this line.

   What is the slope of this line?

   Is this line perpendicular to the line you created in problem 1? Why or why not?

   Is this line also perpendicular to the line you created in problem 2? Why or why not?

5. Perpendicular lines create four angles that each measure _____.

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Station 4

At this station, you will find graph paper and a ruler. Work as a group to construct the graphs and answer the questions.

1. On your graph paper, graph the straight line that passes through the points (–4, 5) and (4, –5).

2. What is the slope of this line?

3. Use the formula for point-slope form to find an equation for this line. Show your work and answer in the space below.

   Point-slope form: $y - y_1 = m(x - x_1)$

   Equation for the line: _____

4. If you were to construct a line parallel to the line in problem 1, what slope would this line have? Explain your answer.

5. How can you use your graph paper and the slope you found in problem 2 to construct a line parallel to the line in problem 1?

   Draw this line on your graph paper.

continued

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6. Use the formula for point-slope form to find an equation for this line. Show your work and answer in the space below.

   Point-slope form: $y - y_1 = m(x - x_1)$

   Equation for the line: _____

7. Construct another parallel line on your graph paper.

8. Use the formula for point-slope form to find an equation for this line. Show your work and answer in the space below.

   Point-slope form: $y - y_1 = m(x - x_1)$

   Equation for the line: _____

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Station Activities Set 2: Perpendicular Lines

Instruction

Goal: To provide opportunities for students to develop concepts and skills related to finding the distance between a point and line, determining whether two lines are perpendicular, and writing equations for perpendicular lines

Common Core State Standards

G–GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*

G–GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

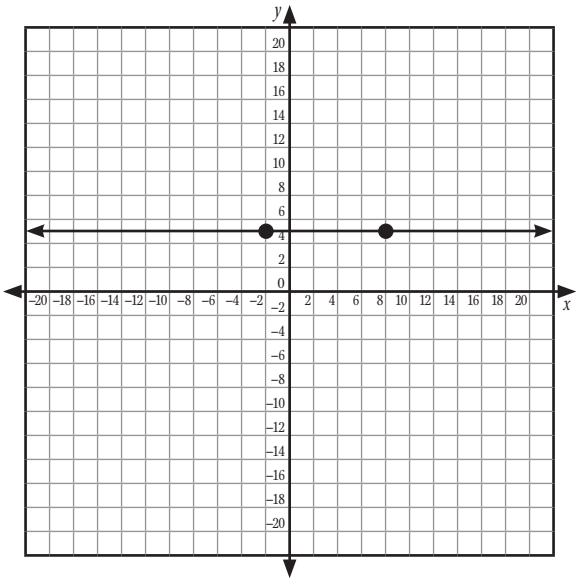
Student Activities Overview and Answer Key

Station 1

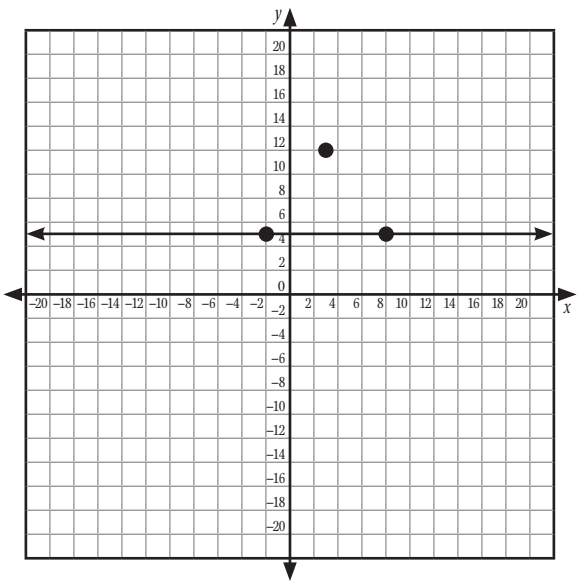
Students will be given graph paper and a ruler. Students will construct a line and a point using the graph paper and ruler. They will construct vertical and horizontal lines. Then they will use a perpendicular line to find the distance between the line and the point.

Answers

1. horizontal line



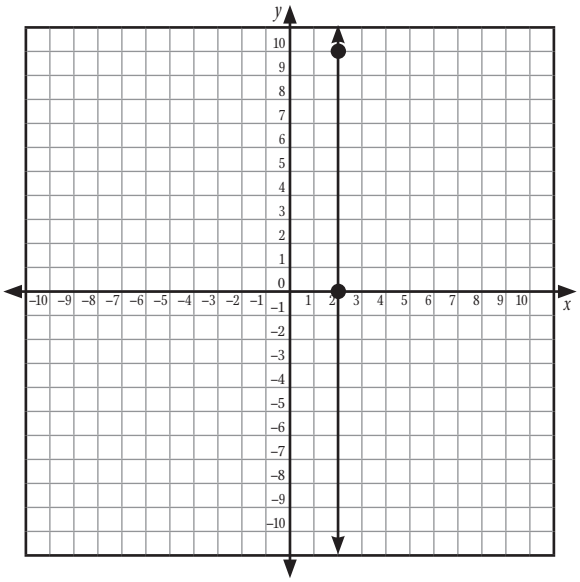
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2. 

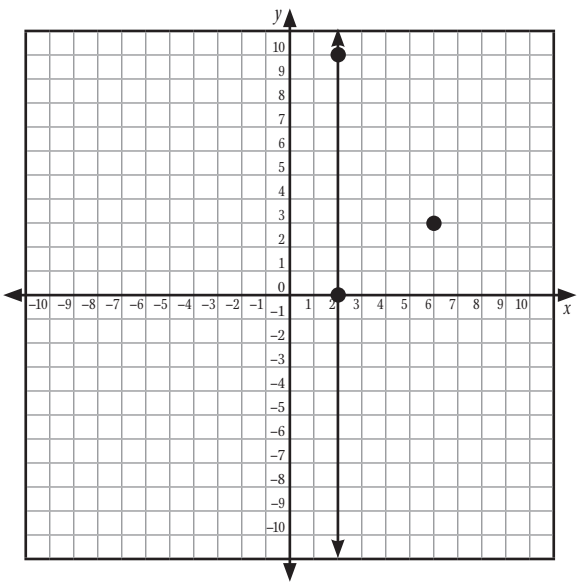
3. Construct a perpendicular line between the horizontal line and the point.

4. Subtracting the *y*-coordinates results in 7 units, which is the vertical distance between the horizontal line and the point.

5. vertical line



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6. 

7. Construct a perpendicular line between the vertical line and the point.
8. The difference in the *x*-coordinates is 4 units, which is the horizontal distance between the vertical line and the point.

Station 2

Students will be given a protractor. Students will use slope to determine whether or not two lines are perpendicular given the lines on a coordinate plane. Then they will use a protractor to determine whether two lines are perpendicular.

