

## Problem-Based Task: Supply and Demand

### Task Overview

#### Focus

How can two functions be compared when one is represented with an equation and one is represented numerically in a table? How are the properties of these functions determined? In this task, students will compare rates of change and the  $y$ -intercepts of two linear functions, which represent the supply and demand of a product, and they will interpret these properties in order to identify when supply exceeds demand.

This activity will provide practice with:

- identifying a rate of change from the slope-intercept form of a linear equation
- calculating the rate of change of a linear function when given coordinates in a table
- determining the  $y$ -intercept from a graph, an equation, and a table of values
- graphing linear functions
- interpreting the solution of a system of linear equations
- analyzing information from a graph of two linear equations

#### Introduction

This task should be used to explore or apply the skill of comparing properties of two functions that are each represented in different formats. After the properties are determined and the functions are graphed, students graphically solve a system of equations to analyze the situation. Students should already be familiar with calculating the slope of a line and graphing linear functions.

Begin by reading the problem and clarifying the meanings of *supply* and *demand*.

**demand**                      the amount of a product that is wanted by customers

**supply**                        the amount of a product that is available

### Facilitating the Task

#### Standards for Mathematical Practice

Many or all of the Standards for Mathematical Practice are addressed through this activity. As students work, reinforce the importance of the following standards:

- **SMP 1:** Make sense of problems and persevere in solving them.

Students will first recognize that this task involves analyzing the relationship between supply and demand. They will make sense of the problem by identifying the two different representations of the functions for supply and demand. Perseverance may be necessary if students have difficulty finding a way to compare the two functions in their given formats.

- **SMP 3:** Construct viable arguments and critique the reasoning of others.

There are several ways that students can compare the two given functions. Students should justify their reasoning to classmates who approached the problem in a different way.

Encourage students to discuss their arguments with each other and explain their reasoning if they do not agree with each other.

- **SMP 4:** Model with mathematics.

Students may recognize that one way that the scenario can be modeled is by graphing both linear functions on the same coordinate axes. They will also see that although linear functions can be modeled in different formats, the properties of the functions can still be compared and analyzed in these different formats, as well as with a graphic representation.

### Addressing Common Errors/Misconceptions

Be aware of common student errors and misconceptions associated with this task:

- incorrectly determining the rate of change

Encourage students to write the slope formula on the top of their papers to use as a reference when working through this task. Remind them to carefully label each of the coordinates with  $x_1$ ,  $y_1$ ,  $x_2$ , and  $y_2$  before substituting them into the formula.

- not comparing the absolute values of the slopes to determine which function is steeper

Remind students that a positive slope does not necessarily mean that the slope is steeper. What determines the steepness of the slope is the absolute value of the rate of change. The graph of the demand function is steeper than the graph of the supply function, even though the demand function has a negative slope and the supply function has a positive slope.

### Monitoring and Coaching

Ask questions as you circulate to monitor student understanding. Suggestions:

- Before students begin the task:

Discuss students' prior knowledge of the concept of supply and demand. Ask, "What are some real-world examples of supply and demand?" (**Sample answer:** "When a new product comes out that is considered to be a desirable item, the price is usually high at first, based on the fact that the item is in great demand and the supply is limited. The price usually drops once the demand is not as great. Some examples include newly released phone models and gaming consoles.")

Discuss the other possible formats in which the two functions can be represented. Point out that the supply function is given as an equation, and the demand function is given as a table of values. Ask, "How else can the functions be represented?" (**Answer:** "Each function can be represented graphically, algebraically, as a table of values, or even as a verbal description. This variety of ways to analyze a function is known as the 'rule of four.'")

