# Introduction

When comparing properties of two functions, it is important to be able to discern the key parts of the functions. For example, it is important to be able to identify the slope of a linear function, whether in a graph, in a table, or in point-slope form. In this lesson, we will compare different functions by analyzing their key features.

## Key Concepts

- When looking at a data set, there are important things to look for when identifying whether a linear, quadratic, or exponential function would be a better choice.
- The average rate of change of a function from $x_1$ to $x_2$ can be found by using the equation $\frac{y_2 - y_1}{x_2 - x_1}$, where $(x_1, y_1)$ and $(x_2, y_2)$ are two points on the graph. You can also use this equation when the function is given in table form.

## Key Concepts, *continued*

- With a data set in a table, if the rate of change between each *x*-value is the same, it is linear. If it is not the same, the function could be either quadratic or exponential.

## Key Concepts, *continued*

- If the function has the same general shape as the equation $y = x^2$, it is quadratic. It is possible the function can be more complicated, but as long as the graph is a parabola when graphed, this function is quadratic. You can also identify quadratic functions by checking the second difference between terms. Recall that the second difference is the difference between the rate of change of consecutive points on the function. For example, let $y = x^2 - 1$. Four consecutive points on this function are (0, –1), (1, 0), (2, 3) and (3, 8). Calculate the rate of change, or first difference, between each consecutive point:

# Key Concepts, *continued*

| *x* | *f*(*x*) | First difference (rate of change) | Second difference |
|---|---|---|---|
| 0 | –1 | — | — |
| 1 | 0 | 1 | — |
| 2 | 3 | 3 | 2 |
| 3 | 8 | 5 | 2 |

## Key Concepts, *continued*

- In a data set, exponential functions will follow the general equation $y = b^x$, where $b$ is a number greater than 0 and not equal to 1. Exponential functions have a common ratio between each $y$-value when $x$-values are evenly spaced. For example, $y = 3^x$ is an exponential function. For inputs of $x = 1, 2$, and 3, $y = 3, 9$, and 27. Notice that each $y$-value is 3 times the previous value.
- Typically, problems that deal with interest and half-lives will be exponential problems.
- Quadratic functions deal with a variable that is multiplied by itself, and many times this will be seen in finding the area of a figure.

# Key Concepts, *continued*

- Linear functions are often used in problems dealing with memberships and sales.
- If the graph of a function is a straight line, it is linear.
- If the graph is a parabola, it is a quadratic function.
- If the graph grows at a very fast rate while the other side approaches a constant value, the function is exponential.

## Key Concepts, *continued*

- You can also examine the end behavior to determine the type of function. If the function is linear, one end approaches negative infinity while the other approaches positive infinity. If the function is parabolic, both ends of the function approach either positive or negative infinity. If the function is exponential, one end approaches either positive or negative infinity while the other approaches a constant value.
- Horizontal and vertical shifts are also important to consider when comparing functions.

# Key Concepts, *continued*

- In cases where the graph or a table of a function is uncertain, you can graph points by substituting *x*-values to get *y*-values.

# Common Errors/Misconceptions

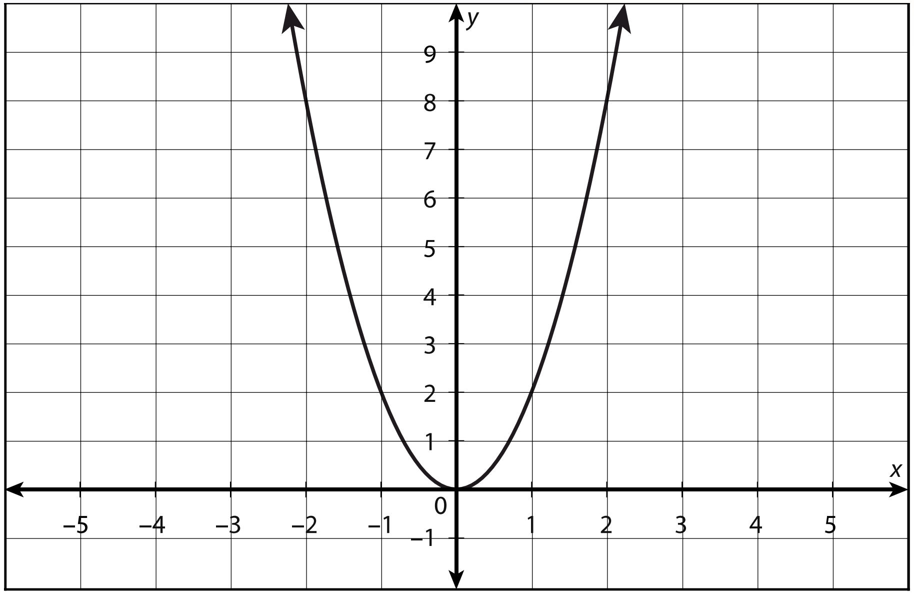
- confusing the shapes of graphs
- swapping values in the slope formula
- incorrectly identifying horizontal and vertical shifts

# Guided Practice

## Example 2

Determine whether the following equation or graph is greater when $x = 2$.

$f(x) = 3^x$



## Guided Practice: Example 2, *continued*

### 1. Identify the model of each function.

Notice that the shape of the graph is a parabola. Since parabolas are graphs of quadratic functions, the graphed function is quadratic. The other function, $f(x) = 3^x$, is characterized by $x$ being the exponent, so this function is exponential.

## Guided Practice: Example 2, *continued*

### 2. Determine the value of the graph at $x = 2$.

When $x = 2$ on the $x$-axis, $y = 8$.

