Comparing Models

Prerequisite Skills

This lesson requires the use of the following skills:

- understanding of slope
- knowing how to find an equation from a graph or table

Introduction

When comparing properties of two functions, it is important to be able to discern the key parts of the functions. For example, it is important to be able to identify the slope of a linear function, whether in a graph, in a table, or in point-slope form. In this lesson, we will compare different functions by analyzing their key features.

Key Concepts

- When looking at a data set, there are important things to look for when identifying whether a linear, quadratic, or exponential function would be a better choice.
- The average rate of change of a function from x_1 to x_2 can be found by using the equation $\frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are two points on the graph. You can also use this equation when the function is given in table form.
- With a data set in a table, if the rate of change between each x-value is the same, it is linear. If it is not the same, the function could be either quadratic or exponential.
- If the function has the same general shape as the equation $y = x^2$, it is quadratic. It is possible the function can be more complicated, but as long as the graph is a parabola when graphed, this function is quadratic. You can also identify quadratic functions by checking the second difference between terms. Recall that the second difference is the difference between the rate of change of consecutive points on the function. For example, let $y = x^2 - 1$. Four consecutive points on this function are (0, –1), (1, 0), (2, 3) and (3, 8). Calculate the rate of change, or first difference, between each consecutive point:

x	***f*(*x*)**	**First difference (rate of change)**	**Second difference**
0	–1	—	—
1	0	1	—
2	3	3	2
3	8	5	2

- In a data set, exponential functions will follow the general equation $y = b^x$, where b is a number greater than 0 and not equal to 1. Exponential functions have a common ratio between each y-value when x-values are evenly spaced. For example, $y = 3^x$ is an exponential function. For inputs of $x = 1$, 2, and 3, $y = 3$, 9, and 27. Notice that each y-value is 3 times the previous value.
- Typically, problems that deal with interest and half-lives will be exponential problems.
- Quadratic functions deal with a variable that is multiplied by itself, and many times this will be seen in finding the area of a figure.
- Linear functions are often used in problems dealing with memberships and sales.
- If the graph of a function is a straight line, it is linear.
- If the graph is a parabola, it is a quadratic function.
- If the graph grows at a very fast rate while the other side approaches a constant value, the function is exponential.
- You can also examine the end behavior to determine the type of function. If the function is linear, one end approaches negative infinity while the other approaches positive infinity. If the function is parabolic, both ends of the function approach either positive or negative infinity. If the function is exponential, one end approaches either positive or negative infinity while the other approaches a constant value.
- Horizontal and vertical shifts are also important to consider when comparing functions.
- In cases where the graph or a table of a function is uncertain, you can graph points by substituting x-values to get y-values.

Common Errors/Misconceptions

- confusing the shapes of graphs
- swapping values in the slope formula
- incorrectly identifying horizontal and vertical shifts