Answers

1. Yes, because the slopes are $m = 1$ and $m = -1$, which are opposite reciprocals.
2. No, because the slopes are $m = 4/5$ and $m = -5/2$, which are not opposite reciprocals.
3. I found the slope of each line. Slopes of perpendicular lines are opposite reciprocals of each other.
4. No, because the angles between the lines are 100° and 80°.
5. Yes, because the angles between the lines are 90°.
6. I measured the angles between the two lines. If all the angles are 90°, then the lines are perpendicular.

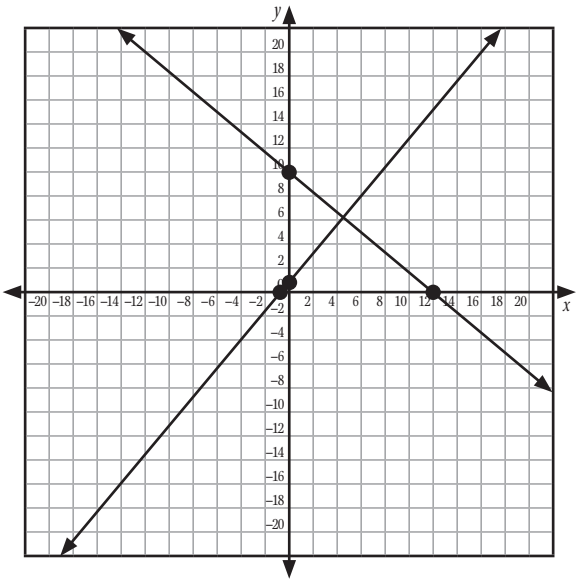
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Instruction

Station 3

Students will be given graph paper and a ruler. Students will construct a straight line. They will find the slope of this line and use it to find the slope of a line perpendicular to it. Then they will use the point-slope form of a linear equation to find the equation of the perpendicular line. They will justify their answer by graphing the perpendicular lines on a coordinate plane.

Answers

1. $m = 2$
2. $m = -1/2$; perpendicular lines have slopes that are opposite reciprocals of each other.
3. $y = -\frac{1}{2}x + 3$
4. $y = \frac{6}{5}x + \frac{4}{5}$
5. 

Station 4

Students are given 18 index cards with different equations written on them. They sort through the index cards to find pairs of parallel and perpendicular lines, and then discuss their strategies for completing the activity.

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Answers

1. Order may vary, but student tables should reflect the following information:

Parallel lines	Perpendicular lines
$y = 7x - 4$ and $y = 7x + 8$	$y = \frac{1}{4}x - 3$ and $y = -4x + 12$
$y = -\frac{1}{2}x - 11$ and $y = -\frac{1}{2}x + 16$	$y = -\frac{11}{2}x - 4$ and $y = \frac{2}{11}x$
$y = 4x + 1$ and $y = 4x$	$y = 3x - 2$ and $y = -\frac{1}{3}x + 4$
$y = 0$ and $y = 12$	$y = x + 4$ and $y = -x - 3$
$y = x$ and $y = x - 9$	

2. Answers will vary. Possible strategy: looking at the slopes for the same slope or negative reciprocals.

Materials List/Setup

Station 1 graph paper; ruler

Station 2 protractor

Station 3 graph paper; ruler

Station 4 18 index cards with the following equations written on them and then shuffled:

$y = 7x - 4$, $y = 7x + 8$, $y = -\frac{1}{2}x - 11$, $y = -\frac{1}{2}x + 16$, $y = 4x + 1$, $y = 4x$, $y = 0$, $y = 12$, $y = x$, $y = x - 9$, $y = \frac{1}{4}x - 3$, $y = -4x + 12$, $y = -\frac{11}{2}x - 4$, $y = \frac{2}{11}x$, $y = 3x - 2$, $y = -\frac{1}{3}x + 4$, $y = x + 4$, $y = -x - 3$

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. How can you find the distance between a point and a horizontal or vertical line?
2. How can you find the distance between a point and a line that is not horizontal or vertical?
3. What are two methods you can use to determine if two lines are perpendicular?
4. How can you write an equation for a line that is perpendicular to a given line and passes through a given point?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. Create a perpendicular line between the point and line. Find the distance of this perpendicular line.
2. Find the equation of the line and write the equation in standard form. Then use the distance formula with the equation and the point to find the distance.
3. Perpendicular lines have opposite reciprocal slopes. Perpendicular lines create four 90° angles.
4. Find the equation of the original line. The slope of the perpendicular line will be the opposite reciprocal of the slope of the given line. Use the point-slope form of a linear equation and the slope of the perpendicular line to find the equation of the perpendicular line.

Possible Misunderstandings/Mistakes

- Not realizing that perpendicular lines have opposite reciprocal slopes
- Not realizing that perpendicular lines create four 90° angles
- Not understanding how to use the point-slope form of a linear equation
- Incorrectly finding the slope of a line as $\frac{\text{run}}{\text{rise}}$ instead of $\frac{\text{rise}}{\text{run}}$

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Station 1

At this station, you will find graph paper and a ruler. As a group, construct an x- and y-axis on the graph paper.

1. Construct a line that passes through (–2, 5) and (8, 5).

   What type of line have you created?

2. On the same graph, plot and label the point (3, 12).

3. How can you use perpendicular lines to find the distance between the line you created in problem 1 and the point you plotted in problem 2?

4. What is the distance between the line you created in problem 1 and the point you plotted in problem 2? Explain your answer.

5. On a new graph, construct a line that passes through (2, 10) and (2, 0).

   What type of line have you created?

6. On the same graph, plot and label the point (6, 3).

continued

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7. How can you use perpendicular lines to find the distance between the line you created in problem 5 and the point you plotted in problem 6?

8. What is the distance between the line you created in problem 5 and the point you plotted in problem 6? Explain your answer.

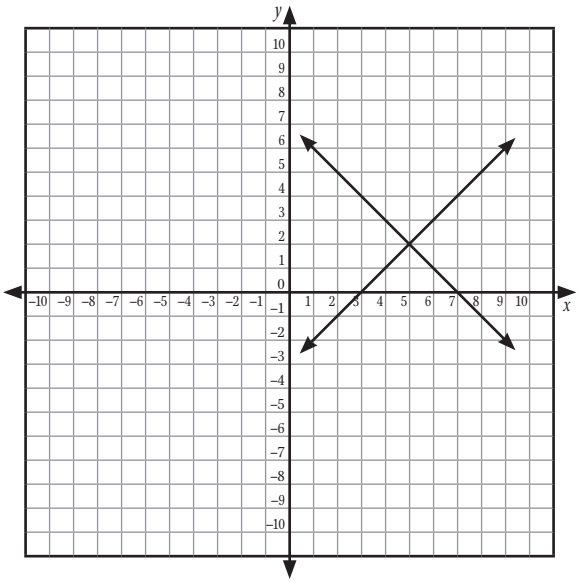
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Station 2

At this station, you will find a protractor.

