Triangle Congruency

Prerequisite Skills

This lesson requires the use of the following skills:

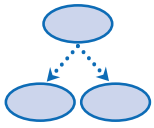
- recognizing transformations performed as a combination of translations, reflections, rotations, dilations, contractions, or stretches
- understanding that rigid motions maintain shape and size of angles and segments

Introduction

If a rigid motion or a series of rigid motions, including translations, rotations, or reflections, is performed on a triangle, then the transformed triangle is congruent to the original. When two triangles are congruent, the corresponding angles have the same measures and the corresponding sides have the same lengths. It is possible to determine whether triangles are congruent based on the angle measures and lengths of the sides of the triangles.

Key Concepts

- To determine whether two triangles are congruent, you must observe the angle measures and side lengths of the triangles.
- When a triangle is transformed by a series of rigid motions, the angles are images of each other and are called corresponding angles.



- Corresponding angles are a pair of angles in a similar position.
- If two triangles are congruent, then any pair of corresponding angles is also congruent.
- When a triangle is transformed by a series of rigid motions, the sides are also images of each other and are called corresponding sides.
- Corresponding sides are the sides of two figures that lie in the same position relative to the figure.
- If two triangles are congruent, then any pair of corresponding sides is also congruent.
- Congruent triangles have three pairs of corresponding angles and three pairs of corresponding sides, for a total of six pairs of corresponding parts.
- If two or more triangles are proven congruent, then all of their corresponding parts are congruent as well. This postulate is known as **Corresponding Parts of Congruent Triangles are Congruent (CPCTC)**. A **postulate** is a true statement that does not require a proof.

- The corresponding angles and sides can be determined by the order of the letters.
- If $\triangle ABC$ is congruent to $\triangle DEF$, the angles of the two triangles correspond in the same order as they are named.
- Use the symbol $\rightarrow$ to show that two parts are corresponding.

  Angle $A \rightarrow$ Angle D; they are equivalent.

  Angle $B \rightarrow$ Angle E; they are equivalent.

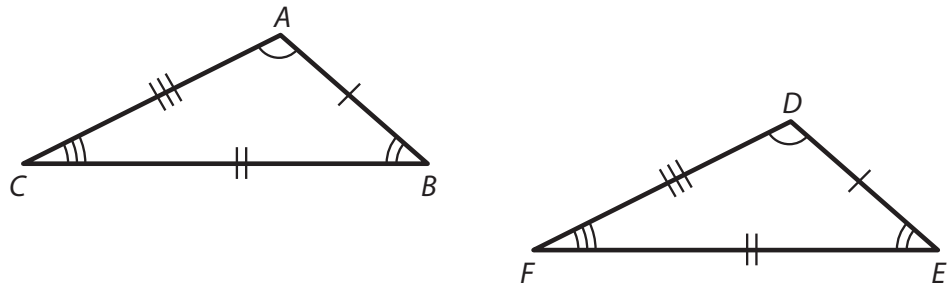
  Angle $C \rightarrow$ Angle F; they are equivalent.
- The corresponding angles are used to name the corresponding sides.

  $\overline{AB} \rightarrow \overline{DE}$; they are equivalent.

  $\overline{BC} \rightarrow \overline{EF}$; they are equivalent.

  $\overline{AC} \rightarrow \overline{DF}$; they are equivalent.
- Observe the diagrams of $\triangle ABC$ and $\triangle DEF$.

$\triangle ABC \cong \triangle DEF$



$\angle A \cong \angle D$ $\quad\quad$ $\overline{AB} \cong \overline{DE}$

$\angle B \cong \angle E$ $\quad\quad$ $\overline{BC} \cong \overline{EF}$

$\angle C \cong \angle F$ $\quad\quad$ $\overline{AC} \cong \overline{DF}$

- By observing the angles and sides of two triangles, it is possible to determine if the triangles are congruent.
- Two triangles are congruent if the corresponding angles are congruent and corresponding sides are congruent.
- Notice the number of tick marks on each side of the triangles in the diagram.

- The tick marks show the sides that are congruent.
- Compare the number of tick marks on the sides of $\triangle ABC$ to the tick marks on the sides of $\triangle DEF$.
- Match the number of tick marks on one side of one triangle to the side with the same number of tick marks on the second triangle.

  $\overline{AB}$ and $\overline{DE}$ each have one tick mark, so the two sides are congruent.

  $\overline{BC}$ and $\overline{EF}$ each have two tick marks, so the two sides are congruent.

  $\overline{AC}$ and $\overline{DF}$ each have three tick marks, so the two sides are congruent.
- The arcs on the angles show the angles that are congruent.
- Compare the number of arcs on the angles of $\triangle ABC$ to the number of arcs on the angles of $\triangle DEF$.
- Match the arcs on one angle of one triangle to the angle with the same number of arcs on the second triangle.

  $\angle A$ and $\angle D$ each have one arc, so the two angles are congruent.

  $\angle B$ and $\angle E$ each have two arcs, so the two angles are congruent.

  $\angle C$ and $\angle F$ each have three arcs, so the two angles are congruent.
- If the sides and angles are not labeled as congruent, you can use a ruler and protractor or construction methods to measure each of the angles and sides.

Common Errors/Misconceptions

- incorrectly identifying corresponding parts of triangles
- assuming corresponding parts indicate congruent parts
- assuming alphabetical order indicates congruence