

Problem-Based Task: Field of Dreams

Task Overview

Focus

How can the criteria for a square be used to prove whether a baseball field is actually a square? Which formulas can be used to calculate the distances between the bases? In this lesson, students will plot points on a coordinate plane representing the locations of the bases and home plate of a baseball diamond, and use these coordinates to draw conclusions about the relationships of the distances between bases and home plate, as well as the relationships of the four corners of the diamond.

This activity will provide practice with:

- plotting points on a coordinate plane
- calculating slopes and distances using the slope formula and the distance formula
- classifying slopes of consecutive sides and opposite sides of a figure
- analyzing pairs of opposite sides of a figure to determine congruency
- determining the relationship between the sides and angles of a figure
- drawing conclusions on classifying a figure based on multiple criteria

Introduction

This task should be used to explore or apply the skill of proving the slope criteria for parallel and perpendicular lines and using them to solve a geometric problem. Students should already be familiar with plotting points in the coordinate plane and calculating slope.

Begin by reading the problem and clarifying the meaning of *distance formula*, *slope formula*, and *square*.

distance formula a formula that states the distance between points (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

slope formula a formula that states the slope given points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$

square a quadrilateral with two pairs of parallel opposite sides, consecutive sides that are perpendicular, and all sides congruent

Facilitating the Task

Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 2:** Reason abstractly and quantitatively.

Students will reason abstractly as they make sense of the relationships between the slopes of the sides of the baseball diamond. They will reason that the slopes will determine if the opposite sides of the diamond are parallel, and if the consecutive sides are perpendicular. They will reason quantitatively as they calculate the slopes of the sides connecting each base and home plate, as well as the distance/length of each side.

- **SMP 4:** Model with mathematics.

Students will recognize that this scenario can be modeled with the use of a coordinate plane that represents the location of each base and home plate of the baseball diamond. They will use the coordinates for each point to calculate the criteria necessary for determining if the baseball diamond is a square.

- **SMP 7:** Look for and make use of structure.

Students will apply the structure of the slope and distance formulas to determine the relationships between the slopes and distances of opposite sides, as well as consecutive sides, of the baseball diamond. They will analyze these results and draw conclusions about the specific type of quadrilateral formed by the locations of the bases and home plate.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- assuming lines are parallel or perpendicular based on appearance only

Remind students to perform the necessary calculations to determine the exact length of the sides and the exact slope of each side so conclusions can be drawn about the relationships between the sides and corners of consecutive sides.

- incorrectly calculating the slope of the line

Have students write down the slope formula and the values for each x_1 , y_1 , x_2 , and y_2 coordinate as they calculate the slope of each side. Remind them that order matters when using the slope formula.

- incorrectly using the x - and y -coordinates in the distance formula

Have students write down the distance formula and the values for each x_1 , y_1 , x_2 , and y_2 coordinate as they calculate the distance between each base and home plate.

- subtracting negative coordinates incorrectly

Review the rules of subtraction of integers with students. Work through an example or two, such as $(5 - (-2)) = (5 + 2)$.

- making determinations about the type of polygon without making all the necessary calculations

Have students write down the specific criteria necessary for a quadrilateral to be identified as a square. Ask students to make a checklist of the criteria as they work through the task to ensure all properties of a square have been identified.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask them, “What are the properties of a square? How is a square different from a rectangle?” (**Answer:** A square has two pairs of parallel opposite sides, consecutive sides that are perpendicular, and all sides congruent. A rectangle has the same properties, with the exception that only opposite sides are congruent.)
- As students are beginning the task, ask them, “Why is it helpful to plot the points on a coordinate plane?” (**Answer:** The coordinate plane can be used to visualize the points that represent each of the bases and home plate. The plane can also be used to label distances and slopes once they are calculated.)
- Once students have completed plotting each point, ask them, “What do you notice about the locations of each base and home plate on the coordinate plane?” (**Answer:** Each point is plotted in a different quadrant, or each quadrant contains a point representing a base or home plate.)
- Before students begin calculating the slopes of each side, ask them, “What does it mean for two sides to be consecutive?” (**Answer:** Consecutive sides are sides that meet at an endpoint or an angle. Consecutive sides share an endpoint.)
- As students are calculating the slopes of the sides of the diamond, ask them to discuss the meaning of parallel lines and perpendicular lines in terms of slope. Ask them, “If a pair of opposite sides of the baseball diamond is parallel, what does it mean about their slopes? If two consecutive sides of the diamond are perpendicular, what does it mean about their slopes?” (**Answer:** If two sides are parallel, they have the same slope. If two consecutive sides are perpendicular, they have slopes that are negative reciprocals of each other.)
- As students are calculating the slopes, ask them, “Suppose you calculated a slope of a side that was -4 , and the other side had a slope of $-\frac{1}{4}$. What can you conclude about the relationship between these sides, and why?” (**Answer:** Although the slopes are reciprocals of each other, they are not negative reciprocals of each other. Therefore, they are neither parallel sides nor perpendicular sides.)

