Problem-Based Task: Medication Levels in the Bloodstream

Coaching Sample Responses

a. What are the asymptotes of the function $f(x)=\frac{10x}{x^2+1}$?

To find the vertical asymptote, set the denominator equal to 0 and solve for x.

$x^2 + 1 = 0$

$x^2 = -1$

$x=\sqrt{-1}$

Since the square root of –1 does not exist in the real number system, there is no vertical asymptote.

For the function $f(x)=\frac{10x}{x^2+1}$, the polynomial in the numerator has a lower degree than the polynomial in the denominator. Therefore, the x-axis ($y = 0$) is the horizontal asymptote.

b. What are the zero(s) and y-intercept of the function?

To find the zero or x-intercept, substitute 0 for $f(x)$ and solve for x.

$f(x)=\frac{10x}{x^2+1}$

$(0)=\frac{10x}{x^2+1}$

$0 = 10x$

$0 = x$

$x = 0$

The zero or x-intercept is (0, 0).

To find the y-intercept, substitute 0 for x and solve for $f(x)$.

$f(x)=\frac{10x}{x^2+1}$

$f(0)=\frac{10(0)}{(0)^2+1}$

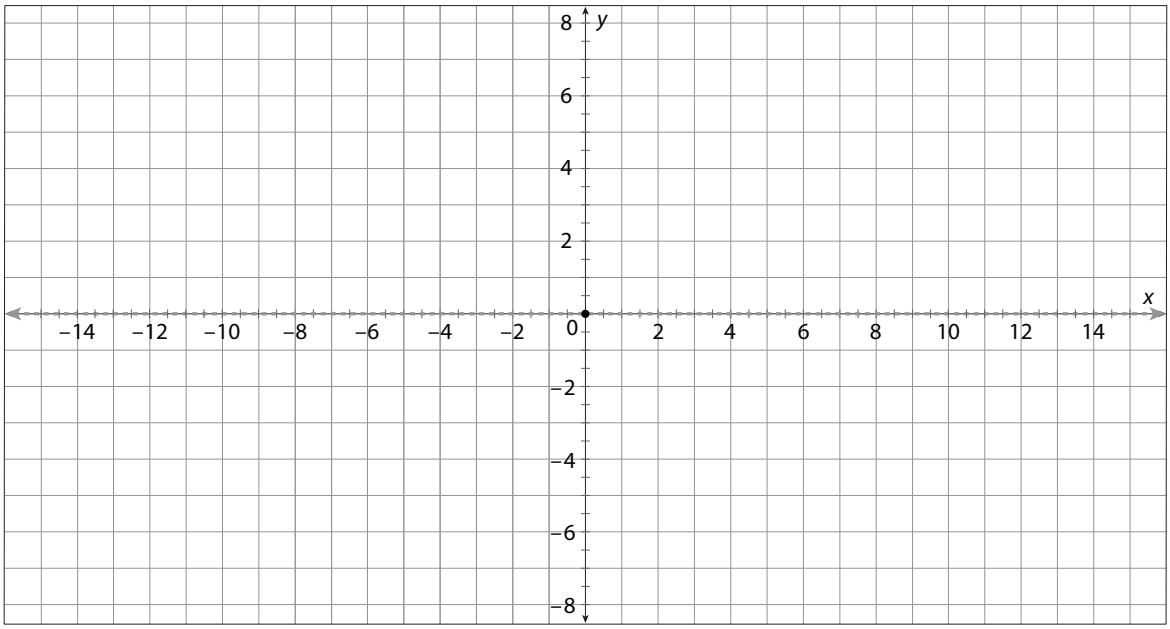
$f(0)=\frac{0}{1}$

$f(0) = 0$

The y-intercept is (0, 0).

c. Use the key features found in parts a and b to graph the function.

Plot the asymptote $y = 0$ and the intercept (0, 0).

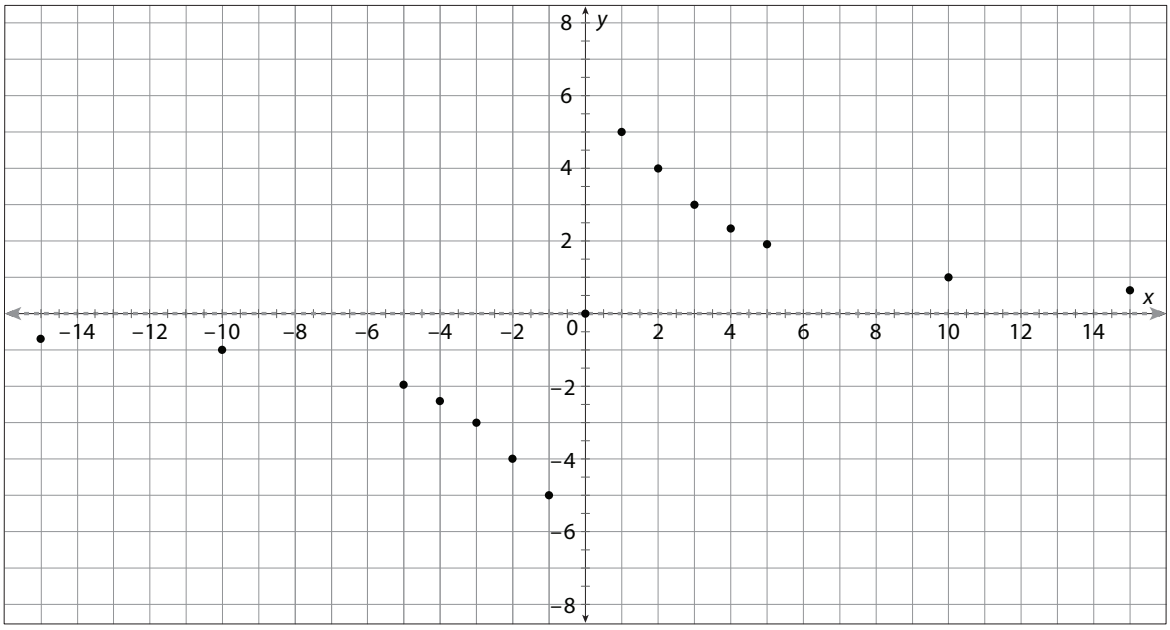


Because the asymptote and intercept do not give much information, multiple points will need to be found by substituting values for x and solving for $f(x)$.

