Mathematical Modeling and Choosing a Model

Unit Assessment

Technology Enhanced

Circle the letter of the best answer.

1. Dry-cleaning machines use solvents to clean clothing. At the end of the rinse cycle, the solvent is extracted from the machine for reuse. The effectiveness of a dry-cleaning solvent decreases by $\frac{1}{3}$ each time it is used. The solvent is given a numerical rating after each use, and when the rating drops below 1 the solvent is replaced. The numerical rating of unused solvent is 18. If a batch of solvent has a rating of 2, which of the following equations models this situation? Let n represent the number of times the solvent has been used.

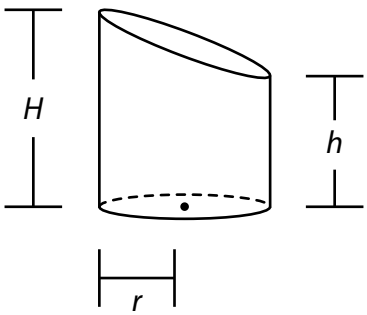
   a. $2 = 18 \cdot 3^{-n}$

   b. $2 = 3 \cdot 18^{-n}$

   c. $2 = 18(1 + 3^{-n})$

   d. $2 = 18 - 3^{n}$

2. A portion of a regular cylinder with heights H and h and a base radius of r is shown in the diagram that follows. What is the correct formula for the volume V of this portion in terms of the lateral area A (the surface area of the shape excluding the top and bottom) if A and V are given by $A = \pi r(H + h)$ and $V = \pi r^2\left(\frac{H+h}{2}\right)$?



   a. $V = \frac{1}{2rA}$

   b. $V = \frac{1}{2}rA$

   c. $V = \frac{r}{2A}$

   d. $V = \frac{2r}{A}$

continued

3. Which function represents a transformation of the parent quadratic function 4 units to the right and 3 units down on a coordinate plane?

   a. $f(x) = x^2 - 8x + 13$

   b. $f(x) = x^2 + 8x + 13$

   c. $f(x) = x^2 - 8x + 19$

   d. $f(x) = x^2 + 8x + 19$

4. Which function is neither odd nor even?

   a. $a(x) = \cos x$

   b. $b(x) = \sin x$

   c. $c(x) = |x|$

   d. $d(x) = 2^x$

5. A sailboat is propelled at a constant rate by wind according to the function $w(t) = 30t$. A shear turbulence force slows down the motion of the boat according to the function $r(t) = -0.05t^2 - 0.5t$. Which function combination represents the net motion of the sailboat relative to the direction of the wind?

   a. $(w + r)(t) = -0.05t^2 + 29.5t$

   b. $(w - r)(t) = 0.05t^2 + 29.5t$

   c. $(w \bullet r)(t) = -1.5t^2 - 15t$

   d. $(w \div r)(t) = -0.001\overline{6}t - 0.01\overline{6}$

6. What is the average rate of change of the function $f(x) = \dfrac{x}{x-1}$ between $x = 0.99$ and $x = 1.01$?

   a. It is infinite because the function is undefined at $x = 1$.

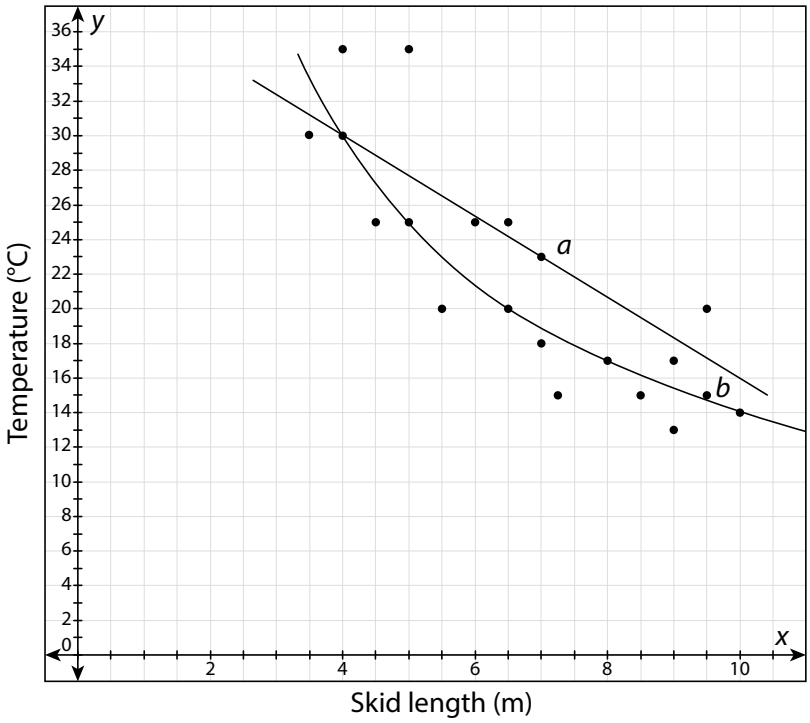
   b. It is undefined because the function is undefined at $x = 1$.

   c. It is 100 regardless of whether the function is undefined at $x = 1$.

   d. It is 0 because the function is undefined at $x = 1$.

continued

7. Skid marks on roads are caused by the heat of friction of a braking car's rubber tires against the road's surface—the heat melts the tires slightly, leaving behind streaks of rubber. The outside temperature can heat or cool the asphalt, which in turn affects the length of a skid mark at the time the vehicle's brakes are applied. The graph shows a linear function *a* and a quadratic function *b* that serve as models for the relationship between data points representing the temperatures that correlate to an increase in skid length. Temperatures are in degrees Celsius and skid lengths are in meters. What do the rates of change of the functions have in common over the interval [6, 8]?



a. Both are equal to 5°C per meter.

