## Problem-Based Task: When Will It Beep?

## Coaching Sample Responses

a. How is the half-life of the radioactive element related to the original amount of radioactivity of the element?

The half-life of the radioactive element is the amount of radioactivity, $R(t)$, that is equal to half of the original amount of radioactivity, $R_0$. Mathematically, this can be written as $R(t) = 0.5 \bullet R_0$.

b. How can you use the half-life data to find the value of the constant $c$ in the exponent of the exponential term?

The results from part a can be used with the given function $R(t) = R_0 \bullet e^{-ct}$ to eliminate one of the variables.

If $R(t) = R_0 \bullet e^{-ct}$ and $R(t) = 0.5 \bullet R_0$, the function can be written as $0.5 \bullet R_0 = R_0 \bullet e^{-ct}$, which simplifies to $0.5 = e^{-ct}$. This can be rewritten as $2 = e^{ct}$ by taking the reciprocal of each side of the equation, which eliminates the negative sign. We are given that $t = 450$ years for this element, so the function becomes $2 = e^{450c}$.

c. How can the inverse of the exponential function be used to simplify finding the answer to part b? Find the value of the constant $c$.

Additional information will be needed to find the value of $c$. The inverse provides the means to do that. A calculator that can compute common and natural logarithms will be needed in order to find the value of $c$.

The equation $2 = e^{450c}$ can be rewritten as $450 \bullet c = \ln 2$.

Therefore, $c = \frac{\ln 2}{450} \approx 0.0015$.

The units of $c$ are "per year" or "$y^{-1}$" since the exponent of $e$ has to be dimensionless.

d. How can you use the result of part c and the information about when radioactive decay will cause the smoke detector to stop working to find how long this will take?

The information regarding the time when the smoke detector will stop working is related to how the half-life was expressed in terms of the original amount of radioactivity.

In this case, the smoke detector will stop working when the radioactivity level, $R(t)$, is down to 0.2% of the original amount, $R_0$. However, 0.2% will need to be converted to a decimal: 0.2% = 0.002.

e. Write an equation using the information in parts c and d, then use it to determine how long it will take for the smoke detector to stop working.

In order to write this equation, recall that the information to solve for $t$ is already known. There is a relationship between $R(t)$ and $R_0$, and the value of $c$ is known.

First, write the function for when $R(t)$ is down to 0.2%.

$$R(t) = 0.002 \bullet R_0$$

Substitute this for $R(t)$ in the original function.

$$0.002 \bullet R_0 = R_0 \bullet e^{-ct}$$

Substitute the value of $c$.

$$0.002 \bullet R_0 = R_0 \bullet e^{-ct}$$

$$0.002 = e^{-0.0015t}$$

Write the inverse natural logarithm function and simplify.

$$-0.0015t = \ln(0.002)$$

$$t = \frac{\ln(0.002)}{-0.0015} \approx 4{,}143 \text{ years}$$

f. How does the answer to part e compare to how often the battery should be changed in a smoke detector?

An annual change of the smoke-detector battery, and the number of years it will take the radioactive element of the smoke detector to decay to the point that the smoke detector stops working, are more than 4,000 years apart.

g. What other factor(s) might explain why the smoke detector's battery needs to be changed annually?

One possible response is that the battery is not related to the decay of the radioactivity element. The battery is used to keep other components of the smoke detector working if there is a household power failure (e.g., the alarm, signal lights).

**Recommended Closure Activity**

Select one or more of the essential questions for a class discussion or as a journal entry prompt.