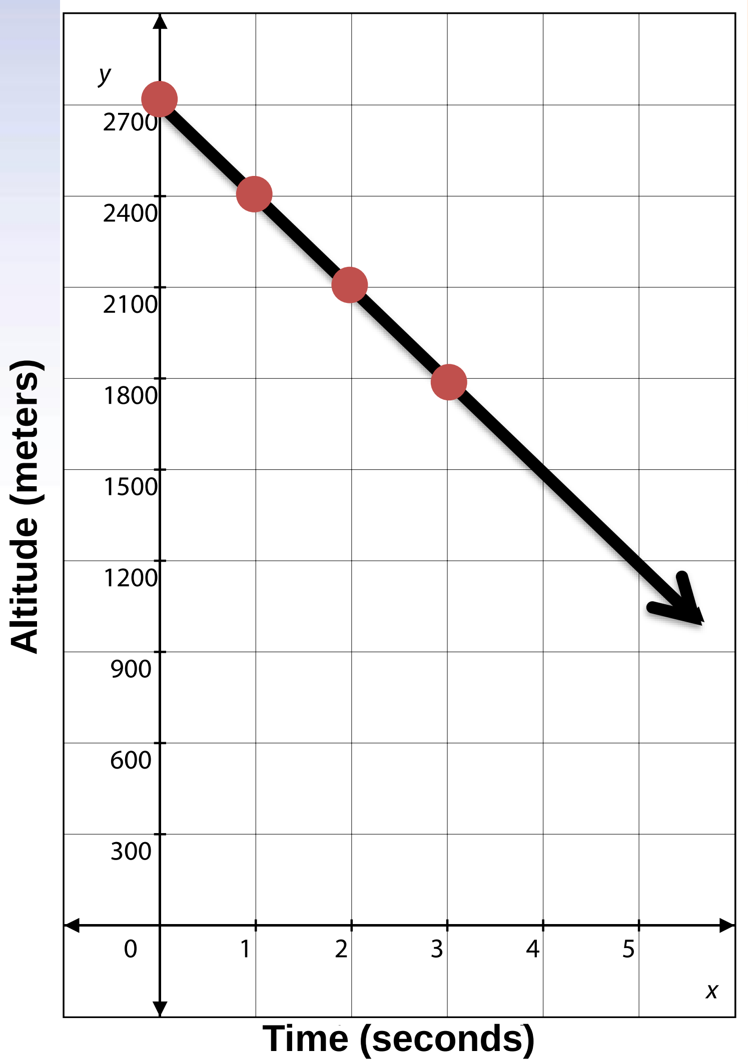
Comparing Linear Functions

Warm-Up

Cecilia took her first parachute jump lesson last weekend.

Her instructor gave her the graph at right, which shows her change in altitude in meters during a 5-second interval.

Use the graph to solve the problems that follow.