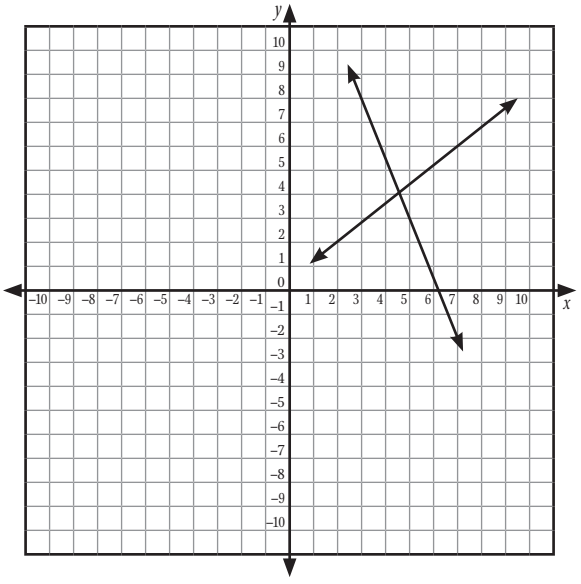
For problems 1 and 2, work as a group to determine whether or not the two lines are perpendicular using the coordinate graph. Do NOT use your protractor.

1.



Are the lines perpendicular? Explain.

2.



Are the lines perpendicular? Explain.

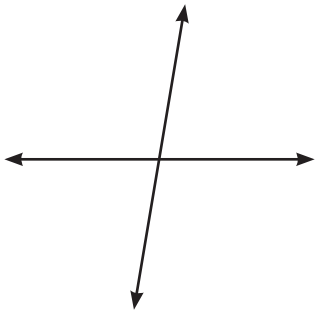
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3. What strategy did you use to determine whether or not the lines are perpendicular?

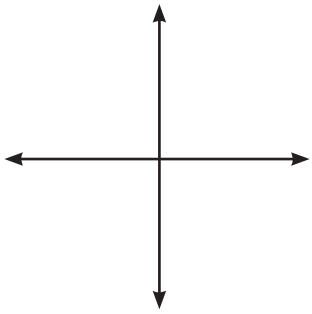
For problems 4 and 5, use your protractor to determine whether or not the lines are perpendicular.

4.



Are the lines perpendicular? Explain.

5.



Are the lines perpendicular? Explain.

6. How did you use your protractor to determine whether or not the lines were perpendicular?

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Station 3

At this station, you will find graph paper and a ruler. As a group, create an *x*- and *y*-axis on the graph paper. Work together to answer the questions.

1. On your graph paper, construct a line that passes through points (–4, 0) and (0, 8).

   What is the slope of this line?

2. What is the slope of a line that is perpendicular to the line you created in problem 1? Explain your answer.

3. Use the point-slope form of an equation, $y - y_1 = m(x - x_1)$, to find the equation of a line that is perpendicular to the line from problem 1, and that passes through the point (–2, 4). Show your work and answer in the space below.

   Equation: _____

4. Write the equation for the line that is perpendicular to $y = -\frac{5}{6}x + 10$ and passes through (1, 2). Show your work and answer in the space below.

   Equation: _____

5. On your graph paper, graph the two lines in problem 4 to justify that the lines are perpendicular.

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Station 4

At this station, you will find 18 index cards. Each index card has an equation written on it. Your job is to pair the equations into sets of parallel and perpendicular lines.

1. Work with your group members to find the parallel and perpendicular lines. Record your results below.

Parallel lines	Perpendicular lines

2. What was your strategy for working through this activity?

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Station Activities Set 3: Coordinate Proof with Quadrilaterals

Instruction

Goal: To provide opportunities for students to develop concepts and skills related to using coordinate geometry to prove properties of congruent, regular, and similar quadrilaterals

Common Core State Standards

G–GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*

G–GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Student Activities Overview and Answer Key

Station 1

Students use geoboards to construct polygons with parallel and perpendicular sides. Then they reflect on the properties of parallel and perpendicular lines, and explain how this knowledge helped them complete the task.

Answers

1. Check student drawings.
2. Check student drawings.
3. Check student drawings.
4. Sample answer: First, we made the parallel or perpendicular lines and then connected them; we needed to know that perpendicular lines meet at right angles and that parallel lines have the same slope.

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Instruction

Station 2

Students will be given graph paper and a ruler. Students will construct a trapezoid using coordinate geometry. They will construct trapezoids that are congruent and not congruent to the original trapezoid. Then they will construct a kite using coordinate geometry and determine whether another kite is congruent to the original kite.

Answers

1. a trapezoid
2. Answers will vary; corresponding sides are congruent.
3. Answers will vary; corresponding sides are NOT congruent.
4. a kite
5. No, because corresponding sides are not congruent; answers will vary.

Station 3

Students will be given graph paper and a ruler. Students will construct a square and a rectangle using coordinate geometry. They will prove that squares are regular quadrilaterals. They will show that a square is also a rhombus and a rectangle. They will show that only quadrilaterals that are squares are classified as regular quadrilaterals.

Answers

1. Square, 4 congruent sides, 4 congruent angles; rhombus, 4 congruent sides; rectangle, all squares are rectangles.
2. rectangle
3. Yes, because all sides are congruent and all angles are congruent.
4. No, because not all sides are congruent.
5. Answers will vary.
6. A regular quadrilateral has four congruent sides and four congruent angles.
7. No, only quadrilaterals that are squares are classified as regular quadrilaterals.

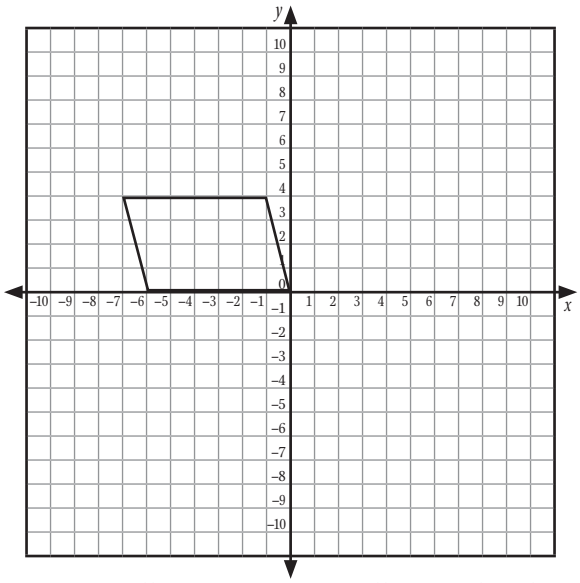
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Instruction

Station 4

Students will be given graph paper and a ruler. Students will construct similar parallelograms, trapezoids, and kites using coordinate geometry. They will explain why different quadrilaterals are similar or not similar. Then they will provide an example of similar quadrilaterals used in the real world.

Answers

1. 

