

## Conceptual Task: Rationalizing Rational Functions

### Exploration Questions Sample Responses

- a. Explain in your own words what was each student supposed to do.

Each student needed to construct a rational function that followed some rules. The function had to have three non-zero invalid values of  $x$ . The function needed 2 or more roots, and to be reported fully simplified.

- b. What are Nicole and Wayne discussing?

Nicole and Wayne are discussing the two functions they created and how well each follows the rules. Nicole has some critiques of Wayne's function.

- c. Explain how you could check to see if a rational expression violates Rule 1.

The invalid values of a rational expression are the  $x$  values for which the denominator is equal to 0. To find these values, the denominator should be fully factored. Set each of the factors equal to 0 and solve for  $x$  to find all the values for which the denominator equals 0. To follow the first rule, a function's denominator must have three factors.

- d. What is Nicole's first critique of Wayne's function? Which rule is he breaking and how does Nicole know?

Nicole tells Wayne that his function does have three invalid values, but one is  $x = 0$ . So, Wayne's function breaks Rule 2. She knows this without having to completely factor Wayne's function's denominator because of the lone  $x$  in the denominator. When  $x$  is equal to 0, that term will make the entire denominator equal to 0.

- e. What is the difference Nicole is talking about when she says, "Mine is different because you put a lone  $x$  in the denominator"?

Wayne is confused because he sees a lone  $x$  in Nicole's function, and he wonders if that breaks Rule 2 as well. She tells him it does not because it makes a difference whether that term is in the numerator or denominator. If the lone  $x$  term is in the numerator, the function will simply equal 0. However, if the lone  $x$  term is in the denominator, the function will be undefined. Therefore,  $x = 0$  is only an invalid value when the denominator contains an  $x$  as a factor.

- f. What is Nicole's second critique of Wayne's function? What is Wayne's argument against Nicole?

Nicole argues that Wayne needs to fully factor his function to check for the number of roots.

Wayne argues that  $x = -2$  is one root and that  $x^2 - 2x - 3$  has two more roots, which gives three roots total.

- g. What would have to be true of Wayne's function to violate Rule 3? Justify your answer.

If the numerator and denominator share any factors, the number of roots of the function will decrease. Once the quadratics on top and bottom are completely factored, if the numerator and denominator share any factors, those factors will cancel each other out.