- As students begin to calculate the rates of change for the two functions, ask the following:  
“What do you know about the rate of change for a linear function?” (**Answer:** “A linear function has a constant rate of change. This means that the slope of the line will be the same between any two points on the line.”)  
“Which has the steeper slope—the supply function or the demand function? How do you know?” (**Answer:** “The demand function has a steeper slope because the absolute value of  $-0.6$  is  $0.6$ , and this is greater than the absolute value of the supply function, which is  $0.3$ .”)
- Ask students, “What do the  $y$ -intercepts of the functions represent in the context of the scenario?” (**Answer:** “The  $y$ -intercepts of the functions represent the initial values for each function. One possible interpretation is that before the tablets first came out, when there were 0 tablets available, they could be priced at \$100 per tablet according to the supply function, which has a  $y$ -intercept of 100. For the demand function, the  $y$ -intercept is 550, which represents the suggested price of \$550 for a tablet before any tablets were available.”)
- If students have difficulty understanding why the  $x$ -variable represents the number of tablets and the  $y$ -variable represents the price per tablet, remind them of the meaning of the terms *independent variable* and *dependent variable*. Ask, “In this scenario, which quantity is dependent on the other?” (**Answer:** “In this scenario, the price of the tablet depends on the number of tablets that are available and in demand. Therefore, the independent variable,  $x$ , is the number of tablets, and the dependent variable,  $y$ , is the price per tablet.”)
- Ask students, “What is the point of intersection of the two functions, and what does the point of intersection represent in the context of the scenario?” (**Answer:** “The two lines intersect at the point  $(500, 250)$ . This represents the fact that when the seller has 500 tablets, each tablet will be priced at \$250.”)
- If students have difficulty understanding what it means when the supply function exceeds the demand function, encourage them to think about real-life examples of products that have been on the market for a long period of time. Discuss how the price decreases for many products as time goes on, since the current demand for those products is lower than when they first came on the market. Ask students, “What does it mean that the supply function for the tablets exceeded the demand function for the tablets?” (**Answer:** “It means that when the supply function exceeds the demand function, the number of tablets available is greater than the number of tablets demanded by the buyers.”)
- Ask students if they have questions about areas of the problem that are not clearly understood, and allow students to clarify these points for each other.

## Debriefing the Task

Compare students' strategies and explanations for calculating the rates of change and the  $y$ -intercepts for both functions. Discuss the comparisons of these properties for the two functions and what this means in the context of the scenario. Focus on the use of precise mathematical language.

## Connecting to Key Concepts

Make explicit connections to key concepts:

- Linear functions can be represented in words or as equations, graphs, or tables.

In this task, students are presented with a linear function represented by an equation, and also a linear function represented by a table, and they may choose to present both of these functions graphically.

- To compare linear functions, determine the rate of change and intercepts of each function.

In this task, students will determine the rate of change and the  $y$ -intercept for both linear functions. Students will compare these properties in order to draw conclusions about the two functions.

- Linear functions are increasing if the rate of change is a positive value.

Students will determine that the rate of change for the supply function is 0.3; therefore, they can conclude that the supply function is increasing.

- Linear functions are decreasing if the rate of change is a negative value.

Students will determine that the rate of change for the demand function is  $-0.6$ ; therefore, they can conclude that the demand function is decreasing.

## Extending the Task

To extend the task, replace one of the linear functions in this problem with an exponential function, and ask students to compare the properties of the functions on specific domain intervals and summarize supply versus demand for this new situation. Discuss how comparing the properties of two linear functions differs from comparing the properties of a linear function and an exponential function.

## Connecting to Standards for Mathematical Practice

Make explicit connections to the Standards for Mathematical Practice described previously for this task.

- **For SMP 1, ASK:** “How did you make sense of the problem or demonstrate perseverance?”  
(**Answer:** “I made sense of the problem by calculating the rates of change for both linear functions, and then graphing them on the same coordinate plane in order to visually compare their properties. I analyzed the properties of each function in order to draw conclusions about supply and demand.”)
- **For SMP 3, ASK:** “How did you construct viable arguments and critique the reasoning of others?” (**Answer:** “I constructed viable arguments by comparing the properties of each function and basing my conclusions about the supply and demand of the tablet on these properties.”)
- **For SMP 4, ASK:** “How did you use mathematics to model this particular scenario?”  
(**Answer:** “I modeled this scenario visually by graphing both functions on the same coordinate plane.”)

## Alternate Strategies or Solutions

Rather than modeling both functions graphically in order to compare them, students may decide to make a table for the first function, and then compare the two tables. Or, they may decide to write an equation for the second function, compare the two equations, and solve the system of equations algebraically. Point out to students that it is often easier to compare the two functions when they are in the same format.

## Technology

Students can use scientific calculators for their calculations of slope, and they can use graphing calculators or other graphing technology when graphing the functions.