

## Conceptual Task: Triangulating a Waterspout

### Exploration Questions Sample Responses

- a. In the angle diagram, what are the parallel lines? How do you know?

The parallel lines are the sight line from Viktor to the location at 10:35 and the sight line from Arlene to the location at 10:40. This is given in the context when it is said that the locations are due east of each observer.

- b. In the angle diagram, what is the transversal? How do you know?

The transversal is defined by the line marked by the waterspout's locations. It represents the path of the waterspout.

- c. There are two triangles in the sight line diagram. One is formed by Viktor, the waterspout's location at 10:30, and the waterspout's location at 10:35. The other is formed by Arlene, the waterspout's location at 10:30, and the waterspout's location at 10:40. What do you notice about these triangles?

Answers may vary. Students should notice that the triangles are similar. Remind students that triangles are similar if they are the same shape, but different sizes. Prompt students to notice this by asking the measure of the angle 10:30, Viktor, and 10:35; the measure of angle 10:30, Arlene, and 10:40; and whether the triangles share any angles. This establishes the *AA* criterion.

- d. Is  $m\angle 4 \cong m\angle 8$ ? How do you know?

Students should recognize the two angles are congruent because they are corresponding angles in similar triangles.

- e. The parallel postulate says that when a set of parallel lines is cut by a transversal, interior angles on the same side of the transversal must sum to  $180^\circ$ . For example,  $m\angle 4 + m\angle 5 = 180^\circ$ . Does this make sense if  $m\angle 4 \cong m\angle 8$ ? Defend your answer.

Answers may vary. Students should recognize the statement is logical because  $\angle 5$  and  $\angle 8$  form a linear pair. Prompt students to notice this by asking what supplementary angles are, and whether  $\angle 5$  and  $\angle 8$  are supplementary. Then ask if  $\angle 5$  and  $\angle 4$  are supplementary, since  $\angle 4$  and  $\angle 8$  are the same. Encourage students to play with specific cases if they get stuck.

- f. What would happen if  $m\angle 4 + m\angle 5 \neq 180^\circ$ ? Would the lines still be parallel? How do you know? Draw an example that shows what would happen.

Answers may vary. Students should recognize that the lines would not be parallel, and would instead intersect to form a triangle.

- g. In the angle diagram,  $\angle 1 \cong \angle 3 \cong \angle 5 \cong \angle 7$  and  $\angle 2 \cong \angle 4 \cong \angle 6 \cong \angle 8$ . What do you notice? Do you think this list will be the same for all sets of parallel lines cut by a transversal?

Answers may vary. Students should notice that the congruent angles alternate around the diagram, and refer to sets of corresponding and vertical angles. This pattern will hold for all sets of parallel lines cut by a transversal.

- h. Viktor isn't 100% sure he was facing east when he observed the waterspout the second time, but he does know the measure of  $\angle 8$ . If Arlene knows the measure of  $\angle 4$ , can they determine whether Viktor was facing east? Defend your answer.

Answers may vary. Students should recognize that knowing  $\angle 4 \cong \angle 8$  is enough to determine whether the sight lines are parallel.

- i. Could you tell if Viktor was facing east if you knew the measures of  $\angle 4$  and  $\angle 5$ ? Defend your answer.

Answers may vary. Students should recognize that knowing  $m\angle 4 + m\angle 5 = 180^\circ$  is enough to determine whether the sight lines are parallel.

- j. Are there other angle pairs that could tell you whether Viktor was facing east? Defend your answer, and give a specific example.

Answers may vary. Students should recognize that knowing the relationship between any set of angles in the diagram (same-side interior angles, same-side exterior angles, alternate interior angles, alternate exterior angles, corresponding angles, etc.) can tell you whether the lines are parallel.