

Defining, Rewriting, and Evaluating Rational Exponents

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Warm-Up

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A population at any time t can be estimated using the equation $p_t = p_0 \cdot (1 + r)^t$, where p_0 is the initial population, r is the annual growth rate, and t is the time in years from now. Today, Tinyville and Littletown both have a population of 10,000 people. Tinyville's yearly growth rate is 2.5% and Littletown's yearly growth rate is 1.8%.

1. Which town will have the larger population after 5 years?
2. What equation can be used to find the approximate population of Littletown after t years?
3. What will be the approximate population of Tinyville in 10 years?



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1. Which town will have the larger population after 5 years?

Since both towns have the **same** population today, each town's **population growth rate** will determine which town has the larger population in 5 years.

Tinyville has a **higher yearly growth rate**, and will have a greater population in 5 years.



2. What equation can be used to find the approximate population of Littletown after t years?

Write an equation using Littletown's starting population and yearly growth rate.

Replace p_0 with 10,000 and r with 0.018 in the equation $p_t = p_0 \cdot (1 + r)^t$. Simplify the equation.

$$p_t = 10,000 \cdot (1 + 0.018)^t$$

$$p_t = 10,000 \cdot (1.018)^t$$

3. What will be the approximate population of Tinyville in 10 years?

Write an **equation** using Tinyville's starting population, its **yearly growth rate**, and the **time** in years.

Replace p_0 with 10,000, r with 0.025, and t with 10 in the equation $p_t = p_0 \cdot (1 + r)^t$ such that

$$p_{10} = 10,000 \cdot (1 + 0.025)^{10}.$$

Evaluate the expression to find p_{10} .

$$p_{10} = 10,000 \cdot (1 + 0.025)^{10}$$

$$p_{10} = 10,000 \cdot (1.025)^{10}$$

$$p_{10} \gg 10,000 \cdot (1.28008)$$

$$p_{10} \gg 12,800.8$$

The approximate population of Tinyville in 10 years will be 12,801 people.



Warm-Up

Defining, Rewriting, and Evaluating Rational Exponents

Instruction



Instruction

Properties of Equality

Introduction

- An **exponent** is a quantity that shows the number of times a factor is being multiplied together in an exponential expression.
- In an expression written in the form a^b , b is the **exponent**.
- So far, the exponents you have worked with have been **whole numbers**, or the set of positive integers and 0: $\{0, 1, 2, 3, \dots\}$.
- **Integers** are positive and negative whole numbers and 0. Exponents can be negative numbers.



Introduction

- They can also be **rational numbers**, which are real numbers that can be expressed as the ratio of two integers.
- **Rational exponents** are simply another way to write radical expressions.
- For example, $\sqrt{x} = x^{\frac{1}{2}}$ and $\sqrt[3]{x} = x^{\frac{1}{3}}$.
- As we will see in this lesson, the rules and properties that apply to integer exponents also apply to rational exponents.



Key Concepts

- An **exponential expression** contains a base and an exponent.
- A **base** is the factor that is being multiplied together in an exponential expression. In other words, it is the quantity that is being raised to a power, and the power is determined by the exponent.
- A **power** is the result of raising a base to an exponent; 32 is a power of 2 since $2^5 = 32$. In the expression 2^5 , 2 is the **base**, 5 is the **exponent**, and the entire expression, 2^5 , is a power.



Key Concepts, *continued*

- A **radical expression** contains a root, which can be shown using the radical symbol, $\sqrt{\quad}$.
- The **root** of a number x is a number that, when multiplied by itself a given number of times, equals x .
- The root of a function is also referred to as the **inverse** of a power, and “**undoes**” the power.

For example, $\sqrt[3]{8} = 2$ and $2^3 = 8$.

Key Concepts, *continued*

- In the radical expression $\sqrt[n]{a^n}$, the *n*th root of the *n*th power of *a* is *a*.
- For example,

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad \text{and} \quad \sqrt[n]{x^m} = x^{\frac{m}{n}}$$

Key Concepts, *continued*

- A **rational exponent** is an exponent that is a rational number.
- A **rational number** is a real number that can be written as $\frac{m}{n}$, where both m and n are integers and $n \neq 0$.
- The **denominator** of the rational exponent is the **root**, and the **numerator** is the **power**.

For example, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$.

Key Concepts, *continued*

- An **exponential equation** is an equation whose independent variable is in the exponent.
- An exponential equation can be written as $y = ab^x$, where x is the independent variable, y is the dependent variable, and a and b are real numbers.
- **Real numbers** are the set of all rational and irrational numbers.
- The **properties of exponents** apply to both integer and rational exponents.



