Triangle Congruence Criteria

Prerequisite Skills

This lesson requires the use of the following skills:

- understanding that rigid motions maintain the shape and size of angles and segments, and that rigid motions include the transformations of reflections, rotations, and translations
- ability to identify corresponding pairs of sides and angles

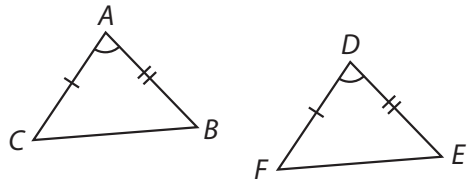
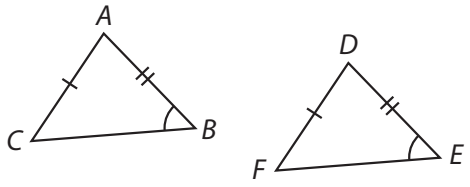
Introduction

When a series of rigid motions is performed on a triangle, the result is a congruent triangle. When triangles are congruent, the corresponding parts of the triangles are also congruent. It is also true that if the corresponding parts of two triangles are congruent, then the triangles are congruent. It is possible to determine if triangles are congruent by measuring and comparing each angle and side, but this can take time. There is a set of congruence criteria that lets us determine whether triangles are congruent with less information.

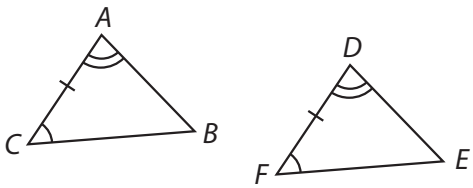
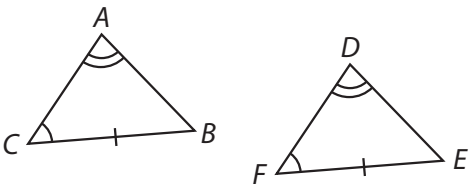
Key Concepts

- The criteria for triangle congruence, known as triangle congruence statements, provide the least amount of information needed to determine if two triangles are congruent.
- Each congruence statement refers to the corresponding parts of the triangles.
- By looking at the information about each triangle, you can determine whether the triangles are congruent.
- The **side-side-side** congruence statement, or **SSS**, states that if three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent.
- If it is known that the corresponding sides are congruent, it is understood that the corresponding angles are also congruent.
- The **side-angle-side** congruence statement, or **SAS**, states that if two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

- The **included angle** is the angle that is between the two congruent sides.

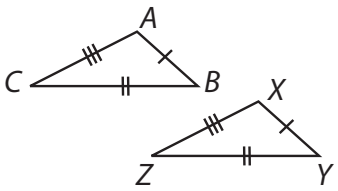
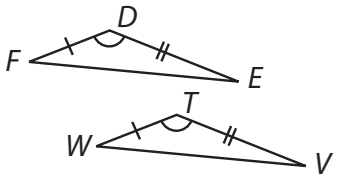
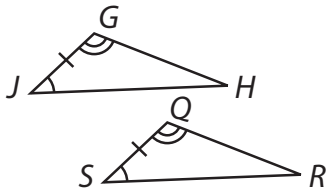
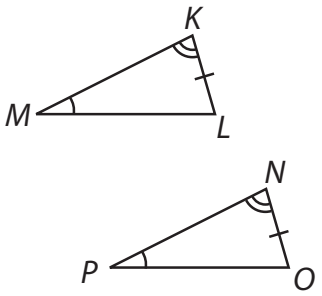
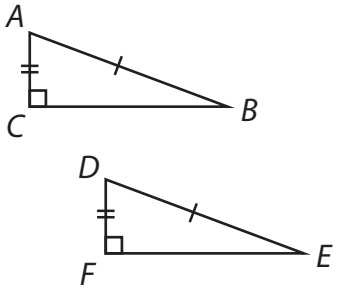
Included angle	Non-included angle
	
$\angle A$ is included between $\overline{CA}$ and $\overline{AB}$. $\angle D$ is included between $\overline{FD}$ and $\overline{DE}$.	$\angle B$ is NOT included between $\overline{CA}$ and $\overline{AB}$. $\angle E$ is NOT included between $\overline{FD}$ and $\overline{DE}$.

- The **angle-side-angle** congruence statement, or **ASA**, states that if two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.
- The **included side** is the side that is between the two congruent angles.

Included side	Non-included side
	
$\overline{AC}$ is included between $\angle C$ and $\angle A$. $\overline{FD}$ is included between $\angle F$ and $\angle D$.	$\overline{CB}$ is NOT included between $\angle C$ and $\angle A$. $\overline{FE}$ is NOT included between $\angle F$ and $\angle D$.

- The **angle-angle-side** congruence statement, or **AAS**, states that if two angles and a non-included side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the triangles are congruent.

- The final triangle congruence statement is the **hypotenuse-leg** theorem, or **HL**. The hypotenuse-leg theorem states that if the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg in another right triangle, then the two triangles are congruent.
- The following table compares these five congruence statements.

Side-Side-Side (SSS)	**Side-Angle-Side (SAS)**	**Angle-Side-Angle (ASA)**
		
$\triangle ABC \cong \triangle XYZ$	$\triangle DEF \cong \triangle TVW$	$\triangle GHJ \cong \triangle QRS$
Angle-Angle-Side (AAS)	**Hypotenuse-Leg (HL)**	
		
$\triangle KLM \cong \triangle NOP$	$\triangle ABC \cong \triangle DEF$	

Common Errors/Misconceptions

- misidentifying included sides and angles, resulting in the wrong congruence statement
- misreading congruency symbols of triangles
- changing the order of named triangles, causing parts to be incorrectly interpreted as congruent