Graphing Rational Functions and Identifying Key Features

Prerequisite Skills

This lesson requires the use of the following skills:

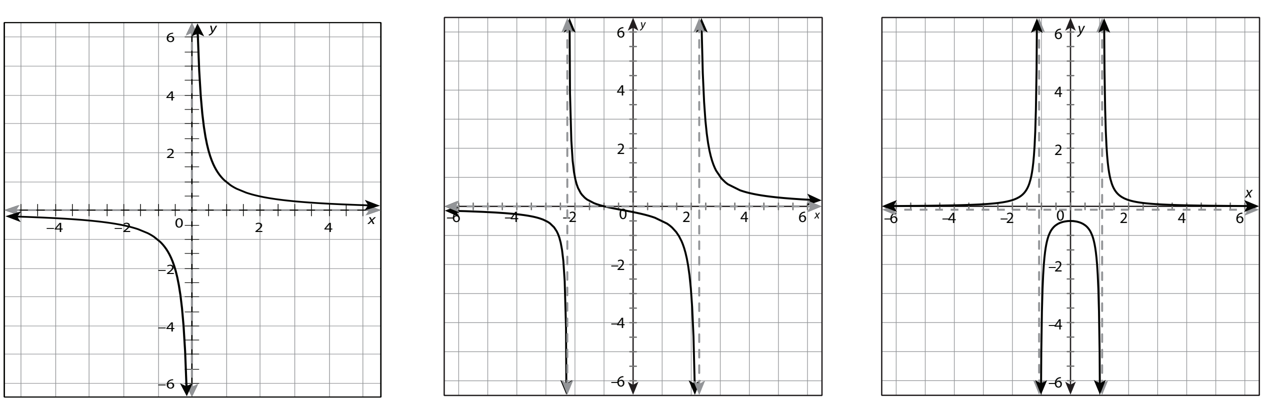
- finding coordinates using an input/output table
- plotting coordinates on a graph
- graphing linear functions
- dividing polynomials
- solving linear and quadratic equations

Introduction

A rational function is a function that is the quotient of two polynomials; it can be written in the form $f(x)=\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$. For example, $f(x)=\frac{x+6}{3x-1}$ is a rational function. In order to graph a rational function, first identify its key features. This information, along with additional points found using an input/output table, can be used to determine the basic shape of the graph.

Key Concepts

- The shape of a rational function will depend on the highest exponent, or degree, of the polynomials in the numerator and denominator. Consider the following examples. These are all graphs of rational functions. Notice that they are in two or more branches and not one smooth curve like the graphs of most functions.



Determining the Asymptotes of a Rational Function

- Notice that in each of these examples, the rational function has **asymptotes**, or lines that the function gets closer and closer to as one of the variables increases or decreases without bound.
- A **vertical asymptote** of the function f is the line $x = a$ where $f(x)$ either increases or decreases without bound as x gets closer to a. To find the vertical asymptote(s) of a rational function, set the denominator equal to 0 and solve for x.
- A **horizontal asymptote** of the function f is the line $y = b$ of the function f where $f(x)$ gets closer to b as x either increases or decreases without bound.
- The method for finding the horizontal asymptote of a rational function differs depending on how the degrees of the polynomials in the numerator and the denominator compare.
- If the numerator has a lower degree than the denominator, then the x-axis is the horizontal asymptote.
- If the numerator and denominator have the same degree, the horizontal asymptote can be found by dividing the coefficient of the highest-degree variable in the numerator by the coefficient of the highest-degree variable in the denominator. For example, in the function $f(x)=\dfrac{6x^2-1}{2x^2-9x+5}$, 6 divided by 2 is 3, so the asymptote is at $y = 3$.

Determining the Zeros and *y*-Intercepts of a Rational Function

- The **zeros** of a function are the x-values at which a function equals 0. Therefore, they are also the points at which the function crosses the x-axis and are also known as the ***x*-intercepts**.
- To find the zeros (or x-intercepts) of a function, replace $f(x)$ or y with 0 and solve for x. Then write the intercept as the coordinate $(x, 0)$.
- The ***y*-intercept** of a function is the point at which the graph crosses the y-axis. To find the y-intercept, replace x with 0 and solve for $f(x)$ or y. Then write the intercept as the coordinate $(0, y)$.
- The table that follows summarizes how to use these key features to graph a rational function.

Graphing a Rational Function
1. Identify and sketch all the asymptotes.
2. Identify and sketch the zero(s) and y-intercept.
3. Plot additional points as needed using an input/output table.
4. Draw the curve of the function.

Identifying the End Behavior of the Function

- The end behavior of a function describes the behavior of the graph as *x* approaches positive infinity and as *x* approaches negative infinity.
- Once a function has been graphed, the graph can be examined to determine the end behavior.
- If the graph appears to be nearly horizontal as *x* approaches negative or positive infinity, then the end behavior is the fact that *y* approaches this *y*-value.
- If instead of approaching a specific *y*-value the graph is constantly increasing or decreasing as *x* approaches negative or positive infinity, then the end behavior is that the function approaches negative or positive infinity.
- The left and right sides of a graph may have different end behaviors; therefore, consider both cases separately.

Common Errors/Misconceptions

- incorrectly applying the process for finding the vertical and/or horizontal asymptote(s)
- incorrectly applying the process for finding the *x*- and/or *y*-intercept(s)
- misidentifying which process to use to find the horizontal asymptote (depending on how the degrees of the numerator and denominator compare)
- not plotting enough points to accurately identify the curve of the function