Key Concepts, *continued*

Properties of Exponents

| Words | Symbols | Numbers |
|--|---|---|
| Zero Exponent Property A base raised to the power of 0 is equal to 1. | $a^0 = 1$ | $12^0 = 1$ |
| Negative Exponent Property A number raised to a negative exponent is equal to the reciprocal of the number raised to a positive power. | $a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}},$ $a \neq 0, n \neq 0$ | $64^{-\frac{2}{3}} = \frac{1}{64^{\frac{2}{3}}} =$ $\frac{1}{(\sqrt[3]{64})^2} = \frac{1}{16}$ |



Key Concepts, *continued*

| Words | Symbols | Numbers |
|---|-----------------------------|---|
| <p>Product of Powers Property</p> <p>To multiply powers with the same base, add the exponents.</p> | $a^m \cdot a^n = a^{m+n}$ | $3^{\frac{1}{4}} \cdot 3^{\frac{7}{4}} = 3^{\frac{1}{4} + \frac{7}{4}} = 3^2 = 9$ |
| <p>Quotient of Powers Property</p> <p>To divide powers with the same base, subtract the exponents.</p> | $\frac{a^m}{a^n} = a^{m-n}$ | $\frac{8^{\frac{4}{9}}}{8^{\frac{1}{9}}} = 8^{\frac{4}{9} - \frac{1}{9}} = 8^{\frac{1}{3}} = 2$ |

Key Concepts, *continued*

| Words | Symbols | Numbers |
|---|---------------------------|--|
| <p>Power of a Power Property</p> <p>To raise one power to another power, multiply the exponents.</p> | $(a^m)^n = a^{m \cdot n}$ | $\left(5^{\frac{2}{3}}\right)^3 = 5^{\frac{2}{3} \cdot 3} = 5^2 = 25$ |
| <p>Power of a Product Property</p> <p>To find the power of a product, raise each factor in the product to the given power.</p> | $(ab)^m = a^m b^m$ | $(25 \cdot 36)^{\frac{1}{2}} = 25^{\frac{1}{2}} \cdot 36^{\frac{1}{2}} = 5 \cdot 6 = 30$ |

Key Concepts, *continued*

| Words | Symbols | Numbers |
|--|--|--|
| <p>Power of a Quotient Property</p> <p>To find the power of a quotient, raise each term in the quotient to the given power.</p> | $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ | $\left(\frac{25}{49}\right)^{\frac{1}{2}} = \frac{25^{\frac{1}{2}}}{49^{\frac{1}{2}}} = \frac{5}{7}$ |

- Either the power or root can be determined first when evaluating an exponential expression with a rational exponent.

Key Concepts, *continued*

- Rational exponents can be reduced to simplest form before evaluating a radical expression.
- Use the absolute value symbol to represent the n th root of a number when n is an even number, because even roots are never negative.
- For example, the square root of x^2 can be written as $(x^2)^{\frac{1}{2}}$, which is equal to $|x|$.



Key Concepts, *continued*

- An **even root** is always **positive**.
- For example, although 2^2 and $(-2)^2$ are both equal to 4, $\sqrt{4}$ is equal to 2, not -2 .
- Sometimes **rational exponents** appear as **decimals**.
For example, $x^{0.25}$ is equal to $x^{\frac{1}{4}}$ or $\sqrt[4]{x}$.



Common Errors/Misconceptions

- not identifying the **denominator** of a rational exponent as being a **root**
- incorrectly **evaluating** an exponential expression with multiple operations

Guided Practice

Example 1

How can the exponential expression $3^{\frac{6}{5}}$ be rewritten as a radical expression?



Instruction

Defining, Rewriting, and Evaluating Rational Exponents

Guided Practice: Example 1, *continued*

1. Identify the power.

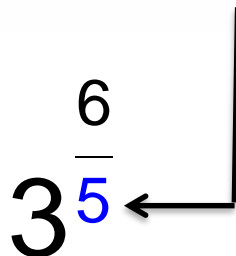
The **power** is the **numerator** of the rational exponent: **6**.

$$3^{\frac{6}{5}}$$

Guided Practice: Example 1, *continued*

2. Identify the root. If the root is even, the solution is the absolute value of the expression.

The **root** is the **denominator** of the rational exponent: **5**.

$$3^{\frac{6}{5}}$$


Since the root is **odd**, it is not necessary to take the absolute value of the expression.

