Interpreting Logarithmic Models

Prerequisite Skills

This lesson requires the use of the following skills:

- writing a logarithmic function from an exponential function and vice versa
- translating what a function represents in a real-world problem
- relating different types of functions to one another in a problem setting
- using the rules of exponentials and logarithms to simplify a function
- identifying the domain and range values of a real-world problem
- relating different types of functions to one another in a real-world scenario

Introduction

Expressing or solving logarithmic functions in terms of exponential function models is one technique for solving real-world problems. One factor in determining which type of function is best in a given situation is how the solution to a problem affects a particular audience. For example, environmental scientists may need to present a study of the acidity or alkalinity of a freshwater pond to citizens at a town hall meeting. The citizens might best understand the results if the logarithm-based pH factor is used to describe the chemical condition of the pond rather than the actual concentration of hydronium or hydroxide ions in a sample of the pond water.

Such initial conditions as the upper and lower bound of a domain are essential to the viability of such models. For example, time is nearly always considered to be a positive quantity that moves in an ever-increasing direction. (There are exceptions to this in some of the leading-edge fields of physics, such as cosmology, but such discussions are generally beyond the scope of a mathematics course at this level.)

The ability to move accurately between a function and its inverse is often important in solving real-world problems that employ logarithms. Also, a thorough mastery of the basic rules of exponents and logarithms is essential for such problems.

Finally, be aware of the potential for a graphical or tabular presentation of a problem to aid in the application of logarithmic functions. In fact, visual models based on real data often provide a more accurate picture of a problem than an algebraic model that does not reveal the restricted domain or range of the problem in the way that a graph or table does.

Key Concepts

- Logarithmic functions have a wide range of applications in real-world problems, such as in the fields of biology, chemistry, ecology, and engineering. Logarithmic functions often provide an alternative approach to the use of exponential functions, which might help to increase the understanding of a problem or its solution.
- The logarithmic function is the inverse of an exponential function and vice versa. Recall two basic steps in writing one as the other:
  - A function's inverse switches the values of the domain and range values. For example, if an ordered pair of a function is (3, 5), then the ordered pair (5, 3) solves the inverse of the function.
  - To find the inverse of a function, replace the domain variable (often x) with the range variable ($f(x)$), and change the range variable to $f^{-1}(x)$.
- Apply the basic rules of exponentials and logarithms, described earlier in this lesson.
- The ability to identify the restricted domain and/or range over which a logarithmic function is defined can mean the difference between finding a solution to a problem and misinterpreting the application of that function to the particulars of the problem setting. This is especially true if a function and its inverse have roles to play in formulating and/or solving the problem.
- A graphing calculator can help with the identification of the restraints on real-world problems. The tables that are created when functions are graphed offer approximations of solutions that cannot be easily found using algebraic methods. Adjust the table and window settings as necessary to reflect the particulars of the problem.

Common Errors/Misconceptions

- neglecting to ensure that the real-world domain and/or range of a problem is compatible with the logarithmic function that models the problem
- incorrectly interpreting the representation of a logarithmic function in a problem
- when working with two or more functions, choosing the less-appropriate function to model a real-world problem and its solution
- misinterpreting a graph or table of a logarithmic function model of a real-world problem