Conceptual Task: Pondering Patterns

Exploration Questions Sample Responses

a. Look at Pattern 1. How many squares are shown in the first figure? In the second figure? In the third figure? Use these numbers to fill in the tables. Repeat this process for each pattern.

In Pattern 1, there are 7 squares in the first figure, 13 squares in the second figure, and 19 squares in the third figure. Some students may notice the pattern right away: x rows of 6 squares, plus one square on the top. See below for table.

In Pattern 2, there are 5 squares in the first figure, 8 squares in the second figure, and 13 squares in the third figure. Some students may notice the pattern right away: x^2 squares in the middle plus 4 squares in the corners. See below for table.

In Pattern 3, there are 5 squares in the first figure, 25 squares in the second figure, and 125 squares in the third figure. Some students may notice the pattern right away: beginning with the 5-star pattern, each new figure is created out of 5 copies of the previous figure. Students may want to use 1, 5, and 25 for the pattern; in this case, their figure number should start at 0 instead of 1.

Pattern 1

Figure #	Squares
1	7
2	13
3	19
4	25
5	31

Pattern 2

Figure #	Squares
1	5
2	8
3	13
4	20
5	29

Pattern 3

Figure #	Squares
1	5
2	25
3	125
4	625
5	3125

b. Use the table to graph your results. What scales would be best for each pattern? What does the graph reveal about the pattern?

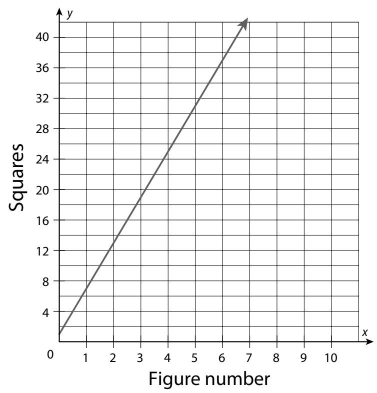
Since all values in all patterns are positive numbers, only the first quadrant is relevant when graphing. While the scale for the independent values can go up by ones, the dependent values will need to be adjusted for each pattern to fit on the graph.

For Pattern 1, the graph reveals a straight line.

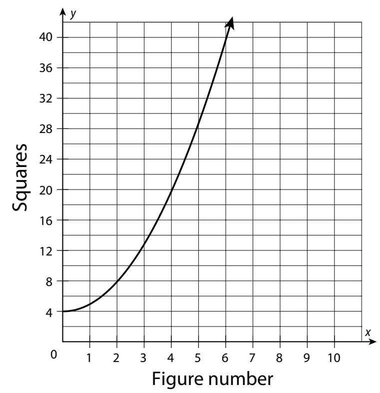
For Pattern 2, the graph reveals an increasing curve. Students may observe that the curve is half of a "U" shape based on the table values.

For Pattern 3, the graph reveals an increasing curve. Students may observe that the graphs of patterns 2 and 3 are both curves. Encourage students to examine the graph together with the table in order to differentiate the third one as an exponential function. Evidence for this is the self-multiplying nature of the pattern and its steep increase.

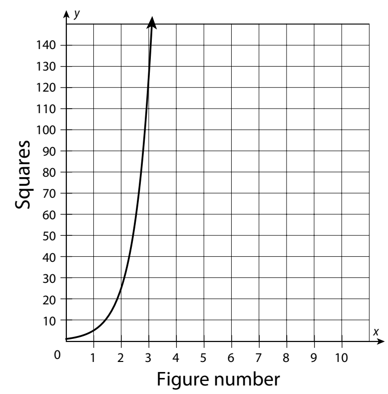
Pattern 1



Pattern 2



Pattern 3



c. What patterns are found in the *y*-values of your tables? How do these help you tell what kind of function each pattern is? Describe how you found each pattern.

For Pattern 1, subtract the dependent variables to get the common difference of 6. A common difference tells us the function is linear. The pattern in each figure increases by one row of 6 each time. This is the slope.

For Pattern 2, there is a second difference as shown in the diagram below:

5 8 13 20 29

+3 +5 +7 +9

+2 +2 +2

This second difference is found by subtracting the values twice. The second difference indicates that this is a quadratic function. From the figures, students might notice the squares in the middle that form perfect square numbers, also indicating a quadratic function.

For the third pattern, the dependent variables are multiplying by 5, indicating this is an exponential function. Students may notice the 5-star pattern that appears in clusters of 5.

