Common Core State Standards Integrated Pathway

Support Supplement

for Mathematics I

$f(x)$ $\angle ABC$ Q_1

Teacher Resource

Unit 2

WALCH
INTEGRATED MATH

This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

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Standards Correlations

Each lesson in the *CCSS Integrated Pathway Support Supplement for Mathematics I* was written specifically to address one or more Common Core State Standards describing prerequisite skills necessary for achieving the standards in the corresponding lesson of *CCSS Integrated Pathway: Mathematics I*. These standards are drawn from the elementary (grades 3–5) and middle-level (grades 6–8) CCSS. Each lesson lists the standards covered in all the sets of Skill Instruction, and each set of Skill Instruction lists the standards addressed in that specific part. In this section, you'll find a comprehensive list mapping the Support resources to the CCSS describing the identified prerequisite skills.

Single asterisks (*) denote Targeted Prerequisite Skills that have been addressed in previous lessons in the Support Supplement. These topics are revisited in abbreviated form to build on prior knowledge and promote skill-building. Double asterisks (**) denote grade-level skills addressed in *CCSS Integrated Pathway: Mathematics I.* These topics are not revisited; instead, references are provided to the relevant instruction in *CCSS Integrated Pathway: Mathematics I* for each grade-level skill.

The Elementary Prerequisite Skills (E-Skills) are italicized for visual distinction from the targeted skills. (*Note*: E-Skills instruction is addressed in the comprehensive appendix.)

Guide to Common Core State Standards Annotation

As you use this resource, you may come across a symbol included with the Common Core standards for some of the lessons and activities. The description of the star symbol is found below, taken verbatim from the Common Core State Standards Initiative website, at www.corestandards.org.

Symbol: ★

Denotes: Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

From http://www.walch.com/CCSS/00003

CCSS INTEGRATED PATHWAY: SUPPORT SUPPLEMENT FOR MATHEMATICS I STANDARDS CORRELATIONS

Unit 2: Linear and Exponential Relationships			
Lesson	**Title**	**Standard(s)**	**Pages**
Lesson 1	**Graphs As Solution Sets and Function Notation: Prerequisite Skills**		
	E-Skill 1: Applying the Order of Operations	*5.OA.1*	*A-2*
	E-Skill 2: Understanding the Coordinate Plane	*5.G.1*	*A-10*
	Skill 1: Solving Equations in Standard Form for *y*	A–CED.4★	U2-3
	Skill 2: Creating Equations from Context	A–CED.2★	U2-16
	Skill 3: Evaluating Negative Exponents	8.EE.1	U2-31
	Skill 4: Substituting Values for Variables*	6.EE.2c	U2-44
	Skill 5: Understanding Domain and Range**	F–IF.1	U2-48
Lesson 2	**Solving Linear Inequalities in Two Variables and Systems of Inequalities: Prerequisite Skills**		
	Skill 1: Graphing Linear Equations in Two Variables*	A–CED.2★	U2-54
	Skill 2: Verifying Whether Inequalities Are True or False	6.EE.5	U2-60
	Skill 3: Creating Equations from Context*	A–CED.2★	U2-82
Lesson 3	**Sequences As Functions: Prerequisite Skills**		
	E-Skill 3: Recognizing Patterns	*3.OA.9*	*A-27*
	Skill 1: Understanding the Properties of Functions	8.F.4	U2-94
	Skill 2: Understanding Function Notation**	F–IF.2	U2-108
Lesson 4	**Interpreting Graphs of Functions: Prerequisite Skills**		
	E-Skill 1: Applying the Order of Operations	*5.OA.1*	*A-2*
	E-Skill 2: Understanding the Coordinate Plane	*5.G.1*	*A-10*
	Skill 1: Graphing Linear Functions from Tables or Equations*	A–CED.2★	U2-113
	Skill 2: Graphing Exponential Functions from Tables or Equations*	A–CED.2★	U2-119
	Skill 3: Understanding Function Notation, Domain, and Independent and Dependent Variables**	F–IF.1	U2-126
	Skill 4: Understanding Slope*	8.EE.5	U2-127
	Skill 5: Interpreting Interval Notation	No standard	U2-134

Lesson	Title	Standard(s)	Pages
Lesson 5	**Analyzing Linear and Exponential Functions: Prerequisite Skills**		
	E-Skill 2: Understanding the Coordinate Plane	*5.G.1*	*A-10*
	Skill 1: Graphing a Function from a Table of Values*	A–CED.2★	U2-152
	Skill 2: Understanding the Rules of Exponents, Including Negative Exponents*	8.EE.1	U2-160
	Skill 3: Recognizing the General Shape of an Exponential Function (Decay or Growth)*	A–CED.2★	U2-165
Lesson 6	**Comparing Functions: Prerequisite Skills**		
	Skill 1: Determining the Slope of Linear Functions*	8.EE.5	U2-174
	Skill 2: Determining the Intercepts of Linear Functions	8.EE.6	U2-182
	Skill 3: Determining the Rate of Change of Exponential Functions**	F–IF.6★	U2-198
	Skill 4: Determining the Intercepts of Exponential Functions**	F–IF.4★	U2-199
	Skill 5: Graphing Functions*	A–CED.2★	U2-200
Lesson 7	**Building Functions: Prerequisite Skills**		
	E-Skill 2: Understanding the Coordinate Plane	*5.G.1*	*A-10*
	Skill 1: Evaluating Exponential Expressions*	8.EE.1	U2-210
	Skill 2: Understanding Independent and Dependent Quantities	6.EE.9	U2-215
Lesson 8	**Operating on Functions and Transformations: Prerequisite Skills**		
	E-Skill 1: Applying the Order of Operations	*5.OA.1*	*A-2*
	Skill 1: Graphing Linear and Exponential Functions*	A–CED.2★	U2-244
	Skill 2: Identifying y-intercepts of Graphs of Functions*	8.EE.6	U2-250

Lesson	Title	Standard(s)	Pages
Lesson 9	**Arithmetic and Geometric Sequences: Prerequisite Skills**		
	E-Skill 1: Applying the Order of Operations	*5.OA.1*	*A-2*
	E-Skill 3: Recognizing Patterns	*3.OA.9*	*A-27*
	E-Skill 4: Multiplying Fractions	*5.NF.4a*	*A-34*
	Skill 1: Adding and Subtracting Signed Numbers	7.NS.1b 7.NS.1c	U2-263
	Skill 2: Identifying Linear Relationships*	8.F.4	U2-281
	Skill 3: Multiplying Signed Numbers	7.NS.2a	U2-286
	Skill 4: Using Exponents*	8.EE.1	U2-298
Lesson 10	**Interpreting Parameters: Prerequisite Skills**		
	Skill 1: Graphing Equations*	A–CED.2★	U2-307
	Skill 2: Writing Linear Equations from Context*	A–CED.2★	U2-313
	Skill 3: Writing Exponential Equations from Context*	A–CED.2★	U2-319

Lesson 1: Graphs As Solution Sets and Function Notation

Instruction

Elementary Prerequisite Skills

This lesson requires the use of the following elementary skill(s) to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*. See the Appendix for material to address the skill(s).

- E-Skill 1: Applying the Order of Operations (5.OA.1), Appendix p. A-2
- E-Skill 2: Understanding the Coordinate Plane (5.G.1), Appendix p. A-10

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*.

Skill 1: Solving Equations in Standard Form for *y* (A–CED.4★)

Common Core State Standard

A–CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*★

Skill 2: Creating Equations from Context (A–CED.2★)

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Skill 3: Evaluating Negative Exponents (8.EE.1)

Common Core State Standard

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

(*continued*)

Skill 4: Substituting Values for Variables* (6.EE.2c)

Common Core State Standard

6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.

Skill 5: Understanding Domain and Range** (F–IF.1)

Common Core State Standard

F–IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

Skill 1: Solving Equations in Standard Form for *y*

SMP	
1 ✓	2 ✓
3	4 ✓
5	6 ✓
7 ✓	8

Common Core State Standard

A–CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*★

Essential Questions

1. What does it mean to solve an equation for *y*?
2. Why would a function need to be solved for *y*?

WORDS TO KNOW

slope-intercept form the form $y = mx + b$, where m is the slope and b is the y-intercept

standard form of a linear function the form $ax + by = c$, where a, b, and c are integers, and a is positive

Recommended Resources

- IXL Learning. "Linear Equations: Solve for *y*."

  http://www.walch.com/rr/04037

  This site features practice problems with immediate feedback, including step-by-step directions for how to complete each problem.

- Purplemath.com. "Solving '$Ax + By = C$' for '$y =$.'"

  http://www.walch.com/rr/04038

  This site gives examples of how to solve an equation for *y* and how the slope-intercept form can be used to find the slope and the intercept of a line.

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Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Have the students create a flowchart of the steps needed to solve an equation for y. For example, distribute copies of the Flowchart graphic organizer found in the Program Overview. Provide students with the equation $9x + 3y = 24$. In the diamond under the word "Start," have students write the given equation. Then have students identify a possible first step to solving the equation for y. One possible first step is to subtract $9x$ from both sides of the equation. In the rectangle to the right of the original equation, have students write "Subtract $9x$ from both sides of the equation."

Next, have students write the new equation, $3y = 24 - 9x$, in the second diamond of the flow chart. Have students identify the next possible step in solving the equation for y. Once identified, have students write this step in the rectangle to the right of the equation. Continue this process until students arrive at the equation solved for y.

Once completed, have students share their flowcharts, noting any differences in the process. Discuss how the processes could vary, but recognize that the results are the same.

Suggestions for Discourse

- Ask students why *every term* of the equation needs to be divided by the coefficient of the y as opposed to just one of the terms.
- The equations presented in Unit 2, Lesson 1 are in standard form, $ax + by = c$. Ask students how standard form is different from slope-intercept form, $y = mx + b$.

Making Connections

In order to find the slope and y-intercept of a line, the equation must be written in slope-intercept form, $y = mx + b$. This means the equation must first be solved for y.

Skill 1: Solving Equations in Standard Form for *y*

Introduction

Often, an equation will be shown in standard form, such as $5x - 2y = 10$. However, for a function to be graphed, its equation may need to be rearranged so it is in a more usable format. Typically, this means solving the equation for y.

Key Concepts

- To solve an equation for y means to rearrange the equation so the y is by itself on the left side of the equal sign. For example, in the equation $y = 2x + 5$, the y is by itself and the rest of the equation shows what y equals.
- The **standard form of a linear function** is written as $ax + by = c$, where a, b, and c are integers and a is positive. In this form, a and b are coefficients and c is the constant term.
- The most common and efficient way to graph this type of equation is to put it in **slope-intercept form**, which is $y = mx + b$, where m is the slope and b is the y-intercept.
- Notice that in order to put the equation in slope-intercept form, the equation must be solved for y.

Solving an Equation for *y*
1. Move the ax term to the other side of the equation by adding its opposite to both sides of the equation. 2. Divide every term on both sides of the equation by b. *Note*: It is best to arrange the final answer so the x-term comes before the constant term. That way, the equation will be in slope-intercept form, $y = mx + b$.

- If the value of y is known, the value of x can be found by substituting the y-value, and then solving the equation for x.

Finding the Value of *x* Given the *y*-value
1. Move any terms that are added to or subtracted from the x-term by performing the opposite operation on both sides of the equation. (For example, if 6 is being added to x, subtract 6 from both sides.) 2. Divide both sides by the coefficient of x.

Guided Practice Skill 1

Example 1

Solve the equation $8x - 2y = 6$ for y.

1. Move the term with x to the other side of the equation by adding its opposite to both sides of the equation.

   The term with x is $8x$.

   Move this term by adding the opposite of $8x$ to both sides of the equation.

   The opposite of $8x$ is $-8x$; therefore, add $-8x$ to both sides of the equation.

$8x - 2y = 6$	Original equation
$8x - 2y + (-8x) = 6 + (-8x)$	Add $-8x$ to both sides of the equation.
$-2y = 6 - 8x$	Simplify.

   The result of adding the opposite of $8x$ to both sides of the equation is $-2y = 6 - 8x$.

2. Divide every term on both sides of the equation by the coefficient of y.

   The coefficient of y is -2. Therefore, divide every term on both sides of the equation by -2.

$-2y = 6 - 8x$	Equation from the previous step
$\frac{-2y}{-2} = \frac{6}{-2} - \frac{8x}{-2}$	Divide every term by -2.
$y = -3 + 4x$	Simplify.
$y = 4x - 3$	Rearrange the terms on the right side so the x-term is listed first.

   Note that it is best to rearrange the terms so the x-term comes before the constant term. That way, the equation is in slope-intercept form.

   The equation $8x - 2y = 6$ solved for y is $y = 4x - 3$.

Example 2

The equation $4x + 6y = 68$ represents a vendor's sales at a ballpark, where x is the number of hot dogs sold and y is the number of hamburgers sold. If the vendor has sold 8 hamburgers, how many hot dogs has he sold?

1. Substitute the given value for y.

   Since y represents the number of hamburgers sold and the vendor has sold 8 hamburgers, substitute 8 for y in the equation.

$4x + 6y = 68$	Original equation
$4x + 6(8) = 68$	Substitute 8 for y.
$4x + 48 = 68$	Simplify.

   The equation is now $4x + 48 = 68$.

2. Solve the equation for x.

   To solve an equation for x, first move any terms that are added to or subtracted from the x-term by performing the opposite operation on both sides of the equation. Then divide both sides by the coefficient of x.

   In this case, 48 needs to be subtracted from both sides of the equation. Then 4, the coefficient of the x-term, needs to be divided from both sides of the resulting equation.

$4x + 48 = 68$	Equation from the previous step
$4x + 48 - 48 = 68 - 48$	Subtract 48 from both sides of the equation.
$4x = 20$	Simplify.
$\frac{4x}{4} = \frac{20}{4}$	Divide both sides by 4.
$x = 5$	Simplify.

   The value of x is 5; therefore, the vendor has sold 5 hot dogs.

Scaffolded Practice Skill 1

Example 1

Solve the equation $8x - 2y = 6$ for y.

1. Move the term with x to the other side of the equation by adding its opposite to both sides of the equation.

2. Divide every term on both sides of the equation by the coefficient of y.

continued

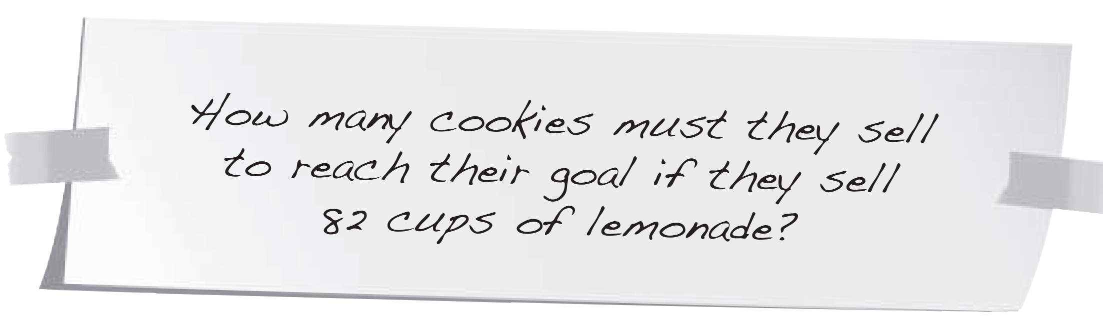
Example 2

The equation $4x + 6y = 68$ represents a vendor's sales at a ballpark, where x is the number of hot dogs sold and y is the number of hamburgers sold. If the vendor has sold 8 hamburgers, how many hot dogs has he sold?

Problem-Based Task Skill 1: Goods for a Good Cause

SMP	
1 ✓	2 ✓
3	4 ✓
5	6 ✓
7 ✓	8

Veronica and Quincy are selling lemonade and cookies to raise money for a local animal shelter, and have a goal to raise \$250. They plan to sell cups of lemonade for \$0.75 each and cookies for \$1.25 each. Write an equation that represents the number of cookies, y, they must sell to reach their goal if they sell x cups of lemonade. How many cookies must they sell to reach their goal if they sell 82 cups of lemonade?



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Problem-Based Task Skill 1: Goods for a Good Cause

Coaching

a. What expression represents the money earned from selling x cups of lemonade?

b. What expression represents the money earned from selling y cookies?

c. What equation represents the total money earned that will equal their goal?

d. How can the equation be rearranged to represent the number of cookies they must sell to reach their goal?

e. How many cookies must they sell to reach their goal if they sell 82 cups of lemonade?

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Problem-Based Task Skill 1: Goods for a Good Cause

Coaching Sample Responses

a. What expression represents the money earned from selling x cups of lemonade?

Each cup of lemonade costs \$0.75. The money earned equals the price of each cup (\$0.75) times the number of cups sold (x).

The expression $0.75x$ represents the money earned from selling x cups of lemonade.

b. What expression represents the money earned from selling y cookies?

Each cookie costs \$1.25. The money earned equals the cost of each cookie (\$1.25) times the number of cookies sold (y).

The expression $1.25y$ represents the money earned from selling y cookies.

c. What equation represents the total money earned that will equal their goal?

The total money earned is the sum of the money earned from the lemonade ($0.75x$) and the money earned from the cookies ($1.25y$). Their goal is \$250, so the sum must equal \$250.

The equation $0.75x + 1.25y = 250$ represents the total money earned that will equal their goal.

d. How can the equation be rearranged to represent the number of cookies they must sell to reach their goal?

The number of cookies is represented by y. Therefore, solve the equation $0.75x + 1.25y = 250$ for y to represent how many cookies they must sell to reach their goal.

Subtract $0.75x$ from both sides, simplify the resulting equation, and then divide every term on both sides of the equation by 1.25.

$$0.75x + 1.25y = 250$$

$$0.75x + 1.25y - 0.75x = 250 - 0.75x$$

$$1.25y = 250 - 0.75x$$

$$\frac{1.25y}{1.25} = \frac{250}{1.25} - \frac{0.75x}{1.25}$$

$$y = 200 - 0.6x$$

The equation $y = 200 - 0.6x$ represents the number of cookies they must sell to reach their goal if they sell x cups of lemonade.

e. How many cookies must they sell to reach their goal if they sell 82 cups of lemonade?

Substitute 82 for x in the equation $y = 200 - 0.6x$, and then solve for y, the number of cookies.

$y = 200 - 0.6(82)$

$y = 200 - 49.2$

$y = 150.8$

A partial cookie cannot be sold, so they must sell 151 cookies in order to reach their goal.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 1: Solving Equations in Standard Form for *y*

For problems 1–6, solve each equation for y.

1. $7x + 6y = -12$

2. $x - 4y = 1$

3. $5x - y = 0$

4. $2x + 3y = -3$

5. $8x - 2y = -10$

6. $x + y = -9$

continued

For problems 7 and 8, find the value of x for the given y-value.

7. $y = 5x - 12$; $y = 6$

8. $5x - 3y = 6$; $y = -2$

For problems 9 and 10, read each scenario and answer the questions.

9. The Andersons are traveling across the country to visit some family members. The equation $y = 62x$ can be used to estimate the number of miles, y, they will have traveled after x hours of driving. About how many hours will it take them to travel 434 miles?

10. The equation $20x - y = -50$ describes the total cost in dollars (y) for a gym membership at Brad's Gym for x months. Kaleigh has paid a total of $210 for her gym membership. For how many months has Kaleigh been attending Brad's Gym?

Skill 2: Creating Equations from Context

SMP
1 ✓ 2 ✓
3 4 ✓
5 6 ✓
7 ✓ 8

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Essential Question

1. What is the difference between the dependent variable and the independent variable?

WORDS TO KNOW

dependent variable labeled on the *y*-axis; the quantity that is based on the input values of the independent variable; the output variable of a function

independent variable labeled on the *x*-axis; the quantity that changes based on values chosen; the input variable of a function

Recommended Resources

- IXL Learning. "Write Variable Expressions to Represent Word Problems."

  http://www.walch.com/rr/04039

  This site includes practice problems for translating real-life situations into variable expressions.

- Shmoop. "High School: Algebra—Creating Equations HSA-CED.A.2."

  http://www.walch.com/rr/04040

  This site provides an explanation of the standard that is useful for both students and teachers, along with sample assignments and aligned resources.

Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

- Provide students with the equation $y = 2x$. Ask students to substitute various values for x and find the corresponding value of y. Then have them graph the corresponding points on a coordinate plane and draw a line through the points. Have students create a new linear equation and again substitute various values of x to find corresponding y-values. Then have students graph the points on a coordinate plane and draw a line through the points. This will help students see how the different x-values produce different y-values.
- Have students create a web of various mathematical words. Instruct them to label the center circle with the phrase "key words," and then add four connecting circles labeled "addition," "subtraction," "multiplication," and "division." Next, instruct students to connect as many words and phrases as they can that mean *addition* to the addition circle (such as *sum* and *added to*). Repeat the process with subtraction, multiplication, and division. Later, students can refer to this organizer as a reference sheet when writing equations.

Suggestions for Discourse

- Ask students how they can determine which variable is the dependent variable and which is the independent variable.
- Ask students to explain how a line on a coordinate plane can represent the equation of a real-life situation. For example, if the equation $y = 6x$ gives the number of miles (y) Andy can run in x hours, how would a graph of the equation $y = 6x$ represent the different possibilities?

Making Connections

Equations in two variables can be graphed on a coordinate plane. When two equations describe the same situation, the intersection of those two equations is the answer to the system. However, before equations can be graphed to find the solution, they must be written.

Skill 2: Creating Equations from Context

Introduction

Many real-life situations can be written as equations. The equations can then be used to analyze the situations or predict future results. Sometimes, the situation has more than one changeable factor, in which case the equation has more than one variable. For example, the owner of a deli could create an equation that tells the amount of money his deli will earn based on the number of sandwiches he sells.

Key Concepts

- When creating an equation with two variables, x is the independent variable and y is the dependent variable.
- The **independent variable** (x) is the input variable, meaning it is the value that starts the situation and produces the other variable. For example, using the deli example from the Introduction, the number of sandwiches sold is the independent variable, because the sandwich sales produce the money earned (and not the other way around). When an equation is graphed, the independent variable is labeled on the x-axis.
- The **dependent variable** (y) is based on the input value of the independent variable, meaning that the dependent variable is produced by (or is dependent on) the independent variable. In the deli example, the money earned is the dependent variable, because it depends on, and is produced by, the sandwich sales.

Key Words to Recognize when Writing Equations

Addition	Subtraction	Multiplication	Division
more	less than	times	each
greater than	fewer	of	quotient
plus	left over	product	divided by
sum	difference	twice	split into
added to	subtracted from	multiplied by	evenly distributed
both	how many more	every	
in all	minus	at this rate	
total	remains		
altogether	decrease		
increase			

Common Formulas

- There are many formulas that are commonly referenced in word problems. The following formulas for distance and interest rate are included:

  distance equals *rate* times *time*, or $d = r \bullet t$

  interest equals *principal* times *rate* times *time*, or $I = P \bullet r \bullet t$

- The principal is the initial or starting amount.

Tables and Graphs

- Tables can be useful for organizing information, especially when two variables are involved.
- To write an equation when given a table, first analyze the table to determine the common relationship between the x- and y-values. (For example, y may always be 7 more than the corresponding x.) Then write an equation that represents the relationship.
- After an equation is written, you can create a table by substituting values for x and solving for y. This table can then be used to create a graph of the function. The graph gives a visual representation of the relationship, and shows various possible values for y depending on the value of x.

Guided Practice Skill 2

Example 1

Victor's car can only go up to 65 mph. Using the variables x and y, create an equation that tells how far he can travel based on the number of hours he drives, assuming he drives at the maximum speed.

1. Determine what the variables x and y will represent.

   x is the independent variable, or the variable that starts the situation.

   y is the dependent variable, or the variable that depends on the independent variable.

   In this situation, the distance depends on the time. Therefore, the time is x (the independent variable), and the distance is y (the dependent variable).

2. Use the formula $d = r \bullet t$ to write an equation for the situation.

   The distance, d, is y. The maximum speed or rate, r, is 65 mph. The time, t, is x. Substitute the values into the formula $d = r \bullet t$.

$d = r \bullet t$	Given formula
$(y) = (65)(x)$	Substitute y for d, 65 for r, and x for t.
$y = 65x$	Simplify.

   The equation $y = 65x$ tells how far Victor can travel based on the number of hours he drives.

Example 2

Vernon Jewelry buys and sells jewelry. Its store policy is to buy necklaces for $60 and sell them for $125. Pairs of earrings are purchased for $35 and then sold for $65. Last week, the store had $3,305 in sales, which came to $1,635 in profit after accounting for the cost of the jewelry. Write an equation that describes the total sales for the week, and a second equation that describes last week's profit.

1. Determine what the variables x and y will represent.

   The two variables in this situation are the number of necklaces sold and the number of pairs of earrings sold.

   In this situation, neither variable is dependent on the other, because the number of necklaces sold does not necessarily affect the number of earrings sold and vice versa. Therefore, it does not matter which is the independent variable (x) and which is the dependent variable (y).

   In this example, use x to represent the number of necklaces sold and y to represent the number of pairs of earrings sold.

2. Write an equation that represents the total sales for the week.

   The total sales last week amounted to $3,305.

   Each necklace sells for $125. Therefore, the total sales from the necklaces is equal to $125 times the number of necklaces sold (x), or $125x$.

   Each pair of earrings sells for $65. Therefore, the total sales from earrings is equal to $65 times the number of pairs of earrings sold (y), or $65y$.

   The total sales ($3,305) is equal to the sum of the sales from the necklaces ($125x$) and the sales of the earrings ($65y$).

   The equation $125x + 65y = 3305$ represents the total sales for the week.

3. Write an equation that represents last week's profit.

   The profit last week was $1,635.

   Each necklace costs $60 and sells for $125. Therefore, the profit on each necklace is $65 ($125 – $60 = $65). The profit from the necklaces is equal to $65 times the number of necklaces sold (x), or $65x$.

   Each pair of earrings costs $35 and sells for $65. Therefore, the profit on each pair of earrings is $30 ($65 – $35 = $30). The profit from the earrings is equal to $30 times the pairs of earrings sold (y), or $30y$.

   The total profit ($1,635) is equal to the sum of the profit from the necklaces ($65x$) and the profit from the earrings ($30y$).

   The equation $65x + 30y = 1635$ represents last week's profit.

Scaffolded Practice Skill 2

Example 1

Victor's car can only go up to 65 mph. Using the variables *x* and *y*, create an equation that tells how far he can travel based on the number of hours he drives, assuming he drives at the maximum speed.

1. Determine what the variables *x* and *y* will represent.

2. Use the formula $d = r \bullet t$ to write an equation for the situation.

continued

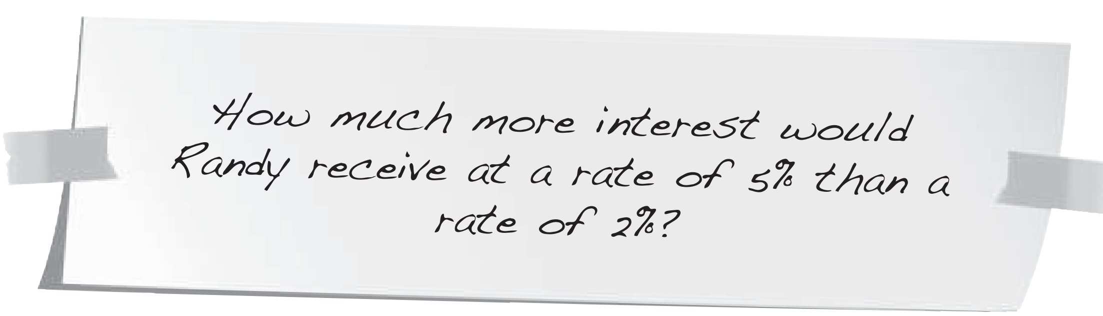
Example 2

Vernon Jewelry buys and sells jewelry. Its store policy is to buy necklaces for $60 and sell them for $125. Pairs of earrings are purchased for $35 and then sold for $65. Last week, the store had $3,305 in sales, which came to $1,635 in profit after accounting for the cost of the jewelry. Write an equation that describes the total sales for the week, and a second equation that describes last week's profit.

Problem-Based Task Skill 2: Piqued Interest

SMP
1 ✓ 2 ✓
3 4 ✓
5 6 ✓
7 ✓ 8

Randy has $7,000 that he wants to invest for 1 year, and is trying to determine how much interest he will make depending on the interest rate he can find. Write an equation to describe the interest he will receive depending on his rate. How much more interest would Randy receive at a rate of 5% than at a rate of 2%?



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Problem-Based Task Skill 2: Piqued Interest

Coaching

a. What formula should be used to create the equation?

b. Which is the dependent variable, and which is the independent variable?

c. What equation can be written to describe the interest Randy will receive?

d. How much interest will Randy receive if he can find an interest rate of 5%?

e. How much interest will Randy receive if he only gets an interest rate of 2%?

f. How much more interest would Randy receive at a rate of 5% than 2%?

Problem-Based Task Skill 2: Piqued Interest

Coaching Sample Responses

a. What formula should be used to create the equation?

The formula that should be used to create the equation is the interest formula $I = P \bullet r \bullet t$, where I is the interest, P is the principal (the amount of money originally invested), r is the interest rate, and t is the time in years.

b. Which is the dependent variable, and which is the independent variable?

Principal and time are fixed amounts in this situation. The interest depends on the rate. Therefore, the rate is the independent variable (x), and the interest is the dependent variable (y).

c. What equation can be written to describe the interest Randy will receive?

Start with $I = P \bullet r \bullet t$ and substitute x for r (the rate), y for I (the interest), 7,000 for P (the principal), and 1 for t (the time).

$I = P \bullet r \bullet t$

$(y) = (7000)(x)(1)$

$y = 7000x$

The equation $y = 7000x$ describes the interest he will receive depending on the rate.

d. How much interest will Randy receive if he can find an interest rate of 5%?

In the equation $y = 7000x$, x is the rate and y is the interest. Substitute 5% for x and solve for y.

Recall that 5% must be converted to a decimal, which is equal to 0.05.

$y = 7000x$

$y = 7000(0.05)$

$y = 350$

If Randy invests the money at a rate of 5%, he will receive $350 in interest.

e. How much interest will Randy receive if he only gets an interest rate of 2%?

In the equation $y = 7000x$, x is the rate and y is the interest. Substitute 2% for x and solve for y.

Recall that 2% must be converted to a decimal, which is equal to 0.02.

$y = 7000x$

$y = 7000(0.02)$

$y = 140$

If Randy invests the money at a rate of 2%, he will receive \$140 in interest.

f. How much more interest would Randy receive at a rate of 5% than 2%?

If Randy invests his money at a rate of 5%, he will receive \$350 in interest.

If he invests his money at a rate of 2%, he will receive \$140 in interest.

Subtract to determine the difference.

$350 - 140 = 210$

Randy would receive \$210 more in interest if he invested at a rate of 5% than if he invested at a rate of 2%.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 2: Creating Equations from Context

Create an equation for each of the following scenarios. Let *x* represent the independent variable and *y* represent the dependent variable.

1. Ben can run at an average pace of 6 mph. Write an equation to describe how far he can go depending on how long he runs.

2. Callie wants to have her birthday party at her favorite restaurant. She has a budget of $350 for the party, and the restaurant charges $20 per guest. Write an equation for the amount of money Callie will have left over, depending on how many people she invites.

3. A farm has twice as many pigs as it has goats. Write an equation to describe the number of pigs on the farm based on the number of goats.

4. The McMillan family vehicle has a 15-gallon gas tank. Write an equation for the cost to fill the tank depending on the price of gas.

5. The concession stand at the middle school sold 45 hamburgers and 42 hot dogs, resulting in total sales of $175.50. Write an equation that describes the total sales.

continued

6. Andrea has to pay a fine of 25 cents ($0.25) per book every day her library books are overdue. Write an equation to describe her total fine if she has 11 books checked out, depending on how many days overdue the books are.

7. Gregory wants to buy as many bags of pretzels as he can for under $30. Write an equation for how many bags he can buy depending on the price per bag.

8. Tanner, who is 10 years old, wants to invest money in a college savings account that pays 4% interest. He plans to take the money out when he goes to college at age 18. Write an equation for how much interest he will earn depending on how much money he invests.

9. Madeline wants to buy a present for her brother. She has $30. Write an equation for how much money she will have left over after she buys the present, depending on its cost.

10. Oliver biked for 2 hours. Write an equation to describe how far he traveled, depending on his average speed.

Skill 3: Evaluating Negative Exponents

SMP	
1 ✓	2 ✓
3	4 ✓
5	6 ✓
7 ✓	8

Common Core State Standard

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Essential Question

1. What does a negative exponent mean?

WORDS TO KNOW

base — the factor being multiplied together in an exponential expression; in the expression a^b, a is the base.

exponent — the number of times a quantity or variable is multiplied together

Recommended Resources

- IXL Learning. "Evaluate Negative Exponents."

  http://www.walch.com/rr/04041

  This site provides practice evaluating negative exponents. Immediate feedback, including an explanation for questions answered incorrectly, is provided.

- MathIsFun.com. "Negative Exponents."

  http://www.walch.com/rr/04042

  This site provides a clear and concise explanation of what negative exponents are and how to work with them.

Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Distribute the Cause and Effect Map graphic organizer from the Program Overview. Have students determine the cause and effect of the various exponent rules. In the first oval under the "Cause" heading, have students write the start of the rule there with an example. For example, write "To multiply powers with the same base…." For this rule, students would write "$a^m \bullet a^n$" and give an example such as $x^5 \bullet x^2$.

Then, in the first box under the "Effect" heading, have students write the end of the rule. In this case, it would be "…add the exponents." Have them also include the result and the answer to the example. The result would be a^{m+n} and the answer to the example would be x^7.

Have students repeat the process for the remaining exponent rules. Provide additional copies of the Cause and Effect Map as needed.

Suggestions for Discourse

- Use an example to demonstrate how an exponent rule works. For example, for the rule "To multiply powers with the same base, add the exponents," use an example such as $x^4 \bullet x^3 = x^{4+3} = x^7$. This can be shown to be true by expanding each power: $x^4 = x \bullet x \bullet x \bullet x$ and $x^3 = x \bullet x \bullet x$. Therefore, $x^4 \bullet x^3 = x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x = x^7$. After showing students how this works, ask them to come up with their own examples and demonstrate them using the same process.
- Have students create examples to explain the remaining exponent rules.

Making Connections

When graphing exponential functions with negative exponents such as $y = 2^{-x}$, students must substitute values for x and solve for y. In order to do this, they must understand how to evaluate negative exponents.

Skill 3: Evaluating Negative Exponents

Introduction

Exponents are a useful concept to master, as they provide a quick way to express repeated multiplication. Exponents can be added, multiplied, divided, and even raised to another power.

Key Concepts

- An **exponent** tells the number of times a quantity or variable is multiplied together. In the expression a^b, b is the exponent.
- The **base** is the factor being multiplied. In the expression a^b, a is the base.
- When a number does not appear to have an exponent, it has an understood exponent of 1.
- Certain rules must be followed when evaluating expressions with exponents.

Exponent Rules

- To multiply powers with the same base, add the exponents.

  $a^m \bullet a^n = a^{m+n}$ For example, $4^5 \bullet 4^3 = 4^{5+3} = 4^8$.

- To divide powers with the same base, subtract the exponents.

  $\frac{a^m}{a^n} = a^{m-n}$ For example, $\frac{4^5}{4^3} = 4^{5-3} = 4^2$.

- Any base raised to the power of 0 is equal to 1.

  $a^0 = 1$ For example, $4^0 = 1$.

- To raise one power to another power, multiply the exponents.

  $(a^m)^n = a^{m \bullet n}$ For example, $(4^5)^3 = 4^{5 \bullet 3} = 4^{15}$.

- To find the power of a product, distribute the exponents.

  $(ab)^m = a^m \bullet b^m$ For example, $(4 \bullet 5)^3 = 4^3 \bullet 5^3$.

- To find the power of a quotient, distribute the exponents.

  $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ For example, $\left(\frac{4}{7}\right)^5 = \frac{4^5}{7^5}$.

Negative Exponents

- It is not proper to leave a negative exponent in an expression. To make a negative exponent positive, take the reciprocal of the power.

  $a^{-m} = \frac{1}{a^m}$; $a \neq 0$ For example, $4^{-5} = \frac{1}{4^5}$.

Guided Practice Skill 3

Example 1

Simplify the expression $\frac{x^5}{x^9}$. Write the expression using only positive exponents, and then show that the two expressions are equal by using the expanded form.

1. Use the exponent rule $\frac{a^m}{a^n} = a^{m-n}$ to simplify the expression.

   Notice in the expression $\frac{x^5}{x^9}$, the base of both the numerator and the denominator is x. To divide powers with the same base, subtract the exponents.

   $$\frac{a^m}{a^n} = a^{m-n}$$

   $$\frac{x^5}{x^9} = x^{5-9} = x^{-4}$$

   The expression $\frac{x^5}{x^9}$ can be simplified to x^{-4}.

2. Use the exponent rule $a^{-m} = \frac{1}{a^m}$ to write the negative exponent as a positive exponent.

   To rewrite a negative exponent as a positive exponent, take the reciprocal of the expression with a positive exponent.

   $$a^{-m} = \frac{1}{a^m}$$

   $$x^{-4} = \frac{1}{x^4}$$

   The expression x^{-4} is equal to $\frac{1}{x^4}$; therefore, the expression $\frac{x^5}{x^9}$ is equal to $\frac{1}{x^4}$.

3. Show that the two expressions are equal by using expanded form.

   Expand the original expression, $\frac{x^5}{x^9}$.

   $$\frac{x^5}{x^9}=\frac{x \bullet x \bullet x \bullet x \bullet x}{x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x}$$

   Cancel out like factors. Each x in the numerator will cancel with an x in the denominator.

   $$\frac{x^5}{x^9}=\frac{x \bullet x \bullet x \bullet x \bullet x}{x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x}=\frac{1}{x \bullet x \bullet x \bullet x}=\frac{1}{x^4}$$

   Notice that using the rules and examining the expanded form produce the same answer, demonstrating the truth of the exponent rules.

Example 2

Simplify the expression $x^4 \bullet y^{-2} \bullet x^7 \bullet z^0$. Write the expression using only positive exponents.

1. Use the exponent rule $a^m \bullet a^n = a^{m+n}$ to simplify the expression.

   To multiply powers with the same base, add the exponents.

   In the expression $x^4 \bullet y^{-2} \bullet x^7 \bullet z^0$, the only terms with the same base are x^4 and x^7. To multiply these terms, add the exponents.

   $$x^4 \bullet x^7 = x^{4+7} = x^{11}$$

   The expression $x^4 \bullet y^{-2} \bullet x^7 \bullet z^0$ can now be simplified to $x^{11} \bullet y^{-2} \bullet z^0$.

2. Use the exponent rule $a^0 = 1$ to further simplify the expression.

   Any value to the 0 power equals 1. Thus, $z^0 = 1$.

   Substitute 1 for z^0.

   $$x^{11} \bullet y^{-2} \bullet z^0 = x^{11} \bullet y^{-2} \bullet 1 = x^{11} \bullet y^{-2}$$

   The expression is now simplified to $x^{11} \bullet y^{-2}$.

3. Use the exponent rule $a^{-m}=\dfrac{1}{a^m}$ to write the negative exponent as a positive exponent.

To rewrite a negative exponent as a positive one, take its reciprocal.

$$a^{-m}=\frac{1}{a^m}$$

$$y^{-2}=\frac{1}{y^2}$$

Substitute $\dfrac{1}{y^2}$ for y^{-2}.

$$x^{11}\bullet y^{-2}=x^{11}\bullet\frac{1}{y^2}=\frac{x^{11}}{y^2}$$

The expression $x^4 \bullet y^{-2} \bullet x^7 \bullet z^0$ can be simplified to $\dfrac{x^{11}}{y^2}$.

Scaffolded Practice Skill 3

Example 1

Simplify the expression $\frac{x^5}{x^9}$. Write the expression using only positive exponents, and then show that the two expressions are equal by using the expanded form.

1. Use the exponent rule $\frac{a^m}{a^n} = a^{m-n}$ to simplify the expression.

2. Use the exponent rule $a^{-m} = \frac{1}{a^m}$ to write the negative exponent as a positive exponent.

3. Show that the two expressions are equal by using expanded form.

continued

Example 2

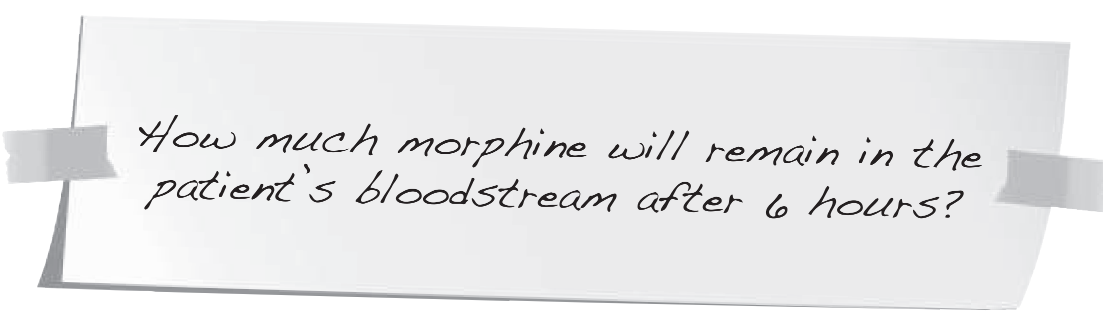
Simplify the expression $x^4 \bullet y^{-2} \bullet x^7 \bullet z^0$. Write the expression using only positive exponents.

Problem-Based Task Skill 3: All That Remains

The half-life of morphine is about 2 hours. This means that after every 2 hours, only half the amount of morphine remains in a patient's bloodstream. Dr. Avenel just gave a patient 20 milligrams of morphine. The equation $y = 20(2^{-0.5x})$ describes how much morphine (y) is left in the patient's bloodstream after x hours. How much morphine will remain in the patient's bloodstream after 6 hours?

SMP

1 ✓	2 ✓
3	4 ✓
5	6 ✓
7 ✓	8



Problem-Based Task Skill 3: All That Remains

Coaching

a. What equation represents the amount of morphine remaining in the patient's bloodstream after 6 hours?

b. How can the negative exponent be rewritten?

c. How much morphine will remain in the patient's bloodstream after 6 hours?

Problem-Based Task Skill 3: All That Remains

Coaching Sample Responses

a. What equation represents the amount of morphine remaining in the patient's bloodstream after 6 hours?

In the equation $y = 20(2^{-0.5x})$, x represents the number of hours the morphine has been in the patient's bloodstream.

The problem is asking how much morphine remains after 6 hours.

Substitute 6 for x.

$y = 20(2^{-0.5x})$

$y = 20[2^{-0.5(6)}]$

Multiply –0.5 and 6. The result is –3.

$y = 20(2^{-3})$

The equation $y = 20(2^{-3})$ represents the amount of morphine left in the patient's bloodstream after 6 hours.

b. How can the negative exponent be rewritten?

To rewrite a negative exponent as a positive one, take its reciprocal.

$$a^{-m} = \frac{1}{a^m}$$

$$2^{-3} = \frac{1}{2^3}$$

Substitute $\frac{1}{2^3}$ for 2^{-3}.

$y = 20(2^{-3})$

$$y = 20 \bullet \frac{1}{2^3}$$

$$y = \frac{20}{2^3}$$

The equation $y = 20(2^{-3})$ can be rewritten as $y = \frac{20}{2^3}$.

c. How much morphine will remain in the patient's bloodstream after 6 hours?

Simplify the expression $\frac{20}{2^3}$.

$$\frac{20}{2^3}=\frac{20}{8}=2.5$$

After 6 hours, about 2.5 milligrams of morphine will remain in the patient's bloodstream.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 3: Evaluating Negative Exponents

For problems 1–3, evaluate each exponential expression.

1. 2^{-5}

2. $\dfrac{1}{10^{-3}}$

3. After 3 rounds of a basketball tournament, there are $64 \bullet 2^{-3}$ teams left in the competition. How many teams remain?

For problems 4–10, simplify each exponential expression. Write the expression with only positive exponents.

4. y^{-9}

5. $a^5 \bullet a^2$

6. $x^0 \bullet y^{-4}$

7. $\dfrac{x^3}{x^8}$

8. $(n^4)^5$

9. $\dfrac{a^5 \bullet b^2}{a \bullet b^6}$

10. $x^4 \bullet (x^{-2})^3$

Instruction

Skill 4: Substituting Values for Variables*

Common Core State Standard

6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\,s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 1, Lesson 1, Skill 2

Guided Practice Skill 4

Example 1

At the school supply store, a bottle of glue is $1.50, a notebook is $5.00, and a pack of ballpoint pens is $3.00. Write an expression to represent each item, then write an expression for the total cost of the three items. What is your total cost if you buy 2 bottles of glue, 1 notebook, and 3 packs of pens?

1. Write an expression that represents the cost of each item.

   First, choose a variable to represent each item.

   Let g represent the number of glue bottles, n represent the number of notebooks, and p represent the number of packs of ballpoint pens.

   Then, write an expression that represents the cost of each item.

   Let $1.5g$ represent the cost of glue bottles.

   Let $5n$ represent the cost of notebooks.

   Let $3p$ represent the cost of packs of ballpoint pens.

2. Write an expression for the total cost of the three items.

   The expression for the total cost of the three items is found by adding the individual expressions for each item.

   The expression for the total cost of the three items is $1.5g + 5n + 3p$.

3. Determine the total cost if you buy 2 bottles of glue, 1 notebook, and 3 packs of pens.

   To find the total cost, evaluate the expression $1.5g + 5n + 3p$ for $g = 2$, $n = 1$, and $p = 3$.

$1.5g + 5n + 3p$	Original expression
$= 1.5(2) + 5(1) + 3(3)$	Substitute 2 for g, 1 for n, and 3 for p.
$= 3 + 5 + 9$	Multiply.
$= 17$	Add.

   The total cost of buying 2 bottles of glue, 1 notebook, and 3 packs of ballpoint pens is $17.

Scaffolded Practice Skill 4

Example 1

At the school supply store, a bottle of glue is $1.50, a notebook is $5.00, and a pack of ballpoint pens is $3.00. Write an expression to represent each item, then write an expression for the total cost of the three items. What is your total cost if you buy 2 bottles of glue, 1 notebook, and 3 packs of pens?

1. Write an expression that represents the cost of each item.

2. Write an expression for the total cost of the three items.

3. Determine the total cost if you buy 2 bottles of glue, 1 notebook, and 3 packs of pens.

Practice Skill 4: Substituting Values for Variables*

For problems 1–3, evaluate each expression for the given values.

1. There are 7 bottles of juice on a store shelf, each priced at $3.50. What is the total amount of money earned if all of the bottles are sold?

2. Use the expression s^3 to find the volume of a cube with a side length of 2.7 inches.

3. Use the expression *lwh* to find the volume of a box if its length is 4 meters, its width is 2 meters, and its height is 6 meters.

