Analyzing Savings Account Options Using Equations and Inequalities

Warm-Up

A bank offers a checking account for an initial deposit of $50. Every month you deposit $15 into the account.

1. Make a table of values that shows the total amount in the account after each month if you saved for one year.
2. Write an algebraic equation that can be used to represent the situation.
3. What do the unknown values in your equation represent?

1. Make a table of values that shows the total amount in the account after each month if you saved for one year.
   - Make a table of values that shows the total amount in the account after each month if you saved for one year.

Number of months	0	1	2	3	4	5	6	7	8	9	10	11	12
Total amount saved	50	65	80	95	110	125	140	155	170	185	200	215	230

2. Write an algebraic equation that can be used to represent the situation.

$$T(m) = 15m + 50$$

- It takes \$50 to start up the account. This is the *y*-intercept or initial value.
- The rate that is saved per month is \$15. This is the slope.
- The equation is written in slope-intercept form.

3. What do the unknown values in your equation represent?
   - m is used to represent the number of months saved.
   - $T(m)$ is used to represent the total amount saved after m months.

Instruction

Introduction

When choosing a savings account, it can be difficult to navigate the wide array of options available on the market. Offers may boast similar annual interest rates, but give wildly different returns based on the compounding period, the presence or absence of fees, and other factors. Equations can be used to help analyze savings options in order to make the best decision.

Key Concepts

- Recall that the amount of simple interest paid on a principal P is $A = Prt$, where A is the balance, t is the number of interest periods, and r is the interest rate for that period.
- Recall that the compound interest formula is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where P is the principal, r is the annual interest rate, n is the number of times interest is compounded in a year, and t is years.

Key Concepts, *continued*

- Recall that a savings account accrues interest.
- If no withdrawals or deposits are made to the account, the savings account balance will grow according to the compound interest formula.
- If withdrawals or deposits are made to the account, the savings account balance can be found by applying the simple interest formula for each compounding period. The compound interest formula cannot be used because it does not take into account the changes to the balance.

Key Concepts, *continued*

- In general, the savings account with the higher interest rate is the better choice. However, this calculation can change if periodic account fees are charged.
- If one account charges account fees and the other does not, the amount deposited in the account matters.
  - If the principal deposit is too low, the account charging no fees is the better choice.
  - If the principal deposit is sufficiently high, the account charging a fee may end up being the better choice.

Key Concepts, *continued*

- One of the best ways to compare account options is to compare their growth over time. This can be done by analyzing the equation, or by using tables and graphs.
- Follow the instructions specific to your calculator model:

On a TI-83/84:

Step 1: Press [Y =].

Step 2: In the Y_1 space, type the equation. Type subsequent equations in Y_2, Y_3, etc.

Step 3: Press [WINDOW] to change the viewing window.

(*continued*)

Key Concepts, *continued*

Step 4: At Xmin, enter the appropriate minimum *x*-value and arrow down 1 level to Xmax.

Step 5: At Xmax, enter the appropriate maximum *x*-value and arrow down 1 level to Xscl.

Step 6: At Xscl, enter the appropriate distance between *x*-axis tick marks and arrow down 1 level to Ymax.

Step 7: At Ymin, enter the appropriate minimum *y*-value and arrow down 1 level to Ymax.

(*continued*)

Key Concepts, *continued*

Step 8: At Ymax, enter the appropriate maximum *y*-value and arrow down 1 level to Yscl.

Step 9: At Yscl, enter the appropriate distance between *y*-axis tick marks.

Step 10: Press [GRAPH].

Key Concepts, *continued*

On a TI-Nspire:

Step 1: Press the [home] key.

Step 2: Arrow over to the graphing icon and press [enter].

Step 3: At the blinking cursor to the right of *f*1(*x*), type the equation.

Step 4: To type additional equations, press [menu], arrow over to 3: Graph Entry/Edit, and select 1: Function. Then repeat Step 3. Do this until all equations have been entered.

(*continued*)

Key Concepts, *continued*

Step 5: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom, and click the center button of the navigation pad.

Step 6: Choose 1: Window settings by pressing the center button.

Step 7: Type in the appropriate Xmin value and press [tab].

Step 8: Type in the appropriate XMax value and press [tab].

