Problem-Based Task: Stained Glass Pattern, Part I

Task Overview

Focus

How can diagonals of quadrilaterals create congruent triangles? How can corresponding parts of triangles be used to prove congruency? In this lesson, students will identify congruent sides and angles of congruent triangles formed by squares and rhombuses in a pattern of stained glass art.

This activity will provide practice with:

- identifying properties of congruent quadrilaterals
- analyzing a pattern of triangles in the context of a scenario
- determining congruent sides and angles of quadrilaterals and triangles
- drawing congruency markings on sides and angles
- justifying congruent triangles based on corresponding parts

Introduction

This task should be used to apply the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Students should already be familiar with recognizing transformations of figures, as well as understanding that rigid motions preserve the size and shape of figures.

Begin by reading the problem and clarifying the meaning of congruent triangles, corresponding angles, and corresponding sides.

congruent triangles	triangles that have the same angle measures and side lengths
corresponding angles	a pair of angles in a similar position
corresponding sides	the sides of two figures that lie in the same position relative to the figures

Facilitating the Task

Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

  Students will recognize that the goal of the task is to identify congruent triangles in a stained glass art pattern. They will determine these congruent triangles based on applying their knowledge about the properties of a square and a rhombus. They will also label and identify the corresponding parts of the congruent triangles.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

  Students will construct viable arguments about which pairs of triangles in the stained glass pattern are congruent based on the identification of congruent corresponding parts of the triangles.

- **SMP 7:** Look for and make use of structure.

  Students will look for patterns in identifying congruent triangles. Specifically, they will determine the corresponding parts of the triangles and conclude that they are congruent based on the properties of the sides and angles of the figures. They will recognize that the same shapes are used to create congruent triangles, and therefore the corresponding sides and angles can be identified as congruent.

Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly identifying corresponding parts of triangles

  Remind students that corresponding angles are angles that are in a similar position in two triangles, and that corresponding sides are sides that lie in the same position relative to the figures.

- assuming that corresponding parts indicate congruent parts

  Remind students that corresponding sides and angles of two triangles must be proven to be congruent by given information, by tick marks for the sides or arc marks for the angles, or by applying properties of shapes to prove congruence.

- assuming that alphabetical order indicates congruence

  Remind students that triangles can be named using any combination and order of letters, so it is important to look at the order of the letters in relation to the corresponding parts of the triangles.

Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task, ask, "What are the properties of a square in regards to sides and angles? What are the properties of a rhombus in regards to sides and angles?" (**Answer:** A square is a quadrilateral with four congruent sides and four right angles. A rhombus is a quadrilateral with four congruent sides and congruent pairs of opposite angles.)
- Before students begin the task, ask, "What do you know about the diagonals of a square and rhombus? What is unique about the diagonals of a square?" (**Answer:** The diagonals of a square and a rhombus bisect the opposite pairs of angles. In a square, the diagonals bisect the opposite pairs of angles and create 45° angles, because each corner of a square is exactly 90°.)
- Ask students, "What must be proven to conclude that any two figures are congruent?" (**Answer:** To conclude that any two figures are congruent, all of the corresponding sides must be the same lengths, and all of the corresponding angles must have the same measures.)
- Ask students, "How do you symbolically label congruent parts on figures? Explain." (**Answer:** Congruent sides are noted by tick marks. One mark on a side of a triangle will indicate congruency to the corresponding side of another triangle with one tick mark. Two tick marks will indicate congruence to the corresponding side that also has two tick marks, etc. Angle measures are noted as congruent by drawing arcs on the angles, as an arc on one triangle matches an arc on the angle of another triangle, two arcs match with two arcs, etc.)
- As students are beginning the task and analyzing the given information, ask them, "It is given that segment *EC* is congruent to segment *BC*. With this information, what are the possible types of triangles that triangle *BEC* could be? Explain." (**Answer:** Since it is given that two sides of triangle *BEC* are congruent, this indicates that the triangle is either an isosceles triangle, which has at least two sides congruent, or an equilateral triangle, which has all three sides congruent.)
- Ask students, "How does a diagonal of a square automatically create two congruent triangles? Explain how the corresponding parts of these two triangles are congruent." (**Answer:** A diagonal of a square divides a square into two congruent triangles, because all six corresponding parts are congruent. The angles that are congruent are the two right angles in the opposite corners, and since a diagonal bisects a 90° angle, the resulting angles are each 45° in both triangles. Since all four sides of a square are congruent, the two legs of the triangle are already congruent, and since the triangles share the third side, which is the diagonal, that side is also congruent for both triangles.)
- If students are having difficulty identifying a pair of congruent triangles, ask them, "How many corresponding parts are there between two triangles?" (**Answer:** Since there are three sides of a triangle and three angles in a triangle, there is a total of six corresponding parts between two triangles.)

