Proving Theorems About Isosceles Triangles

Prerequisite Skills

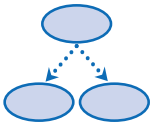
This lesson requires the use of the following skills:

- classifying triangles
- identifying and using vertical angles, supplementary angles, and complementary angles to find unknown angle measures
- applying the Triangle Sum Theorem and the Exterior Angle Theorem to find unknown measures of triangles
- justifying congruence of triangles

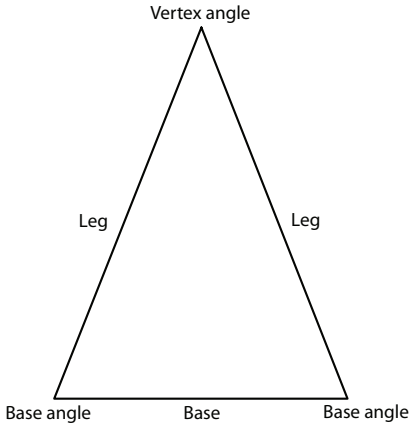
Introduction

Isosceles triangles can be seen throughout our daily lives in structures, supports, architectural details, and even bicycle frames. Isosceles triangles are a distinct classification of triangles with unique characteristics and parts that have specific names. In this lesson, we will explore the qualities of isosceles triangles.

Key Concepts



- Isosceles triangles have at least two congruent sides, called **legs**.
- The angle created by the intersection of the legs is called the **vertex angle**.
- The side that is opposite the vertex angle is the **base** of the isosceles triangle.
- Each of the remaining angles is referred to as a **base angle**. The intersection of a leg and the base of the isosceles triangle creates a base angle.

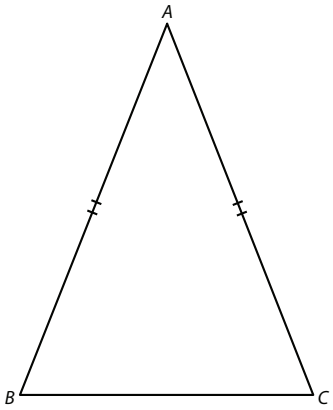


- The following theorem is true of every isosceles triangle.

Theorem

Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite the congruent sides are congruent.



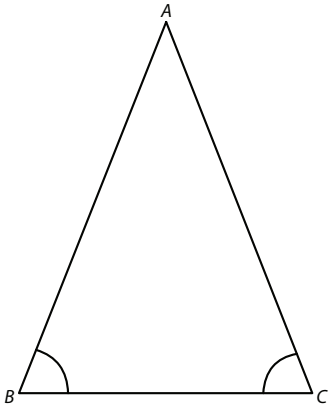
$m\angle B \cong m\angle C$

- If the Isosceles Triangle Theorem is reversed, then that statement is also true.
- This is known as the converse of the Isosceles Triangle Theorem.

Theorem

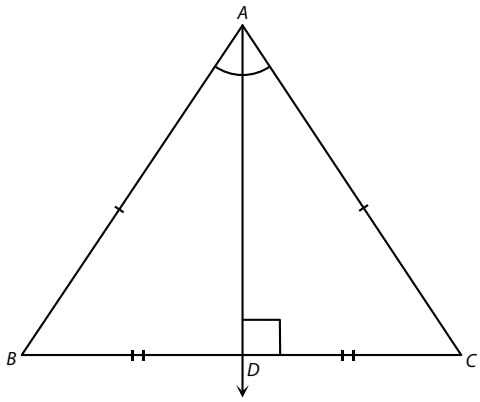
Converse of the Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.



$\overline{AB} \cong \overline{AC}$

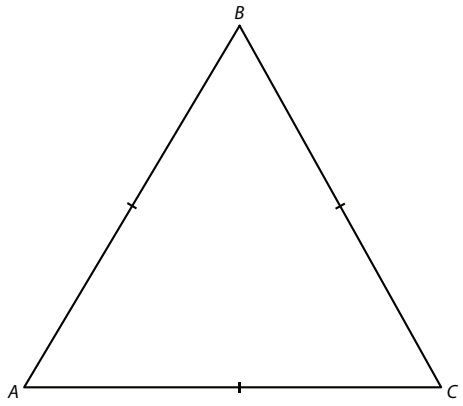
- If the vertex angle of an isosceles triangle is bisected, the bisector is perpendicular to the base, creating two right triangles.
- In the diagram that follows, D is the midpoint of $\overline{BC}$.



- Equilateral triangles are a special type of isosceles triangle, for which each side of the triangle is congruent.
- If all sides of a triangle are congruent, then all angles have the same measure.

Theorem

If a triangle is equilateral then it is **equiangular**, or has equal angles.



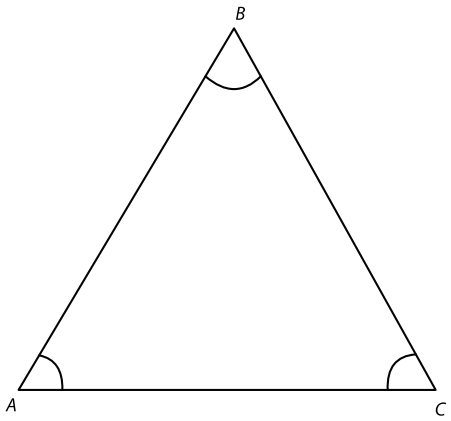
$\angle A \cong \angle B \cong \angle C$

- Each angle of an equilateral triangle measures 60° $(180 \div 3 = 60)$.

- Conversely, if a triangle has equal angles, it is equilateral.

Theorem

If a triangle is equiangular, then it is equilateral.



$\overline{AB} \cong \overline{BC} \cong \overline{AC}$

- These theorems and properties can be used to solve many triangle problems.
- Recall that the **distance formula**, $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, can be used to find distances between points in the coordinate plane. This allows you to find side lengths of triangles whose vertices are given as coordinates.

Common Errors/Misconceptions

- incorrectly identifying parts of isosceles triangles
- not identifying equilateral triangles as having the same properties of isosceles triangles
- incorrectly setting up and solving equations to find unknown measures of triangles
- misidentifying or leaving out theorems, postulates, or definitions when writing proofs