Comparing Linear Functions

Prerequisite Skills

This lesson requires the use of the following skills:

- determining the slopes of linear functions
- determining the intercepts of linear functions

Introduction

Remember that linear functions are first-degree equations that can be written in the form $f(x) = mx + b$, where m is the slope and b is the y-intercept. The slope of a linear function is also the **rate of change** and can be calculated using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$. The y-intercept is the y-coordinate of the point at which the graph intersects the y-axis; it is the value of y when $x = 0$. The x-intercept, if it exists, is the x-coordinate of the point where the graph intersects the x-axis; it is the value of x when $y = 0$. The slope and both intercepts can be determined from tables, equations, and graphs. These features are used to compare linear functions to one another.

Key Concepts

- Linear functions can be represented in words or as equations, graphs, or tables.
- To compare linear functions, determine the rate of change and intercepts of each function.
- Review the following processes for identifying the rate of change and y-intercept of a linear function.

Identifying the Rate of Change and the *y*-intercept from Context
1. Read the problem statement carefully.
2. Look for the information given and make a list of the known quantities.
3. Determine which information tells you the rate of change, or the slope, m. Look for words such as *each*, *every*, *per*, or *rate*.
4. Determine which information tells you the y-intercept, or b. This could be an initial value or a starting value, a flat fee, and so forth.

Identifying the Rate of Change and the *y*-intercept from an Equation

1. Write the equation of the function in slope-intercept form, $f(x) = mx + b$.
2. Identify the rate of change, or the slope, m, as the coefficient of x.
3. Identify the y-intercept, or b, as the constant term in the function.

Identifying the Rate of Change and the *y*-intercept from a Table

1. Choose two points from the table.
2. Assign one point to be (x_1, y_1) and the other point to be (x_2, y_2).
3. Substitute the coordinates into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
4. Identify the y-intercept as the y-coordinate in the ordered pair $(0, y)$. If this coordinate is not given, substitute the slope and the coordinates of any ordered pair from the table into the equation $f(x) = mx + b$ and solve for b.

Identifying the Rate of Change and the *y*-intercept from a Graph

1. Choose two points from the graph.
2. Assign one point to be (x_1, y_1) and the other point to be (x_2, y_2).
3. Substitute the coordinates into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
4. Identify the y-intercept as the y-coordinate of the point where the line intersects the y-axis.

- When presented with functions represented in different ways, it is helpful to rewrite the information using function notation.
- Linear functions are increasing if the rate of change is a positive value.
- Linear functions are decreasing if the rate of change is a negative value.
- The greater the absolute value of the slope, the steeper the line will appear on the graph.
- A rate of change of 0 indicates a horizontal line on a graph.

Common Errors/Misconceptions

- incorrectly determining the rate of change
- not comparing the absolute values of the slopes to determine which function is steeper
- interchanging the x- and y-intercepts