Skill 5: Understanding Domain and Range**

Common Core State Standard

F–IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

This skill has been addressed previously in *CCSS Integrated Pathway: Mathematics I*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

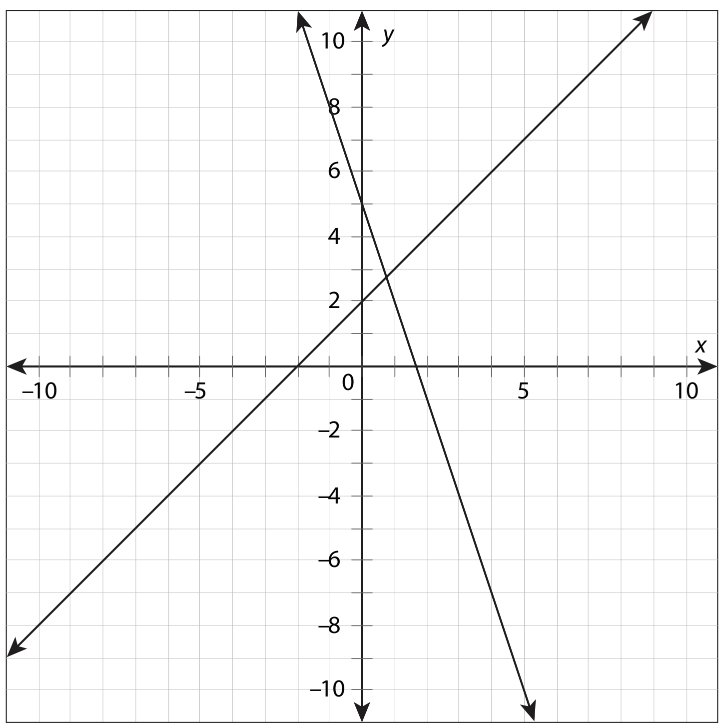
Unit 2, Lesson 1, Sub-lesson 3

Supportive Instructional Strategies for Mathematics I

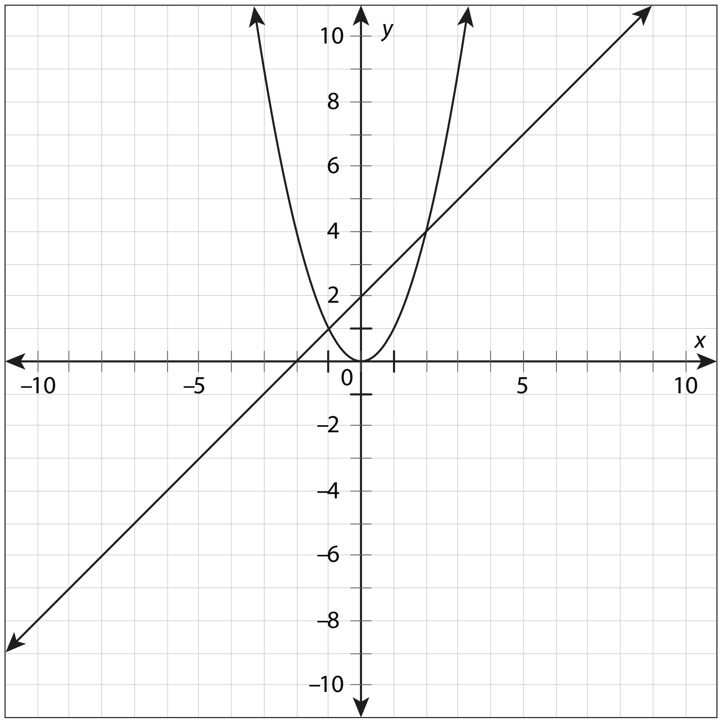
Unit 2 Lesson 1

Suggestions for Graphic Organizers/Manipulatives

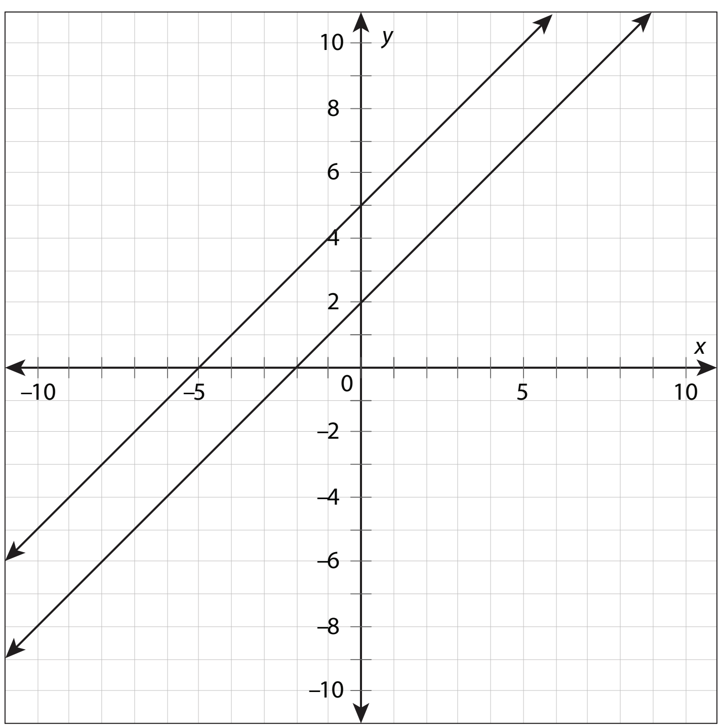
- Have students create input/output tables for various values of the domain of an equation. For example, for the equation $y = x + 4$, have students choose values to input for x and then solve for y. Have them record these values in the input/output table.
- Have students create a cause-and-effect map of the different possible solutions of a system. Have them draw three circles for the different causes and then draw arrows to three boxes for the corresponding effects. In the effects boxes, have them write, "one solution," "more than one solution," and "no solutions," respectively. Then, have students fill in the "cause" circle with the situations that produce either one solution, more than one solution, or no solutions. Have them draw an example picture of each as well.
    - For example, the "one solution" circle could include: "The functions intersect once."



- The "more than one solution" circle could include: "The functions intersect more than once."



- The "no solution" circle could include: "The functions do not intersect at all."



- Have students create a flowchart for the steps for evaluating a function using function notation. For example, for the function $f(2) = 5x – 8$, have students write the original problem in the first diamond of the flowchart. Then have them draw the next box and, in that box, write down the first step; for example, "Substitute 2 for x." In the next diamond, write the new equation; for example, "$f(2) = 5(2) – 8$." Continue the process, writing the steps in the rectangles and the mathematical equation or expression in the diamonds.

Suggestions for Discourse

- Ask students to discuss how the graph of a function is the picture of all the solutions of the function. Ask follow-up questions, such as:
  - "Why do we put arrows on the end of the curve?"
  - "Why does it matter that we draw a straight line accurately?"
  - "If I pick any point on the curve, how is that a solution to the function?"
  - "In a system, why is the point where the graphs intersect the solution to the system?"
- Ask the question, "Why does the vertical line test work?" Ask students if a horizontal line test would work. Have them explain why or why not.
- Ask students why function notation sometimes uses other letters than just f. For example, "Why is there sometimes a $g(x)$ or an $h(x)$?"

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics I*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.
- Ask them to write out, in their own words, how to solve each type of problem.
- Provide a list of sentence frames in which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "I can tell the number of solutions to a graphed system of equations because _______." Or, "When I graph this equation on a coordinate plane, I know it is a function because _______."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- believing the number of solutions an equation has is limited to points seen on the graph

  Discuss how the solutions continue into infinity and why arrows are put on the ends of the curve.

- incorrectly evaluating the equation for different given values

  Practice evaluating expressions, and stress the importance of being careful with the arithmetic and simple algebra (especially negative signs).

- incorrectly plotting ordered pair solutions on a coordinate plane

  Review how to plot ordered pairs, and remind students that the x-value is always listed first, then the y-value (x and y are in alphabetical order).

- believing that all graphs will cross

  Show the students examples of graphs that do not cross and have no solution.

- substituting values incorrectly

  Stress the importance of being careful. It often helps to write the substitutions directly underneath the original equation.

- confusing domain and range

  Quiz the students on the definitions of domain and range. Redefine the words *domain* and *range* whenever you say them. For example, say, "Let's substitute 2 for the domain, or input."

- thinking that if a range value repeats or is the same for a different value in the domain, the relation is not a function (different x-values have the same y-value)

  Use the example of a computer to explain that this is OK. For example, there is more than one way to get the same output on a computer, such as saving a file: The disk icon can be clicked with the cursor, the keyboard command CTRL+S can be pressed, or the menu items File > Save can be chosen. Three different inputs produce the same output, but this is fine, because each input still only has one output, and the computer is not confused. This is also true for a function—as long as each input has only one output, it does not matter.

- thinking function notation means "f times x" instead of "f of x"

  When first explaining the concept, explain that, in this case, the parentheses do not mean multiplication. Explain how the notation is pronounced, then say out loud "f of x" as often as possible as you continue to discuss the lesson and work problems.

- trying to multiply the left side of the function notation

  Repeatedly stress that function notation means you must substitute that value for x.

Lesson 2: Solving Linear Inequalities in Two Variables and Systems of Inequalities

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*.

Skill 1: Graphing Linear Equations in Two Variables* (A–CED.2★)

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Skill 2: Verifying Whether Inequalities Are True or False (6.EE.5)

Common Core State Standard

6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Skill 3: Creating Equations from Context* (A–CED.2★)

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Instruction

Skill 1: Graphing Linear Equations in Two Variables*

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 2

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Guided Practice Skill 1

Example 1

Mrs. Kingston is calculating her students' test scores on a 20-question test with the equation $y = 5x + 2$, where x is the number of correct questions and y is the score. Make a table of values and create a graph of this relationship.

1. Determine values that make the equation true.

   To create a graph of the given equation, at least two points are needed. To find the points, substitute values for x and solve for y. Let's use the values 0, 1, 10, and 20.

   Substitute 0 for x and solve for y.

$y = 5x + 2$	Given equation
$y = 5(0) + 2$	Substitute 0 for x.
$y = 2$	Simplify.

   When $x = 0$, $y = 2$.

   Substitute 1 for x and solve for y.

$y = 5x + 2$	Given equation
$y = 5(1) + 2$	Substitute 1 for x.
$y = 7$	Simplify.

   When $x = 1$, $y = 7$.

   Substitute 10 for x and solve for y.

$y = 5x + 2$	Given equation
$y = 5(10) + 2$	Substitute 10 for x.
$y = 52$	Simplify.

   When $x = 10$, $y = 52$.

   Substitute 20 for x and solve for y.

$y = 5x + 2$	Given equation
$y = 5(20) + 2$	Substitute 20 for x.
$y = 102$	Simplify.

   When $x = 20$, $y = 102$.

2. Organize these values in a table.

   Create a table of the values determined in the previous step, with *x*-values in one column and *y*-values in the other.

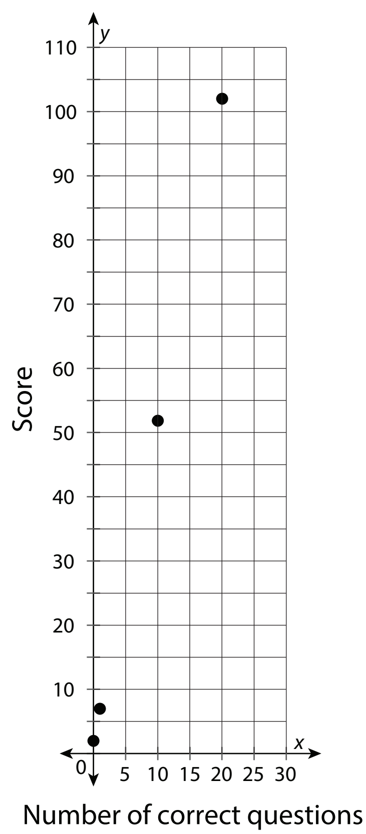
x	*y*
0	2
1	7
10	52
20	102

3. Plot the ordered pairs from the table on a coordinate plane.

   The ordered pairs from the table are (0, 2), (1, 7), (10, 52), and (20, 102).

   Plot these ordered pairs on a coordinate plane.

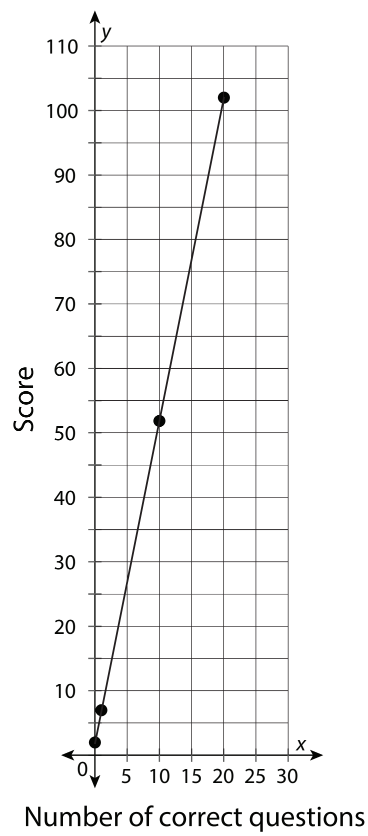
   Label the *x*-axis "Number of correct questions" and the *y*-axis "Score."



4. Draw a line through the points.

   Since the given equation, $y = 5x + 2$, is a linear equation, draw a line through the plotted points.

   Because there are only 20 questions on the test, a student could get from 0 to 20 answers correct. Therefore, the line must start at $x = 0$ and end at $x = 20$.



Scaffolded Practice Skill 1

Example 1

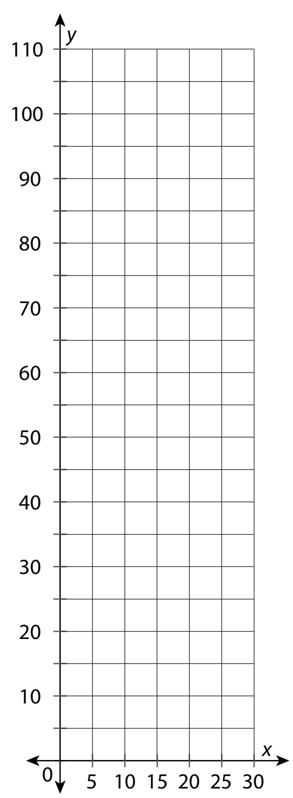
Mrs. Kingston is calculating her students' test scores on a 20-question test with the equation $y = 5x + 2$, where x is the number of correct questions and y is the score. Make a table of values and create a graph of this relationship.

1. Determine values that make the equation true.

2. Organize these values in a table.

x	y

3. Plot the ordered pairs from the table on a coordinate plane.



4. Draw a line through the points.

Practice Skill 1: Graphing Linear Equations in Two Variables*

Graph each of the following relationships on a coordinate plane.

1. $y = \frac{1}{2}x + 3$

2. $y = -5x + 8$

3. Marcy has $50 in her bank account, and hopes to start saving $20 per month. Her balance, *y*, can be found using the equation $y = 20x + 50$, where *x* is the number of months she has been saving. Graph this relationship on a coordinate plane.

Skill 2: Verifying Whether Inequalities Are True or False

SMP	
1 ✓	2 ✓
3 ✓	4 ✓
5 ✓	6
7 ✓	8

Common Core State Standard

6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Essential Questions

1. How can you determine if a given number makes an equation or inequality true?
2. How can you determine if a given coordinate makes a two-variable inequality true?

WORDS TO KNOW

boundary line a line on a coordinate plane that represents an inequality if it was graphed as an equation

boundary value a point on a number line that represents the value that makes both sides of an inequality equal to each other

half plane a region containing all points that has one boundary, a straight line that continues in both directions infinitely

Recommended Resources

- Khan Academy. “Testing Solutions of Equations and Inequalities.”

  http://www.walch.com/rr/04043

  This site provides practice in verifying whether given values make equations and inequalities true.

- Math Planet. “Inequalities.”

  http://www.walch.com/rr/04044

  This site gives an overview on verifying whether a value makes an equation or inequality true or false. It includes a short video clip with an example of how to verify whether a value makes an inequality true.

- Virtual Nerd. “How Do You Determine if an Ordered Pair Is a Solution to a Linear Inequality?”

  http://www.walch.com/rr/04045

  This site features a video about how to verify whether given coordinate points make a two-variable inequality true or false.

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Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Once students have worked through the Guided Practice, distribute the two-circle Venn Diagram from the Program Overview. On the line above the left circle, ask the students to write "Graphed Solution for One-Variable Inequality." On the line above the right circle, ask the students to write "Graphed Solution for Two-Variable Inequality." Ask the students to look at the graphed solutions for the one-variable inequality and two-variable inequality in the Guided Practice. Then have them fill in the diagram with similarities and differences between the ways the solutions are graphed.

Ask volunteers to share their examples. Then discuss the similarities and differences.

- Possible similarities include:
    - both types of inequalities have boundaries
    - both the solution sets are represented by shading
- Possible differences include:
    - using a number line or a coordinate plane and using a circle or a line for the boundary
    - showing when the boundary is included or not
    - having x-values or (x, y) coordinate points for the solutions
    - having the shading of the solution set along the number line or throughout a half plane

By taking the time to more thoroughly understand how the solutions are graphed for these inequalities, it will help students understand how to identify values or coordinate points that will make an inequality true or false.

Suggestions for Discourse

Ask students to think about some real-life examples when inequalities can be used to represent a situation, such as "You can spend at most $25 to buy movie tickets and snacks for two people at the movie theater." When discussing these examples, have students think about how they can determine which values in the situations will make the inequalities true.

Making Connections

Encourage students to connect the concept of verifying whether or not a value makes an equation or inequality true with the process of finding the solution for the equation or inequality.

Skill 2: Verifying Whether Inequalities Are True or False

Introduction

When working with equations and inequalities, it is important to verify whether any values from a set of numbers make the equation or inequality true or false. Determining which values make an equation or inequality true is a key step in the process of learning to solve equations and inequalities.

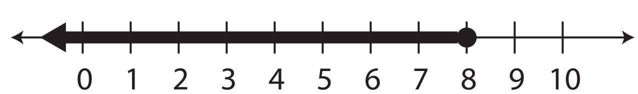
Key Concepts

- To verify whether a value makes a one-variable equation true, substitute the value into the equation and simplify to ensure the left side of the equation is still equal to the right side of the equation.
- An inequality will have a set of solutions that make the inequality true.
- To verify solutions of an inequality, substitute values into the inequality and simplify the left and right sides of the inequality to determine if the inequality is true.
- One way to represent the set of solutions for an inequality is to graph it. There are different types of graphs for one-variable inequalities, for two-variable inequalities, and for systems of two-variable inequalities.

One-Variable Inequalities

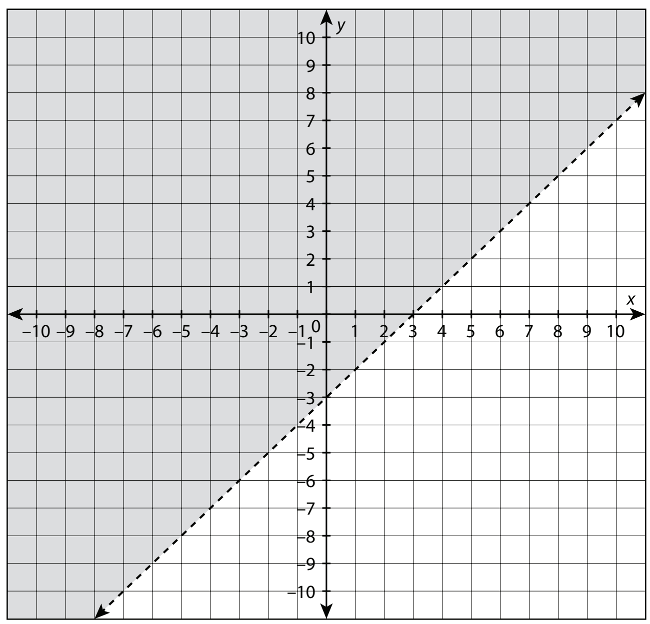
- For a one-variable inequality such as $x < 8$, the solution is graphed on a number line using a circle at the **boundary value**. The boundary value is the point on a number line that represents the value that makes both sides of an inequality equal to each other. Starting at the edge of the circle, shade along the number line in the direction that has values that make the inequality true.

- An inequality symbol that is "less than" (<) or "greater than" (>) does not include the boundary value in the solution, and is shown by drawing an open circle at that boundary value on the number line. An open circle does not have any shading. Notice that the number line for $x < 8$ has an open circle at 8. This is because the inequality uses the "less than" symbol.
- An inequality symbol that is "less than or equal to" (≤) or "greater than or equal to" (≥) *does* include the boundary value in the solution, and is shown by drawing a closed or shaded circle at the boundary value on the number line. For example, this number line shows $x \le 8$.



Two-Variable Inequalities

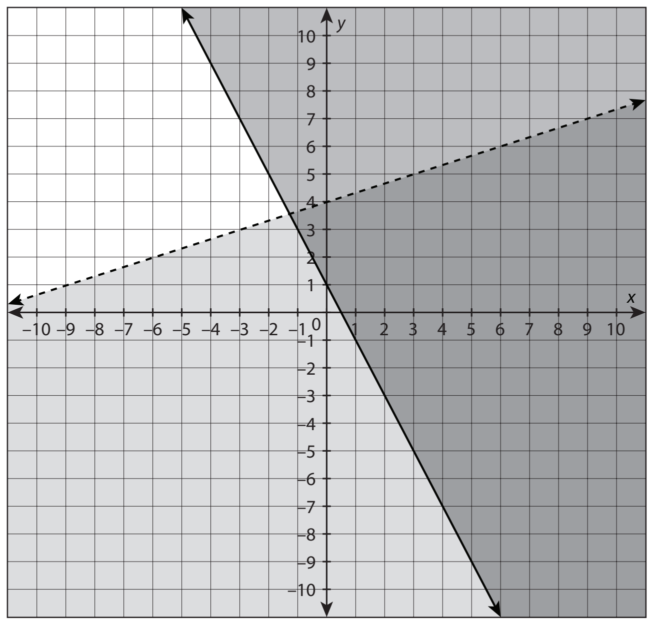
- To verify the solutions for a two-variable inequality such as $y > x - 3$, substitute (x, y) coordinate points into the inequality to determine which coordinate points make the inequality true.
- To represent the solution for a two-variable inequality, solve the inequality for y, then use a coordinate plane to graph the **boundary line**, which is a line on the coordinate plane that represents an inequality if it was graphed as an equation. Finally, shade the **half plane**, or the region containing all points that has one boundary, that represents the solution. The boundary is a straight line that continues in both directions infinitely.



- An inequality symbol that is "less than" (<) or "greater than" (>) does not include the boundary values in the solution, and is shown by drawing a dashed line along the boundary.
- An inequality symbol that is "less than or equal to" (≤) or "greater than or equal to" (≥) *does* include the boundary values in the solution, and is shown by drawing a solid line along the boundary.
- To determine which half plane to shade, select an (x, y) coordinate point on one side of the line and substitute those values into the inequality. If that coordinate point makes the inequality true, shade the half plane on the side of the line that includes the coordinate point. If that coordinate point makes the inequality false, shade the half plane on the side of the line that does not include the coordinate point.

Systems of Two-Variable Inequalities

- To verify the solutions for a system of two-variable inequalities, substitute (x, y) coordinate points into each inequality to determine which coordinate points make all of the inequalities true.
- To represent the solution for a system of two-variable inequalities, graph each inequality on the same coordinate plane. The solution region is where the shaded half planes overlap.



Guided Practice Skill 2

Example 1

Mia wants to buy a new jacket that costs \$40, including taxes. She plans to save \$5 each week. How many weeks will it take Mia to save enough money to buy the jacket?

1. Write an equation to represent the situation.

   Mia plans to save \$5 per week. She needs to save a total of \$40 to buy the jacket.

   Let x represent the number of weeks.

   \$5 saved per week multiplied by x, the number of weeks, will equal the \$40 total.

   This situation is represented by the equation $5x = 40$.

2. Solve the equation.

   Because 5 and x are multiplied, dividing both sides of the equation by 5 will "undo" the multiplication operation and isolate x.

$5x = 40$	Equation from the previous step
$\frac{5x}{5} = \frac{40}{5}$	Divide both sides by 5.
$x = 8$	Simplify.

   The solution to the equation $5x = 40$ is $x = 8$. In other words, it will take Mia 8 weeks to save enough money to buy the jacket.

3. Verify that the solution makes the equation true.

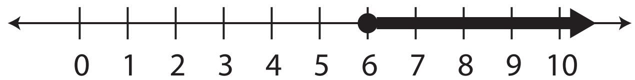
   To verify that it will take Mia 8 weeks to save enough money to buy the jacket, substitute 8 into the equation for x and then solve.

$5x = 40$	Equation from step 1
$5(8) = 40$	Substitute 8 for x.
$40 = 40$	Multiply.

   The result of substituting 8 for x is a true statement, $40 = 40$; therefore, $x = 8$ is the solution to the equation.

Example 2

The shaded number line shown represents the solution set for the inequality $2 + x \geq 8$.



Use both substitution and the number line to determine which values from the set {0, 4, 6, 13} make the inequality true.

1. Use substitution to determine if 0 makes the inequality true.

   Substitute 0 for x in the inequality $2 + x \geq 8$ and simplify.

$2 + x \geq 8$	Original inequality
$2 + (0) \geq 8$	Substitute 0 for x.
$2 \geq 8$	Add.

   The statement $2 \geq 8$ is not true; therefore, 0 does not make the inequality true.

2. Use the number line of the inequality $2 + x \geq 8$ to determine if 0 makes the inequality true.

   Find 0 on the number line:

   It can be seen on the number line of the inequality $2 + x \geq 8$ that 0 is not included in the description of the solution set. Notice that the arrow along the number line extends to the right of the shaded circle and does not pass through 0. Therefore, 0 does not make the inequality true.

3. Use substitution to determine if 4 makes the inequality true.

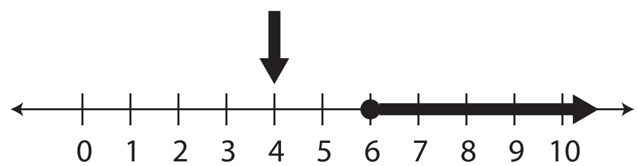
   Substitute 4 for x in the inequality $2 + x \geq 8$ and simplify.

$2 + x \geq 8$	Original inequality
$2 + (4) \geq 8$	Substitute 4 for x.
$6 \geq 8$	Add.

   The statement $6 \geq 8$ is not true; therefore, 4 does not make the inequality true.

4. Use the number line of the inequality $2 + x \geq 8$ to determine if 4 makes the inequality true.

   Find 4 on the number line:



   It can be seen on the number line of the inequality $2 + x \geq 8$ that 4 is not included in the description of the solution set; therefore, 4 does not make the inequality true.

5. Use substitution to determine if 6 makes the inequality true.

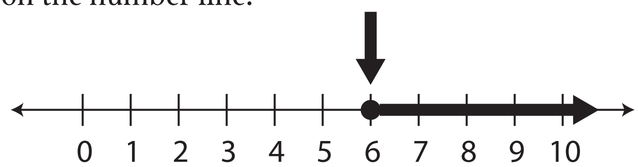
   Substitute 6 for x in the inequality $2 + x \geq 8$ and simplify.

$2 + x \geq 8$	Original inequality
$2 + (6) \geq 8$	Substitute 6 for x.
$8 \geq 8$	Add.

   The statement $8 \geq 8$ is true; therefore, 6 does make the inequality true.

6. Use the number line of the inequality $2 + x \geq 8$ to determine if 6 makes the inequality true.

   Find 6 on the number line:



   It can be seen on the number line of the inequality $2 + x \geq 8$ that 6 is included in the description of the solution set; therefore, 6 does make the inequality true.

7. Use substitution to determine if 13 makes the inequality true.

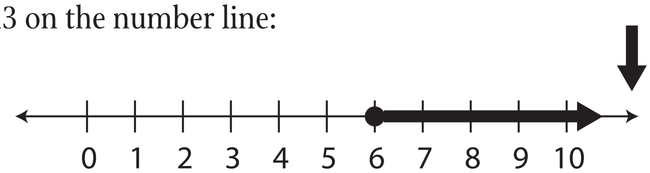
   Substitute 13 for x in the inequality $2 + x \geq 8$ and simplify.

$2 + x \geq 8$	Original inequality
$2 + (13) \geq 8$	Substitute 13 for x.
$15 \geq 8$	Add.

   The statement $15 \geq 8$ is true; therefore, 13 does make the inequality true.

8. Use the number line of the inequality $2 + x \geq 8$ to determine if 13 makes the inequality true.

   Find 13 on the number line:



   The number line only shows values up to 10, but the rightward arrow indicates that the values continue infinitely to the right.

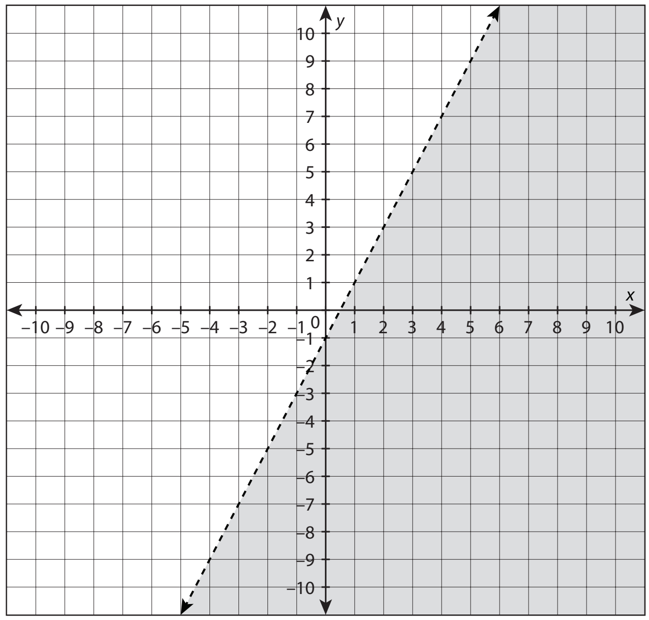
   It can be seen on the number line of the inequality $2 + x \geq 8$ that 13 is included in the description of the solution set; therefore, 13 does make the inequality true.

9. Summarize your results.

   After using substitution and analyzing the number line, the values from the set {0, 4, 6, 13} that make the inequality true are 6 and 13.

Example 3

The shaded half plane in the graph shown represents the solution set for the inequality $y < 2x - 1$.



Use both substitution and the graph of the inequality to determine which values from the set {(–4, 3), (–1, –8), (3, 7), (5, –1)} make the inequality true.

1. Use substitution to determine if (–4, 3) makes the inequality true.

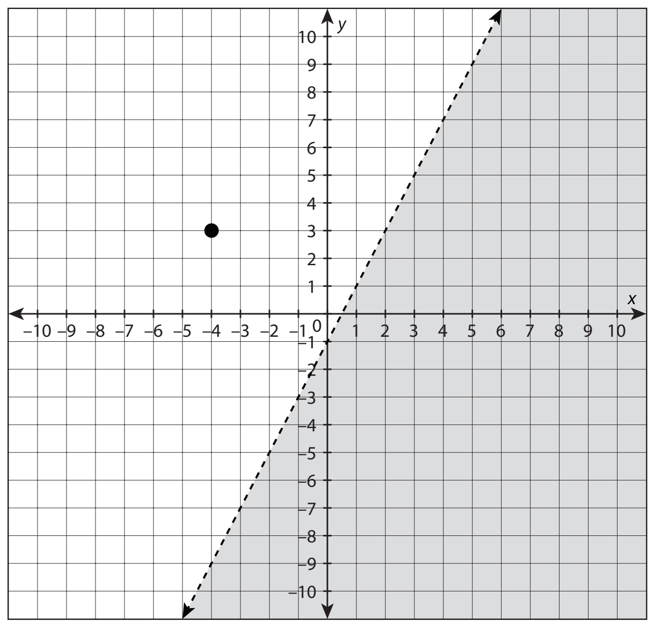
   Substitute –4 for x and 3 for y in the inequality $y < 2x - 1$ and simplify.

$y < 2x - 1$	Original inequality
$(3) < 2(-4) - 1$	Substitute –4 for x and 3 for y.
$3 < -8 - 1$	Multiply.
$3 < -9$	Subtract.

   The statement $3 < -9$ is not true; therefore, (–4, 3) does not make the inequality true.

2. Use the graph of the inequality $y < 2x - 1$ to determine if (–4, 3) makes the inequality true.

   Find (–4, 3) on the graph:



   It can be seen on the graph of the inequality $y < 2x - 1$ that (–4, 3) is not included in the shaded half plane, which represents the solution set; therefore, (–4, 3) does not make the inequality true.

3. Use substitution to determine if (–1, –8) makes the inequality true.

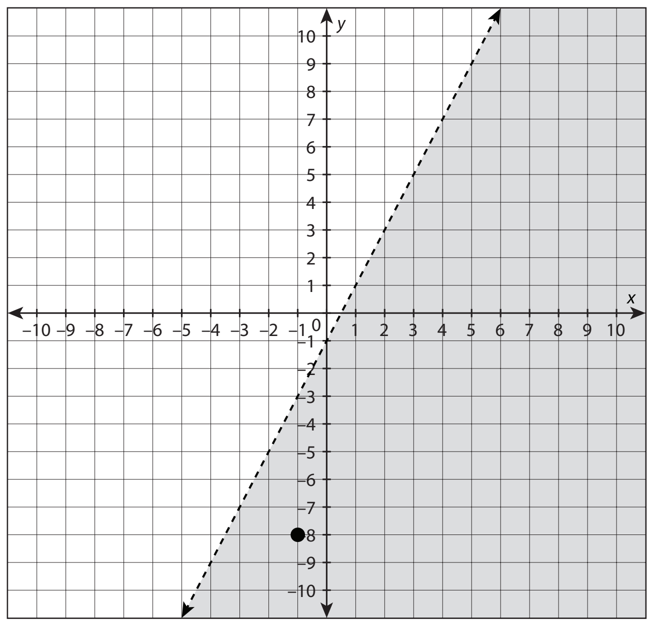
   Substitute –1 for x and –8 for y in the inequality $y < 2x - 1$.

$y < 2x - 1$	Original inequality
$(-8) < 2(-1) - 1$	Substitute –1 for x and –8 for y.
$-8 < -2 - 1$	Multiply.
$-8 < -3$	Subtract.

   The statement $-8 < -3$ is true; therefore, (–1, –8) does make the inequality true.

4. Use the graph of the inequality $y < 2x - 1$ to determine if (–1, –8) makes the inequality true.

   Find (–1, –8) on the graph:



   It can be seen on the graph of the inequality $y < 2x - 1$ that (–1, –8) is included in the shaded half plane, which represents the solution set; therefore, (–1, –8) does make the inequality true.

5. Use substitution to determine if (3, 7) makes the inequality true.

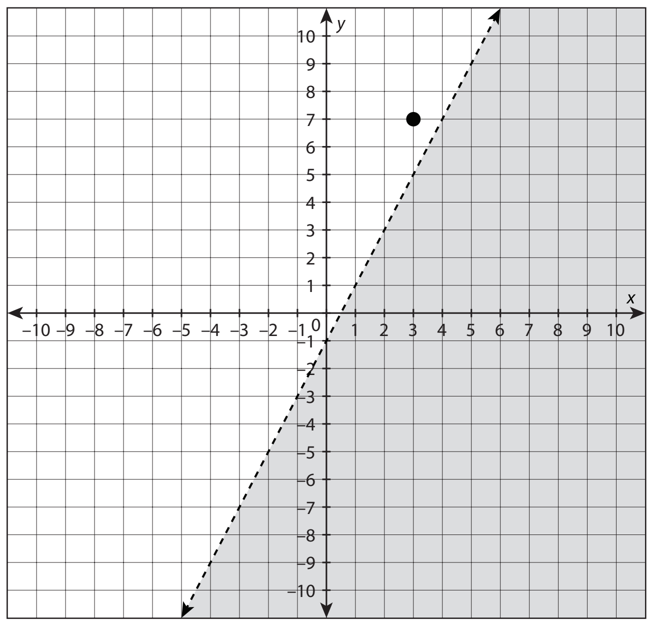
   Substitute 3 for x and 7 for y in the inequality $y < 2x - 1$ and simplify.

$y < 2x - 1$	Original inequality
$(7) < 2(3) - 1$	Substitute 3 for x and 7 for y.
$7 < 6 - 1$	Multiply.
$7 < 5$	Subtract.

   The statement $7 < 5$ is not true; therefore, (3, 7) does not make the inequality true.

6. Use the graph of the inequality $y < 2x - 1$ to determine if (3, 7) makes the inequality true.

   Find (3, 7) on the graph:



   It can be seen on the graph of the inequality $y < 2x - 1$ that (3, 7) is not included in the shaded half plane, which represents the solution set; therefore, (3, 7) does not make the inequality true.

7. Use substitution to determine if (5, –1) makes the inequality true.

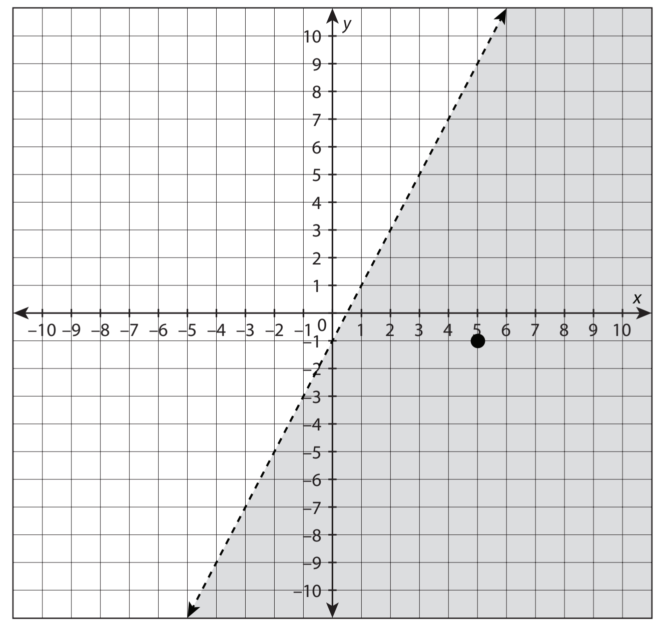
   Substitute 5 for x and –1 for y in the inequality $y < 2x - 1$.

$y < 2x - 1$	Original inequality
$(-1) < 2(5) - 1$	Substitute 5 for x and –1 for y.
$-1 < 10 - 1$	Multiply.
$-1 < 9$	Subtract.

   The statement $-1 < 9$ is true; therefore, (5, –1) does make the inequality true.

8. Use the graph of the inequality $y < 2x - 1$ to determine if (5, –1) makes the inequality true.

   Find (5, –1) on the graph:



   It can be seen on the graph of the inequality $y < 2x - 1$ that (5, –1) is included in the shaded half plane, which represents the solution set; therefore, (5, –1) does make the inequality true.

9. Summarize your results.

   After using substitution and analyzing the graph of the inequality, the values from the set {(–4, 3), (–1, –8), (3, 7), (5, –1)} that make the inequality true are (–1, –8) and (5, –1).

Scaffolded Practice Skill 2

Example 1

Mia wants to buy a new jacket that costs \$40, including taxes. She plans to save \$5 each week. How many weeks will it take Mia to save enough money to buy the jacket?

1. Write an equation to represent the situation.

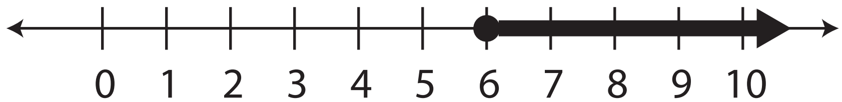
2. Solve the equation.

3. Verify that the solution makes the equation true.

continued

Example 2

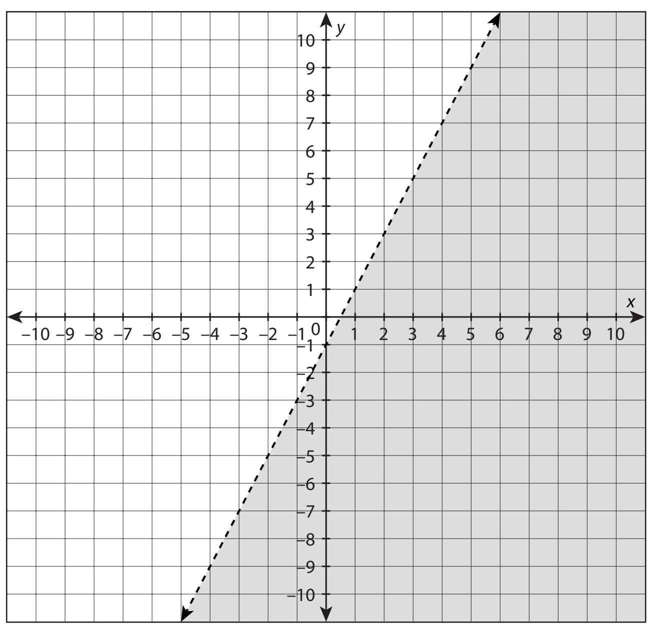
The shaded number line shown represents the solution set for the inequality $2 + x \geq 8$.



Use both substitution and the number line to determine which values from the set {0, 4, 6, 13} make the inequality true.

Example 3

The shaded half plane in the graph shown represents the solution set for the inequality $y < 2x - 1$.



Use both substitution and the graph of the inequality to determine which values from the set {(–4, 3), (–1, –8), (3, 7), (5, –1)} make the inequality true.

Problem-Based Task Skill 2: The Picnic

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 ✓ 6
7 ✓ 8

The second-grade class is having a picnic lunch on the last day of school. One of the teachers, Mrs. Cooper, plans to buy juice boxes and flavored water bottles for the picnic. Juice boxes are sold in packs of 8. Bottles of flavored water are sold in packs of 12. There will be 154 people at the picnic.

Mrs. Cooper needs to determine how many packs of juice boxes and how many packs of flavored water bottles to buy to have enough for everyone to have at least one drink. She represents the situation with the inequality $8x + 12y \geq 154$, where x is the number of packs of juice boxes and y is the number of packs of flavored water bottles.

After looking at the inequality, Mrs. Cooper thinks 10 packs of juice boxes and 5 packs of flavored water bottles will be enough for the picnic. She then asks two other teachers for their opinions. Mr. Jimenez believes 6 packs of juice boxes and 8 packs of flavored water bottles will be enough. Ms. Allen thinks they will need 9 packs of juice boxes and 7 packs of flavored water bottles. Based on the inequality, which teacher correctly determined the number of packs of drinks needed for the picnic?

Based on the inequality, which teacher correctly determined the number of packs of drinks needed for the picnic?

Problem-Based Task Skill 2: The Picnic

Coaching

a. Does Mrs. Cooper's estimation of 10 packs of juice boxes and 5 packs of flavored water bottles make the inequality true?

b. Does Mr. Jimenez's estimation of 6 packs of juice boxes and 8 packs of flavored water bottles make the inequality true?

c. Does Ms. Allen's estimation of 9 packs of juice boxes and 7 packs of flavored water bottles make the inequality true?

d. Which teacher correctly determined the number of packs of drinks needed for the picnic?

Problem-Based Task Skill 2: The Picnic

Coaching Sample Responses

a. Does Mrs. Cooper's estimation of 10 packs of juice boxes and 5 packs of flavored water bottles make the inequality true?

Substitute 10 for x and 5 for y in the inequality $8x + 12y \geq 154$, and then simplify.

$8x + 12y \geq 154$

$8(10) + 12(5) \geq 154$

$80 + 60 \geq 154$

$140 \geq 154$

The statement $140 \geq 154$ is not true; therefore, 10 packs of juice boxes and 5 packs of flavored water bottles do not make the inequality true.

b. Does Mr. Jimenez's estimation of 6 packs of juice boxes and 8 packs of flavored water bottles make the inequality true?

Substitute 6 for x and 8 for y in the inequality $8x + 12y \geq 154$, and then simplify.

$8x + 12y \geq 154$

$8(6) + 12(8) \geq 154$

$48 + 96 \geq 154$

$144 \geq 154$

The statement $144 \geq 154$ is not true; therefore, 6 packs of juice boxes and 8 packs of flavored water bottles do not make the inequality true.

c. Does Ms. Allen's estimation of 9 packs of juice boxes and 7 packs of flavored water bottles make the inequality true?

Substitute 9 for x and 7 for y in the inequality $8x + 12y \geq 154$, and then simplify.

$8x + 12y \geq 154$

$8(9) + 12(7) \geq 154$

$72 + 84 \geq 154$

$156 \geq 154$

The statement $156 \geq 154$ is true; therefore, 9 packs of juice boxes and 7 packs of flavored water bottles do make the inequality true.

d. Which teacher correctly determined the number of packs of drinks needed for the picnic?

Because 9 packs of juice boxes and 7 packs of water bottles are enough drinks for the picnic, Ms. Allen correctly determined the number of packs of drinks needed for the picnic.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 2: Verifying Whether Inequalities Are True or False

For problems 1–6, determine which values or coordinate points from the given sets make the equation or inequality true.

1. Jack wants to run 45 miles this month. He has run 17 miles so far, and wants to know how many more miles he still needs to run. This situation can be represented by the equation $17 + x = 45$, where x is the number of miles Jack still needs to run. Which value, if any, from the set {22, 28, 32, 38} represents the number of miles that Jack still needs to run?

2. Erin is shopping for clothes. She has \$40 to spend. She plans to buy one pair of pants that costs \$14, and she wants to buy some shirts that cost \$6.50 each. The inequality $6.50x + 14 \le 40$ represents this situation, where x is the number of shirts. Which value or values, if any, from the set {1, 2, 3, 4, 5} represent the possible number of shirts that Erin could afford to buy?

3. Given the equation $2x + 3 = 11$, which value or values, if any, from the set {3, 4, 6, 7, 8} make the equation true?

4. Given the inequality $5x - 7 > 38$, which value or values, if any, from the set {4, 7, 9, 11} make the inequality true?

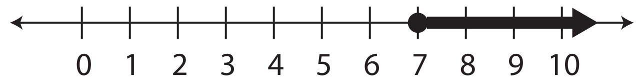
5. Given the inequality $3x - 1 \le 17$, which value or values, if any, from the set {–5, 1, 6, 17} make the inequality true?

6. Given the inequality $y > \frac{1}{2}x + 3$, which coordinates from the set {(–6, 5), (–2, –1), (4, 7), (8, –3)} make the inequality true, if any?

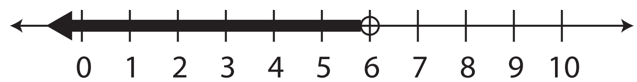
continued

For problems 7–9, the shaded number lines represent the set of solutions for each given inequality. Using the graph of each inequality, list three values that will make the inequality true.

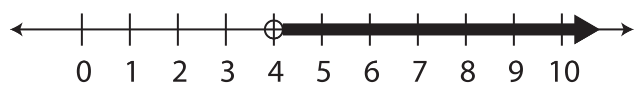
7. $4x - 5 \geq 23$



8. $\frac{1}{3}x + 7 < 9$

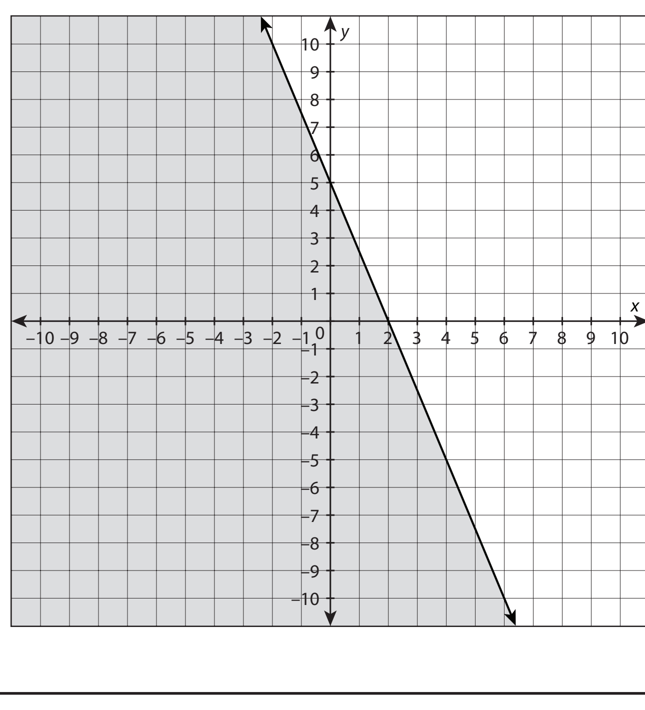


9. $0.5x > 2$



For problem 10, the graph represents the set of solutions for the given inequality. Use the graph to determine three coordinate points that will make the inequality true.

10. $5x + 2y \leq 10$



Skill 3: Creating Equations from Context*

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 2

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Guided Practice Skill 3

Example 1

Rylee is investing $200 in an account that earns 3% interest every year. Write an equation that represents the amount of interest she will earn depending on the number of years she leaves the money in the account. Then create a graph of the equation and use the graph to predict how much interest she will earn in 5 years.

1. Use the formula $I = P \bullet r \bullet t$ to write an equation for this situation.

   In the formula $I = P \bullet r \bullet t$, I is the interest, P is the principal (the amount of money invested), r is the rate (written as a decimal), and t is the time in years.

   Because $200 is the principal, substitute 200 for P.

   The rate is 3%, but this must be written as a decimal. Recall that 3% is equal to 0.03. Substitute 0.03 for r.

   Because the interest depends on the time, t is the independent variable (x) and I is the dependent variable (y). Substitute x for t and y for I.

$I = P \bullet r \bullet t$	Given formula
$(y) = (200) \bullet (0.03) \bullet (x)$	Substitute y for I, 200 for P, 0.03 for r, and x for t.
$y = 6x$	Simplify.

   The equation $y = 6x$ represents the amount of interest (y) Rylee will earn after x years.

2. Determine values that make the equation true.

   Substitute values for x and solve for y. Let's use 0, 1, and 2.

   Substitute 0 for x and solve for y.

$y = 6x$	Equation from the previous step
$y = 6(0)$	Substitute 0 for x.
$y = 0$	Simplify.

   When $x = 0$, $y = 0$.

   Substitute 1 for x and solve for y.

$y = 6x$	Equation from the previous step
$y = 6(1)$	Substitute 1 for x.
$y = 6$	Simplify.

   When $x = 1$, $y = 6$.

