Comparing Linear to Exponential Functions

Warm-Up

Food-poisoning bacteria have to multiply to high numbers in food before there are enough bacteria to make someone sick. When conditions such as type of food, moisture level, temperature, and amount of time passed are ideal, the bacteria can double every 20 minutes.

1. Write a function to represent this scenario.

2. What is the rate of change for the first hour?

3. How many bacteria will there be after 6 hours if the initial number of bacteria is 1?

1. **Write a function to represent this scenario.**

This scenario is represented by an exponential function.

- The initial number of bacteria is 1.
- The bacteria doubles every 20 minutes, so the growth factor is 2.
- The time period is 20 minutes.

The function that represents this scenario is

$$f(x) = 1(2)^{\frac{x}{20}}$$

2. What is the rate of change for the first hour?

- Identify the interval.
- The *x*-value for the start of the interval is 0.
- The *x*-value for the end of the interval is 60, because there are 60 minutes in 1 hour.
- When *x* is 0, $f(x) = 1$. This is stated in the problem.
- To calculate the number of bacteria after 60 minutes, substitute 60 for *x* and evaluate the expression.

$f(x) = 1(2)^{\frac{x}{20}}$ Original function

$f(60) = 1(2)^{\frac{(60)}{20}}$ Substitute 60 for x.

$f(60) = 1(2)^{3}$ Simplify.

$f(60) = 1(8)$

$f(60) = 8$

There are 8 bacteria in the population after 60 minutes.

Calculate the rate of change. Let $(x_1, y_1) = (0, 1)$ and $(x_2, y_2) = (60, 8)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$= \frac{(8) - (1)}{(60) - (0)}$$ Substitute $(0, 1)$ for (x_1, y_1) and $(60, 8)$ for (x_2, y_2).

$$= \frac{7}{60} » 0.12$$ Simplify.

The rate of change for the first hour is approximately 0.12 bacteria per minute.

3. How many bacteria will there be after 6 hours if the initial number of bacteria is 1?

Determine how many minutes are in 6 hours.

$$6 \text{ hours} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}}$$

$$6 \, \cancel{\text{hours}} \cdot \frac{60 \text{ minutes}}{1 \, \cancel{\text{hour}}}$$

$$6 \cdot 60 \text{ minutes}$$

$$360 \text{ minutes}$$

Substitute 360 minutes into the function $f(x) = 1(2)^{\frac{x}{20}}$.

$f(x) = 1(2)^{\frac{x}{20}}$	Original function
$f(360) = 1(2)^{\frac{(360)}{20}}$	Substitute 360 for *x*.
$f(360) = 1(2)^{18}$	Simplify.
$f(360) = 1(262{,}144)$	
$f(360) = 262{,}144$	

After 6 hours, there will be 262,144 bacteria.

Instruction

Introduction

- In previous lessons, linear functions were compared to linear functions and exponential functions to exponential.
- Here, the properties of linear functions will be compared to those of exponential functions.

Key Concepts

- The equation of a linear function can be written in the form $f(x) = mx + b$.
- A factor is one of two or more numbers or expressions that when multiplied produce a given product.
- In the equation $f(x) = mx + b$, m and x are factors.
- As the value of x increases, the value of $f(x)$ will increase at a constant rate.
- The rate of change of linear functions remains constant.

Key Concepts, *continued*

- Exponential functions are written in the form $g(x) = ab^x + k$, where a and k are real numbers, b is the base, and x, the input, is a real number.
- The variable of an exponential function is part of the exponent.
- As the value of x increases, the value of $g(x)$ will increase or decrease by a power of b.

Key Concepts, *continued*

- The rate of change of an exponential function varies depending on the interval observed.
- Graphs of exponential functions where the base is greater than 1 will increase at a faster rate than graphs of linear functions.
- A quantity that increases exponentially will always eventually exceed one that increases linearly.

Common Errors/Misconceptions

- incorrectly determining the rate of change
- not comparing the absolute values of the slopes to determine which function is steeper
- interchanging the *x*- and *y*-intercepts
- assuming the rate of change of a function is linear by only referencing one interval

Guided Practice

Example 1

Which function increases faster, $f(x) = 4x - 5$ or $g(x) = 4^x - 5$.