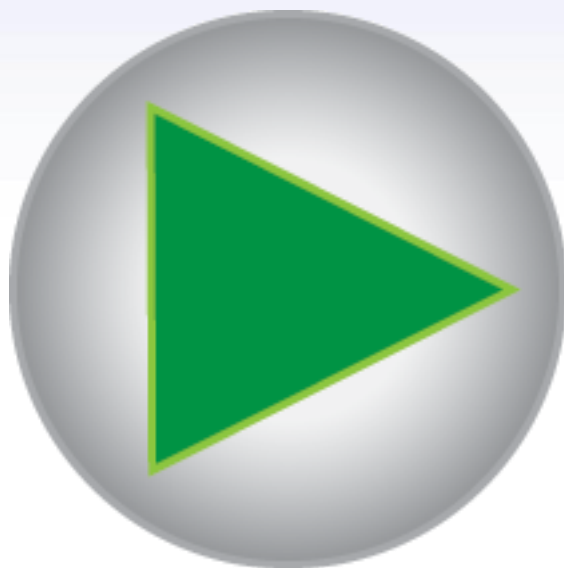
Guided Practice: Example 1, *continued*

3. Rewrite the expression in either of the following forms: $\sqrt[\text{root}]{\text{base}^{\text{power}}}$ or $(\sqrt[\text{root}]{\text{base}})^{\text{power}}$, where the base is the quantity being raised to the rational exponent.

$$3^{\frac{6}{5}} = \sqrt[5]{3^6} = (\sqrt[5]{3})^6$$



Guided Practice: **Example 1, *continued***



Guided Practice

Example 2

How can the radical expression $\sqrt[8]{a^c}$ be rewritten as an exponential expression?

Guided Practice: Example 2, *continued*

1. Identify the numerator of the rational exponent.

The **numerator** is the power: c .

$$\sqrt[8]{a^c}$$

Guided Practice: Example 2, *continued*

2. Identify the denominator of the rational exponent.

The **denominator** is the root: 8.

$$\sqrt[8]{a^c}$$

Guided Practice: Example 2, *continued*

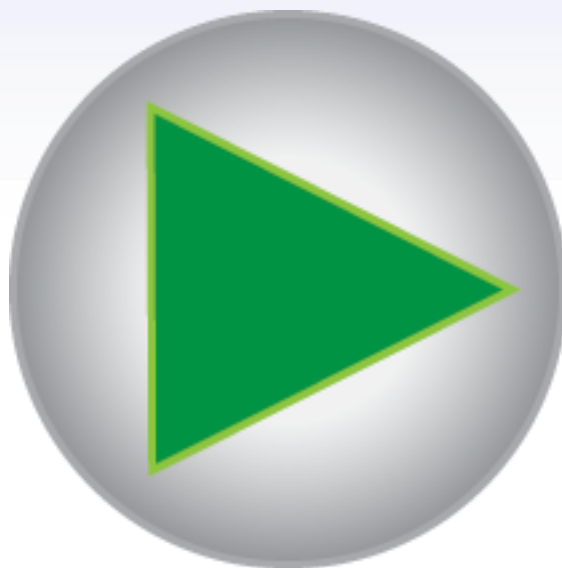
3. Rewrite the expression in the form

$\text{base}^{\text{power}}$ $\sqrt[\text{root}]{\text{base}}$, where the base is the quantity raised to a power and of which the root is being taken.

$$\sqrt[8]{a^c} = a^{\frac{c}{8}}$$



Guided Practice: **Example 2, *continued***



Guided Practice

Example 3

Evaluate the expression $\sqrt[8]{4^{10}}$. Round your answer to the nearest thousandth.

Guided Practice: Example 3, *continued*

1. Evaluate the power.

$$4^{10} = 1,048,576$$

Guided Practice: Example 3, *continued*

2. Find an exact root or approximate root using a calculator.

Use a calculator to approximate the **eighth root** of 1,048,576.

The root, **8**, is even, so we only want the positive value of the root.

$$\sqrt[8]{1,048,576} \gg 5.657$$



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Instruction

Defining, Rewriting, and Evaluating Rational Exponents

Guided Practice

Example 4

A town's population is decreasing. The population in the year 2000 was 4,000, and the population t years after 2000 can be found by using the function $f(t) = 4000(0.96)^t$.

What was the town's approximate population 2.5 years after the year 2000?

Guided Practice: Example 4, *continued*

1. Replace the variable in the equation with the known value.

The variable, t , is the number of years after 2000. To find the approximate population 2.5 years after 2000, replace t with 2.5.

$$f(2.5) = 4000(0.96)^{2.5}$$

Guided Practice: Example 4, *continued*

2. Evaluate the expression by first calculating the power.

The base of the exponential expression is a decimal. In this case, use a calculator to approximate the population for $t = 2.5$. Since the evaluated function is a population, round to the nearest whole number.

$$f(2.5) = 4000(0.96)^{2.5} \gg 4000(0.9030) \gg 3612$$

2.5 years after the year 2000, the town's approximate population was 3,612 people.