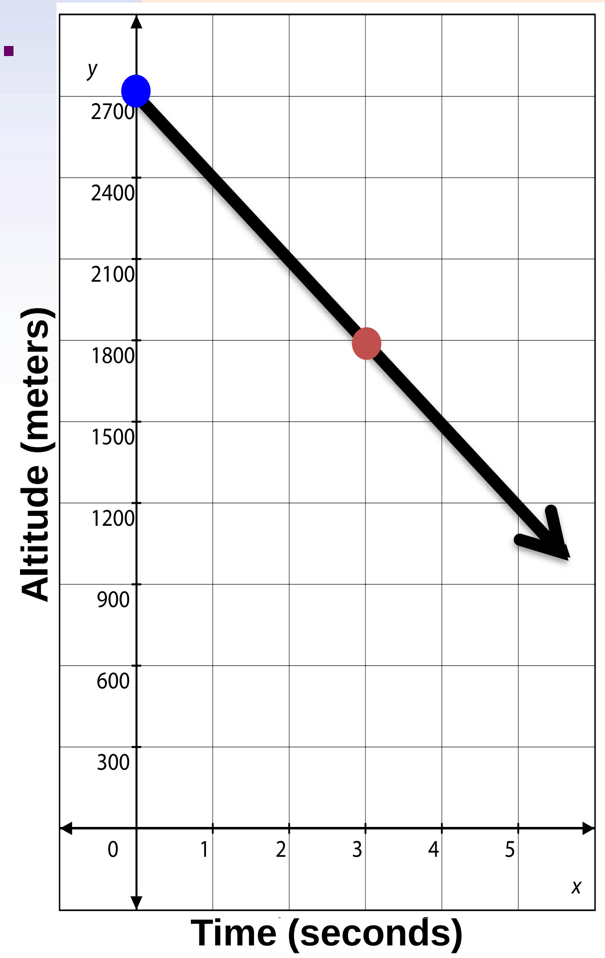


1. Estimate Cecilia’s average rate of change in altitude in meters per second.

2. What are the domain and range for this function?

1. Estimate Cecilia's average rate of change in altitude in meters per second.

- Select two points from the graph and use the slope formula to calculate the rate of change.
- Let $(x_1, y_1) = (1, 2400)$ and $(x_2, y_2) = (3, 1800)$.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope Formula

$$= \frac{(1800) - (2400)}{(3) - (1)}$$ Substitute (1, 2400) for (x_1, y_1) and (3, 1800) for (x_2, y_2).

$$= \frac{-600}{2} = -300$$ Simplify

- The average rate of change is –300 meters per second.
- Cecilia's altitude is decreasing an average of 300 meters each second.

2. What are the domain and range for this function?

- The domain is the set of all possible *x*-values for which the function is defined.
- In this problem, the domain is all real numbers from $x = 0$, when she jumps, to $x = 9$, when she lands on the ground: $0 \le x \le 9$.
- The range is the set of all possible *y*-values for the defined values of *x*.
- In this problem, the range is all real numbers from $y = 2700$, her initial height, to $y = 0$, her altitude when she lands: $0 \le y \le 2700$.

Instruction

Introduction

- Remember that linear functions are first-degree equations that can be written in the form $f(x) = mx + b$, where m is the slope and b is the y-intercept.
- The slope of a linear function is also the rate of change and can be calculated using the formula

.
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- The y-intercept is the y-coordinate of the point at which the graph intersects the y-axis; it is the value of y when $x = 0$.

Introduction, *continued*

- The *x*-intercept, if it exists, is the *x*-coordinate of the point where the graph intersects the *x*-axis; it is the value of x when $y = 0$.
- The slope and both intercepts can be determined from tables, equations, and graphs.
- These features are used to compare linear functions to one another.

Key Concepts

- Linear functions can be represented in words or as equations, graphs, or tables.
- To compare linear functions, determine the rate of change and intercepts of each function.
- Review the following processes for identifying the rate of change and y-intercept of a linear function.

Key Concepts, *continued*

Identifying the Rate of Change and the *y*-intercept from Context
1. Read the problem statement carefully. 2. Look for the information given and make a list of the known quantities. 3. Determine which information tells you the rate of change, or the slope, *m*. Look for words such as *each*, *every*, *per*, or *rate*. 4. Determine which information tells you the *y*-intercept, or *b*. This could be an initial value or a starting value, a flat fee, and so forth.

Key Concepts, *continued*

Identifying the Rate of Change and the *y*-intercept from an Equation
1. Write the equation of the function in slope-intercept form, $f(x) = mx + b$.
2. Identify the rate of change, or the slope, *m*, as the coefficient of *x*.
3. Identify the *y*-intercept, or *b*, as the constant term in the function.

Key Concepts, *continued*

Identifying the Rate of Change and the *y*-intercept from a Table
1. Choose two points from the table.
2. Assign one point to be (x_1, y_1) and the other point to be (x_2, y_2).
3. Substitute the coordinates into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
4. Identify the *y*-intercept as the *y*-coordinate in the ordered pair $(0, y)$. If this coordinate is not given, substitute the slope and the coordinates of any ordered pair from the table into the equation $f(x) = mx + b$ and solve for b.

Key Concepts, *continued*

Identifying the Rate of Change and the *y*-intercept from a Graph
1. Choose two points from the graph.
2. Assign one point to be (x_1, y_1) and the other point to be (x_2, y_2).
3. Substitute the coordinates into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
4. Identify the *y*-intercept as the *y*-coordinate of the point where the line intersects the *y*-axis.

