Problem-Based Task: Future Finances

Coaching Sample Responses

a. How can you determine whether earning \$3,495.25 a month for the remainder of Cole's time on the farm is linear or exponential?

Earning \$3,495.25 a month for the remainder of his time on the farm is a constant pay rate.

If the rate of change of a function remains the same, the function is linear.

b. Write a function to model earning \$3,495.25 a month for the length of Cole's time on the farm.

Linear functions are of the form $f(x) = mx + b$.

- The rate of change for this function is 3,495.25.
- The y-intercept is 0.

The function that models earning an increase of \$3,495.25 a month is $f(x) = 3495.25x$.

c. How can you determine whether earning \$0.01 for the first month and then earning double the previous month's pay for each month afterward is linear or exponential?

Analyze the rates of change over the first few intervals.

- Cole's pay doubles each month.
- The first month, his pay is \$0.01.
- The second month, his pay is \$0.02.
- The third month, his pay is \$0.04.

The rate of change is not the same for each interval of the function, so the function can't be linear.

This is an example of an exponential growth function.

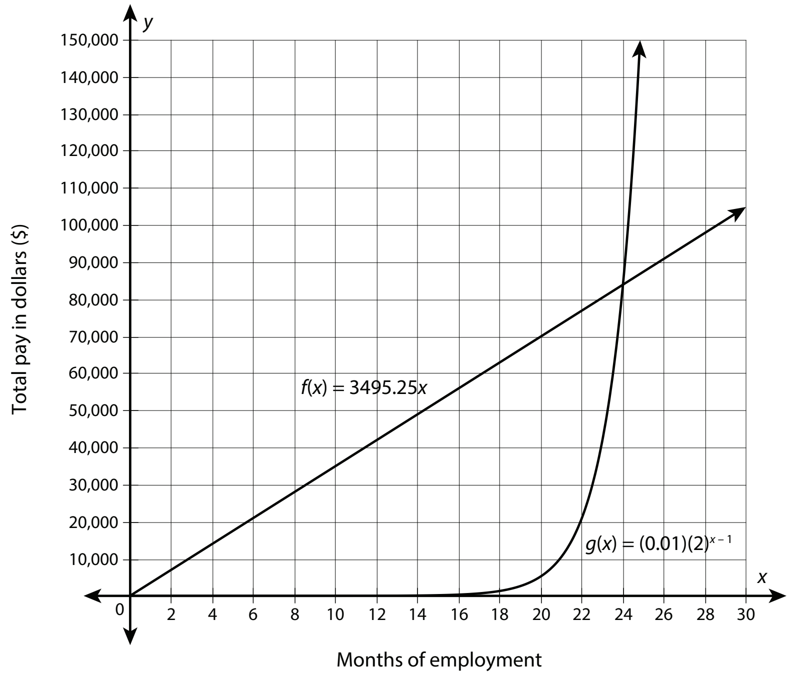
d. Write a function to model earning \$0.01 for the first month and then earning double the previous month's pay each month for the length of Cole's time on the farm.

Exponential growth functions are of the form $g(x) = ab^x$.

- The initial amount is 0.01.
- The rate of growth is 2 because Cole's pay is doubling.

The function that models earning \$0.01 for the first month and double that for each month afterward is $g(x) = (0.01)(2)^{x-1}$ for $x \geq 1$.

e. Graph both functions on the same coordinate plane.



f. When is it a better option to choose \$0.01 for the first month and then double the previous month's pay for every month after?

According to the graph, after about 24 months of employment, the value of the option of earning \$0.01 for the first month and then double the previous month's salary after that becomes greater than the value of the option of being paid \$3,495.25 each month. If Cole plans to be in school for more than two years, choosing to double his pay each month becomes the better option.

g. Is there a point on the graph where choosing either option results in the same payment?

At 24 months, or 2 years, the payments are the same regardless of the option chosen. Up until this point, the better option is to earn \$3,495.25 a month. After this point, the better option is to earn \$0.01 for the first month and then earn double the previous month's pay each month afterward.

h. If Cole finishes school in 21 months and collects his payment then, which is the better option?

The better option before 24 months have passed is the linear function, $f(x) = 3495.25x$. Earnings for this option far exceed the second exponential option at this point. If Cole finishes school in 21 months, he would be better off choosing the first option, the one that increases his pay at the rate of \$3,495.25 a month.

i. If Cole finishes school in 27 months and collects his payment then, which is the better option?

At this point, the exponential model far exceeds the linear model, so Cole should choose the second option, $g(x) = 0.01(2)^{x-1}$ for $x \geq 1$.

j. Which is the better option for Cole?

If Cole plans to take the average amount of time (2 years) to finish his schooling, either option will yield the same earnings. However, if he plans to finish early, he should choose the linear option. If he takes longer than 2 years to finish his schooling, he should choose the exponential option.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.