Problem-Based Task: Pave the Way

Task Overview

Focus

How can the slope criteria for parallel lines be used to write equations of lines that represent the edges of a two-lane road? Which formula can be used to calculate the distance between two points on the edges of the road? In this lesson, students will apply their knowledge of parallel lines and their slopes to create equations of lines given specific points representing the edges of a road. They will also determine if the width of the road meets the minimum required distance by applying the distance formula.

This activity will provide practice with:

- identifying slope values given an equation in slope-intercept form
- substituting values into point-slope form of a linear equation
- writing equations of lines given a point on the line and the slope
- calculating distances between two points using the distance formula
- analyzing results of a calculation to determine if they meet given criteria
- drawing conclusions based on calculations

Introduction

This task should be used to explore or apply the skill of proving the slope criteria for parallel lines and finding the equation of a line parallel to a given line that passes through a given point. Students should already be familiar with slope-intercept form of a linear equation, writing linear equations, and calculating distance using the distance formula.

Begin by reading the problem and clarifying the meaning of distance formula, parallel lines, point-slope form, and slope-intercept form.

distance formula — a formula that states the distance between points (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

parallel lines — lines in a plane that do not share any points and never intersect; parallel lines have the same slope

point-slope form — the equation of a straight line in the form $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) are the coordinates of a given point on the line

slope-intercept form — the equation of a straight line in the form $y = mx + b$, where m is the slope of the line and b is its y-intercept

Facilitating the Task

Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will recognize that the goal of the task is to write equations of the lines that represent the edges of a two-lane road. They will create these equations by applying their knowledge of the slopes of parallel lines, as well as their knowledge of different forms of linear equations such as slope-intercept form and point-slope form. They will also plan a solution pathway to determine if the distance between two points will satisfy a minimum width requirement for the road.

- **SMP 2:** Reason abstractly and quantitatively.

  Students will reason abstractly as they make sense of the relationships and values of the components of the equations of the lines they write. They will reason that since parallel lines have the same slope values, these values will be used in the formula when writing equations of parallel lines. They will reason quantitatively as they perform the algebraic manipulations in the equations that they write, as well as when they use the distance formula to calculate the distance between the two given points representing points on the edge lines of the road.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will construct viable arguments about the process of creating equations of lines representing parallel edges of the road. They will explain their assumptions and calculated results to justify their equations. They will use counter-examples when presenting their arguments to others, justifying their conclusions about the width of the road.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly identifying the slope of the given line

  Have students write down the slope-intercept form, $y = mx + b$, and remind them that the value for m represents the slope of the line.

- incorrectly finding the slope of the line parallel to the given line

  Remind students of the difference in the slope criteria for parallel and perpendicular lines, specifically that parallel lines must have the exact same slope.

- improperly substituting the x- and y-values into the general point-slope equation

  Have students write the point slope formula, $y - y_1 = m(x - x_1)$, then ask them to list the values for each coordinate, (x_1, y_1) and the value for the slope before substituting the values into the formula.

- incorrectly using the x- and y-coordinates in the distance formula

  Have students write down the distance formula and the values for the x_1, y_1, x_2, and y_2 coordinates as they calculate the distance between the two given points, (16, 1) and (4, 21).

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task:
  - Discuss with them the importance of ensuring that edge lines on a road are parallel. Ask them, "Why is it crucial that edges lines on a road be completely parallel?" (**Answer:** If the edge lines were not perfectly parallel, it is possible that one side of the road could have less space than the other side, and as a result, larger cars might not fit in between the lines.)
  - Ask, "What are some examples of values of the width of the road that would meet the law's requirement? Which word or phrase in the task identifies the possible range of values for the width?" (**Answer:** The task states that the law requires the width of a two-lane road must be at least 24 feet. The phrase "at least" means that the width can be 24 feet or more than 24 feet. Some possible values for the width are anything greater than 24 feet, such as 25, 26, 27, etc.)
  - Ask them to clarify the meaning of the slope of the line in terms of the components of the value. Ask, "In this task, the slope of the equation representing the center lane of the road is $\frac{3}{5}$. What do the 3 and the 5 represent?" (**Answer:** The slope is the steepness of a line, or rise over run. For every three units that this line "rises" in a vertical direction, it "runs" five units in a horizontal direction.)
- If students are having difficulty beginning the task, ask them, "What information are you given in the problem that you can visualize by using a coordinate plane?" (**Answer:** The line representing the center of the road, $y = \frac{3}{5}x + 5$, can be graphed on a coordinate plane by first plotting the y-intercept, which is 5, and then plotting the slope, which is $\frac{3}{5}$. Then the coordinates representing points on each edge line of the road can be plotted.)
- As students begin working on the task, ask them, "Is it necessary to find the equations of the lines representing the edges of the road before you find the distance between the two points on the edge lines? In other words, does the information found by writing the equations of the lines affect finding the distance, or the width, between the two given points? Explain." (**Answer:** No. The distance between the two points can be found independently of writing the equations for the two edge lines. The only information necessary for finding the width of the road, or the distance, is the coordinates of the points. This information is given in the original problem.)