Justify your answer with a graph.

Guided Practice: Example 1, *continued*

1. Make a general observation.

$f(x) = 4x - 5$ is a linear function of the form

$f(x) = mx + b$.

The variable x is multiplied by the coefficient 4.

$g(x) = 4^x - 5$ is an exponential function of the form $g(x) = ab^x$.

The variable x is the exponent.

Guided Practice: Example 1, *continued*

2. Create a table of values.

Substitute values for *x* into each function.

$f(x) = 4x - 5$	
x	$f(x)$
–2	–13
–1	–9
0	–5
1	–1
2	3

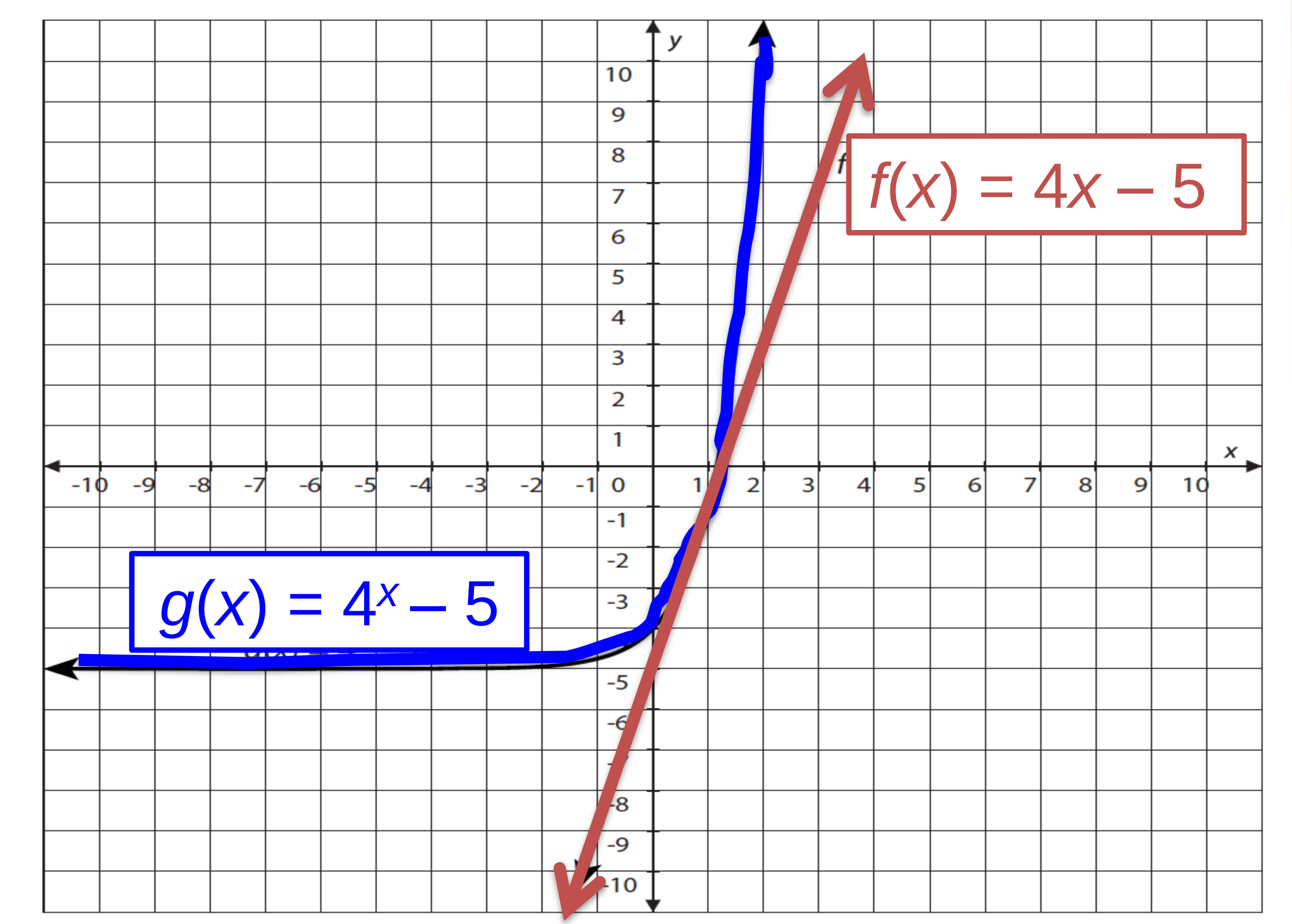
$g(x) = 4^x - 5$	
x	$g(x)$
–2	–4.9375
–1	–4.75
0	–4
1	–1
2	11