2. Answers will vary; answers will vary; they have the same shape, but different sizes.
3. They are similar because they have the same shape, but different sizes.
4. Answers will vary; answers will vary; they are similar because they have the same shape, but different sizes.
5. Answers will vary. Possible answer: scale drawings

Materials List/Setup

Station 1 geoboard; rubber bands

Station 2 graph paper; ruler

Station 3 graph paper; ruler

Station 4 graph paper; ruler

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Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. How can you use coordinate geometry to prove two quadrilaterals are congruent?
2. How can you use coordinate geometry to prove two quadrilaterals are similar?
3. How can you use coordinate geometry to prove a quadrilateral is "regular"?
4. What are some real-world applications of quadrilaterals and coordinate geometry?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. Use the units in the coordinate graph to show that the quadrilaterals have congruent corresponding sides and congruent corresponding angles.
2. Use the units in the coordinate graph to show that the quadrilaterals have the same shape, but are different sizes.
3. Use the units to show that all sides of the quadrilateral are congruent and all angles of the quadrilateral are congruent.
4. Possible answers: scale drawings, art and design, stained glass making, architecture

Possible Misunderstandings/Mistakes

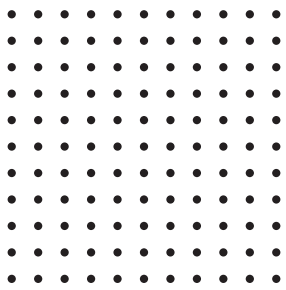
- Not recognizing that the units in a graph can determine the side lengths of quadrilaterals
- Not recognizing that the units in a graph can determine whether two quadrilaterals are congruent or similar
- Not realizing that a "regular" quadrilateral must have four equal sides and four equal angles

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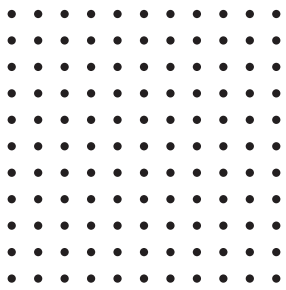
Station 1

At this station, you will find a geoboard and rubber bands.

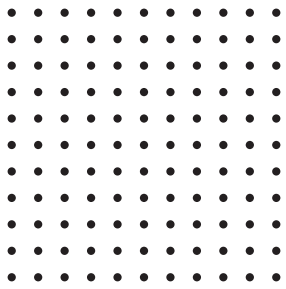
1. With your group, use the geoboard to construct a polygon with only one set of parallel sides. Draw this figure below.



2. Construct a polygon with two sets of parallel sides. Draw this figure below.



3. Construct a polygon with at least one set of perpendicular sides. Draw this figure below.



4. How did you apply what you know about parallel and perpendicular lines to successfully complete this activity?

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Station 2

At this station, you will find graph paper and a ruler. Work as a group to construct the quadrilaterals and answer the questions.

1. On your graph paper, construct a quadrilateral that has vertices (1, 1), (2, 4), (8, 4), and (9, 1).

   What type of quadrilateral did you create?

2. On the same graph, construct a quadrilateral that is congruent to the quadrilateral in problem 1.

   What are the vertices for this new quadrilateral?

   Explain why the quadrilaterals in problems 1 and 2 are congruent.

3. On the same graph, construct a quadrilateral that is NOT congruent to the quadrilateral in problem 1.

   What are the vertices of this new quadrilateral?

   Explain why the quadrilaterals in problems 1 and 3 are NOT congruent.

4. Graph a quadrilateral with vertices (0, 0), (2, 5), (–2, 5), and (0, 7).

   What type of quadrilateral did you create?

continued

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5. Jacob claims that a quadrilateral with vertices (–2, 2), (0, 6), (–2, 8), and (–4, 6) is congruent to the quadrilateral in problem 4. Is Jacob correct? Why or why not? Graph the quadrilaterals on your graph paper to justify your answer.

   If Jacob is incorrect, what vertices would make this quadrilateral congruent to the quadrilateral in problem 4?

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Station 3

At this station, you will find graph paper and a ruler. Work as a group to construct the quadrilaterals and answer the questions.

1. On your graph paper, graph a quadrilateral that has vertices (2, 4), (8, 4), (2, 10), and (8, 10).

   What are three names for this quadrilateral? Justify each name using properties of that type of quadrilateral.

2. On your graph paper, graph a quadrilateral that has vertices (–1, 0), (–4, 0), (–1, 7), and (–4, 7).

   What is the name of this quadrilateral?

3. Is the quadrilateral in problem 1 a regular quadrilateral? Why or why not?

4. Is the quadrilateral in problem 2 a regular quadrilateral? Why or why not?

5. What strategy did you use for problems 1–4?

continued

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6. What is the definition of a regular quadrilateral?

7. Are all rhombi and rectangles regular quadrilaterals? Why or why not?

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Station 4

At this station, you will find graph paper and a ruler. Work together to construct the quadrilaterals and answer the questions.

1. On your graph paper, construct a parallelogram with vertices (0, 0), (–6, 0), (–7, 4), and (–1, 4).

2. On the same graph, construct a similar parallelogram.

   What are the vertices of the parallelogram?

   Why is this new parallelogram similar to the parallelogram in problem 1?

3. On a new graph, construct a trapezoid with vertices (1, 1), (4, 1), (2, 4), and (3, 4). Construct a second trapezoid with vertices (0, 0), (6, 0), (2, –6), and (4, –6).

   Are the two trapezoids congruent or similar? Explain your answer.

4. On a new graph, construct a kite. Construct a second kite that is similar, but not congruent to the first kite.

   What are the vertices of the first kite?

   What are the vertices of the second kite?

   Why are the kites similar?

5. When would you use similar quadrilaterals in the real world?

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APPENDIX

Problem-Based Task Implementation Guides

Introduction

This addendum contains a collection of Implementation Guides (IGs) designed to support the use of the Problem-Based Tasks found throughout this unit.

Each Implementation Guide includes an overview and suggestions for introducing and facilitating the task. These contain specific recommendations for selected Standards for Mathematical Practice (SMPs) to highlight, guidance on addressing common errors and misconceptions associated with the task, and ideas for monitoring and coaching students as they work through the task. Finally, there is guidance for debriefing the task, with special attention to the SMPs and relevant Key Concepts from the instruction, as well as information on suitable extensions and alternate strategies.

Throughout the Implementation Guides, suggestions are provided in a question/prompt format to support your efforts to elicit student thinking and facilitate classroom discourse.

We developed these Implementation Guides for our Problem-Based Tasks at the request of math educators, and with advice and feedback from mathematics supervisors and teachers. Please let us know if they are helpful. Visit us at www.walch.com, follow us on Twitter (@WalchEd), or e-mail suggestions to customerservice@walch.com.

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Guide

Problem-Based Task: Field of Dreams

Common Core State Standards

G–GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point* $\left(1, \sqrt{3}\right)$ *lies on the circle centered at the origin and containing the point (0, 2).*

G–GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Task Overview

Focus

How can the criteria for a square be used to prove whether a baseball field is actually a square? Which formulas can be used to calculate the distances between the bases? In this lesson, students will plot points on a coordinate plane representing the locations of the bases and home plate of a baseball diamond, and use these coordinates to draw conclusions about the relationships of the distances between bases and home plate, as well as the relationships of the four corners of the diamond.