Pattern 1	Pattern 2	Pattern 3
This pattern has a common difference of 6.	This pattern has a second difference of 2.	This pattern has a common ratio of 5.

d. Label the type of function each pattern represents. What clues in the tables and graph let you know what type of function each pattern is?

Note: Students may come back to this question repeatedly as they analyze graphs and tables separately. Specifically, they may return here if they have difficulty differentiating between the function type for Pattern 2 and Pattern 3.

For Pattern 1, the number of squares increases by 6 for each figure. Since the graph is a straight line and has a common difference of 6, this indicates a linear function.

For Pattern 2, there is a second common difference of 2 for the number of squares. Encourage students to examine the table of values and search for a second difference if this is not immediately obvious. Since there is a second difference and the graph is curved, this indicates a quadratic function. Alternatively, students may see the perfect square numbers in the middle of each figure, or view the squares as an area function, as a signal that the pattern is quadratic.

For Pattern 3, the number of squares increases by a factor of 5 in each figure. This is the common ratio. Since the pattern indicates multiplication and there is a sharp increase in the graph, we know this is an exponential function.

Pattern 1	**Pattern 2**	**Pattern 3**
Linear	Quadratic	Exponential
Graph: straight line Table: add 6 each time	Graph: Curved, half a "U" Table: has a second difference of 2; represents increasing area	Graph: curved, steep increase Table: *x*-values multiply by 5

e. What is the starting value for each pattern? Is the starting value the first figure you see or the previous figure? Explain how you arrived at your answer.

To get the starting values for the functions, students may use the tables or graphs. When using the tables, students must remember that the starting values begin with "Figure 0," and therefore must work backward to find the zero term.

For Pattern 1, subtract the common difference of 6 from the first term of 7 to get the starting value of 1.

For Pattern 2, work backward from the first term of 5: subtract 1 to get the starting number of 4. This is easily seen from the graph.

For pattern 3, divide the first term of 5 by the common ratio of 5 to get the starting value of 1.

Pattern 1	**Pattern 2**	**Pattern 3**
Zero term: 1	Zero term: 4	Zero term: 1

f. What is the equation that represents the number of squares in each pattern? How do you know?

Pattern 1 has a constant increase of 6 per term and a starting value of 1 for the zero term. Since pattern 1 is linear, the increase will be the slope and the zero term value will be the intercept. This gives the equation $y = 6x + 1$.

Pattern 2 has a constant second difference of 2, and is therefore quadratic. The zero term is 4, so this must be added to the variable part of the function. Looking at the figures, the pattern is $1 + 4, 4 + 4, 9 + 4, \ldots$ for figures 1, 2, 3, … . Notice that the sequence 1, 4, 9 represents the squares of the sequence 1, 2, 3, which can be represented by x^2. Adding in the zero term, we get the equation $y = x^2 + 4$.

Pattern 3 has a constant common ratio of 5 and a starting value of 1 for the zero term. We also know that pattern 3 is exponential. The common ratio will be the base of the exponent, and the starting value will multiply the base. Since the starting value is 1, the equation is $y = 5^x$.

Pattern 1	Pattern 2	Pattern 3
$y = 6x + 1$	$y = x^2 + 4$	$y = 5x^2$

g. What are some things you know about each type of function? How are the functions the same? How are they different? Use key words in your summary, such as minimum/maximum, intercepts, common difference/ratio, and increasing/decreasing.

Answers may vary. Sample answers:

- **Pattern 1:** Linear functions appear as straight lines. They have equations in the form of $y = mx + b$, where m is the slope, or common difference, and b is the starting point. Like other functions, they can be shown as graphs, equations, or tables. This function is always increasing. It has a slope of 6 and a y-intercept of 1, which is the same as the exponential function. In general, linear functions have no minimum or maximum. However, in this case the minimum is 1 since you can't have a zero or negative number of squares. There is still no maximum because the pattern could extend out to any number of squares.
- **Pattern 2:** This is a quadratic function (as shown by the power of 2). The function has a constant second difference. It opens upward due to the a-value being positive. It increases for $x > 0$ and decreases for $x < 0$. The y-intercept of 4 is higher than the other two functions but is also the minimum point. There is no maximum because the pattern could extend out to any number of squares.
- **Pattern 3:** This is an exponential function (as shown by its multiplication pattern). It has a common ratio of 5 and a y-intercept of 1, like the linear function. This function also always increases, like the linear function. It forms a curve like that of a quadratic function. Its y-values are all positive, just like that of the quadratic function. In general, exponential functions have no minimum or maximum, though they do have asymptotes. However, in this case the minimum is the starting value. There is no maximum because the pattern could extend out to any number of squares.