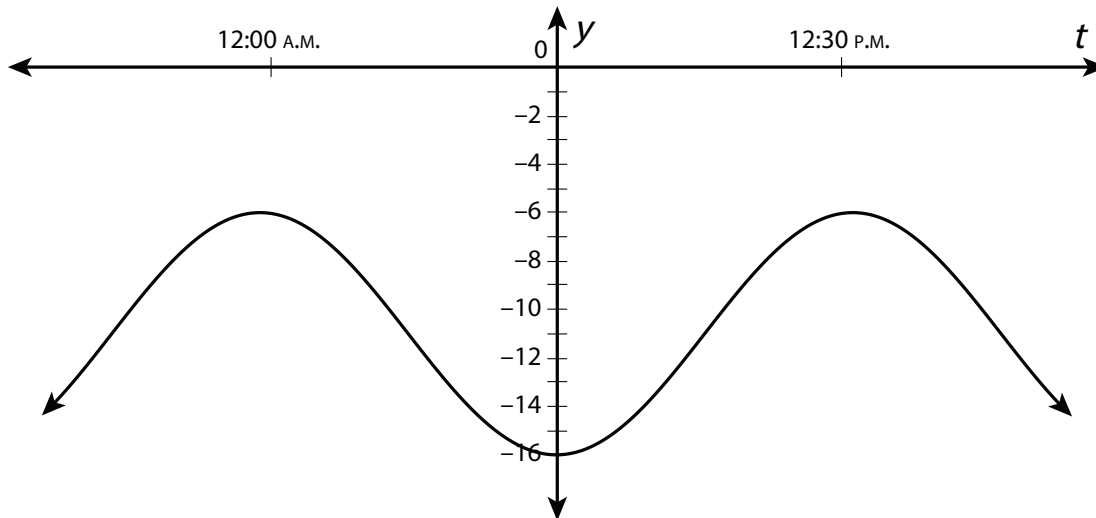


Problem-Based Task: Timing the Tide

Coaching Sample Responses

- a. Sketch the situation on an x - y axis, noting relevant data from the problem on the sketch.



- b. Use the given information to determine three points through which the model must pass.

At $t = 0$, the model should produce the maximum distance of 16 ft from the pier (low tide). Because the pier represents a height of 0 feet, this point can be represented as $(0, -16)$.

At $t = -6.25$ (the time between the first high tide and low tide), the model should produce a distance of -6 ft from the pier. This point can be represented as $(-6.25, -6)$.

At $t = 6.25$ (the time of the second high tide), the model should produce the minimum distance of -6 ft. This can be represented by the point $(6.25, -6)$.

- c. Use the data points from part b to determine the amplitude, period, phase shift, and vertical shift for a cosine model.

- Amplitude: $\frac{16 \text{ ft} - 6 \text{ ft}}{2} = \frac{10 \text{ ft}}{2} = 5 \text{ ft}$

The amplitude is half of the distance between high tide and low tide.

- Period: $12.5 - 0 = 12.5$ hours

The period is the time from one high tide to the next high tide.

- Phase shift: The maximum height of the parent cosine curve, $f(x) = \cos x$, appears at $t = 0$. Because we want to represent the given function with a cosine curve, determine how far the curve was shifted left or right from standard position. It has shifted 6.25 units left or right. Let's use the positive shift, 6.25 hours, which moves the maximum cosine value from $t = 0$ to $t = 6.25$.
- Vertical shift: $-6 \text{ ft} - 5 \text{ ft} = -11 \text{ ft}$
The midline of the curve is 11 feet below the dock's height of 0. This is shifted down 11 units from the horizontal axis.

d. Use the parameters from part c to develop a function that models the situation.

Write the function in the general form of a cosine function, $f(x) = a \cos [b(x - c)] + d$.

$$d(t) = 5 \cos \left[\frac{2\pi}{12.5}(t - 6.25) \right] - 11$$

e. Verify that the model produces the data points identified in part b.

