Creating Rational Equations

Prerequisite Skills

This lesson requires the use of the following skills:

- writing percentages as decimals
- adding, subtracting, multiplying, and dividing with decimals
- determining the least common denominator of fractions
- applying the Distributive Property
- solving linear one-variable equations
- representing comparisons of quantities as ratios and rates
- writing equivalent fractions, some in simplest form
- solving quadratic equations
- factoring quadratic trinomials, including writing them as products of binomials
- multiplying binomials

Introduction

There are many types of equations used in solving mathematics. The type of equation you work with depends largely on the problem situation, the information given, and what you are trying to find out. In this lesson, you will work with rational equations, equations that have a variable in a denominator. Creating rational equations to represent problem situations often involves ratios such as $\frac{\text{20 customers}}{\text{7 employees}}$, and rates such as $\frac{\text{75 miles}}{\text{2.5 gallons}}$. Therefore, solving rational equations frequently uses previously learned skills involving fractions.

Key Concepts

- A **ratio** is a relation between two quantities. A ratio can be expressed in words, fractions, decimals, or as a percentage.
- Examples of ratios include $\frac{2}{5}$, $\frac{\text{15 successes}}{\text{20 attempts}}$, and $\frac{\text{66 students}}{\text{3 teachers}}$.
- A **rate** is a ratio that compares measurements with different kinds of units.

- Examples of rates include $\frac{3\text{ feet}}{2\text{ inches}}$ and $\frac{75\text{ miles}}{2.5\text{ gallons}}$.
- The ratio $\frac{\text{successes}}{\text{attempts}}$ appears in various applications, sometimes using different words, but representing the same concept.
- For example, a shooting percentage in basketball can be written as $\frac{7\text{ successes}}{10\text{ attempts}}$. If you successfully make 7 shots out of 10 attempts, your shooting percentage is: $\frac{7}{10}=0.7=70\%$.
- Another example is batting average in baseball or softball. If you get 3 hits in 8 at-bats, your batting average is $\frac{3\text{ successes}}{8\text{ attempts}}=\frac{3}{8}=0.375$. Batting averages are typically left as a decimal rather than converted to a percent.
- A **rational expression** is an expression made of the ratio of two polynomials, in which a variable appears in the denominator of a polynomial.
- Examples of rational expressions include $\frac{2x}{3y^2}$, $\frac{12}{x}$, and $\frac{x^2+2x-15}{x-3}$.
- A rational expression is in simplest form when it is expressed as a quotient of polynomials whose greatest common factor is 1.
- To simplify a rational expression, factor either the numerator or the denominator or both so that common factors in the numerator and denominator can be identified, and then cancelled out.
- A **rational equation** is an equation that includes the ratio of two rational expressions, in which a variable appears in the denominator of at least one rational expression.
- Examples of rational equations include $\frac{5+x}{20+x}=0.90$, $\frac{12}{20}+\frac{15}{x}=1$, and $\frac{-4}{x^2-2x-3}+2=\frac{x-4}{x-3}$.
- The **least common multiple (LCM)** of two or more polynomials is the common multiple that has the least degree and the least positive constant factor.
- The **least common denominator (LCD)** of two or more fractions is the least common multiple of their denominators.

- Given the fractions $\frac{18}{20}$ and $\frac{5}{x}$, the least common multiple of 20 and x is $20x$, so the least common denominator is $20x$.
- Given the fractions $\frac{20}{x}$ and $\frac{4}{x+8}$, the least common multiple of x and $x + 8$ is $x(x + 8)$, so the least common denominator is $x(x + 8)$.
- A **rational function** is a function that can be written in the form $f(x)=\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$.
- Examples of rational functions include $f(x)=\frac{5+x}{20+x}$, $f(x)=\frac{-4}{x^2-2x-3}$, and $f(x)=\frac{12}{20}+\frac{15}{x}$.
- $f(x)=\frac{12}{20}+\frac{15}{x}$ can be written in the form $f(x)=\frac{p(x)}{q(x)}$:

  $f(x)=\frac{12}{20}+\frac{15}{x}=\frac{x}{x}\bullet\frac{12}{20}+\frac{20}{20}\bullet\frac{15}{x}=\frac{12x+300}{20x}$
- Sometimes an equation representing a problem situation is naturally related to a rational function.
- For example, the function $f(x)=\frac{5+x}{20+x}$ might represent a shooting percentage in basketball. If you want to find the value of x that makes the shooting percentage 50%, then you could solve the equation $0.50=\frac{5+x}{20+x}$.
- Solutions to quadratic functions of the form $ax^2 + bx + c = 0$ can be determined using the quadratic formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.
- An **extraneous solution**, or **extraneous root,** of an equation is a solution of an equation that arises during the solving process, but which is not a solution of the original equation.
- Extraneous solutions sometimes occur when solving rational equations, so it is important to check all apparent solutions by substituting them into the original equation and determining if they make it true.
- When solving rational equations, restrictions on the variable must be considered. Possible solutions that result in 0 as the denominator must be excluded from the solutions of the rational equation since denominators cannot equal 0.

Common Errors/Misconceptions

- mistakenly thinking the time to complete a task by two persons or objects working at different rates is the average of the individual times or half the average of the individual times
- neglecting to check apparent solutions to determine if they make the equation true
- using an equation other than the original equation when checking an apparent solution