Average Rate of Change on a Graph

Prerequisite Skills

This lesson requires the use of the following skills:

- identifying what function model is appropriate for a given situation
- identifying patterns in a function's rate of change
- interpreting key features of functions

Introduction

The world around us is constantly changing. The rate at which things change can vary, and how to model that change takes different forms. Business owners might need quadratic models to determine when profits are increasing or decreasing. Stock market investors may need exponential models to decide which stock is the best investment. In this lesson, you will review patterns of change between different types of functions and then identify the type of function using graphs and rates of change.

Key Concepts

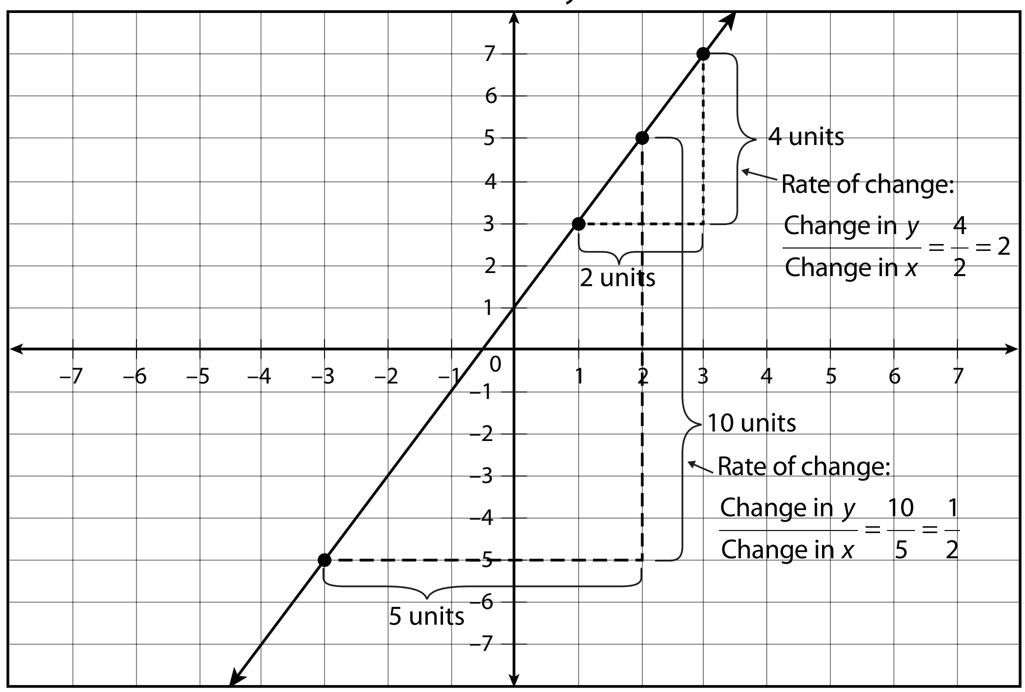
- Recall that **rate of change** is a ratio that describes how much one quantity changes with respect to the change in another quantity. Patterns can help distinguish different rates of change.

Linear Functions

- Linear functions increase or decrease by a common rate, or by the same rate over similar intervals.
- The rate of change in a linear function is referred to as the slope.
- As the value of x increases by 1, $f(x)$ will increase by a constant value.
- The slope of a linear function is constant between any points on the line.
- In a set of data, the change in y when x increases by 1 is called a **first difference**.

- The simplest one-variable form of a **linear function** can be represented as $f(x) = ax + b$, in which a and b are constants.

Function model: $y = 2x + 1$

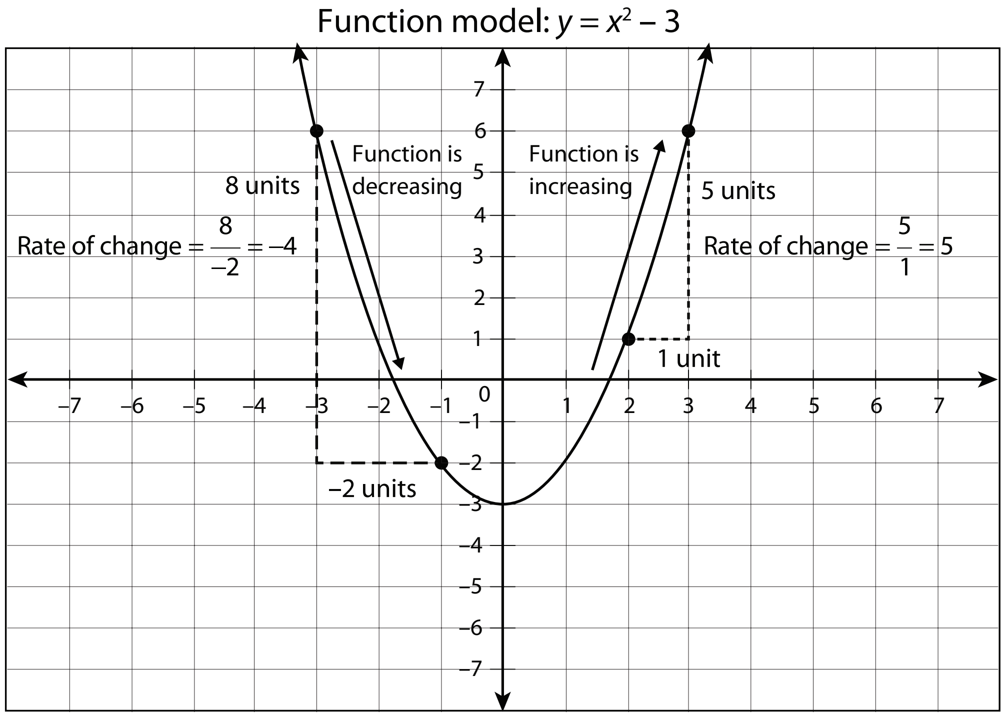


- Linear functions can be limiting as models because the graph never changes directions.