The data points from part b are $(0, -16)$, $(-6.25, -6)$, and $(6.25, -6)$. Input each of these values into the function and solve.

$$d(0) = 5 \cos \left(\frac{2\pi}{12.5}[(0) - 6.25] \right) - 11 = 5 \cos(\pi) - 11 = 5(-1) - 11 = -16$$

$$d(-6.25) = 5 \cos \left(\frac{2\pi}{12.5}[(-6.25) - 6.25] \right) - 11 = 5 \cos(-2\pi) - 11 = 5(1) - 11 = -6$$

$$d(6.25) = 5 \cos \left(\frac{2\pi}{12.5}[(6.25) - 6.25] \right) - 11 = 5 \cos(0) - 11 = 5(1) - 11 = -6$$

The data points identified are all verified with our function.

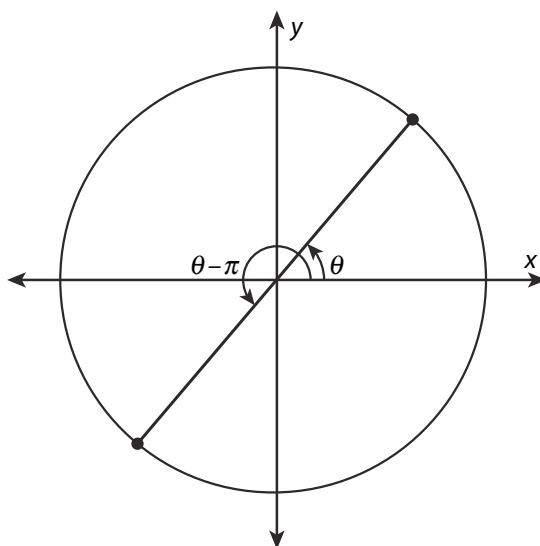
f. Simplify the model.

The argument of the cosine function can be rewritten as $\frac{2\pi}{12.5}t - 2\pi \frac{6.25}{12.5} = \frac{2\pi}{12.5}t - \pi$.

The phase shift in the model, 6.25, is exactly half the period of the function (12.5). Use this information to simplify further.

Use a sketch of the unit circle to visualize and compare the angles for the parent cosine curve that has a phase shift of half the period of the function. The period of the cosine curve is 2π , so half of this is π .

Notice that the two angles, θ and $\theta - \pi$, are symmetric under a rotation of π radians about the origin.



For the cosine function, we are interested in the x -coordinates of the two intercepted points on the unit circle. The symmetry shows us they must have the same magnitude, but opposite signs: $\cos(\theta - \pi) = -\cos(\theta)$. Therefore, the model for our given problem can be simplified by making the same substitution: remove the phase shift constant, and negate the result of the cosine function.

$$d(t) = 5 \cos \left[\frac{2\pi}{12.5}(t - 6.25) \right] - 11$$

$$d(t) = -5 \cos \left(\frac{2\pi}{12.5}t \right) - 11$$

- g. Verify that the simplified model still produces the correct data points.

The data points are $(0, -16)$, $(-6.25, -6)$, and $(6.25, -6)$. Input each of these values into the simplified model and solve.

$$d(0) = -5 \cos \left[\frac{2\pi}{12.5}(0) \right] - 11 = -16$$

$$d(-6.25) = -5 \cos \left[\frac{2\pi}{12.5}(-6.25) \right] - 11 = -5 \cos(-\pi) - 11 = -6$$

$$d(6.25) = -5 \cos \left[\frac{2\pi}{12.5}(6.25) \right] - 11 = -5 \cos(\pi) - 11 = -6$$

All the results remain correct, so the simplified model is equivalent to the original model.

- h. Find the time t corresponding to 9 A.M. on the third day, evaluate the function at this time, and interpret the result.

The time t is measured in hours, beginning at 6:15 A.M. on the first day. (Recall that 6.25 represents the amount of time that has passed since midnight. Converting 6.25 to hours and minutes yields a time of 6:15 A.M.) If we add 48 hours, we arrive at 6:15 A.M. on day three. The given time of 9 A.M. is 2 hours and 45 minutes (2.75 hours) after this, so the value for t at 9 A.M. on day three is $48 + 2.75 = 50.75$.

Evaluating the function argument at this value of t does not lead to a special angle, so a calculator is needed:

$$d(t) = -5 \cos \left[\frac{2\pi}{12.5}(50.75) \right] - 11 \approx -5 \cos(25.510) - 11 \approx -15.64$$

This means that at 9:00 A.M. on the third day, the tide will be about 15.64 ft below the dock.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.