Interpreting Various Forms of Quadratic Functions

Prerequisite Skills

This lesson requires the use of the following skills:

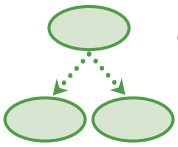
- applying the order of operations
- identifying key features of a quadratic function in standard form

Introduction

Quadratic equations can be written in several forms, including standard form, vertex form, and factored form. While each form is equivalent, certain forms easily reveal different features of the graph of the quadratic function. In this lesson, you will learn to use the various forms of quadratic functions to show the key features of the graph and determine how these key features relate to the characteristics of a real-world situation.

Key Concepts

Standard Form



- Recall that the standard form, or general form, of a quadratic function is written as $f(x) = ax^2 + bx + c$, where a is the coefficient of the quadratic term, b is the coefficient of the linear term, and c is the constant term.
- When a function is written in standard form, the y-intercept is the value of c.
- The vertex of the function can be found by first determining the value of x, $x = \frac{-b}{2a}$, and then finding the corresponding y-value, $y = f\left(\frac{-b}{2a}\right)$.
- The vertex is often written as $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.
- If $a > 0$, the function has a minimum and the graph opens up.
- If $a < 0$, the function has a maximum and the graph opens down.

Vertex Form

- The **vertex form of a quadratic function** is written as $f(x) = a(x - h)^2 + k$.
- In vertex form, the maximum or minimum of the function is identified using the vertex of the parabola, the point (h, k).

- If $a > 0$, the function has a minimum. The minimum is the value of k, the y-coordinate of the vertex.
- If $a < 0$, the function has a maximum. The maximum is the value of k, the y-coordinate of the vertex.
- Because the axis of symmetry goes through the vertex, the axis of symmetry can be identified from vertex form as $x = h$.
- The graph of a quadratic function is symmetric about the axis of symmetry.

Factored Form

- The **factored form**, or **intercept form**, of a quadratic function is written as $f(x) = a(x - p)(x - q)$.
- Recall that the x-intercepts of a function are the x-values when $y = 0$.
- In factored form, the x-intercepts of the function are identified as p and q.
- Recall that the y-intercept of a function is the point at which the function intersects the y-axis.
- To determine the y-intercept, substitute 0 for x and simplify.
- The axis of symmetry can be identified from the factored form since it passes through the midpoint between the x-intercepts. Therefore, the axis of symmetry is $x = \frac{p+q}{2}$.
- To determine the vertex of the parabola, calculate the y-value that corresponds to the x-value of the axis of symmetry.
- If $a > 0$, the function has a minimum and the graph opens up.
- If $a < 0$, the function has a maximum and the graph opens down.

Common Errors/Misconceptions

- confusing the attributes of different forms
- incorrectly identifying x-intercepts of the factored form
- incorrectly identifying the vertex as a maximum or minimum