(*continued*)

Key Concepts, *continued*

Step 9: Leave the XScale set to "Auto." Press [tab] twice to navigate to YMin and enter the appropriate value.

Step 10: Press [tab] to navigate to YMax. Enter the appropriate value. Press [tab] twice to leave YScale set to "Auto" and to navigate to "OK."

Step 11: Press [enter].

Step 12: Press [menu] and select 2: View and 5: Show Grid.

Common Errors/Misconceptions

- incorrectly applying the order of operations
- incorrectly identifying the rate
- forgetting to add 1 after dividing r by n
- forgetting to calculate the number of time periods it takes for a given rate of growth or decay to occur, and simply substituting in the time given

Guided Practice

Example 1

An investment of $1,000 is put into a savings account with an annual interest rate of 3.1%, compounded annually. Write an equation to model this scenario. Graph the equation.

Guided Practice: Example 1, *continued*

1. **Read the problem statement and then reread the problem, determining the known quantities.**

   Principal: $1,000

   Interest rate: 3.1%

Guided Practice: Example 1, *continued*

2. Use the compound interest formula, $A = P\left(1 + \frac{r}{n}\right)^{nt}$, to substitute values.

Before substituting, change the interest rate to a decimal by dividing by 100.

$3.1 \div 100 = 0.031$

Since interest is compounded annually, the number of compounding periods per year is 1, or $n = 1$.

Guided Practice: Example 1, *continued*

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$ Compound interest formula

$$A = 1000\left(1 + \frac{0.031}{1}\right)^{1t}$$ Substitute the given values. Keep the time variable, t, when graphing to indicate the number of years the account is growing.

$$A = 1000(1 + 0.031)^{t}$$ Simplify.

Guided Practice: Example 1, *continued*

3. Use the equation created to make a table of values.

Use values 0 – 5 for *t*, to indicate the number of years the money is in the savings account. Substitute these values into the equation to find the total amounts, *A*.

t	*A(t)*
0	1000
1	1031
2	1063
3	1095.90
4	1129.90
5	1164.90

Guided Practice: Example 1, *continued*

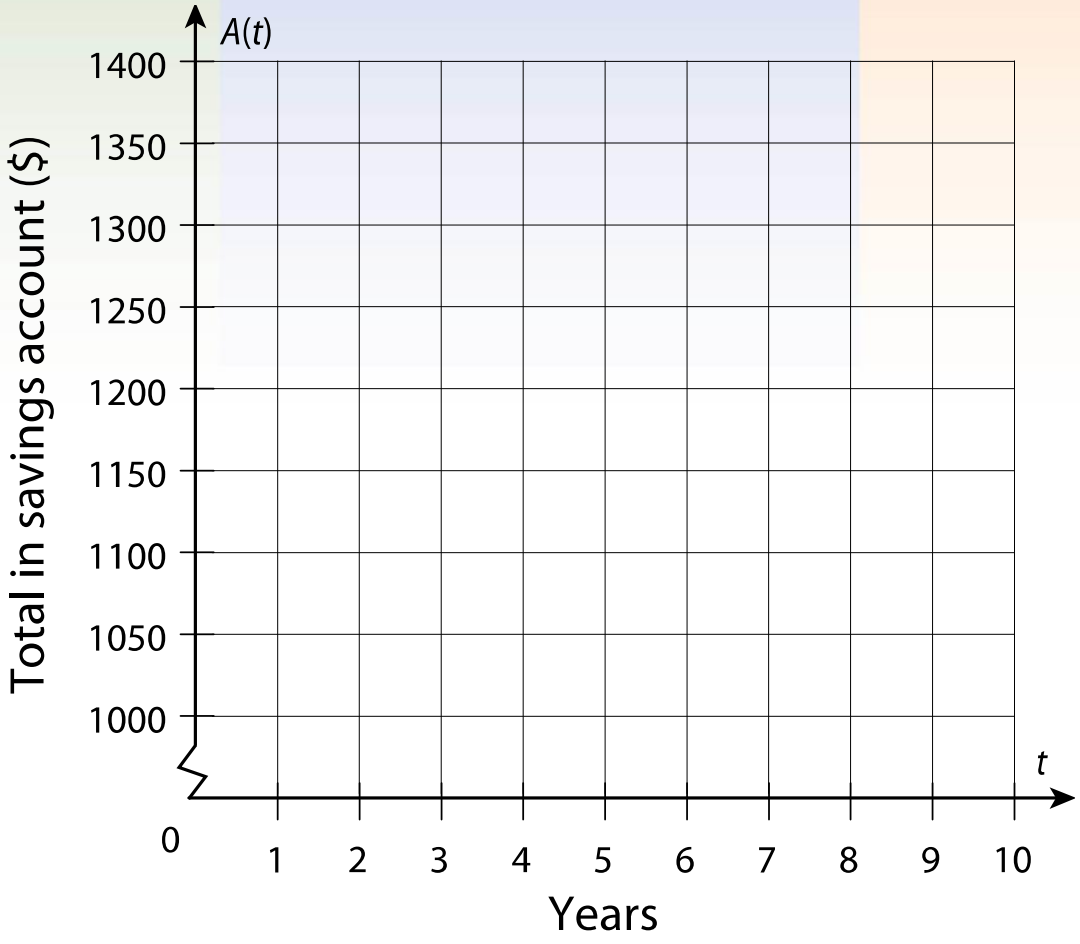
4. Set up the coordinate plane.