This activity will provide practice with:

- plotting points on a coordinate plane
- calculating slopes and distances using the slope formula and the distance formula
- classifying slopes of consecutive sides and opposite sides of a figure
- analyzing pairs of opposite sides of a figure to determine congruency
- determining the relationship between the sides and angles of a figure
- drawing conclusions on classifying a figure based on multiple criteria

Introduction

This task should be used to explore or apply the skill of proving the slope criteria for parallel and perpendicular lines and using them to solve a geometric problem. Students should already be familiar with plotting points in the coordinate plane and calculating slope.

Begin by reading the problem and clarifying the meaning of distance formula, slope formula, and square.

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distance formula	a formula that states the distance between points (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
slope formula	a formula that states the slope given points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$
square	a quadrilateral with two pairs of parallel opposite sides, consecutive sides that are perpendicular, and all sides congruent

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they make sense of the relationships between the slopes of the sides of the baseball diamond. They will reason that the slopes will determine if the opposite sides of the diamond are parallel, and if the consecutive sides are perpendicular. They will reason quantitatively as they calculate the slopes of the sides connecting each base and home plate, as well as the distance/length of each side.

- **SMP 4:** Model with mathematics.

  Students will recognize that this scenario can be modeled with the use of a coordinate plane that represents the location of each base and home plate of the baseball diamond. They will use the coordinates for each point to calculate the criteria necessary for determining if the baseball diamond is a square.

- **SMP 7:** Look for and make use of structure.

  Students will apply the structure of the slope and distance formulas to determine the relationships between the slopes and distances of opposite sides, as well as consecutive sides, of the baseball diamond. They will analyze these results and draw conclusions about the specific type of quadrilateral formed by the locations of the bases and home plate.

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Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- assuming lines are parallel or perpendicular based on appearance only

  Remind students to perform the necessary calculations to determine the exact length of the sides and the exact slope of each side so conclusions can be drawn about the relationships between the sides and corners of consecutive sides.

- incorrectly calculating the slope of the line

  Have students write down the slope formula and the values for each x_1, y_1, x_2, and y_2 coordinate as they calculate the slope of each side. Remind them that order matters when using the slope formula.

- incorrectly using the *x*- and *y*-coordinates in the distance formula

  Have students write down the distance formula and the values for each x_1, y_1, x_2, and y_2 coordinate as they calculate the distance between each base and home plate.

- subtracting negative coordinates incorrectly

  Review the rules of subtraction of integers with students. Work through an example or two, such as $(5 - (-2)) = (5 + 2)$.

- making determinations about the type of polygon without making all the necessary calculations

  Have students write down the specific criteria necessary for a quadrilateral to be identified as a square. Ask students to make a checklist of the criteria as they work through the task to ensure all properties of a square have been identified.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask them, "What are the properties of a square? How is a square different from a rectangle?" (**Answer:** A square has two pairs of parallel opposite sides, consecutive sides that are perpendicular, and all sides congruent. A rectangle has the same properties, with the exception that only opposite sides are congruent.)

- As students are beginning the task, ask them, "Why is it helpful to plot the points on a coordinate plane?" (**Answer:** The coordinate plane can be used to visualize the points that represent each of the bases and home plate. The plane can also be used to label distances and slopes once they are calculated.)

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- Once students have completed plotting each point, ask them, "What do you notice about the locations of each base and home plate on the coordinate plane?" (**Answer:** Each point is plotted in a different quadrant, or each quadrant contains a point representing a base or home plate.)
- Before students begin calculating the slopes of each side, ask them, "What does it mean for two sides to be consecutive?" (**Answer:** Consecutive sides are sides that meet at an endpoint or an angle. Consecutive sides share an endpoint.)
- As students are calculating the slopes of the sides of the diamond, ask them to discuss the meaning of parallel lines and perpendicular lines in terms of slope. Ask them, "If a pair of opposite sides of the baseball diamond is parallel, what does it mean about their slopes? If two consecutive sides of the diamond are perpendicular, what does it mean about their slopes?" (**Answer:** If two sides are parallel, they have the same slope. If two consecutive sides are perpendicular, they have slopes that are negative reciprocals of each other.)
- As students are calculating the slopes, ask them, "Suppose you calculated a slope of a side that was –4, and the other side had a slope of $-\frac{1}{4}$. What can you conclude about the relationship between these sides, and why?" (**Answer:** Although the slopes are reciprocals of each other, they are not negative reciprocals of each other. Therefore, they are neither parallel sides nor perpendicular sides.)
- As students are calculating the distances between each consecutive base, ask them, "Could you have calculated the distances in a different order than given in the task? Would this have made a difference in any results?" (**Answer:** Instead of calculating the distances between each base in order, the distances of each side could have been calculated in any order, since the goal was to determine the length of each side to see if all four sides were the same.)
- After students have completed their calculations of the slopes and distances, ask them to explain their conclusions in terms of presenting the argument that the baseball diamond is indeed a square. Ask them, "How do your calculations prove that the baseball diamond is a square?" (**Answer:** By calculating the slopes of the sides, it can be shown that opposite sides of the diamond are parallel to each other, and consecutive sides are perpendicular to each other, so the consecutive sides form four right angles. By calculating the distances between each base and home plate, it can be shown that all four sides have the same length. These criteria match that of a square.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

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Debriefing the Task

- Ask for volunteers to discuss their strategies and processes for performing the necessary calculations for slope and distance. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Ask students to share their reasoning processes and how they were able to draw conclusions. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of the criteria for a square, and how it relates to slopes and distances.

Connecting to Key Concepts

- To find the slope (steepness of a line), calculate the change in y divided by the change in x using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

  In this task, students will apply the slope formula and calculate the slope of all four sides of the baseball diamond to draw conclusions about the relationships between the opposite sides and consecutive sides.
- To find the distance between two points on the coordinate plane, use the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

  Students will apply the distance formula to find the distance between each base and home plate of the baseball diamond. They will use these values to draw conclusions about the relationships between the distances between each base.
- To prove that two lines are parallel, calculate the slopes of each line. If they are the same, the lines are parallel.

  Students will calculate the slopes of the opposite sides of the diamond to determine if the sides are parallel.
- To prove that two lines are perpendicular, calculate the slopes of each line. If they are negative reciprocal values of each other, the lines are perpendicular.

  Students will calculate the slopes of the consecutive sides of the diamond to determine if the sides are perpendicular.

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Extending the Task

- To extend the task, ask students to work together to create their own scenario of plotting points and identifying a shape (other than a square) based on the relationships between the sides and angles of the shape. For example, ask them to find points that, when plotted, will result in a rectangle or another type of quadrilateral, such as a rhombus. Have them calculate the slopes and distances, and explain their results. Ask for volunteers to share their scenarios.
- Another option for extending the task is to ask students to calculate the distances between opposite corners of the baseball diamond, e.g., the distances between home plate and second base, and the distance between first base and third base. Before students complete the calculations, ask them to predict the results. Discuss how these distances, which are the diagonals of a square, must be the same for any square. Ask students if they can think of any other properties of a square in terms of relationships between sides and angles. (Possibly discuss midpoints and the center of a square.)

