## Problem-Based Task: Calibrating Consoles

## Task Overview

### Focus

What information is necessary to classify a triangle as isosceles? What is true about the side lengths and the angles of an isosceles triangle? In this task, students will find missing side lengths and angle measures in triangles created as part of a real-world scenario in order to determine if the triangles are isosceles.

This activity will provide practice with:

- defining an isosceles triangle
- applying the Pythagorean Theorem and other triangle theorems
- solving equations for missing side lengths and angle measures
- interpreting the dimensions and angle measures of triangles
- classifying triangles based on side lengths and angle measures

### Introduction

This task should be used to explore or apply theorems about isosceles triangles. Students should already be familiar with the Triangle Sum Theorem and how to classify triangles based on side lengths and angle measures. If necessary, review the Pythagorean Theorem and types of triangles (equilateral, isosceles, and scalene).

Begin by reading the problem and clarifying the meaning of the following terms:

**calibrate** to adjust a device so that it functions accurately

**optimal** most desirable; best

## Facilitating the Task

### Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will examine the diagrams to determine the given information and identify the unknown information. They will also perform the necessary calculations to find the missing side lengths and angle measures. Encourage students to first identify which of the triangle theorems they will need to apply in order to prove that the triangles formed are isosceles.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will make conjectures about the triangles drawn in the diagrams, and they will test their conjectures by analyzing the given information. They will construct viable arguments about the types of triangles based on their calculations. Encourage students to discuss their reasoning with each other and discuss if they do not agree.

- **SMP 7:** Look for and make use of structure.

  Students look for and use the known structures of isosceles triangles and of right triangles that can be found within the task's provided diagrams.

### Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly identifying parts of isosceles triangles

  Have students create a generic diagram of an isosceles triangle, and label the legs, base, vertex angle, base angles, and all congruency markings on the diagram. Have them use this diagram as a reference when working through the task.

- incorrectly setting up and solving equations to find unknown measures of triangles

  Remind students that the Triangle Sum Theorem states that the three angles of a triangle have a sum of 180°. Also remind them to check their answers to verify that the three angles do indeed have a sum of 180°.

- not applying the measurement found with the Pythagorean Theorem to both smaller triangles when calculating the third side in Scarlett's diagram

  Remind students that the small unlabeled triangle will have the same corresponding side lengths as the small labeled triangle because of the congruency markings, and because the triangles share a common side.

### Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task:

  Ask them to make a list of the triangle definitions and theorems they think will be applicable to the problem, based on the diagrams. (**Answer:** definition of an isosceles triangle, the Pythagorean Theorem, the Triangle Sum Theorem, and the Converse of the Isosceles Triangle Theorem)

  Ask students to write predictions about whether each controller is calibrated correctly. Ask them to explain their reasoning based only on the diagrams. Encourage them to use mathematics to support or prove their predictions. (**Sample answer:** "I predict that Scarlett's triangle is isosceles because there are congruency markings in the diagram. These markings will allow me to apply the Pythagorean Theorem to the smaller triangles in order to find the missing side lengths. If these sides are the same length, then my prediction is correct." Or, "I predict that Wren's triangle is isosceles because it looks like two sides are the same length. I can check my prediction by using the Triangle Sum Theorem to see if two angles are congruent. If so, then the triangle is isosceles.")

- Ask students, "What information is provided in Scarlett's diagram that allows you to use the Pythagorean Theorem to find the length of each side of the triangle Scarlett created?" (**Answer:** "The angle formed by the segment labeled '12 ft' and the segment labeled '5 ft' is denoted as a right angle, so the two smaller triangles in the diagram must be right triangles. Therefore, the Pythagorean Theorem can be used.")

- If students are having difficulty understanding why the two smaller triangles in Scarlett's diagram are congruent, ask, "Which congruent parts are identified in the two smaller triangles? Which triangle congruency theorem can be used?" (**Answer:** "Both triangles have sides that measure 5 feet and 12 feet, and an included angle that measures 90°. The Side-Angle-Side (SAS) congruency theorem can be used to prove that the two triangles are congruent.")