Determine the labels by reading the problem again. The independent variable is the number of years the money is in the savings account. Label the *x*-axis "years". The dependent variable is the amount of money that will be in the account after each year. Label the *y*-axis "total amount in savings account".

Guided Practice: Example 1, *continued*

The *x*-axis needs a scale that reflects the number of years the account is active. Use increments of 1 year and the scale should go from at least 0 to 5. Since the *y*-axis represents the amount of money in the account starting at $1,000, show a break in the graph from 0 to 1,000 with a zigzag line. Label the *y*-axis from 0 to 1,400, using increments of 50.

Guided Practice: Example 1, *continued*

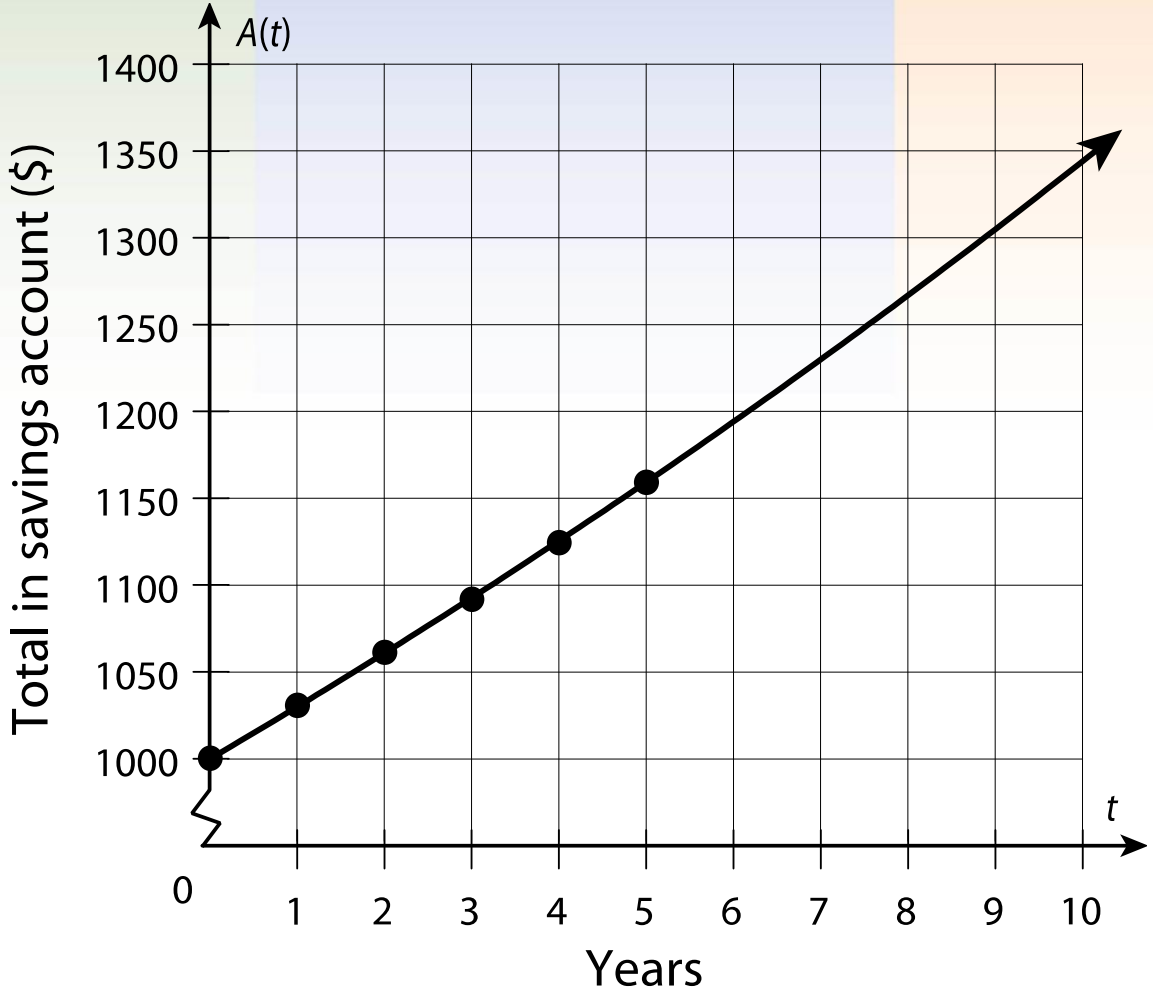


Guided Practice: Example 1, *continued*

5. Plot the points on the coordinate plane and connect the points with a line (curve).

When the points do not lie on a grid line, use estimation to approximate where the point should be plotted. Add an arrow to the right end of the line to show that the curve continues in that direction toward infinity.

Guided Practice: Example 1, *continued*



Guided Practice

Example 2

A bank offers a savings account with 0.6% APR, compounded monthly. However, the account charges a monthly maintenance fee of $3.50. If Saul has $7,500 to invest in a savings account, is this a good deal?

Guided Practice: Example 2, *continued*

1. **Read the problem statement and determine the known quantities.**

   Principal: $7,500

   Interest rate: 0.6% per year

   Maintenance fee: $3.50 per month

Guided Practice: Example 2, *continued*

2. Determine how to find the account balance.