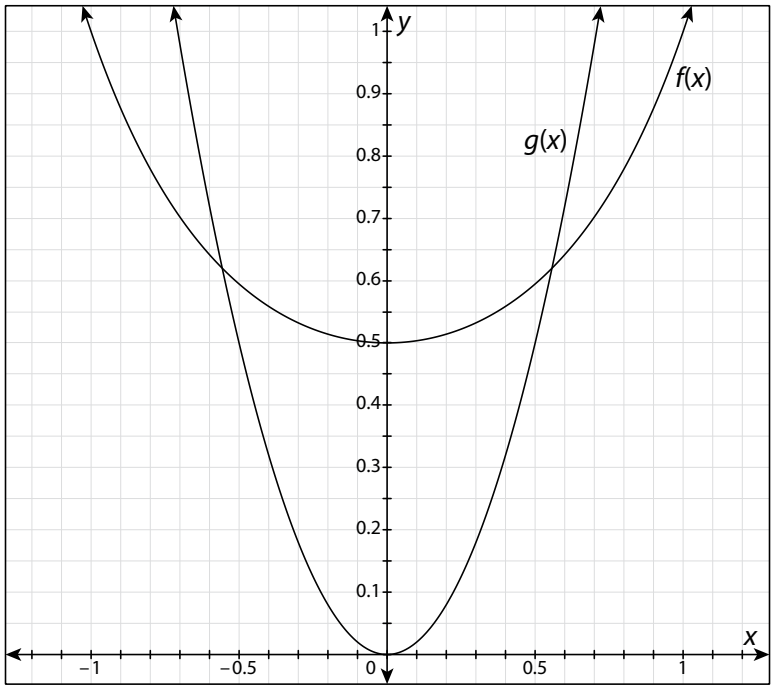
b. Both are equal to –5°C per meter.

c. Both are negative.

d. Both are positive.

continued

8. The graphs of two functions $f(x)=2^{x^2-1}$ and $g(x) = 2x^2$ are shown. If $(\pm a, f(\pm a))$ and $(\pm a, g(\pm a))$ represent the points at which the functions intersect, which of the following is true?



a. over the interval $(-\infty,-a)\cap(a,\infty)$, $f(x) = g(x)$

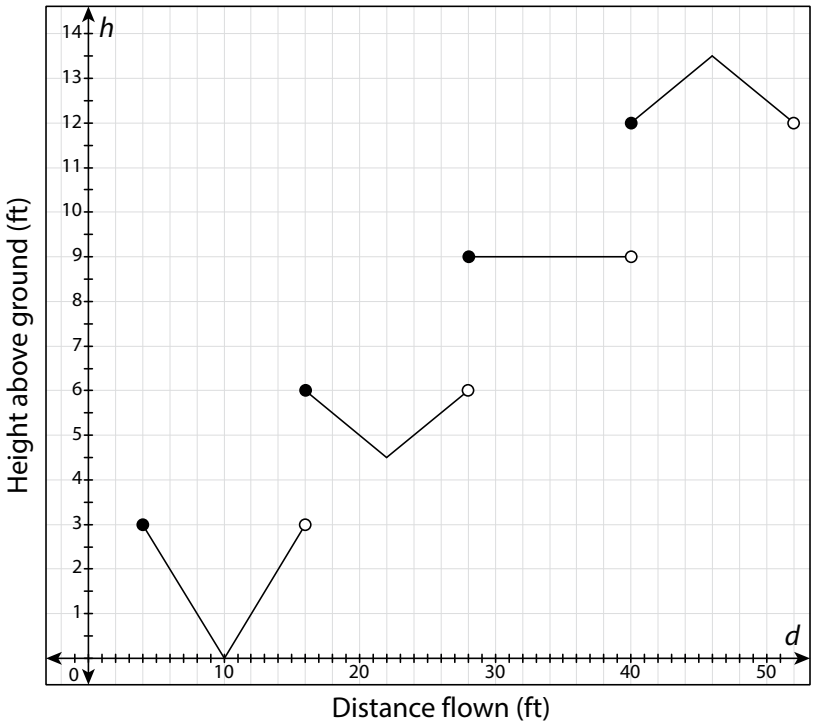
b. over the interval $(-a,0)\cap(0,a)$, $f(x) = 0.5$ and $g(x) = 0$

c. over the interval $[-a, a]$, $g(x) < f(x)$

d. over the interval $(-a, a)$, $g(x) < f(x)$

continued

9. Some members of a school's STEM club came up with a way to model the wing movements of a heron as it takes off from a flooded area where it has been foraging. The model is shown in the following graph, with the *d* and *h* axes representing the bird's distance from its takeoff point and height above ground, respectively, in feet for each of four segments of its flight. Which function model(s) were used to form the mathematical model?