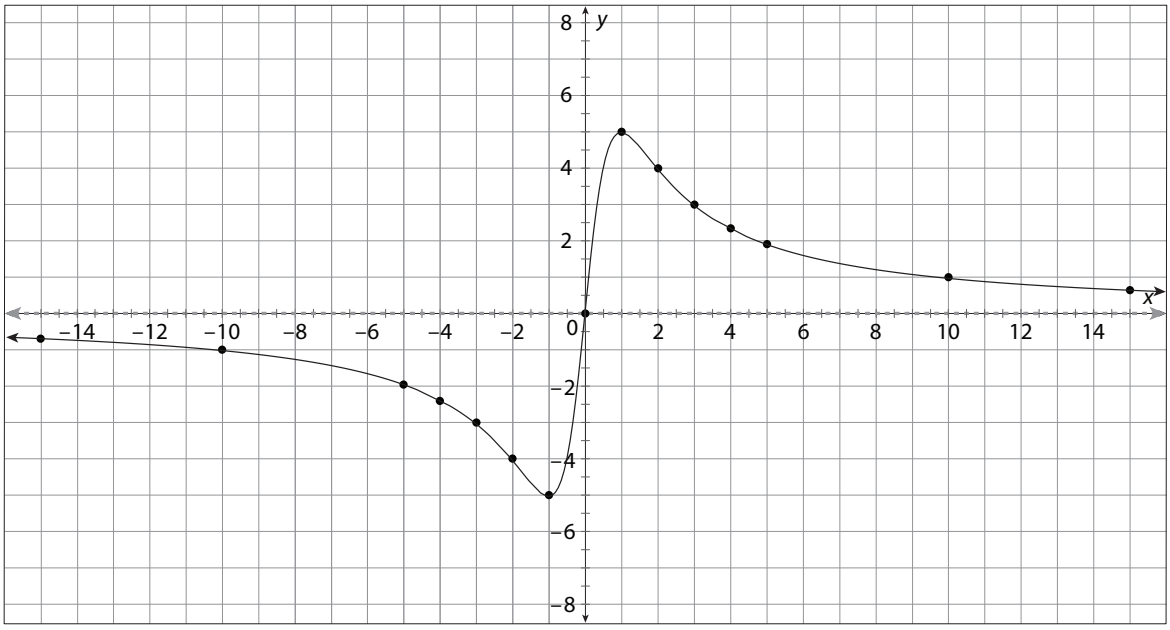
Use an input/output table to find enough points to plot the curve of the function.

x	$f(x)$
1	5
2	4
3	3
4	2.3
5	1.9
10	1.0
15	0.7
–1	–5
–2	–4
–3	–3
–4	–2.3
–5	–1.9
–10	–1.0
–15	–0.7

Plot the additional points.

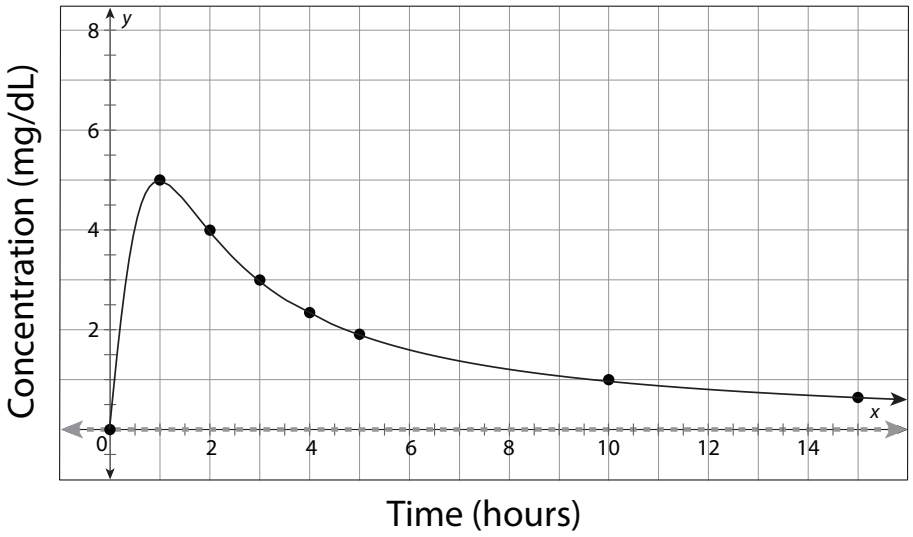


Draw the curve through these points, remembering that the function will get closer and closer to the asymptote as x increases or decreases.



d. What portion of the graph represents the concentration of medication in Andy's bloodstream?

The left side of the plane (Quadrants II and III) represents the part of the function when x is negative. Since time cannot be negative, only the right side of the graph (Quadrants I and IV) represents the concentration of medication in Andy's bloodstream. Also, note that there are no points graphed in Quadrant IV. This indicates that there is a positive amount of medication in Andy's bloodstream.



e. After how many hours will the medication completely leave Andy's system?

Since after time $t = 0$ the function approaches the x-axis but never actually touches it, this means that the concentration will decrease and get closer and closer to 0, but it will never actually reach 0.

Therefore, at least theoretically, the medication will never completely leave Andy's system.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.