Comparing Functions Using Average Rate of Change

Prerequisite Skills

This lesson requires the use of the following skills:

- finding a function's rate of change from an equation, table, or graph
- graphing functions
- interpreting key features of functions

Introduction

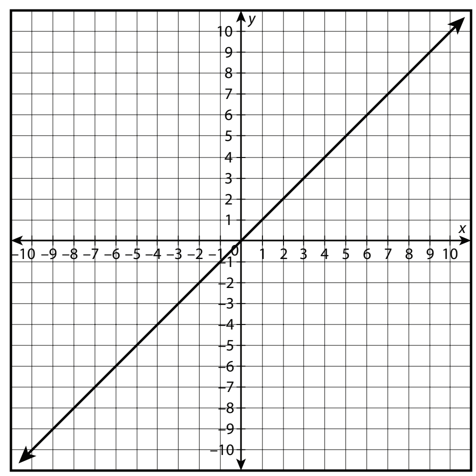
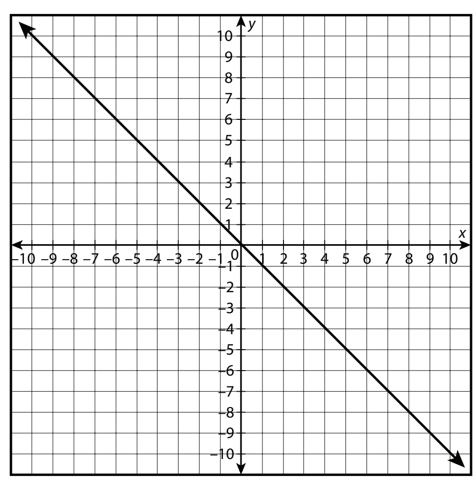
Exponential, quadratic, and linear functions are used to represent a variety of real-world problems. Previously, we calculated average rates of changes for all three types of functions using patterns, tables, and graphs. Here we will focus on end behavior of graphs and how that can help identify the type of function being modeled.

Key Concepts

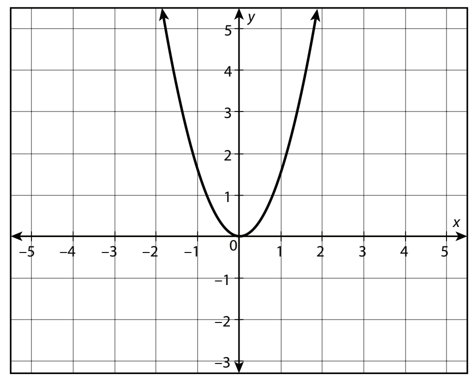
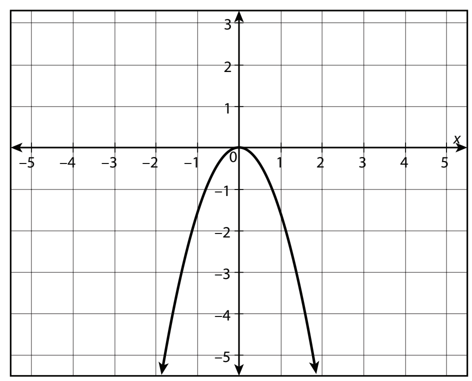
- Linear functions do not change direction and extend in both directions infinitely.
- The rate of change of linear functions remains constant across all intervals.
- Quadratic functions form a parabola (u-shaped graph) where both ends extend infinitely in the same direction.
- Because quadratic graphs change directions, their rate of change is positive on one side and negative on the other.
- Exponential functions increase or decrease rapidly, but they do not change direction.
- The rate of change of exponential functions is not constant across all intervals, but increases or decreases by a constant multiple.
- Exponential functions that increase are known as **exponential growth**. Those that decrease are known as **exponential decay**.
- Exponential functions will begin to level off on one side of the graph, while quadratic functions will continue either increasing or decreasing on both sides of the graph.

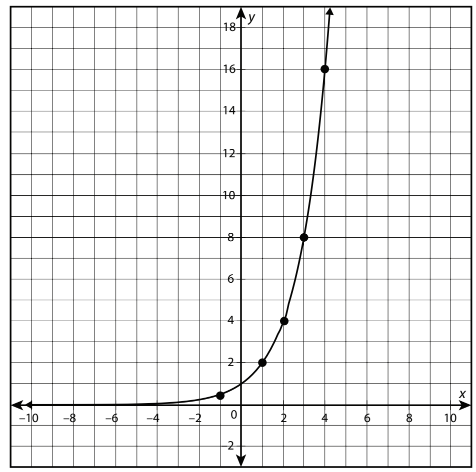
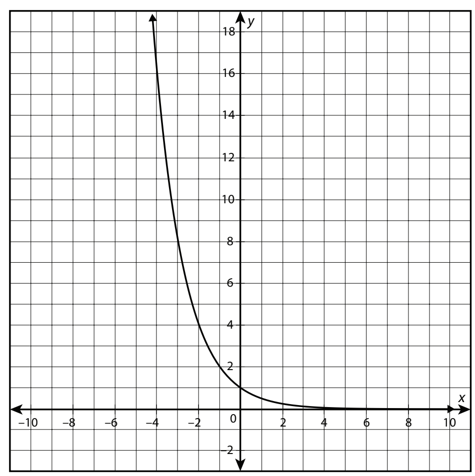
- Because the rate of change of exponential functions increases so rapidly, an exponentially increasing function will always eventually exceed a quadratically increasing function.
- The **end behavior** of a function is the behavior of the graph as x approaches positive or negative infinity.
- The $\rightarrow$ symbol stands for "approaches."