a. absolute value; linear; step

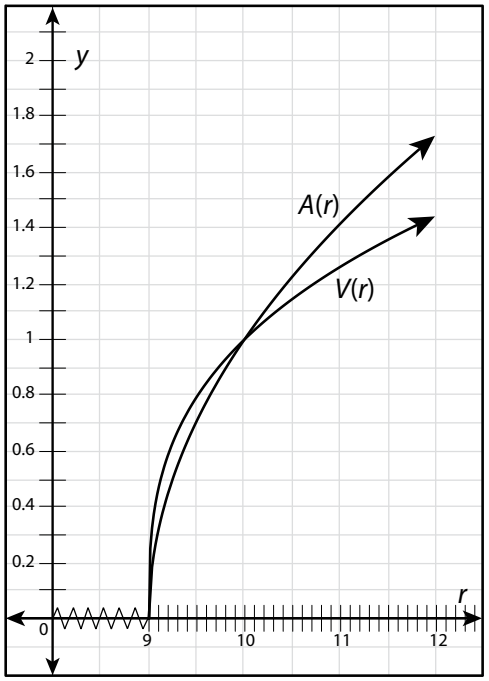
b. absolute value; linear

c. absolute value; step

d. absolute value

continued

10. An astronomer found that the cross-sectional radius in microns of carbon-oxygen-silicon space dust can be modeled by the two functions shown in the following graph. The cube root function $V(r)$ models the volume of a dust particle based on the estimated mass of the dust cloud of which it is a part. The square root function $A(r)$ models the particle's surface area as indicated by light reflectivity. Over what interval of r do the two models give approximately equal results?



a. [0, 3]

b. [9.5, 10.5]

c. $[11, \infty)$

d. $(-\infty, 9]$

11. A solid figure has two cross sections: an isosceles triangle with a base of 8 inches and a height of 5 inches, and a circle with a diameter of 8 inches. Which of the following best describes the solid?

a. It is a cone with a height of 8 inches and a base radius of 5 inches.

b. It is a cone with a height of 4 inches and a base radius of 5 inches.

c. It is a cone with a height of 5 inches and a base radius of 8 inches.

d. It is a cone with a height of 5 inches and a base radius of 4 inches.

continued

12. Which density measurement would best reflect the density of finches in an aviary if there are 75 finches in an enclosure that measures 5 by 10 by 15 meters? (*Note*: 100 meters = 1 hectometer = 10 decameters)

    a. 0.1 finch per cubic meter
    b. 10 finches per cubic meter
    c. 100 finches per cubic decameter
    d. 100,000 finches per cubic hectometer

Use what you have learned about mathematical modeling to solve the following problems.

13. Consider the functions $f(x) = \sin(x + 4)$ and $g(x) = x + 4$. Let $h(x) = f(x) - g(x)$.

    a. Is $h(x)$ a combination, a composition, or both a combination and a composition? Explain your reasoning.

    b. What is the function that represents $h(x)$?

    c. What is the domain of $h(x)$?

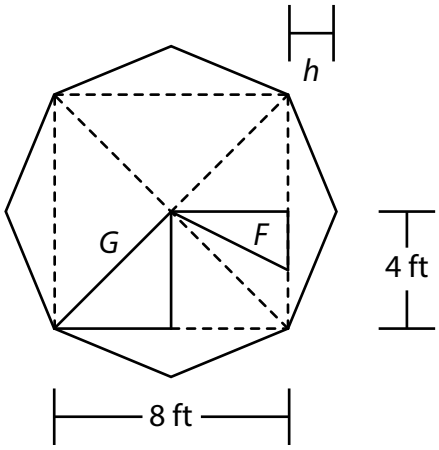
14. An absolute value function has the general form $f(x) = \left|\frac{x+a}{x+b}\right|$.

    a. What is the specific function if $f(4) = 6$, $f(6) = 8$, and $a = 2$?

    b. What is the domain of the specific function?

continued

15. A wilderness-supply manufacturer is designing a simple pyramid-shaped tent with a square base. The central tent pole has a height of 4 feet. The waterproof sheeting that will be used for the tent is in the shape shown in the following overhead view.



To form a pyramid, the sheeting is staked at the outermost end of each edge, *G*. A flap of area *F* is cut into the sheeting as shown, but not removed from it; the bottom edge of the flap has a length of 2 feet. The square base of the pyramid is 8 feet by 8 feet. The four isosceles triangles attached to each side of the square base are needed so that the sheeting fits snugly to the ground around all sides once the sheeting is raised with the tent pole.

a. Find the exact length of the edge, *G*.

b. Find the exact area, *F*, of the flap.

c. If the height *h* of each of the four isosceles triangles is $\sqrt{2}$, what is the exact total area of the waterproof tent sheeting?