

## Conceptual Task: Logs from Trees

### Exploration Questions Sample Responses

- a. What are Umar and Hassan discussing?

Umar is expressing confusion about an analogy for logarithmic functions to Hassan. The analogy is about how the number of people in a generation in a family tree doubles each generation back. Hassan explains the connection between this analogy and logarithmic functions.

- b. According to their discussion, what does a family tree have to do with logarithmic functions?

According to Hassan, a logarithmic function can be used to figure out how many generations back you have to go before the number of ancestors in that generation is a certain size.

- c. Is Umar correct that there is exponential growth involved? Explain your answer.

Yes, he is correct. Since each child has two parents every generation, the number of ancestors in each generation increases by a multiple of two each generation back.

- d. Explain the problem that Hassan asks Umar to solve.

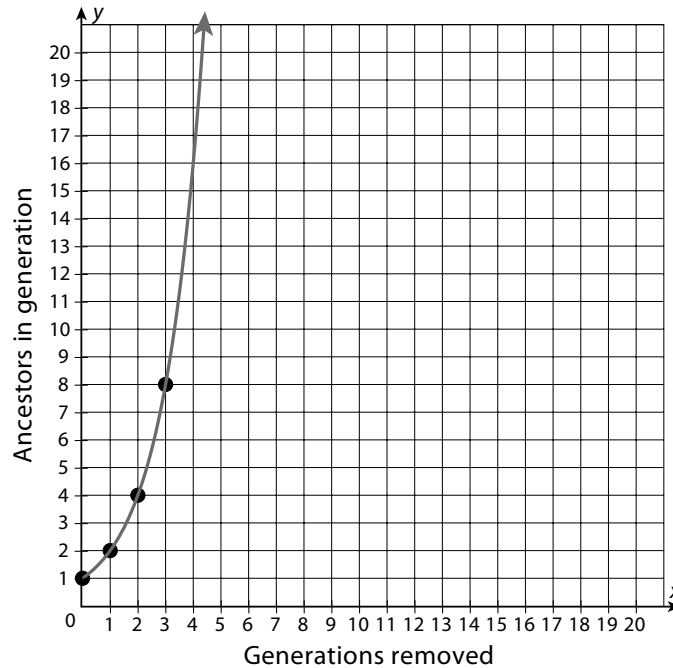
Hassan describes the structure of a family tree, in which each child has two parents. So, each generation back increases in size by a multiple of two. Hassan asks how many generations back you'd have to go for the generation to contain 32 ancestors.

- e. Could a logarithmic function be used to solve the problem above? Explain your answer.

Yes, a logarithm can be used to solve the problem above. The size of the family can be represented by an exponential function. A logarithm will be needed to extract the exponent from that function.

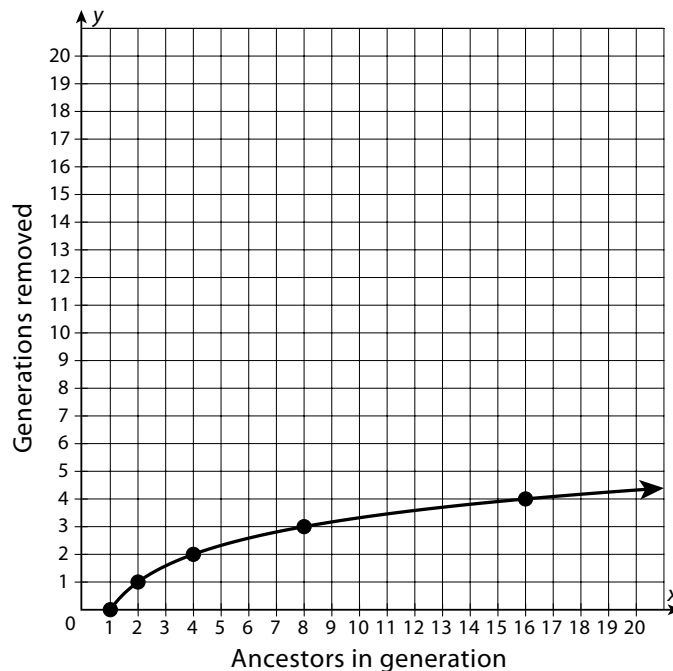
- f. What would a graph of the exponential situation Umar described look like? Create this graph. Be sure to label the axes.

The graph should show at least a few of the points mentioned in the stem:  $(0, 1)$ ,  $(1, 2)$ ,  $(2, 4)$ , etc. Graphs need not be exact, but should have the axes labeled correctly.



- g. What would a graph of the logarithmic situation Hassan described look like? Create this graph. Be sure to label the axes.

The graph should show at least a few of the points mentioned in the stem, with the  $x$ - and  $y$ -coordinates reversed: (1, 0), (2, 1), (4, 2), etc. Graphs need not be exact, but should have the axes labeled correctly. The axes on this graph should be reversed from those of the exponential graph.



- h. What is the connection between exponential growth and logarithms?

Answers may vary. Students should notice that the graphs appear to have the same shape, but reflected across the line  $y = x$ . Students may also notice that the coordinates identified in the first graph have been flipped; that is, the  $x$ - and  $y$ -coordinates have been swapped. Encourage students to observe that the logarithmic function “undoes” exponentiation.