Key Concepts, *continued*

- When presented with functions represented in different ways, it is helpful to rewrite the information using function notation.
- Linear functions are increasing if the rate of change is a positive value.
- Linear functions are decreasing if the rate of change is a negative value.
- The greater the absolute value of the slope, the steeper the line will appear on the graph.
- A rate of change of 0 indicates a horizontal line on a graph.

Common Errors/Misconceptions

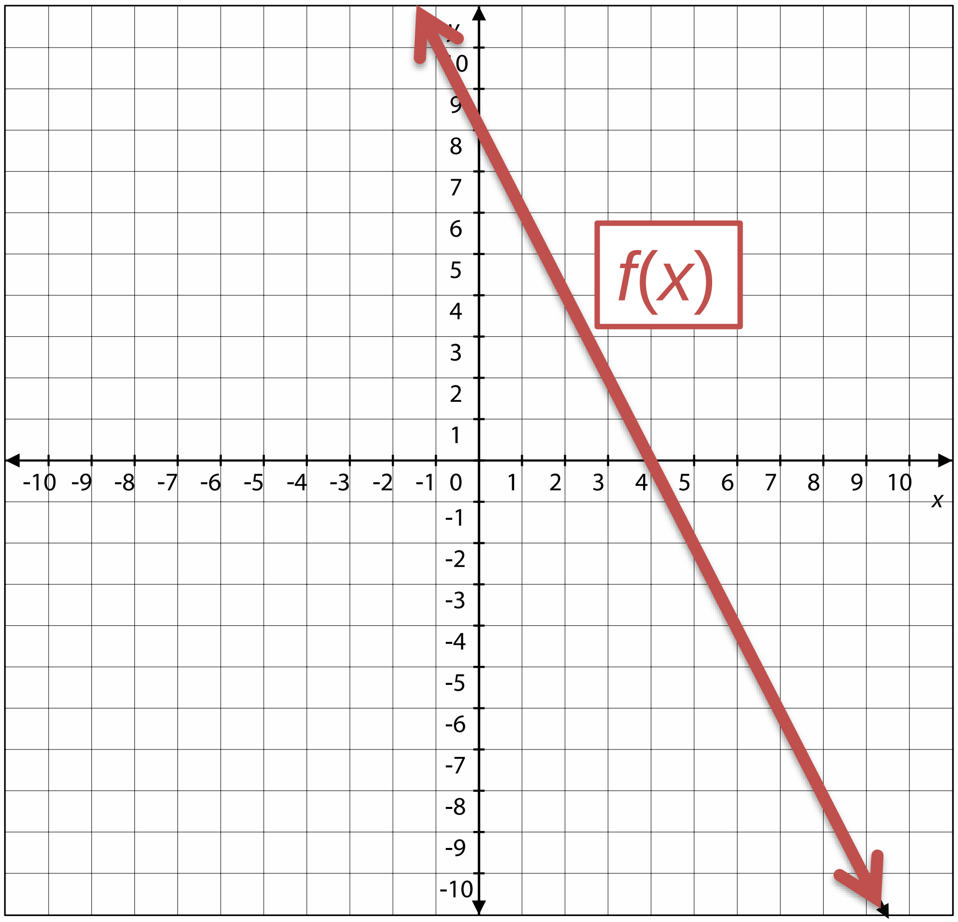
- incorrectly determining the rate of change
- not comparing the absolute values of the slopes to determine which function is steeper
- interchanging the *x*- and *y*-intercepts

Guided Practice

Example 1

The functions $f(x)$ and $g(x)$ are shown on the following slide. Compare the properties of each.

Guided Practice: Example 1, *continued*



x	*g*(*x*)
–2	–10
–1	–8
0	–6
1	–4

Guided Practice: Example 1, *continued*

1. **Identify the rate of change for the first function, *f*(*x*).**

   Let (0, 8) be (x_1, y_1) and (4, 0) be (x_2, y_2).

   Substitute the coordinates into the slope formula.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$= \dfrac{(0) - (8)}{(4) - (0)}$	Substitute (0, 8) for (x_1, y_1) and (4, 0) for (x_2, y_2).
$= \dfrac{-8}{4} = -2$	Simplify.

