Working with Parallel and Perpendicular Lines

Prerequisite Skills

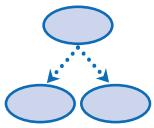
This lesson requires the use of the following skills:

- writing linear equations
- graphing linear equations

Introduction

The slopes of parallel lines are always equal, whereas the slopes of perpendicular lines are always opposite reciprocals. It is important to be able to determine whether lines are parallel or perpendicular, but the creation of parallel and perpendicular lines is also important. In this lesson, you will write the equations of lines that are parallel and perpendicular to a given line through a given point.

Key Concepts



- You can write the equation of a line through a given point that is parallel or perpendicular to a given line if you know the slope of the given line.
- Writing the given equation in slope-intercept form allows you to quickly identify the slope, m, of the equation.
- If the given equation is not in slope-intercept form, take a few moments to rewrite it.

Writing the Equation of a Line Parallel to a Given Line Through a Given Point
1. Rewrite the given equation in slope-intercept form if necessary.
2. Identify the slope of the given line.
3. Write the general point-slope form of a linear equation: $y - y_1 = m(x - x_1)$.
4. Substitute the slope of the given line for m in the general equation.
5. Substitute x and y from the given point into the general equation for x_1 and y_1.
6. Simplify the equation.
7. Rewrite the equation in slope-intercept form if necessary.

- Writing the equation of a line perpendicular to a given line through a given point is similar to writing equations of parallel lines.
- The slopes of perpendicular lines are opposite reciprocals.

Writing the Equation of a Line Perpendicular to a Given Line Through a Given Point

1. Rewrite the given equation in slope-intercept form if necessary.
2. Identify the slope of the given line.
3. Find the opposite reciprocal of the slope of the given line.
4. Write the general point-slope form of a linear equation: $y - y_1 = m(x - x_1)$.
5. Substitute the opposite reciprocal of the given line for m in the general equation.
6. Substitute x and y from the given point into the general equation for x_1 and y_1.
7. Simplify the equation.
8. Rewrite the equation in slope-intercept form if necessary.

- The shortest distance between two points is a line.
- The shortest distance between a given point and a given line is the line segment that is perpendicular to the given line through the given point.

Finding the Shortest Distance Between a Given Point and a Given Line

1. Follow the steps outlined previously to find the equation of the line that is perpendicular to the given line through the given point.
2. Find the intersection between the two lines by substituting the value of y from the second equation in for y in the first equation.
3. Solve for x.
4. Substitute the x-value into the equation of the given line to find the y-value.
5. Find the distance between the given point and the point of intersection of the given line and the perpendicular line using the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Common Errors/Misconceptions

- attempting to identify the slope of the given line without transforming the equation into slope-intercept form
- incorrectly identifying the slope of the given line
- incorrectly finding the slope of the line parallel to the given line
- incorrectly identifying the slope of the line perpendicular to the given line
- improperly substituting the x- and y-values into the general point-slope equation