- As students are calculating the distances between each consecutive base, ask them, “Could you have calculated the distances in a different order than given in the task? Would this have made a difference in any results?” (**Answer:** Instead of calculating the distances between each base in order, the distances of each side could have been calculated in any order, since the goal was to determine the length of each side to see if all four sides were the same.)
- After students have completed their calculations of the slopes and distances, ask them to explain their conclusions in terms of presenting the argument that the baseball diamond is indeed a square. Ask them, “How do your calculations prove that the baseball diamond is a square?” (**Answer:** By calculating the slopes of the sides, it can be shown that opposite sides of the diamond are parallel to each other, and consecutive sides are perpendicular to each other, so the consecutive sides form four right angles. By calculating the distances between each base and home plate, it can be shown that all four sides have the same length. These criteria match that of a square.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask for volunteers to discuss their strategies and processes for performing the necessary calculations for slope and distance. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students’ strategies and ways of justifying responses. Ask students to share their reasoning processes and how they were able to draw conclusions. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of the criteria for a square, and how it relates to slopes and distances.

Connecting to Key Concepts

- To find the slope (steepness of a line), calculate the change in y divided by the change in x using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

In this task, students will apply the slope formula and calculate the slope of all four sides of the baseball diamond to draw conclusions about the relationships between the opposite sides and consecutive sides.

- To find the distance between two points on the coordinate plane, use the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Students will apply the distance formula to find the distance between each base and home plate of the baseball diamond. They will use these values to draw conclusions about the relationships between the distances between each base.

- To prove that two lines are parallel, calculate the slopes of each line. If they are the same, the lines are parallel.

Students will calculate the slopes of the opposite sides of the diamond to determine if the sides are parallel.

- To prove that two lines are perpendicular, calculate the slopes of each line. If they are negative reciprocal values of each other, the lines are perpendicular.

Students will calculate the slopes of the consecutive sides of the diamond to determine if the sides are perpendicular.

Extending the Task

- To extend the task, ask students to work together to create their own scenario of plotting points and identifying a shape (other than a square) based on the relationships between the sides and angles of the shape. For example, ask them to find points that, when plotted, will result in a rectangle or another type of quadrilateral, such as a rhombus. Have them calculate the slopes and distances, and explain their results. Ask for volunteers to share their scenarios.
- Another option for extending the task is to ask students to calculate the distances between opposite corners of the baseball diamond, e.g., the distances between home plate and second base, and the distance between first base and third base. Before students complete the calculations, ask them to predict the results. Discuss how these distances, which are the diagonals of a square, must be the same for any square. Ask students if they can think of any other properties of a square in terms of relationships between sides and angles. (Possibly discuss midpoints and the center of a square.)

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 2, ASK:** “How did you reason abstractly and quantitatively? Which of your strategies represent abstract reasoning?” (**Answer:** I used abstract reasoning as I made sense of the relationships between the slopes of the sides of the baseball diamond by reasoning that the slopes of the sides of the diamond proved the shape of the diamond was a square, as well as the distances of the four sides proved that the diamond was a square.) “Which of your strategies represent quantitative reasoning?” (**Answer:** I reasoned quantitatively when I calculated the slopes of the sides and the distances of the sides of the baseball diamond.)
- **For SMP 4, ASK:** “How did you use mathematics to model this particular scenario?” (**Answer:** I recognized that this scenario could be modeled with the use of a coordinate plane that represents the location of each base and home plate of the baseball diamond.)
- **For SMP 7, ASK:** “How did you look for and make use of structure when solving this problem?” (**Answer:** I made use of structure by applying the slope and distance formulas to determine the relationships between the slopes and distances of opposite sides, as well as consecutive sides, of the baseball diamond.)

Alternate Strategies or Solutions

- Students may choose to use a straightedge to measure the lengths of the sides of the baseball diamond. They may also use a protractor to measure the angles formed by the consecutive sides. Remind them that the slope and distance formulas allow for accurate calculations, while measuring with a straightedge and a protractor can have human measurement errors.

Technology

No technology is needed for this task, although students may want to use scientific calculators to determine the simplified value of the distances, which are given in radical form in the task.