Guided Practice: Example 1, *continued*

1. **Identify the rate of change for the first function, *f*(*x*).**

The rate of change for this function is –2.

Guided Practice: Example 1, *continued*

2. **Identify the rate of change for the second function, *g*(*x*).**

Let (−2, −10) be (x_1, y_1) and (−1, −8) be (x_2, y_2).

Substitute the coordinates into the slope formula.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$= \frac{(-8) - (-10)}{(-1) - (-2)}$	Substitute (−2, −10) for (x_1, y_1) and (−1, −8) for (x_2, y_2).
$= \frac{2}{1} = 2$	Simplify.

Guided Practice: Example 1, *continued*

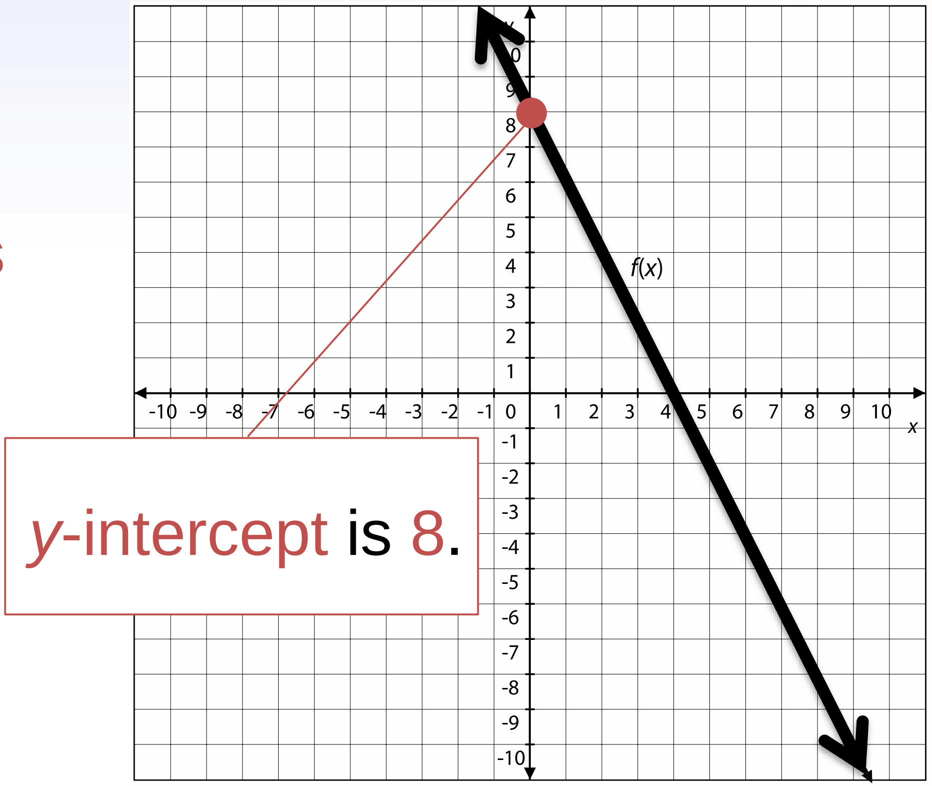
2. **Identify the rate of change for the second function, $g(x)$.**

The rate of change for this function is 2.

Guided Practice: Example 1, *continued*

3. Identify the *y*-intercept of the first function, *f*(*x*).

The graph intersects the *y*-axis at (0, 8).



Guided Practice: Example 1, *continued*

4. Identify the *y*-intercept of the second function, *g*(*x*).

- From the table, we can determine that the function would intersect the *y*-axis where the *x*-value is 0.
- This happens at the point (0, –6).
- The *y*-intercept is –6.

x	*g*(*x*)
–2	–10
–1	–8
0	–6
1	–4

Guided Practice: Example 1, *continued*

5. Compare the properties of each function.

- The rate of change for the first function is –2 and the rate of change for the second function is 2.
- The first function is decreasing and the second is increasing, but the absolute values of the **slopes are equal**, so the lines are **equally steep**.
- The *y*-intercept of the first function is 8, but the *y*-intercept of the second function is –6. The graph of *y*-axis at a **lower** point.