- Once the pairs of congruent triangles have been identified in the pattern, ask students, "How can Mary apply congruent triangles to her project? How will this knowledge help her in terms of creating her stained glass art?" (**Answer:** Mary can use the congruent triangles in her pattern when she is creating a template to use for future patterns. Since she knows that certain pairs of triangles are congruent, she can make cuts to fit her pattern using the congruent triangles.)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

Debriefing the Task

- Ask for volunteers to discuss their strategies and processes for identifying the congruent parts of the triangles, as well as determining which triangles in the pattern are congruent. Encourage students to discuss any difficulties or confusion they experienced when working through the various parts of the task.
- Compare students' strategies and ways of justifying responses, specifically when identifying pairs of congruent triangles. Focus on the use of precise mathematical language and clarity, specifically when referring to the terminology of corresponding parts of triangles.

Connecting to Key Concepts

Make explicit connections to key concepts:

- To determine whether two triangles are congruent, you must observe the angle measures and side lengths of the triangles.

  In this task, students will apply the properties of a square and a rhombus to identify angle measures and side lengths of a number of triangles.

- Congruent triangles have three pairs of corresponding angles and three pairs of corresponding sides, for a total of six pairs of corresponding parts.

  Students will identify and list three pairs of corresponding angles and three pairs of corresponding sides in each pair of triangles that they identify as congruent.

- Two triangles are congruent if the corresponding angles are congruent and corresponding sides are congruent.

  Students will identify and label all corresponding angles and sides between a pair of triangles and determine they are congruent to each other, so they will conclude that the pair of triangles is congruent.

- Tick marks show the sides that are congruent, and arcs on the angles show the angles that are congruent in a pair of triangles.

  In this task, students will draw the tick marks on the sides and the arcs on the angles of the parts of the triangles that are congruent.

Extending the Task

- Ask students to use a ruler to measure the side lengths of the borders of Mary's pattern. Ask them to extend her pattern by drawing squares, rhombuses, and/or triangles that will continue her pattern of congruent figures. Remind students that the figures should not overlap. Encourage them to see how many figures they can draw. Ask for volunteers to share their new designs and discuss their reasoning in their choices for their types of figures.
- Provide students with a blank coordinate plane and a ruler. Ask them to create their own patterns of polygons similar to what was given in the task. Ask them to use a combination of shapes, and encourage them to be creative in creating a pattern in which congruent figures can be identified. Remind them to carefully measure their shapes using the graph as a means of ensuring congruent sides. Ask for volunteers to share their shapes and discuss the congruency in their patterns.

Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** "How did you make sense of the problem or demonstrate perseverance?" (**Answer:** I persevered by recognizing that the goal of the task was to identify congruent triangles in a stained glass art pattern. I determined these congruent triangles based on applying my knowledge about the properties of a square and a rhombus. I also labeled and identified the corresponding parts of the congruent triangles.)
- **For SMP 3, ASK:** "Did you construct viable arguments and critique the reasoning of others?" (**Answer:** I constructed viable arguments about which pairs of triangles in the stained glass pattern are congruent based on the identification of congruent corresponding parts of the triangles.)
- **For SMP 7, ASK:** "How did you look for, and make use of, structure?" (**Answer:** I looked for patterns when I identified congruent triangles. I determined the corresponding parts of the triangles and concluded that they are congruent based on the properties of the sides and angles of the figures. I recognized that the same shapes are used to create congruent triangles, and that therefore the corresponding sides and angles can be identified as congruent.)

Alternate Strategies or Solutions

- Students may choose to redraw the given pattern to break apart the triangles. It may be helpful to reorient the triangles so students can more easily identify the corresponding parts. Encourage students to be careful when labeling the vertices of each triangle with the correct letter to maintain the correspondence between a pair of triangles.
- Students may choose to use a ruler and a protractor to measure each of the angles and sides. Remind them that it is possible to have measurement errors using these techniques. They may also choose to use a compass and a straightedge to measure the sides and angles, using construction methods. Note that construction methods will generally result in more precise and accurate measurements, as opposed to using a ruler and a protractor.

Technology

No technology is needed for this task.