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** I used abstract reasoning as I made sense of the relationships between the slopes of the sides of the baseball diamond by reasoning that the slopes of the sides of the diamond proved the shape of the diamond was a square, as well as the distances of the four sides proved that the diamond was a square.) "Which of your strategies represent quantitative reasoning?" (**Answer:** I reasoned quantitatively when I calculated the slopes of the sides and the distances of the sides of the baseball diamond.)
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** I recognized that this scenario could be modeled with the use of a coordinate plane that represents the location of each base and home plate of the baseball diamond.)
- **For SMP 7, ASK:** "How did you look for and make use of structure when solving this problem?" (**Answer:** I made use of structure by applying the slope and distance formulas to determine the relationships between the slopes and distances of opposite sides, as well as consecutive sides, of the baseball diamond.)

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Alternate Strategies or Solutions

- Students may choose to use a straightedge to measure the lengths of the sides of the baseball diamond. They may also use a protractor to measure the angles formed by the consecutive sides. Remind them that the slope and distance formulas allow for accurate calculations, while measuring with a straightedge and a protractor can have human measurement errors.

Technology

No technology is needed for this task, although students may want to use scientific calculators to determine the simplified value of the distances, which are given in radical form in the task.

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Problem-Based Task: Pave the Way

Common Core State Standard

G–GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Task Overview

Focus

How can the slope criteria for parallel lines be used to write equations of lines that represent the edges of a two-lane road? Which formula can be used to calculate the distance between two points on the edges of the road? In this lesson, students will apply their knowledge of parallel lines and their slopes to create equations of lines given specific points representing the edges of a road. They will also determine if the width of the road meets the minimum required distance by applying the distance formula.

This activity will provide practice with:

- identifying slope values given an equation in slope-intercept form
- substituting values into point-slope form of a linear equation
- writing equations of lines given a point on the line and the slope
- calculating distances between two points using the distance formula
- analyzing results of a calculation to determine if they meet given criteria
- drawing conclusions based on calculations

Introduction

This task should be used to explore or apply the skill of proving the slope criteria for parallel lines and finding the equation of a line parallel to a given line that passes through a given point. Students should already be familiar with slope-intercept form of a linear equation, writing linear equations, and calculating distance using the distance formula.

Begin by reading the problem and clarifying the meaning of distance formula, parallel lines, point-slope form, and slope-intercept form.

distance formula	a formula that states the distance between points (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
parallel lines	lines in a plane that do not share any points and never intersect; parallel lines have the same slope

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point-slope form	the equation of a straight line in the form $y - y_1 = m(x - x_1)$ where m is the slope of the line and (x_1, y_1) are the coordinates of a given point on the line
slope-intercept form	the equation of a straight line in the form $y = mx + b$ where m is the slope of the line and b is its y-intercept.

Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will recognize that the goal of the task is to write equations of the lines that represent the edges of a two-lane road. They will create these equations by applying their knowledge of the slopes of parallel lines, as well as their knowledge of different forms of linear equations such as slope-intercept form and point-slope form. They will also plan a solution pathway to determine if the distance between two points will satisfy a minimum width requirement for the road.

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they make sense of the relationships and values of the components of the equations of the lines they write. They will reason that since parallel lines have the same slope values, these values will be used in the formula when writing equations of parallel lines. They will reason quantitatively as they perform the algebraic manipulations in the equations that they write, as well as when they use the distance formula to calculate the distance between the two given points representing points on the edge lines of the road.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will construct viable arguments about the process of creating equations of lines representing parallel edges of the road. They will explain their assumptions and calculated results to justify their equations. They will use counter-examples when presenting their arguments to others, justifying their conclusions about the width of the road.

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Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly identifying the slope of the given line

  Have students write down the slope-intercept form, $y = mx + b$, and remind them that the value for m represents the slope of the line.

- incorrectly finding the slope of the line parallel to the given line

  Remind students of the difference in the slope criteria for parallel and perpendicular lines, specifically that parallel lines must have the exact same slope.

- improperly substituting the x- and y-values into the general point-slope equation

  Have students write the point slope formula, $y - y_1 = m(x - x_1)$, then ask them to list the values for each coordinate, (x_1, y_1) and the value for the slope before substituting the values into the formula.

- incorrectly using the x- and y-coordinates in the distance formula

  Have students write down the distance formula and the values for the x_1, y_1, x_2, and y_2 coordinates as they calculate the distance between the two given points, (16, 1) and (4, 21).

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task:
  - Discuss with them the importance of ensuring that edge lines on a road are parallel. Ask them, "Why is it crucial that edges lines on a road be completely parallel?" (**Answer:** If the edge lines were not perfectly parallel, it is possible that one side of the road could have less space than the other side, and as a result, larger cars might not fit in between the lines.)
  - Ask, "What are some examples of values of the width of the road that would meet the law's requirement? Which word or phrase in the task identifies the possible range of values for the width?" (**Answer:** The task states that the law requires the width of a two-lane road must be at least 24 feet. The phrase "at least" means that the width can be 24 feet or more than 24 feet. Some possible values for the width are anything greater than 24 feet, such as 25, 26, 27, etc.)
  - Ask them to clarify the meaning of the slope of the line in terms of the components of the value. Ask, "In this task, the slope of the equation representing the center lane of the road is $\frac{3}{5}$. What do the 3 and the 5 represent?" (**Answer:** The slope is the steepness of a line, or rise over run. For every three units that this line "rises" in a vertical direction, it "runs" five units in a horizontal direction.)

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- If students are having difficulty beginning the task, ask them, "What information are you given in the problem that you can visualize by using a coordinate plane?" (**Answer:** The line representing the center of the road, $y = \frac{3}{5}x + 5$, can be graphed on a coordinate plane by first plotting the *y*-intercept, which is 5, and then plotting the slope, which is $\frac{3}{5}$. Then the coordinates representing points on each edge line of the road can be plotted.)
- As students begin working on the task, ask them, "Is it necessary to find the equations of the lines representing the edges of the road before you find the distance between the two points on the edge lines? In other words, does the information found by writing the equations of the lines affect finding the distance, or the width, between the two given points? Explain." (**Answer:** No. The distance between the two points can be found independently of writing the equations for the two edge lines. The only information necessary for finding the width of the road, or the distance, is the coordinates of the points. This information is given in the original problem.)
- Once students have found the equations for the two edge lines, ask them, "If you wanted to graph these equations, how could you graph the *y*-intercepts for each equation, since they are in improper fraction form? What would these intercepts be in decimal form?" (**Answer:** The *y*-intercept for the equation containing the point (16, 1) is $-\frac{43}{5}$. The improper fraction can be changed to a decimal form of -8.6. The *y*-intercept for this line would be plotted with the point (0, –8.6). The *y*-intercept for the equation containing the point (4, 21) is $\frac{93}{5}$. The improper fraction can be changed to a decimal form of 18.6. The *y*-intercept for this line would be plotted with the point (0, 18.6).)
- Ask students, "When the distance between the two points is calculated, the result in radical form is $\sqrt{544}$. Why is it unnecessary to express the simplified result more specifically than 'approximately 23 feet'?" (**Answer:** When the radical is simplified, the result is 23.324, rounded to the nearest thousandth. Since this value is already less than 24 feet, the exact value is insignificant. This value does not meet the minimum requirement for the road width.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