   Substitute 2 for x and solve for y.

$y = 6x$	Equation from the previous step
$y = 6(2)$	Substitute 2 for x.
$y = 12$	Simplify.

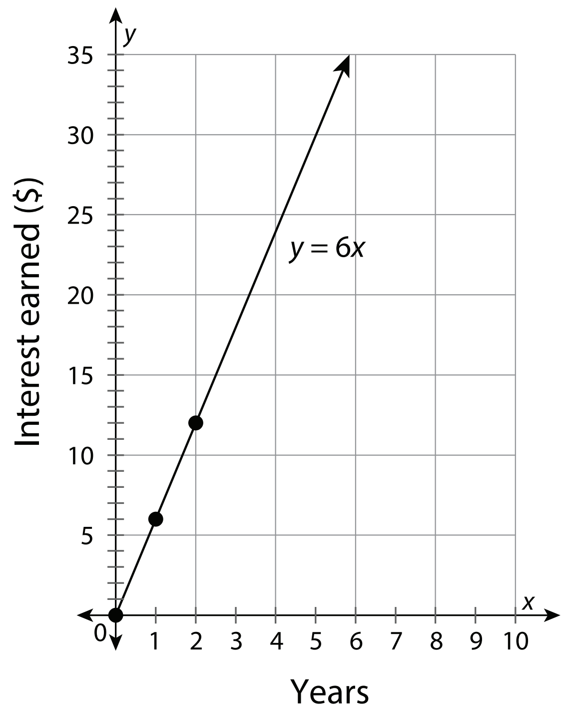
   When $x = 2$, $y = 12$.

3. Organize these values in a table.

x	y
0	0
1	6
2	12

4. Plot the ordered pairs from the table on a coordinate plane and draw a line through the points.

   Plot the ordered pairs (0, 0), (1, 6), and (2, 12) on a coordinate plane and draw a line through the points.



5. Use the graph to predict how much interest she will earn in 5 years.

   The graph shows time in years on the *x*-axis and the amount of interest earned in dollars on the *y*-axis.

   To determine how much interest Rylee will earn in 5 years, locate the value 5 on the *x*-axis. Then, move up on the *y*-axis to find the corresponding *y*-value, which is 30.

   Rylee will earn $30 interest in 5 years.

Scaffolded Practice Skill 3

Example 1

Rylee is investing $200 in an account that earns 3% interest every year. Write an equation that represents the amount of interest she will earn depending on the number of years she leaves the money in the account. Then create a graph of the equation and use the graph to predict how much interest she will earn in 5 years.

1. Use the formula $I = P \bullet r \bullet t$ to write an equation for this situation.

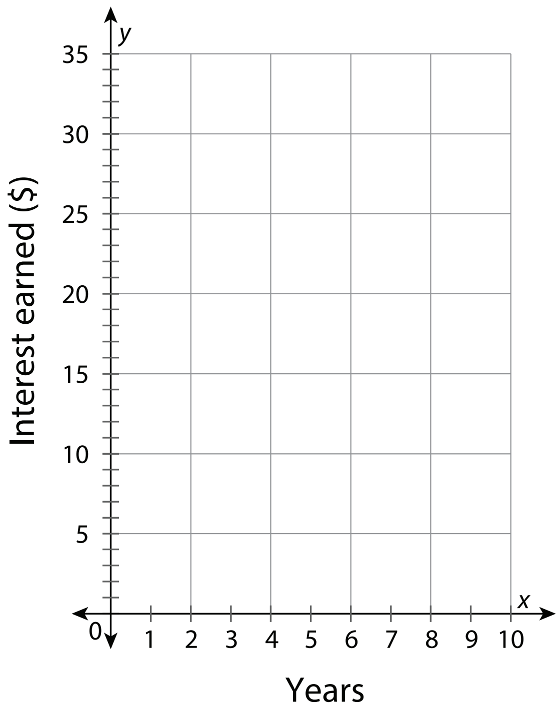
2. Determine values that make the equation true.

3. Organize these values in a table.

x	y

continued

4. Plot the ordered pairs from the table on a coordinate plane and draw a line through the points.



5. Use the graph to predict how much interest she will earn in 5 years.

Practice Skill 3: Creating Equations from Context*

Create an equation that represents each situation.

1. When Jeff goes for a run, he typically runs at a rate of 7 miles per hour (mph). Write an equation that represents how far he will travel depending on how long he runs.

2. Nanette bought 30 candy bars for $0.50 each, and is selling them for $1.50 each. Write an equation that represents Nanette's profit, which is the amount of money she takes home after subtracting her initial costs (the money she spent on candy bars) from the total money she makes selling candy bars.

3. Thomas is filling up his family's inground swimming pool. The pool is 20 feet long and 15 feet wide, and has the same depth throughout. The cost of filling the pool is about 5 cents per cubic foot of water. His father wants him to determine how much it will cost to fill the pool to different depths. Write an equation that represents the cost to fill the pool, depending on how deep the pool is filled.

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Supportive Instructional Strategies for Mathematics I

Unit 2 Lesson 2

Suggestions for Graphic Organizers/Manipulatives

- Distribute the Number Line graphic organizer found in the Program Overview. Have students label the top two number lines from 0 to 10. Then have them write the inequality $x < 9$ above the first number line, and write the inequality $x \geq 3$ above the second number line. Ask the students to graph each inequality on its respective number line. The number lines will look as follows:

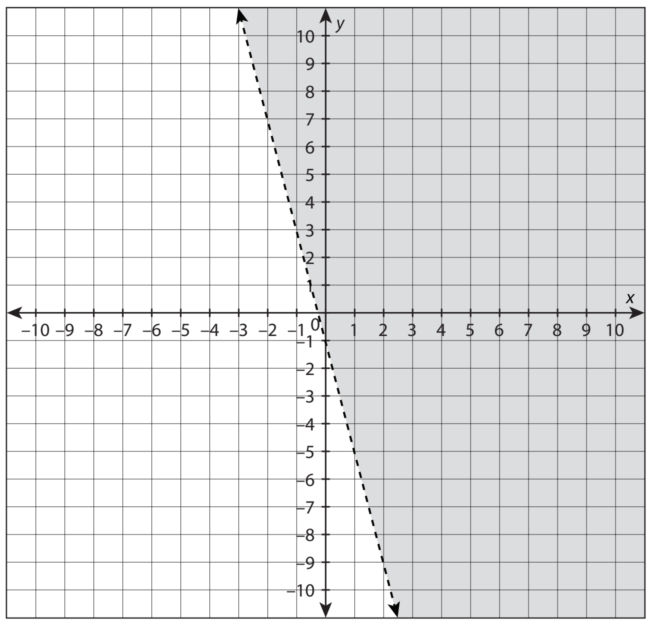
$x < 9$

$x \geq 3$

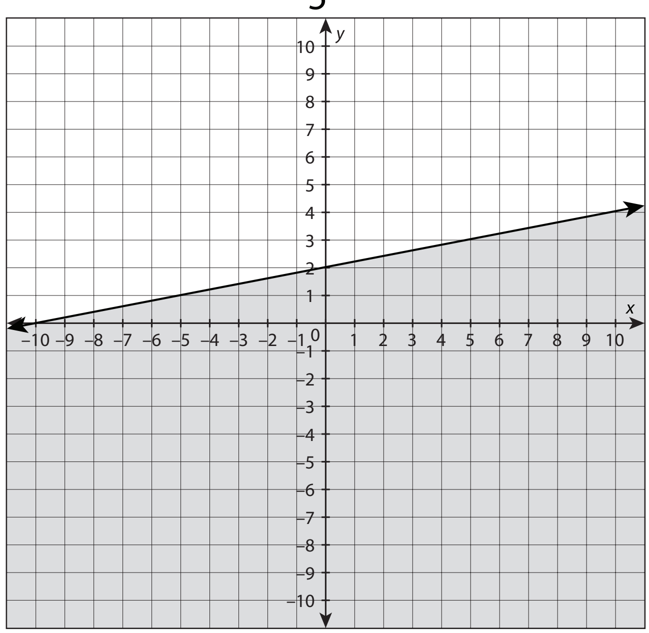
- Ask volunteers to share their number lines and discuss why they drew them as they did. These discussions should include needing to draw a circle at the boundary value, shading the circle if the boundary is included in the solution set, not shading the circle if the boundary is not included in the solution set, and determining which side of the circle to shade along the number line.

- Distribute two of the Coordinate Plane graphic organizers found in the Program Overview. Have students write the inequality $y > -4x - 1$ above the first coordinate plane, and have them write the inequality $y \le \frac{1}{5}x + 2$ above the second coordinate plane. Ask students to graph each inequality on its respective coordinate plane. The coordinate planes will look as follows:

$$y > -4x - 1$$



$$y \le \frac{1}{5}x + 2$$



- Ask volunteers to share their graphs and discuss why they drew their graphs as they did. These discussions should include needing to draw a line at the boundary, drawing a solid line if the boundary is included in the solution set, drawing a dashed line if the boundary is not included in the solution set, and determining which half plane to shade.

Suggestions for Discourse

Present students with the following scenario: "Kylie is going bowling with some friends, and she has $15 to spend. It costs $3 to rent bowling shoes, and $4 for each game of bowling. Kylie wants to figure out how many games she can play without spending more than $15." Ask students to complete the following:

- Write an inequality representing the situation where x is the number of games of bowling.
- Solve the inequality for x to determine the number of games of bowling that Kylie can play.
- Should there be any additional boundaries placed on the solution set? If so, why?
- Graph the solution on a number line.

After students have completed those steps, ask volunteers to share their inequality ($4x + 3 \le 15$), solution ($0 \le x \le 3$ or {0, 1, 2, 3}), and graph. Have those students discuss how they arrived at their answers. Because Kylie cannot play a negative number of games, discuss with students about real-world inequality situations that typically need to have additional boundaries, such as greater than or equal to 0, to avoid having solutions that are not realistic.

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics I*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.
- Use manipulatives where appropriate. For these skills, students could be provided with a ruler and graph paper, for practice with drawing coordinate planes and with graphing equations.

- Provide a list of sentence frames in which students must fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "I knew that 2 was the solution to the equation $3x + 4 = 10$ because _______." Or, "First, I graphed three points on the coordinate plane; the next step was to _______." Or, "The part of this inequality that tells me I am going to make a dashed line for the boundary line when I graph it is the less than symbol."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- choosing an incorrect value for a solution set on the wrong side of the graph or number line

  Reinforce where to look to choose the correct solution set; i.e., practice looking at completed graphs of inequalities and choosing values in the shaded part, or practice choosing values in the solution set on a number line.

- incorrectly substituting a value in an equation or inequality

  Remind students of the order of operations when simplifying an expression, and practice with example problems.

- including a value or coordinate point from a non-included boundary in the solution set

  Reinforce where to look to choose the correct solution set; i.e., practice looking at completed graphs of inequalities and choosing values in the shaded part, or practice choosing values in the solution set on a number line.

- forgetting to include a value or coordinate point when the boundary is included in the solution set

  Have students create a chart with column headings of "included in solution" and "not included in solution;" under the headings would be "closed/shaded circle on number line; solid line on the graph," and "open circle on the number line; dashed line on the graph," respectively.

- confusing the symbols for "less than" (<) and "greater than" (>) when creating equations from context

  Have students create a chart with the headings "less than" and "greater than," and have them write the appropriate symbols and words that represent *less than* and *greater than* in the correct columns for each.

Lesson 3: Sequences As Functions

Instruction

Elementary Prerequisite Skills

This lesson requires the use of the following elementary skill(s) to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*. See the Appendix for material to address the skill(s).

- E-Skill 3: Recognizing Patterns (3.OA.9), Appendix p. A-27

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*.

Skill 1: Understanding the Properties of Functions (8.F.4)

Common Core State Standard

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Skill 2: Understanding Function Notation** (F–IF.2)

Common Core State Standard

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Instruction

Skill 1: Understanding the Properties of Functions

SMP
1 ✓ 2 ✓
3 4 ✓
5 6 ✓
7 ✓ 8

Common Core State Standard

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Essential Questions

1. How can a linear function be represented with an equation?
2. How can the slope and y-intercept of a linear function be determined given the function's equation or graph?

WORDS TO KNOW

dependent variable the quantity that is based on the input values of the independent variable

domain the set of all inputs of a function; the set of x-values that are valid for the function

independent variable the quantity that changes based on values chosen

linear function a function that can be written in the form $y = mx + b$, in which m is the slope, b is the y-intercept, and the graph is a straight line

range the set of all outputs of a function; the set of y-values that are valid for the function

slope the measure of the rate of change of one variable with respect to another variable; $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$; the slope in the equation $y = mx + b$ is m.

***y*-intercept** the point at which a line or curve intersects the y-axis at $(0, y)$; the y-intercept in the equation $y = mx + b$ is b.

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Recommended Resources

- IXL Learning. "Find the Slope of a Graph."

  http://www.walch.com/rr/04046

  This site provides practice with determining the slope of a linear function given the function's graph. Users can enter the answer for the slope of each graph as a proper fraction, an improper fraction, or an integer. For incorrect answers, feedback includes an explanation of how to find the correct slope using two points on the graph.

- WebMATH. "Find the Equation of a Line Given That You Know Two Points It Passes Through."

  http://www.walch.com/rr/04047

  This site allows users to find the equation of a line by entering the *x*- and *y*-coordinates of two points that the graph of the linear function passes through. The equation representing the function is given in the form $y = mx + b$. Also, the site gives a detailed explanation of how to find the values of m and b for the function.

Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Once students have worked through the Guided Practice, distribute the Coordinate Plane graphic organizer found in the Program Overview. Provide blue and red colored pencils or markers. Have students plot the points (–6, –2) and (2, 10), and then draw the line that passes through the points. Have students label each point, using blue to write the x-values and red to write the y-values. Then have them determine the slope and y-intercept of the function that the line represents based on the graph. Have students draw the rest of the slope triangle for the two given points, using blue to draw the horizontal distance and red to draw the vertical distance. Finally, have students write the equation of the function that the line represents in the form $y = mx + b$. The correct equation should be $y=\frac{3}{2}x+7$.

Suggestions for Discourse

Remind students that the equation of any linear function can be found by using two points from a table of values for that function. Show students a table of values for a linear function, and have each student choose any two points. Ask each student to determine the equation of the function by using the two points he or she chose. Then have students report which two points they chose and the equation they found. Based on their results, ask students to draw conclusions about determining the equation for a linear function depending on the points chosen. Guide students to conclude that it does not matter which two points are chosen from the table of values—the equation will be the same.

Making Connections

Explain the similarities between the explicit formula for an arithmetic sequence and the equation of a linear function. Show students step-by-step how the explicit formula $a_n = a_1 + (n - 1)d$ can be rewritten as $a_n = dn + (a_1 - d)$. Point out that this means that the common difference, d, of an arithmetic sequence is similar to a line's slope, and the difference of the first term and the common difference, $a_1 - d$, is similar to a line's y-intercept. Remind students that because an arithmetic sequence is discrete, it does not have a slope and y-intercept.

Skill 1: Understanding the Properties of Functions
Introduction

Functions can be represented as equations or graphs. By examining these representations of a function, it is possible to determine certain information about the situation that the function represents. For example, suppose that the equation $h=-3\frac{2}{5}t+950$ represents a skydiver's height, *h*, above the ground in meters *t* seconds after she opened her parachute. From this equation, it can be determined that the skydiver opened her parachute when she was 950 meters above the ground and that she was falling at a rate of $3\frac{2}{5}$ meters per second.

Key Concepts

- A **linear function** is a function that can be written in the form $y = mx + b$, where m is the slope and b is the y-intercept.
- In the equation $y = mx + b$, x is the **independent variable**, or the variable of a function which has a value that *is not* determined by another variable's value, whereas y is the **dependent variable**, or the variable of a function which has a value that *is* determined by another variable's value.
- All the possible values of a function's independent variable are the function's **domain**, whereas all the possible values of a function's dependent variable are its **range**.
- You can identify a linear relationship by checking to make sure that for each change in the value of the independent variable, the change in the value of the dependent variable remains constant.
- Given any two points that are on the graph of a linear function, the equation that represents the function can be determined, or created.
- The **slope** of a linear function, or the change in the value of a linear function's dependent variable for each one-unit increase in the value of the function's independent variable, can be determined from the equation or graph representing the function.
- The slope is also known as the ratio of the amount of vertical change to the amount of horizontal change, or $\frac{\text{rise}}{\text{run}}$.
- The formula $\frac{y_2 - y_1}{x_2 - x_1}$ can be used to find the slope of a linear function, where (x_1, y_1) and (x_2, y_2) are two points on the line.

Instruction

- A linear function's ***y*-intercept**, or the location where the graph of a linear function crosses the *y*-axis, can also be determined from the equation or graph representing the function.
- Given the equation of a linear function and a specific value of the dependent variable, it is possible to find the corresponding value of the independent variable by isolating the independent variable on one side of the equation.

Guided Practice Skill 1

Example 1

Find the independent variable and the dependent variable in the linear function $y = 6x + 1$.

1. Identify the independent variable of the function.

   The independent variable of a linear function is the variable which has a value that *is not* determined by another variable's value.

   Notice that there are two variables in the equation, x and y. Typically, the independent variable is located on the right side of the equation representing the function. Notice that $y = 6x + 1$ can also be rewritten as $6x + 1 = y$. However, when the variables x and y are used, x is known as the independent variable even when the equation is written with x on the left side.

   Thus, x is the independent variable in the function $y = 6x + 1$.

2. Identify the dependent variable of the function.

   The dependent variable of a linear function is the variable which has a value that *is* determined by another variable's value.

   Usually, the dependent variable is on the left side of the equation representing the function. Again, however, even though $y = 6x + 1$ could be rewritten as $6x + 1 = y$, x would not become the dependent variable. This is because when the variables x and y are used, y is known as the dependent variable.

   Therefore, y is the dependent variable in the function $y = 6x + 1$.

Example 2

The amount of money Garrett will raise for charity by participating in a walkathon can be modeled by the linear function $d = 50m + 120$, for which m is the number of miles he walks and d is the number of dollars he raises. The value 120 represents how much of his own money Garrett donated before the walkathon started. Find the domain and the range of this function.

1. Identify the domain of the function.

   The domain of a linear function is all the possible values of the function's independent variable.

   In this case, the independent variable is m, the number of miles Garrett walks, because its value is not determined by d. That is, the number of miles Garrett walks does not depend on the amount of money he raises—instead, the amount of money he raises depends on the number of miles he walks.

   Garrett cannot walk a negative number of miles, and right when he starts, the number of miles he will have walked is 0. In theory, there is no maximum value for m. Therefore, the domain of the function is all real numbers m such that m is greater than or equal to 0 or, symbolically, $m \geq 0$.

2. Identify the range of the function.

   The range of a linear function is all the possible values of the function's dependent variable.

   In this case, the dependent variable is d, which represents the number of dollars Garrett raises, because its value is determined by m. In other words, the amount of money Garrett raises depends on the number of miles he walks.

   Since the minimum number of miles Garrett can walk is 0, the minimum number of dollars he can raise is $50(0) + 120 = 120$. In theory, there is no maximum. Therefore, the range of the function is all real numbers d such that d is greater than or equal to 120 or, symbolically, $d \geq 120$.

Scaffolded Practice Skill 1

Example 1

Find the independent variable and the dependent variable in the linear function $y = 6x + 1$.

1. Identify the independent variable of the function.

2. Identify the dependent variable of the function.

continued

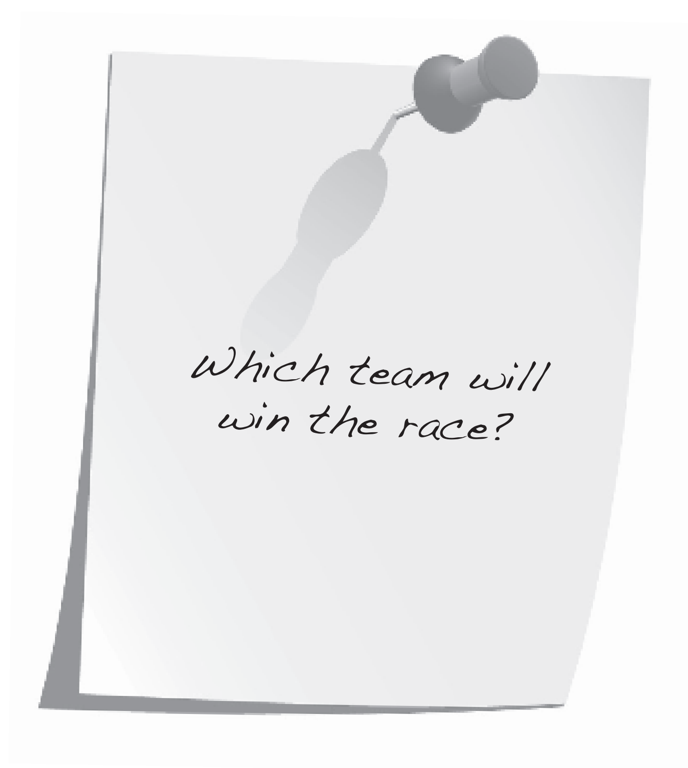
Example 2

The amount of money Garrett will raise for charity by participating in a walkathon can be modeled by the linear function $d = 50m + 120$, for which m is the number of miles he walks and d is the number of dollars he raises. The value 120 represents how much of his own money Garrett donated before the walkathon started. Find the domain and the range of this function.

Problem-Based Task Skill 1: Race to the Finish

SMP
1 ✓ 2 ✓
3 4 ✓
5 6 ✓
7 ✓ 8

Three teams of rowers—the blue team, the green team, and the red team—are competing in a regatta, or boat race. As they approach the finish line, each team has a different distance to go, and each team is traveling at a different constant speed. The blue team's distance in feet from the finish line, D_b, can be modeled by the linear function $D_b = -16t_b + 216$, for which t_b is the time in seconds until the team finishes. The green team's distance can be modeled by $D_g = -22t_g + 275$, and the red team's distance can be modeled by $D_r = -20t_r + 244$. Which team will win the race?



SWB p. 75

Problem-Based Task Skill 1: Race to the Finish

Coaching

a. What will each team's distance from the finish line be when they finish the race?

b. How many seconds will it take for the blue team to finish the race?

c. How many seconds will it take for the green team to finish the race?

d. How many seconds will it take for the red team to finish the race?

e. Which team will win the race?

Problem-Based Task Skill 1: Race to the Finish

Coaching Sample Responses

a. What will each team's distance from the finish line be when they finish the race?

Since each team will be at the finish line when they finish the race, each team's distance from the finish line will be 0 feet.

b. How many seconds will it take for the blue team to finish the race?

To find the number of seconds it will take for the blue team to finish the race, substitute 0 for D_b in the function $D_b = -16t_b + 216$ and solve for t_b.

$D_b = -16t_b + 216$

$(0) = -16t_b + 216$

$0 - 216 = -16t_b + 216 - 216$

$-216 = -16t_b$

$\frac{-216}{-16} = \frac{-16t_b}{-16}$

$t_b = 13.5$

It will take 13.5 seconds for the blue team to finish the race.

c. How many seconds will it take for the green team to finish the race?

To find the number of seconds it will take for the green team to finish the race, substitute 0 for D_g in the function $D_g = -22t_g + 275$ and solve for t_g.

$D_g = -22t_g + 275$

$(0) = -22t_g + 275$

$0 - 275 = -22t_g + 275 - 275$

$-275 = -22t_g$

$\frac{-275}{-22} = \frac{-22t_g}{-22}$

$t_g = 12.5$

It will take 12.5 seconds for the green team to finish the race.

d. How many seconds will it take for the red team to finish the race?

To find the number of seconds it will take for the red team to finish the race, substitute 0 for D_r in the function $D_r = -20t_r + 244$ and solve for t_r.

$D_r = -20t_r + 244$

$(0) = -20t_r + 244$

$0 - 244 = -20t_r + 244 - 244$

$-244 = -20t_r$

$\frac{-244}{-20} = \frac{-20t_r}{-20}$

$t_r = 12.2$

It will take 12.2 seconds for the red team to finish the race.

e. Which team will win the race?

It will take 13.5 seconds for the blue team to finish the race, 12.5 seconds for the green team to finish the race, and 12.2 seconds for the red team to finish the race. Since the red team will finish the race in the least amount of time, they will finish first and win the race.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 1: Understanding the Properties of Functions

For problems 1–10, answer the following questions about the properties of the given linear function.

1. What is the independent variable of the linear function $y = 8x - 15$?

2. What is the dependent variable of the linear function $y = \frac{3}{4}x + 6$?

3. The linear function $b = -0.04p + 8$ models the milliliters of black ink, b, remaining in a printer's ink cartridge after printing p pages. What is the independent variable in the function?

4. The linear function $h = 9.5t + 285$ models an elevator's height, h, in feet, above the ground floor of a skyscraper t seconds since the elevator started rising. What is the dependent variable in the function?

5. The linear function $p = 0.1d + 1$ models the amount of pressure, p, in atmospheres, on a deep-sea diver as she increases her depth, d, in meters. What is the independent variable in the function?

6. What is the domain of the linear function $y = \frac{7}{2}x + 9$?

7. What is the range of the linear function $y = -4x - 3$?

8. The linear function $p = 25c$ models the number of grams of protein, p, in c cups of cottage cheese. What is the domain of the function?

9. It costs $80 to rent out a laser tag arena for a birthday party, plus an additional $150 per hour of use. The linear function $c = 150t + 80$ models the cost, c, in dollars, for t hours of use. What is the range of the function?

10. The linear function $w = 0.1s + 42$ models the weight, w, in pounds, of a sandbox as it is being filled with s cubic inches of sand. What is the domain of the function?

Skill 2: Understanding Function Notation**

Common Core State Standard

F–IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

This skill has been addressed previously in *CCSS Integrated Pathway: Mathematics I*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

Unit 2, Lesson 1, Sub-lesson 4

Instruction

Supportive Instructional Strategies for Mathematics I

Unit 2 Lesson 3

Suggestions for Graphic Organizers/Manipulatives

Distribute the Coordinate Plane graphic organizer found in the Program Overview. At the top of the page, have students list the first ten terms of the arithmetic sequence defined by the explicit formula $a_n = 10 - 2n$, which are 8, 6, 4, 2, 0, –2, –4, –6, –8, and –10. Then have them cross out the "*x*" along the horizontal axis of the coordinate plane, replacing it with a_n, and cross out the "*y*" along the vertical axis, replacing it with *n*. Have students plot the sequence on the coordinate plane, making sure not to connect the points with a line. After students finish plotting the terms, point out that only the first and fourth quadrants of the coordinate plane were used, whereas the second and third quadrants were not. Let students know that when plotting a sequence, the second and third quadrants will never be used.

Suggestions for Discourse

Tell students that one of the most famous sequences in mathematics is the Fibonacci sequence. Show them the first ten terms of the sequence, which are 0, 1, 1, 2, 3, 5, 8, 13, 21, and 34, and have them vote on whether they think the sequence is an arithmetic sequence, a geometric sequence, or neither. Ask some of the students who voted for each of the options to support their answers, and then explain that the sequence is neither an arithmetic sequence nor a geometric sequence, because it has neither a common difference nor a common ratio. Review with students the concept of a recursive formula, have them form groups, and ask each group to try to write a recursive formula for the sequence. If necessary, get them started by giving them $f_1 = 0$ and $f_2 = 1$. Finally, have each group report on their findings and why, and let students know that the rest of the formula is $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$.

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics I*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.
- Elaborate on culture-specific contexts.

- Use manipulatives if appropriate. For example, provide students with graph paper and a ruler.
- Allow students to experiment with creating a coordinate graph by providing several linear and exponential equations. Coach students on making the coordinate plane (with straight lines) and show them how to plot the points and connect them, if needed.
- Provide prompts to support and encourage student responses. For example, either say or write:
  - "You knew that (0, 4) was the *y*-intercept because _______."
  - "First, you graphed the *y*-intercept, and then the next step was to _______."
  - "To find the value of the function when $x = 3$, the first step is to _______."
- Provide a list of sentence frames in which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "The value in this equation that represents the slope is _______." Or, "To find the common difference of this linear function, I will _______ consecutive output values." Or, "This <u>slope</u> is <u>negative</u>, because there is a <u>negative sign</u> in front of the value for <u>*m*</u> in the equation."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- not realizing that a sequence is a function

  Explain to students that the input value of the function defining a sequence is the term number, and the output value is the term.
- not understanding that a sequence is graphed without a line connecting the points

  Give students some real-world examples of objects that are discrete, such as candies that come in a range of colors. Have the students plot the number of each color on a graph. Remind students that when they plot a sequence on a coordinate plane, just as when they plot the number of each color of candy, they do not draw a line through the points.

Lesson 4: Interpreting Graphs of Functions

Instruction

Elementary Prerequisite Skills

This lesson requires the use of the following elementary skill(s) to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*. See the Appendix for material to address the skill(s).

- E-Skill 1: Applying the Order of Operations (5.OA.1), Appendix p. A-2
- E-Skill 2: Understanding the Coordinate Plane (5.G.1), Appendix p. A-10

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*.

Skill 1: Graphing Linear Functions from Tables or Equations* (A–CED.2★)

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Skill 2: Graphing Exponential Functions from Tables or Equations* (A–CED.2★)

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Skill 3: Understanding Function Notation, Domain, and Independent and Dependent Variables** (F–IF.1)

Common Core State Standard

F–IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

(*continued*)

Skill 4: Understanding Slope* (8.EE.5)

Common Core State Standard

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

Skill 5: Interpreting Interval Notation

Common Core State Standard

Note: There is no specific Common Core State Standard for this skill.

Instruction

Skill 1: Graphing Linear Functions from Tables or Equations*

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 2

SWB p. 81

Instruction

Guided Practice Skill 1

Example 1

Melina bought 50 shirts for \$7.83 each and plans to sell them for \$15.00 each. Write an equation that describes how much profit she will make depending on how many shirts she sells. Create a table of values that shows her profit if she sells 10, 30, and 50 shirts, respectively. Then, create a graph of the equation.

1. Write an equation that describes the scenario.

   Melina's profit depends on how many shirts she sells. The number of shirts sold is the independent variable, x, and the profit is the dependent variable, y.

   Melina paid \$7.83 each for 50 shirts. Multiply 50 by \$7.83 to determine how much Melina spent.

   $50 \bullet 7.83 = 391.50$

   Melina spent \$391.50.

   The amount that Melina spent, \$391.50, will be subtracted from her total profit from selling the shirts for \$15.00 each. Therefore, her total profit, y, is \$15.00 times the number of shirts she sells, x, minus her initial cost of \$391.50.

   $y = 15x - 391.5$

   The equation $y = 15x - 391.5$ represents how much Melina's profit, y, will be depending on how many shirts she sells, x.

2. Create a table of values by solving the equation for each number of shirts sold.

Create a table of values that shows the profit she will make if she sells 10, 30, and 50 shirts, respectively, by substituting these values for x and solving for y.

Substitute 10 for x and solve for y.

$y = 15x - 391.5$	Equation
$y = 15(10) - 391.5$	Substitute 10 for x.
$y = -241.5$	Simplify.

If Melina sells 10 shirts, she will lose $241.50.

Substitute 30 for x and solve for y.

$y = 15x - 391.5$	Equation
$y = 15(30) - 391.5$	Substitute 30 for x.
$y = 58.5$	Simplify.

If Melina sells 30 shirts, she will make $58.50.

Substitute 50 for x and solve for y.

$y = 15x - 391.5$	Equation
$y = 15(50) - 391.5$	Substitute 50 for x.
$y = 358.50$	Simplify.

If Melina sells 50 shirts, she will make $358.50.

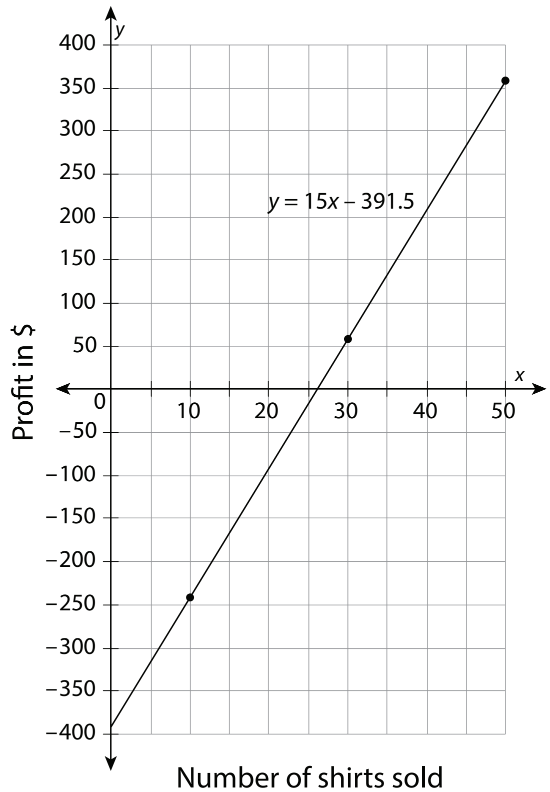
Organize the values of x and y into a table of values.

Number of shirts sold (x)	Profit in $ (y)
10	–241.50
30	58.50
50	358.50

3. Use the table of values to graph the equation.

The ordered pairs given by the table of values are (10, –241.5), (30, 58.5), and (50, 358.5). Graph these points on a coordinate plane and draw a line through them.

Since the number of shirts sold could only be between 0 and 50, start the line at $x = 0$ and end it at $x = 50$.



The graph represents the amount of profit Melina will make depending on the number of shirts she sells.

Scaffolded Practice Skill 1

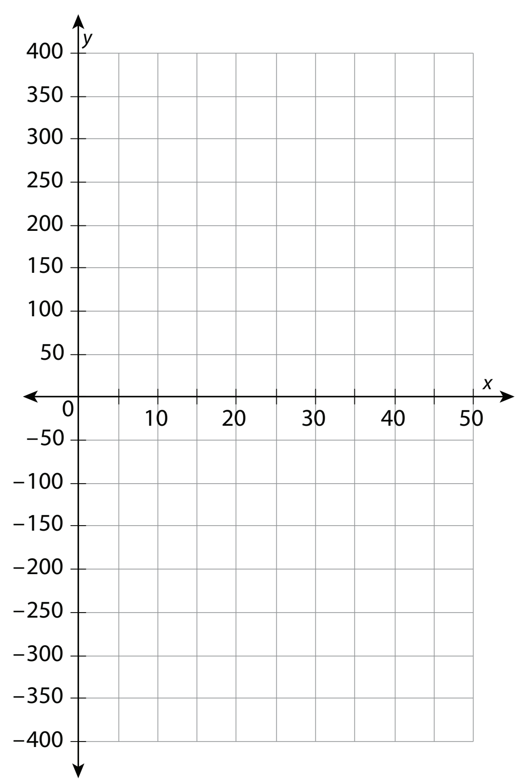
Example 1

Melina bought 50 shirts for $7.83 each and plans to sell them for $15.00 each. Write an equation that describes how much profit she will make depending on how many shirts she sells. Create a table of values that shows her profit if she sells 10, 30, and 50 shirts, respectively. Then, create a graph of the equation.

1. Write an equation that describes the scenario.

2. Create a table of values by solving the equation for each number of shirts sold.

3. Use the table of values to graph the equation.



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Practice Skill 1: Graphing Linear Functions from Tables or Equations*

Use the following information for problems 1–3.

As a server at a restaurant, Elizabeth makes $3.36 an hour plus tips. She is scheduled to work 28 hours this week.

1. The equation $y = 94.08 + x$ represents what Elizabeth's total earnings will be this week depending on the amount of her tips, x. Create a table of values for what her total earnings will be if her total tips for the week are $190, $245, or $335.

2. Use the table to create a graph that represents Elizabeth's total earnings depending on the amount of tips she earns.

3. The equation $y = 110.88 + x$ represents Elizabeth's total earnings this week depending on the amount of her tips, x, if she picks up an extra shift and works 5 more hours. Create a graph that represents this equation.

Skill 2: Graphing Exponential Functions from Tables or Equations*

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 2

SWB p. 89

Guided Practice Skill 2

Example 1

The formula $y = a(1 - r)^t$ can be used to calculate the current value (y) of any item that is depreciating in value. In this formula, a is the original value, r is the rate at which it is depreciating, and t is the time in years. Shiloh just bought a used car for $8,300 that will likely depreciate at a rate of 20% each year. Write an equation that can be used to predict the car's value in the future. Then create a table of values that predicts its value after 2, 5, and 10 years. Use the table to create a graph of the equation.

1. Write an equation that represents the situation.

   In the formula $y = a(1 - r)^t$, a is the original value of the car, r is the rate at which it is depreciating, and t is the time in years.

   Since the original value is $8,300, substitute 8,300 for a.

   Since the rate is 20% and 20% is equal to 0.20, substitute 0.20 for r.

   Since the value is dependent on the number of years that have passed, the number of years (t) is the independent variable (x), and the value (y) is the dependent variable (which will remain "y").

$y = a(1 - r)^t$	Given formula
$y = (8300)[1 - (0.20)]^{(x)}$	Substitute 8,300 for a, 0.20 for r, and x for t.
$y = 8300(0.8)^x$	Simplify.

   The equation that can be used to predict the car's value is $y = 8300(0.8)^x$.

2. Create a table of values by solving the equation for each number of years.

   To create a table of values that predicts the car's value after 2, 5, and 10 years, substitute each of these values for x and solve for y.

   Substitute 2 for x and solve for y.

   $y = 8300(0.8)^x$ Equation
   $y = 8300(0.8)^{(2)}$ Substitute 2 for x.
   $y = 5312$ Simplify.

   After 2 years, the car will be worth $5,312.

   Substitute 5 for x and solve for y.

   $y = 8300(0.8)^x$ Equation
   $y = 8300(0.8)^{(5)}$ Substitute 5 for x.
   $y \approx 2720$ Simplify.

   After 5 years, the car will be worth approximately $2,720.

   Substitute 10 for x and solve for y.

   $y = 8300(0.8)^x$ Equation
   $y = 8300(0.8)^{(10)}$ Substitute 10 for x.
   $y \approx 891$ Simplify.

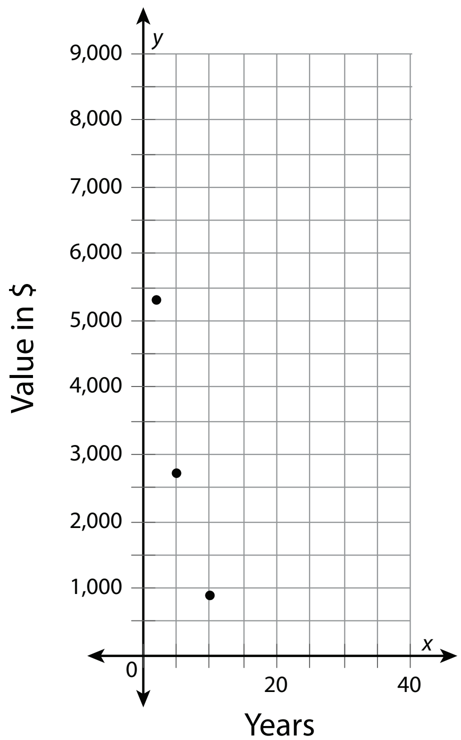
   After 10 years, the car will be worth approximately $891.

   Organize this information into a table of values.

Years (x)	Value in $ (y)
2	5,312
5	2,720
10	891

3. Use the table of values to create a graph of the equation.

   The table of values gives the coordinates (2, 5,312), (5, 2,720), and (10, 891). Graph these values on a coordinate plane. Label the *x*-axis "Years" and the *y*-axis "Value in $."



   Note that these three points do not fully show the shape of the graph. We need to graph more points.

4. Plot additional points to determine the shape of the graph.

   Choose a value for *x* on the left side of the first point (for example, $x = 0$). Then, choose a value for *x* on the right side of the last point (for example, $x = 20$). Substitute 0 and 20 into the equation and find the corresponding *y*-values, so that there are two more points to graph.

   Substitute 0 for *x* and solve for *y*.

$y = 8300(0.8)^x$	Equation
$y = 8300(0.8)^{(0)}$	Substitute 0 for *x*.
$y = 8300$	Simplify.

(*continued*)

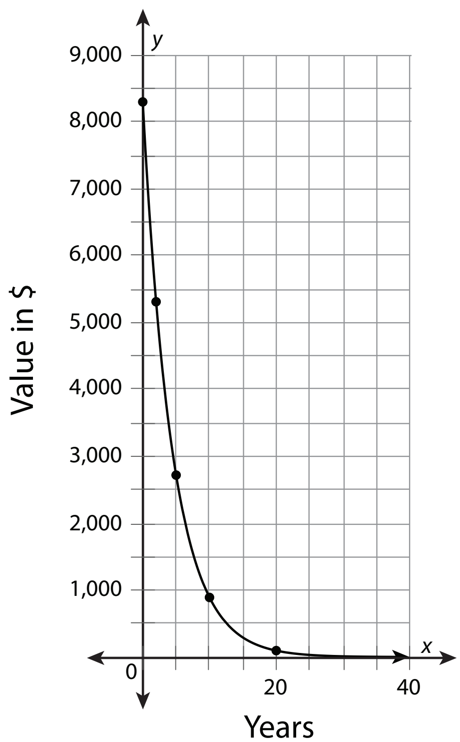
The corresponding coordinate pair is (0, 8,300). Notice that this gives us the original value of the car.

Substitute 20 for *x* and solve for *y*.

$y = 8300(0.8)^x$	Equation
$y = 8300(0.8)^{(20)}$	Substitute 20 for *x*.
$y \approx 96$	Simplify.

The corresponding coordinate pair is approximately (20, 96).

Plot these points on the coordinate plane, then draw a smooth curve through all the points.



Notice that the graph starts at (0, 8,300) because time cannot be negative. The graph approaches 0 as *x*, the number of years, increases. The car becomes less valuable as more time passes.

Scaffolded Practice Skill 2

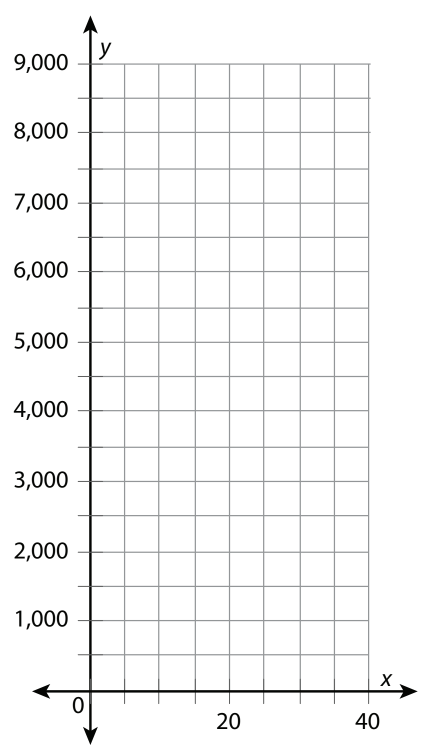
Example 1

The formula $y = a(1 - r)^t$ can be used to calculate the current value (y) of any item that is depreciating in value. In this formula, a is the original value, r is the rate at which it is depreciating, and t is the time in years. Shiloh just bought a used car for \$8,300 that will likely depreciate at a rate of 20% each year. Write an equation that can be used to predict the car's value in the future. Then create a table of values that predicts its value after 2, 5, and 10 years. Use the table to create a graph of the equation.

1. Write an equation that represents the situation.

2. Create a table of values by solving the equation for each number of years.

3. Use the table of values to create a graph of the equation.



4. Plot additional points to determine the shape of the graph.

Practice Skill 2: Graphing Exponential Functions from Tables or Equations*

Use the following information to complete problems 1–3.

Anton bought a brand new car for $32,865. He estimates that the car will depreciate at a rate of 18% each year. The formula $y = a(1 - r)^t$ can be used to calculate the value (y) of any item that is depreciating in value, where a is the original value, r is the rate at which it is depreciating, and t is the time in years.

1. The equation $y = 32{,}865(0.82)^x$ represents the future value of the car. Create a table of values that predicts the value of the car after 1 year, 3 years, and 5 years. Round to the nearest whole dollar.

2. Use the table to create a graph that predicts the future value of the car. *Hint*: Plot additional points as needed to create a more accurate graph.

3. Create an equation that predicts the future value of the car if it actually depreciates at 21% each year instead of 18%. Use this equation to create a graph that predicts the future value of the car with a depreciation rate of 21%.

Skill 3: Understanding Function Notation, Domain, and Independent and Dependent Variables**

Common Core State Standard

F–IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

This skill has been addressed previously in *CCSS Integrated Pathway: Mathematics I*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

Unit 2, Lesson 1, Sub-lesson 3

Skill 4: Understanding Slope*

Common Core State Standard

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 1, Lesson 3, Skill 1

Guided Practice Skill 4

Example 1

Winston's bathtub has a leak, and he has called a plumber for repairs. The plumber charges $24 per hour plus a $50 service fee just for coming to look at the problem. Determine the slope of the line that passes through the points that represent the total cost of 1 hour of work up to 4 hours of work. Explain what the slope means in the context of the problem. Finally, use the slope to determine how many hours the plumber worked if the final bill was $206. Assume there is no sales tax.

1. Create a table to show how the quantities described vary.

   The two quantities described are the number of hours worked and the total cost of the repair.

   The total cost, y, can be determined by multiplying the number of hours worked, x, by 24 and then adding 50 to include the $50 service charge.

   Therefore, the equation that represents this relationship is $y = 24x + 50$.

   Choose several values for the hours worked and calculate the total cost. Let's use 1, 2, 3, and 4.

   $24(1) + 50 = 74$

   $24(2) + 50 = 98$

   $24(3) + 50 = 122$

   $24(4) + 50 = 146$

   Next, organize the results in a table.

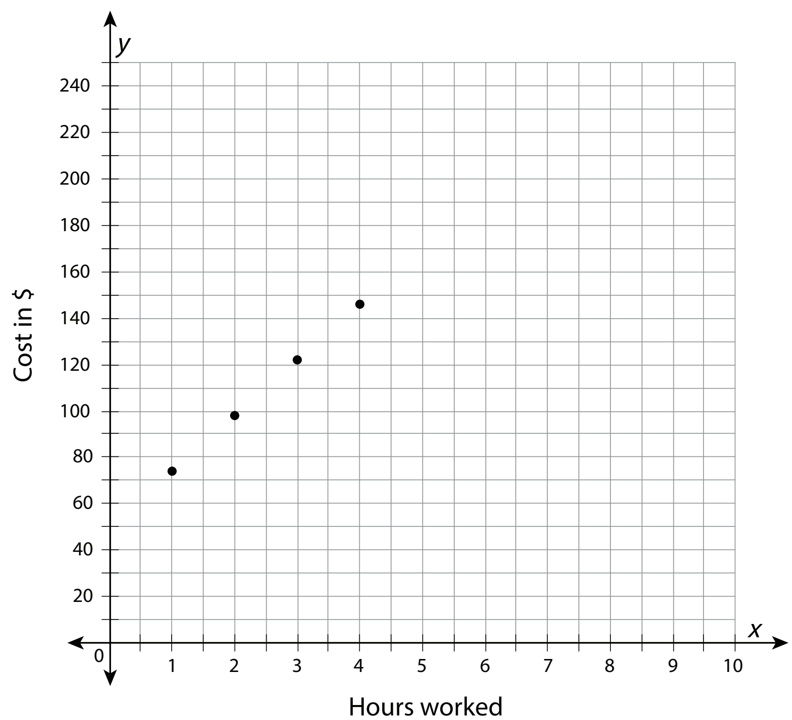
Hours worked (x)	Cost in $ (y)
1	74
2	98
3	122
4	146

2. Graph the relationship.

   Use the table of values to graph the relationship.

   Let *x* represent the number of hours worked and *y* represent the total cost.

   The table of values represents the coordinates (1, 74), (2, 98), (3, 122), and (4, 146).



3. Determine the slope using the slope formula, $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$. Explain what the slope means in the context of the problem.

Determine the slope by substituting values from the graph into the slope formula, $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$. Let (x_1, y_1) be (1, 74) and (x_2, y_2) be (2, 98). Substitute these values into the slope formula to find the slope of the line, and then simplify.

