Using Coordinates to Prove Geometric Theorems with Slope and Distance

Prerequisite Skills

This lesson requires the use of the following skills:

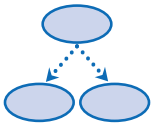
- calculating slope
- writing equations of lines
- using the Pythagorean Theorem

Introduction

It is not uncommon for people to think of geometric figures, such as triangles and quadrilaterals, to be separate from algebra; however, we can understand and prove many geometric concepts by using algebra. In this lesson, you will see how the distance formula originated with the Pythagorean Theorem, as well as how distance between points and the slope of lines can help us to determine specific geometric shapes.

Key Concepts

Calculating the Distance Between Two Points



- To find the distance between two points on a coordinate plane, you have used the Pythagorean Theorem.
  - After creating a right triangle using each point as the endpoints of the hypotenuse, you calculated the vertical height (a) and the horizontal height (b).
  - These lengths were then substituted into the Pythagorean Theorem ($a^2 + b^2 = c^2$) and solved for c.
  - The result was the distance between the two points.
- This is similar to the **distance formula**, which states the distance between points (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- Using the Pythagorean Theorem:
  - Find the length of a: $|y_2 - y_1|$.
  - Find the length of b: $|x_2 - x_1|$.
  - Substitute these values into the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$

$$c^2 = |y_2 - y_1|^2 + |x_2 - x_1|^2$$

$$c = \sqrt{|y_2 - y_1|^2 + |x_2 - x_1|^2}$$

- Using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- We will see in the Guided Practice an example that proves the calculations will result in the same distance.

Calculating Slope

- To find the **slope**, or steepness of a line, calculate the change in y divided by the change in x using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Parallel and Perpendicular Lines

- **Parallel lines** are lines in a plane that do not share any points and never intersect.
  - To prove that two lines are parallel, you can show that the slopes of both lines are equal.
- **Perpendicular lines** are lines that intersect at a right angle (90˚). The slopes of perpendicular lines are always opposite reciprocals.
  - To prove that two lines are perpendicular, you can show that the slopes of both lines are opposite reciprocals.
  - When the slopes of perpendicular lines are multiplied, the result will always be –1.
  - Horizontal and vertical lines are always perpendicular to each other.

Common Errors/Misconceptions

- incorrectly using the x- and y-coordinates in the distance formula
- subtracting negative coordinates incorrectly
- incorrectly calculating the slope of a line
- incorrectly determining the slope of a line that is perpendicular to a given line
- assuming lines are parallel or perpendicular based on appearance only
- making determinations about the type of polygon without making all the necessary calculations