- After students have calculated the length of the legs of Scarlett's triangle, ask, "Why does knowing that the two legs of Scarlett's triangle are congruent allow you to conclude that the triangle is isosceles?" (**Answer:** "The definition of an isosceles triangle states that if two sides of a triangle are congruent, then the triangle is isosceles.")

- Ask, "What theorem can be used to determine the missing angle measure in Wren's diagram? What equation will be used?" (**Answer:** "The Triangle Sum Theorem states that the interior angles of a triangle sum to 180°. The equation is $68 + 44 + x = 180$.")

- To extend students' thinking beyond the task, ask, "Is either triangle equilateral? Why or why not?" (**Answer:** "Neither triangle is equilateral. Scarlett's triangle is not equilateral because an equilateral triangle has three congruent sides, and her triangle has side lengths of 13, 13, and 10. Wren's triangle is not equilateral because an equilateral triangle has three congruent angles, and her triangle has angles measuring 68°, 68°, and 44°.")

- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

## Debriefing the Task

- Ask volunteers to share their initial predictions about whether each controller was calibrated correctly. Encourage students to discuss their reasoning for their predictions, as well as whether they were correct based on the actual results of the task.
- Compare students' strategies and explanations for analyzing the given information and finding the unknown side length in Scarlett's triangle and the unknown angle measure in Wren's triangle. Focus on the use of precise mathematical language.

### Connecting to Key Concepts

Make explicit connections to key concepts:

- Isosceles triangles have at least two congruent sides, called legs.

  In this task, students will identify triangles that may be isosceles, and then determine the lengths of the sides that appear to be congruent to see if they are.

- The angle created by the intersection of the legs is called the vertex angle.

  Students will observe that the vertex angle for both triangles in the task is created by the intersection of the legs of each isosceles triangle.

- The Converse of the Isosceles Triangle Theorem states that if two angles of a triangle are congruent, then the sides opposite those angles are congruent.

  In this task, students will discover that in Wren's triangle, when the unknown angle is calculated, it is congruent to the other base angle. Therefore, the Converse of the Isosceles Triangle Theorem can be applied, and the triangle can be classified as isosceles.

- If the vertex angle of an isosceles triangle is bisected, the bisector is perpendicular to the base, creating two right triangles.

  In Scarlett's triangle, students will observe that two right triangles are created by the bisection of the vertex angle. Students can verify that the angle has been bisected because of the congruency markings on the base of the triangle.

### Extending the Task

To extend the task, have each student create three different triangles with either side lengths or angle measures missing. Ask each student to trade triangles with a partner, and then determine whether their partner's triangles are isosceles. Encourage students to discuss their strategies for creating triangles that are isosceles and those that are not, focusing on correct terminology and use of theorems.

## Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you demonstrate perseverance?" (**Sample answer:** "I persevered by trying several strategies, and when they didn't work, I looked at my notes to remind myself what information is needed to prove that a triangle is isosceles.")
- **For SMP 3, ASK:** "Did you construct viable arguments and did you critique the reasoning of others?" (**Answer:** "I constructed viable arguments by drawing conclusions about the type of triangle based on my analysis of the sides and angles, which allowed me to classify each triangle as isosceles. When another student disagreed with my answers, we looked at each others' work together to figure out who made a mistake.")
- **For SMP 7, ASK:** "How did you look for and make use of structure when solving this problem?" (**Answer:** "I recognized that the diagrams appeared to show isosceles and right triangles, and this structure allowed me to determine which theorems to use to solve the problem.")

## Alternate Strategies or Solutions

- For the diagram of Scarlett's triangle, students may choose to break the large triangle apart in order to examine the two smaller congruent triangles separately. Remind them that the two smaller triangles share the altitude, and that the congruency markings indicate that each of the shorter sides is 5 feet long.
- Some students might choose to redraw the diagrams such that the vertex angle is at the top. Remind them to make sure that the sides and angles are still labeled correctly if they redraw the diagrams.
- For students who have learned about trigonometric ratios, Scarlett's triangle can be shown to be isosceles by using the arctangent function to prove that the two base angles are congruent.

## Technology

Students may use scientific calculators when performing calculations with the Pythagorean Theorem.