End Behavior of Linear and Quadratic Functions
Linear: $y = ax$ Quadratic: $y = ax^2$

Function type	Value of *a* (leading coefficient)	End behavior	Sample graph
Linear	Positive $a > 0$	as $x \rightarrow \infty, f(x) \rightarrow \infty$ as $x \rightarrow -\infty, f(x) \rightarrow -\infty$	$f(x) = x$ 
Linear	Negative $a < 0$	as $x \rightarrow \infty, f(x) \rightarrow -\infty$ as $x \rightarrow -\infty, f(x) \rightarrow \infty$	$f(x) = -x$ 

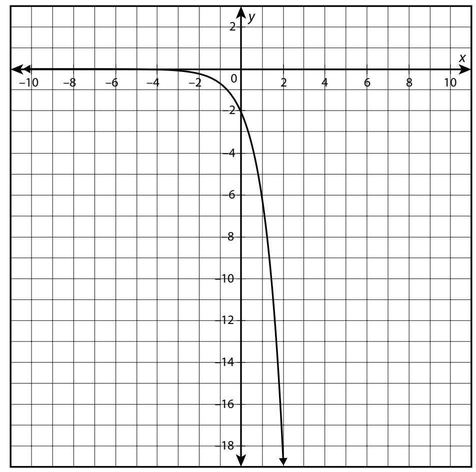
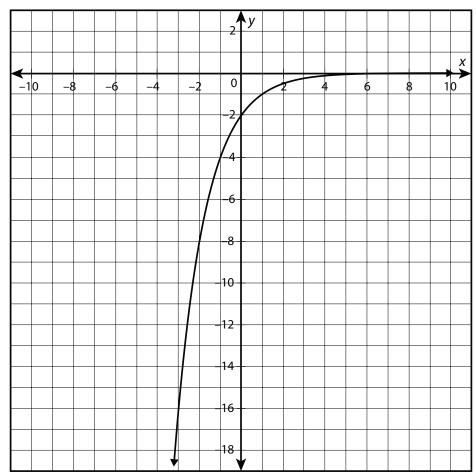
(*continued*)

End Behavior of Linear and Quadratic Functions (*continued*) **Linear: $y = ax$ Quadratic: $y = ax^2$**			
Function type	**Value of *a* (leading coefficient)**	**End behavior**	**Sample graph**
Quadratic	Positive $a > 0$	as $x \to \infty, f(x) \to \infty$ as $x \to -\infty, f(x) \to \infty$	$f(x) = x^2$ 
Quadratic	Negative $a < 0$	as $x \to \infty, f(x) \to -\infty$ as $x \to -\infty, f(x) \to -\infty$	$f(x) = -x^2$ 

End Behavior of Exponential Functions ***y* = *ab*^{x}**			
Value of *a* (leading coefficient)	**Value of *b***	**End behavior**	**Graph**
Positive $a > 0$	$b > 1$	as $x \to \infty, f(x) \to \infty$ as $x \to -\infty, f(x) \to 0$	$f(x) = 2^x$ 
Positive $a > 0$	$0 < b < 1$	as $x \to \infty, f(x) \to 0$ as $x \to -\infty, f(x) \to \infty$	$f(x) = (0.5)^x$ 

(*continued*)

End Behavior of Exponential Functions (*continued*)
***y* = *ab*^{x}**

Value of *a* (leading coefficient)	Value of *b*	End behavior	Graph
Negative $a < 0$	$b > 1$	as $x \to \infty$, $f(x) \to -\infty$ as $x \to -\infty$, $f(x) \to 0$	$f(x) = -2(3^x)$ 
Negative $a < 0$	$0 < b < 1$	as $x \to \infty$, $f(x) \to 0$ as $x \to -\infty$, $f(x) \to -\infty$	$f(x) = -2(0.5)^x$ 

Common Errors/Misconceptions

- confusing the values of x and $f(x)$
- not recognizing the approach to 0 in exponential functions
- forgetting to check the sign of the leading coefficient