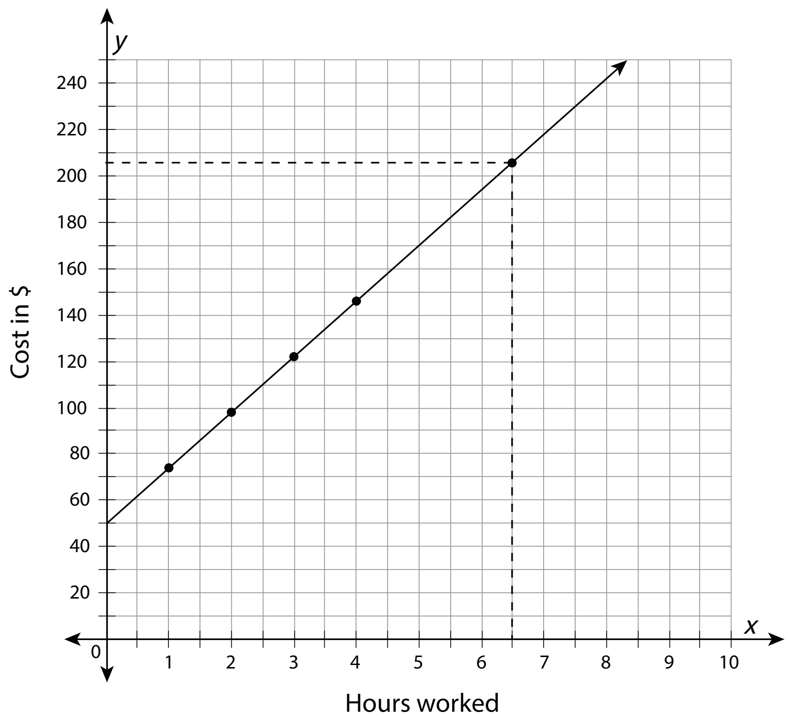
$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$\text{slope} = \frac{(98)-(74)}{(2)-(1)}$	Substitute 98 for y_2, 74 for y_1, 2 for x_2, and 1 for x_1.
$\text{slope} = \frac{24}{1}$	Subtract.
$\text{slope} = 24$	Simplify.

The slope is 24; this represents a rate of $24 per hour worked. This verifies the given information in the problem that the plumber charges $24 per hour.

4. Use the slope to determine how many hours the plumber worked if the final bill was \$206. Plot the corresponding point on your graph.

   To determine the number of hours the plumber worked if the total cost was \$206, examine the graph.

   Find 206 on the y-axis, and then look to the right to determine the corresponding x-coordinate.



   From the graph, it appears that a total cost of \$206 means the plumber worked about 6.5 hours.

   Another way to verify this is to solve an equation for the number of hours worked. The total cost is \$206, so the equation is $24x + 50 = 206$. Solve this equation for x.

$24x + 50 = 206$	Equation
$24x = 156$	Subtract 50 from both sides
$x = 6.5$	Divide both sides by 24.

   This gives the same result found by examining the graph. In this case, plumbers can work a portion of an hour and charge for their time, so this answer is logical.

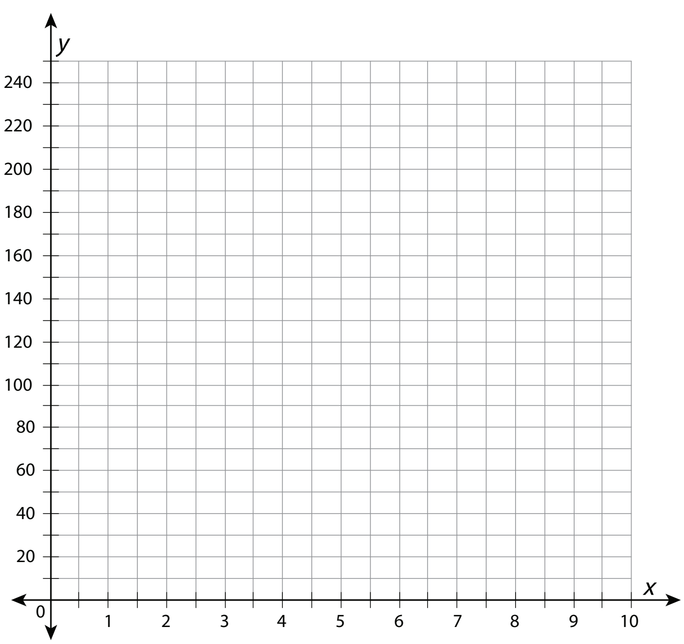
Scaffolded Practice Skill 4

Example 1

Winston's bathtub has a leak, and he has called a plumber for repairs. The plumber charges $24 per hour plus a $50 service fee just for coming to look at the problem. Determine the slope of the line that passes through the points that represent the total cost of 1 hour of work up to 4 hours of work. Explain what the slope means in the context of the problem. Finally, use the slope to determine how many hours the plumber worked if the final bill was $206. Assume there is no sales tax.

1. Create a table to show how the quantities described vary.

2. Graph the relationship.



3. Determine the slope using the slope formula, $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$. Explain what the slope means in the context of the problem.

4. Use the slope to determine how many hours the plumber worked if the final bill was $206. Plot the corresponding point on your graph.

Practice Skill 4: Understanding Slope*

For problems 1 and 2, determine the slope of the line that passes through the given points.

1. (4, 9) and (6, –2)

2. (–3, 7) and (–5, 9)

For problem 3, graph the relationship between the given quantities, and then use the slope of the line to answer the question.

3. A medical school started the new school year with 240 students. After 4 months, only 180 students were still enrolled at the school. How many students dropped out each month, assuming the dropout rate was the same each month?

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Instruction

Skill 5: Interpreting Interval Notation

SMP	
1 ✓	2 ✓
3	4 ✓
5	6 ✓
7	8

Common Core State Standard

Note: There is no specific Common Core State Standard for this skill.

Essential Questions

1. What is interval notation?
2. How can the minimum and maximum values of a function be recorded?

WORDS TO KNOW

brackets	[ ]; when used with intervals, indicate inclusiveness
closed interval	a range of values with the endpoints included
interval	a continuous series of values
interval notation	a way of identifying and recording the values of a solution for an inequality
open interval	a range of values with the endpoints not included
parentheses	( ); when used with intervals, indicate exclusiveness

Recommended Resources

- Coolmath.com. "Interval Notation."

  http://www.walch.com/rr/04048

  This user-friendly site provides a concise explanation of interval notation using inequalities and number lines.

- Oswego City School District Regents Exam Prep Center. "Set-builder & Interval Notation."

  http://www.walch.com/rr/04049

  This site summarizes interval notation, using various examples to describe different possible notations.

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Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Distribute copies of the two-circle Venn Diagram graphic organizer from the Program Overview. Have students use the diagram to explain how interval notation is like and unlike notation that uses inequality symbols. Encourage students to think about the visual representation as well as the meaning of each symbol. Extend the exercise by using the three-circle Venn Diagram organizer to include the graphical representation of these sets of numbers. Through comparison, students can also evaluate which representation they think is easiest to understand and why.

Suggestions for Discourse

Ask students to think about why interval notation might have been developed. Encourage them to think about all that this relatively simple notation communicates as well as how it compares to other notations used to communicate the same information.

Making Connections

Encourage students to connect the use of the phrases *open interval* and *closed interval* with the endpoint circles on graphs of inequalities. These intervals can be represented by a number line graph, an inequality, and interval notation.

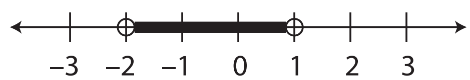
Skill 5: Interpreting Interval Notation

Introduction

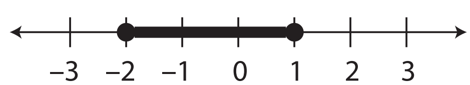
Interval notation is a way of identifying and recording the values of a solution for an inequality. It is used when writing a description of a function that applies to a specific **interval**, or range of continuous values of the independent variable. For example, a function may be increasing when x is greater than or equal to 4. This interval can be represented by an inequality, $x \geq 4$, or by interval notation, $[4, \infty)$, which includes brackets and parentheses. Each part of the notation relays specific information about the interval.

Key Concepts

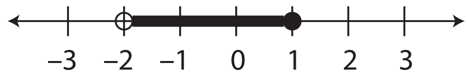
- Intervals are either "open" or "closed" depending on whether the endpoints are included in the interval.
- An **open interval**, (m, n), can be interpreted to mean $m < x < n$. Note that both the inequality symbols as well as the **parentheses** indicate that the endpoints are *not* included in the possible values for x. Although (m, n) has the same notation as an ordered pair, the context of a problem will help determine which purpose the notation has. It can be read, "x is between m and n, exclusive." Exclusive means m and n are not included.
- The endpoints of open intervals are represented with open circles on a number line. For example, the following number line shows the open interval (–2, 1).



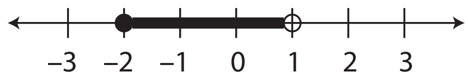
- A **closed interval**, $[m, n]$, can be interpreted to mean $m \leq x \leq n$. In this case, the inequality symbols, which include "or equal to," and the **brackets** indicate that the endpoints *are* included in the possible values for x. It can be read, "x is between m and n, inclusive." Inclusive means m and n are included.
- The endpoints of closed intervals are represented with closed circles on a number line. For example, the following number line shows the closed interval [–2, 1].



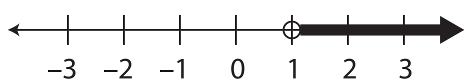
- Some intervals are both open and closed. For example, $(m, n]$ means $m < x \le n$, such that m is not a possible value of x and n is a possible value of x. The following graph of (–2, 1] demonstrates this, using an open circle for –2 and a closed circle for 1.



- Another example, $[m, n)$, means $m \le x < n$, such that m is a possible value of x and n is not a possible value of x. The following graph of [–2, 1) demonstrates this, using a closed circle for –2 and an open circle for 1.



- Intervals that extend forever in a particular direction can be written using an infinity symbol: ∞.
- When an interval extends forever in the negative direction, use the interval notation $(-\infty, n)$ or $(-\infty, n]$, where –∞, or negative infinity, represents the notion of extending forever in the negative direction. Notice that the right side, which uses n, can be open or closed, but the left side, which uses –∞, must be open.
- If the interval extends forever in the positive direction, use the interval notation (m, ∞) or $[m, \infty)$, where ∞, or positive infinity, represents the notion of extending forever in the positive direction. Notice that the left side, which uses m, can be open or closed, but the right side, which uses ∞, must be open.
- Always use parentheses, or open notation, for –∞ and ∞.
- If x can be any real number, the interval notation is $(-\infty, \infty)$.
- The graph of $(1, \infty)$ is shown. The arrow at the right end shows that the interval goes on forever in the positive direction.



Guided Practice Skill 5

Example 1

Write $-10 \le p < 4$ using interval notation.

1. Determine which values are included and which are not.

   The "less than or equal to" symbol, ≤, indicates that –10 is included in the interval. However, 4 is not included because of the "less than" symbol, <.

2. Write the interval using interval notation.

   Interval notation uses parentheses for non-included values and brackets for included values.

   The interval $-10 \le p < 4$ can be written using interval notation as $[-10, 4)$.

Example 2

Use interval notation to identify the interval represented by the following graph.

1. Identify the minimum value and determine whether it is included.

   The arrow on the graph points to the left, which means there is no minimum value. In other words, the interval extends infinitely in the negative direction. Use $-\infty$ to represent that there is no minimum. This value is never included.

2. Identify the maximum value and determine whether it is included.

   The closed circle on 9 indicates that this is the maximum value and it is included.

3. Write the represented interval using interval notation.

   A parenthesis indicates that a value is not included in the interval, whereas a bracket indicates that the value is included. Because $-\infty$ is not included, use a parenthesis. Because 9 is included, use a bracket.

   The graphed interval can be expressed in interval notation as $(-\infty, 9]$.

Example 3

Logan is looking for a part-time job to help pay for his car. He wants to work more than 10 hours a week, but no more than 18 hours a week. Use interval notation to express the number of hours, h, Logan wants to work.

1. Write an inequality to represent the number of hours Logan wants to work.

   Logan wants to work more than 10 hours a week, so $h > 10$, but he wants to work no more than 18 hours a week, so $h \le 18$. The 18 is included because of the phrase "no more than," which indicates the value is part of the interval.

   The interval can be written as $10 < h \le 18$.

2. Write the inequality using interval notation.

   The endpoint of the interval 10 will not be included, but the endpoint 18 will be included; therefore, the inequality can be written as $(10, 18]$.

Scaffolded Practice Skill 5

Example 1

Write $-10 \le p < 4$ using interval notation.

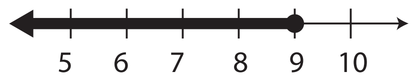
1. Determine which values are included and which are not.

2. Write the interval using interval notation.

continued

Example 2

Use interval notation to identify the interval represented by the following graph.



Example 3

Logan is looking for a part-time job to help pay for his car. He wants to work more than 10 hours a week, but no more than 18 hours a week. Use interval notation to express the number of hours, *h*, Logan wants to work.

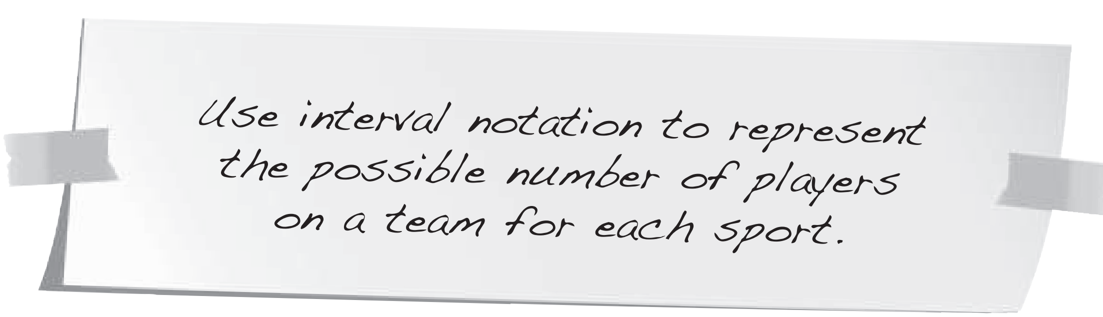
Problem-Based Task Skill 5: Player Limits

SMP	
1 ✓	2 ✓
3	4 ✓
5	6 ✓
7	8

The varsity and junior varsity soccer, baseball, and lacrosse teams at Parkville High School are holding tryouts. Each team has a different minimum and maximum number of players.

- Soccer: There are at least 11 players on a team, but each team has less than 18 players.
- Baseball: Each baseball team has 9 or more players, but at most 25 players.
- Lacrosse: Each team has more than 10 players, but no more than 15 players.

Use interval notation to represent the possible number of players on a team for each sport. Assume that each set contains only integers.



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Problem-Based Task Skill 5: Player Limits

Coaching

a. What is the minimum number of players on a soccer team?

b. What is the maximum number of players on a soccer team?

c. What is the minimum number of players on a baseball team?

d. What is the maximum number of players on a baseball team?

e. What is the minimum number of players on a lacrosse team?

f. What is the maximum number of players on a lacrosse team?

g. Which values are included in each set?

h. Which values are not included in each set?

i. How can the possible numbers of players on each team be represented using interval notation?

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Instruction

Problem-Based Task Skill 5: Player Limits

Coaching Sample Responses

a. What is the minimum number of players on a soccer team?

There are at least 11 players on each soccer team. In other words, there are 11 or more players on each soccer team.

b. What is the maximum number of players on a soccer team?

Each soccer team has less than 18 players.

c. What is the minimum number of players on a baseball team?

There are 9 players or more on each baseball team. In other words, each baseball team has greater than or equal to 9 players.

d. What is the maximum number of players on a baseball team?

There are at most 25 players on each baseball team. In other words, the number of players on each baseball team is less than or equal to 25.

e. What is the minimum number of players on a lacrosse team?

There are more than 10 players on each lacrosse team. In other words, the number of players on each lacrosse team is greater than 10.

f. What is the maximum number of players on a lacrosse team?

There are no more than 15 players on each lacrosse team. In other words, the number of players on each lacrosse team is less than or equal to 15.

g. Which values are included in each set?

The value that is included for soccer is the minimum value, 11.

The values that are included for baseball are 9 and 25.

The value that is included for lacrosse is the maximum value, 15.

h. Which values are not included in each set?

The value that is not included for soccer is the maximum, 18.

Both values, 9 and 25, are included for baseball, so there are no values that are not included.

The value that is not included for lacrosse is 10.

i. How can the possible numbers of players on a team be represented using interval notation?

For soccer, the interval is [11, 18).

For baseball, the interval is [9, 25].

For lacrosse, the interval is (10, 15].

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Name: **Date:**

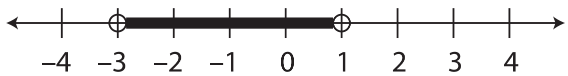
UNIT 2 • LINEAR AND EXPONENTIAL RELATIONSHIPS
Lesson 4: Interpreting Graphs of Functions

Practice Skill 5: Interpreting Interval Notation

Use what you have learned about interval notation to complete problems 1–7.

1. Write $-9 \le n < 0$ using interval notation.

2. Use interval notation to represent the interval shown in the graph.

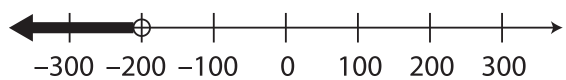


3. For a project, Elle needs a ribbon that is at least 18 inches long, but less than 22 inches long. Use interval notation to express the length of ribbon she needs.

4. Write $25 < p \le 29$ using interval notation.

5. Write $d > -56$ using interval notation.

6. Use interval notation to represent the interval shown in the graph.



7. Carl's family is taking a road trip. He knows he will be in the car for at least 4 hours. Use interval notation to express the time he will be in the car.

For problems 8–10, explain the meaning of each interval in words.

8. [–4, 2)

9. [–30, 20]

10. $(-30, \infty)$

Supportive Instructional Strategies for Mathematics I

Unit 2 Lesson 4

Suggestions for Graphic Organizers/Manipulatives

- Distribute copies of the two-circle Venn Diagram graphic organizer from the Program Overview. Ask students to label the circle on the left "Linear function" and label the circle on the right "Exponential function." Ask students to list specific characteristics of each type of function in the appropriate circles, and then list characteristics that both types of functions have in common in the middle of the diagram. Ask volunteers to share answers for all parts of the diagram. Create a master version so that all answers can be compiled into one organizer.
  - Possible linear function characteristics: *has a constant rate of change, the graph is a line*.
  - Possible exponential function characteristics: *the rate of change varies depending on the interval observed, the graph is a curve*.
  - Possible shared characteristics: *both have an independent and a dependent variable, a table of values can be created, x- and y-intercepts can be found, the rate of change can be found*.

- Provide students with the Coordinate Plane graphic organizer from the Program Overview. Ask students to write the linear function $f(x)=\frac{4}{5}x+2$ at the top of the page and identify the slope from the function. Then, ask them to write the points (–1, –2) and (5, 2) next to the function, and calculate the slope for these points. Have them work with a partner to discuss and then predict which will have a steeper slope: the line of the function $f(x)=\frac{4}{5}x+2$, or the line formed from the two given points. Then, ask them to graph the function and the two points on the same graph. Ask for volunteers to discuss their results, and point out that two ways the slope of a line can be determined are from a function or from two given points.

- Provide students with at least 10 blank flash cards. Ask them to write on one side of each flash card an example of a number, including examples from the entire set of real numbers: natural, whole, integers, rational, and irrational. After the students have created their flash cards, ask them to switch their cards with a partner, and ask the partner to write on the other side of each card all of the categories that the number can belong to. Ask for volunteers to share their examples, and create a master list of examples, discussing why a number can or cannot be classified in one category or more. (For example, the number –5 is an integer and a rational number. It is a rational number because it can be written as a ratio with a denominator of 1: $\frac{-5}{1}$.)

Suggestions for Discourse

- Ask students to work with a partner to list real-life examples in which linear functions would apply, and then list real-life examples in which exponential functions would apply. Ask them to think about the ways in which each type of function increases or decreases and how this would apply to the type of graph for both functions. Then, ask for volunteers to share their ideas, and make a master list compiling all valid examples for each type of function.
- Ask students, "What are four different ways in which linear or exponential functions can be represented?" Guide students in a discussion about functions being represented by equations, tables, graphs, and verbal descriptions.
- Ask students, "Why is it important to identify minimum and maximum values of a function in a real-world problem? What are some real-life scenarios in which the minimum value of a function would be 0?" Divide students into groups of three and ask them to discuss and create a list of examples of real-life problems that would have a minimum value, or domain of a function, greater than 0. Ask students to volunteer their answers.
- Create a game in which students classify characteristics of a function. Provide cards which have, for example, "Given the function, $f(x) = -4x - 2$, which value is the slope? Which value is the y-intercept? How can the x-intercept be found? Is this function increasing or decreasing?"

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics I*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.

- Provide a list of sentence frames in which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "I know this is an exponential function because the shape of the graph is a _______." Or, "In the equation $y = 5x + 4$, 5 is the _______ and 4 is the _______." Or, "This function is linear, because its graph is in the shape of a line."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- believing that exponential functions will eventually touch or intersect an asymptote

  Remind students that an asymptote is a line that a graph gets closer and closer to but never crosses or touches. Provide students with an additional example in the domain that is high in value, to show them that it still does not touch the asymptote.

- incorrectly identifying the type of function as either exponential or linear

  Remind students that a linear function does not contain an exponent, and an exponential function does. Also remind them that the graph of a linear equation is a line, and the graph of an exponential function is a curve.

- misidentifying key features on a graph

  Provide students with a word bank of key vocabulary words such as *x-intercept*, *y-intercept*, *slope*, *asymptote*, *increasing*, and *decreasing*.

- incorrectly choosing the domain for a function

  Remind students that the domain is the *x*-values, or the independent variable in a function, and that the domain values must make sense in the context of the problem.

- incorrectly choosing the values of the indicated interval to calculate the rate of change

  Remind students that the rate of change for a linear function is constant, so when two coordinates are given, label them (x_1, y_1) and (x_2, y_2), and find the slope, which is the rate of change. For an exponential function, substitute each *x*-value back into the original function and find the corresponding *y*-value, and then find the slope of these two coordinates.

- substituting incorrect values into the formula for calculating the rate of change

  Have students label the values as (x_1, y_1) and (x_2, y_2), and then input them into the slope formula.

- assuming the rate of change must remain constant over a period of time regardless of the function

  Remind students that a linear function has a constant rate of change, but an exponential function does not.

- interpreting interval notation as coordinates

  Remind students that although interval notation has the same form as the notation for an ordered pair, or coordinates, the context of the problem will indicate if there is an exact point to be plotted on a graph, or if there is a particular range of values that are being considered, which would indicate interval notation.

Lesson 5: Analyzing Linear and Exponential Functions

Instruction

Elementary Prerequisite Skills

This lesson requires the use of the following elementary skill(s) to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*. See the Appendix for material to address the skill(s).

- E-Skill 2: Understanding the Coordinate Plane (5.G.1), Appendix p. A-10

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*.

Skill 1: Graphing a Function from a Table of Values* (A–CED.2★)

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Skill 2: Understanding the Rules of Exponents, Including Negative Exponents* (8.EE.1)

Common Core State Standard

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Skill 3: Recognizing the General Shape of an Exponential Function (Decay or Growth)* (A–CED.2★)

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Instruction

Skill 1: Graphing a Function from a Table of Values*

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 2

Guided Practice Skill 1

Example 1

The Johnson family plans to travel by car 2,458 miles across the country. Write an equation that can be used to find how long the trip will take depending on their average speed. Then create a table of values that represents how long the trip will take if they drive an average speed of 55 miles per hour (mph), 60 mph, or 70 mph. Use the table to create a graph of the equation. Recall that the distance formula is $d = rt$, where d is the distance, r is the rate or speed, and t is the time.

1. Create an equation that describes the situation.

   Time, t, depends on the speed, r. Therefore, speed is the independent variable and time is the dependent variable.

   The formula for distance is rate (or speed) multiplied by time. The distance (d) of the trip is 2,458 miles.

   Since t is the unknown variable in the problem, substitute 2,458 miles for d and solve the resulting equation for t by dividing both sides by r.

$d = rt$	Formula for distance
$(2458) = rt$	Substitute 2458 for d.
$\frac{2458}{r} = t$	Divide both sides of the equation by r.
$t = \frac{2458}{r}$	Rearrange the equation.

   The equation $t = \frac{2458}{r}$ can be used to find how long the trip will take depending on the Johnsons' average speed.

2. Use the equation to create a table of values.

   To create a table of values that indicates how long the trip will take if the Johnsons drive at an average speed of 55 mph, 60 mph, or 70 mph, substitute each of these values into the equation for *r* and solve for *t*.

   Substitute 55 for *r* and solve.

$t=\dfrac{2458}{r}$	Equation from the previous step
$t=\dfrac{2458}{(55)}$	Substitute 55 for *r*.
$t\approx 44.7$ hours	Simplify.

   If the Johnsons average 55 mph, the trip will take approximately 44.7 hours.

   Substitute 60 for *r* and solve.

$t=\dfrac{2458}{r}$	Equation from the previous step
$t=\dfrac{2458}{(60)}$	Substitute 60 for *r*.
$t\approx 41.0$ hours	Simplify.

   If the Johnsons average 60 mph, the trip will take approximately 41.0 hours.

   Substitute 70 for *r* and solve.

$t=\dfrac{2458}{r}$	Equation from the previous step
$t=\dfrac{2458}{(70)}$	Substitute 70 for *r*.
$t\approx 35.1$ hours	Simplify.

   If the Johnsons average 70 mph, the trip will take approximately 35.1 hours.

(*continued*)

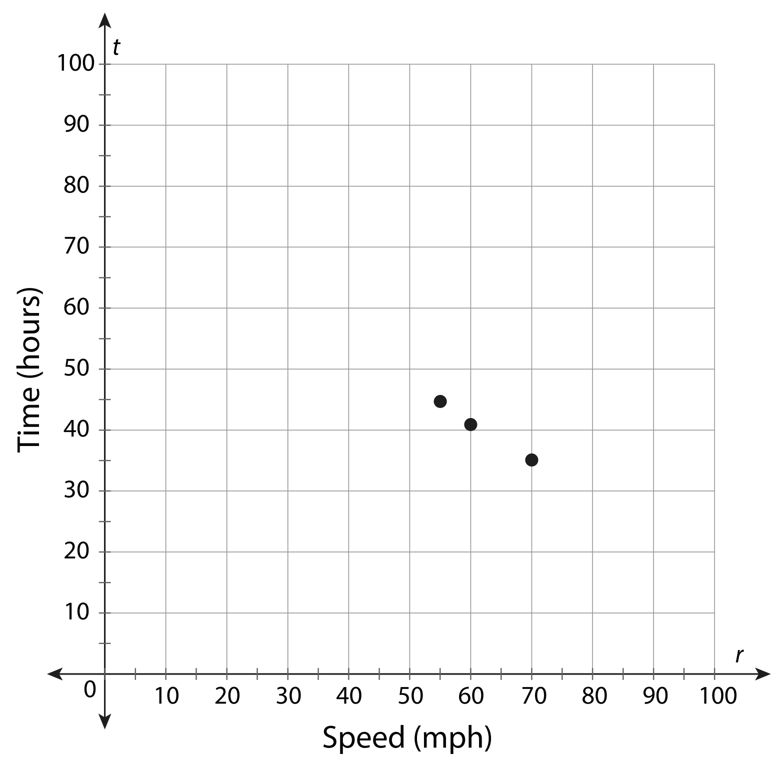
Arrange this information into a table of values.

Average speed (*r*)	Time (*t*)
55 mph	44.7 hours
60 mph	41.0 hours
70 mph	35.1 hours

3. Use the table of values to create a graph of the equation.

Using *r* for the horizontal axis and *t* for the vertical axis, then the table of values includes the following coordinates: (55, 44.7), (60, 41.0), and (70, 35.1).

Plot these values on a coordinate plane.



The three points shown are not enough to determine an accurate graph of the equation. We need to graph more points.

4. Plot additional points to determine the shape of the graph.

In order to get a better representation of the equation, find more coordinates by substituting other values into the formula for r, such as 35 mph, 45 mph, and 90 mph, then solve for t.

Substitute 35 for r and solve.

$t=\frac{2458}{r}$	Equation from the first step
$t=\frac{2458}{(35)}$	Substitute 35 for r.
$t \approx 70.2$ hours	Simplify.

Plot the resulting coordinate pair, (35, 70.2), on the graph.

Substitute 45 for r and solve.

$t=\frac{2458}{r}$	Equation from the first step
$t=\frac{2458}{(45)}$	Substitute 45 for r.
$t \approx 54.6$ hours	Simplify.

Plot the resulting coordinate pair, (45, 54.6), on the graph.

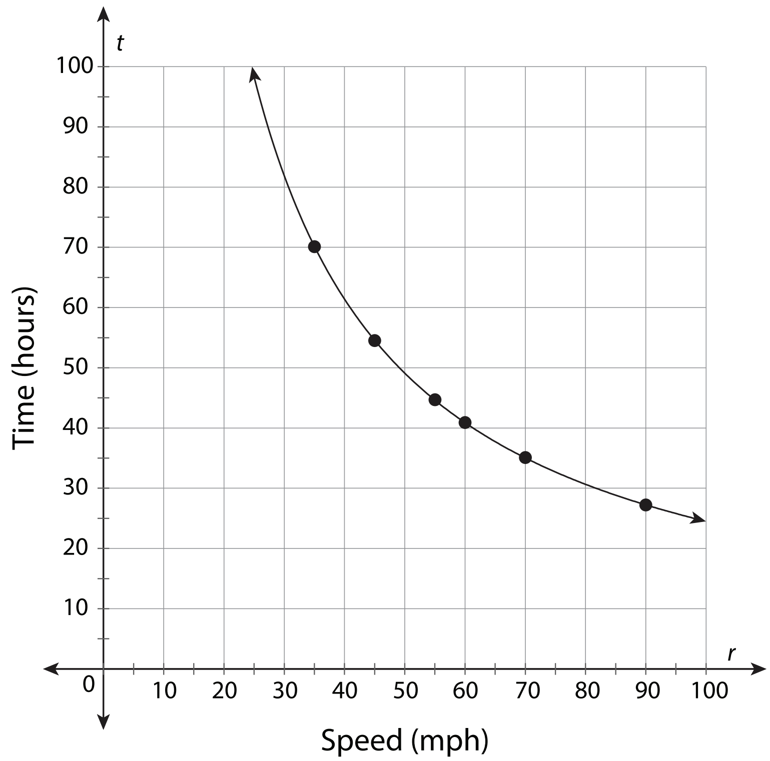
Substitute 90 for r and solve.

$t=\frac{2458}{r}$	Equation from the first step
$t=\frac{2458}{(90)}$	Substitute 90 for r.
$t \approx 27.3$ hours	Simplify.

Plot the resulting coordinate pair, (90, 27.3), on the graph. (Of course, driving at a speed of 90 mph is both illegal and unsafe, but graphing this high r-value will give us a better understanding of the end behavior of the graphed equation.)

(*continued*)

Draw a smooth curve through all the points.



The graph represents how long the trip will take depending on the average driving speed.

Scaffolded Practice Skill 1

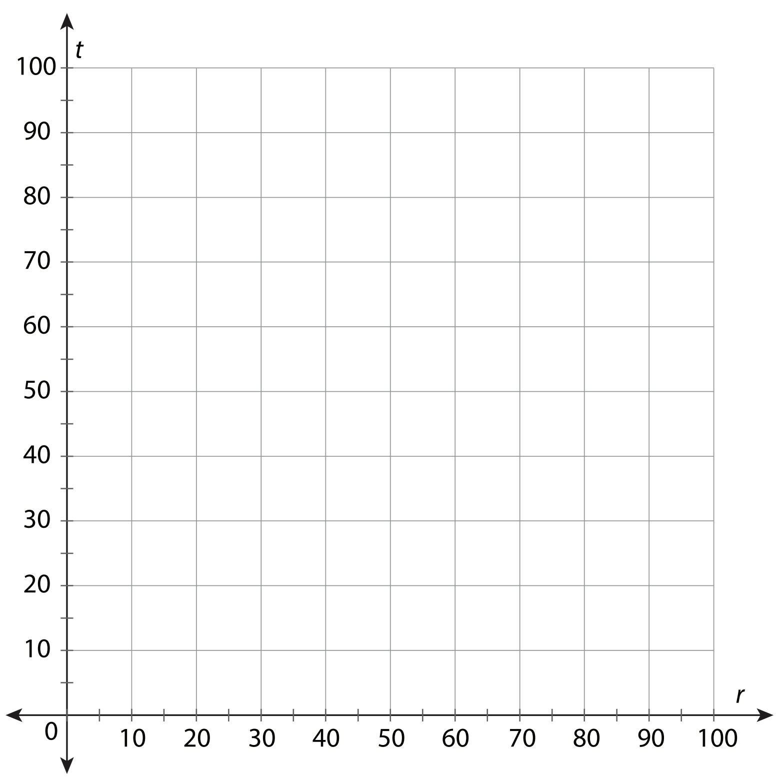
Example 1

The Johnson family plans to travel by car 2,458 miles across the country. Write an equation that can be used to find how long the trip will take depending on their average speed. Then create a table of values that represents how long the trip will take if they drive an average speed of 55 miles per hour (mph), 60 mph, or 70 mph. Use the table to create a graph of the equation. Recall that the distance formula is $d = rt$, where d is the distance, r is the rate or speed, and t is the time.

1. Create an equation that describes the situation.

2. Use the equation to create a table of values.

3. Use the table of values to create a graph of the equation.



4. Plot additional points to determine the shape of the graph.

Practice Skill 1: Graphing a Function from a Table of Values*

Use the following information to complete problems 1–3.

Brandon is trying to plan out today's running route. He wants to run for 45 minutes after school. He knows that the formula for distance is $d = rt$, where d is the distance, r is the rate or speed, and t is the time (in hours).

1. Create a table of values that tells how far Brandon will run (in miles) if his average speed is 4.5 mph, 6.2 mph, or 7.0 mph.

2. Create a graph that shows how far in miles Brandon will run depending on his average speed in mph.

3. Create a graph that shows how far in miles Brandon will run depending on his average speed in mph if he decides to run for 90 minutes instead of 45 minutes.

SWB p. 119

Instruction

Skill 2: Understanding the Rules of Exponents, Including Negative Exponents*

Common Core State Standard

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 3

Guided Practice Skill 2

Example 1

Simplify $\dfrac{x^{-3} \bullet x^{7} \bullet \left(y^{4}\right)^{3}}{y^{20}}$ as much as possible so that it has only positive exponents.

1. Use the exponent rule $(a^m)^n = a^{m \bullet n}$ to begin simplifying the expression.

   When a power is raised to another power, multiply the exponents.

   $\dfrac{x^{-3} \bullet x^{7} \bullet \left(y^{4}\right)^{3}}{y^{20}}$ Given expression

   $= \dfrac{x^{-3} \bullet x^{7} \bullet y^{4 \bullet 3}}{y^{20}}$ Rewrite $\left(y^{4}\right)^{3}$ as a multiplication of the exponents.

   $= \dfrac{x^{-3} \bullet x^{7} \bullet y^{12}}{y^{20}}$ Multiply the exponents.

   The expression $\dfrac{x^{-3} \bullet x^{7} \bullet \left(y^{4}\right)^{3}}{y^{20}}$ can be simplified to $\dfrac{x^{-3} \bullet x^{7} \bullet y^{12}}{y^{20}}$.

2. Use the exponent rule $a^m \bullet a^n = a^{m+n}$ to further simplify the expression.

   Notice that x^{-3} and x^{7} have the same base, x. When powers with the same base are multiplied, add the exponents.

   $\dfrac{x^{-3} \bullet x^{7} \bullet y^{12}}{y^{20}}$ Expression from the previous step

   $= \dfrac{x^{-3+7} \bullet y^{12}}{y^{20}}$ Rewrite $x^{-3} \bullet x^{7}$ as an addition of exponents.

   $= \dfrac{x^{4} \bullet y^{12}}{y^{20}}$ Add the exponents.

   The expression $\dfrac{x^{-3} \bullet x^{7} \bullet y^{12}}{y^{20}}$ can be simplified to $\dfrac{x^{4} \bullet y^{12}}{y^{20}}$. All the exponents are positive, but we can simplify this result even more.

3. Use the exponent rule $\frac{a^m}{a^n} = a^{m-n}$ to further simplify the expression.

Notice that y^{12} and y^{20} have the same base, y. When powers with the same base are divided, subtract the exponents.

$\frac{x^4 \bullet y^{12}}{y^{20}}$	Expression from the previous step
$= x^4 \bullet y^{12-20}$	Rewrite $\frac{y^{12}}{y^{20}}$ as a subtraction of exponents.
$= x^4 \bullet y^{-8}$	Subtract the exponents.

The expression $\frac{x^4 \bullet y^{12}}{y^{20}}$ can be simplified to $x^4 \bullet y^{-8}$. This result has a negative exponent, which we need to rewrite as a positive exponent.

4. Use the exponent rule $a^{-m} = \frac{1}{a^m}$ to write the negative exponent as a positive exponent.

To rewrite a negative exponent as a positive one, take its reciprocal.

The reciprocal of y is $\frac{1}{y}$. Written with a positive exponent, y^{-8} is $\frac{1}{y^8}$.

$x^4 \bullet y^{-8}$	Expression from the previous step
$= x^4 \bullet \left(\frac{1}{y^8}\right)$	Substitute $\frac{1}{y^8}$ for y^{-8}.
$= \frac{x^4}{y^8}$	Simplify.

The expression $x^4 \bullet y^{-8}$ can be rewritten as $\frac{x^4}{y^8}$. It cannot be simplified any further because there are no common factors. So, the expression $\frac{x^{-3} \bullet x^7 \bullet (y^4)^3}{y^{20}}$ is equal to $\frac{x^4}{y^8}$ when fully simplified.

Scaffolded Practice Skill 2

Example 1

Simplify $\dfrac{x^{-3} \bullet x^{7} \bullet \left(y^{4}\right)^{3}}{y^{20}}$ as much as possible so that it has only positive exponents.

1. Use the exponent rule $(a^m)^n = a^{m \bullet n}$ to begin simplifying the expression.

2. Use the exponent rule $a^m \bullet a^n = a^{m+n}$ to further simplify the expression.

3. Use the exponent rule $\dfrac{a^m}{a^n} = a^{m-n}$ to further simplify the expression.

4. Use the exponent rule $a^{-m} = \dfrac{1}{a^m}$ to write the negative exponent as a positive exponent.

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Practice Skill 2: Understanding the Rules of Exponents, Including Negative Exponents*

For problems 1–3, simplify each expression as much as possible, and write it with only positive exponents.

1. $(x^{-6})^{-3}$

2. $\dfrac{x^3 \bullet \left(y^4\right)^{-2}}{y^3 \bullet x^6} \bullet y^{-5}$

3. $\dfrac{\left(x^2\right)^6 \bullet y^5}{x^0 \bullet y^{-4}} \bullet y^7$

Instruction

Skill 3: Recognizing the General Shape of an Exponential Function (Decay or Growth)*

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 2

SWB p. 131

Guided Practice Skill 3

Example 1

The formula $y = a(1 \pm r)^t$ can be used to calculate the current value, y, of any item that is steadily increasing (+) or decreasing (–) in value. In this formula, a is the original value, r is the rate at which it is increasing or decreasing, and t is the time in years.

The Quincy family bought a house for $153,000, and since then the house has increased in value by an average of 1% each year. Write an equation that can be used to find the value of the house based on the number of years that have passed.

1. Determine if this situation represents an exponential growth or exponential decay problem.

   The value of the house is increasing. Therefore, this is an exponential growth situation, and a plus sign (+) should be used in the formula.

   So, the formula will have the format $y = a(1 + r)^t$.

2. Determine the values of a, r, and t.

   The value of the house depends on the amount of time, t, that has passed. Therefore, t is the independent variable, x.

   a is the original value, or $153,000.

   r is the rate, which is 1%. Recall that 1% equals 0.01. Therefore, $r = 0.01$.

   To summarize, $t = x$, $a = 153,000$, and $r = 0.01$.

3. Substitute the values into the formula to find the equation for the value of the house.

   Recall from step 1 that the formula $y = a(1 + r)^t$ can be used to calculate the value of the house.

   Substitute the values from step 2 ($t = x$, $a = 153{,}000$, and $r = 0.01$) into this equation.

$y = a(1 + r)^t$	Exponential growth function
$y = (153{,}000)[1 + (0.01)]^{(x)}$	Substitute 153,000 for *a*, 0.01 for *r*, and *x* for *t*.
$y = 153{,}000(1.01)^x$	Simplify.

   The equation $y = 153{,}000(1.01)^x$ can be used to find the value of the house, *y*, depending on the number of years that have passed, *x*.

Scaffolded Practice Skill 3

Example 1

The formula $y = a(1 \pm r)^t$ can be used to calculate the current value, y, of any item that is steadily increasing (+) or decreasing (–) in value. In this formula, a is the original value, r is the rate at which it is increasing or decreasing, and t is the time in years.

The Quincy family bought a house for $153,000, and since then the house has increased in value by an average of 1% each year. Write an equation that can be used to find the value of the house based on the number of years that have passed.

1. Determine if this situation represents an exponential growth or exponential decay problem.

2. Determine the values of a, r, and t.

3. Substitute the values into the formula to find the equation for the value of the house.

SWB p. 133

Practice Skill 3: Recognizing the General Shape of an Exponential Function (Decay or Growth)*

The formula $y = a(1 \pm r)^t$ can be used to calculate the current amount (y) of any item that is steadily increasing (+) or decreasing (–). In this formula, a is the original amount, r is the rate at which it is increasing or decreasing, and t is the time in years. Use this information to complete problems 1–3.

1. The population of Fairview has been steadily increasing by 5% each year to reach its current population of 49,000 people. Write an equation that can be used to predict Fairview's future population, assuming this trend continues.

2. A neighboring town, Oakton, currently has 10,000 more people, but its population has been steadily decreasing at a rate of 3%. Write an equation that predicts the future population of Oakton, assuming this trend continues.

3. Create separate graphs that show the future population projections for Fairview and Oakton, assuming the population trends continue.

SWB p. 135

Supportive Instructional Strategies for Mathematics I

Unit 2 Lesson 5

Suggestions for Graphic Organizers/Manipulatives

- Provide flash cards for students to use for vocabulary words. Ask students to write the vocabulary word on one side of the card and the definition on the other side. Words include: *linear, exponential, end behavior, asymptote, x-intercept, y-intercept, slope,* and *quadrant*. Ask students to quiz themselves first, then quiz a partner.
- Distribute copies of the two-circle Venn Diagram graphic organizer from the Program Overview. Ask students to label the circle on the left "Linear function" and label the circle on the right "Exponential function." Ask students to list specific characteristics of each type of function in the appropriate circles, with characteristics that both functions have in common listed in the middle. Ask volunteers to share answers for the three parts of the Venn diagram. Create a master version so all answers can be compiled into one organizer and displayed in the classroom.
- Provide students with a ruler and blank graph paper. Ask students to use the ruler to draw a coordinate plane with an x- and y-axis. Then give them the linear equation $y = 2x$. Ask students to create an x-y table with the x-coordinates $\{-3, -2, -1, 0, 1, 2, 3\}$, find the corresponding y-values, and plot the points on the graph. Use the ruler to connect the points and make a line. Then provide students the exponential equation $y = 2^x$. Ask students to create an x-y table with the same x-coordinates as for the linear equation, find the corresponding y-values, and plot the points on the same graph as the linear equation. Ask students to connect the dots in order to make a smooth curve.

Suggestions for Discourse

- Present students with the equation $y = 3x + 1$. Ask the following questions:
  - "Can the type of function be determined without plotting any points and graphing? Explain."
  - "What is the first step in graphing this equation?"
  - "How does the shape of the graph determine whether an equation is linear or exponential?"
- Ask students, "Where have you heard the word *slope* before? What does *slope* mean?" Then ask students to discuss real-life examples of slope. Ask students to connect the types of slopes to different types of graphs of linear equations.

- Ask students, "How can the *x*- and *y*-intercepts be used to identify points on a graph?" Discuss how an equation given in slope-intercept form can be helpful for graphing an equation by providing two specific points, as opposed to creating random points in an *x*-*y* table.
- Create a game in which students have to classify parts of a graph. Provide cards that have questions such as, "In the equation, $y = -4x - 2$, which value is the slope? Which value is the *y*-intercept?"

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics I*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.
- Provide a list of sentence frames in which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "I know this is an exponential function because the shape of the graph is a _______." Or, "This <u>slope</u> is <u>negative</u>, because there is a <u>negative sign</u> in front of the value for <u>*m*</u> in the linear equation."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- incorrectly plotting points

  Review how to plot ordered pairs and remind students that the *x*-value is always listed first, and then the *y*-value (in other words, *x* and *y* in alphabetical order). Provide students extra practice by giving them a blank graph and a list of 10 points to graph.

- mistaking the *y*-intercept for the *x*-intercept, and vice versa

  Ask students to write on a flash card: "The *x*-intercept is $(x, 0)$, and the *y*-intercept is $(0, y)$." Remind them that the word *intercept* means to "cross over" or "intersect," so the *x*-intercept is the point where the graph intersects the *x*-axis, and the *y*-intercept is the point where the graph intersects the *y*-axis.

- confusing the value of a function for its corresponding *x*-coordinate

  Remind students that *x* is the independent variable and *y* is the dependent variable, so the *x*-value must be input into the equation, and the *y*-value (function value) is the result. Also remind students that the *x*-value comes first when graphing.

- being unable to identify key features of a linear model

  Have students create a word bank in which all key components and vocabulary words pertaining to a linear model are listed. Also, create a visual of a linear equation on a graph, then write the equation on the graph and label the graph's key features.

- not being able to identify key features of an exponential model

  Have students create a word bank in which all key components and vocabulary words pertaining to an exponential model are listed. Also, create a visual of an exponential equation on a graph, then write the equation on the graph and label the graph's key features.

Lesson 6: Comparing Functions

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*.

Skill 1: Determining the Slope of Linear Functions* (8.EE.5)

Common Core State Standard

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

Skill 2: Determining the Intercepts of Linear Functions (8.EE.6)

Common Core State Standard

8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Skill 3: Determining the Rate of Change of Exponential Functions** (F–IF.6★)

Common Core State Standard

F–IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Skill 4: Determining the Intercepts of Exponential Functions** (F–IF.4★)

Common Core State Standard

F–IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

Skill 5: Graphing Functions* (A–CED.2★)

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Instruction

Skill 1: Determining the Slope of Linear Functions*

Common Core State Standard

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 1, Lesson 3, Skill 1

SWB p. 139

Guided Practice Skill 1

Example 1

Quick Driver Rental Cars is running a monthly special. For each rental car the company offers, it charges a set fee of $25, then $0.20 per mile driven. Graph the line that represents the total cost for a minimum of 15 miles driven to a maximum of 75 miles driven. Determine the slope of this line, and then use this slope to find the number of miles the car was driven if the customer was charged $45. Assume there is no sales tax.

1. Identify the two quantities in the problem and create a table to show the relationship between them.

   The two quantities described are the number of miles driven and the total cost of the car rental.

   The total cost can be determined by multiplying the number of miles driven by 0.20, and then adding 25 to include the $25 rental charge.

   Choose values for the miles driven, between and including 15 and 75, and calculate the total cost. Let's use 15, 30, 45, 60, and 75.

   $0.20(15) + 25 = 28$

   $0.20(30) + 25 = 31$

   $0.20(45) + 25 = 34$

   $0.20(60) + 25 = 37$

   $0.20(75) + 25 = 40$

   Next, organize the results in a table.

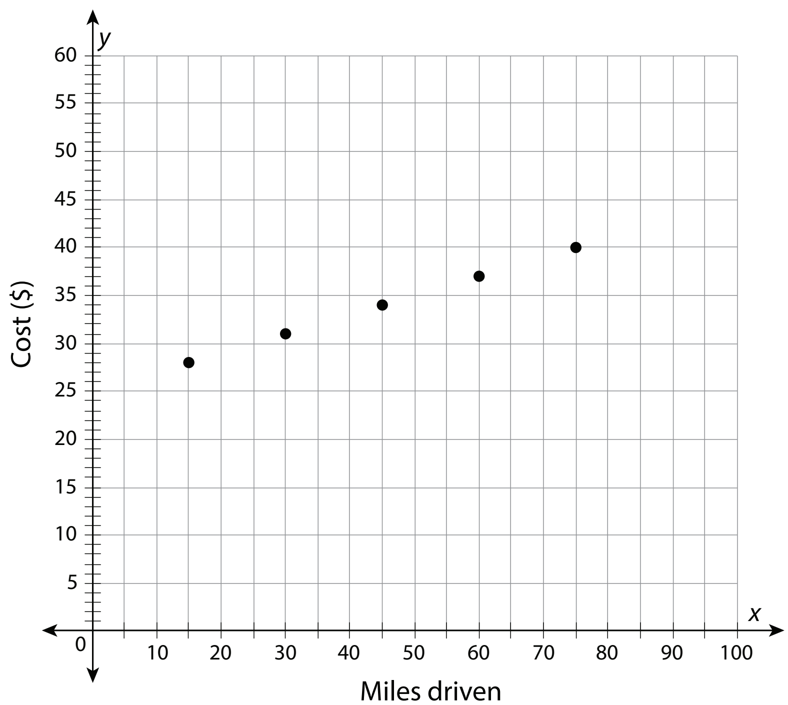
Miles driven	Cost ($)
15	28
30	31
45	34
60	37
75	40

2. Graph the relationship.

   Use the table of values to graph the relationship.