Since a maintenance fee is charged to the account, the compound interest formula cannot be used. Instead, the simple interest formula must be used to calculate the monthly interest. Then the maintenance fee will be subtracted.

Guided Practice: Example 2, *continued*

3. Calculate the account balance for 1 year.

This will require applying the simple interest formula to the new balance for each month. First, find the monthly interest rate. This is done by dividing the annual interest rate by 12.

$$0.6 \div 12 = 0.05\%$$

The monthly interest rate is 0.05%, or 0.0005.

Guided Practice: Example 2, *continued*

Next, apply the simple interest formula, $A = Prt$, to find the amount of interest paid on the principal. Note that $r = 0.0005$, $t = 1$, and $P = 7{,}500$ to start. Add this to the balance, and subtract the maintenance fee of $3.50. The following table shows the results of the calculations.

Guided Practice: Example 2, *continued*

Month	Starting balance (P)	Interest (Pr)	Maintenance fee (F)	New balance ($P + Pr - F$)
1	$7,500.00	$3.75	$3.50	$7,500.25
2	$7,500.25	$3.75	$3.50	$7,500.50
3	$7,500.50	$3.75	$3.50	$7,500.75
4	$7,500.75	$3.75	$3.50	$7,501.00
5	$7,501.00	$3.75	$3.50	$7,501.25
6	$7,501.25	$3.75	$3.50	$7,501.50
7	$7,501.50	$3.75	$3.50	$7,501.75
8	$7,501.75	$3.75	$3.50	$7,502.00
9	$7,502.00	$3.75	$3.50	$7,502.25
10	$7,502.25	$3.75	$3.50	$7,502.50
11	$7,502.50	$3.75	$3.50	$7,502.75
12	$7,502.75	$3.75	$3.50	$7,503.00

Guided Practice: Example 2, *continued*

4. Determine whether this offer is a good deal.

The account yields about $3.75 each month in interest payments, but charges $3.50 each month in maintenance fees. This yields a net of $0.25 per month. This account offer is probably not a great deal, since it yields $3.00 per year, but Saul would need to compare this to other savings options to make a final determination.