# Guided Practice: Example 2, *continued*

## 3. Determine the value of the equation.

Substitute 2 into the equation $y = 3^x$: $y = 3^2 = 9$.

## Guided Practice: Example 2, *continued*

### 4. Compare values.

The value of the graph at $x = 2$ is 8, while the value of the equation at $x = 2$ is 9. Therefore, the equation is greater when $x = 2$.

Exponential functions grow extremely fast, and will eventually outgrow both linear and quadratic functions in the end that is approaching infinity. It is important
to note, however, that exponential functions are not greater for all values of $x$, especially
when $x$ is a small value.

# Guided Practice: Example 2, *continued*

# Guided Practice

## Example 4

Two analysts at a company have collected data on the sales performance for the year. The first analyst describes his data with an equation, while the second analyst puts his data into a table. They both describe sales growth as "exponential," and this makes the board of directors very happy. In each model, *y* is the revenue in thousands of dollars, while *x* is the quarter of the year. Are they correct in saying their data describe exponential functions? If not, what are they? What would an exponential function look like?

$$y = 2x^2$$

| *x* | *y* |
|---|---|
| 1 | 5 |
| 2 | 10 |
| 3 | 15 |
| 4 | 20 |

# Guided Practice: Example 4, *continued*

## 1. Check if the data is exponential.

Exponential functions are characterized by $f(x) = b^x$, where $b$ is a constant. They also have a constant ratio between $y$-values when the $x$-values are equally spaced. The equation does not match this description. Check if the table has a constant ratio between the $y$-values. We'll use the points (1, 5), (2, 10), and (3, 15), since the $x$-values for these points are equally spaced.

$$10 \div 5 = 2$$

$$15 \div 10 = 1.5$$

The table does not have a constant ratio between consecutive $y$-values, so it does not represent an exponential function.

## Guided Practice: Example 4, *continued*

### 2. Determine what kind of function the equation represents.

We know that the equation is not exponential, so we must check if it is linear or quadratic. It cannot be linear, because in linear functions, the $x$ term is always to the first power, not the second. Therefore, it is a quadratic function, because the $x$ term is to the second power.

## Guided Practice: Example 4, *continued*

### 3. Determine the type of function for the second data set.

Again, we must check if the function is linear or quadratic. The increase in consecutive $y$-values is 5 for every $x$. This means the slope does not change; therefore, the function must be linear.

## Guided Practice: Example 4, *continued*

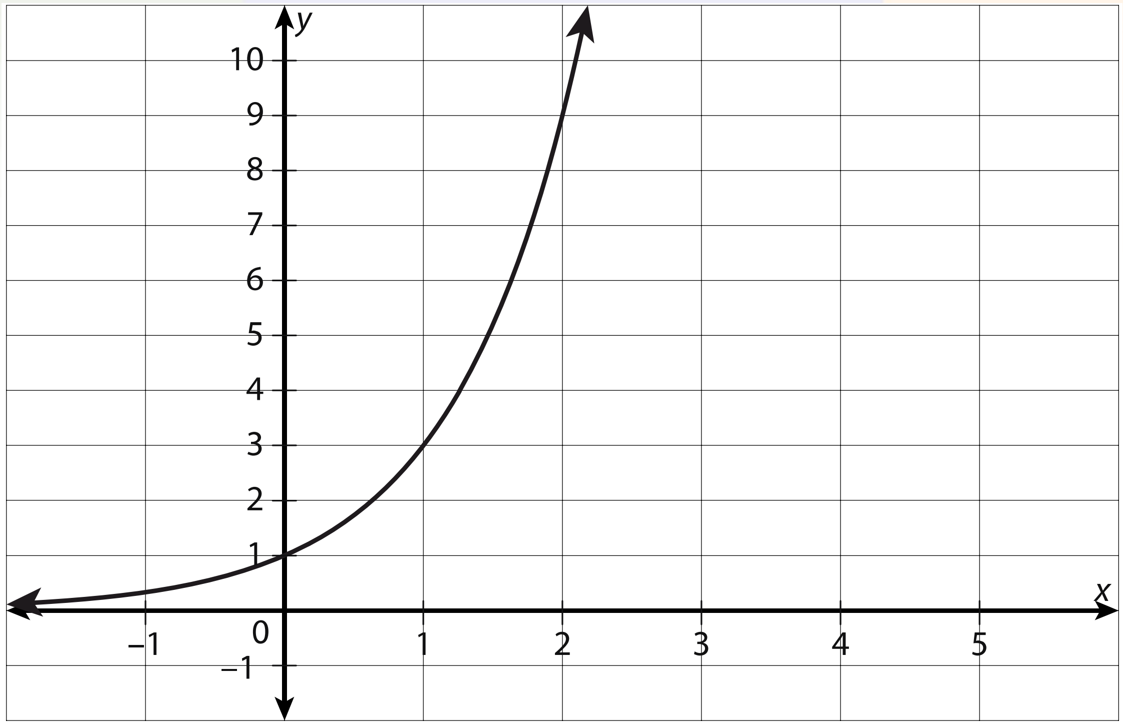
### 4. Show an example of an exponential function.

As an equation, an exponential function would take the basic form of $y = b^x$, where $b$ is a constant greater than 0 that is not equal to 1. In a table format, the following is an example of $y = 3^x$:

| | | | | |
|---|---|---|---|---|
| ***x*** | 1 | 2 | 3 | 4 |
| ***y*** | 3 | 9 | 27 | 81 |

# Guided Practice: Example 4, *continued*

The table values are shown in the following graph:



# Guided Practice: Example 4, *continued*