

Problem-Based Task: Radio Interference

Coaching Sample Responses

- a. What information will determine whether the transmission time should be changed?

The transmission time should only be rescheduled if the airplane is going to interrupt the signal during the important broadcast. To determine this, we need to analyze the system of equations to figure out when the plane will cross the signal.

- b. How can the path of the airplane be represented?

The path of the airplane can be represented by a parabolic function.

- c. How can the path of the radio signal be represented?

The path of the radio signal can be represented by an absolute value function.

- d. What information is needed to represent the path of the plane?

To write a parabolic function, we need to know either the a -, b -, and c -values of the general form $f(x) = ax^2 + bx + c$, or the a -value and the vertex (h, k) of the vertex form $f(x) = a(x - h)^2 + k$.

From the given information, we know the a -value is $-\frac{1}{324}$. When we scan the information again, we notice that a high point in the path is given as well, indicating that we should use the vertex form. Because the vertex of the path is 44 km below the reflection point of the radio signal, we choose $(0, 6)$ as our vertex. Substitute the a -value and the vertex into the vertex form for a parabola, $f(x) = a(x - h)^2 + k$, to find the equation of the airplane's path.

$$f(x) = a(x - h)^2 + k$$

$$f(x) = -\frac{1}{324}(x - 0)^2 + 6$$

$$f(x) = -\frac{1}{324}x^2 + 6$$

The equation that models the path of the plane is $f(x) = -\frac{1}{324}x^2 + 6$.

- e. What information is needed to represent the path of the radio signal?

For an absolute value function, we need to know the a -value and the vertex (h, k) of the general form $g(x) = a|x - h| + k$.

The problem states that the signal reflects off the ionosphere, which is 50 km above the surface of the earth. This point will be the vertex. Because the reflection point is directly above the parabola vertex, and because we chose 0 as the h -coordinate for the vertex earlier, we choose $(0, 50)$ as the coordinates for the reflection point. Substitute $(0, 50)$ into the general form of an absolute value function:

$$g(x) = a|x - h| + k$$

$$g(x) = a|x - 0| + 50$$

$$g(x) = a|x| + 50$$

Therefore, $g(x) = a|x| + 50$ is the form of our absolute value function. Now we need to solve for a .

We can find a by substituting an (x, y) pair that lies in the path of the radio signal and solving for a . We know that the transmitter and receiver are 40 km apart, which means that $(-20, 0)$ and $(20, 0)$ lie on the path. Substitute $(20, 0)$ into $g(x) = a|x| + 50$:

$$g(x) = a|x| + 50$$

$$0 = a|20| + 50$$

$$0 = 20a + 50$$

$$-50 = 20a$$

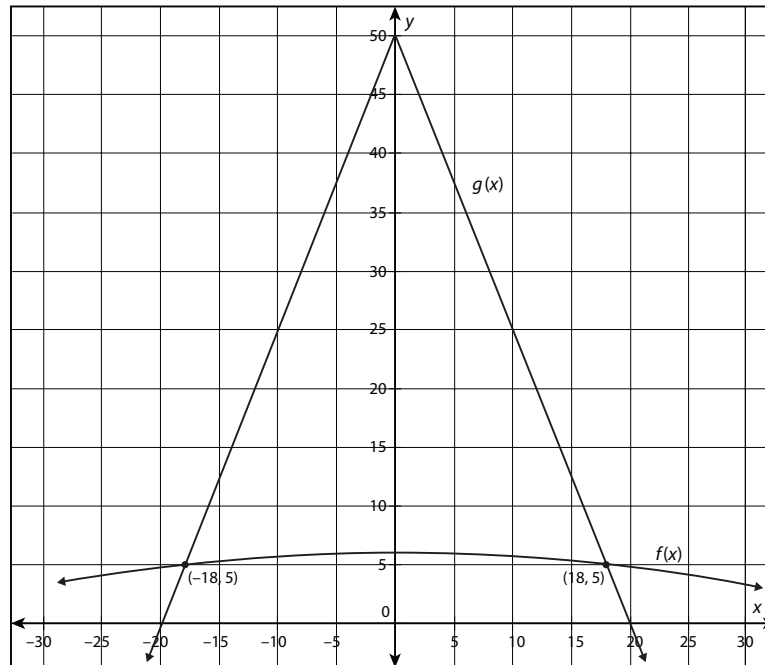
$$a = -\frac{50}{20}$$

$$a = -\frac{5}{2}$$

Since $a = -\frac{5}{2}$, the path of the radio signal can be modeled by the function $g(x) = -\frac{5}{2}|x| + 50$.

- f. What information is needed in order to figure out when the plane blocks the radio signal?

The plane will block the radio signal when it crosses the path of transmission. Therefore, we need to find out where the two paths intersect. One way to do this is by plotting the paths on a graph and seeing where they intersect. You can do this by hand, or you can use a graphing calculator to graph the two functions and find the intersection points.



By graphing the two functions, we see that the intersection points are $(-18, 5)$ and $(18, 5)$. This means that the plane crosses the radio signal 18 miles away from the reflection point on either side.

g. How can we convert a distance to time?

We know that the plane is traveling at a ground speed of about 3 km/minute, and that it crosses the vertex at 23:00 hours. Therefore, we can divide the distance from the crossing point to the vertex by the speed of the aircraft to figure out how long it takes the plane to cross that distance:

$$18 \text{ km} \cdot \frac{1 \text{ min}}{3 \text{ km}} = 6 \text{ min}$$

So, the plane will cross the radio signal at 6 minutes before and after 23:00 hours. In other words, the signal will be interrupted at 22:54 hours and again at 23:06 hours. Because the message will be 10 minutes long and is scheduled to begin transmitting at 23:00, it would be interrupted at the 6-minute mark. Therefore, the start time should be rescheduled for either 22:55 or 23:07, which avoids the interference while changing the time only slightly.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.