Guided Practice

Example 3

Jenne is comparing two bank options:

- **Option 1**: 5.1% APR, compounded monthly; $50 minimum deposit
- **Option 2**: 1.4% APR, compounded monthly; $100 minimum deposit

Suppose Jenne makes only the minimum deposit for whichever option she chooses. Find equations to model each option, then graph them. Use the graph to determine which account will have the greater yield after 1, 5, and 10 years. Will one option always be better than the other in this situation?

Guided Practice: Example 3, *continued*

1. **Find an expression for the growth of the deposit in Option 1.**

   The formula for calculating the value of a deposit that accrues compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where P is the principal, r is the interest rate, t is years, and n is the number of times the interest is compounded per year.

Guided Practice: Example 3, *continued*

The principal amount in option 1 is \$50, and the annual interest rate is 5.1%. The interest is compounded monthly. This gives the formula

$$A = 50\left(1 + \frac{0.051}{12}\right)^{12t}.$$

Guided Practice: Example 3, *continued*

2. Find an expression for the growth of the deposit in Option 2.

The formula for calculating the value of a deposit that accrues compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where P is the principal, r is the interest rate, t is years, and n is the number of times the interest is compounded per year.

Guided Practice: Example 3, *continued*

The principal amount in option 2 is $100, and the annual interest rate is 1.5%. The interest is compounded monthly. This gives the formula

$$A = 100\left(1 + \frac{0.015}{12}\right)^{12t}.$$

Guided Practice: Example 3, *continued*

3. Create a table of values for each equation.

We will need to see the account yields after 1, 5, and 10 years. Evaluate each equation for $t = 0, 1, 2, \ldots 10$. Substitute these values into each equation to find the total amounts.

Guided Practice: Example 3, *continued*

t	Option 1:	Option 2:
0	50.00	100.00
1	52.61	101.51
2	55.36	103.04
3	58.25	104.60
4	61.29	106.18
5	64.49	107.78
6	67.86	109.41
7	71.40	111.06
8	75.13	112.74
9	79.05	114.44
10	83.17	116.17

Guided Practice: Example 3, *continued*

4. Set up the coordinate plane.

Determine the labels by reading the problem again. The independent variable is the number of years the money is in the savings accounts. Label the *x*-axis "years."

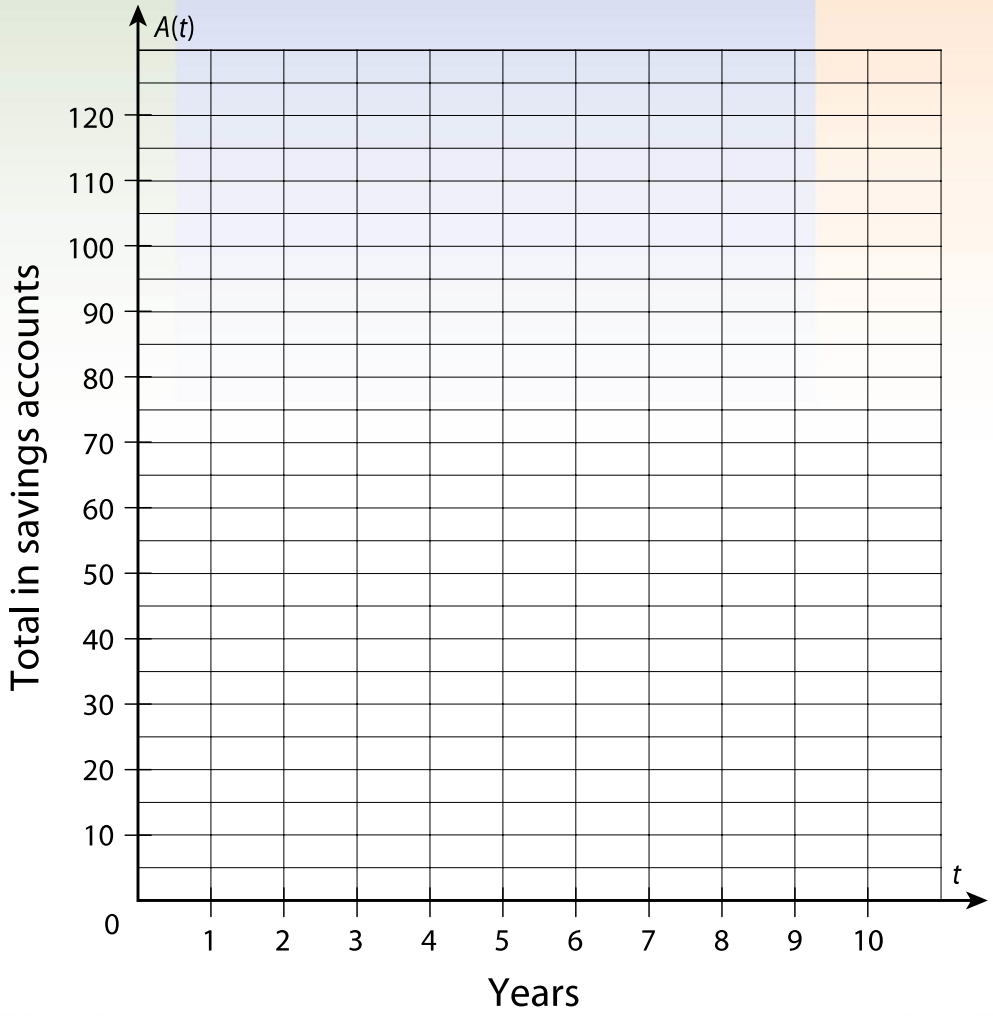
The dependent variable is the amount of money that will be in the accounts after each year. Label the *y*-axis "Total amount in savings accounts."