   Because the total cost depends on the number of miles driven, the miles driven will be the independent variable, *x*, and the total cost will be the dependent variable, *y*.

   Let *x* represent the number of miles driven and *y* represent the total cost.



3. Determine the slope and what it means in the context of the problem.

   Determine the slope by using the slope formula, $\text{slope}=\dfrac{y_2-y_1}{x_2-x_1}$. Any two points on the graph can be chosen to substitute into the slope formula. For example, let (x_1, y_1) be (15, 28) and (x_2, y_2) be (30, 31). Substitute these values into the slope formula, and then simplify.

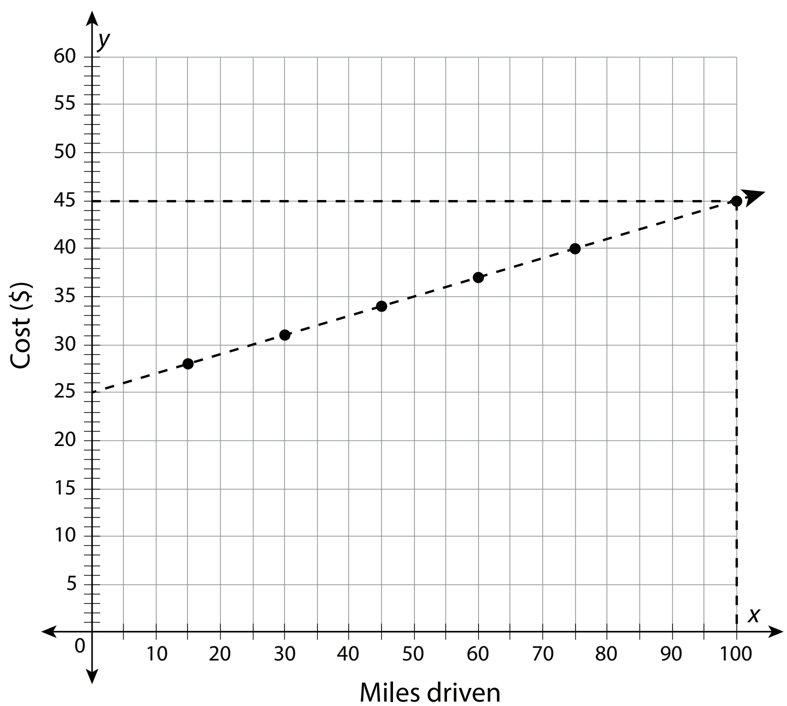
$\text{slope}=\dfrac{y_2-y_1}{x_2-x_1}$	Slope formula
$\text{slope}=\dfrac{(31)-(28)}{(30)-(15)}$	Substitute 31 for y_2, 28 for y_1, 30 for x_2, and 15 for x_1.
$\text{slope}=\dfrac{3}{15}$	Subtract.
slope = 0.20	Simplify.

   The slope is 0.20, or $0.20 per mile driven. This verifies the given information in the problem that the rental company charges $0.20 per mile driven.

4. Use the slope to determine the number of miles the car was driven if the customer was charged $45.

   The number of miles driven for $45 can be estimated from the graphed values.

   Draw a line connecting the points, extending the line out on the right. Find 45 on the *y*-axis, then locate the corresponding *x*-coordinate.



   From the graph, it appears a total cost of $45 means the car was driven about 100 miles.

   To verify this estimated value, the exact number of miles can be found by using the equation that represents this scenario: $0.20x + 25 = 45$. Solving for x yields 100, which is the same as the value found by estimating from the graph. In the context of this problem, this means that if a customer was charged $45, that person must have driven the car a total of 100 miles.

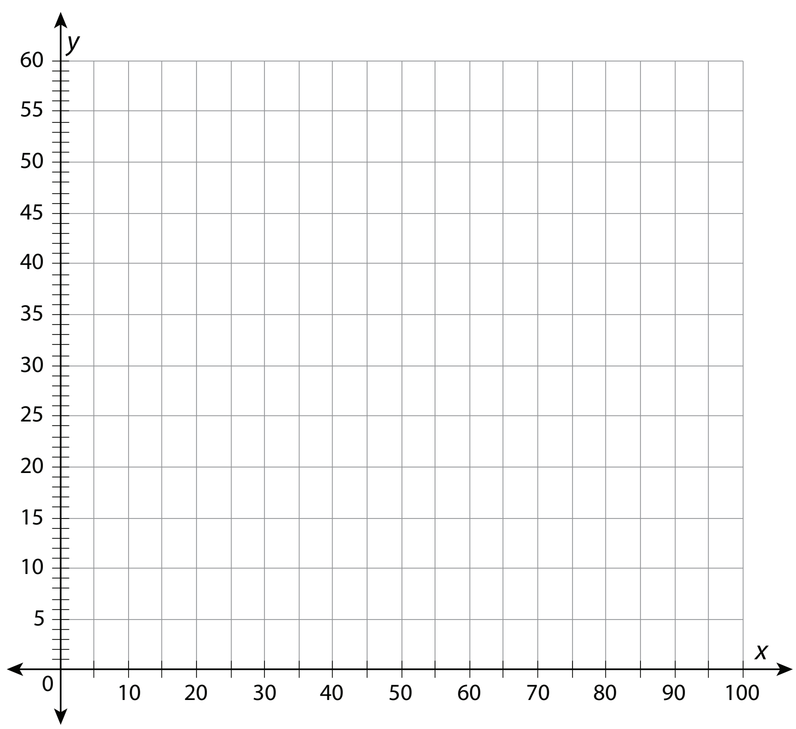
Scaffolded Practice Skill 1

Example 1

Quick Driver Rental Cars is running a monthly special. For each rental car the company offers, it charges a set fee of $25, then $0.20 per mile driven. Graph the line that represents the total cost for a minimum of 15 miles driven to a maximum of 75 miles driven. Determine the slope of this line, and then use this slope to find the number of miles the car was driven if the customer was charged $45. Assume there is no sales tax.

1. Identify the two quantities in the problem and create a table to show the relationship between them.

2. Graph the relationship.



continued

3. Determine the slope and what it means in the context of the problem.

4. Use the slope to determine the number of miles the car was driven if the customer was charged $45.

Practice Skill 1: Determining the Slope of Linear Functions*

For problems 1 and 2, determine the slope of the line that passes through the given points.

1. (3, –5) and (6, 1)

2. (–4, 2) and (–1, 0)

For problem 3, graph the relationship between the given quantities, then use the slope of the line to answer the question.

3. Kutter ordered the same gift for several of his friends at an online store. The gift cost $4, and he paid a flat fee of $9.99 for shipping. Graph the relationship between the number of gifts and the total cost. Then determine how many gifts he bought if his total cost was $49.99. Assume there is no sales tax.

SWB p. 143

Instruction

Skill 2: Determining the Intercepts of Linear Functions

SMP	
1 ✓	2 ✓
3 ✓	4 ✓
5 ✓	6 ✓
7	8

Common Core State Standard

8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Essential Questions

1. How can the equation of a line in standard form or point-slope form be converted into slope-intercept form?
2. What are some ways in which the x- or y-intercept of a line can be found?

WORDS TO KNOW

linear function a function that can be written in the form $y = mx + b$, in which m is the slope, b is the y-intercept, and the graph is a straight line

point-slope form the form $y - y_1 = m(x - x_1)$, where m is the slope, and (x_1, y_1) is a point on the line

slope the measure of the rate of change of one variable with respect to another variable; $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$; the slope in the equation $y = mx + b$ is m

slope-intercept form the form $y = mx + b$, where m is the slope and b is the y-intercept

standard form the form $ax + by = c$, where a, b, and c are integers, and a is positive

***x*-intercept** the point at which the line intersects the x-axis at $(x, 0)$

***y*-intercept** the point at which the line intersects the y-axis at $(0, y)$

SWB p. 147

Recommended Resources

- Khan Academy. "Multiple Examples of Constructing Linear Equations in Slope-Intercept Form."

  http://www.walch.com/rr/04050

  This video gives multiple examples of constructing linear equations in slope-intercept form. It shows how to write the slope-intercept form of a linear equation given the slope and *y*-intercept of a line, the slope and a point on a line, two points on a line, and the graph of a line. It ends by relating the slope-intercept form of a linear equation to function notation.

- Texas Instruments. "Finding the *x*-Intercepts of a Function Using the TI-83 Family, TI-84 Plus Family, and TI-Nspire Handheld in TI-84 Plus Mode."

  http://www.walch.com/rr/04051

  The article gives step-by-step instructions on how to find the *x*-intercept of a linear equation by using the TI-83/84 graphing calculator. It includes the specific example of finding the *x*-intercept of the line $y = 2x - 7$. It also gives screenshots so that users can follow along on their own calculators.

Instruction

Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

- Once students have worked through the Guided Practice, distribute the Coordinate Plane graphic organizer found in the Program Overview. Ask students to draw the line that passes through the points (2, 0) and (8, –7). Then have them draw a triangle with the vertices *A* (8, –7), *B* (8, 0), and *C* (2, 0). Then have them draw a similar triangle with the vertices *C* (2, 0) and *D* (0, 0), and the vertex *E* at the *y*-intercept of the line. Finally, have them set up a proportion based on the two similar triangles and then solve it to find the exact value of the line's *y*-intercept. The proportion would be $\frac{7}{6}=\frac{ED}{2}$, and solving the proportion for *ED* would give a *y*-intercept of $\frac{7}{3}$.
- Demonstrate to students how to enter and graph a linear function and how to access the zero feature on the TI-83/84 or TI-Nspire graphing calculator, or walk them through the appropriate steps. Once the zero feature is selected and the graph is displayed again, ask the students what information they think the calculator is requesting.
    - For example, on the TI-83/84 graphing calculator, ask them what they think "Left Bound," "Right Bound," and "Guess" mean. Have the calculator find the *x*-intercept of the line, then go back and access the zero feature again, this time moving the cursor to different positions for Left Bound, Right Bound, and Guess. Check whether the calculator gives the same *x*-intercept as before and repeat the process, seeing if there are any scenarios that do not give the correct *x*-intercept, such as choosing a Left Bound that is to the right of the Right Bound.

Suggestions for Discourse

- Ask students, "In your own words, what does *slope* mean in terms of real-world contexts? Where would you see examples of lines that have slope?" Ask students to list examples and/or describe scenarios of slope in real-world contexts. Ask for volunteers to share their responses, then discuss and create a master list of the examples.

- Provide students with an example of a table of values that represents the amount of money a person can earn if they mow lawns at $15 per lawn. (Possible table values: (0, 0), (1, 15), (2, 30), (3, 45), (4, 60), and (5, 75).) Ask the students to work with a partner to graph the values, and then answer the following questions, having volunteers share their responses at the end:
    - What is the slope of this graph?
    - What are the *x*- and *y*-intercepts?
    - How much money could a person earn by mowing 8 lawns?
    - How many lawns would a person have to mow if they wanted to earn $180?
- Ask students to work with a partner. Ask each person to create a linear equation in slope-intercept form, and then give the equation to his or her partner. Ask each student to identify the slope, *x*-intercept, and *y*-intercept for the equation, and also explain how to find each.
- Ask students, "What words in a word problem would help you identify a value as a rate of change?" Ask students to volunteer their responses, which should include the following: *per, each, every, rate*.

Making Connections

Point out that the intercepts of linear functions are often compared, but usually are not explicitly given. Often, to compare the *x*- or *y*-intercepts of two or more linear functions, it is first necessary to find the intercepts by converting the functions' equations to slope-intercept form, substituting 0 into the functions for *x* or *y*, graphing the functions, or using the zero feature on a graphing calculator. Once the intercepts of the functions are found, it is possible to determine if the *x*- or *y*-intercept of one function is less than, equal to, or greater than that of another.

Skill 2: Determining the Intercepts of Linear Functions

Introduction

A linear function, which is a function that has a graph that is a straight line, has certain properties. Two such properties are the location where the graph of the function crosses the *x*-axis, and the location where the graph of the function crosses the *y*-axis. Knowing these locations can help determine important information in real-world contexts. For example, suppose that the equation $y = -275x + 825$ models the depth, *y*, in feet, of a submarine *x* minutes after it begins its rise to the surface. Finding where the function crosses the *x*-axis will tell us the number of minutes the submarine must rise before it reaches the surface of the water.

Key Concepts

Linear Functions

- **Linear functions** form a straight line when graphed, and can be written in the form $y = mx + b$, in which *m* is the slope and *b* is the *y*-intercept.
- **Slope** is the measure of the rate of change of one variable with respect to another variable. The formula for finding slope is $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$.
- Two important properties of linear functions are the *x*- and *y*-intercepts.
- The ***x*-intercept** is the point at which the line intersects the *x*-axis at (*x*, 0).
- The ***y*-intercept** is the point at which the line intersects the *y*-axis at (0, *y*).

Various Forms of Linear Functions

- One way to find the slope and *y*-intercept of a linear function is to convert its equation into **slope-intercept form**, or the form $y = mx + b$, where *m* is the slope and *b* is the *y*-intercept.
- A common method of finding the *x*-intercept of a linear function is to substitute 0 for *y* in the equation of the function, and then solve for *x*.
- The equation of a linear function is sometimes given in **standard form**, or the form $ax + by = c$, where *a*, *b*, and *c* are integers, and *a* is positive. To convert standard form into slope-intercept form, solve the equation for *y*.
- When a linear function is graphed in a coordinate plane, it is possible to find the exact value of one of its intercepts by using similar triangles to write and solve a proportion.
- If two points that a line passes through are known, these two points can be used to find the equation of the line.
- The equation of a linear function is sometimes given in **point-slope form**, or the form $y - y_1 = m(x - x_1)$, where *m* is the slope and (x_1, y_1) is a point on the line. To convert point-slope form into slope-intercept form, solve for *y*.

Guided Practice Skill 2

Example 1

Given the linear function $y=\frac{1}{3}x-\frac{2}{3}$, determine the slope and *y*-intercept of the line.

1. Find the values of *m* and *b*.

   The equation $y=\frac{1}{3}x-\frac{2}{3}$ is in slope-intercept form, or the form $y = mx + b$, where *m* is the coefficient of *x*, and *b* is a constant.

   In this equation, $m=\frac{1}{3}$ and $b=-\frac{2}{3}$.

2. Determine the slope of the line.

   For an equation in slope-intercept form, *m* is the slope, and *b* is the *y*-intercept. Therefore, because $m=\frac{1}{3}$ in the equation $y=\frac{1}{3}x-\frac{2}{3}$, $\frac{1}{3}$ is the slope of the line.

3. Determine the *y*-intercept of the line.

   For an equation in slope-intercept form, *m* is the slope, and *b* is the *y*-intercept. Therefore, because $b=-\frac{2}{3}$ in the equation $y=\frac{1}{3}x-\frac{2}{3}$, $-\frac{2}{3}$ is the *y*-intercept of the line.

Example 2

Given the linear function $y = -4x + \frac{1}{2}$, determine the x-intercept of the line.

1. Substitute 0 for y.

   Because the x-intercept of a line is on the x-axis of a coordinate plane, the y-coordinate of the point where it occurs is 0.

   Therefore, to find the x-intercept of the line $y = -4x + \frac{1}{2}$, substitute 0 for y in the equation to get $0 = -4x + \frac{1}{2}$.

2. Solve the equation for x.

   Solve the equation $0 = -4x + \frac{1}{2}$ for x to determine the x-coordinate of the x-intercept.

$0 = -4x + \frac{1}{2}$	Equation from the previous step
$0 - \frac{1}{2} = -4x + \frac{1}{2} - \frac{1}{2}$	Subtract $\frac{1}{2}$ from both sides.
$-\frac{1}{2} = -4x$	Simplify.
$\frac{-\frac{1}{2}}{-4} = \frac{-4x}{-4}$	Divide both sides by –4.
$\frac{1}{8} = x$	Simplify.

   The value of x is $\frac{1}{8}$.

3. Determine the x-intercept of the line.

The x-intercept is the point at which the line intersects the x-axis at $(x, 0)$.

The line passes through the point $\left(\frac{1}{8}, 0\right)$, so the x-intercept of the line is $\frac{1}{8}$.

Example 3

A customer at a copy shop has $6.00 remaining on a prepaid card. Black-and-white copies cost $0.12 each, and color copies cost $0.20 each. The equation $12x + 20y = 600$ models this situation, where x is the number of black-and-white copies and y is the number of color copies the customer can make by using the card. What is the maximum number of color copies the customer can make?

1. Convert the equation into slope-intercept form.

The equation is in standard form, $ax + by = c$, where a, b, and c are integers, and a is positive.

To convert it into slope-intercept form, $y = mx + b$, solve for y.

$12x + 20y = 600$	Given equation
$12x + 20y - 12x = 600 - 12x$	Subtract $12x$ from both sides.
$20y = 600 - 12x$	Simplify.
$\frac{20y}{20} = \frac{600 - 12x}{20}$	Divide both sides by 20.
$y = 30 - 0.6x$	Simplify.
$y = -0.6x + 30$	Rewrite in slope-intercept form.

The equation $y = -0.6x + 30$ is now in slope-intercept form.

2. Find the values of m and b.

The equation $y = -0.6x + 30$ is in slope-intercept form, or the form $y = mx + b$, where m is the coefficient of x, and b is a constant.

In the equation, $m = -0.6$ and $b = 30$.

3. Determine the y-intercept of the line represented by the equation.

   For an equation in slope-intercept form, m is the slope, and b is the y-intercept. Therefore, because $b = 30$ in the equation, 30 is the y-intercept of the line.

4. Determine the maximum number of color copies the customer can make.

   The customer can make the maximum number of color copies when the number of black-and-white copies made is 0. The variable x represents the number of black-and-white copies made, so the value that represents the maximum number of color copies is the y-intercept of the equation, because it occurs at $x = 0$.

   Substitute 0 for x in the original equation and solve for y.

$12x + 20y = 600$	Original equation
$12(0) + 20y = 600$	Substitute 0 for x.
$20y = 600$	Multiply.
$y = 30$	Divide both sides by 20.

   The maximum number of color copies the customer can make is 30.

Scaffolded Practice Skill 2

Example 1

Given the linear function $y=\frac{1}{3}x-\frac{2}{3}$, determine the slope and *y*-intercept of the line.

1. Find the values of *m* and *b*.

2. Determine the slope of the line.

3. Determine the *y*-intercept of the line.

continued

Example 2

Given the linear function $y = -4x + \frac{1}{2}$, determine the *x*-intercept of the line.

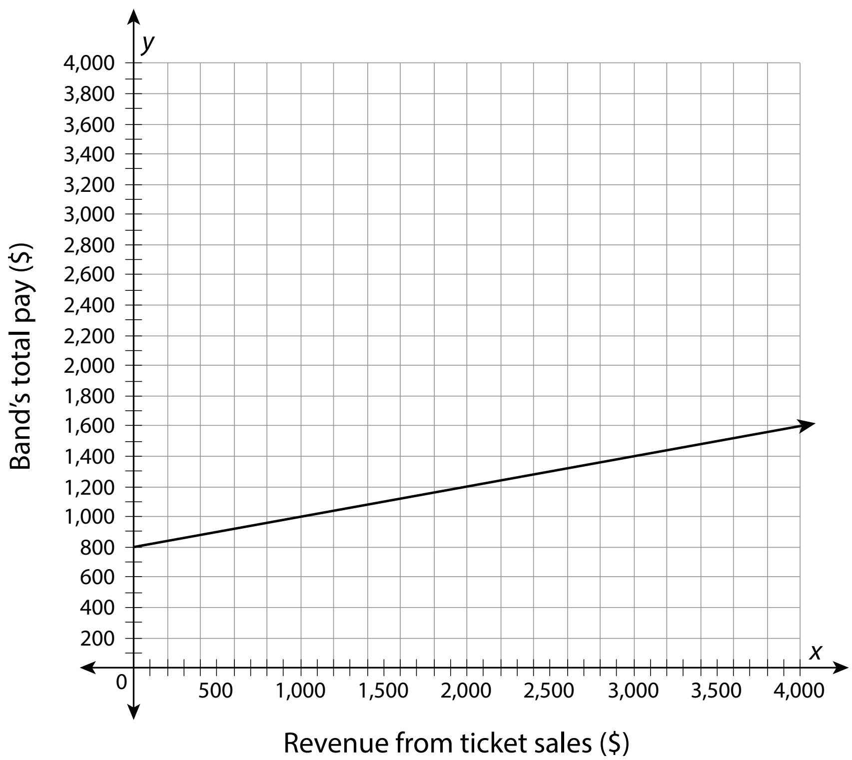
Example 3

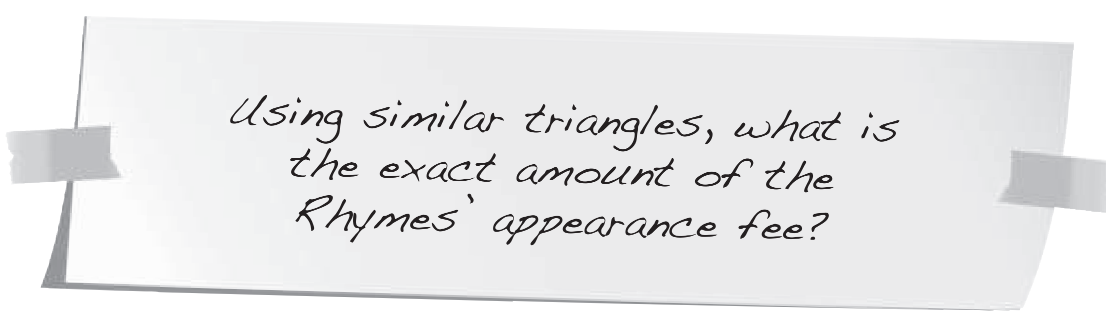
A customer at a copy shop has $6.00 remaining on a prepaid card. Black-and-white copies cost $0.12 each, and color copies cost $0.20 each. The equation $12x + 20y = 600$ models this situation, where *x* is the number of black-and-white copies and *y* is the number of color copies the customer can make by using the card. What is the maximum number of color copies the customer can make?

Problem-Based Task Skill 2: Rhyme Time

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 ✓ 6 ✓
7 8

A new hip-hop band, the Lilac Rhymes, just booked its first concert. The Rhymes will be paid an appearance fee by the promoter, plus a percentage of the revenue from all tickets sold. The following graph shows how much the Rhymes can expect to be paid based on the amount of ticket revenue generated. Using similar triangles, what is the exact amount of the Rhymes' appearance fee?





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Problem-Based Task Skill 2: Rhyme Time

Coaching

a. Suppose a triangle were drawn with the vertices *A* (3,500, 1,500), *B* (3,500, 1,000), and *C* (1,000, 1,000). What would be the length of *AB*?

b. Using the same triangle, what would be the length of *BC*?

c. Suppose a similar triangle were drawn with vertex *E* at the *y*-intercept of the line, and the vertices *D* (0, 1,000) and *C* (1,000, 1,000). What would be the length of *DC*?

d. What proportion could be set up to solve for *ED*?

e. What would be the length of *ED*?

f. What is the exact amount of the Rhymes' appearance fee?

Problem-Based Task Skill 2: Rhyme Time

Coaching Sample Responses

a. Suppose a triangle were drawn with the vertices *A* (3,500, 1,500), *B* (3,500, 1,000), and *C* (1,000, 1,000). What would be the length of *AB*?

To find the length of *AB*, subtract the *y*-coordinate of vertex *B* (1,000) from the *y*-coordinate of vertex *A* (1,500).

$1500 - 1000 = 500$

The length of *AB* would be 500.

b. Using the same triangle, what would be the length of *BC*?

To find the length of *BC*, subtract the *x*-coordinate of vertex *C* (1,000) from the *x*-coordinate of vertex *B* (3,500).

$3500 - 1000 = 2500$

The length of *BC* would be 2,500.

c. Suppose a similar triangle were drawn with vertex *E* at the *y*-intercept of the line, and the vertices *D* (0, 1,000) and *C* (1,000, 1,000). What would be the length of *DC*?

To find *DC*, subtract the *x*-coordinate of vertex *D* (0) from the *x*-coordinate of vertex *C* (1,000).

$1000 - 0 = 1000$

The length of *DC* would be 1,000.

d. What proportion could be set up to solve for *ED*?

In similar triangles *ABC* and *EDC*, side *AB* corresponds to side *ED*, and side *BC* corresponds to side *DC*. Therefore, because *AB* = 500, *BC* = 2500, and *DC* = 1000, you can set up the proportion $\frac{500}{2500}=\frac{ED}{1000}$ to solve for *ED*.

e. What would be the length of *ED*?

Solve the proportion for the length of *ED*.

$$\frac{500}{2500}=\frac{ED}{1000}$$

$$2500 \bullet ED = 500 \bullet 1000$$

$$2500 \bullet ED = 500{,}000$$

$$\frac{2500 \bullet ED}{2500}=\frac{500{,}000}{2500}$$

$$ED = 200$$

The length of *ED* would be 200.

f. What is the exact amount of the Rhymes' appearance fee?

To find the exact amount of the Rhymes' appearance fee, determine the *y*-intercept of the line. Because *ED* = 200, and because the coordinates of point *D* are (0, 1,000), the coordinates of point *E* must be (0, (1000 – 200)), or (0, 800), so the *y*-intercept is 800. Therefore, the fee is $800.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 2: Determining the Intercepts of Linear Functions

For problems 1–9, find the requested intercept.

1. the y-intercept of $y = 13x - 3$

2. the x-intercept of $y = 8x + 2$

3. the y-intercept of $y = -5x + 11$

4. the x-intercept of $y = 6x - 1$

5. the y-intercept of $y = \frac{2}{3}x - 9$

6. the x-intercept of $y = -12x + 4$

7. the y-intercept of $y = 7x - 8$

8. the x-intercept of $y = -3x - 15$

9. the y-intercept of $y = 10x - \frac{1}{2}$

For problem 10, read the scenario and use the information in it to answer the questions.

10. The equation $y = -15x + 180$ models the number of gallons of water, y, in a reef tank x minutes after it has started being drained. What intercept of the graph gives the number of minutes it will take the reef tank to drain? According to the intercept, how many minutes will it take to empty the reef tank?

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Instruction

Skill 3: Determining the Rate of Change of Exponential Functions**

Common Core State Standard

F–IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

This skill has been addressed previously in *CCSS Integrated Pathway: Mathematics I*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

Unit 2, Lesson 4, Sub-lesson 2

Instruction

Skill 4: Determining the Intercepts of Exponential Functions**

Common Core State Standard

F–IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

This skill has been addressed previously in *CCSS Integrated Pathway: Mathematics I*. Refer to the following sub-lesson(s) to find Essential Questions, Words to Know, Recommended Resources, and Key Concepts for this skill.

Unit 2, Lesson 4, Sub-lesson 1

Skill 5: Graphing Functions*

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 2

Guided Practice Skill 5

Example 1

Sergio just got a new job that pays \$9.15 an hour plus 3% commission on all his sales. If Sergio works 40 hours per week, write an equation that represents what his total wages will be depending on how much he sells. Then create a graph of the equation.

1. Determine how much Sergio makes each week before his commission.

   Sergio works 40 hours a week and is paid \$9.15 an hour.

   Multiply 40 by 9.15 to determine how much he makes each week, not counting commission.

   $40 \bullet 9.15 = 366$

   Sergio makes \$366 a week before commission.

2. Write an equation that represents Sergio's total wages depending on how much he sells.

   Because Sergio's total wages depend on his sales, his sales will be the independent variable, x, and his total wages will be the dependent variable, y.

   Sergio receives a 3% commission, which means that 3% of his total sales will be added to his paycheck. Therefore, his commission will be 3% times his sales. Recall that 3% is equal to 0.03, and his sales are represented by x.

   Therefore, Sergio's commission can be written as $0.03x$.

   His total wages, y, will equal his total hourly pay, \$366, plus his commission, $0.03x$.

   $y = 366 + 0.03x$

   The equation $y = 366 + 0.03x$ represents Sergio's total wages depending on his sales.

3. Create a table of values for this equation.

To graph the equation, first create a table of values by substituting values for x and solving for y.

Substitute 0 for x and solve for y.

$y = 366 + 0.03x$	Equation
$y = 366 + 0.03(0)$	Substitute 0 for x.
$y = 366 + 0$	Multiply.
$y = 366$	Simplify.

If Sergio makes \$0 in sales, his total wages will be \$366.

Substitute 100 for x and solve for y.

$y = 366 + 0.03x$	Equation
$y = 366 + 0.03(100)$	Substitute 100 for x.
$y = 366 + 3$	Multiply.
$y = 369$	Simplify.

If Sergio makes \$100 in sales, his total wages will be \$369.

Substitute 500 for x and solve for y.

$y = 366 + 0.03x$	Equation
$y = 366 + 0.03(500)$	Substitute 500 for x.
$y = 366 + 15$	Multiply.
$y = 381$	Simplify.

If Sergio makes \$500 in sales, his total wages will be \$381.

Arrange this information into a table of values.

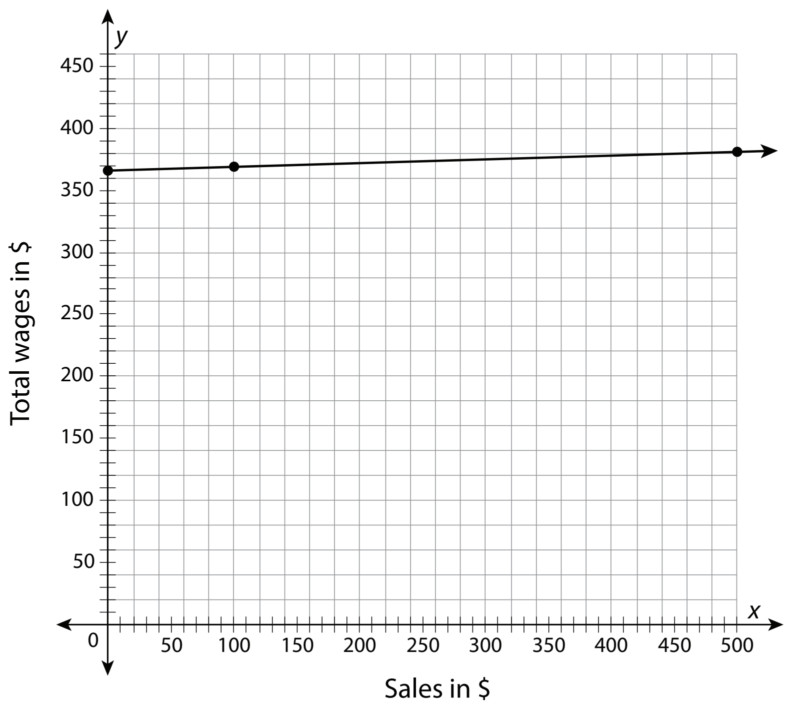
Sales in \$ (x)	Total wages in \$ (y)
0	366
100	369
500	381

4. Use the table of values to create a graph of the equation.

   The ordered pairs from the table are (0, 366), (100, 369), and (500, 381).

   Plot these ordered pairs on a coordinate plane, and connect them with a smooth line.

   Label the *x*-axis "Sales in \$" and the *y*-axis "Total wages in \$."



   Because it is impossible to have sales less than \$0, the line does not exist when x is less than 0.

Scaffolded Practice Skill 5

Example 1

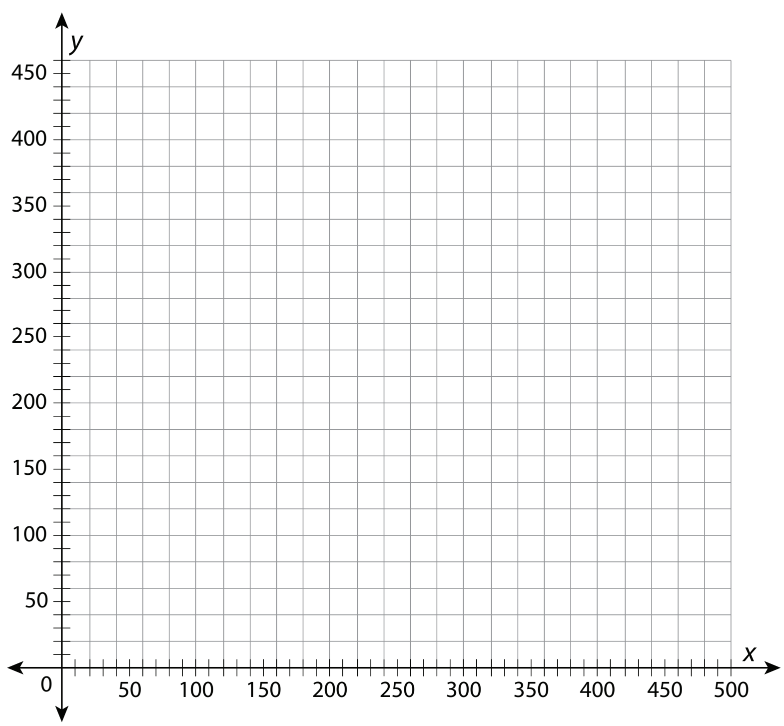
Sergio just got a new job that pays $9.15 an hour plus 3% commission on all his sales. If Sergio works 40 hours per week, write an equation that represents what his total wages will be depending on how much he sells. Then create a graph of the equation.

1. Determine how much Sergio makes each week before his commission.

2. Write an equation that represents Sergio's total wages depending on how much he sells.

3. Create a table of values for this equation.

4. Use the table of values to create a graph of the equation.



Practice Skill 5: Graphing Functions*

Use the information that follows to complete problems 1–3.

Vanessa is baking cookies for her class bake sale. Her recipe, which makes 2 dozen cookies, calls for 1 cup of chocolate chips, 2 cups of flour, and $1\frac{1}{2}$ cups of sugar.

1. The equation $y=\frac{1}{2}x$ represents the number of cups of chocolate chips Vanessa needs depending on how many dozen cookies she plans to make. Create a graph that represents this equation.

2. Write and graph an equation that represents how many cups of sugar she needs depending on how many dozen cookies she plans to make.

3. Write and graph the equation that represents the total amount of flour she needs depending on how many dozen cookies she plans to make if she also needs an additional 3 cups of flour to make a cake.

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Instruction

Supportive Instructional Strategies for Mathematics I

Unit 2 Lesson 6

Suggestions for Graphic Organizers/Manipulatives

- Once students have worked through the Guided Practice, distribute the Coordinate Plane graphic organizer found in the Program Overview. At the top of the page, ask students to write the linear function $f(x)=\frac{4}{5}x+2$. Then, in the coordinate plane, have them plot the points (–8, –3) and (8, 9), and draw a line through the points. Ask them which linear function—the one given by the equation or the one graphed—has the greater rate of change, and which has the greater *y*-intercept. Because the graphed linear function has a slope of $\frac{3}{4}$ and a *y*-intercept of 3, the linear function given by the equation has the greater rate of change, and the graphed linear function has the greater *y*-intercept.
- Provide students with a two-circle Venn diagram. Ask students to label the circle on the left "Linear function" and label the circle on the right "Exponential function." Ask them to create a specific example of each type of function. Then, ask them to list specific characteristics of each type of function in the appropriate circles, and list characteristics that both types of functions have in common in the middle of the Venn diagram. Ask volunteers to share answers for the three parts of the diagram, and create a master Venn diagram so all answers can be compiled into one organizer.
    - Possible linear function characteristics: *has a constant rate of change, the graph is a line*.
    - Possible exponential function characteristics: *the rate of change varies depending on the interval observed, the graph is a curve*.
    - Possible shared characteristics: *both have an independent and a dependent variable, a table of values can be created, the x- and y-intercepts can be found, the rate of change can be found*.

Suggestions for Discourse

- Write the equations of some linear functions on slips of paper and place them in a bowl. Then have students form groups, and show the class the graph of a linear function. Have each group draw one slip of paper, and ask the groups to decide if the rate of change and *y*-intercept of the linear function they drew are less than or greater than those of the linear function represented by the graph. Ask the groups to explain their answers to the rest of the class.

- Next, write the equations of some exponential functions on slips of paper and place them in the bowl. Show the entire class a table of values representing an exponential function, and have each group draw one of the slips of paper. Ask the groups to decide if the average rate of change over a specified interval and the *y*-intercept of the exponential function they drew are less than or greater than those of the exponential function represented by the table. Again, ask the groups to explain their answers to the rest of the class.
- Have students work together with a partner to come up with a list of real-life examples in which linear functions would apply, then a list of real-life examples in which exponential functions would apply. Ask them to think about ways in which each type of function increases or decreases, and how this would apply to the type of graph for both functions. Then ask for volunteers to share their ideas, and make a master list compiling all valid examples for each type of function.
- Ask students, "What are four different ways in which linear or exponential functions can be represented?" Guide students in a discussion about functions being represented by equations, tables, graphs, and verbal descriptions.

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics I*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.
- Elaborate on culture-specific contexts.
- Provide a list of sentence frames in which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "The reason I know this is an exponential function is because the shape of the graph is a _______." Or, "In the equation $y = 5x + 4$, the 5 is the _______, and the 4 is the _______." Or, "This function is <u>linear</u> because its graph is in the shape of a <u>line</u>."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- incorrectly determining the rate of change

  Remind students that *rate of change* means the vertical change is divided by the horizontal change, and that a linear function's rate of change will always be the same, while an exponential function's rate of change will vary.

- assuming that a positive slope will be steeper than a negative slope

  Explain to students that the sign of a linear function's slope indicates whether its graph rises or falls from left to right, but that the sign has nothing to do with the rate at which the graph rises or falls.

- interchanging the *x*- and *y*-intercepts

  Remind students that because the *x*-intercept of a function occurs at a point on the *x*-axis, the *y*-coordinate of the point where it occurs will always be 0. Likewise, because the *y*-intercept of a function occurs at a point on the *y*-axis, the *x*-coordinate of the point where it occurs will always be 0.

Lesson 7: Building Functions

Instruction

Elementary Prerequisite Skills

This lesson requires the use of the following elementary skill(s) to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*. See the Appendix for material to address the skill(s).

- E-Skill 2: Understanding the Coordinate Plane (5.G.1), Appendix p. A-10

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*.

Skill 1: Evaluating Exponential Expressions* (8.EE.1)

Common Core State Standard

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Skill 2: Understanding Independent and Dependent Quantities (6.EE.9)

Common Core State Standard

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

Skill 1: Evaluating Exponential Expressions*

Common Core State Standard

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 3

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Guided Practice Skill 1

Example 1

Use properties of exponents to evaluate the exponential expression $3^{-5} \bullet 9$.

1. Rewrite the exponential expression to have a positive exponent.

   To rewrite a negative exponent as a positive exponent, take the reciprocal of the base with a positive exponent. For example, x^{-a} is written as $\frac{1}{x^a}$.

   $x^{-a} = \frac{1}{x^a}$ General form for rewriting a negative exponent

   $3^{-5} = \frac{1}{3^5}$ Rewrite 3^{-5} as the reciprocal of 3 with a positive exponent.

   3^{-5} is equal to the expression $\frac{1}{3^5}$.

2. Evaluate the result, then use it to rewrite the original exponential expression.

   The exponential expression 3^5 can be rewritten as $3 \bullet 3 \bullet 3 \bullet 3 \bullet 3$, which is equal to 243; therefore, $\frac{1}{3^5}$ is equal to $\frac{1}{243}$.

   The expression $3^{-5} + 9$ can be written as $\frac{1}{243} \bullet 9$.

3. Simplify the rewritten expression.

   To simplify the expression $\frac{1}{243} \bullet 9$, first multiply $\frac{1}{243}$ by 9.

   The product of $\frac{1}{243}$ and 9 is $\frac{9}{243}$.

   Both the numerator and denominator are divisible by 9, so the expression can be simplified further: $\frac{9}{243} = \frac{1}{27}$.

   Therefore, the expression $3^{-5} \bullet 9$ is equal to $\frac{1}{27}$.

Scaffolded Practice Skill 1

Example 1

Use properties of exponents to evaluate the exponential expression $3^{-5} \cdot 9$.

1. Rewrite the exponential expression to have a positive exponent.

2. Evaluate the result, then use it to rewrite the original exponential expression.

3. Simplify the rewritten expression.

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Practice Skill 1: Evaluating Exponential Expressions*

Evaluate each exponential expression.

1. 4^{-3}

2. $2 \bullet 10^{-3}$

3. When Kiel takes medication, the equation $y = 500(2^{-0.7x})$ tells how many milligrams of the medication are left in his body after x hours. How much of the medication will be left 4 hours after he takes it? Round to the nearest tenth of a milligram.

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Instruction

Skill 2: Understanding Independent and Dependent Quantities

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 ✓ 6 ✓
7 ✓ 8 ✓

Common Core State Standard

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Essential Questions

1. How can variables be used to represent two quantities in a real-world problem that change in relationship to one another?
2. How can equations, tables, and graphs be used to represent the relationship between independent and dependent variables?

WORDS TO KNOW

dependent variable labeled on the *y*-axis; the quantity that is based on the input values of the independent variable; the output variable of a function

equation a mathematical sentence that uses an equal sign (=) to show that two quantities are equal

independent variable labeled on the *x*-axis; the quantity that changes based on values chosen; the input variable of a function

variable a letter used to represent a value or unknown quantity that can change or vary

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Recommended Resources

- IXL Learning. “Write Linear Functions.”

  http://www.walch.com/rr/04052

  This site provides practice with writing linear functions given a problem scenario, and includes a table showing the relationship between independent and dependent variables. Immediate feedback is provided, with step-by-step explanations for finding the correct linear function when an incorrect answer is given.

- Khan Academy. “Dependent and Independent Variables.”

  http://www.walch.com/rr/04053

  This site provides practice understanding the difference between independent and dependent variables in real-world situations, along with how to represent each problem scenario using an equation, a table, and/or a graph.

Instruction

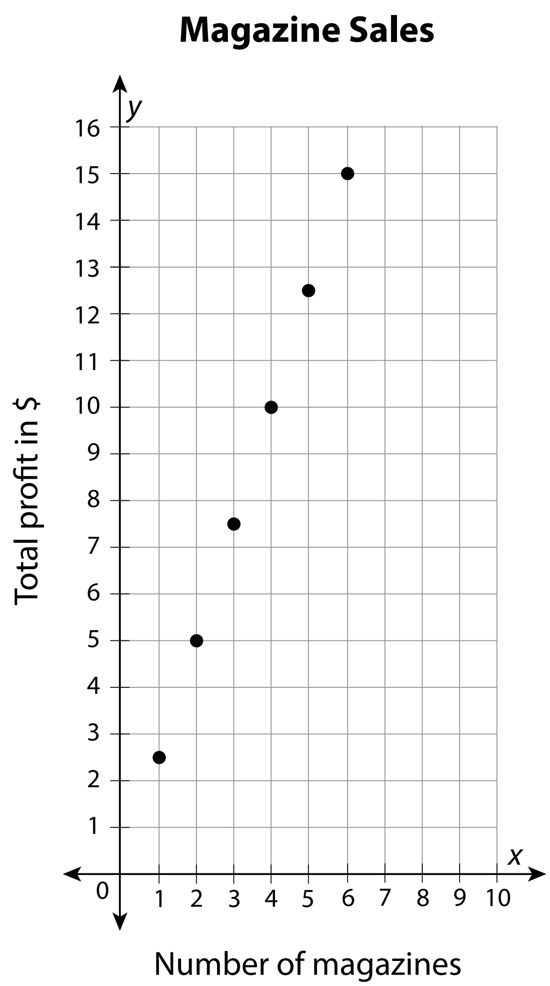
Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Once students have completed the Guided Practice, distribute the Line Graph graphic organizer. Write the following scenario on the board for the students:

> "A store sells magazines for $2.50 each. Explain the relationship between the number of magazines sold and the total profit with an equation, a table, and a graph."

First, discuss with the class how to tell which quantity is independent and which quantity is dependent. Then ask the students to choose a variable to represent each quantity. Let the students work on their own to represent the scenario with an equation, a table, and a graph. Remind the students to label the *x*-axis, the *y*-axis, and the title of the graph. The graph should look similar to the following:



Ask volunteers to share and explain their equation, which should be similar to $y = 2.5x$, their table of values, and their graph. Additional time and practice spent on identifying quantities and creating equations, tables, and graphs will enhance students' understanding of the relationships between independent and dependent quantities.

Suggestions for Discourse

Ask students to think about situations that have two quantities that change in relation to one another. Give students the example that if a person earns $8 per hour at a job, then the more hours that person works, the more money that person will earn. When comparing the number of hours worked to the total amount of money earned, discuss which quantity in the scenario is independent, which quantity is dependent, and why. Explain that the number of hours worked is the independent quantity because the number of hours worked can be any value greater than or equal to 0, but the amount of money earned is determined by multiplying $8 by the number of hours the person worked, so the amount of money earned is the dependent quantity.

Making Connections

Encourage students to connect the concept that an independent quantity in a scenario can be any reasonable value chosen for the scenario. For example, the number of movie tickets purchased must be a whole number since it is not possible to buy a negative number of tickets or a fraction of a ticket. The dependent quantity is then the result of a calculation involving the independent variable. For example, the total cost to see a movie is calculated by taking the cost per ticket and multiplying it by the number of movie tickets purchased. Therefore, the dependent quantity depends on a calculation performed with the independent quantity.

Skill 2: Understanding Independent and Dependent Quantities

Introduction

There are many situations in which two quantities being compared change in relationship to one another. For example, the distance you travel during a road trip varies over time. Or, the money a worker earns varies based on the number of hours worked. In these situations, the dependent quantity can be expressed in terms of an independent quantity. Understanding this relationship between the independent and dependent quantities, along with how to represent these situations with equations, tables, and graphs, is an important step in learning how to find solutions to real-world problems.

Key Concepts

- When two quantities in a problem scenario change in relationship to one another, one quantity, the **dependent variable**, is dependent upon the other quantity, the **independent variable**.
- In these situations, variables are used to represent the quantities being compared. **Variables** are letters used to represent values or unknown quantities that can change or vary in expressions or equations.
- The relationship between independent and dependent variables can be represented as equations, tables, and graphs.

Equations

- An **equation** is a mathematical sentence that uses an equal sign (=) to show that two quantities are equal.
- The independent variable in an equation can change, whereas the dependent variable is affected by changes made to the independent variable.
- For example, in the equation $y = 5x$, when $x = 1$, then $y = 5(1)$, which simplifies to $y = 5$. When $x = 2$, then $y = 5(2)$, which simplifies to $y = 10$. For this equation, as the value of x increases, the value of y also increases. However, the value of y depends on changes in the value of x, so y is the dependent variable and x is the independent variable.
- Other letters besides x and y can be used to represent the independent and dependent quantities in an equation to make it easier to associate each variable with a quantity.
- For example, if books cost $4 each and we want to find the total cost for a certain number of books, let c represent the total cost and let b represent the number of books. Since each book costs $4, the total cost will be equal to 4 times the number of books purchased, which can be written as the equation $c = 4b$.

Tables

- Tables comparing independent and dependent variables typically have two columns, with the independent quantity in the left column and the dependent quantity in the right column. The title of each column should describe the variable. For example, in the equation $c = 4b$, the independent quantity is the number of books (b), so a good title for the left column is "Number of books (b)." The dependent quantity is the total cost in dollars (c), so a good title for the right column is "Total cost in \$ (c)."

Number of books (b)	**Total cost in \$ (c)**

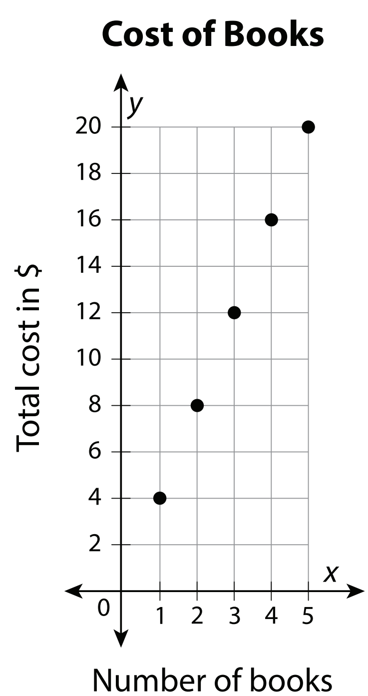
- To fill in each row of the table, start by choosing a reasonable value for the independent variable. For example, a reasonable number of books to start with is $b = 1$. Substitute 1 for b in the equation $c = 4b$ to calculate the corresponding value for c. In this case, $c = 4(1)$, which simplifies to $c = 4$. The first row of the table will then have 1 for b and 4 for c. For the second row, when $b = 2$, substitute 2 for b into the equation: $c = 4(2) = 8$. The second row of the table will then have 2 for b and 8 for c. Fill in the rest of the table following the same steps, using different values of b to calculate the corresponding value for c.