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Debriefing the Task

- Ask for volunteers to discuss their strategies and processes for determining the equations of the lines representing the two edge lines on the road. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Ask students to share their reasoning processes, including how they determined which values to substitute into the formulas, and how they were able to draw conclusions based on their results of applying the distance formula. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of the components of a linear equation, such as slope and *y*-intercept, as well as connecting the distance between the two points to its representation of the width of the road in the task.

Connecting to Key Concepts

The slopes of parallel lines are always equal.

- Students will apply the slope from the given equation representing the center of the road to the equations for each of the edge lines. Since the slope values must be the same, the slope of $\frac{3}{5}$ will be used throughout the equations.
- To write the equation of a line through a given point that is parallel to a given line if you know the slope of the given line, substitute the slope, m, and the given point, (x_1, y_1), into the point-slope form, $y - y_1 = m(x - x_1)$.
- In this task, students will apply the point-slope form by substituting the known values for slope and a point on the line. They will write equations in slope-intercept form that represent the parallel edge lines of a two-lane road.
- To find the distance between two points on the coordinate plane, use the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- Students will apply the distance formula to find the distance between the two given points. They will use this value to draw conclusions about the width of the road, specifically whether or not this width meets the requirements of the law.

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Extending the Task

- To extend the task, ask students to work together to identify new points on the edge lines of the road that will satisfy the requirement that the two-lane road must be a minimum of 24 feet wide. Ask students to develop a process of selecting points other than choosing random points and performing trial-and-error calculations. Have students brainstorm their ideas with each other, reminding them that their new points must also work in the equations of the lines that have a slope of $\frac{3}{5}$. Have them write down their new equations, along with their calculations. Ask for volunteers to share their results and justify their new, acceptable road width value.
- Another option for extending the task is to ask students to create a square using the given points on the edge lines of the road. Have them identify the two new vertices of the square, located on the line that represents the center of the road. Discuss strategies to determine the coordinates of these new points and how to determine that the four corners of the square are perpendicular (e.g., slope criteria for perpendicular lines).

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** I persevered by first recognizing that the goal of the task is to write equations of the lines that represent the edges of a two-lane road. I planned my solution pathway by using the given information and writing the equations using the point-slope form. I also assessed the reasonableness of my results and how they related to the goal of the task.)
- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** I used abstract reasoning as I made sense of the relationships and values of the components of the equations of the lines I wrote to represent the edge lines of the road. I reasoned that the slopes of parallel lines will be the same, and used this fact in my equations.) Ask, "Which of your strategies represent quantitative reasoning?" (**Answer:** I used quantitative reasoning as I algebraically manipulated the equations I wrote so that I could get them in slope-intercept form. I also calculated the distance between two points using the distance formula.)
- **For SMP 3, ASK:** "How did you construct viable arguments and critique the reasoning of others?" (**Answer:** I constructed viable arguments about the process of creating equations of lines representing parallel edges of the road by explaining the results of my equations and why I chose the values I did for the slope. I also used counter-examples of values to demonstrate which values are valid for the width of the road.)

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Alternate Strategies or Solutions

- Students may choose to graph the given information (the two points and equation for the center line of the road) on a coordinate plane. Encourage them to ensure the scales on the *x*- and *y*-axes are large enough to accommodate the given points. Students may also choose to draw their own sketches of a two-lane road with the center line and the points on the edge lines. This may help students who are visual learners and benefit from seeing a "picture" of the problem.

Technology

Students can use scientific calculators or graphing calculators for extending the task activities.

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Problem-Based Task: Building Fences

Common Core State Standard

G–GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Task Overview

Focus

How can perimeter and area formulas be used to find the distance around a polygon and the area of a polygon? Which formula can be used to calculate the distances between each side of a polygon?

In this lesson, students will use coordinates representing the locations of the vertices of a rectangular-shaped baseball field to calculate the amount of fencing needed to enclose the field, and will also determine the area of the field.

This activity will provide practice with:

- calculating distances between two points using the distance formula
- determining values necessary for perimeter and area of a polygon
- applying perimeter and area formulas to a real-world scenario
- calculating perimeter and area of a polygon
- interpreting results of calculations to real-world units of measurement

Introduction

This task should be used to explore or apply the skill of using coordinates to compute perimeters and areas of rectangles using the distance formula and perimeter and area formulas. Students should already be familiar with finding the distance between two points on the coordinate plane using the distance formula. They should also be familiar with finding the perimeter and area of basic polygons.

Begin by reading the problem and clarifying the meaning of area, distance formula, perimeter, and rectangle.

area	the amount of space inside the boundary of a two-dimensional figure
distance formula	the formula that states the distance between (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
perimeter	the distance around a two-dimensional figure
rectangle	a two-dimensional figure with four sides; opposite sides are congruent and parallel, and all four corners are right angles

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Facilitating the Task

Common Core Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they make sense of, and make the connection between, the coordinates given in the problem to those of the corners of a rectangle. They will reason that the distances between each consecutive corner represent the length of a side of the rectangle. They will extend this reasoning to using the values for the side lengths in perimeter and area formulas to accomplish the task. They will reason quantitatively as they perform the calculations using the distance formula to identify the distance between each side of the rectangle. They will also reason quantitatively as they substitute values into the perimeter and area formulas.

- **SMP 4:** Model with mathematics.

  Students will recognize that this scenario can be modeled using the distance formula, and the perimeter and area formulas. They will use the coordinates representing each corner of the fence to determine the length of each side of the rectangular-shaped fence. They will use these distances to find the perimeter of the fence so that the amount of fencing needed can be determined. They will also use the side lengths in the formula for the area of a rectangle, which is the amount of space in the fenced-in field.

- **SMP 6:** Attend to precision.

  Students will apply the definitions of the perimeter and area of a polygon to determine the amount of fencing needed to enclose a baseball field, as well as the amount of space inside the fenced-in field. Encourage students to use the correct terminology of the perimeter and area as well as the correct units of measurement as they apply to the task—specifically, how many feet each unit on the map represents.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- adding x-values and y-values rather than subtracting them when using the distance formula

  Have students write down the distance formula and values for each x_1, y_1, x_2, and y_2 coordinate as they calculate the distance between each base and home plate. Remind them to pay close attention to the signs used in the formula.

