Conceptual Task: Solution Squabble

Exploration Questions Sample Responses

a. Look at each step for Avi and Ben's solutions. Were there any errors made?

No errors were made in Avi and Ben's solutions. Both students' work is correct. Some students may think there are wrong steps in step 2 or 3 because they may be used to doing this step in their heads.

b. Explain what each student did for each step.

Have students explain what is happening in each step.

Avi's work

1. $2x^2 - 12x + 7 = 0$	1. Given
2. $2\left(x^2 - 6x + \_\_\_\_\_\_\right) - \_\_\_\_\_ + 7 = 0$	2. Factor out the 2 from $2x^2 - 12$.
3. $2\left(x^2 - 6x + \left(\frac{6}{2}\right)^2\right) - 2\left(\frac{6}{2}\right)^2 + 7 = 0$	3. Make a perfect square by doing the calculation $\left(\frac{b}{2}\right)^2$; here, you must multiply the second calculation by the number outside the parentheses.
4. $2\left(x^2 - 6x + 9\right) - 18 + 7 = 0$	4. Simplify.
5. $2(x-3)^2 - 11 = 0$	5. Factor the quadratic inside the parentheses. Simplify outside the parentheses.
6. $2(x-3)^2 = 11$	6. Add 11 to both sides.
7. $(x-3)^2 = \frac{11}{2}$	7. Divide both sides by 2.
8. $\sqrt{(x-3)^2} = \sqrt{\frac{11}{2}}$	8. Take the square root of both sides.
9. $x - 3 = \pm\frac{\sqrt{22}}{2}$	9. Rationalize the denominator. Simplify as necessary.
10. $x = 3 \pm \frac{\sqrt{22}}{2}$	10. Add 3 to both sides.

Ben's work

1. $2x^2 - 12x + 7 = 0$	1. Given
2. $x = \dfrac{12 \pm \sqrt{(-12)^2 - 4(2)(7)}}{2(2)}$	2. Substitute $a = 2$, $b = -12$, and $c = 7$ into the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
3. $x = \dfrac{12 \pm \sqrt{144 - 56}}{4}$	3. Simplify calculations under the radical sign.
4. $x = \dfrac{12}{4} \pm \dfrac{\sqrt{88}}{4}$	4. Divide both terms by 4.
5. $x = 3 \pm \dfrac{2\sqrt{22}}{4}$	5. Simplify the radical.
6. $x = 3 \pm \dfrac{\sqrt{22}}{2}$	6. Simplify.

c. Which method would you choose? Why would you choose that method?

Student answers may vary. Some students will choose the quadratic formula because it takes fewer steps. Someone might choose completing the square because they cannot remember the quadratic formula, or they understand that process better.

d. Look at the first few steps in each solution. What do the solutions have in common?

Students should observe that the b-value in both solutions is squared.

Some students may also observe that the seven is in the same location within each equation.

e. What other commonalities can someone find between the solutions? How might these methods be related?

Other observations that should be made are that both solutions simplify the radical, but at different times in the process. Also, both solutions divide by the a-value at some point. The quadratic formula divides in the beginning, while completing the square divides at the end. The ultimate goal is the students to realize these methods are related and the quadratic formula is a derivation of completing the square.