Working with Rational Exponents

Prerequisite Skills

This lesson requires the use of the following skills:

- evaluating expressions using the order of operations
- evaluating exponential expressions involving integer exponents
- rewriting fractions in the simplest form
- rewriting mixed fractions as improper fractions

Introduction

An **exponent** is a quantity that shows the number of times a factor is being multiplied together in an exponential expression. In an expression written in the form a^b, b is the exponent. So far, the exponents you have worked with have been **whole numbers**, or the set of positive integers and 0: {0, 1, 2, 3, ...}. **Integers** are positive and negative whole numbers and 0. Exponents can be negative numbers. They can also be **rational numbers**, which are real numbers that can be expressed as the ratio of two integers. Rational exponents are simply another way to write radical expressions. For example, $\sqrt{x} = x^{\frac{1}{2}}$ and $\sqrt[3]{x} = x^{\frac{1}{3}}$.

As we will see in this lesson, the rules and properties that apply to integer exponents also apply to rational exponents.

Key Concepts

- An **exponential expression** contains a base and a power/exponent. A **base** is the factor that is being multiplied together in an exponential expression. In other words, it is the quantity that is being raised to a power, and the power is determined by the exponent.
- A **power** is the result of raising a base to an exponent; 32 is a power of 2 since $2^5 = 32$. In the expression 2^5, 2 is the base, 5 is the exponent, and the entire expression, 2^5, is a power.
- A **radical expression** contains a root, which can be shown using the radical symbol, $\sqrt{}$. The **root** of a number x is a number that, when multiplied by itself a given number of times, equals x.
- The root of a function is also referred to as the inverse of an exponent, and "undoes" the exponent. For example, $\sqrt[3]{8} = 2$ and $2^3 = 8$.
- In the radical expression $\sqrt[n]{a^n}$, the nth root of the nth power of a is a, if $a \geq 0$. For example, $\sqrt[3]{2^3} = 2$.
- Roots can be expressed using rational exponents instead of radical symbols. For example, $\sqrt[n]{a} = a^{\frac{1}{n}}$ and $\sqrt[n]{x^m} = x^{\frac{m}{n}}$.

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- A rational exponent is an exponent that is a rational number.
- A rational number is a real number that can be written as $\frac{m}{n}$, where both m and n are integers and $n \neq 0$.
- The denominator of the rational exponent is the root, and the numerator indicates the power. For example, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$.
- An **exponential equation** is an equation whose independent variable is in the exponent. An exponential equation can be written as $y = ab^x$, where x is the independent variable, y is the dependent variable, and a and b are real numbers. **Real numbers** are the set of all rational and irrational numbers.
- The properties of exponents apply to both integer and rational exponents.

Properties of Exponents

Words	Symbols	Numbers
Zero Exponent Property A base raised to the power of 0 is equal to 1.	$a^0 = 1$	$12^0 = 1$
Negative Exponent Property A number raised to a negative exponent is equal to the reciprocal of the number raised to a positive power.	$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}}, a \neq 0, n \neq 0$	$64^{-\frac{2}{3}} = \frac{1}{64^{\frac{2}{3}}} = \frac{1}{\left(\sqrt[3]{64}\right)^2} = \frac{1}{16}$
Product of Powers Property To multiply powers with the same base, add the exponents.	$a^m \bullet a^n = a^{m+n}$	$3^{\frac{1}{4}} \bullet 3^{\frac{7}{4}} = 3^{\frac{1}{4}+\frac{7}{4}} = 3^2 = 9$
Quotient of Powers Property To divide powers with the same base, subtract the exponents.	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{8^{\frac{4}{9}}}{8^{\frac{1}{9}}} = 8^{\frac{4}{9}-\frac{1}{9}} = 8^{\frac{1}{3}} = 2$

(*continued*)

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Power of a Power Property To raise one power to another power, multiply the exponents.	$(a^m)^n = a^{m \bullet n}$	$\left(5^{\frac{2}{3}}\right)^3 = 5^{\frac{2}{3} \bullet 3} = 5^2 = 25$
Power of a Product Property To find the power of a product, raise each factor in the product to the given power.	$(ab)^m = a^m b^m$	$(25 \bullet 36)^{\frac{1}{2}} = 25^{\frac{1}{2}} \bullet 36^{\frac{1}{2}} = 5 \bullet 6 = 30$
Power of a Quotient Property To find the power of a quotient, raise each term in the quotient to the given power.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{25}{49}\right)^{\frac{1}{2}} = \frac{25^{\frac{1}{2}}}{49^{\frac{1}{2}}} = \frac{5}{7}$

- Either the power or root can be determined first when evaluating an exponential expression with a rational exponent.
- Rational exponents can be reduced to simplest form before evaluating a radical expression.
- Use absolute value when simplifying expressions with an even root or variable roots, if the base contains a variable or negative number. For example, the square root of x^2 can be written as $(x^2)^{\frac{1}{2}}$, which is equal to $|x|$.
- An even root is always positive. For example, although 2^2 and $(-2)^2$ are both equal to 4, $\sqrt{4}$ is equal to 2, not –2.
- Sometimes rational exponents appear as decimals. For example, $x^{0.25}$ is equal to $x^{\frac{1}{4}}$ or $\sqrt[4]{x}$.

Common Errors/Misconceptions

- not identifying the denominator of a rational exponent as being a root
- incorrectly evaluating an exponential expression with multiple operations

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