Problem-Based Task: When Is the Next Dose Due?

Coaching Sample Responses

a. Using synthetic substitution, what is $M(3)$?

By the Remainder Theorem, $M(3)$ should be the remainder when you divide the polynomial, $M(t) = -x^3 + 5x^2 + 3x + 18$, by $x - 3$.

$$\begin{array}{r|cccc} 3 & -1 & 5 & 3 & 18 \\ & & -3 & 6 & 27 \\ \hline & -1 & 2 & 9 & \boxed{45} \end{array}$$

The remainder is 45; therefore, using synthetic substitution, $M(3) = 45$.

b. Using synthetic substitution, what is $M(4)$?

By the Remainder Theorem, $M(4)$ should be the remainder when you divide the polynomial, $M(t) = -x^3 + 5x^2 + 3x + 18$, by $x - 4$.

$$\begin{array}{r|cccc} 4 & -1 & 5 & 3 & 18 \\ & & -4 & 4 & 28 \\ \hline & -1 & 1 & 7 & \boxed{46} \end{array}$$

The remainder is 46; therefore, using synthetic substitution, $M(4) = 46$.

c. Using synthetic substitution, what is $M(5)$?

By the Remainder Theorem, $M(5)$ should be the remainder when you divide the polynomial, $M(t) = -x^3 + 5x^2 + 3x + 18$, by $x - 5$.

$$\begin{array}{r|cccc} 5 & -1 & 5 & 3 & 18 \\ & & -5 & 0 & 15 \\ \hline & -1 & 0 & 3 & \boxed{33} \end{array}$$

The remainder is 33; therefore, using synthetic substitution, $M(5) = 33$.

d. Using synthetic substitution, what is $M(6)$?

By the Remainder Theorem, $M(6)$ should be the remainder when you divide the polynomial, $M(t) = -x^3 + 5x^2 + 3x + 18$, by $x - 6$.

$$\begin{array}{r|cccc} 6 & -1 & 5 & 3 & 18 \\ & & -6 & -6 & -18 \\ \hline & -1 & -1 & -3 & \boxed{0} \end{array}$$

The remainder is 0; therefore, using synthetic substitution, $M(6) = 0$.

e. For each value of t given in parts a–d, what does a remainder mean in terms of the context of the problem?

A remainder of a value other than 0 indicates that the medication is still in the bloodstream.

f. What does a remainder of 0 mean in the context of the problem?

A remainder of 0 means that the medication is no longer in the bloodstream.

g. After how many hours is the medication completely eliminated from the bloodstream?

The medication is completely eliminated from the bloodstream after 6 hours.

h. Why would the package directions suggest taking a second dose after 4–6 hours?

Comparing the remainders for $M(3)$, $M(4)$, and $M(5)$ shows that the amount of medication in the bloodstream peaks at the 4-hour mark, then starts to decrease, meaning the medication is wearing off. Taking medication before 4 hours have passed could be harmful, since the first dose is still building up in the bloodstream. If the second dose is taken after the first dose starts wearing off but before it is completely eliminated from the bloodstream, then the patient is less likely to feel the effects of the ailment as the first dose wears off, since some medication will still be in the bloodstream.

i. Are these directions accurate?

Yes, the directions are accurate. Taking a second dose after 4 hours have passed but before 6 hours have passed will lessen the effects of the ailment, without resulting in an overdose.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.