Guided Practice: Example 1, *continued*

3. Graph both functions on the same coordinate plane.

Use the tables of values created in order to plot both functions.

Guided Practice: Example 1, *continued*



Guided Practice: Example 1, *continued*

4. Identify the approximate point where $f(x)$ is greater than $g(x)$.

The graph of $f(x) = 4x – 5$ appears to be steeper than the graph of $g(x) = 4x – 5$ until the point (1, –1).

At this point, the graphs intersect and $f(x) = g(x)$.

Once x is greater than 1, the graph of $g(x) = 4x – 5$ becomes steeper.

From there, $g(x) = 4x – 5$ increases faster than $f(x) = 4x – 5$.

Guided Practice

Example 2

At approximately what point does the value of $f(x)$ **exceed** the value of $g(x)$ if $f(x)=2(4)^{\frac{x}{20}}$ and $g(x) = 0.5x$?

Justify your answer with a graph.

Guided Practice: Example 2, *continued*

1. Make a general observation.

$f(x) = 2(4)^{\frac{x}{20}}$ is an exponential function of the form $g(x) = ab^x$.

The variable *x* is part of the exponent.

$g(x) = 0.5x$ is a linear function of the form $f(x) = mx + b$.

The variable *x* is multiplied by the coefficient 0.5.

Guided Practice: Example 2, *continued*

2. Create a table of values.

Substitute values for *x* into each function.