Guided Practice: Example 1, *continued*

Guided Practice

Example 2

Your employer has offered two pay scales for you to choose from.

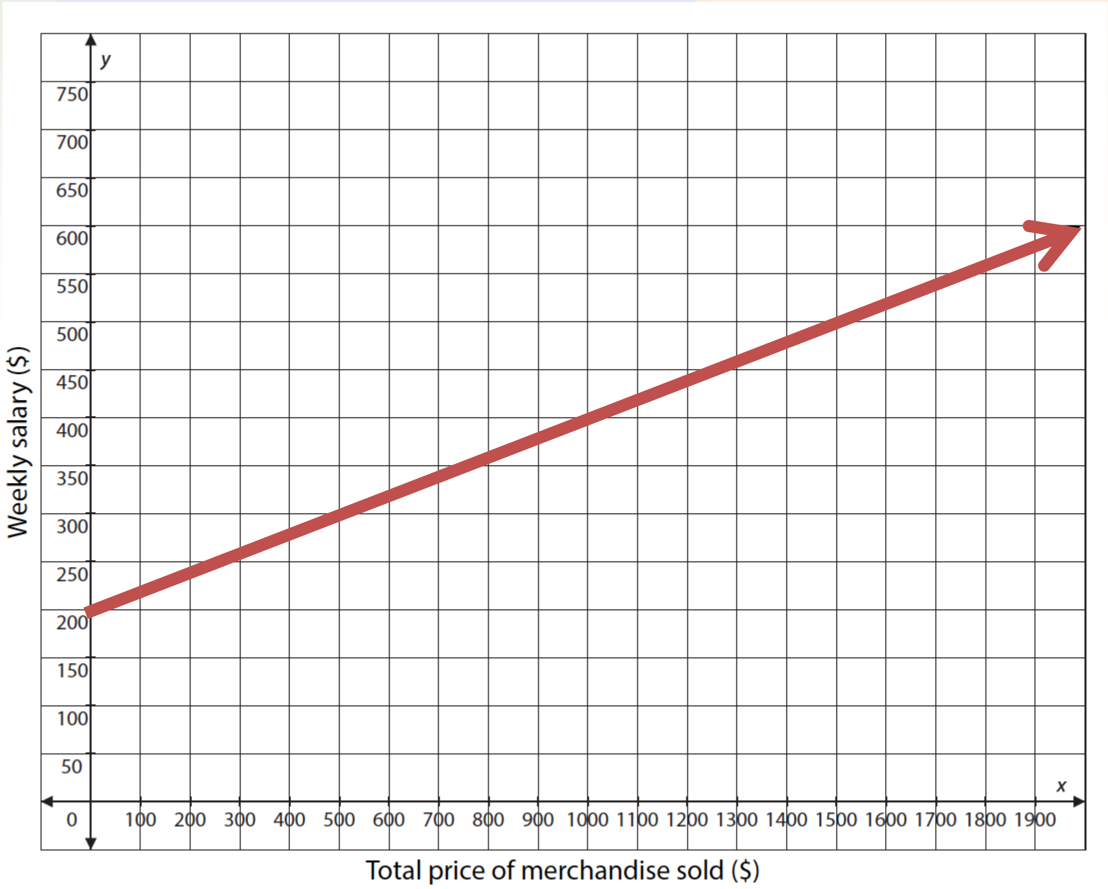
The first option is to receive a base salary of $250 a week plus 15% of the price of any merchandise you sell.

The second option is represented in the graph, where *x* represents the price of the merchandise sold and *y* represents your weekly salary.

Compare the properties of the functions.

Guided Practice

Example 2



Guided Practice: Example 2, *continued*

1. Identify the rate of change for the first function.

- Determine which information tells you the rate of change, or the slope, *m*.
- You are told that your employer will pay you 15% of the price of the merchandise you sell.
- This information is the rate of change for this function and can be written as 0.15.

Guided Practice: Example 2, *continued*

2. Identify the *y*-intercept of the first function.

- Your employer has offered a base salary of $250 per week.

- 250 is the *y*-intercept of the function.

Guided Practice: Example 2, *continued*

3. **Identify the rate of change for the second function.**

Let (0, 200) be (x_1, y_1) and (500, 300) be (x_2, y_2).