Number of books (b)	**Total cost in \$ (c)**
1	4
2	8
3	12
4	16
5	20

- This table of values can be used to create a graph for the equation.

Graphs

- In a graph, the independent quantity is represented along the *x*-axis and the dependent quantity is represented along the *y*-axis. The axes are labeled to describe each quantity being represented.
- For a problem scenario based on a real-world context, the graph is generally drawn in the first quadrant since the scenario more than likely deals with positive independent and dependent quantities. For example, it is impossible to buy a negative number of books, or spend a negative amount of money on the books.
- Each row in the table created for $c = 4b$ provides a coordinate point that can be graphed on the coordinate plane to represent the relationship. For example, the values in the first row, 1 and 4, can be written as the coordinate point (1, 4).
- When graphing a problem scenario for which it is possible to use fractions for the independent quantity, draw a line through the points. For example, it is possible to spend 2.5 hours on a task, so a graph with time as the independent variable would have a line through the points.
- If the only possible values for the independent quantity are whole numbers, such as with people or entire objects, the graph would only show the points, without a line through them. For example, the graph of the equation for the cost of books, $c = 4b$, does not have a line through the points because it is impossible to buy a fraction of a book. Notice that the points on this graph match the table values.



Guided Practice Skill 2

Example 1

On a road trip, Jameson drove his car at an average speed of 60 miles per hour. Explore the relationship between the total distance that Jameson traveled and the time that he spent driving. Identify the independent and dependent quantities, and choose variables to represent them. Then, create an equation, a table, and a graph for the situation.

1. Identify the independent and dependent quantities.

   Jameson drove his car at 60 miles per hour, which means that the total distance in miles that Jameson traveled depends on the time in hours that he spent driving.

   Therefore, the time is the independent quantity, and the distance is the dependent quantity.

2. Choose variables to represent the independent and dependent quantities.

   Since "time" starts with the letter "t," let the time be represented by the variable t.

   Since "distance" starts with the letter "d," let the distance be represented by the variable d.

3. Write an equation to represent the relationship between the independent and dependent variables.

   The distance in miles (d) that Jameson traveled is calculated by multiplying his rate (60 miles per hour) by the time in hours (t) that he spent driving.

   Therefore, the relationship can be represented by the equation $d = 60t$.

4. Create a table of values to represent the relationship between the independent and dependent variables.

Use the left column of the table for the values of the independent variable, the time in hours (t).

Use the right column of the table for the values of the dependent variable, the total distance in miles (d).

To fill in the table, choose values for t to substitute into the equation $d = 60t$. Let's start with 0.

For the first row of the table, let $t = 0$:

$d = 60t$

$d = 60(0)$

$d = 0$

So, the values in the first row will be 0 and 0.

For the second row of the table, let $t = 1$:

$d = 60t$

$d = 60(1)$

$d = 60$

The values in the second row will be 1 and 60.

Continue with other values of t, as shown.

Time in hours (t)	Total distance in miles (d)
0	0
1	60
2	120
3	180
4	240
5	300

Based on the table, as the time increases, the total distance also increases.

5. Use the table to create a graph that represents the relationship between the independent and dependent variables.

   The independent quantity, the time in hours (*t*), is plotted along the *x*-axis of the coordinate plane.

   The dependent quantity, the total distance in miles (*d*), is plotted along the *y*-axis.

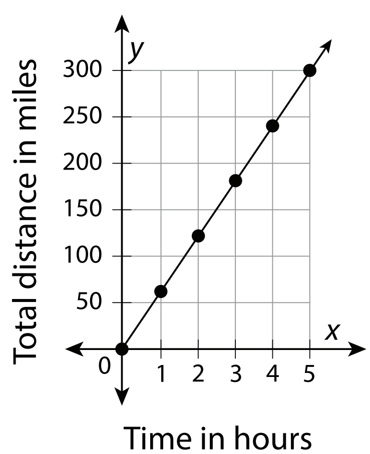
   Since it is not possible to have a negative amount of time, the *x*-axis starts at 0. Since it is not possible to travel a negative distance, the *y*-axis also starts at 0.

   Label the *x*-axis as "Time in hours" and label the *y*-axis as "Total distance in miles." The title of the graph can be "Distance Traveled over Time."

   Each row in the table of values provides a coordinate point that can be graphed on the coordinate plane to represent the relationship.

   Since Jameson drove at a constant rate of speed, and since it is possible for Jameson to drive for a fractional number of hours (such as 2.5 hours) instead of only a whole number of hours, draw a line through the coordinate points to show how the total distance traveled was constantly changing.

Distance Traveled over Time



   This graph shows that as the time increases, the total distance traveled also increases. This confirms the relationship shown in the table of values.

Example 2

At a grocery store, mini seedless watermelons cost \$3.50 each. Explore the relationship between the total cost to buy watermelons and the number of watermelons purchased. Identify the independent and dependent quantities, and choose variables to represent them. Then, create an equation, a table, and a graph to represent the situation.

1. Identify the independent and dependent quantities.

   Watermelons cost \$3.50 each, which means that the total cost to buy watermelons depends on the number of watermelons purchased.

   Therefore, the number of watermelons is the independent quantity, and the total cost is the dependent quantity.

2. Choose variables to represent the independent and dependent quantities.

   Since "watermelons" starts with the letter "w," let the number of watermelons purchased be represented by the variable w.

   Since "cost" starts with the letter "c," let the total cost be represented by the variable c.

3. Write an equation to represent the relationship between the independent and dependent variables.

   The total cost (c) is calculated by multiplying the cost per watermelon (\$3.50) by the number of watermelons purchased (w).

   Therefore, the relationship can be represented by the equation $c = 3.50w$.

4. Create a table of values to represent the relationship between the independent and dependent variables.

   Use the left column of the table for the values of the independent variable, the number of watermelons purchased (w).

   Use the right column of the table for the values of the dependent variable, the total cost (c).

   To fill in the table, choose values for w to substitute into the equation $c = 3.50w$. Let's start with 1.

   For the first row of the table, let $w = 1$:

   $c = 3.50w$

   $c = 3.50(1)$

   $c = 3.50$

   So, the values in the first row will be 1 and 3.50.

   For the second row, let $w = 2$:

   $c = 3.50w$

   $c = 3.50(2)$

   $c = 7.00$

   The values in the second row will be 2 and 7.00.

   Continue filling out the table with other values of w, as shown.

Number of watermelons (w)	Total cost in $ (c)
1	3.50
2	7.00
3	10.50
4	14.00
5	17.50

   Based on the table, as the number of watermelons increases, the total cost also increases.

Instruction

5. Use the table to create a graph that represents the relationship between the independent and dependent variables.

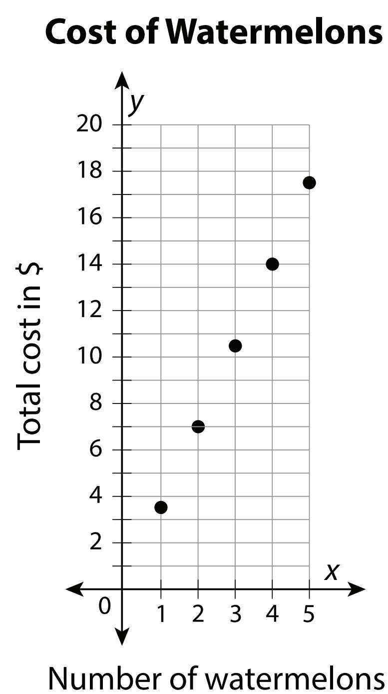
   The independent quantity, the number of watermelons purchased (*w*), is plotted along the *x*-axis of the coordinate plane. The dependent quantity, the total cost in dollars (*c*), is plotted along the *y*-axis.

   Since it is not possible to have a negative number of watermelons, the *x*-axis starts at 0. Since it is not possible to have a negative total cost, the *y*-axis also starts at 0.

   Label the *x*-axis as "Number of watermelons" and label the *y*-axis as "Total cost in $." The title of the graph can be "Cost of Watermelons."

   Each row in our table of values provides a coordinate point that can be graphed on the coordinate plane to represent the relationship.

   Since the problem scenario deals with whole watermelons, instead of fractions of watermelons, do not draw a line through the points.



   This graph shows that as the number of watermelons increases, the total cost also increases. This confirms the relationship shown in the table of values.

Example 3

A barrel that was filled with 40 gallons of water is leaking water at a constant rate of 5 gallons per hour. Explore the relationship between the total gallons of water left in the barrel and the number of hours the water has been leaking from the barrel. Identify the independent and dependent quantities, and choose variables to represent them. Then, create an equation, a table, and a graph to represent the situation.

1. Identify the independent and dependent quantities.

   The barrel is leaking water at a rate of 5 gallons per hour. This means that the total gallons of water left in the barrel depends on the number of hours that the water has been leaking.

   Therefore, the number of hours is the independent quantity, and the dependent quantity is the total gallons of water left in the barrel.

2. Choose variables to represent the independent and dependent quantities.

   Since "hours" starts with the letter "h," let the number of hours be represented by the variable h.

   Since "gallons" starts with the letter "g," let the total gallons of water left in the barrel be represented by the variable g.

3. Write an equation to represent the relationship between the independent and dependent variables.

   The total gallons of water left in the barrel (g) is calculated by multiplying the rate at which water is leaking (5 gallons per hour) by the number of hours (h) and then subtracting that value from the initial 40 gallons.

   Therefore, the relationship can be represented by the equation $g = 40 - 5h$.

4. Create a table of values to represent the relationship between the independent and dependent variables.

Use the left column of the table for the values of the independent variable, the number of hours (h).

Use the right column of the table for the values of the dependent variable, the total gallons of water in the barrel (g).

To fill in the table, choose values for h to substitute into the equation $g = 40 – 5h$. Let's start with 0.

For the first row of the table, let $h = 0$:

$g = 40 – 5h$

$g = 40 – 5(0)$

$g = 40 – 0 = 40$

So, the values in the first row will be 0 and 40.

For the second row of the table, let $h = 1$:

$g = 40 – 5h$

$g = 40 – 5(1)$

$g = 40 – 5 = 35$

The values in the second row will be 1 and 35.

Continue with other values of h, as shown.

Number of hours (h)	**Total gallons (g)**
0	40
1	35
2	30
3	25
4	20
5	15
6	10
7	5
8	0

Based on the table, as the number of hours increases, the total gallons of water in the barrel decreases.

5. Use the table to create a graph representing the relationship between the independent and dependent variables.

   The independent quantity, the number of hours (h), is plotted along the x-axis of the coordinate plane.

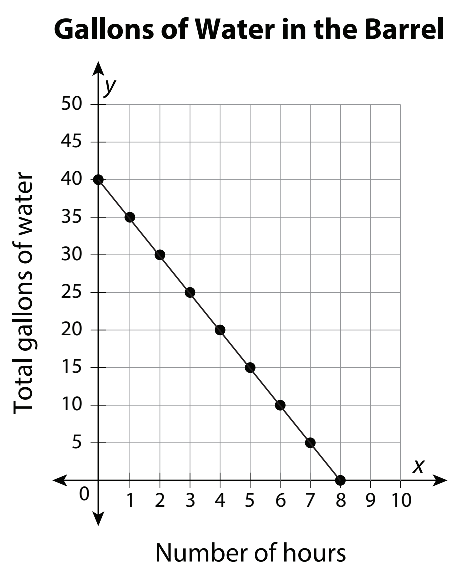
   The dependent quantity, the total gallons of water in the barrel (g), is plotted along the y-axis.

   Since it is not possible to have a negative number of hours, the x-axis starts at 0. Since it is not possible to have a negative number of gallons of water, the y-axis also starts at 0.

   Label the x-axis as "Number of hours" and label the y-axis as "Total gallons of water." The title of the graph can be "Gallons of Water in the Barrel."

   Each row in our table of values provides a coordinate point that can be graphed on the coordinate plane to represent the relationship.

   Since the water is leaking from the barrel at a constant rate, and since it is possible to have a fractional number of hours (such as 2.5 hours) instead of only a whole number of hours, draw a line through the coordinate points to show how the remaining amount of water was constantly changing.



   This graph shows that as the number of hours increases, the total gallons of water in the barrel decreases. This confirms the relationship shown in the table of values.

Scaffolded Practice Skill 2

Example 1

On a road trip, Jameson drove his car at an average speed of 60 miles per hour. Explore the relationship between the total distance that Jameson traveled and the time that he spent driving. Identify the independent and dependent quantities, and choose variables to represent them. Then, create an equation, a table, and a graph for the situation.

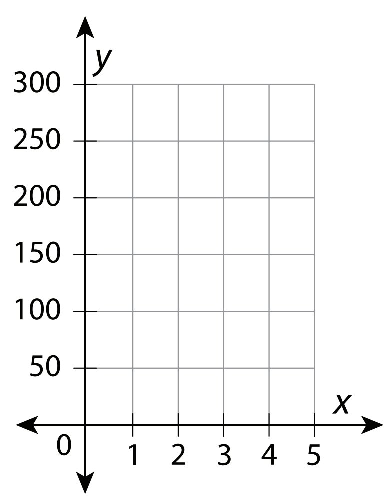
1. Identify the independent and dependent quantities.

2. Choose variables to represent the independent and dependent quantities.

3. Write an equation to represent the relationship between the independent and dependent variables.

4. Create a table of values to represent the relationship between the independent and dependent variables.

5. Use the table to create a graph that represents the relationship between the independent and dependent variables.



continued

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Example 2

At a grocery store, mini seedless watermelons cost $3.50 each. Explore the relationship between the total cost to buy watermelons and the number of watermelons purchased. Identify the independent and dependent quantities, and choose variables to represent them. Then, create an equation, a table, and a graph to represent the situation.

Example 3

A barrel that was filled with 40 gallons of water is leaking water at a constant rate of 5 gallons per hour. Explore the relationship between the total gallons of water left in the barrel and the number of hours the water has been leaking from the barrel. Identify the independent and dependent quantities, and choose variables to represent them. Then, create an equation, a table, and a graph to represent the situation.

Problem-Based Task Skill 2: Saving for a Laptop

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 ✓ 6 ✓
7 ✓ 8 ✓

Brandon is starting college in 8 months and wants to buy a new laptop before his classes start. The laptop that Brandon wants to buy costs $1,185, including tax. He decides to start saving $150 per month to put toward the purchase of the new laptop. Write an equation and create a table to represent the relationship between the number of months and the total amount of money saved. Will Brandon have saved enough money to buy the laptop in 8 months?

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Problem-Based Task Skill 2: Saving for a Laptop

Coaching

a. What variables can be used to represent the quantities in the relationship?

b. What equation represents the relationship between the number of months and the total amount of money saved?

c. Use the equation to create a table that represents the relationship between the number of months and the total amount of money saved.

d. Use the table to determine if Brandon will have saved enough money to buy the laptop in 8 months.

Problem-Based Task Skill 2: Saving for a Laptop

Coaching Sample Responses

a. What variables can be used to represent the quantities in the relationship?

The variables for each quantity are arbitrary, but since "months" starts with the letter "m," let the number of months that Brandon saves money be represented by the variable m.

Since "total" starts with the letter "t," let the total amount of money that Brandon has saved be represented by the variable t.

b. What equation represents the relationship between the number of months and the total amount of money saved?

The total amount of money saved (t) is calculated by multiplying the amount of money saved per month ($150) by the number of months (m).

Therefore, the relationship can be represented by the equation $t = 150m$.

c. Use the equation to create a table that represents the relationship between the number of months and the total amount of money saved.

List values for the independent variable, m, in the left column, and values for the dependent variable, t, in the right column.

Since the problem scenario specifies a time frame of 8 months, fill in the left column with values of m from 1 through 8. Substitute these values into $t = 150m$ to fill in the right column.

The resulting table is shown.

Number of months (m)	Total saved in $ (t)
1	150
2	300
3	450
4	600
5	750
6	900
7	1,050
8	1,200

d. Use the table to determine if Brandon will have saved enough money to buy the laptop in 8 months.

The table shows that Brandon will have saved $1,200 by the eighth month. The laptop only costs $1,185, so he will have enough money.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 2: Understanding Independent and Dependent Quantities

For problems 1 and 2, identify which variable is the independent variable and which is the dependent variable. Then explain the relationship between the variables.

1. Jared rides his bike at a speed of 15 miles per hour. Let h represent the number of hours spent biking and let d represent the total distance traveled.

2. Gretchen can build 3 birdhouses per week. Let w represent the number of weeks and let b represent the total number of birdhouses built.

For problems 3–6, write an equation to represent the relationship between the two variables.

3. A bakery sells pies for $6 each. Let p represent the number of pies purchased and let c represent the total cost to buy those pies.

4. A teddy bear factory can produce 40 teddy bears per hour. Let h represent the number of hours and let t represent the total number of teddy bears made.

continued

5. A recipe calls for $\frac{1}{2}$ cup of raisins for each batch of oatmeal raisin muffins. Let *b* represent the number of batches of muffins made, and let *c* represent the total cups of raisins used.

6. Omar is using a garden hose to fill up his pool with water. The hose pours 4.5 gallons of water into the pool each minute. Let *m* represent the number of minutes and let *g* represent the total gallons of water in the pool.

Use the information given in each problem to complete problems 7–10.

7. A street vendor in New York sells shirts for $7.50 each. Let *s* represent the number of shirts purchased and let *c* represent the total cost to buy those shirts. Complete the table to show the total cost based on the number of shirts purchased.

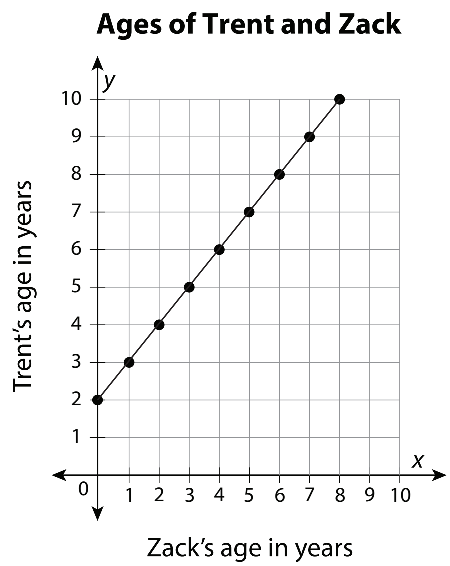
Number of shirts (*s*)	Total cost in $ (*c*)
1	
2	
3	
4	
8	
10	

continued

8. Kelsey bought a new car for $20,000. Every year, the value of the car decreases by $1,500. Let *t* represent the time in years since Kelsey bought the car, and let *v* represent the value of the car. Complete the table to show the value of the car based on the time in years since Kelsey bought it.

Time in years (*t*)	Car value in $ (*v*)
1	
2	
3	
4	
5	

9. The graph shows the relationship between Trent's age and Zack's age. Use the data in the graph to write an equation to represent this relationship. Let *z* represent Zack's age in years and let *t* represent Trent's age in years.



10. A store sells jackets for $25 each. The table shows the store's total profit for selling a given number of jackets. Use the table to create a graph that represents this relationship. Use the labels "Number of jackets" and "Total profit in $" along each appropriate axis.

Number of jackets (*j*)	Total profit in $ (*p*)
1	25
2	50
3	75
4	100
5	125

Supportive Instructional Strategies for Mathematics I

Unit 2 Lesson 7

Suggestions for Graphic Organizers/Manipulatives

Provide each student with a flash card. Ask the students to think of a problem scenario involving two quantities that change in relationship to one another. Have each student write that scenario on the flash card, along with the variables that represent those two quantities. Then pair up students in the class. Have students exchange their flash card with a partner. Ask them to represent the problem scenario on the flash card they've been given as an equation, a table, and a graph. Once both students in each pair are done, ask them to share their answers with each other and to discuss the relationship between the variables. Then ask volunteers to share their flash card scenarios with the class, along with the equation, table, and graph they created to represent it, in order to make sure students understand how to represent these relationships.

Suggestions for Discourse

Ask students to think about problem scenarios in which two quantities change in relationship to each other and how those scenarios are represented by equations. Discuss with students the differences in the types of equations that would be used to represent problems where the dependent quantity increases as the independent quantity increases (likely involving addition or multiplication by a positive number), and problems where the dependent quantity decreases as the independent quantity increases (likely involving subtraction or multiplication by a negative number). As students become more comfortable with the types of equations that represent the different situations, they should better understand how to write an equation to represent these relationships.

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics I*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.

- Provide a list of sentence frames for which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "I know that the variable *d* is the dependent variable in this problem because its value depends on how many hours the person drives." Or, "Three ways I can show the relationship between two quantities are to write an _______, create a _______, and create a _______."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- confusing linear and exponential functions when writing a function rule for a problem situation

  Remind students to look for the key words or phrases in a problem that indicate an exponential increase or decrease, such as *increasing exponentially* or *decaying at an exponential rate*, and words or phrases that indicate a linear function, such as *constant rate*.

- believing that $f(x)$ means "f times x;" i.e., not understanding that $f(x)$ represents the dependent variable for the function

  Continually reinforce with students that $f(x)$ means "function of x" and not "f times x." Also, write the start of the function as both $f(x)$ = and y = to help solidify that $f(x)$ is just another way to represent finding the y-value, which students are already likely comfortable doing.

- confusing the term *explicit function* with the explicit formula for a sequence

  Remind students that an *explicit function* is a function in which the dependent variable can be written in terms of the independent variable, and an *explicit formula* is a formula that allows for any term in a sequence to be calculated.

- looking for a common difference between dependent quantities when the independent values are not one unit apart

  Remind students that a common difference can only be found when the independent values are the same distance apart. Remind them to check for the difference between each independent value to make sure it is the same.

- incorrectly calculating the slope

  Have students write the slope formula at the top of their papers, and write "rise over run" as another way to help them remember how to find the slope. Remind them that it is important to keep the order of the coordinates the same when substituting them into the slope formula.

- trying to match an exponential graph to a linear equation

  Remind students that the graph of an exponential equation will be a curve and the equation will follow the general form of $y = ab^x$. The graph of a linear equation will be a straight line and the equation will follow the general form of $y = mx + b$.

- trying to match a linear graph to an exponential equation

  Remind students that the graph of a linear equation will be a straight line and the equation will follow the general form of $y = mx + b$. The graph of an exponential equation will be a curve and the equation will follow the general form of $y = ab^x$.

Lesson 8: Operating on Functions and Transformations

Elementary Prerequisite Skills

This lesson requires the use of the following elementary skill(s) to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*. See the Appendix for material to address the skill(s).

- E-Skill 1: Applying the Order of Operations (5.OA.1), Appendix p. A-2

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*.

Skill 1: Graphing Linear and Exponential Functions* (A–CED.2★)

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Skill 2: Identifying *y*-intercepts of Graphs of Functions* (8.EE.6)

Common Core State Standard

8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Skill 1: Graphing Linear and Exponential Functions*

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 2

Guided Practice Skill 1

Example 1

Mr. Russell wants to invest in a mutual fund that earns interest that is compounded annually. He hopes to earn an 8% return on his investment. The formula $A = P(1 + r)^t$ can be used to find the balance (A), where P is the principal or the original amount invested, r is the interest rate written as a decimal, and t is the time in years. Write an equation that predicts what Mr. Russell's balance will be in future years if he invests $8,000 today. Then graph the equation.

1. Use the formula to write an equation for Mr. Russell's balance.

   Use the formula $A = P(1 + r)^t$, where P is the principal, r is the interest rate, and t is the time in years.

   Because his future balance depends on the time that has passed, the time, t, will be the independent variable (x), and his future balance, A, will be the dependent variable (y).

   The principal or original amount, P, is $8,000.

   The interest rate, r, is 8%, which is equal to 0.08.

   Substitute these values into the formula.

$A = P(1 + r)^t$	Given formula
$(y) = (8000)[1 + (0.08)]^{(x)}$	Substitute y for A, 8000 for P, 0.08 for r, and x for t.
$y = 8000(1.08)^x$	Simplify.

   The formula $y = 8000(1.08)^x$ predicts Mr. Russell's future balance.

2. Create a table of values for this equation.

To graph this equation, first create a table of values by substituting values for x and solving for y. Let's use 0, 2, 5, and 10 for x.

Substitute 0 for x and solve for y.

$y = 8000(1.08)^x$	Equation
$y = 8000(1.08)^{(0)}$	Substitute 0 for x.
$y = 8000$	Simplify.

When $x = 0$, $y = 8000$. Recall that the interest is compounded once a year. This means that in "year 0," Mr. Russell will not have earned any interest yet, so his balance will be his original amount, $8,000.

Substitute 2 for x and solve for y.

$y = 8000(1.08)^x$	Equation
$y = 8000(1.08)^{(2)}$	Substitute 2 for x.
$y = 9331.20$	Simplify.

When $x = 2$, $y = 9331.20$. After 2 years, Mr. Russell's balance will be $9,331.20.

Substitute 5 for x and solve for y.

$y = 8000(1.08)^x$	Equation
$y = 8000(1.08)^{(5)}$	Substitute 5 for x.
$y \approx 11,754.62$	Simplify.

When $x = 5$, $y \approx 11,754.62$. After 5 years, Mr. Russell's balance will be approximately $11,754.62.

Substitute 10 for x and solve for y.

$y = 8000(1.08)^x$	Equation
$y = 8,000(1.08)^{(10)}$	Substitute 10 for x.
$y \approx 17,271.40$	Simplify.

When $x = 10$, $y \approx 17,271.40$. After 10 years, Mr. Russell's balance will be approximately $17,271.40.

(*continued*)

Organize this information into a table of values.

Years (x)	**Balance in \$ (y)**
0	8,000
2	9,331.20
5	11,754.62
10	17,271.40

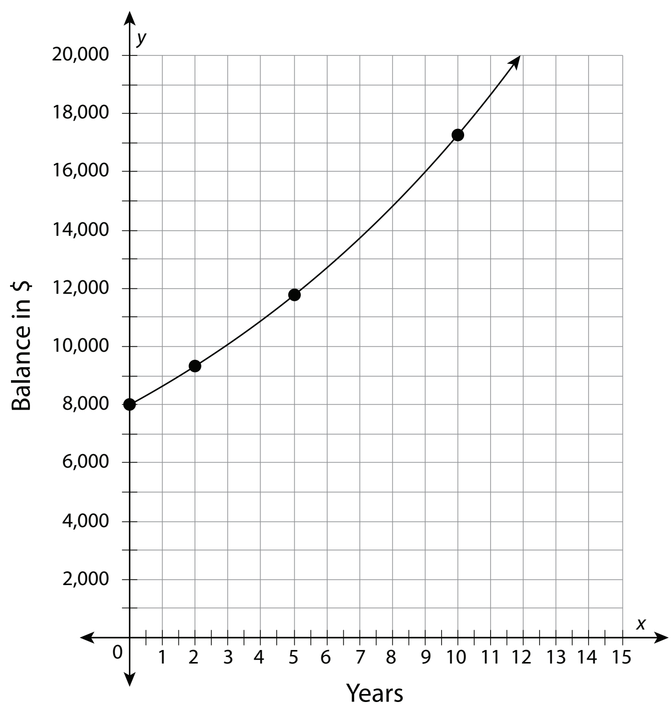
3. Use the table of values to create a graph of the equation.

   The ordered pairs from the table are (0, 8,000), (2, 9,331.20), (5, 11,754.62), and (10, 17,271.40).

   Plot these ordered pairs on a coordinate plane and draw a smooth curve through them.

   Since the problem only refers to positive amounts of money, and time cannot be negative, all of the values will be in the first quadrant.

   Label the x-axis "Years" and the y-axis "Balance in \$."



Scaffolded Practice Skill 1

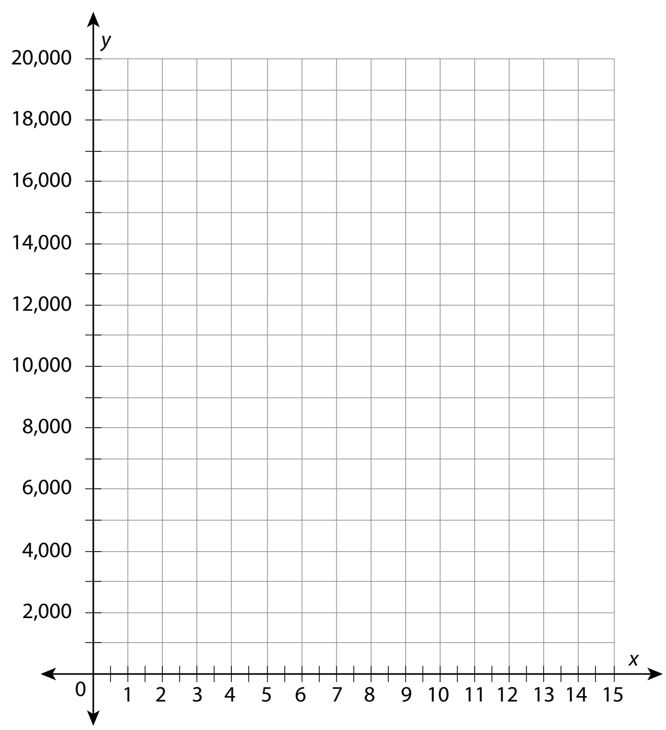
Example 1

Mr. Russell wants to invest in a mutual fund that earns interest that is compounded annually. He hopes to earn an 8% return on his investment. The formula $A = P(1 + r)^t$ can be used to find the balance (A), where P is the principal or the original amount invested, r is the interest rate written as a decimal, and t is the time in years. Write an equation that predicts what Mr. Russell's balance will be in future years if he invests $8,000 today. Then graph the equation.

1. Use the formula to write an equation for Mr. Russell's balance.

2. Create a table of values for this equation.

3. Use the table of values to create a graph of the equation.



Practice Skill 1: Graphing Linear and Exponential Functions*

For problems 1–3, use the following information:

> Brad won $1,500 in an art contest and is considering investing all of the money in an account that earns 7% interest compounded quarterly. The formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ calculates the balance (A) for compounding interest, where P is the principal or the original amount invested, r is the interest rate written as a decimal, n is the number of times the interest is compounded each year, and t is the time in years.

1. Write an equation that predicts the future balance of the account.

2. Use the equation to create a graph that predicts the future balance of the account.

3. Brad found a second account that earns 6% interest, but is compounded monthly instead of quarterly. Create a graph that predicts the future balance of the second account.

Instruction

Skill 2: Identifying *y*-intercepts of Graphs of Functions*

Common Core State Standard

8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

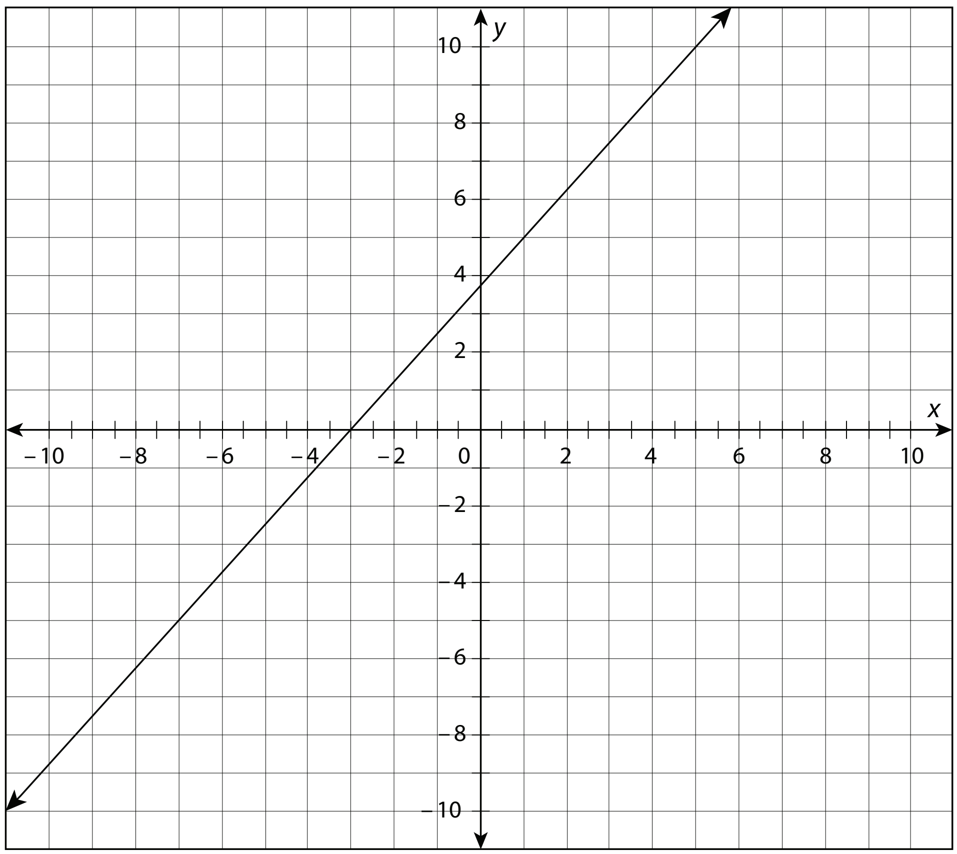
This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 6, Skill 2

Guided Practice Skill 2

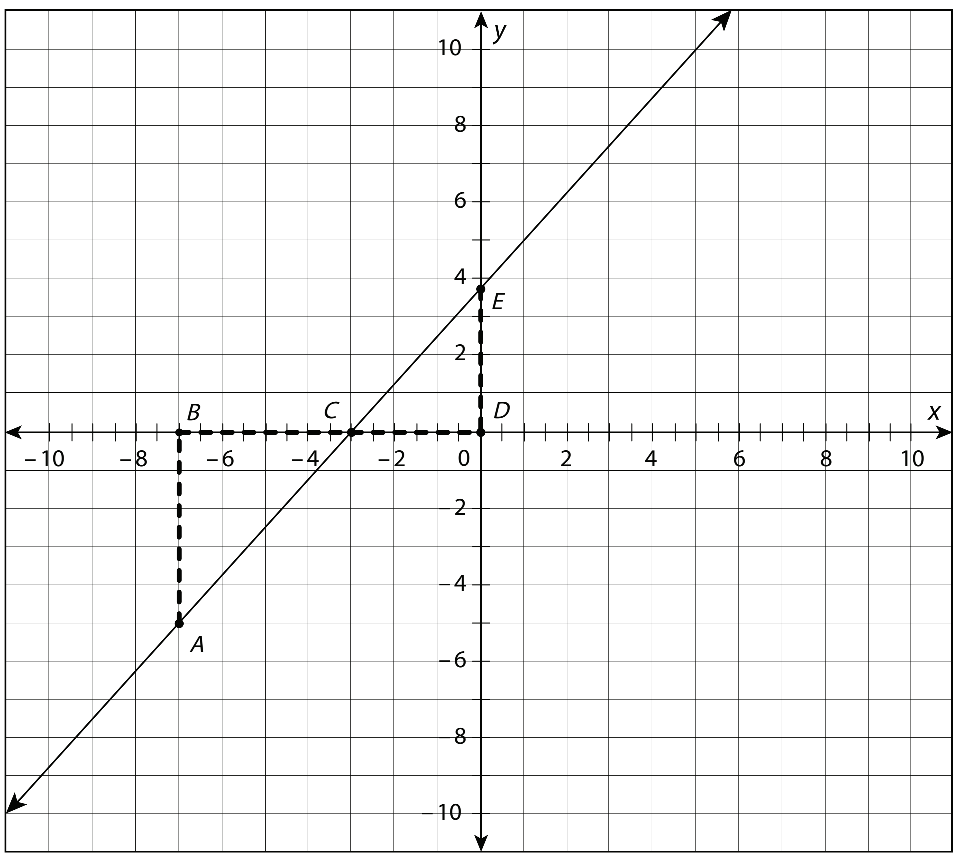
Example 1

The following graph of a linear function passes through the points (–7, –5) and (–3, 0). Use what you know about slope and the properties of similar triangles to determine the exact value of the *y*-intercept of the function.



1. Draw similar triangles, using the graphed line and the *x*- and *y*-axes to form the sides where possible.

   The slope is the same between any two points on the line, so the change in the value of the *y*-coordinates of two points over the change in the value of the *x*-coordinates will remain constant no matter which two points are chosen. This means that in the following figure, $\frac{AB}{BC}=\frac{ED}{DC}$, so triangles *ABC* and *EDC* are similar.



2. Calculate the exact length of the vertical and horizontal sides of the similar triangles using the coordinates of their endpoints, for the endpoints whose exact coordinates can be found by looking at the graph.

The horizontal sides are *BC* and *DC*.

To find the lengths of these sides, subtract the *x*-values of the endpoints.

The coordinates of points *B* and *C* are (–7, 0) and (–3, 0), respectively. Subtract the smaller *x*-value, –7, from the larger *x*-value, –3:

$BC = -3 - (-7) = 4$

The coordinates of points *D* and *C* are (0, 0) and (–3, 0), respectively. Subtract the smaller *x*-value, –3, from the larger *x*-value, 0:

$DC = 0 - (-3) = 3$

Therefore, side *BC* is 4 units long and side *DC* is 3 units long.

The vertical sides are *AB* and *ED*.

To find the lengths of these sides, subtract the *y*-values of the endpoints.

The coordinates of points *A* and *B* are (–7, –5) and (–7, 0), respectively. Subtract the smaller *y*-value, –5, from the larger *y*-value, 0:

$AB = 0 - (-5) = 5$

The exact coordinates of point *E* are unknown, so we cannot use subtraction to find the exact length of *ED*. But, remember that these two triangles are similar—that means all three pairs of corresponding sides are proportionate in length.

Recall that in step 1, we found that $\frac{AB}{BC} = \frac{ED}{DC}$ because of the properties of slope. Because we know the lengths of three of these sides—*AB*, *BC*, and *DC*—we can use the proportion $\frac{AB}{BC} = \frac{ED}{DC}$ to find the exact length of *ED*.

3. Use proportions to determine the exact length of any sides whose exact endpoints could not be determined from the graph.

Substitute the known values into the proportion $\frac{AB}{BC}=\frac{ED}{DC}$.

The length of AB is 5, the length of BC is 4, and the length of DC is 3.

Substituting these values into the proportion gives $\frac{5}{4}=\frac{ED}{3}$.

Solve the proportion $\frac{5}{4}=\frac{ED}{3}$ for ED.

$\frac{5}{4}=\frac{ED}{3}$	Given proportion
$4 \bullet ED = 5 \bullet 3$	Cross multiply.
$4 \bullet ED = 15$	Multiply.
$\frac{4 \bullet ED}{4}=\frac{15}{4}$	Divide both sides by 4.
$ED=\frac{15}{4}$	Simplify.

The length of ED is $\frac{15}{4}$, or 3.75 units.

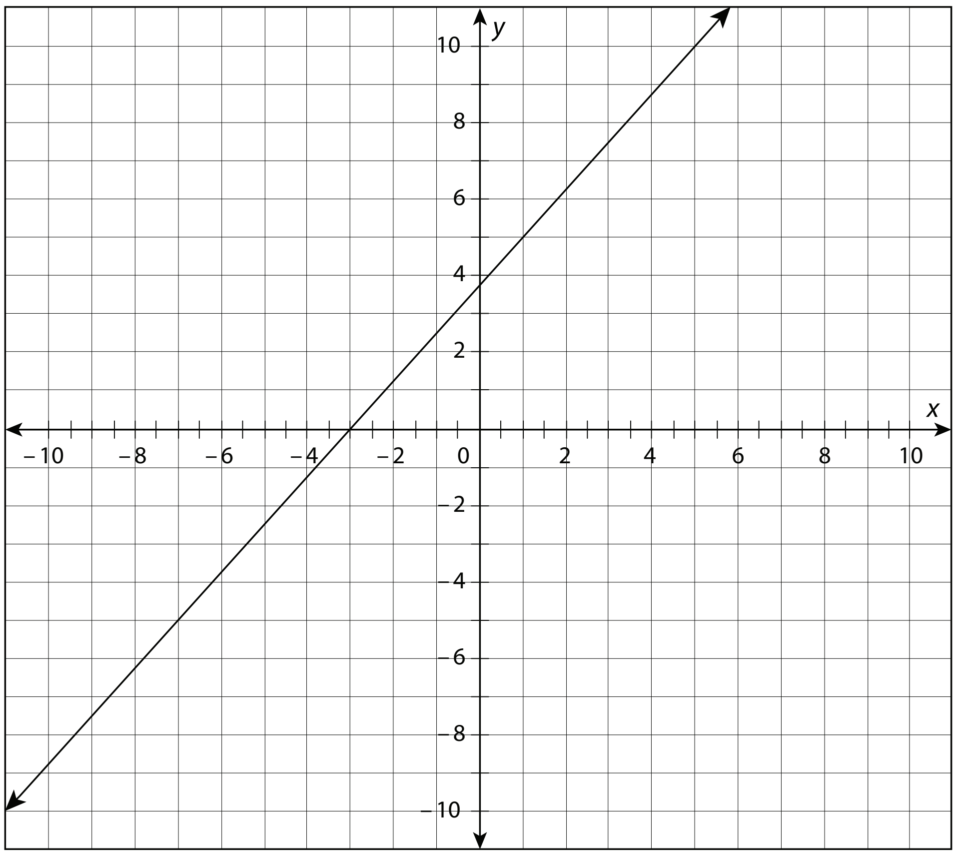
4. Determine the exact value of the y-intercept of the function.

Because $ED = 3.75$ and the coordinates of point D are (0, 0), the coordinates of point E must be (0, 3.75). Therefore, the exact value of the y-intercept of the function is 3.75.

Scaffolded Practice Skill 2

Example 1

The following graph of a linear function passes through the points (–7, –5) and (–3, 0). Use what you know about slope and the properties of similar triangles to determine the exact value of the *y*-intercept of the function.



1. Draw similar triangles, using the graphed line and the *x*- and *y*-axes to form the sides where possible.

2. Calculate the exact length of the vertical and horizontal sides of the similar triangles using the coordinates of their endpoints, for the endpoints whose exact coordinates can be found by looking at the graph.

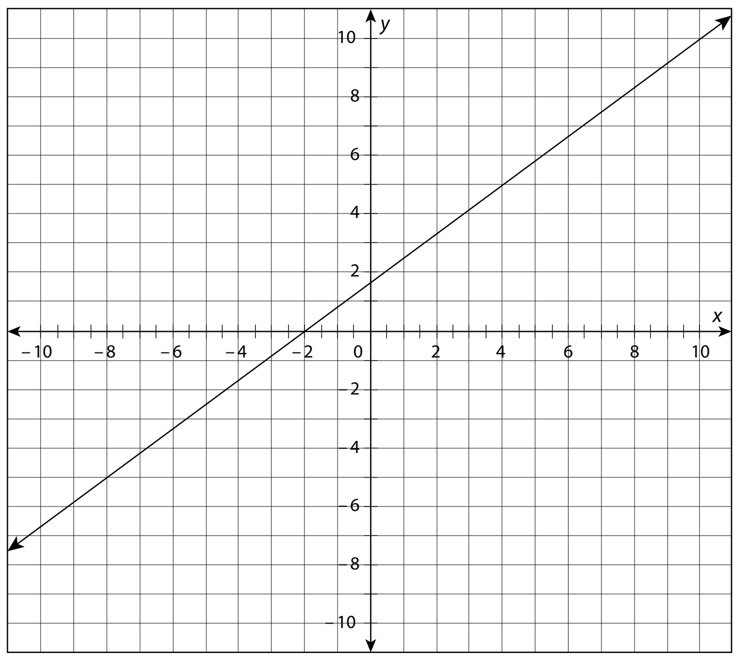
3. Use proportions to determine the exact length of any sides whose exact endpoints could not be determined from the graph.

4. Determine the exact value of the *y*-intercept of the function.

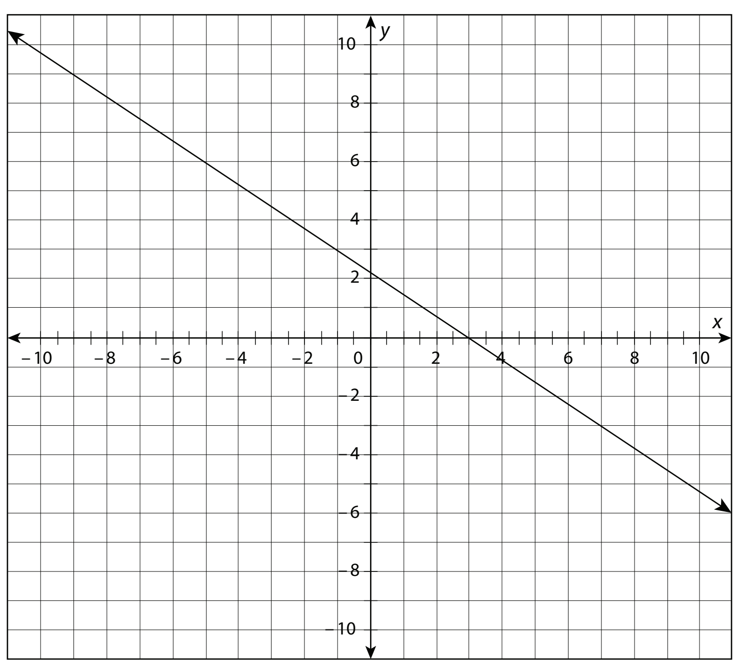
Practice Skill 2: Identifying *y*-intercepts of Graphs of Functions*

For problems 1–3, use what you know about slope and the properties of similar triangles to find the exact value of the *y*-intercept for each graphed function.

1.



2.



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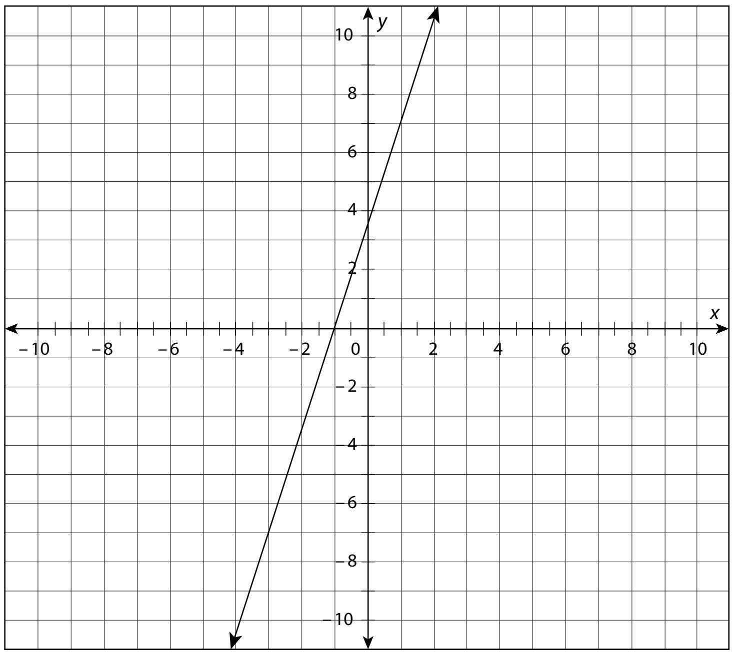
3.



Supportive Instructional Strategies for Mathematics I

Unit 2 Lesson 8

Suggestions for Graphic Organizers/Manipulatives

- Provide students with a two-circle Venn diagram. Ask them to label the circle on the left "$f(x) = x + 2$" and label the circle on the right "$g(x) = x^2 + 2$." Ask them to list specific characteristics of each function in the appropriate circles, with characteristics that both functions have in common listed in the middle. Ask volunteers to share answers for the three parts of the Venn diagram. Create a master copy with all answers compiled into one organizer.
    - Possible characteristics of $f(x) = x + 2$: *it is a linear function, the graph is a line, its slope is 1*.
    - Possible characteristics of $g(x) = x^2 + 2$: *it is an exponential function, the graph is a curve*.
    - Possible shared characteristics: *both have parameters, both have x as the independent variable and the output [f(x) and g(x)] as the dependent variable, both have a vertical shift of positive 2.*
- Provide students with a ruler and blank graph paper. Ask them to use the ruler to draw an x- and y-axis. Then give them the following three linear functions: $f(x) = 3x$, $g(x) = 3x + 1$, and $h(x) = 3x - 1$. For each function, ask them to create an x-y table with the x-coordinates $\{-3, -2, -1, 0, 1, 2, 3\}$, and then find the corresponding y-values and plot the points on the graph, so that all three functions will be graphed on the same graph. Use the ruler to connect the points and make the lines. Then ask students to write down some conclusions about the three graphs. Ask for volunteers to discuss their conclusions about the similarities and differences of the three graphs.
- Provide students with 10 blank flash cards. Ask them to create five examples of linear functions and five examples of exponential functions. Then ask them to shuffle the cards and switch with a partner. Ask the partner to write either "linear" or "exponential" on the back of each card, and also write the value of the vertical shift, if one exists for the function.