$f(x)=2(4)^{\frac{x}{20}}$	
x	$f(x)$
0	2
2	2.30
4	2.64
6	3.03

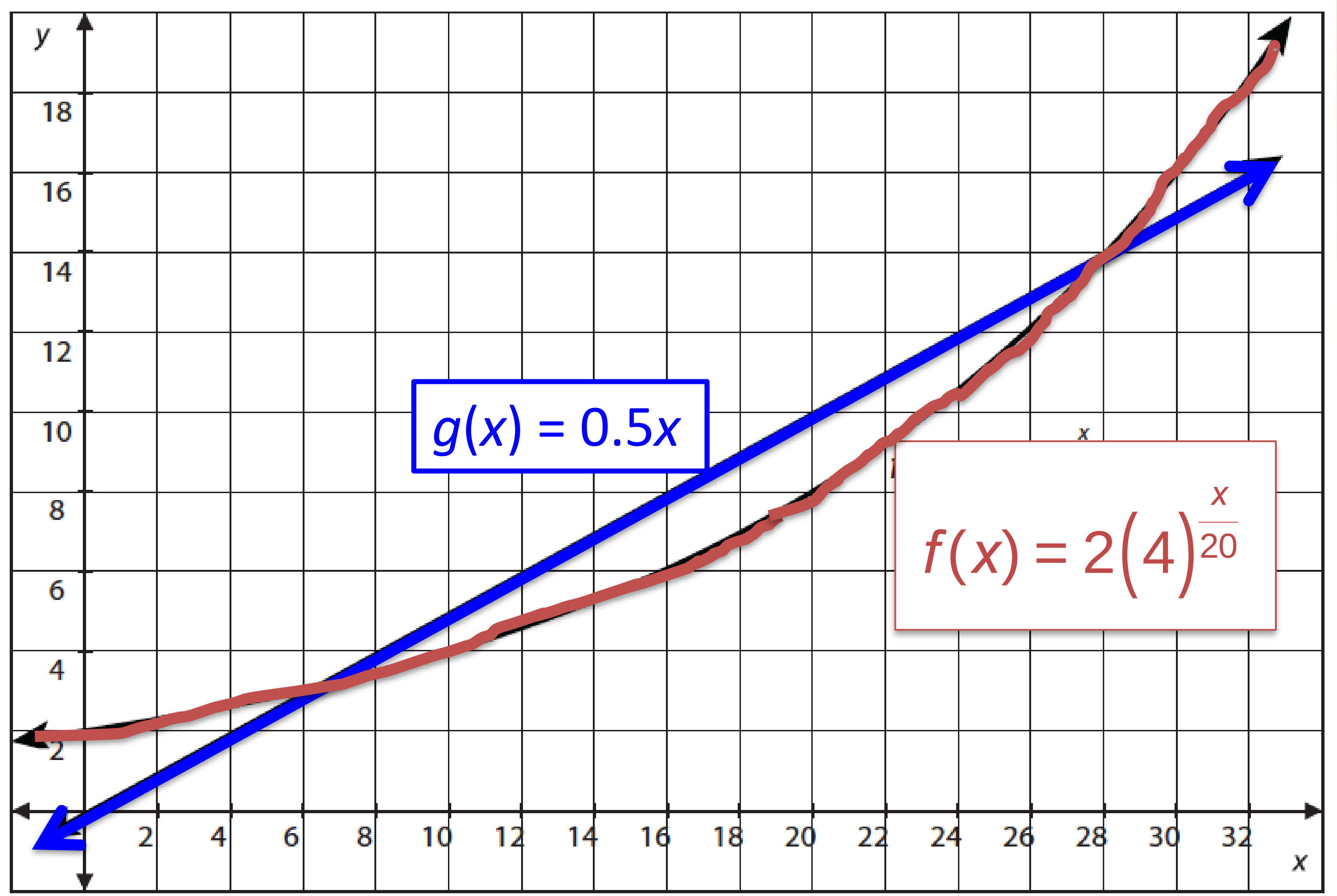
$g(x) = 0.5x$	
x	$g(x)$
0	0
2	1
4	2
6	3

Guided Practice: Example 2, *continued*

3. Graph both functions on the same coordinate plane.

Use the tables of values created in order to plot both functions.

Guided Practice: Example 2, *continued*



Guided Practice: Example 2, *continued*

4. Identify the approximate point where $f(x)$ is greater than $g(x)$.

It can be seen from the graph that both functions are equal where x is approximately equal to 28.

When x is greater than 28, $f(x)$ is greater than $g(x)$.

✓

Guided Practice: Example 2, *continued*

Guided Practice

Example 3

Lena has been offered a job with two salary options. The first option is modeled by the function $f(x) = 500x + 31{,}000$, where $f(x)$ is her salary in dollars after x years. The second option is represented by the function $g(x) = 29{,}000(1.04)^x$, where $g(x)$ is her salary in dollars after x years.

If Lena is hoping to keep this position for at least 5 years, which salary option should she choose? Support your answer with a graph.

Guided Practice: Example 3, *continued*

1. Make a general observation.

$f(x) = 500x + 31{,}000$ is a linear function of the form $f(x) = mx + b$.

The variable x is multiplied by the coefficient 500 and added to the constant 31,000.

$g(x) = 29{,}000(1.04)^x$ is an exponential function of the form $g(x) = ab^x$.

The variable x is the exponent.

Guided Practice: Example 3, *continued*

Use the two equations to create a table of values.

Substitute the same values for *x* into each function.

