Conceptual Task: Engineering Polynomials

Exploration Questions Sample Responses

a. What is being discussed here?

An engineer is describing to a client how they are going to use polynomials to model the shape of his coaster. The client has some concerns that the engineer addresses.

b. What do you think the engineer means by "control the hills and valleys of the coaster"? What does this have to do with polynomials?

The hills and valleys of a polynomial are the turns. The number of turns in a polynomial can be controlled by changing its degree.

c. Recall the factored form of a polynomial function is $f(x) = a(x - x_1)(x - x_2) \ldots (x - x_n)$. What effect would changing $a, x_1, x_2, \ldots x_n$ have on the graph? How would this change the shape of the coaster tracks?

Changing a vertically stretches or shrinks the graph. This would affect the height of the hills. Changing $x_1, x_2, \ldots x_n$ moves the x-intercepts. This would affect the length of the hills.

d. What is the client's concern? What does it have to do with polynomials?

The client is concerned about modeling the ends of the roller coaster. The client knows that the end behaviors of polynomials can ascend to infinity or descend to negative infinity, but their roller coaster has a special start and end shape.

e. How does the engineer respond? How would this address the client's concern?

The engineer explains that it is possible for more than one polynomial to be used. A polynomial that accurately models the ends can be attached to a polynomial that models the body of the coaster at a specified point.

f. Are there any potential problems with the engineer's solution? How could this be addressed?

If the curve the engineer stitches in is not chosen carefully, the coaster could end up with a "spike" in the racks where the two graphs join at an angle.

g. Suppose the client wants to be able to control the height of each hill. How would this be implemented? Explain any technical difficulties that might arise.

The engineer could use the same stitching trick to create a different polynomial for each hill. The same requirement of avoiding sharp points in the graph would still apply, and would be harder to implement the more pieces the coaster has.

h. What do you think are some advantages or disadvantages of using polynomials to model the shape of a roller coaster?

Answers will vary. Sample answers:

Advantages	Disadvantages
Simple form	The higher the degree, the more chaotic the shape becomes.
Well understood	Can't control individual hills without stitching models together
Somewhat flexible	Likely to need to stitch models together to account for flat areas in the track