- confusing perimeter and area formulas

  Have students write down each formula, along with a definition for each in their own words. Remind them that "perimeter" is the distance around an object, and "area" is the space inside the object.

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- incorrectly stating the answer as units, or stating the answer without any units

  Remind students that since this task models a real-world scenario, the results must contain units of measurement specified in the problem. Have them reread the task to identify the conversion of units to feet.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task:
  - Discuss the context of the task. Ask them, "What mathematical term is used to describe the amount of fencing needed around the baseball field?" (**Answer:** The mathematical term used to describe the distance around an object is "perimeter".)
  - Ask them, "What is the difference between the perimeter of a rectangle and the area of the rectangle in terms of definitions and formulas?" (**Answer:** The perimeter of a rectangle is the distance around the rectangle, and it can be found by calculating the sum of all four sides. The area is the amount of space inside the rectangle, and it can be found by multiplying the base of the rectangle by the height of the rectangle.)
- As students are beginning the task, ask them, "What can you do with the coordinates to get a visual representation of the baseball field?" (**Answer:** The points can be plotted on a coordinate plane. The plane can also be used to label the distances that represent the side lengths of the rectangular-shaped fence once they are calculated.)
- As students are working on the task, discuss the variables that represent the sides of the rectangular-shaped field, as well as how the variables are used in the formulas for perimeter and area. Ask them, "What other variables can be used besides b and h? What are some different formulas for perimeter and area?" (**Answer:** The variables l and w can be used to represent the length and width of a rectangle. A formula that can be used to represent perimeter is $l + l + w + w$, or $2l + 2w$. Another formula that can be used for area is $l \bullet w$.)
- As students are calculating the distances of each side of the rectangle, ask them, "What type of result could you get that would indicate you made an error in your calculations? Explain." (**Answer:** If the answer is negative, an error was made in the calculations. Since the sides represent lengths, they must be positive values.)
- Ask students, "What is the minimum number of distance calculations for the rectangle? Why is it not necessary to calculate the lengths of all four sides? Explain." (**Answer:** Since the fence that will enclose the baseball field is rectangular shaped, the opposite sides of the field are congruent, as per the definition of a rectangle. Only the lengths of two consecutive sides need to be calculated, as those values will repeat for the respective opposite sides.)

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- Once students have completed their calculations and have determined that the perimeter of the field is 56.92 units, ask them, "How do you convert this value to a value that represents the real-world context of the scenario given in the task?" (**Answer:** The problem states that each unit represents 10 feet, so 56.92 is multiplied by 10 to get a result of 569.2 feet.)
- Ask students, "If the perimeter of the rectangle is 569.2 feet, explain how many feet of fencing should be bought to ensure the baseball field is completely enclosed. Is the value rounded up or down?" (**Answer:** To ensure the field is completely enclosed, 569.2 feet should be rounded up to 570 feet so that there is more fencing. If the value was rounded down to 569, there would not be enough fencing.)
- As students determine the area of the field, ask them, "Why are the units for perimeter expressed in terms of only the unit type itself (e.g., feet), but the area is represented in terms of the unit type squared (e.g., $feet^2$)?" (**Answer:** Area is expressed in square units because it represents two dimensions (base and height, or length and width), whereas perimeter only represents one dimension of a figure.)
- Ask students, "If the area of the rectangle is 200 $units^2$, explain how the area of the field is calculated so it represents the context of the scenario." (**Answer:** Since the problem states that each unit represents 10 feet, each square unit represents 10-by-10, or 100, square feet. Therefore, the value of the area of the rectangle, 200, is multiplied by 100 for a result of 20,000 $feet^2$.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask for volunteers to discuss their strategies and processes for performing the necessary calculations for the lengths of the sides, as well as the calculations for perimeter and area. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Ask students to share their reasoning processes, including how they determined which values to substitute into the perimeter and area formulas, and how they were able to apply the results in terms of the real-world context of the task. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of the components of the sides of the rectangle and how these sides represent the variables in the formulas.

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Connecting to Key Concepts

- Situations in which you would need to calculate perimeter include finding the amount of linear feet needed to fence a yard or a garden, determining the amount of trim needed for a room, or finding the amount of concrete needed to create a barrier around a statue.

  In this task, students are asked to use given coordinates to find the perimeter of a rectangular-shaped fence that will be used to enclose a baseball field.

- The perimeter is the sum of the lengths of all the sides of a polygon.

  Students will determine the length of each side of the rectangle by using the given coordinates in the distance formula. Then they will find the sum of the four lengths.

- The final answer must include the appropriate label for length (units, feet, inches, meters, centimeters, etc.)

  After calculating the perimeter and area, students will use the conversion given in the task that 1 unit is equal to 10 feet. They will then compute the actual number of feet of fencing needed, as well as the area of the field enclosed by the fence, in feet.

- The area of a rectangle is found using the formula $Area = (base)(height)$.

  Students will apply the area formula after determining the values representing the base and height of the rectangle.

- The lengths of the base and height can be found using the distance formula.

  Students will use the given coordinates and determine the values of the base and height of the rectangle by using the distance formula.

Extending the Task

- To extend the task, ask students to work together to determine another method of proving that the given coordinates represent the vertices of a rectangle. Ask them to apply their knowledge about slopes of parallel and perpendicular lines and prove that opposite sides of the rectangle are parallel and that all four corners are right angles. Ask for volunteers to share their processes, calculations, and results.

- Another option for extending the task is to ask students to work together to create their own real-world scenario where it is necessary to calculate the perimeter and area of a figure. Ask them to use a polygon other than a rectangle. Have them explain the problem, procedure, and results in specific steps on paper. Ask for volunteers to share their scenarios.

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Guide

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** I used abstract reasoning as I made the connection between the coordinates given in the problem to those of the corners of a rectangle. I reasoned that the distances between each consecutive corner represent the length of a side of the rectangle.) "Which of your strategies represent quantitative reasoning?" (**Answer:** I used quantitative reasoning as I calculated the distance between two points using the distance formula and used these distances as the sides of the rectangle. I then substituted the values into the perimeter and area formulas.)
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** I recognized that this scenario could be modeled using the distance formula, and the perimeter and area formulas. I used the coordinates representing each corner of the fence to determine the length of each side of the rectangular-shaped fence, and I used these lengths in the perimeter and area formulas.)
- **For SMP 6, ASK:** "How did you make sure you attended to precision?" (**Answer:** I attended to precision by using correct math terminology in my explanations and explaining my reasoning in a mathematically correct and precise way. Specifically, I referred to the base and height of the rectangle when using the perimeter and area formulas.)

Alternate Strategies or Solutions

- Students may choose to graph the given points on a coordinate plane. Encourage them to ensure the scales on the *x*- and *y*-axes are large enough to accommodate the given points. Students may also choose to draw their own sketches of a baseball field with a rectangle representing the fence around the field. This may help students who are visual learners and benefit from seeing a "picture" of the problem.

Technology

Students can use scientific calculators to help in their simplification of the radicals in the task.

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