Substitute the coordinates into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(300) - (200)}{(500) - (0)}$$ Substitute (0, 200) for (x_1, y_1) and (500, 300) for (x_2, y_2).

$$= \frac{100}{500} = \frac{1}{5} = 0.2$$ Simplify.

Guided Practice: Example 2, *continued*

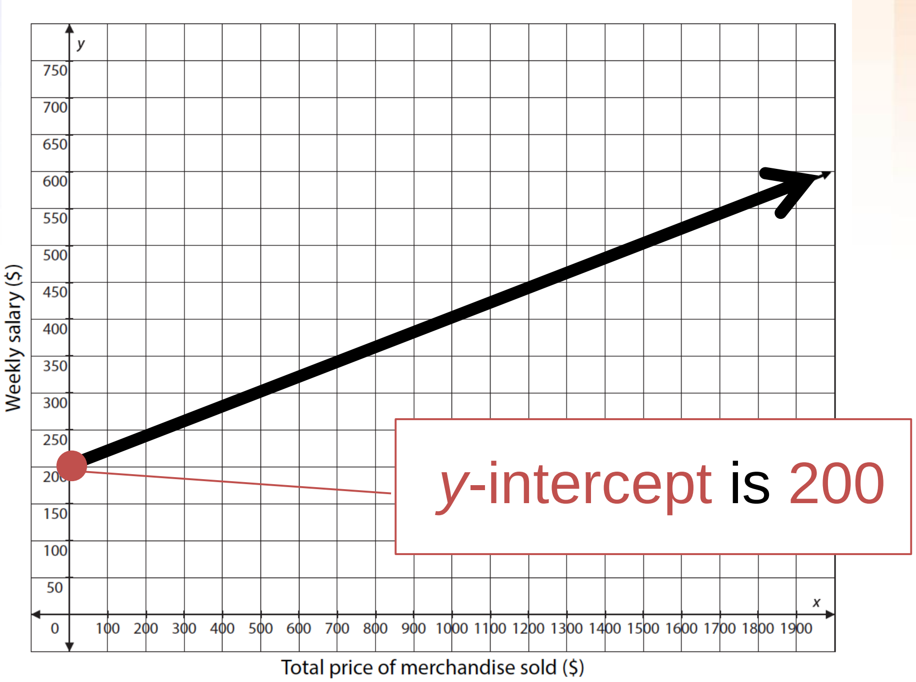
3. Identify the rate of change for the second function.

The rate of change for this function is 0.2.

Guided Practice: Example 2, *continued*

4. Identify the *y*-intercept as the *y*-coordinate of the point where the line intersects the *y*-axis.

The graph intersects the *y*-axis at (0, 200).



Guided Practice: Example 2, *continued*

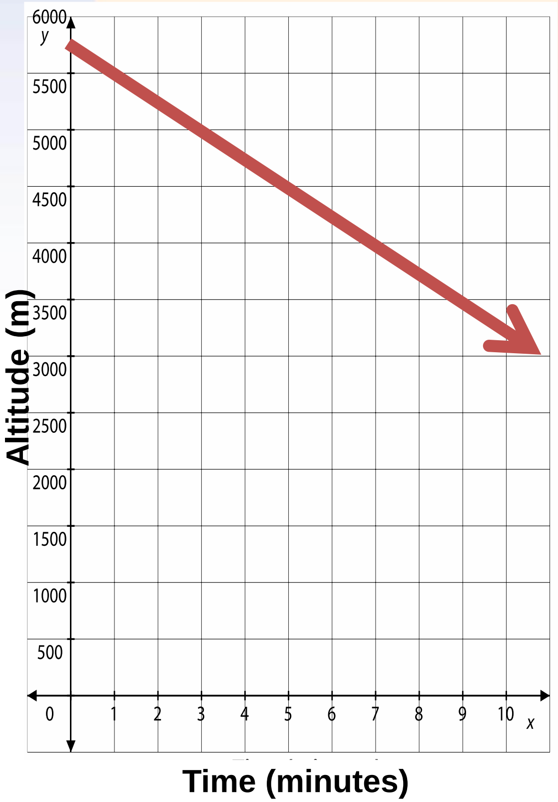
5. Compare the properties of each function.