Suggestions for Discourse

- Ask students to work with a partner to list real-life examples in which linear functions would apply, and then list real-life examples in which exponential functions would apply. Ask students to think about the ways in which each type of function increases or decreases and how this applies to the type of graph for both functions. Then, ask for volunteers to share their ideas, and make a master list compiling all valid examples for each type of function.
- Ask students, "Why is the order of operations important when performing operations on functions?" Show an example of adding or subtracting two functions for which the order of operations is used. Discuss how incorrectly applying the order of operations could lead to an incorrect function combination.

- Ask students, "Without graphing, what is the difference between the linear functions $f(x) = 4x + 3$ and $g(x) = 4x - 3$? In other words, how would they compare to each other on a graph?" Then, encourage students to use the vocabulary term *vertical shift* to discuss how adding 3 shifts the parent graph of "$y = 4x$" up 3 units, and how subtracting 3 shifts the parent graph of "$y = 4x$" down 3 units.

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics I*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.
- Provide a list of sentence frames in which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "I know this is an exponential function because the shape of the graph is a _______." Or, "In the equation $y = 5x + 4$, 5 is the _______, and 4 is the _______." Or, "This function is <u>linear</u>, because its graph is in the shape of a <u>line</u>."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- incorrectly using the Distributive Property when subtracting two expressions

  Provide basic examples of using the Distributive Property with subtraction, as well as the reverse of it. Remind students that the factor outside the parentheses must be distributed to each term inside the parentheses, and that the signs of each term must be carried throughout.

- incorrectly using the order of operations when multiplying or dividing two expressions

  Have students write "PEMDAS" at the top of their papers, and remind them that multiplication and division operations are performed in order from left to right, no matter which of the two operations comes first.

- incorrectly using the Distributive Property when multiplying two expressions

  Provide basic examples of using the Distributive Property with multiplication, as well as the reverse of it. Remind students that the factor outside the parentheses must be distributed to each term inside the parentheses, and that the signs of each term must be carried throughout.

- mistaking vertical shift for horizontal shift

  Remind students that *vertical* means "up or down," and *horizontal* means "left or right." Remind them that the vertical shift is the constant added to or subtracted from the original function.

- mistaking a y-intercept for the value of the vertical translation

  Remind students that the y-intercept is the point where the graph crosses the y-axis, and the vertical translation is the number of units that the graph is shifted either up or down from the original function.

- incorrectly graphing linear or exponential functions

  Remind students that a linear function does not have an exponent and its graph is a straight line, and that an exponential function does have an exponent and its graph is a curve.

- incorrectly combining like terms when changing a function rule

  Remind students that like terms must have the same variables and powers. Provide basic examples of combining like terms, and include a variety of variables and powers so that students can see terms that are alike and terms that are not alike.

Lesson 9: Arithmetic and Geometric Sequences

Instruction

Elementary Prerequisite Skills

This lesson requires the use of the following elementary skill(s) to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*. See the Appendix for material to address the skill(s).

- E-Skill 1: Applying the Order of Operations (5.OA.1), Appendix p. A-2
- E-Skill 3: Recognizing Patterns (3.OA.9), Appendix p. A-27
- E-Skill 4: Multiplying Fractions (5.NF.4a), Appendix p. A-34

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*.

Skill 1: Adding and Subtracting Signed Numbers (7.NS.1b, 7.NS.1c)

Common Core State Standards

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Skill 2: Identifying Linear Relationships* (8.F.4)

Common Core State Standard

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

(*continued*)

Skill 3: Multiplying Signed Numbers (7.NS.2a)

Common Core State Standard

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

Skill 4: Using Exponents* (8.EE.1)

Common Core State Standard

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Instruction

Skill 1: Adding and Subtracting Signed Numbers

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 6 ✓
7 ✓ 8

Common Core State Standards

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Essential Questions

1. How are signed numbers added and subtracted?
2. How can sums and differences of signed numbers be applied to real-world problems?

WORDS TO KNOW

absolute value	a number's distance from 0 on a number line
additive inverse	a number added to a given number that results in 0
opposite number	a number with the same absolute value as a given number, but the opposite sign
signed number	a number preceded by a negative sign (–) to indicate a negative value or by a plus sign (+) to indicate a positive value. The plus sign is usually implied and not shown to indicate a positive value.

SWB p. 201

Recommended Resources

- IXL Learning. "Add and Subtract Integers."

  http://www.walch.com/rr/04054

  This site provides practice with adding and subtracting signed numbers. Immediate feedback is provided and users are shown how to correctly add or subtract the numbers when an incorrect answer is given.

- Oswego City School District Regents Exam Prep Center. "Adding Signed Numbers."

  http://www.walch.com/rr/04055

  This site provides instruction and examples on the process of adding signed numbers.

- Oswego City School District Regents Exam Prep Center. "Subtracting Signed Numbers."

  http://www.walch.com/rr/04056

  This site provides instruction and examples on the process of subtracting signed numbers.

Recommended Instructional Strategies for Skill Development

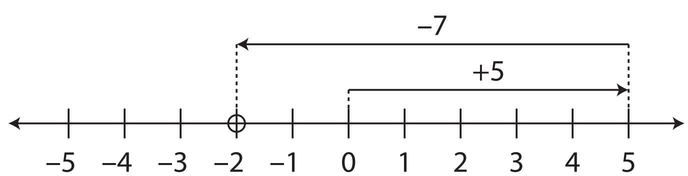
Suggestions for Graphic Organizers/Manipulatives

Once students have completed the Guided Practice, distribute the Number Line graphic organizer. Have students label each of the four number lines from –5 to 5 by ones. Then ask the students to use the number lines to show how to find the result for each of the following expressions: 5 + (–7), –3 + 8, –1 + (–4), and –2 – (–6). Ask for volunteers to share their number lines and discuss why they drew them as they did. These discussions should include:

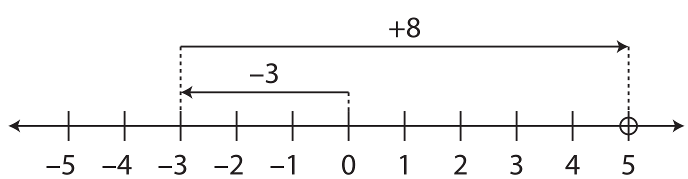
- drawing the arrows the length of the absolute values of the numbers in the expression
- drawing the arrows to the left or right depending on whether the original number was negative or positive
- starting the second arrow at the end of the first arrow
- having the answer located at the end of the second arrow

The number lines should look as follows:

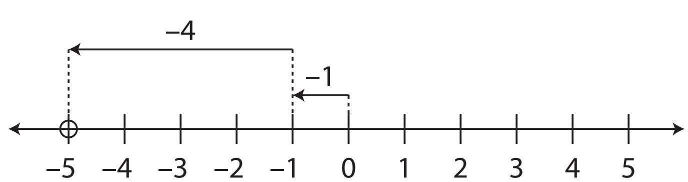
5 + (–7)



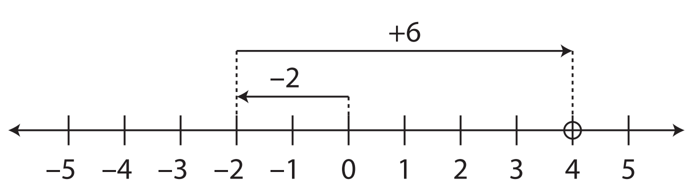
–3 + 8



–1 + (–4)



–2 – (–6) = –2 + 6



Suggestions for Discourse

Ask students to think about real-life examples in which positive and negative numbers can represent situations, and use the examples to create a master list that can be displayed in the classroom. These examples could include negative temperature readings, money in a bank account/being overdrawn, a submarine's distance below sea level, situations such as adding water to a pool but then having a certain amount of that water leak out, or earning and losing points while playing a board game. When discussing these examples, have students think about how they can determine when a value in a situation should be represented as a positive or negative number.

Making Connections

Encourage students to connect the concept of subtraction with adding the additive inverse, or opposite, of the subtracted number. This may help them better visualize how to find the result when the number being subtracted is negative.

Skill 1: Adding and Subtracting Signed Numbers

Introduction

Negative numbers in real life can occur in many cases, such as in situations in which money is spent, distance is traveled in the opposite direction, or items are given to others. Understanding how to add and subtract positive and negative numbers is an important step in learning how to solve these situations in real-life contexts.

Key Concepts

Adding Signed Numbers

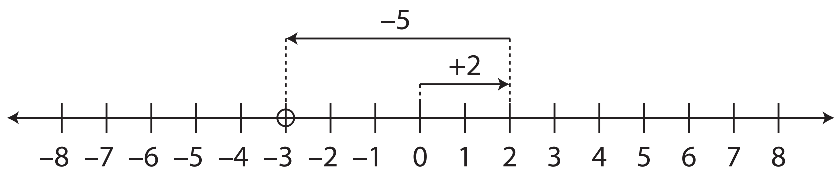
- **Signed numbers** are numbers preceded by a negative sign (–) to indicate a negative value or by a plus sign (+) to indicate a positive value. The plus sign is usually implied and not shown to indicate a positive value.
- To add two signed numbers with opposite signs, such as 2 + (–5), first find the **absolute value**, or distance from 0, of each number: |2| = 2 and |–5| = 5. Then find the difference between the larger absolute value and the smaller absolute value: 5 – 2 = 3. Determine which absolute value was larger, 5 > 2, and use the original sign of that number as the sign of the result: 2 + (–5) = –3.
- To add two signed numbers with the same sign, such as –1 + (–3), first find the absolute of each number: |–1| = 1 and |–3| = 3. Then find the sum of those absolute values: 1 + 3 = 4. Use the original sign of both numbers as the sign in front of the sum: –1 + (–3) = –4.

Using a Number Line to Add Signed Numbers

- A number line can also be used to help visualize the result when adding signed numbers.
- To represent the sum of signed numbers on a number line, first find the absolute value of each number in the expression. Then start at 0 and draw an arrow the length of that first absolute value to the right if the number was originally positive, or to the left if the number was originally negative.
- Next, start at the end of the first arrow and just above it draw a new arrow that is the length of that second absolute value to the right or left, depending on if the original sign of the second number was positive or negative. The location of the end of that second arrow is the value of the sum.

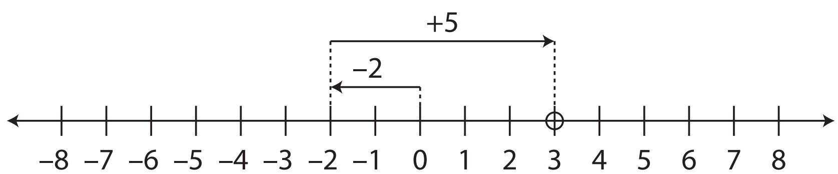
- For example, given 2 + (–5), the number line would look as follows:

2 + (–5)



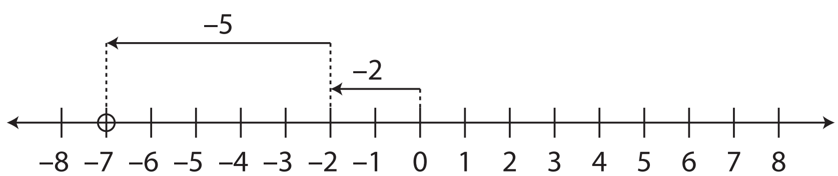
- Given –2 + 5, the number line would look as follows:

–2 + 5



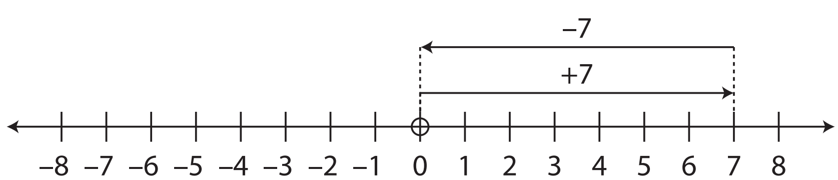
- Given –2 + (–5), the number line would look as follows:

–2 + (–5)



- Given any number, its **opposite number** is a number with the same absolute value but the opposite sign. A number and its opposite number have a sum of 0, such as 7 + (–7) = 0. Together, these numbers are called **additive inverses**.

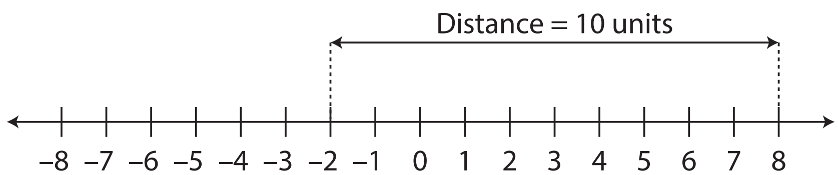
7 + (–7)



Subtracting Signed Numbers

- To subtract signed numbers, rewrite the expression as adding the additive inverse of the subtracted number. For example, –8 – 3 becomes –8 + (–3), because the additive inverse of 3 is –3. The expression 5 – (–12) becomes 5 + 12, because the additive inverse of –12 is 12.
- On a number line, the distance between any two numbers is equal to the absolute value of their difference, such as the distance between 8 and –2 is 10 units given that 8 – (–2) = 8 + 2 = 10.

8 – (–2)



Guided Practice Skill 1

Example 1

Simplify the expression 4 + (–7). Use a number line to verify the result.

1. Find the absolute value of each number in the expression.

   The absolute value of 4 is written as |4| and is equal to 4.

   The absolute value of –7 is written as |–7| and is equal to 7.

2. Find the difference of the two absolute values.

   Find the difference between the larger absolute value and the smaller absolute value.

   The larger absolute value is 7 and the smaller absolute value is 4; therefore the difference is 7 – 4, which is equal to 3.

3. Use the original sign of the larger absolute value as the sign of the result.

   Because 7 is the larger absolute value and its original sign was negative, the sign of the result will also be negative. So the result is –3.

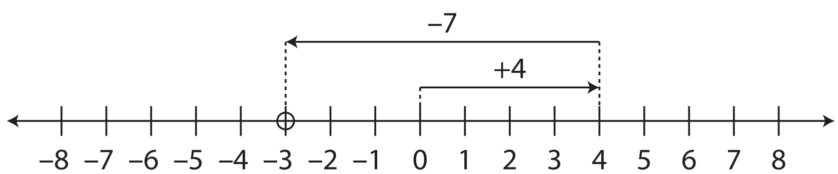
   Therefore, the expression 4 + (–7) simplifies to –3.

4. Draw the expression 4 + (–7) on a number line to verify the result.

   The first number in the expression is positive 4. Draw an arrow starting at 0 that extends a length of 4 units to the right. It ends at 4.

   The second number in the expression is negative 7. Starting just above the end of the first arrow (at 4), draw another arrow that extends a length of 7 units to the left.

   The arrow will end at –3.



   It can be seen on the number line that the expression 4 + (–7) is equal to –3.

Example 2

Simplify the expression –2 – (–3). Use a number line to verify the result.

1. Rewrite the expression as adding the additive inverse of the subtracted number.

   Recall that a number and its opposite number have a sum of 0, and are additive inverses.

   In the expression –2 – (–3), the value –3 is being subtracted. The additive inverse of –3 is 3 since these two numbers sum to 0: –3 + 3 = 0.

   Rewrite the given expression (subtracting a negative number) as adding the additive inverse.

–2 – (–3)	Original expression
–2 + 3	Change subtraction to addition and write the additive inverse of –3.

   The expression –2 – (–3) can be rewritten using the additive inverse as –2 + 3.

2. Find the absolute value of each number in the rewritten expression.

   The rewritten expression is –2 + 3.

   The absolute value of –2 is written as |–2| and is equal to 2.

   The absolute value of 3 is written as |3| and is equal to 3.

3. Find the difference of the two absolute values.

   Find the difference between the larger absolute value and the smaller absolute value.

   The larger absolute value is 3 and the smaller absolute value is 2; therefore the difference is 3 – 2, which is equal to 1.

4. Use the sign of the larger absolute value from the expression –2 + 3 as the sign of the result.

   Because 3 is the larger absolute value and its sign in the rewritten expression –2 + 3 was positive, the sign of the result will also be positive. So the result is positive 1.

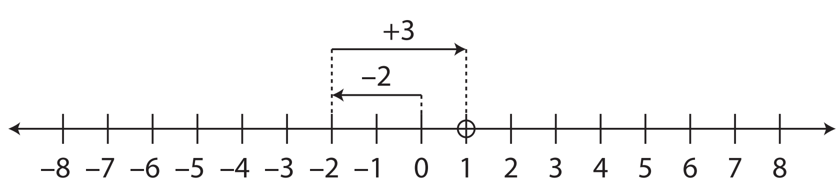
   Therefore, the expression –2 + 3 simplifies to 1.

5. Draw the expression –2 + 3 on a number line to verify the result.

   The first number in the expression is negative 2. Draw an arrow starting at 0 that extends a length of 2 units to the left. It ends at –2.

   The second number in the expression is positive 3. Starting just above the end of the first arrow (at –2), draw another arrow that extends a length of 3 units to the right.

   The arrow will end at 1.



   It can be seen on the number line that the rewritten expression –2 + 3 is equal to 1. Likewise, since –2 + 3 = –2 – (–3), the original expression –2 – (–3) also simplifies to 1.

Example 3

Starting from a ranger station, Justin hikes east 8.5 miles along a nature trail. Later in the day, he turns around and hikes west along the same path to return to the ranger station. After completing 4.7 miles of the return hike, Justin stops to take a break. How many miles does Justin still need to hike to get back to the ranger station? Use a number line to verify the result.

1. Write an expression to represent the situation.

   Justin hikes east for 8.5 miles, which is represented by 8.5 because east is a positive direction on a number line.

   Justin hikes west for 4.7 miles when he returns, which is represented by –4.7 because west is a negative direction on a number line.

   The situation can be represented by the expression 8.5 + (–4.7).

2. Find the absolute value of each number in the expression.

   The absolute value of 8.5 is written as |8.5| and is equal to 8.5.

   The absolute value of –4.7 is written as |–4.7| and is equal to 4.7.

3. Find the difference of the two absolute values.

   Find the difference between the larger absolute value and the smaller absolute value.

   The larger absolute value is 8.5 and the smaller absolute value is 4.7. The difference is 8.5 – 4.7, which is equal to 3.8.

4. Use the original sign of the larger absolute value as the sign of the result.

   Because 8.5 is the larger absolute value and its original sign was positive, the sign of the result will be positive. So, the result is positive 3.8.

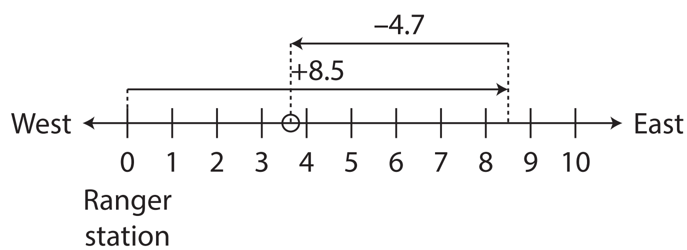
   Therefore, the expression 8.5 + (–4.7) simplifies to 3.8.

5. Draw the expression 8.5 + (–4.7) on a number line to verify the result.

The first number in the expression is positive 8.5. Draw an arrow starting at 0 that extends a length of 8.5 units to the right. It ends at 8.5.

The second number in the expression is negative 4.7. Starting just above the end of the first arrow (at 8.5), draw another arrow that extends a length of 4.7 units to the left.

The arrow will end at 3.8.



It can be seen on the number line that Justin still needs to hike 3.8 miles to get back to the ranger station.

Scaffolded Practice Skill 1

Example 1

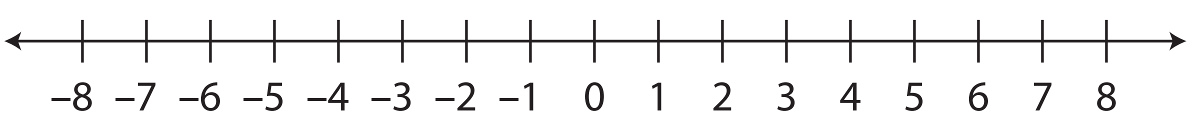
Simplify the expression 4 + (–7). Use a number line to verify the result.

1. Find the absolute value of each number in the expression.

2. Find the difference of the two absolute values.

3. Use the original sign of the larger absolute value as the sign of the result.

4. Draw the expression 4 + (–7) on a number line to verify the result.



continued

Example 2

Simplify the expression –2 – (–3). Use a number line to verify the result.

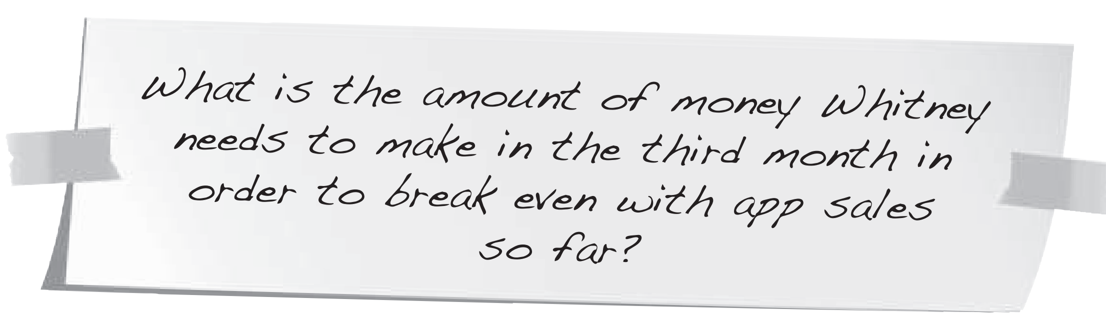
Example 3

Starting from a ranger station, Justin hikes east 8.5 miles along a nature trail. Later in the day, he turns around and hikes west along the same path to return to the ranger station. After completing 4.7 miles of the return hike, Justin stops to take a break. How many miles does Justin still need to hike to get back to the ranger station? Use a number line to verify the result.

Problem-Based Task Skill 1: App Sales

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 6 ✓
7 ✓ 8

Whitney recently created a new smartphone app. As a result of the initial costs and fees of producing the app, she did not make any money, or profit, selling the app during the first two months of the app's release. For the first month, Whitney had a loss of $324. During the second month of sales, she had a loss of $97. She hopes that in the third month, she will sell enough apps to break even—that is, she hopes to earn enough money to cover her losses. What is the amount of money Whitney needs to make in the third month in order to break even with app sales so far?



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Problem-Based Task Skill 1: App Sales

Coaching

a. What was the total amount of money Whitney lost in the first two months?

b. What dollar amount represents "breaking even"? Explain what "breaking even" means in terms of app sales for the three months.

c. How much money would Whitney need to make in the third month to make up for her losses in the first two months?

Problem-Based Task Skill 1: App Sales

Coaching Sample Responses

a. What was the total amount of money Whitney lost in the first two months?

The amount of loss in the first month of app sales was \$324, or –324.

The amount of loss in the second month was \$97, or –97.

The total amount of loss for the two months is $-324 + (-97) = -421$.

Whitney lost a total of \$421 in the first two months.

b. What dollar amount represents "breaking even"? Explain what "breaking even" means in terms of app sales for the three months.

"Breaking even" means Whitney has not lost money or made money. Therefore, to break even, the dollar amount needs to be \$0. Whitney needs a total profit of \$0 after three months in order to break even.

c. How much money would Whitney need to make in the third month to make up for her losses in the first two months?

Let x represent the amount of sales for the third month. The equation $-421 + x = 0$ represents the amount needed for Whitney to break even. Solve the equation to determine the amount needed.

$-421 + x = 0$

$x = 0 - (-421)$

$x = 0 + 421$

$x = 421$

Whitney would need to make \$421 in the third month in order to break even with app sales so far.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 1: Adding and Subtracting Signed Numbers

For problems 1–8, simplify each expression.

1. $12 + (-8)$
2. $-\frac{1}{10} - \frac{7}{10}$
3. $-9 + 14$
4. $\frac{3}{7} - \left(-\frac{2}{7}\right)$
5. $-4 + (-17)$
6. $-6 - (-2)$
7. $11.4 + (-13.9)$
8. $-15 + 3$

For problems 9 and 10, write an expression to represent the given scenario and then simplify the expression.

9. While playing a board game, Josie lost 7 points on her first turn. She then earned 5 points on her second turn. What is Josie's current point total?

10. Lee is recording January temperatures in Calgary, Alberta, Canada, for a project. One day, the low temperature was –14°F. The high temperature that day was 9°F. What is the difference between those high and low temperatures?

Instruction

Skill 2: Identifying Linear Relationships*

Common Core State Standard

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 3, Skill 1

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Guided Practice Skill 2

Example 1

A watering can for a garden weighs 1.8 pounds when empty, and a quart of water weighs 2.1 pounds. Is there a linear relationship between the number of quarts of water in the can and the total weight of the can plus the water?

1. Determine the total weight of the can plus the water for increasing amounts of water.

   The total weight of the can plus the water when there are 0 quarts of water in the can is the same as the weight of the can when empty. This is given as 1.8 pounds.

   When there is 1 quart of water in the can, the total weight is $1.8 + 2.1(1) = 3.9$ pounds.

   When there are 2 quarts of water in the can, the total weight is $1.8 + 2.1(2) = 6$ pounds.

   When there are 3 quarts of water in the can, the total weight is $1.8 + 2.1(3) = 8.1$ pounds.

   This information can be organized in a table.

Quarts of water	Total weight (pounds)
0	1.8
1	3.9
2	6
3	8.1

2. Decide if a linear relationship exists between the number of quarts of water in the can and the total weight.

   From 0 quarts of water in the can to 1 quart, the total weight increases by 3.9 – 1.8 = 2.1 pounds.

   Also, from 1 quart to 2 quarts, the total weight increases by 6 – 3.9 = 2.1 pounds.

   From 2 quarts to 3 quarts, the total weight increases by 8.1 – 6 = 2.1 pounds.

   Notice that the total weight increases by the same amount, 2.1 pounds, each time 1 quart of water is added. In other words, the weight increases at a constant rate (2.1) as the water increases incrementally (in increments of 1).

   This information can be organized in a table.

Quarts of water	Total weight (pounds)	Weight change (pounds)
0	1.8	—
1	3.9	3.9 – 1.8 = 2.1
2	6	6 – 3.9 = 2.1
3	8.1	8.1 – 6 = 2.1

   Because the total weight increases at a constant rate as the quarts of water in the can increase incrementally, the relationship is linear.

Scaffolded Practice Skill 2

Example 1

A watering can for a garden weighs 1.8 pounds when empty, and a quart of water weighs 2.1 pounds. Is there a linear relationship between the number of quarts of water in the can and the total weight of the can plus the water?

1. Determine the total weight of the can plus the water for increasing amounts of water.

Quarts of water	Total weight (pounds)

2. Decide if a linear relationship exists between the number of quarts of water in the can and the total weight.

Quarts of water	Total weight (pounds)	Weight change (pounds)

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Practice Skill 2: Identifying Linear Relationships*

Use the given information to determine if each problem describes a linear relationship. Write “yes” or “no” for your answers.

1. An Internet café charges a one-time fee of $2.50 to use a computer plus $0.05 per minute for each minute that a customer spends online. Is there a linear relationship between the number of minutes a customer spends online and the total amount of money she must pay?

2. When it was founded, a large town’s population was 20,000 people. The population has been increasing at a rate of 10% per year. Is there a linear relationship between the number of years since the town was founded and the town’s population?

3. A kerosene lamp had 100 milliliters of fuel in its tank when it was first lit. Since then, it has been burning 8 milliliters of fuel per minute. Is there a linear relationship between the number of minutes the lamp has been lit and the number of milliliters of fuel in its tank?

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Instruction

Skill 3: Multiplying Signed Numbers

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 6 ✓
7 ✓ 8

Common Core State Standard

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

Essential Questions

1. How are signed numbers multiplied?
2. How can the product of signed numbers be used to model real-world problems?

WORDS TO KNOW

signed number a number preceded by a negative sign (–) to indicate a negative value or by a plus sign (+) to indicate a positive value. The plus sign is usually implied and not shown to indicate a positive value.

Recommended Resources

- LearnZillion. "Prove That a Negative Times a Negative Equals a Positive."

  http://www.walch.com/rr/04057

  This site features additional resources and a video that explains why multiplying two negative values together results in a positive value.

- MathIsFun.com. "Multiplying Negatives."

  http://www.walch.com/rr/04058

  This site gives an overview of multiplying signed numbers, as well as examples. It also provides links to practice questions at the bottom of the page.

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Recommended Instructional Strategies for Skill Development

Suggestions for Graphic Organizers/Manipulatives

Once students have worked through the Guided Practice, have them create a three-column chart with various combinations of positive and negative numbers and their product. Have students reserve the last four rows of the table to write the general combinations and the product of positive and negative numbers. For example:

Value 1	Value 2	Product
3	2	6
3	–2	–6
–3	2	–6
–3	–2	6
+	+	+
+	–	–
–	+	–
–	–	+

Ask volunteers to share their examples. Then have students summarize the rules in their own words.

Suggestions for Discourse

Ask students to think about real-life examples when positive and negative signed numbers would need to be multiplied to represent a situation. These examples could include situations such as owing a company $20 per month for 6 months (-20×6), or a pool leaking 3 gallons of water per hour for 4 hours (-3×4). When discussing these examples, have the students think about how they can determine when a value in a situation should be represented as a positive number or as a negative number.

Making Connections

Encourage students to make the connection that the product of two negative numbers is positive to satisfy the properties of operations, specifically the Distributive Property. Discuss with students the following example:

- Start with an equation that multiplies any negative number by 0.

$-3 \times 0 = 0$	Any value multiplied by 0 equals 0 due to the Zero Product Property.

- Due to the Additive Inverse Property, any number and its opposite added together equal 0. Therefore, using substitution, replace 0 with the sum of any number and its opposite.

$-3 \times [2 + (-2)] = 0$	Substitute [2 + (–2)] for 0 due to the Additive Inverse Property.

- Apply the Distributive Property to the left side of the equation.

$(-3)(2) + (-3)(-2) = 0$	Distribute –3.

- For this equation to equal 0, it must be that (–3)(2) and (–3)(–2) are additive inverses.
- The product of two numbers with opposite signs is negative, so $(-3)(2) = -6$.
- This means that (–3)(–2) must equal a positive 6 to have the equation be equal to 0 with $-6 + 6 = 0$.
- Therefore, the product of two negative numbers is positive.

Skill 3: Multiplying Signed Numbers

Introduction

Multiplying signed numbers is necessary to properly interpret the products that describe real-world contexts. For example, to find the total amount of money deducted monthly from a checking account over a period of time, multiply the deduction (negative) by the number of months (positive). Or, to find how much you would pay for a premium cable package over the next year, multiply the monthly cost (positive) by 12 months (positive).

Key Concepts

- **Signed numbers** are numbers preceded by a negative sign (–) to indicate a negative value or by a plus sign (+) to indicate a positive value. It is important to note that the plus sign is usually implied and not shown.
- When multiplying two numbers with opposite signs, the product is negative, such as $8 \times (-3) = -24$ or $-9 \times 7 = -63$.
- When multiplying two numbers with the same sign, the product is positive, such as $13 \times 2 = 26$ or $-6 \times (-5) = 30$.
- When multiplying more than two signed numbers:
  - If there is an odd number of negative numbers, the product will be negative, such as $-8 \times (-1) \times (-2) = -16$.
  - If there is an even number of negative numbers, the product will be positive, such as $-5 \times 3 \times (-4) = 60$.
- The rules can be summarized as follows:

Sign of value 1	×	Sign of value 2	=	Sign of product
+	×	+	=	+
+	×	–	=	–
–	×	+	=	–
–	×	–	=	+

Guided Practice Skill 3

Example 1

Simplify the expression $9 \times (-5)$.

1. Find the absolute value of each number in the expression.

   The absolute value of 9 is written as $|9|$ and is equal to 9.

   The absolute value of –5 is written as $|-5|$ and is equal to 5.

2. Multiply the absolute values.

   The product of 9 and 5 can be written as 9×5 and is equal to 45.

3. Determine the sign of the product.

   Because 9 and –5 have opposite signs, the product is negative. Therefore, $9 \times (-5)$ is equal to –45.

Example 2

Simplify the expression $-4 \times (-7)$.

1. Find the absolute value of each number in the expression.

   The absolute value of –4 is written as $|-4|$ and is equal to 4.

   The absolute value of –7 is written as $|-7|$ and is equal to 7.

2. Multiply the absolute values.

   The product of 4 and 7 can be written as 4×7 and is equal to 28.

3. Determine the sign of the product.

   Because –4 and –7 have the same signs, the product is positive. Therefore, $-4 \times (-7)$ is equal to 28.

Example 3

During a drought, a park ranger regularly measured the water level in a nearby lake for 8 weeks. Each week, the water level fell 1.5 inches. What was the total change in the water level by the end of 8 weeks?

1. Write an expression to represent the situation.

   The water level fell 1.5 inches each week, which is represented by –1.5.

   The water level was measured for 8 weeks.

   The situation can be represented by the expression -1.5×8.

2. Find the absolute value of each number in the expression.

   The absolute value of –1.5 is written as $|-1.5|$ and is equal to 1.5.

   The absolute value of 8 is written as $|8|$ and is equal to 8.

3. Multiply the absolute values.

   The product of 1.5 and 8 can be written as 1.5×8 and is equal to 12.

4. Determine the sign of the product.

   Because -1.5×8 multiplies two numbers with opposite signs, the product is negative.

   Therefore, -1.5×8 is equal to –12.

   The change in the water level was –12 inches, which means the water level in the lake fell 12 inches by the end of 8 weeks. ✓

Scaffolded Practice Skill 3

Example 1

Simplify the expression $9 \times (-5)$.

1. Find the absolute value of each number in the expression.

2. Multiply the absolute values.

3. Determine the sign of the product.

continued

Example 2

Simplify the expression $-4 \times (-7)$.

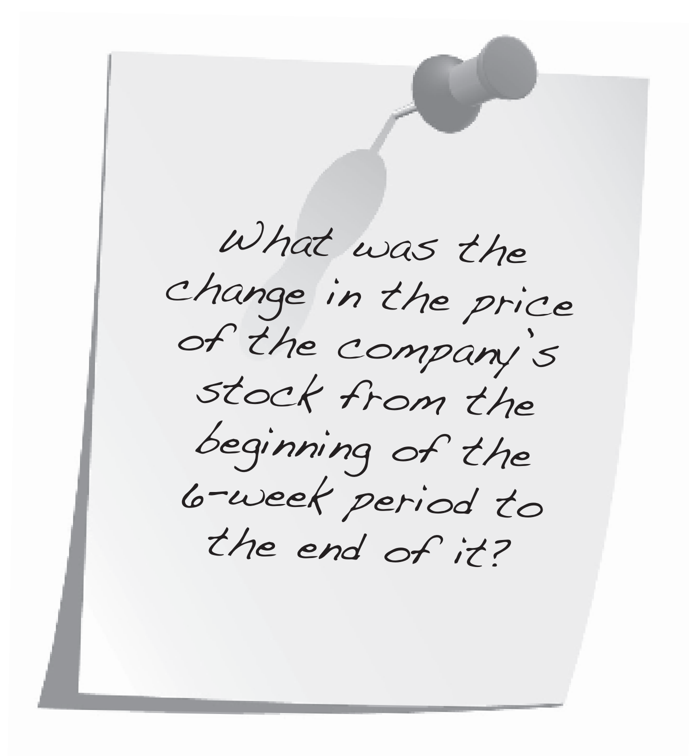
Example 3

During a drought, a park ranger regularly measured the water level in a nearby lake for 8 weeks. Each week, the water level fell 1.5 inches. What was the total change in the water level by the end of 8 weeks?

Problem-Based Task Skill 3: Stock Price

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 6 ✓
7 ✓ 8

After a company reported a decrease in its sales, the price of the company's stock began to drop by $0.25 every day the stock market was open, Monday through Friday. The company continued to struggle for 6 weeks before reporting that sales had improved. At that point, the stock price stopped dropping. During that difficult 6-week period, the company's stock price continued to drop by $0.25 per day each weekday. What was the change in the price of the company's stock from the beginning of the 6-week period to the end of it?



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Problem-Based Task Skill 3: Stock Price

Coaching

a. What number can be used to represent the amount that the stock price changed per weekday?

b. What was the change in the company's stock price each week?

c. What was the change in the price of the company's stock from the beginning of the 6-week period to the end of it?

Problem-Based Task Skill 3: Stock Price

Coaching Sample Responses

a. What number can be used to represent the amount that the stock price changed per weekday?

The company's stock price dropped by \$0.25 per weekday, so –0.25 represents the price change per weekday.

b. What was the change in the company's stock price each week?

The stock price dropped \$0.25 per day each weekday, represented by –0.25.

There are 5 weekdays in a week.

The weekly change in the stock price was -0.25×5.

The absolute values of these terms are 0.25 and 5.

The product of 0.25 and 5 is 1.25.

Because -0.25×5 multiplies two numbers with opposite signs, the product is negative, or –1.25.

The change in the company's stock price each week is equal to –\$1.25.

c. What was the change in the price of the company's stock from the beginning of the 6-week period to the end of it?

The change in the stock price was –\$1.25 per week.

The stock price dropped for 6 weeks.

Therefore, the overall change in the stock price for those 6 weeks was -1.25×6.

The absolute values of these terms are 1.25 and 6.

The product of 1.25 and 6 is 7.5.

Because -1.25×6 multiplies two numbers with opposite signs, the product is negative, or –7.5.

The change in the price of the company's stock from the beginning to the end of that 6-week period was –\$7.50.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.

Practice Skill 3: Multiplying Signed Numbers

For problems 1–8, simplify each expression.

1. $5.2 \times (-4)$

2. $(-3) \times (-16)$

3. $-\frac{1}{2} \times \frac{5}{8}$

4. -7×10

5. $6 \times (-2.5)$

6. $-21 \times (-2)$

7. $4 \times (-9) \times 3$

8. $-8.7 \times 5 \times (-2)$

For problems 9 and 10, read each scenario. Then write an expression to represent the scenario and simplify the expression.

9. Catherine went scuba diving in the ocean. After she got into the water, she descended at a rate of 25 feet per minute. What was Catherine's change in water depth relative to the surface of the water after 3 minutes?

10. Jamal used his debit card to buy 4 shirts that each cost $8.75. After that purchase, what was the change to the amount of money in Jamal's bank account? Assume that there is no sales tax.

SWB p. 225

Instruction

Skill 4: Using Exponents*

Common Core State Standard

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 3

Guided Practice Skill 4

Example 1

Write the simplified expression of $\dfrac{x^2 \bullet x^{-5} \bullet \left(y^3\right)^8}{y^0}$ using only positive exponents.

1. Use the exponent rule $a^0 = 1$ to simplify the expression.

   Any value to the 0 power is equal to 1. Thus, the denominator of y^0 is equal to 1.

   Substitute 1 for y^0.

$\dfrac{x^2 \bullet x^{-5} \bullet \left(y^3\right)^8}{y^0}$	Original expression
$=\dfrac{x^2 \bullet x^{-5} \bullet \left(y^3\right)^8}{1}$	Rewrite y^0 as 1.
$= x^2 \bullet x^{-5} \bullet \left(y^3\right)^8$	Rewrite the fraction.

   The expression $\dfrac{x^2 \bullet x^{-5} \bullet \left(y^3\right)^8}{y^0}$ can be simplified to $x^2 \bullet x^{-5} \bullet \left(y^3\right)^8$.

2. Use the exponent rule $(a^m)^n = a^{m \bullet n}$ to further simplify the expression.

   When a power is raised to another power, multiply the exponents.

$x^2 \bullet x^{-5} \bullet \left(y^3\right)^8$	Expression from the previous step
$= x^2 \bullet x^{-5} \bullet y^{3 \bullet 8}$	Rewrite $\left(y^3\right)^8$ as a multiplication of exponents.
$= x^2 \bullet x^{-5} \bullet y^{24}$	Multiply the exponents.

   The expression $x^2 \bullet x^{-5} \bullet \left(y^3\right)^8$ can be simplified to $x^2 \bullet x^{-5} \bullet y^{24}$.

3. Use the exponent rule $a^m \bullet a^n = a^{m+n}$ to further simplify the expression.

Notice that x^2 and x^{-5} have the same base, x. When powers with the same base are multiplied, add the exponents.

$x^2 \bullet x^{-5} \bullet y^{24}$	Expression from the previous step
$= x^{2+(-5)} \bullet y^{24}$	Rewrite $x^2 \bullet x^{-5}$ as an addition of exponents.
$= x^{-3} \bullet y^{24}$	Add the exponents.

The expression $x^2 \bullet x^{-5} \bullet y^{24}$ can be simplified to $x^{-3} \bullet y^{24}$.

4. Use the exponent rule $a^{-m} = \frac{1}{a^m}$ to write the expression with positive exponents.

To rewrite a negative exponent as a positive exponent, take the reciprocal of the expression with the negative exponent.

Using the reciprocal, the expression with the negative exponent, x^{-3}, can be rewritten as $\frac{1}{x^3}$.

$x^{-3} \bullet y^{24}$	Expression from the previous step
$= \frac{1}{x^3} \bullet y^{24}$	Rewrite x^{-3} as $\frac{1}{x^3}$.
$= \frac{y^{24}}{x^3}$	Rewrite the expression as a fraction.

The expression $x^{-3} \bullet y^{24}$ can be simplified to $\frac{y^{24}}{x^3}$; therefore, the expression $\frac{x^2 \bullet x^{-5} \bullet \left(y^3\right)^8}{y^0}$ can be simplified to $\frac{y^{24}}{x^3}$.

Scaffolded Practice Skill 4

Example 1

Write the simplified expression of $\frac{x^2 \bullet x^{-5} \bullet \left(y^3\right)^8}{y^0}$ using only positive exponents.

1. Use the exponent rule $a^0 = 1$ to simplify the expression.

2. Use the exponent rule $(a^m)^n = a^{m \bullet n}$ to further simplify the expression.

3. Use the exponent rule $a^m \bullet a^n = a^{m+n}$ to further simplify the expression.

4. Use the exponent rule $a^{-m} = \frac{1}{a^m}$ to write the expression with positive exponents.

Practice Skill 4: Using Exponents*

Simplify each expression and write it with only positive exponents.

1. $(y^5)^{-9}$

2. $\dfrac{x^2 \bullet \left(y^2\right)^8}{x^7 \bullet y^4} \bullet x^{-3}$

3. The expression $64 \bullet 2^{-x}$ represents the number of teams left in the NCAA college basketball tournament after x rounds. Simplify this expression and write it using only positive exponents.

Supportive Instructional Strategies for Mathematics I

Unit 2 Lesson 9

Suggestions for Graphic Organizers/Manipulatives

- Provide flash cards that include a variety of addition and subtraction expressions with signed numbers. Ask the students to look at the cards and try to determine whether each expression will have a positive or negative answer without finding the answer. Have the students sort the cards into two piles—one for expressions they think will have a positive answer and one for those they think will have a negative answer. Then have students find the answers to each flash card to verify whether or not they guessed correctly, and, if necessary, move the cards into the correct piles. Have the students look for characteristics that indicate whether the answers will be positive or negative. Ask volunteers to share their answers, such as:
    - When adding opposite signed numbers, the answer is positive if the larger absolute value in the expression was positive and negative if the larger absolute value was negative.
    - When adding two negative numbers, the answer will always be negative.
- Provide flash cards that have a variety of multiplication expressions with signed numbers, and ask each student to take one card. Then ask the students to think of a real-life context that can be described by that multiplication expression. Ask volunteers to share their answers to ensure that students understand when a situation can be represented with the product of signed numbers.

Suggestions for Discourse

- Present students with the following sequence of numbers: 20, 13, 6, –1, –8. Ask them the following questions:
    - Starting with the second term in the sequence, what is the difference between each term in the sequence and the term before it?
    - Is the difference between all of those consecutive terms in the sequence the same? If so, what is that common difference? If not, is there a pattern with the difference?
    - Based on that difference, what will be the next term in the sequence of numbers?
- Present students with the following sequence of numbers: 1, –2, 4, –8, 16. Ask them the following questions:
    - What number is multiplied by each term in the sequence to get to the next term?
    - Is the same number multiplied through the sequence to get to the next term? If so, what is that common ratio? If not, is there a pattern with the ratios?
    - What will be the next term in the sequence of numbers?

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics I*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.
- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding.
- Provide students with a word bank during discussions and/or explanations.
- Use manipulatives if appropriate. For example, provide students with tables. Allow students to organize the results to sequences in tabular form to recognize differences and ratios.
- Create a synonym word bank or chart that can be referred to and viewed during the lesson or discussions, so students can become more familiar and comfortable with everyday vocabulary words. For example, create a word bank for *difference* that includes words such as *subtract* and *minus*. Similarly, the word bank for *product* could include *times* and *multiply*.

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- identifying a non-arithmetic sequence as arithmetic

  Reinforce to students that the difference between terms must be the same in an arithmetic sequence. Give students a list of examples and non-examples of arithmetic sequences.
- defining the common difference, d, in a decreasing sequence as a positive number

  Remind students to first determine if the terms in the sequence are increasing or decreasing before they find the difference.
- incorrectly using the Distributive Property when finding the nth term with the explicit formula

  Provide some basic examples of simplifying expressions with the Distributive Property. Reinforce that the factor on the outside of the parentheses must be distributed to each term inside the parentheses.
- identifying a non-geometric sequence as geometric

  Show students several examples of how to check the common ratio between each term—if each term is multiplied or divided by the same number to get to the next term, it is a geometric sequence.

- defining the constant ratio, *r*, in a decreasing sequence as a number greater than 1

  Provide students with several examples that show the results of multiplying by a number greater than 1. Then, show the contrast by multiplying by a number less than 1.

- incorrectly using the order of operations when finding the *n*th term in a geometric sequence

  Have students write the acronym "PEMDAS" for reference, and ask them to think about it each time they are simplifying an expression with multiple steps.

- forgetting to identify the first term when defining a geometric sequence recursively

  As a first step, have students write down the words "first term =" each time when determining a geometric sequence.

- forgetting to identify the first term when defining an arithmetic sequence recursively

  As a first step, have students write down the words "first term =" each time when determining an arithmetic sequence.

Lesson 10: Interpreting Parameters

Instruction

Targeted Prerequisite Skills

This lesson explores the following skill(s) necessary to meet the standards addressed in *CCSS Integrated Pathway: Mathematics I*.

Skill 1: Graphing Equations* (A–CED.2★)

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Skill 2: Writing Linear Equations from Context* (A–CED.2★)

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Skill 3: Writing Exponential Equations from Context* (A–CED.2★)

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Instruction

Skill 1: Graphing Equations*

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 2

SWB p. 237

Guided Practice Skill 1

Example 1

A restaurant has three round tables of different diameters—5 feet, 7 feet, and 9 feet. The owner wants to add a strip of veneer around the outside edge of each table. Write an equation that represents the total length of veneer (in feet) needed for any table, depending on the table's diameter. Then, create a table of values and a graph. Use the formula for the circumference of a circle, $C = \pi d$, where C is the circumference (the distance around the circle) and d is the diameter. Recall that the value of π is approximately 3.14.

1. Write an equation that represents the total length (in feet) of veneer needed depending on the diameter of the table.

   The owner wants to put veneer around the edge of each table. Because each table is circular, the edge represents the circumference of a circle. So, the length of the veneer is equal to the circumference.

   The equation for the circumference of a circle is $C = \pi d$, where C is the circumference, d is the diameter, and π is approximately equal to 3.14.

   The total length of veneer depends on the table's diameter. Therefore, the diameter, d, is the independent variable, x; the total length of veneer, C, is the dependent variable, y.

   Substitute the variables x and y into the formula for the circumference of a circle. Use 3.14 for π.

$C = \pi d$	Formula for circumference
$(y) = (3.14)(x)$	Substitute y for C, 3.14 for π, and x for d.
$y = 3.14x$	Simplify.

   The equation $y = 3.14x$ represents the total length of veneer needed depending on the diameter of the table.

2. Create a table of values for the equation.

Since we are given that the diameters of the tables are 5 feet, 7 feet, and 9 feet, let the values of x be 5, 7, and 9. Substitute these values into the equation and solve for y.

Substitute 5 for x and solve for y.

$y = 3.14x$ Equation from the previous step
$y = 3.14(5)$ Substitute 5 for x.
$y = 15.7$ Simplify.

