

Problem-Based Task: Golden Rectangles and the Golden Ratio

Coaching Sample Responses

- a. What is true about corresponding sides of similar polygons?

They are proportional.

- b. You know that $BCQP \sim ABCD$. How can you complete the proportion $\frac{AB}{BC} = \frac{?}{?}$, using \overline{AD} and \overline{BP} ?

$$\frac{AB}{BC} = \frac{AD}{BP}$$

- c. You know $AD = 1$ and $AB = x$. What is the length of \overline{BC} ? Explain.

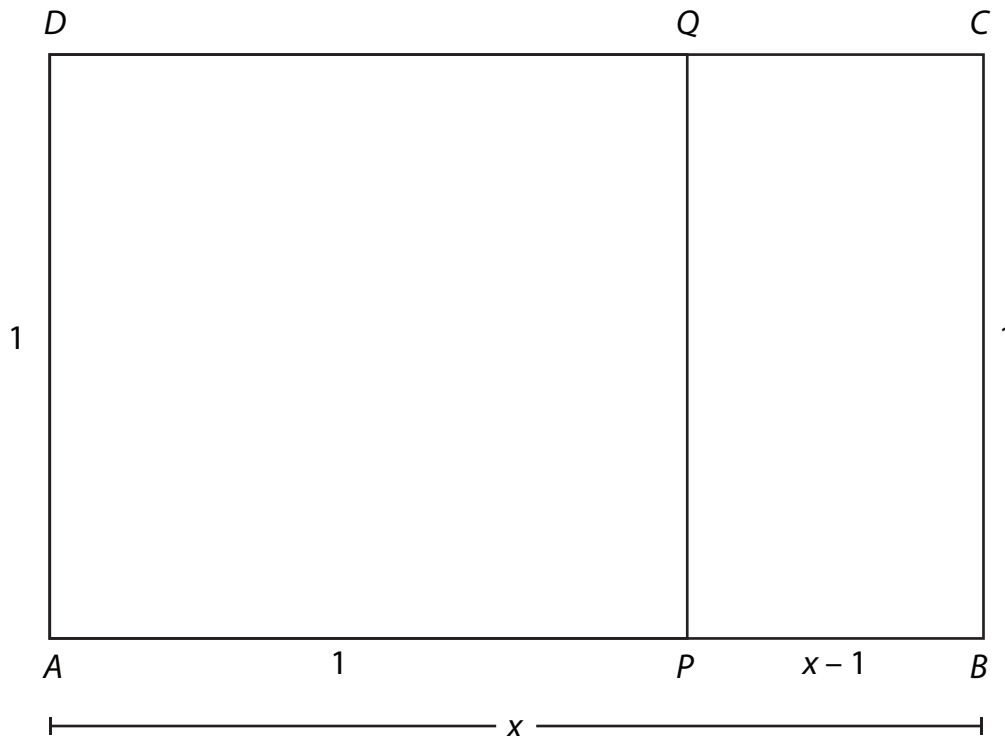
$BC = AD$ because opposite sides of a rectangle are congruent; therefore, $BC = 1$.

- d. What is the length of \overline{AP} ? Explain.

$AD = 1$ and $APQD$ is a square; therefore, $AP = 1$.

- e. What is the length of \overline{BP} ? Explain your answer and then sketch the given diagram, labeling lengths \overline{AB} , \overline{AD} , \overline{AP} , \overline{BC} , and \overline{BP} .

$AB = x$, $AP = 1$, and \overline{BP} is the length remaining when 1 is subtracted from x ; therefore, $BP = x - 1$.



- f. How can you use the proportion from part b to find the value of x ? Show your work and explain your steps.

$$\frac{AB}{BC} = \frac{AD}{BP}$$

Write the proportion from part b.

$$\frac{(x)}{(1)} = \frac{(1)}{(x-1)}$$

Substitute the values and expressions previously identified.

$$(x-1)\left(\frac{x}{1}\right) = (x-1)\left(\frac{1}{x-1}\right)$$

Multiply both sides by the LCD.

$$(x-1)(x) = 1$$

Simplify.

$$x^2 - x = 1$$

Distribute.

$$x^2 - x - 1 = 0$$

Subtract 1 from both sides.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$x = \frac{1 \pm \sqrt{1+4}}{2}$$

Simplify.

$$x = \frac{1 + \sqrt{5}}{2} \text{ or } x = \frac{1 - \sqrt{5}}{2}$$

The value of $\frac{1 - \sqrt{5}}{2}$ is negative, so it cannot represent the length of a rectangle.

The length of rectangle $ABCD$ is $x = \frac{1 + \sqrt{5}}{2}$, which is the golden ratio.

- g. What is a decimal approximation of the golden ratio?

$\frac{1 + \sqrt{5}}{2}$ is approximately equal to 1.618.

- h. What does the decimal approximation of the golden ratio tell you about the shape of any golden rectangle?

The length is about 1.618 times the width.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.