Problem-Based Task: It's Electric!

Task Overview

Focus

How are key characteristics of a polynomial function identified from a graph of the function? How can a graph of a polynomial function provide information about a real-world scenario? In this task, students will analyze the graph of a polynomial function and identify characteristics of the function that will help make predictions about the growth of demand for electricity in the United States from 2040 to 2050.

This activity will provide practice with:

- analyzing a graph of a polynomial function
- identifying turning points of a graph
- classifying turning points as local maximum or local minimum
- representing the roots of a function as ordered pairs
- describing end behavior of a function
- making predictions about the behavior of a function based on its characteristics

Introduction

This task should be used to explore or to apply the skill of graphing polynomial functions, as well as analyzing the graphs of polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. Students should already be familiar with recognizing the format of a polynomial function, identifying intercepts of many types of functions, and identifying ordered pairs on a graph.

Begin by reading the problem and clarifying the meaning of *end behavior*, *local maximum*, *local minimum*, *root*, and *turning point*.

end behavior — the behavior of the graph as x approaches positive or negative infinity

local maximum — the greatest value of a function for a particular interval of the function; also known as a *relative maximum*

local minimum — the least value of a function for a particular interval of the function; also known as a *relative minimum*

root — the x-intercept of a function; also known as *zero*

turning point — a point where the graph of the function changes direction, from sloping upward to sloping downward or vice versa

Facilitating the Task

Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will recognize that the goal is to analyze a graph of a polynomial function representing information about the growth of demand for electricity in the United States, and predict whether they expect the growth of electricity to increase or decrease between 2040 and 2050. Specifically, they will look at the end behavior, turning points, and the roots of the polynomial function on the graph in order to make a valid prediction about what the graph will look like in the interval that represents the years between 2040 and 2050. They will also describe the behavior of the graph using specific points on the graph that represent the turning points and roots. Their predictions about the graph's behavior between 2040 and 2050 will be based their analysis of these characteristics of the graph.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will construct viable arguments and justify their choices for predicting whether or not they expect the growth of demand for electricity in the United States to increase or decrease between the years 2040 and 2050. They will examine and analyze the given graph of the polynomial function which represents the approximate growth since 1950 and includes projections through 2040. When presenting their arguments to others, they will explain key characteristics of the graph, specifically the turning points, roots, and end behavior, and they will justify their expectations about the growth between 2040 and 2050 based these characteristics.

- **SMP 4:** Model with mathematics.

  Students will recognize that this scenario can be modeled with the graph of a polynomial function representing the growth of demand for electricity in the United States from 1950 and projected through 2040. They will analyze the specific characteristics of the graph which will provide information about the future behavior of the graph. This analysis will allow them to make a prediction about their expectations for the growth of demand for electricity from 2040 to 2050.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- confusing the roots of the function for the turning points

  Review the definition of roots and turning points of a function with students. Remind them that the roots are also the x-intercepts of a function, and these are the points where the graph intersects the x-axis. Also remind them that turning points are the points where the graph changes direction.

- incorrectly classifying local minimum points for local maximum points and vice versa

  Have students write the definitions for local minimum and maximum points. Discuss the meanings of the words "minimum" and "maximum" and how they relate to the points on the graph where the slope of the graph changes direction.

- mistakenly identifying the roots of the function as (0, 2003) and (0, 2011) instead of (2003, 0) and (2011, 0)

  Review the process for writing an ordered pair to represent a point on a graph. Remind students that the *x*-coordinate is first in an ordered pair, and it is found by moving left or right on the *x*-axis. Then the *y*-coordinate is found. On the graph, the roots lie directly on the *x*-axis, therefore the *x*-coordinate is represented by the point where the graph crosses the *x*-axis.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, discuss their initial thoughts about accomplishing the goal of the task. Ask them, "Based solely on briefly examining the given graph of the polynomial function representing the growth of demand for electricity since 1950, what can you predict about the growth of demand for 2040 through 2050?" (**Answer:** Answers may vary. Some students may think the demand will increase since the graph is already sloping upwards at the year 2040. Some students may think the demand will decrease, because there could be another turning point at the year 2040, since are several turning points from the previous years.)
- Ask students, "Why do you think the growth of demand for electricity in the United States was much higher from 1950 through 1970, compared to the overall trend of decreasing growth after 1970?" (**Answer:** Answers may vary. In general, electricity was the most well-known form of power in those earlier decades. After 1970, alternate power sources were being discovered and implemented around the United States.)
- Before students begin the task, ask them to classify the type of function shown on the graph. Ask them, "Based on the graph of the polynomial function, is this an even or an odd-degree polynomial? How can you tell?" (**Answer:** The function represented in the graph is an even-degree polynomial, because both ends of the function extend upward.)
- Ask students, "Why is only the first quadrant represented in this problem? What does each axis represent in the context of the scenario?" (**Answer:** Since the scenario represents the percent of growth of demand for electricity, the values representing the independent variable, *t* for time, are positive. Therefore the first quadrant is used, since time is not negative. The *x*-axis represents the time in years, and the *y*-axis represents the percentage of growth of electricity demand.)