When $x = 5$, $y = 15.7$.

Substitute 7 for x and solve for y.

$y = 3.14x$ Equation from the previous step
$y = 3.14(7)$ Substitute 7 for x.
$y = 21.98$ Simplify.

When $x = 7$, $y = 21.98$.

Substitute 9 for x and solve for y.

$y = 3.14x$ Equation from the previous step
$y = 3.14(9)$ Substitute 9 for x.
$y = 28.26$ Simplify.

When $x = 9$, $y = 28.26$.

Organize this information into a table of values.

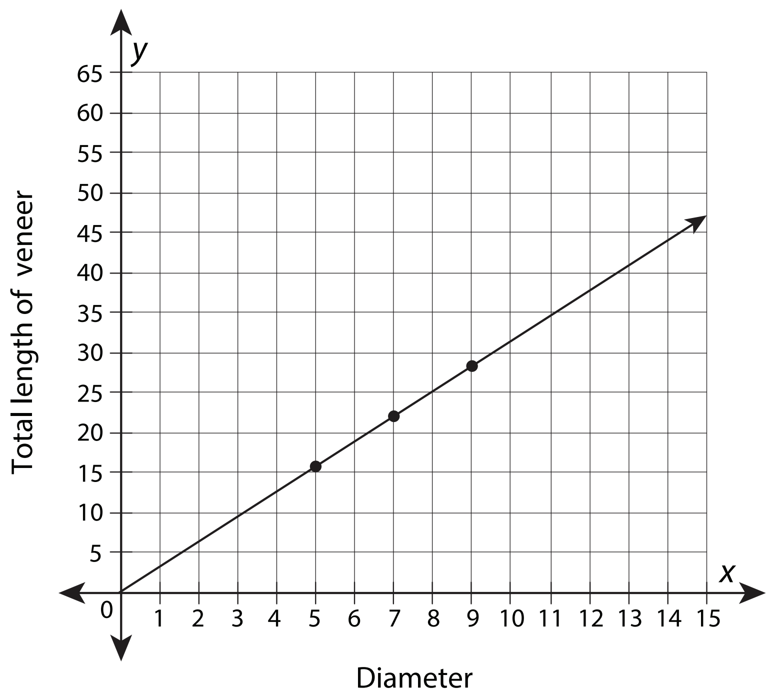
Diameter (x)	Total length of veneer (y)
5	15.7
7	21.98
9	28.26

3. Use the table of values to create a graph of the equation.

On a coordinate plane, plot the ordered pairs from the table of values.

The ordered pairs are (5, 15.7), (7, 21.98), and (9, 28.26).

Draw a line through the points. Since the table diameters and veneer lengths cannot be negative, do not draw the line where x is negative.



The graph represents the total length of veneer needed depending on the diameter of the table.

Scaffolded Practice Skill 1

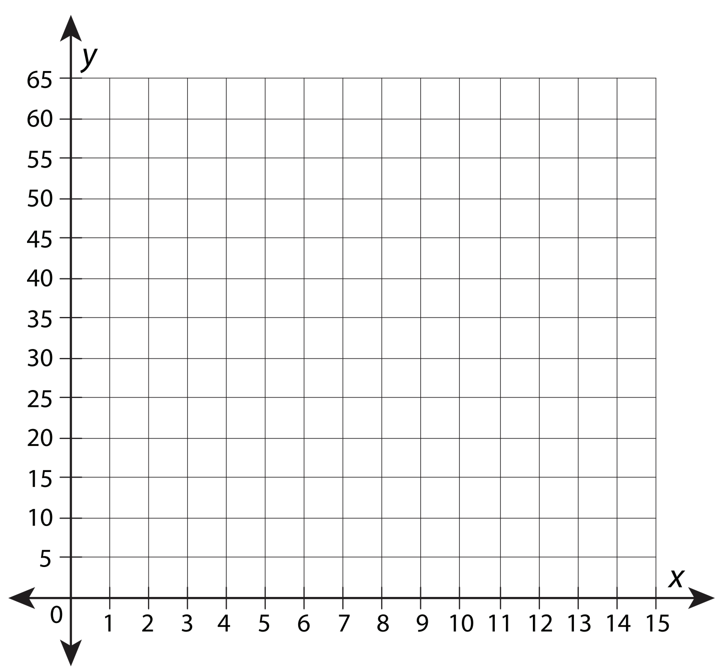
Example 1

A restaurant has three round tables of different diameters—5 feet, 7 feet, and 9 feet. The owner wants to add a strip of veneer around the outside edge of each table. Write an equation that represents the total length of veneer (in feet) needed for any table, depending on the table's diameter. Then, create a table of values and a graph. Use the formula for the circumference of a circle, $C = \pi d$, where C is the circumference (the distance around the circle) and d is the diameter. Recall that the value of π is approximately 3.14.

1. Write an equation that represents the total length (in feet) of veneer needed depending on the diameter of the table.

2. Create a table of values for the equation.

3. Use the table of values to create a graph of the equation.



SWB p. 239

Practice Skill 1: Graphing Equations*

Use the following information to complete problems 1–3.

Mrs. Cole gave her students a test with 30 questions. She grades each test and writes the number of incorrect answers at the top.

1. The equation $y = 100 - 3x$ represents the scores the students could get on the test if Mrs. Cole subtracts 3 points (from 100) for every incorrect answer. Create a graph that shows the scores the students will get based on the number of incorrect answers. Let x represent the number of incorrect answers and y represent the total test score.

2. If Mrs. Cole decides to award 3.5 points for every *correct* answer, what equation could be used to find the score based on the number of *incorrect* answers? Use the equation to create a graph that shows the students' scores based on the number of incorrect answers if each correct answer is worth 3.5 points. Let x represent the number of incorrect answers and y represent the total test score.

3. If Mrs. Cole decides to determine the scores by simply calculating the percentage of correct answers (100% being a perfect score), what equation could be used to find the scores depending on the number of incorrect answers? Let x represent the number of incorrect answers and y represent the total test score.

SWB p. 241

Skill 2: Writing Linear Equations from Context*

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 2

SWB p. 245

Guided Practice Skill 2

Example 1

On Saturdays, Isaac mows lawns for neighbors on his street. It normally takes him about 30 minutes to mow a lawn, and about 10 minutes total to take out and put away the lawn mower from the shed at his house. Write an equation that describes the total time Isaac will spend mowing lawns depending on how many lawns he mows. Use the equation to create a table of values. Then create a graph of the equation.

1. Write an equation for the given situation.

   Because the total time depends on the number of lawns mowed, the number of lawns should be the independent variable, x, and the total time should be the dependent variable, y.

   The total time, y, is equal to 30 minutes times the number of lawns, or $30x$, plus a constant time of 10 minutes to take out the lawn mower and put it away, or +10. So, the equation $y = 30x + 10$ represents the total time Isaac will spend mowing lawns depending on how many lawns he mows.

2. Create a table of values for the equation.

   To create a table of values, substitute values for x and solve for y.

   Substitute 1 for x and solve for y.

$y = 30x + 10$	Equation from the previous step
$y = 30(1) + 10$	Substitute 1 for x.
$y = 40$	Simplify.

   If Isaac mows 1 lawn, it will take him 40 minutes.

   Substitute 2 for x and solve for y.

$y = 30x + 10$	Equation from the previous step
$y = 30(2) + 10$	Substitute 2 for x.
$y = 70$	Simplify.

   If Isaac mows 2 lawns, it will take him 70 minutes.

   Substitute 3 for x and solve for y.

$y = 30x + 10$	Equation the previous step
$(3) + 10$	Substitute 3 for x.
$y = 100$	Simplify.

   If Isaac mows 3 lawns, it will take him 100 minutes.

   Organize this information into a table of values.

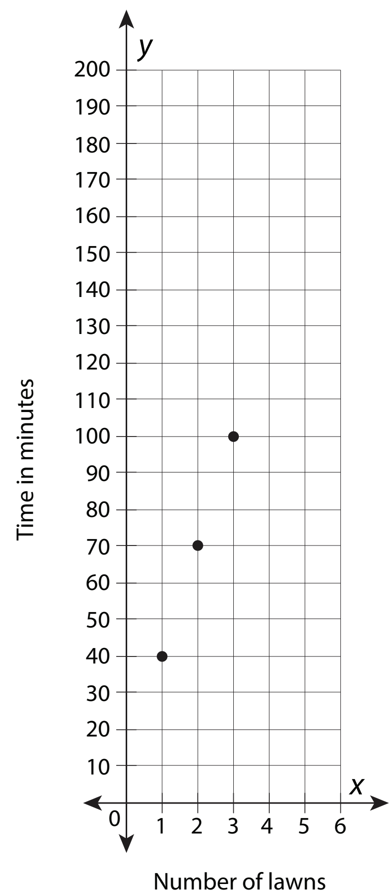
Number of lawns (x)	Time in minutes (y)
1	40
2	70
3	100

3. Use the table of values to create a graph that models the scenario.

   On a coordinate plane, plot the ordered pairs from the table of values.

   The ordered pairs are (1, 40), (2, 70), and (3, 100).

   Isaac is unlikely to mow partial lawns; therefore, it is only necessary to plot whole number values of x.



   The graph represents the total time Isaac will spend on this task depending on how many lawns he mows.

Scaffolded Practice Skill 2

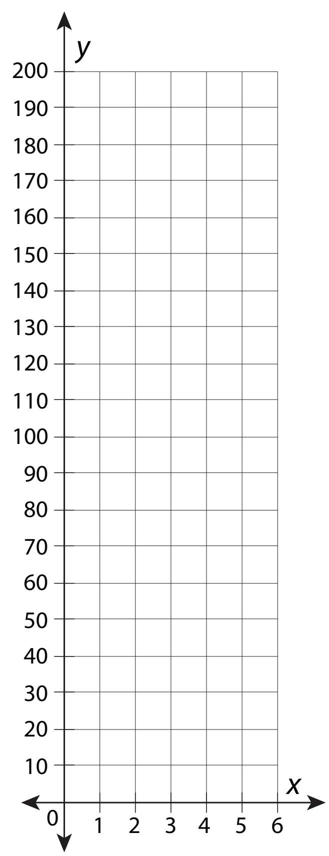
Example 1

On Saturdays, Isaac mows lawns for neighbors on his street. It normally takes him about 30 minutes to mow a lawn, and about 10 minutes total to take out and put away the lawn mower from the shed at his house. Write an equation that describes the total time Isaac will spend mowing lawns depending on how many lawns he mows. Use the equation to create a table of values. Then create a graph of the equation.

1. Write an equation for the given situation.

2. Create a table of values for the equation.

3. Use the table of values to create a graph that models the scenario.



SWB p. 247

Practice Skill 2: Writing Linear Equations from Context*

Use the following information to complete problems 1–3.

Francesca normally takes the bus to and from work. A one-way fare costs $1.85.

1. Write an equation that calculates Francesca's total travel costs for the month depending on how many days she works.

2. Write an equation that calculates Francesca's total travel costs for the month depending on how many days she works if her mom drives her to and from work twice during the month.

3. Once Francesca gets her license, she can drive her mom's car to work as long as she pays for the gas she uses. Her job is 3.5 miles away and her mom's car gets 20 miles per gallon on average. Write an equation that calculates Francesca's daily travel costs depending on the cost per gallon of gas.

Instruction

Skill 3: Writing Exponential Equations from Context*

Common Core State Standard

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

This skill has been addressed previously in this Support Supplement. Refer to the following lesson to find Essential Questions, Words to Know, Recommended Resources, Recommended Instructional Strategies, Key Concepts, and a Problem-Based Task for this skill.

Unit 2, Lesson 1, Skill 2

SWB p. 253

Guided Practice Skill 3

Example 1

A city's population has been steadily decreasing at the rate of 2% each year. The latest figures show that 85,000 people currently live in the city. Use the general equation $y = a(1 - r)^t$ to create an equation that predicts what the city's population will be in future years if this trend continues. In the equation, y is the future population, a is the current population, r is the rate at which the population is decreasing (written as a decimal), and t is the time in years. Create a table of values, then graph the equation.

1. Use the general equation to write a specific equation to predict the city's future population.

   Because the future population depends on how much time has passed, the time, t, is the independent variable, x. The future population, y, is the dependent variable and will remain y.

   The current population, a, is 85,000.

   The rate, r, is 2%, which is equal to 0.02.

   Substitute these values into the general formula $y = a(1 - r)^t$.

$y = a(1 - r)^t$	General formula
$y = (85{,}000)[1 - (0.02)]^{(x)}$	Substitute 85,000 for a, 0.02 for r, and x for t.
$y = 85{,}000(0.98)^x$	Simplify.

   The equation $y = 85{,}000(0.98)^x$ predicts the population in future years.

2. Create a table of values for the equation.

To create a table of values, substitute values for x and solve for y.

Substitute 0 for x and solve for y.

$y = 85{,}000(0.98)^x$	Equation from the previous step
$y = 85{,}000(0.98)^{(0)}$	Substitute 0 for x.
$y = 85{,}000$	Simplify.

After 0 years have passed, the population is 85,000. In other words, "year 0" represents the city's current population.

Substitute 2 for x and solve for y.

$y = 85{,}000(0.98)^x$	Equation from the previous step
$y = 85{,}000(0.98)^{(2)}$	Substitute 2 for x.
$y \approx 81{,}634$	Simplify.

After 2 years, the population will be approximately 81,634.

Substitute 5 for x and solve for y.

$y = 85{,}000(0.98)^x$	Equation from the previous step
$y = 85{,}000(0.98)^{(5)}$	Substitute 5 for x.
$y \approx 76{,}833$	Simplify.

After 5 years, the population will be approximately 76,833.

Substitute 10 for x and solve for y.

$y = 85{,}000(0.98)^x$	Equation from the previous step
$y = 85{,}000(0.98)^{(10)}$	Substitute 10 for x.
$y \approx 69{,}451$	Simplify.

After 10 years, the population will be approximately 69,451.

Substitute 20 for x and solve for y.

$y = 85{,}000(0.98)^x$	Equation from the previous step
$y = 85{,}000(0.98)^{(20)}$	Substitute 20 for x.
$y \approx 56{,}747$	Simplify.

After 20 years, the population will be approximately 56,747.

(*continued*)

Organize this information into a table of values.

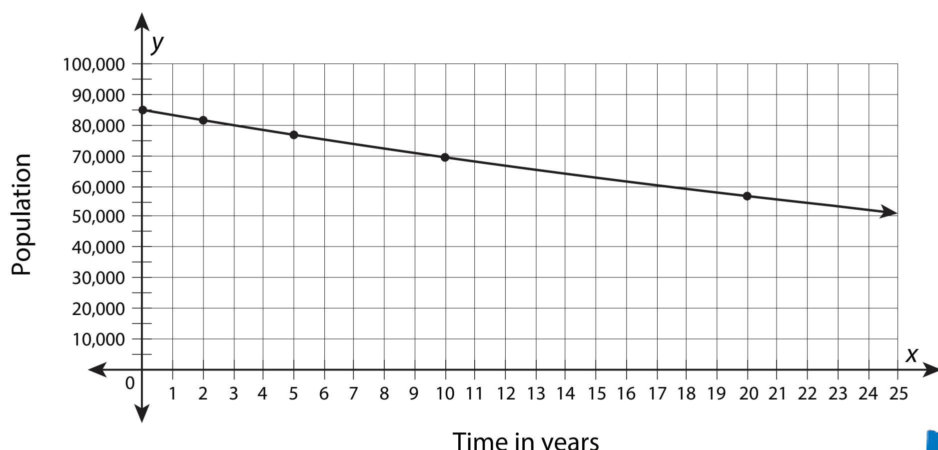
Time in years (x)	Population (y)
0	85,000
2	81,634
5	76,833
10	69,451
20	56,747

3. Use the table of values to create a graph of the equation.

On a coordinate plane, plot the ordered pairs from the table of values.

The ordered pairs are (0, 85,000), (2, 81,634), (5, 76,833), (10, 69,451), and (20, 56,747).

Draw a line through the points. Since the number of years cannot be negative, do not draw the line where x is negative.



The graph represents the predicted population in future years.

Scaffolded Practice Skill 3

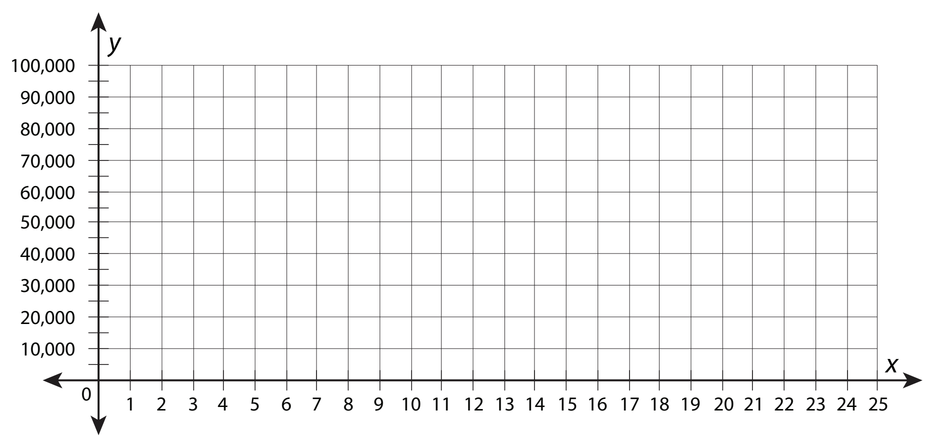
Example 1

A city's population has been steadily decreasing at the rate of 2% each year. The latest figures show that 85,000 people currently live in the city. Use the general equation $y = a(1 - r)^t$ to create an equation that predicts what the city's population will be in future years if this trend continues. In the equation, y is the future population, a is the current population, r is the rate at which the population is decreasing (written as a decimal), and t is the time in years. Create a table of values, then graph the equation.

1. Use the general equation to write a specific equation to predict the city's future population.

2. Create a table of values for the equation.

3. Use the table of values to create a graph of the equation.



Practice Skill 3: Writing Exponential Equations from Context*

Use the following information to complete problems 1–3.

Beverlee won $2,500 in a poetry contest, and is considering investing all the money in an account that earns 6.5% interest compounded quarterly. The formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ calculates the balance (A) for compounding interest, where P is the principal or the original amount invested, r is the interest rate written as a decimal, n is the number of times the interest is compounded each year, and t is the time in years.

1. Write an equation that predicts the future balance of the account.

2. Beverlee found a second account that only earns 5.5% interest but is compounded monthly. Write an equation that predicts her future balance if she decides to put the money in this account instead.

3. Write an equation that predicts the future balance if Beverlee decides that, instead of investing all her winnings, she will give away 15%, keep 10% to spend, and invest the remaining amount in the original account that earns 6.5% interest compounded quarterly.

Supportive Instructional Strategies for Mathematics I

Unit 2 Lesson 10

Suggestions for Graphic Organizers/Manipulatives

- Provide students with a blank Venn diagram. Ask them to label the circle on the left "Linear equation" and label the circle on the right "Exponential equation." Ask them to list specific characteristics of each type of equation in the appropriate circles, and then list common characteristics in the middle. Ask volunteers to share answers for the three parts of the Venn diagram. Create a master Venn diagram so that all answers can be compiled into one organizer.
    - Possible linear equation characteristics: *it has parameters of "m" for the slope and "b" for the y-intercept, the graph forms a line, it is used for situations where there is a steady/constant increase or decrease for different input values*.
    - Possible exponential equation characteristics: *it has parameters of "b" for the growth factor and "k" for the vertical shift, the graph forms a curve, it is used for situations where there is not a steady increase or decrease for different input values*.
    - Possible shared characteristics: *both have parameters, "x" is the independent variable, "y" is the dependent variable*.
- Provide students with two blank Frayer models. Ask them to label one "Linear equation" and label the other "Exponential equation." Ask them to define each term on the respective sheet, and then list all of the characteristics of each type of equation. Next, ask them to create examples of where each type of equation can be used, or seen, in the real world, as well as "non-examples" for each type. Then, pair students with a partner and ask them to share and expand their Frayer models based on their partner's model. Ask for volunteers to share the content of the four categories from their models. Create a master Frayer model of each type so that all answers can be compiled into one organizer.
- Provide students with a ruler and blank graph paper. Ask them to use the ruler to draw an x- and y-axis. Then give them the linear equation $y = 2x + 1$. Ask them to use the equation to create an x-y table with the x-coordinates $\{-3, -2, -1, 0, 1, 2, 3\}$, and then find the corresponding y-values and plot the points on the graph. Have them use the ruler to connect the points and make a line. Then, give them the exponential equation $y = 2^x + 1$. Ask students to create an x-y table with the same x-coordinates as for the linear equation, and then find the corresponding y-values. Have them plot the points on the same graph as the linear equation, and connect the dots with a smooth curve. Ask for volunteers to identify the values for the parameters of each type of equation.

Suggestions for Discourse

- Ask students, "Why is it necessary to have at least two points in order to graph a linear equation? Why is it necessary to graph at least five points for an exponential equation?" Discuss and remind students that a line can be drawn between any two points, so that is all that is necessary to determine the graph of a linear equation. However, with an exponential equation, in order to determine the shape of the curve, at least five points should be graphed. (And the more points plotted, the more accurate the graph of the curve will be.)
- Ask students to write the words "Linear" and "Exponential" on a sheet of paper. Then ask, "What words do you see 'hidden' in these words?" Discuss how the root word "line" in "Linear" can help in remembering that the graph of a linear equation is a line, and the root word "exponent" in "Exponential" can help in remembering that an exponential equation will have an exponent in it.
- Ask students to explain why sometimes only the first quadrant on the coordinate plane is used when graphing linear or exponential equations that have a real-world context. Encourage students to think about and then list quantities that cannot be negative. Then, ask them to list some real-world quantities that can be negative. Create a master list of these positive and negative quantities. Some positive quantities include: *time*, *earning money*, *traveling a certain distance*. Some negative quantities include: *temperatures below 0°*, *losing money*, *distance below sea level*, *losing yards in a football game*.
- Create a game in which students have to classify an equation as linear or exponential, and then identify parts of linear and exponential equations. Provide cards which have either a linear or exponential equation on them. Then, have the students work with a partner to see who can answer the most questions about the equations correctly in the least amount of time. Sample questions:
  - Is the equation linear or exponential?
  - Which values are the parameters?
  - Which variable is the independent variable?
  - Which value is the dependent variable?

Suggestions for English Language Learners

- Refer students to the English/Spanish glossaries found in the back of the Student Resource and Student Workbook for *CCSS Integrated Pathway: Mathematics I*.
- Provide scaffolding by using the Coaching Questions along with the Problem-Based Task.
- Discuss the multiple Guided Practices.

- Provide extra practice using ExamView.
- Encourage the use of diagrams or illustrations to demonstrate understanding. Provide students with a graph of a linear equation and a graph of an exponential equation side by side so that they can see the differences between the two.
- Provide students with a word bank during discussions and/or explanations.
- Provide a list of sentence frames in which students are asked to fill in the blanks or underline the key components of a sentence to show their understanding of the vocabulary and concepts. For example, write, "I know this is a linear function because the shape of the graph is a _______." Or, "In the equation $y = 3x - 9$, 3 is the _______ and 9 is the _______." Or, "This equation is exponential, because its graph is in the shape of a curve."

Addressing Common Errors/Misconceptions

The following student errors and/or misconceptions are commonly associated with the topics addressed in this lesson. Monitor and correct students' work as needed.

- not understanding the difference between variables and parameters in a function

  Have students write the basic forms of a linear and exponential function. Remind students that in a linear equation, *m* and *b* are the parameters and *x* is the variable, and in an exponential equation, *b* and *k* are the parameters and *x* is the variable.
- mistaking the slope for the *y*-intercept or vice versa

  Remind students that the slope, *m*, is the parameter next to the variable, *x*, and the *y*-intercept, *b*, is the parameter all by itself in the equation.
- forgetting to identify the vertical shift in the context of an exponential problem

  Ask students to write the general form of an exponential equation, and then highlight (or underline) "*k*" and write "vertical shift" next to it.
- when reading a word problem, not being able to identify the parameters in the context of the problem

  Have students make a list of information presented in the problem, or a list of the values given, and then have them label each piece of the problem as a "parameter" or a "variable."

Answer Key

Lesson 1: Graphs As Solution Sets and Function Notation

Practice Skill 1: Solving Equations in Standard Form for *y*, p. U2-14

1. $y=-\frac{7}{6}x-2$
2. $y=\frac{1}{4}x-\frac{1}{4}$
3. $y = 5x$
4. $y=-\frac{2}{3}x-1$
5. $y = 4x + 5$
6. $y = -x - 9$
7. $x=3\frac{3}{5}$ or $x = 3.6$
8. $x = 0$
9. 7 hours
10. 8 months

Practice Skill 2: Creating Equations from Context, p. U2-29

1. $y = 6x$
2. $y = 350 - 20x$
3. $y = 2x$
4. $y = 15x$
5. $45x + 42y = 175.50$
6. $y = 2.75x$
7. $y=\frac{30}{x}$
8. $y = 0.32x$
9. $y = 30 - x$
10. $y = 2x$

Practice Skill 3: Evaluating Negative Exponents, p. U2-43

1. $\frac{1}{32}$
2. 1,000
3. 8 teams
4. $\frac{1}{y^9}$
5. a^7
6. $\frac{1}{y^4}$
7. $\frac{1}{x^5}$
8. n^{20}
9. $\frac{a^4}{b^4}$
10. $\frac{1}{x^2}$

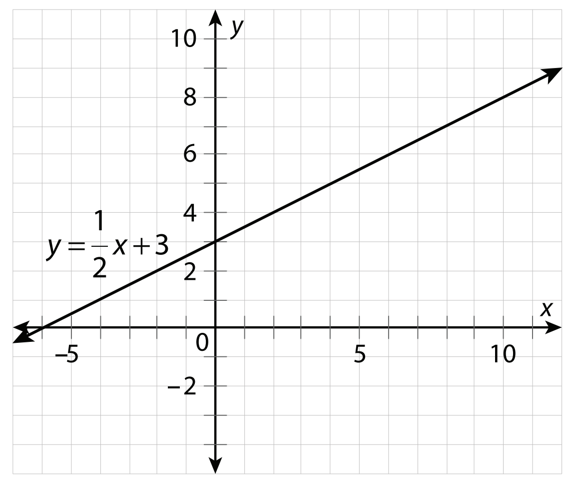
Practice Skill 4: Substituting Values for Variables*, p. U2-47

1. \$28.00
2. 19.683 in^3
3. 48 m^3

Lesson 2: Solving Linear Inequalities in Two Variables and Systems of Inequalities

Practice Skill 1: Graphing Linear Equations in Two Variables*, p. U2-59

1.



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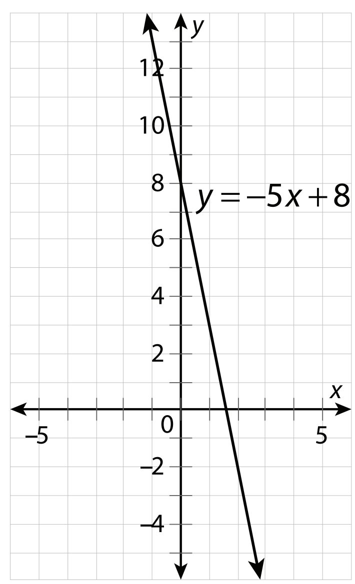

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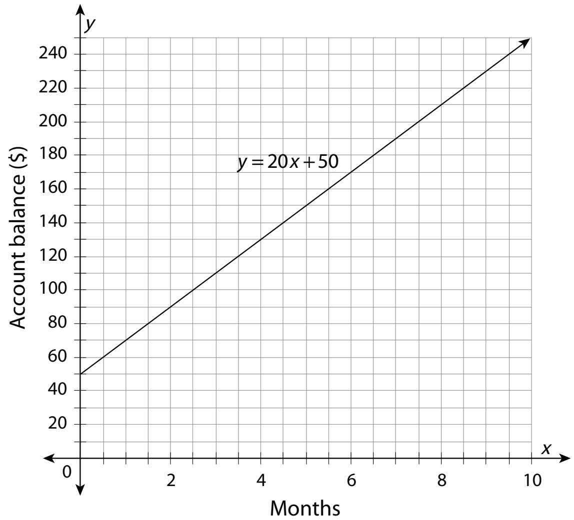


Practice Skill 2: Verifying Whether Inequalities Are True or False, p. U2-80

1. 28
2. 1, 2, 3, and 4
3. 4
4. 11
5. –5, 1, and 6
6. (–6, 5) and (4, 7)
7. any three values that are greater than or equal to 7
8. any three values that are less than 6
9. any three values that are greater than 4
10. any three coordinate points located in the shaded region, including the boundary line

Practice Skill 3: Creating Equations from Context*, p. U2-88

1. $y = 7x$
2. $y = 1.5x - 15$
3. $y = 15x$

Lesson 3: Sequences As Functions

Practice Skill 1: Understanding the Properties of Functions, p. U2-107

1. x
2. y
3. p
4. h
5. d
6. all real numbers
7. all real numbers
8. all real numbers greater than or equal to 0
9. all real numbers greater than or equal to 80
10. all real numbers greater than or equal to 0

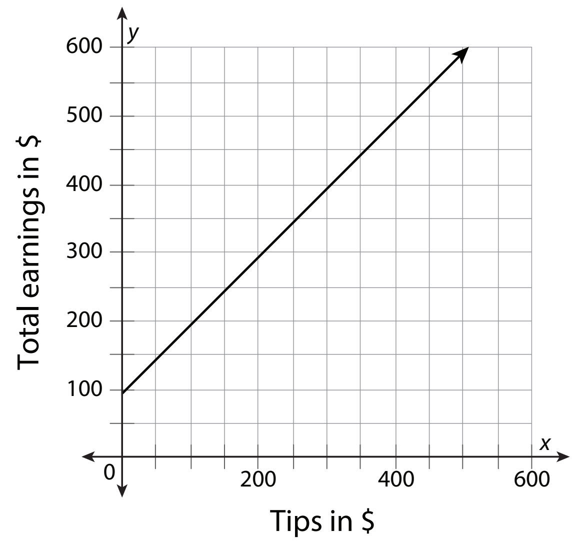
Lesson 4: Interpreting Graphs of Functions

Practice Skill 1: Graphing Linear Functions from Tables or Equations*, p. U2-118

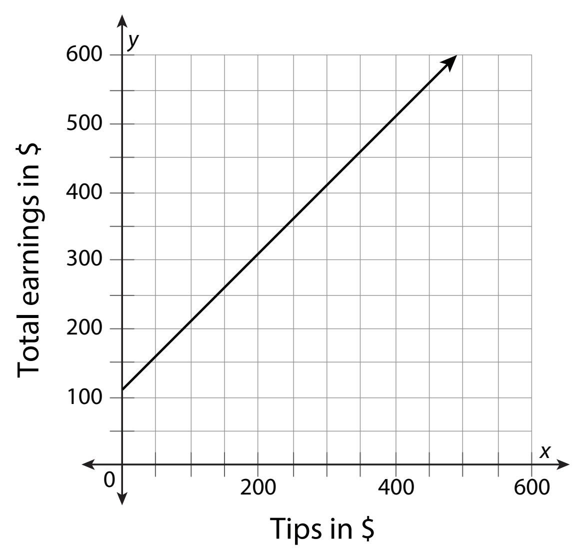
1.

Tips in $ (x)	Total earnings in $ (y)
190	284.08
245	339.08
335	429.08

2.



3.

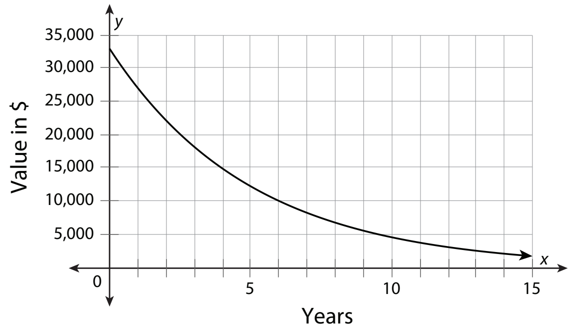


Practice Skill 2: Graphing Exponential Functions from Tables or Equations*, p. U2-125

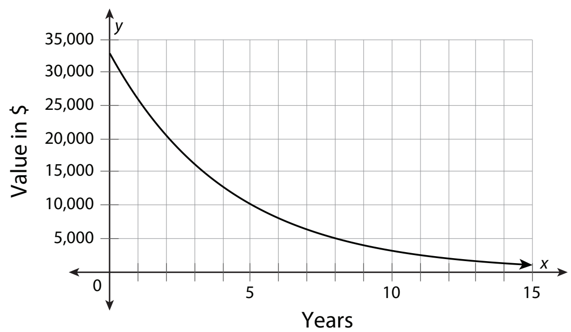
1.

Years (x)	Value in $ (y)
1	26,949
3	18,121
5	12,184

2.

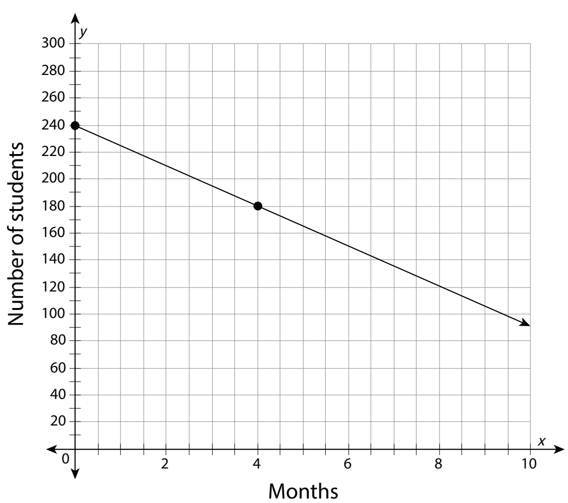


3.



Practice Skill 4: Understanding Slope*, p. U2-133

1. slope $= -\dfrac{11}{2}$
2. slope = –1
3. 15 students



Practice Skill 5: Interpreting Interval Notation, p. U2-146

1. [–9, 0)
2. (–3, 1)
3. [18, 22)
4. (25, 29]
5. (–56, ∞)
6. (–∞, –200]
7. [4, ∞)
8. the set of all real numbers that are greater than or equal to –4 and less than 2
9. the set of all real numbers that are greater than or equal to –30 and less than or equal to 20
10. the set of all real numbers that are greater than –30

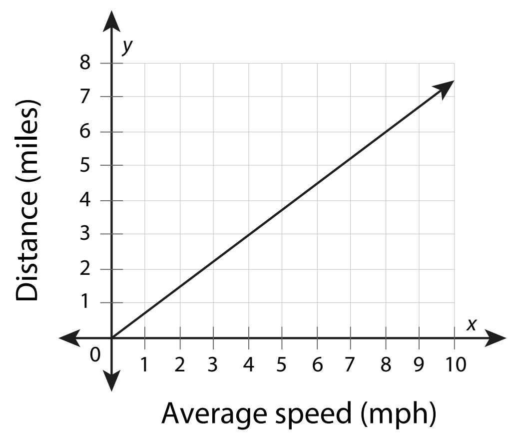
Lesson 5: Analyzing Linear and Exponential Functions

Practice Skill 1: Graphing a Function from a Table of Values*, p. U2-159

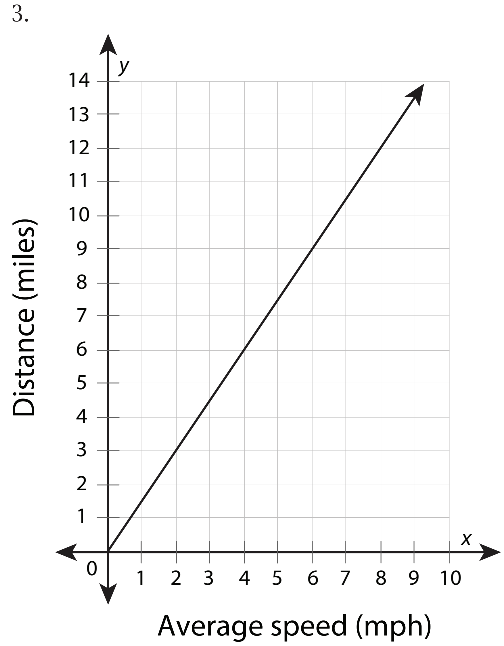
1.

Average speed (mph)	Distance (miles)
4.5	3.375
6.2	4.65
7.0	5.25

2.



3.

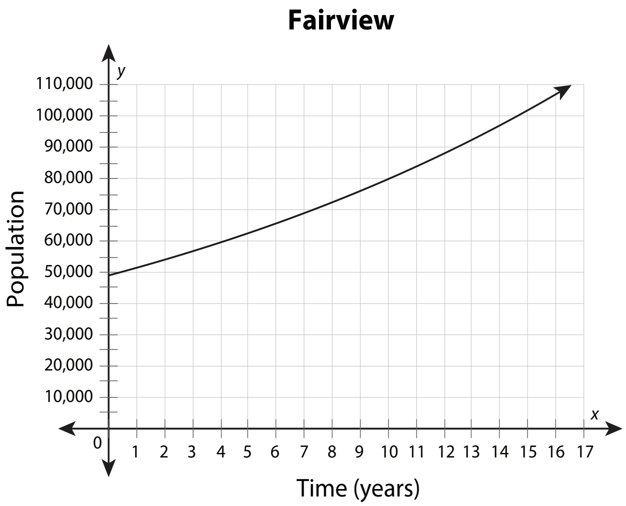


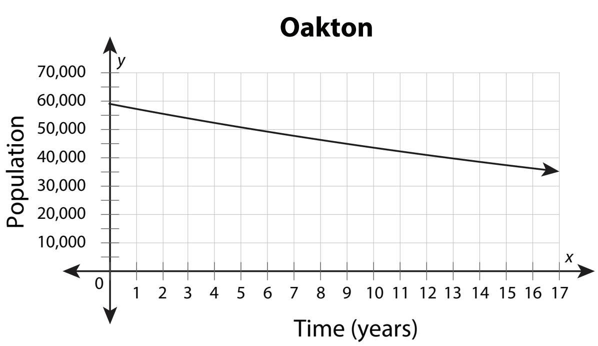
Practice Skill 2: Understanding the Rules of Exponents, Including Negative Exponents*, p. U2-164

1. x^{18}
2. $\dfrac{1}{x^3 \bullet y^{16}}$
3. $x^{12} \bullet y^{16}$

Practice Skill 3: Recognizing the General Shape of an Exponential Function (Decay or Growth)*, p. U2-169

1. $y = 49{,}000(1.05)^x$
2. $y = 59{,}000(0.97)^x$
3.





Lesson 6: Comparing Functions

Practice Skill 1: Determining the Slope of Linear Functions*, p. U2-181

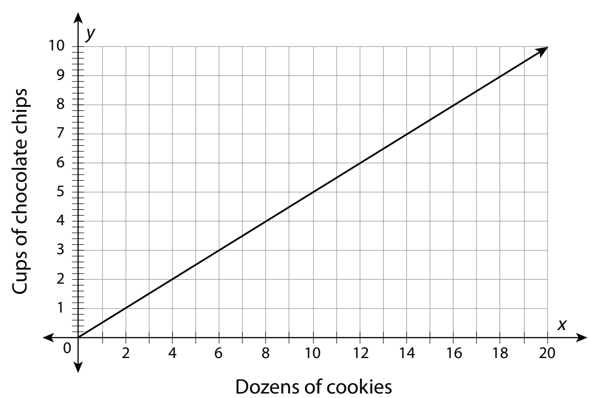
1. 2
2. $-\dfrac{2}{3}$
3. 10 gifts

Practice Skill 2: Determining the Intercepts of Linear Functions, p. U2-197

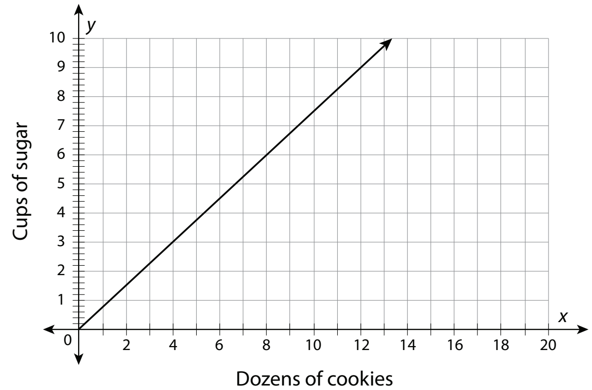
1. –3
2. $-\dfrac{1}{4}$
3. 11
4. $\dfrac{1}{6}$
5. –9
6. $\dfrac{1}{3}$
7. –8
8. –5
9. $-\dfrac{1}{2}$
10. *x*-intercept; 12 minutes

Practice Skill 5: Graphing Functions*, p. U2-205

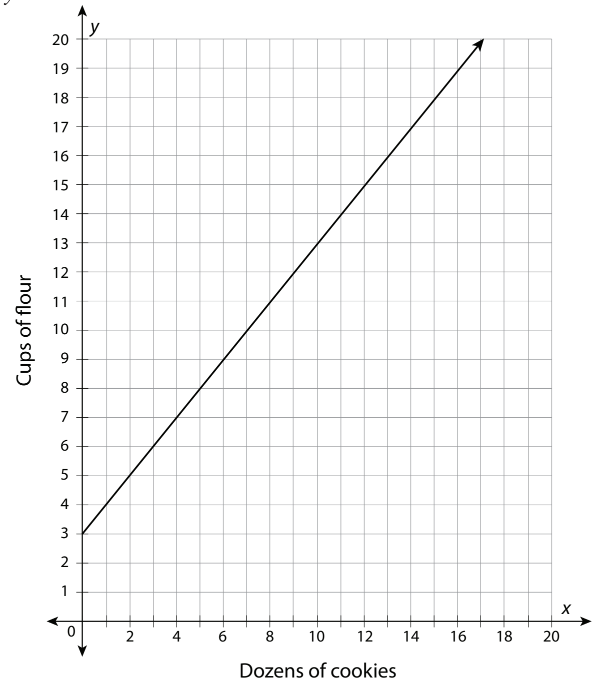
1.



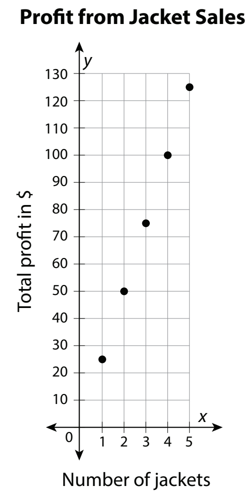
2. $y = \dfrac{3}{4}x$



3. $y = x + 3$



10.



Lesson 7: Building Functions

Practice Skill 1: Evaluating Exponential Expressions*, p. U2-214

1. $\frac{1}{64}$
2. $\frac{1}{500}$
3. 71.8 mg

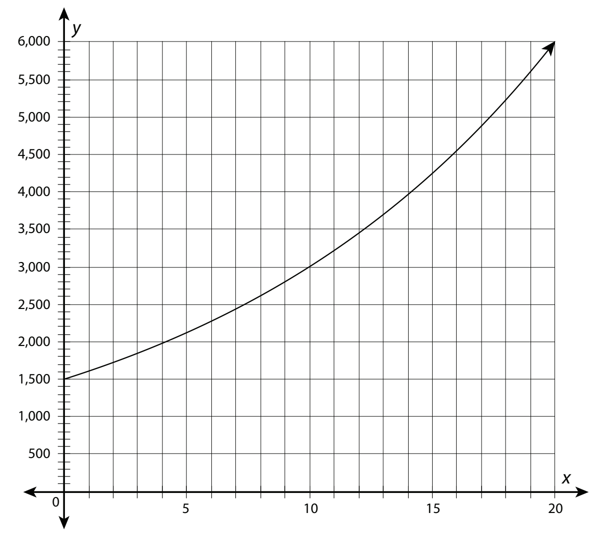
Practice Skill 2: Understanding Independent and Dependent Quantities, p. U2-237

1. h = independent variable, d = dependent variable; as h increases, d also increases
2. w = independent variable, b = dependent variable; as w increases, b also increases
3. $c = 6p$
4. $t = 40h$
5. $c = \frac{1}{2}b$
6. $g = 4.5m$
7. values for c: \$7.50, \$15.00, \$22.50, \$30.00, \$60.00, \$75.00
8. values for v: \$18,500, \$17,000, \$15,500, \$14,000, \$12,500
9. $t = 2 + z$ or $t = z + 2$

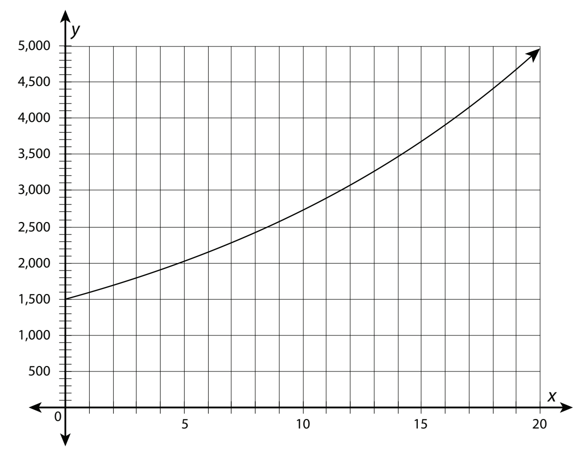
Lesson 8: Operating on Functions and Transformations

Practice Skill 1: Graphing Linear and Exponential Functions*, p. U2-249

1. $y = 1500(1.0175)^{4x}$
2.



3.



Practice Skill 2: Identifying *y*-intercepts of Graphs of Functions*, p. U2-256

1. $y=\frac{5}{3}$
2. $y=\frac{9}{4}$
3. $y=\frac{7}{2}$

Lesson 9: Arithmetic and Geometric Sequences

Practice Skill 1: Adding and Subtracting Signed Numbers, p. U2-280

1. 4
2. $-\frac{4}{5}$
3. 5
4. $\frac{5}{7}$
5. –21
6. –4
7. –2.5
8. –12
9. –7 + 5; –2 points
10. 9 – (–14); 23°F

Practice Skill 2: Identifying Linear Relationships*, p. U2-285

1. yes
2. no
3. yes

Practice Skill 3: Multiplying Signed Numbers, p. U2-297

1. –20.8
2. 48
3. $-\frac{5}{16}$
4. –70
5. –15
6. 42
7. –108
8. 87
9. –75 feet
10. –$35

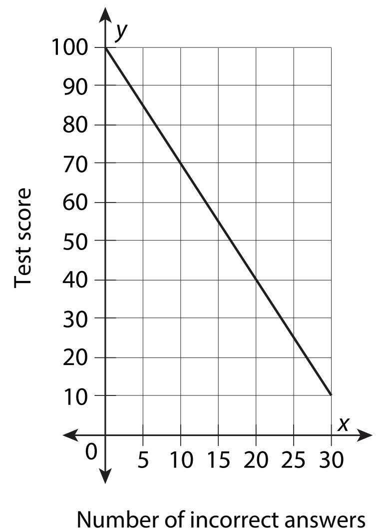
Practice Skill 4: Using Exponents*, p. U2-302

1. $\frac{1}{y^{45}}$
2. $\frac{y^{12}}{y^{8}}$
3. $\frac{64}{2^{x}}$

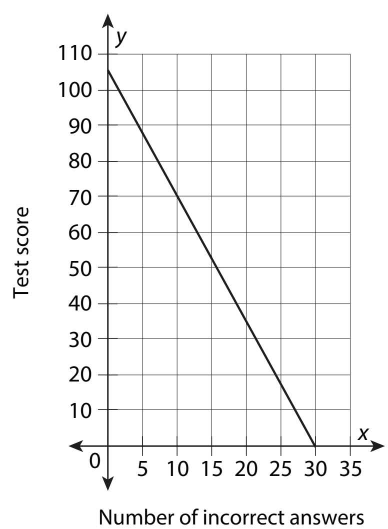
Lesson 10: Interpreting Parameters

Practice Skill 1: Graphing Equations*, p. U2-312

1.



2. $y = 3.5(30 - x)$ or $y = 105 - 3.5x$



3. The following are acceptable equations: $y=\frac{(30-x)}{30}\bullet 100$, $y=\frac{(300-10x)}{3}$, or $y=100-\frac{10x}{3}$.

Practice Skill 2: Writing Linear Equations from Context*, p. U2-318

1. $y = 3.70x$
2. $y = 3.70(x - 2)$ or $y = 3.70x - 7.40$
3. $y=\frac{7x}{20}$ or $y = 0.35x$

Practice Skill 3: Writing Exponential Equations from Context*, p. U2-324

1. $y = 2500(1.01625)^{4x}$
2. $y = 2500(1.055)^{12x}$
3. $y = 1875(1.01625)^{4x}$