- Once students have found the equations for the two edge lines, ask them, "If you wanted to graph these equations, how could you graph the *y*-intercepts for each equation, since they are in improper fraction form? What would these intercepts be in decimal form?" (**Answer:** The *y*-intercept for the equation containing the point (16, 1) is $-\frac{43}{5}$. The improper fraction can be changed to a decimal form of -8.6. The *y*-intercept for this line would be plotted with the point (0, –8.6). The *y*-intercept for the equation containing the point (4, 21) is $\frac{93}{5}$. The improper fraction can be changed to a decimal form of 18.6. The *y*-intercept for this line would be plotted with the point (0, 18.6).)
- Ask students, "When the distance between the two points is calculated, the result in radical form is $\sqrt{544}$. Why is it unnecessary to express the simplified result more specifically than 'approximately 23 feet'?" (**Answer:** When the radical is simplified, the result is 23.324, rounded to the nearest thousandth. Since this value is already less than 24 feet, the exact value is insignificant. This value does not meet the minimum requirement for the road width.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask for volunteers to discuss their strategies and processes for determining the equations of the lines representing the two edge lines on the road. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Ask students to share their reasoning processes, including how they determined which values to substitute into the formulas, and how they were able to draw conclusions based on their results of applying the distance formula. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of the components of a linear equation, such as slope and *y*-intercept, as well as connecting the distance between the two points to its representation of the width of the road in the task.

Connecting to Key Concepts

The slopes of parallel lines are always equal.

- Students will apply the slope from the given equation representing the center of the road to the equations for each of the edge lines. Since the slope values must be the same, the slope of $\frac{3}{5}$ will be used throughout the equations.
- To write the equation of a line through a given point that is parallel to a given line if you know the slope of the given line, substitute the slope, m, and the given point, (x_1, y_1), into the point-slope form, $y - y_1 = m(x - x_1)$.
- In this task, students will apply the point-slope form by substituting the known values for slope and a point on the line. They will write equations in slope-intercept form that represent the parallel edge lines of a two-lane road.
- To find the distance between two points on the coordinate plane, use the formula $\sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$.
- Students will apply the distance formula to find the distance between the two given points. They will use this value to draw conclusions about the width of the road, specifically whether or not this width meets the requirements of the law.

Extending the Task

- To extend the task, ask students to work together to identify new points on the edge lines of the road that will satisfy the requirement that the two-lane road must be a minimum of 24 feet wide. Ask students to develop a process of selecting points other than choosing random points and performing trial-and-error calculations. Have students brainstorm their ideas with each other, reminding them that their new points must also work in the equations of the lines that have a slope of $\frac{3}{5}$. Have them write down their new equations, along with their calculations. Ask for volunteers to share their results and justify their new, acceptable road width value.
- Another option for extending the task is to ask students to create a square using the given points on the edge lines of the road. Have them identify the two new vertices of the square, located on the line that represents the center of the road. Discuss strategies to determine the coordinates of these new points and how to determine that the four corners of the square are perpendicular (e.g., slope criteria for perpendicular lines).

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** I persevered by first recognizing that the goal of the task is to write equations of the lines that represent the edges of a two-lane road. I planned my solution pathway by using the given information and writing the equations using the point-slope form. I also assessed the reasonableness of my results and how they related to the goal of the task.)
- **For SMP 2, ASK:** "How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?" (**Answer:** I used abstract reasoning as I made sense of the relationships and values of the components of the equations of the lines I wrote to represent the edge lines of the road. I reasoned that the slopes of parallel lines will be the same, and used this fact in my equations.) Ask, "Which of your strategies represent quantitative reasoning?" (**Answer:** I used quantitative reasoning as I algebraically manipulated the equations I wrote so that I could get them in slope-intercept form. I also calculated the distance between two points using the distance formula.)
- **For SMP 3, ASK:** "How did you construct viable arguments and critique the reasoning of others?" (**Answer:** I constructed viable arguments about the process of creating equations of lines representing parallel edges of the road by explaining the results of my equations and why I chose the values I did for the slope. I also used counter-examples of values to demonstrate which values are valid for the width of the road.)

Alternate Strategies or Solutions

- Students may choose to graph the given information (the two points and equation for the center line of the road) on a coordinate plane. Encourage them to ensure the scales on the x- and y-axes are large enough to accommodate the given points. Students may also choose to draw their own sketches of a two-lane road with the center line and the points on the edge lines. This may help students who are visual learners and benefit from seeing a "picture" of the problem.

Technology

Students can use scientific calculators or graphing calculators for extending the task activities.