- The rate of change for the second function is greater than the first function. You will get paid more for the amount of merchandise you sell.
- The *y*-intercept of the first function is greater than the second. You will get a higher base pay with the first function.
- In the first function, you would receive a higher base salary, but get paid less for the amount of merchandise you sell.
- In the second function, you would receive a lower base salary, but get paid more for the merchandise you sell.

Guided Practice

Example 3

- Two airplanes are in flight.
- The function $f(x) = 400x + 1200$ represents the altitude in meters, $f(x)$, of one airplane after x minutes.
- The graph represents the altitude of the second airplane after x minutes.
- Compare the properties of the functions.



Guided Practice: Example 3, *continued*

1. **Identify the rate of change for the first function.**
   - The function, $f(x) = 400x + 1200$, is written in $f(x) = mx + b$ form.
   - The rate of change for the function is 400.

Guided Practice: Example 3, *continued*

2. Identify the *y*-intercept for the first function.

The *y*-intercept of the first function is 1,200, as stated in the equation, $f(x) = 400x + 1200$.

Guided Practice: Example 3, *continued*

3. **Identify the rate of change for the second function.**

Choose two points from the graph.

Let (0, 5750) be (x_1, y_1) and (5, 4500) be (x_2, y_2).

Substitute the coordinates into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(4500) - (5750)}{(5) - (0)}$$ Substitute (0, 5750) for (x_1, y_1) and (5, 4500) for (x_2, y_2).

$$= \frac{-1250}{5} = -250$$ Simplify.

Guided Practice: Example 3, *continued*

3. Identify the rate of change for the second function.

The rate of change for this function is –250.

Guided Practice: Example 3, *continued*

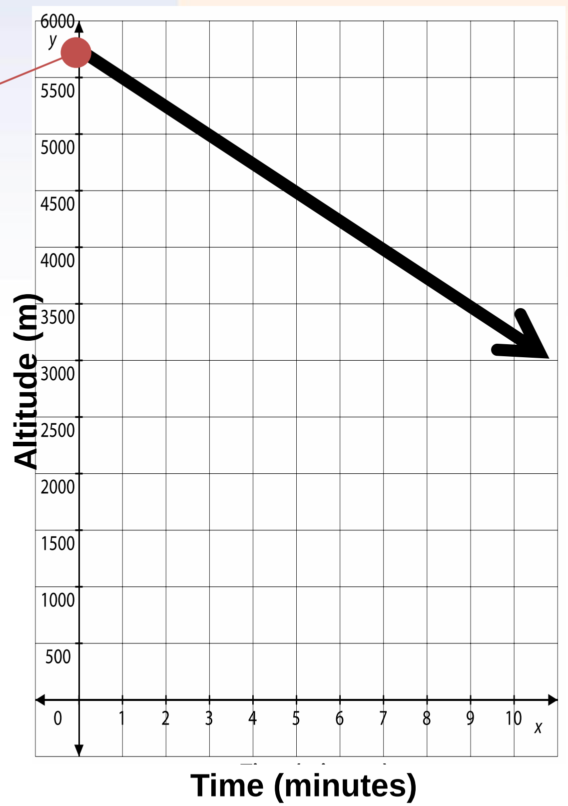
4. Identify the *y*-intercept of the second function as the *y*-coordinate of the point where the line intersects the *y*-axis.

The graph intersects the *y*-axis at (0, 5750), so the *y*-intercept is 5,750.

Guided Practice: Example 3, *continued*

3. **Identify the *y*-intercept of the first function, *f*(*x*).**

y-intercept is 5750.



The graph intersects the *y*-axis at (0, 5750).

Guided Practice: Example 3, *continued*

5. Compare the properties of each function.

- The absolute value of the slope for the first function is greater than the absolute value of the slope for the second function.
- The slope for the first function is also positive, whereas the slope for the second function is negative.
- The first airplane is ascending at a **faster rate** than the second airplane is descending.

Guided Practice: Example 3, *continued*

5. Compare the properties of each function.

- The *y*-intercept of the second function is greater than the first.
- The second airplane is higher in the air than the first airplane at that moment.

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Guided Practice: Example 3, *continued*