

Comparing Exponential Functions

Prerequisite Skills

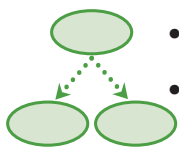
This lesson requires the use of the following skills:

- determining the rate of change of exponential functions
- determining the intercepts of exponential functions

Introduction

Exponential functions are functions that can be written in the form $f(x) = ab^x + k$, where a and k are real numbers, b is the base, x is the input value, and $f(x)$ is the output. In a growth or decay formula, the **growth factor** is the factor by which a quantity increases or decreases over time. The rate of change of an exponential function can be calculated using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, over a specified interval. An interval is a continuous set of real numbers between two given numbers. The y -intercept is the y -coordinate of the point at which the function intersects the y -axis. Both the rate of change and y -intercept can be determined from tables, equations, and graphs. Exponential functions can also be compared to one another using these features.

Key Concepts



- Exponential functions can be represented in words or as equations, graphs, or tables.
- To compare exponential functions, determine the rate of change and the intercepts of each function. Review the following processes for identifying the rate of change and the y -intercept of an exponential function.

Identifying the Rate of Change and the y -intercept from Context

1. Determine the interval to be observed.
2. Create a table of values by choosing appropriate x -values, substituting them into the equation for the function, and solving for $f(x)$.
3. Choose two points from the table.
4. Assign one point to be (x_1, y_1) and the other point to be (x_2, y_2) .
5. Substitute the coordinates into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
6. The result is the rate of change for the interval between the two points.
7. Determine which information tells you the y -intercept. This could be an initial value or a starting value, a flat fee, and so forth.

Identifying the Rate of Change and the y -intercept from Exponential Equations

1. Determine the interval to be observed.
2. Determine (x_1, y_1) by identifying the initial x -value of the interval and substituting it into the function.
3. Solve for $f(x)$.
4. Determine (x_2, y_2) by identifying the ending x -value of the interval and substituting it into the function.
5. Solve for $f(x)$.
6. Substitute (x_1, y_1) and (x_2, y_2) into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to calculate the rate of change.
7. Determine the y -intercept by substituting 0 for x and solving for $f(0)$.

Identifying the Rate of Change and the y -intercept from a Table

1. Determine the interval to be observed.
2. Assign one point to be (x_1, y_1) and the other point to be (x_2, y_2) .
3. Substitute the values into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
4. The result is the rate of change for the interval between the two points.
5. Identify the y -intercept as the y -coordinate in the ordered pair $(0, y)$.

Identifying the Rate of Change and the y -intercept from a Graph

1. Determine the interval to be observed.
2. Identify (x_1, y_1) as the initial point of the interval.
3. Identify (x_2, y_2) as the ending point of the interval.
4. Substitute (x_1, y_1) and (x_2, y_2) into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to calculate the rate of change.
5. Identify the y -intercept as the y -coordinate in the ordered pair $(0, y)$.

- Exponential functions are increasing if the rate of change is a positive value.
- Exponential functions are decreasing if the rate of change is a negative value.
- The function whose rate of change has the greater absolute value over a given interval has a steeper graph.

Common Errors/Misconceptions

- incorrectly determining the rate of change
- assuming that a positive slope will be steeper than a negative slope
- not comparing the absolute values of the slopes to determine which function is steeper
- incorrectly applying the order of operations
- using the exponential growth model instead of exponential decay and vice versa