

# Graphing Polynomial Functions

## Essential Questions

1. What are the different ways to graph quadratic equations?
2. What are key features of polynomial functions?
3. How can the degree and sign of the leading coefficient of a polynomial function be used to determine the end behavior of that function?
4. How can the degree and sign of the leading coefficient of a polynomial function be used to determine the number of turns of that function?
5. How can you use the number of sign changes in a function to determine the number and type of real zeros of that function?
6. How can synthetic substitution be used to find the value of a function?
7. How are factors, zeros, and  $x$ -intercepts of a polynomial function related?

SMP

1 ✓ 2 ✓

3 ✓ 4 ✓

5 ✓ 6 ✓

7 ✓ 8 ✓

## WORDS TO KNOW

### axis of symmetry

the vertical line that passes through the vertex and divides the parabola into two mirror images.



### complex conjugate

the complex number that when multiplied by another complex number produces a value that is wholly real; the complex conjugate of  $a + bi$  is  $a - bi$

### Complex Conjugate Theorem

Let  $p(x)$  be a polynomial with real coefficients. If  $a + bi$  is a root of the equation  $p(x) = 0$ , where  $a$  and  $b$  are real and  $b \neq 0$ , then  $a - bi$  is also a root of the equation.

### degree of a polynomial

the value of the exponent of the variable raised to the highest power in a polynomial function

### depressed polynomial

the result of dividing a polynomial by one of its binomial factors

### directrix of a parabola

a line that is perpendicular to the axis of symmetry of a parabola and that is in the same plane as both the parabola and the focus of the parabola; the fixed line referenced in the definition of a parabola.

### domain

the set of all possible inputs of the function.

### end behavior

the behavior of the graph as  $x$  approaches positive or negative infinity

<b>even-degree polynomial function</b>	a polynomial function in which the highest exponent is an even number. Both ends of the graph of an even-degree polynomial function will extend in the same direction, either upward or downward.
<b>factor of a polynomial</b>	any polynomial that divides evenly into the function $p(x)$
<b>Factor Theorem</b>	The binomial $x - a$ is a factor of the polynomial $p(x)$ if and only if $p(a) = 0$ .
<b>factored form of a quadratic function</b>	the intercept form of a quadratic equation, written as $f(x) = a(x - p)(x - q)$ , where $p$ and $q$ are the $x$ -intercepts of the function; also known as the <i>intercept form of a quadratic function</i>
<b>focus of a parabola</b>	a fixed point on the interior of a parabola that is not on the directrix of the parabola but is on the same plane as both the parabola and the directrix; the fixed point referenced in the definition of a parabola.
<b>Fundamental Theorem of Algebra</b>	If $p(x)$ is a polynomial function of degree $n \geq 1$ with complex coefficients, then the related equation $p(x) = 0$ has at least one complex solution (root).
<b>integer</b>	a number that is not a fraction or decimal
<b>Integer Zero Theorem</b>	If the coefficients of a polynomial function are integers such that $a_n = 1$ and $a_0 \neq 0$ , then any rational zeros of the function must be factors of $a_0$ .
<b>Irrational Root Theorem</b>	If a polynomial $p(x)$ has rational coefficients and $a + b\sqrt{c}$ is a root of the polynomial equation $p(x) = 0$ , where $a$ and $b$ are rational and $\sqrt{c}$ is irrational, then $a - b\sqrt{c}$ is also a root of $p(x) = 0$ .
<b>local maximum</b>	the greatest value of a function for a particular interval of the function; also known as a <i>relative maximum</i>
<b>local minimum</b>	the least value of a function for a particular interval of the function; also known as a <i>relative minimum</i>
<b>multiplicity (of a zero)</b>	the number of times a zero of a polynomial function occurs
<b>odd-degree polynomial function</b>	a polynomial function in which the highest exponent is an odd number. One end of the graph of an odd-degree polynomial function will extend upward and the other end will extend downward.
<b>parabola</b>	the U-shaped graph of a quadratic function; the set of all points that are equidistant from a fixed line, called the directrix, and fixed point not on that line, called the focus.
<b>polynomial function</b>	a function with a general form of $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x^1 + a_0$ , where $a_1$ is a rational number, $a_n \neq 0$ , and $n$ is a nonnegative integer and the highest degree of the polynomial

