Graphing Logarithmic Functions

Prerequisite Skills

This lesson requires the use of the following skills:

- calculating the intercept(s) and zero(s) of a logarithmic function
- determining the value of a function as the domain values approach the upper and lower bounds of the domain
- comparing logarithmic functions by describing what happens to the function values as the domain values vary in a specific way

Introduction

The functions $f(x) = a \bullet \log_b c$, $g(x) = a + b \bullet \log_c d$, and $h(x) = a \bullet \log_d c$ are "families" of logarithmic functions. The graphs of logarithmic functions exhibit patterns that can help in identifying these functions when the algebraic function is not present.

Key Concepts

- Logarithmic functions or the logarithmic terms within logarithmic functions are equal to 0 when their arguments equal 1. Recall that the argument is the result of raising the base of a logarithm to the power of the logarithm, so that b is the argument of the logarithm $\log_a b = c$.
- For example, let's look at $f(x) = 30 \bullet \ln x$ at $x = 1$: $f(1) = 30 \bullet \ln 1 = 30 \bullet 0 = 0$.
- For another example, let's look at the function $g(x) = -250 + 6 \bullet \log_4 (x + 4)$. Evaluate when $x = -3$:

  $g(-3) = -250 + 6 \bullet \log_4 [(-3) + 4]$

  $g(-3) = -250 + 6 \bullet \log_4 (1)$

  $g(-3) = -250 + 0 = -250$
- Notice that the argument for each example was equal to 1.
- The values of logarithmic functions or their logarithmic terms approach positive or negative infinity as the argument of the logarithmic term approaches 0. For example, in the function $f(x) = -\ln x$, $f(0.1)$ is about 2.3, but $f(0.001)$ is almost 7. If $x = 10^{-12}$, $f(x)$ is almost 28. The function value increases continuously as the value of x decreases.
- A graphing calculator will also show this function behavior. By looking at a variety of function values of interest (such as very small values of x), it is possible to see trends in the changing values of the function. Examine very small values of x by adjusting the axis scales or using the calculator's trace feature.

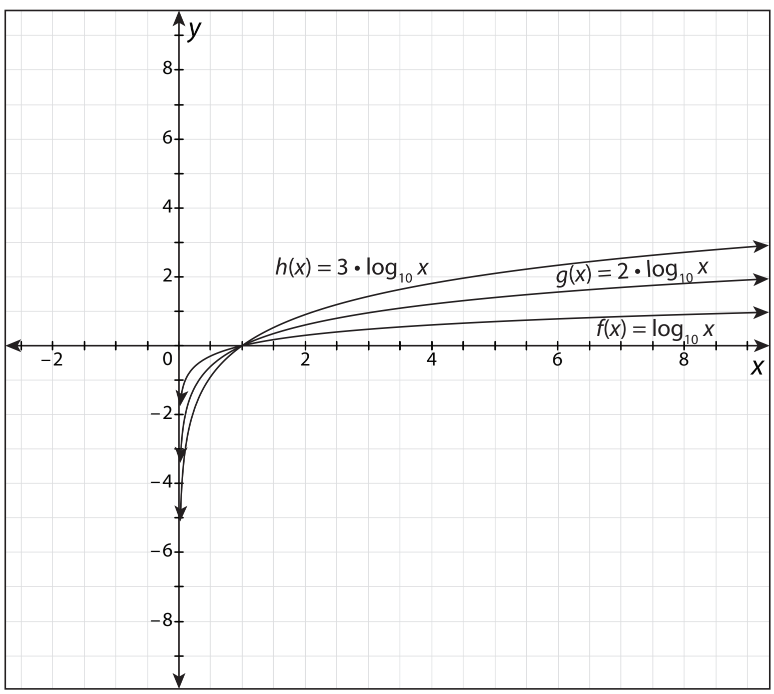
- Similarly, the value of logarithmic functions or their logarithmic terms as x becomes very large, positively or negatively, can be seen by substituting values for x or by using a calculator to calculate the values.
- For example, for the function $g(x) = \log x$, the domain is $(0,+\infty)$. As x becomes very large, the value of $g(x)$ increases, too, but at a much slower rate. The value of $g(10^3) = 3$, but the value of $g(10^{200}) = 200$. If you suspect the function value of $g(x)$ has an upper bound, try to find the highest value of the function.
- Follow these basic rules to compare logarithmic functions. However, use caution when defining domains: positive, negative, or zero domain values can result in an undefined function, or change the ordering in a comparison of function values.

Powers, Products, Quotients, Roots, and Sums of Logarithmic Functions

- Families of logarithmic functions are grouped according to the operations shown in the equations of the functions. In real-world problems, you can calculate such combined operations with logarithmic functions best by approximation techniques or with calculators. Each example that follows shows how different operations affect the graph of a logarithmic function.