- As students are working through the task and analyzing the turning points, ask them, “Based on what you know about the relationship between the number of turning points and the degree of the polynomial, what is the smallest possible degree for this polynomial function? Explain.” (**Answer:** Since there are seven turning points, the smallest possible degree of the function is 8, because the maximum number of turning points is one less than the degree of a polynomial. The polynomial could have a higher degree, but it is at least an eighth-degree polynomial since there are seven turning points.)
- Ask students, “Explain the turning points in terms of the slope of the graph. Use a specific point from the graph to support your explanation.” (**Answer:** A turning point on the graph indicates a change in slope. If the graph was sloping upwards, then it will be sloping downwards at a turning point. For example, from 1960 to 1970, the growth of demand was sloping upwards. At the point (1970, 8), the graph now slopes downwards.)
- As students are working through the task, ask them to examine the graph in terms of intervals. Ask them, “From the years 1950 through 2000, what pattern do you notice about the turning points in this time interval? Explain what this means in the context of the scenario.” (**Answer:** From 1950 through 2000, there are four turning points which occur approximately every ten years. This means that about every ten years, the growth of demand for electricity changes from either increasing to decreasing, or the demand changes from decreasing to increasing.)
- Ask students to analyze the behavior of the graph at the year 2010. Ask them, “What value is represented when the graph went below zero in the year 2010?” (**Answer:** In 2010, the growth of demand for electricity was approximately –1%.)
- Ensure students’ understanding of the roots of an equation. Ask them, “What does it mean that this polynomial function has two roots? Explain in the context of the scenario and also using proper function terminology.” (**Answer:** There are two points on the graph where the graph intersects the x-axis, which are called the zeros or roots of the function. In the context of the scenario, this represents the years in which the growth of demand for electricity was 0. In function terminology, the roots of a function represent the values that are solutions to the equation of the function, or in other words, the values that satisfy the equation.)
- As students are working to describe the end behavior of the function, ask them, “Why do you think it is difficult to accurately predict the end behavior of the function?” (**Answer:** One reason that it is difficult to accurately predict the end behavior of the function is that there are seven turning points represented on the graph, which means that the growth for demand of electricity has changed direction several times over the course of the years since 1950. One decade the growth increases, and then the next decade the growth decreases. Based on this behavior, it is difficult to predict if the growth will increase or decrease after the year 2040.)

- Ask students to share their thoughts about alternate power sources. Ask them, "What other types of power sources are available today that might cause the growth of demand for electricity to decrease as the years go by?" (**Answer:** Some common alternate power sources are wind power and solar power. Some students may discuss biofuels, tidal and wave energy, and hydro power.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask for volunteers to discuss their initial thoughts and predictions about if they expect the growth of electricity demand in the United States to increase or decrease between 2040 and 2050. Discuss their general observations about the shape of the graph, and the general trends of the graph. Ask them to explain how they identified the turning points and classified which ones were local minimum and which ones were local maximum points. Also ask them to explain how they identified the ordered pairs which represented the roots of the function. Ask for volunteers to explain how the compilation of the characteristics of the graph of the function led them to their predicted expectations about the growth of demand for the years of 2040 to 2050. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses. Ask students to share their reasoning process and methods of analyzing the turning points, roots, and end behavior of the polynomial function. Encourage them to discuss which characteristics of the graph had more weight in their overall expectation for the growth of demand for 2040 – 2050. Focus on the use of precise mathematical language and clarity, specifically when discussing the behavior of the function, such as the local minimum and local maximum turning points, as well as the roots of the function.

