Conceptual Task: String Games

Exploration Questions Sample Responses

a. What would happen if you flattened $\angle A$ out, so that $m\angle A = 180°$? Would the triangle still exist? What would you be left with?

Students should realize that flattening $\angle A$ will flatten the triangle, turning it into a line segment. Students may notice that the length of the segment will be 7.5 inches. Students may manipulate a loop of string themselves to investigate the behavior of $\angle A$.

b. What would happen if you folded $\angle A$ up, so that $m\angle A = 0°$? Would the triangle still exist? What would you be left with?

Students should realize that folding up $\angle A$ will also flatten the triangle, turning it into a line segment. This time, the segment will be vertical. Students may notice that the length of the segment will be 7.5 inches. Students may manipulate a loop of string themselves to investigate the behavior of $\angle A$.

c. What would happen to $\angle B$ and $\angle C$ if you flattened out $\angle A$? As $\angle A$ gets bigger, what happens to $m\angle B$ and $m\angle C$?

Students should realize that $m\angle B$ and $m\angle C$ get smaller and smaller as $\angle A$ gets bigger; both will approach 0° before vanishing as the triangle flattens. Students may manipulate a loop of string themselves to investigate this behavior.

d. What would happen to $\angle B$ and $\angle C$ if you fold up $\angle A$? As $\angle A$ gets smaller, what happens to $m\angle B$ and $m\angle C$? Are they getting closer to a particular value?

Students should realize that $m\angle B$ and $m\angle C$ get bigger as $\angle A$ gets bigger; both will approach 90° before vanishing as the triangle flattens. Students may manipulate a loop of string themselves to investigate this behavior.

e. What do you think $m\angle A + m\angle B + m\angle C$ might equal? Why? Defend your answer.

Students should realize that 180° seems to be the measure of sum at the extreme cases (180° + 0° + 0°, 0° + 90° + 90°), and extrapolate that this may be true of the intermediate cases as well.

f. Consider the diagram of $\triangle ABC$, where a line parallel to $\overline{BC}$ has been drawn through A. Identify the parallel lines and the transversals.

The parallel lines are given to be the grey line and the line segment $\overline{BC}$. There are two transversals: $\overline{AB}$ and $\overline{AC}$.

g. Consider the diagram from part f. What is the relationship between $\angle B$ and $\angle 1$? Between $\angle C$ and $\angle 2$? How do you know?

Students should realize $\angle B$ and $\angle 1$ are alternate interior angles, and therefore congruent. The same logic applies to $\angle C$ and $\angle 2$.

h. What does $m\angle A + m\angle 1 + m\angle 2$ equal? Why? Defend your answer.

Students should realize this sum is equal to $180°$ because $\angle 1$, $\angle A$, and $\angle 2$ together form a line.

i. Does this agree with the sum $m\angle A + m\angle B + m\angle C$? Why? Defend your answer.

Students should realize that the two sums are the same because of the already established congruency of $\angle B$ with $\angle 1$ and $\angle C$ with $\angle 2$. They should also connect it to their previous response about the sum $m\angle A + m\angle B + m\angle C$. Encourage students to consider the difference between the investigation with the string, which relied on intuition, and the investigation using already established geometric theorems. Ask students which argument makes more sense, and which one is more convincing.