Quadratic Functions

- If a quantity gets larger first, and then starts to get smaller, the graph will change directions.
- The graph of a quadratic function is a **parabola**, a U-shaped model.
- The rate of change of a quadratic function varies over different intervals.
- Because the graph changes directions, the rate of change on one side of the parabola will be increasing while the rate of change on the other side of the parabola will be decreasing.
- There is neither a constant rate of change nor a constant multiple in a quadratic function.
- In a quadratic model, the change in the second differences is constant.
- The general form of the quadratic function is $f(x) = ax^2 + bx + c$, in which a, b, and c are constants. The rate of change of a quadratic function with differ across intervals.
- The value of a determines whether the quadratic has a maximum or a minimum.
- If a is negative, the quadratic function will have a maximum.

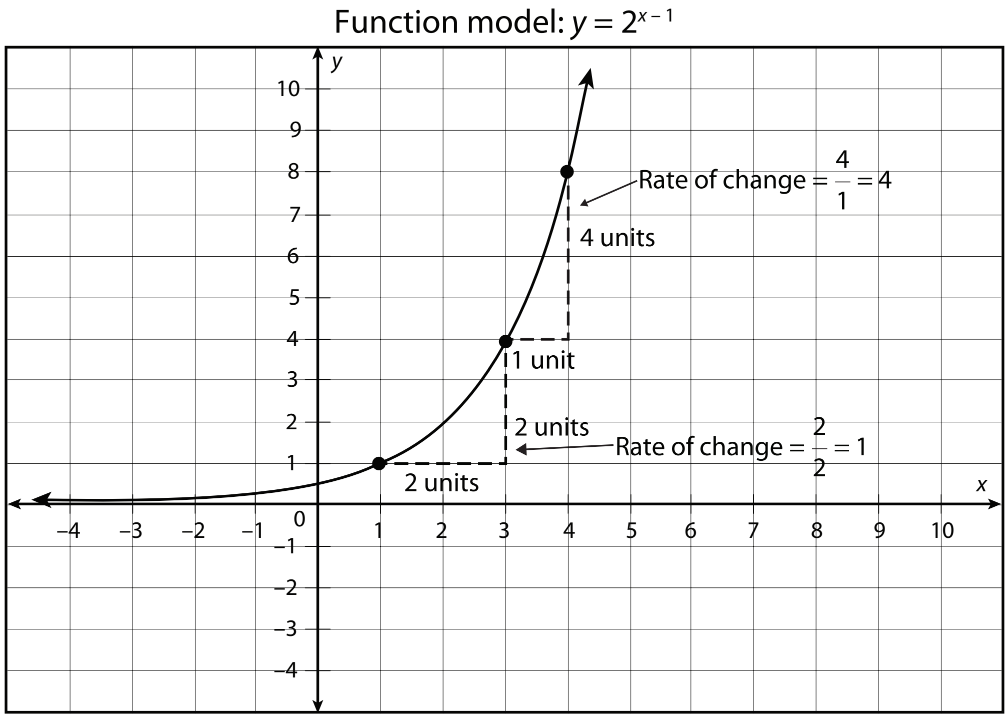
- If *a* is positive, the quadratic function will have a minimum.



Exponential Functions

- Quantities that grow or decay very quickly can often be best modeled by an exponential function.
- Exponential functions either increase or decrease, but they do not change direction.
- The pattern of rate of change of an exponential model shows an increase or decrease by a constant multiple.
- Exponential functions increase or decrease by a constant multiple or divisor, or by a constant percentage.
- There is a constant multiple in the rate of change between *y*-values.
- As the value of *x* increases, the value of $f(x)$ will increase by a multiple of *b*.
- Exponential functions have the general form $f(x) = ab^{cx}$, in which *a*, *b*, and *c* are constants.
- Graphs of exponential functions of the form $f(x) = ab^x$, where *b* is greater than 1, will increase faster than graphs of linear functions of the form $f(x) = mx + b$.

- A quantity that increases exponentially will always eventually exceed a quantity that increases linearly or quadratically.



- Complicated real-world problems are often difficult to model using purely mathematical functions with simple parameters. It is crucial to think critically and compare function models to describe a real-world problem. Selecting an appropriate model is often more important than a thorough knowledge of the functions' characteristics and how they apply to a problem.

Common Errors/Misconceptions

- confusing exponential functions with quadratic functions
- forgetting that the rate of change is not constant across exponential and quadratic functions