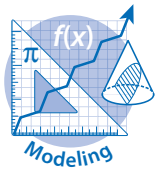
Name: **Date:**

Conceptual Task: Rational Decisions



SMP	
1 ✓	2
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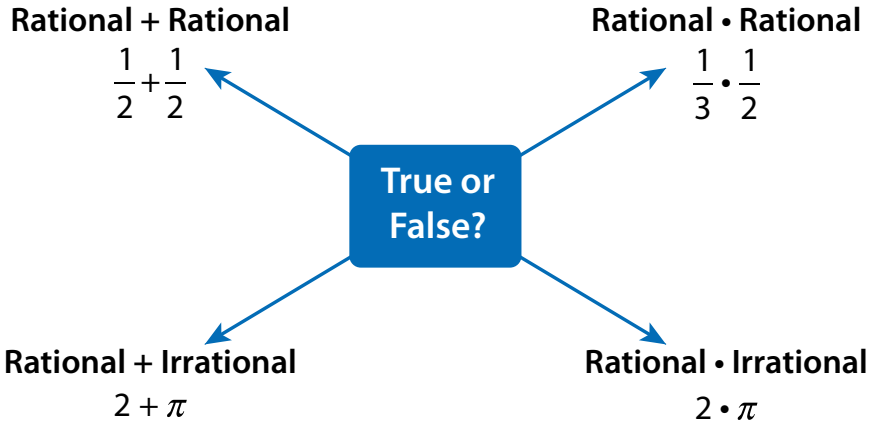
Joe is considering the following statements:

- The sum of two rational numbers is rational.
- The product of two rational numbers is rational.
- The sum of a rational number and an irrational number is irrational.
- The product of a rational number and an irrational number is irrational.

Are these statements true? Why or why not?



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Exploration Questions

a. Recall that rational numbers can be written as a ratio of integers. Will the numerator and denominator of a product of rational numbers also be integers? How do you know?

b. Will the numerator and denominator of a sum of rational numbers also be integers? How do you know?

c. Recall that if you write a rational number as a decimal, the decimal either ends or repeats. The decimal 0.101001000100001 ... is an irrational number. Why?

d. Add 1 to 0.101001000100001 ... . Is the result still irrational? Why?

e. Multiply 0.101001000100001 ... by 2. Is the result still irrational? Why?

f. Consider the sum $a + b$, where a is rational and b is irrational. What happens if you subtract a? Is the result rational or irrational?

g. Joe is convinced that the sum of a rational and irrational number can be rational. How could you prove or disprove his claim?

h. Can you use a similar argument to show that the product of a rational number and an irrational number is irrational? Explain. (You need not write a proof.)

i. The expressions $\sqrt{2} + \sqrt{2}$ and $\sqrt{2} \bullet \sqrt{3}$ are both irrational. Will the sum or product of two irrational numbers always be irrational? Or can it be rational? Provide a specific example.

j. Formulate additional statements about the sums and products of irrational numbers to add to the list from the problem scenario.