The *x*-axis needs a scale that reflects the number of years the accounts are active. Use increments of 1 year. The scale should go from 0 to at least 10.

Guided Practice: Example 3, *continued*

Since the *y*-axis represents the amount of money in the account, it should have a range of at least 50 to 116, though the minimum value can be less than 50 and the maximum value can be more than 116 to make graphing easier. Increments of 5 or 10 will work nicely for this range. The following is an example of a coordinate plane set up for this problem.

Guided Practice: Example 3, *continued*



Guided Practice: Example 3, *continued*

5. Plot the points on the coordinate plane for each equation, connecting them with a curve.

When the points do not lie on a grid line, which is true for the majority of the points in this situation, approximate the places where each point should be plotted.

You can also use a graphing calculator to create the graph of the equation. Follow the directions appropriate to your calculator model to graph the equations.

Guided Practice: Example 3, *continued*

On a TI-83/84:

Step 1: Press [Y =].

Step 2: In the Y_1 space, type the first equation:

$$Y_1 = 50\left(1+\frac{0.051}{12}\right)^{12t}$$

Step 3: In the Y_2 space, type the second equation:

$$Y_2 = 100\left(1+\frac{0.015}{12}\right)^{12t}$$

(*continued*)

Guided Practice: Example 3, *continued*

Step 4: Press [WINDOW] to change the viewing window.

Step 5: At Xmin, enter [0] and arrow down 1 level to Xmax.

Step 6: At Xmax, enter [10] and arrow down 1 level to Xscl.

Step 7: At Xscl, enter [1] and arrow down 1 level to Ymax.

(*continued*)

Guided Practice: Example 3, *continued*

Step 8: At Ymin, enter [0] and arrow down 1 level to Ymax.

Step 9: At Ymax, enter [120] and arrow down 1 level to Yscl.

Step 10: At Yscl, enter [5].

Step 11: Press [GRAPH].

Guided Practice: Example 3, *continued*

On a TI-Nspire:

Step 1: Press the [home] key.

Step 2: Arrow over to the graphing icon and press [enter].

Step 3: At the blinking cursor to the right of $f1(x)$, type the first equation, $f1(x) = 50\left(1 + \frac{0.051}{12}\right)^{12t}$, and press [enter].

(*continued*)

Guided Practice: Example 3, *continued*

Step 4: Press [menu], arrow over to 3: Graph Entry/ Edit, and select 1: Function. At the blinking cursor to the right of $f2(x)$, type the second equation, $f2(x) = 100\left(1 + \frac{0.015}{12}\right)^{12t}$, and press [enter].

(*continued*)

Guided Practice: Example 3, *continued*

Step 5: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom, and click the center button of the navigation pad.

Step 6: Choose 1: Window settings by pressing the center button.

Step 7: Type in the appropriate Xmin value, [0], and press [tab].