Connecting to Key Concepts

- To determine the **end behavior** of a polynomial function, or the behavior of the graph as x approaches positive or negative infinity, consider the highest degree of the polynomial and its coefficient, ax^n.

  In this task, students will be examining the graph of a polynomial function and making predictions about its end behavior based on the shape of the graph as the amount of time, in years, increases.
- A **turning point** of a function is a point where the graph of the function changes from sloping upward to sloping downward or, alternatively, from sloping downward to sloping upward.

  Students will identify the turning points on the graph, and they will specify these points as ordered pairs. Specifically, they will identify 7 turning points on the graph.

- A turning point corresponds to a **local maximum**, the greatest value of a function for a particular interval of the function, or a **local minimum,** the least value of a function for a particular interval of the function.

  Students will examine each of the seven turning points and classify each one as either a local maximum, or a local minimum, based on whether the graph increases or decreases after the point.

- The highest degree of the polynomial determines the maximum number of **roots**, or *x*-intercepts of a function.

  Students will examine the graph of the polynomial function and recognize that there are two roots, or *x*-intercepts on the graph. They will specify these roots with ordered pairs which each have a zero for the *y*-coordinate, indicating that the graph intersects the *x*-axis.

Extending the Task

- To extend the task, ask students to work with a partner and create an eighth-degree polynomial function with a positive leading coefficient. Encourage them to use coefficients which are small. After each pair of students has written an eighth-degree polynomial, ask them to graph the function using their graphing calculators or other graphing software. Ask them to identify and write down all turning points and roots of their function. Ask for volunteers to share the graphs of their functions, and note if any pair of students was able to graph a function similar to the one in the task. If so, discuss the similarities and differences of their graphs to the one in the task.
- Another option for extending the task is to ask students to work together and use graph paper to sketch three graphs: an eighth-degree polynomial function with a negative leading coefficient, an odd-degree function with a positive leading coefficient, and an odd-degree function with a negative leading coefficient. Ask for volunteers to share the sketches of their three graphs, and discuss the ends of each graph and how the direction of these ends determines which type of polynomial function is represented.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** I made sense of the problem by first recognizing that the goal of the task is to analyze a graph of a polynomial function representing information about the growth of demand for electricity in the United States, and predict whether I expect the growth of electricity to increase or decrease between 2040 and 2050. Specifically, I looked at the end behavior, turning points, and the roots of the polynomial function on the graph so I could make a valid prediction about what the graph will look like in the interval which represents the years between 2040 and 2050. I described the behavior of the graph using specific points on the graph which represent the turning points and roots. My prediction about the graph's behavior between 2040 and 2050 was based on my analysis of these characteristics of the graph.)

- **For SMP 3, ASK:** "How did you construct viable arguments and critique the reasoning of others?" (**Answer:** I constructed viable arguments by explaining my choices for predicting whether or not I expect the growth of demand for electricity in the United States to increase or decrease between the years 2040 and 2050. First I examined and analyzed the given graph of the polynomial function which represents the approximate growth since 1950 and includes projections through 2040. Then I explained the specific characteristics of the graph, specifically the turning points, roots, and end behavior, and then I justified my expectations about the growth between 2040 and 2050 based on my analysis of these characteristics.)
- **For SMP 4, ASK:** "How did you use mathematics to model this particular scenario?" (**Answer:** I used mathematics to model this particular scenario by recognizing that this scenario could be modeled with the graph of a polynomial function representing the growth of demand for electricity in the United States from 1950 and projected through 2040. I analyzed the specific characteristics of the graph which provided information about the future behavior of the graph. This analysis allowed me to make a prediction about my expectations for the growth of demand for electricity from 2040 to 2050.)

Alternate Strategies or Solutions

Some students may choose to sketch the given graph on their own graph paper and extend the *x*-axis out to include the year 2050. This may allow them to sketch their prediction for the behavior of the graph after 2040, and they may try to determine a pattern in the turning points from 1950 to 2040 and use this pattern to predict their expectation for the growth of demand for electricity from 2040 to 2050.

Technology

Students can use graphing calculators or other graphing software in this task.