$f(x) = a \bullet \log_b c$

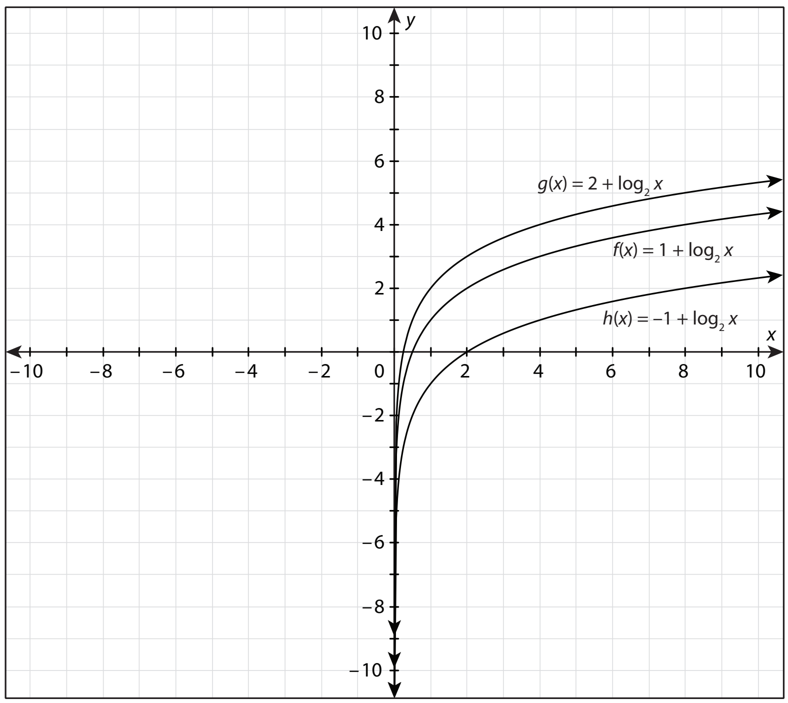
- Compare the graphs of three functions of the form $f(x) = a \bullet \log_b c$.



- All three graphs pass through the point (1, 0) because any number raised to the 0 power (the y-value) is equal to 1 (the x-value). The coefficient in front of each logarithm multiplies the logarithmic value at that value of x by the magnitude of the coefficient. This means that $h(x) = 3 \bullet f(x) = 3 \bullet \log x$, and $g(x) = 2 \bullet f(x) = 2 \bullet \log x$.

$f(x) = a + b \bullet \log_c d$

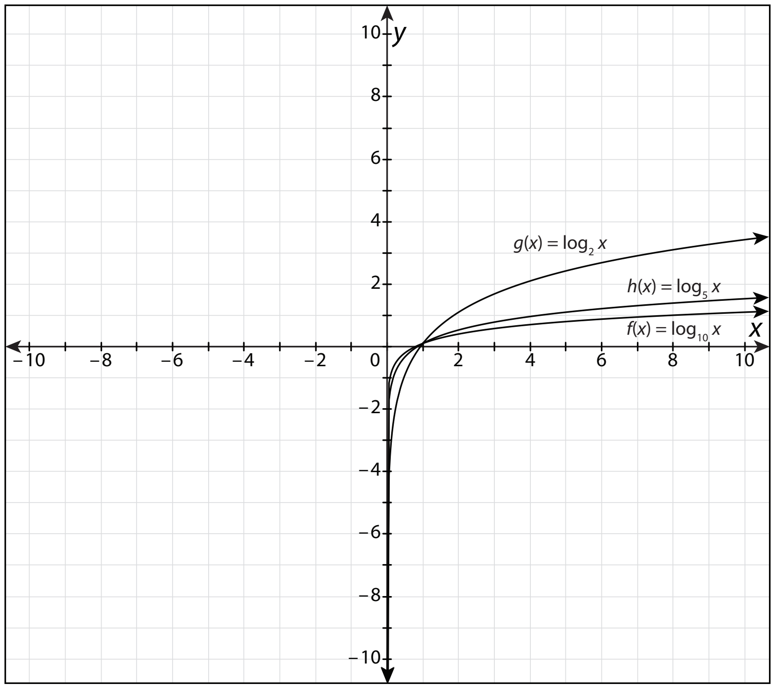
- Observe the graphs of three functions of the form $f(x) = a + b \bullet \log_c d$.



- All three graphs are continuously increasing across the domain $(0, +\infty)$. For any value of x in the domain, the y-values are related by the inequality $g(x) > f(x) > h(x)$. The x-intercepts are determined by the constant added to $\log_2 x$.

$f(x) = a \bullet \log_b c$ and $g(x) = a \bullet \log_d c$

- Compare the graphs of three functions with different bases.



- All three functions contain the point (1, 0), since the logarithm of any base to the power of 0 is equal to 1. As the graphs show, the functions are related by the inequality $g(x) > h(x) > f(x)$ when $x > 1$. Comparing the bases of the three functions reveals that they are ordered in the opposite "direction" from the functions: $2 < 5 < 10$ for $g(x) > h(x) > f(x)$.

Common Errors/Misconceptions

- confusing the domain and range in a logarithmic function problem
- using an argument in a logarithmic term that is less than or equal to 0
- failing to check all of the domain and extreme-value options in graphing one or more logarithmic functions; e.g., intercepts and function values for upper and lower bounds of domains