Step 8: Type in the appropriate XMax value, [10], and press [tab].

(*continued*)

Guided Practice: Example 3, *continued*

Step 9: Leave the XScale set to "Auto." Press [tab] twice to navigate to YMin and enter [0].

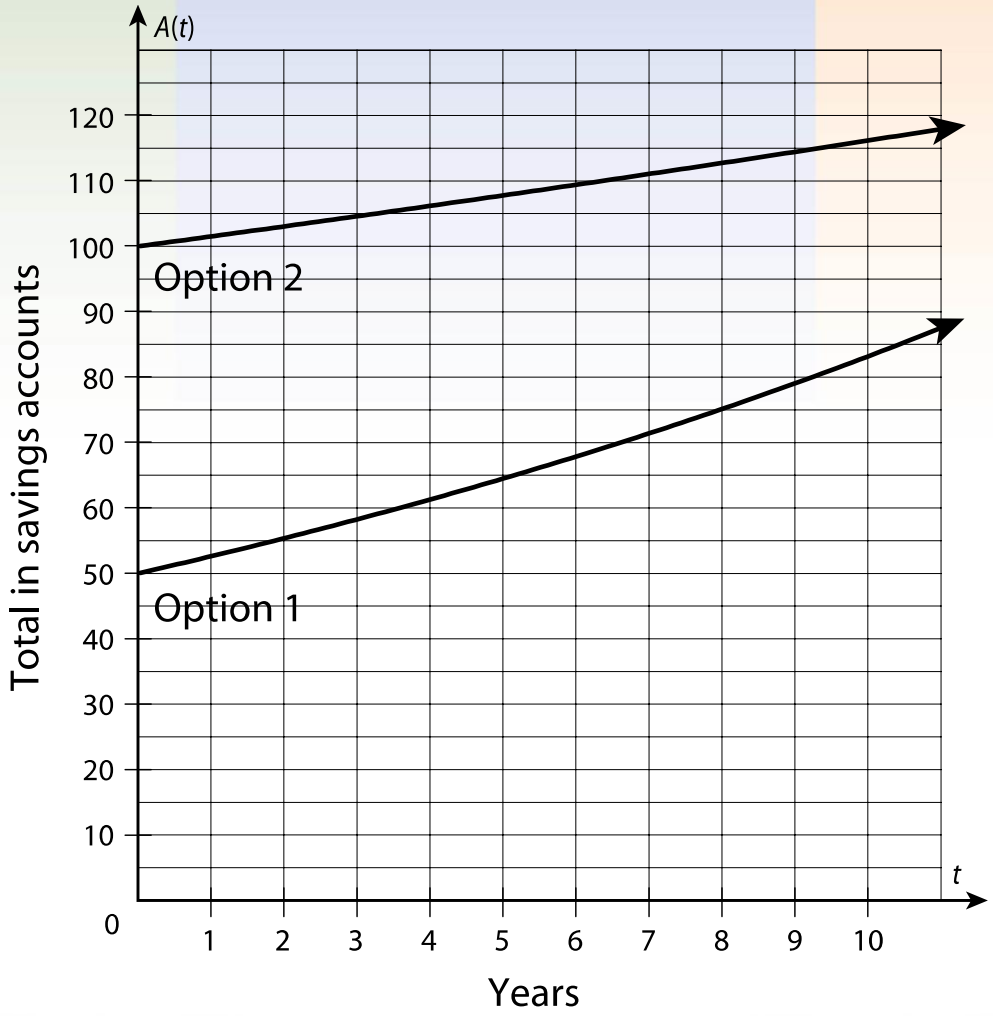
Step 10: Press [tab] to navigate to YMax. Enter [120]. Press [tab] twice to leave YScale set to "Auto" and to navigate to "OK."

Step 11: Press [enter].

Step 12: Press [menu] and select 2: View and 5: Show Grid.

The resulting graph should resemble the following:

Guided Practice: Example 3, *continued*



Guided Practice: Example 3, *continued*

6. Determine which account will have the greater yield after 1, 5, and 10 years. Will one option always be better than the other in this situation?

Option 2 has the greater amount after 1, 5, and 10 years. However, the graph for Option 1 is growing faster than the graph of Option 2. Option 1 would be better if the same amount was invested in each account.