$f(x) = 500x + 31{,}000$	
x	$f(x)$
0	31,000
2	32,000
4	33,000
6	34,000

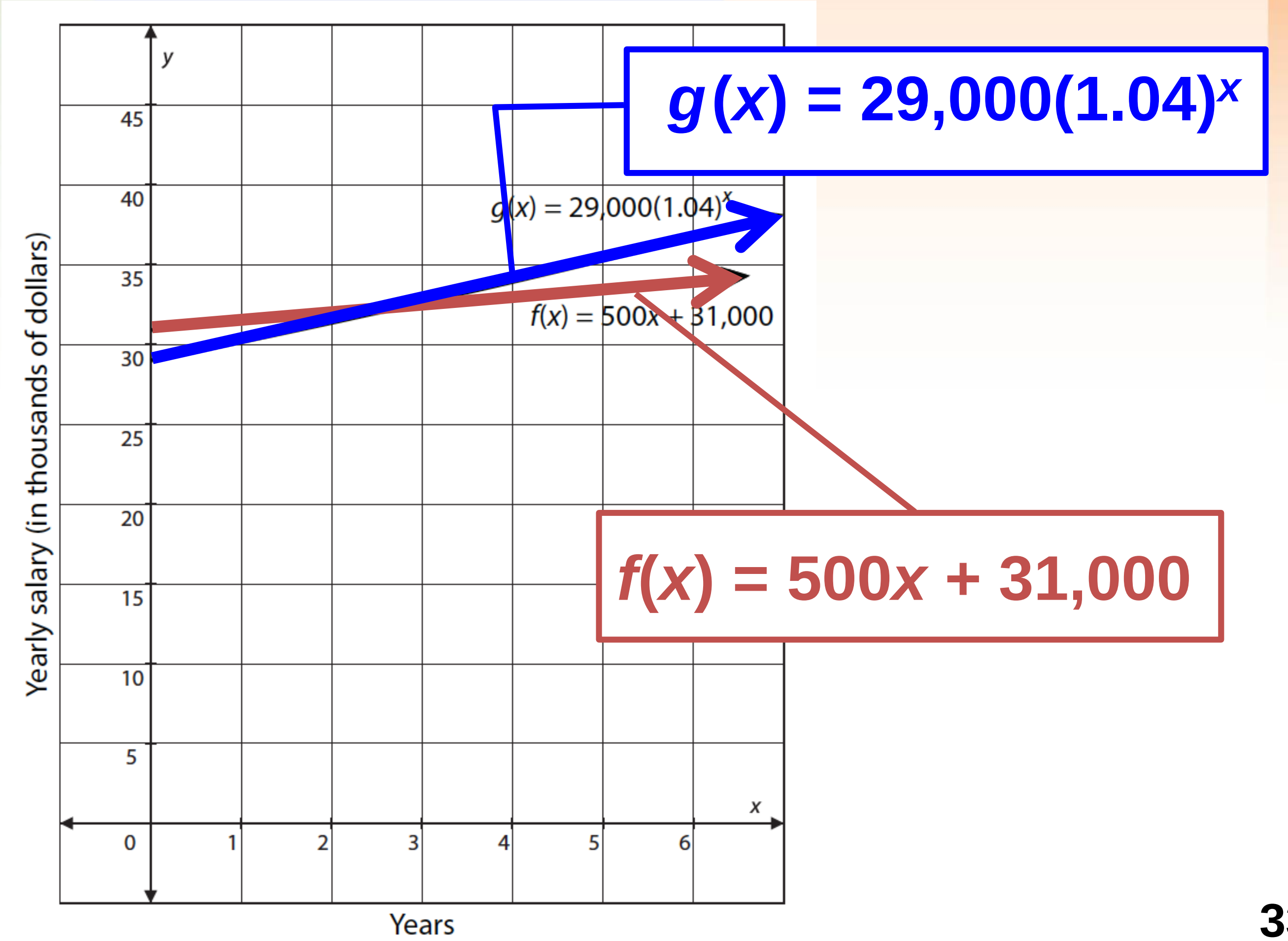
$g(x) = 29{,}000(1.04)^x$	
x	$g(x)$
0	29,000
2	31,366.40
4	33,925.90
6	36,694.25

Guided Practice: Example 3, *continued*

2. **Graph both functions on the same coordinate plane.**

Use the tables of values created in order to plot both functions.

Guided Practice: Example 3, *continued*



Guided Practice: Example 3, *continued*

3. **Identify the approximate point where $g(x)$ is greater than $f(x)$.**

It can be seen from the graph that after 3 years, $g(x)$ is **greater** than $f(x)$.

If Lena is hoping to keep this position for at least 5 years, it is in her best interest to choose the salary option modeled by $g(x) = 29{,}000(1.04)^x$. ✓

Guided Practice: Example 3, *continued*