<b>Rational Root Theorem</b>	If the polynomial $p(x)$ has integer coefficients, then every rational root of the polynomial equation $p(x) = 0$ can be written in the form $\frac{p}{q}$ , where $p$ is a factor of the constant term $p(x)$ and $q$ is a factor of the leading coefficient of $p(x)$ .
<b>range</b>	the set of all possible outputs of the function; the set of $y$ -values that are valid for the function
<b>relative maximum</b>	the greatest value of a function for a particular interval of the function; also known as a <i>local maximum</i>
<b>relative minimum</b>	the least value of a function for a particular interval of the function; also known as a <i>local minimum</i>
<b>Remainder Theorem</b>	For a polynomial $p(x)$ and a number $a$ , dividing $p(x)$ by $x - a$ results in a remainder of $p(a)$ , so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ .
<b>repeated root</b>	a polynomial function with a root that occurs more than once
<b>root</b>	the $x$ -intercept of a function; also known as <i>zero</i>
<b>standard form of a quadratic function</b>	a quadratic function written as $f(x) = ax^2 + bx + c$ , where $a$ is the coefficient of the quadratic term, $b$ is the coefficient of the linear term, and $c$ is the constant term
<b>standard form of an equation of a parabola</b>	$(x - h)^2 = 4p(y - k)$ for parabolas that open up or down; $(h, k)$ is the vertex and $p$ is the minimum distance from the focus to the directrix
<b>synthetic division</b>	a shorthand way of dividing a polynomial by a linear binomial
<b>synthetic substitution</b>	the process of using synthetic division to evaluate a function by using only the coefficients
<b>turning point</b>	a point where the graph of the function changes direction, from sloping upward to sloping downward or vice versa
<b>vertex form of a quadratic function</b>	a quadratic function written as $f(x) = a(x - h)^2 + k$ , where the vertex of the parabola is the point $(h, k)$ ; the form of a quadratic equation where the vertex can be read directly from the equation
<b>vertex of a parabola</b>	the highest or the lowest point on the graph; usually written $(h, k)$
<b><math>x</math>-intercept</b>	the $x$ -coordinate of a point where a line or curve intersects the $x$ -axis
<b><math>x</math>-intercepts</b>	the $x$ -values where the parabola intersects the $x$ -axis, also known as the zeroes, or the roots, of the function
<b><math>y</math>-intercept</b>	the $y$ -value where the parabola intersects the $y$ -axis
<b>zero</b>	the $x$ -intercept of a function; also known as <i>root</i>

## Recommended Resources

- Desmos. “Parabola-Focus-Directrix.”

<http://www.walch.com/rr/10017>

This page provides a graph of a parabola showing the focus and directrix. Values for the focus and vertex can be manipulated to see the effect on the graph.

- Hotmath.com. “Rational Zeros and the Fundamental Theorem of Algebra.”

<http://www.walch.com/rr/00165>

This site provides numerous practice problems centering on determining the zeros of a function using the Fundamental Theorem of Algebra. Users may click to show hints and step-by-step guidance toward solutions.

- Illuminations. “Function Matching.”

<http://www.walch.com/rr/00166>

Given a graphed function, users manipulate this interactive applet to recreate the graph of the identical function. Users can choose the type of function to match, or match a random function generated by the site.

- MathIsFun.com. “Remainder Theorem and Factor Theorem.”

<http://www.walch.com/rr/00167>

This site summarizes both the Remainder Theorem and the Factor Theorem, and features examples as well as practice problems with worked solutions.

- Purplemath.com. “Conics: Parabolas: Introduction.”

<http://www.walch.com/rr/10018>

This site provides an overview of the focus-directrix method for graphing parabolas.

- Purplemath.com. “Solving Polynomials.”

<http://www.walch.com/rr/00168>

This site offers a summary of solving large polynomials in addition to information on factoring and graphing polynomial functions.